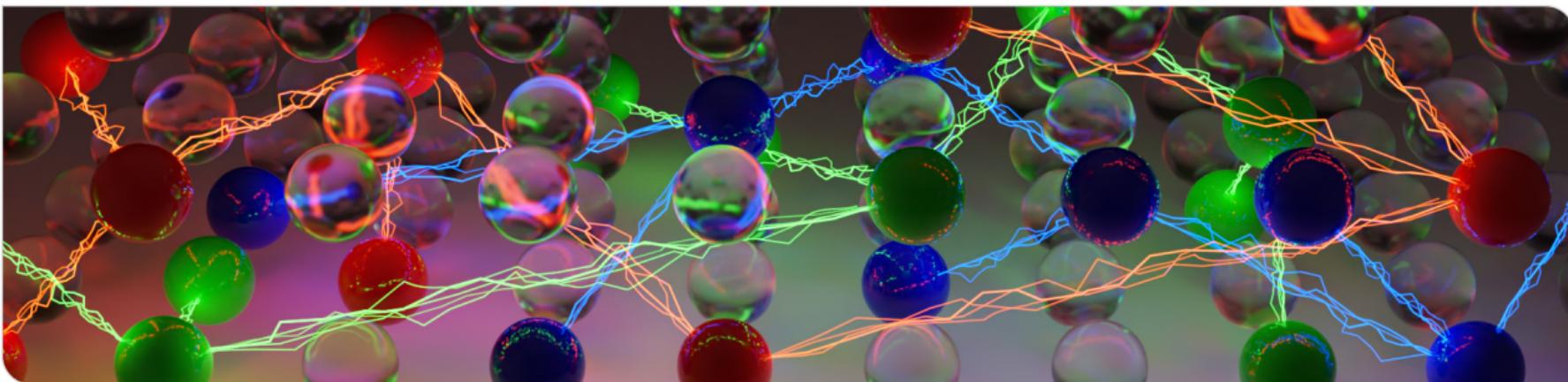


Practical SAT Solving

Lecture 11: Application Highlights

Markus Iser, Dominik Schreiber, Tomáš Balyo | July 1, 2024



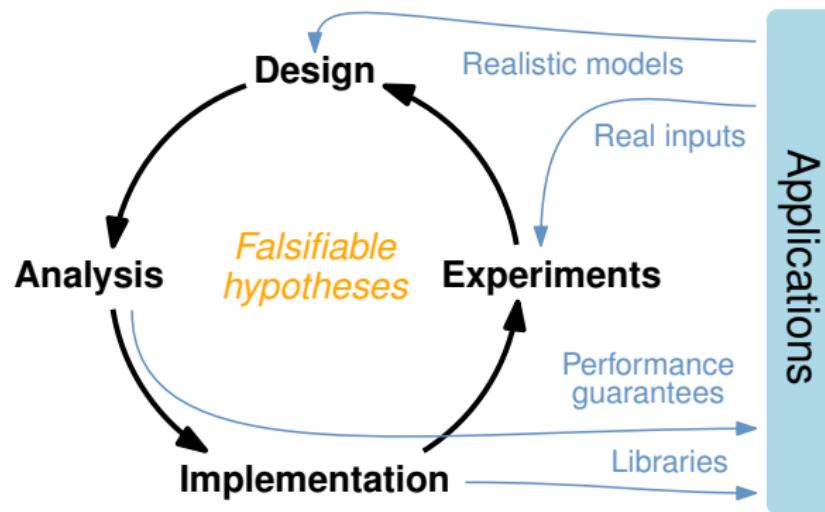
Why consider the Application View?

Result, instance family		1st author	Speedups of MALLOBSAT over KISSAT-MAB_HyWALK
SAT	Hypertree decomposition	Schidler	3, 5, 5, 5, 5, 7, 8, 9, 12, 13
SAT	Hamilton circle	Heule	4, 4, 7, 11, 17, 20, 21, 22, 24, 31, 33, 36, 42
SAT	Tree decomposition	Ehlers	5, 7, 87
UNSAT	Cellular automata	Chowdhury	5, 8, 8, 9, 9, 10, 22, 22, 66
⋮			
UNSAT	Relativized pigeon hole	Elffers	277, 542, 638
UNSAT	Bioinformatics	Bonet	292, 717
UNSAT	Balanced random	Spence	321, 388
SAT	Sum of three cubes	Riveros	384, 509, 1018, 3345
SAT	Circuit multiplication	Shunyang	17, 31, 32, 62, 105, 119, 213, 254, 393, 741, 746, 1401, 2650
UNSAT	Perfect matchings	Reeves	119, 413, 12439, 108593, 217099

3072 cores (128 machines) of SuperMUC-NG · 400 problems from Int. SAT Competition 2021
 Only instances with seq. time \geq 60 s · Only families with \geq 2 instances

Why consider the Application View?

- Perform **sound algorithm engineering** with **realistic applications** in the loop
- Understand application-specific solver behavior, needs, shortcomings
- Advance the positive feedback loop between solvers and applications



SAT Competition Benchmarks – “Real” Applications?



5355 problems,
138 families + 72 unknown

Font size \propto # problems

via wordclouds.com,
GBD (benchmark-database.de)

Overview

Selected (!) Application Highlights of SAT

- Recap: Planning
- Bounded Model Checking
- Combinational Equivalence Checking
- Analyzing Cryptographic Building Blocks
- Multi Agent Path Finding
- Explainable AI: Learning decision trees
- Train scheduling with disruptions via MaxSAT

Remark

This slide set contains [many literature references](#)—please follow them according to your interest!

Recap: Planning

- **World state s :** Set of Boolean features
 - Assert **initial state** s_I via unit clauses for **time step 0**
 - Assert **goal state features** g via unit clauses for **time step k**
- **Action:** Template for valid state transitions $s_x \rightsquigarrow s_{x+1}$
 - Executing an action at step k implies its **preconditions** at step k
 - Executing an action at step k implies its **effects** at step $k + 1$
- **Plan:** Valid sequence of actions leading from s_I to a goal state
 - Sequence of action variables **set to true** in satisfying assignment
- Anything else to encode?



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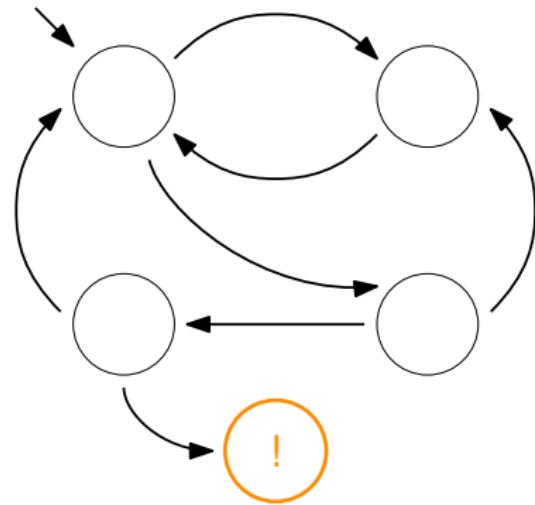
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 - Sequence of action variables **set to true** in satisfying assignment
- Anything else to encode? **Frame axioms – don't let the solver hallucinate causeless state changes!**



From Planning to Verification

Let's use our SAT-based planning to **check the correctness of a system!**

- World state features \equiv State features of our system
- Actions \equiv Valid **transitions between states**
- Goals \equiv

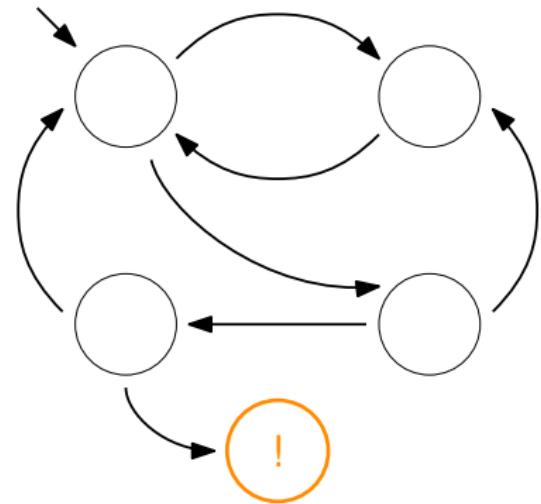


A snack machine or an electronic component or a C program or ...

