Network Theory - Nodal Analysis

There are two basic methods that are used for solving any electrical network: **Nodal analysis** and **Mesh analysis**. In this chapter, let us discuss about the **Nodal analysis** method.

In Nodal analysis, we will consider the node voltages with respect to Ground. Hence, Nodal analysis is also called as **Node-voltage method**.

Procedure of Nodal Analysis

Follow these steps while solving any electrical network or circuit using Nodal analysis.

- **Step 1** − Identify the **principal nodes** and choose one of them as **reference node**. We will treat that reference node as the Ground.

- **Step 2** − Label the **node voltages** with respect to Ground from all the principal nodes except the reference node.

- **Step 3** − Write **nodal equations** at all the principal nodes except the reference node. Nodal equation is obtained by applying KCL first and then Ohm's law.

- **Step 4** − Solve the nodal equations obtained in Step 3 in order to get the node voltages.

Now, we can find the current flowing through any element and the voltage across any element that is present in the given network by using node voltages.

Example

Find the current flowing through 20 Ω resistor of the following circuit using **Nodal analysis**.
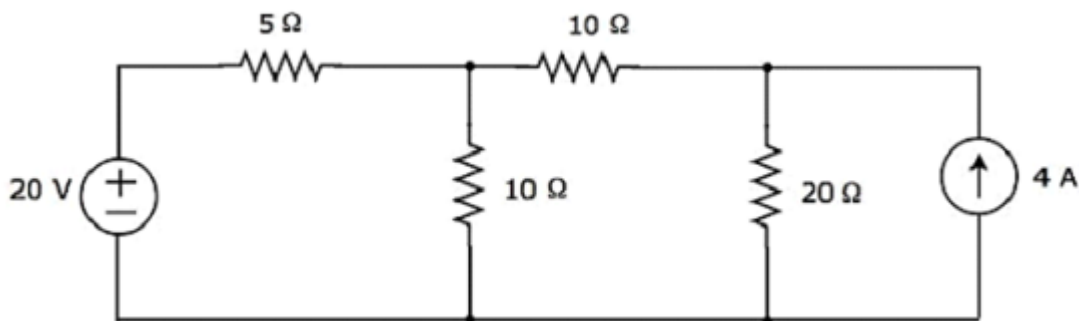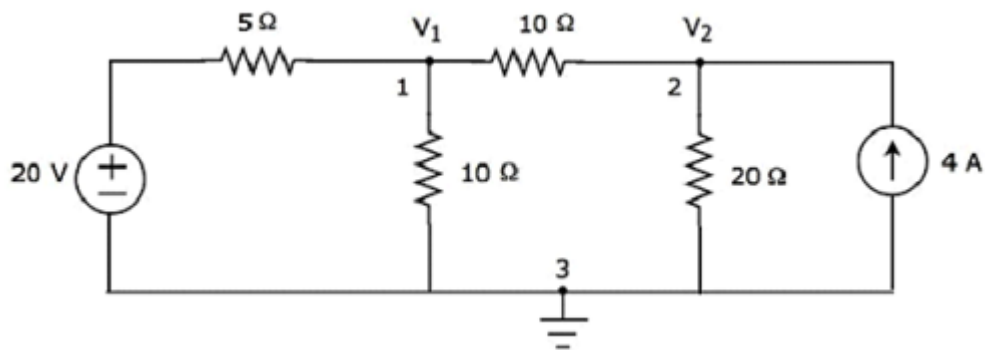

Step 1 − There are **three principle nodes** in the above circuit. Those are labelled as 1, 2, and 3 in the following figure.
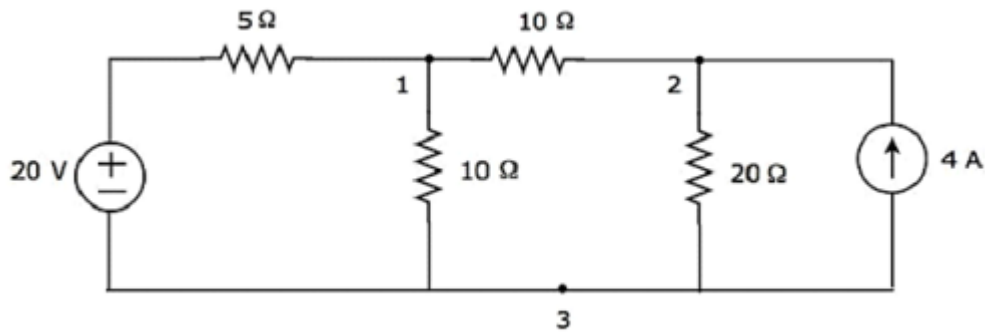
In the above figure, consider **node 3** as reference node (Ground).

Step 2 − The node voltages, V_1 and V_2, are labelled in the following figure.

In the above figure, V_1 is the voltage from node 1 with respect to ground and V_2 is the voltage from node 2 with respect to ground.

Step 3 − In this case, we will get **two nodal equations**, since there are two principal nodes, 1 and 2, other than Ground. When we write the nodal equations at a node, assume all the currents are leaving from the node for which the direction of current is not mentioned and that node's voltage as greater than other node voltages in the circuit.

The **nodal equation** at node 1 is

$$\frac{V_1 - 20}{5} + \frac{V_1}{10} + \frac{V_1 - V_2}{10} = 0$$

$$\Rightarrow \frac{2V_1 - 40 + V_1 + V_1 - V_2}{10} = 0$$

$$\Rightarrow 4V_1 - 40 - V_2 = 0$$

$$\Rightarrow V_2 = 4V_1 - 40 \qquad\qquad \textbf{Equation 1}$$

The **nodal equation** at node 2 is

$$-4 + \frac{V_2}{20} + \frac{V_2 - V_1}{10} = 0$$

$$\Rightarrow \frac{-80 + V_2 + 2V_2 - 2V_2}{20} = 0$$

$$\Rightarrow 3V_2 - 2V_1 = 80 \qquad\qquad \textbf{Equation 2}$$

Step 4 − Finding node voltages, V_1 and V_2 by solving Equation 1 and Equation 2.

Substitute Equation 1 in Equation 2.

$$3(4V_1 - 40) - 2V_1 = 80$$

$$\Rightarrow 12V_1 - 120 - 2V_1 = 80$$

$$\Rightarrow 10V_1 = 200$$

$$\Rightarrow V_1 = 20V$$

Substitute V_1 = 20 V in Equation1.

$$V_2 = 4(20) - 40$$

$$\Rightarrow V_2 = 40V$$

So, we got the node voltages V_1 and V_2 as **20 V** and **40 V** respectively.

Step 5 − The voltage across 20 Ω resistor is nothing but the node voltage V_2 and it is equal to 40 V. Now, we can find the current flowing through 20 Ω resistor by using Ohm's law.

$$I_{20\Omega} = \frac{V_2}{R}$$

Substitute the values of V_2 and R in the above equation.

$$I_{20\Omega} = \frac{40}{20}$$

$$\Rightarrow I_{20\Omega} = 2A$$

Therefore, the current flowing through 20 Ω resistor of given circuit is **2 A**.

Note − From the above example, we can conclude that we have to solve 'n' nodal equations, if the electric circuit has 'n' principal nodes (except the reference node). Therefore, we can choose Nodal analysis when the **number of principal nodes** (except reference node) is less than the number of meshes of any electrical circuit.