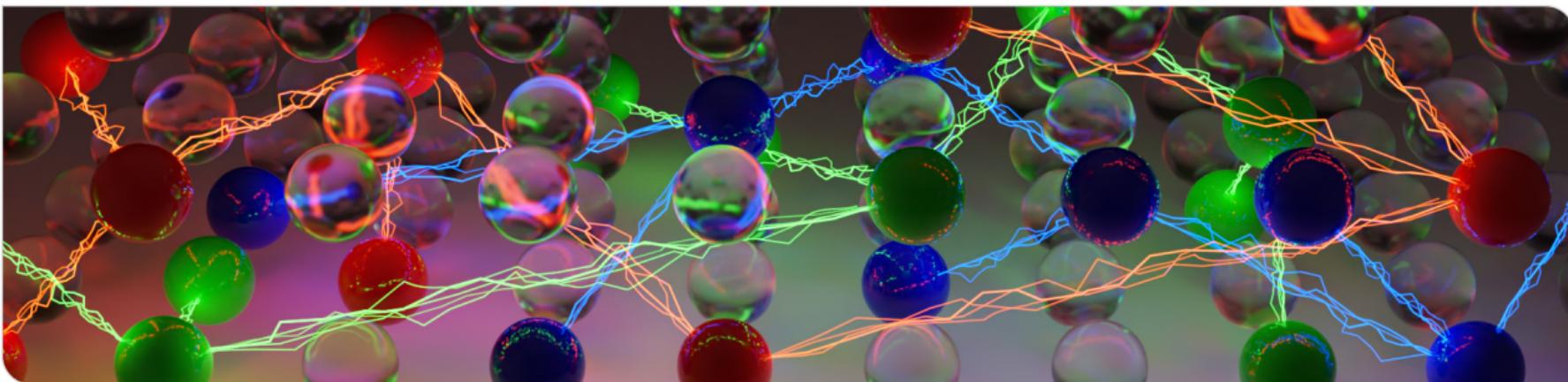


Practical SAT Solving

Lecture 4

Markus Iser, Dominik Schreiber, Tomáš Balyo | May 06, 2024



Results of Feedback Round 1

Result 1: Grades

- Content/Relevance: $2 \times 5p, 4 \times 4p$
- Quality: $1 \times 5p, 4 \times 4p, 1 \times 3p$

Result 2: Aspects

- Positive: practical, discussions, examples, application oriented, exercises, small group size, C++
- Negative: C++, “winner takes all” in competitions, explanation of sequential counter encoding, technical terms not introduced

Lessons learned

- Better care of technical terms introduction
- Split distribution of bonus points in competitions
- Some Python examples in exercises

Overview

Recap. Lecture 4: Heuristics and Modern SAT Solving 1

- Decision Heuristics, Restart Strategies, Phase Saving
- Modern SAT Solving 1: Conflict Analysis / Clause Learning

Regarding Lecture 5: Parallel SAT Solving 1

This topic will be continued on June 10, 2024: Parallel SAT Solving 2.

Today's Topic: Modern SAT Solving 2

- Variable State Independent Decaying Sum (VSIDS) Heuristic
- Efficient Unit Propagation, Watched Literals
- Clause Forgetting
- Preprocessing

Conflict-driven Clause Learning (CDCL) Algorithm

Last Time

- Classic Decision Heuristics
- Restart Strategies
- Clause Learning
- Non-Chronological Backtracking

Today

- Efficient Unit Propagation
- Clause Forgetting
- Modern Decision Heuristics
- Preprocessing

Algorithm 1: CDCL(CNF Formula F , &Assignment $A \leftarrow \emptyset$)

```

1 if not PREPROCESSING then return UNSAT
2 while  $A$  is not complete do
3   UNIT PROPAGATION
4   if  $A$  falsifies a clause in  $F$  then
5     if decision level is 0 then return UNSAT
6     else
7       (clause, level)  $\leftarrow$  CONFLICT-ANALYSIS
8       add clause to  $F$  and backtrack to level
9       continue
10      if RESTART then backtrack to level 0
11      if CLEANUP then forget some learned clauses
12      BRANCHING
13 return SAT

```

Unit Propagation: Cost

Hot Paths in CDCL Solvers

heat	\emptyset per sec. ^a	
Clause Access		Unpredictable memory access: most expensive
Iterate Occurrences		Predictable memory access: array of pointers (hardware prefetching)
Propagation	$\sim 10^6$	Access occurrence-list of yet unpropagated literal
Decision	$\sim 10^3$	
Conflict	$\sim 10^3$	<i>Learn a clause → more to check for propagation</i>
Restart	$\sim 10^{-1}$	
Cleanup		<i>Forget some learned clauses → less to check for propagation</i>

^aOrder of magnitude of average event count per second (in runs of Cadical on a large combined benchmark set)

Unit Propagation

Example: Unit Propagation with Full Occurrence Lists

Trail			Occurrence Lists		Formula	
level	value	reason	idx.	occurrences	addr.	clause
			a	*1	*1	$a \quad b \quad c$
			$\neg a$	*2 *3	*2	$\neg a \quad b \quad \neg c$
			b	*1 *2	*3	$\neg a \quad \neg b \quad c$
			$\neg b$	*3		
			c	*3 *1		
			$\neg c$	*2		

Unit Propagation

Example: Unit Propagation with Full Occurrence Lists

Trail

level	value	reason
1	a	\perp

Occurrence Lists

idx.	occurrences
a	*1
$\neg a$	*2 *3
b	*1 *2
$\neg b$	*3
c	*3 *1
$\neg c$	*2

Formula

addr.	clause
*1	a b c
*2	$\neg a$ b $\neg c$
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Unit Propagation

Example: Unit Propagation with Full Occurrence Lists

Trail			Occurrence Lists		Formula		
level	value	reason	idx.	occurrences	addr.	clause	
1	a	⊥	a	*1	*1	a	c
2	c	⊥	¬a	*2 *3	*2	¬a	¬c
			b	*1 *2	*3	¬a	¬b
			¬b	*3			
			c	*3 *1			
			¬c	*2			

Unit Propagation

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level	value	reason	idx.	occurrences	addr.	clause	
1	a	⊥	a	*1	*1	a	c
2	c	⊥	¬a	*2 *3	*2	¬a	¬c
2	b	*2	b	*1 *2	*3	¬a	¬b
			¬b	*3			
			c	*3 *1			
			¬c	*2			

Unit Propagation: Two Watched Literals

Motivation: Hot Path

heat	\emptyset per sec. ^a	<p>Idea: Reduced occurrence tracking by only keeping the following invariant:</p> <p>Each yet unsatisfied clause is watched by, i.e., in the occurrence list of, two of its unassigned literals.</p> <p>Reasoning: less literals watched → shorter occurrence lists → less clause accesses → fast unit propagation</p>
Clause Access		
Iterate Occurrences		
Propagation	$\sim 10^6$	

^aOrder of magnitude of average event count per second (in runs of Cadical on a large combined benchmark set)

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Clause Access		
Iterate Occurrences		
Propagation	$\sim 10^6$	

