

## 1.

# Basic Concepts

## Theory at a Glance (For GATE, IES & PSUs)

### Intensive and Extensive Properties

**Intensive property:** Whose value is independent of the size or extent i.e. mass of the system. These are, e.g., pressure  $p$  and temperature  $T$ .

**Extensive property:** Whose value depends on the size or extent i.e. mass of the system (upper case letters as the symbols). e.g., Volume, Mass (V, M). If mass is increased, the value of extensive property also increases. e.g., volume  $V$ , internal energy  $U$ , enthalpy  $H$ , entropy  $S$ , etc.

**Specific property:** It is a special case of an intensive property. It is the value of an extensive property per unit mass of system. (Lower case letters as symbols) eg: specific volume, density ( $v, \rho$ ).

### Thermodynamic System and Control Volume

- In our study of thermodynamics, we will choose a *small part of the universe* to which we will apply the laws of thermodynamics.  
We call this subset a **SYSTEM**.
- The thermodynamic system is analogous to the free body diagram to which we apply the laws of mechanics, (i.e. Newton's Laws of Motion).
- **The system is a macroscopically identifiable collection of matter on which we focus our attention** (e.g., the water kettle or the aircraft engine).

### Concept of Continuum

The concept of continuum is a kind of idealization of the continuous description of matter where the properties of the matter are considered as continuous functions of space variables. Although any matter is composed of several molecules, the concept of continuum assumes a continuous distribution of mass within the matter or system with no empty space, instead of the actual conglomeration of separate molecules.

Describing a fluid flow quantitatively makes it necessary to assume that flow variables (pressure, velocity etc.) and fluid properties vary continuously from one point to another. Mathematical descriptions of flow on this basis have proved to be reliable and treatment of fluid medium as a continuum has firmly become established.

For example density at a point is normally defined as

$$\rho = \lim_{\Delta V \rightarrow 0} \left( \frac{m}{\Delta V} \right)$$

Here  $\Delta V$  is the volume of the fluid element and  $m$  is the mass

If  $\Delta V$  is very large  $\rho$  is affected by the inhomogeneities in the fluid medium. Considering another extreme if  $\Delta V$  is very small, random movement of atoms (or molecules) would change their number at different times. In the continuum approximation point density is defined at the smallest magnitude of  $\Delta V$ , before statistical fluctuations become significant. This is called continuum limit and is denoted by  $\Delta V_c$ .

$$\rho = \lim_{\Delta V \rightarrow \Delta V_c} \left( \frac{m}{\Delta V} \right)$$

One of the factors considered important in determining the validity of continuum model is molecular density. It is the distance between the molecules which is characterised by mean free path ( $\lambda$ ). It is calculated by finding statistical average distance the molecules travel between two successive collisions. If the mean free path is very small as compared with some characteristic length in the flow domain (i.e., the molecular density is very high) then the gas can be treated as a continuous medium. If the mean free path is large in comparison to some characteristic length, the gas cannot be considered continuous and it should be analysed by the molecular theory.

A dimensionless parameter known as Knudsen number,  $K_n = \lambda / L$ , where  $\lambda$  is the mean free path and  $L$  is the characteristic length. It describes the degree of departure from continuum.

Usually when  $K_n > 0.01$ , the concept of continuum does not hold good.

Beyond this critical range of Knudsen number, the flows are known as  
slip flow ( $0.01 < K_n < 0.1$ ),

transition flow ( $0.1 < K_n < 10$ ) and

free-molecule flow ( $K_n > 10$ ).

However, for the flow regimes considered in this course,  $K_n$  is always less than 0.01 and it is usual to say that the fluid is a continuum.

Other factor which checks the validity of continuum is the elapsed time between collisions. The time should be small enough so that the random statistical description of molecular activity holds good.

In continuum approach, fluid properties such as density, viscosity, thermal conductivity, temperature, etc. can be expressed as continuous functions of space and time.

## **System**

### **Definition**

- **System:** A quantity of matter in space which is analyzed during a problem.

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- **Surroundings:** Everything external to the system.
- **System Boundary:** A separation present between system and surrounding.

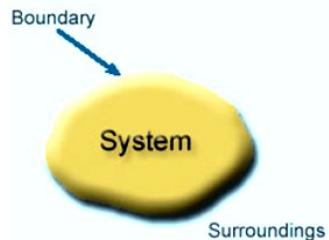
Classification of the system boundary:-

- Real solid boundary
- Imaginary boundary

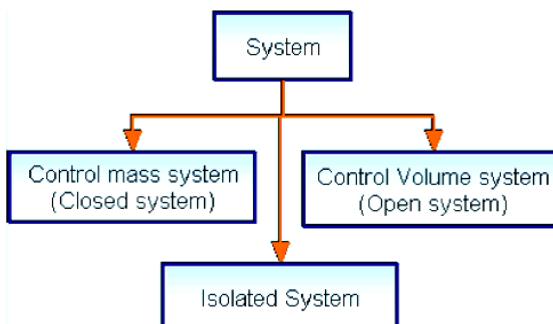
The system boundary may be further classified as:

- Control Mass System.
- Control Volume System.

The choice of boundary depends on the problem being analyzed.



## Types of System



### Closed System (Control Mass System)

1. It's a system of **fixed mass** with **fixed identity**.
2. This type of system is usually referred to as "**closed system**".
3. There is no mass transfer across the system boundary.
4. Energy transfer may take place into or out of the system.

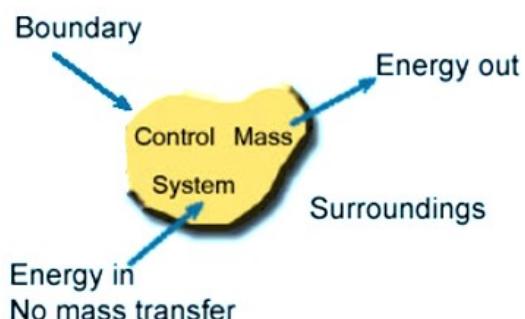


Fig. A Control Mass System  
or Closed System

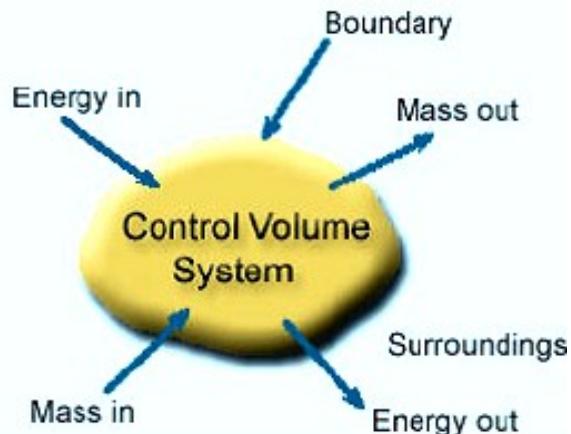
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## Open System (Control Volume System)

1. Its a system of **fixed volume**.
2. This type of system is usually referred to as "**open system**" or a "**control volume**".
3. Mass transfer can take place across a control volume.
4. Energy transfer may also occur into or out of the system.
5. A control volume can be seen as a fixed region across which mass and energy transfers are studied.
6. Control Surface – Its the boundary of a control volume across which the transfer of both mass and energy takes place.
7. The mass of a control volume (open system) may or may not be fixed.
8. When the net influx of mass across the control surface equals zero then the mass of the system is fixed and vice-versa.
9. The identity of mass in a control volume always changes unlike the case for a control mass system (closed system).
10. Most of the engineering devices, in general, represent an open system or control volume.



**Fig. A Control Volume System  
or Open System**

### Example:

- **Heat exchanger** - Fluid enters and leaves the system continuously with the transfer of heat across the system boundary.
- **Pump** - A continuous flow of fluid takes place through the system with a transfer of mechanical energy from the surroundings to the system.

## Isolated System

1. It is a system of **fixed mass** with **same identity and fixed energy**.
2. No interaction of mass or energy takes place between the system and the surroundings.
3. In more informal words an isolated system is like a closed shop amidst a busy market.

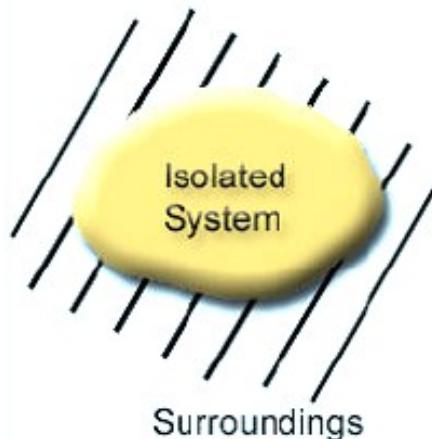


Fig. An Isolated System

## Quasi-Static Process

The processes can be restrained or unrestrained

We need restrained processes in practice.

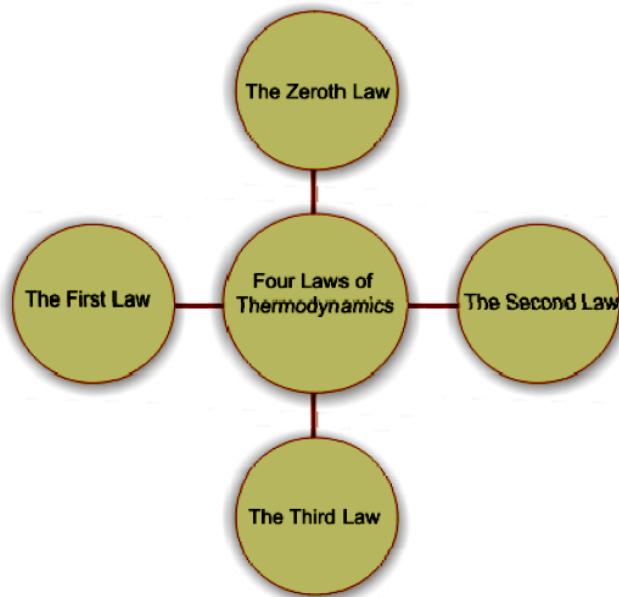
A quasi – static process is one in which

- The deviation from thermodynamic equilibrium is infinitesimal.
- All states of the system passes through are equilibrium states.
- If we remove the weights slowly one by one the pressure of the gas will displace the piston gradually. It is **quasistatic**.
- On the other hand if we remove all the weights at once the piston will be kicked up by the gas pressure. (This is unrestrained expansion) but we don't consider that the work is done – because it is not in a sustained manner.
- In both cases the systems have undergone a change of state.
- Another e.g., if a person climbs down a ladder from roof to ground, it is a quasistatic process. On the other hand if he jumps then it is not a quasistatic process.



Fig. A quasi – static process

## Laws of Thermodynamics



- The Zeroth Law deals with thermal equilibrium and provides a means for measuring temperatures.
- The First Law deals with the conservation of energy and introduces the concept of internal energy.
- The Second Law of thermodynamics provides with the guidelines on the conversion heat energy of matter into work. It also introduces the concept of entropy.
- The Third Law of thermodynamics defines the absolute zero of entropy. The entropy of a pure crystalline substance at absolute zero temperature is zero.

## Summation of 3 Laws

- Firstly, there isn't a meaningful temperature of the source from which we can get the full conversion of heat to work. Only at infinite temperature one can dream of getting the full 1 kW work output.
- Secondly, more interestingly, there isn't enough work available to produce 0 K. In other words, 0 K is unattainable. This is precisely the Third law.
- Because, we don't know what 0 K looks like, we haven't got a starting point for the temperature scale!! That is why all temperature scales are at best empirical.

### You can't get something for nothing:

To get work output you must give some thermal energy.

### You can't get something for very little:

To get some work output there is a minimum amount of thermal energy that needs to be given.

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You can't get every thing:

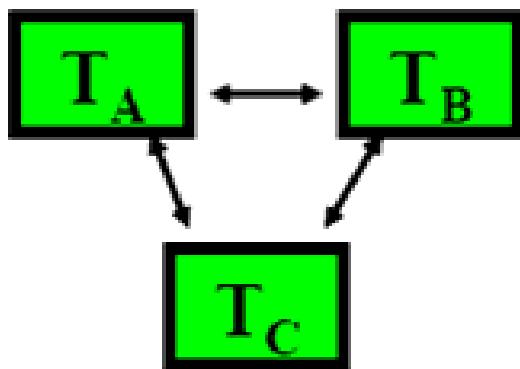
However much work you are willing to give 0 K can't be reached.

Violation of all 3 laws:

Try to get everything for nothing.

## Zeroth Law of Thermodynamics

- If two systems (say A and B) are in thermal equilibrium with a third system (say C) separately (that is A and C are in thermal equilibrium; B and C are in thermal equilibrium) then they are in thermal equilibrium themselves (that is A and B will be in thermal equilibrium).



- All temperature measurements are based on Zeroth law of thermodynamics

## International Temperature Scale

To provide a standard for temperature measurement taking into account both theoretical and practical considerations, the International Temperature Scale (ITS) was adopted in 1927. This scale has been refined and extended in several revisions, most recently in 1990. *The International Temperature Scale of 1990 (ITS-90)* is defined in such a way that the temperature measured on it conforms with the thermodynamic temperature, the unit of which is the kelvin, to within the limits of accuracy of measurement obtainable in 1990. The ITS-90 is based on the assigned values of temperature of a number of reproducible *fixed points* (Table). Interpolation between the fixed-point temperatures is accomplished by formulas that give the relation between readings of standard instruments and values of the ITS. In the range from 0.65 to 5.0 K, ITS-90 is defined by equations giving the temperature as functions of the vapor pressures of particular helium isotopes. The range from 3.0 to 24.5561 K is based on measurements using a helium constant-volume gas thermometer. In the range from 13.8033 to 1234.93 K, ITS-90 is defined by means of certain platinum resistance thermometers. Above 1234.9 K the temperature is defined using *Planck's equation for blackbody radiation* and measurements of the intensity of visible-spectrum radiation the absolute temperature

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**TABLE** Defining Fixed Points of the International Temperature Scale of 1990

T (K)	Substance	State
3 to 5	He	Vapor pressure point
13.8033	e-H <sub>2</sub>	Triple point
≈17	e-H <sub>2</sub>	Vapor pressure point
≈20.3	e-H <sub>2</sub>	Vapor pressure point
24.5561	Ne	Triple point
54.3584	O <sub>2</sub>	Triple point
83.8058	Ar	Triple point
234.3156	Hg	Triple point
273.16	H <sub>2</sub> O	Triple point
302.9146	Ga	Melting point
429.7485	In	Freezing point
505.078	Sn	Freezing point
692.677	Zn	Freezing point
933.473	Al	Freezing point
1234.93	Ag	Freezing point
1337.33	Au	Freezing point
1357.77	Cu	Freezing point

scale. The absolute temperature scale is also known as Kelvin temperature scale. In defining the Kelvin temperature scale also, the triple point of water is taken as the standard reference point. For a Carnot engine operating between reservoirs at temperature  $\theta$  and  $\theta_{tp}$ ,  $\theta_{tp}$  being the triple point of water arbitrarily assigned the value of **273.16 K**.

**Time Constants:** The time constant is the amount of time required for a thermocouple to indicate 63.2% of step change in temperature of a surrounding media. Some of the factors influencing the measured time constant are sheath wall thickness, degree of insulation compaction, and distance of junction from the welded can on an ungrounded thermocouple. In addition, the velocity of a gas past the thermocouple probe greatly influences the time constant measurement.

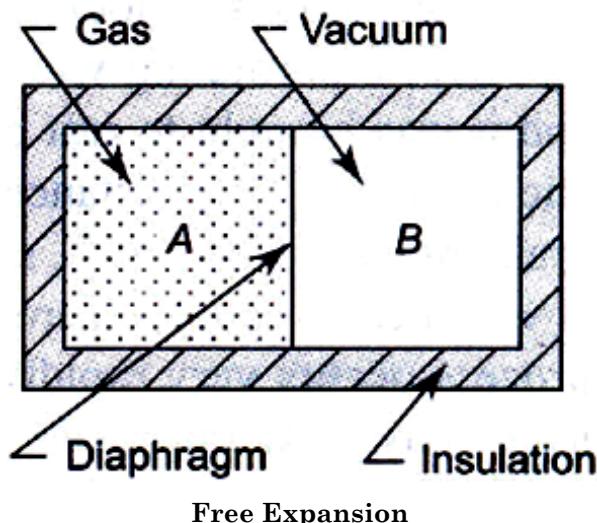
In general, time constants for measurement of gas can be estimated to be ten times as long as those for measurement of liquid. The time constant also varies inversely proportional to the square root of the velocity of the media.

## Work a path function

Work is one of the basic modes of energy transfer. The work done by a system is a path function, and not a point function. **Therefore, work is not a property of the system, and it cannot be said that the work is possessed by the system.** It is an interaction across the boundary. What is stored in the system is energy, but not work. A decrease in energy of the system appears as work done. **Therefore, work is energy in transit and it can be identified only when the system undergoes a process.**

## Free Expansion with Zero Work Transfer

**Free Expansion** Let us consider an insulated container (Figure) which is divided into two compartments *A* and *B* by a thin diaphragm. Compartment *A* contains a mass of gas, while compartment *B* is completely evacuated. If the diaphragm is punctured, the gas in *A* will expand into *B* until the pressures in *A* and *B* become equal. This is known as free or unrestrained expansion. The process of free expansion is **irreversible**. Also work done is zero during free expansion.



## $pdV$ -work or Displacement Work

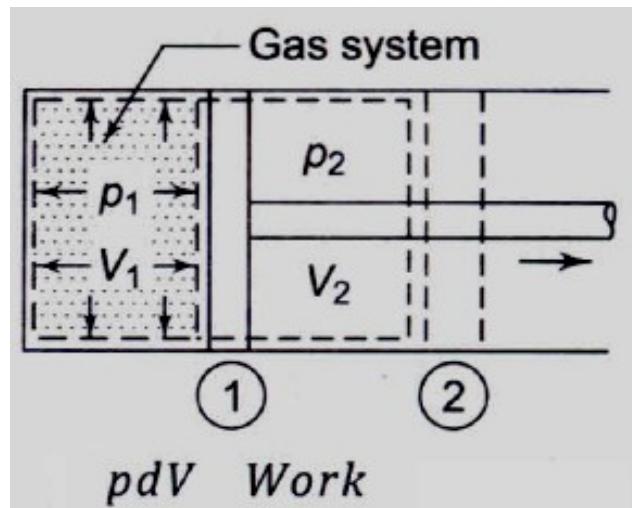
Let the gas in the cylinder (Figure shown in below) be a system having initially the pressure  $p_1$  and volume  $V_1$ . The system is in thermodynamic equilibrium, the state of which is described by the coordinates  $p_1, V_1$ . The piston is the only boundary which moves due to gas pressure. Let the piston move out to a new final position 2, which is also a thermodynamic equilibrium state specified by pressure  $p_2$  and volume  $V_2$ . At any intermediate point in the travel of the piston, let the pressure be  $p$  and the volume  $V$ . This must also be an equilibrium state, since macroscopic properties  $p$  and  $V$

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significant only.



for equilibrium states. When the piston moves an infinitesimal distance  $dl$ , and if 'a' be the area of the piston, the force  $F$  acting on the piston  $F = p.a.$  and the infinitesimal amount of work done by the gas on the piston.

$$dW = F \cdot dl = padl = pdV$$

where  $dV = adl$  = infinitesimal displacement volume. The differential sign in  $dW$  with the line drawn at the top of it will be explained later.

When the piston moves out from position 1 to position 2 with the volume changing from  $V_1$  to  $V_2$ , the amount of work  $W$  done by the system will be

$$W_{1-2} = \int_{V_1}^{V_2} pdV$$

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The magnitude of the work done is given by the area under the path 1-2, as shown in Fig. Since  $p$  is at all times a thermodynamic coordinate, all the states passed through by the system as the volume changes from  $V_1$  to  $V_2$  must be equilibrium states, and the path 1-2 must be quasi-static. The piston moves infinitely slowly so that every state passed through is an equilibrium state.

The integration  $\int p dV$  can be performed only on a **quasi-static path**.

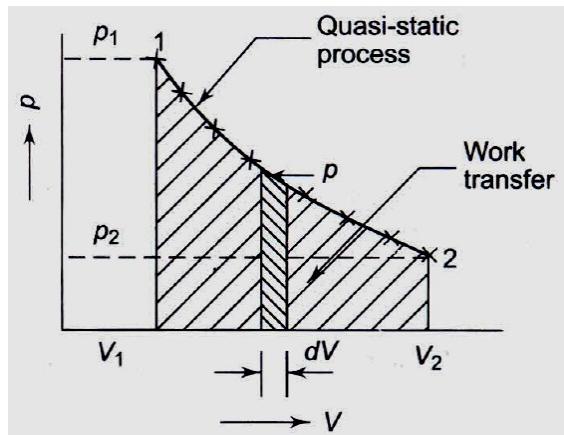


Fig. Quasi-Static  $p dV$  Work

## Heat Transfer-A Path Function

Heat transfer is a **path function**, that is, the amount of heat transferred when a system changes from state 1 to state 2 depends on the intermediate states through which the system passes, i.e. its path. Therefore  $dQ$  is an inexact differential, and we write

$$\int_1^2 dQ = Q_{1-2} \text{ or } {}_1Q \neq Q_1 - Q_2$$

The displacement work is given by

$$W_{1-2} = \int_1^2 dW = \int_1^2 pdV \neq W_2 - W_1$$

## PROBLEMS & SOLUTIONS

### Example 1

In a closed system, volume changes from  $1.5\text{ m}^3$  to  $4.5\text{ m}^3$  and heat addition is  $2000\text{ kJ}$ . Calculate the change in internal energy given the pressure volume relation as

$$p = \left( V^2 + \frac{10}{V} \right) \text{ Where } p \text{ is in kPa and } V \text{ is in m}^3.$$

**Solution:**

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$$\begin{aligned} \text{Work done} &= \int_V^V p dV = \int_{V_1}^{V_2} \left( V^2 + \frac{10}{V} \right) dV \\ &= \left[ \frac{1}{3} (V_2^3 - V_1^3) + 10 \ln \frac{V_2}{V_1} \right] \\ &= \left[ \frac{1}{3} (4.5^3 - 1.5^3) + 10 \ln \frac{4.5}{1.5} \right] \\ &= \left[ \frac{1}{3} (91.125 - 3.375) + 10 \ln 3 \right] \\ &= [29.250 + 10.986] = 40.236 \text{ kJ} \end{aligned}$$

**First Law of Thermodynamics:-**

$$Q = W + \Delta U$$

$$2000 = 40.236 + \Delta U$$

$$\therefore \Delta U = 2000 - 40.236 = 1959.764 \text{ kJ}$$

## Example 2.

A fluid is contained in a cylinder piston arrangement that has a paddle that imparts work to the fluid. The atmospheric pressure is 760 mm of Hg. The paddle makes 10,000 revolutions during which the piston moves out 0.8m. The fluid exerts a torque of 1.275 N-m on the paddle. What is net work transfer, if the diameter of the piston is 0.6m?

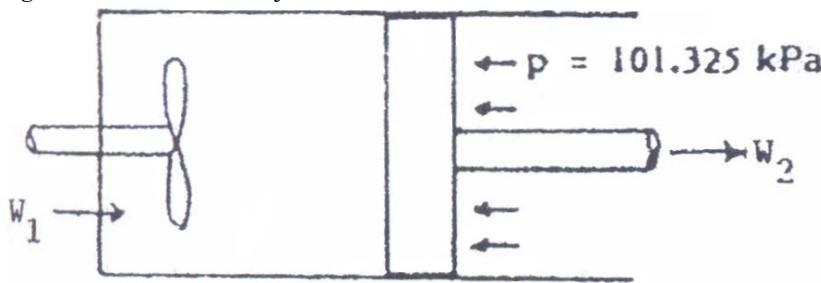
**Solution:**

Work done by the stirring device upon the system

$$W_1 = 2\pi TN$$

$$= 2\pi \times 1.275 \times 10000 \text{ N-m} = 80 \text{ kJ}$$

This is negative work for the system.



(Fig.)

Work done by the system upon the surroundings.

$$W_2 = p.dV = p.(A \times L)$$

$$= 101.325 \times \frac{\pi}{4} (0.6)^2 \times 0.80 = 22.9 \text{ kJ}$$

This is positive work for the system. Hence the net work transfer for the system.

$$W = W_1 + W_2 = -80 + 22.9 = -57.1 \text{ kJ.}$$

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## ASKED OBJECTIVE QUESTIONS (GATE, IES, IAS)

### Previous 20-Years GATE Questions

GATE-1.	List-I		List II		[GATE-1998]				
A.	Heat to work		1. Nozzle						
B.	Heat to lift weight		2. Endothermic chemical reaction						
C.	Heat to strain energy		3. Heat engine						
D.	Heat to electromagnetic energy		4. Hot air balloon/evaporation						
5.	Thermal radiation								
6.	Bimetallic strips								
Codes:	A (a) 3 (c) 3	B (a) 4 (c) 6	C (a) 6 (c) 4	D (a) 5 (d) 2	A (b) 3 (d) 1	B (b) 4 (d) 2	C (b) 5 (d) 3	D (b) 6 (d) 4	

### Open and Closed systems

GATE-2.	An isolated thermodynamic system executes a process, choose the correct statement(s) form the following	[GATE-1999]
(a)	No heat is transferred	
(b)	No work is done	
(c)	No mass flows across the boundary of the system	
(d)	No chemical reaction takes place within the system	

GATE-2a.	Heat and work are	[GATE-2011]
(a)	intensive properties	
(c)	point functions	

### Quasi-Static Process

GATE-3.	A frictionless piston-cylinder device contains a gas initially at 0.8 MPa and 0.015 m <sup>3</sup> . It expands quasi-statically at constant temperature to a final volume of 0.030 m <sup>3</sup> . The work output (in kJ/kg) during this process will be: [GATE-2009]	
(a) 8.32	(b) 12.00	(c) 554.67

(d) 8320.00

### Free Expansion with Zero Work Transfer

GATE-4.	A balloon containing an ideal gas is initially kept in an evacuated and insulated room. The balloon ruptures and the gas fills up the entire room. Which one of the following statements is TRUE at the end of above process?	
(a)	The internal energy of the gas decreases from its initial value, but the enthalpy remains constant	[GATE-2008]
(b)	The internal energy of the gas increases from its initial value, but the enthalpy remains constant	
(c)	Both internal energy and enthalpy of the gas remain constant	
(d)	Both internal energy and enthalpy of the gas increase	

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GATE-5. Air is compressed adiabatically in a steady flow process with negligible change in potential and kinetic energy. The Work done in the process is given by:

[GATE-1996, IAS-2000]

- (a)  $-\int P dv$       (b)  $+\int P dv$       (c)  $-\int v dp$       (d)  $+\int v dp$

## pdV-work or Displacement Work

GATE-6. In a steady state steady flow process taking place in a device with a single inlet and a single outlet, the work done per unit mass flow rate is given by

$$\omega = - \int_{\text{inlet}}^{\text{outlet}} v dp, \text{ where } v \text{ is the specific volume and } p \text{ is the pressure. The}$$

expression for  $w$  given above:

[GATE-2008]

- (a) Is valid only if the process is both reversible and adiabatic  
(b) Is valid only if the process is both reversible and isothermal  
(c) Is valid for any reversible process

$$(d) \text{ Is incorrect; it must be } \omega = - \int_{\text{inlet}}^{\text{outlet}} v dp$$

GATE-7. A gas expands in a frictionless piston-cylinder arrangement. The expansion process is very slow, and is resisted by an ambient pressure of 100 kPa. During the expansion process, the pressure of the system (gas) remains constant at 300 kPa. The change in volume of the gas is 0.01 m<sup>3</sup>. The maximum amount of work that could be utilized from the above process is:

[GATE-2008]

- (a) 0 kJ      (b) 1 kJ      (c) 2 kJ      (d) 3 kJ

GATE-8. For reversible adiabatic compression in a steady flow process, the work transfer per unit mass is:

[GATE-1996]

$$(a) \int pdv      (b) \int vdp      (c) \int Tds      (d) \int sdT$$

## Previous 20-Years IES Questions

IES-1. Which of the following are intensive properties? [IES-2005]

1. Kinetic Energy      2. Specific Enthalpy  
3. Pressure      4. Entropy

Select the correct answer using the code given below:

- (a) 1 and 3      (b) 2 and 3      (c) 1, 3 and 4      (d) 2 and 4

IES-2. Consider the following properties: [IES-2009]

1. Temperature      2. Viscosity  
3. Specific entropy      4. Thermal conductivity

Which of the above properties of a system is/are intensive?

- (a) 1 only      (b) 2 and 3 only      (c) 2, 3 and 4 only      (d) 1, 2, 3 and 4

IES-2a. Consider the following: [IES-2007, 2010]

1. Kinetic energy      2. Entropy

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3. Thermal conductivity                  4. Pressure

Which of these are intensive properties?

- (a) 1, 2 and 3 only                  (b) 2 and 4 only  
(c) 3 and 4 only                  (d) 1, 2, 3 and 4

IES-3. Which one of the following is the extensive property of a thermodynamic system? [IES-1999]

- (a) Volume                  (b) Pressure                  (c) Temperature                  (d) Density

IES-4. Consider the following properties: [IES-2009]

1. Entropy                  2. Viscosity  
3. Temperature                  4. Specific heat at constant volume

Which of the above properties of a system is/are extensive?

- (a) 1 only                  (b) 1 and 2 only                  (c) 2, 3 and 4                  (d) 1, 2 and 4

IES-4a Consider the following: [IES-2010]

1. Temperature                  2. Viscosity  
3. Internal energy                  4. Entropy

Which of these are extensive properties?

- (a) 1, 2, 3 and 4                  (b) 2 and 4 only  
(c) 2 and 3 only                  (d) 3 and 4 only.

IES-4(i). In highly rarefied gases, the concept of this loses validity [IES-2012]

- (a) Thermodynamic equilibrium                  (b) continuum  
(c) stability                  (d) Macroscopic view point

## Thermodynamic System and Control Volume

IES-5. Assertion (A): A thermodynamic system may be considered as a quantity of working substance with which interactions of heat and work are studied.

Reason (R): Energy in the form of work and heat are mutually convertible.

- (a) Both A and R are individually true and R is the correct explanation of A

- (b) Both A and R are individually true but R is NOT the correct explanation of A

- (c) A is true but R is false

[IES-2000]

- (d) A is false but R is true

IES-5a A control volume is [IES-2010]

- (a) An isolated system

- (b) A closed system but heat and work can cross the boundary

- (c) A specific amount of mass in space

- (d) A fixed region in space where mass, heat and work can cross the boundary of that region

## Open and Closed systems

IES-6. A closed thermodynamic system is one in which [IES-1999, 2010, 2011]

- (a) There is no energy or mass transfer across the boundary

- (b) There is no mass transfer, but energy transfer exists

- (c) There is no energy transfer, but mass transfer exists

- (d) Both energy and mass transfer take place across the boundary, but the mass transfer is controlled by valves

IES-7	<b>Isothermal compression of air in a Stirling engine is an example of</b>	[IES-2010]		
	(a) Open system			
	(b) Steady flow diabatic system			
	(c) Closed system with a movable boundary			
	(d) Closed system with fixed boundary			
IES-8.	<b>Which of the following is/are reversible process(es)?</b>	[IES-2005]		
	1. Isentropic expansion			
	2. Slow heating of water from a hot source			
	3. Constant pressure heating of an ideal gas from a constant temperature source			
	4. Evaporation of a liquid at constant temperature			
	<b>Select the correct answer using the code given below:</b>			
	(a) 1 only	(b) 1 and 2	(c) 2 and 3	(d) 1 and 4
IES-9.	<b>Assertion (A): In thermodynamic analysis, the concept of reversibility is that, a reversible process is the most efficient process.</b>	[IES-2001]		
	<b>Reason (R): The energy transfer as heat and work during the forward process as always identically equal to the energy transfer as heat and work during the reversal or the process.</b>			
	(a) Both A and R are individually true and R is the correct explanation of A			
	(b) Both A and R are individually true but R is NOT the correct explanation of A			
	(c) A is true but R is false			
	(d) A is false but R is true			
IES-9a	<b>Which one of the following represents open thermodynamic system?</b>			
	(a) Manual ice cream freezer	(b) Centrifugal pump	[IES-2011]	
	(c) Pressure cooker	(d) Bomb calorimeter		
IES-10.	<b>Ice kept in a well insulated thermo flask is an example of which system?</b>			
	(a) Closed system	(b) Isolated systems	[IES-2009]	
	(c) Open system	(d) Non-flow adiabatic system		
IES-10a	<b>Hot coffee stored in a well insulated thermos flask is an example of</b>			
	(a) Isolated system	(b) Closed system	[IES-2010]	
	(c) Open system	(d) Non-flow diabatic system		
IES10b	<b>A thermodynamic system is considered to be an isolated one if</b>	[IES-2011]		
	(a) Mass transfer and entropy change are zero			
	(b) Entropy change and energy transfer are zero			
	(c) Energy transfer and mass transfer are zero			
	(d) Mass transfer and volume change are zero			
IES-10c.	<b>Match List I with List II and select the correct answer using the code given below the lists:</b>	[IES-2011]		

List I

List II

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- |                                                                                           |                  |
|-------------------------------------------------------------------------------------------|------------------|
| A. Interchange of matter is not possible in a                                             | 1. Open system   |
| B. Any processes in which the system returns to its original condition or state is called | 2. System        |
| C. Interchange of matter is possible in a                                                 | 3. Closed system |
| D. The quantity of matter under consideration in thermodynamics is called                 | 4. Cycle         |
- Code: A      B      C      D      A      B      C      D
- (a) 2      1      4      3      (b) 3      1      4      2  
(c) 2      4      1      3      (d) 3      4      1      2

## Zeroth Law of Thermodynamics

IES-11. Measurement of temperature is based on which law of thermodynamics?

[IES-2009]

- (a) Zeroth law of thermodynamics      (b) First law of thermodynamics  
(c) Second law of thermodynamics      (d) Third law of thermodynamics

IES-12. Consider the following statements:

[IES-2003]

1. Zeroth law of thermodynamics is related to temperature
2. Entropy is related to first law of thermodynamics
3. Internal energy of an ideal gas is a function of temperature and pressure
4. Van der Waals' equation is related to an ideal gas

Which of the above statements is/are correct?

- (a) 1 only      (b) 2, 3 and 4      (c) 1 and 3      (d) 2 and 4

IES-13. Zeroth Law of thermodynamics states that

[IES-1996, 2010]

- (a) Two thermodynamic systems are always in thermal equilibrium with each other.
- (b) If two systems are in thermal equilibrium, then the third system will also be in thermal equilibrium with each other.
- (c) Two systems not in thermal equilibrium with a third system are also not in thermal equilibrium with each other.
- (d) When two systems are in thermal equilibrium with a third system, they are in thermal equilibrium with each other.

## International Temperature Scale

IES-14. Which one of the following correctly defines 1 K, as per the internationally accepted definition of temperature scale?

[IES-2004]

- (a) 1/100<sup>th</sup> of the difference between normal boiling point and normal freezing point of water
- (b) 1/273.15<sup>th</sup> of the normal freezing point of water
- (c) 100 times the difference between the triple point of water and the normal freezing point of water
- (d) 1/273.15<sup>th</sup> of the triple point of water

IES-14(i). The constant volume gas thermometer works on the principle that

[IES-2012]

- (a) at low pressure, the temperature of the gas is independent of its pressure at constant volume

- (b) at high pressure, the temperature of the gas is independent of its pressure at constant volume  
(c) at low pressure, the temperature of the gas is proportional to its pressure at constant volume  
(d) at high pressure, the temperature of the gas is proportional to its pressure at constant volume
- IES-15.** In a new temperature scale say  ${}^{\circ}\rho$ , the boiling and freezing points of water at one atmosphere are  $100{}^{\circ}\rho$  and  $300{}^{\circ}\rho$  respectively. Correlate this scale with the Centigrade scale. The reading of  $0{}^{\circ}\rho$  on the Centigrade scale is: [IES-2001]  
(a)  $0^{\circ}\text{C}$       (b)  $50^{\circ}\text{C}$       (c)  $100^{\circ}\text{C}$       (d)  $150^{\circ}\text{C}$
- IES-16.** Assertion (a): If an alcohol and a mercury thermometer read exactly  $0^{\circ}\text{C}$  at the ice point and  $100^{\circ}\text{C}$  at the steam point and the distance between the two points is divided into 100 equal parts in both thermometers, the two thermometers will give exactly the same reading at  $50^{\circ}\text{C}$ . [IES-1995]  
Reason (R): Temperature scales are arbitrary.  
(a) Both A and R are individually true and R is the correct explanation of A  
(b) Both A and R are individually true but R is NOT the correct explanation of A  
(c) A is true but R is false  
(d) A is false but R is true
- IES-17.** Match List-I (Type of Thermometer) with List-II (Thermometric Property) and select the correct answer using the code given below the [IES 2007]  

List-I	List-II
A. Mercury-in-glass	1. Pressure
B. Thermocouple	2. Electrical resistant
C. Thermistor	3. Volume
D. Constant volume gas	4. Induced electric voltage

Codes:	A	B	C	D	A	B	C	D	
	(a)	1	4	2	3	(b)	3	2	4
	(c)	1	2	4	3	(d)	3	4	2
- IES-18.** Pressure reaches a value of absolute zero [IES-2002]  
(a) At a temperature of  $-273\text{ K}$   
(b) Under vacuum condition  
(c) At the earth's centre  
(d) When molecular momentum of system becomes zero
- IES-19.** The time constant of a thermocouple is the time taken to attain: [IES-1997, 2010]  
(a) The final value to be measured  
(b) 50% of the value of the initial temperature difference  
(c) 63.2% of the value of the initial temperature difference  
(d) 98.8% of the value of the initial temperature difference

## Work a Path Function

- IES-20.** Assertion (A): Thermodynamic work is path-dependent except for an adiabatic process. [IES-2005]

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**Reason(R): It is always possible to take a system from a given initial state to any final state by performing adiabatic work only.**

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is NOT the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

**IES-20a Work transfer between the system and the surroundings**

[IES-2011]

- (a) Is a point function
- (b) Is always given by  $\int pdV$
- (c) Is a function of pressure only
- (d) Depends on the path followed by the system

**IES-20b. Statement (I): Thermodynamic work is given as the product of an intensive thermodynamic property and the differential of an extensive property. [IES-2012]**

**Statement (II): Work is not a thermodynamic property.**

## Free Expansion with Zero Work Transfer

**IES-21. There is no work transfer involved in this process**

[IES-2012]

- (a) Adiabatic expansion
- (b) Isothermal expansion
- (c) Polytropic expansion
- (d) free expansion

**IES-21(i). Match items in List-I (Process) with those in List-II (Characteristic) and select the correct answer using the codes given below the lists:**

**List-I**

- A. Throttling process
- B. Isentropic process
- C. Free expansion
- D. Isothermal process

**List-II**

- 1. No work done
- 2. No change in entropy
- 3. Constant internal energy
- 4. Constant enthalpy

[IES-2001]

**Codes:**

**A**

**B**

**C**

**D**

**A**

**B**

**C**

**D**

(a)

4

2

1

1

2

4

3

(c)

4

3

1

1

3

4

2

**IES-22. The heat transfer, Q, the work done W and the change in internal energy U are all zero in the case of**

[IES-1996]

- (a) A rigid vessel containing steam at 150°C left in the atmosphere which is at 25°C.
- (b) 1 kg of gas contained in an insulated cylinder expanding as the piston moves slowly outwards.
- (c) A rigid vessel containing ammonia gas connected through a valve to an evacuated rigid vessel, the vessel, the valve and the connecting pipes being well insulated and the valve being opened and after a time, conditions through the two vessels becoming uniform.
- (d) 1 kg of air flowing adiabatically from the atmosphere into a previously evacuated bottle.

# Basic Concepts

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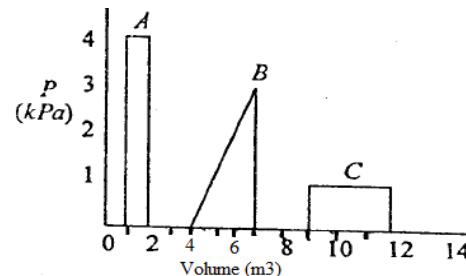
## pdV-work or Displacement Work

**IES-23.** One kg of ice at  $0^{\circ}\text{C}$  is completely melted into water at  $0^{\circ}\text{C}$  at 1 bar pressure. The latent heat of fusion of water is 333 kJ/kg and the densities of water and ice at  $0^{\circ}\text{C}$  are  $999.0 \text{ kg/m}^3$  and  $916.0 \text{ kg/m}^3$ , respectively. What are the approximate values of the work done and energy transferred as heat for the process, respectively?

[IES-2007]

- (a)  $-9.4 \text{ J}$  and  $333.0 \text{ kJ}$       (b)  $9.4 \text{ J}$  and  $333.0 \text{ kJ}$   
 (c)  $-333.0 \text{ kJ}$  and  $-9.4 \text{ J}$       (d) None of the above

**IES-24.** Which one of the following is the correct sequence of the three processes A, B and C in the increasing order of the amount of work done by a gas following ideal-gas expansions by these processes?



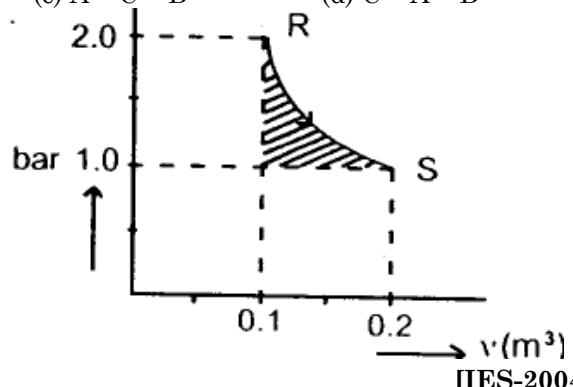
[IES-2006]

**IES-25.** An ideal gas undergoes an isothermal expansion from state R to state S in a turbine as shown in the diagram given below:

The area of shaded region is 1000 Nm. What is the amount of turbine work done during the process?

- (a) 14,000 Nm      (b) 12,000 Nm  
 (c) 11,000 Nm      (d) 10,000 Nm

- (c) A - C - B      (d) C - A - B



[IES-2004]

**IES-26.** Assertion (A): The area 'under' curve on  $pV$  plane,  $\int pdv$  represents the work of reversible non-flow process.

[IES-1992]

Reason (R): The area 'under' the curve  $T-s$  plane  $\int Tds$  represents heat of any reversible process.

- (a) Both A and R are individually true and R is the correct explanation of A  
 (b) Both A and R are individually true but R is NOT the correct explanation of A  
 (c) A is true but R is false  
 (d) A is false but R is true

**IES-27.** If  $\int pdv$  and  $-\int vdp$  for a thermodynamic system of an Ideal gas on valuation give same quantity (positive/negative) during a process, then the process

undergone by the system is:

[IES-2003]

- (a) Isomeric      (b) Isentropic      (c) Isobaric      (d) Isothermal

**IES-28.** Which one of the following expresses the reversible work done by the system (steady flow) between states 1 and 2? [IES-2008]

- (a)  $\int_1^2 pdv$       (b)  $-\int_1^2 vdp$       (c)  $-\int_1^2 pdv$       (d)  $\int_1^2 vdp$

## Heat Transfer-A Path Function

**IES-29.** Assertion (A): The change in heat and work cannot be expressed as difference between the end states. [IES-1999]

Reason (R): Heat and work are both exact differentials.

- (a) Both A and R are individually true and R is the correct explanation of A  
 (b) Both A and R are individually true but R is NOT the correct explanation of A  
 (c) A is true but R is false  
 (d) A is false but R is true

## Previous 20-Years IAS Questions

## Thermodynamic System and Control Volume

**IAS-1.** The following are examples of some intensive and extensive properties:

- |                    |                     |            |
|--------------------|---------------------|------------|
| 1. Pressure        | 2. Temperature      | [IAS-1995] |
| 3. Volume          | 4. Velocity         |            |
| 5. Electric charge | 6. Magnetisation    |            |
| 7. Viscosity       | 8. Potential energy |            |

Which one of the following sets gives the correct combination of intensive and extensive properties?

- | Intensive      | Extensive  |
|----------------|------------|
| (a) 1, 2, 3, 4 | 5, 6, 7, 8 |
| (b) 1, 3, 5, 7 | 2, 4, 6, 8 |
| (c) 1, 2, 4, 7 | 3, 5, 6, 8 |
| (d) 2, 3, 6, 8 | 1, 4, 5, 7 |

## Zeroth Law of Thermodynamics

**IAS-2.** Match List-I with List-II and select the correct answer using the codes given below the lists: [IAS-2004]

**List-I**

- A. Reversible cycle
- B. Mechanical work
- C. Zeroth Law

**List-II**

- 1. Measurement of temperature
- 2. Clapeyron equation
- 3. Clausius Theorem

D. Heat

- 4. High grade energy
- 5. 3<sup>rd</sup> law of thermodynamics
- 6. Inexact differential

<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a)	3	4	1	6	(b)	2	6	1
(c)	3	1	5	6	(d)	1	4	5

**IAS-3. Match List-I with List-II and select the correct answer:** [IAS-2000]**List-I**

- A. The entropy of a pure crystalline substance is zero at absolute zero temperature
- B. Spontaneous processes occur in a certain direction
- C. If two bodies are in thermal equilibrium with a third body, then they are also in thermal equilibrium with each other
- D. The law of conservation of energy

**List-II**

- 1. First law of thermodynamics
- 2. Second law of thermodynamics
- 3. Third law of thermodynamics
- 4. Zeroth law of thermodynamics.

<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a)	2	3	4	1
(c)	3	2	4	1

<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(b)	3	2	1
(d)	2	3	1

## International Temperature Scale

**IAS-4. A new temperature scale in degrees N is to be defined. The boiling and freezing on this scale are 400°N and 100°N respectively. What will be the reading on new scale corresponding to 60°C?** [IAS-1995]

- (a) 120°N
- (b) 180°N
- (c) 220°N
- (d) 280°N

## Free Expansion with Zero Work Transfer

**IAS-5. In free expansion of a gas between two equilibrium states, the work transfer involved** [IAS-2001]

- (a) Can be calculated by joining the two states on p-v coordinates by any path and estimating the area below
- (b) Can be calculated by joining the two states by a quasi-static path and then finding the area below
- (c) Is zero
- (d) Is equal to heat generated by friction during expansion.

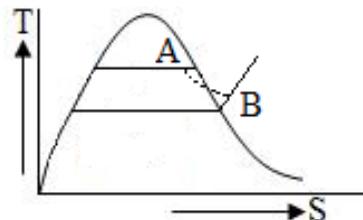
**IAS-6. Work done in a free expansion process is:** [IAS-2002]  
(a) Positive      (b) Negative      (c) Zero      (d) Maximum

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- IAS-7. In the temperature-entropy diagram of a vapour shown in the given figure, the thermodynamic process shown by the dotted line AB represents  
(a) Hyperbolic expansion  
(b) Free expansion  
(c) Constant volume expansion  
(d) Polytropic expansion



[IAS-1995]

- IAS-8. If  $\int pdv$  and  $-\int vdp$  for a thermodynamic system of an Ideal gas on valuation give same quantity (positive/negative) during a process, then the process undergone by the system is:  
(a) Isomeric                    (b) Isentropic                    (c) Isobaric                    (d) Isothermal

- IAS-9. For the expression  $\int pdv$  to represent the work, which of the following conditions should apply?

[IAS-2002]

- (a) The system is closed one and process takes place in non-flow system  
(b) The process is non-quasi static  
(c) The boundary of the system should not move in order that work may be transferred  
(d) If the system is open one, it should be non-reversible

- IAS-10. Air is compressed adiabatically in a steady flow process with negligible change in potential and kinetic energy. The Work done in the process is given by:

[IAS-2000, GATE-1996]

- (a)  $-\int pdv$                     (b)  $+\int pdv$                     (c)  $-\int vdp$                     (d)  $+\int vdp$

- IAS-11. Match List-I with List-II and select the correct answer using the codes given below the lists:

[IAS-2004]

List-I

- A. Bottle filling of gas  
B. Nernst Simon Statement  
C. Joule Thomson Effect  
D.  $\int pdv$

List-II

1. Absolute Zero Temperature  
2. Variable flow  
3. Quasi-Static Path  
4. Isentropic Process  
5. Dissipative Effect  
6. Low grade energy  
7. Process and temperature during phase change.

Codes:

A      B      C      D

(a)     6      5      4      3

(c)     2      5      7      4

A      B      C      D

(b)     2      1      4      3

(d)     6      1      7      4

# Basic Concepts

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## pdV-work or Displacement Work

IAS-13. Thermodynamic work is the product of

[IAS-1998]

- (a) Two intensive properties
- (b) Two extensive properties
- (c) An intensive property and change in an extensive property
- (d) An extensive property and change in an intensive property

## Heat Transfer-A Path Function

IAS-14. Match List-I (Parameter) with List-II (Property) and select the correct answer using the codes given below the lists:

List-I				List-II				[IAS-1999]	
Codes:	A	B	C	D	A	B	C	D	
(a)	3	2	4	1	(b)	3	2	1	4
(c)	2	3	4	1	(d)	2	3	1	4

# Basic Concepts

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## Answers with Explanation (Objective)

### Previous 20-Years GATE Answers

GATE-1. Ans. (a)

GATE-2. Ans. (a, b, c) For an isolated system no mass and energy transfer through the system.  
 $dQ=0, dW=0, \therefore dE=0$  or  $E=\text{Constant}$

GATE-2a. Ans. (d)

$$\begin{aligned}\text{GATE-3. Ans. (a) Iso-thermal work done (W)} &= RT_1 \ln\left(\frac{V_2}{V_1}\right) \\ &= P_1 V_1 \ln\left(\frac{V_2}{V_1}\right) \\ &= 800 \times 0.015 \times \ln\left(\frac{0.030}{0.015}\right) \\ &= 8.32 \text{ kJ/kg}\end{aligned}$$

GATE-4. Ans. (c) It is free expansion. Since vacuum does not offer any resistance, there is no work transfer involved in free expansion.

$$\text{Here, } \int_1^2 \delta w = 0 \quad \text{and } Q_{1-2} = 0 \text{ therefore } Q_{1-2} = \Delta U + W_{1-2} \text{ so, } \Delta U = 0$$

GATE-5. Ans. (c) For closed system  $W = + \int pdv$ , for steady flow  $W = - \int vdp$

GATE-6. (c)

GATE-7. Ans. (c)  $W = \text{Resistance pressure. } \Delta V = p \times \Delta V = 100 \times 0.01 \text{ kJ} = 1 \text{ kJ}$  this is required for atmosphere, means against atmosphere we have to do this. but question is that how much we can use. Now we use a connecting rod so that it can generate 200 KPa pressure extra. so that total resistance pressure is 300 KPa. now total work done = Total Resistance pressure.  $\Delta V = p \times \Delta V = 300 \times 0.01 \text{ kJ} = 3 \text{ kJ}$ . 1 KJ for atmosphere. 2 KJ for use.

GATE-8. Ans. (b)  $W = - \int vdp$

### Previous 20-Years IES Answers

IES-1. Ans. (b)

IES-2. Ans. (d) **Intensive property:** Whose value is independent of the size or extent i.e. mass of the system.

**Specific property:** It is a special case of an intensive property. It is the value of an extensive property per unit mass of system (Lower case letters as symbols) e.g., specific volume, density ( $v, \rho$ ).

**IES-2a.** **Ans.** (c) Kinetic energy  $\frac{1}{2}mv^2$  depends on mass, Entropy  $\text{kJ}/\text{k}$  depends on mass so Entropy is extensive property but specific entropy  $\text{kJ}/\text{kg K}$  is an intensive property.

**IES-3.** **Ans. (a)** Extensive property is dependent on mass of system. Thus volume is extensive property.

**IES-4.** **Ans. (a)** **Extensive property:** Whose value depends on the size or extent i.e. mass of the system (upper case letters as the symbols) e.g., Volume, Mass ( $V, M$ ). If mass is increased, the value of extensive property also increases.

**IES-4a Ans.** (d) The properties like temperature, viscosity which are Independent of the MASS of the system are called Intensive property

**IES-4(i). Ans. (b)**

**IES-5. Ans. (d)**

- But remember 100% heat can't be convertible to work but 100% work can be converted to heat. It depends on second law of thermodynamics.
- A thermodynamic system is defined as a definite quantity of matter or a region in space upon which attention is focused in the analysis of a problem.
- The system is a macroscopically identifiable collection of matter on which we focus our attention

**IES-5a Ans. (d)**

**IES-6.** **Ans. (b)** In closed thermodynamic system, there is no mass transfer but energy transfer exists.

**IES-7. Ans. (c)**

**IES-8. Ans. (d)** Isentropic means reversible adiabatic. Heat transfer in any finite temp difference is irreversible.

**IES-9. Ans. (a)** The energy transfer as heat and work during the forward process as always identically equal to the energy transfer is heat and work during the reversal or the process is the correct reason for maximum efficiency because it is conservative system.

**IES-9a. Ans. (b)**

**IES-10. Ans. (b)** Isolated System - in which there is no interaction between system and the surroundings. It is of fixed mass and energy, and hence there is no mass and energy transfer across the system boundary.

**IES-10a** **Ans. (a)**

**IES-10b.** **Ans. (c)**

**IES-10c.** **Ans. (d)**

**IES-11.** **Ans. (a)** All temperature measurements are based on Zeroth law of thermodynamics.

**IES-12.** **Ans. (a)** Entropy - related to second law of thermodynamics.

Internal Energy ( $u$ ) =  $f(T)$  only (for an ideal gas)

Van der Wall's equation related to  $\Rightarrow$  real gas.

**IES-13. Ans. (d)**

**IES-14. Ans. (d)**

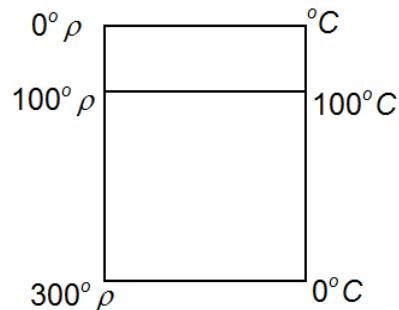
**IES-14(i). Ans. (c)**

# Basic Concepts

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**Chapter 1**

IES-15. Ans. (d)  $\frac{0-300}{100-300} = \frac{C-0}{100-0} \Rightarrow C=150^{\circ}\text{C}$



IES-16. Ans. (b) Both A and R are correct but R is not correct explanation for A. Temperature is independent of thermometric property of fluid.

IES-17. Ans. (d)

IES-18. Ans. (d) But it will occur at absolute zero temperature.

IES-19. Ans. (c) **Time Constants:** The time constant is the amount of time required for a thermocouple to indicated 63.2% of step change in temperature of a surrounding media. Some of the factors influencing the measured time constant are sheath wall thickness, degree of insulation compaction, and distance of junction from the welded cap on an ungrounded thermocouple. In addition, the velocity of a gas past the thermocouple probe greatly influences the time constant measurement. In general, time constants for measurement of gas can be estimated to be ten times as long as those for measurement of liquid. The time constant also varies inversely proportional to the square root of the velocity of the media.

IES-20. Ans. (c)

IES-20a Ans. (d)

IES-20b. Ans. (b) pdv

IES-21. Ans. (d)

IES-21(i). Ans. (a)

IES-22. Ans. (c) In example of (c), it is a case of free expansion heat transfer, work done, and changes in internal energy are all zero.

IES-23. Ans. (a) Work done ( $W$ ) =  $P \Delta V = 100 \times (V_2 - V_1) = 100 \times \left( \frac{m}{\rho_2} - \frac{m}{\rho_1} \right)$   
 $= 100 \text{ kPa} \times \left( \frac{1}{999} - \frac{1}{916} \right) = -9.1 \text{ J}$

IES-24. Ans. (d)  $W_A = \int pdV = 4 \times (2-1) = 4 \text{ kJ}$

$$W_B = \int pdV = \frac{1}{2} \times 3 \times (7-4) = 4.5 \text{ kJ}$$

$$W_C = \int pdV = 1 \times (12-9) = 3 \text{ kJ}$$

IES-25. Ans. (c) Turbine work = area under curve R-S

$$= \int pdV = 1 \text{ bar} \times (0.2 - 0.1) \text{ m}^3 + 1000 \text{ Nm} \\ = 10^5 \times (0.2 - 0.1) \text{ Nm} + 1000 \text{ Nm} = 11000 \text{ Nm}$$

IES-26. Ans. (b)

IES-27. Ans. (d) Isothermal work is minimum of any process.

# Basic Concepts

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$$pv = mRT$$

$$pdv + vdp = 0 \quad [\because T \text{ is constant}]$$

$$\int pdv = -\int vdp$$

IES-28. Ans. (b) For steady flow process, reversible work given by  $-\int vdp$ .

IES-29. Ans. (c) A is true because change in heat and work are path functions and thus can't be expressed simply as difference between the end states. R is false because both work and heat are inexact differentials.

## Previous 20-Years IAS Answers

IAS-1. Ans. (c) Intensive properties, i.e. independent of mass are pressure, temperature, velocity and viscosity. Extensive properties, i.e. dependent on mass of system are volume, electric charge, magnetisation, and potential energy. Thus correct choice is (c).

IAS-2. Ans. (a)

IAS-3. Ans. (c)

IAS-4. Ans. (d) The boiling and freezing points on new scale are  $400^\circ N$  and  $100^\circ N$  i.e. range is  $300^\circ N$  corresponding to  $100^\circ C$ . Thus conversion equation is

$$^\circ N = 100 + 3 \times ^\circ C = 100 + 3 \times 60 = 100 + 180 = 280^\circ N$$

IAS-5. Ans. (c)

IAS-6. Ans. (c) Since vacuum does not offer any resistance, there is no work transfer involved in free expansion.

IAS-7. Ans. (b)

IAS-8. Ans. (d) Isothermal work is minimum of any process.

IAS-9. Ans. (a)

IAS-10. Ans. (c) For closed system  $W = +\int pdv$ , for steady flow  $W = -\int vdp$

IAS-12. Ans. (b) Start with D.  $\int PdV$  only valid for quasi-static path so choice (c) & (d) out. Automatically C-4 then eye on A and B. Bottle filling of gas is variable flow so A-2.

IAS-13. Ans. (c)  $W = \int pdv$  where pressure (p) is an intensive property and volume (v) is an extensive property

IAS-14. Ans. (a) Pressure is intensive property but such option is not there.

# First Law of Thermodynamics

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2.

## First Law of Thermodynamics

### Theory at a Glance (For GATE, IES & PSUs)

#### First Law of Thermodynamics

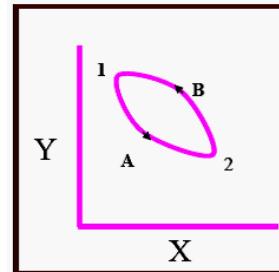
##### Statement:

- When a closed system executes a complete cycle the sum of heat interactions is equal to the sum of work interactions.

Mathematically

$$\bullet (\Sigma Q)_{\text{cycle}} = (\Sigma W)_{\text{cycle}}$$

The summations being over the entire cycle.



##### Alternate statement:

When a closed system undergoes a cycle the cyclic integral of heat is equal to the cyclic integral of work.

Mathematically

$$\oint \delta Q = \oint \delta W$$

In other words for a two process cycle

$$Q_{A1-2} + Q_{B2-1} = W_{A1-2} + W_{B2-1}$$

#### Internal Energy – A Property of System

$$Q_{A1-2} = \int_1^2 \delta Q$$

Along path A

$$\int_1^2 (\delta Q - \delta W) + \int_2^1 (\delta Q - \delta W) = 0$$

Along path A                                    Along path B

Which can be written as

# First Law of Thermodynamics

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$$\int_1^2 (\delta Q - \delta W)_{\text{Along path A}} - \int_1^2 (\delta Q - \delta W)_{\text{Along path B}} = 0$$

$$\int_1^2 (\delta Q - \delta W)_{\text{Along path A}} = \int_1^2 (\delta Q - \delta W)_{\text{Along path B}}$$

- Since A and B are arbitrarily chosen, the conclusion is, as far as a process is concerned (A or B) the difference  $\delta Q - \delta W$  remains a constant as long as the initial and the final states are the same. The difference depends only on the end points of the process. Note that Q and W themselves depend on the path followed. But their difference does not.
- This implies that the difference between the heat and work interactions during a process is a property of the system.
- This property is called the energy of the system. It is designated as E and is equal to some of all the energies at a given state.

We enunciate the FIRST LAW for a process as

$$\delta Q - \delta W = dE$$

E consists of



$$E = U + KE + PE$$

U -internal energy

KE - the kinetic energy

PE - the potential energy

For the whole process A



$$Q - W = E_2 - E_1$$

Similarly for the process B



$$Q - W = E_1 - E_2$$

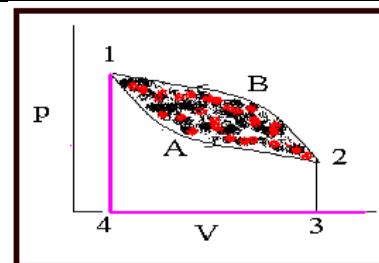
- **An isolated system which does not interact with the surroundings  $Q = 0$  and  $W = 0$ .**  
Therefore, E remains constant for such a system.
- Let us reconsider the cycle 1-2 along path A and 2-1 along path B as shown in fig.
- Work done during the path A = Area under 1-A-2-3-4
- Work done during the path B = Area under 1-B-2-3-4
- Since these two areas are not equal, the net work interaction is that shown by the shaded area.

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- The net area is 1A2B1.
- Therefore some work is derived by the cycle.
- First law compels that this is possible only when there is also heat interaction between the system and the surroundings.
- In other words, if you have to get work out, you must give heat in.**
- Thus, the first law can be construed to be a statement of conservation of energy - in a broad sense.
- In the example shown the area under curve A < that under B
- The cycle shown has negative work output or it will receive work from the surroundings. Obviously, the net heat interaction is also negative. This implies that this cycle will heat the environment. (as per the sign convention).
- For a process we can have  $Q = 0$  or  $W = 0$
- We can extract work without supplying heat (**during a process**) but sacrificing the energy of the system.
- We can add heat to the system without doing work (**in process**) which will go to increasing the energy of the system.
- Energy of a system is an extensive property**



The internal energy depends only upon the initial and final states of the system. Internal energy of a substance does not include any energy that it may possess as a result of its **macroscopic** position or movement. That is why in SFEE (Steady flow energy equation)  $C^2/2$  and  $gz$  is there.

Recognize that  $h = u + pv$  from which  $u_2 + p_2 v_2 = h_2$

and similarly  $u_1 + p_1 v_1 = h_1$

$$\begin{aligned} Q-W &= [(h_2 + C_2^2/2 + gZ_2) - (h_1 + C_1^2/2 + gZ_1)] \quad \text{or} \\ &= [(h_2 - h_1) + (C_2^2/2 - C_1^2/2) + g(Z_2 - Z_1)] \end{aligned}$$

[Where  $C$  = Velocity (m/s),  $h$  = Specific enthalpy (J/kg),  $z$  = elevation (m)]

## But Remember:

Microscopic view of a gas is a collection of particles in random motion. Energy of a particle consists of **translational energy, rotational energy, vibrational energy** and **specific electronic energy**. All these energies summed over all the particles of the gas, form the specific internal energy,  $e$ , of the **gas**.

# First Law of Thermodynamics

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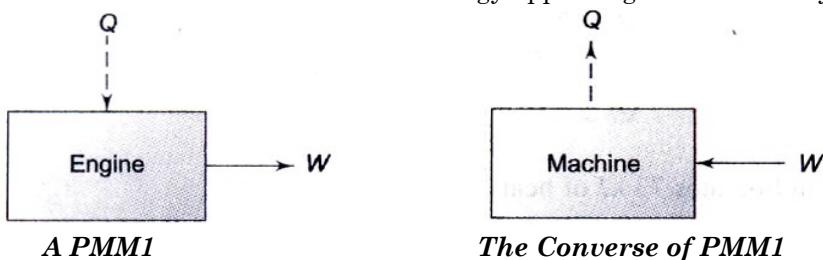
Chapter 2

## Perpetual Motion Machine of the First Kind-PMM1

The first law states the general principle of the conservation of energy. Energy is neither created nor destroyed, but only gets transformed from one form to another.

There can be no machine which would continuously supply mechanical work without some other form of energy disappearing simultaneously (Fig. shown in below). Such a fictitious machine is called a perpetual motion machine of the first kind, or in brief, PMM1. A PMM1 is thus **impossible**.

The converse of the above statement is also true, i.e. there can be no machine which would continuously consume work without some other form of energy appearing simultaneously (Fig.).



## Enthalpy

The enthalpy of a substance H is defined as

$$H = U + PV$$

It is an **extensive property** of a system and its unit is **kJ**.

Specific Enthalpy

$$h = u + pv$$

It is an **intensive property** of a system and its unit is **kJ/kg**.

Internal energy change is equal to the heat transferred in a constant volume process involving no work other than pdv work. It is possible to derive an expression for the heat transfer in a constant pressure process involving no work other than pdv work. In such a process in a closed stationary system of unit mass of a pure substance.

$$dQ = du + pdv$$

At constant pressure

$$\begin{aligned} pdv &= d(pv) \\ \text{Therefore } (dQ)_P &= du + d(pv) \\ \text{or } (dQ)_P &= d(u+pv) \\ \text{or } (dQ)_P &= dh \end{aligned}$$

Where  $H = U + PV$  is the enthalpy, a property of system.

Specific enthalpy  $h = H/m$ ,  $\text{kJ/kg}$  and also  $h = u + pdv$

Where  $h = \text{specific enthalpy, kJ/kg}$

# First Law of Thermodynamics

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$u$  = specific internal energy, kJ/kg

$dv$  = change in specific volume, m<sup>3</sup>/kg.

