A Non-cooperative Game-theoretic Distributed Control Approach for Power Quality Compensators

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Keywords

≪Power quality≫, ≪Distributed Generation≫, ≪Active Filter≫, ≪Grid forming≫, ≪Microgrid≫

Abstract

This paper demonstrates that the Game Theory (GT) can be an effective tool for implementing a distributed control scheme for coordinating the compensation efforts of power quality compensators (*PQCs*) feeding a common unbalanced load. A non-cooperative game is formulated where each *PQCs* minimises its own interests, defined as the power losses that incur each *PQC* in carrying unbalanced power from its connection point in the system to the point of common coupling (*PCC*). By doing this, the whole system will move until it reaches a global equilibrium (the so-called Nash equilibrium). The power losses are calculated based on the conservative power theory (CPT), allowing the implementation of the proposal in the natural *abc* reference frame. A comparison (via simulations) between the proposed non-cooperative distributed scheme and a cooperative distributed approach based on the consensus theory shows that the proposed non-cooperative game compensates the *PCC* with fewer overall losses than the consensus-based cooperative approach, improving the efficiency of the whole compensation system.

Introduction

Power quality compensators (*PQCs*) are widely used in electrical systems to improve the power quality at one or more points of electrical systems. This power quality enhancement can be performed in electrical variables such as powers, voltages and currents. Focusing on the compensation of unbalanced and distorted currents, a typical solution is to place power quality compensators (*PQCs*) in shunt connection at the point where the power quality needs to be improved: named the point of common coupling (*PCC*) in this paper. In this case, *PQCs* will inject unwanted currents at the *PCC*; therefore, the compensation of currents at the *PCC* is fulfilled. This configuration is usual in isolated AC microgrids (MGs), as shown in Fig. 1. In this type of electrical system, usually, there is a grid-forming converter in charge of imposing the voltage and frequency in the MG. In contrast, other converters operate in grid-follower mode, meaning their behaviour is approximately like a current source. Note that power converters allow the integration of the distributed generation units into the MG. In this scenario, grid-follower converters are sized to provide the nominal power based on the potential of power harvesting from the distributed energy sources. However, due to the intermittence of natural resources, it is expected that the nominal VA capacity of power converters will not be fully used for long periods; therefore, this available VA capacity can be used for the grid-following converters to compensate for unbalanced and distorted currents at the *PCC*, making them works as *PQCs*. In this case, and considering the system shown in Fig. 1, grid-follower converters can inject unwanted currents at the *PCC* and, therefore, improve the quality of the currents seen by the grid former converter. These grid-follower converters, operating as *PQCs*, need to be coordinated to determine their respective compensation efforts.

The coordination of a compensation system of *PQCs* using a centralised control approach has been widely used and reported in the literature [1, 2, 3]. In this approach, there is a central controller in charge of receiving all the information from the *PQCs* converters, determining the compensation effort of each one of them, and finally sending this information to activate the *PQC* function of the converters. This approach has some disadvantages, such as susceptibility to single-point failures, the computational capability of the central controller increases with the number of compensating devices, and thereby high-cost control platforms are required. For this reason, the distributed control approach has been getting attention from researchers in recent years. This approach does not require a central controller as the control effort is distributed among local controllers (LCs) placed on the converters. In this case, LCs operate autonomously and cooperatively to obtain global objectives. It has advantages over the centralised approach: better reliability, flexibility, scalability, and plug-and-play operation [4]. This type of control scheme can be classified as a cooperative distributed control approach, as all the LCs work collaboratively, achieving global objectives. Recently in [5, 6], cooperative control schemes based on the consensus theory have been proposed to coordinate the compensation effort of converters placed on MGs, showing promising results. However, a collaborative distributed approach may not be the best solution to calculate the compensation efforts for the converters if aspects such as losses in the distribution lines are considered. Based on this point, this paper proposes a non-cooperative distributed control approach based on forcing the *PQCs* to play a Nash Equilibrium [7] of a game with suitable cost functions. In this game, each *PQC* chooses the proportion of the compensation effort while minimising the costs: one cost related to the power losses that face each *PQC* to compensate the PCC and a suitable penalty if the compensation is not achieved. Unlike the cooperative approach, where all the *PQCs* work collaboratively, converters have their own interests in the proposed non-cooperative distributed approach; therefore, they act to minimise their respective objective functions. By doing that, the global system will reach the so-called Nash equilibrium (NE).