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Clause Access		
Iterate Occurrences		
Propagation	$\sim 10^6$	

^aOrder of magnitude of average event count per second (in runs of Cadical on a large combined benchmark set)

Unit Propagation

Example: Unit Propagation with Two Watched Literals

Trail

level	value	reason

Two Watched Literals

idx.	occurrences
a	*1
$\neg a$	*2 *3
b	*1 *2
$\neg b$	*3
c	
$\neg c$	

Formula

addr.	clause
*1	$a \quad b \quad c$
*2	$\neg a \quad b \quad \neg c$
*3	$\neg a \quad \neg b \quad c$

Unit Propagation

Example: Unit Propagation with Two Watched Literals

Trail

level	value	reason
1	a	\perp

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Unit Propagation

Example: Unit Propagation with Two Watched Literals

Trail

level	value	reason
1	a	\perp
2	c	\perp

Two Watched Literals

idx.	occurrences
a	*1
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Unit Propagation

Example: Unit Propagation with Two Watched Literals

Trail

level	value	reason
1	a	\perp
2	c	\perp
2	b	*2

Two Watched Literals

idx.	occurrences
a	*1
$\neg a$	
b	*1 *2
$\neg b$	*3
c	*3
$\neg c$	*2

Formula

addr.	clause
*1	a b c
*2	$\neg c$ b $\neg a$
*3	c $\neg b$ $\neg a$

Unit Propagation: Two Watched Literals

Two Watched Literals: Optimizations

heat	\emptyset per sec. ^a	Invariant: Each yet unsatisfied clause is watched by two of its unassigned literals. \rightarrow Reduced Load in Occurrence Tracking
Clause Access		Optimization 1: Keep watched literals the first two in clause \rightarrow Alternative: Store watched literals in other location Note: What happens if clauses are kept in shared memory for parallel solving?
Iterate Occurrences		Optimization 2: Also keep a literal of each clause directly in occurrence list \rightarrow Skip clause access if that literal is satisfied
Propagation	$\sim 10^6$	

^aOrder of magnitude of average event count per second (in runs of Cadical on a large combined benchmark set)

Recap

Unit Propagation

- Hottest path in CDCL solvers
- Two watched literals per clause suffice for unit propagation (and conflict detection)
- Other optimizations: keep watched literals first in clause, keep a literal of each clause directly in occurrence list

Next Up

Clause Forgetting

Clause Forgetting

Motivation

Clause learning is most important pruning strategy in CDCL solvers.

Problems:

- Without restrictions the number of clauses grows exponentially (Remember: naive Saturation Algorithm)
- Risk of running out of memory
- Slows down unit propagation

Idea:

- Periodically forget some learned clauses
- Keep only important learned clauses
- Use heuristics to estimate clause importance

Clause Forgetting

Periodic Clause Forgetting: Heuristics

- **Clause Size**

Keep short clauses

- **Least Recently Used (LRU)**

Keep clauses which were reasons in recent conflicts: clause activity (moving average)

- **Literal Block Distance (LBD)**

Keep clauses with a low number of decision levels^a

^aPredicting Learnt Clauses Quality in Modern SAT Solvers, Audemard & Simon (IJCAI 2009)

Forgetting Heuristic: Literal Block Distance (LBD)

“Impact of Community Structure on SAT Solver Performance”, Newsham et al., SAT 2014

Take home: LBD correlates with number of touched communities

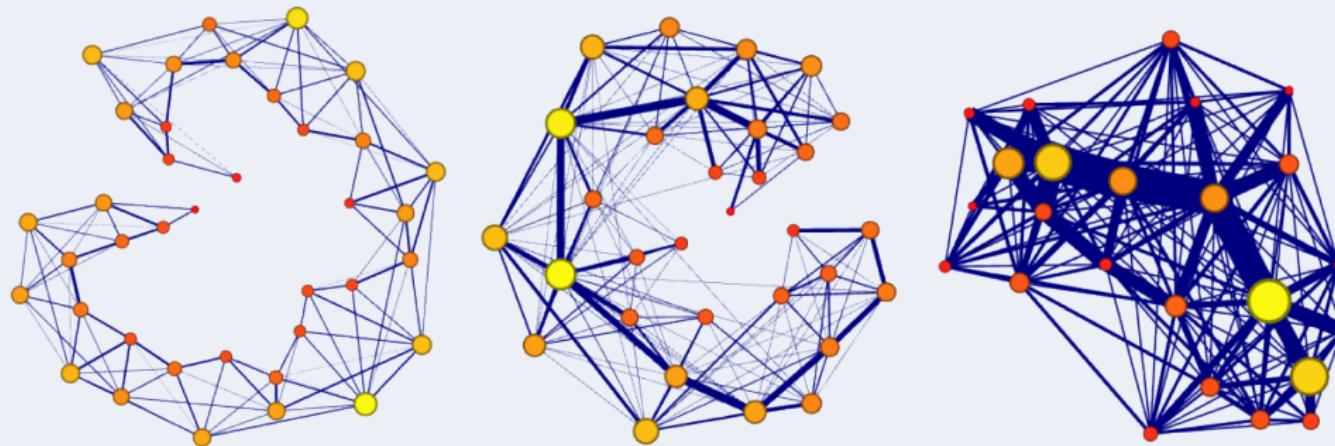


Image Source: “Community Structure in Industrial SAT Instances”, Ansotegui et al., AIJ 2019

Clause Forgetting: Modern Hybrid Approach

Three-Tier Clause Management

Manage clauses differently in three tiers:

- core, LBD: permanently store clauses of $LBD \leq k$ (core-cut value, 3 in practice)
- mid-tier, LRU: clauses stay here if used in recent conflicts
- local, LRU: keep fixed number of clauses (say 5000) of highest activity

History

- core and local tier introduced in SWDiA5BY (Chanseok Oh, 2014)
- mid-tier introduced in CoMinisatPS (Chanseok Oh, 2015)
- “Between SAT and UNSAT: The Fundamental Difference in CDCL SAT” (Chanseok Oh, 2015)
- Note: MapleCOMSPS (2016) is a CoMinisatPS fork

Recap

So far

- Efficient Unit Propagation
- Clause Forgetting Heuristics:
 - Size, LRU, LBD
 - LBD correlation with communities
 - Three-Tier Clause Management

Next Up

Modern Branching Heuristics

Variable State-Independent Decaying Sum (VSIDS)

Implemented in most CDCL solvers. First presented in SAT solver Chaff.¹

VSIDS Heuristic

Compute score for each variable, select variable with highest score:

- Initialize variable score (with zero or use some static heuristic)
- New conflict clause c : Score is incremented for all variables in c
- Periodically, divide all scores by a constant

¹Chaff: Engineering an efficient SAT solver (Moskewicz et al., 2001)

VSIDS Heuristic

VSIDS Example

Initial F :

$\{x_1, x_4\}$
 $\{x_1, \overline{x_3}, \overline{x_8}\}$
 $\{x_1, x_8, x_{12}\}$
 $\{x_2, x_{11}\}$
 $\{\overline{x_7}, \overline{x_3}, x_9\}$
 $\{\overline{x_7}, x_8, \overline{x_9}\}$
 $\{x_7, x_8, \overline{x_{10}}\}$