### Specific heat at constant volume

The specific heat of a substance at constant volume  $C_v$  is defined as the rate of change of specific internal energy with respect to temperature when the volume is held constant, i.e.,

$$C_v = \left( \frac{\partial u}{\partial T} \right)_v$$

For a constant volume process

$$(\Delta u)_v = \int_{T_i}^{T_f} C_v dT$$

The first law may be written for a closed stationary system composed of a unit mass of a pure substance.

$$\begin{aligned} Q &= \Delta u + W \\ \text{or } dQ &= du + dW \end{aligned}$$

For a process in the absence of work other than pdv work

$$dW = pdv$$

Therefore  $dQ = du + pdv$

Therefore, when the volume is held constant

$$\begin{aligned} (Q)_v &= (\Delta u)_v \\ (Q)_v &= \int_{T_i}^{T_f} C_v dT \end{aligned}$$

Since  $u$ ,  $T$  and  $v$  are properties,  $C_v$  is a property of the system. The product  $\dot{m}C_v$  is called the **heat capacity** at constant volume (J/K).

### Specific heat at constant pressure

The specific heat at constant pressure  $C_p$  is defined as the rate of change of specific enthalpy with respect to temperature when the pressure is held constant.

$$C_p = \left( \frac{\partial h}{\partial T} \right)_P$$

For a constant pressure process

$$(\Delta h)_P = \int_{T_i}^{T_f} C_p dT$$

The first law for a closed stationary system of unit mass

$$dQ = du + pdv$$

$$\text{Again, } h = u + pv$$

$$\begin{aligned} \text{Therefore } dh &= du + pdv + vdp \\ &= dQ + vdp \end{aligned}$$

$$\text{Therefore } dQ = dh - vdp$$

$$\text{Therefore } (dQ)_P = dh$$

# First Law of Thermodynamics

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$$\text{or } (Q)_p = (\Delta h)_p$$

∴ Form above equations

$$(Q)_P = \int_{T_1}^{T_2} C_p dT$$

$C_p$  is a property of the system, just like  $C_v$ . The heat capacity at constant pressure is equal to  $m C_p$  (J/K).

## Application of First Law to Steady Flow Process S.F.E.E

S.F.E.E. per unit mass basis

$$(i) \quad h_1 + \frac{C_1^2}{2} + gZ_1 + \frac{dQ}{dm} = h_2 + \frac{C_2^2}{2} + gZ_2 + \frac{dW}{dm}$$

[ $h$ ,  $W$ ,  $Q$  should be in J/Kg and  $C$  in m/s and  $g$  in m/s<sup>2</sup>]

$$(ii) \quad h_1 + \frac{C_1^2}{2000} + \frac{gZ_1}{1000} + \frac{dQ}{dm} = h_2 + \frac{C_2^2}{2000} + \frac{gZ_2}{1000} + \frac{dW}{dm}$$

[ $h$ ,  $W$ ,  $Q$  should be in KJ/Kg and  $C$  in m/s and  $g$  in m/s<sup>2</sup>]

S.F.E.E. per unit time basis

$$\begin{aligned} & w_1 \left( h_1 + \frac{C_1^2}{2} + z_1 g \right) + \frac{dQ}{dt} \\ &= w_2 \left( h_2 + \frac{C_2^2}{2} + z_2 g \right) + \frac{dW_x}{dt} \end{aligned}$$

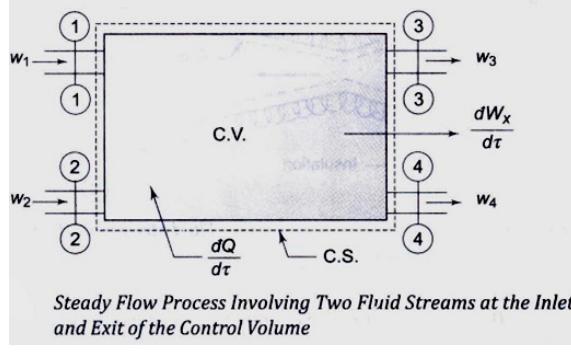
Where,  $w$  = mass flow rate (kg/s)

**Steady Flow Process Involving Two Fluid Streams at the Inlet and Exit of the Control Volume**

# First Law of Thermodynamics

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**Mass balance**

$$\frac{AC_1}{v_1} + \frac{AC_2}{v_2} = \frac{AC_3}{v_3} + \frac{AC_4}{v_4}$$

Where  $v$  = specific volume ( $\text{m}^3/\text{kg}$ )

**Energy balance**

$$\begin{aligned} & w_1 \left( h_1 + \frac{C_1^2}{2} + Z_1 g \right) + w_2 \left( h_2 + \frac{C_2^2}{2} + Z_2 g \right) + \frac{dQ}{dt} \\ &= w_3 \left( h_3 + \frac{C_3^2}{2} + Z_3 g \right) + w_4 \left( h_4 + \frac{C_4^2}{2} + Z_4 g \right) + \frac{dW_x}{dt} \end{aligned}$$

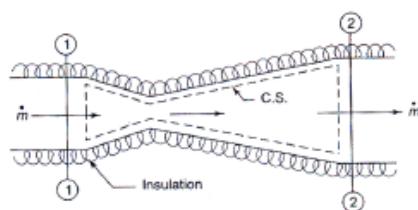
**Some example of steady flow processes:-**

The following examples illustrate the applications of the steady flow energy equation in some of the engineering systems.

**Nozzle and Diffuser:**

A nozzle is a device which increases the **velocity or K.E.** of a fluid at the expense of its pressure drop, whereas a diffuser increases the pressure of a fluid at the expense of its K.E. Figure show in below a nozzle which is insulated. The steady flow energy equation of the control surface gives

$$h_1 + \frac{C_1^2}{2} + Z_1 g + \frac{dQ}{dm} = h_2 + \frac{C_2^2}{2} + Z_2 g + \frac{dW_x}{dm}$$



*Steady Flow Process Involving Two Fluid Streams at the Inlet and Exit of the Control Volume*

**Fig.**

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Here  $\frac{dQ}{dm} = 0$ ,  $\frac{dW_x}{dm} = 0$ , and the change in potential energy is zero. The equation reduces to

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2} \quad (\text{a})$$

The continuity equation gives

$$w = \frac{A_1 C_1}{v_1} = \frac{A_2 C_2}{v_2} \quad (\text{b})$$

When the inlet velocity or the 'velocity of approach'  $V_1$  is small compared to the exit velocity  $V_2$ , Equation (a) becomes

$$h_1 = h_2 + \frac{C_2^2}{2}$$

$$\text{or } C_2 = \sqrt{2(h_1 - h_2)} m/s$$

where  $(h_1 - h_2)$  is in J/kg.

Equations (a) and (b) hold good for a diffuser as well.

## Throttling Device:

When a fluid flows through a constricted passage, like a partially opened valve, an orifice, or a porous plug, there is an appreciable drop in pressure, and the flow is said to be throttled. Figure shown in below, the process of throttling by a prettily opened value on a fluid flowing in an insulated pipe. In the steady-flow energy equation-

$$\frac{dQ}{dm} = 0, \quad \frac{dW_x}{dm} = 0$$

And the changes in P. E. are very small and ignored. Thus, the S.F.E.E. reduces to

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$$

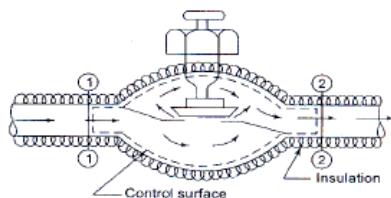


Fig. 6 .Flow Through a Valve  
(Fig.- Flow Through a Valve)

Often the pipe velocities in throttling are so low that the K. E. terms are also negligible. So

$$h_1 = h_2$$

or the enthalpy of the fluid before throttling is equal to the enthalpy of the fluid after throttling.

## Turbine and Compressor:

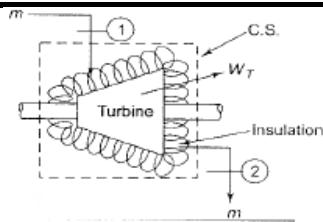
Turbines and engines give positive power output, whereas compressors and pumps require power input.

For a turbine (Fig. below) which is well insulated, the flow velocities are often small, and the K.E. terms can be neglected. The S.F.E.E. then becomes

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**Fig. 7 . Flow Through a Turbine**  
(Fig.-. Flow through a Turbine)

$$h_1 = h_2 + \frac{dW_x}{dm}$$

$$\text{or } \frac{W_x}{m} = h_1 - h_2$$

The enthalpy of the fluid increase by the amount of work input.

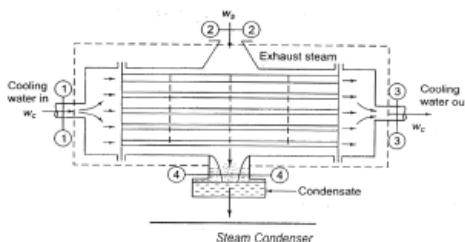
### Heat Exchanger:

A heat exchanger is a device in which heat is transferred from one fluid to another. Figure shown in below a steam condenser where steam condense outside the tubes and cooling water flows through the tubes. The S.F.E.E for the C.S. gives

$$w_c h_1 + w_s h_2 = w_c h_3 + w_s h_4$$

$$\text{or, } w_s(h_2 - h_4) = w_c(h_3 - h_1)$$

Here the K.E. and P.E. terms are considered small, there is no external work done, and energy exchange in the form of heat is confined only between the two fluids, i.e. there is no external heat interaction or heat loss.



**Fig. -**

Figure (shows in below) a steam desuperheater where the temperature of the superheated steam is reduced by spraying water. If  $w_1$ ,  $w_2$ , and  $w_3$  are the mass flow rates of the injected water, of the steam entering, and of the steam leaving, respectively, and  $h_1$ ,  $h_2$ , and  $h_3$  are the corresponding enthalpies, and if K.E. and P.E. terms are neglected as before, the S.F.E.E becomes

$$w_1 h_1 + w_2 h_2 = w_3 h_3$$

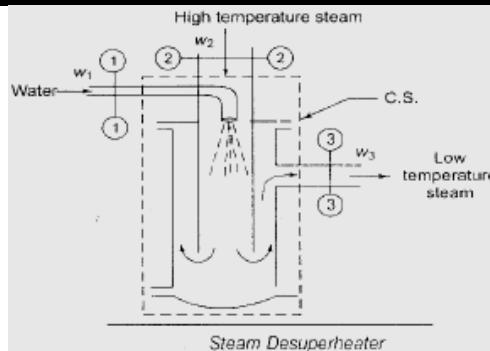
and the mass balance gives

$$w_1 + w_2 = w_3$$

# First Law of Thermodynamics

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The above law is also called as **steady flow energy equation**. This can be applied to various practical situations as work developing system and work absorption system. Let the mass flow rate unity.

### (1) Work developing systems

(a) Water turbines

In this case  $Q = 0$  and  $\Delta U = 0$  and equation becomes

$$p_1 v_1 + z_1 g + \frac{C_1^2}{2} = z_2 g + p_2 v_2 + W$$

(b) Steam or gas turbines

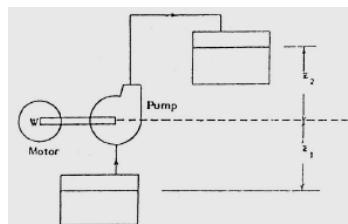
In this case generally  $\Delta Z$  can be assumed to be zero and the equation becomes

$$W = (h_1 - h_2) + \left( \frac{C_1^2 - C_2^2}{2} \right) + \Delta Q$$

### (2) Work absorbing systems

(a) Centrifugal water pump

The system is shown in the Figure below



**Fig.**

In this system  $Q = 0$  and  $\Delta U = 0$ ; the energy equation now becomes,

$$p_1 v_1 + z_1 g + W = z_2 g + p_2 v_2 + \frac{C_2^2}{2}$$

(b) **Centrifugal compressor** – In this system  $\Delta z = 0$  and the equation becomes,

$$\frac{C_1^2}{2} + h_1 + W - Q = \frac{C_2^2}{2} + h_2$$

(c) **Blowers** – In this case we have  $\Delta z = 0$ ,  $p_1 v_1 = p_2 v_2$  and  $Q = 0$ ; now the energy simplifies to

# First Law of Thermodynamics

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$$u_1 + W = u_2 + \frac{C_2}{2} \text{ as } C_2 \gg G$$

(d) **Fans** – In fans the temperature rise is very small and heat loss is neglected (i.e.)  $\Delta h = 0$ ,  $q = 0$  and hence the energy equation for fans becomes,

$$W = \frac{C_2}{2}$$

(e) **Reciprocating compressor** – In a reciprocating compressor  $\Delta KE$  and  $\Delta PE$  are negligibly small. Energy equation applied to a reciprocating compressor is

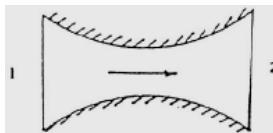
$$\begin{aligned} h_1 - Q &= h_2 - W \\ \text{or} \quad W &= Q + (h_2 - h_1) \end{aligned}$$

### (3) Non-work developing and absorbing systems

(a) **Steam boiler** – In this system we neglect  $\Delta Z$ ,  $\Delta KE$  and  $W$  (i.e.)  $\Delta Z = \Delta KE = W = 0$ ; the energy equation for a boiler becomes  $Q = (h_2 - h_1)$

(b) **Steam condenser** – In this system the work done is zero and we can also assume  $\Delta Z$  and  $\Delta KE$  are very small. Under steady conditions the change in enthalpy is equal to heat lost by steam.  $Q = (h_1 - h_2)$  and this heat is also equal to the change in enthalpy of cooling water circulated (i.e.) the heat lost by steam will be equal to heat gained by the cooling water.

(c) **Steam nozzle:**



In this system we can assume  $\Delta Z$  and  $W$  to be zero and heat transfer which is nothing but any possible heat loss also zero.

The energy equation for this case becomes.

$$\begin{aligned} h_1 + \frac{C_1^2}{2} &= h_2 + \frac{C_2^2}{2} \\ \text{or} \quad C_2 &= \sqrt{C_1^2 + 2(h_1 - h_2)} \end{aligned}$$

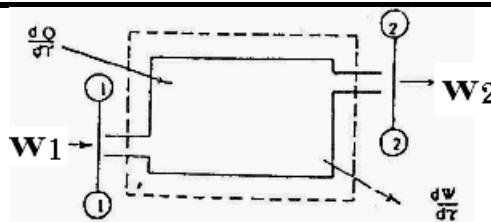
### (vii) Unsteady Flow Analysis:-

Many flow processes, such as filling up and evacuating gas cylinders, are not steady. Such processes can be analyzed by the control volume technique. Consider a device through which a fluid is flowing under non-steady state conditions (Figure-shown in below). The rate at which the mass of fluid within the control volume is accumulated is equal to the net rate of mass flow across the control surface, as given below:

# First Law of Thermodynamics

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**Fig.**

$$\frac{dm}{dt} = u_1 - u_2 = \frac{dm_1}{dt} - \frac{dm_2}{dt}$$

Where  $m_v$  is the mass of fluid within the control volume at any instant.

Over any finite period of time

$$\Delta m_v = \Delta m_1 - \Delta m_2$$

The rate of accumulation of energy within the control volume is equal to the net rate of energy flow across the control surface. If  $E_v$  is the energy of fluid within the control volume at any instant,

Rate of energy increase = Rate of energy inflow – Rate of energy outflow.

$$\frac{dE_v}{dt} = u_1 \left( h_1 + \frac{C_1^2}{2} + Z_1 g \right) + \frac{dQ}{dt} - u_2 \left( h_2 + \frac{C_2^2}{2} + Z_2 g \right) - \frac{dW}{dt} \quad \text{equation...A}$$

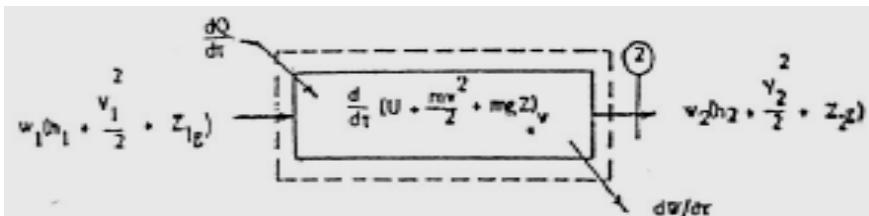
$$E_v = \left( U + \frac{mC^2}{2} + mgZ \right)_v$$

Where  $m$  is the mass of fluid in the control volume at any instant

$$\begin{aligned} \therefore \frac{dE_v}{dt} &= \frac{d}{dt} \left( U + \frac{mC^2}{2} + mgZ \right)_v = \\ & \left( h_1 + \frac{C_1^2}{2} + Z_1 g \right) \frac{dm_1}{dt} + \frac{dQ}{dt} - \left( h_2 + \frac{C_2^2}{2} + Z_2 g \right) \frac{dm_2}{dt} - \frac{dW}{dt} \end{aligned} \quad (\text{equation.....B})$$

Following Figure shows all these energy flux quantities. For any time interval, equation (B) becomes

$$\Delta E_v = Q - W + \int \left( h_1 + \frac{C_1^2}{2} + Z_1 g \right) dm_1 - \int \left( h_2 + \frac{C_2^2}{2} + Z_2 g \right) dm_2$$



**Fig.**

Equation (A) is general energy equation. For steady flow,

$$\frac{dE_v}{dt} = 0$$

and the equation reduces. For a closed system  $w_1 = 0$ ,  $w_2 = 0$ , then from equation (A),

$$\frac{dE_v}{dt} = \frac{dQ}{dt} - \frac{dW}{dt}$$

Or  $dE_v = dQ - dW$  or  $dQ = dE + dW$

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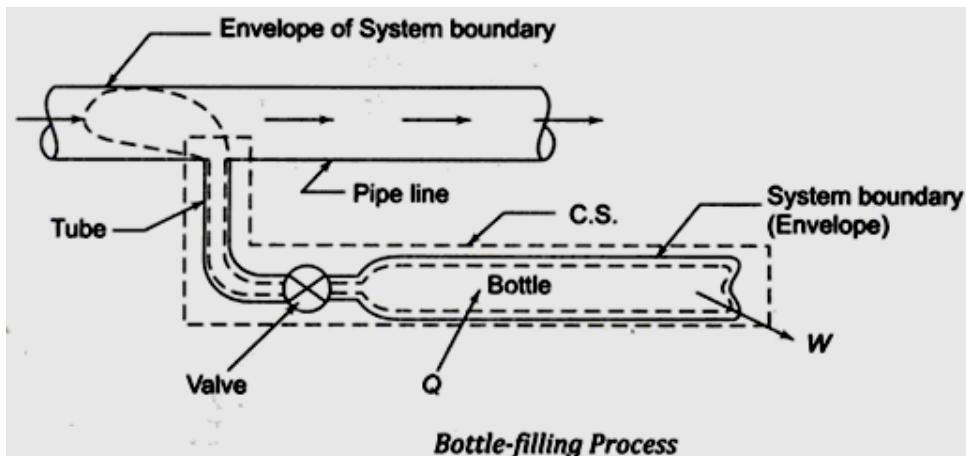
Chapter 2

Flow Processes

## Example of a variable flow problem:

Variable flow processes may be analyzed either by the system technique or the control volume technique, as illustrated below.

Consider a process in which a gas bottle is filled from a pipeline (Figure shown in below). In the beginning the bottle contains gas of mass  $m_1$  at state  $p_1, T_1, v_1, h_1$  and  $u_1$ . The valve is opened and gas flows into the bottle till the mass of gas in the bottle is  $m_2$  at state  $p_2, T_2, v_2, h_2$  and  $u_2$ . The supply to the pipeline is very large so that the state of gas in the pipeline is constant at  $p_p, T_p, v_p, h_p, u_p$  and  $v_p$ .



**System Technique:** Assume an envelope (which is extensible) of gas in the pipeline and the tube which would eventually enter the bottle, as shown in Figure above.

Energy of the gas before filling.

$$E_1 = m_1 u_1 + (m_2 - m_1) \left( \frac{C_p}{2} + u_p \right)$$

Where  $(m_2 - m_1)$  is the mass of gas in the pipeline and tube which would enter the bottle.

$$E_2 = m_2 u_2$$

$$\Delta E = E_2 - E_1 = m_2 u_2 - m_1 u_1 \left( \frac{C_p}{2} + u_p \right)$$

The P.E. terms are neglected. The gas in the bottle is not in motion, and so the K.E. terms have been omitted.

Now, there is a change in the volume of gas because of the collapse of the envelope to zero volume. Then the work done

$$\begin{aligned} W &= p_p (V_2 - V_1) \\ &= p_p [0 - (m_2 - m_1)v_p] \\ &= -(m_2 - m_1)p_p v_p \end{aligned}$$

Using the first for the process

$$Q = \Delta E + W$$

# First Law of Thermodynamics

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$$= m_2 u_2 - m_1 u_1 - (m_2 - m_1) \left[ \frac{C_p^2}{2} + u_p \right] - (m_2 - m_1) p_p v_p$$

$$= m_2 u_2 - m_1 u_1 - (m_2 - m_1) \left( \frac{C_p^2}{2} + h_p \right)$$

which gives the energy balance for the process.

**Control Volume Technique:** Assume a control volume bounded by a control surface as shown in Figure above, Applying the energy equation in this case, the following energy balance may be written on a time rate basis -

$$\frac{dE_v}{dt} = \frac{dQ}{dt} + \left( h_p + \frac{C_p^2}{2} \right) \frac{dm}{dt}$$

Since  $h_p$  and  $C_p$  are constant, the equation is integrated to give for the Total process

$$\Delta E_v = Q + \left( h_p + \frac{C_p^2}{2} \right) (m_2 - m_1)$$

Now

$$\Delta E_v = U_2 - U_1 = m_2 u_2 - m_1 u_1$$

$$Q = m_2 u_2 - m_1 u_1 - \left( h_p + \frac{C_p^2}{2} \right) (m_2 - m_1)$$

## Discharging and Charging a Tank

Let us consider a tank discharging a fluid into a supply line (Figure). Since  $dW_x = 0$  and  $dm_{in} = 0$ , applying first law to the control volume,

$$dU_V = dQ + \left( h + \frac{C^2}{2} + gz \right)_{out} dm_{out}$$

Assuming K.E. and P.E. of the fluid to be small and  $dQ = 0$

$$d(mu) = hdm$$

$$mdu + udm = udm + pv dm$$

$$\frac{dm}{m} = \frac{du}{pv}$$

Again  $V = vm = \text{const.}$

$$vdm + mdv = 0$$

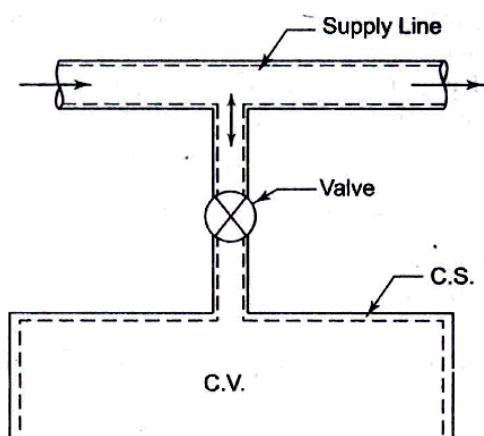
$$\text{or } \frac{dm}{m} = -\frac{dv}{v}$$

$$\frac{du}{pv} = -\frac{dv}{v}$$

$$d(u + pv) = 0$$

$$\text{or } dQ = 0$$

which shows that the process is adiabatic and



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quasi-static.

*Charging and Discharging a Tank*

For charging the tank

$$\int(hdm)_{in} = \Delta U_V = m_2 u_2 - m_1 u_1 \\ m_p h_p = m_2 u_2 - m_1 u_1$$

where the subscript  $p$  refers to the constant state of the fluid in the pipeline. If the tank is initially empty,  $m_1 = 0$ .

$$m_p h_p = m_2 u_2$$

Since

$$m_p = m_2 \\ h_p = u_2$$

If the fluid is an ideal gas, the temperature of the gas in the tank after it is charged is given by

$$c_p T_p = c_v T_2 \\ \text{or} \\ T_2 = \gamma T_p$$

## PROBLEMS & SOLUTIONS

### Example 1

The work and heat transfer per degree of temperature change for a closed system is given by

$$\frac{dW}{dT} = \frac{1}{30} \text{ kJ/}^\circ\text{C}, \frac{dQ}{dT} = \frac{1}{10} \text{ kJ/}^\circ\text{C}$$

Calculate the change in internal energy as its temperature increases from  $125^\circ\text{C}$  to  $245^\circ\text{C}$ .

**Solution:**

$$dW = \frac{dT}{30}$$

$$W = \int_{T_1}^{T_2} \frac{dT}{30} = \frac{1}{30} (T_2 - T_1) = \frac{1}{30} (245 - 125)$$

$$dQ = \frac{dT}{10}$$

$$Q = \int_{T_1}^{T_2} \frac{dT}{10} = \frac{1}{10} (245 - 125) = 12 \text{ kJ}$$

Applying First Law of Thermodynamics

$$Q = W + \Delta U$$

$$\Delta U = Q - W = 12 - 4 = 8 \text{ kJ.}$$

### Example 2

Air expands from 3 bar to 1 bar in a nozzle. The initial velocity is 90 m/s. the initial temperature is  $150^\circ\text{C}$ . Calculate the velocity of air at the exit of the nozzle.

**Solution:**

The system in question is an open one. First Law of Thermodynamics for an open system gives

$$w_1 \left[ h_1 + \frac{C_1^2}{2} + Z_1 g \right] + Q = w_2 \left[ h_2 + \frac{C_2^2}{2} + Z_2 g \right] + W$$

# First Law of Thermodynamics

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Since the flow is assumed to be steady.

$$W_1 = W_2$$

Flow in a nozzle is adiabatic flow.

Hence  $Q = 0$

Also  $W = 0$

The datum can be selected to pass through axis; then

$Z_1 = Z_2$ .

Hence

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$$

$$(or) \frac{C_2^2}{2} = (h_1 - h_2) + \frac{C_1^2}{2}$$

and

$$T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{\gamma-1}{\gamma}}$$

$\gamma$  for air = 1.4

$$T_1 = 150 + 273 = 423$$

$$\therefore T_2 = 423 \left( \frac{1}{3} \right)^{0.4/1.4} = 309 K$$

For air  $C_p = 1.005 \text{ kJ/kg}^\circ\text{C}$

$C_v = 0.718 \text{ kJ/kg}^\circ\text{C}$ .

$R = 287 \text{ J/kg K} = 0.287 \text{ kJ/kg K}$

S.F.E.E. : - We have  $(h_1 - h_2) = C_p(T_1 - T_2)$

$$\frac{C_2^2}{2} = (h_1 - h_2) + \frac{C_1^2}{2} = 1.005 \times 10^3 \times (423 - 309) + \frac{90^2}{2}$$

or,  $C_2 = 487 \text{ m/s}$ .

### Example 3

An evacuated cylinder fitted with a valve through which air from atmosphere at 760 mm Hg and  $25^\circ\text{C}$  is allow to fill it slowly. If no heat interaction is involved, what will be the temperature of air in the bottle when the pressure reaches 760 mm Hg?

Use the following:

(1) Internal energy of air  $u = u_0 + 0.718T \text{ kJ/kg}$  where

T is temperature in  $^\circ\text{C}$ .

(2)  $R = 0.287 \text{ kJ/kg K}$ .

**Solution:**

Applying first law, ignoring potential and kinetic energy terms, to the vessel as control volume.

# First Law of Thermodynamics

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$$Q + m_1 h_i = m_e h_e + m_2 u_2 - m_1 u_i + W$$

Here  $Q=0$ ,  $W=0$ ,  $m_e=0$  (no mass leaving from control vol.)

$m_1=0$  (evacuated)  $\therefore m_2=m_1$

$$\therefore h_i = u_2$$

$$h_i = u_i + pv = u_0 + 0.718T_i + 0.287(T_i + 273)$$

$$= u_0 + 0.718 \times 25 + 0.287 \times 298$$

$$= u_0 + 103.48 \text{ kJ/kg} = u_2$$

$$\text{or } u_2 - u_0 = 103.48 \text{ kJ/kg}$$

$$u_2 = u_0 + 0.718T_2$$

$$T_2 = \frac{u_2 - u_0}{0.718} = \frac{103.48}{0.718} = 144.2^\circ\text{C}$$

## Example 4

A system whose mass is 4.5 kg undergoes a process and the temperature changes from  $50^\circ\text{ C}$  to  $100^\circ\text{C}$ . Assume that the specific heat of the system is a function of temperature only. Calculate the heat transfer during the process for the following relation ship.

$$c_n = 1.25 + \frac{80}{t+160} \text{ kJ/kg}^\circ\text{C} \quad [\text{t is in } {}^\circ\text{C}]$$

**Solution:**

$$\begin{aligned} {}_1 Q_2 &= \int_{50}^{100} mc_n dt = 4.5 \int_{50}^{100} \left( 1.25 + \frac{80}{t+160} \right) dt \\ &= 4.5 \left\{ \int_{50}^{100} 1.25 dt + \int_{50}^{100} \frac{dt}{0.0125t+20} \right\} \\ &= 4.5 \left\{ [1.25t]_{50}^{100} + \left[ \frac{1}{0.0125} \ln(0.0125t+20) \right]_{50}^{100} \right\} \\ &= 4.5 \left\{ [1.25 \times 50] + \left[ \frac{1}{0.0125} (\ln(1.25+20) - \ln(0.625+2)) \right] \right\} \\ &= 4.5 \left\{ 62.5 + \frac{1}{0.0125} \ln \frac{3.25}{2.625} \right\} = 358 \text{ kJ} \end{aligned}$$

# First Law of Thermodynamics

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## ASKED OBJECTIVE QUESTIONS (GATE, IES, IAS)

### Previous 20-Years GATE Questions

#### Application of First Law to Steady Flow Process S.F.E.E

##### Common Data for Questions Q1 and Q2:

The inlet and the outlet conditions of steam for an adiabatic steam turbine are as indicated in the figure. The notations are as usually followed.

$$\begin{aligned} h_1 &= 3200 \text{ kJ/kg} \\ V_1 &= 160 \text{ m/s} \\ Z_1 &\approx 10 \text{ m} \\ P_1 &= 3 \text{ MPa} \end{aligned}$$

[GATE-2009]

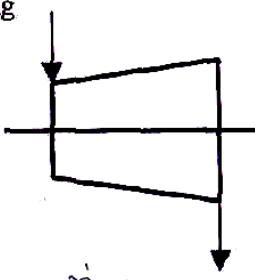


figure.

$$\begin{aligned} h_2 &= 2600 \text{ kJ/kg} \\ V_2 &= 100 \text{ m/s} \\ Z_2 &= 6 \text{ m} \\ P_2 &= 70 \text{ kPa} \end{aligned}$$

- GATE-1. If mass flow rate of steam through the turbine is 20 kg/s the power output of the turbine (in MW) is: [GATE-2009]

(a) 12.157                      (b) 12.941                      (c) 168.001                      (d) 168.785

- GATE-2. Assume the above turbine to be part of a simple Rankine cycle. The density of water at the inlet to the pump is 1000 kg/m<sup>3</sup>. Ignoring kinetic and potential energy effects, the specific work (in kJ/kg) supplied to the pump is:

[GATE-2009]

(a) 0.293                      (b) 0.351                      (c) 2.930                      (d) 3.510

- GATE-3. The following four figures have been drawn to represent a fictitious thermodynamic cycle, on the p-v and T-s planes.

[GATE-2005]

P

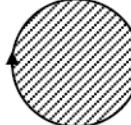


Figure 1

T

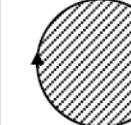


Figure 2

P

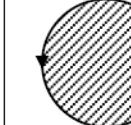


Figure 3

T

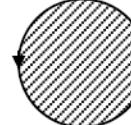


Figure 4

According to the first law of thermodynamics, equal areas are enclosed by

- (a) Figures 1 and 2    (b) Figures 1 and 3    (c) Figures 1 and 4    (d) Figures 2 and 3

# First Law of Thermodynamics

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## Internal Energy – A Property of System

GATE-4. A gas contained in a cylinder is compressed, the work required for compression being 5000 kJ. During the process, heat interaction of 2000 kJ causes the surroundings to be heated. The change in internal energy of the gas during the process is: [GATE-2004]

- (a) - 7000 kJ      (b) - 3000 kJ      (c) + 3000 kJ      (d) + 7000 kJ

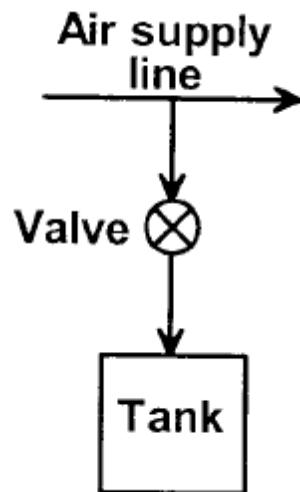
GATE-4a. The contents of a well-insulated tank are heated by a resistor of  $23\Omega$  in which 10 A current is flowing. Consider the tank along with its contents as a thermodynamic system. The work done by the system and the heat transfer to the system are positive. The rates of heat ( $Q$ ), work ( $W$ ) and change in internal energy ( $\Delta U$ ) during the process in kW are [GATE-2011]

- (a)  $Q = 0, W = -2.3, \Delta U = +2.3$       (b)  $Q = +2.3, W = 0, \Delta U = +2.3$   
(c)  $Q = -2.3, W = 0, \Delta U = -2.3$       (d)  $Q = 0, W = +2.3, \Delta U = -2.3$

## Discharging and Charging a Tank

GATE-5. A rigid, insulated tank is initially evacuated. The tank is connected with a supply line through which air (assumed to be ideal gas with constant specific heats) passes at 1 MPa, 350°C. A valve connected with the supply line is opened and the tank is charged with air until the final pressure inside the tank reaches 1 MPa. The final temperature inside the tank

- (A) Is greater than 350°C  
(B) Is less than 350°C  
(C) Is equal to 350°C  
(D) May be greater than, less than, or equal to 350°C, depending on the volume of the tank



## Previous 20-Years IES Questions

## First Law of Thermodynamics

IES-1. Which one of the following sets of thermodynamic laws/relations is directly involved in determining the final properties during an adiabatic mixing process? [IES-2000]

- (a) The first and second laws of thermodynamics  
(b) The second law of thermodynamics and steady flow relations  
(c) Perfect gas relationship and steady flow relations  
(d) The first law of thermodynamics and perfect gas relationship

# First Law of Thermodynamics

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- IES-2.** Two blocks which are at different states are brought into contact with each other and allowed to reach a final state of thermal equilibrium. The final temperature attained is specified by the [IES-1998]

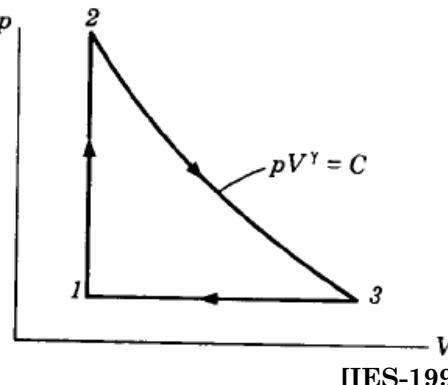
- (a) Zeroth law of thermodynamics      (b) First law of thermodynamics  
 (c) Second law of thermodynamics      (d) Third law of thermodynamics

- IES-3.** For a closed system, the difference between the heat added to the system and the work done by the system is equal to the change in [IES-1992]

- (a) Enthalpy      (b) Entropy  
 (c) Temperature      (d) Internal energy

- IES-4.** An ideal cycle is shown in the figure. Its thermal efficiency is given by

- (a)  $1 - \frac{\left(\frac{V_3}{V_1} - 1\right)}{\left(\frac{P_2}{P_1} - 1\right)}$       (b)  $1 - \frac{1}{\gamma} \frac{\left(\frac{V_3}{V_1} - 1\right)}{\left(\frac{P_2}{P_1} - 1\right)}$   
 (c)  $1 - \gamma \frac{(V_3 - V_1)}{(P_2 - P_1)} \frac{P_1}{V_1}$       (d)  $1 - \frac{1}{\gamma} \frac{(V_3 - V_1)}{(P_2 - P_1)} \frac{P_1}{V_1}$



[IES-1998]

- IES-5.** Which one of the following is correct?

[IES-2007]

The cyclic integral of  $(\partial Q - \partial W)$  for a process is:

- (a) Positive      (b) Negative      (c) Zero      (d) Unpredictable

- IES-6.** A closed system undergoes a process 1-2 for which the values of  $Q_{1-2}$  and  $W_{1-2}$  are +20 kJ and +50 kJ, respectively. If the system is returned to state, 1, and  $Q_{2-1}$  is -10 kJ, what is the value of the work  $W_{2-1}$ ? [IES-2005]

- (a) + 20 kJ      (b) -40 kJ      (c) -80 kJ      (d) +40 kJ

- IES-7.** A gas is compressed in a cylinder by a movable piston to a volume one-half of its original volume. During the process, 300 kJ heat left the gas and the internal energy remained same. What is the work done on the gas? [IES-2005]

- (a) 100 kNm      (b) 150 kNm      (c) 200 kNm      (d) 300 kNm

- IES-8.** In a steady-flow adiabatic turbine, the changes in the internal energy, enthalpy, kinetic energy and potential energy of the working fluid, from inlet to exit, are -100 kJ/kg, -140 kJ/kg, -10 kJ/kg and 0 kJ/kg respectively. Which one of the following gives the amount of work developed by the turbine? [IES-2004]

- (a) 100 kJ/kg      (b) 110 kJ/kg      (c) 140 kJ/kg      (d) 150 kJ/kg

# First Law of Thermodynamics

## S K Mondal's

## Chapter 2

- IES-9.** Gas contained in a closed system consisting of piston cylinder arrangement is expanded. Work done by the gas during expansion is 50 kJ. Decrease in internal energy of the gas during expansion is 30 kJ. Heat transfer during the process is equal to: [IES-2003]

(a) -20 kJ                         (b) +20 kJ                         (c) -80 kJ                         (d) +80 kJ

- IES-10.** A system while undergoing a cycle [IES-2001]  
A – B – C – D – A has the values of heat and work transfers as given in the Table:

Process	Q kJ/min	W kJ/min
A–B	+687	+474
B–C	-269	0
C–D	-199	-180
D–A	+75	0

The power developed in kW is, nearly,

(a) 4.9                             (b) 24.5                             (c) 49                                 (d) 98

- IES-11.** The values of heat transfer and work transfer for four processes of a thermodynamic cycle are given below: [IES-1994]

Process	Heat Transfer (kJ)	Work Transfer (kJ)
1	300	300
2	Zero	250
3	-100	-100
4	Zero	-250

The thermal efficiency and work ratio for the cycle will be respectively.

(a) 33% and 0.66    (b) 66% and 0.36.    (c) 36% and 0.66    (d) 33% and 0.36.

- IES-12.** A tank containing air is stirred by a paddle wheel. The work input to the paddle wheel is 9000 kJ and the heat transferred to the surroundings from the tank is 3000 kJ. The external work done by the system is: [IES-1999]

(a) Zero                             (b) 3000 kJ                             (c) 6000 kJ                             (d) 9000 kJ

## Internal Energy – A Property of System

- IES-13.** For a simple closed system of constant composition, the difference between the net heat and work interactions is identifiable as the change in [IES-2003]

(a) Enthalpy                             (b) Entropy                             (c) Flow energy                             (d) Internal energy

- IES-14.** Assertion (A): The internal energy depends on the internal state of a body, as determined by its temperature, pressure and composition. [IES-2006]

Reason (R): Internal energy of a substance does not include any energy that it may possess as a result of its macroscopic position or movement.

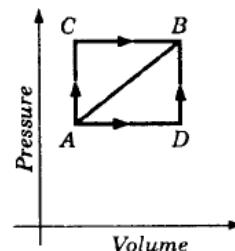
- (a) Both A and R are individually true and R is the correct explanation of A  
 (b) Both A and R are individually true but R is NOT the correct explanation of A  
 (c) A is true but R is false  
 (d) A is false but R is true

# First Law of Thermodynamics

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- IES-15.** Change in internal energy in a reversible process occurring in a closed system is equal to the heat transferred if the process occurs at constant: [IES-2005]  
 (a) Pressure                                 (b) Volume                                 (c) Temperature                                 (d) Enthalpy
- IES-16.** 170 kJ of heat is supplied to a system at constant volume. Then the system rejects 180 kJ of heat at constant pressure and 40 kJ of work is done on it. The system is finally brought to its original state by adiabatic process. If the initial value of internal energy is 100 kJ, then which one of the following statements is correct? [IES-2004]  
 (a) The highest value of internal energy occurs at the end of the constant volume process  
 (b) The highest value of internal energy occurs at the end of constant pressure process.  
 (c) The highest value of internal energy occurs after adiabatic expansion  
 (d) Internal energy is equal at all points
- IES-17.** 85 kJ of heat is supplied to a closed system at constant volume. During the next process, the system rejects 90 kJ of heat at constant pressure while 20 kJ of work is done on it. The system is brought to the original state by an adiabatic process. The initial internal energy is 100 kJ. Then what is the quantity of work transfer during the process? [IES-2009]  
 (a) 30 kJ                                     (b) 25 kJ                                     (c) 20 kJ                                     (d) 15 kJ
- IES-17a.** A closed system receives 60 kJ heat but its internal energy decreases by 30 kJ. Then the work done by the system is [IES-2010]  
 (a) 90 kJ                                     (b) 30 kJ                                     (c) -30 kJ                                     (d) -90 kJ
- IES-17b.** If the work done on a closed system is 20kJ/kg, and 40kJ/kg heat is rejected from the system, its internal energy decreases by [IES-2012]  
 (a) 20kJ/kg                                     (b) 60kJ/kg                                     (c) -20kJ/kg                                     (d) -60kJ/kg
- IES-18.** A system undergoes a process during which the heat transfer to the system per degree increase in temperature is given by the equation: [IES-2004]  
 $dQ/dT = 2 \text{ kJ}/^{\circ}\text{C}$  The work done by the system per degree increase in temperature is given by the equation  $dW/dT = 2 - 0.1 T$ , where  $T$  is in  $^{\circ}\text{C}$ . If during the process, the temperature of water varies from  $100^{\circ}\text{C}$  to  $150^{\circ}\text{C}$ , what will be the change in internal energy?  
 (a) 125 kJ                                     (b) -250 kJ                                     (c) 625 kJ                                     (d) -1250 kJ
- IES-19.** When a system is taken from state A to state B along the path A-C-B, 180 kJ of heat flows into the system and it does 130 kJ of work (see figure given):  
 How much heat will flow into the system along the path A-D-B if the work done by it along the path is 40 kJ?  
 (a) 40 kJ                                             (b) 60 kJ                                             (c) 90 kJ                                             (d) 135 kJ
- IES-20.** The internal energy of a certain system is a function of temperature alone and is given by the formula  $E = 25 + 0.25t$  kJ. If this system executes a process for



[IES-1997]

# First Law of Thermodynamics

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which the work done by it per degree temperature increase is 0.75 kJ/K, then the heat interaction per degree temperature increase, in kJ, is: [IES-1995]  
(a) -1.00                          (b) -0.50                          (c) 0.50                          (d) 1.00

- IES-21. When a gas is heated at constant pressure, the percentage of the energy supplied, which goes as the internal energy of the gas is: [IES-1992]  
(a) More for a diatomic gas than for triatomic gas  
(b) Same for monatomic, diatomic and triatomic gases but less than 100%  
(c) 100% for all gases  
(d) Less for triatomic gas than for a diatomic gas

## Perpetual Motion Machine of the First Kind-PMM1

- IES-22. Consider the following statements: [IES-2000]  
1. The first law of thermodynamics is a law of conservation of energy.  
2. Perpetual motion machine of the first kind converts energy into equivalent work.  
3. A closed system does not exchange work or energy with its surroundings.  
4. The second law of thermodynamics stipulates the law of conservation of energy and entropy.

Which of the statements are correct?

- (a) 1 and 2                          (b) 2 and 4                          (c) 2, 3 and 4                          (d) 1, 2 and 3

## Enthalpy

- IES-23. Assertion (A): If the enthalpy of a closed system decreases by 25 kJ while the system receives 30 kJ of energy by heat transfer, the work done by the system is 55 kJ. [IES-2001]  
Reason (R): The first law energy balance for a closed system is (notations have their usual meaning)  $\Delta E = Q - W$   
(a) Both A and R are individually true and R is the correct explanation of A  
(b) Both A and R are individually true but R is NOT the correct explanation of A  
(c) A is true but R is false  
(d) A is false but R is true

- IES-23a. For steady flow through an insulated horizontal constant diameter pipe, this property remains constant [IES-2012]  
(a) Enthalpy                          (b) Internal energy                          (c) Entropy                          (d) Volume

## Application of First Law to Steady Flow Process S.F.E.E

- IES-24. Which one of the following is the steady flow energy equation for a boiler?

- (a)  $h_1 + \frac{V_1^2}{2gJ} = h_2 + \frac{V_2^2}{2gJ}$                           (b)  $Q = (h_2 - h_1)$  [IES-2005]  
(c)  $h_1 + \frac{V_1^2}{2gJ} + Q = h_2 + \frac{V_2^2}{2gJ}$                           (d)  $W_s = (h_2 - h_1) + Q$

# First Law of Thermodynamics

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- IES-25. A 4 kW, 20 litre water heater is switched on for 10 minutes. The heat capacity  $C_p$  for water is 4 kJ/kg K. Assuming all the electrical energy has gone into heating the water, what is the increase of the water temperature? [IES-2008]  
 (a) 15°C (b) 20°C (c) 26°C (d) 30°C

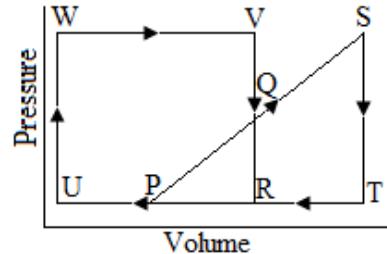
## Discharging and Charging a Tank

- IES-26. An insulated tank initially contains 0.25 kg of a gas with an internal energy of 200 kJ/kg .Additional gas with an internal energy of 300 kJ/kg and an enthalpy of 400 kJ/kg enters the tank until the total mass of gas contained is 1 kg. What is the final internal energy(in kJ/kg) of the gas in the tank? [IES-2007]  
 (a) 250 (b) 275 (c) 350 (d) None of the above

## Previous 20-Years IAS Questions

- IAS-1. A system executes a cycle during which there are four heat transfers:  $Q_{12} = 220$  kJ,  $Q_{23} = -25$  kJ,  $Q_{34} = -180$  kJ,  $Q_{41} = 50$  kJ. The work during three of the processes is  $W_{12} = 15$  kJ,  $W_{23} = -10$  kJ,  $W_{34} = 60$  kJ. The work during the process 4 - 1 is:  
 (a) - 230 kJ (b) 0 kJ (c) 230 kJ (d) 130 kJ [IAS-2003]

- IAS-2. Two ideal heat engine cycles are represented in the given figure. Assume  $VQ = QR$ ,  $PQ = QS$  and  $UP = PR = RT$ . If the work interaction for the rectangular cycle (WVUR) is 48 Nm, then the work interaction for the other cycle PST is:  
 (a) 12Nm (b) 18 Nm (c) 24 Nm (d) 36 Nm [IAS-2001]



- IAS-3. A reversible heat engine operating between hot and cold reservoirs delivers a work output of 54 kJ while it rejects a heat of 66 kJ. The efficiency of this engine is:  
 (a) 0.45 (b) 0.66 (c) 0.75 (d) 0.82 [IAS-1998]

- IAS-4. If a heat engine gives an output of 3 kW when the input is 10,000 J/s, then the thermal efficiency of the engine will be:  
 (a) 20% (b) 30% (c) 70% (d) 76.7% [IAS-1995]

- IAS-5. In an adiabatic process, 5000J of work is performed on a system. The system returns to its original state while 1000J of heat is added. The work done during the non-adiabatic process is:  
 (a) + 4000J (b) - 4000J (c) + 6000J (d) - 6000J [IAS-1997]

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- IAS-6.** In a thermodynamic cycle consisting of four processes, the heat and work are as follows: [IAS-1996]

$$Q: +30, -10, -20, +5$$

$$W: +3, 10, -8, 0$$

The thermal efficiency of the cycle will be:

- (a) Zero                    (b) 7.15%                    (c) 14.33%                    (d) 28.6%

- IAS-7.** Match List-I (Devices) with List-II (Thermodynamic equations) and select the correct answer using the codes below the lists: [IAS-1996]

**List-I**

- A. Turbine
- B. Nozzle
- C. Valve
- D. Compressor

**Codes:**      A      B      C      D  
 (a)      4      3      2      1  
 (c)      4      3      1      2

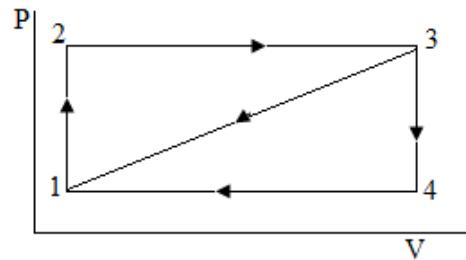
**List-II**

- 1.  $W = h_2 - h_1$
- 2.  $h_1 = h_2$
- 3.  $h_1 = h_2 + V^2/2$
- 4.  $W = h_1 - h_2$

	A	B	C	D
(b)	2	3	1	4
(d)	3	2	4	1

- IAS-8.** Given that the path 1-2-3, a system absorbs 100kJ as heat and does 60kJ work while along the path 1-4-3 it does 20kJ work (see figure given). The heat absorbed during the cycle 1-4-3 is:

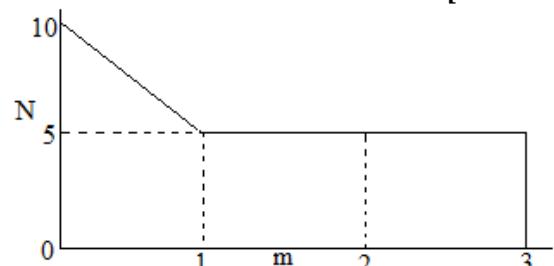
- (a) - 140 KJ                    (b) - 80 kJ  
 (c) - 40kJ                    (d) + 60 kJ



[IAS 1994]

- IAS-9.** The given figure shows the variation of force in an elementary system which undergoes a process during which the plunger position changes from 0 to 3 m. If the internal energy of the system at the end of the process is 2.5 J higher, then the heat absorbed during the process is:

- (a) 15 J                    (b) 20 J

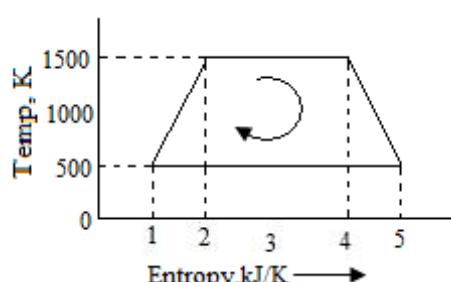


- (c) 25 J                    (d) 30 J

[IAS-1994]

- IAS-10.** The efficiency of a reversible cyclic process undergone by a substance as shown in the given diagram is:

- (a) 0.40                    (b) 0.55  
 (c) 0.60                    (d) 0.80



[IAS 1994]

# First Law of Thermodynamics

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## Internal Energy – A Property of System

- IAS-11. Which one of the following is the correct expression for change in the internal energy for a small temperature change  $\Delta T$  for an ideal gas? [IAS-2007]

(a)  $\Delta U = C_v \times \Delta T$

(b)  $\Delta U = C_p \times \Delta T$

(c)  $\Delta U = \frac{C_p}{C_v} \times \Delta T$

(d)  $\Delta U = (C_p - C_v) \times \Delta T$

- IAS-12. The heat transferred in a thermodynamic cycle of a system consisting of four processes is successively 0, 8, 6 and -4 units. The net change in the internal energy of the system will be: [IAS-1999]

(a) - 8

(b) Zero

(c) 10

(d) -10

- IAS-13. During a process with heat and work interactions, the internal energy of a system increases by 30 kJ. The amounts of heat and work interactions are respectively [IAS-1999]

(a) - 50 kJ and - 80 kJ

(b) -50 kJ and 80 kJ

(c) 50 kJ and 80 kJ

(d) 50 kJ and - 80 kJ

- IAS-14. A mixture of gases expands from  $0.03 \text{ m}^3$  to  $0.06 \text{ m}^3$  at a constant pressure of 1 MPa and absorbs 84 kJ of heat during the process. The change in internal energy of the mixture is: [IAS 1994]

(a) 30 kJ

(b) 54 kJ

(c) 84 kJ

(d) 114 kJ

- IAS-15. In an adiabatic process 6000 J of work is performed on a system. In the non-adiabatic process by which the system returns to its original state 1000J of heat is added to the system. What is the work done during non-adiabatic process? [IAS-2004]

(a) + 7000 J

(b) - 7000 J

(c) + 5000 J

(d) - 5000 J

## Enthalpy

- IAS-16. The fundamental unit of enthalpy is: [IAS 1994]

(a)  $MLT^{-2}$

(b)  $ML^{-2}T^{-1}$

(c)  $ML^2T^{-2}$

(d)  $ML^3T^{-2}$

## Application of First Law to Steady Flow Process S.F.E.E

- IAS-17. In a test of a water-jacketed compressor, the shaft work required is 90 kN-m/kg of air compressed. During compression, increase in enthalpy of air is 30 kJ/kg of air and increase in enthalpy of circulating cooling water is 40 kJ/kg of air. The change in velocity is negligible. The amount of heat lost to the atmosphere from the compressor per kg of air is: [IAS-2000]

(a) 20kJ

(b) 60kJ

(c) 80 kJ

(d) 120kJ

# First Law of Thermodynamics

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- IAS-18. When air is compressed, the enthalpy is increased from 100 to 200 kJ/kg. Heat lost during this compression is 50 kJ/kg. Neglecting kinetic and potential energies, the power required for a mass flow of 2 kg/s of air through the compressor will be:

(a) 300 kW      (b) 200 kW      (c) 100 kW      (d) 50 kW

[IAS-1997]

## Variable Flow Processes

- IAS-19. Match List-I with List-II and select the correct answer using the codes given below Lists:

List-I

- A. Bottle filling of gas  
B. Nernst Simon statement  
C. Joule Thomson effect  
D.  $\int pdv$

List-II

1. Absolute zero temperature  
2. Variable flow  
3. Quasistatic path  
4. Isenthalpic process  
5. Dissipative effect  
6. Low grade energy  
7. Process and temperature during phase change

Codes:

	A	B	C	D	A	B	C	D
(a)	6	5	4	3	(b)	2	1	4
(c)	2	5	7	4	(d)	6	1	7

- IAS-20. A gas chamber is divided into two parts by means of a partition wall. On one side, nitrogen gas at 2 bar pressure and 20°C is present. On the other side, nitrogen gas at 3.5 bar pressure and 35°C is present. The chamber is rigid and thermally insulated from the surroundings. Now, if the partition is removed,

- (a) High pressure nitrogen will get throttled  
(b) Mechanical work, will be done at the expense of internal energy  
(c) Work will be done on low pressure nitrogen  
(d) Internal energy of nitrogen will be conserved

[IAS-1997]

# First Law of Thermodynamics

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## Answers with Explanation (Objective)

### Previous 20-Years GATE Answers

GATE-1. Ans. (a)  $h_1 + \frac{C_1^2}{2000} + \frac{gZ_1}{1000} + \frac{dQ}{dm} = h_2 + \frac{C_2^2}{2000} + \frac{gZ_2}{1000} + \frac{dW}{dm}$

$$3200 + \frac{160^2}{2000} + \frac{9.81 \times 10}{1000} = 2600 + \frac{100^2}{2000} + \frac{9.81 \times 6}{1000} + \frac{dW}{dm}$$

$$600 + 7.8 + 0.04 = + \frac{dW}{dm}$$

GATE-2. Ans. (c)

$$W = V(P_2 - P_1) = \frac{1}{1000} (3000 - 70) \times \text{kJ/kg} = 293$$

GATE-3. Ans. (a) Fig-1 & 2 both are power cycle, so equal areas but fig-3 & 4 are reverse power cycle, so area is not meant something.

GATE-4. Ans. (c)

$$dQ = du + dw$$

$$Q = u_2 - u_1 + W \quad \text{or} \quad -2000 = u_2 - u_1 - 5000 \quad \text{or} \quad u_2 - u_1 = 3000 \text{ kJ}$$

GATE-4a. Ans. (a)  $Q = 0, W = -2.3, \Delta U =$

$$+2.3$$

Tank is well insulated so  $Q = 0$

Work is given to the system in the form of electric current.

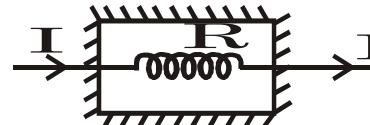
$$\text{So, } W = -I^2 R = -10^2 \times 23 = -2300 \text{ W} = -2.3 \text{ kW}$$

By 1<sup>st</sup> Law of Thermodynamics

$$Q_{1-2} = U_2 - U_1 + W_{1-2}$$

$$0 = U_2 - U_1 - 2.3$$

$$\Delta U = 2.3 \text{ kW}$$



GATE-5. Ans (a) The final Temp. ( $T_2$ ) =  $\sqrt{T_1}$

### Previous 20-Years IES Answers

IES-1. Ans. (a) If we adiabatically mix two liquid then perfect gas law is not necessary. But entropy change in the universe must be calculated by Second law of thermodynamics. Final entropy of the system is also a property. That's why we need second law.

IES-2. Ans. (b) Using conservation of energy law we may find final temperature.

IES-3. Ans. (d) From First law of thermodynamics, for a closed system the net energy transferred as heat  $Q$  and as work  $W$  is equal to the change in internal energy,  $U$ , i.e.  $Q - W = U$

# First Law of Thermodynamics

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**IES-4. Ans. (c)** Total heat addition is constant volume heat addition,  $Q_2 = C_v(T_2 - T_1)$

Total heat rejection is constant pressure heat rejection,  $Q_{31} = C_p(T_3 - T_1)$

Now from equation of state

$$\frac{P_1}{T_1} = \frac{P_2}{T_2} \quad (\because v = \text{const.}) \quad \text{or} \quad T_2 = \frac{P_2}{P_1} \times T_1$$

$$\text{and} \quad \frac{V_1}{T_1} = \frac{V_3}{T_3} \quad (\because p = \text{const.}) \quad \text{or} \quad T_3 = \frac{V_3}{V_1} \times T_1$$

$$\text{Efficiency, } \eta = 1 - \frac{Q_{31}}{Q_2} = 1 - \frac{C_p(T_3 - T_1)}{C_v(T_2 - T_1)} = 1 - \gamma \frac{(T_3 - T_1)}{(T_2 - T_1)}$$

$$\text{or } \eta = 1 - \gamma \frac{\left( \frac{V_3}{V_1} \times T_1 - T_1 \right)}{\left( \frac{P_2}{P_1} \times T_1 - T_1 \right)} = 1 - \gamma \frac{(V_3 - V_1) P_1}{(P_2 - P_1) V_1}$$

**IES-5. Ans. (c)** It is  $du = dQ - dW$ , as  $u$  is a thermodynamic property and its cyclic integral must be zero.

**IES-6. Ans. (b)**  $\sum dQ = \sum dW$       or  $Q_{1-2} + Q_{2-1} = W_{1-2} + W_{2-1}$

$$\text{or } 20 + (-10) = 50 + W_{2-1} \quad \text{or} \quad W_{2-1} = -40 \text{ kJ}$$

**IES-7. Ans. (d)**  $dQ = du + dw$  as  $u = \text{const.}$

Therefore  $du = 0$  or  $dQ = dw = 300 \text{ kNm}$

**IES-8. Ans. (d)**

$$Q - W_x = \Delta \left( h + \frac{V^2}{2} + gz \right)$$

$$0 - W_x = -140 - 10 + 0$$

$$\text{or } W_x = 150 \text{ kJ/kg}$$

Change of internal energy = -100 kJ/kg is superfluous data.

**IES-9. Ans. (b)**  $Q = \Delta E + \Delta W$

$\Delta E = -30 \text{ kJ}$  (decrease in internal energy)

$\Delta W = + 50 \text{ kJ}$  (work done by the system)

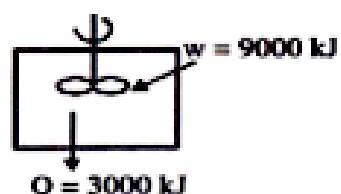
$$Q = -30 + 50 = + 20 \text{ kJ}$$

**IES-10. Ans. (a)** Net work =  $\sum dW = 474 - 180 \text{ kJ/min} = 294 \text{ kJ/min} = 294/60 \text{ kJ/s} = 4.9 \text{ kW}$

**IES-11. Ans. (b)**  $\eta_h = \frac{\text{Workdone}}{\text{heatadded}} = \frac{300 - 100}{300} = 0.66$

$$\text{Workratio} = \frac{\sum (+w) - \sum (-w)}{\sum (+w)} = \frac{550 - 350}{550} = 0.36$$

**IES-12. Ans. (a)** This is a case of constant volume process or an isochoric process. By performing work on the system temperature can be raised. In an irreversible constant volume process, the system doesn't perform work on the surrounding at the expense of its internal energy.



# First Law of Thermodynamics

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**IES-13. Ans. (d)**

**IES-14. Ans. (a)** The internal energy depends only upon the initial and final states of the system.

Internal energy of a substance does not include any energy that it may possess as a result of its **macroscopic** position or movement. That is why in SFEE  $v^2/2$  and  $gz$  is there.

Recognise that  $h=u+pv$  from which  $u_2 + p_2 v_2 = h_2$  and similarly

$$u_1 + p_1 v_1 = h_1$$

$$q - w = [(h_2 + C_2^2/2 + gZ_2) - (h_1 + C_1^2/2 + gZ_1)] \quad \text{or}$$

$$= [(h_2 - h_1) + (C_2^2/2 - C_1^2/2) + g(Z_2 - Z_1)]$$

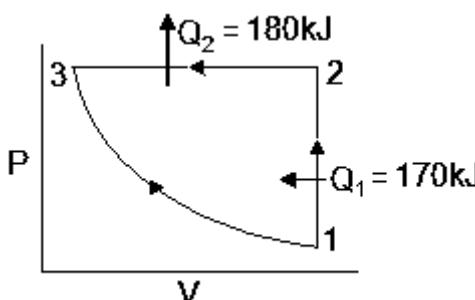
If internal energy include position or movement then why this  $v^2/2$  and  $gz$  terms is there.

**Bur Remember:**

Microscopic view of a gas is a collection of particles in random motion. Energy of a particle consists of **translational energy**, **rotational energy**, **vibrational energy** and **specific electronic energy**. All these energies summed over all the particles of the gas, form the specific internal energy,  $e$ , of the gas.

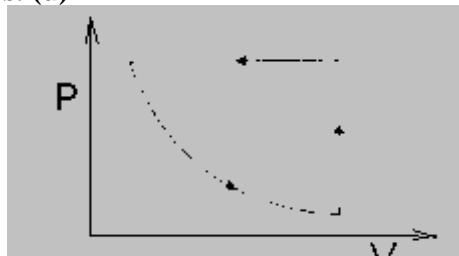
**IES-15. Ans. (b)**  $dQ = dU + pdV$  if  $V$  is constant  $(dQ)_v = (dU)_v$

**IES-16. Ans. (a)**



$$\begin{aligned} Q_2 &= 180 \text{ kJ} = \Delta U + \Delta W = \Delta U + (-40) \\ \therefore U_1 &= 100 \text{ kJ}, U_2 = 100 + 170 = 270 \text{ kJ}, \\ U_3 &= 270 - 180 + 40 = 130 \text{ kJ} \end{aligned}$$

**IES-17. Ans. (d)**



For the process 1-2

$$dQ = +85, dW = 0$$

For the process 2-3

$$dQ = -90 \text{ kJ}, dW = -20 \text{ kJ}$$

For the process 3-1

$$dQ = 0, dW = ?$$

For a cyclic process

$$\sum dQ = \sum dW$$

$$\Rightarrow 85 - 90 + 0 = 0 - 20 + dW$$

$$\Rightarrow -5 = -20 + dW$$

$$\Rightarrow dW = -20 + 5 = +15 \text{ kJ}$$

**IES-17a. Ans. (a)**

$$dQ = dU + dW$$

$$\text{or } 60 = -30 + dW$$

$$\text{or } dW = 90 \text{ kJ}$$

# First Law of Thermodynamics

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IES-17b.Ans. (c)

IES-18. Ans. (c)

$$dQ = du + dw$$

$$2dt = du + (2 - 0.1T)dT$$

$$\text{or } \int du = \int 0.1TdT = \frac{0.1}{2} \times [T^2]_{100}^{150} = \frac{0.1}{2} [150^2 - 100^2] = 625 \text{ kJ}$$

IES-19. Ans. (c) Change of internal energy from A to B along path ACB = 180 - 130 = 50 kJ. It will be same even along path ADB.  $\therefore$  Heat flow along ADB = 40 + 50 = 90 kJ.

