Structuralism and Semantic Glue

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Abstract

Structuralist positions in the philosophy of physics and mathematics have a complicated relationship with Hilary Putnam's model-theoretic arguments. One reason to think that Putnam's permutation argument motivates structuralism is that the hole argument about spacetime, which structuralists about physics take to support their position, has been claimed to be a version of Putnam's permutation argument (Liu, 1996, Rynasiewicz, 1994). I argue against this claim. However, as Demopoulos and Friedman (1985) have claimed, Newman's objection to structuralism *is* closely analogous to Putnam's argument.[1]

1. Introduction

Structuralist theories are currently thriving in the philosophy of mathematics and physics. The structuralist position has a complicated relationship with Putnam's model-theoretic arguments, aspects of which I aim to examine and clarify here. First, the model-theoretic arguments are frequently claimed to support or motivate structuralism. However, the contention is complicated by a second way in which the arguments are implicated in the debate over structuralism. This is by way of an identification between the permutation argument and an argument that proved terminal for an early version of structuralism defended by Russell. Here I will assess one particular line of thought that indicates that Putnam's permutation argument can be used to support structuralism; this is the claim that the hole argument, in the philosophy of spacetime, is really just Putnam's permutation argument in a mathematically complicated guise. I will then attempt to reconcile this assessment with the contrary thesis that something very much like Putnam's permutation argument undermines structuralism.

Putnam's model-theoretic arguments consist of the permutation argument, first presented to the American Philosophical Association in 1976 (Putnam, 1978, pp.123-141) and treated more fully in *Reason, Truth, and History* (1981, Ch.2 and Appendix), and the 'Skolemite' argument (1980). Here, I will be interested in the simpler of the two: the permutation argument.

[1]Forthcoming in *Proceedings to Philosophy in an Age of Science,* Alan Berger (ed.), Oxford University Press

2. Scientific structuralism

Structuralism about a particular domain is the thesis that all that we know of that domain is its structure, where a structure is a network of relations considered independently of any objects (or "nodes") which might be related by them. Because it seeks to establish that we can only have knowledge of *structure*, structuralism is typically supported by underdetermination arguments. Elsewhere (forthcoming) I have argued against mathematical structuralists' claims that structuralism in mathematics is a version of the philosophical position that Putnam took his model-theoretic arguments to establish. Here, I consider the bearing of Putnam's arguments on structuralism about physics. I will look first at the claim that the hole argument about spacetime, which supports structuralism about spacetime points, is the same as the permutation argument. And then I will turn to the question whether Newman's objection, which undermines structuralism, is itself a version of the permutation argument as has been frequently claimed; see, for instance, Demopoulos and Friedman (1985, pp.634-5), Demopoulos (2003, p.385), Lewis (1984, p.224, n.9) and van Fraassen (2008, p.239 n.5).

Structuralism in the philosophy of science, or structural realism as it is also known, is scientific realism about the *structure* of the world. In particular, the claim is that all that we know through science is structure, typically construed as a network of relations. Structural realism claims that, in particular, we cannot know anything about the *nature* of the fundamental objects whose structure science describes.

Structural realism was proposed by Worrall (1989) as a means of responding to two arguments about the subject-matter and reference of scientific theories, which seem to impose constraints from opposite directions. The first argument is the insight that the evident predictive success of scientific theories would be inexplicable, even miraculous, were the theories not tracking reality. This is Putnam's "no miracles" argument against antirealism in ontology (Putnam, 1975b, p.73). The second argument is the "pessimistic meta-induction". This begins with the premise that the progression of accepted scientific theories to date has not been cumulative but revolutionary, involving "radical shifts" in theoretical content. On this basis, arguing by induction, we should expect our current and future scientific theories to be "ontologically false" (Worrall, 1989, p.109). Thus, the first argument tells us that, barring a miracle, the reason why our current science is so predictively successful must be because it is corresponding to reality. But the second argument tells us that we have good reason to believe that current scientific theory has *not* been successful in settling which objects make up the world. Worrall suggested (*Ibid.* p.117) that the problems can be simultaneously satisfied[2] if we hold that scientific theories track the structure of reality, and that while not all content is preserved under theory-change, description of structure is. Worrall finds the roots of this thesis, which he calls structural realism, in Poincaré (1905). Re-

[2] In fact, Putnam is one of two philosophers that Worrall cites as having responded jointly to both the "no miracles" *and* the pessimistic meta-induction. The other is Boyd (1973). Putnam is widely credited with the former argument, which is presented in (Putnam, 1975a, p.73), but it is less well-known that he also proposed a version of the pessimistic meta-induction, in (Putnam, 1978, p.25). Worrall (1989, p.112) claims that his structural realism better captures the "valid intuitions" that underlie Putnam's attempted resolution.

lated views are also found in Russell (1927), Grover Maxwell (1970a; 1970b), Schlick (1918), Carnap (1928) and Cassirer (1937; 1944).

