ELECTRIC MACHINES
ASHFAQ HUSAIN

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Second Edition

ASHFAQ HUSAIN

DHANPAT RAI & Co.

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Transformer – I

1.1 PRINCIPLE OF TRANSFORMER OPERATION

A transformer is a static device which consists of two or more stationary electric circuits interlinked by a common magnetic circuit for the purpose of transferring electrical energy between them. The transfer of energy from one circuit to another takes place without a change in frequency.

Consider two coils 1 and 2 wound on a simple magnetic circuit as shown in Fig. 1.1. These two coils are insulated from each other and there is no electrical connection between them. Let T_1 and T_2 be the number of turns in coils 1 and 2 respectively. When a source of alternating voltage V_1 is applied to coil 1, an alternating current I_0 flows in it. This alternating current produces an alternating flux Φ_M in the magnetic circuit. The mean path of this flux is shown in Fig. 1.1 by

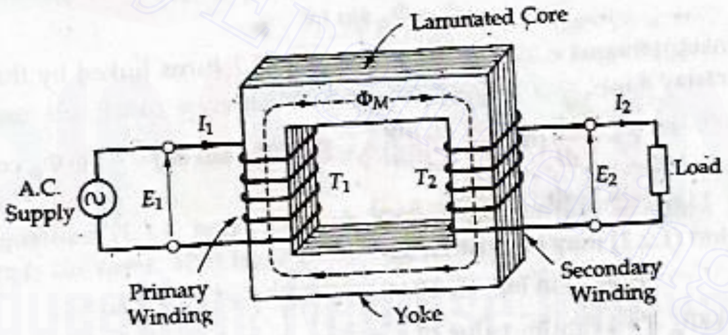


Fig. 1.1. *Arrangement of a simple transformer.*

the dotted line. This alternating flux links the turns T_1 of coil 1 and induces in them an alternating voltage E_1 by **self-induction.** Let us make the following simplifying *assumptions* for an *ideal transformer* :

(*a*) There are no losses either in the electric circuits or in the magnetic circuit.

(*b*) The whole of the magnetic flux Φ is confined to the magnetic circuit, so that there is no leakage flux.

(*c*) The permeability of the core is infinite.

Thus, *all* the flux produced by coil 1 also linkes T_2 turns of coil 2 and induces in them a voltage E_2 **by mutual induction**. If coil 2 is connected to a load then an alternating current will flow through it and energy will be delivered to the load. Thus, electrical energy is transferred from coil 1 to coil 2 by a common magnetic circuit. *Since there is no relative motion between the coils, the frequency of the induced voltage in coil 2 is exactly the same as the frequency of the applied voltage to coil 1.*

Coil 1 which receives energy from the source of a.c. supply is called the *primary coil* or *primary winding* or simply the *primary*. Coil 2, which is connected to load and delivers energy to the load, is called the *secondary coil* or *secondary winding* or simply the *secondary*. The circuit symbol for a two-winding transformer is shown in Fig. 1.2. The two vertical bars are used to signify tight magnetic coupling between the windings.

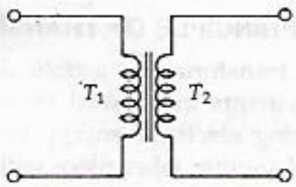


Fig. 1.2. *Circuit symbol for a 2-winding transformer.*

1.2 E.M.F. EQUATION OF A TRANSFORMER

Let the flux at any instant be given by

$$\Phi = \Phi_m \sin \omega t \tag{1.2.1}$$

The instantaneous e.m.f. induced in a coil of T turns linked by this flux is given by Faraday's law as

$$e = -\frac{d}{dt}(\Phi T) = -T\frac{d\Phi}{dt} = -T\frac{d}{dt}(\Phi_m \sin \omega t) = -T\omega\,\Phi_m \cos \omega t$$

$$= T\omega\Phi_m \sin(\omega t - \pi/2) \tag{1.2.2}$$

Equation (1.2.2) may be written as

$$e = E_m \sin(\omega t - \pi/2) \tag{1.2.3}$$

where $E_m = T\omega\Phi_m$ = maximum value of e.

For a sine wave, the r.m.s. value of e.m.f. is given by

$$E_{rms} = E = E_m/\sqrt{2}$$

$$\therefore \quad E = \frac{T\omega\Phi_m}{\sqrt{2}} = \frac{T(2\pi f)\,\Phi_m}{\sqrt{2}}$$

or

$$\boxed{E = 4.44\,\Phi_m f T} \tag{1.2.4}$$

Equation (1.2.4) is called the *e.m.f. equation of a transformer.*

The e.m.f. induced in each winding of the transformer can be calculated from its e.m.f. equation. Let subscripts 1 and 2 be used for primary and secondary quantities. The primary r.m.s. voltage is

$$\boxed{E_1 = 4.44\, \Phi_m f T_1} \tag{1.2.5}$$

The secondary r.m.s. voltage is

$$\boxed{E_2 = 4.44\, \Phi_m f T_2} \tag{1.2.6}$$

where Φ_m is the maximum value of flux in webers (Wb), f is the frequency in hertz (Hz) and E_1 and E_2 are in volts.

If B_m = maximum flux density in the magnetic circuit (core) in tesla (T).

A = area of cross-section of the core in square metres (m^2)

then
$$B_m = \frac{\Phi_m}{A} \tag{1.2.7}$$

It should be noted that

1 tesla (T) = 1 Wb/m^2.

The winding with higher number of turns will have a high voltage and is called the high-voltage (*hv*) winding. The winding with the lower number of turns is called the low-voltage (*lv*) winding.

1.3 VOLTAGE RATIO AND TURNS RATIO

The ratio E/T is called **voltage per turn.**

From Eq. (1.2.5), primary volts per turn,

$$\frac{E_1}{T_1} = 4.44\, \Phi_m f \tag{1.3.1}$$

From Eq. (1.2.6), secondary volts per turn

$$\frac{E_2}{T_2} = 4.44\, \Phi_m f \tag{1.3.2}$$

Equations (1.3.1) and (1.3.2) show that ***the voltage per turn in both the windings is the same.*** That is,

$$\boxed{\frac{E_1}{T_1} = \frac{E_2}{T_2}} \tag{1.3.3}$$

Also,
$$\boxed{\frac{E_1}{E_2} = \frac{T_1}{T_2}} \tag{1.3.4}$$

The ratio $\frac{T_1}{T_2}$ is called *turns ratio.*

The ratio of primary to secondary turns $\left(\frac{T_1}{T_2}\right)$ which equals the ratio of primary to secondary induced voltages $\left(\frac{E_1}{E_2}\right)$, indicates how much the primary voltage is lowered or raised. The turn ratio, or the induced voltage ratio, is called the **transformation ratio** and is denoted by the symbol a. Thus,

$$\boxed{a = \frac{E_1}{E_2} = \frac{T_1}{T_2}} \tag{1.3.5}$$

In a practical voltage transformer, there is a very small difference between the terminal voltage and the induced voltage. Therefore, we can assume that $E_1 = V_1$, and $E_2 = V_2$. Equation (1.3.5) is modified as

$$\boxed{\frac{V_1}{V_2} = \frac{T_1}{T_2} = a} \tag{1.3.6}$$

If a voltage ratio or turns ratio is specified, this is always put in the order input : output, which is primary : secondary. It is to be noted from Eq. (1.3.6) that almost any desired voltage ratio can be obtained by adjusting the number of turns.

1.4 STEP-UP AND STEP-DOWN TRANSFORMERS

A transformer in which the output (secondary) voltage is greater than its input (primary) voltage is called a **step-up transformer.**

A transformer in which the output (secondary) voltage is less than its input (primary) voltage is called a **step-down transformer.**

The same transformer can be used as a step-up transformer or a step-down transformer depending on the way it is connected in the circuit. When the transformer is used as a step-up transformer, the low voltage winding is the primary. In a step-down transformer, the high-voltage winding is the primary.

A transformer may receive energy at one voltage and deliver it at the same voltage. Such a transformer is called a **one-to-one (1 : 1) transformer.** For a 1 : 1 transformer $T_1 = T_2$ and $|E_1| = |E_2|$. Such a transformer is used to isolate two circuits.

