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A Two-Degree-of-Freedom Current Loop Parameter Tuning Method Based on Bandwidth and Phase Margin

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Abstract--**The traditional current loop tuning method is usually completed based on pole-zero cancellation (PZC) in permanent magnet synchronous motor (PMSM) system. However, it is difficult to take both bandwidth and margin optimization into consideration. In view of the limitation of PZC method, this paper proposes a two-degree-of-freedom current loop tuning method based on bandwidth and phase margin(PM). The design criterion of proposed self-tuning algorithm is making the two indexes meet the preset value and maximizing the gain margin(GM) at the same time. The effectiveness of the proposed scheme are verified by simulation experimental comparison.**

Index Terms--bandwidth, current loop, phase margin, two-degree-of-freedom

I. Introduction

In recent years, with the rapid development of industrial automation, the PMSM AC servo system with the advantages of high control accuracy and tracking characteristics [1-2] has gradually become the focus of attention, and is widely used in many industrial occasions with high position control accuracy requirements. In the research of servo system control, the self-tuning [3-4] of controller parameters is always a hot topic that cannot be avoided. It can avoid a large number of complicated manual debugging, and has better robustness to different systems. It has been widely favored by people, and has become the focus and difficulty of AC servo system research at this stage.

In the classical control architecture of PMSM servo system, the current loop [5] is the innermost loop of its three-loop, so its control effect directly affects the overall performance of the servo system. Since the internal electrical parameters of the current loop are not easily affected by external factors, the tuning method based on the internal model [6-7] is more suitable for the controller design of the current loop. The research content of this paper is also based on this.

Common current loop model-based tuning methods mainly include pole placement and pole-zero cancellation [8-9]. The common point of them is to establish an accurate mathematical model of the current loop system through certain equivalence and simplification based on the electrical parameters obtained in advance, and finally design the parameters of the controller based on the model. The classical PZC method uses the zero point of the controller to offset the electric time constant pole of the plant of the current loop. Therefore, the design only needs to adjust the controller gain to complete the tuning process, which has the advantages of simple principle and easy implementation. However, because the integral coefficient of the controller is determined by the electrical time constant and cannot be adjusted artificially, it is essentially a one-dimensional optimization controller design method. The conventional PZC method determines the controller gain according to the bandwidth optimization method. This method is more suitable for the current loop system simplified into a first-order link. However, with the development of servo system research, the requirement for the accuracy of the equivalent mathematical model becomes higher and higher. It is difficult to meet the accuracy requirements only by equivalent to the first-order link. In the design process of the controller, the influence of delay, coupling [10] and other factors need to be considered. In this case, the two-degree-of-freedom controller design method has a wider scope of application, which can obtain the optimal controller parameters under the premise of satisfying the two-dimensional preset index.

In this paper, a two-degree-of-freedom current loop controller design method based on PM and bandwidth index is proposed. At the same time, the delay of the current loop system and the d and q axis coupling factors are taken into account. Finally, the two-dimensional optimization design of the controller parameters is carried out through the preset PM and bandwidth [11] requirements. The purpose is to maximize the GM of the system while satisfying the preset index, so as to take into account the dynamic performance and stability performance of the system. The latter part of the paper compares the time domain performance and frequency domain performance of the proposed method with the classical PZC method through theoretical analysis and simulation experiments, and verifies the feasibility and practicability of the proposed method. Finally, considering the influence of inaccurate electrical parameters on the tuning process, the parameter robustness of this two-degree-of-freedom tuning method is also verified.

II. Current Loop Model of PMSM

The common closed-loop system diagram of PMSM current control is shown in Fig. 1. In the control system in Fig. 1, current values i_d and i_q in the synchronous rotating coordinate system are selected as feedback quantities for current control.

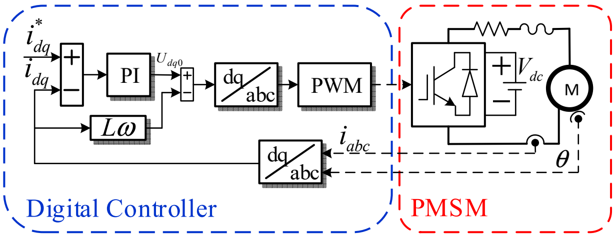

Fig. 1. Block diagram of the current control closed-loop.

