Circumventing the Coarse-Grained Limit: Floquet-Engineered Cooling Cycles for Superconducting Cavities

Research Proposal based on Vashaee & Abouie (2025)

January 3, 2026

Abstract

Passive quantum refrigeration using stochastic streams of correlated ancillas is constrained by a fundamental thermodynamic bound derived from the coarse-grained detailed balance of the interaction [Vashaee & Abouie, arXiv:2512.06996]. We propose to circumvent this "stochastic limit" by replacing the random Poissonian stream with **numerically discovered, coherent Floquet cooling cycles**. By transitioning to a structured, periodic interaction protocol, we exploit coherent interference effects—specifically non-vanishing commutators in the Floquet-Magnus expansion—to selectively suppress heating transitions while enhancing cooling pathways. We outline a rigorous three-tier validation method using Deep Reinforcement Learning (DRL) benchmarked against Gradient Ascent Pulse Engineering (GRAPE), incorporating realistic noise models (TLS defects, $1/f$ noise, finite T_1/T_2). We aim to demonstrate that these "digital" cycles can achieve steady-state temperatures significantly lower than the analytic limits of the stochastic model while maintaining a favorable coefficient of performance (COP) within the 1 K stage energy budget.

1 Motivation: The Stochastic Bottleneck

The Vashaee-Abouie model establishes a framework for cooling a microwave cavity using a stream of correlated qubit pairs. However, the cooling limit is bounded by the **coarse-grained Lindblad generator** derived from the random Poissonian arrival of pairs. The effective steady-state generator is given by [1]:

$$\mathcal{L}_{stream}\rho \propto R\phi^2 \left[r_2^{(2)}\mathcal{D}[a]\rho + r_1^{(2)}\mathcal{D}[a^\dagger]\rho\right] \tag{1}$$

where the ratio of cooling ($r_2^{(2)}$) to heating ($r_1^{(2)}$) rates is fixed by the thermal and correlated state of the ancilla pair [1]. This stochastic approach effectively "washes out" phase information, treating the interaction as a purely incoherent thermodynamic resource.

The Opportunity: In modern circuit QED (cQED), interactions are triggered by high-precision clocks. We propose to replace the *stochastic* stream with a *periodic* (Floquet) unitary sequence $U_{cycle}(t)$. By optimizing the timing, detuning $\Delta(t)$, and coupling $g(t)$, we can engineer an effective Hamiltonian where heating matrix elements interfere destructively.

2 Theoretical Framework: Floquet-Magnus Expansion

To demonstrate the physical mechanism for circumventing the stochastic limit, we consider a periodic protocol of duration T_{cycle}. The evolution is governed by the stroboscopic map $\mathcal{E}_{cycle} = \mathcal{T}\exp\left(\int_0^{T_{cycle}} \mathcal{L}(t)dt\right)$. Using the Floquet-Magnus expansion, the effective generator

$\mathcal{L}_{eff}$ approximates to:

$$\mathcal{L}_{eff} \approx \frac{1}{T_{cycle}} \int_0^{T_{cycle}} \mathcal{L}(t) dt + \frac{1}{2T_{cycle}} \int_0^{T_{cycle}} \int_0^{t_1} [\mathcal{L}(t_1), \mathcal{L}(t_2)] \, dt_2 \, dt_1 + \dots \tag{2}$$

In the stochastic limit, the **commutator terms** average to zero. In a coherent cycle, we engineer $[\mathcal{L}(t_1), \mathcal{L}(t_2)] \neq 0$ to renormalize the transition rates.

Hypothesis 1. *A periodic protocol beats the stochastic limit if and only if the interaction Hamiltonian is non-commuting at different times within the cycle (*$[H_{int}(t_1), H_{int}(t_2)] \neq 0$*), allowing for a coherent violation of the effective detailed balance.*

3 Methodology: Three-Tier Simulation & Optimization

We employ a "Physics-Informed AI" approach, benchmarking Soft Actor-Critic (SAC) against standard optimal control (GRAPE). Validation occurs in three tiers of increasing rigor.

