1 Some properties of the RSI formula

The value of RSI over some period w is:

$$RSI = 100 - \frac{100}{1 + \frac{gains}{losses}}$$

Where gains $= \frac{\sum \text{daily gains}}{\text{count of daily gains}}$, losses $= \frac{\sum \text{daily losses}}{\text{count of daily losses}}$, are both positive values.

We observe that

$$\sum \text{daily gains} = \frac{1}{2} \times \left(\sum |\text{daily move}| + \sum \text{daily move}\right)$$

$$\sum \text{daily losses} = \frac{1}{2} \times \left(\sum |\text{daily move}| - \sum \text{daily move}\right)$$

Therefore

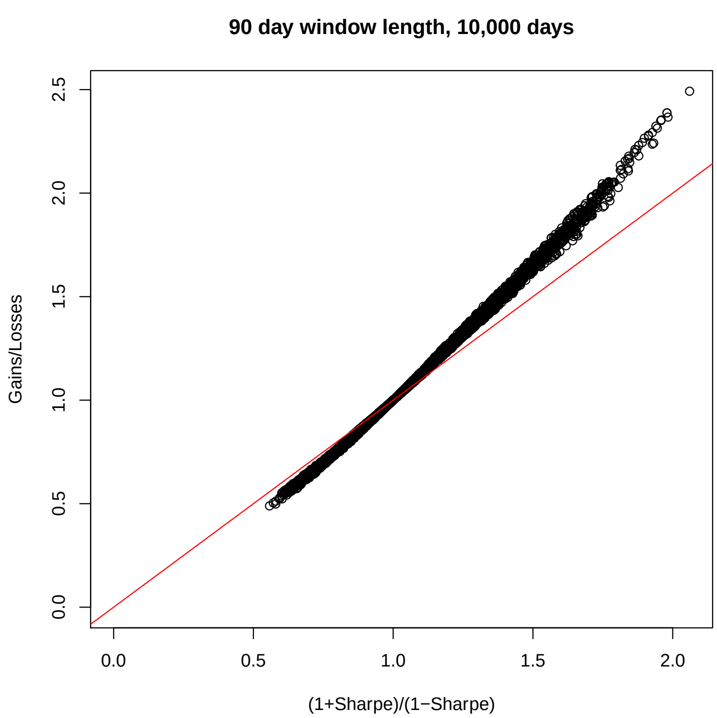
$$\frac{\sum \text{daily gains}}{\sum \text{daily losses}} = \frac{\sum |\text{daily move}| + \sum \text{daily move}}{\sum |\text{daily move}| - \sum \text{daily move}}$$

$$\frac{\sum \text{daily gains}}{\sum \text{daily losses}} = \frac{1 + \frac{\sum dailymove}{\sum |dailymove|}}{1 - \frac{\sum dailymove}{\sum |dailymove|}}$$

And because $\frac{\sum \text{daily move}}{\sum |\text{daily move}|}$ is approximately $C_0 \times \sqrt{\text{length}(w)} \times \frac{\text{return}}{\text{volatility}}$, we get:

$$\frac{\sum \text{daily gains}}{\sum \text{daily losses}} \approx \frac{1 + C_1 \times \text{sharpe ratio}}{1 - C_1 \times \text{sharpe ratio}}$$

We can verify this by running a simulation for 10000 samples and a 90 day window:



In addition, we observe that

$$\text{count of daily gains} = \sum \frac{|\text{daily move}| + \text{daily move}}{2 \times |\text{daily move}|}$$

$$\text{count of daily losses} = \sum \frac{|\text{daily move}| - \text{daily move}}{2 \times |\text{daily move}|}$$

$$\frac{\text{count of daily losses}}{\text{count of daily gains}} = \frac{\sum 1 - \frac{\text{daily move}}{|\text{daily move}|}}{\sum 1 + \frac{\text{daily move}}{|\text{daily move}|}} = \frac{N - \sum \frac{\text{daily move}}{|\text{daily move}|}}{N + \sum \frac{\text{daily move}}{|\text{daily move}|}} = \frac{N - \sum \text{sign(daily move)}}{N + \sum \text{sign(daily move)}} = \frac{1 - \overline{\text{sign(daily move)}}}{1 + \overline{\text{sign(daily move)}}}$$