In the synchronous rotating coordinate system, due to the role of Park transformation, a voltage coupling term will be inevitably introduced to PMSM. The stator voltage equation is:

$$\begin{cases} U_d = Ri_d + \dfrac{d}{dt}\psi_d - \omega_r \psi_q \\ U_q = Ri_q + \dfrac{d}{dt}\psi_q + \omega_r \psi_d \end{cases} \tag{1}$$

Where U_d and U_q are d and q axis components of stator voltage respectively; i_d and i_q are d and q axis components of stator current ; ψ_d and ψ_q are d and q axis components of stator flux linkage; ω_r is the angular velocity of the rotating magnetic field. The stator flux equation is:

$$\begin{cases} \psi_d = L_d i_d + \psi_f \\ \psi_q = L_q i_q \end{cases} \tag{2}$$

Where L_d and L_q are d and q axis components of inductance respectively; ψ_f is the permanent magnet flux linkage. By substituting equation (2) into equation (1), the stator voltage equation is:

$$\begin{cases} U_d = Ri_d + L_d \dfrac{d}{dt} i_d - \omega_r L_q i_q \\ U_q = Ri_q + L_q \dfrac{d}{dt} i_q + \omega_r (L_d i_d + \psi_f) \end{cases} \tag{3}$$

In order to completely decouple the d and q axis of the current loop, a common method is to introduce a voltage compensation term which compensates the coupling part through the current feedback value. The compensated d and q axis voltage U_{d0} and U_{q0} are only related to their own axis component and no longer contain the coupling term. The expression is:

$$\begin{cases} U_{d0} = U_d + \omega_r L_q i_q = Ri_d + L_d \dfrac{d}{dt} i_d \\ U_{q0} = U_q - \omega_r (L_d i_d + \psi_f) = Ri_q + L_q \dfrac{d}{dt} i_q \end{cases} \tag{4}$$

The basic structure diagram of current loop decoupling is shown in Fig. 2.

The Laplace transform of equation (4) is obtained:

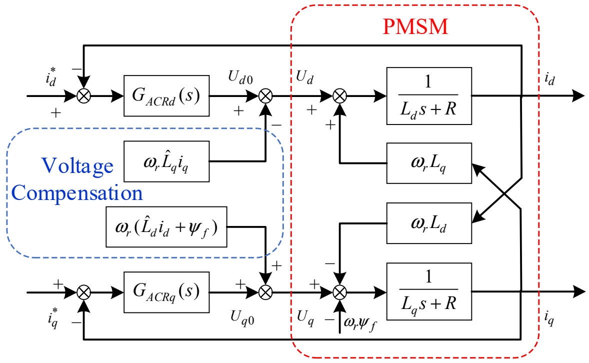

Fig. 2. Diagram of the current loop decoupling based on voltage compensation.

$$\begin{cases} U_{d0}(s) = Ri_d(s) + L_d i_d(s)s \\ U_{q0}(s) = Ri_q(s) + L_q i_q(s)s \end{cases} \tag{5}$$

For surface-mounted permanent magnet synchronous motor (SPMSM), the values of the d and q axis components of the inductance are the same, that is, L_d=L_q=L, so the equivalent transfer function of the d and q axis of the current loop can be obtained:

$$\begin{cases} G_{pd}(s) = \dfrac{i_d(s)}{U_{d0}(s)} = \dfrac{1}{Ls+R} \\ G_{pq}(s) = \dfrac{i_q(s)}{U_{q0}(s)} = \dfrac{1}{Ls+R} \end{cases} \tag{6}$$

Due to the influence of discretization and pulse width modulation, the delay time T_d is inevitably artificially introduced into the system. When corresponding to the frequency domain through Laplace transform, the e^{-sT_d} term will be introduced. The final equivalent transfer function is as follows:

$$\begin{cases} G_{pd}(s) = \dfrac{1}{Ls+R} e^{-sT_d} \\ G_{pq}(s) = \dfrac{1}{Ls+R} e^{-sT_d} \end{cases} \tag{7}$$

According to the classical single acquisition pattern, the general delay time T_d=1.5/f_s, and the later theoretical analysis and simulation experiments are also based on this.