IES-20. Ans. (d)  $dQ = dE + dW$

$$\text{or } \frac{dQ}{dt} = \frac{dE}{dt} + \frac{dW}{dt}$$

$$\text{Given: } E = 25 + 0.25t \text{ kJ and } \frac{dW}{dt} = 0.75 \text{ kJ/k}$$

$$\text{then } \frac{dE}{dt} = 0.25 \text{ kJ/K}$$

$$\text{Therefore } \frac{dQ}{dt} = \frac{dE}{dt} + \frac{dW}{dt} = 0.25 + 0.75 \text{ kJ/K} = 1.00 \text{ kJ/K}$$

IES-21. Ans. (a)

IES-22. Ans. (a) A closed system does exchange work or energy with its surroundings. option '3' is wrong. 4. "The law of conservation of entropy" is imaginary so option '4' is also wrong.

IES-23. Ans. (a)

IES-23a. Ans. (a)

$$\text{IES-24. Ans. (b)} \quad h_1 + \frac{v_1^2}{2} + g z_1 + \frac{dQ}{dm} = h_2 + \frac{v_2^2}{2} + g z_2 + \frac{dw}{dm} = 0$$

For boiler  $v_1, v_2$  is negligible and  $z_1 = z_2$  and  $\frac{dw}{dm} = 0$

$$\text{or } \frac{dQ}{dm} = (h_2 - h_1)$$

IES-25. Ans. (d)

$$mC_p \Delta T = (4 \times 10 \times 60)$$

$$\Rightarrow 20 \times 4 \times \Delta T = 2400$$

$$\Rightarrow \Delta T = 30^\circ\text{C}$$

IES-26. Ans. (c) Enthalpy of additional gas will be converted to internal energy.

$$U_f = m_i u_i + (m_f - m_i) h_p = 0.25 \times 200 + (1 - 0.25) \times 400 = 350 \text{ kJ}$$

As total mass = 1kg,  $u_f = 350 \text{ kJ/kg}$

**Note:** You cannot simply use adiabatic mixing law here because it is not closed system. This is a problem of variable flow process. If you calculate in following way it will be wrong.

Final internal energy of gas(mixture) is

$$u = \frac{m_1 u_1 + m_2 u_2}{m_1 + m_2}$$

$$u = \frac{(0.25 \text{ kg}) \left( 200 \frac{\text{kJ}}{\text{kg}} \right) + (0.75 \text{ kg}) \left( 300 \frac{\text{kJ}}{\text{kg}} \right)}{(0.25 + 0.75) \text{ kg}}$$

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$$u = 275 \frac{\text{kJ}}{\text{kg}}$$

It is valid for closed system only.

## Previous 20-Years IAS Answers

IAS-1. Ans. (b)  $\sum dQ = \sum dW$  or  $220 - 25 - 180 + 50 = 15 - 10 + 60 + W_{4-1}$

IAS-2. Ans. (c) Area under p-v diagram is represent work.

$$\text{Areas } \Delta \text{PTS} = \frac{1}{2} \text{Area } \square(\text{WVUR}) \therefore \text{Work PTS} = \frac{1}{2} \times 48 = 24 \text{ Nm}$$

IAS-3. Ans. (a)  $\eta = \frac{\text{work output}}{\text{Heat input}} = \frac{\text{work output}}{\text{work output} + \text{heat rejection}} = \frac{54}{54+66} = 0.45$

IAS-4. Ans. (b) Thermal efficiency  $= \frac{W}{Q} = \frac{3 \times 10^3 \text{ watts}}{10,000 \text{ J/s}} = 0.3 = 30\%$

IAS-5. Ans. (c)

$$Q_{1-2} = (U_2 - U_1) + W_{1-2}$$

$$\text{or } 0 = (U_2 - U_1) + (-5000) \text{ or } (U_2 - U_1) = 5000 \text{ J}$$

$$Q_{2-1} = (U_1 - U_2) + W_{2-1}$$

$$\text{or } W_{2-1} = Q_{2-1} - (U_1 - U_2) = Q_{2-1} + (U_2 - U_1) = 1000 + 5000 = 6000 \text{ J}$$

IAS-6. Ans. (c) Net work output =  $3 + 10 - 8 = 5$  unit and Heat added =  $30 + 5 = 35$  unit

$$\text{Therefore efficiency, } \eta = \frac{5}{35} \times 100\% = 14.33\%$$

IAS-7. Ans. (a)

IAS-8. Ans. (d)  $Q_{123} = U_{13} + W_{123}$  or,  $100 = U_{13} + 60$  or,  $U_{13} = 40 \text{ kJ}$

$$\text{And } Q_{143} = U_{13} + W_{143} = 40 + 20 = 60 \text{ kJ}$$

IAS-9. Ans. (b) Total work  $= 5 \times 3 + \frac{1}{2} \times 5 \times 1 = 17.5 \text{ J}$  or  $dW = du + dQ = 2.5 + 17.5 = 20 \text{ J}$

IAS-10. Ans. (c) Efficiency  $= \frac{\text{Area under 500 and 1500}}{\text{Area under 0 and 1500}}$

$$= \frac{\frac{1}{2} \times \{(5-1) + (4-2)\} \times (1500 - 500)}{\frac{1}{2} \times \{(5-1) + (4-2)\} \times (1500 - 500) + (5-1) \times 500} = \frac{3000}{5000} = 0.6$$

IAS-11. Ans. (a)

IAS-12. Ans. (b) Internal energy is a property of a system so  $\oint du = 0$

IAS-13. Ans. (a)  $dQ = du + dW$  if  $du = +30 \text{ kJ}$  then  $dQ = -50 \text{ kJ}$  and  $dW = -80 \text{ kJ}$

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**IAS-14. Ans. (b)  $\delta W = du + \delta W = du + pdV$**

$$\text{or } 84 \times 10^3 \text{ J} = du + 1 \times 10^6 \times (0.06 - 0.03) = du + 30 \text{ kJ} \text{ or } du = 83 - 30 = 54 \text{ kJ}$$

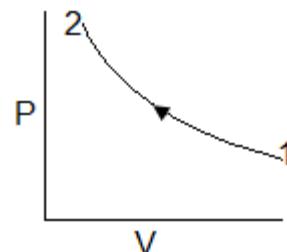
**IAS-15. Ans. (a)  $Q_{1-2} = U_2 - U_1 + W_{1-2}$**

$$\text{Or } 0 = U_2 - U_1 - 6000$$

$$\text{or } U_2 - U_1 = +6000$$

$$Q_{2-1} = U_1 - U_2 + W_{2-1}$$

$$\text{or } W_{2-1} = Q_{2-1} - (U_1 - U_2) \\ = 1000 + 6000 = 7000 \text{ J}$$



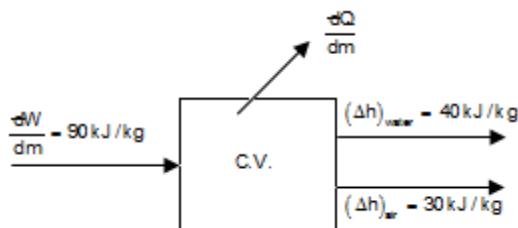
**IAS-16. Ans. (c)**

**IAS-17. Ans. (a) Energy balance gives as**

$$\frac{dW}{dm} = (\Delta h)_{\text{air}} + (\Delta h)_{\text{water}} + \frac{dQ}{dm}$$

$$\text{or } \frac{dQ}{dm} = 90 - 30 - 40$$

$$= 20 \text{ kJ/kg of air compressed}$$



**IAS-18. Ans. (a)**

$$m(h_1) + \frac{dQ}{dt} = m(h_2) + \frac{dW}{dt}$$

$$\text{or } \frac{dW}{dt} = m(h_1 - h_2) + \frac{dQ}{dt} = 2 \times (100 - 200) - 50 \times 2 = -300 \text{ kW}$$

i.e. 300 kW work have to given to the system

**IAS-19. Ans. (b)**

**IAS-20. Ans. (a)**

## 3.

# Second Law of Thermodynamics

## Theory at a Glance (For GATE, IES & PSUs)

The first law of thermodynamics states that a certain energy balance will hold when a system undergoes a change of state or a thermodynamic process. But it does not give any information on whether that change of state or the process is at all feasible or not. The first law cannot indicate whether a metallic bar of uniform temperature can spontaneously become warmer at one end and cooler at the other. All that the law can state is that if this process did occur, the energy gained by one end would be exactly equal to that lost by the other. *It is the second law of thermodynamics which provides the criterion as to the probability of various processes.*

## Regarding Heat Transfer and Work Transfer

- (a) Heat transfer and work transfer are the *energy interactions*. A closed system and its surroundings can interact in two ways: by heat transfer and by work transfer. Thermodynamics studies how these interactions bring about property changes in a system.
- (b) The same effect in a closed system can be brought about either by heat transfer or by work transfer. Whether heat transfer or work transfer has taken place depends on what constitutes the system.
- (c) **Both heat transfer and work transfer are boundary phenomena.** Both are observed at the boundaries of the system, and both represent energy crossing the boundaries of the system.
- (d) ***It is wrong to say 'total heat' or 'heat content' of a closed system, because heat or work is not a property of the system.*** Heat, like work, cannot be stored by the system. Both heat and work are the energy in transit.
- (e) Heat transfer is the energy interaction due to temperature difference only. All other energy interactions may be termed as work transfer.
- (f) **Both heat and work are path functions and inexact differentials.** The magnitude of heat transfer or work transfer depends upon the path the system follows during the change of state.
- (g) Heat transfer takes place according to second law of thermodynamics as it tells about the direction and amount of heat flow that is possible between two reservoirs.

# Second Law of Thermodynamics

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## Qualitative Difference between Heat and Work

- Thermodynamic definition of work:

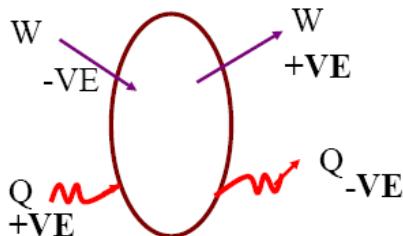
Positive work is done by a system when the **sole effect external** to the system **could be** reduced to the rise of a weight.

- Thermodynamic definition of heat:

It is the **energy in transition** between the system and the surroundings by virtue of the difference in temperature.

## Sign Conventions

- Work done BY the system is **+ve**
- Obviously work done ON the system is **-ve**
- Heat given TO the system is **+ve**
- Obviously Heat rejected by the system is **-ve**



Heat and work are not completely interchangeable forms of energy. When work is converted into heat, we always have

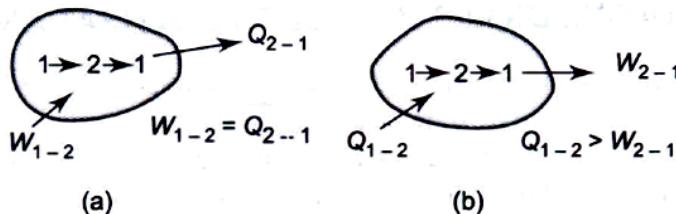
$$W \equiv Q$$

but when heat is converted into work in a complete closed cycle process

$$Q \geq W$$

The arrow indicates the direction of energy transformation.

**Work** is said to be a **high grade energy** and **heat** is **low grade energy**. The complete conversion of low grade energy into high grade energy in a cycle is impossible.



*Qualitative Distinction between Heat and Work*

HEAT and WORK are NOT **properties** because they depend on the

# Second Law of Thermodynamics

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path and end states.

HEAT and WORK are **not properties** because their net change in a cycle is not zero.

Heat and work are **inexact differentials**. Their change cannot be written as differences between their end states.

Thus  $\int_1^2 \delta Q \neq Q_2 - Q_1$  and is shown as  $Q_1 - Q_2$

Similarly  $\int_1^2 \delta W \neq W_2 - W_1$  and is shown as  $W_1 - W_2$

**Note.** The operator  $\delta$  is used to denote inexact differentials and operator  $d$  is used to denote exact differentials.

## Kelvin-Planck Statement of Second Law

There are two statements of the second law of thermodynamics, the *Kelvin-Planck statement*, and the *Clausius statement*. The Kelvin-Planck statement pertains to **heat engines**.

The Clausius statement pertains to **refrigerators/heat pumps** .

### Kelvin-Planck statement of second law

It is impossible to construct a device (engine) operating in a cycle that will produce no effect other than extraction of heat from a single reservoir and convert all of it into work.

Mathematically, Kelvin-Planck statement can be written as:

$$W_{\text{cycle}} \leq 0 \text{ (for a single reservoir)}$$

### Clausius' statement of second law

It is impossible to transfer heat in a cyclic process from low temperature to high temperature without work from external source.

### Reversible and Irreversible Processes

A process is reversible with respect to the system and surroundings if the system and the surroundings can be restored to their respective initial states by reversing the direction of the process, that is, by reversing the heat transfer and work transfer. The process is irreversible if it cannot fulfill this criterion.

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## Clausius' Theorem

Let a system be taken from an equilibrium state  $i$  to another equilibrium state  $f$  by following the reversible path  $i-f$  (Figure). Let a reversible adiabatic  $i-a$  be drawn through  $i$  and another reversible adiabatic  $b-f$  be drawn through  $f$ . Then a reversible isothermal  $a-b$  is drawn in such a way that the area under  $i-a-b-f$  is equal to the area under  $i-f$ . Applying the first law for

Process  $i-f$

$$Q_{if} = U_f - U_i + W_{if}$$

Process  $i-a-b-f$

$$Q_{iabf} = U_f - U_i + W_{iabf}$$

Since

$$W_{if} = W_{iabf}$$

$$\begin{aligned}\therefore Q_{if} &= Q_{iabf} \\ &= Q_{ia} + Q_{ab} + Q_{bf}\end{aligned}$$

Since  $Q_{ia} = 0$  and  $Q_{bf} = 0$

$$Q_{if} = Q_{ab}$$

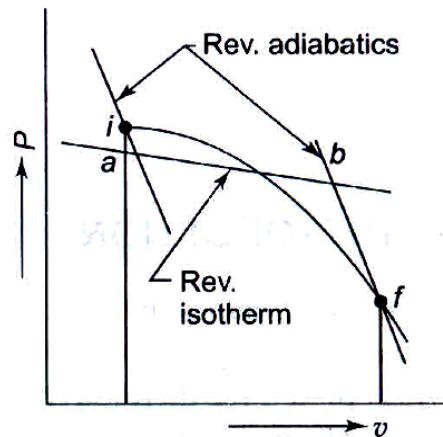


Fig. Reversible Path Substituted  
by Two Reversible Adiabatics  
and a Reversible Isothermal

Heat transferred in the process  $i-f$  is equal to the heat transferred in the isothermal process  $a-b$ .

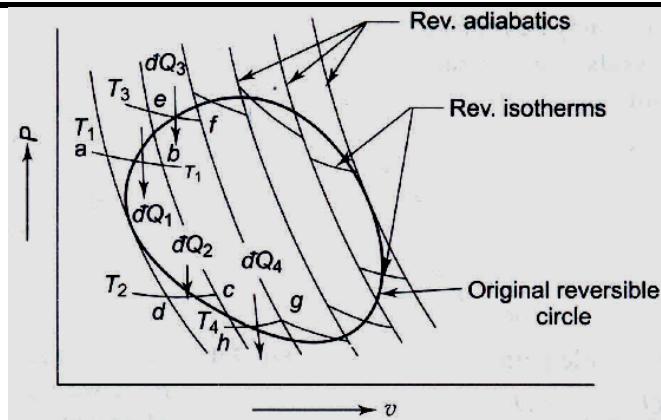
Thus any reversible path may be substituted by a reversible zigzag path, between the same end states, consisting of a reversible adiabatic followed by a reversible isothermal and then by a reversible adiabatic, such that the heat transferred during the isothermal process is the same as that transferred during the original process.

Let a smooth closed curve representing a reversible cycle (Fig below.) be considered. Let the closed cycle be divided into a large number of strips by means of reversible adiabatics. Each strip may be closed at the top and bottom by reversible isothermals. The original closed cycle is thus replaced by a zigzag closed path consisting of alternate adiabatic and isothermal processes, such that the heat transferred during all the isothermal processes is equal to the heat transferred in the original cycle. Thus the original cycle is replaced by a large number of Carnot cycles. If the adiabatics are close to one another and the number of Carnot cycles is large, the saw-toothed zig-zag line will coincide with the original cycle.

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**Fig. A Reversible Cycle Split into a Large Number of Carnot Cycles**

For the elemental cycle  $abcd$   $dQ_1$  heat is absorbed reversibly at  $T_1$ , and  $dQ_2$  heat is rejected reversibly at  $T_2$

$$\frac{dQ_1}{T_1} = \frac{dQ_2}{T_2}$$

If heat supplied is taken as positive and heat rejected as negative

$$\frac{dQ_1}{T_1} + \frac{dQ_2}{T_2} = 0$$

Similarly, for the elemental cycle  $efgh$

$$\frac{dQ_3}{T_3} + \frac{dQ_4}{T_4} = 0$$

If similar equations are written for all the elemental Carnot cycles, then for the whole original cycle

$$\frac{dQ_1}{T_1} + \frac{dQ_2}{T_2} + \frac{dQ_3}{T_3} + \frac{dQ_4}{T_4} + \dots = 0$$

or

$$\oint_R \frac{dQ}{T} = 0$$

**The cyclic integral of  $dQ/T$  for a reversible cycle is equal to zero.** This is known as *Clausius' theorem*. The letter  $R$  emphasizes the fact that the equation is valid only for a reversible cycle.

# Second Law of Thermodynamics

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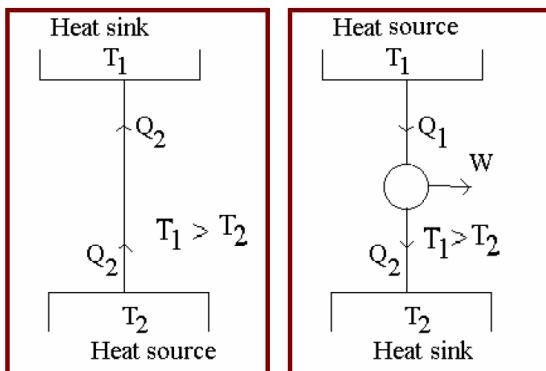
Thus the original cycle is replaced by a large number of Carnot cycles. If the adiabatics are close to one another and the number of Carnot cycles is large, the saw-toothed zig-zag line will coincide with the original cycle.

## Refrigerator and Heat Pump [with RAC]

### Equivalence of Kelvin-Planck and Clausius Statements

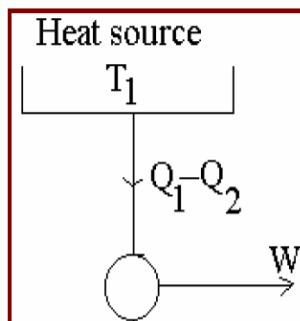
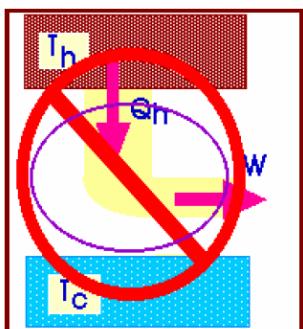
II Law basically a negative statement (*like most laws in society*). The two statements look distinct. We shall prove that violation of one makes the other statement violation too.

**Let us suspect the Clausius statement**-it *may be* possible to transfer heat from a body at colder to a body at hotter temperature without supply of work



Let us have a heat engine operating between  $T_1$  as source and  $T_2$  as a sink. Let this heat engine reject exactly the same  $Q_2$  (as the pseudo-Clausius device) to the reservoir at  $T_2$ . To do this an amount of  $Q_1$  needs to be drawn from the reservoir at  $T_1$ . There will also be a  $W = Q_1 - Q_2$ .

Combine the two. The reservoir at  $T_2$  has not undergone any change ( $Q_2$  was taken out and by pseudo-Clausius device and put back by the engine). Reservoir 1 has given out a net  $Q_1 - Q_2$ . We got work output of  $W$ .  $Q_1 - Q_2$  is converted to  $W$  with no net heat rejection. This is violation of Kelvin-Planck statement.

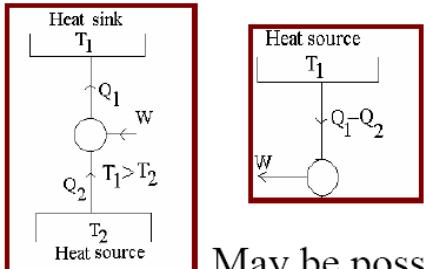


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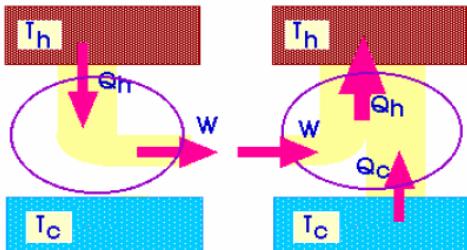
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- Let us assume that Clausius statement is true and suspect Kelvin-Planck statement



May be possible?



Pseudo Kelvin Planck engine requires only  $Q_1 - Q_2$  as the heat interaction to give out  $W$  (because it does not reject any heat) which drives the Clausius heat pump

**Combining the two yields:**

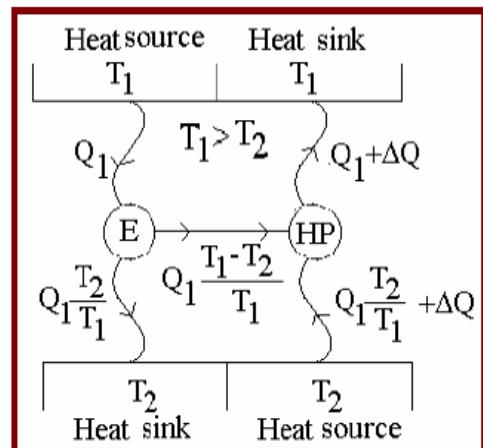
- The reservoir at  $T_1$  receives  $Q_1$  but gives out  $Q_1 - Q_2$  implying a net delivery of  $Q_2$  to it.
- $Q_2$  has been transferred from  $T_2$  to  $T_1$  without the supply of any work!!
- A violation of Clausius statement*

**Moral:** If an engine/refrigerator violates one version of II Law, it violates the other one too.

All reversible engine operating between the **same two fixed temperatures** will have the **same  $\eta$  and COP**.

If there exists a reversible engine/ or a refrigerator which can do better than that, it will violate the Clausius statement.

Let us presume that the HP is super efficient!! For the same work given out by the engine E, it can pick up an extra  $\Delta Q$  from the low temperature source and deliver over to reservoir at  $T_1$ . The net effect is this extra has  $\Delta Q$  been transferred from  $T_2$  to  $T_1$  with no external work expenditure. *Clearly, a violation of Clausius statement!!*



### SUM UP

- Heat supplied =  $Q_1$ ; Source temperature =  $T_1$ ; Sink temperature =  $T_2$
- Maximum possible efficiency =  $W/Q_1 = (T_1 - T_2)/T_1$
- Work done =  $W = Q_1(T_1 - T_2)/T_1$

# Second Law of Thermodynamics

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## Carnot Engine with same efficiency or same work output

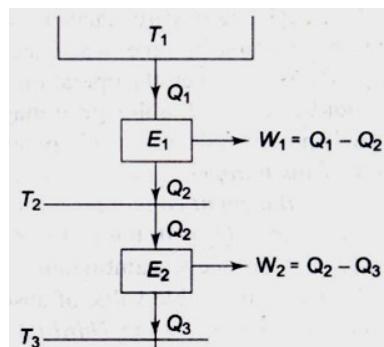
Since,

$$\frac{T_1}{T_2} = \frac{Q}{Q}$$

$$\frac{T_1 - T_2}{T_2} = \frac{Q - Q}{Q}$$

$$T_1 - T_2 = (Q - Q) \frac{T_2}{Q}$$

$$T_2 - T_3 = (Q - Q) \frac{T_3}{Q} = (Q - Q) \frac{T_2}{Q}$$



- For same work output

$$W_1 = W_2$$

$$Q_1 - Q_2 = Q_2 - Q_3$$

$$T_1 - T_2 = T_2 - T_3$$

- For same efficiency

$$\eta = \eta$$

$$\text{or } 1 - \frac{T_2}{T_1} = 1 - \frac{T_3}{T_2}$$

$$\text{or } T_2 = \sqrt{T_1 \times T_3}$$

- Which is the more effective way to increase the efficiency of a Carnot engine: to increase  $T_1$  keeping  $T_2$  constant; or to decrease  $T_2$  keeping  $T_1$  constant?

The efficiency of a Carnot engine is given by

$$\eta = 1 - \frac{T_2}{T_1}$$

If  $T_2$  is constant

$$\left( \frac{\partial \eta}{\partial T_1} \right)_{T_2} = \frac{T_2}{T_1^2}$$

AS  $T_1$  increases,  $\eta$  increases, and the slope  $\left( \frac{\partial \eta}{\partial T_1} \right)_{T_2}$  decreases (Figure). If  $T_1$  is constant,

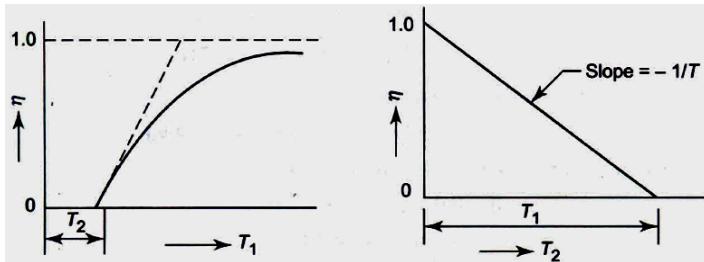
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$$\left(\frac{\partial \eta}{\partial T_2}\right)_{T_1} = -\frac{1}{T_1}$$

As  $T_2$  decreases,  $\eta$  increases, but the slope  $\left(\frac{\partial \eta}{\partial T_2}\right)_{T_1}$  remains constant (Figure).



Also  $\left(\frac{\partial \eta}{\partial T_1}\right)_{T_2} = \frac{T_2}{T_1}$  and  $\left(\frac{\partial \eta}{\partial T_2}\right)_{T_1} = -\frac{T_1}{T_2}$

Since  $T_1 > T_2, \left(\frac{\partial \eta}{\partial T_2}\right)_{T_1} > \left(\frac{\partial \eta}{\partial T_1}\right)_{T_2}$

So, the more effective way to increase the efficiency is to decrease  $T_2$ . Alternatively, let  $T_2$  be decreased by  $\Delta T$  with  $T_1$  remaining the same

$$\eta_1 = 1 - \frac{T_2 - \Delta T}{T_1}$$

If  $T_1$  is increased by the same  $\Delta T$ ,  $T_2$  remaining the same

$$\eta_2 = 1 - \frac{T_2}{T_1 + \Delta T}$$

Then

$$\begin{aligned} \eta_1 - \eta_2 &= \frac{T_2}{T_1 + \Delta T} - \frac{T_2 - \Delta T}{T_1} \\ &= \frac{(T_1 - T_2)\Delta T + (\Delta T)^2}{T_1(T_1 + \Delta T)} \end{aligned}$$

Since  $T_1 > T_2, (\eta_1 - \eta_2) > 0$

The more effective way to increase the cycle efficiency is **to decrease  $T_2$** .

# Second Law of Thermodynamics

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## Irreversibility due to dissipative effects

**Friction:** Friction is always present in moving device. We can reduce friction but can never be completely eliminated. The continual motion of a movable device in the complete absence of friction is known as **perpetual motion of the third kind.**

## PROBLEMS & SOLUTIONS

### Example 1.

An inventor claims that his petrol engine operating between temperatures of 2000°C and 600°C will produce 1 kWhr consuming 150g of petrol having 45000 kJ/kg calorific value. Check the validity of the claim.

#### Solution:

By Carnot's theorem, the thermal efficiency of a reversible cycle engine which cannot be exceeded is given by

$$\eta_{\max} = \frac{T_1 - T_2}{T_1} = \frac{2273 - 873}{2273} = 0.616 \text{ or } 61.6\%$$

Actual thermal efficiency is given by

$$\eta = \frac{1 \times 10^3 \times 3600}{0.15 \times 45000 \times 10^3} = 0.53 \text{ or } 53\%$$

Since actual efficiency is less than the maximum obtainable, the inventor's claim is feasible.

### Example 2.

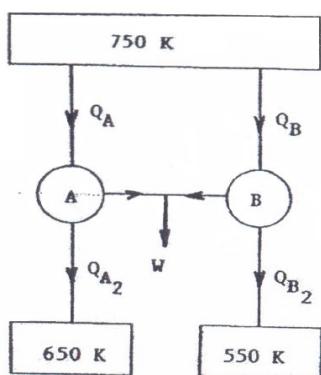
Two reversible engines takes 2400 kJ per minute from a reservoir at 750 K and develops 400 kJ of work per minute when executing complete cycles. The engines reject heat two reservoirs at 650 K and 550 K. Find the heat rejected to each sink.

#### Solution:

$$Q_A + Q_B = 2400$$
$$Q_{A_2} + Q_{B_2} = 2000$$

$$\frac{Q_A - Q_{A_2}}{Q_A} = \frac{750 - 650}{750} = 0.1333$$

$$Q_A = 1.1539 Q_{A_2}$$



(Fig.)

# Second Law of Thermodynamics

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Similarly

$$Q_B = 1.3636 Q_{B_2}$$

$$\text{i.e. } 1.1539 Q_{A_2} + 1.3636 Q_{B_2} = 2400 \text{ kJ}$$

$$Q_{A_2} = 2000 - Q_{B_2}$$

$$\text{i.e. } 1.1539 \times (2000 - Q_{B_2}) + 1.3636 Q_{B_2} = 2400 \text{ kJ.}$$

$$0.2091 Q_{B_2} = 92$$

$$\therefore Q_{B_2} = 440 \text{ kJ and } Q_{A_2} = 1560 \text{ kJ}$$

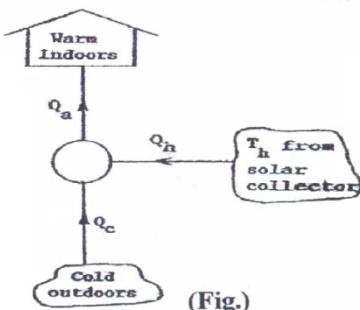
### Example 3.

A solar powered heat pump receives heat from a solar collector at temperature  $T_h$ , uses the entire energy for pumping heat from cold atmosphere at temperature ' $T_c$ ' to a room at temperature ' $T_a$ '. The three heat transfer rates are  $Q_h$ ,  $Q_a$  and  $Q_c$  respectively. Derive an expression for the minimum ratio  $Q_h/Q_c$ , in terms of the three temperatures.

If  $T_h = 400 \text{ K}$ ,  $T_a = 300 \text{ K}$ ,  $T_c = 200 \text{ K}$ ,  $Q_c = 12 \text{ kW}$ , what is the minimum  $Q_h$ ? If the collector captures  $0.2 \text{ kW/m}^2$ , what is the minimum collector area required?

#### Solution:

Let  $Q_h$ ,  $Q_a$  and  $Q_c$  be the quantity of heat transferred from solar collector, room and atmosphere respectively.



$$Q_a = Q_h + Q_c$$

$$\text{or, } Q_a - Q_c = Q_h$$

$$COP_{HP} = \frac{T_a}{T_a - T_c} \text{ and also } \frac{Q_a}{Q_a - Q_c} = \frac{Q_h + Q_c}{Q_h}$$

$$\therefore \frac{Q_h + Q_c}{Q_h} = \frac{T_a}{T_a - T_c}$$

$$\frac{Q_c}{Q_h} = \frac{T_a - T_c + T_c}{T_a - T_c} = \frac{T_c}{T_a - T_c}$$

$$\therefore \frac{Q_h}{Q_c} = \frac{(T_a - T_c)}{T_c}$$

$$Q_h = \frac{Q_c \times (T_a - T_c)}{T_c} = \frac{12 \times (300 - 200)}{200} = 6 \text{ kW}$$

$$\text{Area} = \frac{6}{0.2} = 30 \text{ m}^2$$

# Second Law of Thermodynamics

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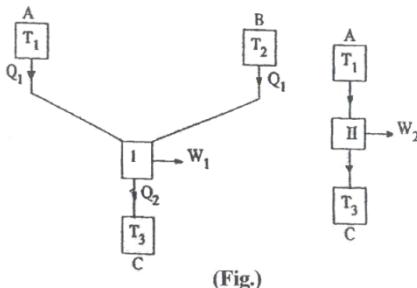
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## Example 4.

A reversible engine works between 3 thermal reservoirs A, B and C. The engine absorbs an equal amount of heat from the reservoirs A and B, maintained at temperatures of  $T_1$  and  $T_2$  respectively and rejects heat to the thermal reservoir C maintained at  $T_3$ . The efficiency of this engine is  $\alpha$  times the efficiency of reversible engine operating between reservoirs A and C only. Show that

$$\frac{T_1}{T_2} = (2\alpha - 1) + \frac{2T_1}{T_3}(1 - \alpha)$$

**Solution:**



(Fig.)

$$\eta_1 = \frac{W_1}{2Q}$$

$$\eta_2 = \frac{T_1 - T_3}{T_1}$$

$$\eta_1 = \alpha \eta_2 = \alpha \left( \frac{T_1 - T_3}{T_1} \right)$$

The cycle is reversible so  $\Delta S = 0$

$$\therefore \frac{Q_1}{T_1} + \frac{Q_2}{T_2} = \frac{Q}{T_3}$$

$$\text{Also, } 2Q = W_1 + Q$$

Combining above equations we have

$$\frac{Q_1}{T_1} + \frac{Q_2}{T_2} = \frac{2Q - W_1}{T_3}$$

$$\frac{W_1}{2Q} = \alpha \left( \frac{T_1 - T_3}{T_1} \right)$$

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$$W_l = 2Q\alpha \left( \frac{T_1 - T_3}{T_1} \right)$$

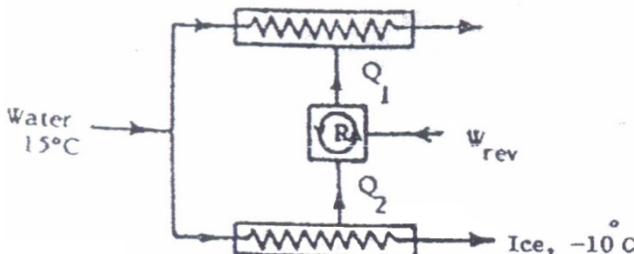
$$\frac{Q}{T_1} + \frac{Q}{T_2} = \frac{2Q}{T_3} \left( 1 - \frac{\alpha(T_1 - T_3)}{T_1} \right)$$

$$\frac{1}{T_1} + \frac{1}{T_2} = \frac{2}{T_3} - \frac{2\alpha}{T_1} + \frac{2\alpha}{T_3}$$

$$\frac{T_1}{T_2} = \frac{2T_1}{T_3}(1-\alpha) + (2\alpha-1)$$

**Example 5.**

Ice is to be made from water supplied at 15°C by the process shown in figure. The final temperature of the ice is -10°C, and the final temperature of the water that is used as cooling water in the condenser is 30°C.



**(Fig.)**

Determine the minimum work required to produce 1000 kg of ice.

**Solution:**

Quantity of ice produced = 1000 kg.

Specific heat of ice = 2070 J/kg K

Specific heat of water = 4198 J/kg K

Latent heat of ice = 335 kJ/kg

$$\text{Mean temperature of ice bath} = \frac{15+(-10)}{2} = 25^\circ C$$

$$\text{Mean temperature of condenser bath} = \frac{15+30}{2} = 22.5^\circ C$$

$$Q_2 = 1000 \times 4198(15 - 2.5) + 1000 \times 335 \times 10^3 + 1000 \times 2070(2.5 + 10) = 413.35 \text{ MJ}$$

For a reversible refrigerator system

$$1 - \frac{T_L}{T_H} = 1 - \frac{Q_2}{Q} ; \frac{T_L}{T_H} = \frac{Q_2}{Q} ;$$

$$Q = Q_2 \frac{T_H}{T_L} = 413.35 \times 10^6 \left( \frac{22.5+273}{25+273} \right) = 443.36 \text{ MJ}$$

$$\text{Minimum work required} = Q_1 - Q_2 = 443.36 - 413.35 = 30.01 \text{ MJ.}$$

# **Second Law of Thermodynamics**

**S K Mondal's**

**Chapter 3**

# Second Law of Thermodynamics

## S K Mondal's

## Chapter 3

### **ASKED OBJECTIVE QUESTIONS (GATE, IES, IAS)**

#### **Previous 20-Years IES Questions**

- IES-1.** Which one of the following is correct on basis of the second law of thermodynamics? [IES 2007]  
(a) For any spontaneous process, the entropy of the universe increases  
(b)  $\Delta S = q_{rev}/T$  at constant temperature  
(c) Efficiency of the Stirling cycle is more than that of a Carnot cycle  
(d)  $\Delta E = q + w$   
(The symbols have their usual meaning)
- IES-2.** Assertion (A): Second law of thermodynamics is called the law of degradation of energy. [IES-1999]  
Reason (R): Energy does not degrade each time it flows through a finite temperature difference.  
(a) Both A and R are individually true and R is the correct explanation of A  
(b) Both A and R are individually true but R is NOT the correct explanation of A  
(c) A is true but R is false  
(d) A is false but R is true
- IES-3.** Heat transfer takes place according to [IES-1996]  
(a) Zeroth Law of Thermodynamics  
(b) First Law of Thermodynamics  
(c) Second Law of Thermodynamics  
(d) Third Law of Thermodynamics.
- IES-3a.** Consider the following statements: [IES-2010]  
1. Slow heating of water from an electric heater.  
2. Isentropic expansion of air.  
3. Evaporation of a liquid from a heat source at the evaporation temperature.  
4. Constant pressure heating of a gas by a constant temperature source.  
Which of these processes is/are reversible?  
(a) 3 only  
(b) 2 and 3 only  
(c) 2 and 4 only  
(d) 1, 2, 3 and 4
- IES-3b.** Consider the following statements: [IES-2010]  
1. Boiling of water from a heat source at the same boiling temperature.  
2. Theoretical isothermal compression of a gas.  
3. Theoretical polytropic compression process with heat rejection to atmosphere.  
4. Diffusion of two ideal gases into each other at constant pressure and temperature.  
Which of these processes are irreversible?  
(a) 1, 2, 3 and 4  
(b) 1 and 4 only  
(c) 2, 3 and 4 only  
(d) 3 and 4 only

# Second Law of Thermodynamics

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## Kelvin-Planck Statement of Second Law

IES-4. Consider the following statements:

The definition of

1. Temperature is due to Zero<sup>th</sup> Law of Thermodynamics.
2. Entropy is due to First Law of Thermodynamics.
3. Internal energy is due to Second Law of Thermodynamics.
4. Reversibility is due to Kelvin-Planck's statement.

Of these statements

- |                            |                            |
|----------------------------|----------------------------|
| (a) 1, 2 and 3 are correct | (b) 1, 3 and 4 are correct |
| (c) 1 alone is correct     | (d) 2 alone is correct     |

[IES-1993]

## Clausius' Statement of the Second Law

IES-5. Assertion (A): Heat cannot spontaneously pass from a colder system to a hotter system without simultaneously producing other effects in the surroundings.

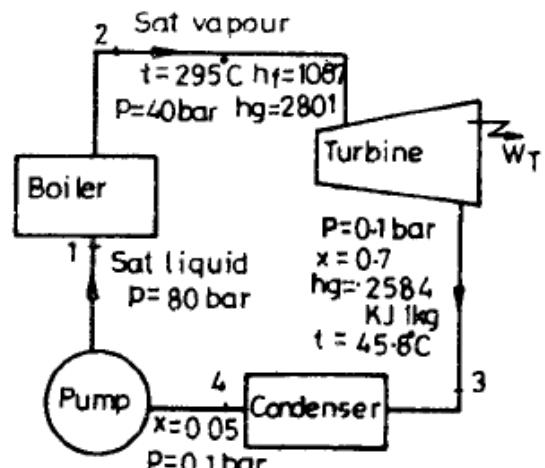
Reason (R): External work must be put into heat pump so that heat can be transferred from a cold to a hot body. [IES-1999]

- (a) Both A and R are individually true and R is the correct explanation of A  
(b) Both A and R are individually true but R is NOT the correct explanation of A  
(c) A is true but R is false  
(d) A is false but R is true

## Clausius' Theorem

IES-6. A steam power plant is shown in figure,

- (a) The cycle violates first and second laws of thermodynamics.
- (b) The cycle does not satisfy the condition of Clausius inequality.
- (c) The cycle only violates the second laws of thermodynamics
- (d) The cycle satisfies the Clausius inequality



[IES-1992]

IES-7. An inventor says that his new concept of an engine, while working between temperature limits of 27°C and 327°C rejects 45% of heat absorbed from the source. His engine is then equivalent to which one of the following engines?

- (a) Carnot engine  
(b) Diesel engine  
(c) An impossible engine  
(d) Ericsson engine

[IES-2009]

# Second Law of Thermodynamics

S K Mondal's

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- IES-7a. An inventor states that his new engine rejects to the sink 40% of heat absorbed from the source while the source and sink temperatures are 327°C and 27°C respectively. His engine is therefore equivalent to [IES-2010]  
(a) Joule engine (b) Stirling engine  
(c) Impossible engine (d) Carnot engine
- IES-7b. A heat engine is supplied with 2512kJ/min of heat at 650°C. Heat rejection with 900kJ/min takes place at 100°C. This type of heat engine is [IES-2012]  
(a) ideal (b) irreversible (c) impossible (d) practical
- IES-7c. An inventor states that his new conceptual engine, while operating between temperature limits of 377°C and 27°C, will reject 50% of heat absorbed from the source. What type of cycle will this engine have? [IES-2012]  
(a) Carnot cycle (b) Stirling cycle (c) Impossible cycle (d) Possible cycle

## Equivalence of Kelvin-Planck and Clausius Statements

- IES-8. Assertion (A): Efficiency of a reversible engine operating between temperature limits  $T_1$  and  $T_2$  is maximum. [IES-2002]  
Reason (R): Efficiency of a reversible engine is greater than that of an irreversible engine.  
(a) Both A and R are individually true and R is the correct explanation of A  
(b) Both A and R are individually true but R is NOT the correct explanation of A  
(c) A is true but R is false  
(d) A is false but R is true

## Carnot Engine with same efficiency or same work output

- IES-9. A reversible engine operates between temperatures  $T_1$ , and  $T_2$ . The energy rejected by this engine is received by a second reversible engine at temperature  $T_2$  and rejected to a reservoir at temperature  $T_3$ . If the efficiencies of the engines are same then the relationship between  $T_1$ ,  $T_2$  and  $T_3$  is given by: [IES-2002]  
(a)  $T_2 = \frac{(T_1+T_3)}{2}$  (b)  $T_2 = \sqrt{(T_1^2+T_3^2)}$  (c)  $T_2 = \sqrt{T_1 T_3}$  (d)  $T_2 = \frac{(T_1+2T_3)}{2}$
- IES-10. A reversible engine operates between temperatures 900 K &  $T_2$  ( $T_2 < 900$  K), & another reversible engine between  $T_2$  & 400 K ( $T_2 > 400$  K) in series. What is the value of  $T_2$  if work outputs of both the engines are equal? [IES-2005]  
(a) 600 K (b) 625 K (c) 650 K (d) 675 K

- IES-10a. An engine operates between temperature limits of 900 K and  $T_2$  and another between  $T_2$  and 400 K. For both to be equally efficient, the value of  $T_2$  will be  
(a) 700 K (b) 600 K (c) 750 K (d) 650 [IES-2010]

- IES-11. Two reversible engine operate between thermal reservoirs at 1200 K,  $T_2$ K and 300 K such that 1<sup>st</sup> engine receives heat from 1200 K reservoir and rejects heat to thermal reservoir at  $T_2$ K, while the 2<sup>nd</sup> engine receives heat from thermal

# Second Law of Thermodynamics

## S K Mondal's Chapter 3

reservoir at  $T_2$ K and rejects heat to the thermal reservoir at 300 K. The efficiency of both the engines is equal. [IES-2004]

What is the value of temperature  $T_2$ ?

- (a) 400 K                          (b) 500 K                          (c) 600 K                          (d) 700 K

IES-12. A series combination of two Carnot's engines operate between the temperatures of 180°C and 20°C. If the engines produce equal amount of work, then what is the intermediate temperature? [IES-2009]

- (a) 80°C                            (b) 90°C                            (c) 100°C                            (d) 110°C

IES-13. An engine working on Carnot cycle rejects 40% of absorbed heat from the source, while the sink temperature is maintained at 27°C, then what is the source temperature? [IES-2009]

- (a) 750°C                            (b) 477°C                            (c) 203°C                            (d) 67.5°C

IES-13a A Carnot engine rejects 30% of absorbed heat to a sink at 30° C. The temperature of the heat source is [IES-2010]

- (a) 100° C                            (b) 433° C                            (c) 737° C                            (d) 1010° C

IES-14. A reversible heat engine rejects 50 percent of the heat supplied during a cycle of operation. If this engine is reversed and operates as a heat pump, then what is its coefficient of performance? [IES-2009]

- (a) 1.0                              (b) 1.5                              (c) 2.0                              (d) 2.5

IES-15. A heat engine is supplied with 250 kJ/s of heat at a constant fixed temperature of 227°C; the heat is rejected at 27°C, the cycle is reversible, then what amount of heat is rejected? [IES-2009]

- (a) 250 kJ/s                        (b) 200 kJ/s                        (c) 180 kJ/s                        (d) 150 kJ/s

IES-16. One reversible heat engine operates between 1600 K and  $T_2$  K, and another reversible heat engine operates between  $T_2$ K and 400 K. If both the engines have the same heat input and output, then the temperature  $T_2$  must be equal to: [IES-1993]

- (a) 1000                            (b) 1200                            (c) 1400                            (d) 800

IES-17. The continual motion of a movable device in absence of friction [IES-2012]

- (a) violates the first law of thermodynamics  
(b) violates the second law of thermodynamics  
(c) is the perpetual motion of the second kind  
(d) is the perpetual motion of the third kind

# Second Law of Thermodynamics

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## Previous 20-Years IAS Questions

### Kelvin-Planck Statement of Second Law

IAS-1. Assertion (A): No machine would continuously supply work without expenditure of some other form of energy. [IAS-2001]

Reason (R): Energy can be neither created nor destroyed, but it can only be transformed from one form into another.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is NOT the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

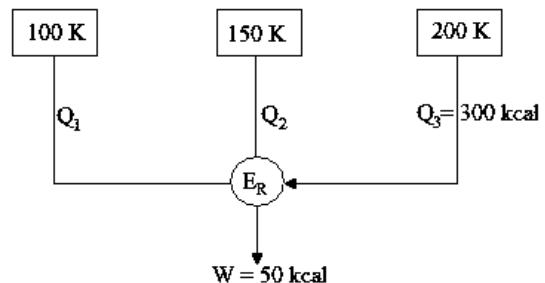
### Equivalence of Kelvin-Planck and Clausius Statements

IAS-2. A heat engine is supplied with 250 KJ/s of heat at a constant fixed temperature of 227°C. The heat is rejected at 27°C. The cycle is reversible, if the amount of heat rejected is: [IAS-1995]

- (a) 273 KJ/s
- (b) 200 KJ/s
- (c) 180 KJ/s
- (d) 150 KJ/s.

IAS-3. A reversible engine En as shown in the given figure draws 300 kcal from 200 K reservoir and does 50 kcal of work during a cycle. The sum of heat interactions with the other two reservoir is given by:

- (a)  $Q_1 + Q_2 = +250 \text{ kcal}$
- (b)  $Q_1 + Q_2 = -250 \text{ kcal}$
- (c)  $Q_1 + Q_2 = +350 \text{ kcal}$
- (d)  $Q_1 + Q_2 = -350 \text{ kcal}$



[IAS-1996]

### Carnot Engine with same efficiency or same work output

IAS-4. Consider the following statements: [IAS-2007]

1. Amount of work from cascaded Carnot engines corresponding to fixed temperature difference falls as one goes to lower absolute level of temperature.
2. On the enthalpy-entropy diagram, constant pressure lines diverge as the entropy increases.

Which of the statements given above is/are correct?

- (a) 1 only
- (b) 2 only
- (c) Both 1 and 2
- (d) Neither 1 nor 2

IAS-5. In a cyclic heat engine operating between a source temperature of 600°C and a sink temperature of 20°C, the least rate of heat rejection per kW net output of the engine is: [IAS 1994]

- (a) 0.460 kW
- (b) 0.505 kW
- (c) 0.588 kW
- (d) 0.650 kW

## **Answers with Explanation (Objective)**

### **Previous 20-Years IES Answers**

**IES-1. Ans. (a)**

**IES-2. Ans. (c)** A is true but R is false.

**IES-3. Ans. (c)** Heat transfer takes place according to second law of thermodynamics as it tells about the direction and amount of heat flow that is possible between two reservoirs.

**IES-3a.** **Ans. (b)** All spontaneous processes are irreversible. Statement-1 and statement-4 heat is transferred with a finite temperature difference they are irreversible.

**IES-3b.** **Ans. (d)** Any natural process carried out with a finite temperature gradient is an irreversible process. All spontaneous processes are irreversible. Statement -4 is a spontaneous process.

**IES-4. Ans. (c)** Out of 4 definitions given, only first definition is correct and balance three are wrong.

**IES-5. Ans. (b)** A and R are true. A is the Clausius statement of second law of thermodynamics. Spontaneously means without change in surroundings.

If question comes like following then answer will be (a).

Assertion (A): External work must be put into heat pump so that heat can be transferred from a cold to a hot body.

Reason (R): Heat cannot spontaneously pass from a colder system to a hotter system without simultaneously producing other effects in the surroundings.

**IES-6. Ans. (d)**

$$\text{IES-7. Ans. (c)} \text{ Carnot efficiency of engine} = \eta = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{600} = 0.5$$

But according to the inventor's Claim

Efficiency of engine =  $1 - 0.45 = 0.55$

$\therefore$  Efficiency of Actual Engine cannot be greater than Carnot efficiency. So this is an impossible engine.

**IES-7a. Ans. (c)** We know Carnot efficiency

$$\eta_{\text{carnot}} = 1 - \frac{T_2}{T_1} \Rightarrow 1 - \frac{300}{600} = \frac{1}{2} = 0.5$$

$\eta_{\text{carnot}} = 50\%$  But inventor claims 60% efficiency (means 40% heat rejection). It is then impossible.

**IES-7b. Ans. (c)**

**IES-7c. Ans. (d)**

**IES-8. Ans. (a)**

**IES-9. Ans. (c)**

**IES-10. Ans. (c)** Figure from another question

$$W_1 = W_2$$

$$\text{or } Q_1 - Q_2 = Q_2 - Q_3 \text{ or } T_1 - T_2 = T_2 - T_3 \text{ or } T_2 = \frac{T_1 + T_3}{2} = \frac{900 + 400}{2} = 650\text{K}$$

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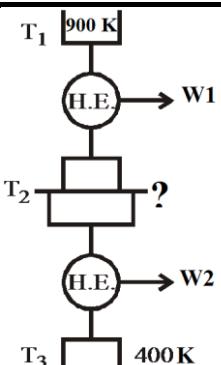
**IES-10a.**

**Ans. (b)**

When equally efficiency

$$1 - \frac{T_2}{T_1} = 1 - \frac{T_3}{T_2}$$

$$\text{or } T_2 = \sqrt{T_1 T_3} = \sqrt{900 \times 400} = 600 \text{ K}$$

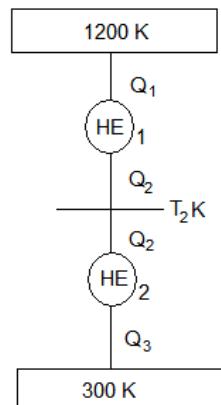


**IES-11. Ans. (c)**

$$\eta_1 = \eta_2$$

$$\text{or } 1 - \frac{T_2}{1200} = 1 - \frac{300}{T_2}$$

$$\text{or } T_2 = \sqrt{1200 \times 300} = 600 \text{ K}$$



**IES-12. Ans. (c)**

Source Temperature =  $T_1$ ,

Intermediate Temperature =  $T$

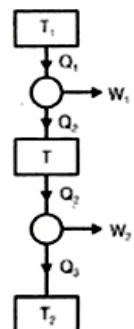
Sink Temperature =  $T_2$

$$\therefore W_1 = W_2$$

$$Q_1 \left(1 - \frac{T}{T_1}\right) = Q_2 \left(1 - \frac{T_2}{T}\right)$$

$$\Rightarrow \frac{T_1}{T} \left(1 - \frac{T}{T_1}\right) = \left(1 - \frac{T_2}{T}\right) \Rightarrow \frac{T_1}{T} - 1 = 1 - \frac{T_2}{T}$$

$$\Rightarrow \frac{T_1 + T_2}{T} = 2 \quad \Rightarrow T = \frac{T_1 + T_2}{2} = \frac{180 + 20}{2} = 100^\circ\text{C}$$

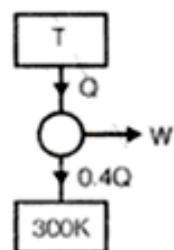


**IES-13. Ans. (b)** Sink temperature =  $27^\circ\text{C} = 27 + 273 = 300\text{K}$

It is given that engine rejects 40% of absorbed heat from the source

For a Carnot cycle engine  $\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$

$$\frac{Q}{T} = \frac{0.4Q}{300} \Rightarrow T = \frac{300}{0.4} = 750\text{K} = 477^\circ\text{C}$$



# Second Law of Thermodynamics

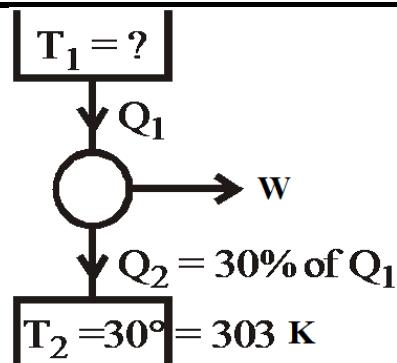
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**IES-13a Ans. (c)**

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

$$or T_1 = T_2 \times \frac{Q_1}{Q_2} = 303 \times \frac{Q_1}{0.3Q_1} = 1010K = 737^\circ C$$



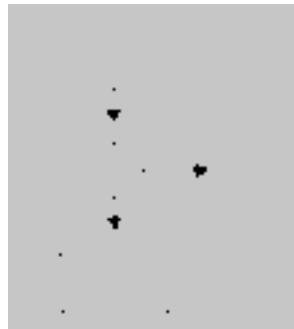
**IES-14. Ans. (c)  $\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$**

$$\Rightarrow \frac{Q_1}{T_1} = \frac{0.5Q_1}{T_2} \Rightarrow \frac{T_2}{T_1} = 0.5$$

If the engine is reversed and operated as the Heat Pump.

Then COP coefficient of performance

$$= \frac{T_1}{T_1 - T_2} = \frac{1}{1 - \frac{T_2}{T_1}} = \frac{1}{1 - 0.5} = 2$$



**IES-15. Ans. (d)** Heat supplied by the Heat Engine =  $Q_1 = 250$  kJ/sec

Source temperature =  $227^\circ C = 500$  K

Sink temperature =  $27^\circ C = 300$  K

$$\frac{250}{500} = \frac{Q_2}{300} \Rightarrow Q_2 = 250 \times 0.6 = 150 \text{ kJ/sec}$$

**IES-16. Ans. (d)** Two reversible heat engines operate between limits of  $1600\text{K}$  and  $T_2$ ;  $T_2$  and  $400\text{K}$

Both have the same heat input and output,

$$\text{i.e. } \frac{T_1 - T_2}{T_1} \text{ is same for both or } \frac{1600 - T_2}{1600} = \frac{T_2 - 400}{T_2} \text{ or } T_2 = 800\text{K}$$

**IES-17. Ans. (d)**

## Previous 20-Years IAS Answers

**IAS-1. Ans. (a)**

$$\text{IAS-2. Ans. (d) } \frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

**IAS-3. Ans. (b)  $\sum Q = \sum W$**

$$300 + Q_1 + Q_2 = 50$$

# Second Law of Thermodynamics

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**IAS-4. Ans. (b)** For reversible cycle

$$\frac{T_1}{Q} = \frac{T_2}{Q} = \frac{T_3}{Q}$$

$$\text{or } \frac{T_1 - T_2}{T_2} = \frac{Q - Q}{Q}$$

$$\text{or } T_1 - T_2 = (Q - Q) \times \frac{T_2}{Q}$$

$$\text{Similarly } T_2 - T_3 = (Q - Q) \times \frac{T_3}{Q}$$

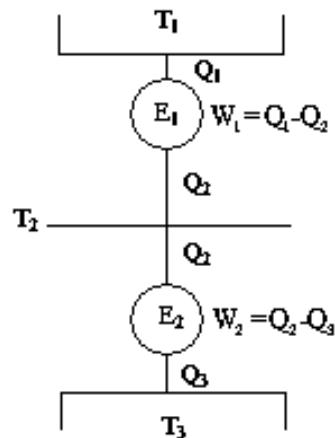
$$\text{If } T_1 - T_2 = T_2 - T_3 \text{ then } Q - Q = Q - Q$$

$$\text{or } W_1 = W_2$$

**IAS-5. Ans. (b)** Reversible engine has maximum efficiency where  $\frac{Q}{T_1} = \frac{Q}{T_2} = \frac{Q - Q}{T_1 - T_2} = \frac{W}{T_1 - T_2}$

Therefore least heat rejection per kW net output,

$$Q_2 = \frac{W}{T_1 - T_2} \times T_2 = \frac{1}{873 - 293} \times 293 = 0.505 \text{ kW}$$



## 4.

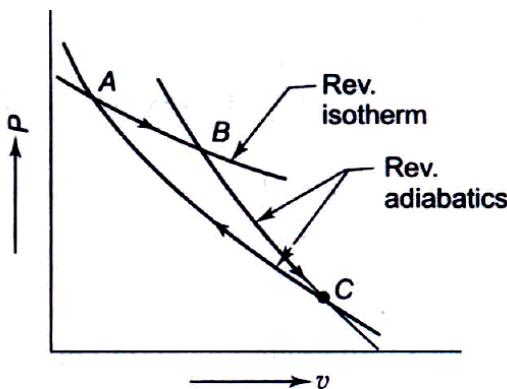
## Entropy

### Theory at a Glance (For GATE, IES & PSUs)

#### Two Reversible Adiabatic Paths cannot Intersect Each Other

Let it be assumed that two reversible adiabatics  $AC$  and  $BC$  intersect each other at point  $C$  (Fig.). Let a reversible isotherm  $AB$  be drawn in such a way that it intersects the reversible adiabatics at  $A$  and  $B$ . The three reversible processes  $AB$ ,  $BC$ , and  $CA$  together constitute a reversible cycle, and the area included represents the net work output in a cycle. But such a cycle is impossible, since net work is being produced in a cycle by a heat engine by exchanging heat with a single reservoir in the process  $AB$ , which violates the Kelvin-Planck statement of the second law. Therefore, the assumption of the intersection of the reversible adiabatics is Fig. wrong. *Through one point, there can pass only one reversible adiabatic.*

Since two constant property lines can never intersect each other, it is inferred that a reversible adiabatic path must represent some property, which is yet to be identified.



*Assumption of Two Reversible Adiabatics Intersecting Each Other*

# Entropy

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## The Property of Entropy

$$ds = \left( \frac{-dQ}{T} \right)_{\text{Reversible}}$$

- $dS$  is an exact *differential* because S is a point function and a property. The subscript *R* in  $dQ$  indicates that heat  $dQ$  is transferred *reversibly*.

$$\cdot s = \frac{S}{m} \text{ J/kg K}$$

- **It is an extensive property**, and has the unit  $J/K$ . The specific entropy is an intensive property and has unit  $J/kgK$
- The change of entropy may be regarded as a measure of the rate of availability of heat for transformation into work.

If the system is taken from an initial equilibrium state *i* to a final equilibrium state *f* by an *irreversible path*, since entropy is a point or state function, and the entropy change is independent of the path followed, the non-reversible path is to be replaced by a reversible path to integrate for the evaluation of entropy change in the irreversible process

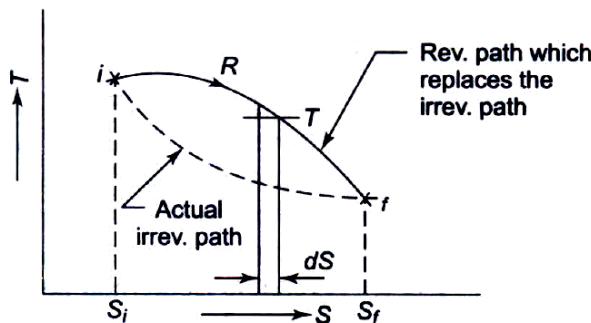
$$S_f - S_i = \int_i^f \frac{dQ_{\text{rev}}}{T} = (\Delta S)_{\text{irrev path}}$$

Integration can be performed only on a reversible path.

# Entropy

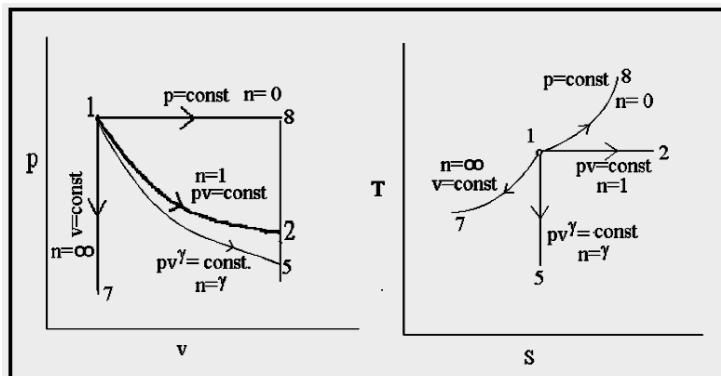
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*Integration can be Done Only on a Reversible Path*

## Temperature-Entropy Plot



## The Inequality of Clausius

Then for any cycle

$$\oint \frac{dQ}{T} \leq \oint ds$$

Since entropy is a property and the cyclic integral of any property is zero

$$\oint \frac{dQ}{T} \leq 0$$

This equation is known as the *inequality of Clausius*. It provides the criterion of the reversibility of a cycle.

If  $\oint \frac{dQ}{T} = 0$ , the cycle is **reversible**,

$\oint \frac{dQ}{T} < 0$ , the cycle is **irreversible and possible**

# Entropy

S K Mondal's

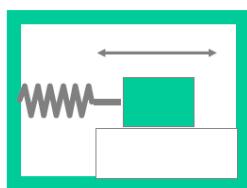
Chapter 4

$$\oint \frac{dQ}{T} > 0,$$

the cycle is **impossible**, since it violates the second law.

## Entropy Change in an Irreversible Process

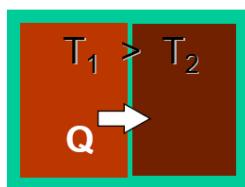
**motion with friction**



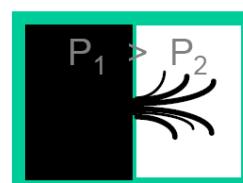
**spontaneous chemical reaction**



**heat transfer**



**unrestrained expansion**



**mixing**



Flow of current through a resistance – when a battery discharges through a resistance heat is dissipated. You can't recharge the battery by supplying heat back to the resistance element!!

Pickpocket

!!!Marriage!!!!.....are irreversible Process.

## Applications of Entropy Principle

$$(S_1 - S_2)_{\text{irreversible}} > (S_1 - S_2)_{\text{reversible}}$$

An irreversible process generates more entropy than a reversible process.

An irreversible engine can't produce more work than a reversible one.

- **An irreversible heat pump will always need more work than a reversible heat pump.**
- An irreversible expansion will produce less work than a reversible expansion
- **An irreversible compression will need more work than a reversible compression.**

# Entropy

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## Maximum Work Obtainable from two Finite Bodies at temperatures $T_1$ and $T_2$

Let us consider two identical finite bodies of constant heat capacity at temperature  $T_1$  and  $T_2$  respectively,  $T_1$  being higher than  $T_2$ . If the two bodies are merely brought together into thermal contact, delivering no work, the final temperature  $T_f$  reached would be the maximum

$$T_f = \frac{T_1 + T_2}{2}$$

If a heat engine is operated between the two bodies acting as thermal energy reservoirs (shown in Fig. below), part of the heat withdrawn from body 1 is converted to work  $W$  by the heat engine, and the remainder is rejected to body 2. The lowest attainable final temperature  $T_f$  corresponds to the delivery of the largest possible amount of work, and is associated with a reversible process.

As work is delivered by the heat engine, the temperature of body 1 will be decreasing and that of body 2 will be increasing. When both the bodies attain the final temperature  $T_f$ , the heat engine will stop operating. Let the bodies remain at constant pressure and undergo no change of phase.

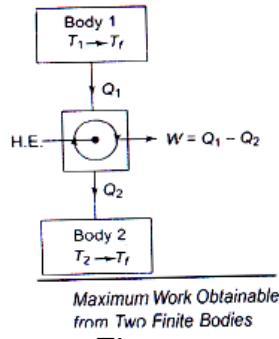


Fig.

Total heat withdrawn from body 1

$$Q_1 = C_p(T_1 - T_f)$$

Where  $C_p$  is the heat capacity of the two bodies at constant pressure.

Total heat rejected to body 2

$$Q_2 = C_p(T_f - T_2)$$

∴ Amount of total work delivered by the heat engine

$$\begin{aligned} W &= Q_1 - Q_2 \\ &= C_p(T_1 + T_2 - 2T_f) \end{aligned}$$

For given values of  $C_p$ ,  $T_1$  and  $T_2$ , the magnitude of work  $W$  depends on  $T_f$ . Work obtainable will be maximum when  $T_f$  is minimum.

