

# Stability Analysis and Optimal Control Design for Dual-Loop Voltage-Controlled Grid-Connected Inverters

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**Abstract**-- This paper analyzes the stability of digitally dual-loop voltage-controlled inverters with consideration of grid impedance. It is revealed that both the digital delay and controller affect the system stability through adjusting the phase at the resonant frequency of the LCL filter. Then an optimal control for inner current loop is proposed to deal with different switching frequencies. Through tuning the feedback gain of inner current loop, the control scheme can cope with different switching frequencies or delays to keep system stable. Then the stable regions presented as the value range of current feedback gain are derived. Simulation and experiment validate the stability analysis and effectiveness of the proposed control.

**Index Terms**—voltage-controlled inverter, dual-loop control, digital delay, stability analysis.

## I. INTRODUCTION

With grid-forming ability, the voltage-controlled inverters have advantages over current-controlled inverters when connected to weak grid[1],[2]. The research on grid-connected voltage-controlled inverters such as virtual synchronous generator and virtual synchronous machine[3],[4], have aroused great attention. Although the upper layer controls are different, the basic control is the same, which is the high-quality and effective voltage control. The existing research on the stability of voltage-controlled inverters mainly focus on the standalone mode, barely consider the effect of grid impedance. The LC filter of voltage-controlled inverter is changed to LCL filter when connected to the inductive grid, which lead to the deviation of resonant frequency.

There are mainly two voltage regulation controls for voltage-controlled inverters: 1)single-loop voltage control[5-7] and 2)dual-loop voltage-current control.[8-10] The single-loop voltage control is simple but suffers from the issues of constrained loop bandwidth and lack of overcurrent protection.[10] On the other hand, by introducing the inner current loop, the dual-loop voltage-current can provide enhanced damping and solve the issues of single-loop voltage control. It has been shown that the inner current loop is equivalent to a virtual impedance in parallel with the filter capacitor or in series with the filter inductor[11-13]. However, the virtual impedance is affected by the digital delay and the real part of the virtual impedance may become negative and cause instability[12]. It is revealed that the negative damping occurs above the one-sixth of sampling frequency( $f_s/6$ ).

Therefore, the dual-loop voltage-current control is constrained in the case of pulse ratio, which is caused by the increase of digital delay.[6]

To fill the gap of stability analysis of voltage-controlled grid-connected inverters and solve the deficiency of dual-loop voltage-current control under low pulse ratio, this paper analyzes the stability of digitally dual-loop voltage-controlled inverters with consideration of grid impedance. Based on the stability analysis, an optimal control for inner current loop is proposed to deal with different switching frequencies.

The remaining parts of this paper are organized as follows: Section II presents the topology and control scheme of the dual-loop voltage-controlled grid-connected inverters considering delay and grid inductance. Section III introduces the stability analysis and optimal control of the inner current loop, the stable regions in different switching frequencies are derived. Section IV shows the simulation and experiment results. The conclusions are drawn in the last section.

## II. DUAL-LOOP CONTROL STRUCTURE OF VOLTAGE-CONTROLLED GRID-CONNECTED INVERTERS

Fig.1 shows the topology and control scheme of voltage-controlled grid-connected inverter. The grid voltage is assumed balanced so per-phase diagrams used for analysis can be depicted in Fig.1(b).  $L_m$  and  $C$  are the inductor and capacitor of the LC filter to attenuate the switching harmonics. The grid is assumed inductive and  $L_g$  is the grid impedance. The LC filter and grid inductance

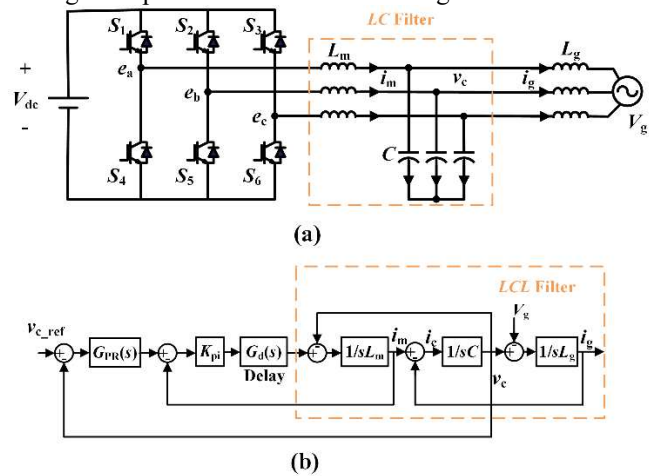


Fig. 1. Dual-loop voltage-controlled grid-connected inverters. (a)Topology. (b) Control scheme

constitute the LCL filter. The parasitic resistance of the inductor and capacitor is neglected to consider the worst case scenario.