Based on the discussion above, this paper shows that the problem of calculating the compensation efforts of the grid-following converters shown in Fig. 1 can be formulated in terms of the Nash Equilibrium of a game, generating a non-cooperative distributed control approach for coordinating the *PQCs*. This approach yields the compensation efforts for each converter, where the losses in the lines can be considered, improving the efficiency in managing the whole *PQC* system. The performance of the proposed non-cooperative approach will be compared with that obtained using the cooperative method discussed in [5, 6]. Finally, the conservative power theory (CPT) [5] is used to calculate the compensation currents that needs to be injected by the *PQCs* into the *PCC* to achieve the compensation.

Non-Cooperative Distributed Approach for Calculating the Compensation Efforts of *PQC* converters

Focusing on the MG shown in Fig. 1 and using the CPT, the load current i_{load}, can be decomposed as the sum of a balanced current and an unbalanced current (see Fig. 1). In this case, the *PQC* system must inject i_u at the *PCC* to achieve that the current injected by the grid forming converter will only be the balanced load current component i_b (see Fig. 1). In this paper, it is assumed that *PQCs* displayed in Fig. 1 have enough VA power capability to inject any unwanted current into the MG. Based on that, and considering that the switch sw_1 is closed in Fig. 1, yield (1).

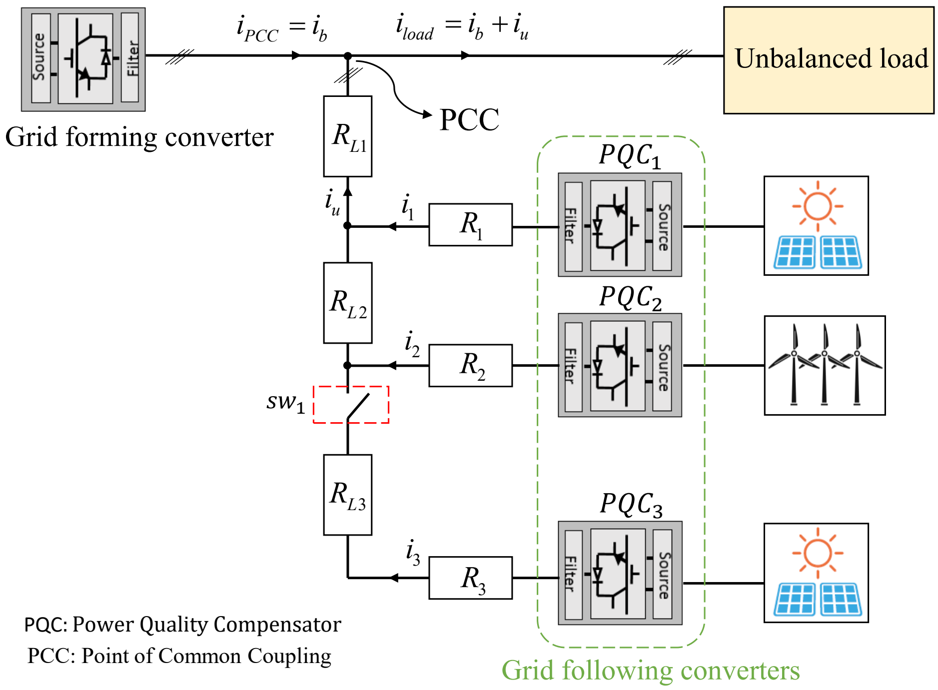

Fig. 1: Isolated three-phase three-wire AC microgrid considered in this work.

$$i_1 + i_2 + i_3 = i_u \cdot (n_1 + n_2 + n_3) = i_u \quad , \quad where \quad \sum_{h=1}^{3} n_h = 1 \tag{1}$$

