Convex Hull

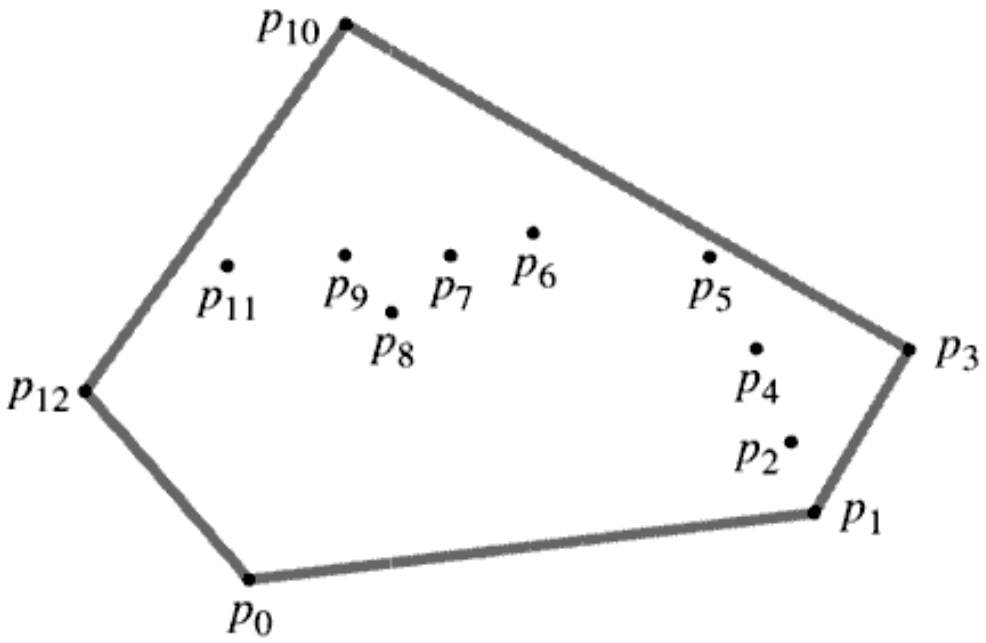
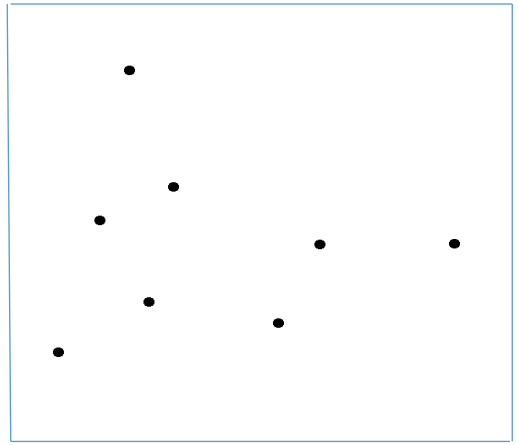


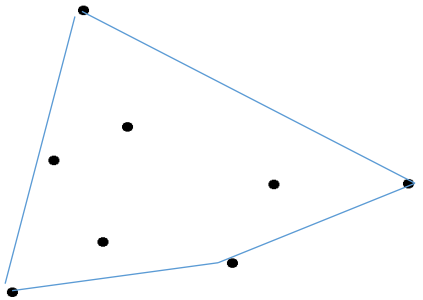
Figure 33.6 A set of points $Q = \{p_0, p_1, \ldots, p_{12}\}$ with its convex hull $\mathrm{CH}(Q)$ in gray.

Applications

- Approximation
  - rough sketch of data



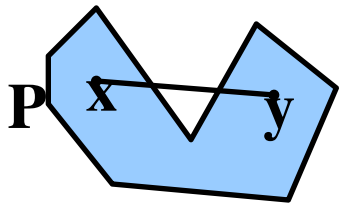
Bounding Rectangle (Envelope)



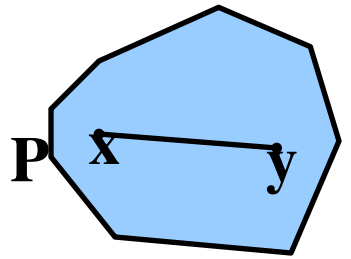
- Given observations of animal locations,
  find its home range.

Convex vs. Concave

- A polygon P is convex if for every pair of points x and y in P, the line xy is also in P; otherwise, it is called concave.



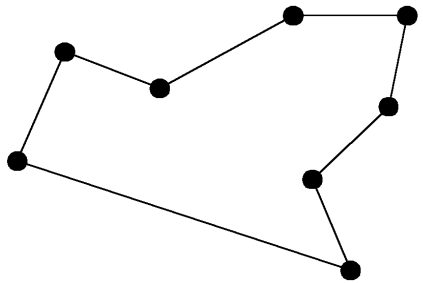
concave



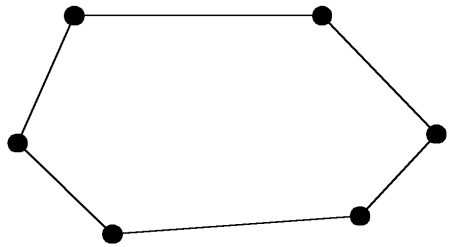
convex

The convex hull problem

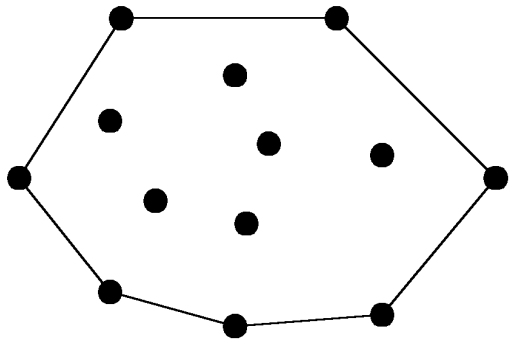
concave polygon:



convex polygon:



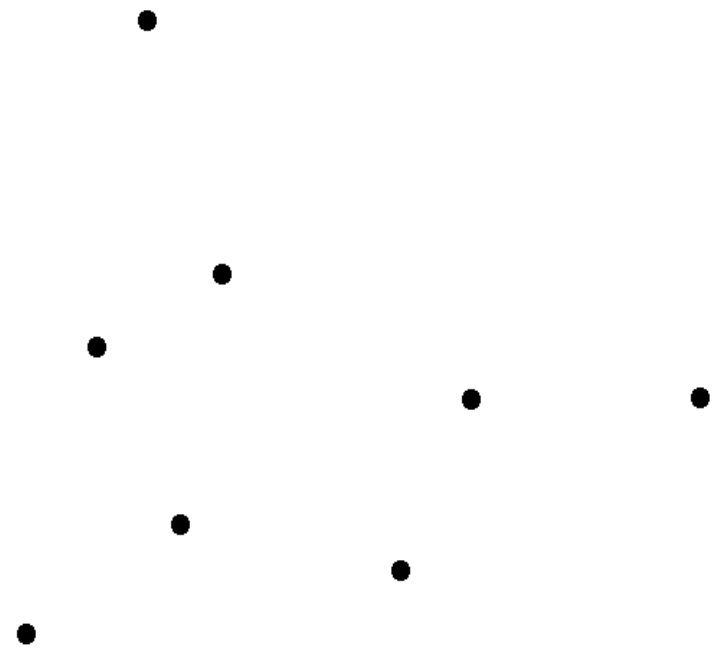
- The convex hull of a set of planar points is the smallest convex polygon containing all of the points.



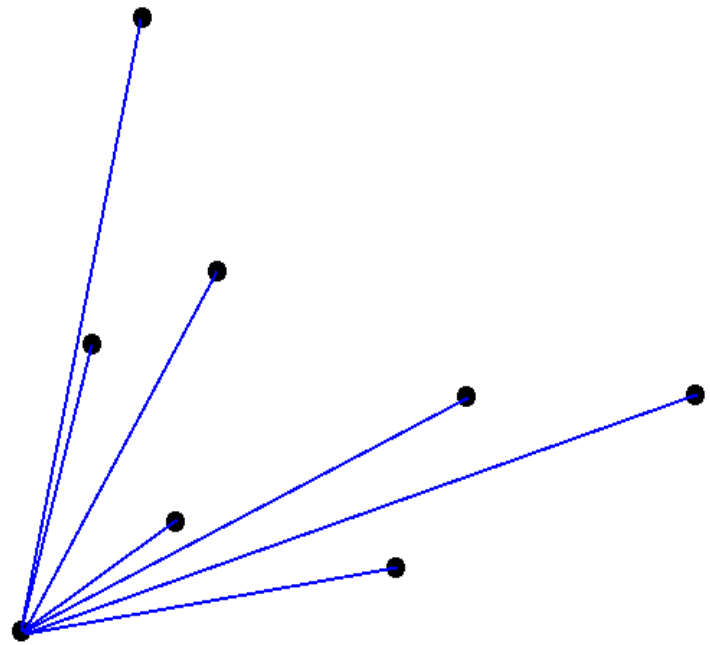
Graham's Scan

- Start at point guaranteed to be on the hull. (the point with the minimum y value)
- Sort remaining points by polar angles of vertices relative to the first point.
- Go through sorted points, keeping vertices of points that have left turns and dropping points that have right turns.

