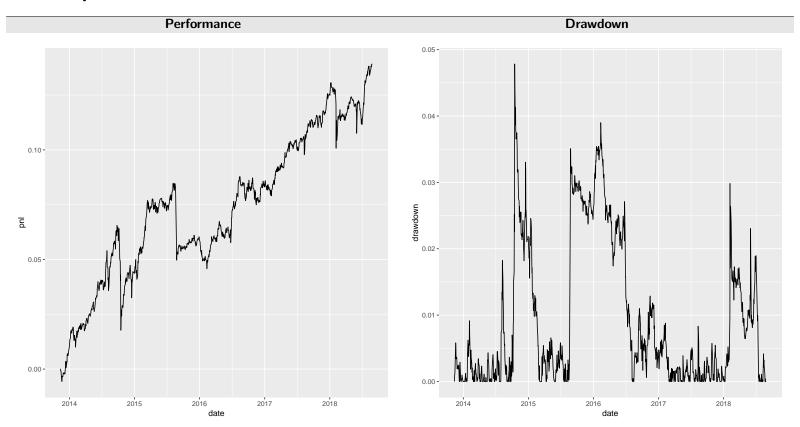
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1 DOF performance



- We wish to investigate the cause of infrequent, relatively large drawdowns.
- In addition, we are looking for a simple, intuitive proxy that can be used to understand DOF performance.
- Moreover, we investigate 3 different mitigation strategies: sizing, stop-loss and a positive tail-risk overlay.

2 Conclusions: Stop-loss vs return quality

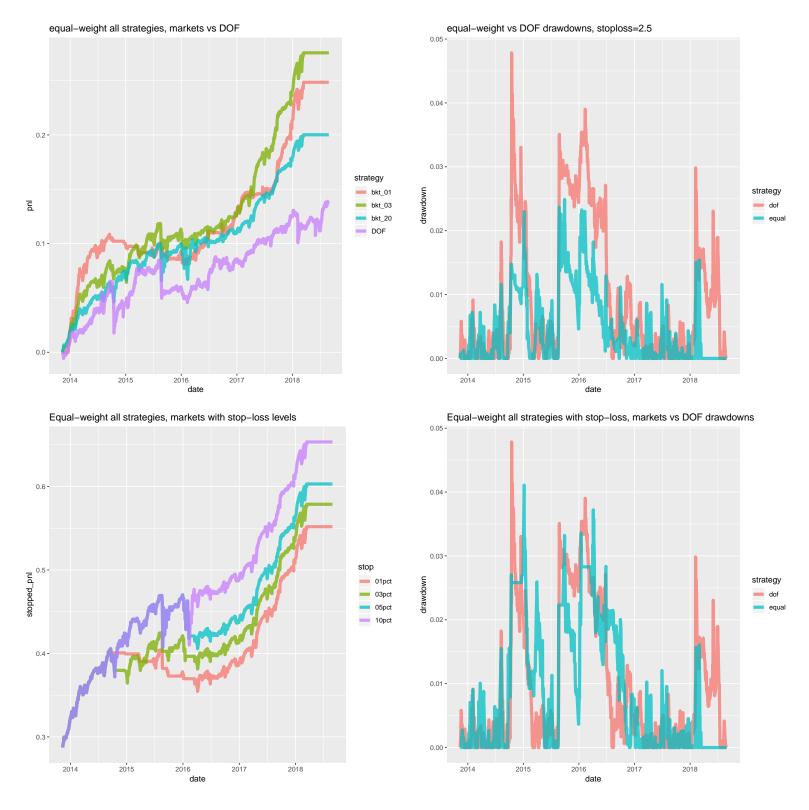
"Small positions with wide stops" are better than "Large positions with tight stops"

In general, we would expect stop-losses to reduce return quality. The intuition behind this expectation is that post-selloff rebounds are among the highest-sharpe periods in a risk-premium strategy's life. Stop-losses limit drawdowns but also deny us participation in rebounds. On balance, return quality reduction dominates. This means that we are better off reducing position sizes rather than tightening stops.

