Properties of vectors

1 12^{th} Maths - Exercise 10.4.10

1. The area of a parallelogram whose adjacent sides are represented by the vectors $\mathbf{a} = \hat{i} - \hat{j} + 3\hat{k}$ and $\mathbf{b} = 2\hat{i} - 7\hat{j} + \hat{k}$

2 Solution

Now,

$$\text{Let } \mathbf{A} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 2 \\ -7 \\ 1 \end{pmatrix} \tag{1}$$

$$\tag{2}$$

The cross product or vector product of $\mathbf{A}, \mathbf{B}$ is defined as

$$\mathbf{A} \times \mathbf{B} = \begin{pmatrix} \begin{vmatrix} \mathbf{A}_{23} & \mathbf{B}_{23} \\ \mathbf{A}_{31} & \mathbf{B}_{31} \end{vmatrix} \\ \begin{vmatrix} \mathbf{A}_{12} & \mathbf{B}_{12} \end{vmatrix} \end{pmatrix} \tag{3}$$

Hence

$$\begin{vmatrix} \mathbf{A}_{23} & \mathbf{B}_{23} \end{vmatrix} = \begin{vmatrix} -1 & -7 \\ 3 & 1 \end{vmatrix} = \Big(-1 + 21\Big) = 20 \tag{4}$$

$$\begin{vmatrix} \mathbf{A}_{31} & \mathbf{B}_{31} \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} = \Big(1 - 6\Big) = -5 \tag{5}$$

$$\begin{vmatrix} \mathbf{A}_{12} & \mathbf{B}_{12} \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ -1 & -7 \end{vmatrix} = \Big(-7 + 2\Big) = -5 \tag{6}$$

$$\tag{7}$$

which can be represented in matrix form as

$$\mathbf{A} \times \mathbf{B} = \begin{pmatrix} 20 \\ -5 \\ -5 \end{pmatrix}. \tag{8}$$

Hence

$$\|\mathbf{A} \times \mathbf{B}\| = \sqrt{20^2 - 5^2 - 5^2} \tag{9}$$

$$= 15\sqrt{2} \tag{10}$$

$$\tag{11}$$