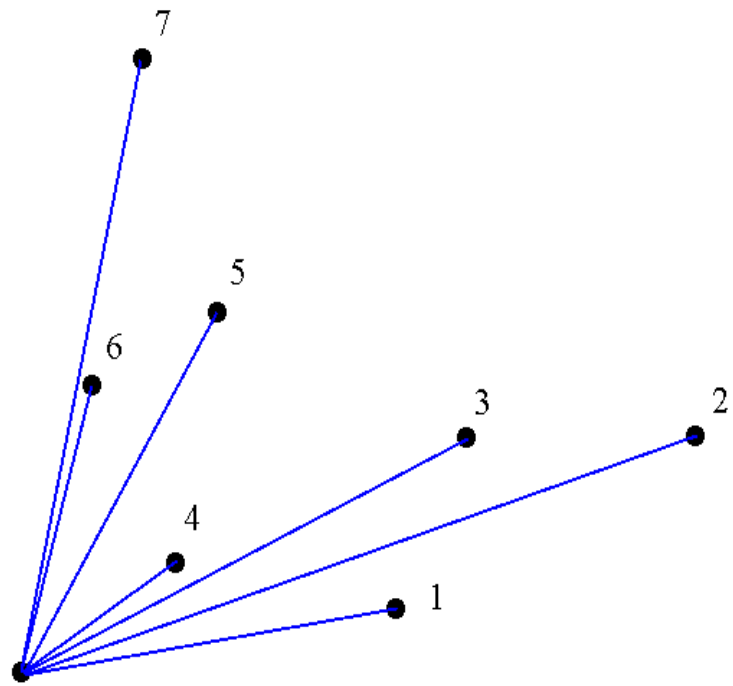
Graham’s Scan



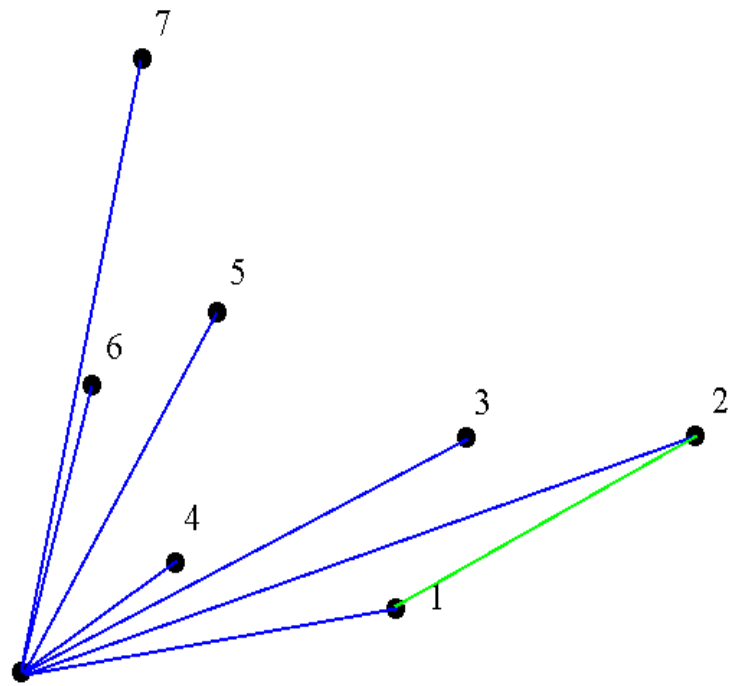
Graham’s Scan



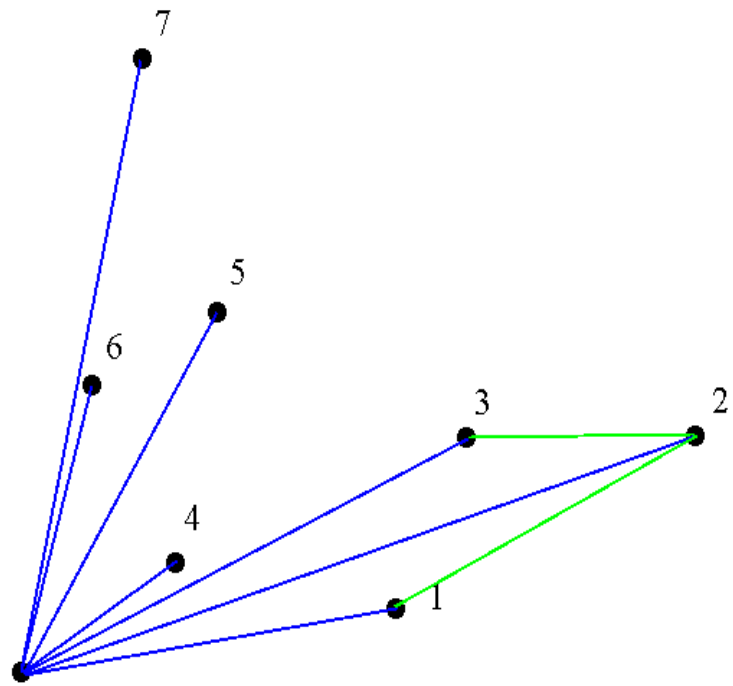
Graham's Scan



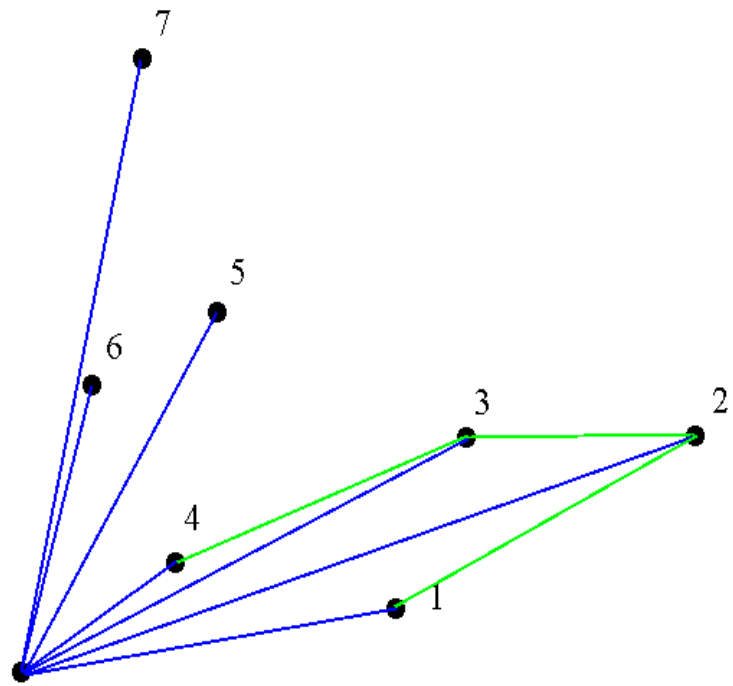
Graham's Scan



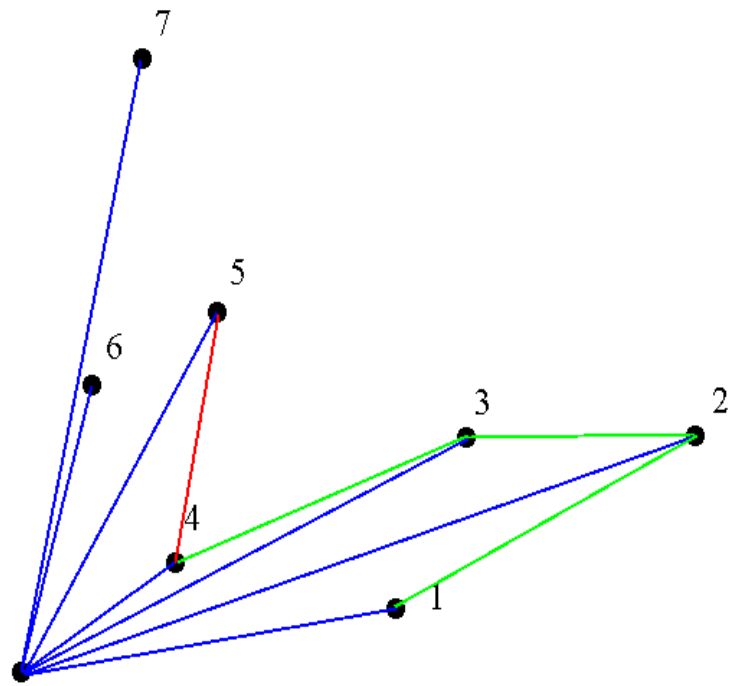
Graham's Scan



