Electrical Quantity Division Principles

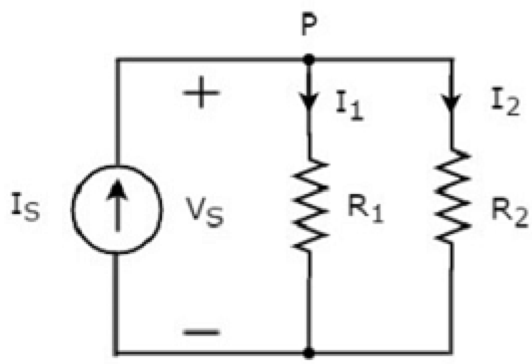
In this chapter, let us discuss about the following two division principles of electrical quantities.

- Current Division Principle
- Voltage Division Principle

Current Division Principle

When two or more passive elements are connected in parallel, the amount of current that flows through each element gets **divided** (shared) among themselves from the current that is entering the node.

Consider the following **circuit diagram**.



The above circuit diagram consists of an input current source $\mathbf{I_S}$ in parallel with two resistors $\mathbf{R_1}$ and $\mathbf{R_2}$. The voltage across each element is $\mathbf{V_S}$. The currents flowing through the resistors $\mathbf{R_1}$ and $\mathbf{R_2}$ are $\mathbf{I_1}$ and $\mathbf{I_2}$ respectively.

The **KCL equation** at node **P** will be

$$I_S = I_1 + I_2$$

- Substitute $I_1 = \frac{V_S}{R_1}$ and $I_2 = \frac{V_S}{R_2}$ in the above equation.

$$I_S = \frac{V_S}{R_1} + \frac{V_S}{R_2} = V_S\left(\frac{R_2 + R_1}{R_1 R_2}\right)$$

$$\Rightarrow V_S = I_S \left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

- Substitute the value of V_S in $I_1 = \frac{V_S}{R_1}$.

$$I_1 = \frac{I_S}{R_1} \left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

$$\Rightarrow I_1 = I_S \left(\frac{R_2}{R_1 + R_2}\right)$$

- Substitute the value of V_S in $I_2 = \frac{V_S}{R_2}$.

$$I_2 = \frac{I_S}{R_2} \left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

$$\Rightarrow I_2 = I_S \left(\frac{R_1}{R_1 + R_2}\right)$$

From equations of I_1 and I_2, we can generalize that the current flowing through any passive element can be found by using the following formula.

$$I_N = I_S \left(\frac{Z_1 \| Z_2 \| \ldots \| Z_{N-1}}{Z_1 + Z_2 + \ldots + Z_N}\right)$$

This is known as **current division principle** and it is applicable, when two or more passive elements are connected in parallel and only one current enters the node.

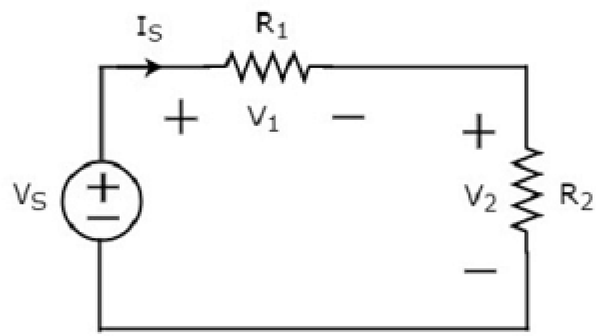
Where,

- I_N is the current flowing through the passive element of N^{th} branch.
- I_S is the input current, which enters the node.
- $Z_1, Z_2, \ldots, Z_N$ are the impedances of 1^{st} branch, 2^{nd} branch, …, N^{th} branch respectively.

Voltage Division Principle

When two or more passive elements are connected in series, the amount of voltage present across each element gets **divided** (shared) among themselves from the voltage that is available across that entire combination.

Consider the following **circuit diagram**.



The above circuit diagram consists of a voltage source, V_S in series with two resistors R_1 and R_2. The current flowing through these elements is I_S. The voltage drops across the resistors R_1 and R_2 are V_1 and V_2 respectively.

The **KVL equation** around the loop will be

$$V_S = V_1 + V_2$$

- Substitute $V_1 = I_S R_1$ and $V_2 = I_S R_2$ in the above equation

$$V_S = I_S R_1 + I_S R_2 = I_S (R_1 + R_2)$$

$$I_S = \frac{V_S}{R_1 + R_2}$$

- Substitute the value of I_S in $V_1 = I_S R_1$.

$$V_1 = (\frac{V_S}{R_1 + R_2}) R_1$$

$$\Rightarrow V_1 = V_S (\frac{R_1}{R_1 + R_2})$$

- Substitute the value of I_S in $V_2 = I_S R_2$.

$$V_2 = (\frac{V_S}{R_1 + R_2})R_2$$

$$\Rightarrow V_2 = V_S(\frac{R_2}{R_1 + R_2})$$

From equations of V_1 and V_2, we can generalize that the voltage across any passive element can be found by using the following formula.

$$V_N = V_S(\frac{Z_N}{Z_1 + Z_2 + \ldots + Z_N})$$

This is known as **voltage division principle** and it is applicable, when two or more passive elements are connected in series and only one voltage available across the entire combination.

Where,

- V_N is the voltage across Nth passive element.
- V_S is the input voltage, which is present across the entire combination of series passive elements.
- Z_1,Z_2, …,Z_3 are the impedances of 1st passive element, 2nd passive element, …, Nth passive element respectively.