Network Theory - Superposition Theorem

Superposition theorem is based on the concept of linearity between the response and excitation of an electrical circuit. It states that the response in a particular branch of a linear circuit when multiple independent sources are acting at the same time is equivalent to the sum of the responses due to each independent source acting at a time.

In this method, we will consider only **one independent source** at a time. So, we have to eliminate the remaining independent sources from the circuit. We can eliminate the voltage sources by shorting their two terminals and similarly, the current sources by opening their two terminals.

Therefore, we need to find the response in a particular branch **'n' times** if there are 'n' independent sources. The response in a particular branch could be either current flowing through that branch or voltage across that branch.

Procedure of Superposition Theorem

Follow these steps in order to find the response in a particular branch using superposition theorem.

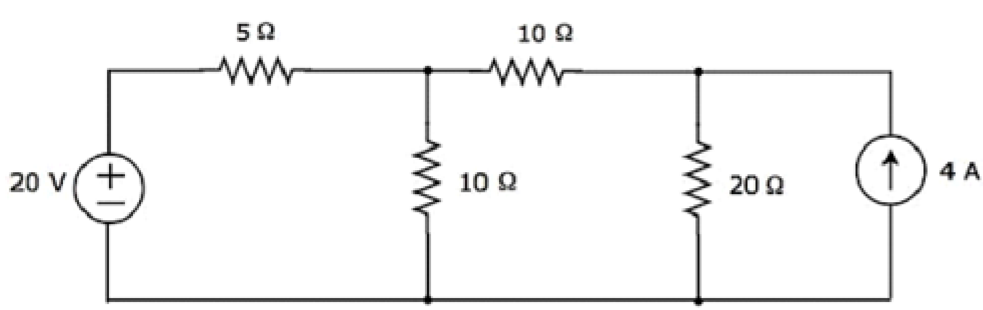
Step 1 − Find the response in a particular branch by considering one independent source and eliminating the remaining independent sources present in the network.

Step 2 − Repeat Step 1 for all independent sources present in the network.

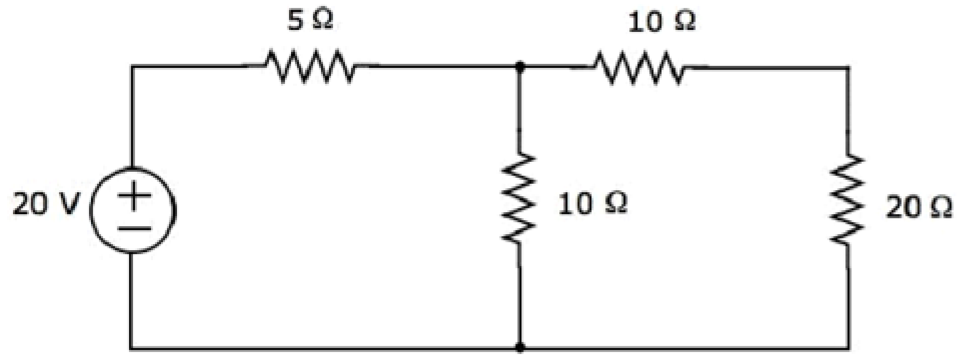
Step 3 − Add all the responses in order to get the overall response in a particular branch when all independent sources are present in the network.

Example

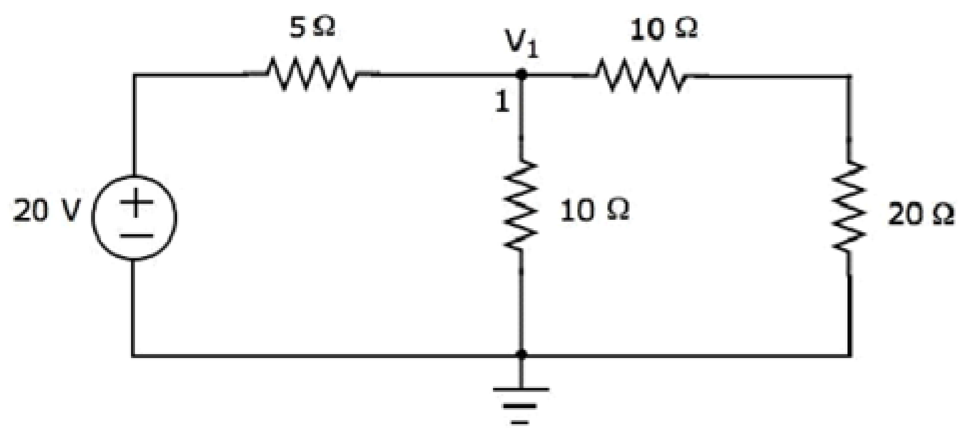
Find the current flowing through 20 Ω resistor of the following circuit using **superposition theorem**.



Step 1 − Let us find the current flowing through 20 Ω resistor by considering only **20 V voltage source**. In this case, we can eliminate the 4 A current source by making open circuit of it. The modified circuit diagram is shown in the following figure.



There is only one principal node except Ground in the above circuit. So, we can use **nodal analysis** method. The node voltage V_1 is labelled in the following figure. Here, V_1 is the voltage from node 1 with respect to ground.