Now, for body 1, entropy change  $\Delta S_1$  is given by

$$\Delta S_1 = \int_{T_1}^{T_f} C_p \frac{dT}{T} = C_p \ln \frac{T_f}{T_1}$$

For body 2, entropy change  $\Delta S_2$  would be

$$\Delta S_2 = \int_{T_2}^{T_f} C_p \frac{dT}{T} = C_p \ln \frac{T_f}{T_2}$$

Since the working fluid operating in the heat engine cycle does not undergo any entropy change,  $\Delta S$  of the working fluid in heat engine =  $\oint dS = 0$ . Applying the entropy principle

# Entropy

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$$\Delta S_{\text{univ}} \geq 0$$

$$\therefore C_p \ln \frac{T_f}{T_1} + C_p \ln \frac{T_f}{T_2} \geq 0$$

$$C_p \ln \frac{T_f^2}{T_1 T_2} \geq 0$$

From above Eq. for  $T_f$  to be a minimum

$$C_p \ln \frac{T_f^2}{T_1 T_2} = 0$$

$$\text{or} \quad \ln \frac{T_f^2}{T_1 T_2} = 0 = \ln 1$$

$$\therefore T_f = \sqrt{T_1 T_2}$$

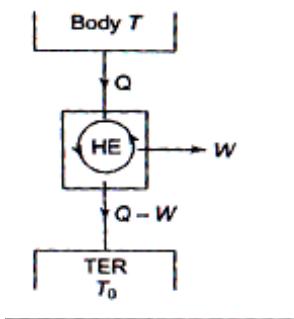
For  $W$  to be a maximum,  $T_f$  will be  $\sqrt{T_1 T_2}$ . From above Equation

$$W_{\max} = C_p (T_1 + T_2 - 2\sqrt{T_1 T_2}) = C_p (\sqrt{T_1} - \sqrt{T_2})^2$$

The final temperatures of the two bodies, initially at  $T_1$  and  $T_2$ , can range from  $(T_1 + T_2)/2$  with no delivery of work to  $\sqrt{T_1 T_2}$  with maximum delivery of work.

- Maximum Work Obtainable from a Finite Body and a TER :-**

Let one of the bodies considered in the previous section be a thermal energy reservoir. The finite body has a thermal capacity  $C_p$  and is at temperature  $T$  and the TER is at temperature  $T_0$ , such that  $T > T_0$ . Let  $(Q - W)$ . Then



*Maximum Work Obtainable  
When One of the Bodies  
is a TER*

**Fig. –**

Maximum Work Obtainable When One of the Bodies is a TER.

# Entropy

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$$\Delta S_{\text{Body}} = \int_T^{T_0} C_p \frac{dT}{T} = C_p \ln \frac{T_0}{T}$$

$$\Delta S_{\text{HE}} = \oint ds = 0$$

$$\Delta S_{\text{IER}} = \frac{Q-W}{T_0}$$

$$\therefore \Delta S_{\text{univ}} = C_p \ln \frac{T_0}{T} + \frac{Q-W}{T_0}$$

By the entropy principle,

$$\Delta S_{\text{univ}} \geq 0$$

$$C_p \ln \frac{T_0}{T} + \frac{Q-W}{T_0} \geq 0$$

$$\text{or } C_p \ln \frac{T_0}{T} \geq \frac{W-Q}{T_0}$$

$$\text{or } \frac{W-Q}{T_0} \leq C_p \ln \frac{T_0}{T}$$

$$\text{or } W \leq Q + T_0 C_p \ln \frac{T_0}{T}$$

$$\therefore W_{\max} = Q + T_0 C_p \ln \frac{T_0}{T}$$

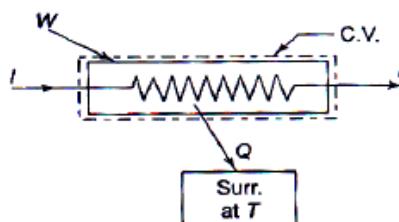
$$\text{or } W_{\max} = C_p \left[ (T - T_0) - T_0 \ln \frac{T}{T_0} \right]$$

- Processes Exhibiting External Mechanical Irreversibility :-**

**[i] Isothermal Dissipation of Work :-** Let us consider the isothermal dissipation of work through a system into the internal energy of a reservoir, as in the flow of an electric current through a resistor in contact with a reservoir (Fig.in below.) At steady state, the internal energy of the resistor and hence its temperature is constant. So, by first law:

$$W = Q$$

The flow of current represents work transfer. At steady state the work is dissipated isothermally into heat transfer to the surroundings. Since the surroundings absorb  $Q$  unit of heat at temperature  $T$ ,



*External Mechanical  
Irreversibility*

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(Fig. )

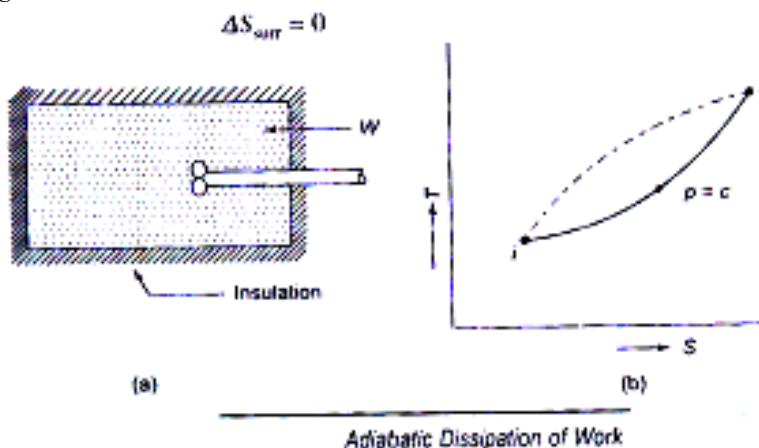
$$\Delta S_{sur} = \frac{Q}{T} = \frac{W}{T}$$

At steady state,  $\Delta S_{sys} = 0$

$$\therefore \Delta S_{univ} = \Delta S_{sys} + \Delta S_{sur} = \frac{W}{T}$$

The irreversible process is thus accompanied by an entropy increase of the universe.

**[ii] Adiabatic Dissipation of Work :-** Let  $W$  be the stirring work supplied to a viscous thermally insulated liquid, which is dissipated adiabatically into internal energy increase of the liquid, the temperature of which increases from  $T_i$  to  $T_f$  ( shown in fig below). Since there is no flow of heat to or from the surroundings.



(Fig. )

To calculate the entropy change of the system, the original irreversible path (dotted line) must be replaced by a reversible one between the same end states,  $i$  and  $f$ . Let us replace the irreversible performance of work by a reversible isobaric flow of heat from a series of reservoirs ranging from  $T_i$  to  $T_f$  to cause the same change in the state of the system. The entropy change of the system will be

$$\Delta S_{sys} = \int_{R^i}^f \frac{dQ}{T} = \int_{R^i}^f \frac{C_p dT}{T} = C_p \ln \frac{T_f}{T_i}$$

where  $C_p$  is the heat capacity of the liquid.

$$\Delta S_{univ} = \Delta S_{sys} + \Delta S_{sur} = C_p \ln \frac{T_f}{T_i}$$

## Entropy Generation

- Irreversible Processes increase the entropy of the universe.
- Reversible Processes do not effect the entropy of the universe.
- Impossible Processes decrease the entropy of the universe.

$$\Delta S_{universe} = 0$$

# Entropy

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Entropy Generation in the universe is a measure of lost of work.

$$\Delta S_{\text{Universe}} = \Delta S_{\text{System}} + \Delta S_{\text{Surroundings}}$$

The losses will keep increasing.

The sin keeps accumulating.

Damage to environment keeps increasing.

When the entropy of the universe goes so high, then some one has to come and set it right. **HE SAYS HE WILL COME.** Every religion confirms this.

**Let us all wait.**

Cheer up, things are not that bad yet!!

## Entropy and Direction: The Second Law a Directional law of Nature

The second law indicates the **direction** in which a process takes place. *A process always occurs in such a direction as to cause an increase in the entropy of the universe.*

# Summary

1. Clausius theorem:  $\oint \left( \frac{dQ}{T} \right)_{\text{rev.}} = 0$

2.  $S_f - S_i = \int_i \frac{dQ_{\text{rev}}}{T} = (\Delta S)_{\text{irrev. path.}}$

Integration can be performed only on a reversible path.

3. Clausius Inequality:  $\oint \frac{dQ}{T} \leq 0$

4. At the equilibrium state. The system is at the peak of the entropy hill. (isolated)

5.  $Tds = du + Pdv$

6.  $Tds = dh - Vdp$

7. Famous relation  $S = K \ln W$       where  $K$  = Boltzmann constant  
 $W$  = thermodynamic probability

8. General case of change of entropy of a Gas.

$$S_2 - S_1 = m \left\{ C_v \ln \frac{P_2}{P_1} + C_p \ln \frac{V_2}{V_1} \right\}$$

Initial condition of gas  $P_1, V_1, T_1, S_1$  and

Final condition of gas  $P_2, V_2, T_2, S_2$

9. Process and property change Table:

# Entropy

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	s	T	p	v
Isothermal expansion	↑	—	↓	↑
Isothermal compression	↓	—	↑	↓
Isentropic compression	—	↑	↑	↓
Isentropic expansion	—	↓	↓	↑
Isochoric heating	↑	↑	↑	—
Isochoric cooling	↓	↓	↓	—
Isobaric heating/expansion	↑	↑	—	↑
Isobaric cooling/compression	↓	↓	—	↓

## PROBLEMS & SOLUTIONS

### Example 1.

5 kg of air is compressed in a reversible polytrophic process from 1 bar and 40°C to 10 bar with an index of compression 1.25. Calculate the **entropy change** during the process.

**Solution:-**

$$\frac{T_2}{T_1} = \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}}$$

$$T_2 = T_1 \left( \frac{P_2}{P_1} \right)^{\frac{n-1}{n}} = (273+40) \left( \frac{10}{1} \right)^{\frac{0.25}{1.25}}$$

$$\text{Therefore } T_2 = 496 \text{ K}$$

$$\begin{aligned} \phi_2 - \phi_1 &= C_p \int_1^2 \frac{dT}{T} - R \int_1^2 \frac{dp}{p} = C_p \int_1^2 \frac{dT}{T} - R \int_1^2 \frac{dp}{p} \\ &= 1.005 \ln \left( \frac{T_2}{T_1} \right) - R \ln \left( \frac{P_2}{P_1} \right) \\ &= 1.005 \ln \frac{496}{313} - 0.287 \ln(10) \\ &= 0.4627 - 0.6608 \\ &= -0.1981 \text{ kJ/kg K} \end{aligned}$$

Total change in entropy =  $5 \times (-0.1981) = -0.9905 \text{ kJ/K (Reduction)}$ .  
(Hence heat is rejected during the process).

### Example 2.

Two compartments of an insulated vessel each of  $3 \text{ m}^3$  contain air at 0.7 MPa, 95°C and 0.35 MPa, 205°C. If the removed, find the **change in entropy**, if the two portions mix completely and adiabatically.

**Solution:**

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$$m_1 = \frac{p_1 V_1}{RT_1} = \frac{0.7 \times 10^6 \times 3}{287 \times (273 + 95)} = 19.883 \text{ kg}$$

$$m_2 = \frac{p_2 V_2}{RT_2} = \frac{0.35 \times 10^6 \times 3}{287 \times (273 + 205)} = 7.654 \text{ kg}$$

Assuming specific heat to be constant

$$T_f = \frac{C_v(m_1 T_1 + m_2 T_2)}{C_v(m_1 + m_2)} = 398.6 \text{ K} = 125.6^\circ\text{C}$$

$$\Delta S_1 = m_1 C_v \ln \frac{T_f}{T_1} = 19.883 \times 1005 \times \ln \frac{398.6}{368} = 1595 \text{ J/K}$$

$$\Delta S_2 = 7.654 \times 1005 \times \ln \frac{398.6}{478} = -1396.6 \text{ J/K}$$

$$\Delta S = 1595 - 1397 = 198 \text{ J/K}$$

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## ASKED OBJECTIVE QUESTIONS (GATE, IES, IAS)

### Previous 20-Years GATE Questions

#### Applications of Entropy Principle

- GATE-1. A 1500 W electrical heater is used to heat 20 kg of water ( $C_p = 4186 \text{ J/kg K}$ ) in an insulated bucket, from a temperature of 30°C to 80°C. If the heater temperature is only infinitesimally larger than the water temperature during the process, the change in entropy for heater is..... J/k and for water ..... J/K.

[GATE-1994]

#### Entropy Generation in a Closed System

- GATE-1A An ideal gas of mass  $m$  and temperature  $T_1$  undergoes a reversible isothermal process from an initial pressure  $P_1$  to final pressure  $P_2$ . The heat loss during the process is  $Q$ . The entropy change  $\Delta S$  of the gas is

- (a)  $mR\ln\left(\frac{P_2}{P_1}\right)$     (b)  $mR\ln\left(\frac{P_1}{P_2}\right)$     (c)  $mR\ln\left(\frac{P_2}{P_1}\right) - \frac{Q}{T_1}$     (d) zero

[GATE-2012]

Data for Q2 and Q3 are given below. Solve the problems and choose correct answers.

Nitrogen gas (molecular weight 28) is enclosed in a cylinder by a piston, at the initial condition of 2 bar, 298 K and 1 m<sup>3</sup>. In a particular process, the gas slowly expands under isothermal condition, until the volume becomes 2m<sup>3</sup>. Heat exchange occurs with the atmosphere at 298 K during this process.

- GATE-2. The work interaction for the Nitrogen gas is: [GATE-2003]  
(a) 200 kJ    (b) 138.6 kJ    (c) 2 kJ    (d) -200 kJ

- GATE-3. The entropy change for the Universe during the process in kJ/K is: [GATE-2003]  
(a) 0.4652    (b) 0.0067    (c) 0    (d) -0.6711

- GATE-4. If a closed system is undergoing an irreversible process, the entropy of the system [GATE-2009]  
(a) Must increase    (b) Always remains constant  
(c) Must decrease    (d) Can increase, decrease or remain constant

#### Entropy and Direction: The Second Law a Directional law of Nature

- GATE-5. One kilogram of water at room temperature is brought into contact with a high temperature thermal reservoir. The entropy change of the universe is:  
(a) Equal to entropy change of the reservoir [GATE-2010]

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- (b) Equal to entropy change of water
- (c) Equal to zero
- (d) Always positive

## Common Data for Questions GATE-6 and GATE-7:

In an experimental set-up, air flows between two stations P and Q adiabatically. The direction of flow depends on the pressure and temperature conditions maintained at P and Q. The conditions at station P are 150 kPa and 350 K. The temperature at station Q is 300 K.

The following are the properties and relations pertaining to air:

Specific heat at constant pressure,  $c_p = 1.005 \text{ kJ/kg K}$ ;

Specific heat at constant volume,  $c_v = 0.718 \text{ kJ/kg K}$ ;

Characteristic gas constant,  $R = 0.287 \text{ kJ/kg K}$ .

Enthalpy,  $h = c_p T$

Internal energy,  $u = c_v T$

**GATE-6.** If the pressure at station Q is 50 kPa, the change in entropy  $(S_Q - S_P)$  in kJ/kg

K is

(a) -0.155

(b) 0

(c) 0.160

[GATE-2011]

(d) 0.355

**GATE-7.** If the air has to flow from station P to station Q, the maximum possible value of pressure in kPa at station Q is close to

[GATE-2011]

(a) 50

(b) 87

(c) 128

(d) 150

## Previous 20-Years IES Questions

### Two Reversible Adiabatic Paths cannot intersect Each Other

**IES-1.** The relation  $dS = \frac{dQ}{T}$ , where s represents entropy, Q represents heat and T represents temperature (absolute), holds good in which one of the following processes?

[IES-2009]

- (a) Reversible processes only
- (b) Irreversible processes only
- (c) Both reversible and irreversible processes
- (d) All real processes

**IES-2.** Which of the following statement is correct?

[IES-2008]

- (a) The increase in entropy is obtained from a given quantity of heat transfer at a low temperature.
- (b) The change in entropy may be regarded as a measure of the rate of the availability of heat for transformation into work.
- (c) The entropy represents the maximum amount of work obtainable per degree drop in temperature
- (d) All of the above

**IES-2a.** A heat engine receives 1000 kW of heat at a constant temperature of 285°C and rejects 492 kW of heat at 5°C. Consider the following thermodynamic cycles in this regard:

[IES-2000]

- 1. Carnot cycle
- 2. Reversible cycle
- 3. Irreversible cycle

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Which of these cycles could possible be executed by the engine?

- (a) 1 alone      (b) 3 alone      (c) 1 and 2      (d) None of 1, 2 and 3

## The Property of Entropy

**IES-3.** Assigning the basic dimensions to mass, length, time and temperature respectively as M, L, T and  $\theta$  (Temperature), what are the dimensions of entropy?  
 [IES-2007]

- (a)  $M LT^{-2} \theta$       (b)  $M L^2 T^{-1} \theta^{-1}$       (c)  $M L^2 T^{-2} \theta^{-1}$       (d)  $M L^3 T^{-2} \theta^{-1}$

**IES-4.** A Carnot engine operates between  $327^\circ C$  and  $27^\circ C$ . If the engine produces 300 kJ of work, what is the entropy change during heat addition?  
 [IES-2008]

- (a) 0.5 kJ/K      (b) 1.0 kJ/K      (c) 1.5 kJ/K      (d) 2.0 kJ/K

**IES-4a.** When a system reaches the state of equilibrium, the following property assumes its maximum value  
 [IES-2012]

- (a) Availability      (b) Entropy      (c) Gibbs function      (d) Helmholtz function

## Temperature-Entropy Plot

**IES-4b.** Isentropic flow is  
 [IES-2011]

- (a) Irreversible adiabatic flow  
 (b) Reversible adiabatic flow  
 (c) Ideal fluid flow  
 (d) Frictionless reversible flow

**IES-5.** A system comprising of a pure substance executes reversibly a cycle 1 -2 -3 -4 -1 consisting of two isentropic and two isochoric processes as shown in the Fig. 1.

Which one of the following is the correct representation of this cycle on the temperature - entropy coordinates?

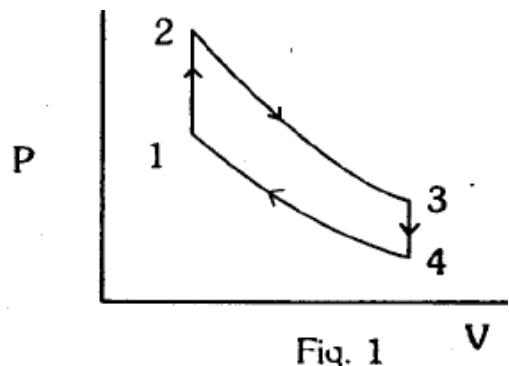
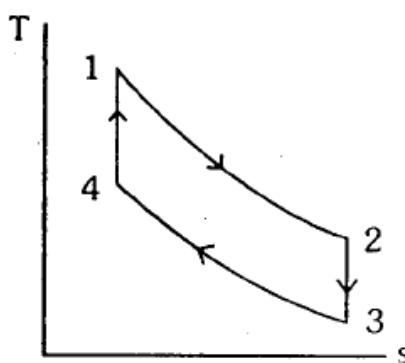
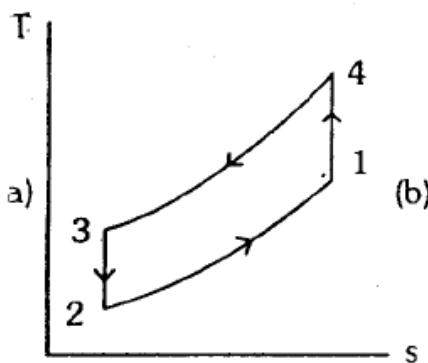


Fig. 1

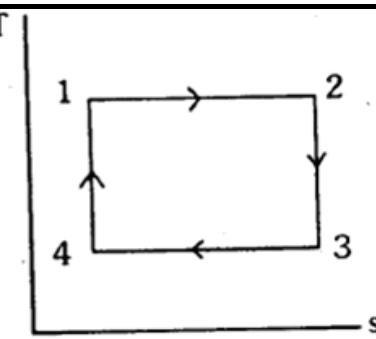
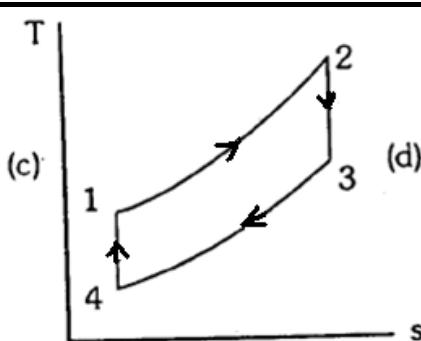
[IES-2002]



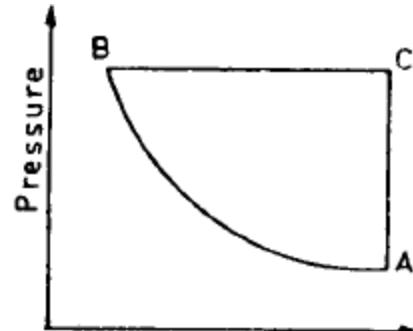
# Entropy

**S K Mondal's**

**Chapter 4**

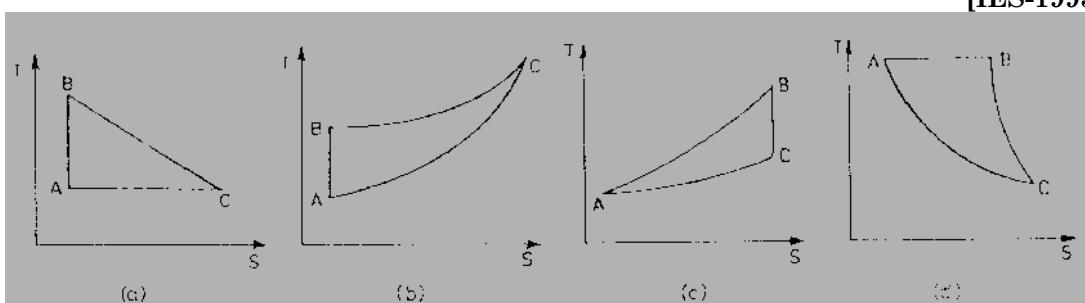


- IES-6.** A cycle of pressure – volume diagram is shown in the given Fig. I. Same cycle on temperature-entropy diagram will be represented by:

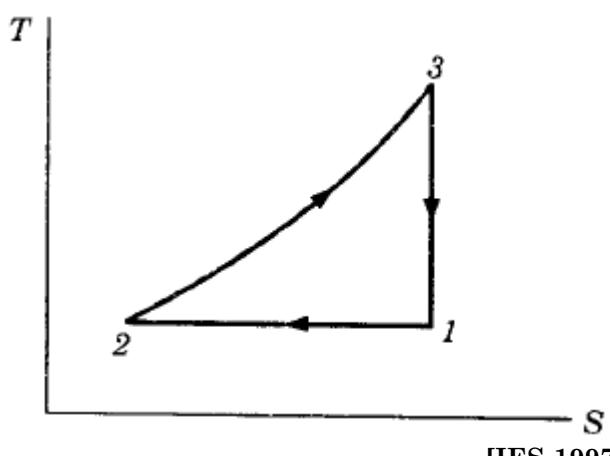


**Fig. 1.**

[IES-1995]



- IES-7.** An ideal air standard cycle is shown in the given temperature-entropy diagram.



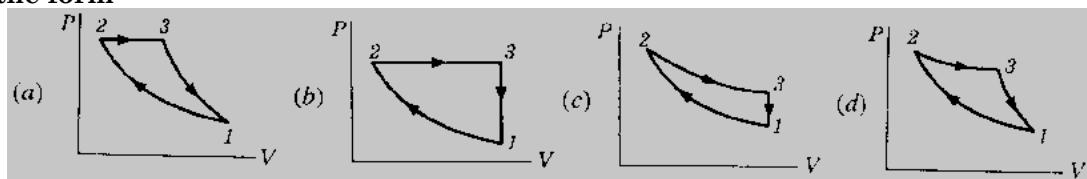
[IES-1997]

# Entropy

**S K Mondal's**

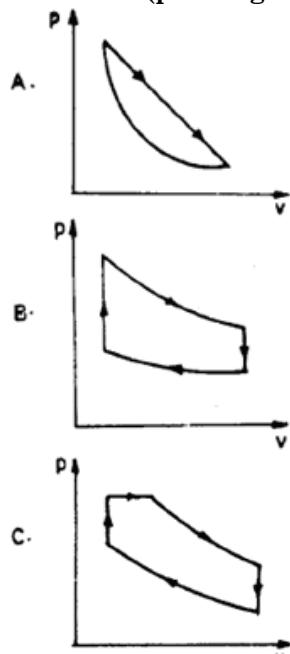
**Chapter 4**

The same cycle, when represented on the pressure-volume coordinates takes the form



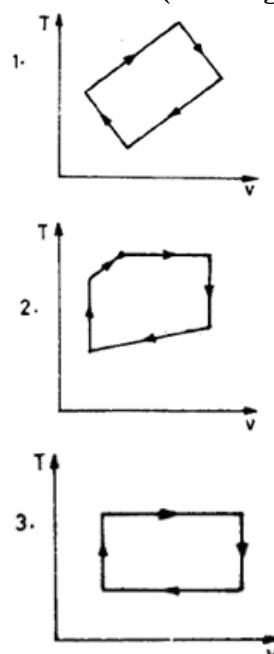
- IES-8.** Match figures of Column-I with those given in Column-II and select given below the columns: [IES-1994]

**Column-I (p-v diagram)**



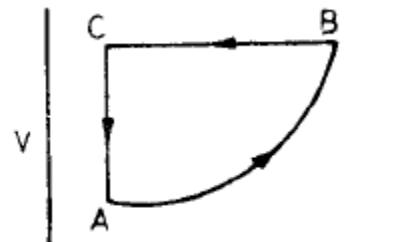
Codes:	A	B	C
(a)	1	2	3
(c)	3	1	2

**Column-II (T-s diagram)**



Codes:	A	B	C
(b)	2	3	1
(d)	3	2	1

- IES-9.** A cyclic process ABCD shown in the V-T diagram performed with a constant mass of an ideal gas. The process of p-V diagram will be as shown in

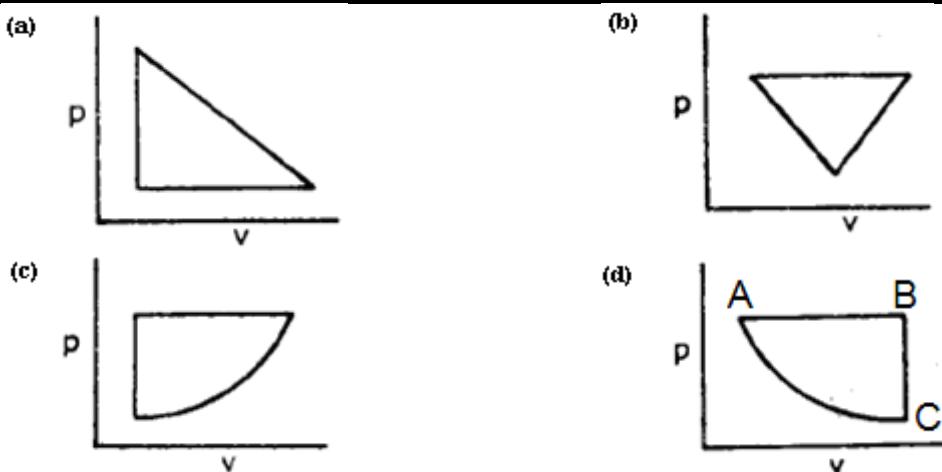


[IES-1992]

# Entropy

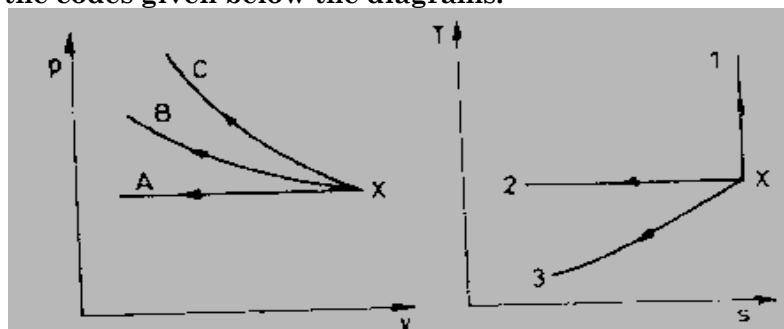
**S K Mondal's**

**Chapter 4**



- IES-10.** Three processes are represented on the p-v and T-s diagrams in the following figures. Match processes in the two diagrams and select the correct answer using the codes given below the diagrams:

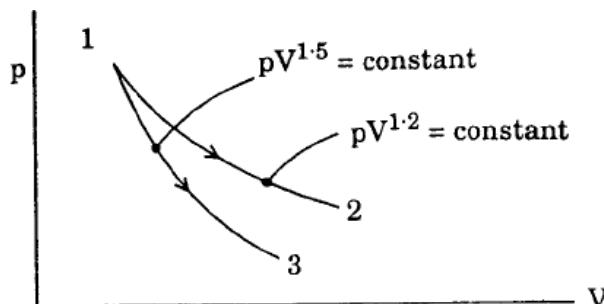
[IES-1994]



Codes:	A	B	C	(a)	A	B	C
(a)	1	2	3	(b)	2	3	1
(c)	3	2	1	(d)	1	3	2

- IES-11.** Two polytropic processes undergone by a perfect gas are shown below in the pressure-volume co-ordinates.

[IES-2008]

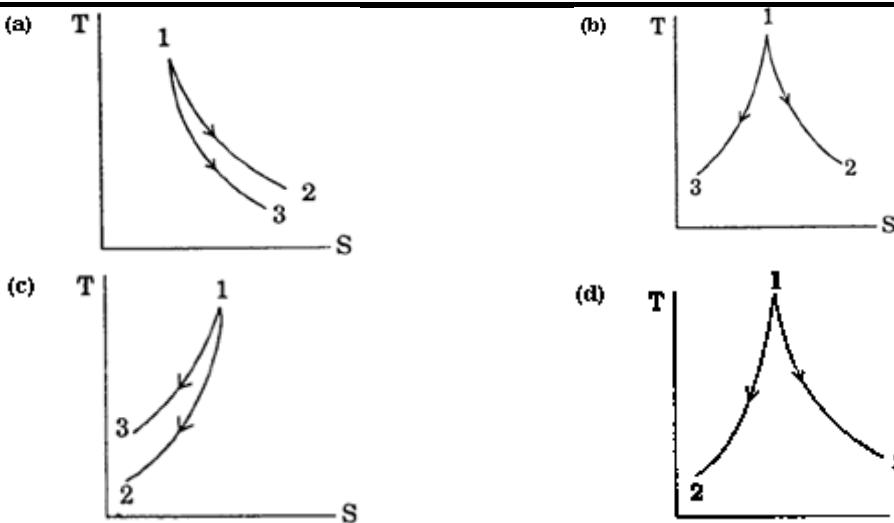


Which representation shows correctly the above processes on the temperature-entropy co-ordinates?

# Entropy

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**IES-12.** Assertion (A): If a graph is plotted for absolute temperature as a function of entropy, the area under the curve would give the amount of heat supplied.

Reason (R): Entropy represents the maximum fraction of work obtainable from heat per degree drop in temperature. [IES-1998]

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is NOT the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

## The Inequality of Clausius

**IES-13.** For real thermodynamic cycle:

$$(a) \oint \frac{dQ}{T} > 0 \text{ but } < \infty \quad (b) \oint \frac{dQ}{T} < 0 \quad (c) \oint \frac{dQ}{T} = 0 \quad (d) \oint \frac{dQ}{T} = \infty \quad [\text{IES-2005}]$$

**IES-14.** For a thermodynamic cycle to be irreversible, it is necessary that [IES-1998]

$$(a) \oint \frac{dQ}{T} = 0 \quad (b) \oint \frac{dQ}{T} < 0 \quad (c) \oint \frac{dQ}{T} > 0 \quad (d) \oint \frac{dQ}{T} \geq 0$$

**IES-15.** For an irreversible cycle:

$$(a) \oint \frac{dQ}{T} \leq 0 \quad (b) \oint \frac{dQ}{T} > 0 \quad (c) \oint \frac{dQ}{T} < 0 \quad (d) \oint \frac{dQ}{T} \geq 0 \quad [\text{IES-1994, 2011}]$$

**IES-16.** If a system undergoes an irreversible adiabatic process, then (symbols have usual meanings) [IES-1997]

$$(a) \int \frac{dQ}{T} = 0 \text{ and } \Delta S > 0 \quad (b) \int \frac{dQ}{T} = 0 \text{ and } \Delta S = 0 \\ (c) \int \frac{dQ}{T} > 0 \text{ and } \Delta S = 0 \quad (d) \int \frac{dQ}{T} < 0 \text{ and } \Delta S < 0$$

# Entropy

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## Entropy Change in an Irreversible Process

- IES-17. Consider the following statements:

[IES-1998]

In an irreversible process

1. Entropy always increases.
2. The sum of the entropy of all the bodies taking part in a process always increases.
3. Once created, entropy cannot be destroyed.

Of these statements

- (a) 1 and 2 are correct  
(c) 2 and 3 are correct

- (b) 1 and 3 are correct  
(d) 1, 2 and 3 are correct

- IES-18. Consider the following statements:

[IES-1997]

When a perfect gas enclosed in a cylinder piston device executes a reversible adiabatic expansion process,

1. Its entropy will increase
2. Its entropy change will be zero
3. The entropy change of the surroundings will be zero

Of these statements

- (a) 1 and 3 are correct  
(c) 2 and 3 are correct

- (b) 2 alone is correct  
(d) 1 alone is correct

- IES-19. A system of 100 kg mass undergoes a process in which its specific entropy increases from 0.3 kJ/kg-K to 0.4 kJ/kg-K. At the same time, the entropy of the surroundings decreases from 80 kJ/K to 75 kJ/K. The process is:

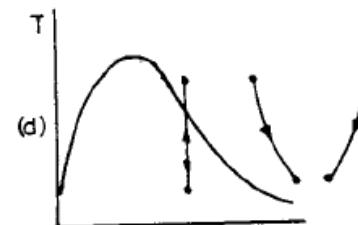
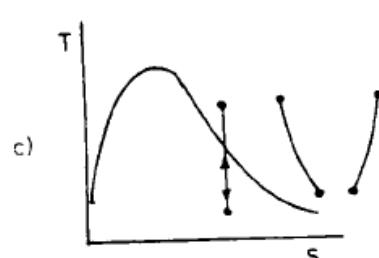
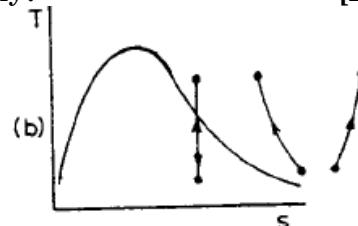
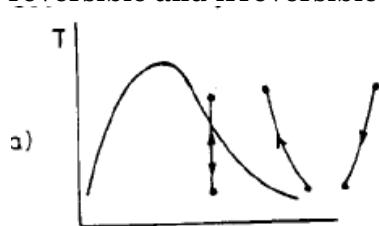
[IES-1997]

- (a) Reversible and isothermal  
(c) Reversible

- (b) Irreversible  
(d) Impossible

- IES-20. Which one of the following temperature entropy diagrams of steam shows the reversible and irreversible processes correctly?

[IES-1996]



## Applications of Entropy Principle

- IES-21. A Carnot engine operates between 27°C and 327°C. If the engine produces 300 kJ of Work, What is the entropy change during heat addition?

[IES-2005]

# Entropy

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(a) 0.5 kJ/K

(b) 1.0 kJ/K

(c) 1.5 kJ/K

(d) 2.0 kJ/K

- IES-22. The entropy of a mixture of ideal gases is the sum of the entropies of constituents evaluated at: [IES-2005]

(a) Temperature and pressure of the mixture

(b) Temperature of the mixture and the partial pressure of the constituents

(c) Temperature and volume of the mixture

(d) Pressure and volume of the mixture

- IES-23. The heat added to a closed system during a reversible process is given by  $Q = \alpha T + \beta T^2$ , where  $\alpha$  and  $\beta$  are constants. The entropy change of the system as its temperature changes from  $T_1$  to  $T_2$  is equal to: [IES-2000]

(a)  $\alpha + \beta(T_2 - T_1)$

(b)  $\left[ \alpha(T_2 - T_1) + \frac{\beta}{2}(T_2^2 - T_1^2) \right] / T_1$

(c)  $\left[ \frac{\alpha}{2}(T_2^2 - T_1^2) + \frac{\beta}{2}(T_2^3 - T_1^3) \right] / T_1^2$

(d)  $\alpha \ln\left(\frac{T_2}{T_1}\right) + 2\beta(T_2 - T_1)$

- IES-24. One kg of air is subjected to the following processes: [IES-2004]

1. Air expands isothermally from 6 bar to 3 bar.

2. Air is compressed to half the volume at constant pressure

3. Heat is supplied to air at constant volume till the pressure becomes three fold

In which of the above processes, the change in entropy will be positive?

(a) 1 and 2

(b) 2 and 3

(c) 1 and 3

(d) 1, 2 and 3

- IES-25. A reversible heat engine receives 6 kJ of heat from thermal reservoir at temperature 800 K, and 8 kJ of heat from another thermal reservoir at temperature 600 K. If it rejects heat to a third thermal reservoir at temperature 100 K, then the thermal efficiency of the engine is approximately equal to: [IES-2002]

(a) 65%

(b) 75%

(c) 80%

(d) 85%

- IES-26. A reversible engine exceeding 630 cycles per minute drawn heat from two constant temperature reservoirs at 1200 K and 800 K rejects heat to constant temperature at 400 K. The engine develops work 100kW and rejects 3200 KJ heat per minute. The ratio of heat drawn from two reservoirs  $\frac{Q_{1200}}{Q_{800}}$  is nearly. [IES-1992]

(a) 1

(b) 1.5

(c) 3

(d) 10.5

- IES-27. In which one of the following situations the entropy change will be negative

(a) Air expands isothermally from 6 bars to 3 bars

[IES-2000]

(b) Air is compressed to half the volume at constant pressure

(c) Heat is supplied to air at constant volume till the pressure becomes three folds

(d) Air expands isentropically from 6 bars to 3 bars

# Entropy

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## Entropy and Direction: The Second Law a Directional law of Nature

- IES-28. A mass M of a fluid at temperature  $T_1$  is mixed with an equal mass of the same fluid at temperature  $T_2$ . The resultant change in entropy of the universe is:

[IES-1992]

- (a) Zero      (b) Negligible      (c) Always negative      (d) Always positive

- IES-29 Increase in entropy of a system represents [IES-2011]

- (a) Increase in availability of energy      (b) Increase in temperature  
(c) Decrease in pressure      (d) Degradation of energy

## Previous 20-Years IAS Questions

### Two Reversible Adiabatic Paths cannot Intersect Each Other

- IAS-1. Which one of the following is the correct statement?

[IAS-2007]

Two adiabatic will:

- (a) Intersect at absolute zero temperature  
(b) Never intersect  
(c) Become orthogonal at absolute zero temperature  
(d) Become parallel at absolute zero temperature

### The Property of Entropy

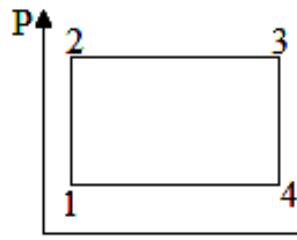
- IAS-2. Heat flows between two reservoirs having temperatures 1000 K and 500 K, respectively. If the entropy change of the cold reservoir is 10 kJ/K, then what is the entropy change for the hot reservoir?

[IAS-2004]

- (a) -10 kJ/K      (b) -5 kJ/K      (c) 5 kJ/K      (d) 10 kJ/K

### Temperature-Entropy Plot

- IAS-3. An ideal cycle is shown in the given pressure-volume diagram:



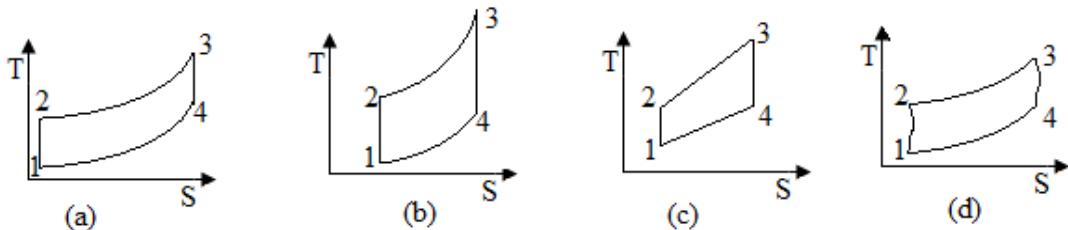
[IAS-1997]

# Entropy

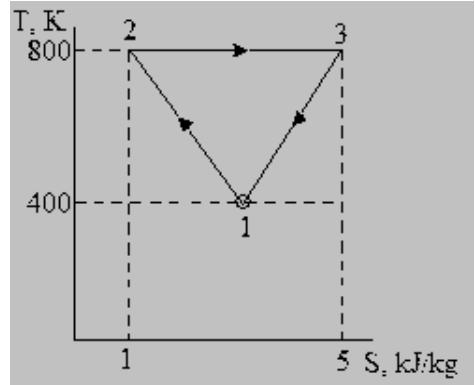
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The same cycle on temperature-entropy diagram will be represented as:



- IAS-4.** The thermal efficiency of the hypothetical heat engine cycle shown in the given figure is:  
 (a) 0.5                          (b) 0.45  
 (c) 0.35                          (d) 0.25

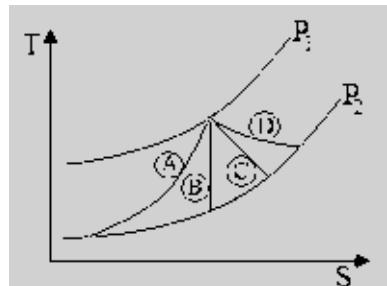


[IAS-2000]

- IAS-5.** Which one of the following pairs best expresses a relationship similar to that expressed in the pair "pressure-volume" for a thermodynamic system undergoing a process?  
 (a) Enthalpy-entropy                                  (b) Pressure-enthalpy  
 (c) Pressure-temperature                                (d) Temperature-entropy

[IAS-1995]

- IAS-6.** An ideal gas contained in a rigid tank is cooled such that  $T_2 <$  and  $P_2 < P_1$ . In the given temperature entropy diagram, this process path is represented by the line labelled.  
 (a) A                                                         (b) B  
 (c) C                                                         (d) D



[IAS-1999]

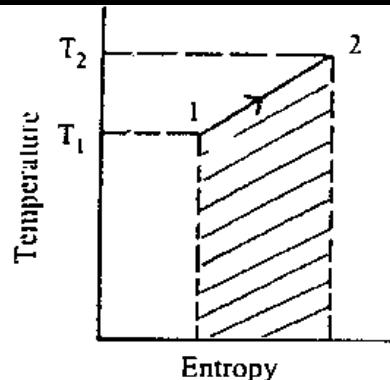
# Entropy

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**IAS-7.** In the T-S diagram shown in the figure, which one of the following is represented by the area under the curve?

- (a) Total work done during the process
- (b) Total heat absorbed during the process
- (c) Total heat rejected during the process
- (d) Degree of irreversibility



[IAS-2004]

## The Inequality of Clausius

**IAS-8.** Clausius inequality is stated as

- |                          |                          |                                    |                                       |
|--------------------------|--------------------------|------------------------------------|---------------------------------------|
| (a) $\oint \delta Q < 0$ | (b) $\oint \delta Q = 0$ | (c) $\oint \delta \frac{Q}{T} > 0$ | (d) $\oint \delta \frac{Q}{T} \leq 0$ |
|--------------------------|--------------------------|------------------------------------|---------------------------------------|

**IAS-9.** For real thermodynamic cycle:

- |                                             |                              |                              |                                   |
|---------------------------------------------|------------------------------|------------------------------|-----------------------------------|
| (a) $\oint \frac{dQ}{T} > 0$ but $< \infty$ | (b) $\oint \frac{dQ}{T} < 0$ | (c) $\oint \frac{dQ}{T} = 0$ | (d) $\oint \frac{dQ}{T} = \infty$ |
|---------------------------------------------|------------------------------|------------------------------|-----------------------------------|

**IAS-10(i).** If a system undergoes an irreversible adiabatic process, then (symbols have usual meanings)

$$(b) \int \frac{dQ}{T} = 0 \text{ and } \Delta S = 0$$

$$(c) \int \frac{dQ}{T} > 0 \text{ and } \Delta S = 0$$

$$(d) \int \frac{dQ}{T} < 0 \text{ and } \Delta S < 0$$

**IAS-10(ii).** A cyclic heat engine receives 600 kJ of heat from a 1000 K source and rejects 450 kJ to a 300 K sink. The quantity  $\oint \frac{dQ}{T}$  and efficiency of the engine are respectively

- |                      |                       |
|----------------------|-----------------------|
| (a) 2.1 kJ/K and 70% | (b) -0.9 kJ/K and 25% |
|----------------------|-----------------------|

- |                        |                       |
|------------------------|-----------------------|
| (c) + 0.9 kJ/K and 70% | (d) -2.1 kJ/K and 25% |
|------------------------|-----------------------|

[IAS-2001]

## Applications of Entropy Principle

**IAS-11.** Which one of the following statements is not correct?

[IAS-2003]

- (a) Change in entropy during a reversible adiabatic process is zero
- (b) Entropy increases with the addition of heat
- (c) Throttling is a constant entropy expansion process
- (d) Change in entropy when a gas is heated under constant pressure given by

$$S_2 - S_1 = mC_p \log_e \frac{T_2}{T_1}$$

**IAS-12.** Assertion (A): Entropy change for a reversible adiabatic process is zero.

# Entropy

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**Reason (R): There is no heat transfer in an adiabatic process.**

[IAS 1994]

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is NOT the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

## Entropy Generation in a Closed System

**IAS-13. 1600 kJ of energy is transferred from a heat reservoir at 800 K to another heat reservoir at 400 K. The amount of entropy generated during the process would be:**

[IAS-2000]

- (a) 6 kJ/k
- (b) 4 kJ/k
- (c) 2kJ/k
- (d) Zero

**IAS-14. An electric motor of 5 kW is subjected to a braking test for 1 hour. The heat generated by the frictional forces in the process is transferred to the surroundings at 20°C. The resulting entropy change will be:**

[IAS-1998]

- (a) 22.1 kJ/K
- (b) 30.2 kJ/K
- (c) 61.4 kJ/K
- (d) 82.1 kJ/K

## Entropy and Direction: The Second Law a Directional law of Nature

**IAS-15.  $M_1$  kg of water at  $T_1$  is isobarically and adiabatically mixed with  $M_2$  kg of water at  $T_2$  ( $T_1 > T_2$ ). The entropy change of the universe is:**

[IAS-2004]

- (a) Necessarily positive
- (b) Necessarily negative
- (c) Always zero
- (d) Negative or positive but not zero

**IAS-16. In which one of the following processes is there an increase in entropy with no degradation of energy?**

[IAS-1996]

- (a) Polytropic expansion
- (b) Isothermal expansion
- (c) Isochoric heat addition
- (d) Isobaric heat addition

# Entropy

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## Answers with Explanation (Objective)

### Previous 20-Years GATE Answers

GATE-1. Ans. -11858 J/K, 12787 J/K.

GATE-1A Ans. (b)

$$\text{GATE-2. Ans. (b)} \quad w_{1-2} = mRT \ln\left(\frac{v_2}{v_1}\right) = pV \ln\left(\frac{v_2}{v_1}\right) = 200 \times 1 \times \ln\left(\frac{2}{1}\right) \text{ kJ} = 138.6 \text{ kJ}$$

GATE-3. Ans. (c) It is reversible process so  $(\Delta S)_{\text{universe}} = 0$

GATE-4. Ans. (d)

GATE-5. Ans. (d) It is a case of spontaneous process i.e. irreversibility involved that so why entropy change of the universe is positive.

GATE-6. Ans. (c)

$$T_Q = 300 \text{ K}, \quad P_Q = 50 \text{ kPa}$$

$$T_P = 350 \text{ K}, \quad P_P = 150 \text{ kPa}$$

$$S_Q - S_P = c_p \ln \frac{T_Q}{T_P} - R \ln \frac{P_Q}{P_P}$$

$$S_Q - S_P = 1.005 \ln\left(\frac{300}{350}\right) - 0.287 \ln\left(\frac{50}{150}\right)$$

$$S_Q - S_P = 0.160 \text{ kJ/kg-K}$$

GATE-7. Ans. (b) If air has to flow from station P to station Q adiabatically means no entropy change in surroundings, then

$$S_Q - S_P \geq 0$$

$$\Rightarrow c_p \ln \frac{T_Q}{T_P} - R \ln \frac{P_Q}{P_P} \geq 0$$

$$\Rightarrow 1.005 \ln\left(\frac{300}{350}\right) - 0.287 \ln\left(\frac{P_Q}{150}\right) \geq 0$$

$$\Rightarrow -0.287 \ln\left(\frac{P_Q}{150}\right) \geq 0.15492$$

$$\Rightarrow P_Q \leq 87.43$$

So, Ans. is  $P_Q = 87 \text{ kPa}$



$$\Rightarrow -0.15492 - 0.287 \ln\left(\frac{P_Q}{150}\right) \geq 0$$

$$\Rightarrow \ln\left(\frac{P_Q}{150}\right) \leq -0.53979$$

# Entropy

**S K Mondal's**

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## **Previous 20-Years IES Answers**

**IES-1. Ans. (a)** Remember  $dS = \left(\frac{dQ}{T}\right)_{Rev}$

**IES-2. Ans. (b)** The change of entropy may be regarded as a measure of the rate of availability of heat for transformation into work.

**IES-2a Ans. (b)**  $\dot{S}_{gen} = -\frac{1000}{(285+273)} + \frac{492}{(5+273)} = -0.02233 kW/K$

So cycle is impossible Cycle

See in both the case Carnot Cycle and Reversible cycle entropy change of the Universe will be zero.

Irreversible cycle entropy change will be positive.

**IES-3. Ans. (c)**

**IES-4. Ans. (b)**

$$\frac{Q_1}{T_1} = \frac{Q_1}{T_1} = \frac{Q_1 - Q_2}{T_1 - T_2} = \frac{W}{T_1 - T_2}$$

$$Q_1 = 600 \text{ kJ}$$

The entropy change during heat addition

$$= \frac{Q_1}{T_1} = \frac{600}{600} = 1 \text{ kJ/K}$$

**IES-4a. Ans. (b)**

**IES-4b. Ans. (b)**

**IES-5. Ans. (c)**

**IES-6. Ans. (b)**

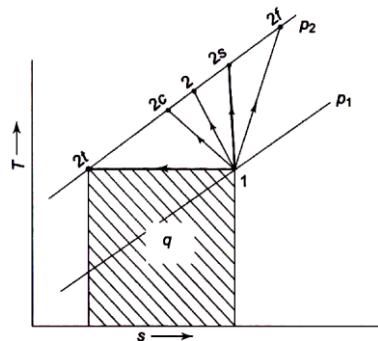
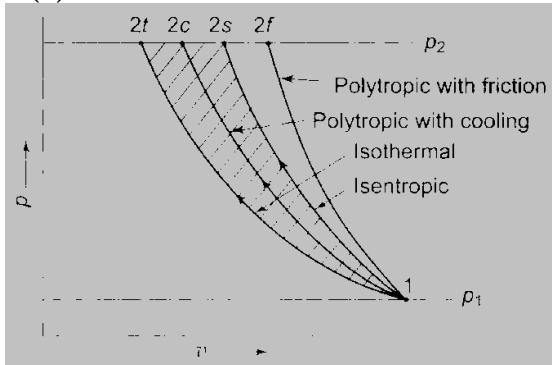
**IES-7. Ans. (b)**

**IES-8. Ans. (c)**

**IES-9. Ans. (d)** AB constant pressure heat addition.

**IES-10. Ans. (c)** XA constant pressure heat rejection. XB = const. temp. heat rejection. XC = isentropic heat rejection.

**IES-11. Ans. (b)**



**IES-12. Ans. (b)**

**IES-13. Ans. (b)**

**IES-14. Ans. (b)**

**IES-15. Ans. (c)**

# Entropy

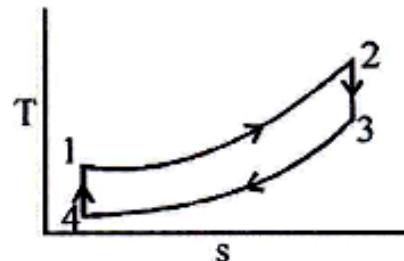
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**IES-16.** Ans. (a)  $\int \frac{dQ}{T} = 0$  does not necessarily mean reversible process. If  $dQ = 0$ .

**IES-17.** Ans. (c) In irreversible heat rejection process entropy decreases. In an irreversible process entropy of the universe always increases i.e. sum of system + surroundings will increase.

Consider the process 3–4 if it is irreversible process then also entropy will decrease.



**IES-18.** Ans. (c) In reversible adiabatic expansion, entropy change is zero and no change in entropy of surroundings.

**IES-19.** Ans. (b) Entropy increase in process =  $100(0.4 - 0.3) = 10 \text{ kJ/kg}$

Entropy change of surroundings =  $5 \text{ kJ/K}$

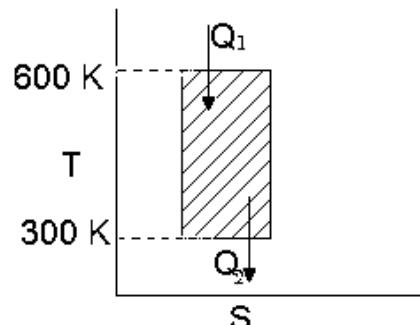
Thus net entropy increases and the process is irreversible.

**IES-20.** Ans. (c) In reversible process entropy change is zero and in four figures it is represented by straight vertical line. However, in irreversible process, entropy increases in all processes (expansion as well as compression).

**IES-21.** Ans. (b)

$$(T_1 - T_2)\Delta S = W$$

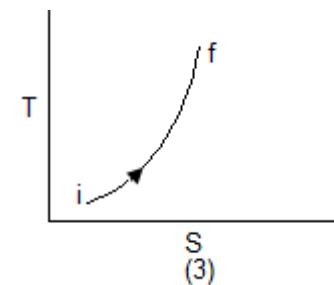
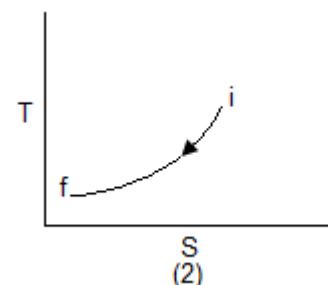
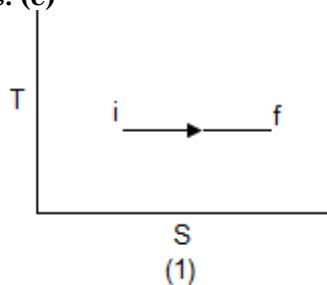
$$\text{or } \Delta S = \frac{300}{600-300} = 1 \text{ kJ/k}$$



**IES-22.** Ans. (c)

$$\text{IES-23. Ans. (d)} \quad \int_{T_1}^T \frac{dQ}{T} = \int_{T_1}^T \frac{\alpha + 2\beta T}{T} dT = \alpha \ln\left(\frac{T_2}{T_1}\right) + 2\beta(T_2 - T_1)$$

**IES-24.** Ans. (c)



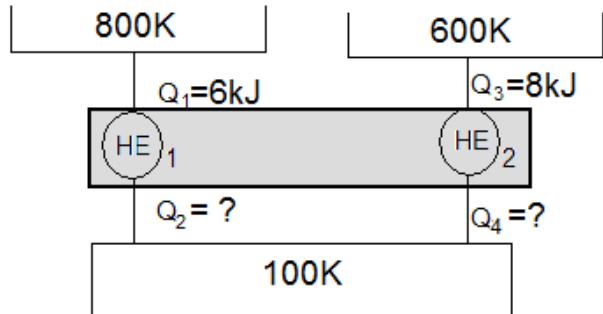
**IES-25.** Ans. (d)

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$$\begin{aligned}\frac{6}{800} &= \frac{Q_2}{100} \\ \frac{8}{600} &= \frac{Q_4}{100} \\ \eta &= 1 - \frac{Q_2 + Q_4}{Q_1 + Q_3} \\ &= 1 - \frac{\frac{6}{100} + \frac{8}{100}}{\frac{6}{800} + \frac{8}{600}} = 0.85\end{aligned}$$



**IES-26. Ans. (d)** Refer to given figure, as given

$$\begin{aligned}\text{Engine work developed} &= 100 \text{ kW} \\ &= 100 \times 1000 \times 60 \\ &= 6 \times 10^6 \text{ J/min.}\end{aligned}$$

$$\begin{aligned}\text{Thus, } Q_s &= \text{total heat supplied} \\ &= 6 \times 10^6 + 3.2 \times 10^6 \\ &= 9.2 \times 10^6 \text{ J/min.}\end{aligned}$$

Let reservoir at 1200 K supply  $Q_{s1}$  J/min.

Therefore reservoir at 800 K will supply.  $Q_{s2} = 9.2 \times 10^6 - Q_{s1}$

Also, by data the engine is a reversible heat engine completing 600 cycles/min. and therefore entropy change after every complete cycle is zero.

$$\text{Thus, } \frac{Q_{s1}}{1200} + \frac{Q_{s2}}{800} - \frac{Q_R}{400} = 0$$

$$\text{or } \frac{Q_{s1}}{1200} + \frac{9.2 \times 10^6 - Q_{s1}}{800} - \frac{6 \times 3.2 \times 10^6}{400} = 0$$

$$\frac{2Q_{s1} + 3(9.2 \times 10^6 - Q_{s1}) - 6 \times 3.2 \times 10^6}{2400} = 0$$

$$\text{or } Q_{s1} = 3 \times 9.2 \times 10^6 - 6 \times 3.2 \times 10^6$$

$$= 8.4 \times 10^6 \text{ J/min}$$

$$Q_{s2} = 9.2 \times 10^6 - 8.4 \times 10^6$$

$$= 0.8 \times 10^6 \text{ J/min}$$

$$\text{Hence ratio} = \frac{8.4}{0.8} = 10.5$$

**IES-27. Ans. (b)** It is isobaric compression.

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	S	T	p	v
Isothermal expansion	↑	—	↓	↑
Isothermal compression	↓	—	↑	↓
Isentropic compression	—	↑	↑	↓
Isentropic expansion	—	↓	↓	↑
Isochoric heating	↑	↑	↑	—
Isochoric cooling	↓	↓	↓	—
Isobaric heating/expansion	↑	↑	—	↑
Isobaric cooling/compression	↓	↓	—	↓

**IES-28. Ans. (d)**

$$\therefore \text{Change in entropy} = 2 MC \log \frac{(T_1 + T_2)}{2\sqrt{T_1 T_2}}$$

In  $\frac{(T_1 + T_2)}{2\sqrt{T_1 T_2}}$ , the term  $\frac{T_1 + T_2}{2}$  is arithmetic mean and  $\sqrt{T_1 T_2}$  is geometric mean. As arithmetic mean is always greater than geometric mean the expression  $\frac{(T_1 + T_2)}{2} - \sqrt{T_1 T_2}$  is always positive. Hence the change in entropy is always positive.

**IES-29 Ans. (d)**

## Previous 20-Years IAS Answers

**IAS-1. Ans. (b)**

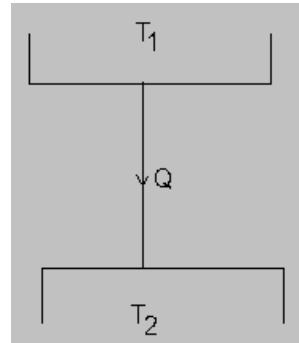
**IAS-2. Ans. (b)**

$$\Delta S_2 = \frac{+Q}{500} = 10$$

$$\text{or } Q = 5000 \text{ kJ}$$

$$\Delta S_1 = \frac{-Q}{1000} = \frac{-5000}{1000} = -5 \text{ kJ/k}$$

∵ Heat added to the system is +ive  
 Heat rejected from the system is -ive



**IAS-3. Ans. (d)**

$$\text{IAS-4. Ans. (d)} \eta = \frac{\text{Workdone}}{\text{Heatadded}} = \frac{\text{area}1-2-3}{\text{areaundercurve}2-3} = \frac{\frac{1}{2} \times (5-1) \times (800-400)}{(5-1) \times 800} = 0.25$$

**IAS-5. Ans. (d)** That so why we are using p-v or T-s diagram.

**IAS-6. Ans. (a)**

**IAS-7. Ans. (b)**

**IAS-8. Ans. (d)**

**IAS-9. Ans. (b)**

# Entropy

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IAS-10(i). Ans. (a)  $\int \frac{dQ}{T} = 0$  does not necessarily mean reversible process. If  $dQ = 0$ .

IAS-10(ii). Ans. (b)  $\eta = \frac{Q - \dot{Q}}{Q} = 1 - \frac{\dot{Q}}{Q} = 1 - \frac{450}{600} = 0.25 = 25\%$

IAS-11. Ans. (c) Throttling is a constant enthalpy expansion process.

IAS-12. Ans. (b)

IAS-13. Ans. (c) Entropy generated  $= ds_{at 400K} - ds_{at 800K} = \frac{dQ}{400} - \frac{dQ}{800} = \frac{1600}{400} - \frac{1600}{800} = 2 \text{ kJ/K}$

IAS-14. Ans. (c)  $\Delta S = \frac{\Delta Q}{T} = \frac{5 \times 3600}{293} \text{ kJ/K} = 61.4 \text{ kJ/K}$

IAS-15. Ans. (a)

IAS-16. Ans. (b)

**5.****Availability, Irreversibility****Theory at a Glance (For GATE, IES & PSUs)**

That part of the low grade energy which is available for conversion is referred to as *available energy*, while the part which, according to the second law, must be rejected, is known as *unavailable energy*.

**Availability**

The availability of a given system is defined as the maximum useful work that can be obtained in a process in which the system comes to equilibrium with the surroundings or attains a dead state. Clearly, the availability of a system depends on the condition of the system as well as those of the surroundings.

**Availability:**

- *Yields the maximum work producing potential or the minimum work requirement of a process*
- *Allows evaluation and quantitative comparison of options in a sustainability context*

Availability = Maximum possible work-Irreversibility

$$W_{\text{useful}} = W_{\text{rev}} - I$$

**Irreversibility**

The actual work done by a system is always less than the idealized reversible work, and the difference between the two is called the irreversibility of the process.

$$I = W_{\max} - W$$

This is also sometimes referred to as '*degradation*' or '*dissipation*'.

**Availability and Irreversibility****1. Available Energy (A. E.)**

$$\begin{aligned} W_{\max} &= Q \left( 1 - \frac{T_0}{T_1} \right) = m c_p \int_{T_0}^{T_1} \left( 1 - \frac{T_0}{T} \right) dT \\ &= (T_1 - T_0) \Delta S \end{aligned}$$

Where  $T_0$  is surroundings temperature.

# Availability, Irreversibility

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$$= u_1 - u_2 - T_0(S_1 - S_2)$$

(For closed system), it is NOT ( $\phi_1 - \phi_2$ ) because change of volume is present there.

$$= h_1 - h_2 - T_0(S_1 - S_2)$$

(For steady flow system), it is ( $A_1 - A_2$ ) as in steady state no change in volume in C.V.

(i.e. change in availability in steady flow)

## 2. Decrease in Available Energy

$$= T_0[\Delta S - \Delta S]$$

Take  $\Delta S$  and  $\Delta S$  both positive quantity.