Ladyman (1998) disambiguated Worrall's thesis by distinguishing *epistemic* versions of structural realism, which hold that science can only yield knowledge about the structure of some otherwise unknowable objects, from *ontic* versions of structural realism. Ontic structuralism realism adds to the thesis that science only tells us about structure, an ontological claim which accounts for its epistemic humility. At the strongest, this ontological thesis is eliminativist about objects; it is the claim that all that there is to be described by science is structure. On this account, there *are no* unknowable objects that are arranged in structures.[3] Weaker, non-eliminativist ontic structural realist theses claim that the domain is *primarily* structural in the sense that the relations have ontological priority over the objects,[4] or, that at any rate there is no ontological priority of objects over relations.[5]

Ontic structural realism is claimed to have the advantage over epistemic structural realism that it resolves certain problems arising from modern physics. One particular such problem is posed by the hole argument, which was adapted by Earman and Norton (1987) from a problem Einstein grappled with during the formulation of general relativity. Earman and Norton use the argument to criticize the substantivalist position about spacetime points—although, as I shall go on to show, the hole argument requires further metaphysical presuppositions that go beyond what the substantivalist is committed to. Substantivalism is the metaphysical position committed to the reality of absolute space. Sklar defines it as the "postulation of the independent reality of space as a kind of substance" (1974, p.161), a definition that goes back to Newton (1729).

3. The hole argument

I will first review the hole argument and the claim that it supports structuralism, then explain how it has been argued to be a version of Putnam's permutation argument.

The hole argument turns on the issue of whether spacetime exists independently of the events and matter in it, or, instead, depends on them for its existence. It relies on the fact that the theory of general relativity has models which have surplus mathematical structure. The hole argument ascribes to the substantivalist the view that this surplus structure does in fact represent something of physical significance, namely the metrical properties of spacetime points. Since the surplus structure is independent of the theory of general relativity, Earman and Norton conclude that the hole argument generates a "very radical form of indeterminism" for the substantivalist (1987, p.516).

A model of spacetime according to general relativity consists of a 4-dimensional pseudo-Riemannian manifold, M, together with a metric tensor, g, and an energy-momentum tensor $\mathscr{T}$ of the matter field. The resulting triple $\mathscr{M} := \langle M, g, \mathscr{T}\rangle$ is then

[3] See Ladyman (1998, p.422).
[4] See French and Ladyman (2003).
[5] See Pooley (2006), Esfeld and Lam (2008).

a *structured manifold*. To avoid confusion between M and $\mathscr{M}$, I will refer to points of the manifold M independently of the structure as "bare" manifold points. It is the full structured manifold $\mathscr{M}$ which models a physical "world" consistent with general relativity (GR).

The field equations for GR tell us how the energy-momentum tensor $\mathscr{T}$ influences the metric tensor g. As differential equations, the GR field equations have the mathematical property of being generally covariant. This means that distinct solutions to the field equations can take the form of different spreadings of the dynamical fields of the theory over a given manifold. Suppose that $\mathscr{M} := \langle M, g, \mathscr{T} \rangle$ is a solution to the equations, and $d*$ is a diffeomorphism on M.[6] Let $d*\mathscr{M} := \langle M, d*g, d*\mathscr{T} \rangle$ represent the structured manifold which results from dragging both g and $\mathscr{T}$ along under $d*$. General covariance entails that if $\mathscr{M}$ is a solution to the GR equations then so is $d*\mathscr{M}$.

The hole argument begins by considering whether, and how, spacetime *itself* might be represented in a model of the GR equations. A natural idea is to take the bare manifold M considered in isolation from the fields on it to represent spacetime. This is to let points of the manifold represent spacetime points, and the fields on the manifold to represent the spatiotemporal contents of spacetime. This hypothesis about representation is known as *manifold substantivalism*, or the "container view" of spacetime models (Norton, 2008) since it treats the manifold M as a container for its matter contents, which can be shaken up within it. The move to treat the bare manifold as representing spacetime is recommended by the fact that the metric tensor field, which represents the gravitational field as well as encoding physical distances, can be seen as a physical field *on* spacetime (Earman and Norton, 1987, p.519). Earman and Norton interpret substantivalism as committed to manifold substantivalism (pp.518-9), however this premise about representational convention has been challenged by substantivalists (see note 8 below).

In addition to a commitment to manifold substantivalism, Earman and Norton took the substantivalist's belief in absolute space, in the context of General Relativity, to entail that spacetime points are individuated independently of the fields that act on spacetime. Talk of individuation is somewhat opaque, but for the purposes of this presentation we will not need to clarify that talk, because the hole argument doesn't put pressure on the premise of point-individuation itself, but rather on a consequence of it that Earman and Norton call "an acid test of substantivalism drawn from Leibniz" (1987, p.521). Leibniz famously proposed to the Newtonian, Clarke, that whoever believes the "chimerical supposition of the reality of space in itself" is forced to concede a difference between our world and the counterfactual situation in which everything to the east has been permuted with that to the west (Leibniz, 1716, III, 5.). Relying on the same inference, Earman and Norton's "acid test" characterizes substantivalists as committed to the belief that worlds that are exactly alike except for a difference

[6] A diffeomorphism is a kind of mapping between manifold points. It is defined to be an isomorphism of structured manifolds that is smooth, invertible and has a smooth inverse.

in the absolute spatio-temporal properties they attribute to some particular points of spacetime are different worlds.