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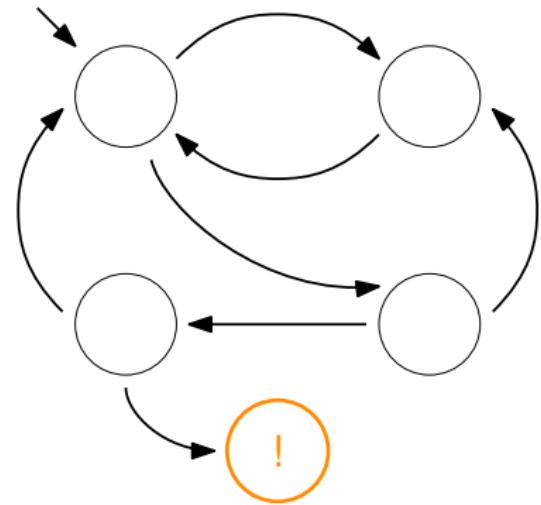


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- **Unsatisfiability** \equiv system is always correct?

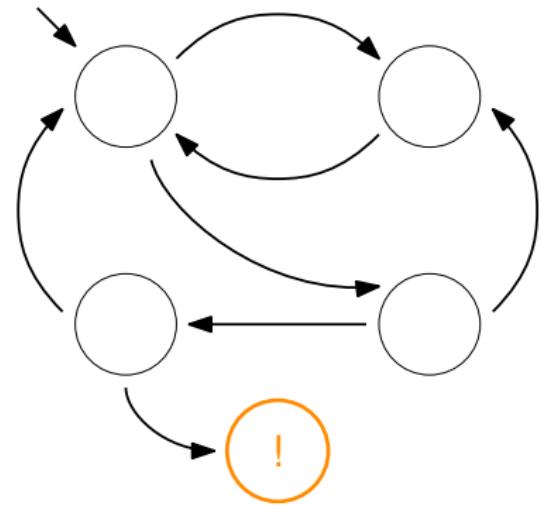


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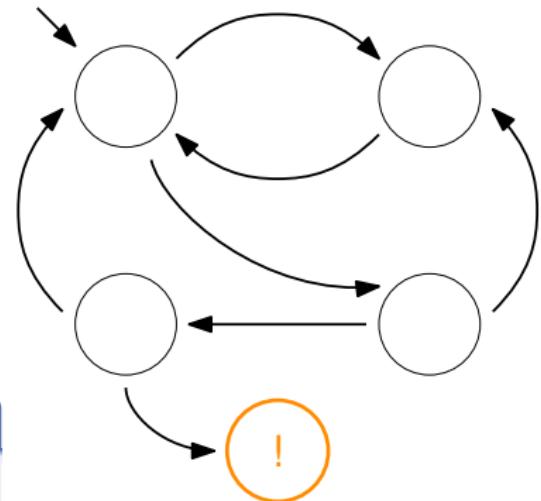
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Bounded Model Checking

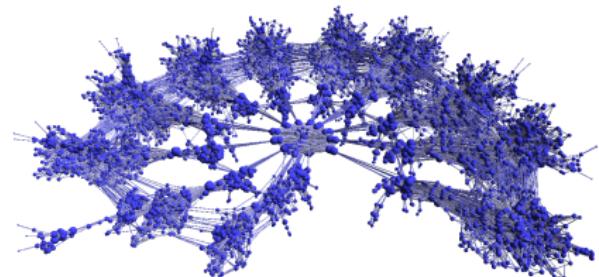
- Encode and check transition system for $k = 1, 2, \dots$
- Satisfying assignment \equiv counter example!
- Crucial tool for hardware and software verification [1]



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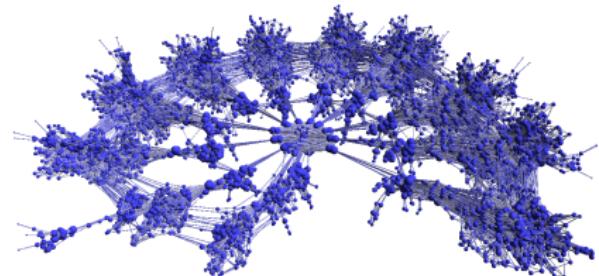
Bounded Model Checking

- Invented by Clarke & Biere in ~2000 [2], mostly replacing **BDD-based model checking**
- Uses state transition system based on **temporal logic** (LTL, CTL, ...)
- Applications: Computer-aided design (CAD), software verification, invariant checking, bug detection, ... [1]



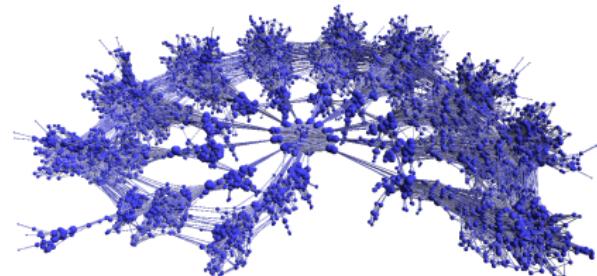
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- One of the **most essential real-world applications** of SAT
 - Pushed industrial interest in SAT solvers in 2000s
 - Actively influenced solver design and algorithms
 - Some of the **largest, structurally most distinct** benchmarks



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- Examples for BMC @ KIT:
 - **Low-Level Bounded Model Checker** (LLBMC) [3] (C program verification)
 - Verification of Java contracts [4] (see right)
 - Cryptography [5]



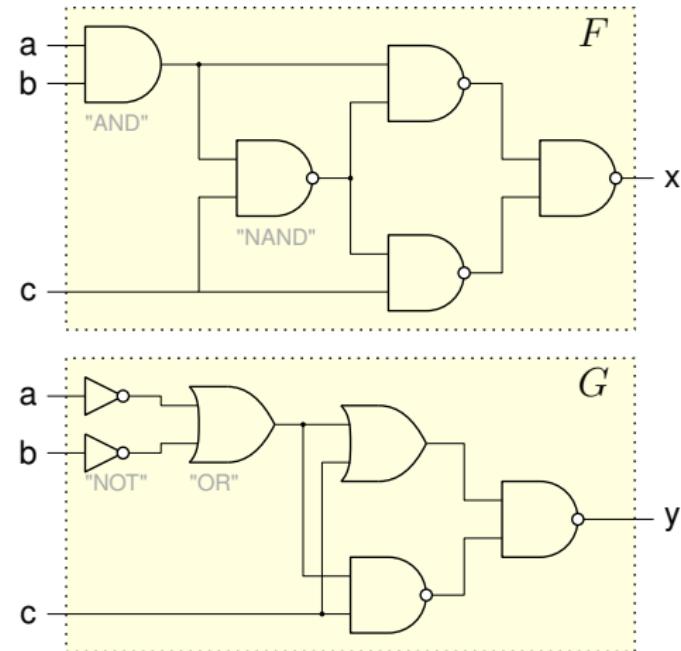
```

/*
 * @ requires 0 <= x1;
 * @ ensures \result == x1 * x2;
 * @ assignable \nothing;
 */
public int mult(int x1, int x2) {
    int res = 0;
    /*@ loop_invariant 0 <= i && i <= x1 && res == i * x2;
     * @ decreases x1 - i;
     * @ assignable \nothing;
     */
    for (int i = 0; i < x1; ++i) res += x2;
    return res;
}


```

Combinational Equivalence Checking

- Given: two **combinational circuits**
stateless “input→output” circuit, no feedback
- Question: are the circuits **logically equivalent?**
- Right example: Is $F(a, b, c) \equiv G(a, b, c)$?
- How to solve with SAT?