III. Classical Pole-zero Cancellation Method

It can be seen from equation (7) that the current loop model of permanent magnet synchronous motor is usually equivalent to a low-pass link plus a delay link:

$$G_p(s) = \frac{1}{R(1+\tau_e s)} e^{-sT_d} \tag{8}$$

In which $\tau_e = L/R$. The current loop controller is usually designed in PI form:

$$G_c(s) = k_p(1+\frac{k_i}{s}) = k_p \frac{1+\tau_i s}{\tau_i s} \tag{9}$$

In which $\tau_i = 1/k_i$. If the tuning method based on pole-zero cancellation is adopted, when $\tau_i = \tau_e$, the open-loop transfer function of the plant is:

$$G_{co}(s) = G_c(s)G_p(s) = \frac{k_p}{\tau_e R}\frac{1}{s}e^{-sT_d} \tag{10}$$

In this way, the parameter tuning problem is simplified to the optimal bandwidth search problem, which simplifies the tuning problem. But the optimal characteristics of one-dimensional parameters (bandwidth) does not mean that it is the overall optimal situation.

According to (10), the PM γ can be obtained:

$$\begin{cases} \gamma = \pi/2 - T_d\omega_c \\ 1 = \dfrac{k_p}{L\omega_c} \end{cases} \tag{11}$$

And the GM h is:

$$\begin{cases} 10^{-\frac{h}{20}} = \dfrac{k_p}{L}\dfrac{1}{\omega_g} \\ 0 = \pi/2 - T_d\omega_g \end{cases} \tag{12}$$

Combine equations (11) and (12):

$$\begin{cases} \omega_c = \dfrac{1}{T_d}(\dfrac{\pi}{2} - \gamma) \\ \gamma = \dfrac{\pi}{2} - \dfrac{\pi}{2}10^{-\frac{h}{20}} \end{cases} \tag{13}$$

As can be seen from (13), when PZC is used for current loop tuning, the phase margin γ, gain margin h and cut-off frequency ω_c correspond to each other one by one, and the corresponding relationship among the three is shown in Fig. 3. The problem brought by this is that when any two indicators are defined, the system after tuning always cannot accurately reach the preset index value, thus causing unnecessary bandwidth reduction or introducing the risk of instability. The current control system with one degree of freedom cannot make its performance reach two-dimensional optimal state while following the preset value.

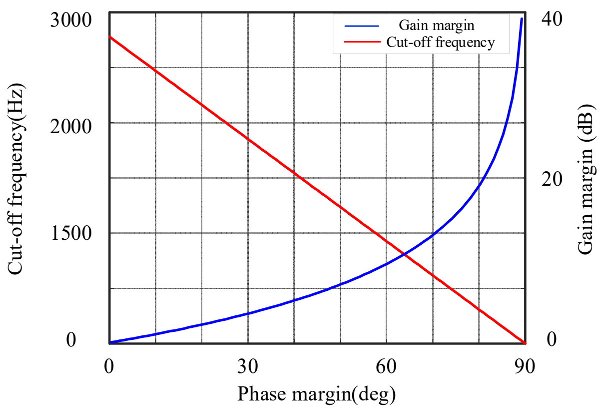

Fig. 3. Relationship curve of γ, h and ω_c in pole-zero cancellation.

IV. Two-Degree-of-Freedom Parameter Design Method

The design of controller is a result of comprehensive consideration. In order to design k_p and k_i at the same time, we need to consider the influence of PM, GM and bandwidth. The model current loop tuning method based on phase margin γ and bandwidth ω_b will be introduced below.