As we have seen, Earman and Norton's substantivalist holds both manifold substantivalism and the belief that, *contra* Leibniz, absolute spatio-temporal differences yield physically distinct situations. Now, because the field equations for GR are generally covariant, there exist pairs of manifolds related by a non-trivial diffeomorphism $d*$ that are both solutions to the GR equations. Moreover, given such manifolds $\mathscr{M}$ and $d*\mathscr{M}$ there will be manifold points m of M whose metric field properties are not properties had by the image of m under diffeomorphism, $d*(m)$. If manifold points do represent spacetime points and diffeomorphisms represent translations of spacetime points, then it follows that for Earman and Norton's substantivalist the structured manifolds $\mathscr{M}$ and $d*\mathscr{M}$ will represent worlds that differ in the spatial properties of certain spacetime points. This is to say, the diffeomorphic manifolds are representations of *physically distinct worlds*. (Earman and Norton, 1987, p.515, pp.518-9; see also Norton 1988, p.59). The claim that diffeomorphically related models of GR represent the same physical situation is called Leibniz equivalence. So, Earman and Norton take the substantivalist to be committed to the denial of Leibniz equivalence. They go on to identify two problems that arise from this belief.

First, note that the worlds represented by diffeomorphic manifolds must be observationally indistinguishable, irrespective of whether or not Leibniz equivalence is true, since diffeomorphisms on spacetime manifolds preserve observables (Earman and Norton, 1987, p.522). And so, if the worlds represented are maintained to be distinct, then the distinction appears to be one without a difference. On this basis of this, Earman and Norton formulate a "verificationist dilemma" for the substantivalist, forcing him to either give up his substantivalism or to accept that there are observationally equivalent states of affairs. But not every distinction posited in metaphysics need have a correlate in physical theory, and so while the verificationist dilemma is sound, it needn't compel the substantivalist to reject substantivalism.

Earman and Norton's second dilemma is the more forceful of the two, and has become known as the hole argument. It is intended to force the substantivalist to either give up substantivalism, or to accept that GR is *radically indeterministic*. The hole argument establishes this as follows. First, let $\mathscr{M} := \langle M, g, \mathscr{T} \rangle$ be a structured manifold that is a solution to the GR equations, and let H be an open region of the manifold M. Call H the "hole". Relative to H we can define a "hole diffeomorphism", d_H which is the identity on $M \setminus H$, and is non-trivial on H. Since $\mathscr{M}$ is a solution to the field equations, $d_H * \mathscr{M}$ is also a solution by general covariance. As before, $\mathscr{M}$ and , $d_H * \mathscr{M}$ are observationally indistinguishable. Suppose that the manifolds admit foliation by space-like hypersurfaces, so that we can talk about temporally ordered events in the spacetime(s) they model. It follows that if manifold substantivalism and the denial of Leibniz equivalence are true, determinism must be false. For manifolds $\mathscr{M}$ and $d_H * \mathscr{M}$ assign the very same metrical properties to pairs of manifold points m and $d_H * (H)$ which fall *earlier* in time than the points in H and $d_H * (m)$ respectively. Yet, pairs of points in the hole, $n \in H$ and $d_H * (n) \in d_H * (H)$, have *different* metric properties, because the diffeomorphism acts non-trivially on this region. And so any form of determinism which entails that the properties of past states fix those at future states is ruled out, because $\mathscr{M}$ and $d_H * \mathscr{M}$ represent two worlds which are identical up to a moment in time, but diverge in the properties of spacetime points after that time.

Earman and Norton's final premise is that "if [determinism] fails, it should fail for a reason of physics, not because of a commitment to substantival properties which can be eradicated without affecting the empirical consequences of the theory" (1987, p.524). As a principle of naturalistic philosophy, this premise is hard to argue with—note that Earman and Norton don't stipulate that determinism is, or ought to be, true. The conclusion of the hole argument is that substantivalism is unacceptable because it comes at the cost of claiming that that our best spacetime theory fails to be deterministic for metaphysical reasons.

In the following section, I will show that the hole argument's real focus is not substantivalism *per se*, but some additional metaphysical baggage that the substantivalist position has historically been associated with. Nevertheless, the hole argument is generally taken to support structural realism as an alternative to substantivalism. The inference made by those who defend structural realism is as follows: to avoid the underdetermination of properties of spacetime points that arises from the covariance of General Relativity, we should deny that spacetime points have primitive identity. Renouncing a commitment to the primitive identity of some objects renders meaningless questions about whether *one of them* could have had the properties of *another* one, because with no primitive identity, there is no more to the objects than the properties they possess.[7] The alternative to primitive identity is contextual identity, according to which objects depend for their identity on the pattern of properties and relations distributed across them. Crucially for the hole argument, this pattern is preserved under diffeomorphism. The structuralist response to the hole argument recommends adopting contextual identity for spacetime points, and is advocated by Hoefer (1996, p.11), Stachel (2002), Ladyman (1998) and Ladyman and Ross (2007, p.143).

4. The hole argument is Putnam's permutation argument

Several commentators have claimed that the hole argument just *is* Putnam's permutation argument. Thus, Rynasewicz made the point that:

> The hole argument can be recast as a special instance of [... the] permutation arguments [...] familiar to the readers of Quine, Davidson, and the 'new' Putnam. (Rynasiewicz, 1994, p.419)

Likewise, Liu argued that the hole argument is really Putnam's argument about referential indeterminacy:

[7] Stachel argues, in a different context, that Putnam's permutation argument relys on such a notion of primitive identity (2005), the implication being that denying primitive identity is a means of avoiding the permutation argument's conclusion.