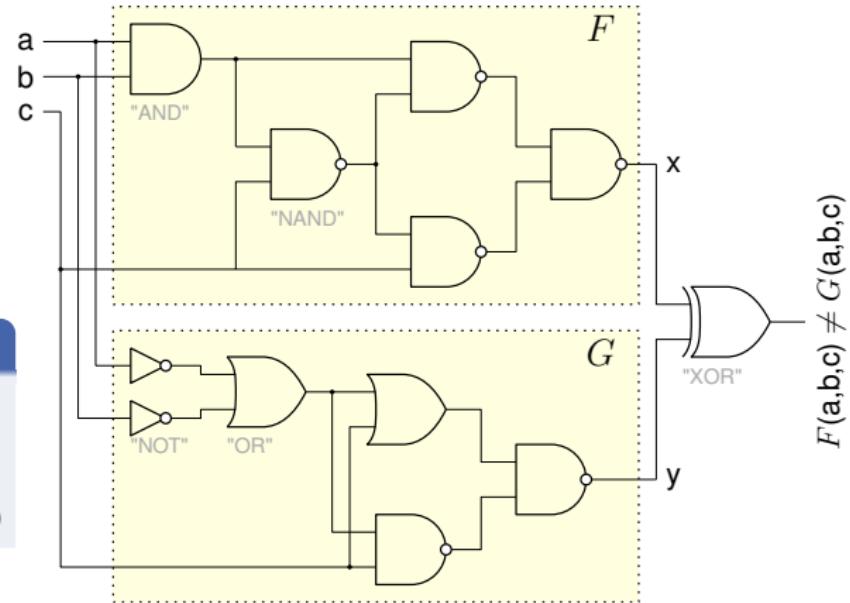


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Miter Formula

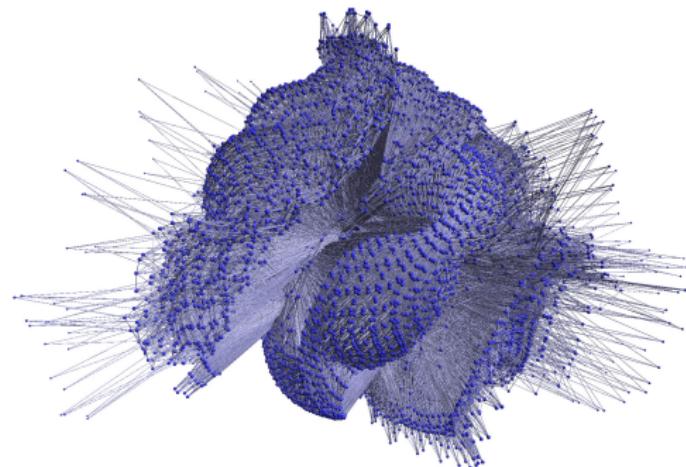
- Encode F, G relative to **shared input bits**
- Assert $x \neq y$ (multi-bit output: $\bigvee_{x_i, y_i} x_i \neq y_i$)
- Satisfiable $\Leftrightarrow (F \not\equiv G)$ **(why this way?)**



CEC: How Hard Can It Be?

Two Miter examples from SAT Competition 2023

- Instance A: 260k variables, 850k clauses
 - Circuits are **not equivalent**
 - Solved in 1.33s by KISSAT_MAB_PROP-NO_SYM [6]

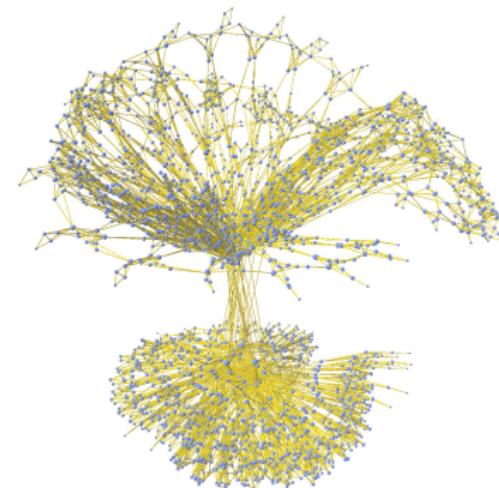


Nodes = variables;
Edges = common clause(s);
variables contracted by factor ≈ 16

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- Instance B: 4k variables, 13k clauses
 - Circuits are **equivalent**
 - **Unsolved** by sequential solvers within 5000 s
 - Solved by some **parallel solvers** :)



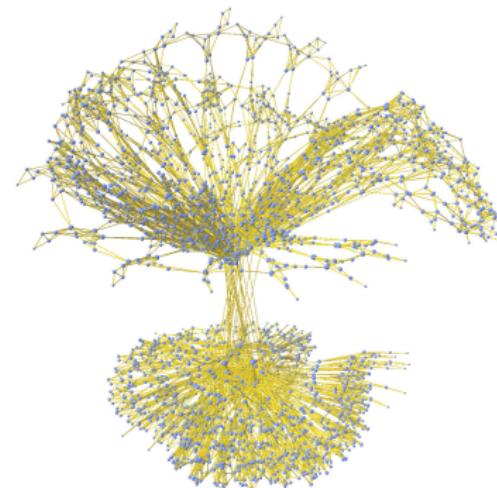
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Generally: co-NP-complete, can require **very large proofs**

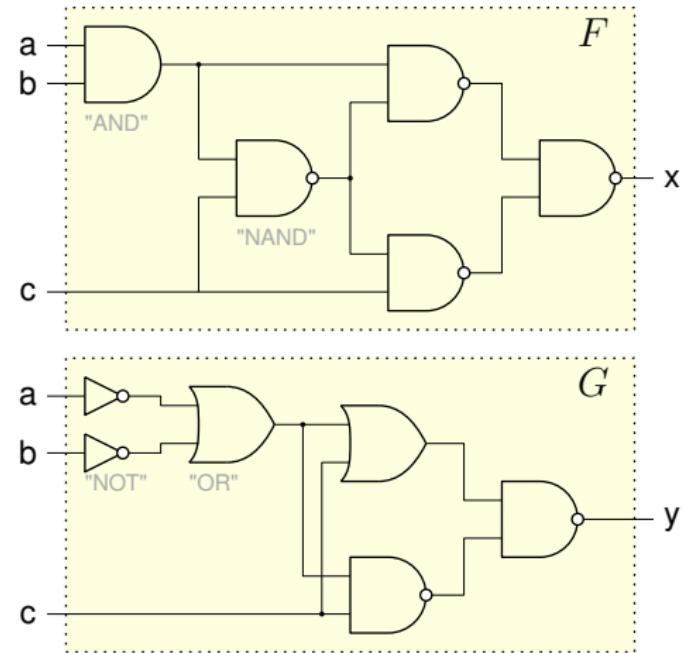


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CEC Techniques [7]

