Two-Port Parameter Conversions

In the previous chapter, we discussed about six types of two-port network parameters. Now, let us convert one set of two-port network parameters into other set of two port network parameters. This conversion is known as two port network parameters conversion or simply, **two-port parameters conversion**.

Sometimes, it is easy to find one set of parameters of a given electrical network easily. In those situations, we can convert these parameters into the required set of parameters instead of calculating these parameters directly with more difficulty.

Now, let us discuss about some of the two port parameter conversions.

Procedure of two port parameter conversions

Follow these steps, while converting one set of two port network parameters into the other set of two port network parameters.

- **Step 1** − Write the equations of a two port network in terms of desired parameters.
- **Step 2** − Write the equations of a two port network in terms of given parameters.
- **Step 3** − Re-arrange the equations of Step2 in such a way that they should be similar to the equations of Step1.
- **Step 4** − By equating the similar equations of Step1 and Step3, we will get the desired parameters in terms of given parameters. We can represent these parameters in matrix form.

Z parameters to Y parameters

Here, we have to represent Y parameters in terms of Z parameters. So, in this case Y parameters are the desired parameters and Z parameters are the given parameters.

Step 1 − We know that the following set of two equations, which represents a two port network in terms of **Y parameters**.

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

We can represent the above two equations in **matrix** form as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$ **Equation 1**

Step 2 − We know that the following set of two equations, which represents a two port network in terms of **Z parameters**.

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

We can represent the above two equations in **matrix** form as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Step 3 − We can modify it as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$ **Equation 2**

Step 4 − By equating Equation 1 and Equation 2, we will get

$$\begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}^{-1}$$

$$\Rightarrow \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \frac{\begin{bmatrix} Z_{22} & -Z_{12} \\ -Z_{21} & Z_{11} \end{bmatrix}}{\Delta Z}$$

Where,

$$\Delta Z = Z_{11}Z_{22} - Z_{12}Z_{21}$$

So, just by doing the **inverse of Z parameters matrix**, we will get Y parameters matrix.

Z parameters to T parameters

Here, we have to represent T parameters in terms of Z parameters. So, in this case T parameters are the desired parameters and Z parameters are the given parameters.

Step 1 − We know that, the following set of two equations, which represents a two port network in terms of **T parameters**.

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

Step 2 − We know that the following set of two equations, which represents a two port network in terms of **Z parameters**.

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

Step 3 − We can modify the above equation as

$$\Rightarrow V_2 - Z_{22}I_2 = Z_{21}I_1$$

$$\Rightarrow I_1 = \left(\frac{1}{Z_{21}}\right)V_2 - \left(\frac{Z_{22}}{Z_{21}}\right)I_2$$

Step 4 − The above equation is in the form of $I_1 = CV_2 - DI_2$. Here,

$$C = \frac{1}{Z_{21}}$$

$$D = \frac{Z_{22}}{Z_{21}}$$

Step 5 − Substitute I_1 value of Step 3 in V_1 equation of Step 2.

$$V_1 = Z_{11}\{\left(\frac{1}{Z_{12}}\right)V_2 - \left(\frac{Z_{22}}{Z_{21}}\right)I_2\} + Z_{12}I_2$$

$$\Rightarrow V_1 = \left(\frac{Z_{11}}{Z_{21}}\right)V_2 - \left(\frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}\right)I_2$$

Step 6 − The above equation is in the form of $V_1 = AV_2 - BI_2$. Here,

$$A = \frac{Z_{11}}{Z_{21}}$$

$$B = \frac{Z_{11}Z_{22} - Z_{12}Z_{21}}{Z_{21}}$$

Step 7 − Therefore, the **T parameters matrix** is

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{Z_{11}}{Z_{21}} & \frac{Z_{11}Z_{22}-Z_{12}Z_{21}}{Z_{21}} \\ \frac{1}{Z_{21}} & \frac{Z_{22}}{Z_{21}} \end{bmatrix}$$

Y parameters to Z parameters

Here, we have to represent Z parameters in terms of Y parameters. So, in this case Z parameters are the desired parameters and Y parameters are the given parameters.

Step 1 − We know that, the following matrix equation of two port network regarding Z parameters as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$ **Equation 3**

Step 2 − We know that, the following matrix equation of two port network regarding Y parameters as

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Step 3 − We can modify it as

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$ **Equation 4**

Step 4 − By equating Equation 3 and Equation 4, we will get

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1}$$

$$\Rightarrow \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \frac{\begin{bmatrix} Y_{22} & -Y_{12} \\ -Y_{21} & Y_{11} \end{bmatrix}}{\Delta Y}$$

Where,

$$\Delta Y = Y_{11}Y_{22} - Y_{12}Y_{21}$$

So, just by doing the **inverse of Y parameters matrix**, we will get the Z parameters matrix.

Y parameters to T parameters

Here, we have to represent T parameters in terms of Y parameters. So, in this case, T parameters are the desired parameters and Y parameters are the given parameters.

Step 1 − We know that, the following set of two equations, which represents a two port network in terms of **T parameters**.

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

Step 2 − We know that the following set of two equations of two port network regarding Y parameters.