In general, when $\tau_i \neq \tau_e$, according to (8) and (9), it can be concluded that the open-loop transfer function of the plant is:

$$G_{co}(s) = G_c(s)G_p(s) = \frac{k_p k_i}{Rs}(\frac{1+\tau_i s}{1+\tau_e s})e^{-sT_d} \tag{14}$$

According to (14), the open-loop amplitude and open-loop phase can be expressed as follows:

$$A(\omega) = \frac{k_p k_i}{R}\frac{1}{\omega}\sqrt{\frac{\tau_i^2\omega^2+1}{\tau_e^2\omega^2+1}} \tag{15}$$

$$\theta(\omega) = \arctan(\tau_i\omega) - \arctan(\tau_e\omega) - T_d\omega - \pi/2 \tag{16}$$

According to (15) and (16), the PM γ can also be obtained:

$$\gamma = \pi/2 + \arctan(\tau_i\omega_c) - \arctan(\tau_e\omega_c) - T_d\omega_c \tag{17}$$

$$1 = \frac{k_p k_i}{R}\frac{1}{\omega_c}\sqrt{\frac{\tau_i^2\omega_c^2+1}{\tau_e^2\omega_c^2+1}} \tag{18}$$

Where ω_c is the open-loop cut-off frequency. Because this two-degree-of-freedom tuning method does not set the open-loop transfer function as an integral delay link like PZC, the correspondence between the cut-off frequency ω_c and bandwidth ω_b will become complicated. However, it is still a functional correspondence: when the PM γ and bandwidth ω_b are determined, the open-loop cut-off frequency ω_c can be determined uniquely. The relationship between the closed-loop amplitude and bandwidth of the current loop is:

$$M(\omega_b) = \left[1 + \frac{1}{A^2(\omega_b)} + \frac{2\cos\theta(\omega_b)}{A(\omega_b)}\right]^{-\frac{1}{2}} = 10^{-\frac{3}{20}} \tag{19}$$

In (19), $A(\omega_b)$ and $\theta(\omega_b)$ are the open-loop amplitude and open-loop phase at the bandwidth ω_b respectively:

$$A(\omega_b) = \frac{k_p k_i}{R}\frac{1}{\omega_b}\sqrt{\frac{\tau_i^2\omega_b^2+1}{\tau_e^2\omega_b^2+1}} \tag{20}$$

$$\theta(\omega_b) = \arctan(\tau_i\omega_b) - \arctan(\tau_e\omega_b) - T_d\omega_b - \pi/2 \tag{21}$$

Equation (19) is arranged as:

$$M' A^2(\omega_b) - 2A(\omega_b)\cos\theta(\omega_b) - 1 = 0 \tag{22}$$

where

$$M' = \frac{1 - M^2(\omega_b)}{M^2(\omega_b)} = \frac{1 - 0.708^2}{0.708^2} = 0.995 \tag{23}$$

For the correspondence between the open-loop cut-off frequency, bandwidth and phase margin, many empirical approximate formulas have been proposed in the classical control theory. Some of them are simple and widely used, such as $\omega_b = 1.6\omega_c$, but the error of this approximate formula is relatively large in practical application. Therefore, this paper adopts the method of multi-point fitting to obtain the corresponding relation among the three, and eliminates the fitting error through several iterations. Finally, the convergent k_p and k_i values are obtained.

The data that $\omega_b \in [500, 2000], \omega_c \in [300, 1000], \gamma \in [25, 50]$ were collected and linearly fitted as shown in Fig. 4. The fitting results were as follows:

$$\omega_b = -254.4 + 2.401\gamma + 2.052\omega_c \tag{24}$$

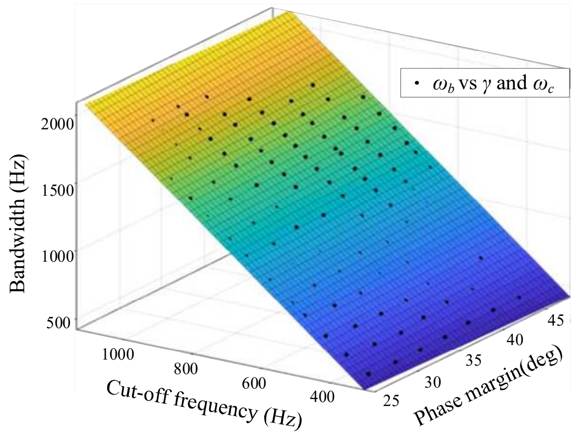

Fig. 4. Fitting results of ω_b, ω_c and γ.