Improving CEC performance: Try to merge equivalent sub-circuits

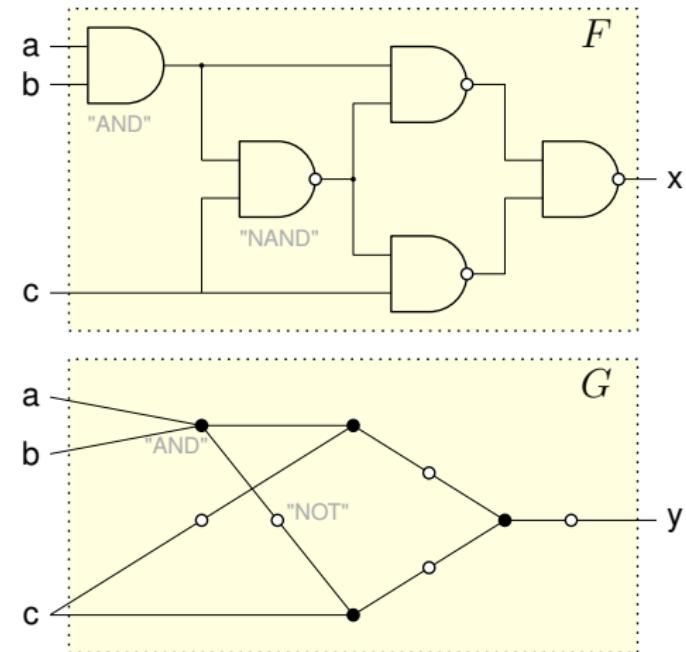
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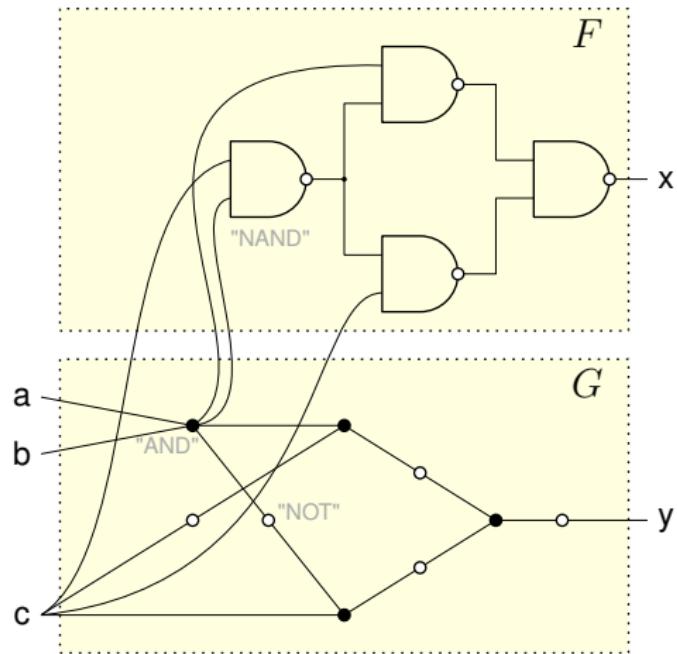
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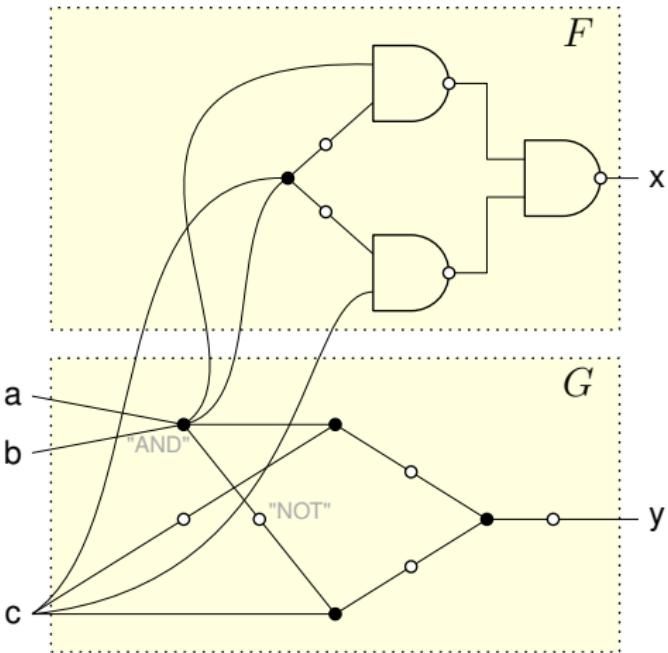
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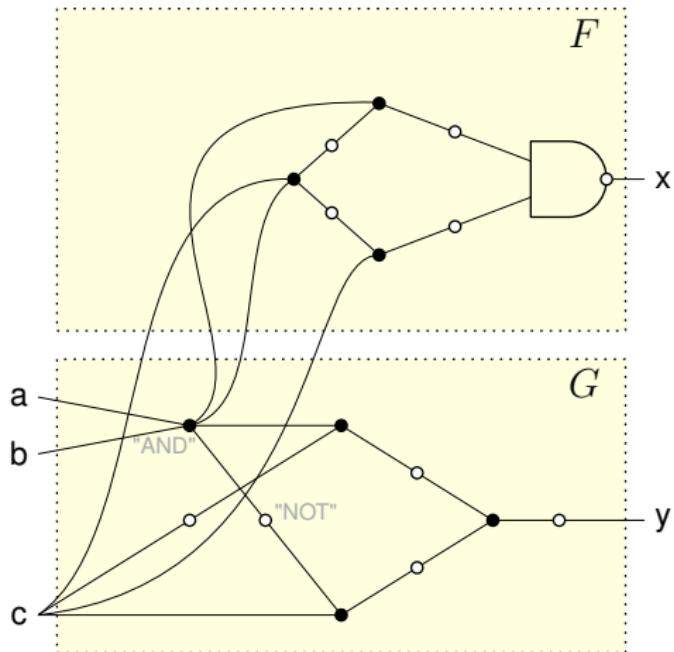
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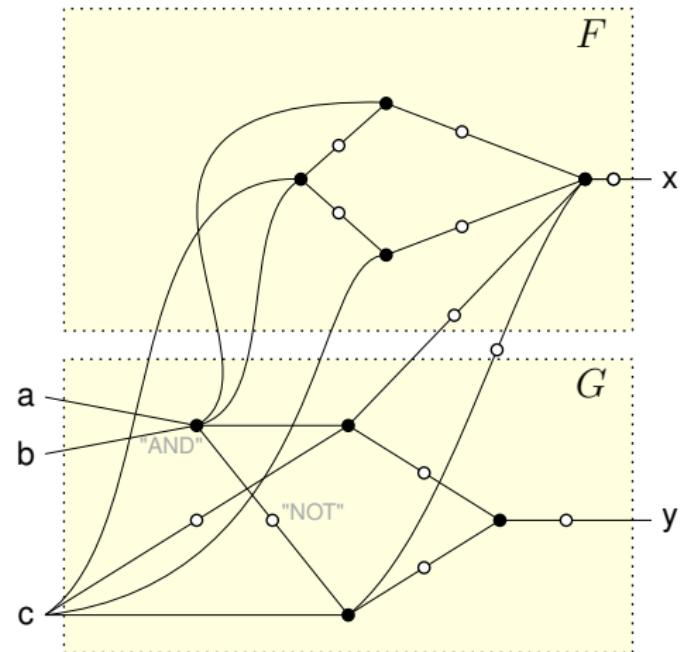
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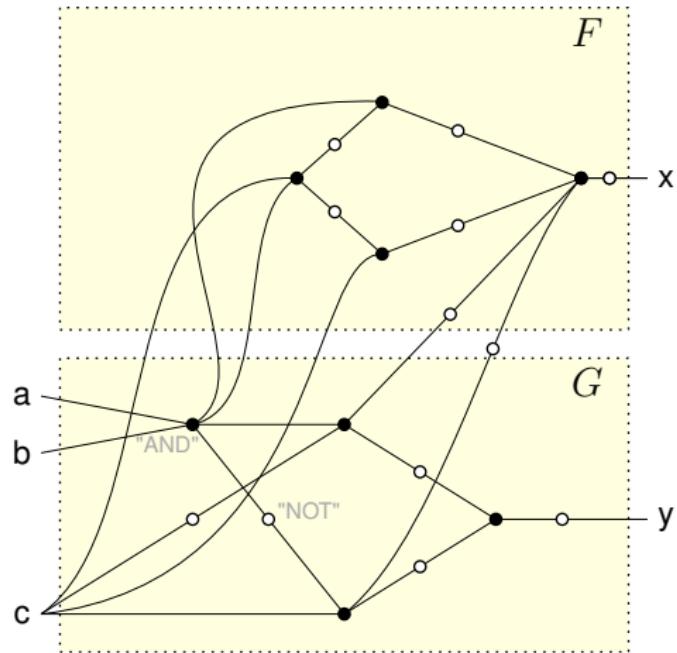
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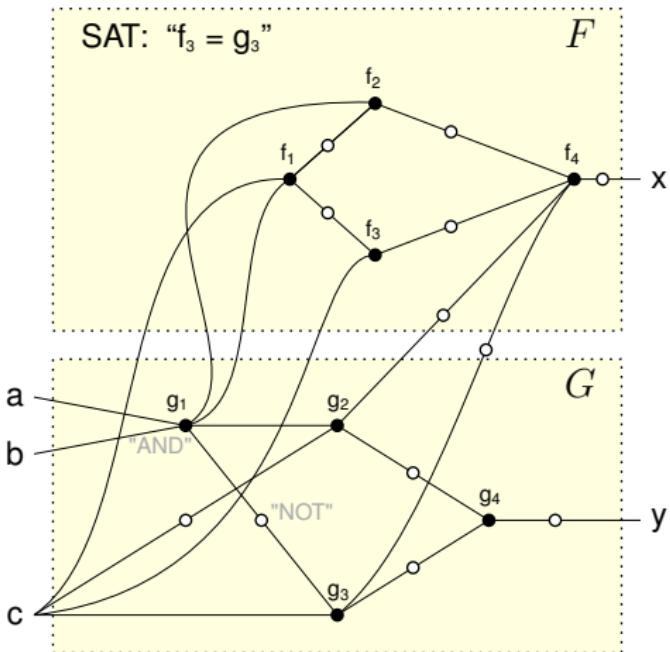
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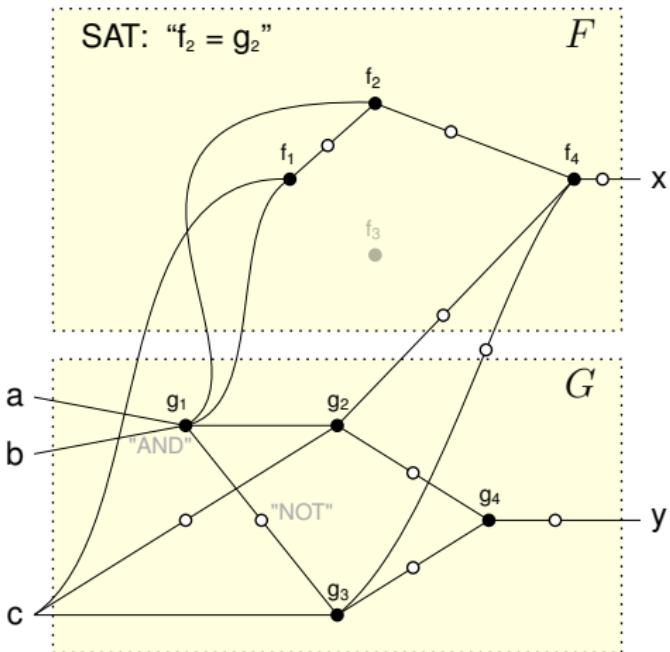
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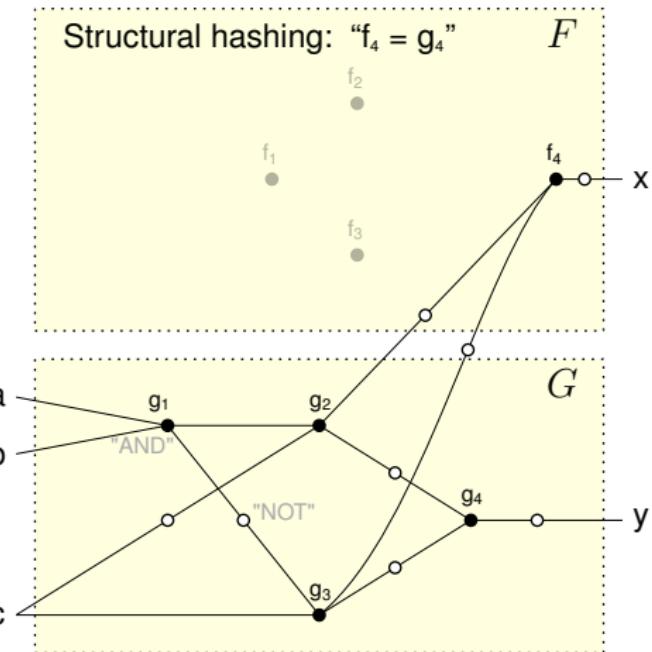
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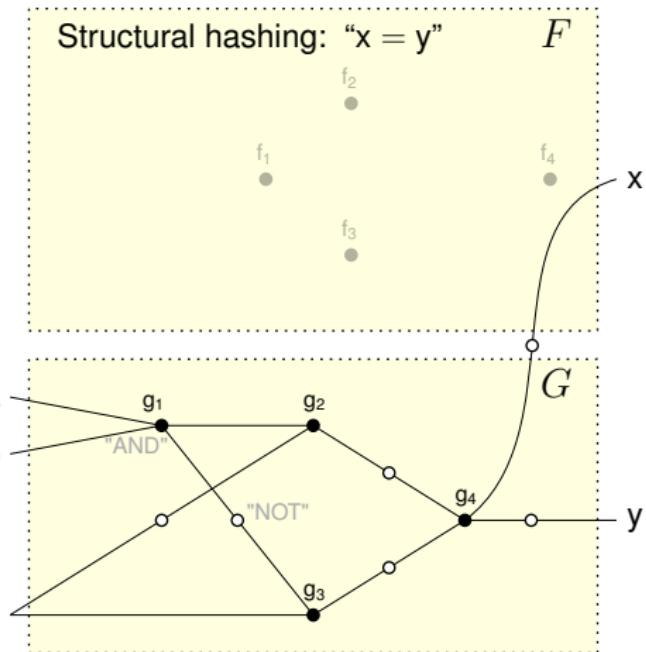
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CEC: Remarks

