Earthquake Forecasting with ETAS.inlabru

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Introduction

- **Temporal Hawkes process**

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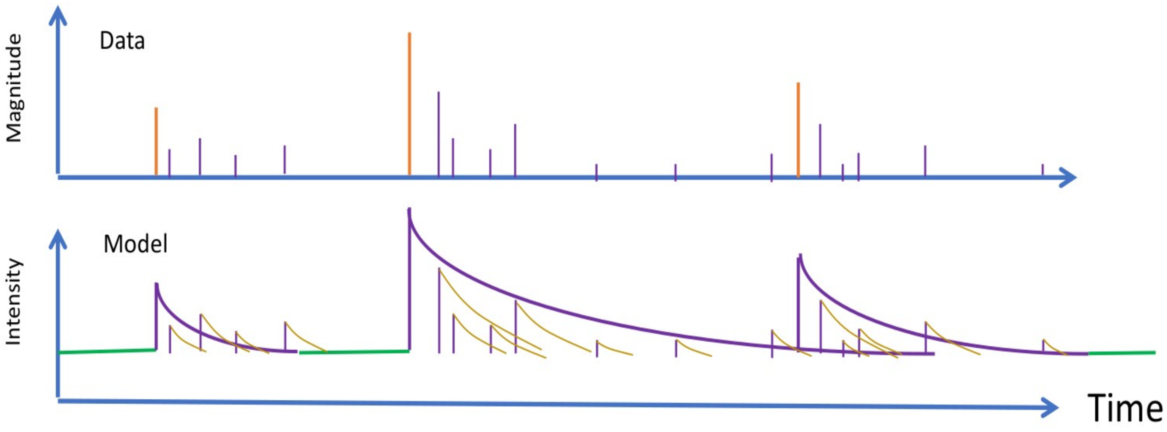
Triggering Function

History of the process

Background Rate



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- **Epidemic-Type Aftershock Sequence (ETAS) Model**

$$\lambda(t,m|H_t)=\left(\mu+\sum_{(t_h,m_h)\in H_t} K e^{\alpha(m_h-M_0)}\left(\frac{t-t_h}{c}+1\right)^{-p}\right)\pi(m)$$

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Hawkes process for occurrence times

Parameters of the model:

- $\mu \geq 0$ Background rate
- $K \geq 0$ Productivity
- $\alpha \geq 0$ Magnitude scaling
- $c \geq 0$ Time offset
- $p \geq 1$ Aftershocks decay

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Magnitude distribution

Gutenberg-Richter Law

$$\log_{10} N(m) = a - bm$$

Or, equivalently,

$$m - M_0 \sim Expo(\beta)$$

where $\beta = \frac{2}{3} b$

Log-Likelihood approximation

$$L = \underbrace{-\Lambda(T_1, T_2)}_{\int_{T_1}^{T_2} \mu\, dt + \sum_{(t_h, m_h) \in H} \int_{T_1}^{T_2} g(t, t_h)\, dt} + \sum_{(t_i, m_i) \in H} \log \lambda(t_i | H_{t_i})$$