The root mean square error (RMSE) of the fitting is 16.76Hz, which is completely acceptable for the bandwidth of about 2kHz. In this tuning method, the PM and bandwidth are taken as inputs, and the cut-off frequency can be obtained from (24), k_i can also be obtained by substituting ω_c into (17) :

$$k_i = \frac{1}{\tau_i} = \frac{\omega_c}{\tan(\gamma + \omega_c T_d + \tan^{-1}(\omega_c \tau_e) - \pi/2)} \tag{25}$$

Substituting the obtained k_i ($1/\tau_i$) into (21), we can get:

$$\theta = \arctan(\tau_i \omega_b) - \arctan(\tau_e \omega_b) - T_d \omega_b - \frac{\pi}{2} \tag{26}$$

Substituting the obtained θ into (22), we can get:

$$A(\omega_b) = \frac{2\cos\theta + 2\sqrt{\cos^2\theta + M'}}{2M'} \tag{27}$$

And then we can figure out k_p from equation (20):

$$k_p = A(\omega_b) / \left(\frac{k_i}{R}\frac{1}{\omega_b}\sqrt{\frac{\tau_i^2\omega_b^2 + 1}{\tau_e^2\omega_b^2 + 1}}\right) \tag{28}$$

Due to the existence of fitting error, the k_p and k_i values obtained at this time are not exactly optimal solutions. They need to be introduced into the iterative process and the exact convergence solution can be obtained finally.

Substitute the initial values k_p and k_i back to equation (18) to get a new open-loop cut-off frequency ω_c':

$$\omega_c' = \sqrt{\frac{(K^2\tau_i^2 - 1) + \sqrt{(K^2\tau_i^2 - 1)^2 + 4\tau_e^2 K^2}}{2\tau_e^2}} \tag{29}$$

where $K = k_p k_i / R$.

The new cut-off frequency ω_c' is closer to the real one than previous ω_c obtained by (24). A newer and more precise k_p and k_i can be obtained if we replace ω_c with the newer ω_c' and repeat the previous calculation. So we iterated it until finally we got the convergent controller parameter value. The flow diagram of the iterative program is shown in Fig. 5.

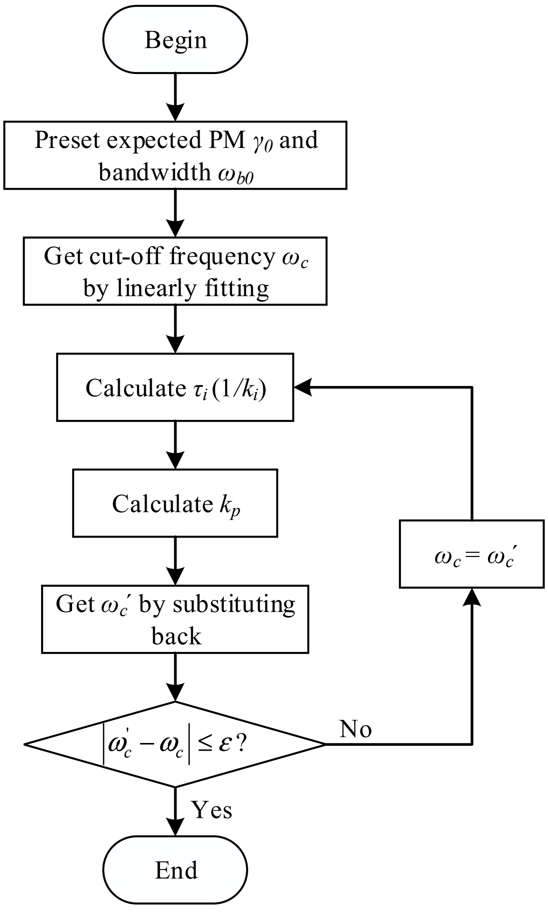

Fig. 5. Flow diagram of the iterative program.