- Important cornerstone of **Electronic Design Automation** [8]
 - can be used to validate **implementation** based on **specification**
 - other EDA techniques: model checking, **Automated Test Pattern Generation (ATPG)**
- Crucial “**intrinsically Boolean**” benchmark problem throughout history of SAT solving
 - Every SAT competition features mitters
- SAT sweeping originally proposed for **bounded model checking** [9]
- Gate recognition and merging now a form of general inprocessing for **any formula**, connected to variable elimination [10]

Analyzing Cryptographic Building Blocks

Cryptanalysis = analyze, attempt to “break” cryptographic building blocks to test, advance them

- Building blocks: Stream ciphers ($(\text{msg}, \text{key}) \mapsto \text{encrypted msg}$) [11], hash functions [12], ...
- **Algebraic cryptanalysis**: try to build equations relating output to input [11]
 - SAT solver should support [XOR clauses](#)
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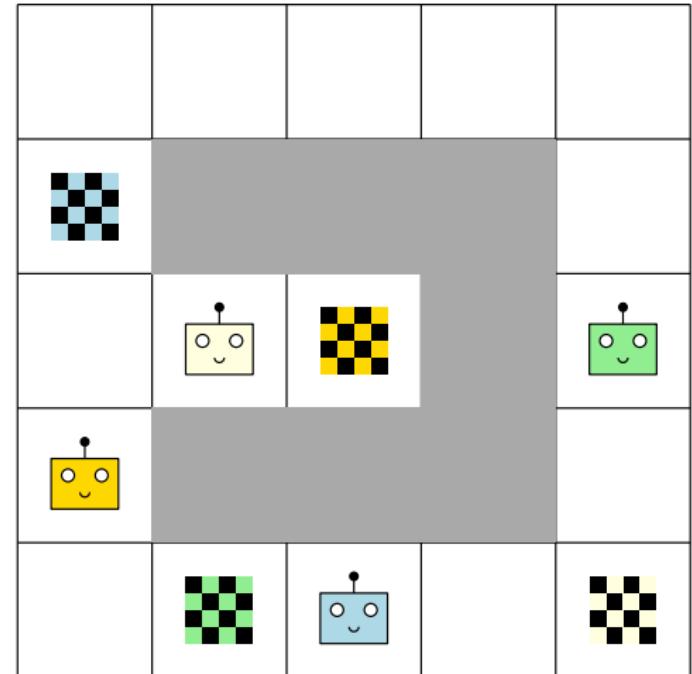
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- Cross-application use of SAT techniques:
 - Cryptanalysis via SMT solving [15]
 - Cryptanalysis via bounded model checking [16]
 - Hash function analysis also used in algorithm design [17]

“it turns out that the highly combinatorial nature of the problem is not well suited for linear solvers, and that SAT solvers are a better fit for this type of problem” —Dobraunig et al. after trying MILP for ASCON [12]

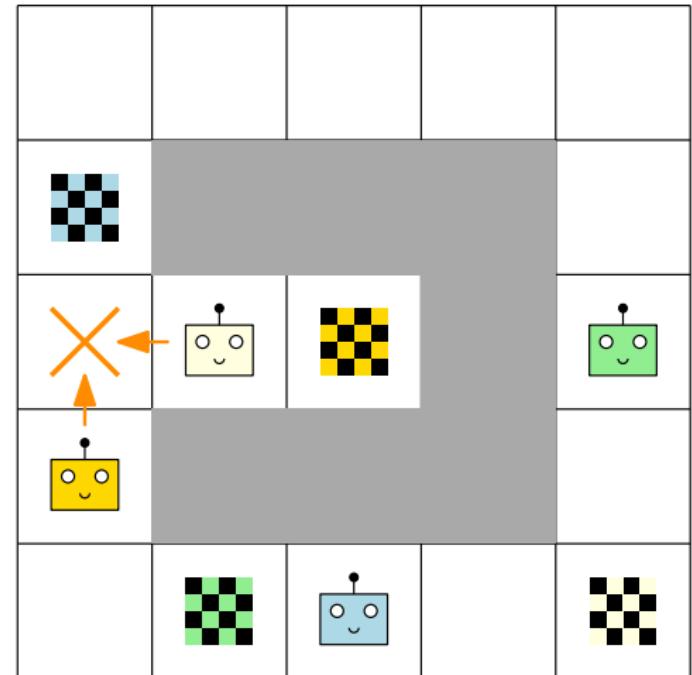
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- Discretized 2D grid of positions, n cooperative agents
- Discretized time steps: move 0-1 cells per time step
- Per agent: Initial position and goal position