In this work, the compensation efforts (n_1, n_2 and n_3) of the *PQC* compensation system are calculated in a distributed fashion by the non-cooperative approach proposed in this paper. This is achieved by formulating a non-cooperative game among the *PQCs*. To this end, it is necessary to set the individual cost of each *PQC* of participating or not participating in the game. In this work, the cost associated with a given *PQC* is quantified as the losses incurred by the *PQC* in carrying the compensation power from its place into the MG to the *PCC*. In this sense, the losses are calculated based on the conservative power theory, which provides a framework to calculate unbalanced and distorted currents in the natural *abc* reference frame. For the sake of clarity, this section is subdivided into the following subsections: (i) A brief overview of the Conservative Power Theory (CPT), focusing on its use to calculate losses in the distribution lines, (ii) the introduction of the proposed non-cooperative game considering only two *PQCs*, and finally, (iii) the extension of the proposed non-cooperative distributed control approach considering *N PQCs*. These sections are discussed as follows.

Theoretical Background

According to the CPT [5], the instantaneous vector current (i) is split into, balanced active current (i_b^a), balanced reactive current (i_b^r), unbalanced current (i_u), and void current (i_v). So, the current, i is expressed as:

$$i = i_b^a + i_b^r + i_u + i_v \tag{2}$$

By definition, the collective *RMS* current can be split into:

$$I^2 = I_b^{a^2} + I_b^{r^2} + I_u^2 + I_v^2 \tag{3}$$

Note that each current term is orthogonal to each others. Thus, multiplying the collective *RMS* current and voltage, the apparent power (A) can be decomposed into:

$$A^2 = V^2 I^2 = V^2 I_b^{a^2} + V^2 I_b^{r^2} + V^2 I_u^2 + V^2 I_v^2 \tag{4}$$

where $P = V I_b^a$ is the active power, $Q = V I_b^r$ is the reactive power, $N = V I_u$ is the unbalance power, and $D = V I_v$ is void (distortion) power. More information about the current and power components defined by the CPT can be found in [5]. Apart from P, all power terms characterise a non-ideal aspect of the load behaviour. The global performance index is the power factor:

$$\lambda = \frac{P}{A} = \frac{I_b^a}{I} = \frac{I_b^a}{\sqrt{I_b^{a^2} + I_b^{r^2} + I_u^2 + I_v^2}} \tag{5}$$

Note that, to provide active power (P) at a given load, any deviation of power from a balanced purely resistive load increases line losses. Thus, from (4) and active power (P) definition, the minimum line losses results:

$$P_{min}^{loss} = R_{line} I_{min}^2 = R_{line} I_b^{a^2} \tag{6}$$

On the other hand, considering the load current (3), the total loss is given by:

$$P_{total}^{loss} = R_{line} I_{load}^2 = R_{line} (I_b^{a^2} + I_b^{r^2} + I_u^2 + I_v^2) = P_{min}^{loss} (\frac{1}{\lambda})^2 \tag{7}$$

Finally, we can define the line utilisation factor as the ratio between the total losses and minimum losses:

$$\rho_{line} = \frac{1}{\lambda^2} = \frac{P_{total}^{loss}}{P_{min}^{loss}} \tag{8}$$

The above equation shows how the harmonics, unbalance and reactive power increase the line utilisation factor, enlarging line losses. Moreover, $\rho_{line} = 1$ only in the case of balanced resistive loads (current waveforms proportional to voltage waveforms).

The MG in Fig. 1 shows the case considered in this paper, i.e., where the grid following converters injected only unbalanced currents to the PCC for compensation purposes. In this scenario, the power loss associated with this compensation is given by: $P^{loss} = R_{line} I_u^2$. In the subsequent sections, the operating principle of the proposed non-cooperative distributed strategy for this situation is explained in detail.

Proposed non-cooperative game considering two *PQC* converters

In this section, we analyse the proposed game with two *PQCs* (the switch sw_1 in Fig. 1 is open). By doing this, the main ideas and concepts behind the proposed game can be easily presented and discussed, and then its extension to more complex networks will be more understandable. It is worth remembering that this game aims to determine n_1 and n_2 associated with PQC_1 and PQC_2 respectively, that satisfy (1) (considering $n_3 = 0$), and at the same time, minimise the losses in carrying the compensation power from the *PQCs* to the *PCC*. The formulation of the proposed non-cooperative game is detailed as follows.

