Lab 6: Non-Linearity and its effects in communication systems

EE340: Prelab Reading material for Experiment 6

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Non-Linear Systems

- Linear Systems: Satisfy superposition principle
- However, any practical system is non-linear (amount of non-linearity may vary)
- Non-linearity results in generation of "new frequency components" – i.e. frequency components that are not there at the input of the system.
- Memory-less non-linearity can be modeled as:

  $$y(t) = a_1x(t) + a_2x^2(t) + a_3x^3(t) + a_4x^4(t)...$$

- Memory-less means present output depends only on the present input (also see Appendix – last slide)

Effects of Non-Linearity

Consider a simplified non-linear system described by

$$y(t) = a_1 x(t) + a_2 x^2(t) + a_3 x^3(t)$$

For $x(t) = Acos(\omega t)$,

$$y(t) = \tfrac{1}{2} a_2 A^2 + (a_1 + \tfrac{3}{4} a_3 A^2) Acos(\omega t) + \tfrac{1}{2} a_2 A^2 cos(2\omega t) + \tfrac{1}{4} a_3 A^3 cos(3\omega t)$$

Important observations:

- **Second order non-linearity (a$_2$ coefficient)** :

  $$\boxed{\frac{1}{2} a_2 A^2 (1 + cos(2\omega t)}$$

  Adds DC + 2nd harmonic

- **Third order non-linearity (a$_3$ coefficient)** :

  $$\boxed{(a_1 + \frac{3}{4} a_3 A^2) Acos(\omega t) + \frac{1}{4} a_3 A^3 cos(3\omega t) \quad Gain: \; (a_1 + \frac{3}{4} a_3 A^2)}$$

  Adds 3rd harmonic

  Gain becomes input amplitude (A) dependent. Also, a_3 is generally negative $=>$ gain compresion with increasing A

Consider a non-linear system described by

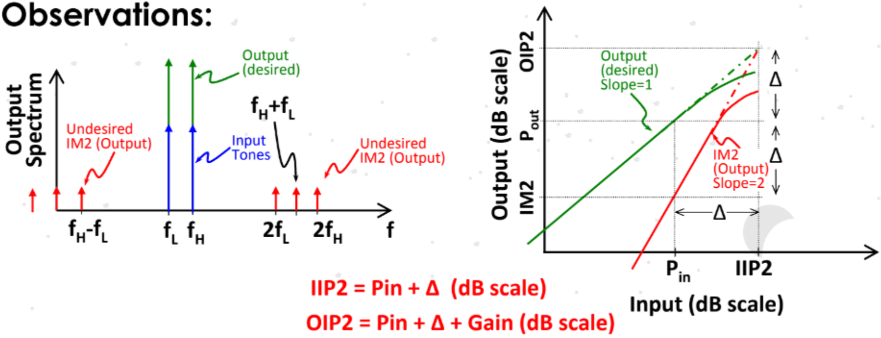
$$y(t) = a_1 x(t) + a_2 x^2(t)$$

For $x(t) = A(cos\omega_1 t + cos\omega_2 t)$,

$$\boxed{y(t) = a_2 A^2 + a_1 A\left(\cos(\omega_1 t) + \cos(\omega_2 t)\right) + a_2 A^2\left(\frac{\cos(2\omega_1 t) + \cos(2\omega_2 t)}{2} + \cos((\omega_1 - \omega_2)t) + \cos((\omega_1 + \omega_2)t)\right)}$$

The undesired spectral components generated due to the second order non- linearity coefficent a_2 at frequencies 0, $2\omega_1$, $2\omega_2$, $(\omega_1 - \omega_2)$ and $(\omega_1 + \omega_2)$ are called IM2 (Inter-Modulation products due to 2^{nd} order non-linearity) components.

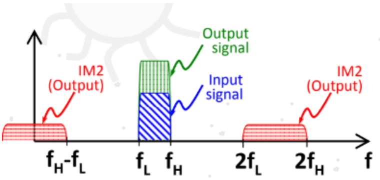
Observations:



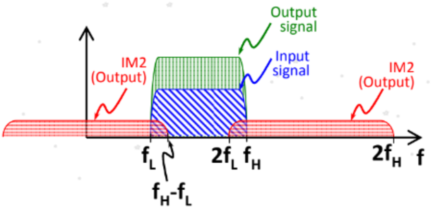
where

- 2nd-order intercept point (IP2) is where the asymptotes for the 2nd-order intermodulation product and the fundamental cross.
- IIP2 is the input power and OIP2 is the output power corresponding to the intercept point.
- Pin is the given input signal's power.

More Observations:



(a) Sub-Octave



(b) Multi-Octave

Sub-Octave: $f_H < 2f_L$
(i.e. BW $< f_L$)
-> No in-band IM2 distortion
out-of-band IM2 components,
can easily be filtered out
-> DC components can sometimes
cause amplifier saturation

Multi-Octave: $f_H > 2f_L$
(i.e. BW $> f_L$)
-> In-band IM2 distortion present
can't be filtered out

