Exercises

February 18, 2015

1.

Solve numerically the following equation:

$$U'' + U - \frac{Gm}{h^2} = 0$$

2.

From

$$\frac{\dot{r}\left(\vec{r}\cdot\vec{r}\right) - \left(\vec{r}\cdot\dot{\vec{r}}\right)\vec{r}}{r^3},$$

derive

$$\left(\frac{\left(\vec{r}\times\dot{\vec{r}}\right)\times\vec{r}}{r^3}\right),$$

using

$$\vec{A}\times\left(\vec{B}\times\vec{C}\right) = \vec{A}\left(\vec{B}\cdot\vec{C}\right) - \left(\vec{A}\cdot\vec{B}\right)\vec{C}.$$

3.

From

$$GM\left(\frac{\vec{r}}{r}+\vec{e}\right)\cdot\vec{r} = \left(\dot{\vec{r}}\times\vec{h}\right)\cdot\vec{r}$$

derive

$$\begin{aligned} GM\left(\frac{\vec{r}}{r}+\vec{e}\right)\cdot\vec{r} &= \left(\vec{r}\times\dot{\vec{r}}\right)\cdot\vec{h}, \\ &= h^2. \end{aligned}$$

using

$$\left(\vec{A}\times\vec{B}\right)\cdot\vec{C} = \left(\vec{C}\times\vec{A}\right)\cdot\vec{B}.$$

4.

From

$$\vec{f} = GM\vec{e} = \dot{\vec{r}} \times \left(\vec{r} \times \dot{\vec{r}}\right) - GM\frac{\vec{r}}{r}$$

derive

$$GM\vec{e} = \left[v^2 - \frac{GM}{r}\right]\vec{r} - \left(\dot{\vec{r}} \cdot \vec{r}\right)\dot{\vec{r}}$$

5.

What is the Vernal point?

6.

Solve numerically the Kepler's equation:

$$E - e\sin E = M,$$

for $M = 1$ with 12 decimal digits of precision.

7.

Since

$$\cos f = \frac{\cos E - e}{1 - e\cos E}$$

compute explicitly

$$\sin f$$

and

$$\tan f,$$

in order to get

$$\tan\frac{f}{2} = \sqrt{\frac{1+e}{1-e}}\tan\left(\frac{E}{2}\right).$$

8.

Calculate

$$\frac{d\hat{l}}{dt} = \frac{d\Omega}{dt}\begin{pmatrix} -\sin\Omega \\ \cos\Omega \\ 0 \end{pmatrix}.$$

9.

Find the five osculating elements:

$$\begin{aligned}
\frac{da}{dt} &= \frac{2}{n\sqrt{1-e^2}}\left[Re\sin f + \frac{SP}{r}\right] \\
\frac{de}{dt} &= \frac{\sqrt{1-e^2}}{na}\left[Re\sin f + S\left(\cos f + \cos E\right)\right] \\
\frac{di}{dt} &= \frac{r\cos\phi}{na^2\sqrt{1-e^2}}W \\
\frac{d\Omega}{dt} &= \frac{r\sin\phi}{na^2\sqrt{1-e^2}\sin i}W \\
\frac{d\omega}{dt} &= -\cos i\frac{d\Omega}{dt} + \frac{\sqrt{1-e^2}}{nae}\left[R\cos f + S\left(1+\frac{r}{P}\right)\sin f\right]
\end{aligned}$$