

# State of Charge Prediction of Lithium-Ion Batteries Based on Artificial Neural Networks and Reduced Data

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## Keywords

«Battery», «State of charge», «Machine learning», «Deep Learning», «Battery Management Systems (BMS)».

## Abstract

Lithium-ion batteries (LIBs) are the key technology for the electrification of the transport sector. Since LIBs have a complex, electrochemical structure, it is a challenge to accurately determine the condition, which is crucial for safety and efficiency during operation. This paper presents the forecasting of the state-of-charge (SOC) of a LIB based on machine learning (ML) algorithms. Data from battery simulation and augmented data are additionally used to train the models. To reduce the dimension of the feature matrix, a singular value decomposition is performed. A multi-layer perceptron (MLP) and a convolutional neural network (CNN) are compared to a linear regression. The impact of the augmented data on the prediction accuracies and the reliabilities of the models is analyzed. The lowest test error is achieved using the CNN with augmented data with a root mean square error (RMSE) of 1.78 %. The results show the applicability of data-driven models for the SOC prediction and the optimization potential using data augmentation techniques.

## Introduction

The need to reduce greenhouse gas emissions and, consequently, the government policy requirements substantiate the relevance of the electrification of the transport sector. The move towards renewable energy is a key challenge for leading automotive manufactures [1]. In this context, lithium-ion batteries (LIBs) have taken a predominant role in the automotive industry due to their high energy density and their long lifespan [2, 3]. The dominant role in automotive applications over the next decade is predicted [4]. To reduce costs, achieve a high driving range, and increase the overall efficiency of the battery, it is crucial to determine the condition of the battery [5]. In operation, this is taken over by the Battery Management System (BMS), which ensures the safety and balanced charge and discharge cycles by monitoring and controlling the battery cells [6]. Among other functions, the BMS is responsible for the estimation of the state of charge (SOC), which is the ratio of the remaining capacity to its full capacity [7]. A direct measurement is not possible due to the complex electrochemical structure of a LIB and the

varying characteristics under different working conditions [8]. The approaches to estimate the SOC can be separated in three areas. The first one are physical models like coulomb counting [9], open-circuit voltage (OCV) correction method [10, 11], or other physical models to approximate the electrochemical connections [12]. The second one are model-based estimations with mainly the usage of Kalman filters [13, 14]. The third one are data-driven models by means of machine learning (ML) or other statistical algorithms. Applied methods are, for example, support vector machines (SVMs) [15] and primarily artificial neural networks (ANNs) [16]. The models vary between deep neural networks [17] over recurrent neural networks (RNN) [18, 19] to convolutional neural networks (CNNs) [20, 21]. The data-driven models show a high accuracy without the need for a detailed electrochemical approximation. While keeping the computing costs moderate, the correlation between battery parameters and the SOC can be determined. Nevertheless, ML models are highly dependent on their input data. As the working conditions of a battery are extremely diverse, it is important to have a sufficient data basis. The costly and highly time-consuming battery tests exacerbate the problem. Possible relief, instead of real world tests, might come in the form of simulation data and data augmentation.

In this work, simulation data of a electrical equivalent model (ECM) and methods of data augmentation are used to enrich the data basis for training different ML models. Further, the dimensionality of the input data is reduced to optimize the models. A linear regression is used as the benchmark model and compared to different forms of ANNs. Based on the test results with a real-world data set, the most suitable model to predict the SOC is identified.

## Methodology

### Data pre-processing

According to the knowledge discovery in databases process (KDD), one important step is the data pre-processing, which includes data collection, cleaning, accounting for time-sequence, and transformation [22]. Different sources of data are combined in this work. A battery cell test system is used to obtain voltage and current values of a LIB of the type Samsung INR18650-30Q. This data is complemented by simulation data with an ECM with a voltage source  $U_{OCV}$ , the internal ohmic resistance  $R_i$ , and two RC-pairs [23]. The model is selected for high accuracy simulating the battery cell while keeping the computing time low. The ECM is shown in Fig. 1.

