Assignment - 4.

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1. If $t_1(n) \in O\, g(C_1(n))$ and $t_2(n) \in O(g\,2(n))$, then $t_1(n) + t_2(n)$ $\in O$ $(\max \{ g_1(n), g_2(n) \})$. Prove the assertions.

$\Rightarrow$ $g(n) = \max\{g_1(n) \& g_2(n)\}$

$\Rightarrow g_1(n) \leq g(n)$ and $g_2(n) \leq g(n)$

$\Rightarrow$ $t_1(n) + t_2(n) \leq C_1\, g_1(n) + C_2\, g_2(n)$

$\leq c_1 \cdot g(n) + c_2 \cdot g(n)$

$= (c_1 + c_2)\, g(n)$

Let $c = c_1 + c_2$.

$t_1(n) + t_2(n) \leq c\; g(n)$

$t_1(n) + t_2(n) \in O(g(n))$

$= O(\max(g_1(n); g_2(n)\})$

2. Time complexity of recurrence relation.

$T(n) = 2T(n/2) + 1$

$T(n) = aT(n/b) + f(n)$

$a = 2$, $b = 2$ $f(n) = 1$ $\quad | \; n^{\log_b a} = n^{\log_a b} = n$

$f(n) = 1 = n^0$

$f(n) = O(n^2)$

when $c < \log_b a$ $(0 < 1)$ by max 1 of Master theorem.

$t(n) = O(n \log_a 2) = \Theta(n)$

3. Find time complexity of $T(n) = 2T(n-1)$

$T(n) = 2T(n-1) = 2(2T(n-2))$

$= 2^2 \; T(n-2)$

$\vdots$

$= 2^{n-1} \; T(1)$

$T(1) = c$

$\Rightarrow T(n) = 2^{n-1} \cdot c$

$= O(2^n)$

4. Show that $f(n) = n^2 + 3n + 5n \; O(n^2)$

$f(n) \le c \cdot n^2$ when c is constant. and n in large.

$\therefore f(n) = n^2 + 3n + 5 \le n^2 + 3n^2 + 5n^2 = 9n^2$

So $f(n) \le 9n^2$ for $c = 9$

Hence $f(n)$ in $O(n^2)$

5. Prove that $g(n) = n^3 + n^2 + 4n + 12$ in $\Omega(n^3)$

$g(n) \ge c \cdot n^3$ where $c > 0$

n is large

$g(n) = n^3 + n^2 + 4n + 12 \ge n^2$

$\Rightarrow g(n) \ge 1 \cdot n^3 \Rightarrow g(n)$ is $\Omega(n^3)$

6. Determine wheather $h(n) = hn^2 + 3nh \; \Theta(n^2)$ or not.

$h(n) = 4n^2 + 3n \le 4n^2 + 3n^2 = 7n^2$

so $h(n)$ is $O(n^2)$

lower bond So $h(n)$ is $\Omega(n^2)$

So $n(n)$ is $O(n^2)$

h is both $\Omega(n^2)$ and $O(n^2) \Rightarrow$ it is $\Theta(n^2)$

$f(n) = n^3 + 2n^2 + n$ and $g(n) = n^2$, $f(n) = \Omega(g(n))$

is ~~true~~ or false.

for large n.

$$f(n) = n^3 + 2n^2 + n \geq n^3$$

$\therefore g(n) = n^2$, $n^3 \geq n^2$ for large n

Hence $f(n) = \Omega(g(n))$ is true.

9. Determine whether $h(n) = n\log n + n$ is $\Theta(n \log n)$

upper bond

$$h(n) = n\log n + n \leq n\log n + n\log n$$
$$= 2n\log n.$$

So $h(n)$ is $O(n \log n)$

Lower bond $(n \log n)$

$$h(n) = n \log n + n \geq \log n.$$

So, $h(n)$ is $\Omega(n \log n)$

$\Rightarrow \Theta(n \log n)$.

10. Recurrence relation $T(n) = 4T(n/2) + n^2$

By master theorem,

$$T(n) = 4T(n/2) + n^2$$

$a = 4$ $b = 2$ $f(n) = n^2$

then $n^{\log_b a} = n^{\log_2 4} = n^2$

Since $f(n) = \Theta(n^2)$ where $n^{\log_b a}$ in box 2 of Master theorem.

$$T(n) = \Theta(n^2 \log n)$$