_					
		Return	quality vs	Stop-loss	level
		stop_loss	sharpe	calmar	proxy
	1	0	0.42	46.77	equal weight
	2	0.83	0.45	36.18	equal weight
	3	1.67	0.47	20.85	equal weight
	4	2.5	0.5	16.68	equal weight
	5	3.33	0.53	10.41	equal weight
	6	4.17	0.57	8.37	equal weight
	7	5	0.64	6.52	equal weight
	8	5.83	0.7	5.69	equal weight
	9	6.67	0.75	5.39	equal weight
	10	7.5	0.79	5.08	equal weight
	11	8.33	0.86	4.45	equal weight
	12	9.17	0.92	4.03	equal weight
	13	10	0.96	3.72	equal weight
	14	10.83	1.02	3.25	equal weight
	15	11.67	1.07	2.92	equal weight
	16	12.5	1.1	2.83	equal weight
	17	13.33	1.12	2.84	equal weight
	18	14.17	1.19	2.61	equal weight
	19	15	1.19	2.62	equal weight
	20	15.83	1.17	2.77	equal weight
	21	16.67	1.2	2.58	equal weight
	22	17.5	1.23	2.48	equal weight
	23	18.33	1.2	2.54	equal weight
	24	19.17	1.21	2.57	equal weight
	25	20	1.22	2.53	equal weight

3 Weighted strategy backtests

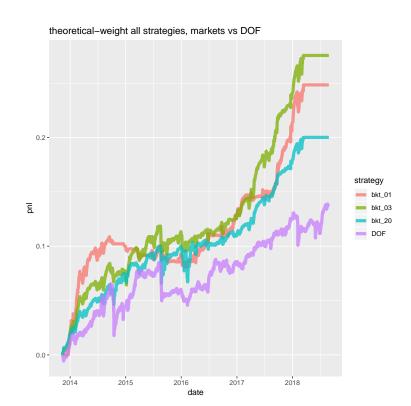
3.1 equal weights

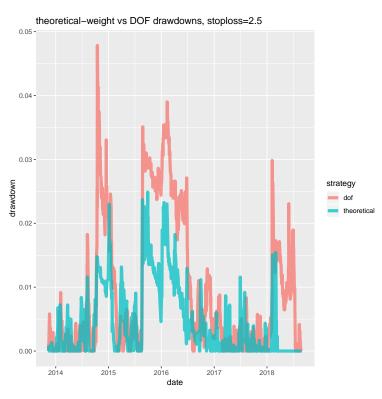


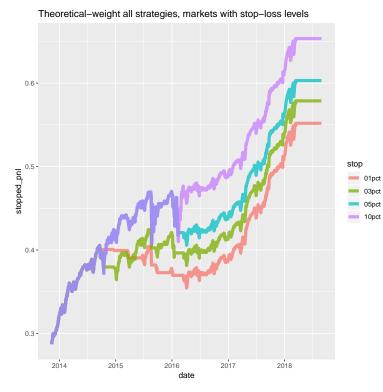
Note the effect of a 3% stop-loss on drawdowns. The drawdown depth decreases, but drawdown length increases.

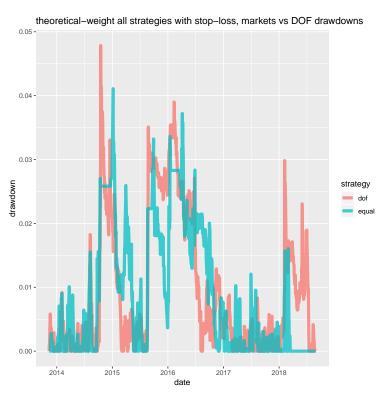
3.2 theoretical weights

Portfolio composition	
Zero Cost Call Spread Collars	33 %
Zero Cost Put Spread Collars	0 %
Zero Cost Risk Reversals	0 %
Zero Cost Put Spread Ratios	33 %
Short Straddles	33 %