EXAMPLE 1.1. *A 3300/250 V, 50 Hz, single-phase transformer is built on a core having an effective cross-sectional area of 125 cm² and 70 turns on the low-voltage winding. Calculate (a) the value of the maximum flux density, (b) the number of turns on the high voltage winding.*

SOLUTION. $E_1 = 3300$ V, $E_2 = 250$ V, $f = 50$ Hz

$$A = 125\ \text{cm}^2 = 125 \times 10^{-4}\ \text{m}^2$$

$$E_2 = 4.44\,\Phi_m f T_2 = 4.44\, B_m A f T_2$$

$$B_m = \frac{E_2}{4.44\, A f T_2}$$

$$= \frac{250}{4.44 \times 125 \times 10^{-4} \times 50 \times 70} = 1.289 \text{ teslas (T)}$$

$$\frac{E_1}{E_2} = \frac{T_1}{T_2}$$

$$T_1 = \frac{E_1}{E_2} \times T_2 = \frac{3300}{250} \times 70 = 924$$

Example 1.2. *A transformer with 800 primary turns and 200 secondary turns is supplied from a 100 V a.c. supply. Calculate the secondary voltage and the volts per turn.*

Solution. $T_1 = 800,\ T_2 = 200,\ V_1 = 100$ V

$$\frac{V_1}{V_2} = \frac{T_1}{T_2},\quad V_2 = V_1 \times \frac{T_2}{T_1}$$

$$= 100 \times \frac{200}{800} = 25 \text{ V}$$

$$\text{Volts per turn } = \frac{V_1}{T_1} = \frac{100}{800} = 0.125$$

or

$$\text{volts per turn } = \frac{V_2}{T_2} = \frac{25}{200} = 0.125$$

Example 1.3. *A transformer with an output voltage of 4200 V is supplied at 230 V. If the secondary has 2000 turns, calculate the number of primary turns.*

Solution. $V_2 = 4200$ V, $V_1 = 230$ V, $T_2 = 2000$

$$\frac{V_1}{V_2} = \frac{T_1}{T_2}$$

$$T_1 = \frac{T_2 V_1}{V_2}$$

$$= 2000 \times \frac{230}{4200} = 109.52 \text{ turns}$$

In practice, it is not possible for a winding to have part of a turn (that is, turns cannot be fractional). Therefore, the number of turns should be a whole number. In our case we shall take $T_1 = 110$.

1.5 CONSTRUCTION OF SINGLE-PHASE TRANSFORMERS

A single-phase transformer consists of primary and secondary windings put on a magnetic core. Magnetic core is used to confine flux to a definite path. Transformer cores are made from thin sheets (called *laminations*) of high-grade silicon steel. The laminations reduce eddy-current loss and the silicon steel reduces hysteresis loss. The laminations are insulated from one another by heat resistant enamel insulation coating. L-type and E-type laminations are used. The laminations are built up into stack and the joints in the laminations are staggered to minimize airgaps (which require large exciting currents). The laminations are tightly clamped.

There are two basic types of transformer constructions, the core type and the shell type.

1.5.1 Core-type Construction

In the core-type transformer, the magnetic circuit consists of two vertical *legs* or *limbs* with two horizontal sections, called *yokes*. To keep the leakage flux to a minimum, half of each winding is placed on each leg of the core as shown in

Fig. 1.3. The low-voltage winding is placed next to the core and the high-voltage winding is placed around the low-voltage winding to reduce the insulating material required. Thus, the two windings are arranged as *concentric coils*. Such a winding is, therefore, called *concentric winding* or *cylindrical winding*.

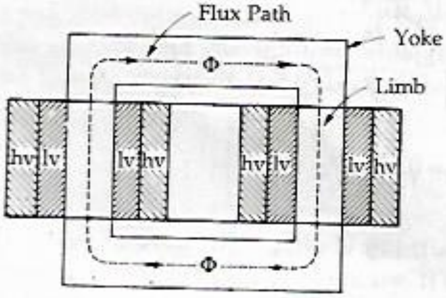


Fig. 1.3. *Core-type transformer.*

1.5.2 Shell-type Transformer

In the shell-type transformer (Fig. 1.4), both primary and secondary windings are wound on the central limb, and the two outer limbs complete the low-reluctance flux paths. Each winding is subdivided into sections. Low-voltage (lv) and high-voltage (hv) subsections are alternately put in the form of a sandwich. Such a winding is, therefore, called *sandwich* or *disc winding*.

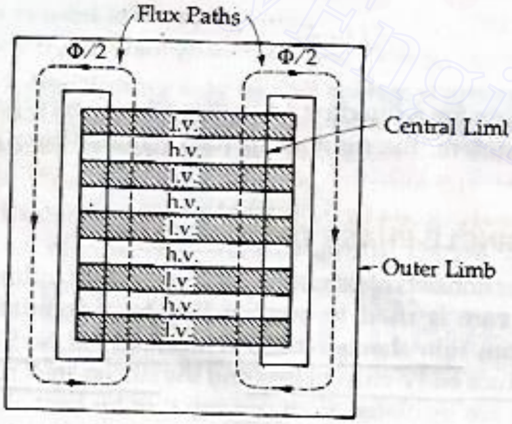


Fig. 1.4. *Shell-type transformer.*

A core must be made up of at least two types of laminations. The laminations for the core-type transformers are of *U* and *I* shape as shown in Fig. 1.5*a*. The *U*-shaped laminations are first stacked together for the required length. Half of the prewound low voltage (*lv*) coil is placed around the limbs. The *lv* coil is further provided with insulation. Then half of the prewound high-voltage (*hv*) coil is placed around the *lv* coil. The core is then closed by the *I*-shaped laminations at the top.

The core for the shell-type transformer is made up of either *U* and *T* shape (Fig. 1.5*b*) or *E* and *I* shape (Fig. 1.5*c*). In this type *T* or *E* shaped laminations are stacked together. The entire prewound low-voltage coil is placed around the central limb and the full prewound high-voltage coil is placed around the low-voltage coil. The core is then closed by *U* or *I* type laminations.

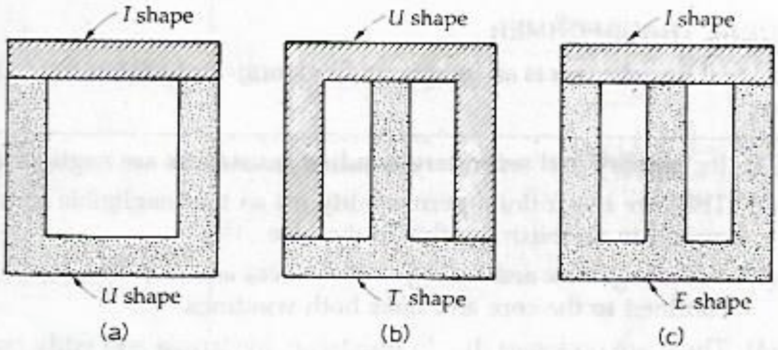


Fig. 1.5. *Core laminations (a) U and I type laminations for the core-type transformer (b) U and T laminations for the shell-type transformer (c) E and I shaped laminations for the shell-type transformer*

Small-core-type transformers are made of rectangular section core limbs with rectangular coils as shown in Fig. 1.6*a*. For economic reasons, the cross section of the core should be circle. Since the circle has the minimum periphery for a given area, the winding which is put around the circular core has minimum length of the mean turn. The volume and, therefore, the cost of the conductor material is reduced. The resistance of the winding is also reduced and, therefore, I^2R loss is reduced. However, a circular core requires the use of large number of laminations of different sizes. For economy, stepped-core arrangement is used. Figure 1.6*b* shows a two-stepped core which is also known as the **cruciform core.** This core requires two sizes of laminations. As the number of steps increases, the number of different sizes of laminations also increases. Three-stepped core (Fig. 1.6*c*) is very commonly used in large transformers. More than three steps may be used in very large transformers.

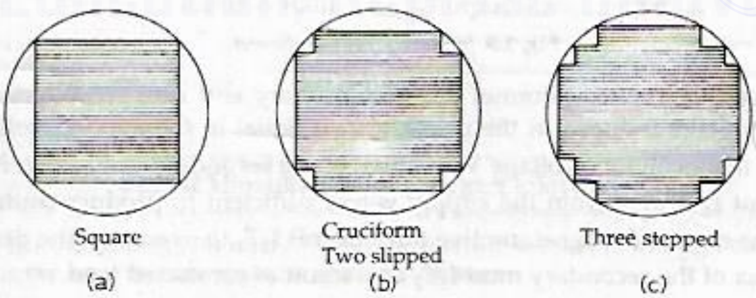


Fig. 1.6. *Square and stepped-core cross-sections*

The core-type transformer is easier to dismantle for repair. The shell-type transformer gives better support against electromagnetic forces between the current-carrying conductors. These forces are of considerable magnitude under short-circuit conditions. Shell-type transformers provide a shorter magnetic path, and hence magnetizing current is lesser than that in the core-type transformer. The natural cooling is poor in a shell-type transformer due to the embedding of the coils.

1.6 IDEAL TRANSFORMER

An **ideal transformer** is an imaginary transformer which has the following properties :

(1) Its primary and secondary winding resistances are negligible.
(2) The core has infinite permeability (μ) so that negligible mmf is required to establish the flux in the core.
(3) Its leakage flux and leakage inductances are zero. The entire flux is confined to the core and links both windings.
(4) There are no losses due to resistance, hysteresis and eddy currents. Thus, the efficiency is 100 per cent.

It is to be noted that practical (commercial) transformer has none of these properties inspite of the fact that its operation is close to ideal.

An ideal iron-core transformer is shown in Fig. 1.7. It consists of two coils *wound in the same direction* on a common magnetic core. The winding connected to the supply, $\mathbf{V}_1$, is called the *primary*. The winding connected to the load, $\mathbf{Z}_1$, is called the *secondary*.