Scores:

4 : x_8
3 : x_1, x_7
2 : x_3
1 : $x_2, x_4, x_9, x_{10}, x_{11}, x_{12}$

VSIDS Heuristic

VSIDS Example

Initial F :

- $\{x_1, x_4\}$
- $\{x_1, \overline{x}_3, \overline{x}_8\}$
- $\{x_1, x_8, x_{12}\}$
- $\{x_2, x_{11}\}$
- $\{\overline{x}_7, \overline{x}_3, x_9\}$
- $\{\overline{x}_7, x_8, \overline{x}_9\}$
- $\{x_7, x_8, \overline{x}_{10}\}$

F with new learned clause added:

- $\{x_1, x_4\}$
- $\{x_1, \overline{x}_3, \overline{x}_8\}$
- $\{x_1, x_8, x_{12}\}$
- $\{x_2, x_{11}\}$
- $\{\overline{x}_7, \overline{x}_3, x_9\}$
- $\{\overline{x}_7, x_8, \overline{x}_9\}$
- $\{x_7, x_8, \overline{x}_{10}\}$
- $\{x_7, x_{10}, \overline{x}_{12}\}$ (new learned clause)

Scores:

- 4 : x_8
- 3 : x_1, x_7
- 2 : x_3
- 1 : $x_2, x_4, x_9, x_{10}, x_{11}, x_{12}$

Scores:

- 4 : $x_8, \cancel{x_7}$
- 3 : x_1
- 2 : $x_3, \cancel{x_{10}}, \cancel{x_{12}}$
- 1 : x_2, x_4, x_9, x_{11}

VSIDS Heuristic

Keep list of variables sorted by score

VSIDS Common Implementation: Binary Heap

- Backtrack: `insert_with_priority` $\in \mathcal{O}(\log n)$
- Branch: `pull_highest_priority_element` $\in \mathcal{O}(\log n)$
- Bump: `increase_key` $\in \mathcal{O}(\log n)$
- Decay: `decay` $\in \mathcal{O}(n)$

Reasoning behind VSIDS

Make heuristics more “focused”

- try to find small unsatisfiable subsets
- prefer variables that occurred in a recent conflict

VSIDS Heuristic

Periodically divide scores to give priority to recently learned clauses

VSIDS Variants

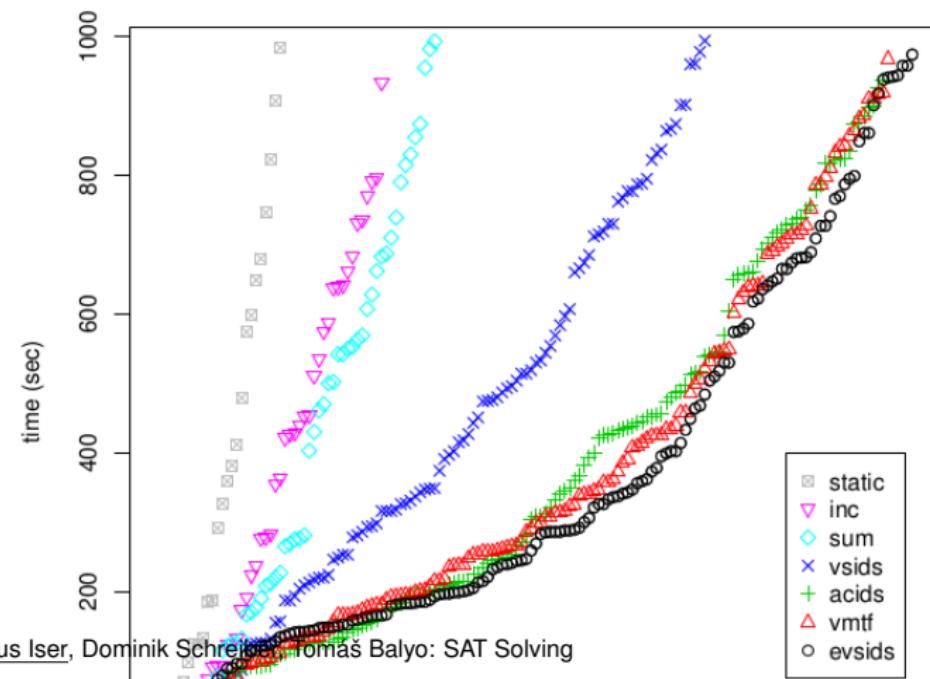
- Chaff (2001)
 - half scores every 256 conflicts (“decay”)
 - sort priority queue after each decay only
- Berkmin (2002)
 - bump all literals in implication graph
 - divide scores by 4
- Minisat (2003)
 - exponential VSIDS (eVSIDS)

Alternatives

- Siege (2004): Variable Move To Front (VMTF)
- HaifaSAT (2008): Clause Move To Front (CMTF)

Comparison of Heuristics

Evaluating CDCL Variable Scoring Schemes (Biere & Fröhlich, 2015)



Recent Hybrid Approaches

Hybrid Approaches

- **Warmup Phase:**
 - MapleCOMSPS (2016): use Learning Rate-based Branching (LRB) in *initial* period, then switch to VSIDS
 - Maple_LCM_Dist (2017): use Distance Heuristic (Dist.) in *initial* period, then switch to VSIDS
- **Reinforcement Learning:** Kissat_MAB (2021)
 - Two-armed Bandit switches between VSIDS and Conflict History-Based (CHB) Heuristic
 - Reward function favors variables that contribute to learning “good” clauses

Recap

So far

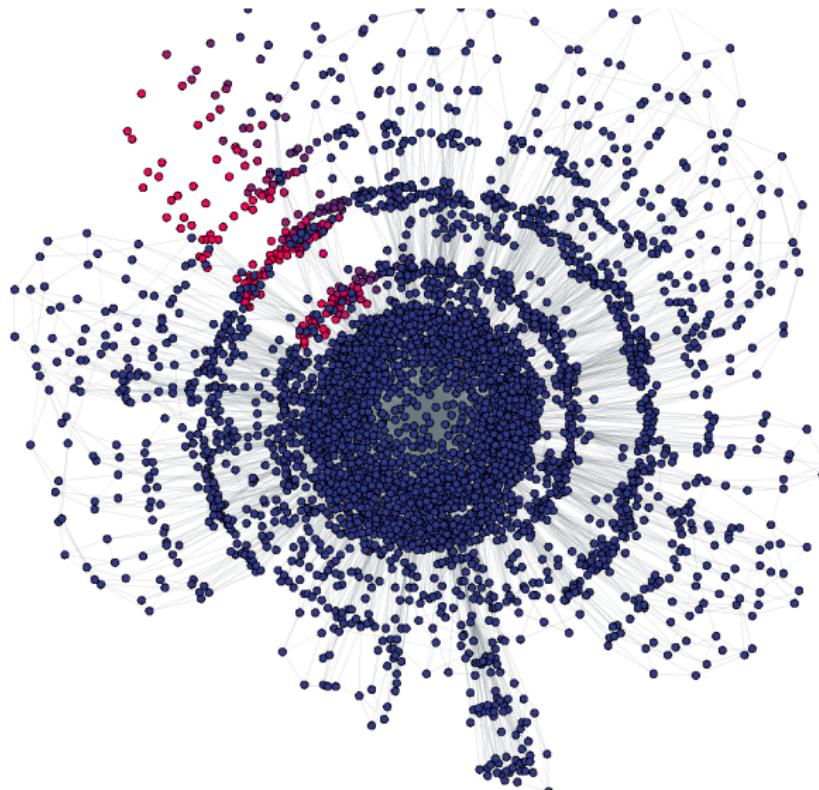
- Unit Propagation
- Clause Forgetting
- Modern Branching Heuristics
 - Mostly VSIDS
 - Hybrid approaches: warmup VSIDS scores, reinforcement learning