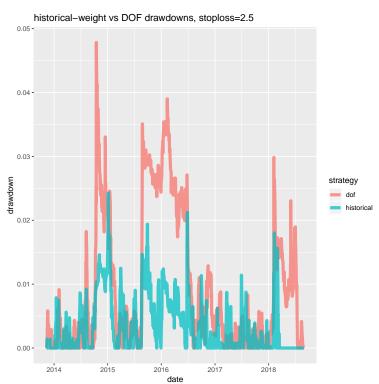




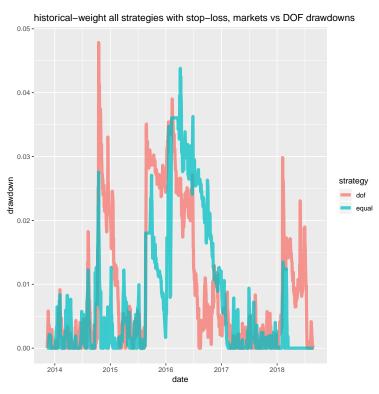
3.3 historical weights

Portfolio composition	
Zero Cost Call Spread Collars	43 %
Zero Cost Put Spread Collars	19 %
Zero Cost Risk Reversals	7%
Zero Cost Put Spread Ratios	15 %
Short Straddles	16 %