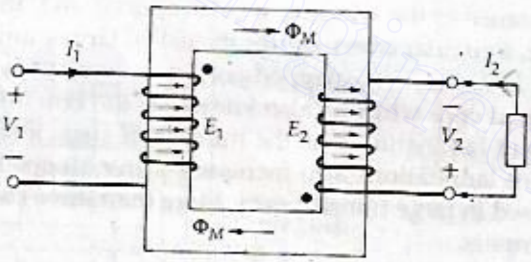


Fig. 1.7. *Ideal iron-core transformer.*

Since the ideal transformer has zero primary and zero secondary impedance, the voltage induced in the primary $\mathbf{E}_1$ is equal to the applied voltage $\mathbf{V}_1$. Similarly, the secondary voltage $\mathbf{V}_2$ is equal to the secondary induced voltage $\mathbf{E}_2$. The current $\mathbf{I}_1$ drawn from the supply is just sufficient to produce mutual flux Φ_M and the required magnetomotive force (mmf) $\mathbf{I}_1 T_1$ to overcome the demagnetizing effect of the secondary mmf $\mathbf{I}_2 T_2$ as a result of connected load.

By Lenz's law $\mathbf{E}_1$ is equal and opposite to $\mathbf{V}_1$. Since $\mathbf{E}_2$ and $\mathbf{E}_1$ are both induced by the same mutual flux, $\mathbf{E}_2$ is in the same direction as $\mathbf{E}_1$ but opposite to $\mathbf{V}_1$. The magnetizing current $\mathbf{I}_\mu$ lags $\mathbf{V}_1$ by 90° and produces Φ_M in phase with $\mathbf{I}_\mu$. $\mathbf{E}_1$ and $\mathbf{E}_2$ lag Φ_M by 90° and are produced by Φ_M. $\mathbf{V}_2$ is equal in magnitude to $\mathbf{E}_2$, and is opposite to $\mathbf{V}_1$. Fig. 1.8 shows the no-load phasor diagram of the ideal transformer.

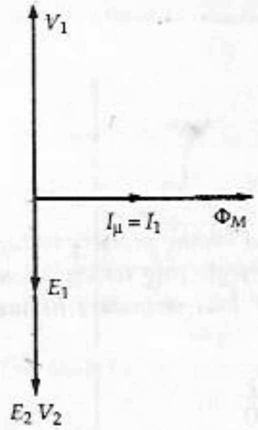


Fig. 1.8. *No-load phasor diagram of an ideal transformer.*

For an ideal transformer, if

$$a = \text{transformation ratio} = \text{turn ratio}$$

then,
$$a = \frac{T_1}{T_2} = \frac{\mathbf{E}_1}{\mathbf{E}_2} = \frac{\mathbf{V}_1}{\mathbf{V}_2} = \frac{\mathbf{I}_2}{\mathbf{I}_1} \tag{1.6.1}$$

$\therefore$
$$\mathbf{I}_1 T_1 = \mathbf{I}_2 T_2 \tag{1.6.2}$$

$$\mathbf{E}_1\mathbf{I}_1 = \mathbf{E}_2\mathbf{I}_2 = \mathbf{S}_2 = \mathbf{S}_1 \tag{1.6.3}$$

$$\mathbf{V}_1\mathbf{I}_1 = \mathbf{V}_2\mathbf{I}_2 = \mathbf{S}_2 = \mathbf{S}_1 \tag{1.6.4}$$

Equation (1.6.2) states that the demagnetizing ampere-turns of the secondary are equal and opposite to the magnetizing mmf of the primary of an *ideal* transformer.

Equation (1.6.3) shows that the voltamperes (apparent power) drawn from the primary supply is equal to the voltamperes (apparent power) transferred to the secondary without any loss in an *ideal* transformer. In other words,

$$\text{input voltamperes} = \text{output voltamperes}$$

Also,
$$\frac{V_1 I_1}{1000} = \frac{V_2 I_2}{1000}$$

$$(kVA)_1 = (kVA)_2 \tag{1.6.5}$$

or
$$\boxed{\text{input kilovoltamperes} = \text{output kilovoltamperes}}$$

Thus, *the kVA input of an ideal transformer is equal to the kVA output. That is, kVA is the same on both the sides of the transformer.*

Example 1.4. *A 25 kVA transformer has a voltage ratio of 3300/400 V. Calculate the primary and secondary currents.*

Solution. $\text{kVA} = \dfrac{V_1 I_1}{1000}$

$$I_1 = \frac{1000 \times \text{kVA}}{V_1} = \frac{1000 \times 25}{3300} = 7.58 \text{ A}$$

Also, $$\text{kVA} = \frac{V_2 I_2}{1000}$$

$$I_2 = \frac{1000 \text{ kVA}}{V_2}$$

$$= \frac{1000 \times 25}{400} = 62.5 \text{ A}$$

EXAMPLE 1.5. *A 125 kVA transformer having primary voltage of 2000 V at 50 Hz has 182 primary and 40 secondary turns. Neglecting losses, calculate (a) the full-load primary and secondary currents, (b) the no-load secondary induced e.m.f. and (c) the maximum flux in the core.*

SOLUTION. (*a*) $$\text{kVA} = \frac{V_1 I_1}{1000} = \frac{V_2 I_2}{1000}$$

$$I_1 = \frac{1000 \times kVA}{V_1} = \frac{1000 \times 125}{2000} = 62.5 \text{ A}$$

$$I_2 T_2 = I_1 T_1$$

$$I_2 = \frac{I_1 T_1}{T_2} = \frac{62.5 \times 182}{40} = 284.4 \text{ A}$$

(*b*) $$\frac{E_2}{T_2} = \frac{E_1}{T_1}$$

$$E_2 = \frac{E_1 T_2}{T_1} = \frac{2000 \times 40}{182} = 439.6 \text{ V}$$

(*c*) $$E_1 = 4.44\, \Phi_m f T_1$$

$$\Phi_m = \frac{E_1}{4.44 f T_1} = \frac{2000}{4.44 \times 50 \times 182} = 0.0495 \text{ Wb}$$

1.7 TRANSFORMER ON NO LOAD

A transformer is said to be on *no load* when the secondary winding is open-circuited. The secondary current is thus zero. When an alternating voltage is applied to the primary, a small current I_0 flows in the *primary*. The current I_0 is called the *no-load current* of the transformer. It is made up of two components I_μ and I_W. The component I_μ is called the *magnetizing component*. It magnetizes the core. In other words, it sets up a flux in the core and therefore I_μ is in phase with Φ_M. The current I_μ is also called *reactive, or wattless component of no-load current*.

The component I_W supplies the hysteresis and eddy-current losses in the core and the negligible I^2R loss in the primary winding. The current I_W is called the *active component or wattful component of no-load current*. It is in phase with the applied voltage V_1. The no-load current I_0 is small of the order of 3 to 5 percent of the rated current of the primary.

1.8 PHASOR DIAGRAM AT NO LOAD

An approximate phasor diagram for a transformer under no-load conditions is shown in Fig. 1.9. The flux Φ_M is taken as the reference phasor.

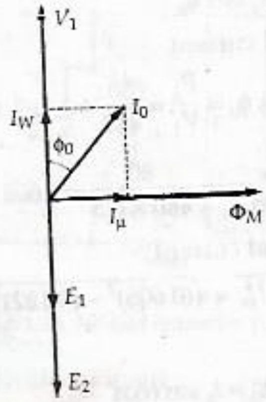


Fig. 1.9. *Phasor diagram at no load.*

For a transformer on no load, we have

$$\Phi = \Phi_M \sin \omega t$$
$$e_1 = E_{1m} \sin (\omega t - \pi/2)$$
$$e_2 = E_{2m} \sin (\omega t - \pi/2)$$

Since $\mathbf{E}_1$ and $\mathbf{E}_2$ are induced by the same flux Φ, they will be in phase with each other. E_2 differs in magnitude from E_1 because $E_2 = E_1 \dfrac{T_2}{T_1} = \dfrac{E_1}{a}$.

The above equations show that $\mathbf{E}_1$ and $\mathbf{E}_2$ lag behind Φ by 90°. If the voltage drops in the primary winding are neglected $\mathbf{E}_1$ will be equal and opposite to the applied voltage $\mathbf{V}_1$. $\mathbf{I}_\mu$ is in phase with Φ and $\mathbf{I}_W$ is in phase with $\mathbf{V}_1$. The phasor sum of $\mathbf{I}_\mu$ and $\mathbf{I}_W$ is $\mathbf{I}_0$. Angle ϕ_0 is called the *no-load power factor angle,* so that the power factor on no load is $\cos \phi_0$.