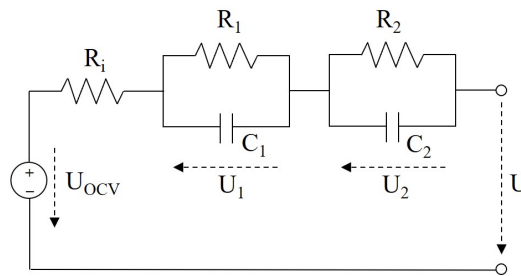


Fig. 1: Second order electrical equivalent model of a battery cell.

During the simulation, the C-rates are varied between 0.1 C and 2 C. To improve the simulation results, the simulation data is transformed by means of the end-of-charge voltage and the cut-off voltage of the real battery. Both values are used for a better approximation of the battery behavior, which can be seen in Fig. 2.

Additionally, a data augmentation method is applied. While the current values are kept constant, the voltage is randomly varied. As starting point, a random difference using a Gaussian distribution is used. All further points are influenced by a random uniform distribution, the current real value, and the previous deviations. Using the Savitzky-Golay-Filter [24], the difference curve is smoothed and added to the initial discharge curve. Following this procedure, a confidence interval around the initial curve is created, where the voltage is randomly varied. Thereby, the differences between the cells should be reproduced.

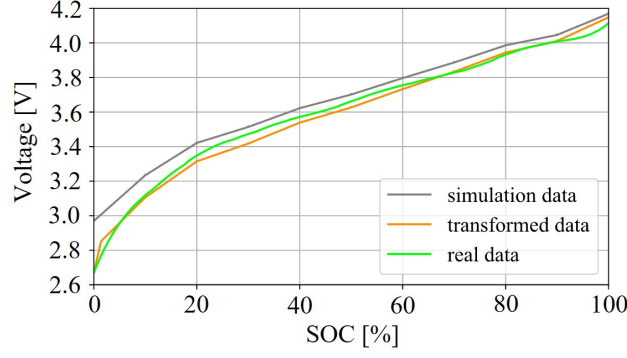


Fig. 2: Data transformation of simulation data with end-of-charge and cut-off voltage in comparison to the real data.

Before the data is further processed, the data is normalized using the standard scaler. Therefore, the standard deviation  $\sigma$  and the mean values  $\bar{x}$  of the data distribution are used to calculate the transformed values  $x_i^n$ , see (1) [25].

$$x_i^n = \frac{x_i - \bar{x}}{\sigma} \quad (1)$$

With a higher number of time steps used for the SOC prediction, the number of features for one data point and, thereby, the dimensionality of the problem is growing. To adequately describe a high dimensional data distribution, the needed data size is increasing exponentially with the dimension. This is referred to as the "curse of dimensionality" [26]. A possible solution are dimensionality reduction techniques. The data is preprocessed using a singular value decomposition (SVD), which is factorization of a matrix  $M$  in two orthogonal matrices  $U$  and  $V$ , and a diagonal matrix  $\Sigma$ , as can be seen in (2) [27].

$$M = U \Sigma V^T \quad (2)$$

In other words, a matrix  $M \in \mathbb{R}^{m \times n}$  is transformed using a rotation, scaling, and another rotation in the following form, displayed in Fig. 3.  $\mu_m$  are the singular values of the matrix  $M$  [28].

$$\begin{array}{c} m \\ \boxed{M} \\ n \end{array} = \begin{array}{c} m \\ \boxed{U} \\ m \end{array} \begin{array}{c} \mu_1 \\ \vdots \\ \mu_m \\ 0 \end{array} \begin{array}{c} n \\ \boxed{V^T} \\ n \end{array}$$

Fig. 3: Graphical representation of a singular value decomposition of a  $m \times n$  matrix  $M$  in two orthogonal and one diagonal matrices.

Based on the used singular values, the number of features can be reduced significantly. The SVD is used for the reduction of the dimension of the input data for the ML models.

