Circles

1 11^{th} Maths - Exercise 11.1.8

1. Find the centre and radius of the given circle $\mathbf{x}^2+\mathbf{y}^2-8\mathbf{x}+10\mathbf{y}-12=0$

2 Solution

The general equation of the circle is

$$\|\mathbf{x}\|^2 + 2\mathbf{u}^\top\mathbf{x} + f = 0 \tag{1}$$

by using above equation

$$\mathbf{x}^2 + \mathbf{y}^2 - 8\mathbf{x} + 10\mathbf{y} - 12 = 0 \tag{2}$$

$$\|\mathbf{x}\|^2 + 2\begin{pmatrix}-4 & 5\end{pmatrix}\mathbf{x} - 12 = 0 \tag{3}$$

Where,

$$\mathbf{u} = -\mathbf{c} \text{ and } f = \|\mathbf{u}\|^2 - r^2 \tag{4}$$

$$\mathbf{u} = \begin{pmatrix}-4\\5\end{pmatrix} \tag{5}$$

$$f = -12 \tag{6}$$

$$\mathbf{c} = \begin{pmatrix}4\\-5\end{pmatrix} \tag{7}$$

$$r^2 = \|\mathbf{u}\|^2 - f \tag{8}$$

$$r^2 = 53, r = \sqrt{53} \tag{9}$$

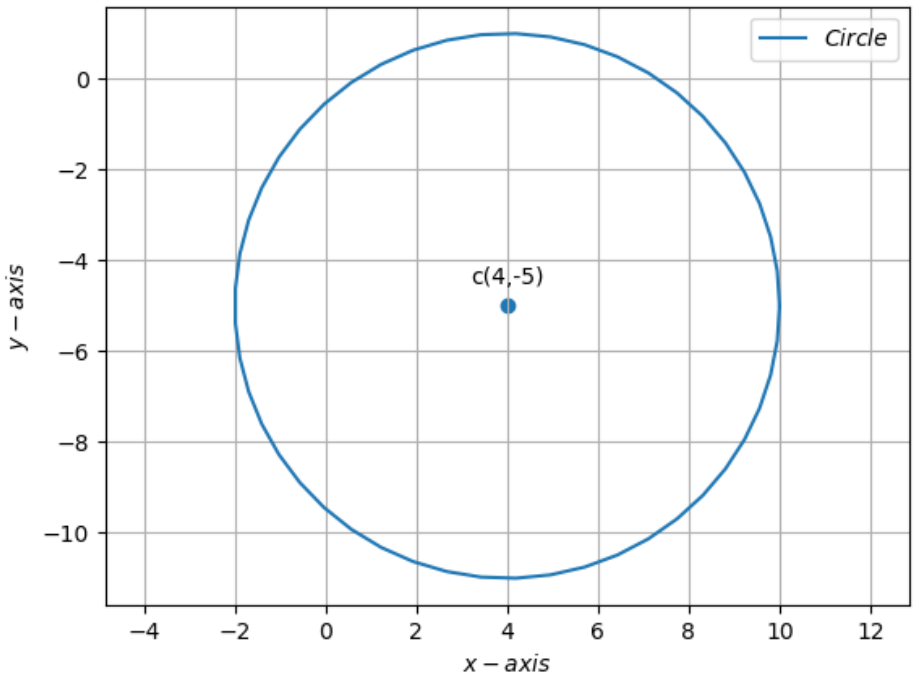

Figure 1

radius of circle is positive so the centre and radius of circle is $\begin{pmatrix} 4 \\ -5 \end{pmatrix}$ and $\sqrt{53}$ respectively