From the phasor diagram of Fig. 1.9.

$$I_W = I_0 \cos \phi_0 \tag{1.8.1}$$
$$I_\mu = I_0 \sin \phi_0 \tag{1.8.2}$$
$$I_0 = \sqrt{I_W^2 + I_\mu^2} \tag{1.8.3}$$
$$\cos \phi_0 = \frac{I_W}{I_0} \tag{1.8.4}$$

Also, core loss $= V_1 I_0 \cos \phi_0 = V_1 I_W$ W (1.8.5)

Magnetizing (reactive) voltamperes

$$= V_1 I_0 \sin \phi_0 = V_1 I_\mu \text{ VAr} \tag{1.8.6}$$

EXAMPLE 1.6. *Find the active and reactive components of no-load current, and the no-load current of a 440/220 V single-phase transformer if the power input to the hv winding is 80 W. The low-voltage winding is kept open. The power factor of the no-load current is 0.3 lagging.*

SOLUTION. $P_o = V_1 I_o \cos\phi_o$

Active component of no-load current

$$I_W = I_o \cos\phi_o = \frac{P_o}{V_1} = \frac{80}{440} = 0.182 \text{ A}$$

No-load current $I_o = \dfrac{P_o}{V_1 \cos\phi_o} = \dfrac{80}{440 \times 0.3} = 0.606$

Reactive component of no-load current

$$I_\mu = \sqrt{I_o^2 - I_W^2} = \sqrt{(0.606)^2 - (0.182)^2} = 0.578$$

ALTERNATIVELY

$$I_\mu = I_o \sin\phi_o = I_o \sin(\cos^{-1}\phi_o)$$
$$= 0.606 \sin(72.54°) = 0.578 \text{ A}$$

EXAMPLE 1.7. *A 230/110 V single-phase transformer takes an input of 350 VA at no load and at rated voltage. The core loss is 110 W. Find (i) the iron-loss component of no-load current, (ii) the magnetizing component of no-load current and (iii) no-load power factor.*

SOLUTION. $V_1 I_o = 350$

$$I_0 = \frac{350}{V_1} = \frac{350}{230} = 1.52 \text{ A}$$

(*i*) No-load p.f. $\cos\phi_o$

$$\text{Core loss} = V_1 I_o \cos\phi_o$$

$$110 = 350 \cos\phi_o, \quad \cos\phi_o = \frac{110}{350} = 0.314$$

(*ii*) Core-loss component of no-load current

$$I_W = I_o \cos\phi_o = \frac{\text{core loss}}{V_1} = \frac{110}{230} = 0.478$$

(*iii*) Magnetizing component of no-load current

$$I_\mu = \sqrt{I_o^2 - I_W^2} = \sqrt{(1.52)^2 - (0.478)^2} = 1.44 \text{ A}$$

1.9 NO-LOAD EQUIVALENT CIRCUIT

The no-load equivalent circuit for the transformer is shown in Fig. 1.10. The actual transformer is replaced by an ideal transformer with a resistance R_0 and an inductive reactance X_0 in parallel with its primary. R_0 represents the core losses and so the current I_W which supplies core loss is shown passing through it. Thus,

$$I_W^2 R_0 = \text{core loss of the actual transformer}$$

The inductive reactance X_0 takes a reactive current equal to the magnetizing current I_μ of the actual transformer.

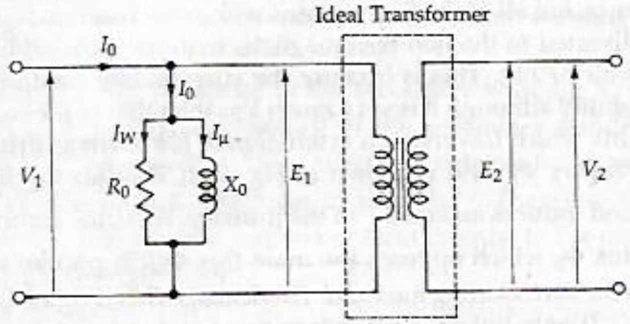


Fig. 1.10. *No-load equivalent circuit for a real transformer.*

From the equivalent circuit

$$\boxed{\begin{array}{l} V_1 = I_W R_0, \; V_1 = I_\mu X_0 \\ \mathbf{I}_0 = \mathbf{I}_\mu + \mathbf{I}_W \end{array}}$$

$$\boxed{\text{Core loss} = I_W^2 R_o = \frac{V_1^2}{R_0}}$$

1.10 PRACTICAL TRANSFORMER

In Section 1.6 the properties of an ideal transformer were given. Certain assumptions were made which are not valid in a practical transformer. For example, in a practical transformer the windings have resistance. The core has finite permeability and there is a leakage of flux. The efficiency of a practical transformer is not 100 per cent due to the losses. Therefore, in a practical transformer we shall consider all these imperfections.

1.11 WINDING RESISTANCE

An ideal transformer is supposed to possess no resistance, but in actual transformer there is always some resistance of primary and secondary windings. The effect of resistance is equivalent to an ideal transformer with resistances connected in series with each winding as shown in Fig. 1.11, where the quantities R_1 and R_2 are the resistances of the primary and secondary windings of the actual transformer.

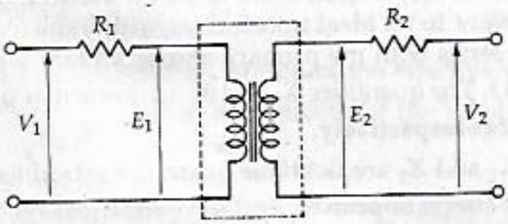


Fig. 1.11. *Winding resistances.*

1.12 LEAKAGE REACTANCE

In an ideal transformer it is assumed that all the flux produced by the primary winding links both the primary and secondary windings. However, in an actual transformer, not all of the flux remains within the magnetic core. A portion of this flux is diverted to the non-ferromagnetic material surrounding the windings (generally air or oil). This is because the surrounding medium also, has a definite permeability although it is very much less than that of the core. This small portion of the flux which traverses an external path is known as **primary leakage flux.** It is denoted by Φ_{L_1} and is shown in Fig. 1.12. The flux Φ_{L_1} links only the primary turns and induces an emf E_{L_1} in the primary. Also, the secondary current I_2 produces a flux Φ_2 which opposes the main flux Φ_M. A portion of this flux is also diverted to the surrounding medium. This leakage flux is called the **secondary leakage flux** Φ_{L_2}. It only links the secondary turns and induces an emf E_{L_2} in the secondary. Thus, each leakage flux links one winding only and it is caused by the current in that winding alone. The flux which passes completely through the core and links *both* windings is called mutual flux and is shown as Φ_M in Fig. 1.12.

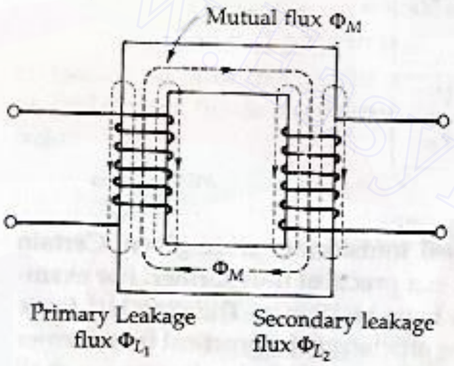


Fig. 1.12.

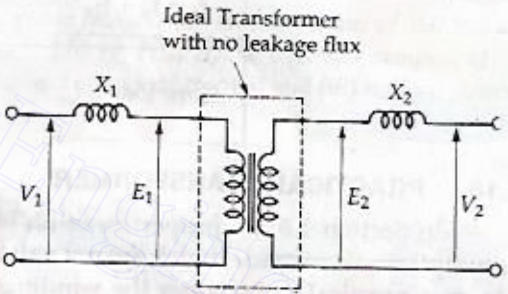


Fig. 1.13. *Representation of leakage fluxes with reactances X_1 and X_2.*

It should be noted that the induced voltages E_{L_1} and E_{L_2} due to leakage fluxes Φ_{L_1} and Φ_{L_2} are different from induced voltages E_1 and E_2 caused by the mutual flux Φ_M. As the leakage flux linking with each winding produces a self-induced emf in that winding, hence the effect of leakage flux is equivalent to an inductance in series with each winding such that the voltage drop in each series inductance is equal to that produced by the leakage flux. In other words, a transformer with magnetic flux leakage is equivalent to an ideal transformer with inductive reactances X_1 and X_2 connected in series with the primary and secondary windings respectively as shown in Fig. 1.13. The quantities X_1 and X_2 are known as **primary and secondary leakage reactances respectively.**

It should be noted that X_1 and X_2 are fictitious quantities introduced as a convenience in representing the effects of primary and secondary leakage fluxes.

1.13 REFERRED VALUES

In order to simplify calculations, it is *theoretically* possible to transfer voltage, current, and impedance of one winding to the other and combine them into single values for each quantity. Thus, we have to work in one winding only which is more convenient.

Let us transfer the resistance of the secondary winding R_2 to the primary side. Suppose that R_2' is the resistance of the secondary winding referred or reflected to the primary winding. This reflected resistance R_2' should produce the same effect in primary as R_2 produces in secondary. Therefore the power consumed by R_2' when carrying the primary current is equal to the power consumed by R_2 due to the secondary current.