## Machine learning methods

### Linear regression

As benchmark model, a linear regression is implemented. Aim is to obtain the linear dependence between the input features based on voltage and current, and the target values SOC. Weight parameters  $w_i$  in combination with the input features  $x_i$  are used to calculate the target value  $y^*$ , see (3) [29].

$$y^* = w_0 + \sum_{i=1}^n w_i x_i \quad (3)$$

The weight parameters are calculated by minimizing a loss function. The loss function  $L$  consists mainly of the squared error between the predicted and the real target values, see (4) [30].

$$L(w) = \frac{1}{2n} \sum_{i=1}^n (y^*(x_{ij}) - y_i)^2 \quad (4)$$

The data points are separated in two intervals for the prediction. The mean value between end-of-charge and cut-off voltage is used as the threshold. Both intervals are trained individually and then combined for the SOC prediction.

### Artificial neural networks

Different types of artificial neural networks (ANNs) are created to predict the SOC. The initial structure was influenced by the information transfer and processing in a biological brain. Artificial neurons are arranged in layers and share information through connections [31]. Based on the structure and the layer formation, different types of ANNs can be separated. A simple form is the multi-layer-perceptron (MLP), which is a feed-forward ANN. The information flow is in one direction from input to output layer by means of several mathematical functions. The information in a neuron is summed up and then used in an activation function, where the output of the neuron is calculated [29]. An artificial neuron is connected to all neurons in the next layer. Two hidden layers are used with eight neurons in the first, and four neurons in the second layer. As activation function, the Rectified Linear Unit (ReLU) function is used for a time efficient training phase.

Another form of ANNs are convolutional neural networks (CNNs), which main application field is image processing, but also the usage for time series analysis is increasing [32]. It consists of a convolution, where a kernel function is used to convolve the input and mostly a pooling layer, where several data points can be pooled in a single data point [31]. The structure is shown in Fig. 4 [33].

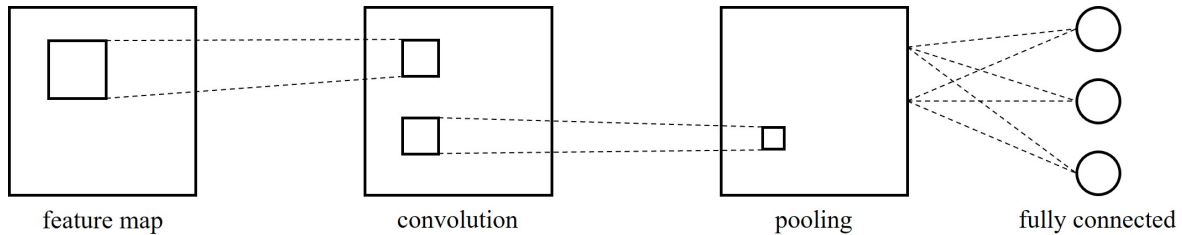


Fig. 4: Traditional structure of a CNN with feature map, convolution layer, pooling layer, and the fully connected layer.

A 1-D CNN is used for the SOC prediction. The convolution filters are moved along the temporal dimension, while analyzing the time-series data. In comparison to the MLP, other processing steps are added to the model, even though the basic structure remains similar. For both types of ANNs, several hyperparameters have to be determined. The hyperparameter space is searched systematically by using a grid search. The parameter combination which minimizes the loss function is selected for further analysis [34].

To evaluate the prediction accuracy, the root mean square error (RMSE) is used for training and test results. Thereby, the target values  $y$  are compared to the predicted values  $y^*$ , see (5) [25].