> [T]he hole argument is really Putnam's argument restricted to spacetime theories. This is because the gauge theorem is about the inscrutability of reference, not about the indeterminacy of possible worlds. Thus, the gauge freedom in spacetime theories expresses a semantic fact rather than one about ontology; and the hole argument is really against the metaphysical determinacy of reference. (Liu, 1996, p.243)

I am highly sceptical of the thesis that the hole argument just *is* Putnam's permutation argument. This is because the hole argument relies on a feature of certain kinds of theory, specifically, those which are generally covariant, and hence whose models exhibit surplus mathematical structure. The fact that the surplus features, which in this case are properties of bare manifold points, are not determined by the theory being modeled, together with the supposition that for the substantivalist these features represent something significant, are what lead to the conclusion of underdetermination. While it is possible to formulate a hole argument for pre-general relativistic theories, such as special relativity and Newtonian spacetime, by re-expressing them in generally covariant form, the feature of general covariance *is* necessary to generate the hole argument's conclusion of underdetermination between different representations of spacetime. Putnam's permutation argument, however, has no such restriction. As Putnam made clear, in response to the objection that his Skolemite argument is a problem just for first-order languages, his *permutation* argument is *wholly general*. For it uses "a technique—permutation of individuals—which applies to second-order logic, modal logic, tensed logic, etc" (Putnam, 1995, p.356, n.11), and is therefore valid for any theory whatsoever.

Stachel has recently presented a simplified "set-theoretic" form of the hole argument, which he claims was "inspir[ed]" (2002, p.231) by Liu and Rynasewicz's thesis that the hole argument is Putnam's permutation argument. Pooley has since elaborated on Stachel's argument to present what he takes to be "the proper set-theoretic analogue of the hole argument" (2006, p.109). I argue that the *set-theoretic* hole argument essentially *is* Putnam's permutation argument, and, moreover, that it fails to preserve the content of the original hole argument. Stachel's set-theoretic hole argument asks, purportedly in analogy with the hole argument, whether two different interpretations of an extensional ensemble, which agree only in part, suffice to pick out *one* unique world, or, on the other hand, whether we need to fully specify an interpretation to pin down a single world (Stachel, 2002, p.244). The set-theoretic hole argument therefore turns on the underdetermination of representation, as does Putnam's permutation argument. Stachel justifies his presentation of the argument by claiming that is obtained from Earman and Norton's by an information-preserving procedure of "abstraction by deletion" (p.235). But the set-theoretic hole argument is not a faithful presentation of the full, differential geometry version of the argument. This is because, for the substantivalist, the original hole argument was supposed to involve underdetermination between models of GR that represented *physically distinct* worlds that differed in the properties ascribed to regions and points of absolute space. But, in the set-theoretic hole argument, the only underdetermination is between two ways of interpreting the theory that is the analogue of GR. So, by making the argument turn on the underdetermination of representation, the set-theoretic hole argument simply begs the question against the substantivalist, for whom there are genuine physical differences that are underdetermined by General Relativity.

I have objected to the tendency to treat Earman and Norton's hole argument as a particular instance of Putnam's permutation argument. However, I think that by isolating the real focus of the hole argument, one reason why comparisons between the two have seemed so compelling becomes apparent. I take it, following Pooley (2006), that what the hole argument targets is the metaphysical thesis of haecceitism, which Earman and Norton mistakenly take to be entailed by substantivalism. Recall that, following Leibniz, Earman and Norton attribute to substantivalism the counterfactual thesis that if everything to the East were swapped with everything to the West, then a physical difference would result. This premise, together with manifold substantivalism, is used to justify the claim that the substantivalist must believe that diffeomorphic models of GR must represent physically distinct worlds, since they are models in which the manifold points have been moved around. Although Leibniz's Newtonian interlocutor, Clarke, took this counterfactual claim to be characteristic of substantivalism, substantivalists have since rejected it. The metaphysical doctrine that there exist possible worlds that are qualitatively identical yet *de re* distinct is called *haecceitism*. The difference that Leibniz believes would result, according to substantivalism, if the universe rotated to swap East with West is a haecceitistic difference. This is because the state of affairs that would result after the permutation is qualitatively identical to the actual world, even though its regions of space have different absolute locations and are therefore *de re* different from the corresponding unpermuted regions in the actual world. For the same reason, the world that Earman and Norton believe would be represented by $d_H * \mathscr{M}$, according to substantivalism, is qualitatively identical yet *de re* distinct from the world represented by $\mathscr{M}$. So, both Leibniz and Earman and Norton take the substantivalist to be committed to haecceitism. But haecceitism is a thesis about modality and identity: there is no reason why it must be accepted by someone who believes in the reality of space, or, today, the reality of spacetime. Anti-haecceitist substantivalism is called "sophisticated substantivalism" by Pooley (2006), and it avoids the force of the hole argument.[8]

Given that it is really haecceitism, not substantivalism, that falls into the target of the hole argument, we can appreciate one similarity between the hole argument and Putnam's model-theoretic argument, although this is not, as Liu, Rynasewich and Stachel believe, because of any semantic feature of the hole argument. Instead, there is a similarity between *haecceitism* and the aspect of metaphysical realism that made

[8]Opting for anti-haecceitist substantivalism is not the only way that the substantivalist may avoid the hole argument's conclusion. A further premise that can be challenged is manifold substantivalism. In the hole argument, in order to obtain the conclusion that the field equations for GR underdetermine certain physically significant properties of spacetime points, it is assumed that a manifold point in one model of GR and its image under diffeomorphism in another model of GR represent the *very same* spacetime point in different spatio-temporal locations. This assumption, manifold substantivalism, is a thesis about how we represent spacetime points. Its denial is perfectly compatible with substantivalism, as the thesis that spacetime exists as a real substance. Maudlin (1990), for instance, argues that we can avoid the hole argument by denying manifold substantivalism.

that position vulnerable to the model-theoretic arguments. To explain this I will need to briefly revise how, as I understand it, the model-theoretic arguments undermine metaphysical realism.