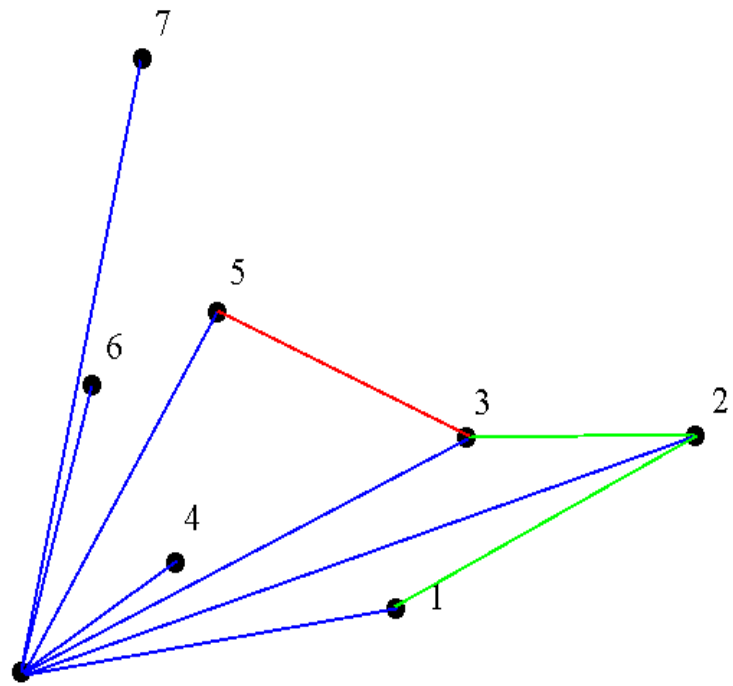
Graham's Scan



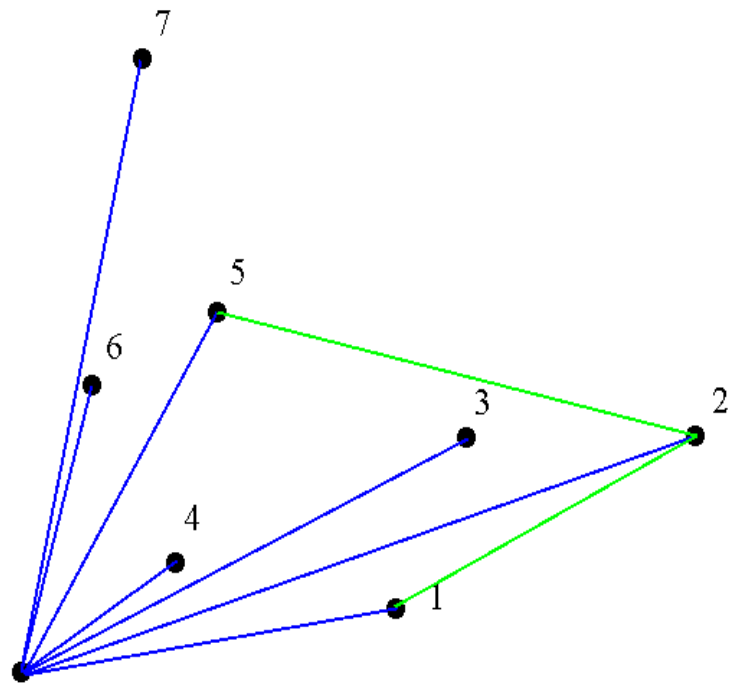
Graham's Scan



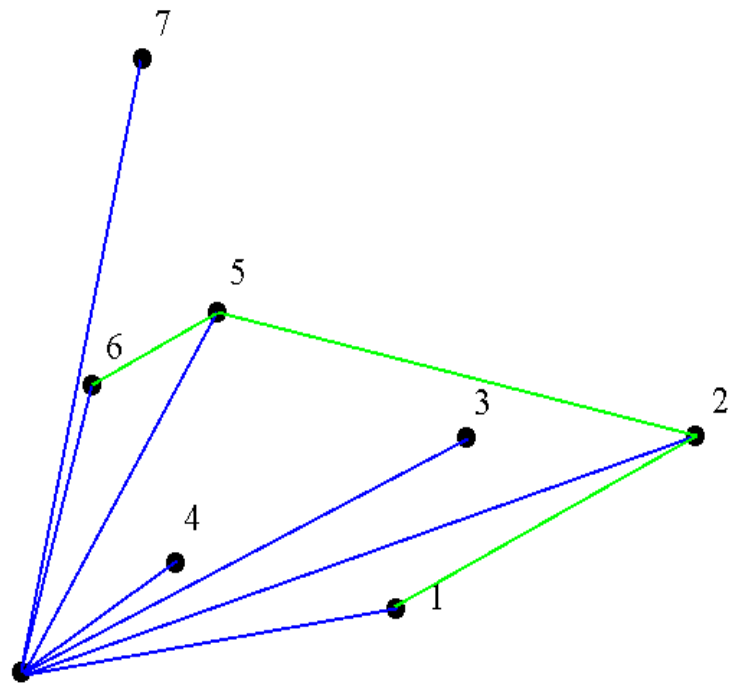
Graham's Scan



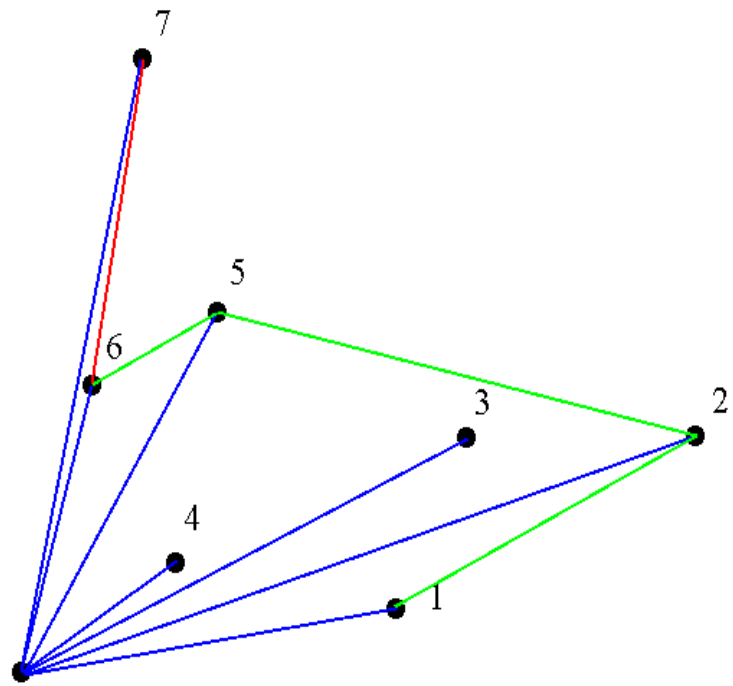
Graham's Scan



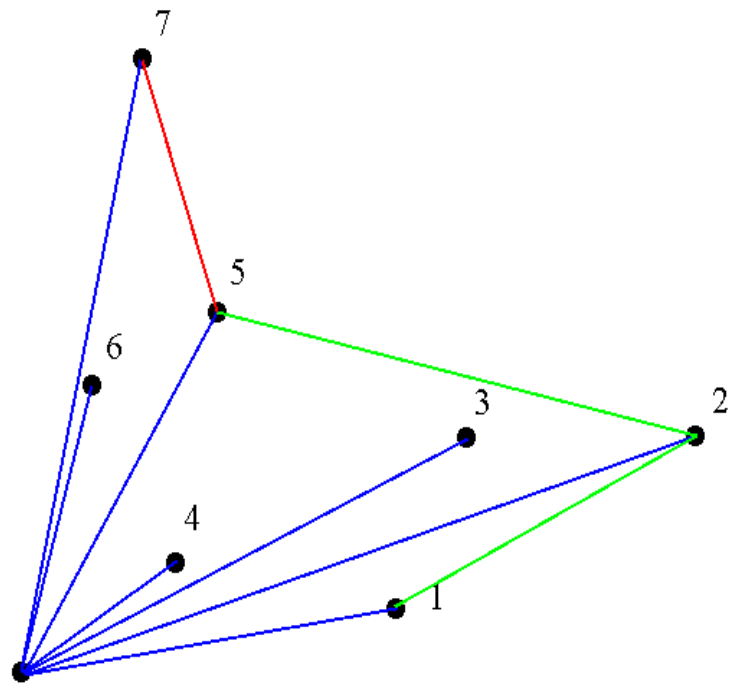
Graham's Scan