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - y_i^*)^2} \quad (5)$$

## Results

Different ML models are trained using simulation and augmented data. These models are tested on a real world dataset to determine the prediction accuracy. The results are evaluated by means of the RMSE with the overall aim to reduce the test error. The models are retrained five times, and the mean errors with the according standard deviation are stated. To reduce the dimension of the input data, a SVD is performed. The dimension of the initial data was 80 and is reduced to 14. This dimension was chosen based on the singular values, which are more similar to each other in higher dimensions. The deviation and, consequently, the impact on the transformation is lower with an increasing number of features. A linear regression is used as the benchmark model and is compared to different types of ANNs. The linear model is separated in two voltage intervals for an improved SOC prediction. The mean train error is 4.52 % with a standard deviation of 0.04, while the test error is 3.80 % with a deviation of 0.08. The mismatch between train and test error can be explained by the self defined current patterns used in the battery simulation and the overall number of outliers in the training set. As explained, the data driven model highly depend on the input data. Nevertheless, battery tests are expensive and time consuming. In reality, expert knowledge is necessary to assess if data points should be included or sorted out. However, it could be shown that the outliers have a higher impact in the evaluation than affecting the model training in a negative way. The results are plotted in Fig. 5. For reasons of comprehensibility, the results picture the SOC plotted over the voltage of the current time step. This is included as one feature in the initial feature map before the dimensionality reduction using the SVD, but it is not the only feature as indicated in the figure.

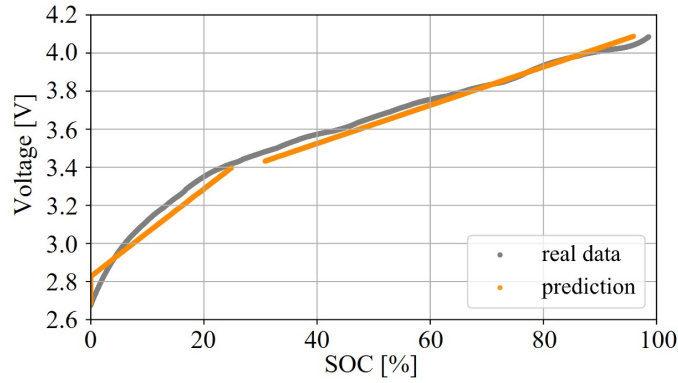


Fig. 5: Test results of the linear regression model separated in two voltage intervals.

Next to the linear regression, a MLP and a CNN are implemented to predict the SOC. The MLP is trained without augmented data, with augmented data five times the initial data, and ten times the initial data. The data is preprocessed using a standard scaler and afterwards a SVD. The training and test errors of the MLP are summarized in Table I.

Table I: Training and test results of the MLP with and without augmented data. The results are mean values of five experiments with the according standard deviation.

	Train	Test
without augmented data	4.40 (1.38) %	2.45 (0.32) %
with augmented data (5x)	4.71 (0.39) %	2.28 (0.32) %
with augmented data (10x)	2.09 (0.09) %	1.83 (0.17) %

Even the model without augmented data has test errors below 3 %. All in all, the test errors can be reduced using augmented data. Especially with ten times the initial data, the RMSE could be decreased to 1.83 % in comparison to a RMSE of 2.45 % without augmented data. Additionally, the model is more robust, which is apparent analyzing the standard deviation of the models. Using more data, the deviation is decreasing and the models became more reliable. To further reduce the prediction error, a CNN is implemented to better approximate the time-series data. The results are shown in Table II.

Table II: Training and test results of the CNN with and without augmented data. The results are mean values of five experiments with the according standard deviation.

	Train	Test
without augmented data	2.70 (0.18) %	2.26 (0.49) %
with augmented data (5x)	2.48 (0.35) %	2.17 (0.31) %
with augmented data (10x)	2.61 (0.46) %	1.78 (0.34) %

An improvement in comparison to the MLP becomes apparent for all types of input data, even though the impact is small. A lowest test error can be identified using the CNN with ten times the initial data with a RMSE of 1.78 %. The standard deviation is slightly higher compared to the MLP, but the trend for lower errors while using more data can be determined. The time frame used for the models is relatively small. A considerable improvement analyzing a larger area of past data is expected for the CNN. The MLP with a comparable simpler architecture fits the needs for this problem. However, the CNN shows more potential for optimization. A comparison of the results of the ANNs is shown in Fig. 6.