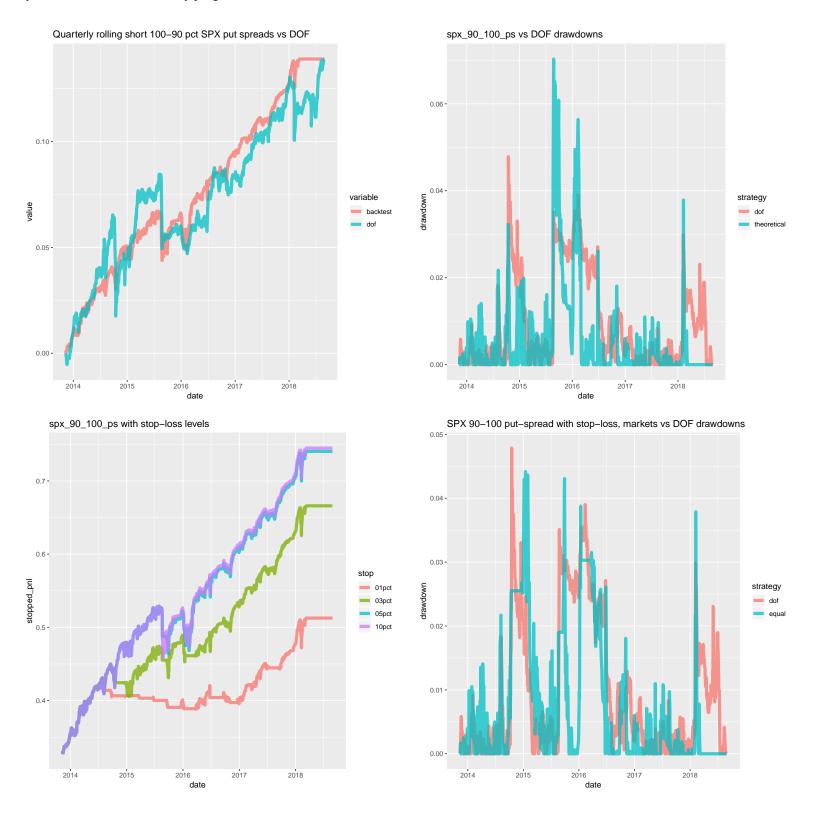






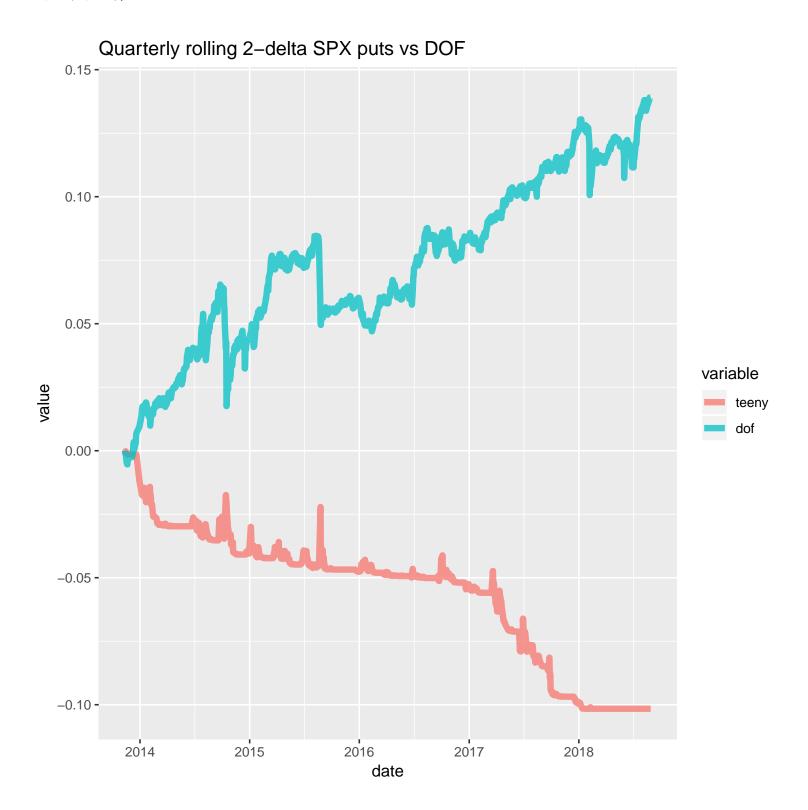
4 A DOF proxy: SPX Q90-100 PS vs DOF performance

We examine a very simple strategy that seems to have similar drawdown performance with DOF. The value of this proxy is that its behaviour is very intuitive and allows for easy judgement of scenario outcomes.



5 Drawdown mitigation: SPX "teenies"

Downside mitigation can be achieved using "teenies", i.e. very low delta puts (50 to 300 bps). These options almost never pay out anything at settlement. However, their value tends to spike during market-wide bouts of liquidation. We can take advantage of this by holding some amount of these and selling them back whever their value spikes by a pre-determined amount. The combined portfolio should have smaller returns but much reduced drawdowns. The key to this risk-mitigation strategy is that we use a "stop-profit" on the tail-protection portfolio rather than a "stop-loss" on the risk-premium portfolio. The reason this works better than stop-losses is that volatility risk premia are very strongly mean-reverting. It is better to buy tail-risk when the market is calm and then sell volatility on the pop rather than stop-out the risk premium collection portfolio on the pop and buy the volatility highs. (tail risk becomes vol risk on a strong selloff, the teenies become 5-10 deltas pretty quickly)

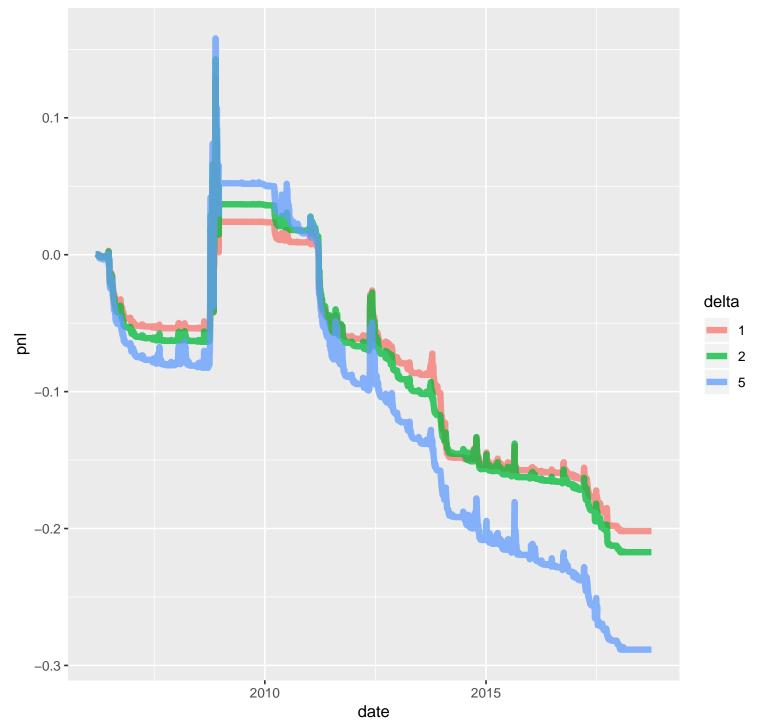


We examine the performanse of a tail-risk hedge during the largest draw-down episodes