As Fig1.(b) shows, the upper layer control(such as droop control and VSG) will synthesize the voltage reference  $v_{c\_ref}$ . Then the filter capacitor voltage  $v_c$  is fed back as the control variable of outer voltage loop while the filter inductor current  $i_m$  is fed back as the control variable of inner current loop. The voltage regulator  $G_{PR}(s)$  adopts proportional- vector resonant controller, which can be expressed as (1)

$$G_{PR}(s) = K_{PR} + \frac{K_{r1}}{s - j\omega_1} \quad (1)$$

Where  $\omega_1$  is the fundamental frequency of output voltage. The vector resonant controller can ensure the voltage tracking at  $\omega_1$  without error. The current regulators is a proportional controller with gain of  $K_{pi}$ .  $G_d(s)$  is the digital delay which is comprised of computation delay and PWM delay and can be expressed as (2)

$$G_d(s) = e^{-1.5T_s \cdot s} \quad (2)$$

Where  $T_s$  is the sampling period. Referring to Fig.1(b), the loop gain can be derived as (3)

$$T_1(s) = \frac{[K_{pi}(s^2 L_g C + 1) + K_{pi} G_{PR}(s) \cdot s L_g] e^{-1.5T_s \cdot s}}{s L_m L_g C + s(L_m + L_g)} \quad (3)$$

$$= \frac{[K_{pi}(s^2 L_g C + 1) + K_{pi} G_{PR}(s) \cdot s L_g] e^{-1.5T_s \cdot s}}{s L_m L_g C (s^2 + \omega_r^2)}$$

Where  $\omega_r$  is the LCL filter resonance angular frequency and can be expressed as (4).

$$\omega_r = \sqrt{\frac{L_m + L_g}{L_m L_g C}} = 2\pi f_r \quad (4)$$

With the loop gain, the stability of the dual-loop voltage-controlled grid-connected inverters can be analyzed.

### III. STABILITY ANALYSIS AND OPTIMAL CONTROL OF DUAL-LOOP VOLTAGE-CONTROLLED GRID-CONNECTED INVERTERS

#### A. Stability Analysis

The loop gain in (3) can be divided into two parts: the numerator and the denominator, which can be expressed as (5)

$$\begin{cases} G_{nu}(s) = [K_{pi}(s^2 L_g C + 1) + K_{pi} G_{PR}(s) \cdot s L_g] e^{-1.5T_s \cdot s} \\ G_{dc}(s) = \frac{1}{s L_m L_g C (s^2 + \omega_r^2)} \end{cases} \quad (5)$$

$G_{dc}(s)$  is the typical characteristic of LCL filter, whose bode plot is shown in Fig.2(a). It can be found that there is a resonant peak with phase crossing  $-180^\circ$  at  $\omega_r$ , which is naturally unstable. Therefore,  $G_{nu}(s)$  plays a role of phase adjustment for  $G_{dc}(s)$  especially the phase at  $\omega_r$  to avoid  $-180^\circ$  phase crossing at the resonant peak.  $G_{nu}(s)$  can also be divided into two parts: the controller part(in square brackets) and the delay part ( $G_d(s)$ ), they can provide

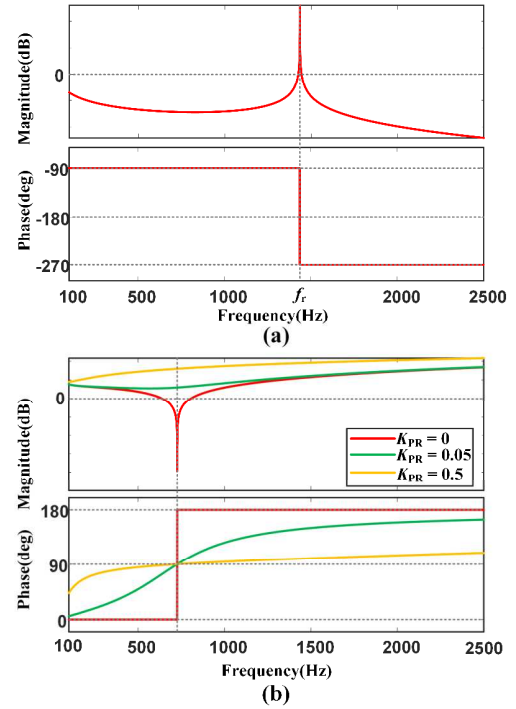


Fig. 2. Bode plot of  $G_{dc}(s)$  and controller parts of  $G_{nu}(s)$  (a)  $G_{dc}(s)$ . (b) Controller parts of  $G_{nu}(s)$  with different  $K_{PR}$