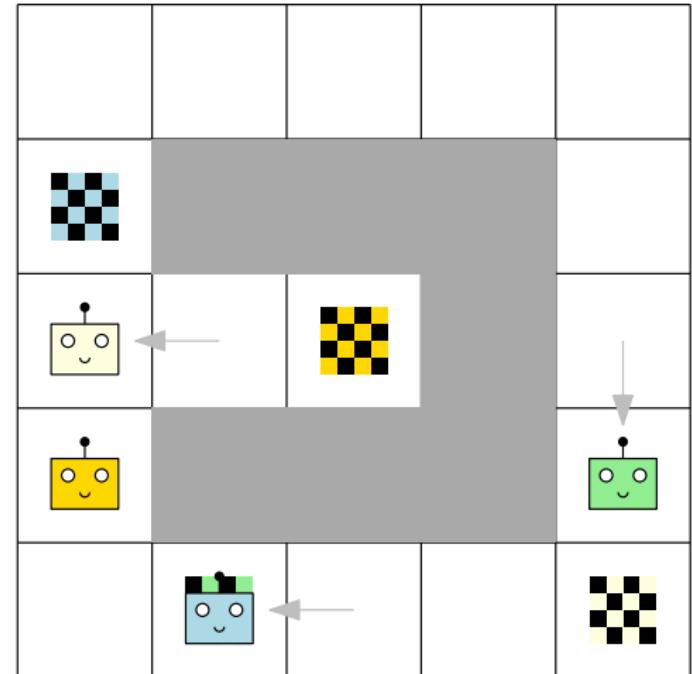
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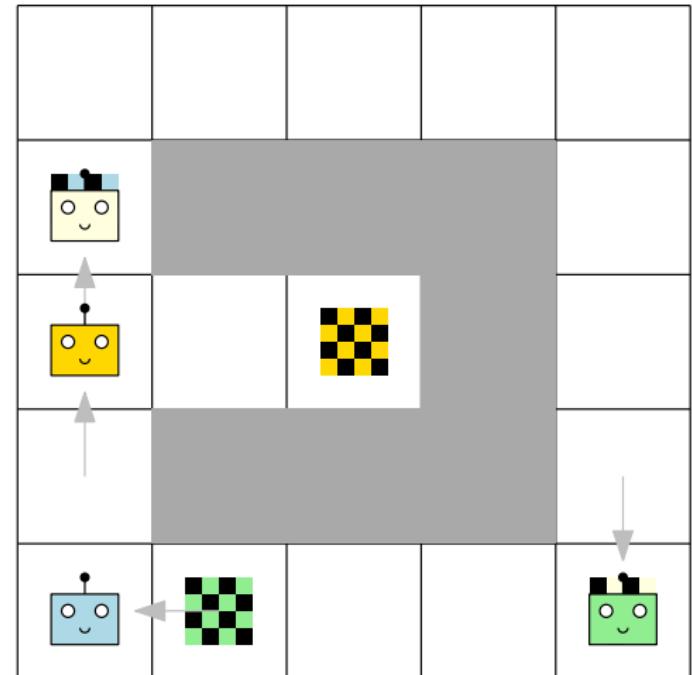
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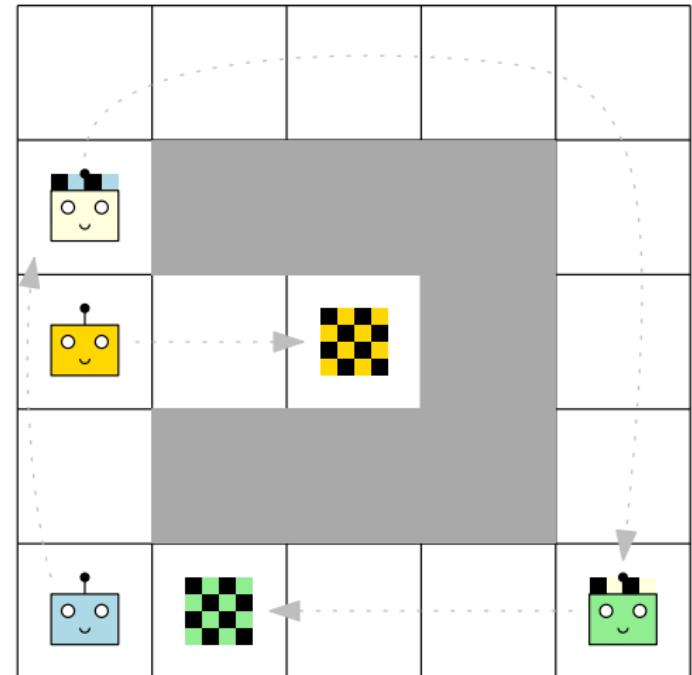
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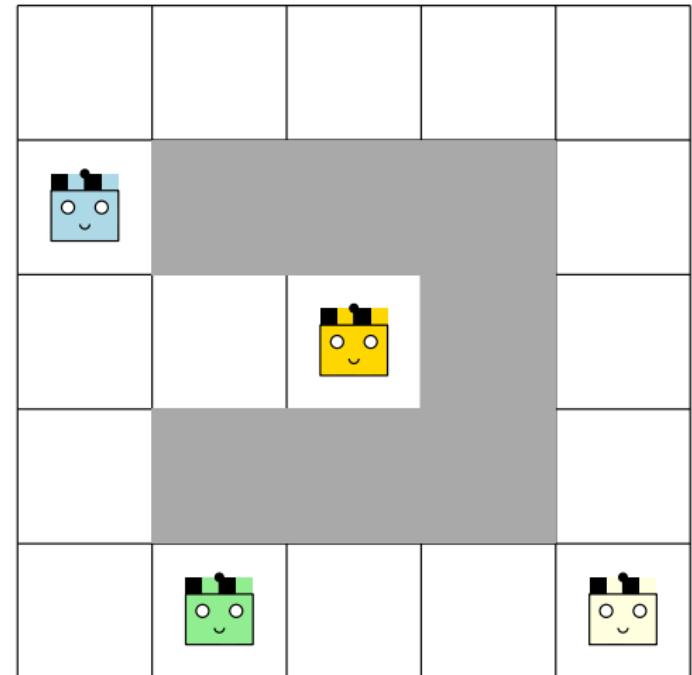
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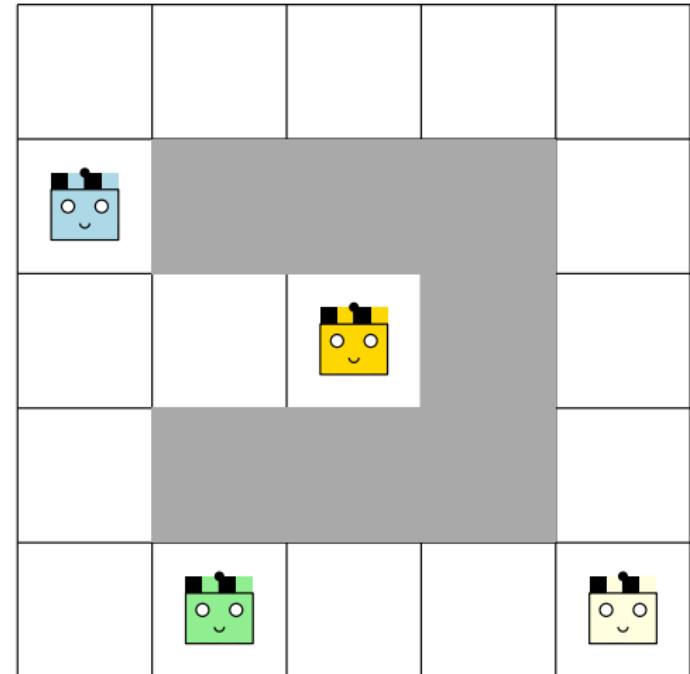