## 3. Availability function

$$A = h - T_0 S + \frac{C^2}{2} + gz$$

For open system

$$\phi = u - T_0 S + P_0 V$$

For closed system

**Availability = maximum useful work.**

For steady flow

$$\text{Availability} = A_1 - A_0 = (h_1 - h_0) - T_0(S_1 - S_0) + \frac{C_1^2}{2} + gz$$

For closed system

$$\text{Availability} = \phi_1 - \phi_0 = u_1 - u_0 - T_0(S_1 - S_0) + P_0(V_1 - V_0)$$

## 4. Unavailable Energy (U.E.)

$$= T_0(S_1 - S_2)$$

## 5. Increase in unavailable Energy = Loss in availability

$$= T_0(\Delta S)_{\text{Univ}}$$

## 6. Irreversibility

# Availability, Irreversibility

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$$I = W_{\max} - W_{actual}$$

$$= T_0 (\Delta S)_{univ}$$

7. Irreversibility rate =  $\dot{I}$  = rate of energy degradation  $S_{gen} = \int_1^{\infty} m ds$
- = rate of energy loss ( $W_{lost}$ )
- =  $T_0 \times \dot{S}_{gen}$  for all processes
8.  $W_{actual} \Rightarrow dQ = du + dW_{act} \rightarrow$  this for closed system  
 $h_1 + \frac{C_1}{2} + gz_1 + \frac{dQ}{dm} = h_2 + \frac{C_2}{2} + gz_2 + \frac{dW_{act}}{dm} \rightarrow$  this for steady flow
9. Helmholtz function

$$\mathbf{F} = \mathbf{U} - \mathbf{TS}$$

10. Gibb's function

$$\mathbf{G} = \mathbf{H} - \mathbf{TS}$$

11. Entropy generation number ( $N_s$ )

$$N_s = \frac{\dot{S}_{gen}}{\dot{m} C_p}$$

12. To calculate  $dS$

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$$(i) \text{ Use } S_2 - S_1 = m \left[ C_v \ln \frac{P_2}{P_1} + C_p \ln \frac{V_2}{V_1} \right]$$

For closed system

$$\begin{aligned} Tds &= du + PdV \\ \text{or} \quad ds &= mC_v \frac{dT}{T} + \frac{P}{T} dN \\ &= mC_v \frac{dT}{T} + mR \frac{dN}{V} \\ \int ds &= mC_v \int \frac{dT}{T} + mR \int \frac{dN}{V} \end{aligned}$$

For steady flow system

$$\begin{aligned} Tds &= dh - VdP \\ \text{or} \quad ds &= mC_p \frac{dT}{T} - \frac{V}{T} dP \quad PV = mRT \\ \int ds &= mC_p \int \frac{dT}{T} - mR \int \frac{dP}{P} \quad \frac{V}{T} = \frac{mR}{P} \end{aligned}$$

But note that

$$\text{and } \begin{cases} Tds = du + PdV \\ Tds = dh - VdP \end{cases} \text{ Both valid for closed system only.}$$

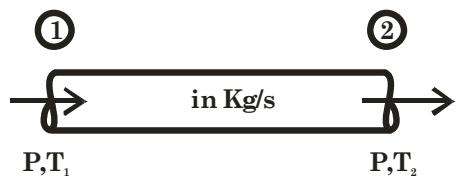
## 13. In Pipe flow Entropy generation rate

$$\begin{aligned} \dot{S}_{gen} &= \dot{S}_{sys} - \frac{\dot{Q}}{T_0} \\ &= \dot{m}(S_2 - S_1) - \frac{\dot{m}C_p(T_2 - T_1)}{T_0} \end{aligned}$$

Due to lack of insulation it may be  $T_1 > T_2$  for hot fluid

or  $T_1 < T_2$  for cold fluid

$$\therefore \text{rate of irreversibility } (\dot{I}) = T_0 \dot{S}_{gen}$$



## 14. Flow with friction

$$\text{Decrease in availability} = \dot{m}RT_0 \frac{\Delta P}{P_1}$$

# Availability, Irreversibility

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## 15. Second Law efficiency

$$\eta_{II} = \frac{\text{Min}^n. \text{exergy intake to perform the given task } (X_{min})}{\text{Actual exergy intake to perform the given task } (X)} = \eta_I / \eta_{carnot}$$
$$X_{min} = W \text{ if work is involved}$$
$$= Q \left( 1 - \frac{T_0}{T} \right) \text{ if Heat is involved.}$$

A common measure on energy use efficiency is the first law efficiency,  $\eta_I$ . The first law efficiency is defined as the ratio of the output energy of a device to the input energy of the device. The first law is concerned only with the quantities of energy, and disregards the forms in which the energy exists. It does not also discriminate between the energies available at different temperatures. It is the second law of thermodynamics which provides a means of assigning a quality index to energy. The concept of available energy or energy provides a useful measure of energy quality.

With this concept it is possible to analyze means of minimizing the consumption of available energy to perform a given process, thereby ensuring the most efficient possible conversion of energy for the required task.

The second law efficiency,  $\eta_{II}$ , of a process is defined as the ratio of the minimum available energy (or energy) which must be consumed to do a task divided by the actual amount of available energy (or energy) consumed in performing the task.

$$\eta_{II} = \frac{\text{minimum exergy intake to perform the given task}}{\text{actual exergy intake to perform the same task}}$$

$$\text{or } \eta_{II} = \frac{A_{min}}{A}$$

where  $A$  is the availability or energy.

A power plant converts a fraction of available energy  $A$  or  $W_{max}$  to useful work  $W$ .

For the desired output of  $W$ ,  $A_{min} = W$  and  $A = W_{max}$ , Here,

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$$I = W_{\max} - W \text{ and } \eta_{II} = \frac{W}{W_{\max}}$$

Now  $\eta_{II} = \frac{W}{Q} = \frac{W}{W_{\max}} \cdot \frac{W_{\max}}{Q}$

$$= \eta_{II} \cdot \eta_{Carot}$$

$\therefore \eta_{II} = \frac{\eta_{II}}{\eta_{Carot}}$

Since  $W_{\max} = Q(1 - \frac{T_0}{T})$ , can also be obtained directly as follows

$$\eta_{II} = \frac{W}{Q\left(1 - \frac{T_0}{T}\right)} = \frac{\eta_{II}}{\eta_{Carot}}$$

If work is involved,  $A_{min} = W$  (desired) and if heat is involved,  $A_{min} = Q\left(1 - \frac{T_0}{T}\right)$ .

The general definition of second law efficiency of a process can be obtained in terms of change in availability during the process:

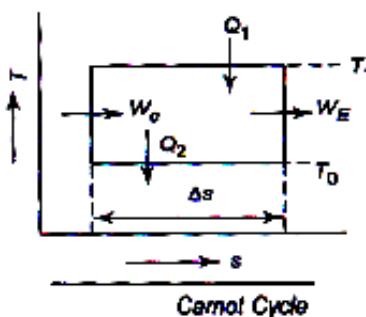
$$\text{Second Law Efficiency} = \eta_{II, \text{Law}} = \frac{\text{Production of availability}}{\text{Destruction of availability}}.$$

## Available Energy Referred to a Cycle

- Decrease in Available Energy when Heat is Transferred through a Finite Temperature Deference :-

Whenever heat is transferred through finite temperature deference, there is a decrease in the availability of energy so transferred.

Let us consider a reversible heat engine operating between  $T_1$  and  $T_0$  (Fig. shown in below) Then



$$Q_1 = T_1 \Delta S, Q_2 = T_0 \Delta S,$$

and  $W = A.E. = (T_1 - T_0) \Delta s$

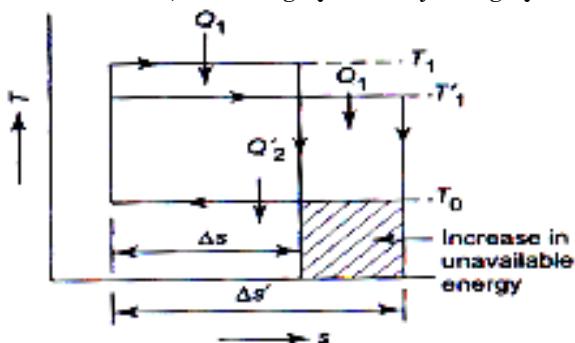
Let us now assume that heat  $Q_1$  is transferred through a finite temperature difference from the reservoir or source at  $T_1$  to the engine absorbing heat at  $T'_1$ , lower turn  $T_1$  (Fig. shown in below).

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The availability of  $Q_1$  as received by the engine at  $T'_1$  can be found by allowing the engine to operate reversible in a cycle between  $T'_1$  and  $T_0$ , receiving  $Q_1$  and rejecting  $Q'_2$ .



Increase in Unavailable Energy Due to Heat Transfer Through a Finite Temperature Difference

$$\text{Now } Q = T_1 \Delta s = T_1 \Delta s'$$

$$\text{Since } T_1 > T'_1, \therefore \Delta s' > \Delta s$$

$$Q_2 = T_0 \Delta s$$

$$Q'_2 = T_0 \Delta s'$$

$$\text{Since } \Delta s' > \Delta s \therefore Q'_2 > Q_2$$

$$\therefore W = Q - Q'_2 = T_1 \Delta s' - T_0 \Delta s'$$

$$\text{and } W = Q - Q_2 = T_1 \Delta s - T_0 \Delta s$$

$$\therefore W < W, \text{ because } Q'_2 > Q_2$$

Available energy lost due to irreversible heat transfer through finite temperature difference between the source and the working fluid during the heat addition process is given by

$$W - W' = Q'_2 - Q_2$$

$$= T_0(\Delta s' - \Delta s)$$

or decrease in A.E. =  $T_0(\Delta s' - \Delta s)$

The decrease in available energy or exergy is thus the product of the lowest feasible temperature of heat rejection and the additional entropy change in the system while receiving heat irreversibly, compared to the case of reversible heat transfer from the same source. The greater is the temperature difference ( $T_1 - T'_1$ ), the greater is the heat rejection  $Q'_2$  and the greater will be the unavailable part of the energy supplied or energy (Fig. above). Energy is said to be degraded each time it flows through a finite temperature difference.

## PROBLEMS & SOLUTIONS

### Example 1

What is the maximum useful work which can be obtained when 100 kJ are abstracted from a heat reservoir at 675 K in an environment at 288 K? What is the loss of useful work if, (a) a temperature drop of 50°C is introduced between the heat source and the heat engine, on the one hand and the heat engine and the heat sink, on the other. (b) the source temperature drops by 50°C

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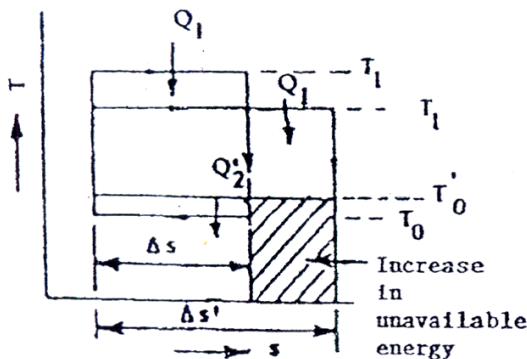
and the sink temperature rises by  $50^\circ\text{C}$  during the heat transfer process according to the linear law

$$\frac{dQ}{dT} = \pm \text{constant?}$$

**Solution:**

Availability =

$$W_{\max} = \left(1 - \frac{T_o}{T_1}\right) Q = \left(1 - \frac{288}{675}\right) \times 100 = 57.33 \text{ kJ}$$

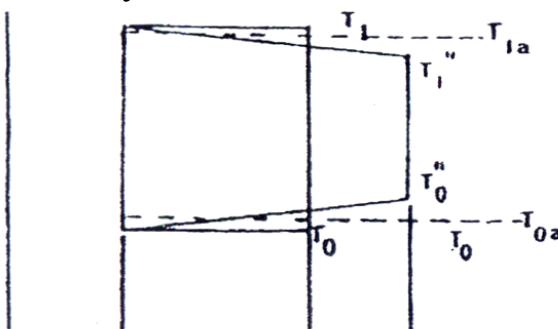


(Fig.)

$$\Delta S' = \frac{Q}{T_1} = \frac{100}{625} = 0.16 \text{ kJ/K}$$

$$W = \Delta S'(T_1 - T_0) = 0.16(625 - 338) = 45.92 \text{ kJ}$$

$$\text{Loss in availability} = 57.33 - 45.92 = 11.41 \text{ kJ.}$$



(Fig.)

Average temperature of source

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$$T_{1a} = \frac{T_1 + T'_1}{2} = 650K$$

$$\text{Average temperature of sink } (T_{0a}) = \frac{T_0 + T'_1}{2} = 313K$$

$$\Delta S'' = \frac{Q}{T_{1a}} = \frac{100}{650} = 0.1538 kJ/K$$

$$W'' = \Delta S''(650 - 313) = 51.83 kJ/K$$

$$\text{Loss in availability} = 57.33 - 51.83 = 5.5 kJ.$$

### Example 2.

A lead storage battery used in an automobile is able to deliver 5.2 MJ of electrical energy. This energy is available for starting the car.

Let compressed air be considered for doing an equivalent amount of work in starting the car. The compressed air is to be stored at 7 MPa, 25°C, what is the volume of the tank that would be required to let the compressed air having an availability of 5.2 MJ? For air  $p v = 0.287 T$ , where T is in K, p in kPa, and v in m<sup>3</sup>/kg.

**Solution:**

Availability is given by

$$a = (u - u_o) - T_o(s - s_o) + p_o(v - v_o)$$

$$v = \frac{0.287 \times 298}{7000} = 0.012218 \text{ m}^3/\text{kg}$$

$$v_o = \frac{0.287 \times 298}{100} = 0.85526 \text{ m}^3/\text{kg}$$

$$a = 0 - 298 \left( 0.287 \ln \frac{7000}{298} - 0.287 \ln \frac{7000}{100} \right) + 100(0.012218 - 0.85526) \\ = 279 \text{ kJ/kg}$$

$$\text{For availability of 5.2 MJ, } m = \frac{5200}{279} = 18.635 \text{ kg}$$

$$V = mv = 18.635 \times 0.012218 = 0.2277 \text{ m}^3$$

### Example 3.

Air enters a compressor in steady flow at 140 kPa, 17°C and 70 m/s and leaves it at 350 kPa, 127°C and 110 m/s. The environment is at 100 kPa, 7°C. Calculate per kg of air (a) the actual amount of work required, (b) the minimum work required, and (c) the irreversibility of the process.

**Solution:**

This is the case of a steady flow system in which the maximum useful work is given by

$$W_{\max \text{ useful}} = \phi_1 - \phi_2 + \frac{C_1^2 - C_2^2}{2}$$

$$= (H_1 - T_o S_1) - (H_2 - T_o S_2) + \frac{C_1^2 - C_2^2}{2}$$

As air is to be considered as a perfect gas,  $h = C_p T$

Hence

$$W_{\max \text{ useful}} = C_p (T_1 - T_2) - T_o (S_1 - S_2) + \frac{C_1^2 - C_2^2}{2}$$

In the above, expression  $(S_1 - S_2)$  is the entropy change. For a perfect gas, when pressure and temperature vary, the entropy change is given by the relation

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$$S_1 - S_2 = C_p \ln \frac{T_1}{T_2} - R \ln \frac{P_1}{P_2}$$

Hence

$$W_{\max \text{ useful}} = C_p(T_1 - T_2) - T_o \left( C_p \ln \frac{T_1}{T_2} - R \ln \frac{P_1}{P_2} \right) + \frac{C_2^2 - C_1^2}{2}$$

Various parameters given are:

$T_1 = 290 \text{ K}$ ;  $T_2 = 400 \text{ K}$ ;  $C_p = 1.005 \text{ kJ/kg K}$

$P_1 = 140 \text{ kPa}$ ;  $P_2 = 350 \text{ kPa}$

$G_1 = 70 \text{ m/s}$ ;  $G_2 = 110 \text{ m/s}$ ;  $T_o = 280 \text{ K}$

(a) Actual work done in case of a steady flow process is given by

$$\begin{aligned} W_{\text{actual}} &= (H_1 - H_2) + \frac{C_2^2 - 110^2}{2} \\ &= 1.005(290 - 400) + \frac{70^2 - 110^2}{2 \times 10^3} = -114.1 \text{ kJ} \end{aligned}$$

The negative sign obviously indicates that work is done on the air in compression.

$$\begin{aligned} \text{(b) Minimum work} &= 1.005(290 - 400) - 280 \times \left[ 1.005 \ln \frac{290}{400} - 0.287 \ln \frac{140}{350} \right] + \frac{70^2 - 110^2}{2 \times 10^3} \\ &= -97.3 \text{ kJ} \end{aligned}$$

$$\text{(c) Irreversibility} = 114.1 - 97.3 = 16.8 \text{ kJ.}$$

#### Example 4.

By how much is the available energy of 5 kg of air increased by heating reversibly at a constant pressure of 1.5 atm. from  $27^\circ\text{C}$  to  $227^\circ\text{C}$  with the lowest available temperature of  $20^\circ\text{C}$ ? ( $C_p$  for air =  $1005 \text{ J/kg}^\circ\text{C}$ ).

**Solution:**

$$\begin{aligned} \text{Change in entropy} &= m C_p \ln \frac{T_2}{T_1} \\ &= 5 \times 1.005 \times \ln \frac{500}{300} = 2.567 \text{ kJ/K} \end{aligned}$$

$$\text{Availability} = T_0 \Delta S = 293 \times 2.567 = 752 \text{ kJ.}$$

# Availability, Irreversibility

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## ASKED OBJECTIVE QUESTIONS (GATE, IES, IAS)

### Previous 20-Years GATE Questions

- GATE-1.** A steel billet of 2000 kg mass is to be cooled from 1250 K to 450 K. The heat released during this process is to be used as a source of energy. The ambient temperature is 303 K and specific heat of steel is 0.5 kJ/kg K. The available energy of this billet is: [GATE-2004]  
 (a) 490.44 MJ                          (b) 30.95 MJ                          (c) 10.35 MJ                          (d) 0.10 MJ

## Availability

- GATE-2.** Availability of a system at any given state is: [GATE-2000]  
 (a) A property of the system  
 (b) The maximum work obtainable as the system goes to dead state  
 (c) The total energy of the system  
 (d) The maximum useful work obtainable as the system goes to dead state
- GATE-3.** A heat reservoir at 900 K is brought into contact with the ambient at 300 K for a short time. During this period 9000 kJ of heat is lost by the heat reservoir. The total loss in availability due to this process is: [GATE-1995]  
 (a) 18000 kJ                            (b) 9000 kJ                            (c) 6000 kJ                            (d) None of the above

## Irreversibility

- GATE-4.** Consider the following two processes: [GATE-2010]  
 a. A heat source at 1200 K loses 2500 kJ of heat to sink at 800 K  
 b. A heat source at 800 K loses 2000 kJ of heat to sink at 500 K  
**Which of the following statements is TRUE?**  
 (a) Process I is more irreversible than Process II  
 (b) Process II is more irreversible than Process I  
 (c) Irreversibility associated in both the processes is equal  
 (d) Both the processes are reversible

### Previous 20-Years IES Questions

## Available Energy

- IES-1.** What will be the loss of available energy associated with the transfer of 1000 kJ of heat from constant temperature system at 600 K to another at 400 K when the environment temperature is 300 K? [IES-2004, IES-2012]  
 (a) 150 kJ                              (b) 250 kJ                              (c) 500 kJ                              (d) 700 kJ

# Availability, Irreversibility

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- IES-2.** What is the loss of available energy associated with the transfer of 1000 kJ of heat from a constant temperature system at 600 K to another at 400 K when the environmental temperature is 300 K? [IES-2008]  
 (a) 150 kJ                  (b) 250 kJ                  (c) 166.67 kJ                  (d) 180kJ

## Available Energy Referred to a Cycle

- IES-3.** A heat source  $H_1$  can supply 6000kJ/min. at 300°C and another heat source  $H_2$  can supply 60000 kJ/min. at 100°C. Which one of the following statements is correct if the surroundings are at 27°C? [IES-2006]  
 (a) Both the heat sources have the same efficiency  
 (b) The first heat source has lower efficiency  
 (c) The second heat source has lower efficiency  
 (d) The first heat source produces higher power

## Availability

- IES-4.** Assertion (A): The change in availability of a system is equal to the change in the Gibbs function of the system at constant temperature and pressure.  
 Reason (R): The Gibbs function is useful when evaluating the availability of systems in which chemical reactions occur. [IES-2006]  
 (a) Both A and R are individually true and R is the correct explanation of A  
 (b) Both A and R are individually true but R is NOT the correct explanation of A  
 (c) A is true but R is false  
 (d) A is false but R is true

- IES-5.** For a steady flow process from state 1 to 2, enthalpy changes from  $h_1 = 400$  kJ/kg to  $h_2 = 100$  kJ/kg and entropy changes from  $s_1 = 1.1$  kJ/kg-K to  $s_2 = 0.7$  kJ/kg-K. Surrounding environmental temperature is 300 K. Neglect changes in kinetic and potential energy. The change in availability of the system is: [IES-2003]  
 (a) 420 kJ/kg                  (b) 300 kJ/kg                  (c) 180 kJ/kg                  (d) 90 kJ/kg

- IES-6.** Availability function for a closed system is expressed as: [IES-2002]  
 (a)  $\phi = u + p_o v - T_o S$                   (b)  $\phi = du + p_o dv - T_o ds$   
 (c)  $\phi = du + p_o dv + T_o ds$                   (d)  $\phi = u + p_o v + T_o S$

- IES-7.** Consider the following statements: [IES-2001]  
 1. Availability is the maximum theoretical work obtainable.  
 2. Clapeyron's equation for dry saturated steam is given by

$$(V_g - V_f) = \frac{dT_s}{dQ} \left[ \frac{h_g - h_f}{T_s} \right]$$

3. A gas can have any temperature at a given pressure unlike a vapour which has a fixed temperature at a given pressure.  
 4. Joule Thomson coefficient is expressed as  $\mu = \left[ \frac{\partial s}{\partial p} \right]_h$

Of these statements

# Availability, Irreversibility

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- |                            |                            |
|----------------------------|----------------------------|
| (a) 1, 2 and 3 are correct | (b) 1, 3 and 4 are correct |
| (c) 2 and 3 are correct    | (d) 1, 2 and 4 are correct |

**IES-8.** 10kg of water is heated from 300 K to 350 K in an insulated tank due to churning action by a stirrer. The ambient temperature is 300 K. In this context, match List-I and List-II and select the correct answer using the codes given below the Lists: [IES-2000]

List-I				List-II			
A. Enthalpy change				1. 12.2 kJ/kg			
B. Entropy change/kg				2. 1968 kJ			
C. Availability/kg				3. 2090 kJ			
D. Loss of availability				4. 656 J/kg-k			
Codes:							
(a) 3				(b) 2			
(c) 3				(d) 2			

**IES-9.** Neglecting changes in kinetic energy and potential energy, for unit mass the availability in a non-flow process becomes  $a = \phi - \phi_0$ , where  $\phi$  is the availability function of the [IES-1998]

- |                     |                         |
|---------------------|-------------------------|
| (a) Open system     | (b) Closed system       |
| (c) Isolated system | (d) Steady flow process |

**IES-10.** Consider the following statements: [IES-1996]

1. Availability is generally conserved
2. Availability can either be negative or positive
3. Availability is the maximum theoretical work obtainable
4. Availability can be destroyed in irreversibility

Of these correct statements are:

(a) 3 and 4	(b) 1 and 2	(c) 1 and 3	(d) 2 and 4
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## Irreversibility

**IES-11.** The irreversibility is defined as the difference of the maximum useful work and actual work:  $I = W_{max,useful} - W_{actual}$ . How can this be alternatively expressed?

- |                                                                         |                                                                         |
|-------------------------------------------------------------------------|-------------------------------------------------------------------------|
| (a) $I = T_o(\Delta S_{system} + \Delta S_{surrounding})$               | (b) $I = T_o(\Delta S_{system} - \Delta S_{surrounding})$ [IES-2005]    |
| (c) $I = T_o(\sqrt{\Delta S_{system}} + \sqrt{\Delta S_{surrounding}})$ | (d) $I = T_o(\sqrt{\Delta S_{system}} - \sqrt{\Delta S_{surrounding}})$ |

**IES-12.** Assertion (A): All constant entropy processes are adiabatic, but all adiabatic processes are not isentropic. [IES-2006]

Reason (R): An adiabatic process which resists the exchange of energy to the surroundings may have irreversibility due to friction and heat conduction.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is NOT the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

**IES-13.** Which of the following statement is incorrect? [IES-1992]

- (a) The greater the pressure difference in throttling the lesser the irreversibility

# Availability, Irreversibility

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- (b) The primary source of internal irreversibility in power is fluid friction in rotary machines.  
(c) The greater the irreversibility, the greater the increase in adiabatic process  
(d) The entropy of the universe is continually on the increase.

### Previous 20-Years IAS Questions

## Available Energy

- IAS-1.** What will be the loss of available energy associated with the transfer of 1000 kJ of heat from constant temperature system at 600 K to another at 400 K when the environment temperature is 300 K? [IAS-1995]  
(a) 150 kJ                   (b) 250 kJ                   (c) 500 kJ                   (d) 700 kJ
- IAS-2.** An inventor claims that heat engine has the following specifications: [IAS-2002]  
Power developed = 50 kW;                                 Fuel burned per hour = 3 kg  
Heating value of fuel = 75,000 kJ per kg;           Temperature limits = 627°C and 27°C  
Cost of fuel = Rs. 30/kg;                                 Value of power = Rs. 5/kWh  
(a) Possible                         (b) Not possible                         (c) Economical                         (d) Uneconomical
- IAS-3.** For a reversible power cycle, the operating temperature limits are 800 K and 300 K. It takes 400 kJ of heat. The unavailable work will be: [IAS-1997]  
(a) 250 kJ                           (b) 150 kJ                           (c) 120 kJ                           (d) 100 kJ

## Quality of Energy

- IAS-4.** Increase in entropy of a system represents [IAS-1994]  
(a) Increase in availability of energy                   (b) Increase in temperature  
(c) Decrease in pressure                                 (d) Degradation of energy

## Availability

- IAS-5.** If  $u$ ,  $T$ ,  $v$ ,  $s$ , and  $p$  refer to internal energy, temperature, volume, entropy, enthalpy and pressure respectively; and subscript 0 refers to environmental conditions, availability function for a closed system is given by: [IAS-2003]  
(a)  $u + P_0 v - T_0 s$       (b)  $u - P_0 v + T_0 s$       (c)  $h + P_0 v - T_0 s$       (d)  $h - P_0 v + T_0 s$
- IAS-6.** Match List-I with List-II and select the correct answer using the codes given below the lists:

List-I	List-II			
A. Irreversibility	1. Mechanical equivalent			
B. Joule Thomson experiment	2. Thermodynamic temperature scale			
C. Joule's experiment	3. Throttling process			
D. Reversible engines	4. Loss of availability			
<b>Codes:</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a)	1	2	3	4
(c)	4	3	2	1
	(b)	1	2	4
	(d)	4	3	1
				2

# Availability, Irreversibility

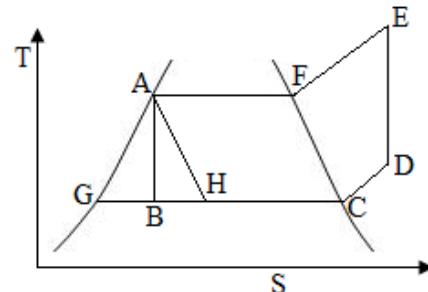
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## Irreversibility

IAS-7. The loss due to irreversibility in the expansion valve of a refrigeration cycle shown in the given figure is represented by the area under the line.

- (a) GB
- (b) AG
- (c) AH
- (d) BH



[IAS-1999]

IAS-8. Assertion (A): When a gas is forced steadily through an insulated pipe containing a porous plug, the enthalpy of gas is the same on both sides of the plug. [IAS-1997]

Reason (R): The gas undergoes an isentropic expansion through the porous plug.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is NOT the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

## Second Law efficiency

IAS-9. Assertion (A): The first-law efficiency evaluates the energy quantity utilization, whereas the second-law efficiency evaluates the energy quality utilization.

Reason (R): The second-law efficiency for a process is defined as the ratio of change of available energy of the source to the change of available energy of the system. [IAS-1998]

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is NOT the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

## **Answers with Explanation (Objective)**

### **Previous 20-Years GATE Answers**

$$\begin{aligned} \text{GATE-1. Ans. (a)} \quad AE &= \int_{T_1}^{T_2} mc_p \left(1 - \frac{T_0}{T}\right) dT = \int_{T_1}^{T_2} mc_p \left[ (T_2 - T_1) - T_0 \ln\left(\frac{T_2}{T_1}\right) \right] \\ &= 2000 \times 0.5 \left[ (1250 - 450) - 303 \ln\left(\frac{1250}{450}\right) \right] = 490 \text{ MJ} \end{aligned}$$

**GATE-2. Ans. (d)** Maximum useful work, i.e. total work minus pdv work. Not maximum work.

**GATE-3. Ans. (d)** The availability of a thermal reservoir is equivalent to the work output of a Carnot heat engine operating between the reservoir and the environment. Here as there is no change in the temperatures of source (reservoir) or the sink (atmosphere), the initial and final availabilities are same. Hence, there is no loss in availability.

**GATE-4. Ans. (b)**

### **Previous 20-Years IES Answers**

$$\text{IES-1. Ans. (b)} \quad \text{Loss of available energy} = T_0 \times (\Delta S)_{\text{univ.}} = 300 \left\{ \frac{1000}{400} - \frac{1000}{600} \right\} \text{ kJ} = 250 \text{ kJ}$$

$$\text{IES-2. Ans. (b)} \quad \text{Loss of Availability} = T_0 \Delta S$$

$$T_0 = 300 \text{ K}$$

$$\Delta S = \frac{Q}{T_1} - \frac{Q}{T_2}$$

$$\therefore \text{Loss of Availability} = 300 \left( \frac{1000}{400} - \frac{1000}{600} \right) = 250 \text{ kJ}$$

$$\text{IES-3. Ans. (c)} \quad \eta = 1 - \frac{T_{\text{source}}}{T_{\text{surroundings}}} \quad \therefore \eta > \eta_b$$

**IES-4. Ans. (b)**

$$\text{IES-5. Ans. (c)} \quad UE = T_0 (s_i - s_2) = 300 \times (1.1 - 0.7) = 120 \text{ kJ/kg}$$

$$\text{Change in availability} = (h_i - h_2) - (UE) = (400 - 100) - 120 = 180 \text{ kJ/kg}$$

**IES-6. Ans. (a)**

**IES-7. Ans. (c)** The availability of a given system is defined as the maximum useful work that can be obtained in a process in which the system comes to equilibrium with the surroundings or attains a dead state. Clearly, the availability of a system depends on the condition of the system as well as those of the surroundings.

# Availability, Irreversibility

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Availability:

- Yields the maximum work producing potential or the minimum work requirement of a process
- Allows evaluation and quantitative comparison of options in a sustainability context

Availability = Maximum possible work-Irreversibility

$$W_{\text{useful}} = W_{\text{rev}} - I$$

**IES-8. Ans. (c)**

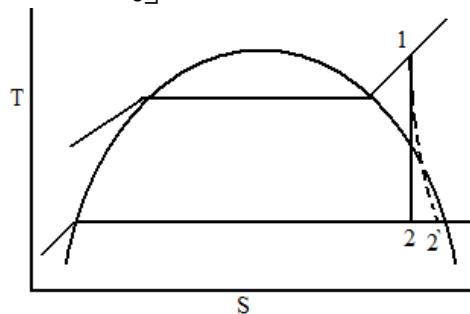
**IES-9. Ans. (a)**

**IES-10. Ans. (a)** Availability is the maximum theoretical work obtainable and it can be destroyed in irreversibility.

**IES-11. Ans. (a)**  $I = T_0 \times (\Delta S)_{\text{universe}} = T_0 \times [\Delta S_{\text{system}} + \Delta S_{\text{surrounding}}]$

**IES-12. Ans. (d)** A is false, For a process due to irreversibility entropy will increase and actual process may be 1-2' but due to heat loss to the surroundings, may 2' coincide with 2 but the process not adiabatic. So, all isentropic process is not adiabatic.

**IES-13. Ans. (a)**



## Previous 20-Years IAS Answers

**IAS-1. Ans. (b)** Loss of available energy  $= T_0 \times (\Delta S)_{\text{univ.}} = 300 \left\{ \frac{1000}{400} - \frac{1000}{600} \right\} \text{kJ} = 250 \text{kJ}$

**IAS-2. Ans. (b)** Maximum possible efficiency ( $\eta_{\text{max}}$ )  $= 1 - \frac{T_2}{T_1} = 1 - \frac{300}{900} = \frac{2}{3}$

Maximum possible Power output with this machine

$$(W_{\text{max}}) = Q \times \eta_{\text{max}} = \frac{3 \times 75000}{3600} \times \frac{2}{3} \text{kW} = 41.67 \text{ KW}$$

So above demand is impossible.

**IAS-3. Ans. (b)** Available part of the heat ( $W_E$ )  $= Q \left( 1 - \frac{T_2}{T_1} \right) = 400 \left( 1 - \frac{300}{800} \right) = 250 \text{ kJ}$

$$\text{Unavailable work } (W_u) = 400 - 250 = 150 \text{ kJ}$$

$$= 2000 \times 0.5 \left[ (1250 - 450) - 303 \ln \left( \frac{1250}{450} \right) \right] = 490 \text{ MJ}_s$$

**IAS-4. Ans. (d)**

**IAS-5. Ans. (a)**

**IAS-6. Ans. (d)**

# Availability, Irreversibility

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**IAS-7. Ans. (d)** Entropy will increase in the process AH is BH.

Therefore Irreversibility ( $I$ ) =  $T_o \times \Delta S$  i.e. area under the line BH.

**IAS-8. Ans. (c)** Expansion through the porous plug is adiabatic as no heat added or rejected to the system. It is not reversible, due to large irreversibility entropy increases so it is not an isentropic process.

**IAS-9. Ans. (c)**  $\eta_l = \frac{\text{minimum energy intake to perform the given task}}{\text{actual energy intake to perform the same task}}$

**6.****Thermodynamic Relations****Theory at a Glance (For GATE, IES & PSUs)**

Adiabatic index ( $\gamma$ ) =  $1 + \frac{2}{N}$  where N is degrees of freedom of molecules

N=3 for monatomic gas

N=5 for diatomic gas

N=6 for try atomic gas.

**Some Mathematical Theorem**

**Theorem 1.** If a relation exists among the variables x, y and z, then z may be expressed as a function of x and y, or

$$dz = \left( \frac{\partial z}{\partial x} \right)_y dx + \left( \frac{\partial z}{\partial y} \right)_x dy$$

then  $dz = M dx + N dy$ .

where z, M and N are functions of x and y. Differentiating M partially with respect to y, and N with respect to x.

$$\left( \frac{\partial M}{\partial y} \right)_x = \frac{\partial^2 z}{\partial x \partial y}$$

$$\left( \frac{\partial N}{\partial x} \right)_y = \frac{\partial^2 z}{\partial y \partial x}$$

$$\left( \frac{\partial M}{\partial y} \right)_x = \left( \frac{\partial N}{\partial x} \right)_y$$

This is the condition of exact (or perfect) differential.

**Theorem 2.** If a quantity f is a function of x, y and z, and a relation exists among x, y and z, then f is a function of any two of x, y and z. Similarly any one of x, y and z may be regarded to be a function of f and any one of x, y and z. Thus, if

$$dx = \left( \frac{\partial x}{\partial f} \right)_y df + \left( \frac{\partial x}{\partial y} \right)_f dy$$

Similarly, if

$$dy = \left( \frac{\partial y}{\partial f} \right)_z df + \left( \frac{\partial y}{\partial z} \right)_f dz$$

Substituting the expression of dy in the preceding equation

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$$dx = \left( \frac{\partial x}{\partial f} \right)_y df + \left( \frac{\partial x}{\partial y} \right)_f \left[ \left( \frac{\partial y}{\partial f} \right)_z df + \left( \frac{\partial y}{\partial z} \right)_f dz \right] \\ = \left[ \left( \frac{\partial x}{\partial f} \right)_y + \left( \frac{\partial x}{\partial y} \right)_f \left( \frac{\partial y}{\partial f} \right)_z \right] df + \left( \frac{\partial x}{\partial y} \right)_f \left( \frac{\partial y}{\partial z} \right)_f dz$$

Again

$$dx = \left( \frac{\partial x}{\partial f} \right)_z df + \left( \frac{\partial x}{\partial z} \right)_f dz \\ \left( \frac{\partial x}{\partial z} \right)_f = \left( \frac{\partial x}{\partial y} \right)_f \left( \frac{\partial y}{\partial z} \right)_f \\ \left( \frac{\partial x}{\partial y} \right)_f \left( \frac{\partial y}{\partial z} \right)_f \left( \frac{\partial z}{\partial x} \right)_f = 1$$

**Theorem 3.** Among the variables x, y, and z any one variable may be considered as a function of the other two. Thus

$$x = x(y, z)$$

$$dx = \left( \frac{\partial x}{\partial y} \right)_z dy + \left( \frac{\partial x}{\partial z} \right)_y dz$$

Similarly,

$$dz = \left( \frac{\partial z}{\partial x} \right)_y dx + \left( \frac{\partial z}{\partial y} \right)_x dy \\ dx = \left( \frac{\partial x}{\partial y} \right)_z dy + \left( \frac{\partial x}{\partial z} \right)_y \left[ \left( \frac{\partial z}{\partial x} \right)_y dx + \left( \frac{\partial z}{\partial y} \right)_x dy \right] \\ = \left[ \left( \frac{\partial x}{\partial y} \right)_z + \left( \frac{\partial x}{\partial z} \right)_y \left( \frac{\partial z}{\partial y} \right)_x \right] dy + \left( \frac{\partial x}{\partial z} \right)_y \left( \frac{\partial z}{\partial x} \right)_y dx \\ = \left[ \left( \frac{\partial x}{\partial y} \right)_z + \left( \frac{\partial x}{\partial z} \right)_y \left( \frac{\partial z}{\partial y} \right)_x \right] dy + dx \\ \therefore \left( \frac{\partial x}{\partial y} \right)_z + \left( \frac{\partial z}{\partial y} \right)_x \left( \frac{\partial x}{\partial z} \right)_y = 0 \\ \left( \frac{\partial x}{\partial y} \right)_z \left( \frac{\partial z}{\partial x} \right)_y \left( \frac{\partial y}{\partial z} \right)_x = -1$$

Among the thermodynamic variables p, V and T. The following relation holds good

$$\left( \frac{\partial p}{\partial V} \right)_T \left( \frac{\partial V}{\partial T} \right)_p \left( \frac{\partial T}{\partial p} \right)_V = -1$$

**Maxwell's Equations:-**

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A pure substance existing in a single phase has only two independent variables. Of the eight quantities  $p$ ,  $V$ ,  $T$ ,  $S$ ,  $U$ ,  $H$ ,  $F$  (Helmholtz function), and  $G$  (Gibbs function) any one may be expressed as a function of any two others.

For a pure substance undergoing an infinitesimal reversible process

- (a)  $dU = TdS - pdV$
- (b)  $dH = dU + pdV + VdP = TdS + Vdp$
- (c)  $dF = dU - TdS - SdT = -pdT - SdT$
- (d)  $dG = dH - TdS - SdT = Vdp - SdT$

Since  $U$ ,  $H$ ,  $F$  and  $G$  are thermodynamic properties and exact differentials of the type

$$dz = M dx + N dy, \text{ then}$$

$$\left( \frac{\partial M}{\partial y} \right)_x = \left( \frac{\partial N}{\partial x} \right)_y$$

Applying this to the four equations

$$\left( \frac{\partial T}{\partial V} \right)_s = - \left( \frac{\partial p}{\partial S} \right)_v$$

$$\left( \frac{\partial T}{\partial P} \right)_s = \left( \frac{\partial V}{\partial S} \right)_p$$

$$\left( \frac{\partial p}{\partial T} \right)_V = \left( \frac{\partial S}{\partial V} \right)_T$$

$$\left( \frac{\partial V}{\partial T} \right)_P = - \left( \frac{\partial S}{\partial p} \right)_T$$

These four equations are known as **Maxwell's equations**.

**(i) Derive:**

$$dS = C_v \frac{dT}{T} + \left( \frac{\partial p}{\partial T} \right)_V dV \quad [\text{IAS-1986}]$$

Let entropy  $S$  be imagined as a function of  $T$  and  $V$ .

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Then  $S = S(T, V)$

$$\text{or } dS = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV$$

multiplying both side by  $T$

$$TdS = T \left(\frac{\partial S}{\partial T}\right)_V dT + T \left(\frac{\partial S}{\partial V}\right)_T dV$$

Since  $T \left(\frac{\partial S}{\partial T}\right)_V = C_V$ , heat capacity at constant volume

and  $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V$  by Maxwell's equation

$$\therefore TdS = C_V dT + T \left(\frac{\partial p}{\partial T}\right)_V dV$$

dividing both side by  $T$

$$dS = C_V \frac{dT}{T} + \left(\frac{\partial p}{\partial T}\right)_V dV \text{ proved}$$

**(ii) Derive:**

$$TdS = C_p dT - T \left(\frac{\partial V}{\partial T}\right)_p dp$$

[IES-1998]

Let entropy  $S$  be imagined as a function of  $T$  and  $p$ .

Then  $S = S(T, p)$

$$\text{or } dS = \left(\frac{\partial S}{\partial T}\right)_p dT + \left(\frac{\partial S}{\partial p}\right)_T dp$$

multiplying both side by  $T$

$$TdS = T \left(\frac{\partial S}{\partial T}\right)_p dT + T \left(\frac{\partial S}{\partial p}\right)_T dp$$

Since  $T \left(\frac{\partial S}{\partial T}\right)_p = C_p$ , heat capacity at constant pressure

and  $\left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_p$  by Maxwell's equation

$$\therefore TdS = C_p dT - T \left(\frac{\partial V}{\partial T}\right)_p dp \quad \text{proved.}$$

**(iii) Derive:**

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$$TdS = C_v dT + T \frac{\beta}{k} dV = C_p dT - TV\beta dp = \frac{k C_p dp}{\beta} + \frac{C_v}{\beta V} dV$$

[IES-2001]

We know that volume expansivity ( $\beta$ ) =  $\frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_p$

and isothermal compressibility ( $k$ ) =  $-\frac{1}{V} \left( \frac{\partial V}{\partial P} \right)_T$

∴ From first  $TdS$  equation

$$TdS = C_v dT + T \left( \frac{\partial P}{\partial T} \right)_V dV$$

$$\frac{\beta}{k} = - \frac{\left( \frac{\partial V}{\partial T} \right)_p}{\left( \frac{\partial V}{\partial P} \right)_T} = - \left( \frac{\partial V}{\partial T} \right)_p \cdot \left( \frac{\partial P}{\partial V} \right)_T$$

$$\text{As } \left( \frac{\partial V}{\partial T} \right)_p \cdot \left( \frac{\partial T}{\partial P} \right)_V \cdot \left( \frac{\partial P}{\partial V} \right)_T = -1$$

$$\therefore - \left( \frac{\partial V}{\partial T} \right)_p \cdot \left( \frac{\partial P}{\partial V} \right)_T = \left( \frac{\partial P}{\partial T} \right)_V$$

$$\text{or } \frac{\beta}{k} = \left( \frac{\partial P}{\partial T} \right)_V$$

$$\therefore TdS = C_v dT + T \cdot \frac{\beta}{k} \cdot dV \quad \text{proved}$$

From second  $TdS$  relation

$$TdS = C_p dT - T \left( \frac{\partial V}{\partial T} \right)_p dp$$

$$\text{as } \beta = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_p$$

$$\therefore \left( \frac{\partial V}{\partial T} \right)_p = V\beta$$

$$\therefore TdS = C_p dT - TV\beta dp \quad \text{proved}$$

Let  $S$  is a function of  $p, V$

$$\therefore S = S(p, V)$$

$$\therefore dS = \left( \frac{\partial S}{\partial p} \right)_V dp + \left( \frac{\partial S}{\partial V} \right)_p dV$$

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Multiply both side by T

$$TdS = T \left( \frac{\partial S}{\partial p} \right)_V dp + T \left( \frac{\partial S}{\partial V} \right)_p dV$$

$$\text{or } TdS = T \left( \frac{\partial S}{\partial T} \cdot \frac{\partial T}{\partial p} \right)_V dp + T \left( \frac{\partial S}{\partial T} \cdot \frac{\partial T}{\partial V} \right)_p dV$$

$$\text{or } TdS = T \left( \frac{\partial S}{\partial T} \right)_V \cdot \left( \frac{\partial T}{\partial p} \right)_V dp + T \left( \frac{\partial S}{\partial T} \right)_p \cdot \left( \frac{\partial T}{\partial V} \right)_p dV$$

$$C_p = T \left( \frac{\partial S}{\partial T} \right)_p \quad \text{and} \quad C_V = T \left( \frac{\partial S}{\partial T} \right)_V$$

$$\therefore TdS = C_p \left( \frac{\partial T}{\partial p} \right)_V dp + C_V \left( \frac{\partial T}{\partial V} \right)_p dV$$

$$\text{From first } \frac{\beta}{k} = \left( \frac{\partial p}{\partial T} \right)_V \quad \text{or} \quad \frac{k}{\beta} = \left( \frac{\partial T}{\partial p} \right)_V$$

$$\therefore TdS = C_p \frac{k}{\beta} dp + C_V \left( \frac{\partial T}{\partial V} \right)_p dV$$

$$\therefore \beta = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_p$$

$$\therefore \left( \frac{\partial T}{\partial V} \right)_p = \frac{1}{\beta V}$$

$$\therefore TdS = \frac{C_p k dp}{\beta} + \frac{C_V}{\beta V} dV \quad \text{proved.}$$

**(iv) Prove that**

$$C_p - C_V = -T \left( \frac{\partial V}{\partial T} \right)_p^2 \cdot \left( \frac{\partial p}{\partial V} \right)_T$$

[IAS-1998]

We know that

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$$TdS = C_p dT - T \left( \frac{\partial V}{\partial T} \right)_p dp = C_v dT + T \left( \frac{\partial p}{\partial T} \right)_V dV$$

$$\text{or } (C_p - C_v) dT = T \left( \frac{\partial V}{\partial T} \right)_p dp + T \left( \frac{\partial p}{\partial T} \right)_V dV$$

$$\text{or } dT = \frac{T \left( \frac{\partial V}{\partial T} \right)_p dp}{C_p - C_v} + \frac{T \left( \frac{\partial p}{\partial T} \right)_V dV}{C_p - C_v} \quad \text{---(i)}$$

since T is a function of p, V

$$T = T(p, V)$$

$$\text{or } dT = \left( \frac{\partial T}{\partial p} \right)_V dp + \left( \frac{\partial T}{\partial V} \right)_p dV \quad \text{---(ii)}$$

comparing (i) & (ii) we get

$$\frac{T \left( \frac{\partial V}{\partial T} \right)_p}{C_p - C_v} = \left( \frac{\partial T}{\partial p} \right)_V \quad \text{and} \quad \frac{T \left( \frac{\partial p}{\partial T} \right)_V}{C_p - C_v} = \left( \frac{\partial T}{\partial V} \right)_p$$

both these give

$$C_p - C_v = T \left( \frac{\partial V}{\partial T} \right)_p \left( \frac{\partial p}{\partial T} \right)_V$$

$$\text{Here } \left( \frac{\partial p}{\partial T} \right)_V \cdot \left( \frac{\partial T}{\partial V} \right)_p \cdot \left( \frac{\partial V}{\partial p} \right)_T = -1 \quad \text{or} \quad \left( \frac{\partial p}{\partial T} \right)_V = - \left( \frac{\partial V}{\partial T} \right)_p \cdot \left( \frac{\partial p}{\partial V} \right)_T$$

$$\therefore C_p - C_v = -T \left( \frac{\partial V}{\partial T} \right)_p^2 \cdot \left( \frac{\partial p}{\partial V} \right)_T \quad \text{proved} \quad \dots \text{Equation(A)}$$

This is a very important equation in thermodynamics. It indicates the following important facts.

(a) Since  $\left( \frac{\partial V}{\partial T} \right)_p^2$  is always positive, and  $\left( \frac{\partial p}{\partial V} \right)_T$  for any substance is negative.  $(C_p - C_v)$  is always positive. Therefore,  $C_p$  is always greater than  $C_v$ .

(b) As  $T \rightarrow 0K, C_p \rightarrow C_v$  or at absolute zero,  $C_p = C_v$ .

(c) When  $\left( \frac{\partial V}{\partial T} \right)_p = 0$  (e.g for water at 4°C, when density is maximum. Or specific volume minimum).

$$C_p = C_v.$$

(d) For an ideal gas,  $pV = mRT$

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$$\left(\frac{\partial V}{\partial T}\right)_p = \frac{mR}{P} = \frac{V}{T}$$

$$\text{and } \left(\frac{\partial p}{\partial V}\right)_T = -\frac{mRT}{V^2}$$

$$\therefore C_p - C_v = mR$$

$$\text{or } c_p - c_v = R$$

Equation (A) may also be expressed in terms of **volume expansively ( $\beta$ )** defined as

$$\beta = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_p$$

and **isothermal compressibility ( $k_T$ )** defined as

$$k_T = -\frac{1}{V} \left( \frac{\partial V}{\partial p} \right)_T$$

$$C_p - C_v = \frac{TV \left[ \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_p \right]^2}{-\frac{1}{V} \left( \frac{\partial V}{\partial p} \right)_T}$$

$$C_p - C_v = \frac{TV\beta^2}{k_T}$$

## Ratio of heat capacities

At constant S, the two TdS equations become

$$C_p dT_s = T \left( \frac{\partial V}{\partial T} \right)_p dp_s$$

$$C_v dT_s = -T \left( \frac{\partial p}{\partial T} \right)_v dV_s$$

$$\frac{C_p}{C_v} = - \left( \frac{\partial V}{\partial T} \right)_p \left( \frac{\partial T}{\partial p} \right)_v \left( \frac{\partial p}{\partial V} \right)_s = \frac{\left( \frac{\partial p}{\partial V} \right)_s}{\left( \frac{\partial p}{\partial V} \right)_T} = \gamma$$

since  $\gamma > 1$ ,

$$\left( \frac{\partial p}{\partial V} \right)_s > \left( \frac{\partial p}{\partial V} \right)_T$$

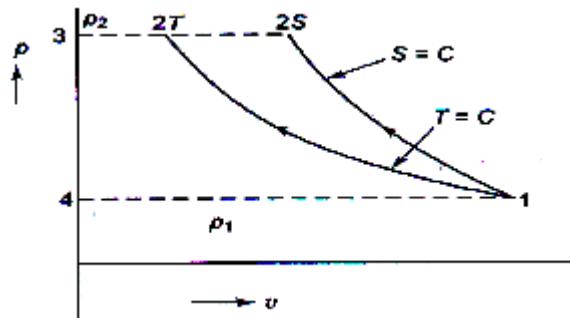
Therefore, the slope of an isentropic is greater than that of an isotherm on p-v diagram (figure below). For reversible and adiabatic compression, the work done is

$$W_s = h_{2s} - h_1 = \int_1^{2s} vdp \\ = \text{Area 1-2s-3-4-1}$$

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**(Fig. Compression Work in Different Reversible Process)**

For reversible and isothermal compression, the work done would be

$$\begin{aligned} W_T &= h_{2T} - h_1 = \int_1^{2T} pdp \\ &= \text{Area } 1-2T-3-4-1 \\ W_T &< W_s \end{aligned}$$

For polytropic compression with  $1 < n < \gamma$ , the work done will be between these two values. So isothermal compression minimum work.

The adiabatic compressibility ( $k_s$ ) is defined as

$$\begin{aligned} \text{or } k_s &= -\frac{1}{V} \left( \frac{\partial V}{\partial p} \right)_s \\ \frac{C_p}{C_v} &= -\frac{1}{V} \left( \frac{\partial V}{\partial p} \right)_T = \gamma \\ &= -\frac{1}{V} \left( \frac{\partial V}{\partial p} \right)_s \\ \gamma &= \frac{k}{k_s} \end{aligned}$$

## Energy Equation

For a system undergoing an infinitesimal reversible process between two equilibrium states, the change of internal energy is

$$dU = TdS - pdV$$

Substituting the first TdS equation

$$dU = C_v dT + T \left( \frac{\partial p}{\partial T} \right)_V dV - pdV$$

$$= C_v dT + \left[ T \left( \frac{\partial p}{\partial T} \right)_V - p \right] dV$$

if  $U = (T, V)$

$$dU = \left( \frac{\partial U}{\partial T} \right)_V dT + \left( \frac{\partial U}{\partial V} \right)_T dV$$

$$\left( \frac{\partial U}{\partial V} \right)_T = T \left( \frac{\partial p}{\partial T} \right)_V - p$$

This is known as energy equation. Two applications of the equation are given below-

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(a) For an ideal gas,  $p = \frac{nRT}{V}$

$$\therefore \left(\frac{\partial p}{\partial T}\right)_V = \frac{nR}{V} = \frac{p}{T}$$

$$\therefore \left(\frac{\partial U}{\partial V}\right)_T = T \cdot \frac{p}{T} - p = 0$$

U does not change when V changes at T = C.

$$\left(\frac{\partial U}{\partial p}\right)_T \left(\frac{\partial p}{\partial V}\right)_T \left(\frac{\partial V}{\partial U}\right)_T = 1$$

$$\left(\frac{\partial U}{\partial p}\right)_T \left(\frac{\partial p}{\partial V}\right)_T = \left(\frac{\partial U}{\partial V}\right)_T = 0$$

$$\text{since } \left(\frac{\partial p}{\partial V}\right)_T \neq 0, \left(\frac{\partial U}{\partial p}\right)_T = 0$$

U does not change either when p changes at T = C. So the internal energy of an ideal gas is a function of temperature only.

Another important point to note is that for an ideal gas

$$pV = nRT \text{ and } T \left(\frac{\partial p}{\partial T}\right)_v - p = 0$$

Therefore

$$dU = C_v dT$$

holds good for an ideal gas in any process (even when the volume changes). But for any other substance

$$dU = C_v dT$$

is true only when the volume is constant and  $dV = 0$

Similarly

$$dH = TdS + Vdp$$

$$\text{and } TdS = C_p dT - T \left(\frac{\partial V}{\partial T}\right)_p dp$$

$$\therefore dH = C_p dT + \left[ V - T \left(\frac{\partial V}{\partial T}\right)_p \right] dp$$

$$\therefore \left(\frac{\partial H}{\partial p}\right)_T = V - T \left(\frac{\partial V}{\partial T}\right)_p$$

As shown for internal energy, it can be similarly proved from Eq. shown in above that the enthalpy of an ideal gas is not a function of either volume or pressure.

$$\left[ i.e. \left(\frac{\partial H}{\partial p}\right)_T = 0 \text{ and } \left(\frac{\partial H}{\partial V}\right)_T = 0 \right]$$

but a function of temperature alone.

Since for an ideal gas,  $pV = nRT$

$$\text{and } V - T \left(\frac{\partial V}{\partial T}\right)_p = 0$$

the relation  $dH = C_p dT$  is true for any process (even when the pressure changes.)

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However, for any other substance the relation  $dH = C_p dT$  holds good only when the pressure remains constant or  $dp = 0$ .

(b) Thermal radiation in equilibrium with the enclosing walls processes an energy that depends only on the volume and temperature. The energy density ( $u$ ), defined as the ratio of energy to volume, is a function of temperature only, or

$$u = \frac{U}{V} = f(T) \text{ only.}$$

The electromagnetic theory of radiation states that radiation is equivalent to a photon gas and it exerts a pressure, and that the pressure exerted by the black body radiation in an enclosure is given by

$$p = \frac{u}{3}$$

Black body radiation is thus specified by the pressure, volume and temperature of the radiation. since.

$$U = uV \text{ and } p = \frac{u}{3}$$
$$\left(\frac{\partial U}{\partial V}\right)_T = u \text{ and } \left(\frac{\partial p}{\partial T}\right)_V = \frac{1}{3} \frac{du}{dT}$$

By substituting in the energy Eq.

$$u = \frac{T}{3} \frac{du}{dT} - \frac{u}{3}$$
$$\therefore \frac{du}{u} = 4 \frac{dT}{T}$$

or  $\ln u = \ln T^4 + \ln b$

or  $u = bT^4$

where  $b$  is a constant. This is known as the **Stefan - Boltzmann Law**.

Since  $U = uV = VbT^4$

$$\left(\frac{\partial U}{\partial T}\right)_V = C_v = 4VbT^3$$
$$\text{and } \left(\frac{\partial p}{\partial T}\right)_V = \frac{1}{3} \frac{du}{dT} = \frac{4}{3} bT^3$$

From the first TdS equation

$$TdS = C_v dT + T \left( \frac{\partial p}{\partial T} \right)_v dV$$
$$= 4VbT^3 dT + \frac{4}{3} bT^4 dV$$

For a reversible isothermal change of volume, the heat to be supplied reversibly to keep temperature constant.

$$Q = \frac{4}{3} bT^4 \Delta V$$

For a reversible adiabatic change of volume

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$$\frac{4}{3}bT^4dV = -4VbT^3dT$$

$$\text{or } \frac{dV}{V} = -3\frac{dT}{T}$$

$$\text{or } VT^3 = \text{const}$$

If the temperature is one-half the original temperature. The volume of black body radiation is to be increased adiabatically eight times its original volume so that the radiation remains in equilibrium with matter at that temperature.

**(v) Prove that**

$$\frac{\beta}{k} = \left( \frac{\partial p}{\partial T} \right)_V \quad \text{and} \quad C_p - C_V = \left\{ p + \left( \frac{\partial U}{\partial V} \right)_T \right\} \left( \frac{\partial V}{\partial T} \right)_p$$

Hence show that

$$C_p - C_V = \frac{\beta^2 TV}{k} \quad [\text{IES-2003}]$$

$$\text{Here } \beta = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_p$$

$$k = -\frac{1}{V} \left( \frac{\partial V}{\partial p} \right)_T$$

$$\therefore \frac{\beta}{k} = -\frac{\left( \frac{\partial V}{\partial T} \right)_p}{\left( \frac{\partial V}{\partial p} \right)_T} = -\left( \frac{\partial V}{\partial T} \right)_p \cdot \left( \frac{\partial p}{\partial V} \right)_T$$

$$\text{we know that } \left( \frac{\partial V}{\partial T} \right)_p \cdot \left( \frac{\partial T}{\partial p} \right)_V \cdot \left( \frac{\partial p}{\partial V} \right)_T = -1$$

$$\text{or } -\left( \frac{\partial V}{\partial T} \right)_p \cdot \left( \frac{\partial p}{\partial V} \right)_T = \left( \frac{\partial p}{\partial T} \right)_V$$

$$\therefore \frac{\beta}{k} = \left( \frac{\partial p}{\partial T} \right)_V \quad \text{proved}$$

$\Rightarrow$  From Tds relations

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$$TdS = C_p dT - T \left( \frac{\partial V}{\partial T} \right)_p dP = C_v dT + T \left( \frac{\partial P}{\partial T} \right)_v dV$$

$$\therefore (C_p - C_v) dT = T \left( \frac{\partial V}{\partial T} \right)_p dP + T \left( \frac{\partial P}{\partial T} \right)_v dV$$

$$\text{or } dT = \frac{T \left( \frac{\partial V}{\partial T} \right)_p}{C_p - C_v} dP + \frac{T \left( \frac{\partial P}{\partial T} \right)_v}{C_p - C_v} dV \quad \text{--- (i)}$$

Since  $T$  is a function of  $(P, V)$

$$T = T(P, V)$$

$$\therefore dT = \left( \frac{\partial T}{\partial P} \right)_V dP + \left( \frac{\partial T}{\partial V} \right)_P dV \quad \text{--- (ii)}$$

Comparing (i) & (ii) we get

$$\frac{T \left( \frac{\partial V}{\partial T} \right)_p}{C_p - C_v} = \left( \frac{\partial T}{\partial P} \right)_V \quad \text{and} \quad \frac{T \left( \frac{\partial P}{\partial T} \right)_V}{C_p - C_v} = \left( \frac{\partial T}{\partial V} \right)_P$$

$$\therefore C_p - C_v = T \left( \frac{\partial V}{\partial T} \right)_p \cdot \left( \frac{\partial P}{\partial T} \right)_V$$

$$\text{as } dU = dQ - pdV$$

$$\therefore dU = TdS - pdV$$

$$\text{or } \left( \frac{\partial U}{\partial V} \right)_T = T \left( \frac{\partial S}{\partial V} \right)_T - p$$

$$\text{or } \left( \frac{\partial U}{\partial V} \right)_T + p = T \left( \frac{\partial S}{\partial V} \right)_T$$

From Maxwell's Third relations

$$\left( \frac{\partial P}{\partial T} \right)_V = \left( \frac{\partial S}{\partial V} \right)_T$$

$$\therefore C_p - C_v = T \left( \frac{\partial V}{\partial T} \right)_p \cdot \left( \frac{\partial P}{\partial T} \right)_V = \left\{ p + \left( \frac{\partial U}{\partial V} \right)_T \right\} \left( \frac{\partial V}{\partial T} \right)_p$$

**(vi) Prove that**

$$\mu = \left( \frac{\partial T}{\partial P} \right)_h = \frac{T^2}{C_p} \left[ \frac{\partial}{\partial T} \left( \frac{V}{T} \right) \right]_p \quad [\text{IES-2002}]$$

**Joule – Thomson co-efficient**

The numerical value of the slope of an isenthalpic on a  $T - p$  diagram at any point is called the Joule – Kelvin coefficient.

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$$\therefore \mu = \left( \frac{\partial T}{\partial p} \right)_h$$

$$\text{Here } dH = TdS + Vdp$$

$$\text{as } TdS = C_p dT - T \left( \frac{\partial V}{\partial T} \right)_p dp$$

$$\therefore dH = C_p dT - T \left( \frac{\partial V}{\partial T} \right)_p dp + Vdp = C_p dT - \left[ T \left( \frac{\partial V}{\partial T} \right)_p - V \right] dp$$

$$\text{if } H = \text{const.} \quad \therefore dH = 0$$

$$\text{so } C_p (dT)_h - \left[ T \left( \frac{\partial V}{\partial T} \right)_p - V \right] (dp)_h = 0$$

$$\therefore \left( \frac{\partial T}{\partial p} \right)_h = \frac{1}{C_p} \left[ T \left( \frac{\partial V}{\partial T} \right)_p - V \right] = \frac{T^2}{C_p} \left[ \frac{1}{T} \cdot \left( \frac{\partial V}{\partial T} \right)_p - \frac{V}{T^2} \right] = \frac{T^2}{C_p} \left[ \frac{\partial}{\partial T} \left( \frac{V}{T} \right) \right]_p$$

## (vii) Derive clausius – clapeyron equation

$$\left( \frac{dp}{dT} \right) = \frac{h_{fg}}{T(v_g - v_f)} \quad \text{and} \quad \frac{dp}{p} = \frac{h_{fg}}{RT^2} dT$$

[IES-2000]

$$\left( \frac{\partial p}{\partial T} \right)_V = \left( \frac{\partial S}{\partial V} \right)_T \quad \text{Maxwells equation}$$

When saturated liquid convert to saturated vapour at constant temperature. During the evaporation, the pr. & T is independent of volume.

$$\therefore \left( \frac{dp}{dT} \right)_{\text{sat}} = \frac{s_g - s_f}{v_g - v_f}$$

$$s_g - s_f = s_{fg} = \frac{h_{fg}}{T}$$

$$\text{or} \quad \left( \frac{dp}{dT} \right)_{\text{sat}} = \frac{h_{fg}}{T(v_g - v_f)}$$

→ It is useful to estimate properties like h from other measurable properties.

→ At a change of phase we may find  $h_{fg}$  i.e. latent heat.

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At very low pressure  $v_g \approx v_{fg}$  as  $v_f$  very small

$$pv_g = RT \quad \text{or} \quad v_g = \frac{RT}{p}$$

$$\therefore \frac{dp}{dT} = \frac{h_{fg}}{T \cdot v_g} = \frac{h_{fg}}{T \cdot \frac{RT}{p}} = \frac{h_{fg} \cdot p}{RT^2}$$

$$\text{or} \quad \frac{dp}{p} = \frac{h_{fg}}{R} \cdot \frac{dT}{T^2}$$

$$\text{or} \quad \ln\left(\frac{p_2}{p_1}\right) = \frac{h_{fg}}{R} \left( \frac{1}{T_1} - \frac{1}{T_2} \right)$$

→ Knowing vapour pressure  $p_1$  at temperature  $T_1$ , we may find out  $p_2$  at temperature  $T_2$ .

## Joule-Kelvin Effect or Joule-Thomson coefficient

The value of the specific heat  $c_p$  can be determined from  $p-v-T$  data and the Joule-Thomson coefficient. The **Joule-Thomson coefficient**  $\mu_J$  is defined as

$$\mu_J = \left( \frac{\partial T}{\partial p} \right)_h$$

Like other partial differential coefficients introduced in this section, the Joule-Thomson coefficient is defined in terms of thermodynamic properties only and thus is itself a property. The units of  $\mu_J$  are those of temperature divided by pressure.

A relationship between the specific heat  $c_p$  and the Joule-Thomson coefficient  $\mu_J$  can be established to write

$$\left( \frac{\partial T}{\partial p} \right)_h \left( \frac{\partial p}{\partial h} \right)_T \left( \frac{\partial h}{\partial T} \right)_p = -1$$

The first factor in this expression is the Joule-Thomson coefficient and the third is  $c_p$ . Thus

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$$c_p = \frac{-1}{\mu_J (\partial p / \partial h)_T}$$

With  $(\partial h / \partial p)_T = 1 / (\partial p / \partial h)_T$

this can be written as

$$c_p = -\frac{1}{\mu_J} \left( \frac{\partial h}{\partial p} \right)_T$$

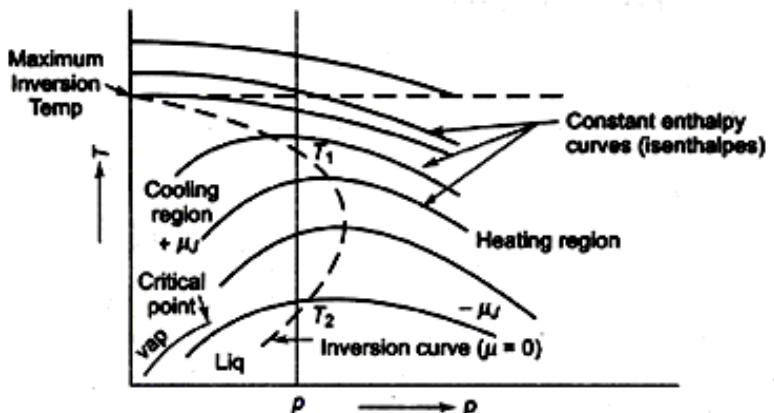
The partial derivative  $(\partial h / \partial p)_T$ , called the **constant-temperature coefficient**, can be eliminated. The following expression results:

$$c_p = \frac{1}{\mu_J} \left[ T \left( \frac{\partial v}{\partial T} \right)_p - v \right]$$

allows the value of  $c_p$  at a state to be determined using  $p-v-T$  data and the value of the Joule-Thomson coefficient at that state. Let us consider next how the Joule-Thomson coefficient can be found experimentally.