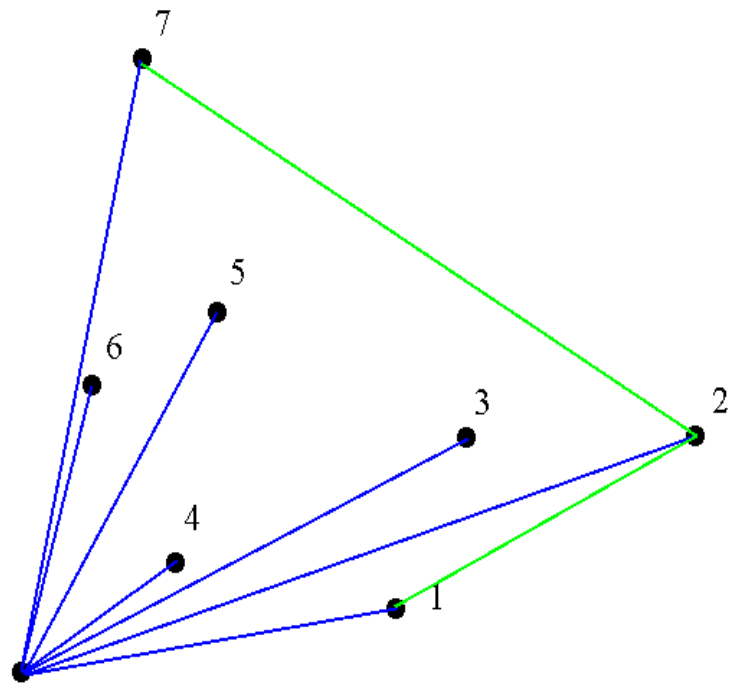
Graham's Scan



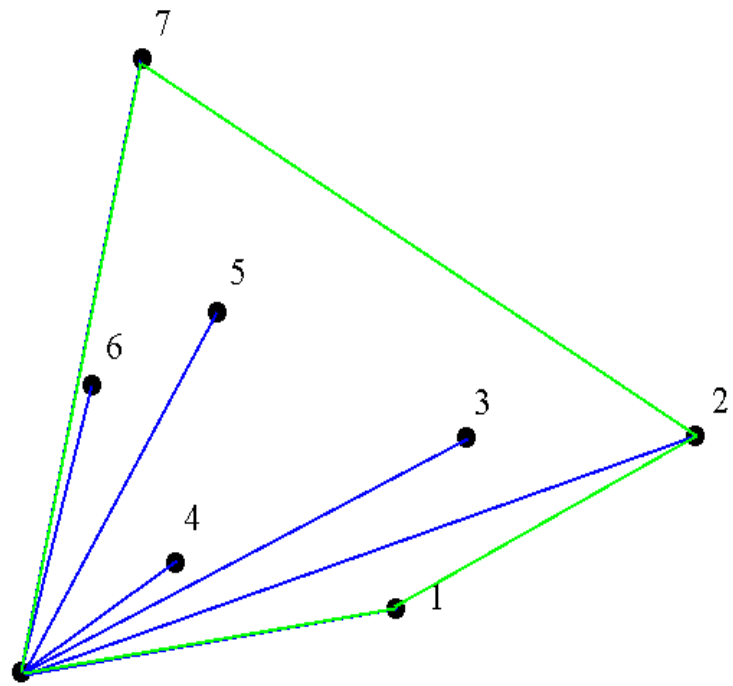
Graham's Scan

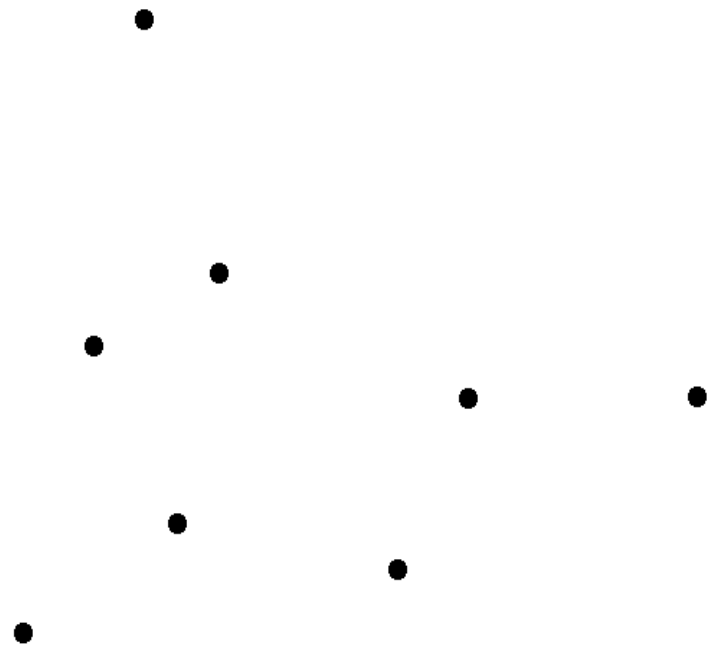


Graham's Scan



Graham's Scan

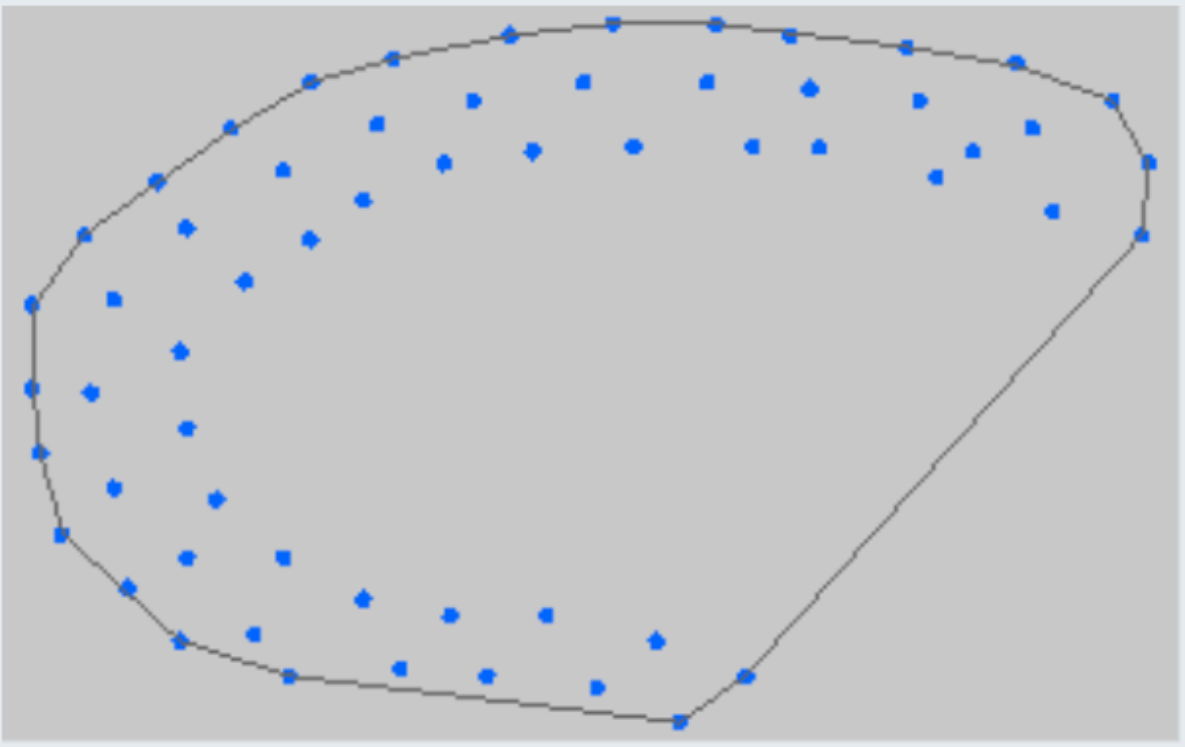


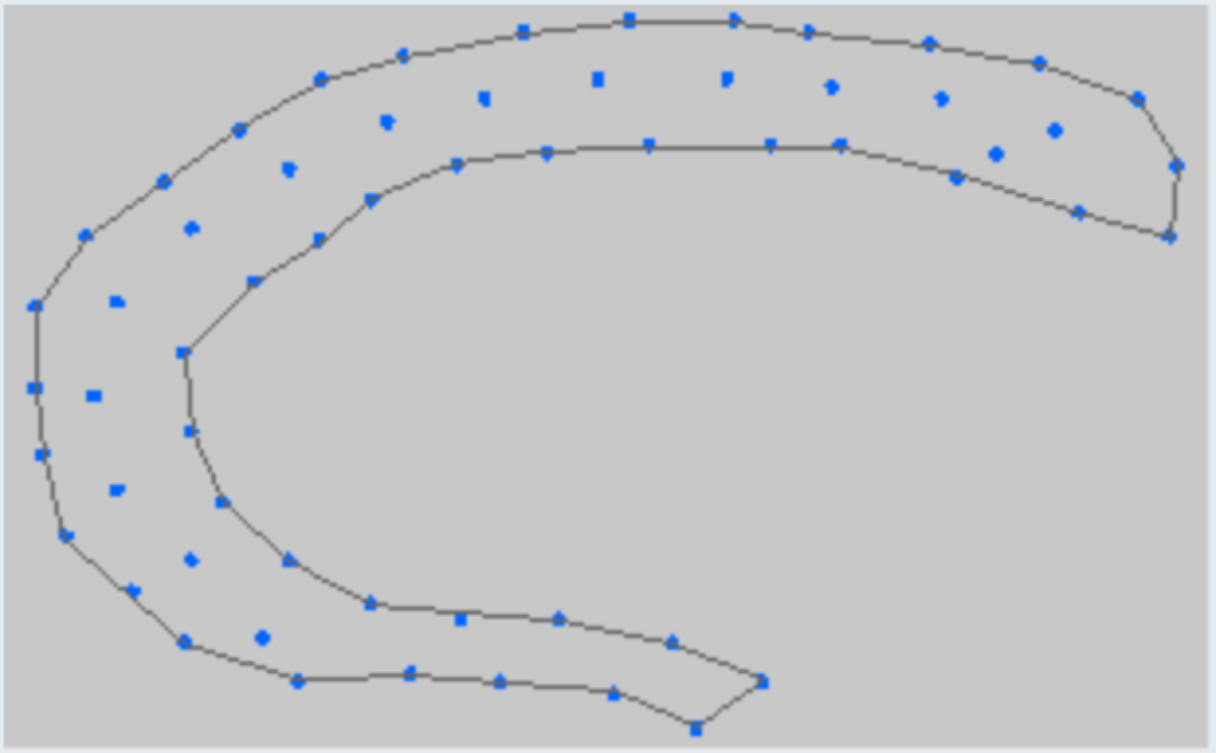


Draw Convex Hull for a set of input points.