	largest DOF drawdown episodes, optimum teeny performance							
	episode	max_draw	date_draw	date_start	date_end	teeny1	teeny2	teeny5
1	63	4.78	2014-10-16	2014-09-25	2015-03-04	1.2	1.39	1.71
2	84	3.9	2016-02-11	2015-08-11	2016-08-05	1.67	2.35	3.59
3	152	2.98	2018-02-06	2018-01-09	2018-07-17	0	0	0
4	54	1.83	2014-08-08	2014-07-30	2014-09-02	0.04	0.08	0.13
5	88	1.29	2016-11-15	2016-08-15	2017-02-28	0.78	0.89	1.08
6	17	0.91	2014-02-04	2014-01-21	2014-02-25	0.5	0.48	0.69
7	118	0.83	2017-08-11	2017-08-08	2017-08-15	0.11	0.16	0
8	73	0.66	2015-05-07	2015-04-13	2015-05-27	0.13	0.15	0.16
9	1	0.58	2013-11-19	2013-11-12	2013-12-06	0.03	0.05	0.08
10	76	0.58	2015-06-30	2015-06-02	2015-07-17	0.42	0.54	0.74

The key metric for choosing a tail hedge is the ratio of premium to potential upside. We expect to rarely be able to recoup any of the premium, since tail options almost never end up paying out anything. From this viewpoint, lower delta options are the best choice. For the examples examined in thise report, 1-delta SPX puts have the best ratio of premium to "price pop" during a selloff.