$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

Step 3 − We can modify the above equation as

$$\Rightarrow I_2 - Y_{22}V_2 = Y_{21}V_1$$

$$\Rightarrow V_1 = \left(\frac{-Y_{22}}{Y_{21}}\right)V_2 - \left(\frac{-1}{Y_{21}}\right)I_2$$

Step 4 − The above equation is in the form of $V_1 = AV_2 - BI_2$. Here,

$$A = \frac{-Y_{22}}{Y_{21}}$$

$$B = \frac{-1}{Y_{21}}$$

Step 5 − Substitute V_1 value of Step 3 in I_1 equation of Step 2.

$$I_1 = Y_{11}\{\left(\frac{-Y_{22}}{Y_{21}}\right)V_2 - \left(\frac{-1}{Y_{21}}\right)I_2\} + Y_{12}V_2$$

$$\Rightarrow I_1 = \left(\frac{Y_{12}Y_{21} - Y_{11}Y_{22}}{Y_{21}}\right)V_2 - \left(\frac{-Y_{11}}{Y_{21}}\right)I_2$$

Step 6 − The above equation is in the form of $I_1 = CV_2 - DI_2$. Here,

$$C = \frac{Y_{12}Y_{21} - Y_{11}Y_{22}}{Y_{21}}$$

$$D = \frac{-Y_{11}}{Y_{21}}$$

Step 7 − Therefore, the **T parameters matrix** is

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \frac{-Y_{22}}{Y_{21}} & \frac{-1}{Y_{21}} \\ \frac{Y_{12}Y_{21} - Y_{11}Y_{22}}{Y_{21}} & \frac{-Y_{11}}{Y_{21}} \end{bmatrix}$$

T parameters to h-parameters

Here, we have to represent h-parameters in terms of T parameters. So, in this case hparameters are the desired parameters and T parameters are the given parameters.

Step 1 − We know that, the following **h-parameters** of a two port network.

$$h_{11} = \frac{V_1}{I_1}, \ when\ V_2 = 0$$

$$h_{12} = \frac{V_1}{V_2}, \ when\ I_1 = 0$$

$$h_{21} = \frac{I_2}{I_1}, \ when\ V_2 = 0$$

$$h_{22} = \frac{I_2}{V_2}, \ when\ I_1 = 0$$

Step 2 − We know that the following set of two equations of two port network regarding **T parameters**.

$V_1 = AV_2 - BI_2$ **Equation 5**

$I_1 = CV_2 - DI_2$ **Equation 6**

Step 3 − Substitute $V_2 = 0$ in the above equations in order to find the two h-parameters, h_{11} and h_{21}

.

$$\Rightarrow V_1 = -BI_2$$

$$\Rightarrow I_1 = -DI_2$$

Substitute, V_1 and I_1 values in h-parameter, h_{11}.

$$h_{11} = \frac{-BI_2}{-DI_2}$$

$$\Rightarrow h_{11} = \frac{B}{D}$$

Substitute I_1 value in h-parameter h_{21}.

$$h_{21} = \frac{I_2}{-DI_2}$$

$$\Rightarrow h_{21} = -\frac{1}{D}$$

Step 4 − Substitute $I_1 = 0$ in the second equation of step 2 in order to find the h-parameter h_{22}.

$$0 = CV_2 - DI_2$$

$$\Rightarrow CV_2 = DI_2$$

$$\Rightarrow \frac{I_2}{V_2} = \frac{C}{D}$$

$$\Rightarrow h_{22} = \frac{C}{D}$$

Step 5 − Substitute $I_2 = \left(\frac{C}{D}\right)V_2$ in the first equation of step 2 in order to find the h-parameter, h_{12}.

$$V_1 = AV_2 - B\left(\frac{C}{D}\right)V_2$$

$$\Rightarrow V_1 = \left(\frac{AD - BC}{D}\right)V_2$$

$$\Rightarrow \frac{V_1}{V_2} = \frac{AD - BC}{D}$$

$$\Rightarrow h_{12} = \frac{AD - BC}{D}$$

Step 6 − Therefore, the h-parameters matrix is

$$\begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} \frac{B}{D} & \frac{AD-BC}{D} \\ -\frac{1}{D} & \frac{C}{D} \end{bmatrix}$$

h-parameters to Z parameters

Here, we have to represent Z parameters in terms of h-parameters. So, in this case Z parameters are the desired parameters and h-parameters are the given parameters.

Step 1 − We know that, the following set of two equations of two port network regarding **Z parameters**.

$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

Step 2 − We know that, the following set of two equations of two-port network regarding **h-parameters**.

$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

Step 3 − We can modify the above equation as

$$\Rightarrow I_2 - h_{21}I_1 = h_{22}V_2$$

$$\Rightarrow V_2 = \frac{I_2 - h_{21}I_1}{h_{22}}$$

$$\Rightarrow V_2 = \square\frac{-h_{21}}{h_{22}}\square I_1 + \square\frac{1}{h_{22}}\square I_2$$

The above equation is in the form of $V_2 = Z_{21}I_1 + Z_{22}I_2. Here,$

$$Z_{21} = \frac{-h_{21}}{h_{22}}$$

$$Z_{22} = \frac{1}{h_{22}}$$

Step 4 − Substitute V_2 value in first equation of step 2.

$$V_1 = h_{11}I_1 + h_{21}\left\{\left(\frac{-h_{21}}{h_{22}}\right)I_1 + \left(\frac{1}{h_{22}}\right)I_2\right\}$$

$$\Rightarrow V_1 = \left(\frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}}\right)I_1 + \left(\frac{h_{12}}{h_{22}}\right)I_2$$

The above equation is in the form of $V_1 = Z_{11}I_1 + Z_{12}I_2$. Here,

$$Z_{11} = \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}}$$

$$Z_{12} = \frac{h_{12}}{h_{22}}$$

Step 5 − Therefore, the Z parameters matrix is

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} \frac{h_{11}h_{22} - h_{12}h_{21}}{h_{22}} & \frac{h_{12}}{h_{22}} \\ \frac{-h_{21}}{h_{22}} & \frac{1}{h_{22}} \end{bmatrix}$$

In this way, we can convert one set of parameters into other set of parameters.