That is,

$$I_2'^2 R_2' = I_2^2 R_2$$

$$R_2' = \left(\frac{I_2}{I_2'}\right)^2 R_2$$

But

$$I_2 T_2 = I_2' T_1$$

$$\therefore \quad \frac{I_2}{I_2'} = \frac{T_1}{T_2} = a$$

and

$$R_2' = \left(\frac{T_1}{T_2}\right)^2 R_2 = a^2 R_2 \qquad (1.13.1)$$

Let X_2' be the reactance of the secondary winding reflected or referred to the primary side. For X_2' to produce the same effect in the primary side as X_2 in the secondary side, each must absorb the same reactive voltamperes (VAr).

$$\text{VAr} = VI \sin\phi = IZ \, . \, I \, . \, \frac{X}{Z} = I^2 X$$

Equating the reactive voltamperes consumed by X_2' and X_2 gives

$$(I_2')^2 X_2' = I_2^2 X_2$$

$$X_2' = \left(\frac{I_2}{I_2'}\right)^2 X_2$$

$$\therefore \quad X_2' = \left(\frac{T_1}{T_2}\right)^2 X_2 = a^2 X_2 \qquad (1.13.2)$$

Let R_{e_1}, X_{e_1}, and Z_{e_1} represent the effective resistance, effective reactance, and effective impedance respectively of the whole transformer referred to the primary, then

$$R_{e_1} = \text{primary resistance} + \text{secondary resistance referred to primary}$$

$$\therefore \quad \boxed{R_{e_1} = R_1 + R_2' = R_1 + R_2\left(\frac{T_1}{T_2}\right)^2 = R_1 + a^2 R_2} \tag{1.13.3}$$

X_{e_1} = primary reactance + secondary reactance referred to primary

$$\therefore \quad \boxed{X_{e_1} = X_1 + X_2' = X_1 + X_2\left(\frac{T_1}{T_2}\right)^2 = X_1 + a^2 X_2} \tag{1.13.4}$$

Z_{e_1} = primary impedance + secondary impedance referred to primary

$$\boxed{Z_{e_1} = Z_1 + Z_2' = Z_1 + Z_2\left(\frac{T_1}{T_2}\right)^2 = Z_1 + a^2 Z_2} \tag{1.13.5}$$

$$\text{Also,} \quad \boxed{Z_{e_1} = \sqrt{R_{e_1}^2 + X_{e_1}^2}, \quad Z_{e_1} = R_{e_1} + jX_{e_1}} \tag{1.13.6}$$

The load impedance Z_L referred to primary is $a^2 Z_L$. It may also be taken into account by adding its resistive and reactive components to R_{e_1} and X_{e_1} respectively.

Equivalent values referred to secondary

The equivalent values referred to secondary can also be found in the same manner. If R_{e_2}, X_{e_2} and Z_{e_2} denote the equivalent resistance, equivalent reactance, and equivalent impedance respectively of the *whole* transformer referred to secondary, then

R_{e_2} = secondary resistance + primary resistance referred to secondary

$$\boxed{R_{e_2} = R_2 + R_1' = R_2 + R_1\left(\frac{T_2}{T_1}\right)^2 = R_2 + \frac{R_1}{a^2}} \tag{1.13.7}$$

X_{e_2} = secondary resistance + primary resistance referred to secondary

$$\boxed{X_{e_2} = X_2 + X_1' = X_2 + X_1\left(\frac{T_2}{T_1}\right)^2 = X_2 + \frac{X_1}{a^2}} \tag{1.13.8}$$

Z_{e_2} = secondary impedance + primary impedance referred to primary.

$$\boxed{Z_{e_2} = Z_2 + Z_1' = Z_2 + Z_1\left(\frac{T_2}{T_1}\right)^2 = Z_2 + \frac{Z_1}{a^2}} \tag{1.13.9}$$

$$\text{Also,} \quad Z_{e_2} = R_{2e} + jX_{e_2}$$

$$Z_{e_2} = \sqrt{R_{2e}^2 + X_{e_2}^2}$$

$$R_{e_1} = R_1 + R_2\left(\frac{T_1}{T_2}\right)^2$$

$$R_{e_1}\left(\frac{T_2}{T_1}\right)^2 = R_1\left(\frac{T_2}{T_1}\right)^2 + R_2 = R_{e_2}$$

$$\boxed{R_{e_2} = R_{e_1}\left(\frac{T_2}{T_1}\right)^2 = \frac{R_{e_1}}{a^2}}$$

Similarly,

$$\boxed{X_{e_2} = X_{e_1}\left(\frac{T_2}{T_1}\right)^2 = \frac{X_{e_1}}{a^2}}$$

and

$$\boxed{Z_{e_2} = Z_{e_1}\left(\frac{T_2}{T_1}\right)^2 = \frac{Z_{e_1}}{a^2}}$$

EXAMPLE 1.8. *A 200 kVA, 1-phase transformer with a voltage ratio of 6350/660 V has the following winding resistances and reactances :*

$$R_1 = 1.56\ \Omega, \quad R_2 = 0.016\ \Omega, \quad X_1 = 4.67\ \Omega, \quad X_2 = 0.048\ \Omega$$

Calculate the resistance and reactance of the transformer referred to the high-voltage winding.

SOLUTION.

$$R_{e_1} = R_1 + R_2\left(\frac{T_1}{T_2}\right)^2 = 1.56 + 0.016\left(\frac{6350}{660}\right)^2 = 3.04\ \Omega$$

$$X_{e_1} = X_1 + X_2\left(\frac{T_1}{T_2}\right)^2 = 4.67 + 0.048\left(\frac{6350}{660}\right)^2 = 9.12\ \Omega$$

EXAMPLE 1.9. *A 1-phase transformer has 180 and 90 turns respectively in its secondary and primary windings. The respective resistances are 0.233 Ω and 0.067 Ω. Calculate the equivalent resistance of (a) the primary in terms of the secondary winding, (b) the secondary in terms of the primary winding, and (c) the total resistance of the transformer in terms of the primary.*

SOLUTION. (*a*) The equivalent resistance of the primary winding in terms of the secondary winding

$$R_1' = R_1\left(\frac{T_2}{T_1}\right)^2 = (0.067)\left(\frac{180}{90}\right)^2 = 0.268\ \Omega$$

(*b*) The equivalent secondary resistance in terms of the primary winding

$$R_2' = R_2\left(\frac{T_1}{T_2}\right)^2 = 0.233\left(\frac{90}{180}\right)^2 = 0.058\ \Omega$$

(c) The total resistance of the transformer in terms of the primary

$$R_{e_1} = R_1' + R_2' = R_1 + R_2\left(\frac{T_1}{T_2}\right)^2$$

$$= 0.067 + \left(\frac{90}{180}\right)^2 \times 0.233 = 0.125\ \Omega$$

EXAMPLE 1.10. *A single-phase 3300/400 V transformer has the following winding resistances and reactances :*

$$R_1 = 0.7\ \Omega,\quad R_2 = 0.011\ \Omega,\qquad X_1 = 3.6\ \Omega,\quad X_2 = 0.045\ \Omega$$

The secondary is connected to a coil having a resistance of 4.5 Ω and inductive reactance 3.2 Ω. Calculate the secondary terminal voltage and the power consumed by the coil.

SOLUTION. Since the results required refer to the secondary side, we use the simplified equivalent circuit in Fig. 1.14, in which

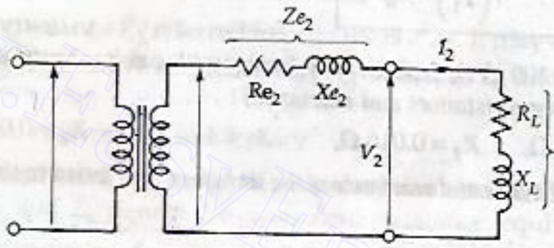


Fig. 1.14.

$$R_{e_2} = R_2 + R_1\left(\frac{T_2}{T_1}\right)^2$$

$$= 0.011 + 0.7\left(\frac{400}{3300}\right)^2 = 0.0213\ \Omega$$

$$X_{e_2} = X_2 + X_1\left(\frac{T_2}{T_1}\right)^2$$

$$= 0.045 + 3.6\left(\frac{400}{3300}\right)^2 = 0.0979\ \Omega$$

$$\mathbf{Z}_{e_2} = R_{e_2} + jX_{e_2} = 0.0213 + j\,0.0979\ \Omega$$

$$\mathbf{Z}_L = 4.5 + j\,3.2\ \Omega = 5.522\,\underline{/35.42^\circ}\ \Omega$$

The total impedance in the secondary circuit is

$$\mathbf{Z} = \mathbf{Z}_{e_2} + \mathbf{Z}_L$$

$$= 0.0213 + j\,0.0979 + 4.5 + j\,3.2$$

$$= 4.5213 + j\,3.2979 = 5.596\,\underline{/36.1^\circ}\ \Omega$$

By Ohm's law

$$I_2 = \frac{400\angle 0^\circ}{Z} = \frac{400\angle 0^\circ}{5.596\,\underline{/36.1^\circ}} = 71.48\,\underline{/-36.1^\circ}\text{ A}$$

Secondary terminal voltage

$$V_2 = I_2 Z_L = (71.48\,\underline{/-36.1^\circ}) \times (5.522\,\underline{/35.42^\circ}) = 394.7\,\underline{/-0.68^\circ}\text{ V}$$

Power consumed by the coil

$$P_L = I_2^2 R_L = (71.48)^2 \times 4.5 = 22992\text{ W} = 22.992\text{ kW}$$

1.14 DERIVATION OF EQUIVALENT CIRCUIT OF A TRANSFORMER

Figure 1.15 shows the complete equivalent circuit of a transformer. An exact equivalent circuit referred to the primary can be deduced as follows :

(1) All secondary resistances and reactances are reflected to the primary as the square of the transformation ratio. That is, the quantities R_2, X_2 and Z_L in the secondary become a^2R_2, a^2X_2 and a^2Z_L respectively, when referred to the primary.

(2) All voltages are reflected from secondary to primary directly as the product of the transformation ratio. That is, V_2 and E_2 in the secondary become aV_2 and aE_2 respectively, when referred to the primary.

