M. Satvi Reddy
192311038
CSA-0688 - DAA

$x(n) = x(n-1) + 5$ for $n > 1$ $x(1) = 0$

$x(n) = x(n-1) + 5 \rightarrow (1)$

Sub $x(n-1)$ in (1)

$x(n) = (x(n-2) + 1) + 5$

$= x(n-2) + 6 \rightarrow (2)$

Sub $x(n-2)$ in (2)

$x(n) = (x(n-3) + 1) + 6$

$= x(n-3) + 7 \rightarrow (3)$

$x(k) = x(n-k) + (n+4)$

$n - k = 0$

$n = k$

$x(n-n) + (n+4)$

$= x(0) + n + 4$

$= 1 + n + 4 = n + 5$

$\Rightarrow O(n) \rightarrow$ Linear.

$x(n) = x(n-1) + 1$

$x(n-1) = x(n-1-1) + 1$

$x(n-2) + 1$

$x(n-2) = x(n-2-1) + 1$

$= x(n-3) + 1$

b) $x(n) = 3x(n-1)$ for $n>1$ $x(1) = 4$

$x(n) = 3x(n-1) \rightarrow (1)$

sub $x(n-1)$ in (1)

$x(n) = 3(3x(n-2)$

$= 9x(n-2) \rightarrow (2)$

$x(n) = 9 \cdot 3x(n-3)$

$= 27\ x(n-3) \rightarrow (3)$

$x(n) = 3x(n-1)$

$x(n-1) = 3x(n-1-1)$

$= 3x(n-2)$

$x(n-2) = 3x(n-2-1)$

$= 3x(n-3)$

$x(k) = k \cdot 3x(n-k)$

$n-k = 0$

$n = k$

$= n \cdot 3x(n-n)$

$= n \cdot 3 \cdot 0$

$= O(n) \rightarrow$ linear.

(2)

a) $T(n) = T(n/2) + 1 \Rightarrow n = 2^k$

$T(n) = T(n/2) + 1$

$T(n) = (T(n/4) + 1) + 1$

$= T(n/4) + 2$

$T(n) = T(n/8) + 1 + 2$

$= T(n/8) + 3$

$T(n) = T(n/2^k) + k$

$n/2^k = 1$

$2^k = n$

$k = \log_2 n$

$T(n) = \log_2 n + 1$

b) $T(n) = \log_2 n + T(n/3) + T(2n/3) + cn$

$T(n) = (\log_2(n/3) + 1) + (\log(2n/3) + 1] + cn$

$= \log_2(n/3) + \log_2(2n/3) + 2 + cn$

$= \log_2 n + \log_2 3 + \log_2 2 + \log_2(n/3) + 2 + cn$

$= \log_2 n - \log_2 3 + \log_2 n - \log_2 3 + 2 + cn$

$T(n) = 2\log_2 n - 2\log_2 3 + 3 + cn \qquad T(n) = O(n)$

3) $F(n) = 2n^2 + 5$

$g(n) = 7n$

$= (2+5)n^2$

$= 7n^2$

$c = 7 \quad g(n) = 7n$

$f(n) \geq g(n)$

$\Rightarrow F(n) = \Omega g(n)$

Hence proved.

4) Fun TowerofHanoi (n, source, destination, auxiliary):

if n==1:

Move disk from source to destination

else:

Tower of Hanoi (n-1, source, auxiliary, destination)

Move dis from source to destination

Tower of Hanoi (n-1, auxiliary, destination, source)

$\Rightarrow T(n) = 2T(n-1) + 1$

$T(n-1) = 2T(n-2) + 1$

$T(n) = 2(2T(n-2)+1) + 1$

$= 4T(n-2) + 3$

$T(n) = 2^2 T(n-2) - (2^2 - 1)$

$T(n) = 2^{n-1} + (2^{n+1} - 1)$

$= 2^n - 1$

$T(n) = O(2^n)$

5)

a) $T(n) = 3T(n/2) + n^2$

$a = 3 \quad b = 2$

$f(n) = n^2$

$\Rightarrow n \log_2 3$

$f(n) \geq n \log_2 3$

$T(n) = \theta(f(n))$

$= \theta(n^2)$

b) $T(n) = T(n/2) / 2^n$

-> Master theorem can't be applied. //