The numerical value of the slope of an isenthalpic on a  $T-p$  diagram at any point is called the **Joule-Kelvin coefficient** and is denoted by  $\mu_J$ . Thus the locus of all points at which  $\mu_J$  is zero is the inversion curve. The region inside the inversion curve where  $\mu_J$  is positive is called the *cooling region* and the region outside where  $\mu_J$  is negative is called the *heating region*. So,

$$\mu_J = \left( \frac{\partial T}{\partial p} \right)_h$$



*Isenthalpic Curves and the Inversion Curve*

## Gibbs Phase Rule

Gibbs Phase Rule determines what is expected to define the state of a system

$$F = C - P + 2$$

F = Number of degrees of freedom (i.e., no. of properties required)

C = Number of components

P = Number of phases

e.g., Nitrogen gas C = 1; P = 1. Therefore, F = 2

- To determine the state of the nitrogen gas in a cylinder two properties are adequate.
- A closed vessel containing water and steam in equilibrium: P = 2, C = 1
- Therefore, F = 1. If any one property is specified it is sufficient.
- A vessel containing water, ice and steam in equilibrium
- P = 3, C = 1 therefore F = 0. The triple point is uniquely defined.

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## **ASKED OBJECTIVE QUESTIONS (GATE, IES, IAS)**

# **Previous 20-Years GATE Questions**

# Maxwell's Equations

- GATE-1.** Which of the following relationships is valid only for reversible processes undergone by a closed system of simple compressible substance (neglect changes in kinetic and potential energy)? [GATE-2007]

(a)  $\delta Q = dU + \delta W$    (b)  $TdS = dU + pdV$    (c)  $TdS = dU + \delta W$    (d)  $\delta Q = dU + pdV$

- GATE-2.** Considering the relationship  $TdS = dU + pdV$  between the entropy (S), internal energy (U), pressure (p), temperature (T) and volume (V), which of the following statements is correct? [GATE-2003]

  - It is applicable only for a reversible process
  - For an irreversible process,  $TdS > dU + pdV$
  - It is valid only for an ideal gas
  - It is equivalent to 1 law, for a reversible process

## Difference in Heat Capacities and Ratio of Heat Capacities

- GATE-3. The specific heats of an ideal gas depend on its [GATE-1996]**

  - (a) Temperature
  - (b) Pressure
  - (c) Volume
  - (d) Molecular weight and structure

- GATE-4. The specific heats of an ideal gas depends on its [GATE-1996]**

  - (a) Temperature
  - (b) Pressure
  - (c) Volume
  - (d) Molecular weight and structure

- GATE-5.** For an ideal gas the expression  $\left| T \left( \frac{\partial S}{\partial T} \right)_p - T \left( \frac{\partial S}{\partial T} \right)_v \right|$  is always equal to:

[GATE-1997]

- $$(a) zero \quad (b) \frac{c_p}{c_v} \quad (c) R \quad (d) RT$$

- GATE-6.** A 2 kW, 40 litre water heater is switched on for 20 minutes. The heat capacity  $C_p$  for water is 4.2 kJ/kg K. Assuming all the electrical energy has gone into heating the water, increase of the water temperature in degree centigrade is:

[GATE-2003]



## Joule-Kelvin Effect or Joule-Thomson coefficient

- GATE-7. Which combination of the following statements is correct?** [GATE-2007]

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- P: A gas cools upon expansion only when its Joule-Thomson coefficient is positive in the temperature range of expansion.
- Q: For a system undergoing a process, its entropy remains constant only when the process is reversible.
- R: The work done by a closed system in an adiabatic process is a point function.
- S: A liquid expands upon freezing when the slope of its fusion curve on Pressure Temperature diagram is negative.
- (a) R and S                         (b) P and Q                         (c) Q, R and S                         (d) P, Q and R

**GATE-8.** A positive value to Joule-Thomson coefficient of a fluid means [GATE-2002]

- (a) Temperature drops during throttling  
(b) Temperature remains constant during throttling  
(c) Temperature rises during throttling  
(d) None of these

**GATE-9.** A gas having a negative Joule-Thompson coefficient ( $\mu < 0$ ), when throttled, will: [GATE-2001]

- (a) Become cooler  
(b) Become warmer  
(c) Remain at the same temperature  
(d) Either be cooler or warmer depending on the type of gas

**GATE-10.** Match 4 correct pairs between List-I and List-II for the questions [GATE-1994]

For a perfect gas:

List-I	List-II
(a) Isobaric thermal expansion coefficient	1. 0
(b) Isothermal compressibility	2. $\infty$
(c) Isentropic compressibility	3. $1/v$
(d) Joule – Thomson coefficient	4. $1/T$
	5. $1/p$
	6. $1/\gamma_p$

## Previous 20-Years IES Questions

### Some Mathematical Theorems

**IES-1.** Given: [IES-1993]

$p$  = pressure,  $T$  = Temperature,  $v$  = specific volume

Which one of the following can be considered as property of a system?

- (a)  $\int pdv$                      (b)  $\int vdp$                      (c)  $\int \left( \frac{dT}{T} + \frac{pdv}{v} \right)$                      (d)  $\int \left( \frac{dT}{T} - \frac{vdp}{T} \right)$

### Maxwell's Equations

**IES-2.** Which thermodynamic property is evaluated with the help of Maxwell equations from the data of other measurable properties of a system? [IES 2007]

- (a) Enthalpy                     (b) Entropy                     (c) Latent heat                     (d) Specific heat

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IES-3. Consider the following statements pertaining to the Clapeyron equation:

1. It is useful to estimate properties like enthalpy from other measurable properties. [IES-2006]
2. At a change of phase, it can be used to find the latent heat at a given pressure.
3. It is derived from the relationship  $\left(\frac{\partial p}{\partial v}\right)_T = \left(\frac{\partial s}{\partial T}\right)_V$

Which of the statements given above are correct?

- (a) 1,2 and 3      (b) Only 1 and 2      (c) Only 1 and 3      (d) Only 2 and 3

IES-3a Clapeyron's equation is used for finding out the

[IES-2011]

- (a) Dryness fraction of steam only
- (b) Entropy of superheater vapour only
- (c) Specific volume at any temperature and pressure
- (d) Total heat of superheated steam only

## TdS Equations

IES-4. T ds equation can be expressed as:

[IES-2002]

- |                                          |                                          |
|------------------------------------------|------------------------------------------|
| (a) $Tds = C_v dT + \frac{T\beta dv}{k}$ | (b) $Tds = C_v dT + \frac{Tdv}{k}$       |
| (c) $Tds = C_v dT + \frac{Tk}{\beta} dv$ | (d) $Tds = C_v dT + \frac{T\beta}{k} dp$ |

IES-5. Which one of the following statements applicable to a perfect gas will also be true for an irreversible process? (Symbols have the usual meanings). [IES-1996]

- (a)  $dQ = du + pdV$       (b)  $dQ = Tds$       (c)  $Tds = du + pdV$       (d) None of the above

IES-6. Consider the following thermodynamic relations:

[IES-2000]

- |                     |                     |
|---------------------|---------------------|
| 1. $Tds = du + pdv$ | 2. $Tds = du - pdv$ |
| 3. $Tds = dh + vdP$ | 4. $Tds = dh - vdP$ |

Which of these thermodynamic relations are correct?

- (a) 1 and 3      (b) 1 and 4      (c) 2 and 3      (d) 2 and 4

## Difference in Heat Capacities and Ratio of Heat Capacities

IES-7. Match List-I (Terms) with List-II (Relations) and select the correct answer using the codes given below the Lists:

[IES-2003]

### List-I (Terms)

- A. Specific heat at constant volume,  $C_v$
- B. Isothermal compressibility  $k_T$

### List-II (Relations)

1.  $\frac{1}{v} \left( \frac{\partial v}{\partial T} \right)_p$
2.  $-T \left( \frac{\partial p}{\partial T} \right)_v \left( \frac{\partial v}{\partial T} \right)_p$

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C. Volume expansivity  $\beta$

$$3. \quad T \left( \frac{\partial \beta}{\partial T} \right)_v$$

D. Difference between specific heats at constant pressure and at constant  $C_p - C_v$

$$4. \quad -\frac{1}{v} \left( \frac{\partial v}{\partial p} \right)_T$$

Codes:

	A	B	C	D	A	B	C	D
(a)	3	4	2	1	(b)	4	1	3
(c)	3	4	1	2	(d)	4	1	2

- IES-8.** Assertion (A): Specific heat at constant pressure for an ideal gas is always greater than the specific heat at constant volume. [IES-2002]

Reason (R): Heat added at constant volume is not utilized for doing any external work.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is NOT the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

- IES-9.** An insulated box containing 0.5 kg of a gas having  $C_v = 0.98 \text{ kJ/kgK}$  falls from a balloon 4 km above the earth's surface. What will be the temperature rise of the gas when the box hits the ground? [IES-2004]

- (a) 0 K
- (b) 20 K
- (c) 40 K
- (d) 60 K

- IES-10.** As compared to air standard cycle, in actual working, the effect of variations in specific heats is to: [IES-1994]

- (a) Increase maximum pressure and maximum temperature
- (b) Reduce maximum pressure and maximum temperature
- (c) Increase maximum pressure and decrease maximum temperature
- (d) Decrease maximum pressure and increase maximum temperature

- IES-11.** The number of degrees of freedom for a diatomic molecule [IES-1992]

- (a) 2
- (b) 3
- (c) 4
- (d) 5

- IES-12.** The ratio  $\frac{C_p}{C_v}$  for a gas with n degrees of freedom is equal to: [IES-1992]

- (a)  $n + 1$
- (b)  $n - 1$
- (c)  $\frac{2}{n} - 1$
- (d)  $1 + \frac{2}{n}$

- IES-13.** Molal specific heats of an ideal gas depend on [IES-2010]

- (a) Its pressure
- (b) Its temperature
- (c) Both its pressure and temperature
- (d) The number of atoms in a molecule

- IES-14.** Assertion (A): Ratio of specific heats  $\frac{C_p}{C_v}$  decreases with increase in temperature. [IES-1996]

Reason (R): With increase in temperature,  $C_p$  decreases at a higher rate than  $C_v$ .

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is NOT the correct explanation of A

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- (c) A is true but R is false
- (d) A is false but R is true

**IES-15.** It can be shown that for a simple compressible substance, the relationship

$$C_p - C_v = -T \left( \frac{\partial V}{\partial T} \right)_p^2 \left( \frac{\partial P}{\partial V} \right)_T \text{ exists.} \quad [\text{IES-1998}]$$

Where  $C_p$  and  $C_v$  are specific heats at constant pressure and constant volume respectively. T is the temperature V is volume and P is pressure.

Which one of the following statements is NOT true?

- (a)  $C_p$  is always greater than  $C_v$ .
- (b) The right side of the equation reduces to R for ideal gas.
- (c) Since  $\left( \frac{\partial P}{\partial V} \right)_T$  can be either positive or negative, and  $\left( \frac{\partial V}{\partial T} \right)_p^2$  must be positive, T must have a sign that is opposite to that of  $\left( \frac{\partial P}{\partial V} \right)_T$
- (d) Is very nearly equal to R for liquid water.

**IES-15a.** The difference between constant pressure specific heat  $C_p$  and constant volume specific heat  $C_v$  for pure substance [IES-2012]

- (a) approaches zero at triple point
- (b) approaches zero as the absolute temperature approaches zero
- (c) is always equal to the gas constant R
- (d) approaches zero at critical point

## Joule-Kelvin Effect or Joule-Thomson coefficient

**IES-16.** Joule-Thomson coefficient is defined as: [IES-1995]

$$(a) \left( \frac{\partial T}{\partial p} \right)_h \quad (b) \left( \frac{\partial h}{\partial p} \right)_T \quad (c) \left( \frac{\partial h}{\partial T} \right)_p \quad (d) \left( \frac{\partial p}{\partial T} \right)_h$$

**IES-16a.** Joule-Thomson coefficient is the slope of [IES-2012]

- (a) constant enthalpy lines on T-s diagram
- (b) constant enthalpy lines on T-p diagram
- (c) inversion curve on T-s diagram
- (d) inversion curve on T-p diagram

**IES-17.** The throttling of certain gasses may be used for getting the refrigeration effect. What is the value of Joule – Thomson coefficient ( $\mu$ ) for such a throttling process? [IES-2007]

- (a)  $\mu = 0$
- (b)  $\mu = 1$
- (c)  $\mu < 1$
- (d)  $\mu > 1$

**IES-17a.** Which gas shows a heating effect in the Joule-Thomson experiment, while undergoing throttling process through a porous plug of cotton wool? [IES-2012]

- (a) Carbon dioxide
- (b) Hydrogen
- (c) Oxygen
- (d) Nitrogen

**IES-18.** Which one of the following is correct? [IES 2007]

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When a real gas undergoes Joule-Thomson expansion, the temperature

- (a) May remain constant (b) Always increases  
(c) May increase or decrease (d) Always decreases

IES-19. Assertion (A): Throttling process for real gases at initial temperature higher than maximum inversion temperature is accompanied by decrease in temperature of the gas. [IES-2003]

Reason (R): Joule-Kelvin coefficient  $\mu_j$  is given  $(\partial T / \partial p)_h$  and should have a positive value for decrease in temperature during throttling process.

- (a) Both A and R are individually true and R is the correct explanation of A  
(b) Both A and R are individually true but R is NOT the correct explanation of A  
(c) A is true but R is false  
(d) A is false but R is true

IES-20. Match List-I (Name of entity) with List-II (Definition) and select the correct answer using the codes given below the lists: [IES-2001]

List-I (Name of entity)

A. Compressibility factor

List-II (Definition)

$$1. -\frac{1}{v} \left( \frac{\partial v}{\partial p} \right)_T$$

B. Joule – Thomson coefficient

$$2. \left( \frac{\partial h}{\partial T} \right)_p$$

C. Constant pressure specific heat

$$3. \left( \frac{\partial T}{\partial p} \right)_h$$

D. Isothermal compressibility

$$4. \left( \frac{p v}{R T} \right)$$

Codes:

A      B      C      D

(a)    2      1      4      3

(c)    2      3      4      1

A      B      C      D

(b)    4      3      2      1

(d)    4      1      2      3

IES-21. Joule – Thomson coefficient is the ratio of [IES-1999]

- (a) Pressure change to temperature change occurring when a gas undergoes the process of adiabatic throttling  
(b) Temperature change to pressure change occurring when a gas undergoes the process of adiabatic throttling  
(c) Temperature change to pressure change occurring when a gas undergoes the process of adiabatic compression  
(d) Pressure change to temperature change occurring when a gas undergoes the process of adiabatic compression

IES-22. The Joule – Thomson coefficient is the [IES-1996]

(a)  $\left( \frac{\partial T}{\partial p} \right)_h$  of pressure-temperature curve of real gases

(b)  $\left( \frac{\partial T}{\partial p} \right)_v$  of temperature-entropy curve of real gases

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(c)  $\left(\frac{\partial h}{\partial s}\right)_T$  of enthalpy-entropy curve of real gases

(d)  $\left(\frac{\partial V}{\partial T}\right)_p$  of pressure-volume curve of real gases

IES-23. Match the following:

List-I

A. Work

B. Heat

C. Internal energy

D. Joule Thomson Coefficient

List-II

1. Point function

2.  $\int T ds$

3.  $\left(\frac{\partial u}{\partial T}\right)_h$

4.  $\int pdv$

Code:

A      B      C      D

(a)    4      2      1      3

(c)    4      1      2      3

A      B      C      D

(b)    1      2      4      3

(d)    2      1      4      3

[IES-1992]

## Clausius-Clapeyron Equation

IES-24. Consider the following statements in respect of the Clausius – Clapeyron equation: [IES-2007]

1. It points to one possible way of measuring thermodynamic temperature.
2. It permits latent heat of vaporization to be estimated from measurements of specific volumes of saturated liquid, saturated vapour and the saturation temperatures at two nearby pressures.
3. It does not apply to changes from solid to the liquid phase and from solid to the Vapour phase.

Which of the statements given above are correct?

- (a) 1, 2 and 3      (b) 1 and 2 only      (c) 2 and 3 only      (d) 1 and 3 only

IES-25. The equation relating the following measurable properties:

[IES-2005]

(i) The slope of saturation pressure temperature line

(ii) The latent heat, and

(iii) The change in volume during phase transformation; is known as:

(a) Maxwell relation

(b) Joules equation

(c) Clapeyron equation

(d) None of the above

IES-26. The variation of saturation pressure with saturation temperature for a liquid is 0.1 bar/K at 400 K. The specific volume of saturated liquid and dry saturated vapour at 400 K are 0.251 and 0.001 m<sup>3</sup>/kg What will be the value of latent heat of vaporization using Clausius Clapeyron equation? [IES-2004]

- (a) 16000 kJ/kg      (b) 1600 kJ/kg      (c) 1000 kJ/kg      (d) 160 kJ/kg

IES-27. If h, p, T and v refer to enthalpy, pressure, temperature and specific volume respectively and subscripts g and f refer to saturation conditions of vapour and liquid respectively then Clausius-Clapeyron equation applied to change of phase from liquid to vapour states is: [IES-1996, 2006]

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$$(a) \frac{dp}{dt} = \frac{(h_g - h_f)}{(v_g - v_f)}$$

$$(b) \frac{dp}{dt} = \frac{(h_g - h_f)}{T(v_g - v_f)}$$

$$(c) \frac{dp}{dt} = \frac{(h_g - h_f)}{T}$$

$$(d) \frac{dp}{dt} = \frac{(v_g - v_f)T}{(h_g - h_f)}$$

- IES-28.** Which one of the following functions represents the Clapeyron equation pertaining to the change of phase of a pure substance? [IES-2002]

(a)  $f(T, p, h_{fg})$       (b)  $f(T, p, h_{fg}, v_{fg})$       (c)  $f(T, p, h_{fg}, s_{fg})$       (d)  $f(T, p, h_{fg}, s_{fg}, v_{fg})$

- IES-29.** The Clapeyron equation with usual notations is given by: [IES-2000]

$$(a) \left(\frac{dT}{dP}\right)_{sat} = \frac{h_{fg}}{Tv_{fg}} \quad (b) \left(\frac{dP}{dT}\right)_{sat} = \frac{h_{fg}}{Tv_{fg}} \quad (c) \left(\frac{dT}{dP}\right)_{sat} = \frac{Th_{fg}}{v_{fg}} \quad (d) \left(\frac{dP}{dT}\right)_{sat} = \frac{Th_{fg}}{v_{fg}}$$

- IES-30.** Clausius-Clapeyron equation gives the 'slope' of a curve in [IES-1999]

(a) p-v diagram      (b) p-h diagram      (c) p-T diagram      (d) T-S diagram

- IES-31.** The thermodynamic parameters are: [IES-1997]

I. Temperature   II. Specific Volume   III. Pressure   IV. Enthalpy   V. Entropy  
The Clapeyron Equation of state provides relationship between:

(a) I and II      (b) II, III and V      (c) III, IV and V      (d) I, II, III and IV

## Gibbs Phase Rule

- IES-32.** Number of components (C), phase (P) and degrees of freedom (F) are related by Gibbs-phase rule as: [IES-2001]

(a)  $C - P - F = 2$       (b)  $F - C - P = 2$       (c)  $C + F - P = 2$       (d)  $P + F - C = 2$

- IES-33.** As per Gibb's phase rule, if number of components is equal to 2 then the number of phases will be: [IES-2002]

(a)  $\leq 2$       (b)  $\leq 3$       (c)  $\leq 4$       (d)  $\leq 5$

- IES-34.** Gibb's phase rule is given by: [IES-1999]

(F = number of degrees of freedom; C = number of components; P = number of phases)

(a)  $F = C + P$       (b)  $F = C + P - 2$       (c)  $F = C - P - 2$       (d)  $F = C - P + 2$

- IES-35.** Gibb's free energy 'c' is defined as: [IES-1999]

(a)  $G = H - TS$       (b)  $G = U - TS$       (c)  $G = U + pV$       (d)  $G = H + TS$

- IES-36.** Which one of the following relationships defines the Helmholtz function F? [IES-2007]

(a)  $F = H + TS$       (b)  $F = H - TS$       (c)  $F = U - TS$       (d)  $F = U + TS$

- IES-37.** Assertion (A): For a mixture of solid, liquid and vapour phases of a pure substance in equilibrium, the number of independent intrinsic properties needed is equal to one. [IES-2005]

Reason(R): The Three phases can coexist only at one particular pressure.

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- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is NOT the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

IES-38. Consider the following statements:

[IES-2000]

- 1. Azeotropes are the mixtures of refrigerants and behave like pure substances.
- 2. Isomers refrigerants are compounds with the same chemical formula but have different molecular structures.
- 3. The formula  $n + p + q = 2m$  is used for unsaturated chlorofluorocarbon compounds (m, n, p and q are the numbers atoms of carbon, hydrogen, fluorine and chlorine respectively).

Which of these statements are correct?

- (a) 1 and 3
- (b) 2 and 3
- (c) 1 and 2
- (d) 1, 2 and 3

## Previous 20-Years IAS Questions

### Maxwell's Equations

IAS-1. According to the Maxwell relation, which of the following is/are correct?

- (a)  $\left(\frac{\partial V}{\partial T}\right)_p = -\left(\frac{\partial S}{\partial P}\right)_T$
- (b)  $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$  [IAS-2007]
- (c)  $\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$
- (d) All of the above

### TdS Equations

IAS-2. Which one of the following expressions for  $T ds$  is true for a simple compressible substance? (Notations have the usual meaning) [IAS-1998]

- (a)  $dh - vdp$
- (b)  $dh + vdp$
- (c)  $dh - pdv$
- (d)  $dh + pdv$

### Difference in Heat Capacities and Ratio of Heat Capacities

IAS-3. The specific heat  $C_p$  is given by:

[IAS-2000]

- (a)  $T\left(\frac{\partial V}{\partial T}\right)_p$
- (b)  $T\left(\frac{\partial T}{\partial S}\right)_p$
- (c)  $T\left(\frac{\partial S}{\partial T}\right)_p$
- (d)  $T\left(\frac{\partial T}{\partial V}\right)_p$

IAS-4. For an ideal gas the expression  $\left[T\left(\frac{\partial S}{\partial T}\right)_p - T\left(\frac{\partial S}{\partial T}\right)_V\right]$  is always equal to:

- (a) zero
- (b)  $\frac{C_p}{C_v}$
- (c)  $R$
- (d)  $RT$  [IAS-2003]

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- IAS-5. Assertion (A): Specific heat at constant pressure for an ideal gas is always greater than the specific heat at constant volume. [IAS-2000]

Reason (R): Heat added at constant volume is not utilized for doing any external work.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is NOT the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

- IAS-6. Match List-I with List-II and select the correct answers using the codes given below the lists. [IAS-2002]

List-I

A. Joule Thomson co-efficient

B.  $C_p$  for monatomic gas

C.  $C_p - C_v$  for diatomic gas

$$D. \left( \frac{\partial U}{\partial T} \right)_v$$

List-II

1.  $\frac{5}{2} R$

2.  $C_v$

3. R

4.  $\left( \frac{\partial T}{\partial P} \right)_h$

Codes:

A

B

C

D

(a)

(b)

(c)

A

B

C

D

- IAS-7. Ratio of specific heats for an ideal gas is given by (symbols have the usual meanings) [IAS-1999]

(a)  $\frac{1}{1 - \frac{R}{C_p}}$

(b)  $\frac{1}{1 - \frac{C_p}{R}}$

(c)  $\frac{1}{1 + \frac{C_p}{R}}$

(d)  $\frac{1}{1 + \frac{R}{C_p}}$

## Joule-Kelvin Effect or Joule-Thomson coefficient

- IAS-8. Which one of the following properties remains unchanged for a real gas during Joule-Thomson process? [IAS-2000]

- (a) Temperature
- (b) Enthalpy
- (c) Entropy
- (d) Pressure

## Clausius-Clapeyron Equation

- IAS-9. If h, p, T and v refer to enthalpy, pressure, temperature and specific volume respectively and subscripts g and f refer to saturation conditions of vapour and liquid respectively then Clausius-Clapeyron equation applied to change of phase from liquid to vapour states is: [IAS-2003]

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$$(a) \frac{dp}{dt} = \frac{(h_g - h_f)}{(v_g - v_f)}$$

$$(b) \frac{dp}{dt} = \frac{(h_g - h_f)}{T(v_g - v_f)}$$

$$(c) \frac{dp}{dt} = \frac{(h_g - h_f)}{T}$$

$$(d) \frac{dp}{dt} = \frac{(v_g - v_f)T}{(h_g - h_f)}$$

- IAS-10.** Which one of the following is the correct statement?

[IAS-2007]

**Clapeyron equation is used for:**

- |                                         |                                       |
|-----------------------------------------|---------------------------------------|
| (a) Finding specific volume of vapour   | (b) Finding specific volume of liquid |
| (c) Finding latent heat of vaporization | (d) Finding sensible heat             |

- IAS-11.** Assertion (A): Water will freeze at a higher temperature if the pressure is increased.

[IAS-2003]

**Reason (R): Water expands on freezing which by Clapeyron's equation gives negative slope for the melting curve.**

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is NOT the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

- IAS-12.** Match List-I with List-II and select the correct answer using the codes given below the lists

[IAS-1994]

**List-I**

- A. Mechanical work
- B.  $\int \frac{dQ}{T} \leq 0$
- C. Zeroth Law
- D. H-TS

**Codes:**

(a)

(b)

(c)

—

**A**

**B**

**C**

**D**

**List-II**

- 1. Clausius-Clapeyron equation
- 2. Gibb's equation
- 3. High grade energy
- 4. Concept of temperature

**A**

**B**

**C**

**D**

(b)

(d)

(d)

(d)

—

—

—

—

2

4

4

2

## Gibbs Phase Rule

- IAS-13.** Which one of the following relationships defines Gibb's free energy G?

[IAS-2007]

- (a)  $G = H + TS$
- (b)  $G = H - TS$
- (c)  $G = U + TS$
- (d)  $G = U - TS$

- IAS-14.** The Gibbs free-energy function is a property comprising

[IAS-1998]

- (a) Pressure, volume and temperature
- (b) Enthalpy, temperature and entropy
- (c) Temperature, pressure and enthalpy
- (d) Volume, enthalpy and entropy

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## Answers with Explanation (Objective)

### Previous 20-Years GATE Answers

GATE-1. Ans. (d)

GATE-2. Ans. (d)

GATE-3. Ans. (a)

GATE-4. Ans. (d)

$$\text{GATE-5. Ans. (c)} \quad T\left(\frac{\partial S}{\partial T}\right)_P = \left(\frac{T\delta S}{\delta T}\right)_P = \left(\frac{dQ}{dT}\right)_P = C_P$$

$$T\left(\frac{\partial S}{\partial T}\right)_V = \left(\frac{T\delta S}{\delta T}\right)_V = \left(\frac{dQ}{dT}\right)_V = C_V$$

$$\therefore T\left(\frac{\partial S}{\partial T}\right)_P - T\left(\frac{\partial S}{\partial T}\right)_V = C_P - C_V = R$$

GATE-6. Ans. (c) Heat absorbed by water = Heat supplied by heater.

$$m_w c_{pw} (\Delta T)_w = P \times t \quad \text{or} \quad 40 \times 4.2 \times (\Delta T)_w = 2 \times 20 \times 60 \quad \text{or} \quad (\Delta T)_w = 14.3^\circ\text{C}$$

GATE-7. Ans. (b)

$$\text{GATE-8. Ans. (a)} \quad \mu = \left(\frac{\partial T}{\partial P}\right)_h \text{ i.e. } \mu > 0, \text{ if } \mu \text{ is (+ive) so } \frac{\partial T}{\partial P} \text{ must be +ive}$$

**GATE-9. Ans. (b)** Joule-Thomson co-efficient  $\left(\frac{\partial T}{\partial P}\right)_h$ . Here,  $\frac{\partial P}{\partial T}$  is -ive and  $\left(\frac{\partial T}{\partial P}\right)_h$  is +ive so  $\frac{\partial T}{\partial P}$  must be +ive so gas will be warmer.

**GATE-10. Ans. (a)** - 4, (b) - 5, (c) - 6, (d) - 1

### Previous 20-Years IES Answers

**IES-1. Ans. (d)** P is a function of v and both are connected by a line path on p and v coordinates.

Thus  $\int pdv$  and  $\int vdp$  are not exact differentials and thus not properties.

If X and Y are two properties of a system, then dx and dy are exact differentials. If the differential is of the form  $Mdx + Ndy$ , then the test for exactness is  $\left[\frac{\partial M}{\partial y}\right]_x = \left[\frac{\partial N}{\partial x}\right]_y$

Now applying above test for

$$\int \left( \frac{dT}{T} + \frac{pdv}{v} \right), \left[ \frac{\partial(1/T)}{\partial v} \right]_T = \left[ \frac{\partial(p/v)}{\partial T} \right]_v = \left[ \frac{\partial(RT/v^2)}{\partial T} \right]_v \text{ or } 0 = \frac{R}{v^2}$$

This differential is not exact and hence is not a point function and hence

$\int \left( \frac{dT}{T} + \frac{pdv}{v} \right)$  is not a point function and hence not a property.

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And for  $\int \left( \frac{dT}{T} - \frac{vdp}{T} \right) \left[ \frac{\partial(1/T)}{\partial p} \right]_T = \left[ \frac{\partial(-v/T)}{\partial T} \right]_P = \left[ \frac{\partial(-R/P)}{\partial T} \right]_P$  or  $0=0$

Thus  $\int \left( \frac{dT}{T} - \frac{vdp}{T} \right)$  is exact and may be written as  $ds$ , where  $s$  is a point function and hence a property

**IES-2. Ans. (a)** From Maxwell relation Clapeyron equation comes.

**IES-3. Ans. (b)** 3 is false. It is derived from the Maxwell's 3<sup>rd</sup> relationship  $\left( \frac{\partial p}{\partial T} \right)_v = \left( \frac{\partial s}{\partial v} \right)_T$

**IES-3a Ans. (c)**

**IES-4. Ans. (a)**

**IES-5. Ans. (c)** The relations in (a) and (b) are applicable for a reversible processes and (c)  $Tds = du + pdV$  is a relation among properties which are independent of the path.

**IES-6. Ans. (b)**

**IES-7. Ans. (c)**

**IES-8. Ans. (a)**

**IES-9. Ans. (c)** Potential energy will converted to heat energy.

$$mgh = mc_v \Delta T \quad \text{or} \quad \Delta T = \frac{gh}{c_v} = \frac{980 \times 4000}{980} = 40K$$

**IES-10. Ans. (b)**

**IES-11. Ans. (d)** A diatomic gas (such as that of oxygen) has six degrees of freedom in all-three corresponding to translator motion, two corresponding to rotatory motion and one corresponding to vibratory motion. Experiments have shown that at ordinary temperatures, the vibratory motion does not occur. Hence, at 27°C, an oxygen molecule has just five degrees of freedom.

**IES-12. Ans. (d)**

$$C_v = \frac{n}{2} R, \quad C_p = \left( \frac{n}{2} + 1 \right) R$$

$$\therefore \frac{C_p}{C_v} = \frac{n+2}{n}$$

**IES-13. Ans. (b)** For ideal gas  $C_p$  and  $C_v$  are constant but mole is depends on the number of atoms in a molecule.

**IES-14. Ans. (c).** A is correct but R is false.

We know that  $C_p = a + KT + K_1T^2 + K_2T^3$

$$C_v = b + KT + K_1T^2 + K_2T^3$$

See  $C_p$  and  $C_v$  both increase with temperature and by same amount. As  $C_p > C_v$  then percentage increase of  $C_p$  is less than  $C_v$ . So  $\frac{C_p}{C_v}$  decreases with temperature.

**IES-15. Ans. (c)** Sign of T must be positive  $\left( \frac{\partial P}{\partial V} \right)_T$  is always negative.

**IES-15a.Ans. (b)**

**IES-16. Ans. (a)**

**IES-16a.Ans. (b)**

**IES-17. Ans. (d)** Actually Joule – Thomson coefficient will be positive.

**IES-17a.Ans. (b)**

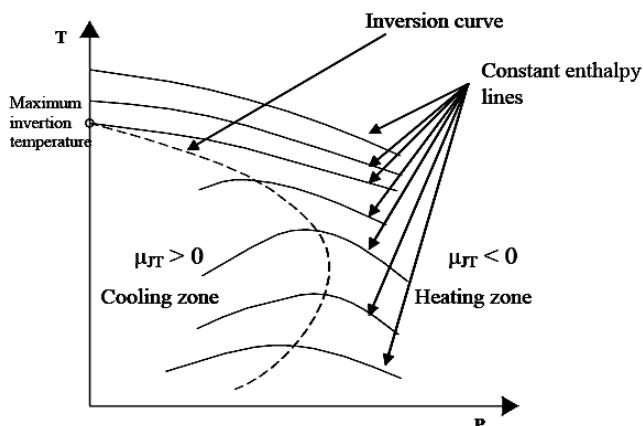
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**IES-18. Ans. (c)** For ideal gas  $\mu = 0$  and for real gas  $\mu$  may be positive ( $N_2, O_2, CO_2$  etc.) or negative ( $H_2$ ).

**IES-19. Ans. (d)** When a real gas which is initially at a temperature lower than the maximum inversion temperature is throttled, its temperature decreases.



**IES-20. Ans. (b)**

**IES-21. Ans. (b)** Joule Thomson coefficient is the ratio of temperature change to pressure change when a gas undergoes adiabatic throttling.

**IES-22. Ans. (a)** The slope of the isenthalpic curve at any point is known as Joule-Thomson coefficient and is expressed as,  $\mu = \left( \frac{\partial T}{\partial P} \right)_h$

**IES-23. Ans. (a)**

**IES-24. Ans. (b)**

**IES-25. Ans. (c)**

$$\text{IES-26. Ans. (c)} \quad \left( \frac{dP}{dT} \right)_{sat} = \frac{h_g}{T(V_g - V_f)}$$

$$\text{or } h_g = T(V_g - V_f) \times \left( \frac{dP}{dT} \right)_{sat} = 400 \times (0.251 - 0.001) \times 0.1 \times 10^5 \text{ J/kg} = 1000 \text{ kJ/kg}$$

**IES-27. Ans. (b)**

**IES-28. Ans. (b)**

**IES-29. Ans. (b)**

**IES-30. Ans. (c)**

**IES-31. Ans. (d)** Clapeyron equation state provides relationship between temperature, specific volume, pressure and enthalpy.

**IES-32. Ans. (d)**

**IES-33. Ans. (c)**

**IES-34. Ans. (d)**  $F = C - P + 2$

**IES-35. Ans. (a)** Gibb's free energy 'G' is defined as  $G = H - TS$ .

**IES-36. Ans. (c)**

**IES-37. Ans. (d)**  $F = C - P + 2$

$$C = 1, P = 3 \text{ or } F = 1 - 3 + 2 = 0$$

**IES-38. Ans. (d)** *Isomers:* Compounds with the same chemical formula but different molecular structure.

# Thermodynamic Relations

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## Previous 20-Years IAS Answers

**IAS-1. Ans. (c)**  $\left(\frac{\partial P}{\partial T}\right)_V = \left(\frac{\partial S}{\partial V}\right)_T$  to memorize Maxwell's relation remember T V P S, -ive and S S V P see highlights.

**IAS-2. Ans. (a)**  $dQ = dh - Vdp$  or  $Tds = dh - Vdp$

**IAS-3. Ans. (c)**  $C_p = \frac{dQ}{dT} = T \left( \frac{\partial S}{\partial T} \right)_p$   $[\because dQ = TdS]$

**IAS-4. Ans. (c)**

$$T \left( \frac{\partial S}{\partial T} \right)_P = \left( \frac{T \partial S}{\partial T} \right)_P = \left( \frac{dQ}{dT} \right)_P = C_p$$

$$T \left( \frac{\partial S}{\partial T} \right)_V = \left( \frac{T \partial S}{\partial T} \right)_V = \left( \frac{dQ}{dT} \right)_V = C_v$$

$$\therefore T \left( \frac{\partial S}{\partial T} \right)_P - T \left( \frac{\partial S}{\partial T} \right)_V = C_p - C_v = R$$

**IAS-5. Ans. (a)** Both A and R correct and R is the correct explanation of A

**IAS-6. Ans. (b)**  $C_p - C_v$  for all ideal gas is R, So C-3, (a) & (c) out. A automatically match 4, and

$$C_p = \frac{\gamma}{\gamma-1} R \text{ for monatomic gas } \gamma = \frac{5}{3}. \text{ So, } \gamma = \frac{5}{2} R$$

**IAS-7. Ans. (a)**  $C_p - C_v = R$  and  $\gamma = \frac{C_p}{C_v} = \frac{C_p}{C_p - R} = \frac{1}{1 - \frac{R}{C_p}}$

**IAS-8. Ans. (b)**

**IAS-9. Ans. (b)**

**IAS-10. Ans. (c)**

**IAS-11. Ans. (a)**

**IAS-12. Ans. (d)**

**IAS-13. Ans. (b)**

**IAS-14. Ans. (b)**

## 7.

# Pure Substances

## Theory at a Glance (For GATE, IES & PSUs)

A pure substance is defined as one that is homogeneous and invariable in chemical composition throughout its mass. The relative proportions of the chemical elements constituting the substance are also constant. Atmospheric air, steam – water mixture and combustion products of a fuel are regarded as pure substances. But the mixture of air and liquid air is not a pure substance, since the relative proportions of oxygen and nitrogen differ in the gas and liquid phases in equilibrium.

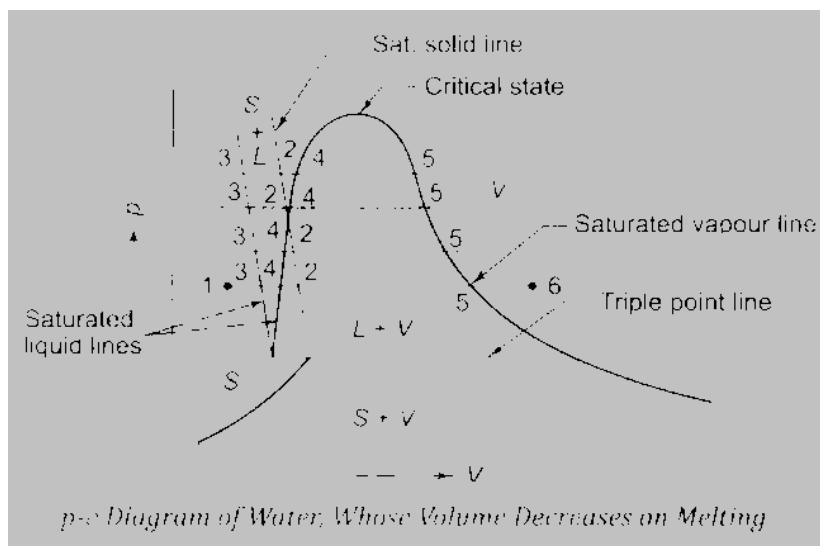
Mixtures are not pure substances. (e.g., Humid air)

**Exception!! Air is treated as a pure substance though it is a mixture of gases.**

In a majority of cases a minimum of two properties are required to define the state of a system. The best choice is an extensive property and an intensive property.

The state of a pure substance of given mass can be fixed by specifying two independent intensive properties, provided the system is in equilibrium. This is known as the '**two-property rule**'. The state can thus be represented as a point on thermodynamic property diagrams. Once any two properties of a pure substance are known, other properties can be determined from the available thermodynamic relations.

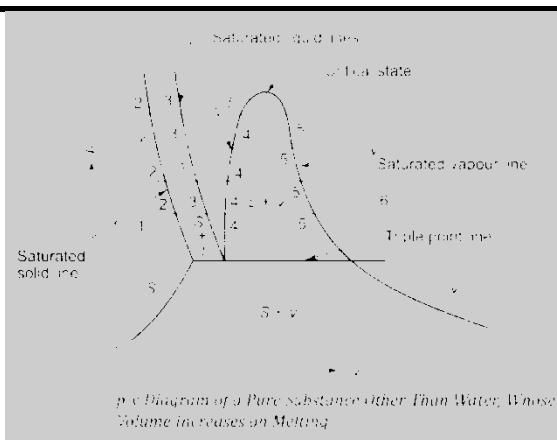
## p-v Diagram for a Pure Substance



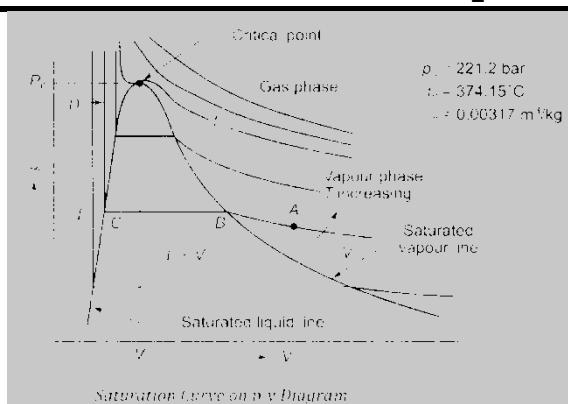
# Pure Substances

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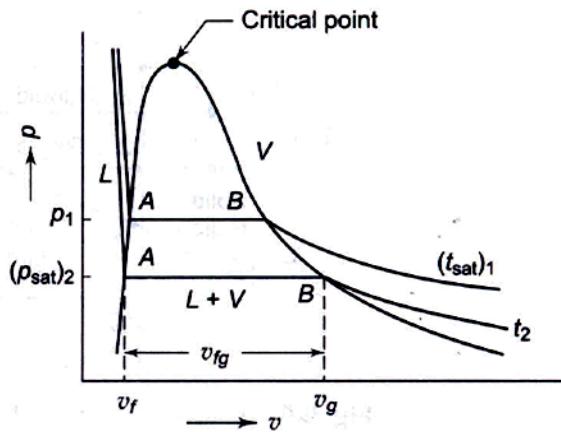
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p-v Diagram of a Pure Substance other Than Water, Whose Volume increases on Melting



Saturation Curve on p-v Diagram



Saturation Pressure and Temperature

## Triple point

Triple point is a point where all the three phases exist in equilibrium.

$u = 0$  for water at triple point. (You can assign any number you like instead of 0).

Since,  $p$  and  $v$  for water at triple point are known you can calculate  $h$  for water at triple point (it will not be zero).

The entropy of saturated water is also chosen to be zero at the **triple point**.

## Gibbs Phase Rule at Triple Point

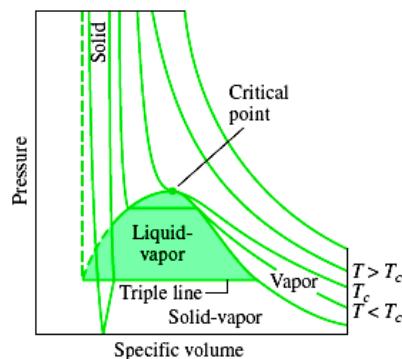
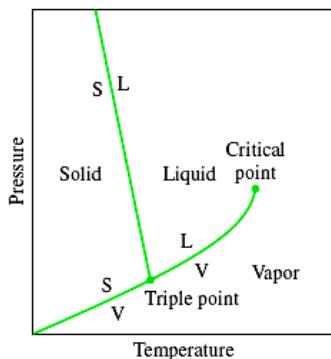
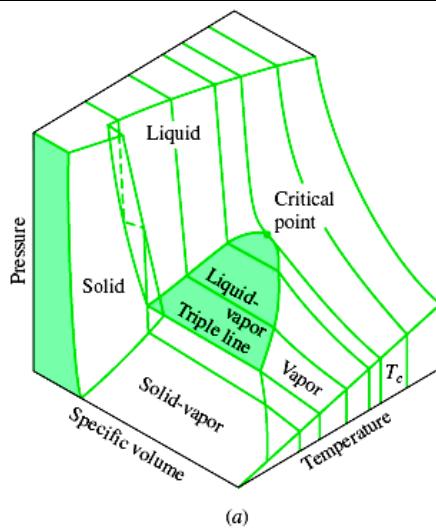
If  $C = 1$ ,  $\phi = 3$ , then  $f = 0$ . The state is thus unique for a substance; and refers to the triple point where all the three phases exist in equilibrium.

- $\ln p = A + B/T$  valid only near the triple point.(Called Antoine's equation)

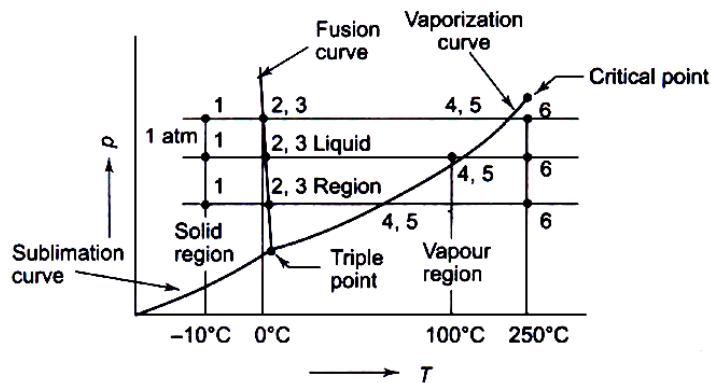
# Pure Substances

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## $p$ - $T$ Diagram for a Pure Substance



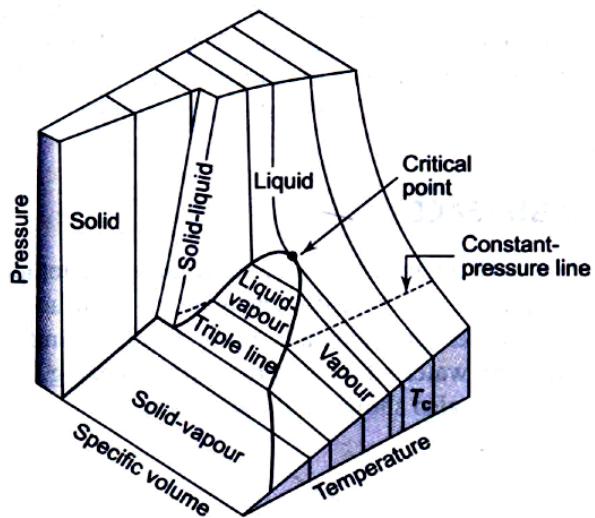
Phase Equilibrium Diagram on  $p$ - $T$  Coordinates

# Pure Substances

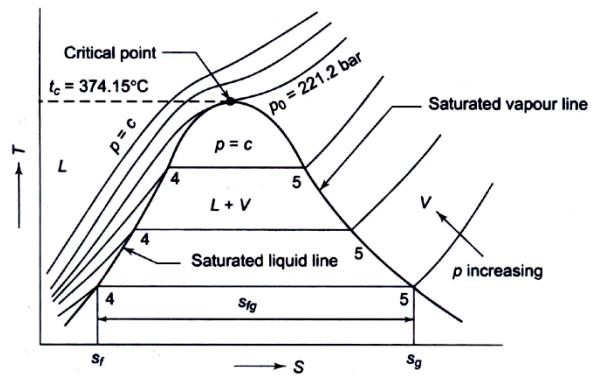
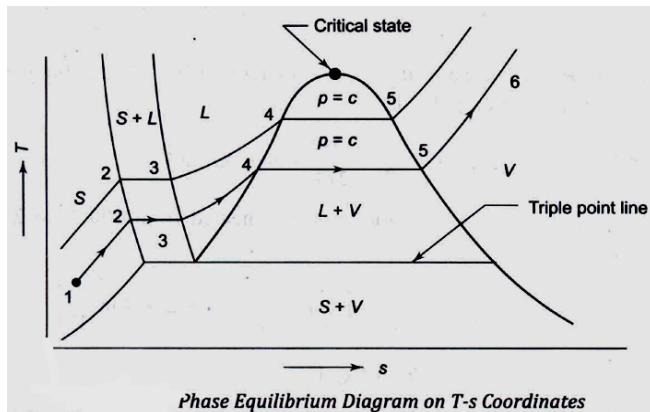
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## $p$ - $v$ - $T$ Surface



## T-s Diagram for a Pure Substance



## Critical Point

A state at which a phase change begins or ends is called a **saturation state**. The dome shaped region composed of the two-phase liquid–vapor states is called the **vapor dome**. The lines bordering the vapor dome are called saturated liquid and saturated vapor lines. At the top of the dome, where the saturated liquid and saturated vapor lines meet, is the **critical point**. The **critical temperature**  $T_c$  of a pure substance is the maximum temperature at which liquid and vapor phases can coexist in equilibrium. The pressure at the critical point is called the **critical pressure**,  $p_c$ . The specific volume at this state is the **critical specific volume**.

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The isotherm passing through the critical point is called the *critical isotherm*, and the corresponding temperature is known as the *critical temperature* ( $t_c$ ). The pressure and volume at the critical point are known as the *critical pressure* ( $P_c$ ) and the *critical volume* ( $v_c$ ) respectively. For water

$$P_c = 221.2 \text{ bar}$$

$$t_c = 374.15^\circ\text{C}$$

$$v_c = 0.00317 \text{ m}^3/\text{kg}$$

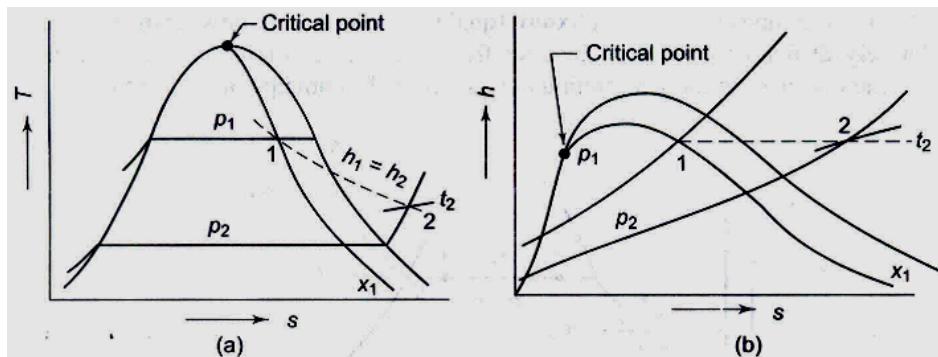
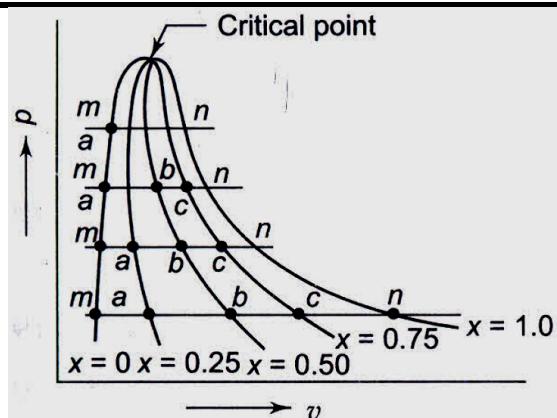


Fig. Critical Point on P-v, T-S, h-s diagram

## Characteristics of the critical point:

1. It is the highest temperature at which the liquid and vapour phases can coexist.
2. At the critical point  $h_{fg}$ ,  $u_{fg}$  and  $v_{fg}$  are zero.
3. Liquid vapour meniscus will disappear.
4. Specific heat at constant pressure is infinite.
  - A majority of engineering applications (e.g., steam based power generation; Refrigeration, gas liquefaction) involve thermodynamic processes close to saturation.
  - For saturated phase often it enthalpy is an important property.
  - Enthalpy-pressure charts are used for refrigeration cycle analysis.
  - Enthalpy-entropy charts for water are used for steam cycle analysis.
  - Note: Unlike pressure, volume and temperature which have specified numbers associated with it, in the case of internal energy, enthalpy (and entropy) only changes are required. Consequently, a base (or datum) is defined - as you have seen in the case of water.
  - For example for NIST steam tables  $u = 0$  for water at triple point. (You can assign any number you like instead of 0). [Don't be surprised if two different sets of steam tables give different values for internal energy and enthalpy].

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- Since,  $p$  and  $v$  for water at triple point are known you can calculate  $h$  for water at triple point (it will not be zero).

## ***h-s Diagram or Mollier Diagram for a Pure Substance***

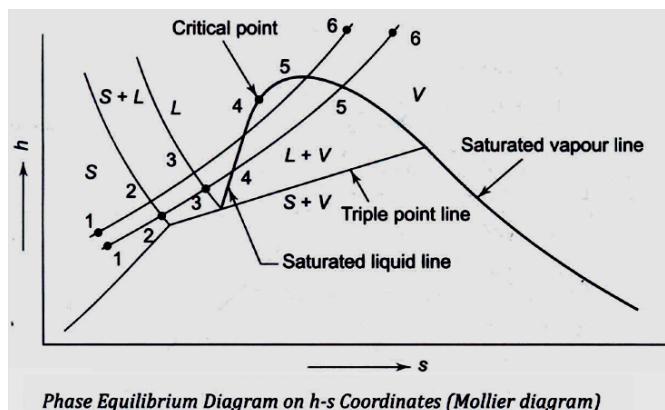
From the first and second laws of thermodynamics, the following property relation was obtained.

$$Tds = dh - vdp$$

or

$$\left(\frac{\partial h}{\partial s}\right)_p = T$$

This equation forms the basis of the *h-s* diagram of a pure substance, also called the **Mollier diagram**. The slope of an isobar on the *h-s* coordinates is equal to the absolute saturation temperature ( $t_{sat} + 273$ ) at that pressure. If the temperature remains constant the slope will remain constant. If the temperature increases, the slope of the isobar will increase.



## **Quality or Dryness Fraction**

- The zone between the saturated liquid and the saturated vapour region is called the two phase region - where the liquid and vapour can co-exist in equilibrium.
- Dryness fraction: It is the mass fraction of vapour in the mixture.
- Normally designated by 'x'.
- On the **saturated liquid** line  $x = 0$
- On the **saturated vapour** line  $x = 1$
- $x$  can have a value only between 0 and 1
- Data tables will list properties at the two ends of saturation.
- To calculate properties in the two-phase region:
  - $p, T$  will be the same as for saturated liquid or saturated vapour.

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$$V = V_g + (1-x)V_f$$

$$h = h_g + (1-x)h_f$$

$$u = u_g + (1-x)u_f$$

AND

$$v = v_f + xV_{fg}$$

$$u = u_f + xU_{fg}$$

$$h = h_f + xh_{fg}$$

$$s = s_f + xS_{fg}$$

## Saturation States

When a liquid and its vapour are in equilibrium at a certain pressure and temperature, only the pressure or the temperature is sufficient to identify the saturation state. If the pressure is given, the temperature of the mixture gets fixed, which is known as the saturation temperature, or if the temperature is given, the saturation pressure gets fixed. Saturated liquid or the saturated vapour has only one independent variable, i.e., only one property is required to be known to fix up the state. A Steam Table (saturated state) give the properties of saturated liquid and saturated vapour. In steam table the independent variable is temperature. At a particular temperature, the values of saturation pressure  $p$ , and states,  $V_g, h_g$  and  $S_g$  refer to the saturated vapour state; and  $V_{fg}, h_{fg}$ , and  $S_{fg}$  refer to the changes in the property values during evaporation (or condensation) at the temperature. where  $V_{fg} = V_g - V_f$  and  $S_{fg} = S_g - S_f$ .

If steam Table the independent variable is pressure. At a particular pressure, the values of saturation temperature  $t$ , and  $v_f, v_g, h_f, h_{fg}, s_f$ , and  $s_g$  are given. whether the pressure or the temperature is given, either steam Table can be conveniently used for computing the properties of saturation states.

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If data are required for intermediate temperature or pressures, linear interpolation is normally accurate. The reason for the two tables is to reduce the amount of interpolation required.

## Liquid vapour Mixtures:

Let us consider a mixture of saturated liquid water and water vapour in equilibrium at pressure  $p$  and temperature  $t$ . The composition of the mixture by mass will be within the vapour dome figure. The properties of the mixture are as given in Article. i.e

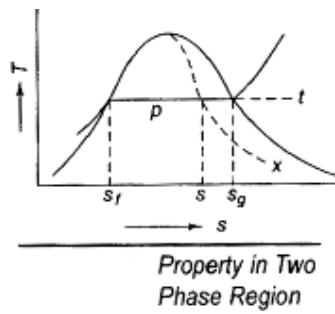


Fig.

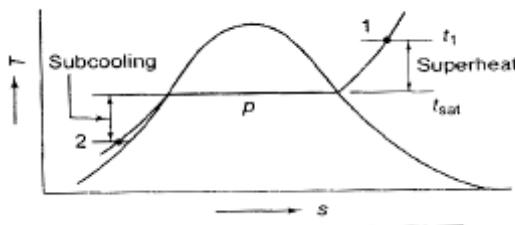
where  $v_f$ ,  $v_{fg}$ ,  $h_{fg}$ ,  $s_f$  and  $s_{fg}$  are the saturation properties at the given pressure and temperature.

If  $p$  or  $t$  and the quality of the mixture are given, the properties of the mixture ( $v$ ,  $u$ ,  $h$  and  $s$ ) can be evaluated from the above equations. Sometimes instead of quality, one of the above equations say specific volume  $v$  and pressure or temperature are given. In that case, the quality of the mixture  $x$  has to be calculated from the given  $v$  and  $p$  or  $t$  and then  $x$  being known, other properties are evaluated.

If  $p$  or  $t$  and the quality of the mixture are given, the properties of the mixture ( $v$ ,  $u$ ,  $h$  and  $s$ ) can be evaluated from the above equations. Sometimes, instead of quality, one of the above properties, say, specific volume  $v$  and pressure or temperature are given in that case, the quality of the mixture  $x$  has to be calculated from the given  $v$  and  $p$  or  $t$  and then  $x$  being known, other properties are evaluated.

## Superheated Vapour:

When the temperature of the vapour is greater than the saturation temperature corresponding to the given pressure, the vapour is said to be superheated (state 1 in figure). The difference between the temperature of the superheated vapour and the saturation temperature at that pressure is called the superheat or the degree of superheat. As shown in figure below, the difference ( $t_1 - t_{\text{sat}}$ ) is the superheat.



(Fig. - Superheat and Sub-cooling.)

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In a superheated vapour at a given pressure, the temperature may have different values greater than the saturation temperature. Steam Table gives the values of the properties (volume, enthalpy, and entropy) of superheated vapour for each tabulated pair of values of pressure and temperature, both of which are now independent. Interpolation or extrapolation is to be used for pairs of values of pressure and temperature not given.

## Compressed Liquid:

When the temperature of a liquid is less than the saturation temperature at the given pressure, the liquid is called compressed liquid (state 2 in figure above). The pressure and temperature of compressed liquid may vary independently, and a table of properties like the superheated vapour table could be arranged, to give the properties at any  $p$  and  $t$ . However, the properties of liquids vary little with pressure. Hence the properties are taken from the saturation tables at the temperature of the compressed liquid. When a liquid is cooled below its saturation temperature at a certain pressure it is said to be sub-cool. The difference in saturation temperature and the actual liquid temperature is known as the degree of Sub-cooling, or simply, Sub-cooling.

## Charts of Thermodynamic Properties

- One of the important properties is the change in enthalpy of phase transition  $h_{fg}$  also called the latent heat of vaporisation or latent heat of boiling. It is equal to  $h_g - h_f$ .
- Similarly  $u_{fg}$  - internal energy change due to evaporation and  $v_{fg}$  - volume change due to evaporation can be defined (but used seldom).
- The saturation phase depicts some very interesting properties:
- The following saturation properties depict a maximum:

1 $T\alpha$	2 $T(\alpha - \alpha_g)$	3 $Th_{fg}$	4 $T_c(p_c - p)$
5 $p(T_c - T)$	6 $p(v_g - v_f)$	7 $T(\rho_c^2 - \rho_f \rho_g)$	8 $h_g$
- The equation relating the pressure and temperature along the saturation is called the vapour pressure curve.
- Saturated liquid phase can exist only between the triple point and the critical point.

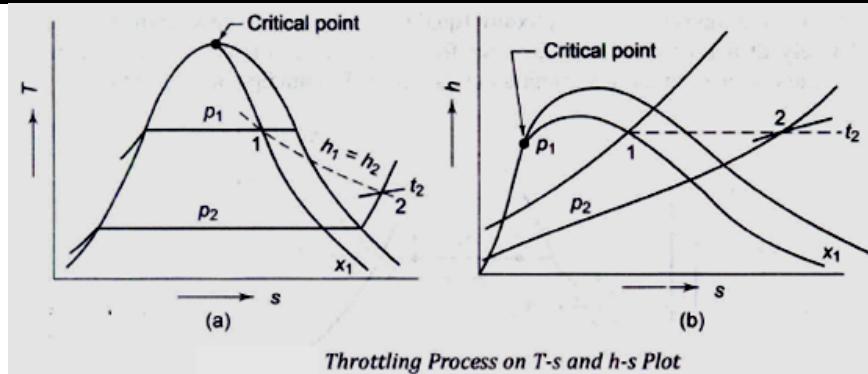
## Measurement of Steam Quality

**Throttling calorimeter:** Throttling calorimeter is a device for determining the quality of a two-phase liquid-vapor mixture.

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Intermediate states are non-equilibrium states not describable by thermodynamic coordinates. The initial state (wet) is given by  $p_1$  and  $x_1$ , and the final state by  $p_2$  and  $t_2$  (superheated). Now

since

$$h_1 = h_2$$

$$H_{fp1} + x_1 h_{fgp1} = h_2$$

or

$$x_1 = \frac{h_2 - h_{fp1}}{h_{fgp1}}$$

With  $p_2$  and  $t_2$  being known,  $h_2$  can be found out from the superheated steam table. The values of  $h_f$  and  $h_{fg}$  are taken from the saturated steam table corresponding to pressure  $p_1$ . Therefore, the quality of the wet steam  $x_1$  can be calculated.

## The Throttling Calorimeter

### Example

A supply line carries a two-phase liquid-vapor mixture of steam at 20 bars. A small fraction of the flow in the line is diverted through a throttling calorimeter and exhausted to the atmosphere at 1 bar. The temperature of the exhaust steam is measured as 120°C. Determine the quality of the steam in the supply line.

### Solution

**Known:** Steam is diverted from a supply line through a throttling calorimeter and exhausted to the atmosphere.

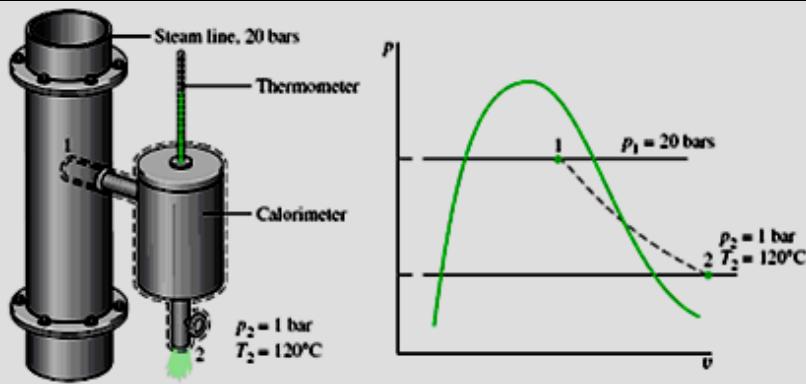
**Find:** Determine the quality of the steam in the supply line.

**Schematic and Given Data:**

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## Assumptions:

1. The control volume shown on the accompanying figure is at steady state.
2. The diverted steam undergoes a throttling process.

**Analysis:** For a throttling process, the energy and mass balances reduce to give  $h_2 = h_1$ . Thus, with state 2 fixed, the specific enthalpy in the supply line is known, and state 1 is fixed by the known values of  $p_1$  and  $h_1$ .

As shown on the accompanying  $p-v$  diagram, state 1 is in the two-phase liquid-vapor region and state 2 is in the superheated vapor region. Thus

$$h_2 = h_1 = h_{f1} + x_1(h_{g1} - h_{f1})$$

Solving for  $x_1$

$$x_1 = \frac{h_2 - h_{f1}}{h_{g1} - h_{f1}}$$

From steam table at 20 bars,  $h_{f1} = 908.79$  kJ/kg and  $h_{g1} = 2799.5$  kJ/kg. At 1 bar and 120°C,  $h_2 = 2766.6$  kJ/kg from steam table. Into the above expression, the quality in the line is  $x_1 = 0.956$  (95.6%).

For throttling calorimeters exhausting to the atmosphere, the quality in the line must be greater than about 94% to ensure that the steam leaving the calorimeter is superheated.

## Throttling

When a fluid flows through a constricted passage, like a partially opened valve, an orifice, or a porous plug, there is an appreciable drop in pressure, and the flow is said to be throttled. Figure shows the process of throttling by a partially opened valve on a fluid flowing in an insulated pipe. In the steady-flow energy equation.

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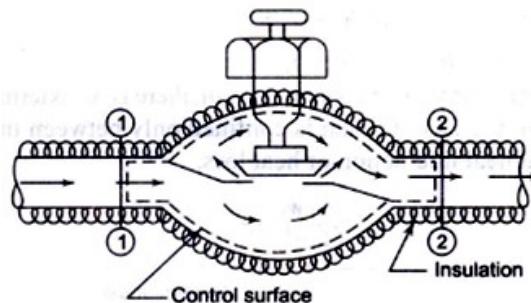
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$$\frac{dQ}{dm} = 0, \quad \frac{dW_x}{dm} = 0$$

and the changes in P.E. are very small and ignored. Thus, the S.F.E.E. reduces to

$$h_1 + \frac{C_1}{2} = h_2 + \frac{C_2}{2}$$

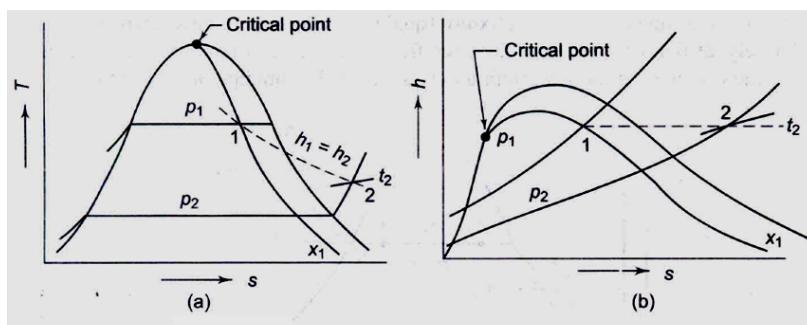


*Flow Through a Valve*

Often the pipe velocities in throttling are so low that the K.E. terms are also negligible. So

$$h_1 = h_2$$

or the enthalpy of the fluid before throttling is equal to the enthalpy of the fluid after throttling.