Pop quiz

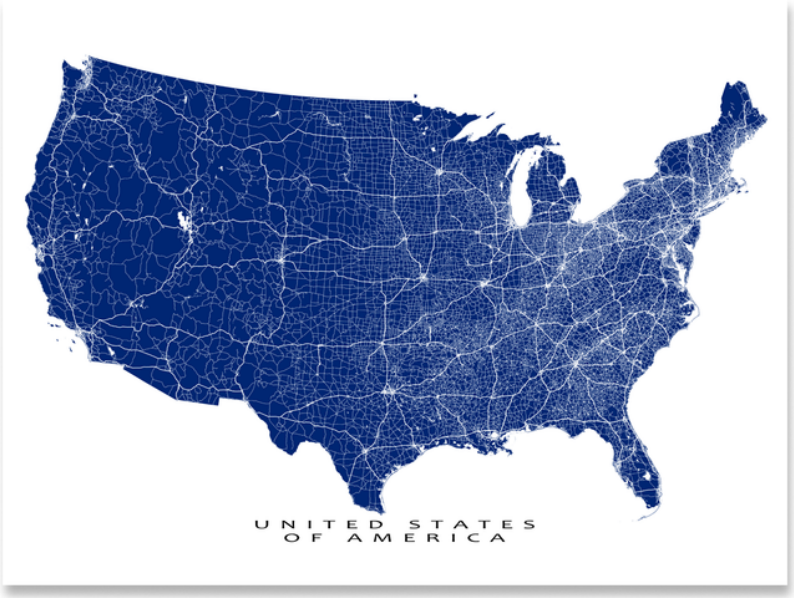
Identify convex hull ?





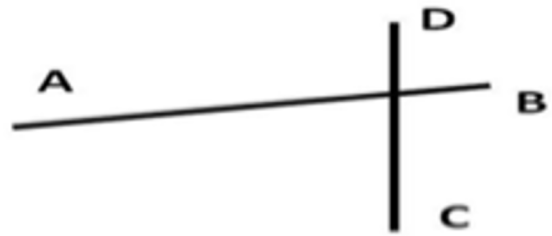
Find which counties a river goes through?

Find the roads intersecting rivers?

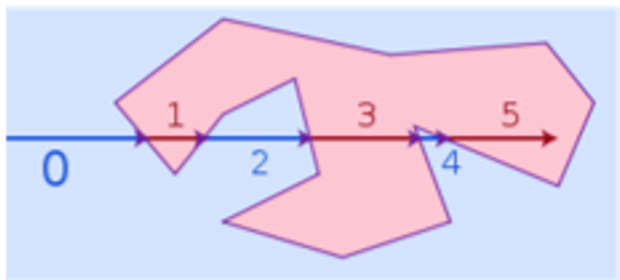


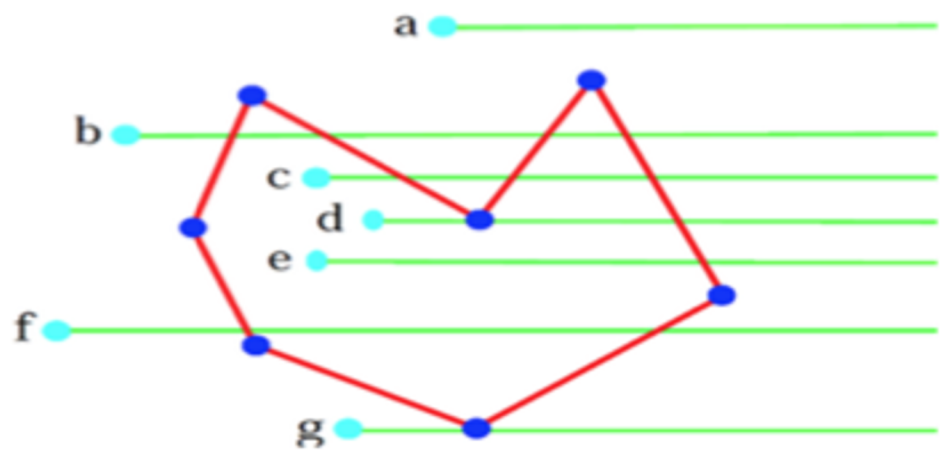
Basics: Intersection and PNP

No intersection

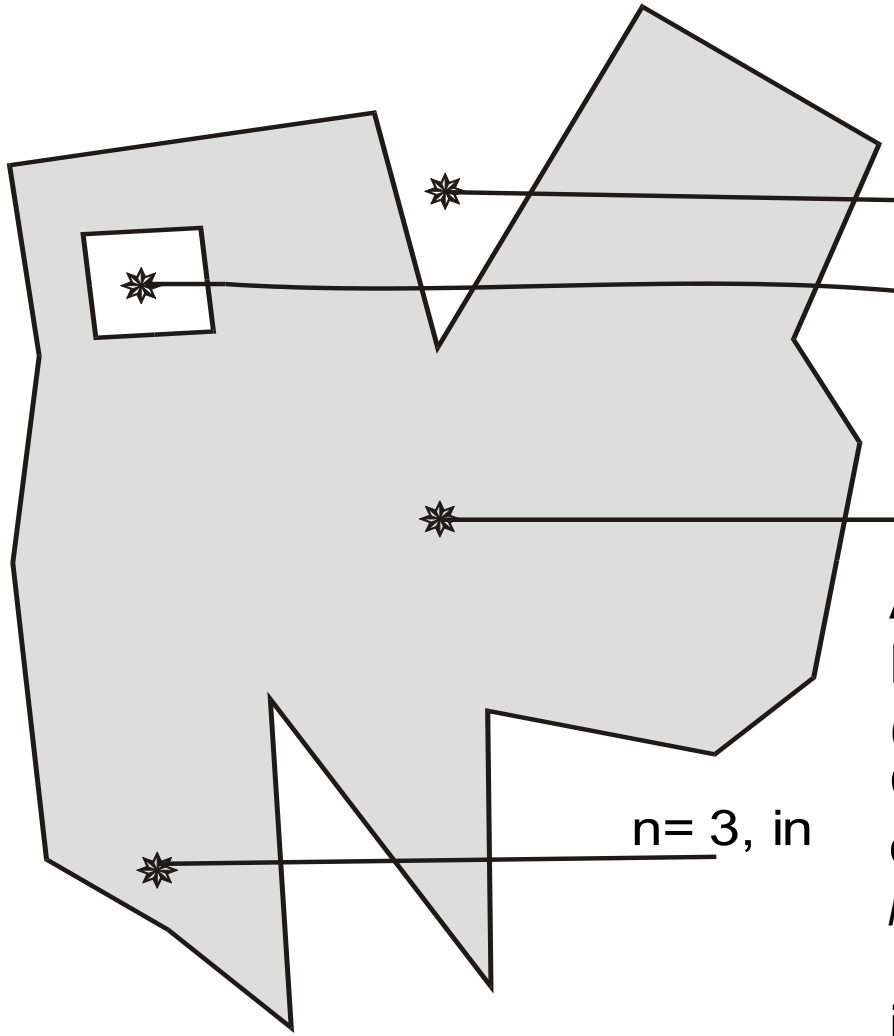


"Real" intersection





Finding the interior: Is a point inside a polygon *(shaded)*?

