

Vector Algebra

1 12th Maths - Exercise 10.3.13

1. If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a} + \vec{b} + \vec{c} = 0$, find the value of $\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}$.

2 Solution

The given vectors \vec{a}, \vec{b} and \vec{c} are unit vectors

The magnitudes vectors $\vec{a}, \vec{b}, \vec{c}$ are

$$\|\vec{a}\| = \sqrt{1^2} = 1 \quad (1)$$

$$\|\vec{b}\| = \sqrt{1^2} = 1 \quad (2)$$

$$\|\vec{c}\| = \sqrt{1^2} = 1 \quad (3)$$

Now we have $\vec{a} + \vec{b} + \vec{c} = 0$

$$\|\vec{a} + \vec{b} + \vec{c}\|^2 = 0^2 \quad (4)$$

$$\|\vec{a}\|^2 + \|\vec{b}\|^2 + \|\vec{c}\|^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0 \quad (5)$$

$$1^2 + 1^2 + 1^2 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0 \quad (6)$$

$$3 + 2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = 0 \quad (7)$$

$$2(\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a}) = -3 \quad (8)$$

$$\vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{c} + \vec{c} \cdot \vec{a} = \frac{-3}{2} \quad (9)$$