

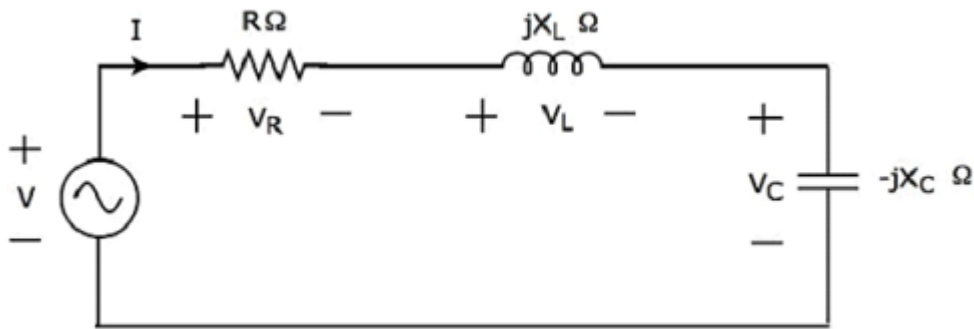
## Network Theory - Series Resonance

**Resonance** occurs in electric circuits due to the presence of energy storing elements like inductor and capacitor. It is the fundamental concept based on which, the radio and TV receivers are designed in such a way that they should be able to select only the desired station frequency.

There are **two types** of resonances, namely series resonance and parallel resonance. These are classified based on the network elements that are connected in series or parallel. In this chapter, let us discuss about series resonance.

### Series Resonance Circuit Diagram

If the resonance occurs in series RLC circuit, then it is called as **Series Resonance**. Consider the following **series RLC circuit**, which is represented in phasor domain.



Here, the passive elements such as resistor, inductor and capacitor are connected in series. This entire combination is in **series** with the input sinusoidal voltage source.

Apply **KVL** around the loop.

$$V - V_R - V_L - V_C = 0$$

$$\Rightarrow V - IR - I(jX_L) - I(-jX_C) = 0$$

$$\Rightarrow V = IR + I(jX_L) + I(-jX_C)$$

$$\Rightarrow V = I[R + j(X_L - X_C)]$$

**Equation 1**

The above equation is in the form of **V = IZ**.

Therefore, the **impedance Z** of series RLC circuit will be

$$Z = R + j(X_L - X_C)$$

## Parameters & Electrical Quantities at Resonance

Now, let us derive the values of parameters and electrical quantities at resonance of series RLC circuit one by one.

### Resonant Frequency

The frequency at which resonance occurs is called as **resonant frequency  $f_r$** . In series RLC circuit resonance occurs, when the imaginary term of impedance Z is zero, i.e., the value of  $X_L - X_C$  should be equal to zero.

$$\Rightarrow X_L = X_C$$

Substitute  $X_L = 2\pi fL$  and  $X_C = \frac{1}{2\pi fC}$  in the above equation.

$$2\pi fL = \frac{1}{2\pi fC}$$

$$\Rightarrow f^2 = \frac{1}{(2\pi)^2 LC}$$

$$\Rightarrow f = \frac{1}{(2\pi)\sqrt{LC}}$$

Therefore, the **resonant frequency  $f_r$**  of series RLC circuit is

$$f_r = \frac{1}{(2\pi)\sqrt{LC}}$$

Where, **L** is the inductance of an inductor and **C** is the capacitance of a capacitor.

The **resonant frequency**  $f_r$  of series RLC circuit depends only on the inductance  $L$  and capacitance  $C$ . But, it is independent of resistance  $R$ .

### Impedance

We got the **impedance**  $Z$  of series RLC circuit as

$$Z = R + j(X_L - X_C)$$

Substitute  $X_L = X_C$  in the above equation.

$$Z = R + j(X_C - X_C)$$

$$\Rightarrow Z = R + j(0)$$

$$\Rightarrow Z = R$$

At resonance, the **impedance**  $Z$  of series RLC circuit is equal to the value of resistance  $R$ , i.e.,  $Z = R$ .

### Current flowing through the Circuit

Substitute  $X_L - X_C = 0$  in Equation 1.

$$V = I[R + j(0)]$$

$$\Rightarrow V = IR$$

$$\Rightarrow I = \frac{V}{R}$$

Therefore, **current** flowing through series RLC circuit at resonance is  $I = \frac{V}{R}$ .

At resonance, the impedance of series RLC circuit reaches to minimum value. Hence, the **maximum current** flows through this circuit at resonance.

### Voltage across Resistor

The voltage across resistor is

$$V_R = IR$$

Substitute the value of  $I$  in the above equation.

$$V_R = \left(\frac{V}{R}\right)R$$

$$\Rightarrow V_R = V$$

Therefore, the **voltage across resistor** at resonance is  $V_R = V$ .

### Voltage across Inductor

The voltage across inductor is

$$V_L = I(jX_L)$$

Substitute the value of  $I$  in the above equation.

$$V_L = \left(\frac{V}{R}\right)(jX_L)$$

$$\Rightarrow V_L = j\left(\frac{X_L}{R}\right)V$$

$$\Rightarrow V_L = jQV$$

Therefore, the **voltage across inductor** at resonance is  $V_L = jQV$  .

So, the **magnitude** of voltage across inductor at resonance will be

$$|V_L| = QV$$

Where **Q** is the **Quality factor** and its value is equal to  $\frac{X_L}{R}$

### Voltage across Capacitor

The voltage across capacitor is

$$V_C = I(-jX_C)$$

Substitute the value of  $I$  in the above equation.

$$V_C = \left(\frac{V}{R}\right)(-jX_C)$$

$$\Rightarrow V_C = -j\left(\frac{X_C}{R}\right)V$$

$$\Rightarrow V_C = -jQV$$

Therefore, the **voltage across capacitor** at resonance is  $V_C = -jQV$  .

So, the **magnitude** of voltage across capacitor at resonance will be

$$|V_C| = QV$$

Where **Q** is the **Quality factor** and its value is equal to  $\frac{X_C}{R}$

**Note** – Series resonance RLC circuit is called as **voltage magnification** circuit, because the magnitude of voltage across the inductor and the capacitor is equal to Q times the input sinusoidal voltage  $V$ .