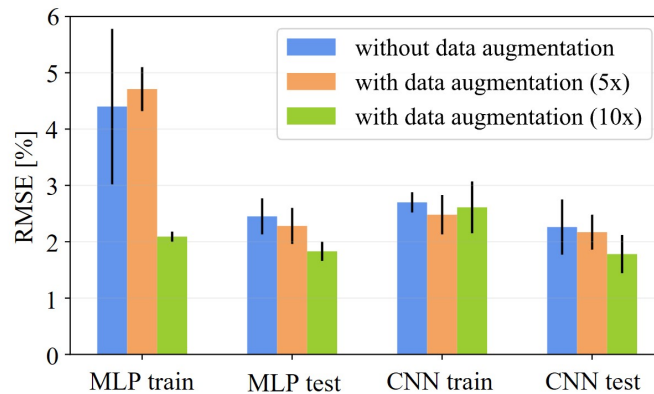


Fig. 6: Results of the ANNs for different input data compared for training and test phase with the corresponding standard deviation.

The MLP without augmented data and with five times the initial data have similar training errors to the linear regression and are not sufficient to predict the SOC. Further, the deviation between training and test error is higher for these models. The MLP with augmented data (ten times) shows a good tradeoff between prediction accuracy and structure complexity compared to the CNN. The overall better prediction ability of the CNN can be identified using the models without augmented data. Both models show the trend of improvement while using a larger data basis. The CNN is more suitable for predicting the SOC of a LIB; however, the optimized MLP shows similar results.

Another considered aspect is the convergence of the models and the impact of data augmentation. The course of the RMSE over the epochs is plotted in Fig. 7.

It is striking that the more data is used, the earlier convergence is reached. While the convergence values are similar, an earlier drop of the model error can be identified using augmented data. A higher improvement can be observed from the change of no augmented data to five times the initial data. The further improvement using ten times the data is also significant. On the one hand, the data augmentation shows potential for optimizing the ML models for SOC prediction, resulting in more robust models and a

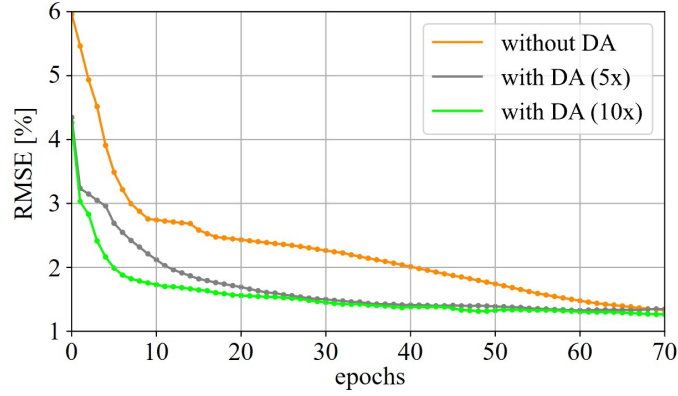


Fig. 7: Comparison of the convergence without augmented data (without DA), with five times the initial data (with DA (5x)), and ten times the initial data (with DA (10x)).

higher prediction accuracy. On the other hand, data augmentation in the field of battery analysis demands expert knowledge and is more complicated than conventional methods. Benefits of data augmentation are the higher robustness of the models and the improved prediction accuracies. The impact is greater for the MLP with the more simple structure, but the trend for model improvement can also be identified using the CNN.

## Conclusion

The move to renewable energy is crucial for environmental protection. LIBs are a key technology to electrify the transport sector. A challenge in operation is the determination of the battery condition. For a safe and efficient usage, it is important to accurately assess the state. Among others, data-driven models show high accuracies predicting the SOC. Disadvantages of these models are the high dependence on their data basis. Because battery tests are time and cost consuming, data augmentation techniques can be used to enrich the data basis. To reduce the experiments to a minimum, simulation and augmented data are used to train different types of ANNs and a linear benchmark model. The data is preprocessed using a standard scaler, and, further, the dimension of the features is reduced using a SVD. All in all, the data augmentation method leads to more reliable models and reduced model errors. The CNN with augmented data reaches a lowest test RMSE of 1.78 % and is therefore slightly favorable in comparison to the MLP with a test error of 1.83 %.

In future work, it is planned to analyze advanced data augmentation methods for a larger timeframe. The optimization potential is expected to be higher for the CNN. Further, it is planned to test the algorithms in an experimental prototype to analyze the applicability in the real world and the transferability for other cell chemistries.

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