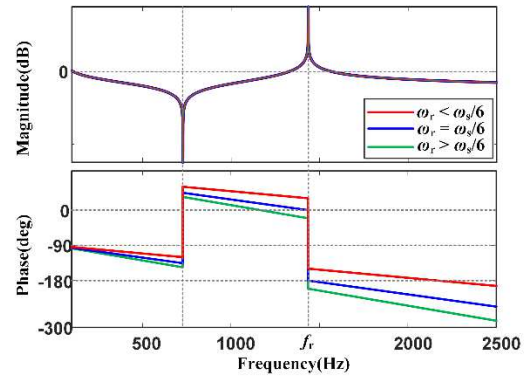


Fig. 3. Bode plot of  $T(s)$  with different switching frequencies

different phase shift for  $G_{dc}(s)$  at  $\omega_r$  and lead to different stable and unstable condition.

With the parameters listed in Table I, Fig.2(b) shows the bode plot of the controller part of  $G_{nu}(s)$  with different  $K_{PR}$ . As shown in Fig.2(b), there is imaginary axis zero for the controller part of  $G_{nu}(s)$  when  $K_{pr}=0$ , and  $180^\circ$  phase lead can be provided above the frequency of the zero in this case. However, with the increase of  $K_{pr}$ , the phase lead will increase below the zero frequency while the phase lead decreases above the zero frequency and the phase lead provided by the controller part will gradually approach  $90^\circ$ . Therefore, if the digital delay is not considered, the phase lead provided by the controller part will adjust the phase of LCL resonant peak to a critical stable state. But the delay part  $G_d(s)$ , on the other hand, will cause phase lag and make the system change from critical stability to instability. Instead, when  $K_{pr} = 0$ , the controller part can provide  $180^\circ$  phase lead for the LCL resonant peak, which extend the phase margin for the phase lag caused by delay. Generally, it can be seen that the proportional part  $K_{pr}$  of



range of  $K_{im}$  to keep system stable in the case of  $\omega_r < \omega_s/6$  can be derived in (9)

$$K_{r1}L_m < K_{im} < \frac{36\omega_s(L_m + L_g) - \omega_s^3 L_m L_g C - 216K_{pi}K_{r1}L_g}{6K_{pi}(36 - \omega_s^2 L_g C)} \quad (9)$$

Define the two thresholds of  $K_{im}$  as (10)

$$\begin{cases} \text{Threshold1} = K_{r1}L_m \\ \text{Threshold2} = \frac{36\omega_s(L_m + L_g) - \omega_s^3 L_m L_g C - 216K_{pi}K_{r1}L_g}{6K_{pi}(36 - \omega_s^2 L_g C)} \end{cases} \quad (10)$$

Fig.5(b) shows the bode plot of  $T(s)$  with different  $K_{im}$  in the case of  $\omega_r > \omega_s/6$ . It can be seen from Fig.6(b) that the system can be possible stable only when  $\omega_0 > \omega_r$ . In this case, the phase lag caused by delay is larger than  $90^\circ$ . Therefore, the phase lag can be utilized to avoid the  $-180^\circ$  crossing at  $\omega_r$  and this can be realized when  $\omega_0 > \omega_r$ . Similarly, the phase will cross  $-180^\circ$  at the frequency  $\omega_c$ . The system can keep stable when the amplitude at the frequency  $\omega_c$  is smaller than 0dB. Therefore, the value range of  $K_{im}$  to keep system stable in the case of  $\omega_r > \omega_s/6$  can be derived in (11)

$$\begin{cases} (K_{im} < \text{Threshold1}) \& (K_{im} < \text{Threshold2}), \left( \sqrt{\frac{1}{L_g C}} > \frac{\omega_s}{6} \right) \\ (\text{Threshold2} < K_{im} < \text{Threshold1}) \& (K_{im} > 0), \left( \sqrt{\frac{1}{L_g C}} < \frac{\omega_s}{6} \right) \end{cases} \quad (11)$$

With the parameters shown in Table I, Fig.6 shows the stable region of system in different switching frequencies. It can be seen from Fig.6 that the stable region is relatively narrow when  $\omega_r > \omega_s/6$  and the value range of  $K_{im}$  is shrink more when  $\omega_r$  is closer to  $\omega_r$ . However, when  $\omega_r < \omega_s/6$ , the stable region will get larger with the increase of switching frequency. Especially, when  $\omega_r = \omega_c$ , there is 'Threshold1 = Threshold2', which means the system is critical stable in this condition and the system can hardly keep stable in this case.