(3) All secondary currents are reflected to the primary inversely as the transformation ratio. Thus, I_2 in the secondary becomes $\frac{I_2}{a}$ when referred to the primary.

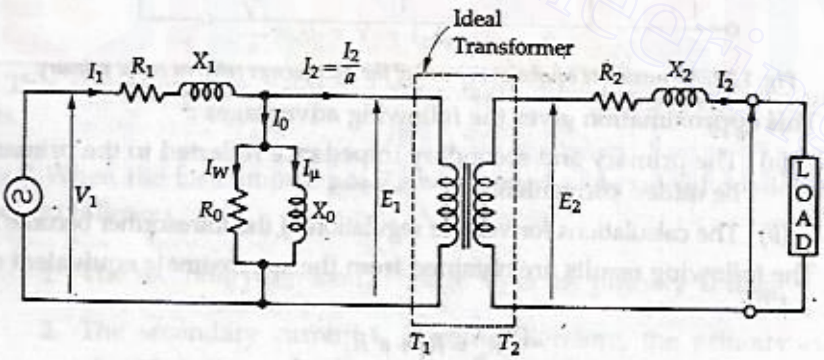


Fig. 1.15. *Complete circuit model (equivalent circuit) of a real transformer.*

Figure 1.16 shows the equivalent circuit with all *secondary* values referred (reflected) to the primary. This circuit is called the **exact equivalent circuit of the transformer referred to the primary.** With this model, it is not possible to add directly the primary impedance $(R_1 + jX_1)$ to the secondary impedance reflected to the primary side.

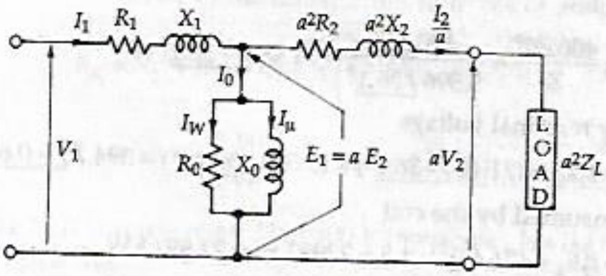


Fig. 1.16. *Exact equivalent circuit of a transformer referred to the primary.*

Figure 1.17 shows the approximate equivalent circuit referred to the primary. The justification for approximation is as follows :

The no-load current I_0 is usually less than 5 per cent of the full-load primary current. The voltage drop produced by I_0 in $(R_1 + jX_1)$ is negligible for practical purposes. Therefore it is immaterial that the shunt branch $R_0 \parallel X_0$ is connected before the primary series impedance $(R_1 + jX_1)$ or after it. The currents I_W and I_μ are not much affected. Therefore the parallel branch $R_0 \parallel X_0$ is connected to the input terminals as shown in Fig. 1.17.

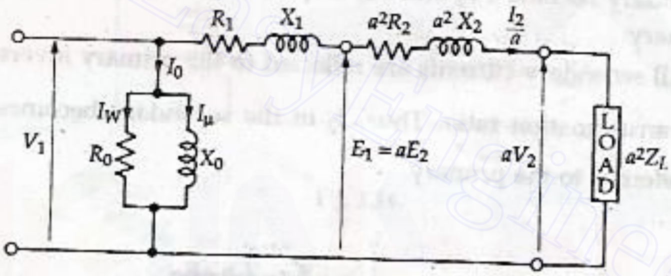


Fig. 1.17. *Approximate equivalent circuit of the transformer referred to the primary.*

This approximation gives the following advantages :

(*a*) The primary and secondary impedance reflected to the primary can be added conveniently.

(*b*) The calculations for voltage regulation of the transformer become easier.

The following results are obtained from the approximate equivalent circuit of Fig. 1.17.

$$R_{e_1} = R_1 + a^2R_2$$

$$X_{e_1} = X_1 + a^2X_2$$

$$\mathbf{Z}_{e_1} = R_{e_1} + jX_{e_1}$$

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{\mathbf{Z}_{e_1} + a^2\mathbf{Z}_L}$$

where $\mathbf{Z}_L$ is the load impedance.

1.15 APPROXIMATE EQUIVALENT CIRCUIT REFERRED TO THE SECONDARY

The approximate equivalent circuit of the transformer referred to the secondary can also be found in the same manner. Here we transfer all primary resistances and reactances to the secondary. The approximate equivalent circuit referred to the secondary is shown in Fig. 1.18. In this circuit,

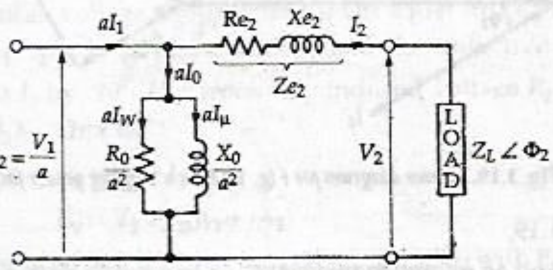


Fig. 1.18. *Approximate equivalent circuit of the transformer referred to the secondary.*

$$R_{e_2} = R_2 + \frac{R_1}{a^2}$$

$$X_{e_2} = X_2 + \frac{X_1}{a^2}$$

$$Z_{e_2} = R_{e_2} + jX_{e_2}$$

$$I_2 = \frac{V_2}{Z_L} = \frac{E_2}{Z_{e_2} + Z_L}$$

$$\frac{V_1}{a} = E_2 = V_2 + I_2 Z_{e_2}$$

This circuit has the following advantages over the previous equivalent circuits :

(*a*) When the load impedance Z_L is removed, the no-load conditions are obtained as follows :

1. The secondary (no-load) voltage V_2 is the primary voltage $\frac{V_1}{a}$.
2. The secondary current I_2 is zero. Therefore, the primary current $aI_1 = aI_0$ is the no-load primary current (reflected to the secondary).

(*b*) When the load Z_L is connected, then the primary applied voltage reflected to the secondary, $\frac{V_1}{a}$, becomes

$$\frac{V_1}{a} = E_2 = V_2 + I_2 Z_{e_2} \qquad (1.15.1)$$

The phasor diagram for Fig. 1.18 is shown in Fig. 1.19.

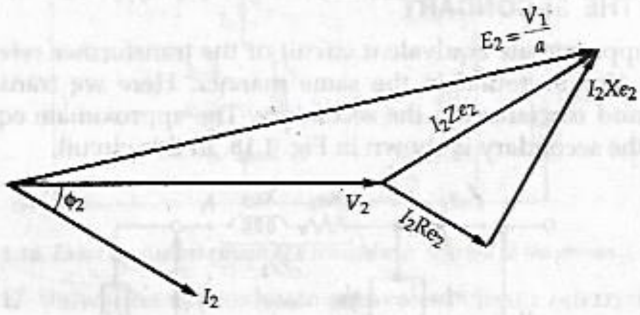


Fig. 1.19. *Phasor diagram for Fig. 1.18 for a lagging power factor load.*

In Fig. 1.19,

$\mathbf{V}_2$ is taken as reference phasor.

$\mathbf{I}_2$ lags $\mathbf{V}_2$ by the power factor angle ϕ_2.

$I_2R_{e_2}$ is in phase with $\mathbf{I}_2$.

$I_2X_{e_2}$ leads $\mathbf{I}_2$ by 90°.

The phasor sum of $\mathbf{I}_2R_{e_2}$ and $\mathbf{I}_2X_{e_2}$ is $\mathbf{I}_2\mathbf{Z}_{e_2}$.

The phasor sum of $\mathbf{V}_2$ and $\mathbf{I}_2\mathbf{Z}_{e_2}$ is $\frac{\mathbf{V}_1}{a}$ (or $\mathbf{E}_2$) and $\frac{\mathbf{V}_1}{a}$ represents the secondary no-load voltage.

Also, $$\mathbf{E}_2 = \frac{\mathbf{V}_1}{a} = \mathbf{V}_2 + \mathbf{I}_2\mathbf{Z}_{e_2} \tag{1.15.2}$$

The significance of Eq. (1.15.2) is that it permits the calculation of the voltage regulation of the transformer in terms of secondary (load) voltages.

1.16 FURTHER SIMPLIFICATION TO APPROXIMATE EQUIVALENT CIRCUIT

Since the no-load current of a transformer is about 3 to 5 per cent of the full-load primary current, its effect can be neglected. Hence for all practical purposes, the parallel circuit containing R_0 and X_0 can be omitted on full load without loss of accuracy. This is the further simplification to the approximate equivalent circuit of the transformer. The simplified circuit is shown in Fig. 1.20.

