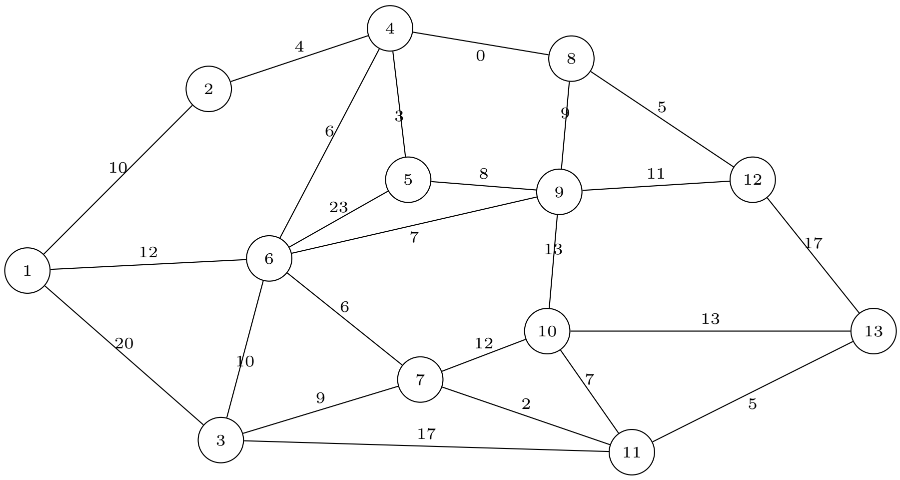
Optimization of Logistics Systems / Logistics Systems Planning I
4. Exercise

Exercise 4.1 (Kruskal's algorithm)

Given the street network in the following figure:



The local energy supplier wants to build a new district heating network. To do this, all nodes in the city must be connected to the district heating network. The energy supplier produces at node 1 and would like to supply all nodes with energy. What does a minimum-cost energy network look like if the cost of installing a line is proportional to the weights on the edges of the graph? Solve the problem using Kruskal's algorithm!

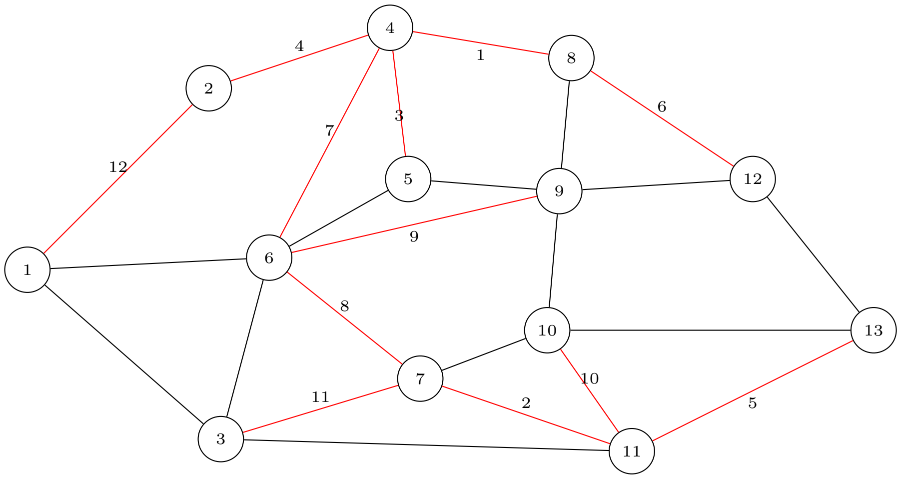
Solution:

Kruskal's algorithm:

1. $T = \emptyset$
2. Terminate if $|T| = |V| - 1$
3. Choose minimal-cost edge e which has not yet been considered
4. If $(V, T \cup \{e\})$ contains no circle, set $T := T \cup \{e\}$. Go to 2!

Sort the edges according to non-decreasing weights:

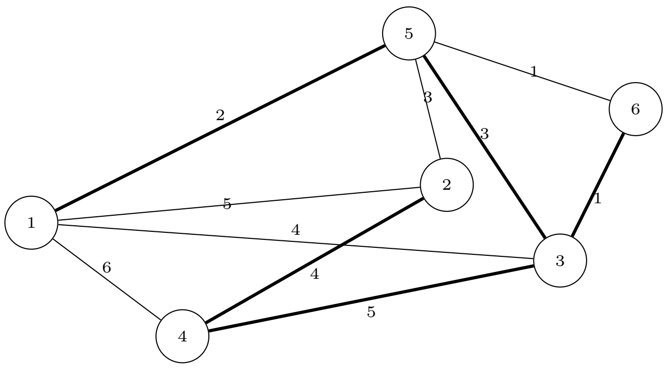
Edge (i,j)	Weight	yes/no	Edge (i,j)	Weight	yes/no
(4,8)	0	yes	(10,11)	7	yes
(7,11)	2	yes	(6,9)	7	yes
(4,5)	3	yes	(5,9)	8	no
(2,4)	4	yes	(8,9)	9	no
(11,13)	5	yes	(3,7)	9	yes
(8,12)	5	yes	(6,3)	10	no
(4,6)	6	yes	(1,2)	10	yes FINISH!
(6,7)	6	yes	(9,12)	11	no



In the figure, edge weights correspond to the **order in which the edges are added to the MST**.

Exercise 4.2 (MST)

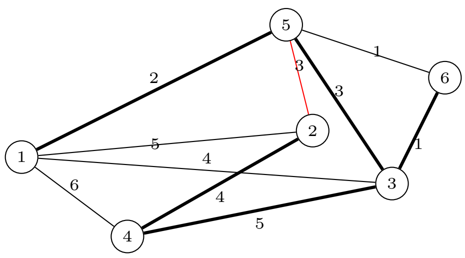
The following graph is given, the edges marked in bold represent a spanning tree:



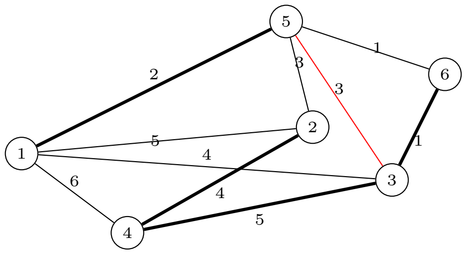
1. Use the path optimality conditions to prove that the graph is not an MST. Give the corresponding path and edges.
2. Use the cut optimality conditions to prove that the graph is not an MST. Give the corresponding path and edges.
3. Construct an MST for the graph.

Solution:

1. Path optimality: For example, non-tree edge $\{2, 5\}$ with cost 3 and associated tree path $2 - 4 - 3 - 5$. Edge $\{4, 3\}$ of the tree has higher cost of 5.



2. Cut optimality: Remove edge $\{3, 5\}$ with cost 3, then the two resulting components are defined by the node sets $\{1, 5\}$ and $\{2, 3, 4, 6\}$. The edge $\{5, 6\}$ from the cut set has smaller cost of 1.



3. Kruskal: (3,6) yes, (5,6) yes, (1,5) yes, (2,5) yes, (3,5) no, (1,3) no, (2,4) yes

⇒ Edge weights correspond to the **order in which the edges are added to the MST**.