The effect of other parameters including the voltage controller parameter  $K_{r1}$  and the current controller parameter  $K_{pi}$  on the stable regions are also analyzed as shown in Fig.8 and Fig.9. It can be shown in Fig.8 that the increase of  $K_{pi}$  generally will reduce the stable regions no matter when  $\omega_r < \omega_s/6$  and  $\omega_r > \omega_s/6$ . Fig.9 shows that the width of the stable region is almost unchanged and shift upwards as a whole with the increase of  $K_{r1}$  when  $\omega_r < \omega_s/6$ , but in the case of  $\omega_r < \omega_s/6$ , the stable region expands to some extent when  $K_{r1}$  increases due to  $\text{Threshold2} < 0$  and '0' plays as the lower limit of the stable region..

Generally, with the stable region presented as the value range of  $K_{im}$ , the optimal control of inner current loop can be realized through tuning the current feedback proportion flexibly based on (9) and (11) to cope with different switching frequencies.

#### IV. SIMULATION AND EXPERIMENTAL RESULTS

To verify the stability analysis and the effectiveness of the optimal control, the parameters shown in Table I are adopted and the simulation and experiment are designed as follows

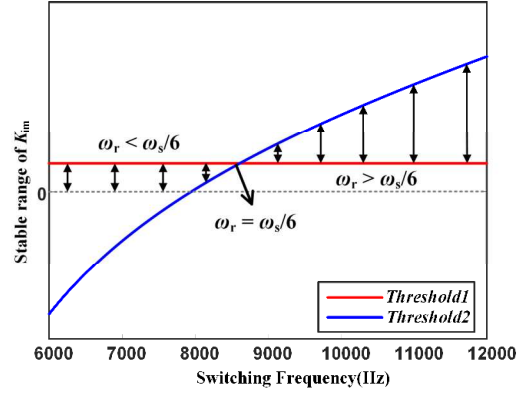


Fig. 6. Stable region with different switching frequencies

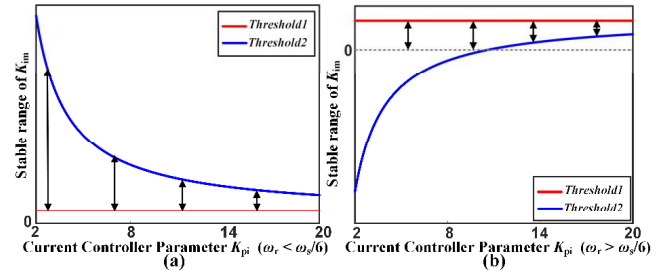


Fig. 7. Stable region with different  $K_{pi}$  (a)  $\omega_r < \omega_s/6$  (b)  $\omega_r > \omega_s/6$

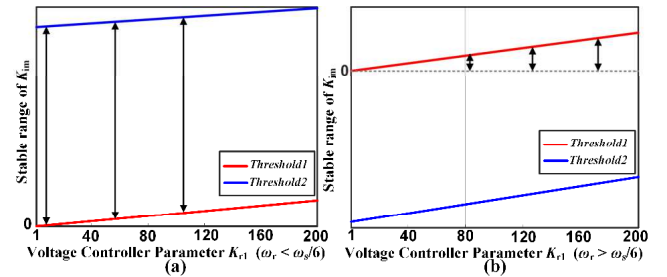


Fig. 8. Stable region with different  $K_{r1}$  (a)  $\omega_r < \omega_s/6$  (b)  $\omega_r > \omega_s/6$

#### A. Simulation Results

It can be seen from Table I that the LCL filter resonant frequency  $f_r$  is 1.435kHz and  $f_c$  is 1.33kHz when switching frequency is 8kHz while  $f_c$  is 2kHz when switching frequency is 12kHz. According to (9) and (11), the value range of  $K_{im}$  to keep system stable should be [0.12, 3.22] when switching frequency is 12kHz and [0, 0.12] when switching frequency is 8kHz. Therefore, the simulation are designed as follows.