Where N is the window size

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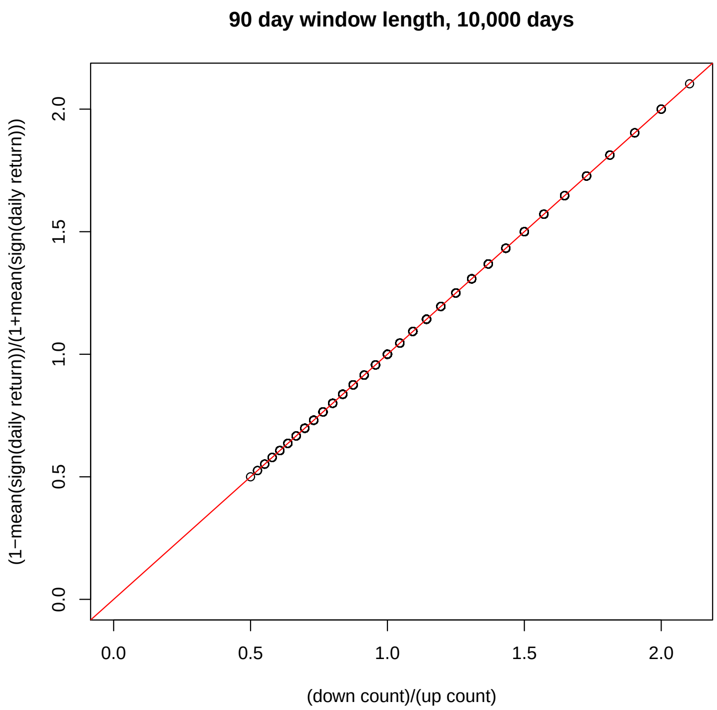


Combining these two observations we get an expression for $\frac{\text{gains}}{\text{losses}}$ which can be computed using averages of statistics over the whole window, i.e. all divisors are the same and equal to N, the number of days in the window:

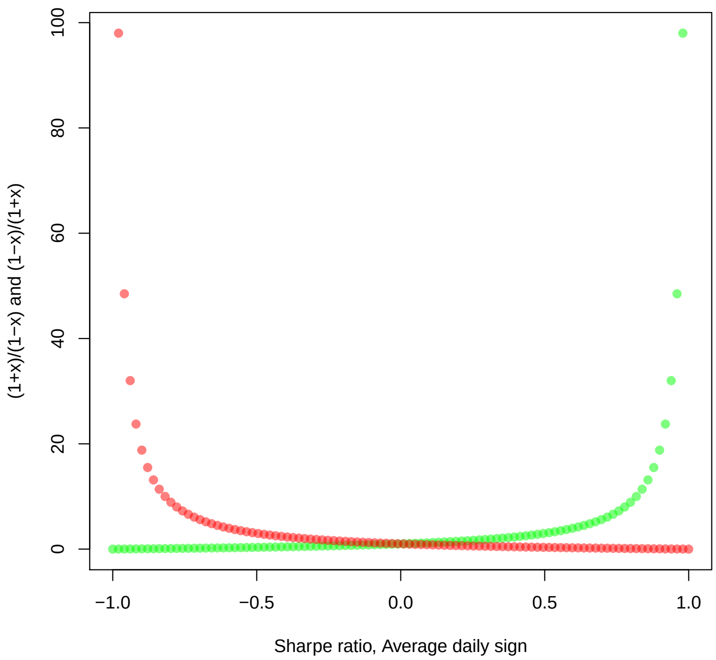
$$\frac{\text{gains}}{\text{losses}} \approx \frac{1+C_1\times\text{sharpe ratio}}{1-C_1\times\text{sharpe ratio}} \times \frac{1-\overline{\text{sign(daily move)}}}{1+\overline{\text{sign(daily move)}}}$$

in conclusion, the ratio $\frac{\text{gains}}{\text{losses}}$ of average gains/losses is the product of two factors. These factors are functions of the **sharpe ratio** and the **average sign**. The functional form is such that the ratio is:

- an increasing function of the **sharpe ratio** over the window
- a decreasing function of the **average sign** over the window

A plot of these factor values and a simulation nicely illustrate this finding:

$\frac{\text{gains}}{\text{losses}}$ factor values



14-day sharpe, mean sign and RSI for simulated returns