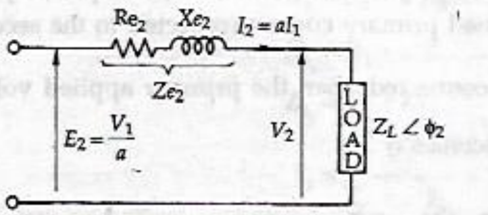


Fig. 1.20. *Simplified equivalent circuit of the transformer referred to secondary.*

1.17 FULL-LOAD PHASOR DIAGRAM

Figure 1.21 shows the phasor diagram for the exact circuit model of transformer of Fig. 1.15. It is assumed that the load is inductive, which is generally the case. Let $\cos\phi_2$ be the power factor of the load (lagging). The phasor $\mathbf{V}_2$ is taken as reference. Since the load power factor is lagging, the secondary current $\mathbf{I}_2$ lags behind $\mathbf{V}_2$ by the power factor angle ϕ_2. The secondary current $\mathbf{I}_2$ flows through R_2 and X_2 and produces voltage drops across them equal to I_2R_2 and I_2X_2. The resistive voltage drop I_2R_2 is in phase with $\mathbf{I}_2$ and the inductive voltage drop I_2X_2 leads the current $\mathbf{I}_2$ by 90°. The secondary induced voltage $\mathbf{E}_2$ is the phasor sum of V_2, I_2R_2 and I_2X_2. That is,

$$\mathbf{E}_2 = \mathbf{V}_2 + \mathbf{I}_2\mathbf{Z}_2 \tag{1.17.1}$$

$$\mathbf{E}_2 = \mathbf{V}_2 + I_2(R_2 + jX_2) \tag{1.17.2}$$

The primary induced voltage $\mathbf{E}_1$ ($= a\mathbf{E}_2$) is in time phase with $\mathbf{E}_2$ because both these voltages are induced by the same flux Φ_M. The flux Φ_M leads $\mathbf{E}_1$ by 90°.

The ampere turns of the secondary I_2T_2 must be balanced by a load component of current I_2 in the primary winding such that $I_2'T_1 = I_2T_2$. The current I_2' is called the *secondary current referred (reflected) to the primary*. Thus, the current I_2' represents the component of the primary current to neutralize the demagnetizing effect of the secondary current. The current $\mathbf{I}_2'\left(= \frac{\mathbf{I}_2}{a}\right)$ is therefore 180° out of phase with $\mathbf{I}_2$.

The current $\mathbf{I}_W$ is in phase opposition with $\mathbf{E}_1$ and $\mathbf{I}_\mu$ leads $\mathbf{E}_1$ by 90°. The no-load current $\mathbf{I}_0$ is the phasor sum of $\mathbf{I}_W$ and $\mathbf{I}_\mu$.

$$\mathbf{I}_0 = \mathbf{I}_W + \mathbf{I}_\mu$$

The total primary current $\mathbf{I}_1$ taken from the supply is the phasor sum of $\mathbf{I}_2'$ and $\mathbf{I}_0$.

$$\mathbf{I}_1 = \mathbf{I}_2' + \mathbf{I}_0$$

$$\mathbf{I}_1 = \frac{\mathbf{I}_2}{a} + \mathbf{I}_0$$

Since $\mathbf{E}_1$ is the voltage induced in the primary winding, it is equal and opposite to the component of the applied voltage at the ideal transformer winding. Let $\mathbf{V}_1'$ be the voltage applied to the primary of the ideal transformer to neutralize the effect of induced voltage $\mathbf{E}_1$. Thus $\mathbf{V}_1'$ is equal and opposite to $\mathbf{E}_1$. The phasor sum of I_1R_1, I_1X_1 and $\mathbf{V}_1'$ is equal to the supply voltage $\mathbf{V}_1$. That is,

$$\mathbf{V}_1 = \mathbf{V}_1' + I_1R_1 + jI_1X_1$$

$$\mathbf{V}_1' = -\mathbf{E}_1$$

The resistive voltage drop I_1R_1 is in phase with $\mathbf{I}_1$ and the inductive voltage drop I_1X_1 leads $\mathbf{I}_1$ by 90°. The angle between $\mathbf{V}_1$ and $\mathbf{I}_1$ is ϕ_1. Thus $\cos\phi_1$ is the power factor on the primary side. Power input to the transformer is given by $V_1I_1\cos\phi_1$.

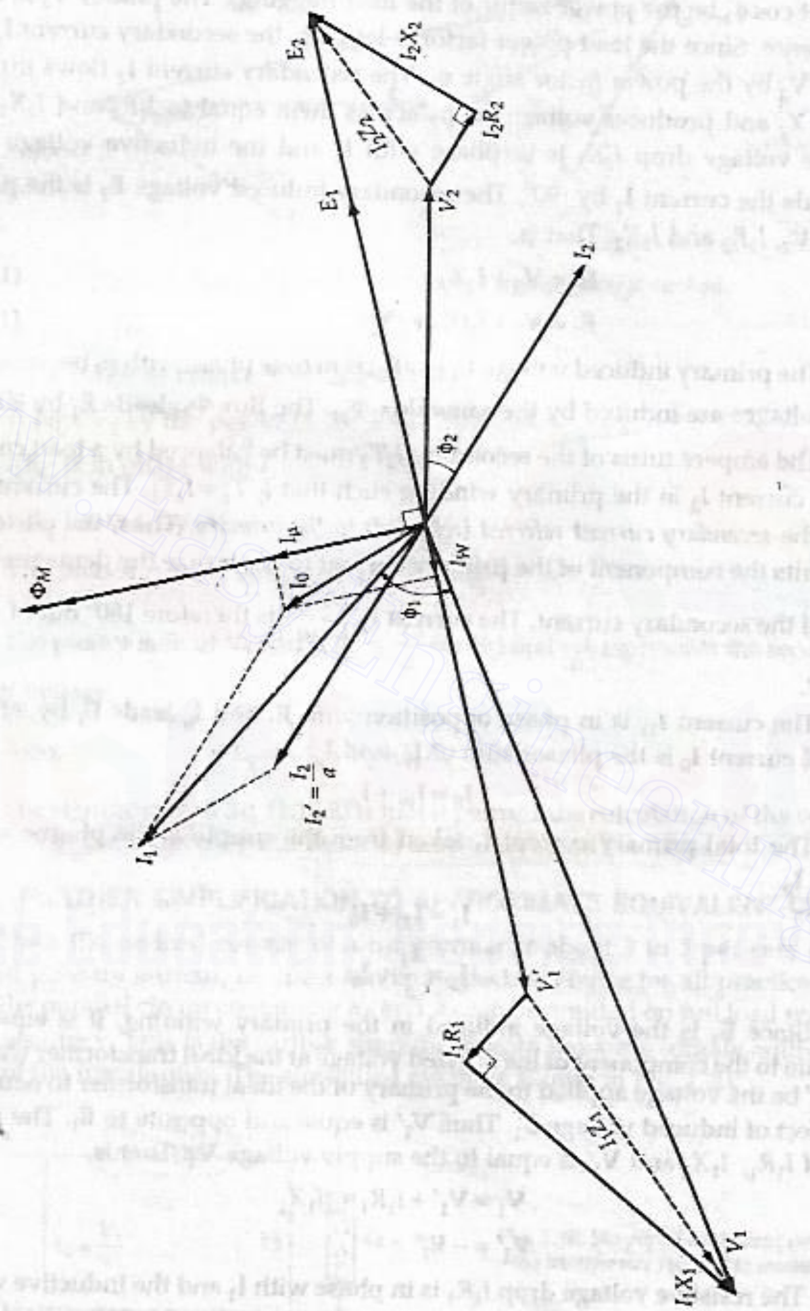


Fig. 1.21. Phasor diagram for the exact equivalent circuit of real transformer of Fig. 1.15.

1.18 VOLTAGE REGULATION OF A TRANSFORMER

Majority of loads connected to the secondary of a transformer are designed to operate at practically constant voltage. However, as the current is taken through the transformer, the load terminal voltage changes because of the voltage drop in the internal impedance of the transformer. The term voltage regulation is used to identify the characteristic of the voltage change in a transformer with loading.

The voltage regulation of a transformer is defined as the arithmetical difference in the secondary terminal voltage between no-load ($I_2 = 0$) and full-rated load ($I_2 = I_{2fl}$) at a given power factor with the same value of primary voltage for both rated load and no-load. It is expressed as either a per unit or a percentage of the rated load voltage. Rated voltage is usually taken to be the nameplate value.

The numerical difference between no-load and full-load voltage is called *inherent voltage regulation.*

Inherent voltage regulation

$$\triangleq |\mathbf{V}_{2nl}| - |\mathbf{V}_{2fl}| \tag{1.18.1}$$

where V_{2fl} = rated secondary terminal voltage at rated load

V_{2nl} = no-load secondary terminal voltage with the same value of primary voltage for both rated load and no load.

The quantities in Eq. (1.18.1) are *magnitudes, not phasors.*

Per-unit voltage regulation at full load

$$\triangleq \left|\frac{|\mathbf{V}_{2nl}| - |\mathbf{V}_{2fl}|}{|\mathbf{V}_{2fl}|}\right|_{|V_1| = \text{constant}} \tag{1.18.2}$$

Percent voltage regulation at full load

$$\triangleq \left|\frac{|\mathbf{V}_{2nl}| - |\mathbf{V}_{2fl}|}{|\mathbf{V}_{2fl}|}\right|_{|V_1| = \text{constant}} \times 100 \tag{1.18.3}$$

The conditions under which the regulation is to be figured are as follows :

(1) Rated voltage, current and frequency.

(2) When regulation is stated without specific reference to the load conditions, rated load is to be understood.

(3) Waveform of voltage should be assumed sinusoidal unless stated otherwise.

(4) Power factor of the load should be mentioned. If the power factor is not specified, its value is to be assumed unity.