PQC_1: This player feeds the *PCC* with a current equal to $i_u \cdot n_1$ where $n_1 \in [0,1]$. In this case, the PQC_1 minimises its cost function (9) that quantifies the losses in carrying $i_u \cdot n_1$ to the *PCC* (see Fig. 1) and an additional cost K_1 if the system is not able to supply all the compensating current (i.e. $n_1+n_2<1$). In (9), I_u is the the collective RMS (norm) value of i_u. From here onwards, we use the notation $\mathbb{1}$ as the indicator function, meaning that $\mathbb{1}_{n_1+n_2<1}$ equals to 1 if $n_1+n_2<1$ and 0 otherwise.

$$\underset{n_1\in[0,1]}{Min} \quad (R_1+R_{L1})\cdot I_u^2\cdot n_1^2 \; +\mathbb{1}_{\{n_1+n_2<1\}}\cdot K_1 \tag{9}$$

Given that the second term involves the decision of the PQC_2, we compute the reaction curve of player PQC_1 given n_2. This reaction curve corresponds to the optimal response of PQC_1 considering that the PQC_2 plays n_2. This reaction curve is computed as:

$$n_1^*(n_2)=\begin{cases}0 & \text{if } K_1<I_u^2\cdot(R_1+R_{L1})(1-n_2)^2\\ 1-n_2 & \text{otherwise.}\end{cases} \tag{10}$$

PQC_2: Similarly to player PQC_1, player PQC_2 minimises its cost function, given by (11). This function quantifies the losses in carrying $i_u\cdot n_2$ ($n_2\in[0,1]$) to the PCC shown in Fig. 1 and the additional cost K_2 if the system is not able to supply all the compensating current (i.e. $n_1+n_2<1$). In (11), I_u corresponds to the collective RMS value of i_u.

$$\underset{n_2\in[0,1]}{Min} \quad (R_2+R_{L2}+R_{L1})\cdot I_u^2\cdot n_2^2 \; +\mathbb{1}_{\{n_1+n_2<1\}}\cdot K_2 \tag{11}$$

Analogously, the reaction curve of PQC_2, considering the PQC_1 plays n_1 is given by (12):

$$n_2^*(n_1)=\begin{cases}0 & \text{if } K_2<I_u^2\cdot(R_2+R_{L2}+R_{L1})(1-n_1)^2\\ 1-n_1 & \text{otherwise.}\end{cases} \tag{12}$$

The Nash Equilibrium (NE) of this game are the points where both reaction curves intersect. Fig. 2 shows the graphical representation of the game between PQC_1 and PQC_2 given by equations (9)-(12). Depending on the parameters of the MG and the cost K_1 and K_2, $PQCs$ may participate or not in this game. Fig. 2 represents the case where there is only one NE: the cost K_1 and K_2 are too high to incentivise the $PQCs$ to inject current to the PCC, so both $PQCs$ do not participate. However, for the proposed game, we manage costs of K to have one or multiple NEs.

In the proposal, we define $\bar{n}_i$ the strategy for player $i=1,2$ that makes the other player to be indifferent between participating or not. Note that $\bar{n}_1=1-\frac{1}{I_u}\sqrt{\frac{K_2}{(R_2+R_{L1}+R_{L2})}}$ is explicitly computed by setting to equality the *if* condition in (12). Following the same procedure, $\bar{n}_2$ is calculated using (10), giving: $\bar{n}_2=1-\frac{1}{I_u}\sqrt{\frac{K_1}{(R_1+R_{L1})}}$.

Note that the game represented in Fig. 2(a) can be managed via the free parameters K_1 and K_2. In this paper, these parameters are set to ensure that the proposed game shown in Fig. 2(a) has an equilibrium point in which both $PQCs$ participate, producing the game illustrated in Fig. 2(b). In that figure, the Nash equilibrium point (n_1^*,n_2^*) is obtained by imposing the conditions: (i) $n_1(\bar{n}_2)=\bar{n}_1$, and (ii) $n_2(\bar{n}_1)=\bar{n}_2$, as shown in Fig. 2(b). By doing this, the following relationship is got: $\sqrt{\frac{K_1}{R_1+R_{L1}}}+\sqrt{\frac{K_2}{(R_2+R_{L2}+R_{L1})}}=I_u$. This equation shows that for a given unbalanced current I_u, a single equilibrium point (for the proposed game) is obtained if K_1 and K_2 are in the surface given by that equation.

