Vector Algebra

1 12^{th} Maths - Exercise 10.2.14

1. Show that the vector $\hat{i}+\hat{j}+\hat{k}$ is equally inclined to the axes OX, OY, and OZ

2 Solution

$$\text{a}=1\hat{i}+1\hat{j}+1\hat{k}$$

$$\text{Let } \mathbf{A} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \tag{1}$$

$$\tag{2}$$

let $\mathbf{A}$ be the given vector The Directional vectors of x, y and z axes are given respectively

$$\mathbf{b_1} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \mathbf{b_2} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \mathbf{b_3} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{3}$$

The magnitudes for $\mathbf{A}$ and directional vectors $\mathbf{e_1}, \mathbf{e_2}, \mathbf{e_3}$ are

$$\|\mathbf{A}\| = \sqrt{1^2+1^2+1^2} = \sqrt{3} \tag{4}$$

$$,\|\mathbf{b_1}\| = \sqrt{1^2+0^2+0^2} = \sqrt{1} \tag{5}$$

$$\|\mathbf{b_2}\| = \sqrt{0^2+1^2+0^2} = \sqrt{1} \tag{6}$$

$$\|\mathbf{b_3}\| = \sqrt{0^2+0^2+1^2} = \sqrt{1} \tag{7}$$

Let

$$\cos\theta_i = \frac{\mathbf{A}^T\mathbf{b}}{\left|\mathbf{A}\right|\left|\mathbf{b}\right|} \tag{8}$$

$$\tag{9}$$

So for different values of $\cos\theta_i$ the direction cosines of vector $\mathbf{A}$ are

$$\cos\theta_1 = \frac{\begin{pmatrix}1 & 1 & 1\end{pmatrix}\begin{pmatrix}1\\0\\0\end{pmatrix}}{\sqrt{3}\sqrt{1}} = \frac{1}{\sqrt{3}} \tag{10}$$

$$\cos\theta_2 = \frac{\begin{pmatrix}1 & 1 & 1\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix}}{\sqrt{3}\sqrt{1}} = \frac{1}{\sqrt{3}} \tag{11}$$

$$\cos\theta_3 = \frac{\begin{pmatrix}1 & 1 & 1\end{pmatrix}\begin{pmatrix}0\\0\\1\end{pmatrix}}{\sqrt{3}\sqrt{1}} = \frac{1}{\sqrt{3}} \tag{12}$$

$$\cos\theta_1 = \cos\theta_2 = \cos\theta_3 = \frac{1}{\sqrt{3}} \tag{13}$$

Hence the given vectors equally inclined to axes OX,OY,OZ