*Throttling Process on T-s and h-s Plot*

Consider a **throttling process** (also referred to as wire drawing process)



There is no work done (rising a weight)

$$W = 0$$

If there is no heat transfer

$$Q = 0$$

Conservation of mass requires that

$$C_1 = C_2$$

Since 1 and 2 are at the same level

$$Z_1 = Z_2$$

From SFEE it follows that

$$h_1 = h_2$$

**Conclusion: Throttling is a constant enthalpy process (isenthalpic process)**

# Pure Substances

## Properties of Pure Substances

### Highlights

#### 1. Triple Point

On P – T diagram it is a **Point**.

On P – V diagram it is a **Line**. Also in T – S diagram it is a **Line**.

On U – V diagram it is a **Triangle**.

#### 2. Triple point of water

$$T = 273.16 \text{ K}$$

$$P = 0.00612 \text{ bar}$$

$$\text{Entropy (s)} = 0$$

$$= 0.01^\circ\text{C}$$

$$= 4.587 \text{ mm of Hg}$$

$$\text{Internal energy (u)} = 0$$

$$\text{Enthalpy} = u + PV = \text{slightly positive.}$$

#### 3. Triple point of CO<sub>2</sub>

$$P \approx 5 \text{ atm} \quad \text{and} \quad T = 216.55 \text{ K}$$

$$= -56.45^\circ\text{C} \text{ that so why sublimation occurred.}$$

#### 4. Critical point

$$\text{For water } p_c = 221.2 \text{ bar} \approx 225.5 \text{ kgf/cm}^2$$

$$t_c = 374.15^\circ\text{C} \approx 647.15 \text{ K}$$

$$v_c = 0.00317 \text{ m}^3/\text{kg}$$

At critical point

$$h_{fg} = 0; \quad v_{fg} = 0; \quad s_{fg} = 0$$

#### 5. Mollier Diagram

$$\text{Basis of the h-s diagram is } \left( \frac{\partial h}{\partial s} \right)_P = T$$

$\left[ \begin{array}{l} \because Tds = dh - vdp \\ \therefore \left( \frac{\partial h}{\partial s} \right)_P = T \end{array} \right]$

∴ The slope of an isobar on the h-s co-ordinates is equal to the absolute saturation temperature at that pressure. And for that isobars on Mollier diagram diverges from one another.

#### 6. Dryness fraction

$$x = \frac{m_v}{m_v + m_L}$$

7.  $v = (1 - x) v_f + xv_g$        $v = v_f + xv_{fg}$   
 $u = (1 - x) u_f + xu_g$        $u = u_f + xu_{fg}$   
 $h = (1 - x) h_f + xh_g$       and       $h = h_f + xh_{fg}$   
 $s = (1 - x) s_f + xs_g$        $s = s_f + xs_{fg}$
8. **Super heated vapour:** When the temperature of the vapour is greater than the saturation temperature corresponding to the given pressure.
9. **Compressed liquid:** When the temperature of the liquid is less than the Saturation temperature at the given pressure, the liquid is called compressed liquid.
10. **In combined calorimeter**  
 $x = x_1 \times x_2$        $x_1$  = from throttle calorimeter.  
 $x_2$  = from separation calorimeter.

# Pure Substances

S K Mondal's

Chapter 7

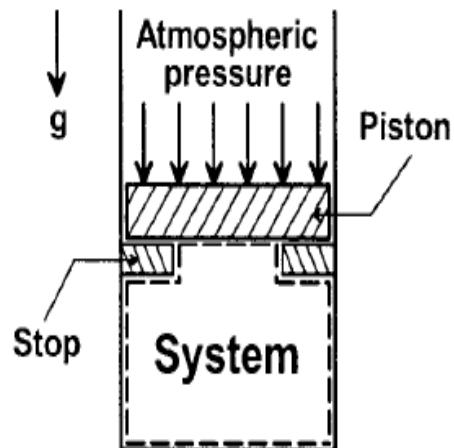
## ASKED OBJECTIVE QUESTIONS (GATE, IES, IAS)

### Previous 20-Years GATE Questions

#### Common data for Question Q1 – Q3

In the figure shown, the system is a pure substance kept in a piston-cylinder arrangement. The system is initially a two-phase mixture containing 1 kg of liquid and 0.03 kg of vapour at a pressure of 100 kPa. Initially, the piston rests on a set of stops, as shown in the figure. A pressure of 200 kPa is required to exactly balance the weight of the piston and the outside atmospheric pressure. Heat transfer takes place into the system until its volume increases by 50%. Heat transfer to the system occurs in such a manner that the piston, when allowed to move, does so in a very slow (quasi-static) quasi-equilibrium process. The thermal reservoir from which heat is transferred to the system has a temperature of 400°C. Average temperature of the system boundary can be taken as 17°C. The heat transfer to the system is 1 kJ, during which its entropy increases by 10 J/K.

Atmospheric pressure.



Specific volumes of liquid ( $v_f$ ) and vapour ( $v_g$ ) phases, as well as values of saturation temperatures, are given in the table below.

Pressure (kPa)	Saturation temperature, $T_{sat}$ (°C)	$v_f$ (m <sup>3</sup> /kg)	$v_g$ (m <sup>3</sup> /kg)
100	100	0.001	0.1
200	200	0.0015	0.002

**GATE-1.** At the end of the process, which one of the following situations will be true?

- (a) Superheated vapour will be left in the system
- (b) No vapour will be left in the system
- (c) A liquid + vapour mixture will be left in the system
- (d) The mixture will exist at a dry saturated vapour state

[GATE-2008]

**GATE-2.** The work done by the system during the process is:

- (a) 0.1 kJ
- (b) 0.2 kJ
- (c) 0.3 kJ

[GATE-2008]

- (d) 0.4 kJ

**GATE-3.** The net entropy generation (considering the system and the thermal reservoir together) during the process is closest to:

- (a) 7.5 J/K
- (b) 7.7 J/K
- (c) 8.5 J/K

[GATE-2008]

- (d) 10 J/K

# Pure Substances

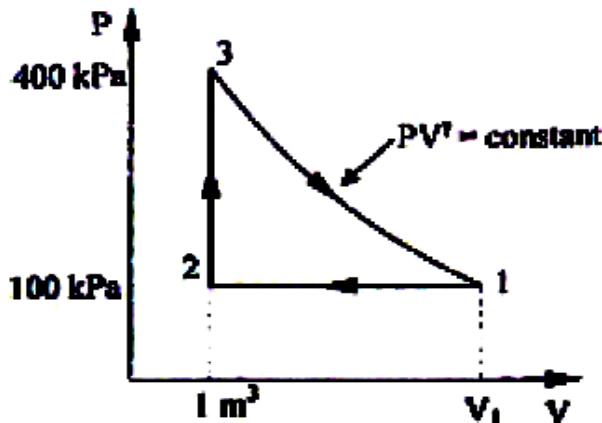
S K Mondal's

Chapter 7

## T-s Diagram for a Pure Substance

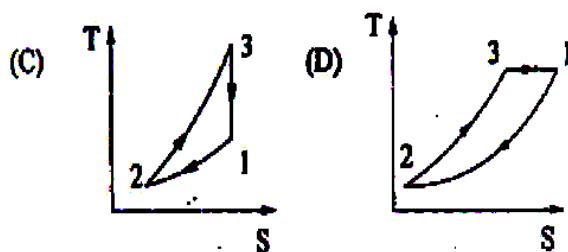
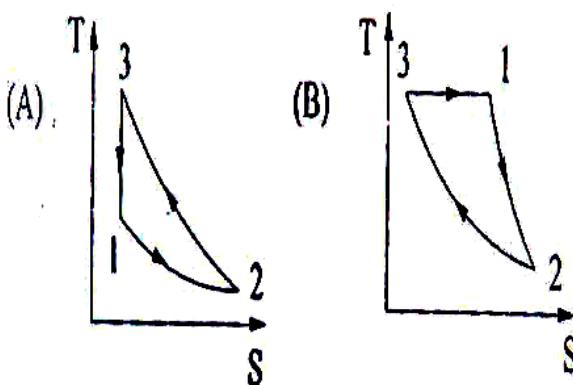
Common Data for Questions GATE-4- GATE-5

A thermodynamic cycle with an ideal gas as working fluid is shown below.



GATE-4. The above cycle is represented on T-S plane by

[GATE-2007]



GATE-5. If the specific heats of the working fluid are constant and the value of specific heat ratio  $\gamma$  is 1.4, the thermal efficiency (%) of the cycle is: [GATE-2007]  
(a) 21      (b) 40.9      (c) 42.6      (d) 59.7

GATE-6. The slopes of constant volume and constant pressure lines in the T-s diagram are..... and..... respectively. [GATE-1994]

# Pure Substances

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## ***h-s Diagram or Mollier Diagram for a Pure Substance***

**GATE-7.** Constant pressure lines in the superheated region of the Mollier diagram have what type of slope? [GATE-1995]

- (a) A positive slope
- (b) A negative slope
- (c) Zero slope
- (d) May have either positive or negative slopes

## **Quality or Dryness Fraction**

**GATE-8.** Consider a Rankine cycle with superheat. If the maximum pressure in the cycle is increased without changing the maximum temperature and the minimum pressure, the dryness fraction of steam after the isentropic expansion will increase. [GATE-1995]

## **Throttling**

**Statement for Linked Answer Questions Q9 & Q10:**

The following table of properties was printed out for saturated liquid and saturated vapour of ammonia. The titles for only the first two columns are available. All that we know is that the other columns (columns 3 to 8) contain data on specific properties, namely, internal energy (kJ/kg), enthalpy (kJ/kg) and entropy (kJ/kgK) [GATE-2005]

T(°C)	P(kPa)						
-20	190.2	88.76	0.3657	89.05	5.6155	1299.5	1418.0
0	429.6	179.69	0.7114	180.36	5.3309	1318.0	1442.2
20	857.5	272.89	1.0408	274.30	5.0860	1332.2	1460.2
40	1554.9	368.74	1.3574	371.43	4.8662	1341.0	1470.2

**GATE-9.** The specific enthalpy data are in columns [GATE-2005]  
(a) 3 and 7      (b) 3 and 8      (c) 5 and 7      (d) 5 and 8

**GATE-10.** When saturated liquid at 40°C is throttled to -20°C, the quality at exit will be [GATE-2005]  
(a) 0.189      (b) 0.212      (c) 0.231      (d) 0.788

**GATE-11.** When wet steam flows through a throttle valve and remains wet at exit  
(a) its temperature and quality increases [GATE-1996]  
(b) its temperature decreases but quality increases  
(c) its temperature increases but quality decreases  
(d) its temperature and quality decreases

**GATE-12.** When an ideal gas with constant specific heats is throttled adiabatically, with negligible changes in kinetic and potential energies [GATE-2000]  
(a)  $\Delta h=0, \Delta T=0$     (b)  $\Delta h>0, \Delta T=0$     (c)  $\Delta h>0, \Delta S>0$     (d)  $\Delta h=0, \Delta S>0$

Where  $h$ ,  $T$  and  $S$  represent respectively, enthalpy, temperature and entropy, temperature and entropy

# Pure Substances

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- GATE-13. One kilomole of an ideal gas is throttled from an initial pressure of 0.5 MPa to 0.1 MPa. The initial temperature is 300 K. The entropy change of the universe is:  
[GATE-1995]  
(a) 13.38 kJ/K      (b) 401.3 kJ/K      (c) 0.0446 kJ/K      (d) -0.0446 kJ/K

## Previous 20-Years IES Questions

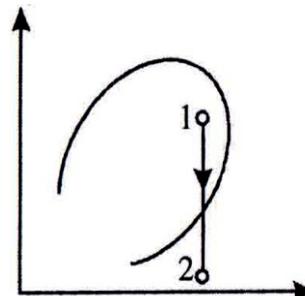
- IES-1. Assertion (A): Water is not a pure substance. [IES-1999]

Reason (R): The term pure substance designates a substance which is homogeneous and has the same chemical composition in all phases.

- (a) Both A and R are individually true and R is the correct explanation of A  
(b) Both A and R are individually true but R is NOT the correct explanation of A  
(c) A is true but R is false  
(d) A is false but R is true

- IES-2. The given diagram shows an isometric cooling process 1-2 of a pure substance. The ordinate and abscissa are respectively

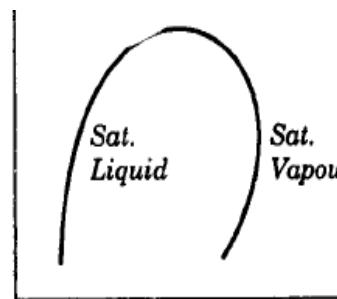
- (a) Pressure and volume  
(b) Enthalpy and entropy  
(c) Temperature and entropy  
(d) Pressure and enthalpy



[IES-1998]

- IES-3. The ordinate and abscissa in the given figure showing the saturated liquid and vapour regions of a pure substance represent:

- (a) Temperature and pressure  
(b) Enthalpy and entropy  
(c) Pressure and volume  
(d) Pressure and enthalpy

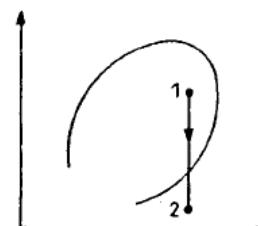


[IES-1997]

- IES-4. The given diagram shows the throttling process of a pure substance.

The ordinate and abscissa are respectively

- (a) Pressure and volume  
(b) Enthalpy and entropy  
(c) Temperature and entropy  
(d) Pressure and enthalpy



[IES-1995]

- IES-4a. Consider the following statements for a throttling process:

1. It is an adiabatic process.

[IES-2012]

# Pure Substances

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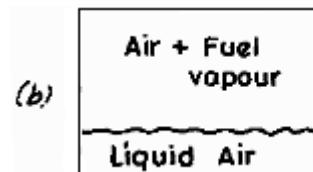
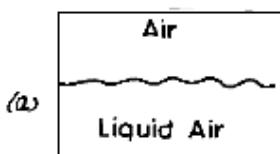
2. There is no work transfer in the process.

3. Entropy increases in throttling process

Which of these statements are correct ?

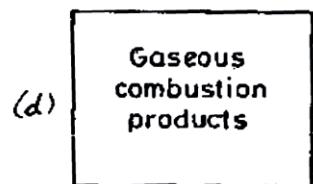
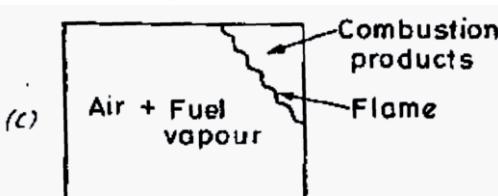
(a) 1, 2 and 3 (b) 1 and 2 only (c) 2 and 3 only (d) 1 and 3 only

IES-5. Which one of the following systems can be considered to be containing a pure substance? [IES-1993]



No Reaction

O<sub>2</sub> : N<sub>2</sub> proportions differ in the gas and liquid phase in equilibrium



IES-6. Consider the following: [IES-2009]

1. Air                  2. Gaseous combustion products                  3. Steam

Which of these are pure substances, assuming there is no phase change?

(a) 1 and 2 only      (b) 1 and 3 only      (c) 2 and 3 only      (d) 1, 2 and 3

IES-7. Assertion (A): At a given temperature, the enthalpy of super-heated steam is the same as that of saturated steam. [IES-1998]

Reason (R): The enthalpy of vapour at lower pressures is dependent on temperature alone.

- (a) Both A and R are individually true and R is the correct explanation of A  
(b) Both A and R are individually true but R is NOT the correct explanation of A  
(c) A is true but R is false  
(d) A is false but R is true

IES-8. Considers the following properties of vapour: [IES-2009]

1. Pressure                  2. Temperature  
3. Dryness fraction                  4. Specific volume

Which of these two properties alone are not sufficient to specify the condition of a vapour?

- (a) 1 and 2      (b) 1 and 3      (c) 2 and 3      (d) 3 and 4

IES-9. Which one of the following is correct? [IES-2008]

The specific volume of water when heated from 0°C

- (a) First increases and then decreases      (b) First decreases and then increases  
(c) Increases steadily      (d) Decreases steadily

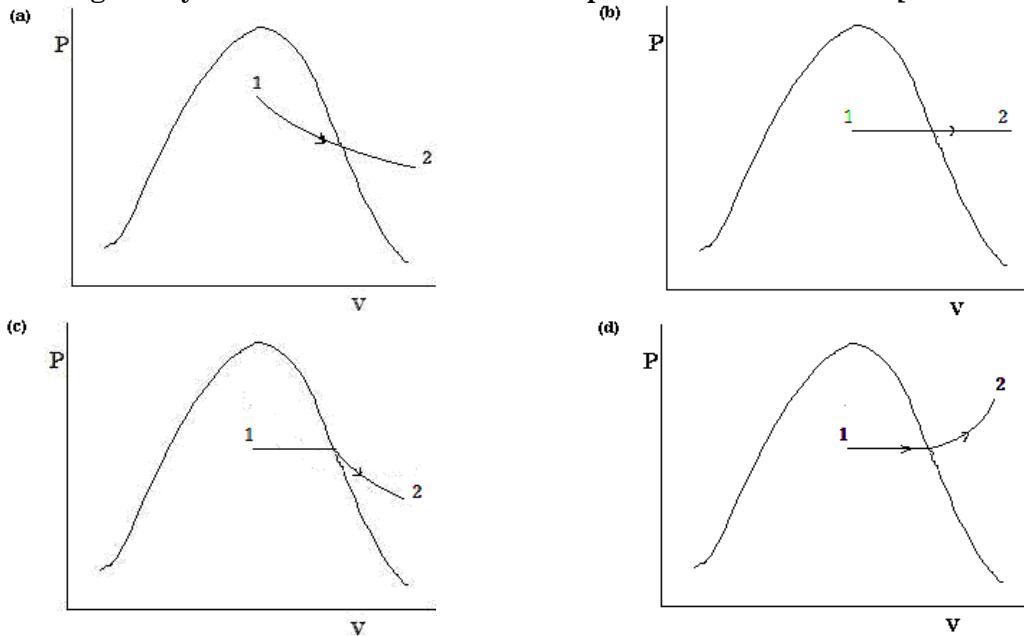
# Pure Substances

S K Mondal's

Chapter 7

## $p-v$ Diagram for a Pure Substance

IES-10. Which  $p-v$  diagram for steam illustrates correctly the isothermal process undergone by wet steam till it becomes superheated? [IES 1995, 2007]



## Triple Point

IES-10a. No substance can exist in the liquid phase in stable equilibrium

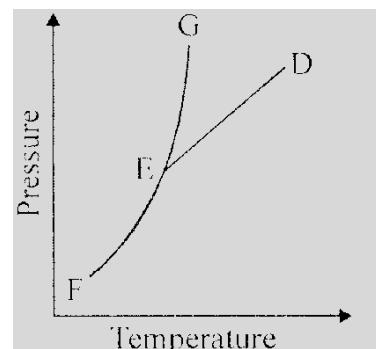
[IES-2012]

- (a) at temperature below the triple point temperature
- (b) at pressure below the triple point pressure
- (c) at pressure above the triple point pressure
- (d) at temperature above the triple point temperature

## $p-T$ Diagram for a Pure Substance

IES-11. Consider the phase diagram of a certain substance as shown in the given figure. Match List-I (Process) with List-II (Curves/lines) and select the correct answer using the codes given below the lists:

List-I (Process)	List-II (Curves/lines)
A. Vaporization	1. EF
B. Fusion	2. EG
C. Sublimation	3. ED



[IES-2001]

Codes:	A	B	C	A	B	C
(a)	1	3	2	(b)	1	2
(c)	3	2	1	(d)	3	1

# Pure Substances

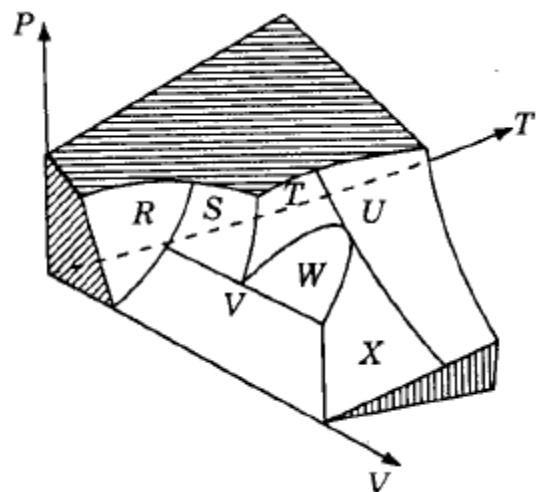
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Chapter 7

## p-v-T Surface

IES-12. The p-v-T surface of a pure substance is shown in the given figure. The two-phase regions are labelled as:

- (a) R, T and X
- (b) S, U and W
- (c) S, W and V
- (d) R, T and V

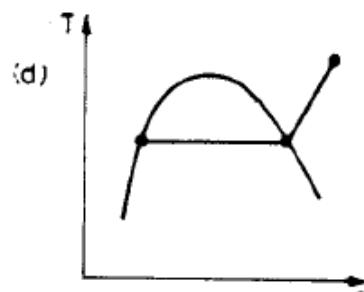
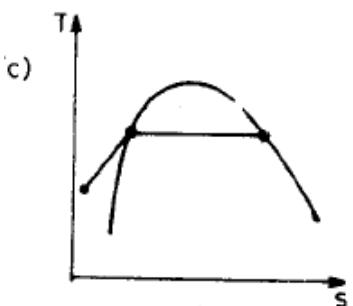
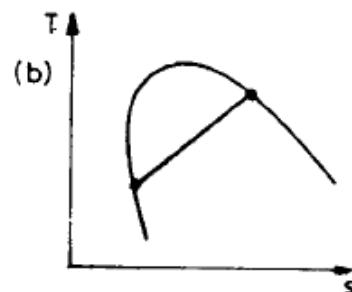
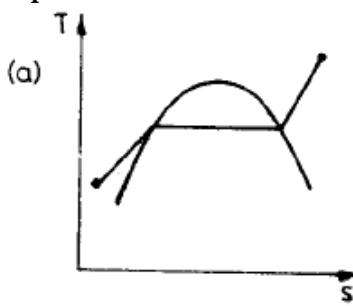


[IES-1999]

## T-s Diagram for a Pure Substance

IES-13. The conversion of water from 40°C to steam at 200°C pressure of 1 bar is best represented as

[IES-1994]



# Pure Substances

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- IES-14.** The following figure shows the T-s diagram for steam. With respect to this figure, match List I with List II and select the correct answer using the codes given below the Lists:

**List-I**

A. Curve I

B. Curve II

C. Curve III

D. Curve IV

Codes: A      B      C      D

(a) 2

(c) 1

**List-II**

1. Saturated liquid line

2. Saturated vapour line

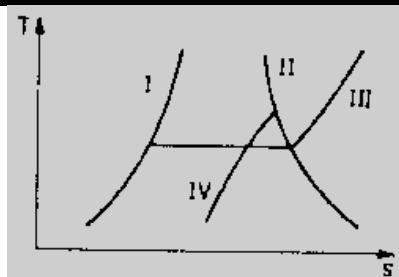
3. Constant pressure line

4. Constant volume line

A      B      C      D

(b) 2

(d) 1



[IES-1994]

(a) 1

(c) 4

(d) 3

(b) 3

(d) 4

(b) 1

(d) 2

## Critical Point

- IES-15.** Which one of the following is correct?

[IES-2008]

At critical point the enthalpy of vaporization is

(a) dependent on temperature only

(b) maximum

(c) minimum

(d) zero

- IES-16.** Consider the following statements about critical point of water:

[IES-1993]

1. The latent heat is zero.

2. The liquid is denser than its vapour.

3. Steam generators can operate above this point.

Of these statements

(a) 1, 2 and 3 are correct

(b) 1 and 2 are correct

(c) 2 and 3 are correct

(d) 1 and 3 are correct

- IES-17.** Which one of the following statements is correct when saturation pressure of a vapour increases?

[IES 2007]

(a) Saturation temperature decreases

(b) Enthalpy of evaporation decreases

(c) Enthalpy of evaporation increases

(d) Specific volume change of phase increases

- IES-18.** Match List I with List II and select the correct answer using the code given below the Lists:

[IES-2005]

**List-I**

A. Critical point

B. Sublimation

C. Triple point

D. Melting

**List-II**

1. All the three phases - solid, liquid and vapour co-exists in equilibrium

2. Phase change from solid to liquid

3. Properties of saturated liquid and saturated vapour are identical

4. Heating process where solid gets directly transformed to gaseous phase

Codes: A      B      C      D

(a) 2

(c) 2

(b) 4

(c) 1

A      B      C      D

(b) 3

(d) 3

(b) 1

(d) 1

(b) 4

(d) 4

(b) 2

(d) 2

- IES-19.** With increase of pressure, the latent heat of steam [IES-2002]  
 (a) Remains same (b) Increases (c) Decreases (d) Behaves unpredictably

- IES-20.** List-I gives some processes of steam whereas List-II gives the effects due to the processes. Match List I with List II, and select the correct answer using the codes given below the lists: [IES-1995]

	List-I				List-II			
	A	B	C	D	A	B	C	D
A.	As saturation pressure increases				1. Entropy increases.			
B.	As saturation temperature increases				2. Specific volume increases.			
C.	As saturation pressure decreases				3. Enthalpy of evaporation decreases.			
D.	As dryness fraction increases				4. Saturation temperature increases.			
Code:	A (a) (c)	B 1 4	C 3 1	D 4 2	A (b) (d)	B 4 2	C 3 4	D 1 3 1

## ***h-s Diagram or Mollier Diagram for a Pure Substance***

- IES-21.** Which one of the following represents the condensation of a mixture of saturated liquid and saturated vapour on the enthalpy-entropy diagram? [IES-2004]

- (a) A horizontal line (b) An inclined line of constant slope  
 (c) A vertical line (d) A curved line

- IES-21a.** The following remarks pertain to the properties of gases and vapours: [IES-2012]  
 1. The equation of state is not by itself sufficient for the calculation of properties such as u, h and s.

2. Throttling process is an adiabatic steady flow process.  
 3. Increase in entropy in wet region is directly proportional to the increase in enthalpy.

Which of these remarks are correct?

- (a) 1, 2 and 3 (b) 1 and 2 only (c) 1 and 3 only (d) 2 and 3 only

## ***Measurement of Steam Quality***

- IES-22.** Saturated liquid at a high pressure  $P_1$  having enthalpy of saturated liquid 1000 kJ/kg is throttled to a lower pressure  $P_2$ . At pressure  $p_2$  enthalpy of saturated liquid and that of the saturated vapour are 800 and 2800 kJ/kg respectively. The dryness fraction of vapour after throttling process is: [IES-2003]

- (a) 0.1 (b) 0.5 (c) 18/28 (d) 0.8

- IES-23.** Consider the following statements regarding the throttling process of wet steam: [IES-2002]

1. The steam pressure and temperature decrease but enthalpy remains constant.
2. The steam pressure decreases, the temperature increases but enthalpy remains constant.
3. The entropy, specific volume, and dryness fraction increase.
4. The entropy increases but the volume and dryness fraction decrease.

# Pure Substances

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Which of the above statements are correct?

- (a) 1 and 4      (b) 2 and 3      (c) 1 and 3      (d) 2 and 4

IES-24. Match List-I (Apparatus) with List-II (Thermodynamic process) and select the correct answer using the code given below the Lists: [IES-2006]

List-I

- A. Separating calorimeter  
B. Throttling calorimeter  
C. Sling psychrometer  
D. Gas thermometer
1. Adiabatic process  
2. Isobaric process  
3. Isochoric process  
4. Isenthalpic process

Codes:

	A	B	C	D	A	B	C	D
(a)	1	3	2	4	(b)	2	4	1
(c)	1	4	2	3	(d)	2	3	1

IES-25. Select the correct answer using the codes given below the Lists: [IES-1998]

List-I

- A. Bomb calorimeter  
B. Exhaust gas calorimeter  
C. Junker gas calorimeter  
D. Throttling calorimeter
1. Pressure  
2. Enthalpy  
3. Volume  
4. Specific heats

Code:

	A	B	C	D	A	B	C	D
(a)	3	4	1	2	(b)	2	4	1
(c)	3	1	4	2	(d)	4	3	2

## Throttling

IES-26. In a throttling process, which one of the following parameters remains constant? [IES-2009]

- (a) Temperature      (b) Pressure      (c) Enthalpy      (d) Entropy

IES-27. Consider the following statements: [IES-2000]

When dry saturated steam is throttled from a higher pressure to a lower pressure, the

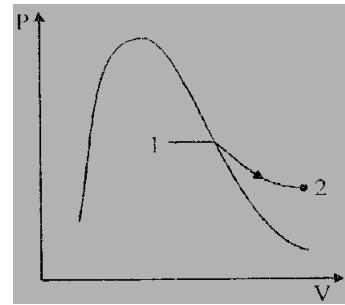
1. Pressure decreases and the volume increases
2. Temperature decreases and the steam becomes superheated
3. Temperature and the dryness fraction increase
4. Entropy increases without any change in enthalpy

Which of these statements are correct?

- (a) 1 and 4      (b) 1, 2 and 4      (c) 1 and 3      (d) 2 and 4

IES-28. The process 1-2 for steam shown in the given figure is

- (a) Isobaric      (b) Isentropic  
(c) Isenthalpic      (d) Isothermal



[IES-2000]

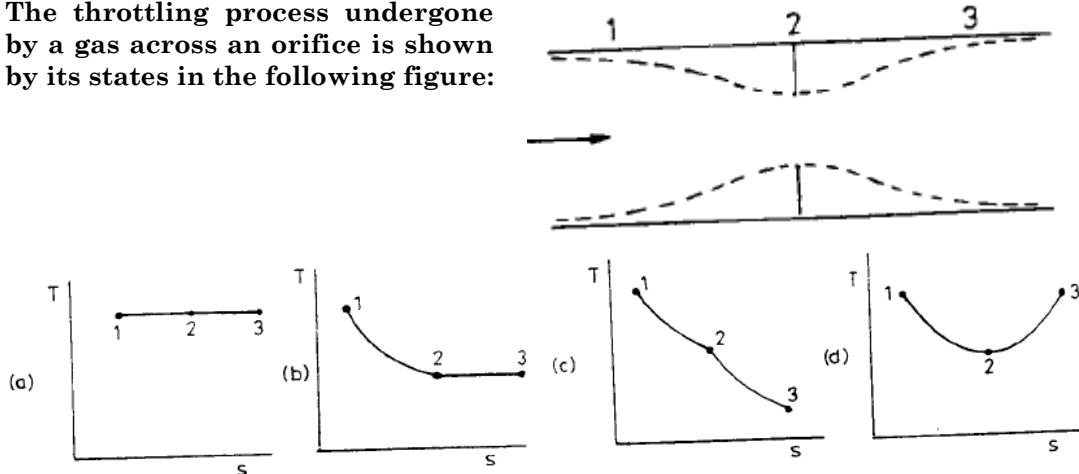
# Pure Substances

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- IES-29.** A fluid flowing along a pipe line undergoes a throttling process from 10 bar to 1 Bar in passing through a partially open valve. Before throttling, the specific volume of the fluid is  $0.5 \text{ m}^3/\text{kg}$  and after throttling is  $2.0 \text{ m}^3/\text{kg}$ . What is the Change in specific internal energy during the throttling process? [IES 2007]
- (a) Zero      (b)  $100 \text{ kJ/kg}$       (c)  $200 \text{ kJ/kg}$       (d)  $300 \text{ kJ/kg}$

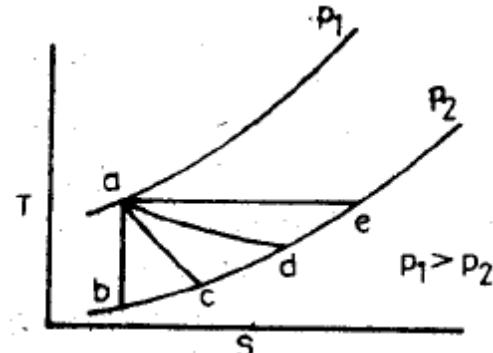
- IES-30.** The throttling process undergone by a gas across an orifice is shown by its states in the following figure:



[IES-1996]

- IES-31.** In the figure shown, throttling process is represented by

- (a) a e      (b) a d  
(c) a c      (d) a b



[IES-1992]

- IES-32.** Match List-I with List-II and select the correct answer using the code given below the lists: [IES-2009]

**List-I**

- A. Isolated system  
B. Nozzle  
C. Throttling device  
D. Centrifugal compressor

**List-II**

1. Energy is always constant
2. Increase in velocity at the expense of its pressure drop
3. Appreciable drop in pressure without any change in energy
4. Enthalpy of the fluid increases by the amount of work input

Codes:	A	B	C	D	A	B	C	D
(a)	4	3	2	1	(b)	1	3	2
(c)	4	2	3	1	(d)	1	2	3

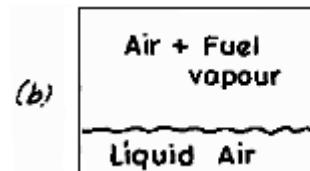
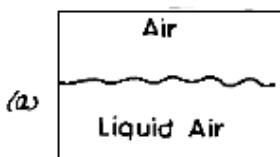
# Pure Substances

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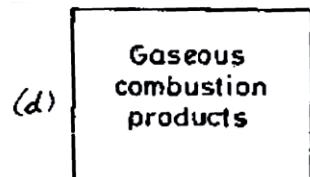
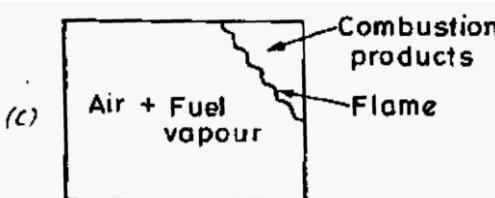
## Previous 20-Years IAS Questions

- IAS-1. Which one of the following systems can be considered to be containing a pure substance? [IAS 1998]



No Reaction

$O_2 : N_2$  proportions differ in the gas and liquid phase in equilibrium



- IAS-2. Assertion (A): On the enthalpy-entropy diagram of a pure substance the constant dryness fraction lines start from the critical point. [IAS-2001]

Reason (R): All the three phases co-exist at the critical point.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is NOT the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

- IAS-3. Assertion (A): Air, a mixture of  $O_2$  and  $N_2$ , is a pure substance. [IAS-2000]

Reason(R): Air is homogeneous in composition and uniform in chemical aggregation.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is NOT the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

- IAS-4. If a pure substance contained in a rigid vessel passes through the critical state on heating, its initial state should be: [IAS-1998]

- (a) Subcooled water
- (b) Saturated water
- (c) Wet steam
- (d) Saturated steam

- IAS-5. Assertion (A): Air is a pure substance but a mixture of air and liquid air in a cylinder is not a pure substance. [IAS-1996]

Reason (R): Air is homogeneous in composition but a mixture of air and liquid air is heterogeneous.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is NOT the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

# Pure Substances

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IAS-6. Assertion (A): Temperature and pressure are sufficient to fix the state of a two phase system. [IAS-1995]

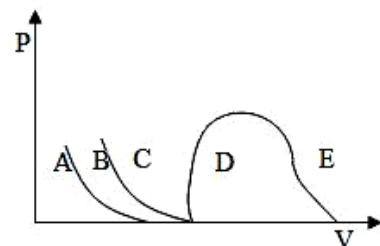
Reason(R): Two independent and intensive properties are required to be known to define the state of a pure substance.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is NOT the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

## p-v Diagram for a Pure Substance

IAS-7. Two-phase regions in the given pressure-volume diagram of a pure substance are represented by

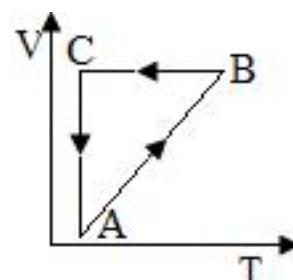
- (a) A, E and F
- (b) B, C and D
- (c) B, D and F
- (d) A, C and E



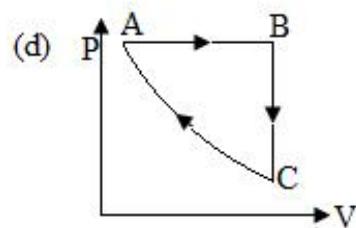
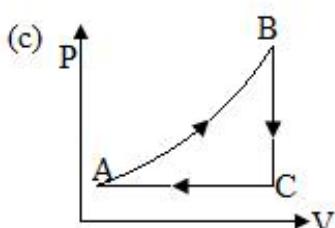
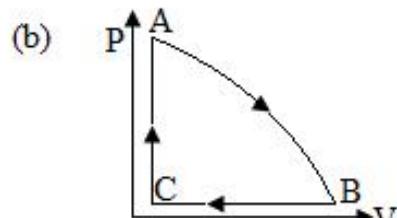
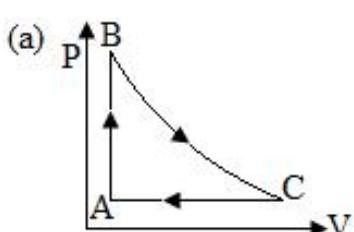
[IAS-1999]

IAS-8. A cyclic process ABC is shown on a V-T diagram in figure.

The same process on a P-V diagram will be represent as:



[IAS-1996]

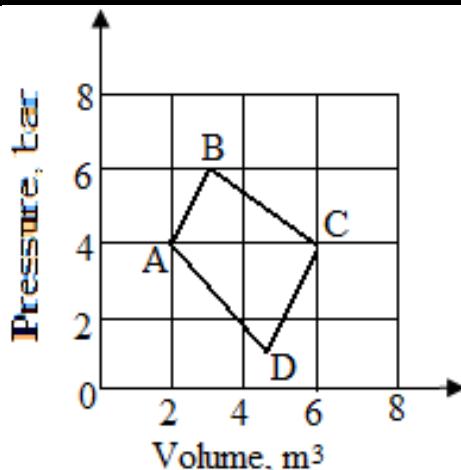


# Pure Substances

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- IAS-9. The network done for the closed shown in the given pressure-volume diagram, is  
(a) 600kN-m      (b) 700kN-m  
(c) 900kN-m      (d) 1000kN-m



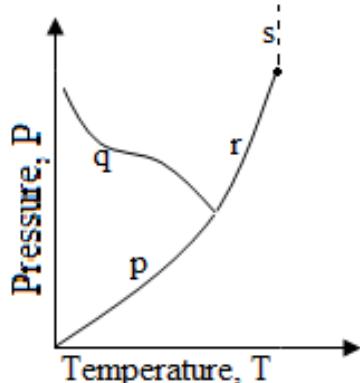
[IAS-1995]

## Triple point

- IAS-10. Triple point temperature of water is:  
(a) 273 K      (b) 273.14 K      (c) 273.15K      (d) 273.16 K [IAS-2000]

## p-T Diagram for a Pure Substance

- IAS-11. In the following P-T diagram of water showing phase equilibrium lines, the sublimation line is:  
(a) p      (b) q  
(c) r      (d) S



[IAS-1998]

## T-s Diagram for a Pure Substance

- IAS-12. Entropy of a saturated liquid at 227°C is 2.6 kJ/kgK. Its latent heat of vaporization is 1800 kJ/kg; then the entropy of saturated vapour at 227°C would be:  
(a) 2.88 kJ/kg K      (b) 6.2 kJ/kg K      (c) 7.93 kJ/kg K      (d) 10.53 kJ/kg K [IAS-2001]

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- IAS-13. Two heat engine cycles (1 - 2 - 3 - 1 and 1' - 2' - 3' - 1') are shown on T-s co-ordinates in

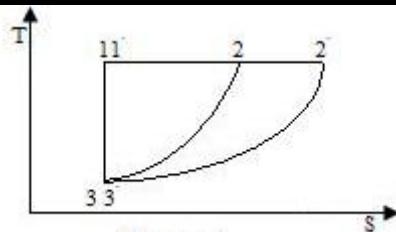
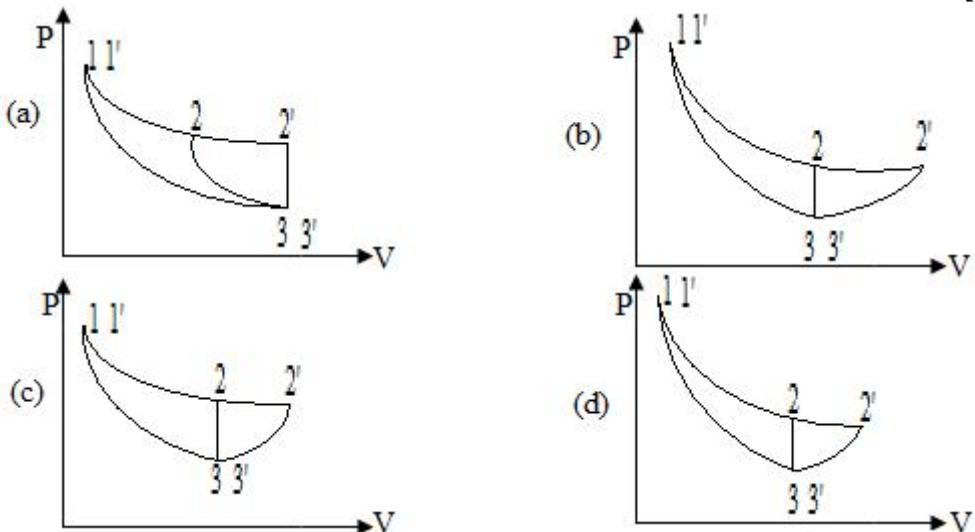
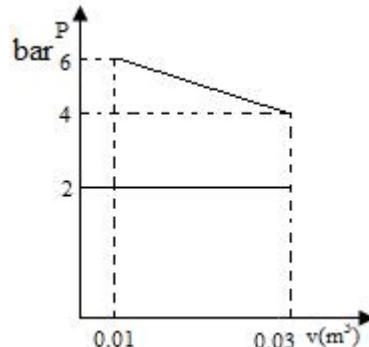


Figure - I

[IAS-1999]



- IAS-14. The mean effective pressure of the thermodynamic cycle shown in the given pressure-volume diagram is:  
 (a) 3.0 bar      (b) 3.5 bar  
 (c) 4.0 bar      (d) 4.5 bar



[IAS-1999]

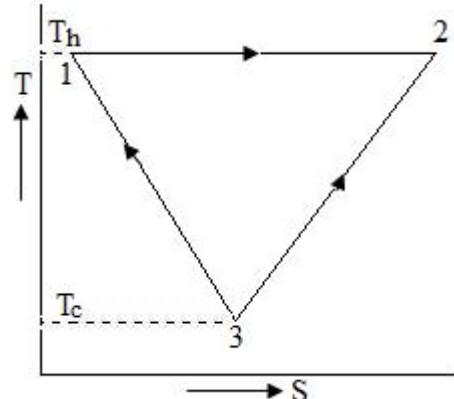
# Pure Substances

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- IAS-15. The given figure shows a thermodynamic cycle on T-s diagram. All the processes are straight lines. The efficiency of the cycle is given by

- (a)  $(0.5 T_h - T_e)/ T_h$
- (b)  $0.5 (T_h - T_e)/ T_h$
- (c)  $(T_h - T_e)/ 0.5 T_h$
- (d)  $(T_h - 0.5 T_e)/ T_h$



[IAS-1996]

## h-s Diagram or Mollier Diagram for a Pure Substance

- IAS-16. Constant pressure lines in the superheated region of the Mollier diagram have what type of slope? [IAS-2007]

- (a) A positive slope
- (b) A negative slope
- (c) Zero slope
- (d) May have either positive or negative slopes

- IAS-17. Assertion (A): In Mollier chart for steam, the constant pressure lines are straight lines in wet region.

Reason (R): The slope of constant pressure lines in wet region is equal to T.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is **not** the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

## Quality or Dryness Fraction

- IAS-18. Dryness fraction of steam means the mass ratio of [IAS-2001]

- (a) Wet steam, to dry steam
- (b) Dry steam to water particles in steam
- (c) Water particles to total steam
- (d) Dry steam to total steam

## Throttling

- IAS-19. Assertion (A): Throttle governing is thermodynamically more efficient than nozzle control governing for steam turbines. [IAS-2000]

Reason (R): Throttling process conserves the total enthalpy.

- (a) Both A and R are individually true and R is the correct explanation of A
- (b) Both A and R are individually true but R is **NOT** the correct explanation of A
- (c) A is true but R is false
- (d) A is false but R is true

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## Answers with Explanation (Objective)

### Previous 20-Years GATE Answers

**GATE-1.** Ans. (a) Initial Volume ( $V_1$ ) =  $0.001 + 0.03 \times 0.1 \text{ m}^3 = 0.004 \text{ m}^3$

Let dryness fraction =  $x$

Therefore  $0.004 \times 1.5 = (1 - x) \times 0.0015 \times 1.03 + x \times 0.002 \times 1.03$

That gives an absurd value of  $x = 8.65$  (It must be less than equal to unity). So vapour is superheated.

**GATE-2.** Ans. (d) Work done = first constant volume heating +  $\int pdV$

$$= 0 + P(V_2 - V_1) = 200 \times (0.006 - 0.004) = 0.4 \text{ kJ}$$

**GATE-3.** Ans. (c)  $(\Delta S = (\Delta S)_{\text{system}} + (\Delta S)_{\text{surroundings}} = 10 - \frac{1000}{(273+400)} = 8.51 \text{ J/K})$

**GATE-4.** Ans. (c)

**GATE-5.** Ans. (b)

**GATE-6.** Ans. Higher, Lower

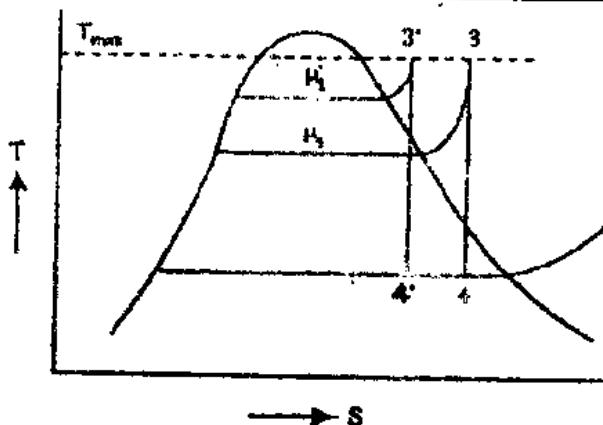
**GATE-7.** Ans. (a) Mollier diagram is a h-s plot.

$$Tds = dh - Vdp \quad \text{or} \quad \left(\frac{\partial h}{\partial s}\right)_P = T = \text{slope}$$

T is always +ive so slope always +ive. Not only this if  $T \uparrow$  then slope  $\uparrow$

**GATE-8.** Ans. False

**Explanation.**  $P'_1 > P_1$ . Dryness fraction after isentropic expansion decreases.



**GATE-9.** Ans. (d)

**GATE-10.** Ans. (b)

$$h_{40} = h_{20} = (1-x)h_{f-20} + xh_g$$

$$\text{or } 371.43 = (1-x)89.05 + x \times 1418.0 \text{ or } x = 0.212$$

# Pure Substances

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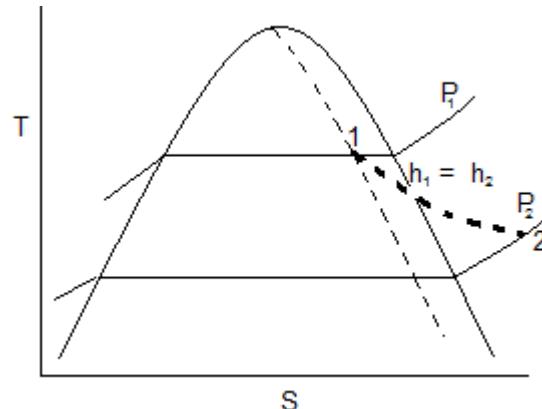
GATE-11. Ans. (b)

GATE-12. Ans. (d)

$$\Delta h = 0$$

$$\Delta S > 0$$

$$\Delta T < 0$$



GATE-13. Ans. (a)  $S_2 - S_1 = C_{pav} \ln \frac{T_2}{T_1} - R_u \ln \frac{P_2}{P_1}$

Change in entropy of the universe =  $-R_u \ln \frac{P_2}{P_1}$

$$= -8.314 \ln \frac{0.1}{0.5} = 13.38 \frac{\text{kJ}}{\text{K}}$$

For an ideal gas change in enthalpy is a function of temperature alone and change in enthalpy of a throttling process is zero.

## Previous 20-Years IES Answers

IES-1. Ans. (d) Water for all practical purpose can be considered as pure substance because it is homogeneous and has same chemical composition under all phases.

IES-2. Ans. (d)

IES-3. Ans. (d) The ordinate and abscissa in given figure are pressure and enthalpy. Such diagram is common in vapour compression refrigeration systems.

IES-4. Ans. (d) The throttling process given in figure is on pressure-enthalpy diagram.

IES-4a. Ans. (a)

IES-5. Ans. (d)

IES-6. Ans. (b) A pure substance is one whose chemical composition does not change during thermodynamic processes.

- Pure Substance is one with uniform and invariant chemical composition.
  - Eg: Elements and chemical compounds are pure substances. (water, stainless steel)
  - Mixtures are not pure substances. (eg: Humid air)
  - Exception!! Air is treated as a pure substance though it is a mixture of gases.
- Gaseous combustion products are a mixture of gases and not a pure substance.

IES-7. Ans. (d)

IES-8. Ans. (a)

IES-9. Ans. (b) The largest density of water near atmospheric pressure is at 4°C.

# Pure Substances

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**IES-10.** Ans. (c) Up to saturation point pressure must be constant. After saturation its slope will be

$$-\text{ve, as } pV = RT \text{ or } pV = \text{const. or } vdp + pdv = 0 \text{ or } \frac{dp}{dv} = -\frac{p}{v}$$

**IES-10a.** Ans. (b)

**IES-11.** Ans. (c)

**IES-12.** Ans. (c)

**IES-13.** Ans. (a)

**IES-14.** Ans. (c)

**IES-15.** Ans.(d) Characteristics of the critical point

1. It is the highest temperature at which the liquid and vapour phases can coexist.
2. At the critical point  $h_{fg}$ ,  $u_{fg}$  and  $v_{fg}$  are zero.
3. Liquid vapour meniscus will disappear.
4. Specific heat at constant pressure is infinite.

**IES-16.** Ans. (d) At critical point, the latent heat is zero and steam generators can operate above this point as in the case of once through boilers.

The density of liquid and its vapour is however same and thus statement 2 is wrong.

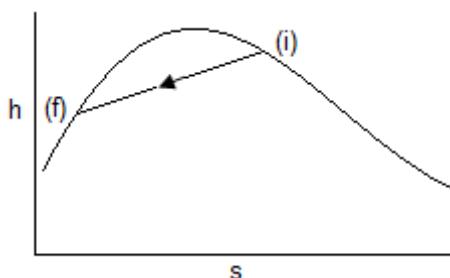
**IES-17.** Ans. (b)

**IES-18.** Ans. (b)

**IES-19.** Ans. (c)

**IES-20.** Ans. (c)

**IES-21.** Ans. (b)



$$TdS = dh - Vdp \quad \text{or} \quad \left(\frac{\partial h}{\partial S}\right)_P = T$$

The slope of the isobar on the h-s diagram is equal to the absolute temp, for condensation T is const so slope is const, but not zero so it is inclined line.

**IES-21a.** Ans. (a) Use Molier Diagram.

**IES-22.** Ans. (a) For throttling process (1-2),  $h_1 = h_2$

$$h_1 = h_f = 1000 \text{ kJ/kg at pressure } P_1$$

$$h_2 = h_f + x(h_g - h_f) \text{ at pressure } P_2$$

$$\therefore 1000 = 800 + x(2800 - 800)$$

$$\text{or } x = 0.1$$

**IES-23.** Ans. (c)

**IES-24.** Ans. (c)

**IES-25.** Ans. (a)

**IES-26.** Ans. (c) Consider a *throttling process* (also referred to as wire drawing process)

# Pure Substances

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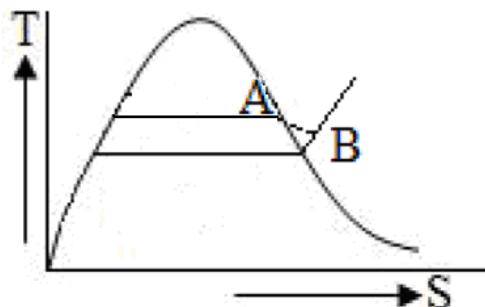
Chapter 7

1 | 2

There is no work done (rising a weight)       $W = 0$   
If there is no heat transfer       $Q = 0$   
Conservation of mass requires that       $C_1 = C_2$   
Since 1 and 2 are at the same level       $Z_1 = Z_2$   
From SFEE it follows that       $h_1 = h_2$

Conclusion: Throttling is a constant enthalpy process (isenthalpic process)

**IES-27. Ans. (b)** Temperature decreases and the steam becomes superheated.



**IES-28. Ans. (d)**

**IES-29. Ans. (d)** Throttling is a isenthalpic process

$$h_1 = h_2 \text{ or } u_1 + p_1 v_1 = u_2 + p_2 v_2 \text{ or } u_2 - u_1 = p_1 v_1 - p_2 v_2 = 1000 \times 0.5 - 100 \times 2 = 300 \text{ kJ/kg}$$

**IES-30. Ans. (d)** The throttling process takes places with enthalpy remaining constant. This process on T-S diagram is represented by a line starting diagonally from top to bottom.

**IES-31. Ans. (b)**

**IES-32. Ans. (d)**

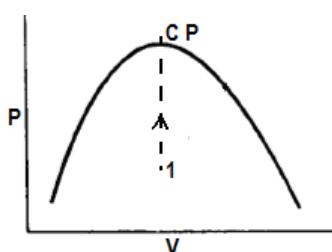
## Previous 20-Years IAS Answers

**IAS-1. Ans. (d)**

**IAS-2. Ans. (c)** Only two phase liquid-vapour is co-exists at the critical point, but at triple point-all three phase are co-exists.

**IAS-3. Ans. (a)** A pure substance is a substance of constant chemical composition throughout its mass.

**IAS-4. Ans. (c)**



**IAS-5. Ans. (a)**

**IAS-6. Ans. (d)** A is false but R is true.

**IAS-7. Ans. (c)**

**IAS-8. Ans. (d)**

# Pure Substances

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**IAS-9. Ans. (d)** Network done is area of closed loop  $ABCD = \text{Area of trapezium } AB32 + \text{Area } BC63 - \text{Area } CD56 - \text{Area } AD52$

$$\begin{aligned} &= \left(\frac{4+6}{2}\right) \times (3-2) + \left(\frac{6+4}{2}\right) \times (6-3) - \left(\frac{1+4}{2}\right) \times (6-5) + \left(\frac{1+4}{2}\right) \times (5-2) \\ &= 5 \times 1 + 5 \times 3 - 2.5 \times 1 - 2.5 \times 3 = 10 \text{ bar m}^3 \\ &= 10 \times 10^5 \frac{\text{N}}{\text{m}^2} \times \text{m}^3 = 10^6 \text{ Nm} = 1000 \text{ kNm} \end{aligned}$$

**IAS-10. Ans. (d)** Remember: Triple point temperature of water = 273.16 K = 0.01°C

**IAS-11. Ans. (a)**

$$\text{IAS-12. Ans. } S_g = S_f + \frac{h_g}{T_{sat}} = 2.6 + \frac{1800}{500} = 6.2 \text{ kJ/kgK}$$

**IAS-13. Ans. (d)**

$$\text{IAS-14. Ans. (a) Work (W)} = (0.03 - 0.01) \times (400 - 200) + \frac{1}{2} \times (600 - 400) \times (0.03 - 0.01) = 6 \text{ kJ}$$

$$W = p_m \times \Delta V \text{ or } p_m = \frac{W}{\Delta V} = \frac{6}{(0.03 - 0.01)} \text{ kPa} = 3 \text{ bar}$$

$$\text{IAS-15. Ans. (b) Work output} = \text{Area } 123 = \frac{1}{2} \times (T_h - T_c) \times (S_2 - S_1)$$

$$\text{Heat added} = \text{Area under } 1-2 = T_h(S_2 - S_1)$$

$$\therefore \eta = \frac{\frac{1}{2}(T_h - T_c)(S_2 - S_1)}{T_h(S_2 - S_1)} = 0.5(T_h - T_c)/T_h$$

**IAS-16. Ans. (a)** Mollier diagram is a h-s plot.

$$TdS = dh - Udp \quad \text{or} \quad \left(\frac{\partial h}{\partial S}\right)_P = T = \text{slope}$$

T is always +ive so slope always +ive. Not only this if  $T \uparrow$  then slope  $\uparrow$

**IAS-17. Ans. (a)** Both A and R are true and R is the correct explanation of A

**IAS-18. Ans. (d)**

**IAS-19. Ans. (d)** If throttle governing is done at low loads, the turbine efficiency is considerably reduced. The nozzle control may then be a better method of governing.

## 8.

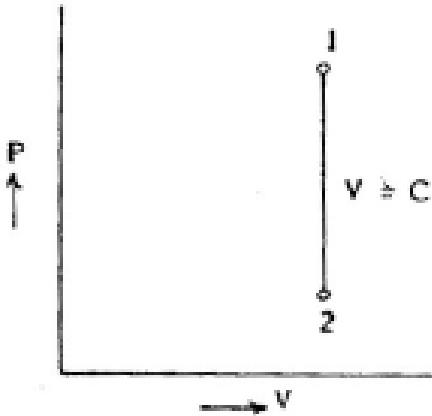
# Properties of Gasses and Gas Mixture

## Theory at a Glance (For GATE, IES & PSUs)

1. The functional relationship among the independent properties, pressure P, molar or specific volume v, and temperature T, is known as '*Equation of state*' i.e. **PV = RT for gases**.
2. A hypothetical gas which obeys the law  $PV = RT$  at all temperatures and pressures is called an '*ideal gas*'
  - An '*ideal gas*' has no forces of intermolecular attraction.
  - The specific heat capacities are constant.
3. '**Real gas**' does not conform to equation of state with complete accuracy. As  $P \rightarrow 0$  or  $T \rightarrow \infty$ , the real gas approaches the ideal gas behaviour.
4. **Joule's law** states that the specific internal energy of a gas depends only on the temperature of the gas and is independent of both pressure and volume.

### PROCESS

- **Constant Volume Process:** – The process is represented on a pv diagram as shown in Figure.



**Fig.**

Here  $v_1 = v_2$   
or  $dv = 0$

Therefore  $pdv = 0$

or work transfer  $W = 0$

# Properties of Gasses & Gas Mix.

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$$\text{Therefore } dQ = dE = du = C_v(T_2 - T_1)$$

$$\text{or } Q_{1-2} = C_v(T_2 - T_1)$$

(i.e.,) during constant volume process, the heat transfer is the change in internal energy. From gas equation

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

We get,

$$P_1 T_2 = P_2 T_1$$

- **Constant Pressure Process:** This process is represented on a pv diagram as shown in Figure. In this process.

$$P_1 = P_2$$

From gas equation we get

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

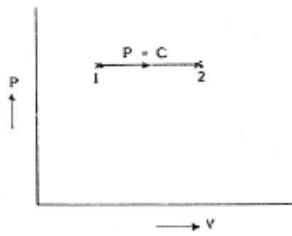


Fig.-

$$(or) \quad u_1 T_2 = u_2 T_1$$

$$dW = pdv = p(v_2 - v_1)$$

$$dQ = dE + dW$$

$$= du + dW$$

$$= d(u + pv)$$

$$= d(h)$$

$$= C_p(T_2 - T_1)$$

(i.e.,) heat transfer is the change in enthalpy in the case of a constant pressure process.

- **Constant temperature (or) Isothermal process:-**

The process is represented in figure(below).

Here  $T_1 = T_2$

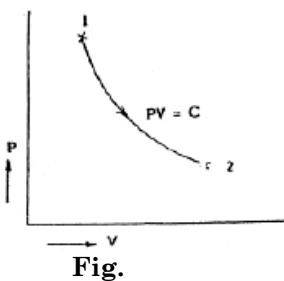
$$dW = pdv$$

$$W = \int p dv = C \int \frac{dv}{V}, \quad \text{since } PV = C$$

# Properties of Gasses & Gas Mix.

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**Fig.**

$$\therefore W = p_1 v_1 \ln \left( \frac{v_2}{v_1} \right) = p_1 v_1 \ln(r)$$

where  $r = \frac{v_2}{v_1}$

Since  $T_1 = T_2$  and  $dT = 0$ .

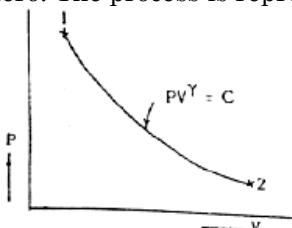
$$du = (u_2 - u_1) = C_v(T_2 - T_1) = 0$$

Therefore, by the First Law of Thermodynamics

$$Q = W = p_1 v_1 \ln(r)$$

**•Adiabatic process:**

This is also called **isentropic process** as entropy in this process will remain constant and heat transfer in this process is also zero. The process is represented in pv diagram shown in Figure below.



**Fig.-**

The equation for this process becomes,

$$p_1 u_1^\gamma = p_2 u_2^\gamma$$

where  $\gamma = \text{ratio of specific heats}$

$$(\text{i.e.,}) \quad \gamma = \frac{C_p}{C_v}$$

For this process  $Q = 0$

$$\text{Hence } 0 = dE + dW = du + dW$$

$$\text{or } dW = -du$$

$$= -mC_v dT$$

$$dW = pdv$$

$$\text{or } pdv = -du = -mC_v dT$$

$$\text{or } pdv + mC_v dT = 0$$

*(equation....1)*

$$\text{But } pv = mRT$$

$$\text{differentiating; } pdv + vdp = mR dT$$

From the above equation,

$$dT = \frac{pdv + vdp}{mR}$$

# Properties of Gasses & Gas Mix.

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In equation-1, value of  $dT$  is substituted.

$$pdv + mC_v \left( \frac{pdv + vdp}{mR} \right) = 0$$

$$R.pdv + C_v (pdv + vdp) = 0$$

But  $R = (C_p - C_v)$

Therefore  $(C_p - C_v) pdv + C_v (pdv + vdp) = 0$

or  $C_v vdp + C_p pdv = 0$

Dividing by  $(C_v p v)$  we get

$$\frac{dp}{p} + \frac{C_p}{C_v} \cdot \frac{dv}{V} = 0$$

But  $\frac{C_p}{C_v} = \gamma$

Therefore  $\frac{dp}{p} + \gamma \frac{dv}{V} = 0$

Integration and rearranging we get,

$$\ln p + \gamma \ln v = C$$

Or  $p v^\gamma = C$

(i.e.)  $p u_1^\gamma = p_2 u_2^\gamma$

From the gas equation  $pv = mRT$ , we can obtain the relationships between pressure and temperature and volume and temperature as given below.

$$\frac{T_1}{T_2} = \left( \frac{v_2}{v_1} \right)^{\gamma-1} = \left( \frac{p_1}{p_2} \right)^{\gamma}$$

Work transfer  $dW = pdv$

$$W = \int_1^2 pdv$$

$$p v^\gamma = C$$

$$p = \frac{C}{v^\gamma}$$

Therefore

$$W = C \int_1^2 \frac{dv}{V^\gamma}$$
$$= C \left[ \frac{v_2^{\gamma+1} - v_1^{\gamma+1}}{\gamma+1} \right]$$

But  $C = p_1 u_1^\gamma = p_2 u_2^\gamma$

$$W = \left( \frac{p_2 v_2 - p_1 v_1}{\gamma+1} \right)$$

$$W = \left( \frac{p_1 u_1 - p_2 u_2}{\gamma-1} \right)$$

Also first law becomes

$$W = -\Delta E = -\Delta U$$

# Properties of Gasses & Gas Mix.

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**General process or polytropic process:** In the adiabatic process the index for V is  $\gamma$ . In a most general case this  $\gamma$  can be replaced by n. The values of n for different processes are indicated below :

if  $n = 0$  the process is constant pressure

if  $n = 1$  the process is isothermal

if  $n = \infty$  the process is constant volume

if  $n = \gamma$  the process is adiabatic.

and for any process other than the above, n becomes a general value n.

All the equations of adiabatic process can be assumed for this process with  $\gamma$  replaced by n.

$$\frac{T_1}{T_2} = \left( \frac{v_2}{v_1} \right)^{n-1} = \left( \frac{P_1}{P_2} \right)^{\frac{n-1}{n}}$$

$$W = \frac{P_1 v_1 - P_2 v_2}{n-1}$$

But  $W =$  is not true as Q is not zero.