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Optimal Approaches to MAPF

- M* algorithm: adjusted A* with collision handling and backtracking
- Conflict Based Search (CBS): route individually; at collision, add constraint to a colliding agent and re-route
- Reduction-based approaches: SAT, MaxSAT, ASP, CSP

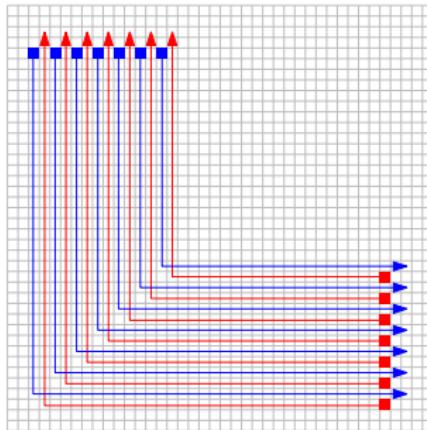


Is SAT-based MAPF worthwhile?

Observation on MAPF [18] (and planning, and scheduling, and probably many other problems . . .):

Direct search-based approaches

perform especially well on **large**,
lightly constrained instances.



SAT-based approaches

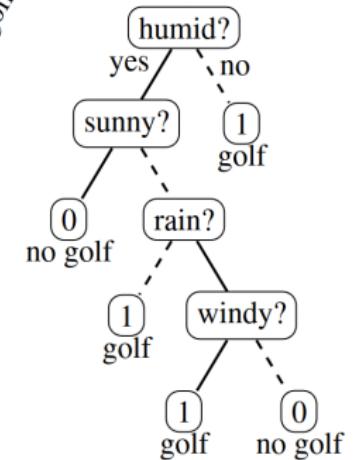
perform especially well on **small-sized**,
highly constrained instances.

13	7	2	4
3	8	15	5
9	12	1	6
14	11	10	

Explainable AI: Learning Decision Trees

- Given: n d -dimensional sample vectors (d features) mapped to a (binary) class
- Task: Learn decision tree classifying all samples
- Explainable classifier (the more shallow the better)

sample	sunny	rain	overcast	temp:mild	temp:hot	temp:cool	humid	windy	play golf
e_1	0	1	0	0	0	1	0	1	1
e_2	0	1	0	1	0	0	0	1	1
e_3	1	0	0	0	1	0	1	1	0
e_4	0	0	1	0	0	1	0	0	1
e_5	1	0	0	1	0	0	0	0	1
e_6	0	1	0	1	0	0	1	0	0
e_7	0	1	0	1	0	0	1	1	1
e_8	0	0	1	0	1	0	1	1	1
e_9	1	0	0	1	0	0	1	1	0
e_{10}	1	0	0	0	0	1	0	1	1
e_{11}	0	0	1	1	0	0	1	0	1
e_{12}	1	0	0	0	1	0	1	0	0



taken from [19]

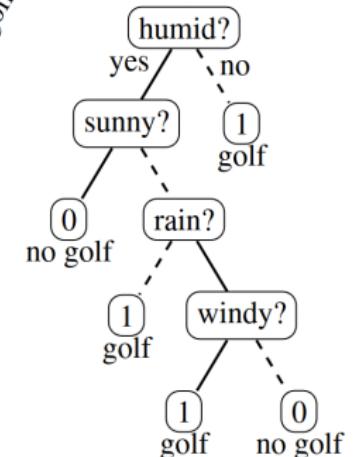
Explainable AI: Learning Decision Trees

- Given: n d -dimensional sample vectors (d features) mapped to a (binary) class
- Task: Learn decision tree classifying all samples
- Explainable classifier (the more shallow the better)

SAT-based approach [20]

- Encode complete binary tree of depth k
- Encode recursively for each node which samples are excluded along its path
- Constrain that 0-leaves exclude all 1-labeled samples and vice versa
- Solver picks a sub-tree and each node's feature

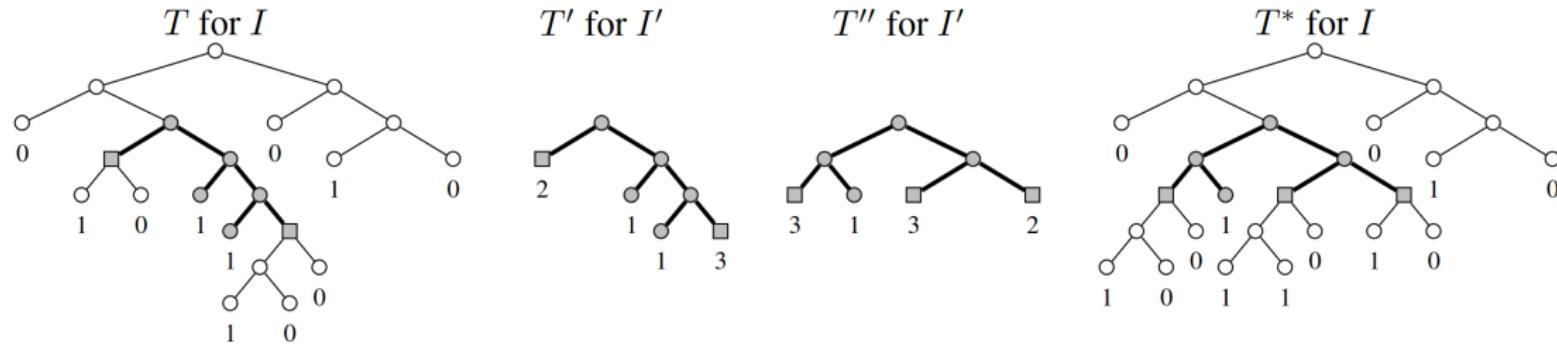
sample	sunny	rain	overcast	temp:mild	temp:hot	temp:cool	humid	windy	play golf
e_1	0	1	0	0	0	1	0	1	1
e_2	0	1	0	1	0	0	0	1	1
e_3	1	0	0	0	1	0	1	1	0
e_4	0	0	1	0	0	1	0	0	1
e_5	1	0	0	1	0	0	0	0	1
e_6	0	1	0	1	0	0	1	0	0
e_7	0	1	0	1	0	0	1	1	1
e_8	0	0	1	0	1	0	1	1	1
e_9	1	0	0	1	0	0	1	1	0
e_{10}	1	0	0	0	0	1	0	1	1
e_{11}	0	0	1	1	0	0	1	0	1
e_{12}	1	0	0	0	1	0	1	0	0



taken from [19]

SAT-based Improvement of Decision Trees [19]

- SAT-based approach slow/infeasible for large data sets
- Better: Construct initial decision tree heuristically, then locally improve sub-trees via SAT
 - Hybrid approach, also beneficial in other contexts, e.g., CEC, planning [21]
- Enables to scale up merits of SAT to arbitrarily large data sets



taken from [19]

More on SAT Solving × Machine Learning

- **Algorithm selection**

Xu, Lin, et al. "SATzilla: portfolio-based algorithm selection for SAT." JAIR 2008.

Eggenberger, Katharina, Marius Lindauer, and Frank Hutter. "Neural networks for predicting algorithm runtime distributions." IJCAI 2018.

- **SAT Solving featuring ML techniques**

Liang, Jia Hui, et al. "Learning rate based branching heuristic for SAT solvers." SAT 2016.

Guo, Wenzuan, et al. "Machine learning methods in solving the boolean satisfiability problem." Machine Intelligence Research (2023).

- **Verify Neural Networks via SAT/SMT solving**

Huang, Xiaowei, et al. "Safety verification of deep neural networks." CAV 2017.

Ehlers, Ruediger. "Formal verification of piece-wise linear feed-forward neural networks." ATVA 2017.

- **Analyze and understand behavior of SAT solvers and instances**

Soos, Mate, Raghav Kulkarni, and Kuldeep S. Meel. "CrystalBall: gazing in the black box of SAT solving." SAT 2019.

Fuchs, Tobias, Jakob Bach, and Markus Iser. "Active Learning for SAT Solver Benchmarking." TACAS 2023.