Next Up

Evolution of formula structure with clause learning

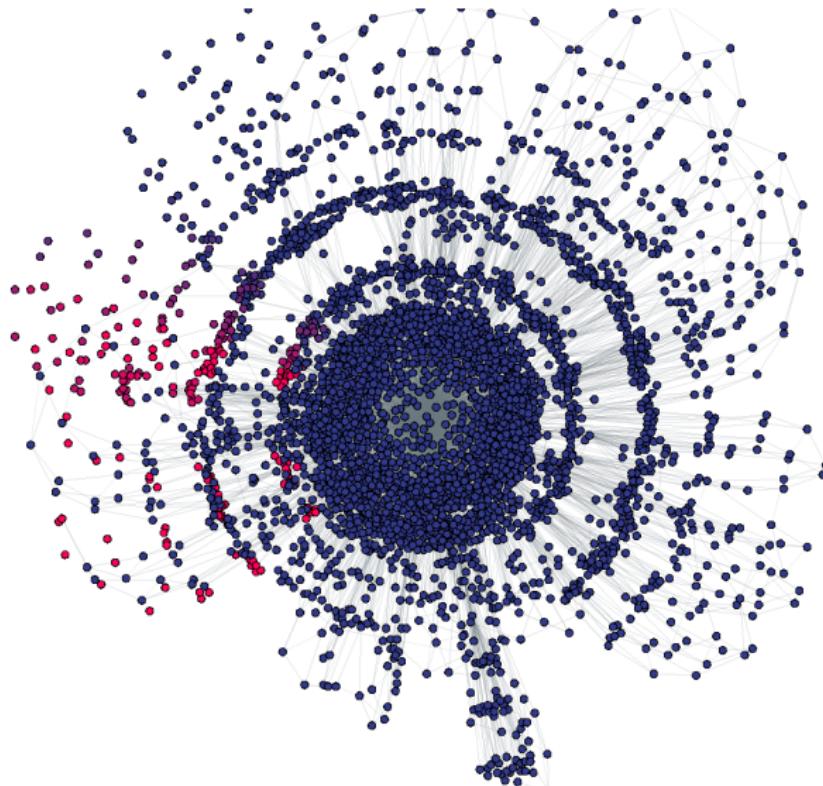
Instance: Aprove (Termination Analysis, SAT)

initial layout, recently active variables after 1000 conflicts



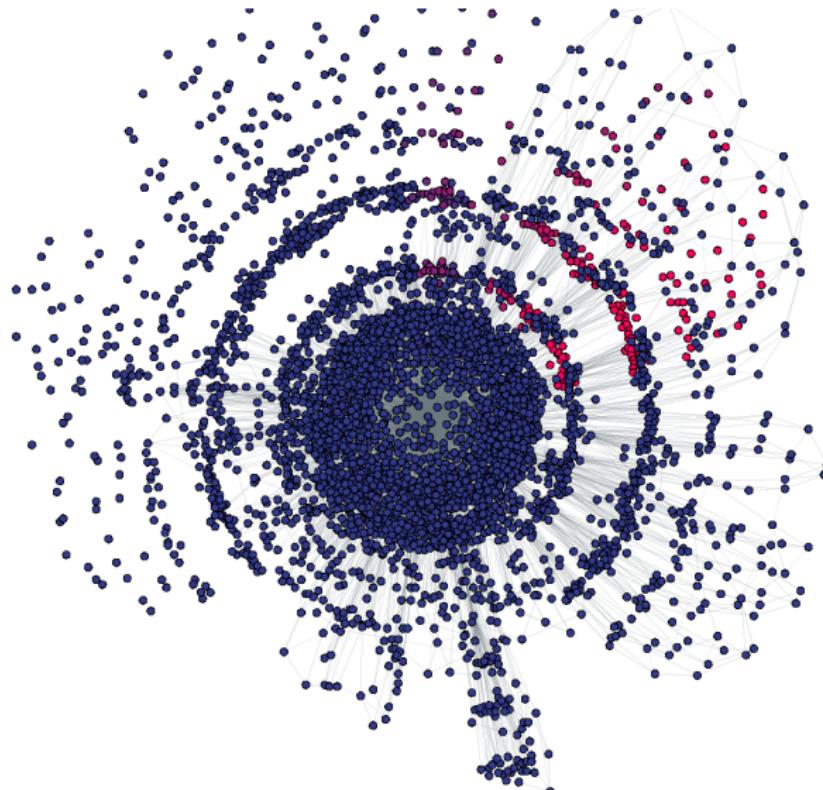
Instance: Aprove (Termination Analysis, SAT)

initial layout, recently active variables after 1690 conflicts



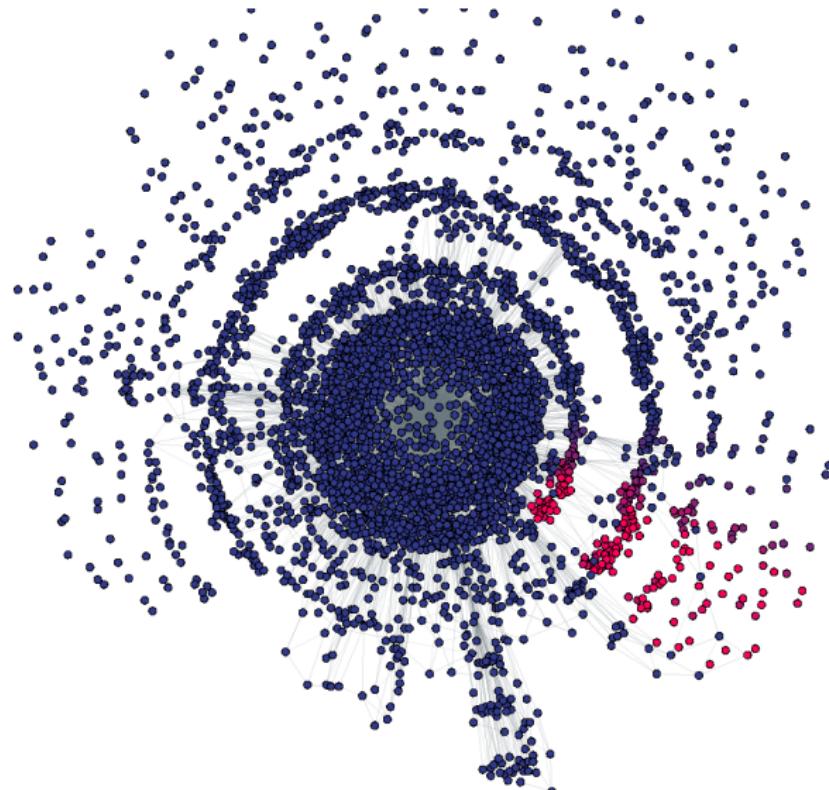
Instance: Aprove (Termination Analysis, SAT)