-> DC components may cause
amplifier saturation

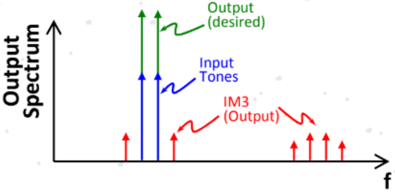
Third Order Non-Linearity

Consider a non-linear system described by

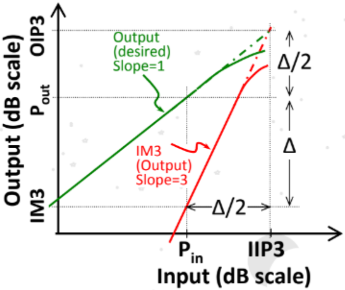
$$y(t) = a_1 x(t) + a_3 x^3(t); \qquad \Rightarrow \text{For} \quad x(t) = A\cos(\omega_1 t) + A\cos(\omega_2 t):$$

$$y(t) = A\left(a_1 + \frac{9a_3A^2}{4}\right)\left(\cos(\omega_1 t) + \cos(\omega_2 t)\right) + \frac{1}{4}a_3A^3\left(\cos(3\omega_1 t) + \cos(3\omega_2 t)\right)$$
$$+\frac{3}{4}a_3A^3\left[\cos((2\omega_1 - \omega_2)t) + \cos((2\omega_1 + \omega_2)t) + \cos((2\omega_2 - \omega_1)t) + \cos((2\omega_2 + \omega_1)t)\right]$$

Observations:

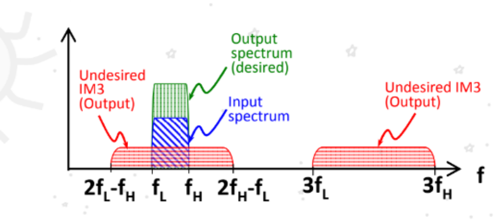


Generates in-band/adjacent band, out-of-band components, but no DC



IIP3 = Pin + Δ/2 (dB scale)

OIP3 = Pin + Δ/2 + Gain (dB scale)



- The figure above shows the undesired spectrum generated by 3^{rd} order non-linearity (i.e. due to non-zero a_3 coefficient).
- The undesired spectrum generated is called IM3 component, i.e. Inter-Modulation products due to 3^{rd} order non-linearity component.
- Due to 3^{rd} or odd order non-linearities (unlike 2^{nd} or even order non-linearities), part of the spectrum is in-band and hence CANNOT be removed by filtering even for narrow-band inputs.
- Therefore, effects of 3^{rd} (or odd) order non-linearities are more difficult to remove in general (than of even order non- linearities).

Calculation Of Intercept Points

For a two-tone source,the general nth-order intercept point is given by

$$IP_n = P_{in} + \frac{P_{in} - P_{IM}}{n-1} \tag{1}$$

where

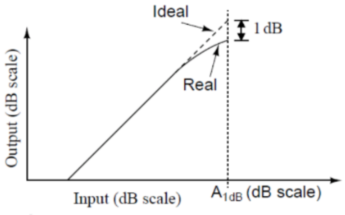
- IP_n is the nth-order intercept point
- P_{in} is the power level of the input tone
- P_{IM} is the power level of the nth-order intermodulation distortion (IMD) product (created by the two tones)
- All the power levels are measured in dBm

For the calculation of IIPn and OIPn, refer to the sections till 4.3 in Calculations of intercept points

Compression Point and Jamming

1-dB compression point: Amplitude (A_{-1dB}) at which gain decreases by 1-dB (without interferer) because a_3 is "almost always" negative.

$$20\log\left[\frac{(a_1+\frac{3}{4}a_3A_{-1dB}^2)A_{-1dB}}{a_1A_{-1dB}}\right] = -1dB \text{ i.e. } A_{-1dB} \approx 0.40\sqrt{\frac{|a_1|}{|a_3|}}$$



$$\text{Compression point in dB} = 20\log(A_{-1dB}) = IIP3(\text{in dB}) - 9.6dB$$

where

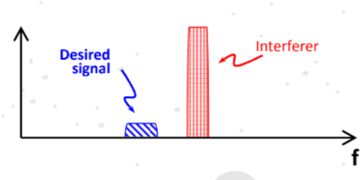
$$IIP3(\text{in dB}) = 20\log\left(\sqrt{\frac{4|a_1|}{3|a_3|}}\right)$$

$$OIP3(\text{in dB}) = IIP3(\text{in dB}) + 20\log(|a_1|)$$

Jamming / Blocking / Desensitization

For $x(t) = Acos(\omega t) + Bcos(\omega_1 t)$,
where $Acos(\omega t)$ is the desired signal and $Bcos(\omega_1 t)$ is the interferer,

$$y(t) = (a_1 + \frac{3}{4}a_3A^2 + \frac{3}{2}a_3B^2)Acos(\omega t) + \text{ other terms}$$



- **Therefore, if interferer amplitude $B >> A$, the receiver is jammed**
- **The transmitter can jam the receiver if they are operating concurrently, for example in full duplex systems (and isolation is poor)**

APPENDIX: Real Systems are not memory-less or linear: Non-linear dynamical behaviour

Transfer function of a dynamic non-linear system

- Very complex, commonly expressed as the Volterra series

$$y(t) = a_0 + \sum_{n=1}^{\infty} \frac{1}{n!} \int_0^{\infty} \ldots . \int_0^{\infty} a_n(\tau_1, \tau_2, \cdots, \tau_n) x(t-\tau_1) x(t-\tau_2) \cdots x(t-\tau_n)\, d\tau_1 d\tau_1 \ldots d\tau_n$$

a_n is called the n^{th} order Volterra kernel.
Therefore, a 2nd order dynamic non-linear system can be modeled as

$$y(t) = a_0 + \int_0^{\infty} a_1(\tau_1) x(t-\tau_1) d\tau_1 + \frac{1}{2} \int_0^{\infty} \int_0^{\infty} a_2(\tau_1, \tau_2) x(t-\tau_1) x(t-\tau_2)\, d\tau_1 d\tau_2$$

✓**Other References** : Reference link