- 1) Case1:  $f_s = 12\text{kHz}$ ,  $K_{im}$  is set to change from 1 to 0.11 at 2.0s and back to 1 at 2.04s
- 2) Case2:  $f_s = 12\text{kHz}$ ,  $K_{im}$  is set to change from 1 to 3.23 at 2.0s and back to 1 at 2.04s
- 3) Case3:  $f_s = 8\text{kHz}$ ,  $K_{im}$  is set to change from 0.1 to 0.13 at 2.0s and back to 1 at 2.04s

The simulation results are shown in Fig.9, in which Fig.9(a) and Fig.9(b) shows the grid current wave  $i_g$  and the oscillation frequency analysis of Case1 and Case3. It can be found that the system turns unstable when  $K_{im}$  changes at 2s and the oscillation frequencies of Case1 and Case3 are both around 1.435kHz, which corresponds to the LCL resonant frequency  $f_r$ . The instability of Case1 can be explained by Fig.5(a) that the change of  $K_{im}$  from 1 to 0.11 results in  $f_i < f_0$  but the phase lag caused by delay is



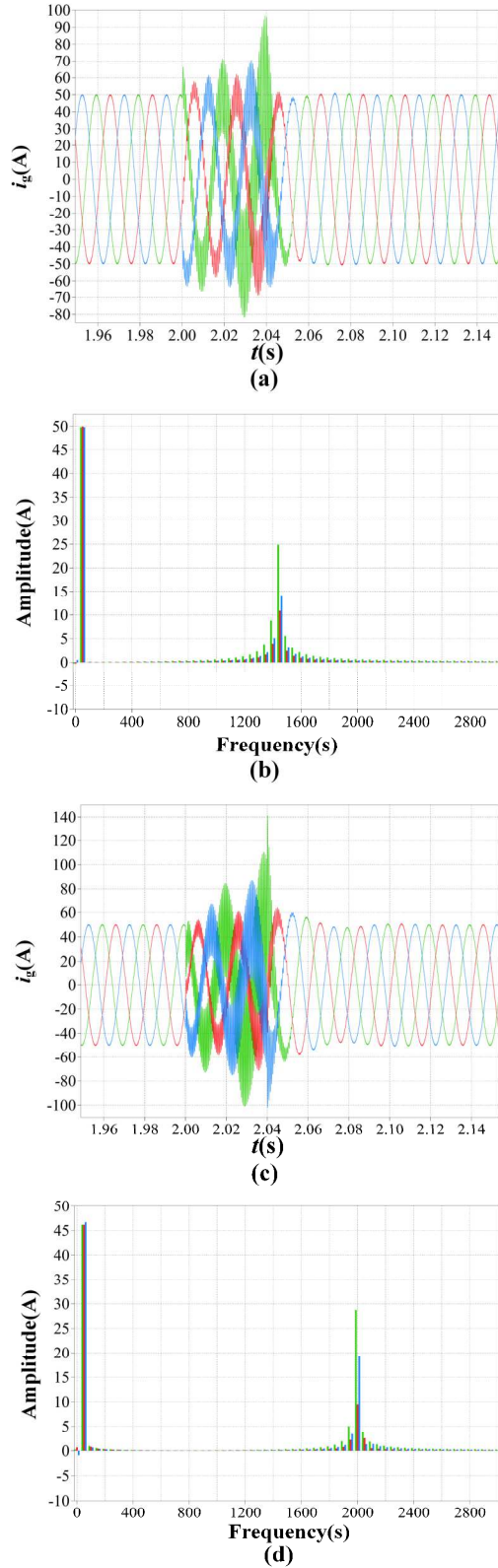


Fig. 9. Simulation results (a) Grid current wave of Case1 and Case3 (b) Oscillation frequency analysis of Case1 and Case3 (c) Grid current wave of Case2 (d) Oscillation frequency analysis of Case2

smaller than  $90^\circ$  thus lead to the  $-180^\circ$  phase crossing at  $f_r$ . The instability of Case3 can be explained by Fig.5(b) that the change of  $K_{im}$  from 0.1 to 0.13 results in  $f_r > f_0$  but the phase lag caused by delay is larger than  $90^\circ$  and

