Vector Algebra

1   12th Maths - Exercise 10.3.13

1. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$, find the value of $\vec{a}\,\vec{b} + \vec{b}\,\vec{c} + \vec{c}\,\vec{a}$.

2   Solution

The given vectors $\mathbf{a}, \mathbf{b}$ and $\mathbf{c}$ are unit vectors

The magnitudes vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are

$$\|\mathbf{a}\| = \sqrt{1^2} = 1 \tag{1}$$

$$\|\mathbf{b}\| = \sqrt{1^2} = 1 \tag{2}$$

$$\|\mathbf{c}\| = \sqrt{1^2} = 1 \tag{3}$$

The Given equation is $\mathbf{a} + \mathbf{b} + \mathbf{c} = 0$

$$\|\mathbf{a} + \mathbf{b} + \mathbf{c}\|^2 = 0^2 \tag{4}$$

$$\|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 + \|\mathbf{c}\|^2 + 2(\mathbf{ab} + \mathbf{bc} + \mathbf{ca}) = 0 \tag{5}$$

$$\|1\|^2 + \|1\|^2 + \|1\|^2 + 2(\mathbf{ab} + \mathbf{bc} + \mathbf{ca}) = 0 \tag{6}$$

$$3 + 2(\mathbf{ab} + \mathbf{bc} + \mathbf{ca}) = 0 \tag{7}$$

$$\mathbf{ab} + \mathbf{bc} + \mathbf{ca} = \frac{-3}{2} \tag{8}$$