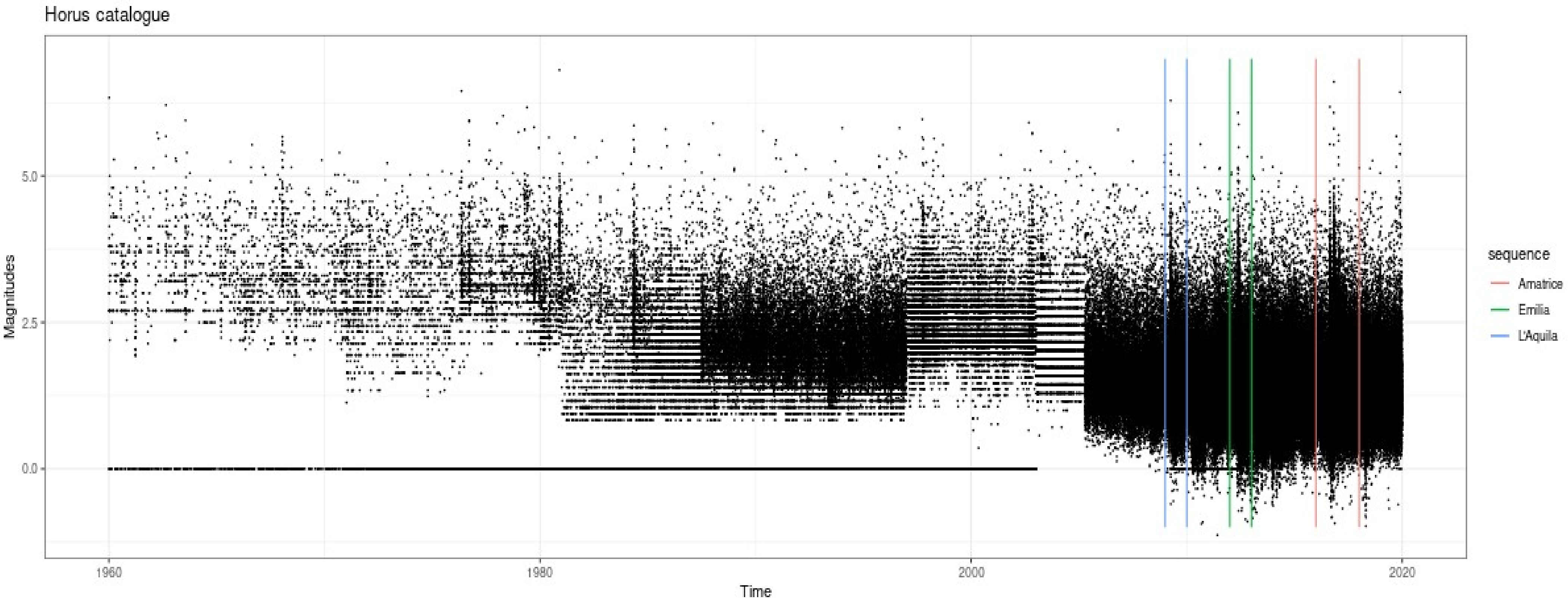
$$\boxed{\int_{T_1}^{T_2} \mu\, dt + \sum_{(t_h, m_h) \in H} \int_{T_1}^{T_2} g(t, t_h)\, dt}$$

$$\boxed{\mu(T_2 - T_1) = \Lambda_0(T_1, T_2)}$$

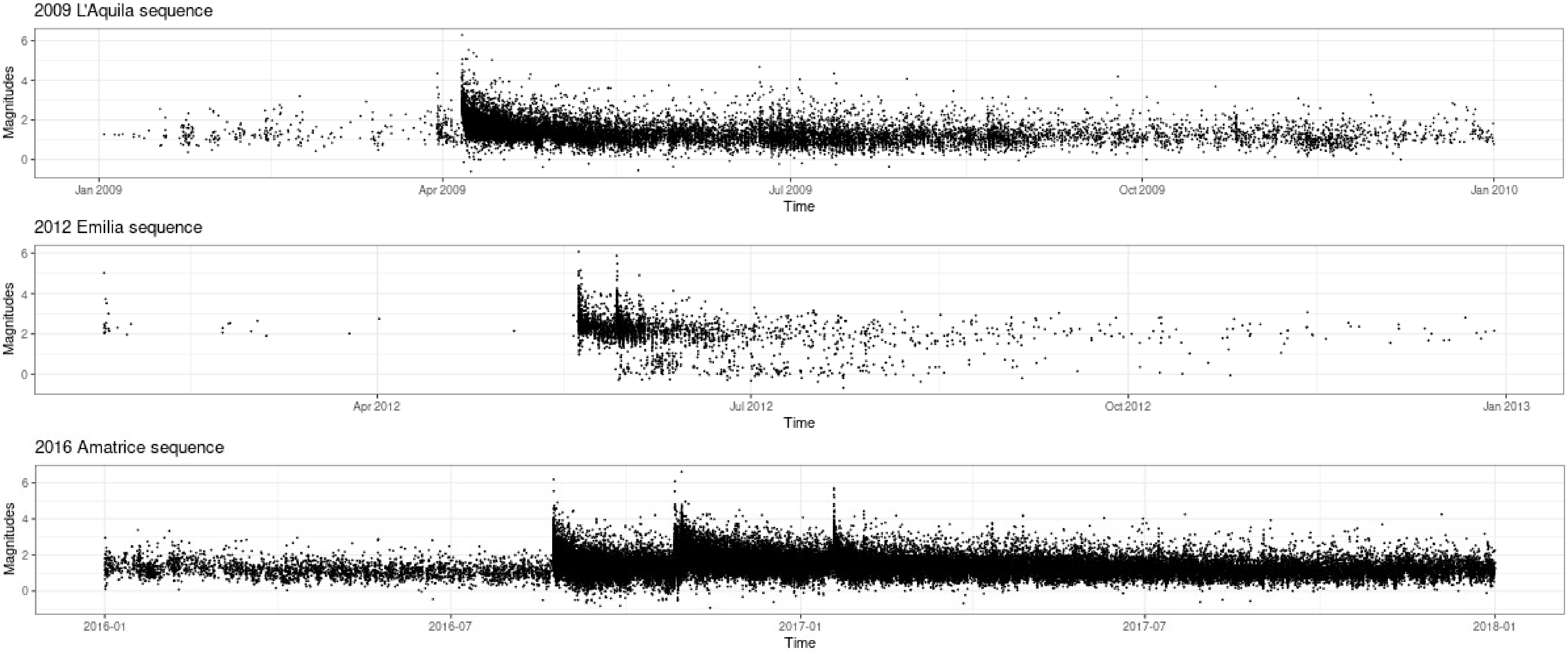
$$\boxed{\sum_{(t_h, m_h) \in H} \sum_{b_j \in B_h} \int_{b_j} g(t, t_h)\, dt = \sum_{(t_h, m_h) \in H} \sum_{b_j \in B_h} \Lambda_h(b_j)}$$