V. Simulation Result and Analysis

The block diagram of the simulation platform is shown in Fig. 1. Parameters of the PMSM set in the simulation are shown in Table 1, and the simulation frequency is set as 10kHz.

A. Comparison of tuning results between the proposed method and the PZC method

The preset bandwidth index is 2000Hz and the PM index is 50deg. Taking this as an example, the comparison between the proposed method and the traditional PZC method is observed. The initial index was input into the iterative program to obtain controller parameters with two-degree-of-freedom tuning. The PM and GM obtained by two-degree-of-freedom tuning were used as the basis to design current-loop controller with PZC method respectively. The open-loop and closed-loop bode diagrams of the three tuned systems were compared, as shown in Fig. 6 and Fig. 7. Table 2 shows the controller parameters and frequency domain performances after tuning. It can be seen that the classical PZC method cannot meet the requirements of bandwidth and PM at the same time. The method proposed in this paper can maximize the GM while both of them are satisfied.

TABLE I PMSM PARAMETERS

Parameter	Value
Phase resistance R	0.98Ω
Phase inductance L	1.11mH
Motor power P_N	750W
Poles p	4
Flux linkage ψ_f	0.056Wb
Bus voltage U_{DC}	72V
Nominal current I_N	7.07A
Nominal torque T_N	2.39N·m
Nominal speed n_0	3000r/min

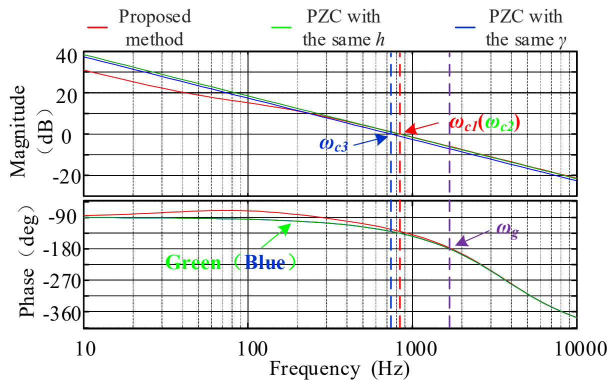

Fig. 6. Open loop Bode diagram of the proposed method and two PZC methods.

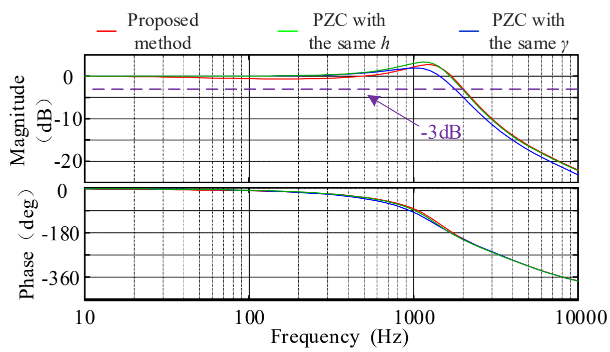

Fig. 7. Closed loop Bode diagram of the proposed method and two PZC methods.

The controller parameters obtained under the three conditions were respectively brought into the simulation system. In the simulation, the reference values of d axis components are set to 0, and i_{qref} uses square wave excitation to observe its time-domain step response. The square wave frequency was set as 50Hz and the amplitude was nominal current I_n. The step response comparison results between the proposed method and two PZC methods are shown in Fig. 8. It can be seen that compared with the other two methods, the overshoot and adjustment time of proposed method in this paper are better.

