

Wakefield Acceleration

February 14, 2023

Our classic particle accelerators span the circumference of two countries, between 27 - 31 km and capable of proton acceleration up to 7 TeV. Wakefield Particle Acceleration enables research to be done in this regard, but within 1 - 2 meters, achieving 1 GeV over 3.3 cm. They are very fast, and have the ability to produce X-Rays. In order to understand Wakefield Acceleration, we have to take into account the physics of lasers (Light Amplified by Stimulated Radiation) the Doppler Effect & General Relativity.

Particle Acceleration begins with a laser. The light particles bounce off a mirror to hit a piece of copper, with enough energy to extract electrons from that piece of copper... then travel through the particle accelerator, affected by the field created by magnets, traveling near the speed of light. Wakefield Acceleration depends on the interaction length of the laser with the plasma, expressed here by the Rayleigh length,

$$\zeta = \frac{\pi \omega_0^2}{\lambda}$$

where ω_0 = where the beam waist will converge and diverge, and λ = the wavelength of the laser. Type of Radiation Frequency Range (Hz) describing the distance between two crests of waves, referred to as the wavelength:

gamma-rays $10^{20} - 10^{24} < 10 - 12m$ (about the size of an atom's nucleus)

x-rays $10^{17} - 10^{20} 1nm - 1pm$ (about the size of an atom)

ultraviolet $10^{15} - 10^{17} 400nm - 1nm$

visible $4 - 7.5 \times 10^{14} 750nm - 400nm$

near-infrared $1 \times 10^{14} - 4 \times 10^{14} 2.5\mu m - 750nm$

infrared $10^{13} - 10^{14} 25\mu m - 2.5\mu m$

microwaves $3 \times 10^{11} - 10^{13} 1mm - 25\mu m$

radio waves $< 3 \times 10^{11} > 1mm$

The acceleration gradient for a linear plasma wave is:

$$E = c \cdot \sqrt{\frac{m_e \cdot n_e}{\epsilon_0}}.$$

where E = the electric field, c = the speed of light in a vacuum, m_e = the mass of the electron, n_e = the plasma electron density in particles per meter cubed, and ϵ_0 = permitted space. If you make the particle turn through magnetism, as opposed to its straight path, it will emit radiation.

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$$

$G_{\mu\nu}$ is the Einstein tensor, $g_{\mu\nu}$ = the metric tensor, $T_{\mu\nu}$ = the stress-energy tensor, Λ = the cosmological constant, and κ = Einstein gravitational constant.

The Einstein tensor is expressed

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$$

$R_{\mu\nu}$ = Ricci curvature tensor, and R = scalar curvature.

Einstein's gravitational constant is expressed as

$$\kappa = \frac{8\pi G}{c^4} \approx 2.076647442844 \times 10^{-43} \text{ N}^{-1}$$

where G = the Newtonian gravitational constant and c = the speed of light in a vacuum.

$E = mc^2$ is how we conceive and express Special Relativity, in the same conversation as General Relativity. Special relativity applies to physical phenomena 'in absentia' of gravity. General relativity expresses the law of gravitation and its relation to force. If an object or particle travels near the speed of light, the observer would have a distorted view of the traveling particle.

The Doppler Effect is defined as $f_o = \frac{v+v_o}{v+v_s} f_s$

f_o = observer frequency v = speed of waves v_o = observer velocity v_s = source velocity f_s = actual frequency

This is why a traveled distance of a centimeter equivalent to the width of two crests of microwaves can be observed millions of times smaller as an X-Ray. To use the example from UCLA lecture, if a cyclist travels near the speed of light past a cop, the cyclist would appear shrunk horizontal to the earth. To the cyclist, the cop would appear horizontally shrunk. Once the Doppler Effect makes its application to this experiment, it affects light the same way it would sound... like a siren changing pitch as it travels relative to us.

Will Boos