The **nodal equation** at node 1 is

$$\frac{V_1 - 20}{5} + \frac{V_1}{10} + \frac{V_1}{10 + 20} = 0$$

$$\Rightarrow \frac{6V_1 - 120 + 3V_1 + V_1}{30} = 0$$

$$\Rightarrow 10V_1 = 120$$

$$\Rightarrow V_1 = 12V$$

The **current flowing through 20 Ω resistor** can be found by doing the following simplification.

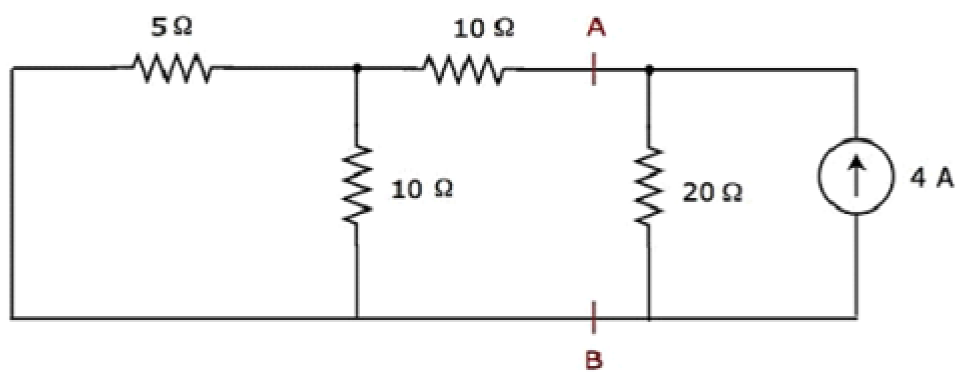
$$I_1 = \frac{V_1}{10+20}$$

Substitute the value of V_1 in the above equation.

$$I_1 = \frac{12}{10+20} = \frac{12}{30} = 0.4A$$

Therefore, the current flowing through 20 Ω resistor is **0.4 A**, when only 20 V voltage source is considered.

Step 2 − Let us find the current flowing through 20 Ω resistor by considering only **4 A current source**. In this case, we can eliminate the 20 V voltage source by making short-circuit of it. The modified circuit diagram is shown in the following figure.

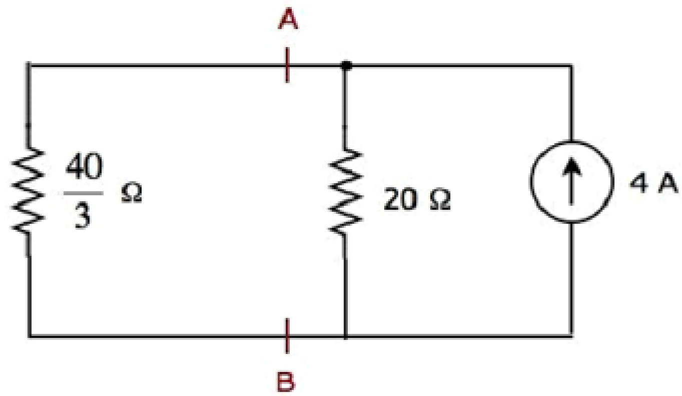


In the above circuit, there are three resistors to the left of terminals A & B. We can replace these resistors with a single **equivalent resistor**. Here, 5 Ω & 10 Ω resistors are connected in parallel and the entire combination is in series with 10 Ω resistor.

The **equivalent resistance** to the left of terminals A & B will be

$$R_{AB} = (\frac{5 \times 10}{5+10}) + 10 = \frac{10}{3} + 10 = \frac{40}{3}\Omega$$

The simplified circuit diagram is shown in the following figure.



We can find the current flowing through 20 Ω resistor, by using **current division principle**.

$$I_2 = I_S\left(\frac{R_1}{R_1 + R_2}\right)$$

Substitute $I_S = 4A,\ R_1 = \frac{40}{3}\Omega$ and $R_2 = 20\Omega$ in the above equation.

$$I_2 = 4\left(\frac{\frac{40}{3}}{\frac{40}{3} + 20}\right) = 4\left(\frac{40}{100}\right) = 1.6A$$

Therefore, the current flowing through 20 Ω resistor is **1.6 A**, when only 4 A current source is considered.

Step 3 − We will get the current flowing through 20 Ω resistor of the given circuit by doing the **addition of two currents** that we got in step 1 and step 2. Mathematically, it can be written as

$$I = I_1 + I_2$$

Substitute, the values of I_1 and I_2 in the above equation.

$$I = 0.4 + 1.6 = 2A$$

Therefore, the current flowing through 20 Ω resistor of given circuit is **2 A**.

Note − We can't apply superposition theorem directly in order to find the amount of **power** delivered to any resistor that is present in a linear circuit, just by doing the addition of powers delivered to that resistor due to each independent source. Rather, we can calculate either total current flowing through or voltage across that resistor by using superposition theorem and from that, we can calculate the amount of power delivered to that resistor using I^2R or $\frac{V^2}{R}$.