TABLE II CONTROLLER PARAMETERS AND PERFORMANCES

Method	Proposed method	PZC based on PM	PZC based on GM
Controller parameter	K_p=5.949(V/A) K_i=57.78 (Hz)	K_p=5.167(V/A) K_i=140.52 (Hz)	K_p=5.826(V/A) K_i=140.52(Hz)
Bandwidth	2000Hz	1780Hz	1970Hz
PM	50deg	50deg	45deg
GM	6	7	6

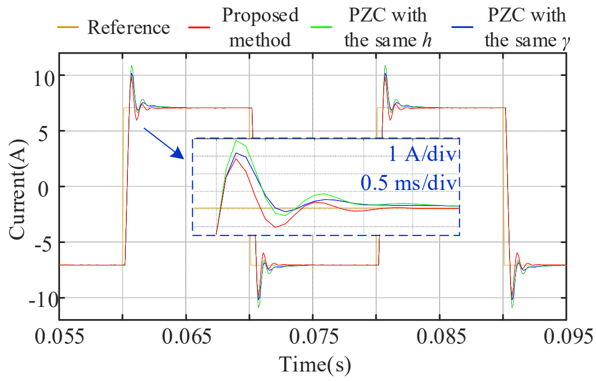

Fig. 8. Comparison of step response of I_q between the proposed method and two PZC methods.

B. Robustness verification of the proposed method to electrical parameters

In most industrial automation occasions, the electrical parameters of PMSM are usually not known and need to rely on a certain parameter identification method. However, the error introduced by the identification process will bring a considerable test to the parameter tuning, so this paper verifies the robustness of the proposed method to the electrical parameters.

In the common PMSM current loop parameter identification method, the identification error of resistance and inductance can usually be guaranteed within 10%. Therefore, ΔR=10%R, ΔL=10%L were taken for simulation experiments and the rest of the simulation parameters remained unchanged. The step response of I_q in the case of inaccurate resistance and inductance was shown in Fig. 9 and Fig. 10 respectively. It can be seen that in the case of inaccurate electrical parameters, the tracking performance of the current loop system can be kept within a certain good range.

VI. CONCLUSIONS

In this paper, a two-degree-of-freedom current loop tuning method based on PM and bandwidth index is proposed. This method introduces the preset PM and bandwidth index into the iterative calculation program to obtain the final controller parameters. Its essential difference from the classical pole-zero cancellation method is that it can maximize the GM while satisfying the PM and bandwidth at the same time, that is, two-dimensional optimal design can be performed. In this paper, the time-domain step response and frequency-domain bode diagram of the two methods are compared and analyzed. The time-domain and frequency-domain performance of the proposed method is better than that of the pole-zero cancellation method. This method has high robustness to the inaccuracy of electrical parameters while taking into account the stability and dynamic performance, and has been successfully verified on the simulation platform.

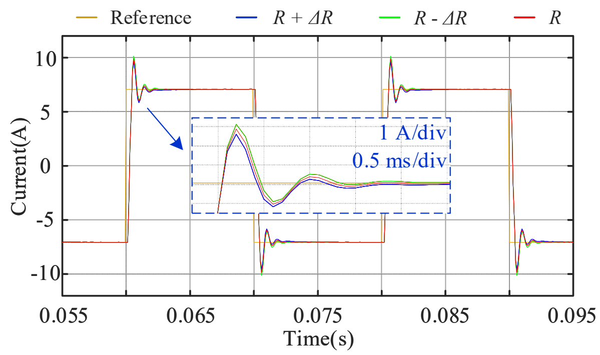

Fig. 9. Step response of I_q in case of inaccurate resistance.

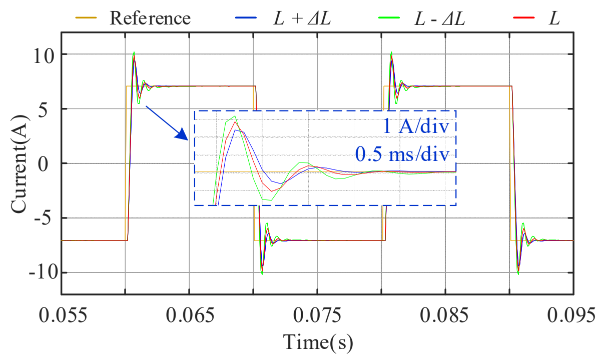

Fig. 10. Step response of I_q in case of inaccurate inductance.

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