$$\tilde{L} = -\exp\left(\overline{\log\left(\Lambda_0(T_1, T_2)\right)}\right) - \sum_{(t_h, m_h) \in H} \sum_{b_j \in B_h} \exp\left(\overline{\log \Lambda_h(b_j)}\right) + \sum_{(t_i, m_i) \in H} \overline{\log\left(\lambda(t_i | H_{t_i})\right)}$$

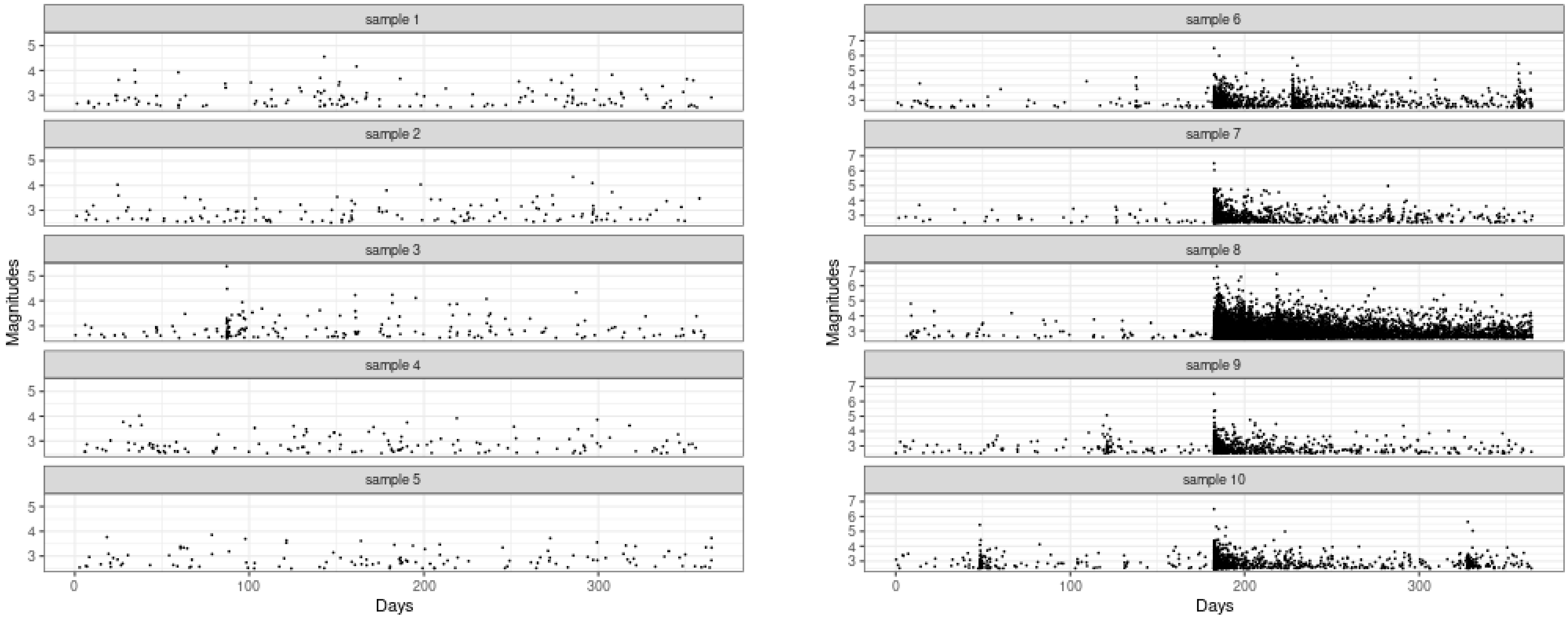
Real Data – Homogenized InstRUmental Seismic catalogue