It must be highlighted that for the MG studied in this paper (see Fig. 1), it is considered that all the $PQCs$ have the same characteristics and nominal power; therefore, it can be assumed that $K_1=K_2=K^*$.

Using this assumption, the relationship discussed in the paragraph above can be rewritten as follows: $K^{*}=I_u^2\cdot\left(\frac{1}{\sqrt{R_1+R_{L1}}}+\frac{1}{\sqrt{R_2+R_{L2}+R_{L1}}}\right)^{-2}$. Thus, at using this latter equation on equations (9)-(12), it can be assured that the proposed game has a single-equilibrium point in which both *PQCs* participate, as shown in Fig. 2(b).

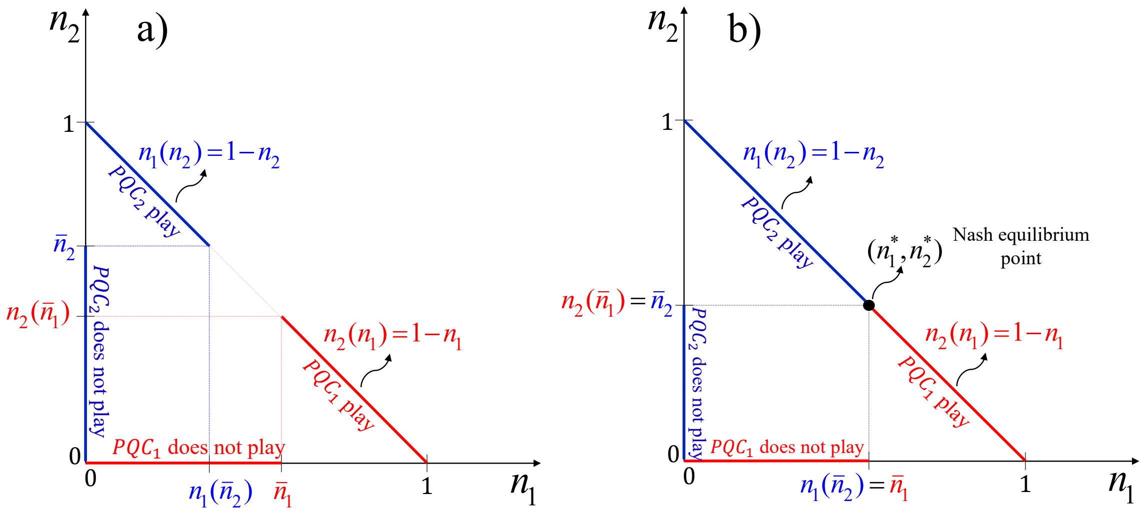

Fig. 2: Reaction curves a) case $K<K^{*}$, b) case $K=K^{*}$

Generalisation of the proposed non-cooperative game

Let us consider that the MG displayed in Fig. 1 has "N" *PQCs* converters that need to be coordinated to supply the compensation current I_u at the *PCC*. In this scenario, the proposed non-cooperative game to coordinate these *PQCs* is as follows: (i) the objective function for the ith *PQC* is given by (13), (ii) the reaction curve of PQC_i is shown in (14), (iii) the critical point until the *PQC* play the game $\bar{n}_i$ is given by (15). Note that these equations consider $K_i=K\,(\forall i=1,\ldots,N)$, meaning that all the *PQCs* have the same characteristics. In this sense, the parameter K that ensure that the game has only an equilibrium point (Nash equilibrium point) is shown in (16).

$$\underset{n_i\in[0,1]}{Min}\quad I_u^2\cdot n_i^2\cdot\left(R_i+\sum_{j=1,j\leq i}^{N}R_{Lj}\right)+\mathbb{1}_{\left\{\sum_{j=1}^{N}n_j<1\right\}}\cdot K_i \tag{13}$$

$$n_i^*(n_{j\neq i})=\begin{cases}0 & \text{if } K_i<I_u^2\cdot\left(R_i+\sum\limits_{j=1,j\leq i}^{N}R_{Lj}\right)\cdot\left(1-\sum\limits_{j=1,j\neq i}^{N}n_j\right)^2\\ 1-\sum\limits_{j=1,j\neq i}^{N}n_j & \text{otherwise.}\end{cases} \tag{14}$$