initial layout, recently active variables after 3090 conflicts



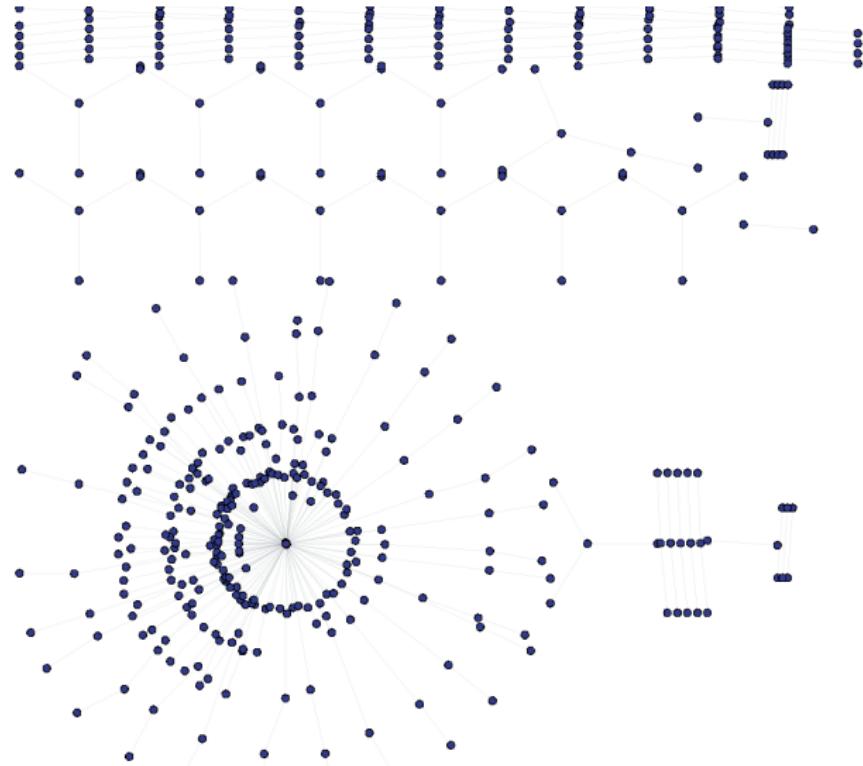
Instance: Aprove (Termination Analysis, SAT)

initial layout, recently active variables after 5000 conflicts



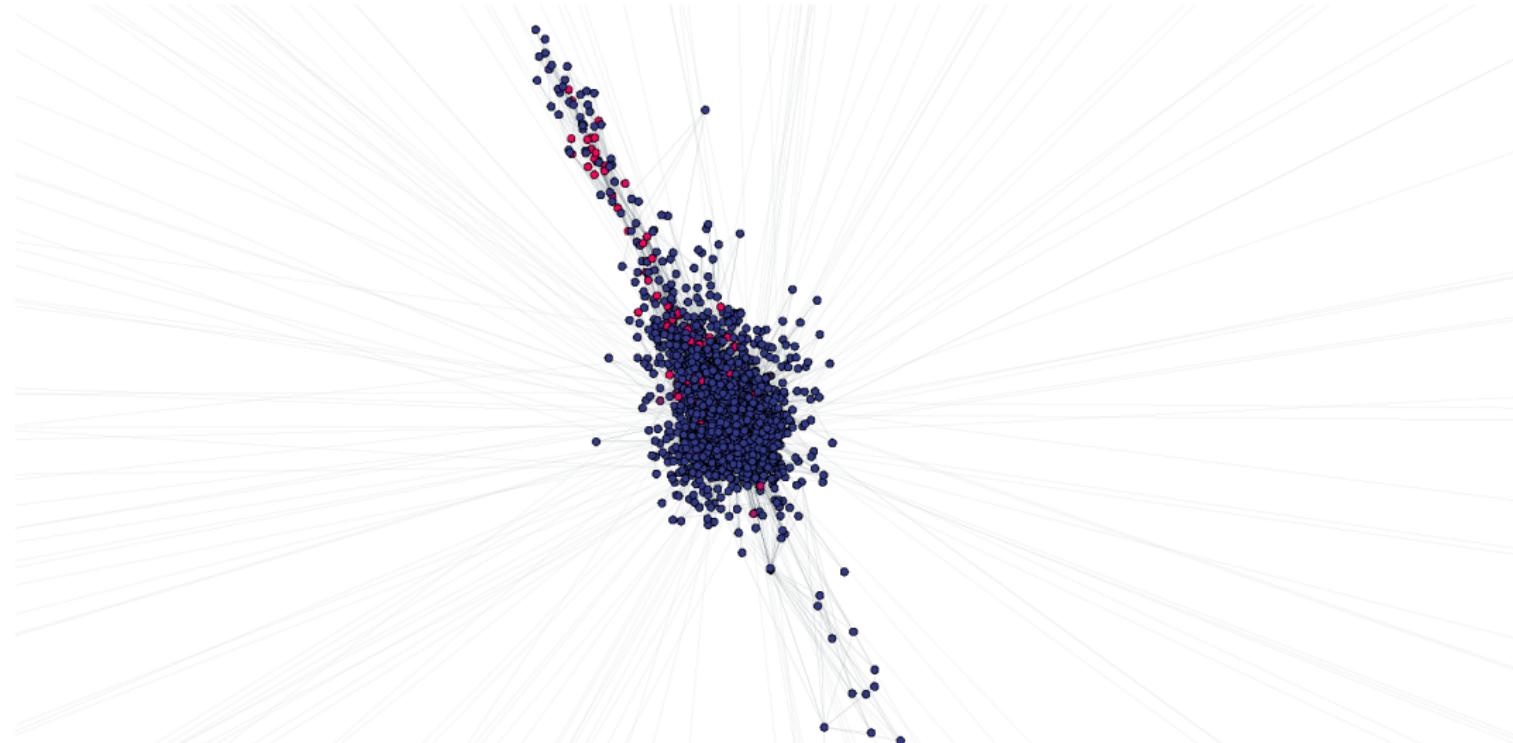
Instance: Aprove (Termination Analysis, SAT)