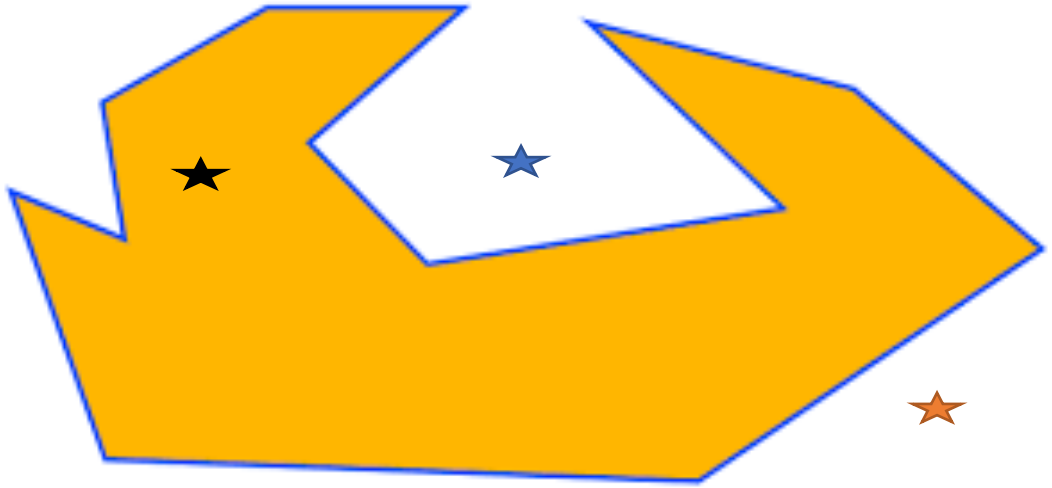


Potential point

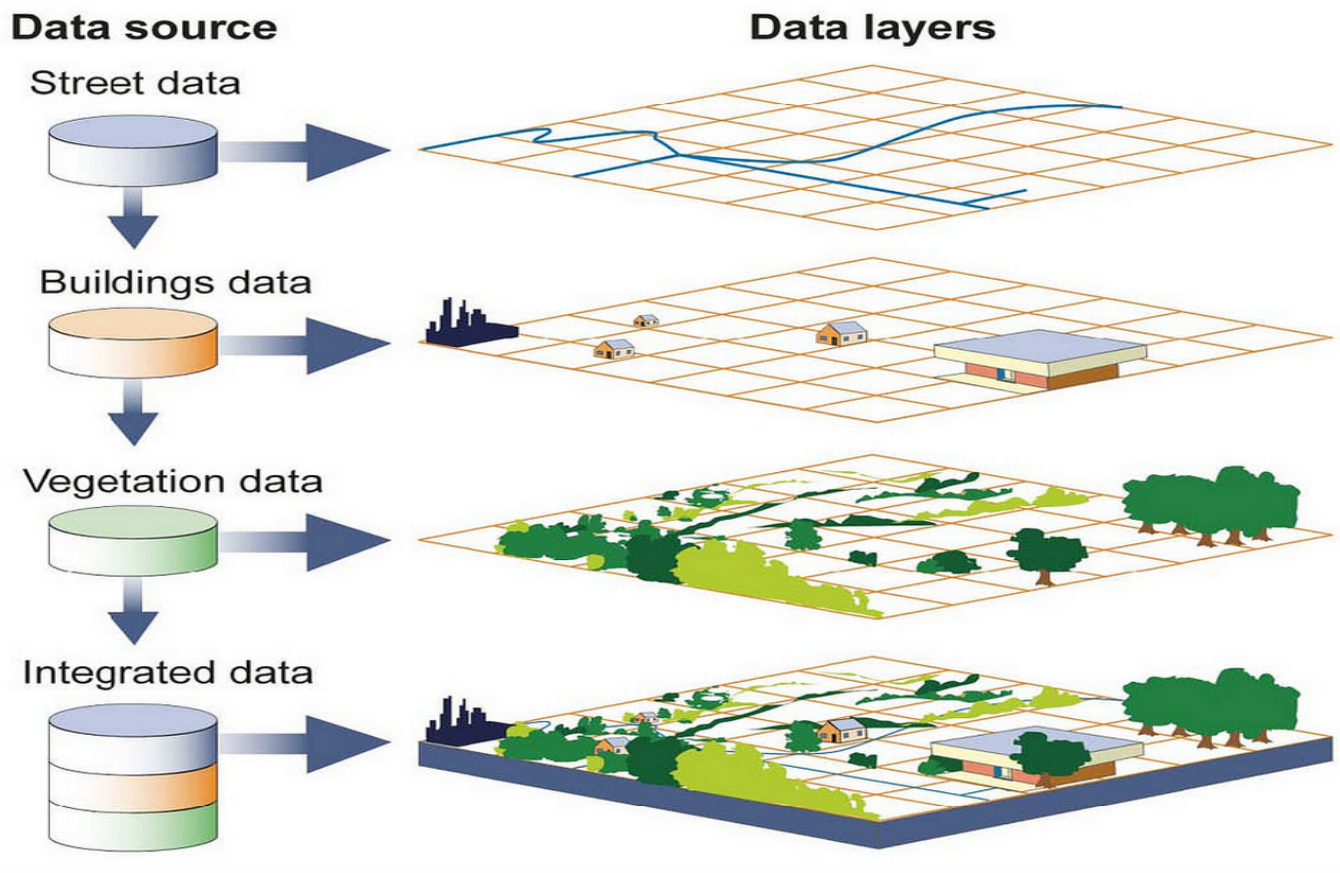
Algorithm:
Pick a direction
(East (right) in the example)
Count line crossings to the outside of convex hull *(shaded polygon)*
If odd number then the point is inside
If even, the point is outside

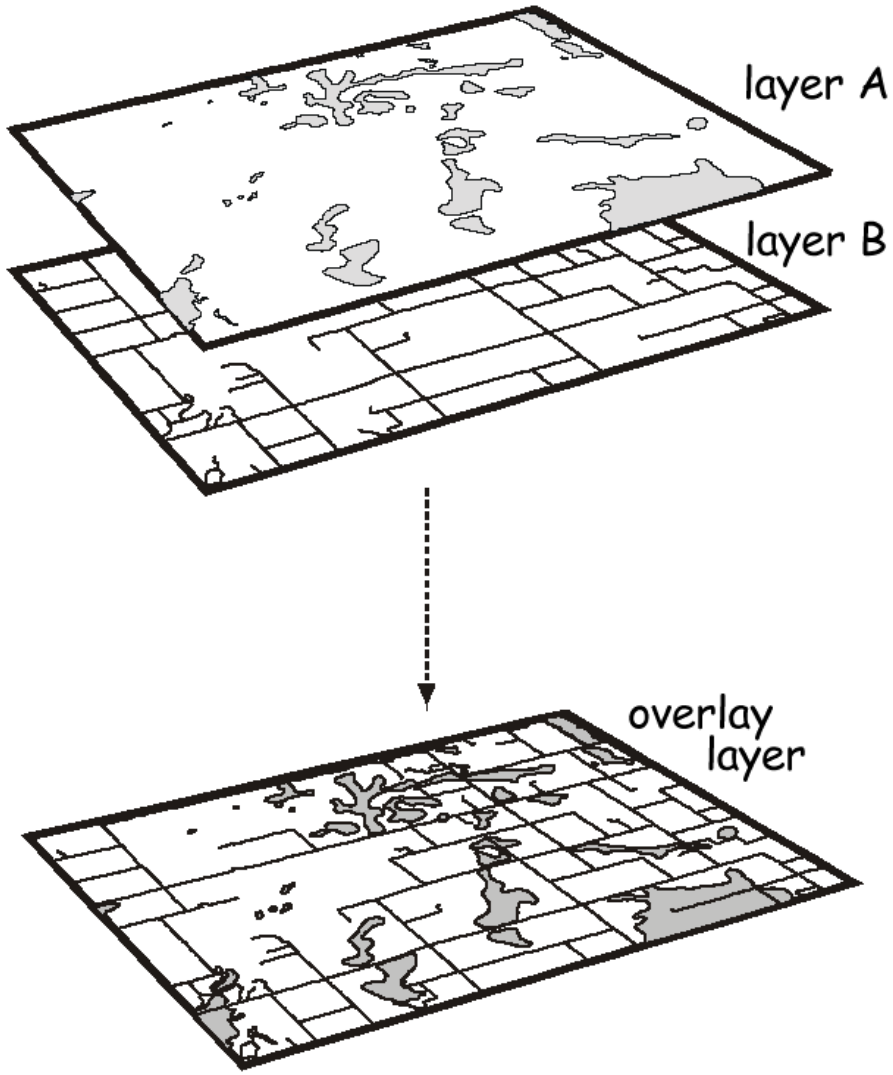
Pop Quiz

- What is n for the three stars?
- Will the algorithm work if the ray is vertical instead of horizontal?