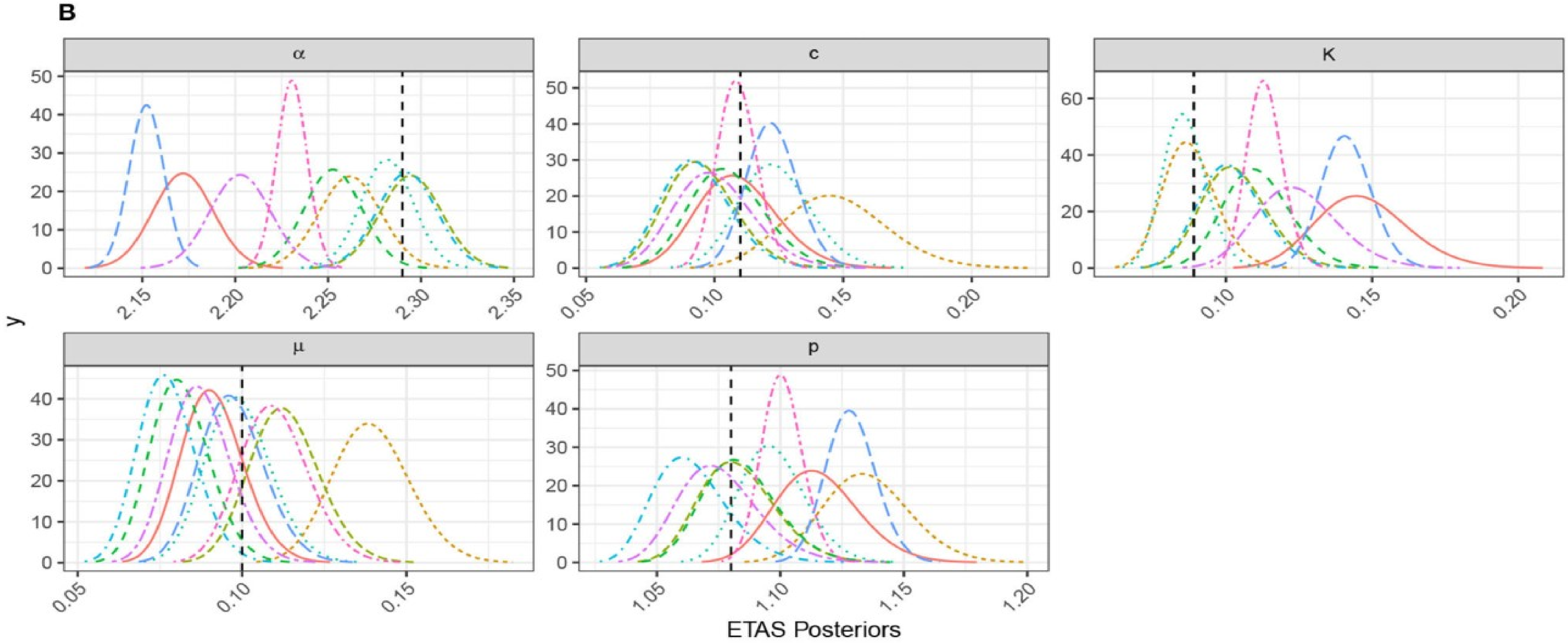
Real Data – Italy Sequences



Synthetic Data



Posteriors from synthetic catalogues



Point of Interest

1. Fit the temporal ETAS model on a large number of synthetic catalogues and study how the posterior changes with respect to earthquake characteristics.
2. Compare the posterior distributons obtained fitting the full model or fixing some parameters to the values used to generate the data.
3. Invetigate the earthquake sequence characteristics that lead to *better* estimates of parameter α .
4. Investigate the interevent time distribution obtained from the data used to fit the model versus synthetic data generated by the fitted model.
5. Assess the forecast accuracy and/or precision of the models, and compare different updating strategies.

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