Metaphysical realism is characterized by three beliefs: that there is a "fixed totality" of objects that make up the world, that there is exactly one true theory about them, and that our theories about the world are true just if they correspond in the uniquely correct way to these objects (Putnam 1981, Ch. 3, 1989, p.352, 1999, p.18, n.41). These three claims commit the metaphysical realist to what Putnam repeatedly calls a "God's Eye" point of view, which is the external perspective required to describe correspondences between the concepts that we use in our theories of world and the "fixed totality" of objects.[9] They also imply a certain understanding of our theories' relation to the world, leading Putnam to introduce metaphysical realism as "a model of the relations of *any* correct theory to all or parts of THE WORLD." (1978, p.123) The way in which our theories must correspond to the world if they are to be true is analogous to, and can be captured by, a model-theoretic assignment function. The reason why this is so is suggested by Putnam's use of a Kantian thesis to differentiate metaphysical realism from internal realism. When Putnam first introduced internal realism, he called it a "softer (and demythologized) Kantianism" (Putnam, 1978, p.138)—a variant of Kant's position "without 'things in themselves'" (*Ibid.* p.6).[10] Putnam says that "the adoption of internal realism is the renunciation of the notion of the 'thing in itself'" (Putnam, 1987, p.36). Metaphysical realism, on the other hand, *does* hold that there is a domain of objects—the "fixed totality"—from which we may be epistemically isolated. Our theories of the world, however, are given in terms of how things are for us. They must correspond, in just the right way, to the things in themselves in order to be true.

The metaphysical realist's distinction between the things that our theories talk in terms of and the "fixed totality" of things in themselves is analogous to a distinction made between two ontologies when modeling. Specifically, there is an analogy between the correspondence between the things for us that our theories are given in terms of and the underlying domain of entities, and the correspondence in model theory between the objects a theory is about—such as the natural numbers—and the domain elements in a model of that theory—for instance, sets. This analogy permits elementary theorems from model theory to be carried over to the metaphysical realist correspondence relation. In particular, what the model-theoretic arguments demonstrate is that there are many correspondences between the two domains. This, together with the metaphysical realist's commitments about our epistemological position, entail that that nothing that the metaphysical realist can do suffices to pick out just one correspon-

[9] Putnam repeatedly claims that the metaphysical realist's beliefs characterize her as an epistemological sceptic: according to the metaphysical realist, but not to the internal realist, it is possible that we really are brains in a vat (Putnam, 1980, p.473, 1981, Ch.1, Ch.3, 1989, p.352-4).

[10] Perhaps puzzlingly, Putnam also often claimed that Kant himself was the first internal realist (1981, pp.56-65; 1987, pp.41-2), despite Kant's belief in things in themselves. This difference between internal realism and Kant's own views vanishes with the adoption of an interpretation of Kant that Putnam came to suggest later on in his internal realist phase. According to this interpretation, "one is not at all committed to a Noumenal World, or even [...] to the intelligibility of thoughts about noumena" (Putnam, 1987, p.41). Moran (2000) gives an assessment of the evidence for such an interpetaion in Kant's writings.

dence.

> The trouble [. . .] is not that correspondences between words or concepts and other entities don't exist, but that *too many* correspondences exist. To pick out just *one* correspondence between words or mental signs and mind-independent things we would have already to have referential access to the mind-independent things. You can't single out a correspondence between two things by just squeezing *one* of them hard (or doing anything else to just one of them); you cannot single out a correspondence between our concepts and the supposed noumenal objects without access to the noumenal objects. (Putnam, 1981, pp.72-3)

In short, the model-theoretic arguments target a position that understands our correspondence with reality in a way that is appropriately analogous to a model-theoretic assignment function, and show that such a correspondence is radically underdetermined.

The point I want to make about how this relates to the hole argument's target of haecceitism, is that, according to one line of thought, haecceitism may have been motivated by reasons that have more to do with the use of models as representations, than with metaphysics. As Kaplan explains, haecceitism requires a "metaphysical reality of sameness and difference which underlies [objects'] clothes" (1975, p.722). In Kaplan's metaphor, the clothes are an object's qualities. They are not limited to the object's spatiotemporal properties, but constitute an exhaustive bundle of the properties that it has. The doctrine of haecceitism gives us the resources to describe counterfactual situations in which one object has another's "clothes" or qualities. It therefore describes a duality between the objects *themselves*,[11] which may be permuted, and the qualitative places that they are permuted between; this duality resembles the metaphysical realist's duality between things in themselves and things for us.