Polygon Overlay



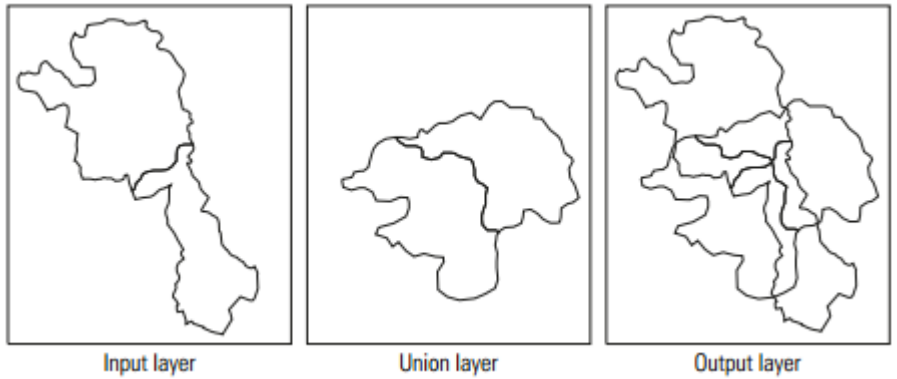


attributes for layer A

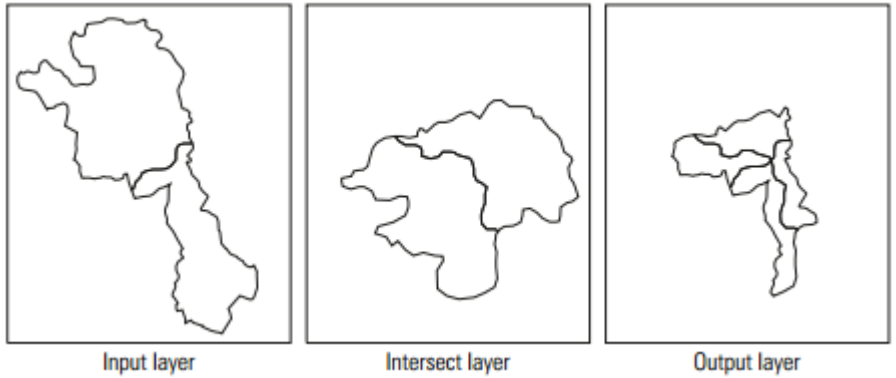
attributes for layer B

overlay attributes, combined
attributes for layers A & B

Overlaying example



Union



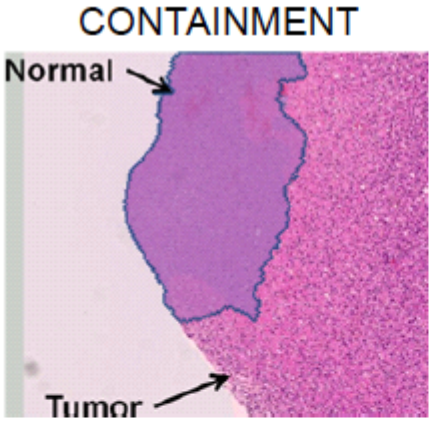
Intersection

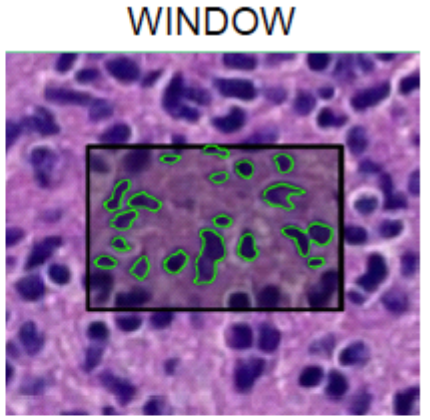
Polygonal map layers

Size	Polygons	Edges	Intersection vertices
70MB	16 K	1.4 M	31 K

Pathology Image Analytics

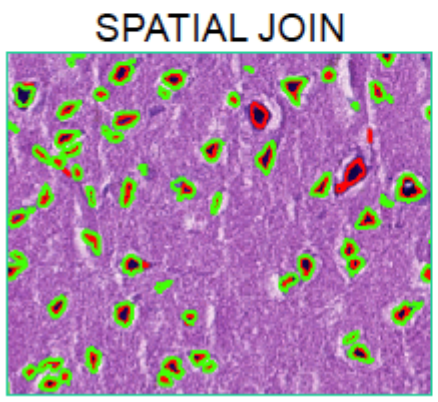
Objects nuclei in tumor Regions ?





Retrieve Nuclei from the window

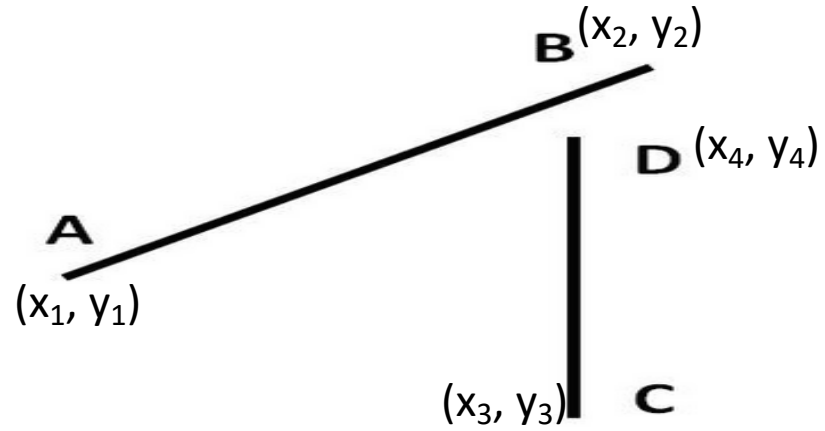
Cross-comparison two results from same image By Jaccard similarity of million by million objects



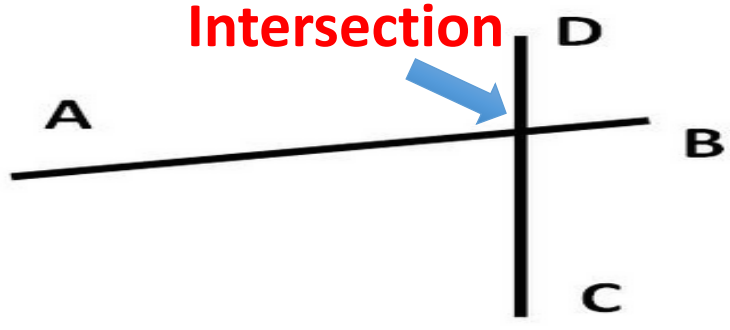
For each stem cell, find the nearest blood vessel