$$\bar{n}_i=\frac{1}{N-1}+\frac{N-2}{N-1}\cdot\frac{\sqrt{K}}{I_u}\cdot\left(R_i+\sum_{j=1,j\leq i}^{N}R_{Lj}\right)^{-0.5}-\frac{1}{N-1}\cdot\frac{\sqrt{K}}{I_u}\cdot\sum_{h=2,h\neq i}^{N}\left(R_h+\sum_{j=1,j\leq h}^{N}R_{Lj}\right)^{-0.5} \tag{15}$$

$$K=I_u^2\cdot\left(\sum_{j=1}^{N}\frac{1}{\sqrt{R_{eq\text{-}j}}}\right)^{-2},\quad\text{where}\quad R_{eq\text{-}j}=R_j+\sum_{h=1,h\leq j}^{N}R_{Lh} \tag{16}$$

Table I: Compensation effort given by the distributed cooperative method [5, 6] (second column), and the proposed distributed non-cooperative approach (third column).

Compensation effort	Cooperative approach [5, 6]	Proposed Non-cooperative approach
n_1	1/3	0.40
n_2	1/3	0.32
n_3	1/3	0.28

Simulation Results

In this section, the proposed non-cooperative game to calculate the compensation efforts for the *PQCs* is validated through simulation work. To this end, the system shown in Fig. 1 is simulated, using PLECS software, considering three *PQCs*, i.e., the switch sw_1 in Fig. 1 is closed during the validation process. The line resistances illustrated in that figure are equal to 1Ω, and the grid forming converter generates a three-phase voltage of 220Vrms and 50Hz. The unbalanced load illustrated in Fig. 1 corresponds to an unbalanced resistive load with the following values: $R_a = 22\Omega$, $R_b = 11\Omega$, and $R_c = 5\Omega$.

Note that, for the case studied in this paper, it is assumed that the three *PQCs* have the same characteristics. Then, if they are managed using the consensus-based distributed control schemes proposed in [5, 6], their compensation efforts will be equal to each other, as shown in the second column of Table I. This result is because all the *PQCs* have the same VA capacity. Therefore, under the cooperative approach [5, 6], all *PQCs* should equally share the compensation effort. In contrast, if the proposed non-cooperative method is used for controlling the *PQCs*, it is found that their compensation efforts are different, as shown in the third column of Table I. In this case, each *PQC* looks to minimise its own interest (losses in carrying compensation power from its connection point to the PCC), reaching the Nash equilibrium, as shown in the third column of Table I.

Fig. 3(a) shows the unbalanced currents that inject each *PQC* to achieve the compensation of i_{PCC} (see Fig. 1), for the case where the compensation effort is calculated by a cooperative approach (second column in Table I). On the other hand, Fig. 3(b) shows the same information but for the case where the compensation efforts are determined by the proposed non-cooperative approach (third column in Table I). From that figure, it can be appreciated that the currents injected by the *PQCs* when they are managed by the cooperative approach [5, 6] are the same (per phase), as they have the same VA power capacity. In contrast, the *PQC* system feeds the PCC with different currents when the proposed non-cooperative approach drives them. In this case, the PQC_1 inject more compensation current to the PCC than the other *PQCs*, as shown in Fig. 3(b). And the PQC_3 inject a lesser compensation current than the rest of the *PQCs*. This latter trend is because the cost, in terms of power losses, is higher for PQC_3 than for others. In addition, the cost for the PQC_1 is smaller than the others; therefore, it feeds the PCC with the highest unbalanced current. The results discussed above show that the proposed distributed non-cooperative approach, differently from the distributed cooperative one, can include criteria of power losses for calculating the compensation efforts of the *PQC* system, which is an advantage of the proposal. This allows achieving the compensation process with an equal loss distribution among the *PQCs*, as shown in Fig. 4(b). In that figure, a comparison of the power losses distribution of the *PQCs* is illustrated for both the cooperative and non-cooperative approaches during the compensation process. While the proposed non-cooperative method compensates the PCC with a similar losses distribution for the *PQCs* (see Fig. 4(b)), the cooperative approach compensates the PCC with an unequal power losses distribution for the *PQCs* (Fig. 4(a)). To sum up, comparing both results shown in Fig. 4, it is concluded that the proposed non-cooperative approach achieves the coordination of the *PQCs* with a better losses distribution than the cooperative approach reported in [5, 6].