By the first law of Thermodynamics,

$$\begin{aligned} Q &= pdv + (u_2 - u_1) \\ &= \frac{P_1 v_1 - P_2 v_2}{n-1} + mC_v(T_2 - T_1) \end{aligned}$$

But by gas equation  $P_1 v_1 = mRT_1$  and  $P_2 v_2 = mRT_2$

$$\begin{aligned} \text{so, } Q &= mC_v(T_2 - T_1) + \frac{mR(T_2 - T_1)}{1-n} \\ &= m(T_2 - T_1) \left( C_v + \frac{R}{1-n} \right) \end{aligned}$$

$$\begin{aligned} \text{But, } Q &= m(T_2 - T_1) \left( C_v + \frac{C_p - C_v}{1-n} \right) \\ &= m \left( C_v + \frac{C_p - C_v}{1-n} \right) (T_2 - T_1) \\ Q &= \left( \frac{C_p - nC_v}{1-n} \right) (T_2 - T_1) \end{aligned}$$

Substituting  $C_p = \gamma C_v$

$$Q = \left( \frac{\gamma - n}{1-n} \right) C_v (T_2 - T_1)$$

Hence the specific heat for a polytropic process is  $C_n = C_v$

$$= C_n \left( \frac{\gamma - n}{1-n} \right) (T_2 - T_1)$$

4. **For minimum work in multistage compression  $P_2 = \sqrt{P_1 P_3}$**

a. Equal pressure ratio i.e.  $\frac{P_2}{P_1} = \frac{P_3}{P_2}$

# Properties of Gasses & Gas Mix.

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- b. Equal discharge temperature i.e.  $T_2 = T_3$
- c. Equal work required for both the stages.

**5. Equation of states for real gas**

- a. Van der waals equation  $(p + \frac{a}{v^2})(v-b) = RT$

The coefficient  $a$  is introduced to account for the existence of mutual attraction between the molecules. The term  $a/v^2$  is called the **force of cohesion**. The coefficient  $b$  is introduced to account for the volumes of the molecules, and is known as **co-volume**.

- b. Beattie Bridgeman equation

$$p = \frac{RT(l-e)(v+B)}{v^2} - \frac{A}{v^2} \quad \text{Where } A = A_0 \left(1 - \frac{a}{v^2}\right)$$

$$B = B_0 \left(1 - \frac{b}{v}\right)$$

$$e = \frac{c}{vT^3}$$

This equation does not give satisfactory results in the critical point region.

- 6.** The ratio  $\frac{PV}{RT}$  is called the **compressibility factor**.

Value of compressibility factor ( $Z$ ) at critical point is 0.375 for Van der waals gas.

**For ideal gas  $z = 1$**

**7. Critical Properties:**

$$a = 3P_c V_c^2, \quad b = \frac{V_c}{3}, \quad \text{and} \quad R = \frac{8}{3} \frac{P_c V_c}{T_c}$$

Where  $P_c$ ,  $V_c$  and  $T_c$  are critical point pressure, volume and temperature respectively.

**At Critical Point**

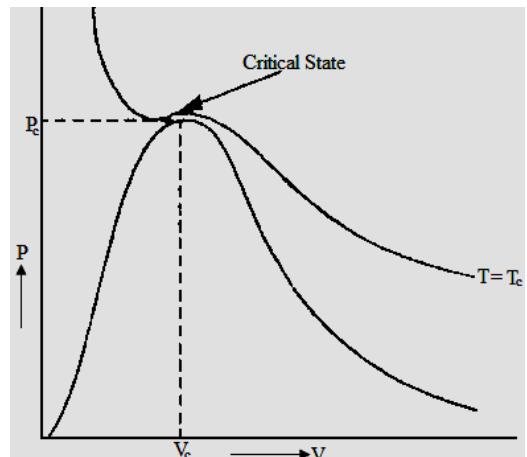
- (i) Three real roots of Vander Waal equation coincide.

$$(ii) \left(\frac{\partial p}{\partial v}\right)_{T_c} = 0 \text{ i.e. Slope of p-v diagram is zero.}$$

$$(iii) \left(\frac{\partial^2 p}{\partial v^2}\right)_{T_c} = 0 \text{ i.e. Change of slope also zero.}$$

$$(iv) \left(\frac{\partial^3 p}{\partial v^3}\right)_{T_c} < 0 \text{ i.e. negative, and equal to } -9p_c$$

- 8. Boyle's Temperature ( $T_B$ )** =  $\frac{a}{bR}$



**Fig. Critical properties on p-v diagram**

# Properties of Gasses & Gas Mix.

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**Boyle's Law** is obeyed fairly accurately up to a moderate pressure and the corresponding temperature is called the Boyle's Temperature.

### 9. Dalton's Law

- The pressure of a mixture of gases is equal to the sum of the partial pressures of the constituents.
- The partial pressure of each constituent is that pressure which the gas would exert if it occupied alone that volume occupied by the mixture at the same temperature.

### 10. Gibbs-Dalton Law

- The internal energy, enthalpy and entropy of a gaseous mixture are respectively equal to the sum of the internal energies, enthalpies, entropies of the constituents.
- Each Constituent has that internal energy, enthalpy and entropy, which it would have if it occupied alone that volume occupied by the mixture at the same temperature.

### 11. Equivalent molecular weight ( $M_e$ ) = $x_1M_1+x_2M_2+\dots+x_nM_n$

$$\text{Equivalent gas constant } (R_e) = x_1R_1+x_2R_2+\dots+x_nR_n$$

$$\text{Equivalent constant volume specific heat } (C_{Ve}) = x_1C_{v1}+x_2C_{v2}+\dots+x_nC_{vn}$$

$$\text{Equivalent constant pressure specific heat } (C_{Pe}) = x_1C_{p1}+x_2C_{p2}+\dots+x_nC_{pn}$$

$$x_i = \frac{m_i}{m} = \text{mass fraction of a constituent}$$

12. The value of Universal Gas constant  $R = 8.3143 \text{ KJ/Kg mole K}$ .

## PROBLEMS & SOLUTIONS

### Example 1.

Ten moles of ethane were confined in a vessel of volume 4.86 litres at 300K. Predict the pressure of gaseous ethane under these conditions with the use of equation of state of Van der Waals when  $a = 5.44 \text{ litre}^2 \text{ atm mol}^{-2}$ ;  $b = 64.3 \text{ millilitre mol}^{-1}$ ;  $R = 0.08205 \text{ atm mol}^{-1} \text{ deg}^{-1}$

#### Solution:

Using van der Waals equation, (for total volume  $V = nv$ )

$$p = \frac{nRT}{V-nb} - \frac{an^2}{V^2} = \frac{10.00 \times 0.08205 \times 300}{4.86 - 10.0 \times 0.0643} - \frac{5.44 \times 10.0^2}{4.86^2}$$
$$= 58.4 - 23 = 35.4 \text{ atm}$$

### Example 2.

A rigid vessel of  $0.3 \text{ m}^3$  volume contains a perfect gas at a pressure of 1 bar. In order to reduce the pressure, it is connected to an extraction pump. The volume flow rate through the pump is  $0.014 \text{ m}^3/\text{min}$ . Assuming the gas temperature remains constant, calculate the time taken to reduce the pressure to 0.35 bar.

#### Solution:

$$m = pV/RT, dm = \frac{Vdp}{RT}$$

$$\text{Mass of gas removed} = \frac{-p(V_e dt)}{RT}$$

Where  $V_e$  is the rate of extraction and  $dt$  is the time interval.

Equating and simplifying

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$$\frac{dp}{p} = -\left(\frac{V_e}{V}\right)dt$$

$$\int_{P_1}^P \frac{dp}{p} = \int_0^t \left(-\frac{V_e}{V}\right)dt$$

$$\ln \frac{P_2}{P_1} = -\left(\frac{V_e}{V}\right)t$$

$$\therefore t = \left(\frac{V}{V_e}\right) \ln \frac{P_1}{P_2}$$

$$= \left(\frac{0.3}{0.014}\right) \times \ln \left(\frac{1.0}{0.35}\right) = 22.5 \text{ min.}$$

### Example 3.

The value of characteristic constant for a gas is 4.1 kJ/kgK and specific heat at constant pressure is 14.28 kJ/kg K. 5 cubic meters of this gas at a pressure of 100 kPa and 20°C are compressed adiabatically to 500 kPa. The compressed gas is then expanded isothermally to original volume. Calculate:

(a) The final pressure of the gas after expansion, (b) The quantity of heat added from the beginning of compression to the end of expansion.

**Solution:**

$$m = \frac{pV}{RT} = \frac{100 \times 10^3 \times 5}{4.1 \times 10^3 \times 293} = 0.4162 \text{ kg}$$

$$C_p = 14.28 \text{ kJ/kg K}$$

$$C_v = C_p - R = 10.18$$

$$\gamma = 1.4028$$

$$V_2 = V_1 \left( \frac{p_1}{p_2} \right)^{\frac{1}{\gamma}} = 5 \times \left( \frac{1}{5} \right)^{\frac{1}{1.4028}} = 1.5874 \text{ m}^3$$

$$p_3 = \frac{V_2}{V_1} \times p_2 = \frac{5 \times 10^5 \times 1.5874}{5} = 158.74 \text{ kPa}$$

$$T_2 = T_1 \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} = 465 \text{ K}$$

$$Q = p_2 V_2 \ln \left( \frac{V_1}{V_2} \right) = 5 \times 10^5 \times 1.5874 \ln \frac{5}{1.5874} \\ = 910.6 \text{ kJ.}$$

# Properties of Gasses & Gas Mix.

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## ASKED OBJECTIVE QUESTIONS (GATE, IES, IAS)

### Previous 20-Years GATE Questions

#### Equation of State of a Gas

GATE-1. Nitrogen at an initial stage of 10 bar, 1 m<sup>3</sup> and 300K is expanded isothermally to a final volume of 2 m<sup>3</sup>. The P-V-T relation is  $\left(P + \frac{a}{V^2}\right)V = RT$ , where  $a > 0$ .

The final pressure will be:

[GATE-2005]

- (a) Slightly less than 5 bar      (b) Slightly more than 5 bar  
(c) Exactly 5 bar      (d) Cannot be ascertained.

#### Adiabatic Process

GATE-2. A mono-atomic ideal gas ( $\gamma = 1.67$ , molecular weight = 40) is compressed adiabatically from 0.1 MPa, 300 K to 0.2 MPa. The universal gas constant is 8.314 kJ kmol<sup>-1</sup>K<sup>-1</sup>. The work of compression of the gas (in kJ kg<sup>-1</sup>) is:

[GATE-2010]

- (a) 29.7      (b) 19.9      (c) 13.3      (d) 0

#### Statement for Linked Answer Questions Q3 & Q4:

A football was inflated to a gauge pressure of 1 bar when the ambient temperature was 15°C. When the game started next day, the air temperature at the stadium was 5°C. Assume that the volume of the football remains constant at 2500 cm<sup>3</sup>.

GATE-3. The amount of heat lost by the air in the football and the gauge pressure of air in the football at the stadium respectively equal [GATE-2006]

- (a) 30.6 J, 1.94 bar      (b) 21.8 J, 0.93 bar      (c) 61.1 J, 1.94 bar      (d) 43.7 J, 0.93 bar

GATE-4. Gauge pressure of air to which the ball must have been originally inflated so that it would equal 1 bar gauge at the stadium is: [GATE-2006]

- (a) 2.23 bar      (b) 1.94 bar      (c) 1.07 bar      (d) 1.00 bar

GATE-5. A 100 W electric bulb was switched on in a 2.5 m × 3 m × 3 m size thermally insulated room having a temperature of 20°C. The room temperature at the end of 24 hours will be [GATE-2006]

- (a) 321°C      (b) 341°C      (c) 450°C      (d) 470°C

# Properties of Gasses & Gas Mix.

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## Isothermal Process

- GATE-6. A frictionless piston-cylinder device contains a gas initially at 0.8 MPa and 0.015 m<sup>3</sup>. It expands quasi-statically at constant temperature to a final volume of 0.030 m<sup>3</sup>. The work output (in J) during this process will be: [GATE-2009]
- (a) 8.32                          (b) 12.00                          (c) 554.67                          (d) 8320.00

## Properties of Mixtures of Gases

- GATE-7. 2 moles of oxygen are mixed adiabatically with another 2 moles of oxygen in a mixing chamber, so that the final total pressure and temperature of the mixture become same as those of the individual constituents at their initial states. The universal gas constant is given as  $R$ . The change in entropy due to mixing, per mole of oxygen, is given by [GATE-2008]
- (A)  $-R\ln 2$                           (B) 0                                  (C)  $R\ln 2$                                   (D)  $R\ln 4$

## Previous 20-Years IES Questions

## Avogadro's Law

- IES-1. Assertion (A): The mass flow rate through a compressor for various refrigerants at same temperature and pressure, is proportional to their molecular weights. [IES-2002]  
Reason (R): According to Avogadro's Law all gases have same number of moles in a given volume of same pressure and temperature.  
(a) Both A and R are individually true and R is the correct explanation of A  
(b) Both A and R are individually true but R is NOT the correct explanation of A  
(c) A is true but R is false  
(d) A is false but R is true

## Ideal Gas

- IES-2. Assertion (A): A perfect gas is one that satisfies the equation of state and whose specific heats are constant. [IES-1993]  
Reason (R): The enthalpy and internal energy of a perfect gas are functions of temperature only.  
(a) Both A and R are individually true and R is the correct explanation of A  
(b) Both A and R are individually true but R is not the correct explanation of A  
(c) A is true but R is false  
(d) A is false but R is true

- IES-3. In a reversible isothermal expansion process, the fluid expands from 10 bar and 2 m<sup>3</sup> to 2 bar and 10m<sup>3</sup>, during the process the heat supplied is 100 kW. What is the work done during the process? [IES-2009]

(a) 33.3 kW                          (b) 100 kW                          (c) 80 kW                          (d) 20 kW

- IES-4. Consider an ideal gas contained in vessel. If intermolecular interaction suddenly begins to act, which of the following happens? [IES-1992]

# Properties of Gasses & Gas Mix.

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- |                           |                                    |
|---------------------------|------------------------------------|
| (a) The pressure increase | (b) The pressure remains unchanged |
| (c) The pressure increase | (d) The gas collapses              |

**IES-5. Which of the following statement is correct?** [IES-1992]

- (a) Boilers are occasionally scrubbed by rapidly and artificially circulating water inside them to remove any thin water film may have formed on their inside
- (b) A sphere, a cube and a thin circular plate of the same mass are made of the same material. If all of them are heated to the same high temperature, the rate of cooling is maximum for the plate and minimum for the sphere.
- (c) One mole of a monoatomic ideal gas is mixed with one mole of diatomic ideal gas. The molar specific heat of the mixture at constant volume is  $2R$ , where  $R$  is the molar gas constant.
- (d) The average kinetic energy of 1 kg of all ideal gases, at the same temperature, is the same.

**IES-6. Consider the following statements:** [IES-2000]

A real gas obeys perfect gas law at a very

- |                     |                  |                 |
|---------------------|------------------|-----------------|
| 1. High temperature | 2. High-pressure | 3. Low pressure |
|---------------------|------------------|-----------------|

Which of the following statements is/are correct?

- |             |             |             |             |
|-------------|-------------|-------------|-------------|
| (a) 1 alone | (b) 1 and 3 | (c) 2 alone | (d) 3 alone |
|-------------|-------------|-------------|-------------|

## Equation of State of a Gas

**IES-7. The correct sequence of the decreasing order of the value of characteristic gas constants of the given gases is:** [IES-1995]

- (a) Hydrogen, nitrogen, air, carbon dioxide
- (b) Carbon dioxide, hydrogen, nitrogen, air
- (c) Air, nitrogen, carbon dioxide, hydrogen
- (d) Nitrogen, air, hydrogen, carbon dioxide

**IES-8. If a real gas obeys the Clausius equation of state  $p(v - b) = RT$  then,** [IES-1992]

$$(a) \left(\frac{\partial u}{\partial v}\right)_T \neq 0 \quad (b) \left(\frac{\partial u}{\partial v}\right)_T = 0 \quad (c) \left(\frac{\partial u}{\partial v}\right)_T = 1 \quad (d) \left(\frac{\partial u}{\partial v}\right)_T = \frac{1}{p}$$

**IES-9. Pressure reaches a value of absolute zero** [IES-2002]

- (a) At a temperature of -273K
- (b) Under vacuum condition
- (c) At the earth's centre
- (d) When molecular momentum of system becomes zero

**IES-9a. Reduced pressure is** [IES-2011]

- |                                             |                   |
|---------------------------------------------|-------------------|
| (a) Always less than atmospheric pressure   | (b) Always unity  |
| (c) An index of molecular position of a gas | (d) Dimensionless |

# Properties of Gasses & Gas Mix.

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## Van der Waals equation

IES-10. Which one of the following is the characteristic equation of a real gas?

[IES-2006]

- (a)  $\left(p + \frac{a}{v^2}\right)(v - b) = RT$       (b)  $\left(p - \frac{a}{v^2}\right)(v + b) = RT$   
(c)  $PV = RT$       (d)  $PV = nRT$

IES-11. Which of the following statement about Van der waal's equation is valid?

- (a) It is valid for all pressure and temperatures  
(b) It represents a straight line on  $PV$  versus  $V$  plot  
(c) It has three roots of identical value at the critical point  
(d) The equation is valid for diatomic gases only.

[IES-1992]

IES-12. The internal energy of a gas obeying Van der Waal's equation

$$\left(P + \frac{a}{v^2}\right)(v - b) = RT, \text{ depends on}$$
 [IES-2000]

- (a) Temperature      (b) Temperature and pressure  
(c) Temperature and specific volume      (d) Pressure and specific volume

IES-13. Van der Waal's equation of state is given by  $\left(P + \frac{a}{v^2}\right)(v - b) = RT$ . The constant

' $b$ ' in the equation in terms of specific volume at critical point  $V_c$  is equal to:

[IES-2003]

(a)  $V_c/3$       (b)  $2 V_c$       (c)  $3 V_c$       (d)  $\frac{8a}{27cR}$

## Compressibility

IES-14. Consider the following statements:

[IES-2007]

1. A gas with a compressibility factor more than 1 is more compressible than a perfect gas.
2. The x and y axes of the compressibility chart are compressibility factor on y-axis and reduced pressure on x-axis.
3. The first and second derivatives of the pressure with respect to volume at critical points are zero.

Which of the statements given above is/are correct?

- (a) 2 and 3 only      (b) 1 and 3 only      (c) 1 and 2 only      (d) 1, 2 and 3

IES-15. Which one of the following statements is correct?

[IES-2007]

- (a) Compressibility factor is unity for ideal gases  
(b) Compressibility factor is zero for ideal gases  
(c) Compressibility factor is lesser than unity for ideal gases

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- (d) Compressibility factor is more than unity for ideal gases

- IES-16. Assertion (A): At very high densities, compressibility of a real gas is less than one. [IES-2006]

Reason (R): As the temperature is considerably reduced, the molecules are brought closer together and thermonuclear attractive forces become greater at pressures around 4 MPa.

- (a) Both A and R are individually true and R is the correct explanation of A  
(b) Both A and R are individually true but R is **not** the correct explanation of A  
(c) A is true but R is false  
(d) A is false but R is true

- IES-17. The value of compressibility factor for an ideal gas may be: [IES-2002]

1. less or more than one    2. equal to one    3. zero    4. less than zero

The correct value(s) is/are given by:

- (a) 1 and 2    (b) 1 and 4    (c) 2 only    (d) 1 only

- IES-18. Assertion (A): The value of compressibility factor, Z approaches zero of all isotherms as pressure p approaches zero. [IES-1992]

Reason (R): The value of Z at the critical points is about 0.29.

- (a) Both A and R are individually true and R is the correct explanation of A  
(b) Both A and R are individually true but R is **not** the correct explanation of A  
(c) A is true but R is false  
(d) A is false but R is true

## Adiabatic Process

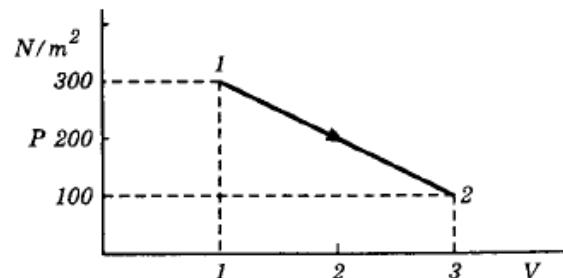
- IES-19. Assertion (A): An adiabatic process is always a constant entropy process.

Reason(R): In an adiabatic process there is no heat transfer. [IES-2005]

- (a) Both A and R are individually true and R is the correct explanation of A  
(b) Both A and R are individually true but R is **not** the correct explanation of A  
(c) A is true but R is false  
(d) A is false but R is true

- IES-20. A control mass undergoes a process from state 1 to state 2 as shown in the given figure. During this process, the heat transfer to the system is 200 KJ. If the control mass returned adiabatically. From state 2 to state 1 by another process, then the work interaction during the return process (in kNm) would be:

- (a) -400    (b) -200  
(c) 200    (d) 400



[IES-1998]

- IES-21. A gas expands from pressure  $P_1$  to pressure  $P_2$  ( $P_2 = P_1/10$ ). If the process of expansion is isothermal, the volume at the end of expansion is  $0.55 \text{ m}^3$ . If the process of expansion is adiabatic, the volume at the end of expansion will be closer to: [IES-1997]

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(a)  $0.45 \text{ m}^3$

(b)  $0.55 \text{ m}^3$

(c)  $0.65 \text{ m}^3$

(d)  $0.75 \text{ m}^3$

- IES-22.** A balloon which is initially collapsed and flat is slowly filled with a gas at  $100 \text{ kPa}$  so as to form it into a sphere of  $1 \text{ m}$  radius. What is the work done by the gas in the balloon during the filling process? [IES-2008]

(a)  $428.9 \text{ kJ}$

(b)  $418.9 \text{ kJ}$

(c)  $420.9 \text{ kJ}$

(d)  $416.9 \text{ kJ}$

- IES-22(i).** In which of the following processes, the heat is fully converted into work? [IES-2012]
- (a) Reversible adiabatic process
  - (b) Reversible isobaric process
  - (c) reversible isometric process
  - (d) Reversible isothermal process

- IES-22(ii)** This process is one in which there is only work interaction between the system and the Surroundings [IES-2012]
- (a) Diabatic process
  - (b) Adiabatic process
  - (c) isothermal process
  - (d) Quasi-static process

## Isothermal Process

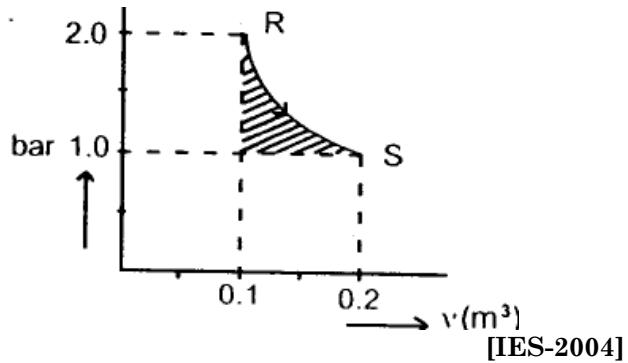
- IES-23.** An ideal gas undergoes an isothermal expansion from state R to state S in a turbine as shown in the diagram given below: The area of shaded region is  $1000 \text{ Nm}$ . What is the amount of turbine work done during the process?

(a)  $14,000 \text{ Nm}$

(b)  $12,000 \text{ Nm}$

(c)  $11,000 \text{ Nm}$

(d)  $10,000 \text{ Nm}$



[IES-2004]

- IES-24.** The work done in compressing a gas isothermally is given by [IES-1997]

$$(a) \frac{\gamma}{\gamma-1} p_1 v_1 \left[ \left( \frac{p_2}{p_1} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right]$$

$$(b) mRT_1 \log_e \left( \frac{p_2}{p_1} \right)$$

$$(c) mc_p(T_2 - T_1) \text{ kJ}$$

$$(d) mRT_1 \left( 1 - \frac{T_2}{T_1} \right) \text{ kJ}$$

- IES-25.** The slope of log P-log V graph for a gas for isothermal change is  $m_1$  and for adiabatic changes is  $m_2$ . If the gas is diatomic gas, then [IES-1992]

(a)  $m_1 < m_2$

(b)  $m_1 > m_2$

(c)  $m_1 + m_2 = 1.0$

(d)  $m_1 = m_2$

- IES-26.** The work done during expansion of a gas is independent of pressure if the expansion takes place [IES-1992]

(a) Isothermally

(b) Adiabatically

(c) In both the above cases

(d) In none of the above cases

- IES-26a** Air is being forced by the bicycle pump into a tyre against a pressure of 4-5 bars. A slow downward movement of the piston can be approximated as

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- (a) Isobaric process  
(c) Throttling process

- (b) Adiabatic process  
(d) Isothermal process

[IES-2011]

- IES-27. Three moles of an ideal gas are compressed to half the initial volume at a constant temperature of 300k. The work done in the process is [IES-1992]  
(a) 5188 J      (b) 2500 J      (c) -2500 J      (d) -5188 J

- IES-28. The change in specific entropy of a system undergoing a reversible process is given by  $s_2 - s_1 = (c_p - c_v) \ln(v_2/v_1)$ . This is valid for which one of the following?  
(a) Adiabatic process undergone by an ideal gas  
(b) Isothermal process undergone by an ideal gas  
(c) Polytropic process undergone by a real gas  
(d) Isobaric phase change from liquid to vapour [IES-2008]

## Polytropic Process

- IES-29. Assertion (A): Though heat is added during a polytropic expansion process for which  $\gamma > n > 1$ , the temperature of the gas decreases during the process.  
Reason (R): The work done by the system exceeds the heat added to the system.  
(a) Both A and R are individually true and R is the correct explanation of A [IES 2007]  
(b) Both A and R are individually true but R is not the correct explanation of A  
(c) A is true but R is false  
(d) A is false but R is true

- IES-30. In a polytropic process, the term  $\left(\frac{\gamma-n}{\gamma-1}\right) \left\{ \frac{p_1 v_1 - p_2 v_2}{(n-1)} \right\}$  is equal to: [IES-2005]  
(a) Heat absorbed or rejected      (b) Change in internal energy  
(c) Ratio of  $T_1/T_2$       (d) Work done during polytropic expansion

- IES-31. The heat absorbed or rejected during a polytropic process is equal to:  
(a)  $\left(\frac{\gamma-n}{\gamma-1}\right)^{1/2} \times \text{work done}$       (b)  $\left(\frac{\gamma-n}{n-1}\right) \times \text{work done}$  [IES-2002]  
(c)  $\left(\frac{\gamma-n}{\gamma-1}\right) \times \text{work done}$       (d)  $\left(\frac{\gamma-n}{\gamma-1}\right)^2 \times \text{work done}$

## Constant Pressure or Isobaric Process

- IES-32. Change in enthalpy in a closed system is equal to the heat transferred, if the reversible process takes place at [IES-2005]  
(a) Temperature      (b) Internal energy      (c) Pressure      (d) Entropy

- IES-33. Which one of the following phenomena occurs when gas in a piston-in-cylinder assembly expands reversibly at constant pressure? [IES-2003]  
(a) Heat is added to the gas  
(b) Heat is removed from the gas  
(c) Gas does work from its own stored energy  
(d) Gas undergoes adiabatic expansion

# Properties of Gasses & Gas Mix.

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Chapter 8

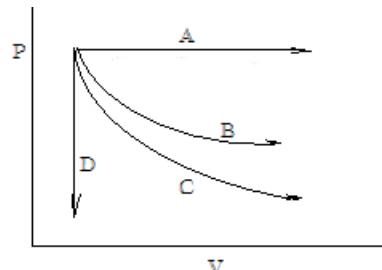
- IES-34. A saturated vapour is compressed to half its volume without changing its temperature. The result is that: [IES-1997]
- (a) All the vapour condenses to liquid
  - (b) Some of the liquid evaporates and the pressure does not change
  - (c) The pressure is double its initial value
  - (d) Some of the vapour condenses and the pressure does not change
- IES-35. An ideal gas at 27°C is heated at constant pressure till its volume becomes three times. [IES-2008, 2012]
- What would be then the temperature of gas?
- (a) 81° C
  - (b) 627° C
  - (c) 543° C
  - (d) 327° C

## Constant Volume or Isochoric Process

- IES-36. In which one of the following processes, in a closed system the thermal energy transferred to a gas is completely converted to internal energy resulting in an increase in gas temperature? [IES-2008]
- (a) Isochoric process
  - (b) Adiabatic process
  - (c) Isothermal process
  - (d) Free expansion
- IES-37. Which one of the following thermodynamic processes approximates the steaming of food in a pressure cooker? [IES-2007]
- (a) Isenthalpic
  - (b) Isobaric
  - (c) Isochoric
  - (d) Isothermal
- IES-37a Assertion (A): The constant pressure lines are steeper than the constant volume lines for a perfect gas on the T-S plane. [IES-2010]  
Reason (R): The specific heat at constant pressure is more than the specific heat at constant volume for a perfect gas.
- (a) Both A and R are individually true and R is the correct explanation of A
  - (b) Both A and R are individually true but R is **not** the correct explanation of A
  - (c) A is true but R is false
  - (d) A is false but R is true

- IES-38. Consider the four processes A, B, C and D shown in the graph given above:

Match List-I (Processes shown in the graph) with List-II (Index 'n' in the equation  $pV^n = \text{Const}$ ) and select the correct answer using the code given below the lists:



[IES-2007, 2011]

List-I

- A. A
- B. B
- C. C
- D. D

Codes:    A            B            C            D  
(a)       4            2            3            1

List-II

- 1. 0
- 2. 1
- 3. 1.4
- 4.  $\infty$

(b)       A            B            C            D  
            1            2            3            4

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(c)      1      3      2      4

(d)      4      3      2      1

- IES-38a** Match List I with List II and select the correct answer using the code given below the lists:

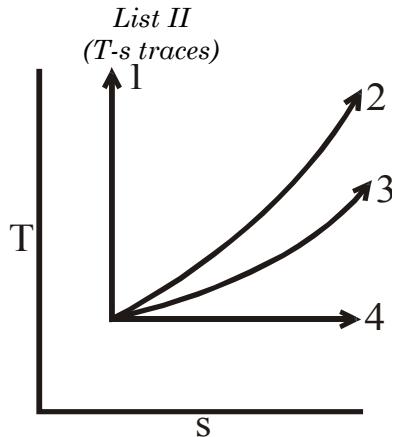
*List I  
(Process index 'n')*

- A. 0
- B. 1.0
- C. 1.4
- D.  $\infty$

**Code :**

	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a)	3	1	4	2
(b)	2	1	4	3
(c)	3	4	1	2
(d)	2	4	1	3

*List II  
(T-s traces)*



[IES-2010]

- IES-39.** Match List-I (process) with List-II (index n in  $PV^n = \text{constant}$ ) and select the correct answers using the codes given below the lists. [IES-1999]

**List-I**

- A. Adiabatic
- B. Isothermal
- C. Constant pressure
- D. Constant volume

**List-II**

1.  $n = \infty$
2.  $n = \frac{C_p}{C_v}$
3.  $n = 1$
4.  $n = \frac{C_p}{C_v} - 1$
5.  $n = 0$

**Codes:**

	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>	<b>A</b>	<b>B</b>	<b>C</b>	<b>D</b>
(a)	2	3	5	4	(b)	3	2	1
(c)	2	3	5	1	(d)	2	5	3

- IES-40.** A system at a given state undergoes change through the following expansion processes to reach the same final volume [IES-1994]

- 1. Isothermal
- 2. Isobaric
- 3. Adiabatic ( $\gamma = 1.4$ )
- 4. Polytropic( $n = 1.3$ )

The correct ascending order of the work output in these four processes is

- (a) 3, 4, 1, 2      (b) 1, 4, 3, 2      (c) 4, 1, 3, 2      (d) 4, 1, 2, 3

- IES-41.** Match the curves in Diagram-I with the curves in Diagram-II and select the correct answer. [IES-1996]

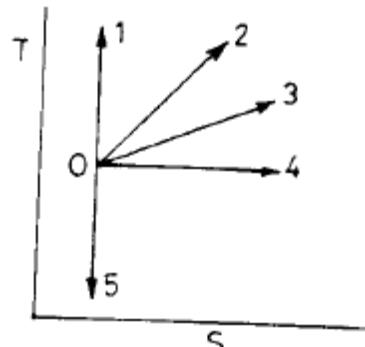
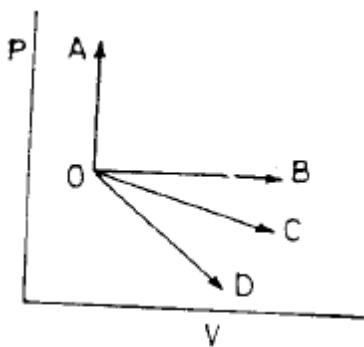
Diagram-I (Process on p-V plane)

Diagram-II (Process on T-s plane)

# Properties of Gasses & Gas Mix.

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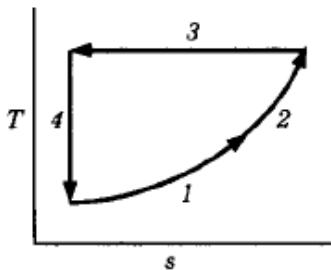
**Code:**

- |     |   |   |   |   |
|-----|---|---|---|---|
| (a) | A | B | C | D |
| (c) | 3 | 2 | 4 | 5 |
|     | 2 | 3 | 4 | 1 |

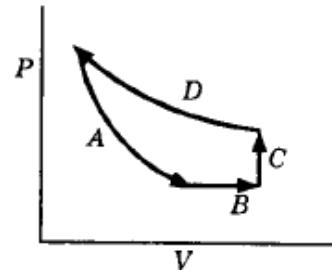
(b)

- |     |   |   |   |   |
|-----|---|---|---|---|
| (d) | A | B | C | D |
|     | 2 | 3 | 4 | 5 |
|     | 1 | 4 | 2 | 3 |

**IES-42.**



**Fig. I**



**Fig. II**

Four processes of a thermodynamic cycle are shown above in Fig.I on the T-s plane in the sequence 1-2-3-4. The corresponding correct sequence of these processes in the p-V plane as shown above in Fig. II will be [IES-1998]

- |             |             |             |             |
|-------------|-------------|-------------|-------------|
| (a) C-D-A-B | (b) D-A-B-C | (c) A-B-C-D | (d) B-C-D-A |
|-------------|-------------|-------------|-------------|

**IES-43.**

Match List-I with List-II and select the correct answer

[IES-1996]

**List-I**

- A. Work done in a polytropic process
- B. Work done in a steady flow process
- C. Heat transfer in a reversible adiabatic process
- D. Work done in an isentropic process

**List-II**

1.  $\int v dp$
2. Zero
3.  $\frac{p_1 V_1 - p_2 V_2}{\gamma - 1}$
4.  $\frac{p_1 V_1 - p_2 V_2}{n - 1}$

**Codes:**

- |     |   |   |   |   |
|-----|---|---|---|---|
| (a) | A | B | C | D |
| (c) | 4 | 1 | 3 | 2 |
|     | 4 | 1 | 2 | 3 |

- |   |   |   |   |
|---|---|---|---|
| A | B | C | D |
|---|---|---|---|

- |     |   |   |   |   |
|-----|---|---|---|---|
| (b) | 1 | 4 | 2 | 3 |
| (d) | 1 | 2 | 3 | 4 |

**IES-44.**

A perfect gas at 27°C was heated until its volume was doubled using the following three different processes separately [IES-2004]

- |                              |                      |                      |
|------------------------------|----------------------|----------------------|
| 1. Constant pressure process | 2 Isothermal process | 3 Isentropic process |
|------------------------------|----------------------|----------------------|

# **Properties of Gasses & Gas Mix.**

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Which one of the following is the correct sequence in the order of increasing value of the final temperature of the gas reached by using the above three different processes?

- (a) 1 – 2 – 3              (b) 2 – 3 – 1              (c) 3 – 2 – 1              (d) 3 – 1 – 2

## **Previous 20-Years IAS Questions**

### **Ideal Gas**

**IAS-1.** Variation of pressure and volume at constant temperature are correlated through [IAS-2002]  
 (a) Charles law        (b) Boyle's law        (c) Joule's Law        (d) Gay Lussac's Law

**IAS-2.** Assertion (A): For a perfect gas, hyperbolic expansion is an isothermal expansion. [IAS-2007]

Reason (R): For a perfect gas,  $\frac{PV}{T} = \text{constant}$ .

- (a) Both A and R are individually true and R is the correct explanation of A  
 (b) Both A and R are individually true but R is not the correct explanation of A  
 (c) A is true but R is false  
 (d) A is false but R is true

**IAS-3.** Variation of pressure and volume at constant temperature are correlated through [IAS-2002]

- (a) Charle's law        (b) Boyle's law        (c) Joule's law        (d) Gay Lussac's law

**IAS-4.** An ideal gas with initial volume, pressure and temperature of  $0.1 \text{ m}^3$ , 1 bar and  $27^\circ\text{C}$  respectively is compressed in a cylinder by a piston such that its final volume and pressure are  $0.04 \text{ m}^3$  and 5 bars respectively, then its final temperature will be: [IAS-2001]

- (a)  $-123^\circ\text{C}$         (b)  $54^\circ\text{C}$         (c)  $327^\circ\text{C}$         (d)  $600^\circ\text{C}$

### **Equation of State of a Gas**

**IAS-5.** The volumetric air content of a tyre at  $27^\circ\text{C}$  and at 2 bars is 30 litres. If one morning, the temperature dips to  $-3^\circ\text{C}$  then the air pressure in the tyre would be: [IAS-2000]

- (a) 1.8 bars        (b) 1.1 bars        (c) 0.8 bars        (d) The same as at  $27^\circ\text{C}$

**IAS-6.** An Ideal gas with initial volume, pressure and temperature of  $0.1 \text{ m}^3$ , 1 bar and  $27^\circ\text{C}$  respectively is compressed in a cylinder by piston such that its final volume and pressure 0.04  $\text{m}^3$  and 5 bar respectively, then its final temperature will be: [IAS-2001]

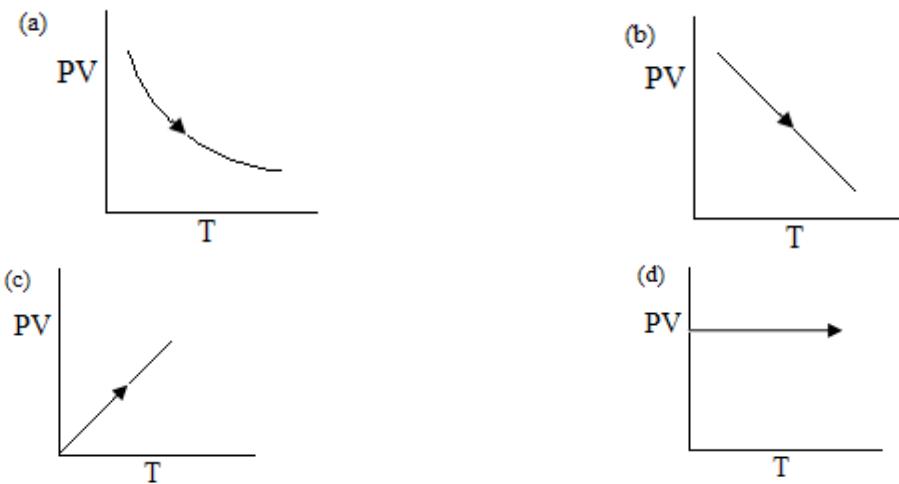
- (a)  $-123^\circ\text{C}$         (b)  $54^\circ\text{C}$         (c)  $327^\circ\text{C}$         (d)  $600^\circ\text{C}$

**IAS-7.** Which one of the following PV-T diagrams correctly represents the properties of an ideal gas? [IAS-1995]

# Properties of Gasses & Gas Mix.

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## Van der Waals Equation

- IAS-8. If a gas obeys van der Waals' equation at the critical point, then  $\frac{RT_c}{p_c v_c}$  is equal to which one of the following [IAS-2004; 2007]  
(a) 0 (b) 1 (c) 1.5 (d) 2.67
- IAS-9. In Van der Waal's gas equation  $\left(P + \frac{a}{v^2}\right)(v - b) = RT$  ( $R$  = Universal gas constant) the unit of 'b' is: [IAS-1997]  
(a) Liter/mole°C (b) m<sup>3</sup>/mole (c) kg-liter/mole (d) Dimensionless
- IAS-10. A higher value of Van der waal's constant for a gas indicates that the [IAS-2003]  
(a) Molecules of the gas have smaller diameter  
(b) Gas can be easily liquefied  
(c) Gas has higher molecular weight  
(d) Gas has lower molecular weight

## Critical Properties

- IAS-11. The mathematical conditions at the critical point for a pure substance are represented by: [IAS-1999]
- (a)  $\frac{\delta p}{\delta v} < 0, \frac{\delta^2 p}{\delta v^2} = 0$  and  $\frac{\delta^3 p}{\delta v^3} = 0$       (b)  $\frac{\delta p}{\delta v} = 0, \frac{\delta^2 p}{\delta v^2} < 0$  and  $\frac{\delta^3 p}{\delta v^3} = 0$   
(c)  $\frac{\delta p}{\delta v} = 0, \frac{\delta^2 p}{\delta v^2} = 0$  and  $\frac{\delta^3 p}{\delta v^3} < 0$       (d)  $\frac{\delta p}{\delta v} = 0, \frac{\delta^2 p}{\delta v^2} = 0$  and  $\frac{\delta^3 p}{\delta v^3} = 0$

# Properties of Gasses & Gas Mix.

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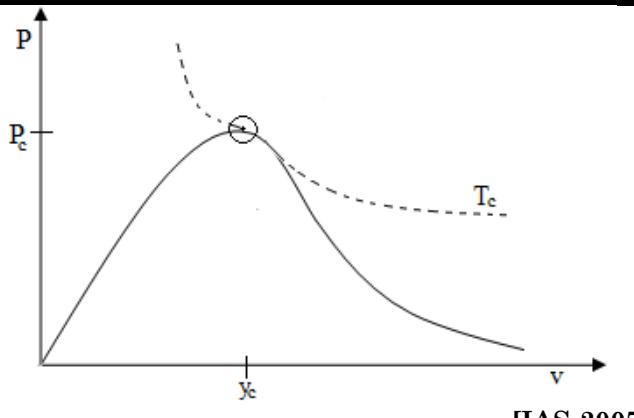
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- IAS-12. In the above figure,  $y_c$  corresponds to the critical point of a pure substance under study. Which of the following mathematical conditions applies/apply at the critical point?

(a)  $\left(\frac{\partial P}{\partial v}\right)_{T_c} = 0$

(b)  $\left(\frac{\partial^2 P}{\partial v^2}\right)_{T_c} = 0$

(c)  $\left(\frac{\partial^3 P}{\partial v^3}\right)_{T_c} < 0$



[IAS-2007]

- (d) All of the above

## Adiabatic Process

- IAS-13. Consider the following statements:

[IAS-2007]

1. During a reversible non-flow process, for the same expansion ratio, work done by a gas diminishes as the value of n in  $PV^n = C$  increases.
2. Adiabatic mixing process is a reversible process.

Which of the statements given above is/are correct?

- (a) 1 only      (b) 2 only      (c) Both 1 and 2      (d) Neither 1 nor 2

## Isothermal Process

- IAS-14. Identify the process of change of a close system in which the work transfer is maximum.

[IAS-2003]

- (a) Isothermal      (b) Isochoric      (c) Isentropic      (d) Polytrop

- IAS-15. In a reversible isothermal expansion process, the fluid expands from 10 bar and  $2 \text{ m}^3$  to 2 bar and  $10 \text{ m}^3$ . During the process the heat supplied is at the rate of 100 kW. What is the rate of work done during the process?

[IAS-2007]

- (a) 20 kW      (b) 35 kW      (c) 80 kW      (d) 100 kW

- IAS-16. In respect of a closed system, when an ideal gas undergoes a reversible isothermal process, the

[IAS-2000]

- (a) Heat transfer is zero      (b) Change in internal energy is equal to work transfer  
(c) Work transfer is zero      (d) Heat transfer is equal to work transfer

## Constant Pressure or Isobaric Process

- IAS-17. For a non-flow constant pressure process the heat exchange is equal to:

- (a) Zero      (b) The work done      (c) The change in internal energy      (d) The change in enthalpy

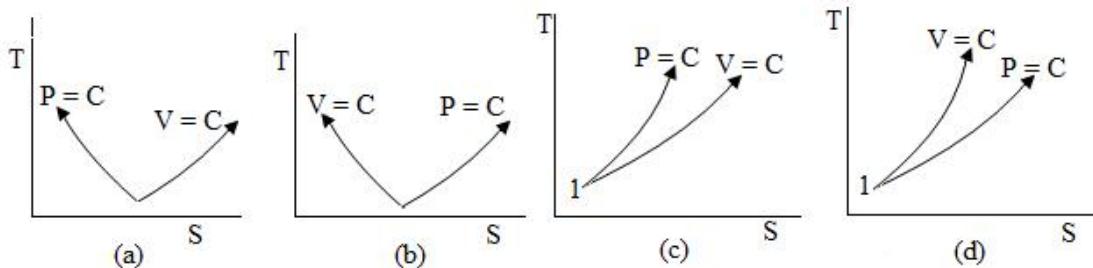
# Properties of Gasses & Gas Mix.

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## Constant Volume or Isochoric Process

- IAS-18. An ideal gas is heated (i) at constant volume and (ii) at constant pressure from the initial state 1. Which one of the following diagrams shows the two processes correctly? [IAS-1996]



- IAS-19. One kg of a perfect gas is compressed from pressure  $P_1$  to pressure  $P_2$  by  
 1. Isothermal process      2. Adiabatic process      3. The law  $PV^{1.4} = \text{constant}$   
 The correct sequence of these processes in increasing order of their work requirement is: [IAS-2000]

(a) 1, 2, 3      (b) 1, 3, 2      (c) 2, 3, 1      (d) 3, 1, 2

- IAS-20. Match List-I with List-II and select the correct answer using the codes given below the Lists: [IAS-1997]

List-I

- A. Constant volume process
- B. Constant pressure process
- C. Constant temperature process
- D. Constant entropy process

List-II

- 1.  $\frac{dP}{dV} = -\frac{P}{V}$
- 2.  $\frac{dP}{dV} = -\frac{\gamma P}{V}$
- 3.  $\frac{dT}{ds} = -\frac{T}{C_v}$
- 4.  $\frac{dT}{ds} = -\frac{T}{C_p}$

Codes:

A

B

C

D

(a) 3      2      1      4

(c) 3      4      1      2

A

B

C

D

(b) 2      4      3      1

(d) 1      3      4      2

## Properties of Mixtures of Gases

- IAS-21. If  $M_1, M_2, M_3$ , be molecular weight of constituent gases and  $m_1, m_2, m_3$ ... their corresponding mass fractions, then what is the molecular weight  $M$  of the mixture equal to? [IAS-2007]

(a)  $m_1 M_1 + m_2 M_2 + m_3 M_3 + \dots$

(b)  $\frac{1}{m_1 M_1 + m_2 M_2 + m_3 M_3 + \dots}$

(c)  $\frac{1}{m_1 M_1} + \frac{1}{m_2 M_2} + \frac{1}{m_3 M_3} + \dots$

(d)  $\left( \frac{m_1}{M_1} \right) + \left( \frac{m_2}{M_2} \right) + \left( \frac{m_3}{M_3} \right) + \dots$

# **Properties of Gasses & Gas Mix.**

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**IAS-22. The entropy of a mixture of pure gases is the sum of the entropies of constituents evaluated at [IAS-1998]**

- (a) Temperature and pressure for the mixture
- (b) Temperature of the mixture and the partial pressure of the constituents
- (c) Temperature and volume of the mixture
- (d) Pressure and volume of the mixture

# Properties of Gasses & Gas Mix.

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## Answers with Explanation (Objective)

### Previous 20-Years GATE Answers

GATE-1. (b). Let no of mole = n

Initial  $P_1 = 10 \text{ bar}$

Final  $P_2 = ?$

$$V_1 = \left(\frac{1}{n}\right) m^3/\text{mole}$$

$$V_2 = \left(\frac{2}{n}\right) m^3/\text{mole}$$

$T_1 = 300\text{K}$

$T_2 = 300\text{K} = T_1 = T$  (say)

$$\therefore (P_1 + a/v_1^2) v_1 = (P_2 + a/v_2^2) v_2$$

$$\Rightarrow (10 + an^2) \times (1/n) = (P_2 + an^2/4) \times (2/n)$$

$$\Rightarrow 2P_2 = 10 + an^2 - an^2/2 = 10 + an^2/2$$

$$\Rightarrow P_2 = 5 + an^2/4$$

As  $a > 0 \therefore P_2$  is slightly more than 5 bar.

GATE-2. Ans. (a)  $wd = \frac{1}{\gamma-1} (P_2 V_2 - P_1 V_1)$

WKT

$$PV = mRT$$

$$\therefore P_1 V_1 = \frac{R}{m} T_1 \quad [R = 8.314 \text{ kJ kmol}^{-1} \text{ K}^{-1}]$$

$$0.1 \times 10^3 \times V_1 = \frac{8.314}{40} \times 300$$

$$\therefore V_1 = 0.623 \text{ m}^3$$

$$\left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}} = \frac{V_2}{V_1} \Rightarrow V_2 = 0.41 \text{ m}^3$$

$$\therefore W.d = \frac{(2 \times 0.41 - 0.1 \times 0.623) \times 10^3}{0.67} = 29.7$$

GATE-3. Ans. (d) Heat lost =  $n C_v d_T$

GATE-4. Ans. (c)

GATE-5. Ans. (c) Heat produced by electric bulb in 24 hr. =  $100 \times 24 \times 60 \times 60 \text{ J} = 8640 \text{ kJ}$

Volume of air =  $25 \times 3 \times 3 = 225 \text{ m}^3$

Density ( $\rho$ ) =  $1.24 \text{ kg/m}^3$

$$\Delta Q = mC_v \Delta t \quad \text{or} \quad \Delta t = \frac{\Delta Q}{mC_v} = \frac{8640}{225 \times 1.24 \times 0.716} = 430^\circ\text{C} \quad \therefore t = 430 + 20 = 450^\circ\text{C}$$

GATE-6. Ans. (a) Iso-thermal work done (W) =  $RT_1 \ln\left(\frac{V_2}{V_1}\right)$

# Properties of Gasses & Gas Mix.

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$$= P_1 V_1 \ln\left(\frac{V_2}{V_1}\right)$$

$$= 800 \times 0.015 \times \ln\left(\frac{0.030}{0.015}\right) = 8.32 \text{ kJ/Kg}$$

**GATE-7. Ans. (b)** Remember if we mix 2 mole of oxygen with another 2 mole of *other gas* the volume will be doubled for first and second constituents  $\Delta S = nR \ln \frac{V_{total}}{V_{initial}} = 2R \ln 2 \therefore$

Total Entropy change =  $4R \ln 2$  So, Entropy change per mole =  $R \ln 2$ . And it is due to diffusion of one gas into another.

## Previous 20-Years IES Answers

**IES-1. Ans. (a)** Both A and R correct and R is the correct explanation of A

**IES-2. Ans. (b)** For perfect gas, both the assertion A and reason R are true. However R is not the explanation for A. A provides definition of perfect gas. R provides further relationship for enthalpy and internal energy but can't be reason for definition of perfect gas.

**IES-3. Ans. (b)** As internal energy is a function of temperature only. In isothermal expansion process no temperature change therefore no internal energy change. A Reversible isothermal expansion process is constant internal energy process i.e.  $dU = 0$

$$\therefore dQ = dU + dW$$

$$\therefore dQ = dW (\because dU = 0)$$

**.. Work done during the process = 100kW**

**IES-4. Ans. (a)**

**IES-5. Ans. (d)** (a) True. A water film, if formed, will act as a very poor conductor of heat and will not easily let the heat of the furnace pass into the boiler. An oil film if present, is even worse than water film and the formation of such films inside the boiler must be avoided.

(b) Since the mass and material are the same, the volumes must also be the same. For the same volume, the surface area of the plate is the greatest and that of the sphere is the least. The rate of loss of heat by radiation being proportional to the surface area, the plate cools the fastest and the sphere the slowest.

(c) True, for a monoatomic gas,  $C_v = \frac{3}{2} R$  and for a diatomic gas,  $C_v = \frac{5}{2} R$ .

Since the mixture has two moles, the value of  $C_v$  for the mixture =  $\frac{1}{2} \left( \frac{3}{2} R + \frac{5}{2} R \right) = 2 R$

(d) False, The average kinetic energy of 1 g of an ideal gas =  $\frac{3}{2} \frac{RT}{M}$

Where M is the molecular weight of the gas and it is different gases, as the value of M will be different.

**IES-6. Ans (b)** In Perfect gas intermolecular attraction is zero. It will be only possible when intermolecular distance will be too high. High temperature or low pressure or both cause high intermolecular distance so choice 1 and 3.

**IES-7. Ans. (a)** The correct sequence for decreasing order of the value of characteristic gas constants is hydrogen, nitrogen, air and carbon dioxide.

**IES-8. Ans. (b)**

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**IES-9.** Ans. (d) we know that  $P = \frac{1}{3} \rho C^2$  If momentum is zero then C must be zero. Hence P

would be zero. That will occur at absolute zero temperature. But note here choice (a) has in defined temp.  $-273K$  which is imaginary temp.

**IES-9a.** Ans. (d)

**IES-10.** Ans. (a)

**IES-11.** Ans. (c)

**IES-12.** Ans. (b) Joule's law states that for an Ideal gas internal energy is a function of temperature only.  $u = f(T)$ . But this is not Ideal gas it is real gas.

**IES-13.** Ans. (a) We know that at critical point

$$a = 3P_c V_c^2 ; b = V_c/3 \text{ and } R = \frac{8P_c V_c}{3T_c}$$

**IES-14.** Ans. (a) 1 is false. At very low pressure, all the gases shown have  $z \approx 1$  and behave nearly perfectly. At high pressure all the gases have  $z > 1$ , signifying that they are more difficult to compress than a perfect gas (for a given molar volume, the product  $pv$  is greater than  $RT$ ). Repulsive forces are now dominant. At intermediate pressure, must gasses have  $Z < 1$ , including that the attractive forces are dominant and favour compression.

**IES-15.** Ans. (a)

**IES-16.** Ans. (d)

**IES-17.** Ans. (c)

**IES-18.** Ans. (d)

**IES-19.** Ans. (d)

**IES-20.** Ans. (b) During adiabatic process, work done = change in internal energy.

Since control man (so case of closed system). Intercept of path on X-axis is the work done by the process.

$$W = \text{area of } \Delta A_1 2 + \text{area of } \square A_2 C B$$

$$W = \frac{1}{2} \times (3 - 1) \times 200 + 100 \times (3 - 1) \\ = 200 + 200 = 400 \text{ kJ}$$

$$W = \frac{1}{2} (300 - 100) \times 2 + 100 \times 2 \\ = 200 + 200 = 400 \text{ kJ}$$

$\therefore$  From 1  $\rightarrow$  2.

$$U_1 + Q = U_2 + W.$$

$$U_1 - U_2 = W - Q = 400 - 200 = 200 \text{ kJ.}$$

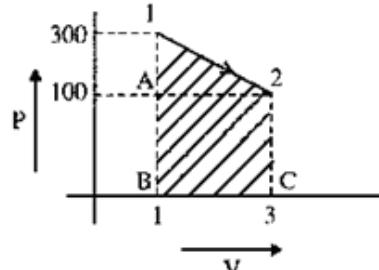
From 2  $\rightarrow$  1

Work done will be same

Since adiabatic So  $Q = 0$

$$U_2 + Q = U_1 + W$$

$$W = U_2 - U_1 = -(U_1 - U_2) = -200 \text{ kJ.}$$



**IES-21.** Ans. (a) For isothermal process,  $p_1 v_1 = p_2 v_2$ , or  $p_1 v_1 = \frac{p_1}{10} \times 0.55$ ,  $v_1 = 0.055 \text{ m}^3$

For adiabatic process

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$$p_1 v_1^{1.4} = p_2 v_2^{1.4}, \text{ or } p_1 (0.055)^{1.4} = \frac{p_1}{10} \times v_2^{1.4} \text{ or } v_2 = 0.055 \sqrt[4]{10} = 0.45 m^3$$

IES-22. Ans. (b) Work done by the gas during filling process =  $-\int v dp$   
 $= \left(\frac{4}{3}\pi r^3\right)(100) = 418.9 \text{ kJ}$

IES-22(i) Ans. (d)

IES-22(ii) Ans. (b)

IES-23. Ans. (c) Turbine work = area under curve R-S =  $\int P dv$   
 $= 1 \text{ bar} \times (0.2 - 0.1) m^3 + 1000 \text{ Nm}$   
 $= 10^5 \times (0.2 - 0.1) \text{ Nm} + 1000 \text{ Nm} = 11000 \text{ Nm}$

IES-24. Ans. (b)

IES-25. Ans. (a) PV = constant, C

$$\Rightarrow \log P + \log V = \log C$$

$$m_1 = -1$$

$$PV^4 = C$$

$$\Rightarrow \log P + q \log V = \log C$$

$$m_2 = -q = -1.4$$

$$\therefore m_2 > m_1$$

IES-26. Ans. (d)

IES-26a Ans. (d)

IES-27. Ans. (d) Since the temperature remains constant, the process is isothermal.

$$\begin{aligned} \therefore \text{Work-done in the process, } W &= 2.303 nRT \log \left( \frac{V_2}{V_1} \right) \\ &= 2.303 \times 3 \times 8.315 \times 8.315 \times 300 \log \left( \frac{1}{2} \right) \\ &= -5188 \text{ J.} \end{aligned}$$

The negative sign indicates that work is done on the gas.

IES-28. Ans. (b)  $Tds = du + pdv$

$$\Rightarrow Tds = C_V dT + pdv$$

$$\Rightarrow ds = C_V \frac{dT}{T} + \frac{P}{T} dv$$

$$\Rightarrow ds = C_V \frac{dT}{T} + \frac{R}{V} dv$$

Integrating the above expression

$$\Rightarrow S_2 - S_1 = C_V \ln \frac{T_2}{T_1} + R \ln \frac{V_2}{V_1}$$

For isothermal process undergone by ideal gas.

$$\Rightarrow S_2 - S_1 = (C_P - C_V) \ln \frac{V_2}{V_1}$$

IES-29. Ans. (a)

IES-30. Ans. (a)

IES-31. Ans. (c)

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IES-32. Ans. (c)  $dQ = du + pdv + pd\ln p - vdp = d(u + pv) - vdp = dh - vdp$

if  $dp=0$  or  $p=\text{const}$  these for  $(dQ)_p = (dh)_p$

IES-33. Ans. (a)

	s	T	p	v
Isothermal expansion	↑	—	↓	↑
Isothermal compression	↓	—	↑	↓
Isentropic compression	—	↑	↑	↓
Isentropic expansion	—	↓	↓	↑
Isochoric heating	↑	↑	↑	—
Isochoric cooling	↓	↓	↓	—
Isobaric heating/expansion	↑	↑	—	↑
Isobaric cooling/compression	↓	↓	—	↓

IES-34. Ans. (d) By compressing a saturated vapour, its vapours condense and pressure remains unchanged. Remember it is not gas.

IES-35. Ans. (b)  $\frac{V_1}{T_1} = \frac{V_2}{T_2} \Rightarrow \frac{V_1}{(273+27)} = \frac{3V_1}{T_2}$   
 $\Rightarrow T_2 = 300 \times 3 = 900 \text{ K} = 627^\circ\text{C}$

IES-36. Ans. (a) **Constant volume (isochoric) process:** An example of this process is the heating or cooling of a gas stored in a rigid cylinder. Since the volume of the gas does not change, no external work is done, and work transferred  $\Delta W$  is zero. Therefore from 1st law of thermodynamics for a constant volume process:

$$\begin{aligned} W_2 &= 0 \\ Q_2 &= \int_1^2 dU = U_2 - U_1 \end{aligned}$$

IES-37. (c) In a pressure cooker, the volume of the cooker is fixed so constant volume process but for safety some of steam goes out to maintain a maximum pressure. But it occurs after proper steaming.

IES-37a Ans. (d)

IES-38. Ans. (b)

IES-38a Ans. (c)

IES-39. Ans. (c)

IES-40. Ans. (a)

IES-41. Ans. (b)

IES-42. Ans. (d)

IES-43. Ans. (c)

IES-44. Ans. (c) Perfect gas

$$T_1 = 27 + 273 = 300 \text{ K}$$

$$V_1 = \text{initial volume}$$

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$$V_2 = \text{final volume}, V_2 = 2V_1$$

$$T_2 = ?$$

(i) Constant Pressure Process

$$V \propto T$$

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

$$\frac{V_1}{T_1} = \frac{2V_1}{T_2}$$

$$\Rightarrow T_2 = 2T_1 = 600\text{K}$$

(ii) Isothermal Process

$$T = \text{Constant}$$

$$T_1 = T_2 = 300\text{K}$$

(iii) Isentropic Process

$$T_1 V_1^{r-1} = T_2 V_2^{r-1}$$

$$T_2 = T_1 \left( \frac{V_1}{V_2} \right)^{r-1} = T_1 \left( \frac{V_1}{2V_1} \right)^{1.4-1}$$

$$T_2 = 0.757 T_1 = 227.35\text{K}$$

## Previous 20-Years IAS Answers

**IAS-1. Ans. (b) Boyle's law:** It states that volume of a given mass of a perfect gas varies inversely as the absolute pressure when temperature is constant.

**IAS-2. Ans. (a)**

**IAS-3. Ans. (b)**

**IAS-4. Ans. (c)**  $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$  or  $T_2 = \frac{P_2 V_2}{P_1 V_1} \times T_1 = \frac{5 \times 0.04}{1 \times 0.1} \times (300) = 600\text{K} = 327^\circ\text{C}$

**IAS-5. Ans. (a)** Apply equation of states

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2} [\because V_1 = V_2] \quad \text{or } P_2 = P_1 \times \frac{T_2}{T_1} = 2 \times \frac{(273 - 3)}{(273 + 27)} = 1.8 \text{ bar}$$

**IAS-6. Ans. (c)** Apply equation of states  $\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$  or  $T_2 = \frac{P_2 V_2}{P_1 V_1} \times T_1$

$$\therefore T_2 = \left( \frac{5}{1} \right) \times \left( \frac{0.04}{0.1} \right) \times (273 + 27) = 600\text{K} = 327^\circ\text{C}$$

**IAS-7. Ans. (c)** For an ideal gas  $PV = MRT$  i.e.  $P$  and  $T$  follow direct straight line relationship, which is depicted in figure (c).

**IAS-8. Ans. (d)**  $a = 3 p_c V_c^2$ ,  $b = \frac{V_c}{3}$ ,  $R = \frac{8 P_c V_c}{3 T_c}$

**IAS-9. Ans. (b)** According to dimensional homogeneity law unit of molar-volume and 'b' must be same. i.e.  $\text{m}^3/\text{mole}$

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**IAS-10. Ans. (b)**

**IAS-11. Ans. (c)**

**IAS-12. Ans. (d) Van der Waals equation**

$$\left(P + \frac{a}{V^2}\right)(V - b) = RT \quad \text{or} \quad P = \frac{RT}{V - b} - \frac{a}{V^2}$$

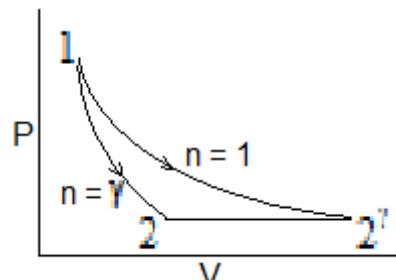
$$\text{At critical point } a = 3p_c V_c^2, b = \frac{V_c}{3}, R = \frac{8P_c V_c}{3T_c}$$

$$\left(\frac{\partial P}{\partial V}\right)_{T=T_c} = \frac{-RT_c}{(V_c - b)^2} + \frac{2a}{V_c^3} = 0$$

$$\left(\frac{\partial^2 P}{\partial V^2}\right)_{T=T_c} = \frac{2RT_c}{(V_c - b)^3} - \frac{6a}{V_c^4} = 0$$

$$\text{And } \left(\frac{\partial^3 P}{\partial V^3}\right)_{T=T_c} = -\frac{6RT_c}{(V_c - b)^4} - \frac{24a}{V_c^5} = -9p_c \text{ i.e.-ive}$$

**IAS-13. Ans. (a)** In adiabatic mixing there is always increase in entropy so large amount of irreversibility is these.



**IAS-14. Ans. (c)**

**IAS-15. Ans. (d)** For reversible isothermal expansion heat supplied is equal to work done during the

$$\text{process and equal to } Q = W = mRT_1 \ln\left(\frac{V_2}{V_1}\right)$$

$\therefore$  Temperature constant so no change in internal energy  $dQ = dU + dW; dU = 0$   
Therefore  $dQ = dW$ .

**IAS-16. Ans. (d)** In reversible isothermal process temperature constant. No change in internal energy. So internal energy constant  $dQ = \delta U + \delta W$  as  $\delta U = 0, dQ = dW$

**IAS-17. Ans. (d)**

**IAS-18. Ans. (d)**

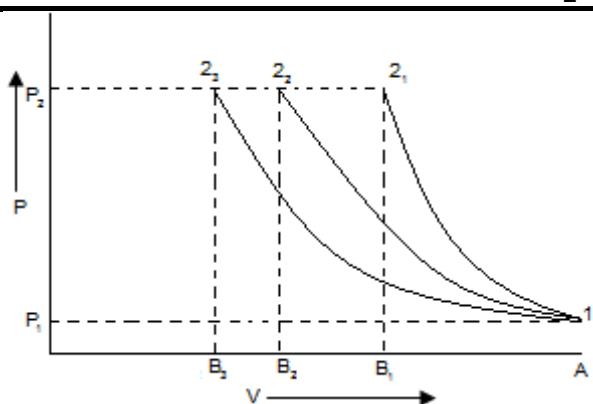
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IAS-19. Ans. (b) Work requirement

1. Isothermal – area under  $12_1B_1A$
2. Adiabatic – area under  $12_2B_2A$
3.  $pv^{1.1} = c$  – area under  $12_3B_3A$



IAS-20. Ans. (c)

IAS-21. Ans. (a)

IAS-22. Ans. (b)