Fig. **Digital pathology images**

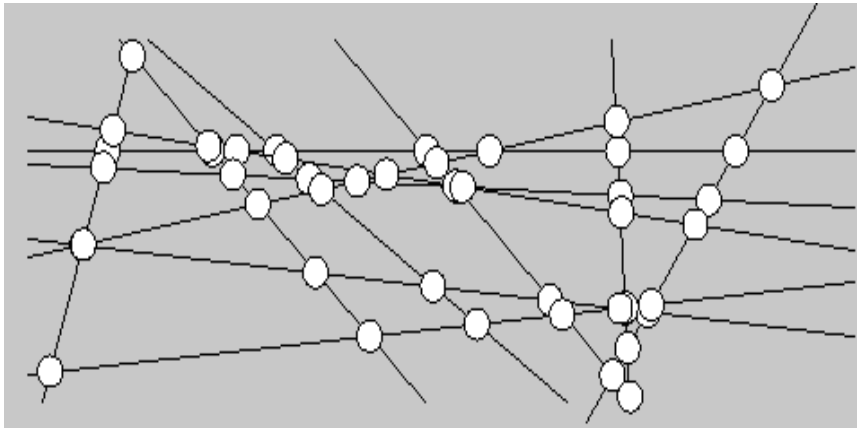
Line segments

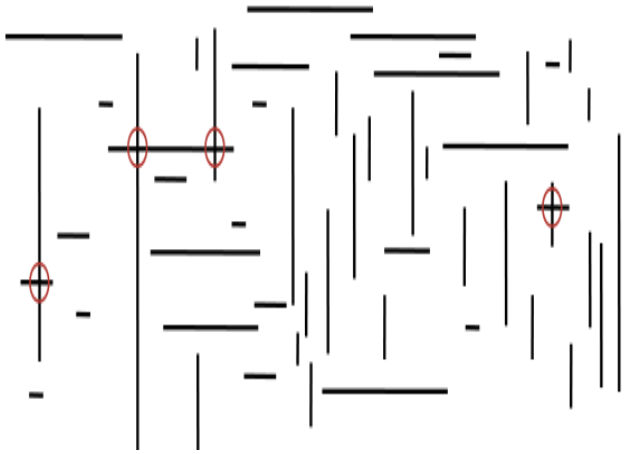


No intersection

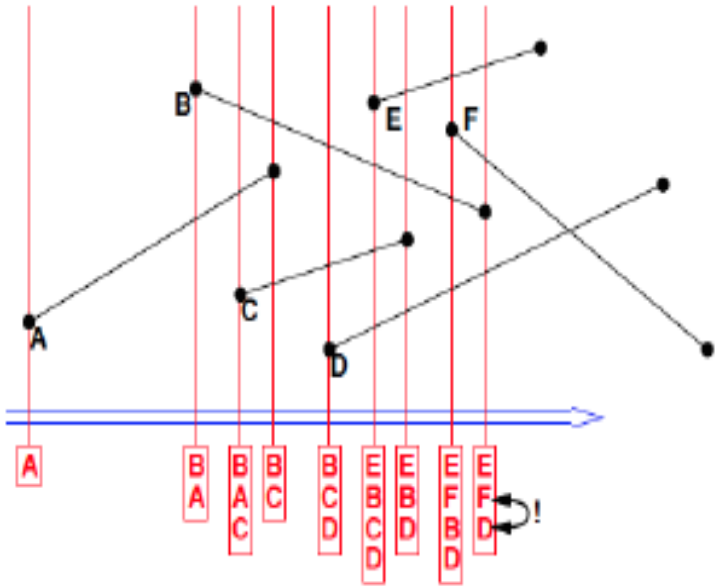


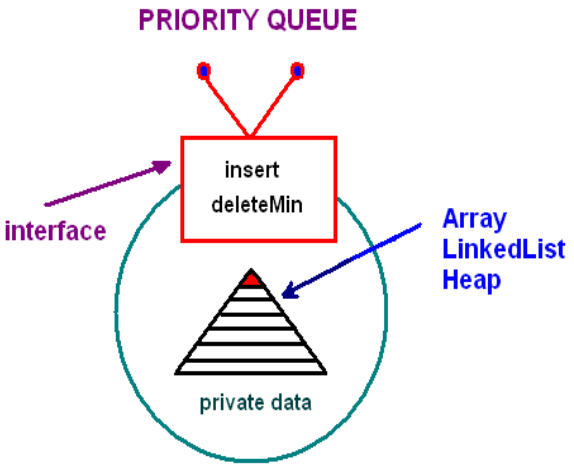
"Real" intersection

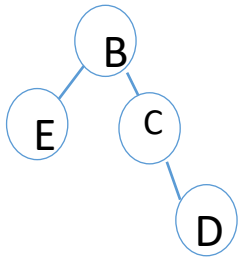




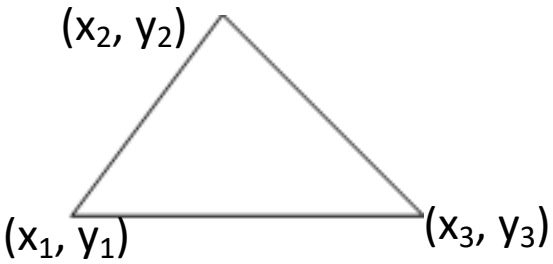
Sweep line algorithm

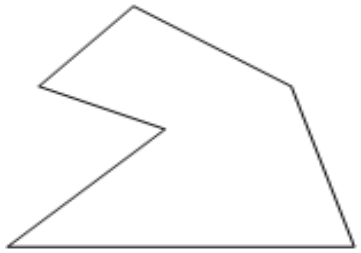


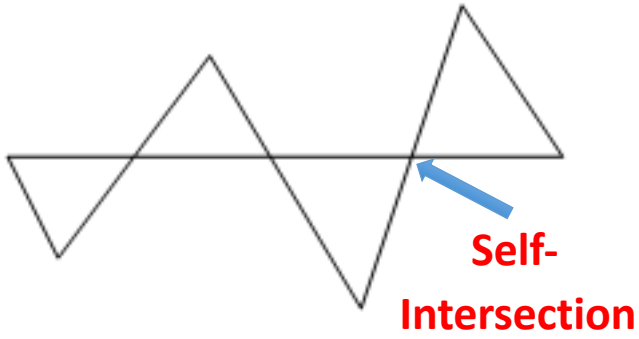




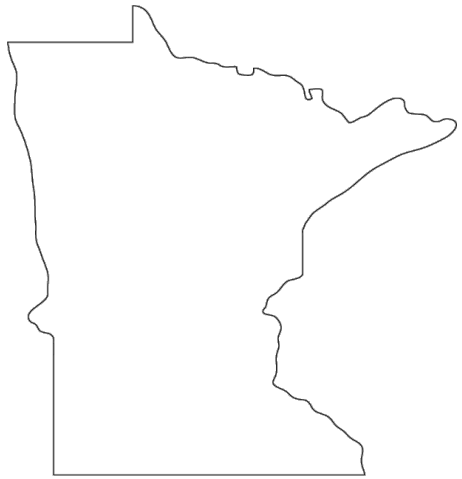
Polygons



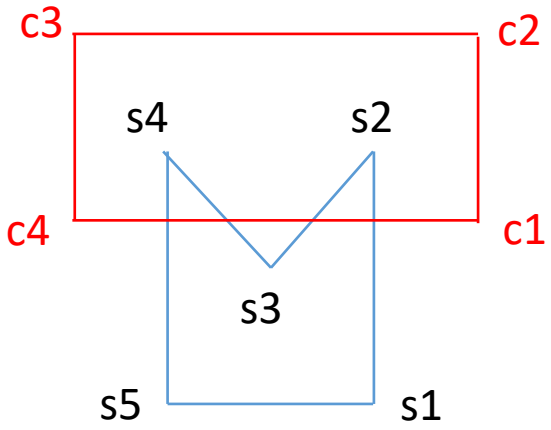




Self-Intersecting polygon



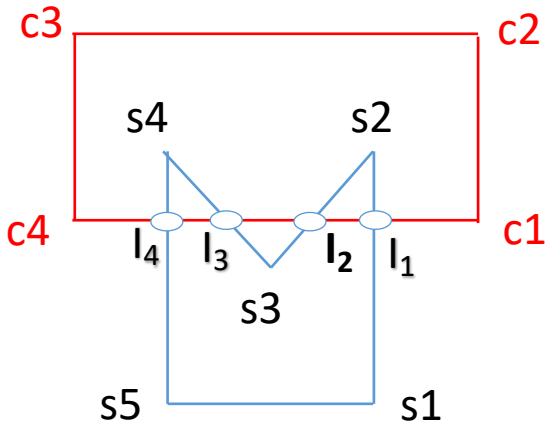
Simple Clipping examples



Input Polygons:

S = {s1, s2, s3, s4, s5}

C = {c1, c2, c3, c4}

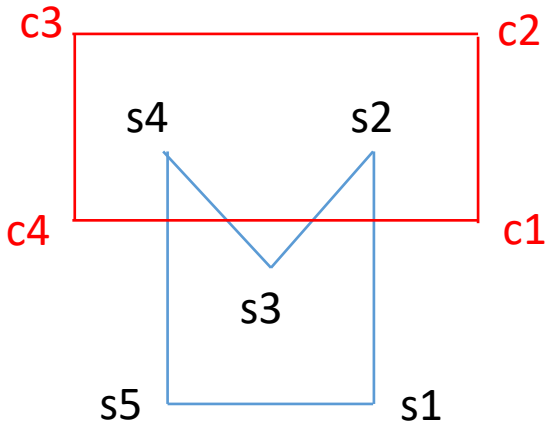


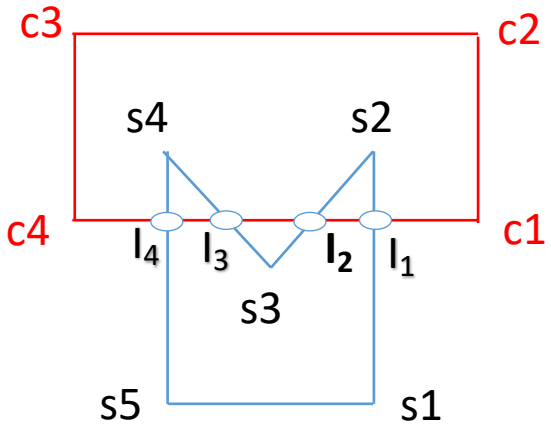
S = {s1, s2, s3, s4, s5}

C = {c1, c2, c3, c4}

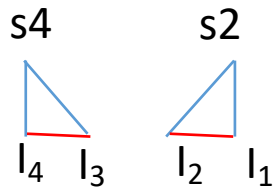
I = {I1, I2, I3, I4}

Simple Clipping examples

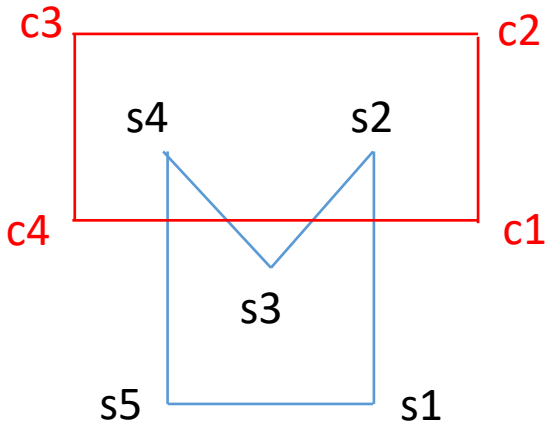


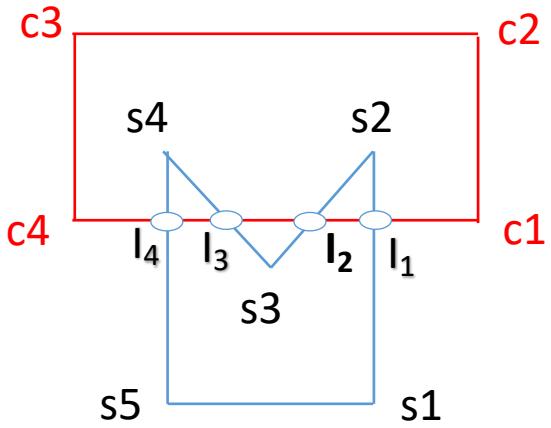


Intersection(S, C)

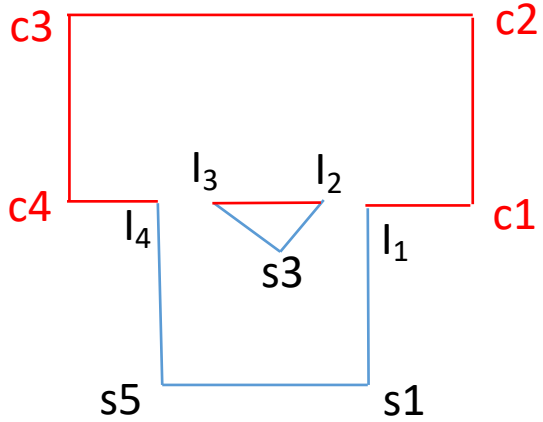


Simple Clipping examples





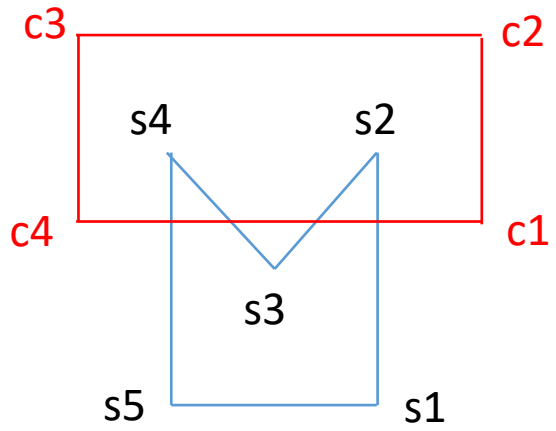
Union (S,C)



Simple Clipping examples

S = {s1, s2, s3, s4, s5}

C = {c1, c2, c3, c4}



Intersection (S, C)

Output polygons:

O_1 = {s4, I_4, I_3},
O_2 = {s2, I_2, I_1}

Union (S, C)

Output polygons:

O_3 = {c1, c2, c3, c4, I_4, s_5, s_1, I_1}
O_4 = {I_2, I_3, s3}