relayout after 6000 conflicts



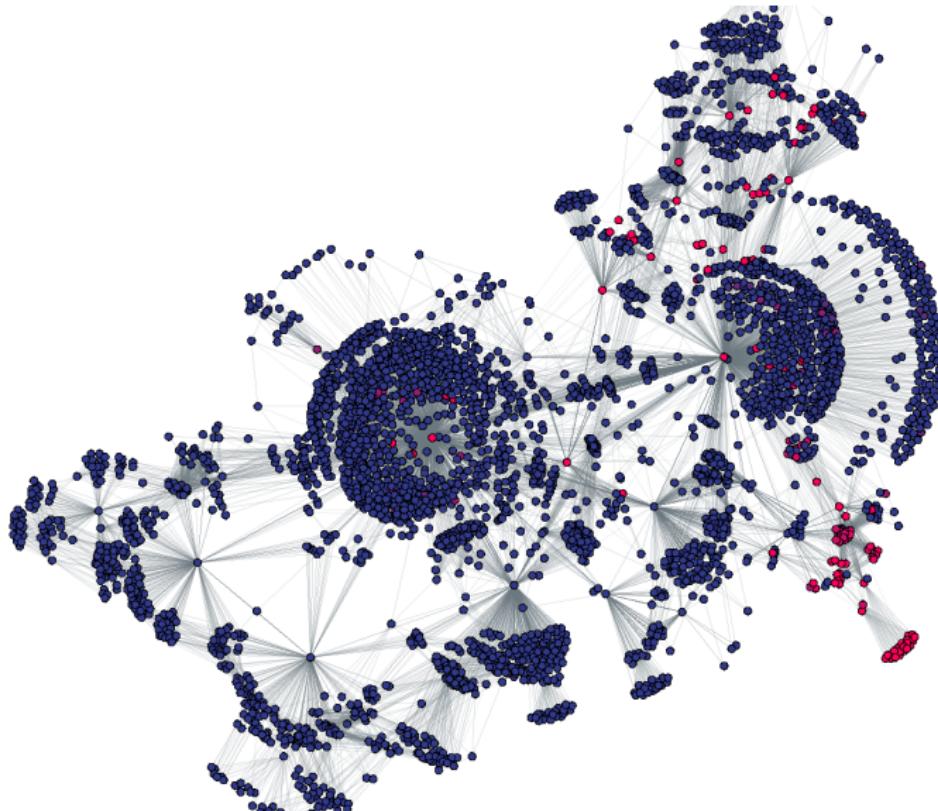
Instance: Aprove (Termination Analysis, SAT)

core after 52500 conflicts



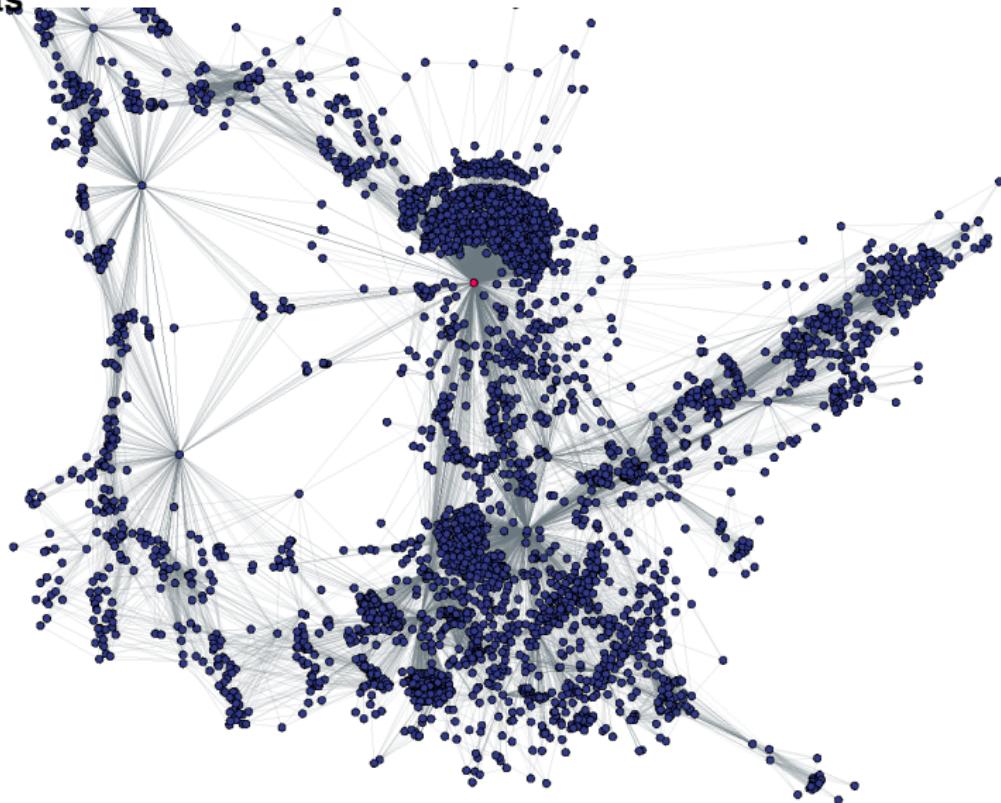
Instance: Newton SMT (SV Competition, SAT)

initial layout

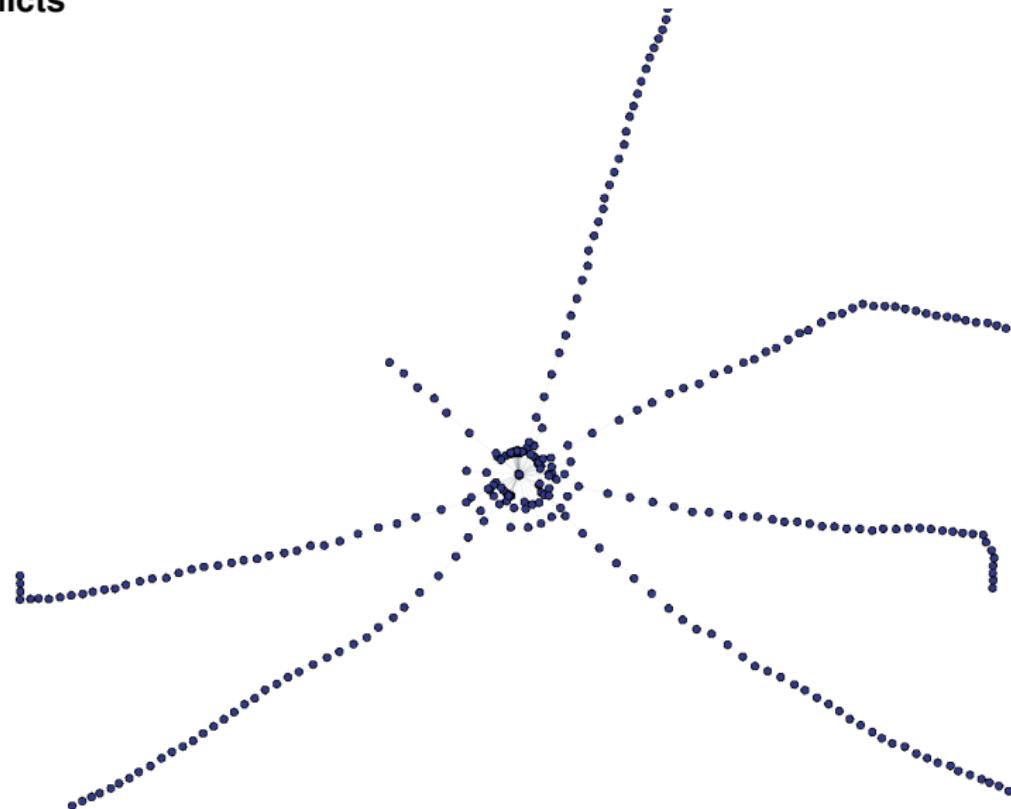


Instance: Newton SMT (SV Competition, SAT)