Kaplan has suggested that the uncritical use of models has lead to some philosophers inadvertently becoming sympathetic to haecceitism, just because they have got so used to the idea of an *underlying realm of objects* that are independent of our theory about the world—that is, domain elements of models. Kaplan describes the problem:

> [T]he use of models as representatives of possible worlds has become so natural for logicians that they sometimes take seriously what are really only artifacts of the model. In particular, they are led almost unconsciously to adopt a *bare particular metaphysics*. Why? Because the model so nicely separates the bare particular from its clothing. The elements of the universe of discourse of a model have an existence which is quite independent of whatever properties the model happens to tack onto them. (1967, p.97)

Although the domain elements' properties do not affect what is being represented, they do make for distinct, isomorphic models. Kaplan's speculation is that these differences between models might "unconsciously" make plausible the idea that some corresponding physical difference is being represented by them, in addition to the theory being modeled. Of course, Kaplan's point is that this is no reason to adopt haecceitism in the

[11] In the terminology of haecceitism, these are called the "haecceities".

first place. To do so would be to derive a metaphysical thesis from what is really just a convention about representation.[12]

The models, in Kaplan's remark, are models of possible worlds. But the same warning that metaphysically substantial theses about identity may be illegitimately inferred from conventions about representation was also made by Maudlin (1990) in the specific context of models of spacetime. Recall that a general-relativistic model of spacetime is a structured manifold, expressed by the triple $\mathscr{M} := \langle M, g, \mathscr{T} \rangle$. As remarked above, there is a distinction to be drawn between the bare manifold, M, and the structured manifold, $\mathscr{M}$. The points of $\mathscr{M}$ have more properties than those of M: they have metrical properties, as encoded by g. A point of M does not come with these properties, although it is mathematically possible for it to have such properties if additional structure is defined on it. Maudlin's reply to the hole argument is to insist that the bare manifold points do not represent *anything* in models of spacetime, thereby denying Earman and Norton's premise of manifold substantivalism. Maudlin justified this claim by appeal to a Newtonian thesis of essentialism, which says that *all* spatiotemporal properties are essential to a spacetime point, and so, contra haecceitism, there just couldn't be two worlds that involve the same spacetime point having different spatiotemporal properties. Maudlin also offered a reason why Earman and Norton were led to think otherwise. His hypothesis was that in models of spacetime, which are, after all, just mathematical objects, the manifold points' metrical properties are *contingent*. This is to say, since the structured manifold is a bare set endowed with a topology and a metric, speaking somewhat metaphorically, the metrical properties could have been distributed on the set in a different way. Yet, the structured manifold that resulted would have been the very same mathematical object up to diffeomorphism (Maudlin, 1990, p.541). Earman and Norton's mistake is in simply inferring that the contingency with which the bare manifold points have their metrical properties is carried over to what those points and properties represent. Maudlin cautions that,

> One must keep in mind that the (abstract) ontological structure of the mathematical representation may suggest, but does not entail, an analogous metaphysical analysis of the physical structure that it represents. (Maudlin, 1990, pp.540-1)

Haecceitism is responsible for the underdetermination demonstrated by the hole argument. Both Kaplan and Maudlin attribute the adoption of haecceitism to the illegitimate assumption that the physical world has something like the "peculiar abstract ontology of mathematical representations" (1990, p.543). The underdetermination demonstrated by the *model-theoretic* arguments arises likewise from the thesis that theories' correspondence to reality is analogous to the assignment function of a mathematical representation.

5. Newman's objection to structuralism

As the previous section explains, Putnam originally used his model-theoretic arguments as a *reductio* of metaphysical realism. But underdetermination arguments turning on

[12]Pooley recommends the same explanation as Kaplan for the adoption of haecceitism (2006, p.103).

the premise that reference can be treated as a model-theoretic assignment function predate Putnam's, and have been used to achieve ostensibly different ends. One was made by the Cambridge mathematician Max Newman (1928) as a *reductio* of Russell's early structural realism of (1927).

Since Newman's objection was brought to bear on the current debate over structuralism by Demopolous and Friedman (1985), the consensus has been that Newman's argument and Putnam's are closely related. Demopolous and Friedman were the first to note the similarities between the two (*Ibid*, pp.634-5).[13] Lewis noted the same point (1984, p.224, n.9), and, more recently, van Frassen has maintained that Newman's objection is "at the heart of Putnam's paradox" (2008, p.239 n.5) and they are "essentially the same problem" (*Ibid.* p.231).

Demopoulos and Friedman raised the problem of how to understand the relationship between Putnam's and Newman's arguments, given that the former is intended to have a much more general target.

> It may be that Newman's argument contains an important observation about realism in *general*—not merely about *structural* realism. The recent work of Putnam, to which we have drawn attention, is intended to suggest that it does. But we have been unable to find or construct a clear statement of the connection. (Demopoulos and Friedman, 1985, pp.634-5)

I will summarize Newman's objection, and then argue for an answer to Demopoulos and Friedman's problem.

Newman's objection applies to epistemic structural realism, and was originally leveled at Russell's early version of that position, defended in (1927). Russell defended an inferential account of scientific knowledge, according to which we infer (at most) the structure of the real objects in the world on the basis of the relations between the percepts the real objects cause us to experience.

Russell made the notion of a theory's commitment only to structure mathematically precise by drawing on his earlier formalization of the notion of structural similarity in *Principia Mathematica*. He defined a relation's structure as what is shared between all relations whose extensions can be given the same graph up to a bijection, where a graph is an abstract representation of an extension (Russell, 1927, pp.249-51). It follows that the *structure of* a relation, unlike the relation itself, does not depend on its extension, but only on the "shape" of its extension:

> It is clear that the "structure" of the relation does not depend upon the particular terms that make up the field [i.e., extension] of the relation. The field may be changed without changing the structure, and the structure may be changed without changing the field. (Russell, 1919, p.61)

[13] Demopoulos wrote elsewhere that Newman's objection "lies at the basis of the formulation of the earliest and simplest of Hilary Putnam's 'model-theoretic arguments'" (2003, p.385).

