EXERCISE 13.4

1. State which of the following are not the probability distributions of a random variable. Give reasons for your answer

X	0	1	2
P(X)	0.4	0.4	0.2

i

X	0	1	2	3	4
P(X)	0.1	0.5	0.2	-0.1	0.3

ii

Y	-1	0	1
P(Y)	0.6	0.1	0.2

iii

Z	3	2	1	0	-1
P(Z)	0.3	0.2	0.4	0.1	-0.05

iv

2. An urn contains 5 red and 2 black balls. Two balls are randomly drawn. Let X represent the number of black balls. What are the possible values of X? Is X a random variable ?

3. Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 6 times. What are possible values of X?

4. Find the probability distribution of

i number of heads in two tosses of a coin.

ii number of tails in the simultaneous tosses of three coins.

iii number of heads in four tosses of a coin.

5. Find the probability distribution of the number of successes in two tosses of a die, where a success is defined as

   i number greater than 4

   ii six appears on at least one die

6. From a lot of 30 bulbs which include 6 defectives, a sample of 4 bulbs is drawn at random with replacement. Find the probability distribution of the number of defective bulbs.

7. A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed twice, find the probability distribution of number of tails.

8. A random variable X has the following probability distribution

   Determine

X	0	1	2	3	4	5	6	7
P(X)	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2 + K$

   i

   ii k

   iii P($X < 3$)

   iv P($X > 6$)

   v P($0 < X < 3$)

9. The random variable X has a probability distribution P(X) of the following form, where k is some number :

$$P(x) = \begin{cases} k, & \text{if } x = 0 \\ 2k, & \text{if } x = 1 \\ 3k, & \text{if } x = 2 \\ 0, & otherwise \end{cases}$$

   a Determine the value of k.

   b Find P ($X < 2$), P ($X \leq 2$), P($X \geq 2$)

10. Find the mean number of heads in three tosses of a fair coin.

11. Two dice are thrown simultaneously. If X denotes the number of sixes, find the expectation of X.

12. Two numbers are selected at random (without replacement) from the first six positive integers. Let X denote the larger of the two numbers obtained. Find E(X).

13. Let X denote the sum of the numbers obtained when two fair dice are rolled. Find the variance and standard deviation of X.

14. A class has 15 students whose ages are 14, 17, 15, 14, 21, 17, 19, 20, 16, 18, 20, 17, 16, 19 and 20 years. One student is selected in such a manner that each has the same chance of being chosen and the age X of the selected student is recorded. What is the probability distribution of the random variable X? Find mean, variance and standard deviation of X.

15. In a meeting, 70A member is selected at random and we take X = 0 if he opposed, and X = 1 if he is in favour. Find E(X) and Var (X).

Choose the correct answer in each of the following:

16. The mean of the numbers obtained on throwing a die having written 1 on three faces, 2 on two faces and 5 on one face is

    a 1

    b 2

    c 5

    d $\frac{8}{3}$

17. Suppose that two cards are drawn at random from a deck of cards. Let X be the number of aces obtained. Then the value of E(X) is

    A $\frac{37}{221}$

    B $\frac{5}{13}$

    C $\frac{1}{13}$

    D $\frac{2}{13}$