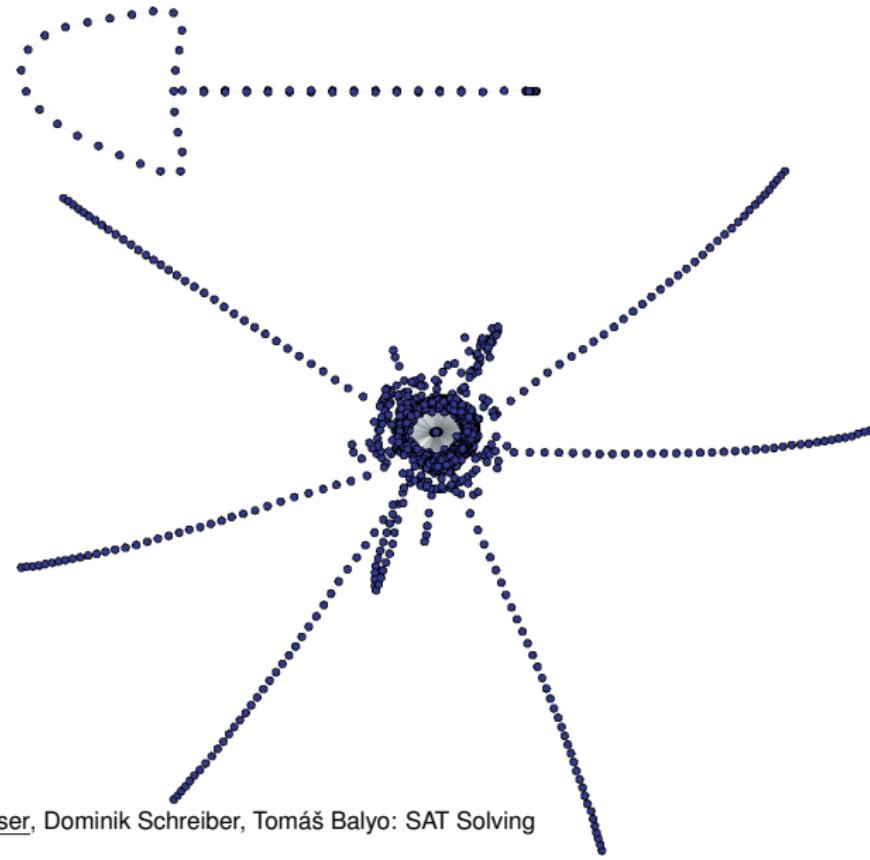
after 10000 conflicts



Instance: Newton SMT (SV Competition, SAT) after 1000000 conflicts

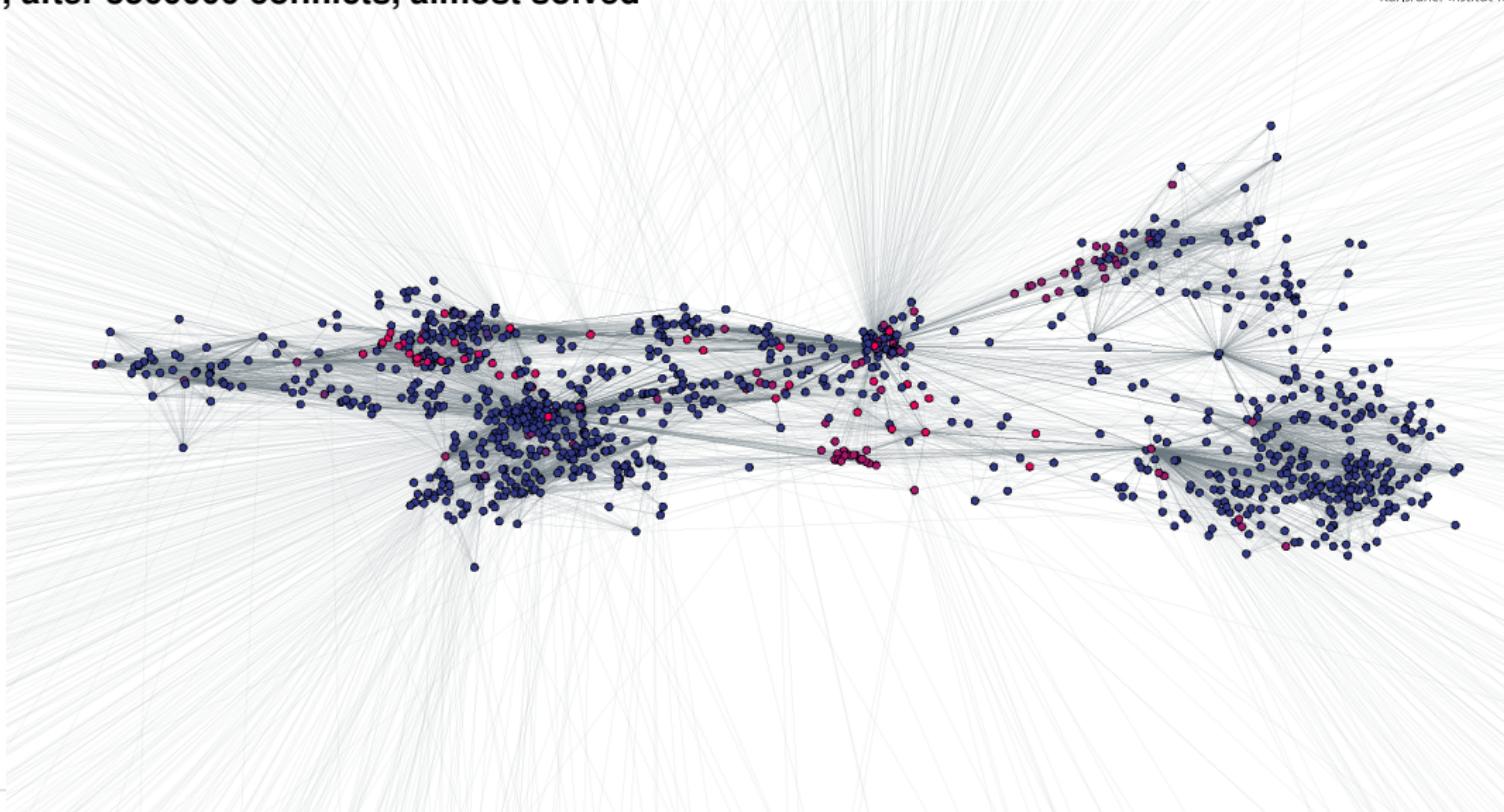


Instance: Newton SMT (SV Competition, SAT) after 3000000 conflicts



Instance: Newton SMT (SV Competition, SAT)

core, after 3500000 conflicts, almost solved



Preprocessing

General Idea

Conjecture: Smaller problems are easier to solve

⇒ Try to reduce the size of the input formula by (polynomial time) simplification procedures.