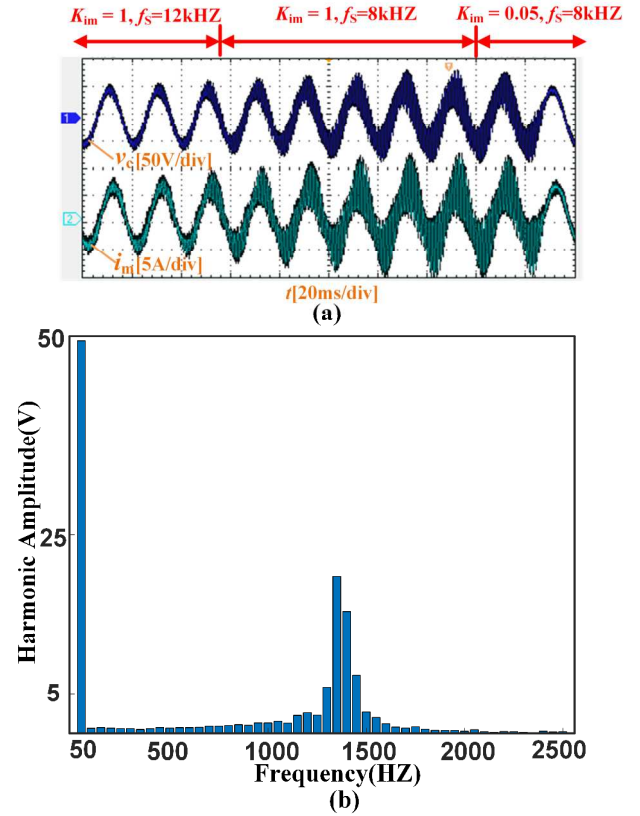


Fig. 10. Experiment results (a) Waveform of capacitor voltage and inverter-side inductor current (b) Oscillation frequency analysis of capacitor voltage

exceed the phase margin provided at  $f_0$ , also leading to  $-180^\circ$  phase crossing at  $f_r$ .

Fig.9(c) and Fig.9(d) shows the grid current  $i_g$  wave and the oscillation frequency analysis of Case1. Similarly, the instability occurs at 2.0s but the oscillation frequencies are around 2kHz, which is corresponding to  $f_c$  when  $f_s = 12 \text{ kHz}$ . The instability can be explained by Fig.5(a) that the phase crosses  $-180^\circ$  at  $f_c$  and the change of  $K_{im}$  from 1 to 3.23 will increase the magnitude to larger than 0dB at  $f_c$ , which causes the instability at  $f_c$ .

### B. Experiment Results

To validate the effectiveness of the optimal control when coping with different switching frequencies, the experiments are designed as follows

1)The inner current feedback proportion  $K_{im}$  is set to 1 and the switching and sampling frequency  $f_s$  is set to change from 12kHz to 8kHz to verify the negative effect of low switching frequency on the stability of dual-loop voltage-current control.

2)Then the  $K_{im}$  is set to change from 1 to 0.05 to verify the effectiveness of tuning  $K_{im}$  to stabilize the system in the case of low switching frequency.

The experimental results are shown Fig10, in which Fig.10(a) presents the waveforms of capacitor voltage  $v_c$  and inverter-side inductor current  $i_m$  while Fig.10(b) is the oscillation frequency analysis of  $v_c$ . It can be seen from Fig.10(a) that when  $K_{im}=1, f_s=12 \text{ kHz}$ , there are  $f_r < f_s/6$  and  $f_0 < f_r$ , thus the phase lead provided by  $f_0$  can help stabilize the system. However, when  $K_{im}=1, f_s=8 \text{ kHz}$ , there are

$f_r > f_s/6$  and  $f_0 < f_r$ , the increase of digital delay will lead to instability. Fig10(b) shows that the main harmonic frequencies are near the LCL filter resonant frequency  $f_r$ , which indicated the  $-180^\circ$  phase crossing at  $f_r$ . Then the  $K_{im}$  is changed from 1 to 0.05 and there is  $f_0 < f_r$ , the phase lag caused by digital delay help avoid  $-180^\circ$  crossing at  $f_r$  instead and the system turns stable.

## V. CONCLUSIONS

This paper analyzes the stability of digitally dual-loop voltage-controlled inverters with consideration of grid impedance. It is revealed that both the digital delay and controller affect the system stability through adjusting the phase at the resonant frequency of the LCL filter. It is found that the dual-loop voltage-controlled inverter may have stability problem in the case of low switching frequency due to the increase of digital delay. Then an optimal control for inner current loop is proposed and can deal with different switching frequency through tuning the feedback proportion of current. The stable regions presented as the value range of current feedback proportion are derived in different switching frequencies. Simulation and experiment validate the stability analysis and effectiveness of the optimal control

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