Tail hedges: 1, 2 and 5 deltas

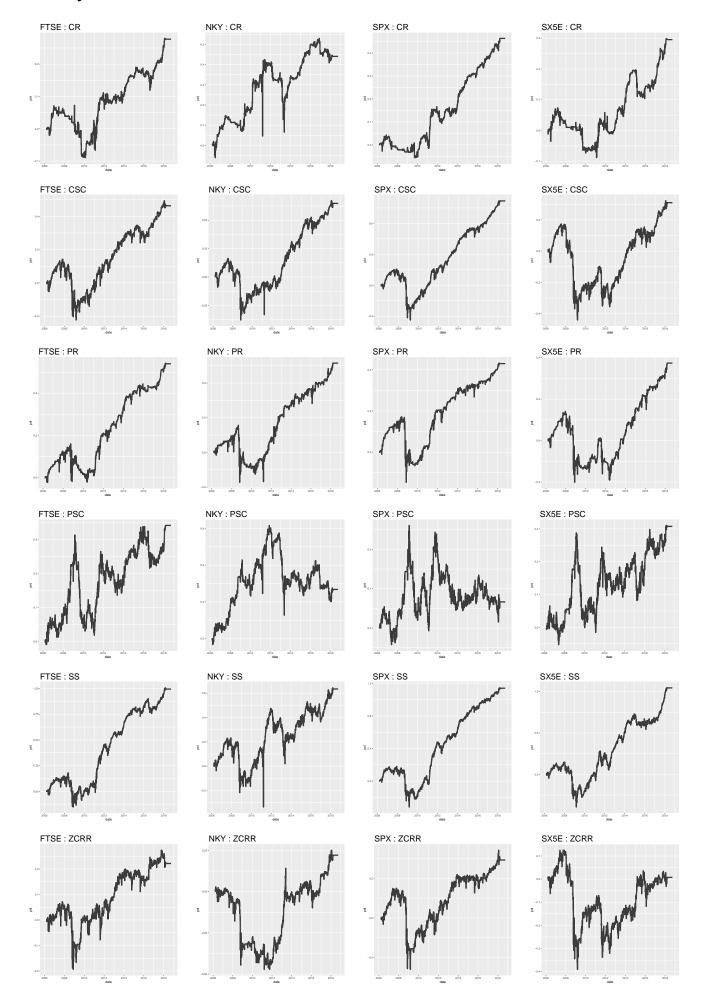


6 All backtests

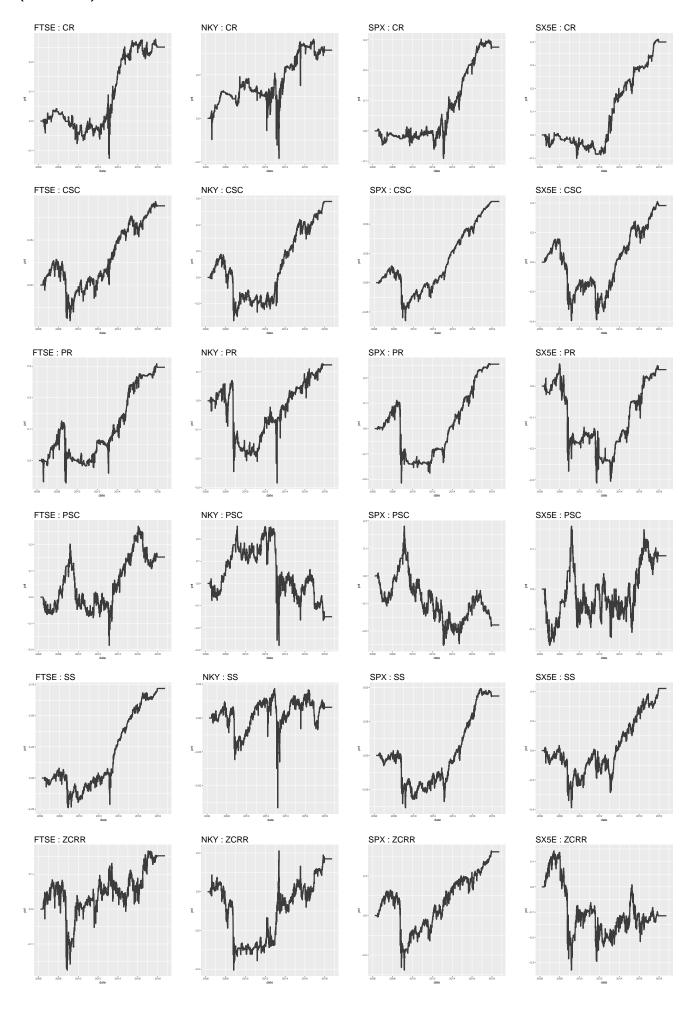


7 Strategies vs market backtests

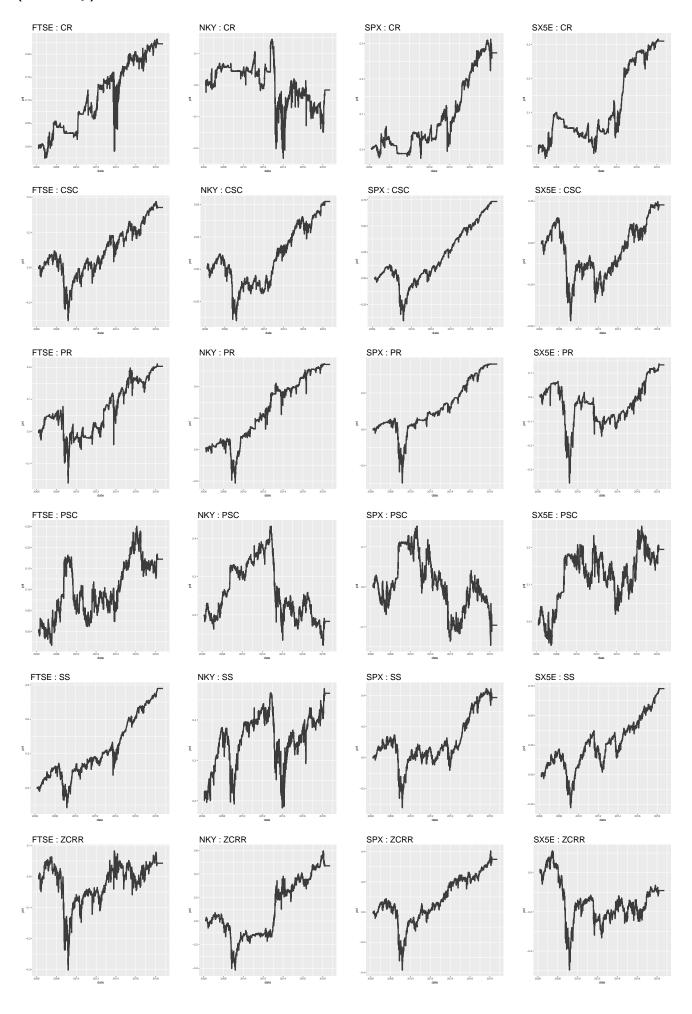
7.1 3m maturity



7.2 6m (Jun, Dec) maturities

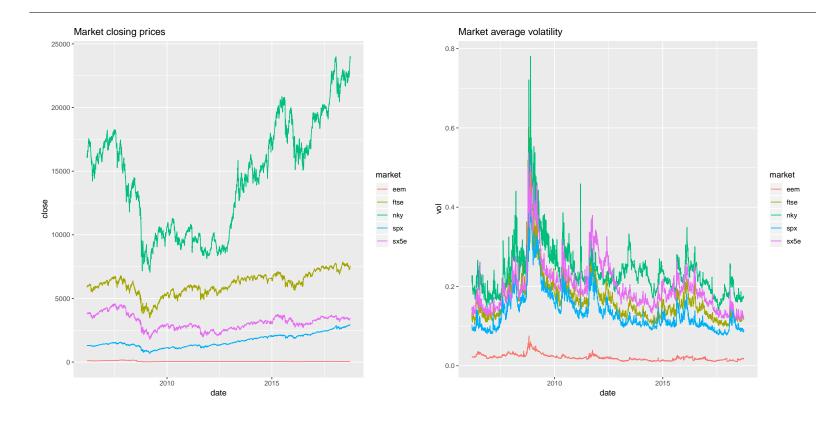


7.3 6m (Mar, Sep) maturities



8 Dataset

We use historical volatility surfaces from JPM's OptionMetrics database. Average volatility, closing prices by market shown below.



9 Backtest algorithm

9.1 backtest inputs

A backtest requires the following inputs:

- 1. volatility surface
- 2. schedule
- 3. payoff

9.2 volatility surface input

The volatility surface input is an R data.table containing volatility marks for all markets, strikes and maturities. The data is stored in "long format", so every volatility mark occupies a single row. The schema is as follows:

vol	atility surface
Date	reval date
Strike	option strike
Days	option maturity
${\tt ImpliedVol}$	implied volatility
ClosePrice	closing price underlying
market	market name

9.3 shedule input

The schedule input is an R data.table containing one row for each date and market we want bactested p&l for. In addition to these data items, the shedule contains the market's close, the average volatility and the current roll period number, start date, end date and days left. Options are assumed to expire on the last day of their respective roll period.

shedule				
date	trading date			
market	market			
close	closing price			
vol	average vol across all strikes and maturities			
roll	roll period			
start	roll period starting date			
end	roll period end date			
maturity	days until end of current roll period			

9.4 payoff input

the payoff is an R data.table containing one row for each of the payoffs we intend to roll on each roll date. For each payoff, the table contains a valuation model and a roll function. The valuation model is used to compute option values on each day in the roll period. The roll function is used to compute option strikes on roll dates. The valuation model is calld for each day in the shedule. The roll function is called only on roll period start days

	payoff
model	<pre>function(spot,strike,maturity,rate,volatility)</pre>
strike	<pre>function(spot, volatility)</pre>

