

Vector Algebra

1 12th Maths - Exercise 10.3.13

1. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$, find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

2 Solution

The given vectors \mathbf{a}, \mathbf{b} and \mathbf{c} are unit vectors

The magnitude of vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are

$$\|\mathbf{a}\| = \sqrt{1^2} = 1 \quad (1)$$

$$\|\mathbf{b}\| = \sqrt{1^2} = 1 \quad (2)$$

$$\|\mathbf{c}\| = \sqrt{1^2} = 1 \quad (3)$$

The Given equation is

$$\mathbf{a} + \mathbf{b} + \mathbf{c} = 0 \quad (4)$$

$$(\mathbf{a} + \mathbf{b} + \mathbf{c})^2 = 0^2 \quad (5)$$

$$\mathbf{a}^\top \mathbf{a} + \mathbf{b}^\top \mathbf{b} + \mathbf{c}^\top \mathbf{c} + 2(\mathbf{a}^\top \mathbf{b} + \mathbf{b}^\top \mathbf{c} + \mathbf{c}^\top \mathbf{a}) \implies 0 \quad (6)$$

$$1^2 + 1^2 + 1^2 + 2(\mathbf{a}^\top \mathbf{b} + \mathbf{b}^\top \mathbf{c} + \mathbf{c}^\top \mathbf{a}) \implies 0 \quad (7)$$

$$3 + 2(\mathbf{a}^\top \mathbf{b} + \mathbf{b}^\top \mathbf{c} + \mathbf{c}^\top \mathbf{a}) \implies 0 \quad (8)$$

$$\mathbf{ab} + \mathbf{bc} + \mathbf{ca} \implies \frac{-3}{2} \quad (9)$$