Train Scheduling with Disruptions via MaxSAT [22]

Can SAT Solving fix the Deutsche Bahn?

Train Scheduling with Disruptions via MaxSAT [22]

Can SAT Solving fix the Deutsche Bahn?

No.

Train Scheduling with Disruptions via MaxSAT [22]

Can SAT Solving fix the Deutsche Bahn?

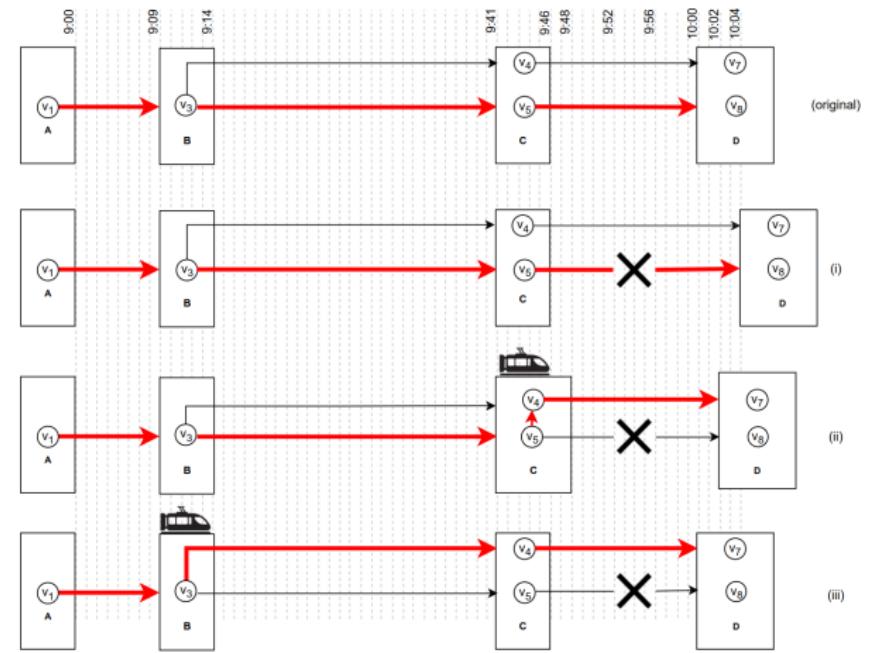
No.

But it might make the Swiss trains run even better!

Train Scheduling with Disruptions via MaxSAT [22]

Train Scheduling Optimization Problem (TSOP)

- **Route** trains through predefined stations
- **Schedule** train timetable subject to time and resource constraints



taken from [22]

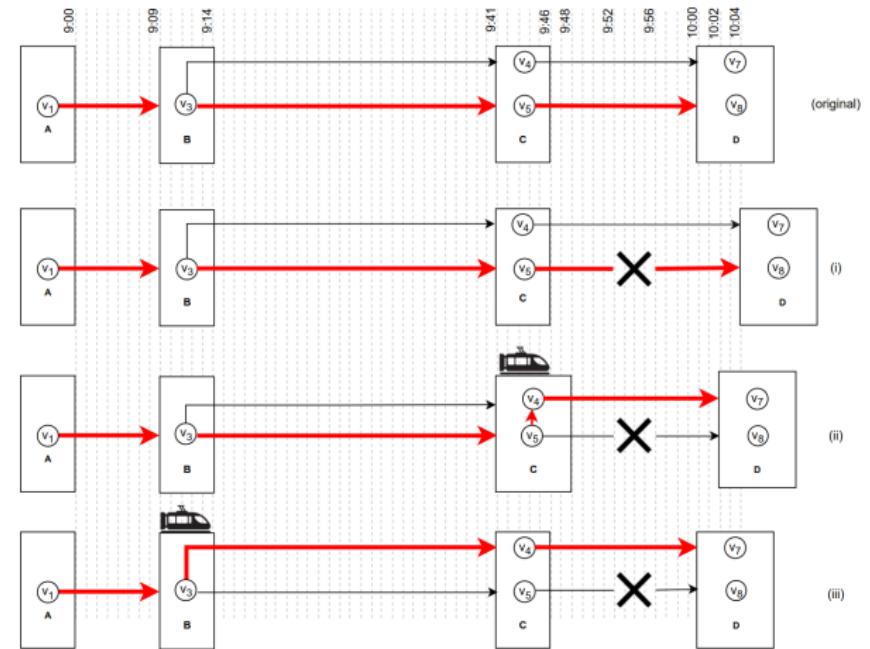
Train Scheduling with Disruptions via MaxSAT [22]

Train Scheduling Optimization Problem (TSOP)

- **Route** trains through predefined stations
- **Schedule** train timetable subject to time and resource constraints

TSOP Under Disruption (TSOPUD)

- Numerous **disruptions**: slowdown, train blocked, track blocked, staffing / rolling stock
- Reroute, reschedule to **minimize delays**



taken from [22]

Train Scheduling with Disruptions via MaxSAT [22]

Train Scheduling Optimization Problem (TSOP)

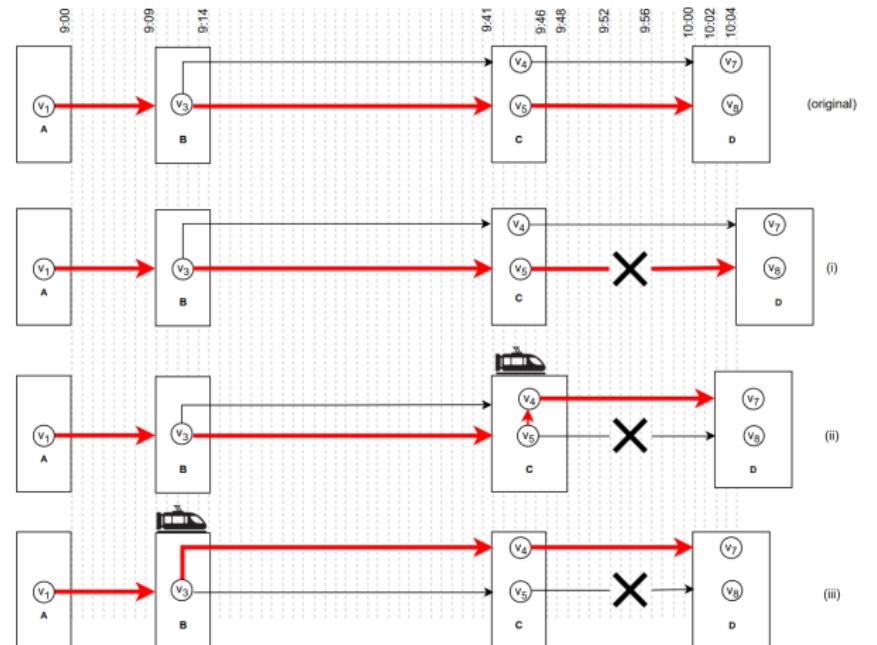
- Route trains through predefined stations
- Schedule train timetable subject to time and resource constraints

TSOP Under Disruption (TSOPUD)

- Numerous disruptions: slowdown, train blocked, track blocked, staffing / rolling stock
- Reroute, reschedule to minimize delays

MaxSAT-based approach

- Encode time requirements as hard clauses, route/train cost as soft clauses
- Relax timings incrementally until feasible
- Add disruption(s), relax timings as needed



taken from [22]

Application Highlights: Takeaways

- SAT is an (actually!) **essential and well-established tool** for
 - software & hardware verification
 - electronic design automation
 - specific cryptanalysis tasks.
- Indicators for a problem to best use SAT for?
 - large portion of “**intrinsically Boolean**” constraints
 - **combinatorial search space** / set of decisions, **NP-hard (or harder)**
 - problem description **not too large**
- **Cross-application** techniques and insights:
 - Often promising to **hybridize** SAT with direct (search) methods: use SAT to resolve **difficult cores**
 - **Positive feedback loop** between solver techniques and applications
 - **Cross-fertilization** between different applications (e.g., BMC, SMT)
- **Many more**, sometimes important, **applications** of SAT we didn’t talk about
 - Automated test pattern generation, graph theory, theorem proving, social choice theory, puzzle solving, knowledge compilation, combinatorial design theory, ...

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