9.5 backtest workflow

- start with shedule
- (not vectorized) compute table of strikes for each roll date (date is equal to start of roll period) by calling roll function. Since roll dates are months appart, the roll function is called relatively infrequently and does not need to be optimized.
- LEFT JOIN option strikes on roll period. strikes are constant over the whole roll period. We use the data.table X[Y] idiom to achieve this, it is very fast.
- (vectorized) compute maturity, strike pillars surrounding contract economics. We use the findInterval function for this, which is very fast
- LEFT JOIN surface volatilities on surrounding pillars. data.table's X[Y] idiom is used here too.
- (vectorized) perform biliear interpolation. This is done on the whole data.table in one go since the biliear interpolation formula is vectorizeable.
- (vectorized) call valuation model, store reval. The model needs to support vectorized operation, there is a single call with vectors of spot, vol, maturity, strike and rate as inputs.

The calculations outlined are vectorizeable. This means that the backtest is quite fast. We can backtest a single option on a single market over a 20-year period in about 0.5 seconds.

9.6 volatility interpolation

The interpolate_vol function takes a reval date, a strike and a maturity as inputs and performs volatility intepolation on a single-market volatility surface. This calculation needs to be performed for every day, every market and every strike in the strategy. This means that the number of interpolations can run into the hundreds of thousands .It is therefore really important to be as efficient as possible. This means we need to vectorize operations as much as we can. The calculation as shown works well for a single market. To do intepolation over multiple markets (for example from a multi-market shedule), we will use the data.table keyby feature to split the schedule by market and perform the interpolation on every subset separately, inside the data.table's J expression.

```
interpolate_vol<-function(date, strike, days, vsurf)</pre>
  # vector of strike pillars in vol surface
  strikes<-sort(unique(vsurf$Strike))</pre>
  # vector of maturity pillars in vol surface
  maturities<-sort(unique(vsurf$Days))</pre>
  # volatility surface pillars surrounding contract
  # strike and expiry days
  option_dets<-data.table(
    date=date,
    lo_strike=strikes[findInterval(strike,strikes)],
    hi_strike=strikes[findInterval(strike,strikes)+1],
    lo_mat=maturities[findInterval(days,maturities)],
   hi_mat=maturities[findInterval(days,maturities)+1]
  # left join lower left corner's volatility
  vol_ll<-merge(
    x=option_dets,
    y=vsurf,
    by.x=c("date","lo_strike","lo_mat"),
    by.y=c("Date","Strike","Days")
  ) $ Implied Vol
  # left join higher left corner's volatility
  vol_lh<-merge(</pre>
    x=option_dets,
    v=vsurf.
    by.x=c("date","lo_strike","hi_mat"),
    by.y=c("Date","Strike","Days")
  ) $ Implied Vol
  # left join right lower corner's volatility
  vol_hl<-merge(
    x=option_dets,
    y=vsurf,
    by.x=c("date","hi_strike","lo_mat"),
    by.y=c("Date","Strike","Days")
  ) $ Implied Vol
  # left join right higher corner's volatility
  vol_hh<-merge(
   x=option_dets,
    y=vsurf,
    by.x=c("date","hi_strike","hi_mat"),
    by.y=c("Date","Strike","Days")
  ) $ Implied Vol
  # coordinates of contract strike, expiry inside surrounding pillar points
  t_strike <- (strike - option_dets $lo_strike) / (option_dets $hi_strike - option_dets $lo_strike)
  t_mat<-(days-option_dets$lo_mat)/(option_dets$hi_mat-option_dets$lo_mat)
  # bilinear interpolation formula,
  # result is vector of volatilities
  rowSums(cbind(
    vol_ll*(1-t_strike)*(1-t_mat),
    vol_hl*(t_strike)*(1-t_mat),
    vol_lh*(1-t_strike)*(t_mat),
    vol_hh*(t_strike)*(t_mat)
  ))
}
```

9.7 Unit tests

We perform a number of backtests for selected strategies on synthetic data to ensure the backtest engine is working as expected.