Fig. 5 shows the current at the PCC (i_{PCC}) when the *PQCs* are enabled. Note that this behaviour is the same no matter what of the two strategies shown in Fig. 4 is used as in both cases, the compensation of i_{PCC} is fully achieved: The main difference between them is how the *PQCs* are coordinated. In this sense, the non-cooperative allocation (calculation of the compensation efforts n_1, n_2 and n_3) has one main advantage: it is not coordinated by a central controller, so under that allocation, none of the PQC have the incentive of deviating by themselves without any further regulation.

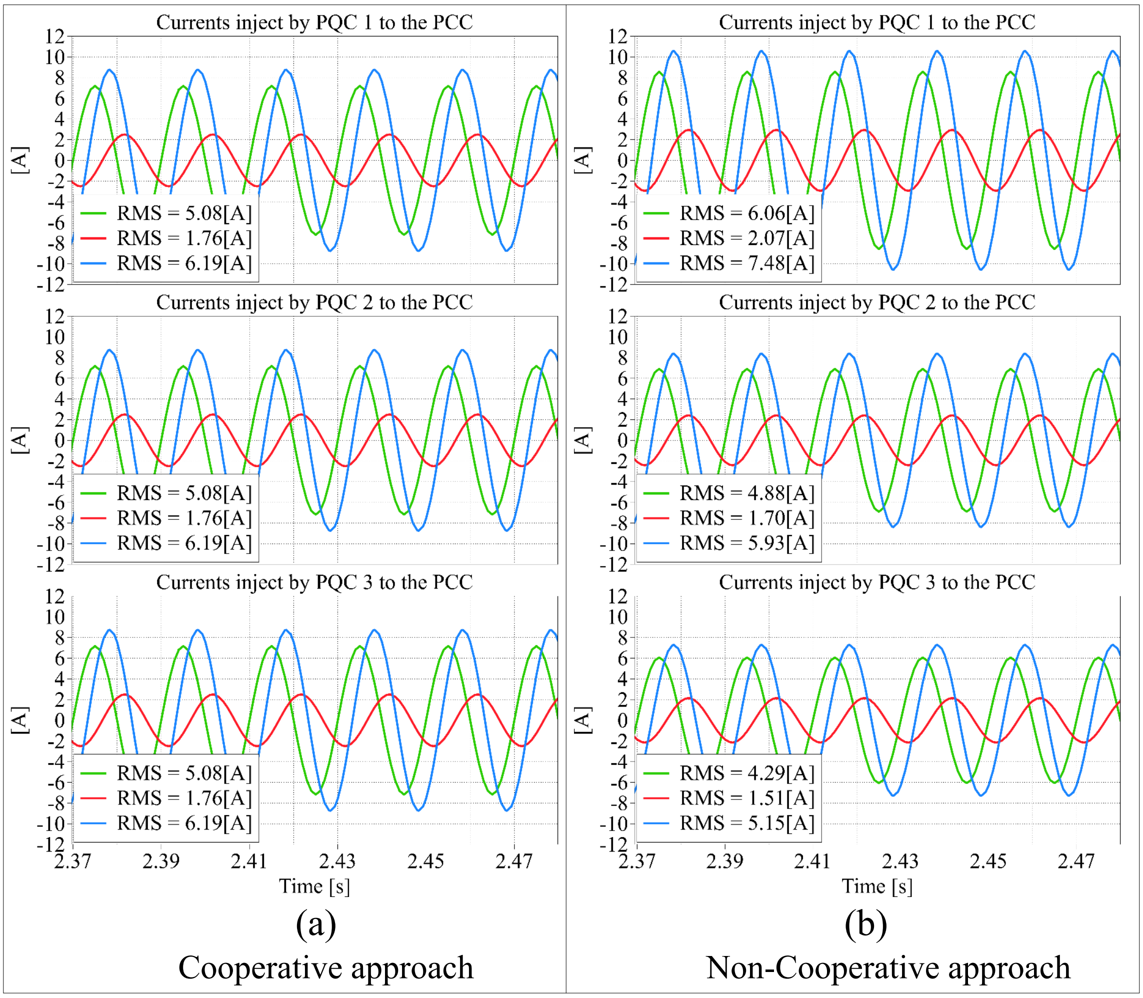

Fig. 3: Currents at the PCC before and after activating the *PQC* system.