- Subsumption
- Self-subsuming resolution
- Variable elimination
- Blocked clause elimination
- Failed literal elimination
- Autarkies

Subsumption

- Definition: A clause D subsumes a clause C iff $D \subseteq C$
- We also say that clause C is subsumed by D
- To check satisfiability, subsumed clauses are irrelevant: $\forall D \subseteq C, D \models C$

Example

$\{a, b\}$ subsumes $\{a, b, c\}$ and $\{a, b, d\}$

Self-Subsuming Resolution

Combination of Subsumption and Resolution

Self-Subsuming Resolution

- Definition: Let C, D be clauses and \otimes_x the resolution operator on variable x . If $C \otimes D \subseteq C$ then C is said to be *self-subsumed by D with respect to x* .
- The resolvent of C and another clause D subsumes C

Example

- $C := \{\neg b, \neg e, f, \neg h\}$
- $D := \{\neg b, \neg e, \neg f\}$
- $E := C \otimes_f D = \{\neg b, \neg e, \neg h\}$
- The clause C is subsumed by the resolvent E

Variable Elimination

Variable Elimination

- Definition:
 - Let $S_x, S_{\bar{x}} \subset F$ be the sets of clauses containing x resp. \bar{x}
 - Let $R = \{C \otimes_x D \mid C \in S_x, D \in S_{\bar{x}}\}$ be the set of all resolvents on x
 - Modify F such that $F' := (F \setminus (S_x \cup S_{\bar{x}})) \cup R$
- The formulas F and F' are *satisfiability equivalent* (not equivalent)

Bounded Variable Elimination (BVE)

- Simulate variable elimination, but replace clauses only if number of clauses decreases (due to tautological resolvents)
- Most important simplification technique today

Blocked Clauses

Definition

A clause $(I \vee C)$ is blocked in F by I if either I is *pure* in F or if for every clause $(\neg I \vee D)$ in F the resolvent $(C \vee D)$ is a *tautology*.

Example

$$F := (a \vee b) \wedge (a \vee \neg b \vee \neg c) \wedge (\neg a \vee c)$$

First clause is not blocked, second is blocked by both a and $\neg c$, third is blocked by c .

Blocked Clause Elimination

Removal of an arbitrary blocked clause preserves satisfiability. Blocked clause elimination (BCE) has a unique fixpoint.

Blocked Clauses and Gates

Left-Totality of Gate Encodings

Given a gate G with output variable o and its clausal encoding E , it holds that for every clause $C \in E$ either $o \in C$ or $\bar{o} \in C$ (part 1) and all resolvents in $E[o] \otimes_o E[\bar{o}]$ are tautologic (part 2).

Proof of Part 1

Assume that there is a clause $C \in E$ such that $o \notin \text{vars}(C)$. It follows that there exists an assignment to input variables which falsifies C for any assignment to o . This contradicts left-totality.

Proof of Part 2

Let R be a non-tautological resolvent in $E[o] \otimes_o E[\bar{o}]$. By Definition of resolution, it holds that $o \notin \text{vars}(R)$ and $E \models R$. This contradicts left-totality.

Variable Elimination and Gates

Property of Gate Encodings

- Resolving the clauses of a gate results in tautological clauses
- Example: Tseitin encoding of gate $x = \text{AND}(y, z)$ results in the clauses $\{\neg x, y\}, \{\neg x, z\}, \{x, \neg y, \neg z\}$

Idea: Variable Elimination for Gate Encoding G

- Split formula F into $F = G \cup R$, where G are the gate clauses
- Apply variable elimination:

$$\begin{aligned}
 S' &= (G_x \cup R_x) \otimes (G_{\bar{x}} \cup R_{\bar{x}}) \\
 &= (G_x \otimes R_{\bar{x}}) \cup (R_x \otimes G_{\bar{x}}) \cup (R_x \otimes R_{\bar{x}}) \cup (\cancel{G_x \otimes G_{\bar{x}}}) \\
 &= (G_x \otimes R_{\bar{x}}) \cup (R_x \otimes G_{\bar{x}}) \cup (\cancel{R_x \otimes R_{\bar{x}}}) \\
 &= (R_x \otimes G_{\bar{x}}) \cup (R_{\bar{x}} \otimes G_x)
 \end{aligned}$$

- $(G_x \otimes R_{\bar{x}}) \cup (R_x \otimes G_{\bar{x}}) \models (R_x \otimes R_{\bar{x}})$ (Why? → Exercise)

Failed Literal Elimination

Definition

If $F \wedge \{I\} \vdash_{UP} \perp$, i.e., applying unit propagation on $F \wedge \{I\}$ derives UNSAT, replace F by $F \wedge \{\neg I\}$.

Generalization

If $(F \setminus \{C\}) \wedge \neg C \vdash_{UP} \perp$, remove C from F .

Autarkies

Definition

Given a partial assignment σ and a formula F , a clause $C \in F$ is *touched by σ* if it contains the negation of a literal assigned in σ . A clause is *satisfied by σ* if it contains a literal assigned to *True* by σ . If all touched clauses are satisfied then σ is an *autarky*.

All clauses touched by an autarky can be removed.

Autarky-based Clause Removal

The partial assignment $\sigma = \{\neg a, \neg c\}$ is an autarky for $F := \{\neg a, b\}, \{\neg a, c\}, \{a, \neg b, \neg c\}, \{b, d\}, \dots$ (more clauses without a and c)

Preprocessing Techniques that do not Reduce the Problem Size

- Bounded Variable Addition (BVA) a.k.a. Extension rule
- Some formulas have refutations of exponential size in the resolution calculus, but of polynomial size in extended resolution, e.g., pigeonhole formulas, mutilated chessboard, ...

Extended Resolution

Extended resolution adds a second rule to the resolution calculus, the Extension Rule. The idea is to introduce new variables as conjunction of existing literals, $x_{\text{new}} \leftrightarrow l_1 \wedge l_2$. As a rule for formulas in CNF:

$$\overline{(\neg x_{\text{new}} \vee l_1) \wedge (\neg x_{\text{new}} \vee l_2) \wedge (x_{\text{new}} \vee \neg l_1 \vee \neg l_2)}$$

Inprocessing

Idea: Interleave search and preprocessing

- Preprocessing can be extremely beneficial
- Most solvers in SAT competitions use bounded variable elimination, subsumption and self-subsuming resolution
- Problem: Many preprocessing techniques, though polynomial, require considerable time
- Possible Solutions:
 - Interrupt preprocessing techniques after some time
 - Resume preprocessing between restarts
 - Limit preprocessing time in relation to search

The End.

Recap

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