3.1 Tier 1: Fast Stroboscopic Map (Discovery)

- **Physics:** Discrete Kraus maps representing instantaneous unitary kicks + free evolution.
- **Purpose:** Rapid exploration of cycle period T and number of steps N.
- **Benchmark:** RL results compared against random search and gradient-descent baselines.

3.2 Tier 2: Full Time-Dependent Unitary (Physics Validation)

- **Physics:** Integration of the time-dependent Schrödinger equation (TDSE).
- **Critical Check:** Includes **counter-rotating terms** and non-secular effects ignored in the RWA limit of the base paper [1]. This ensures the cooling advantage is physical and not an artifact of RWA breakdown.
- **Controls:** Continuous modulation of coupling $g(t)$ and detuning $\Delta(t)$.

3.3 Tier 3: The "Dirty" Model (Robustness)

- **Physics:** Full open-system dynamics with realistic noise defined in Table 2 of [1].
- **Noise Sources:** Finite qubit T_1/T_2; TLS spectral defects (detuning holes); Reset errors $(1-3\%$ residual population).
- **Goal:** Prove robustness against $\pm 10\%$ parameter drift.

4 Thermodynamic Viability: The Energy Budget

To address thermodynamic concerns, we explicitly model the **Coefficient of Performance (COP)**: $\eta = \dot{Q}_{cool}/P_{input}$.

- **Cooling Power:** $\dot{Q}_{cool} = \hbar\omega_{cav}\Gamma_{\downarrow}^{eff}\langle n\rangle$.
- **Input Power:** $P_{input} = P_{ctrl} + P_{reset}$.
- **Constraint:** $P_{input} \ll P_{1K\_stage} \approx$ 100–500 mW [1].

We will demonstrate that the control pulses (integrated $\Omega^2(t)$) add negligible heat load compared to the 1 K stage capacity.

5 Proposed Results

1. **Circumventing the Limit:** A comparison plot of T_{cav} vs. Detuning Δ.
   - *Baseline:* Analytic Poisson limit (Eq. 37 in [1]).
   - *Floquet:* AI-discovered protocol. (Must show $T_{Floquet} < T_{Stochastic}$).
2. **The Recipe:** Explicit waveform diagrams $\{g(t), \Delta(t)\}$ for experimental AWG implementation.
3. **Thermodynamic Cost:** A plot of Entropy Efficiency (η) vs. Cooling Rate, proving the protocol is thermodynamically sound.

Extension: New Ideas & Implementation Candidates

Based on recent critiques, we identify the following high-value extensions that are feasible within the current simulation framework:

1. **The "No-Go" Theorem Verification:** We will numerically verify that if the cycle Hamiltonian commutes with itself at all times ($[H(t), H(t')] = 0$), the RL agent *cannot* beat the stochastic limit. This provides a negative control case that strengthens the physics claim.
2. **Adaptive Detuning ($\Delta(t)$):** Instead of fixed detuning, the control scheme will allow dynamic modulation of the qubit frequency. This creates "interference bands" in the frequency domain, akin to Floquet topological insulators, potentially blocking phonon absorption channels entirely.
3. **The Entropy-per-Joule Metric:** We will introduce a specific figure of merit: $\eta_S = \Delta S_{cav}/W_{pulse}$, quantifying the bits of entropy removed per Joule of control work. This elevates the work from "engineering" to "quantum thermodynamics."
4. **Hybrid Semi-Feedback Loop:** Implementation of a "weak-measurement" step at the end of every N cycles. This allows the protocol to switch between two distinct Floquet cycles based on a coarse readout, bridging the gap between open-loop control and full feedback cooling.

References

[1] D. Vashaee and J. Abouie, "Quantum Correlation Assisted Cooling of Microwave Cavities Below the Ambient Temperature," arXiv:2512.06996 (2025).