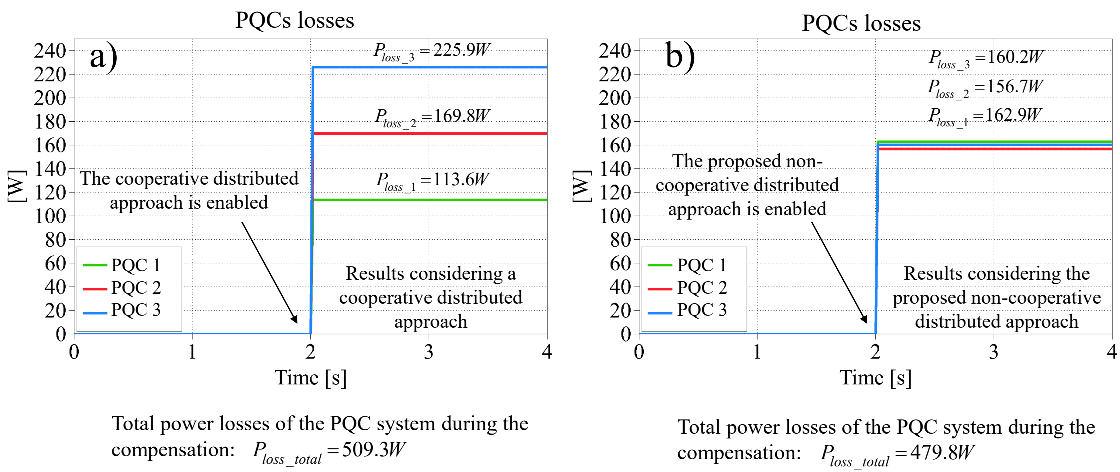

Fig. 4: (a) Results obtained when the cooperative control scheme proposed in [5, 6] is used to coordinate the *PQC* converters, (b) Results obtained when the proposed non-cooperative control is used.

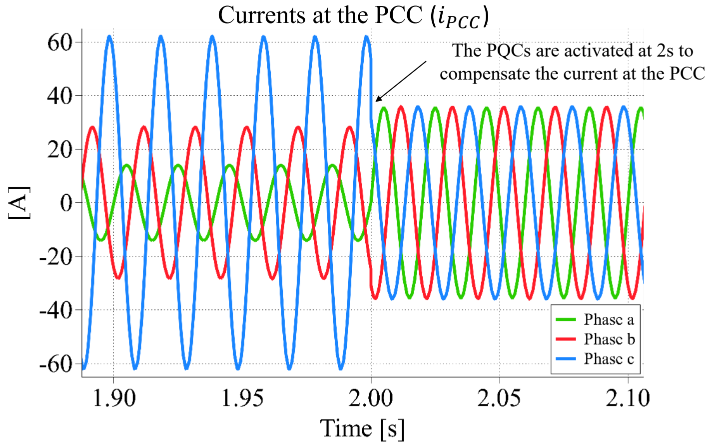

Fig. 5: Currents at the PCC before and after activating the *PQC* system.

Conclusions

This paper demonstrated that the compensation effort of a set of power quality compensators (*PQCs*) can be calculated using the proposed distributed non-cooperative method based on the game theory. Also, a comparison between the non-cooperative approach and the distributed cooperative approach reported in [5, 6] (based on the consensus theory) was performed. From that comparison, it was found that different from the collaborative method; the non-cooperative method can calculate the compensation effort for the *PQCs* considering criteria such as the line power losses, which is an advantage of the proposal and could bring economic benefits for the entire *PQC* system. In future work, we aim to focus on two extensions of this setting. First, to study the case where each PQC has a limited capacity b_i, meaning that each strategy satisfies $n_i \leq b_i$ under the assumption that $\sum_i b_i \geq 1$. Note that, if $b_i \leq \bar{n}_i$ in Equation (15) the analysis above is still valid. Secondly, extending this approach to more complex networks where multiple PCC are considered.

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