It follows from Russell's definition of the structure of a relation that it specifies only the second-order properties of its *relata*. For instance, the structure of a relation will specify whether or not the relation is transitive, its arity, perhaps the cardinality of its extension, and so on. Russell's epistemic structural realism thus grants very impoverished knowledge of the world, restricted to only the logical properties of the relations in the world. Adherents of this strong brand of epistemic structural realism would hold that the most that science licenses belief in is a second-order sentence of the form $\exists X_1 \exists X_2 \ldots \exists X_m \Delta(X_1 X_2 \ldots X_m)$, in which the only non-logical terms are existentially-quantified variables.

Contemporary epistemic structural realists hold that we can know *some* objects and relations directly; in general these are the observable ones, although see note 15 below. In accordance with this idea, epistemic structural realists typically hold that the best way to express their commitment only to the structure of the world is to believe only the *second-order Ramsey sentence* of a theory about the world, defined as follows. If our theory of the world has the form $\Gamma(t_1, \ldots, t_n, o_1, \ldots, o_k)$, where the t_i are theoretical terms and the o_j are observational terms,[14] then the Ramsey sentence of Γ is formed by replacing all the t_i with variables—either first- or second-order, depending on whether they replace a constant of relation symbol—and then existentially quantifying over the resulting formula.[15] This process is Ramsification. It results in a second-order sentence (assuming that at least one predicate symbol or relation symbol occurs in Γ), $\mathscr{R}(\Gamma)$, of the following form

$$\mathscr{R}(\Gamma) := \exists X_1 \ldots \exists X_n [\Gamma(X_1, \ldots X_m, o_1, \ldots, o_k)]$$

Note that the sentence above, licensed by Russellian epistemic structural realism, is a limiting instance of Ramsey sentence. The Ramsey sentence of a scientific theory doesn't tell us much about the referents of the theoretical terms, but it does tell us something. In particular, the Ramsey sentence encodes the same kind of information about them that Russell's abstract notion of a structure told us: the finite cardinality of some relation's extension, whether some relations are transitive, their arities, and so on.

The mathematician Max Newman raised a fundamental objection to Russell's early epistemic structural realism that affects any version of the position that limits our knowledge of the world to a Ramsified theory. The problem Newman raised concerns the ability of theories to latch onto *even the structure* of the physical world that they aim to describe. It does so by demonstrating that a domain that is not *already* carved up into privileged relations and subsets will satisfy any Ramsified theory whatsoever, so long as the theory does not impose constraints on cardinality that are incompatible with the cardinality of the domain. In other words, Newman's objection is that, beyond saying something about a theory's observable objects, a Ramsey sentence determines

[14] A theoretical term is any non-logical symbol that refers to an unobservable object, property or relation.

[15] Ainsworth distinguishes between two versions of epistemic structural realism: one brand which urges that terms for unobservable objects be Ramsified, and another which Ramsifies terms for objects that are not "internal", which is to say, those that do not have an immediate counterpart in our internal, phenomenal experience (2009, pp.139-40).

no more than a cardinality constraint on the domain on which it is true. And so, as a theory of the world, a Ramsey sentence is almost entirely trivial. The reason why a Ramsey sentence can only determine the cardinality of the domain it describes is that a Ramsey sentence only contains non-logical terms for observable objects, and describes the unobservable ones with quantified variables. But in order to assign objects and sets to the first and second-order variables of the Ramsey sentence, all it takes is for the domain to be large enough. Then we can assign members of the powerset of the domain to the second-order variables for monadic predicates, and members of Cartesian products of these to the second-order variables for relations.[16] This will make the Ramsey sentence come out true. Any domain that meets the cardinality constraints imposed by the Ramsey sentence will therefore satisfy the Ramsey sentence whenever its second-order quantifiers have unrestricted range. Newman thus objected to Russell's epistemological theory as follows:

> [N]o important information about the aggregate *A*, except its cardinal number, is contained in the statement that *there exists* a system of relations, with *A* as field [domain], whose structure is an assigned one. (1928, p.140).

Consequently, Newman's objection is generally taken to show that given epistemic structural realism, the content of scientific theories is trivialized.

I contend that Newman's objection to epistemic structural realism is just like Putnam's permutation argument against metaphysical realism. Putnam's permutation argument shows that simply knowing *that* a theory is true of a given domain—again, of the right size—does not determine the reference relation for the theory's sub-sentential terms. In making their respective arguments, Newman and Putnam are each relying on the same combinatorial fact that there are many assignment functions on a domain that will make a given theory true of it. Putnam used this fact to make his *negative* claim that no unique correspondence between objects and sub-sentential terms can be uniquely determined by stipulating which sentences are true. Newman tells us the extent of what *can* be determined by stipulating that a Ramsified theory is true: the domain's cardinality, and nothing else.

6. Demopolous and Friedman's puzzle

Demopolous and Friedman raise an interesting question that I touched on earlier (1985, pp.634-5). They ask how to formulate the connection between Newman's objection and Putnam's argument , given that the former is aimed at structural realism, whereas the latter aspires to a goal that is altogether wider in scope. I believe that we can go some way to answering Demopolous and Friedman's question. First, note that Newman's objection doesn't affect the eliminativist ontic structural realist, but instead relies on the epistemic inaccessibility of the (unobservable) objects that science aims at describing. Both Putnam's argument and Newman's objection are used to target strong epistemic

[16] Equally, because it is possible for the Ramsey sentence to describe an upper bound on the number of individuals, in some cases we must also ensure that the domain is not *too* big. Of course, if the theory is not committed to a finite number of individuals then, by the upwards Löwenheim-Skolem Theorem there is no upper bound on the cardinality of the domain.

humility theses that tell us that there is a bedrock of objects that are either possibly unknowable, in the case of Putnam's target of metaphysical realism, or actually unknowable, in the case of Newman's target of epistemic structural realism. Putnam's and Newman's arguments demonstrate that these respective positions cannot avoid a radical form of underdetermination, for on neither position are theories able to *determine their subject matter*. So, Newman's objection does address a wider target than epistemic structural realism—it aims at any realist position committed to the strong epistemic humility thesis of metaphysical realism.

One immediate reason to question my contention that Newman's and Putnam's arguments are essentially the same is that Newman's objection applies to *Ramsified* theories, whereas Putnam's permutation argument is valid for theories of any form. I want to claim that this difference is less significant than it may first appear. This is because, as I shall now explain, Putnam's permutation argument forces the metaphysical realist to treat her theories *as if* they were Ramsified.

The epistemic structural realist believes, with the metaphysical realist, that there is a fixed totality of objects in the world which are the target of scientific theories. Epistemic structural realism claims that we are unable to know anything about at least the unobservable portion of these, beyond their structure. Because we occupy such a restricted epistemic position, the epistemic structural realist believes that we cannot refer to the objects in question,[17] and expresses this belief by committing herself only to Ramsified theories.

[17] Worrall's structural realism is supported by his prior commitment to the referential indeterminacy of theoretical terms, especially given a causal theory of reference (Worrall, 2007, pp.148-9). In order to single out the referents of theoretical terms, Worrall claims, we would need to have some access to them independently of the theories they feature in. But, because the referents are unobservable by hypothesis, we do not have this. Worrall's description of the problem echoes McDowell's (1994) metaphor of "standing outside" one's language, that Putnam also uses frequently to describe the metaphysical realist account of reference. Worrall writes,

> It is just a fantasy (given credence by unthinking reflection on orthodox logical semantics) that we can "stand outside" of our theories and directly compare terms in them with a reality that we can access directly without any theory. No one *really* believes this, though many act (and even sometimes write) as if they do. But once this is recognised as the fantasy it is, then there just is no question but that the Ramsey sentence of T [...] captures the full cognitive content of T. (Worrall, 2007, pp.148-9)

Because he already accepts radical underdetermination of reference for theoretical terms, Worrall does not so much bite the bullet of Newman's objection as wholeheartedly endorse it. Russell, it seems, also positively denied that there is any determinate reference relation:

> The reason that [knowing the structure of relations] is important is that it represents, much more nearly than might be supposed, the state of our knowledge of nature. We know that certain scientific propositions—which, in the most advanced sciences, are expressed in mathematical symbols—are more or less true of the world, but we are very much at sea as to the interpretation to be put upon the terms which occur in these propositions. We know much more [...] about the *form* of nature than about the *matter*. Accordingly, what we really know when we enunciate a law of nature is only that there is probably *some* interpretation of our terms which will make the law approximately true. (Russell, 1919, p.55, emphasis added)

Putnam believes that the metaphysical realist cannot rule out the possibility that she is in just as impoverished an epistemic situation as the epistemic structural realist, and that this is what makes her susceptible to the underdetermination conclusion of the model-theoretic arguments. This is borne out by what Putnam says in one of the many places in which he makes his "just more theory" reply to the metaphysical realist. Putnam's just more theory reply is a response to the objection that, even though there are many different possible correspondence relations that could make our theory come out true, just *one* of them tracks the *actual* relationship that names have to their bearers. This relationship might be causal, for instance. Putnam just denies that the metaphysical realist can appeal to a constraint like this, for the reason that, in doing so the metaphysical realist is "*ignoring [her] epistemological position*" (Putnam, 1983, p.xi, emphasis added). The metaphysical realist's "epistemological position" is such that it is entirely possible that she is, in principle, ignorant of the "fixed totality" of object (including causal relations, and other relations that may be responsible for reference-fixing) to which her theory must correspond in order to be true. And, if she *is* epistemically isolated from the objects in the world, then she cannot rely on the ability to determinately refer to them. To summarize, the epistemic position that the epistemic structural realist positively believes herself to be in, and commits herself to by Ramsifying her theories of the world, is the *same position* as that which Putnam thinks the metaphysical realist is *unable to rule out*.

7. Conclusion

I have argued that the hole argument cannot be understood as a version of Putnam's permutation argument, as Liu and Rynaseiwicz allege. Despite this, the hole argument's target, haecceitism, and the permutation argument's target, metaphysical realism, are similar so far as the former is claimed to be inferred from the structure of mathematical models, and, according to Putnam, the metaphysical realist conception of our relation to the world so much resembles the assignment function of a mathematical model that we are justified in carrying over theorems about modeling to it. Finally, I have argued that Newman's objection has a similar target as the model-theoretic arguments do: namely, the epistemic humility thesis that there are objects beyond our epistemic access.

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