The voltage regulation is an important measure of transformer performance. The limits of voltage variation are specified in terms of voltage regulation. For example, transformers in public supply systems must be so adjusted that the voltage at the terminals of the consumers must not exceed ± 5%.

1.19 VOLTAGE REGULATION IN TERMS OF PRIMARY VALUES

Per unit voltage regulation

$$= \frac{|\mathbf{V}_{2nl}| - |\mathbf{V}_{2fl}|}{|\mathbf{V}_{2fl}|}$$

$$\mathbf{V}_{2nl} = \frac{\mathbf{V}_1}{a}$$

$\therefore$ *pu* voltage regulation

$$= \frac{\left|\frac{\mathbf{V}_1}{a}\right| - |\mathbf{V}_{2fl}|}{|\mathbf{V}_{2fl}|} \quad (1.19.1)$$

It is assumed that V_1 is adjusted so that the rated voltage is obtained at the secondary terminals under given load conditions.

1.20 CALCULATION OF VOLTAGE REGULATION

The voltage regulation of a transformer can be calculated in terms of its circuit parameters. The approximate equivalent circuit of the transformer referred to the secondary is shown in Fig. 1.22.

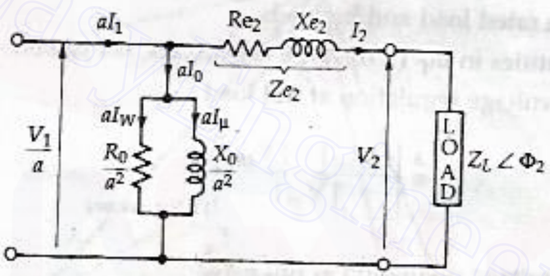


Fig. 1.22. Approximate equivalent circuit of the transformer referred to the secondary.

By KVL, $$\frac{\mathbf{V}_1}{a} = \mathbf{V}_2 + \mathbf{I}_2 \mathbf{Z}_{e_2}$$

Since $I_2 Z_{e_2}$ depends on the power factor of the load, the regulation depends on the load power factor. In order to calculate regulation the following steps are used :

(1) Take V_2 as reference phasor

$$\therefore \quad \mathbf{V}_2 = V_2 \angle 0^\circ = V_2 + j0$$

(2) Write $\mathbf{I}_2$ in phasor form

For lagging power-factor $\cos \phi_2$

$$\mathbf{I}_2 = I_2 \angle -\phi_2 = I_2 \cos \phi_2 - jI_2 \sin \phi_2$$

For leading power-factor $\cos \phi_2$

$$\mathbf{I}_2 = I_2 \angle +\phi_2 = I_2 \cos \phi_2 + jI_2 \sin \phi_2$$

For unity power factor

$$\mathbf{I}_2 = I_2 \angle 0^\circ = I_2 + j0$$

(3) Calculate $\mathbf{Z}_{e_2}$

$$\mathbf{Z}_{e_2} = R_{e_2} + jX_{e_2}$$

(4) Calculate

$$\mathbf{V}_{2nl} = \frac{\mathbf{V}_1}{a}$$

$$\frac{\mathbf{V}_1}{a} = V_2 \angle 0^\circ + \mathbf{I}_2\mathbf{Z}_{e_2}$$

(5) Calculate the voltage regulation

$$= \frac{\left|\frac{\mathbf{V}_1}{a}\right| - |\mathbf{V}_{2fl}|}{|\mathbf{V}_{2fl}|} \text{ pu}$$

It is to be noted that the *magnitudes* of the voltages, not their phasor values determine the regulation.

1.21 VOLTAGE REGULATION AT LAGGING POWER FACTOR

$$\mathbf{V}_2 = V_2 \angle 0^\circ = V_2 + j0$$

$$\mathbf{I}_2 = I_2 \angle -\phi_2 = I_2 \cos\phi_2 - jI_2 \sin\phi_2$$

$$Z_{e_2} = R_{e_2} + jX_{e_2}$$

$$\mathbf{E}_2 = \frac{\mathbf{V}_1}{a} = \mathbf{V}_2 + \mathbf{I}_2 Z_{e_2}$$

$$= V_2 + j0 + (I_2 \cos\phi_2 - jI_2 \sin\phi_2)(R_{e_2} + jX_{e_2})$$

$$= (V_2 + I_2R_{e_2} \cos\phi_2 + I_2X_{e_2} \sin\phi_2) + j(I_2X_{e_2} \cos\phi_2 - I_2R_{e_2} \sin\phi_2)$$

$$|\mathbf{E}_2| = \sqrt{[(V_2 + I_2R_{e_2} \cos\phi_2 + I_2X_{e_2} \sin\phi_2)^2 + (I_2X_{e_2} \cos\phi_2 - I_2R_{e_2} \sin\phi_2)^2]}$$

Figure 1.23 shows the phasor diagram for lagging power factor $\cos\phi_2$. Here $\mathbf{V}_2$ is taken as reference phasor. $\mathbf{I}_2$ lags behind $\mathbf{V}_2$ by ϕ_2. $\mathbf{I}_2\mathbf{R}_{e_2}$ is in phase with $\mathbf{I}_2$ and $I_2X_{e_2}$ leads $\mathbf{I}_2$ by 90°. The phasor sum of $\mathbf{V}_2$, $I_2R_{e_2}$ and $I_2X_{e_2}$ is $\mathbf{E}_2 \left(= \frac{\mathbf{V}_1}{a}\right)$.

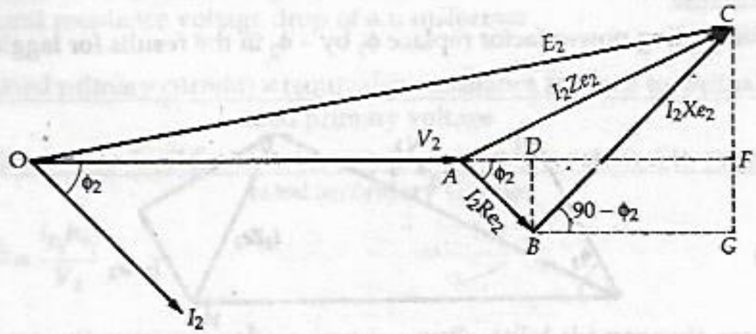


Fig. 1.23. Phasor diagram of the transformer referred to the secondary.

The expression for E_2 can also be found from the phasor diagram of Fig. 1.23.

From Δ OFC,

$$OC^2 = OF^2 + CF^2 = (OA + AD + DF)^2 + (CG - FG)^2$$

$$E_2 = \sqrt{[(V_2 + I_2R_{e_2}\cos\phi_2 + I_2X_{e_2}\sin\phi_2)^2 + (I_2X_{e_2}\cos\phi_2 - I_2R_{e_2}\sin\phi_2)^2]}$$

Per unit regulation

$$= \frac{E_2 - V_2}{V_2} = \frac{\frac{V_1}{a} - V_2}{V_2}$$

1.22 APPROXIMATE REGULATION AT LAGGING POWER FACTOR

Since the term $(I_2X_{e_2}\cos\phi_2 - I_2R_{e_2}\sin\phi_2)$ is small compared with the term $(V_2 + I_2R_{e_2}\cos\phi_2 + I_2X_{e_2}\sin\phi_2)$, it can be neglected. That is,

$$I_2X_{e_2}\cos\phi_2 - I_2R_{e_2}\sin\phi_2 = 0,$$

The approximate value of E_2 is therefore given by

$$E_2 \simeq V_2 + I_2R_{e_2}\cos\phi_2 + I_2X_{e_2}\sin\phi_2$$

Approximate per-unit regulation for lagging power factor $\cos\phi_2$ is

$$\frac{E_2 - V_2}{V_2} = \frac{I_2R_{e_2}\cos\phi_2 + I_2X_{e_2}\sin\phi_2}{V_2}$$

1.23 VOLTAGE REGULATION AT LEADING POWER FACTOR

For leading power factor $\cos\phi_2$

$$\mathbf{I}_2 = I_2\angle + \phi_2 = I_2\cos\phi_2 + jI_2\sin\phi_2$$

$$\mathbf{E}_2 = \frac{\mathbf{V}_1}{a} = \mathbf{V}_2 + \mathbf{I}_2\mathbf{Z}_{e_2} = V_2 + j0 + (I_2\cos\phi_2 + jI_2\sin\phi_2)(R_{e_2} + jX_{e_2})$$

$$E_2 = \sqrt{[(V_2 + I_2R_{e_2}\cos\phi_2 - I_2X_{e_2}\sin\phi_2)^2 + (I_2X_{e_2}\cos\phi_2 + I_2R_{e_2}\sin\phi_2)^2]}$$

ALTERNATIVELY

For leading power factor replace ϕ_2 by $-\phi_2$ in the results for lagging power factor.

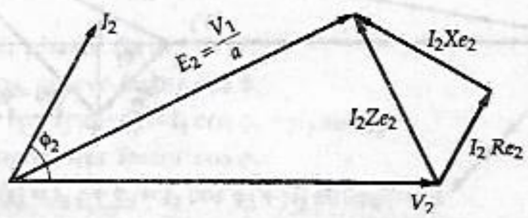


Fig. 1.24

The expression for E_2 can also be obtained from the phasor diagram of the transformer referred to the secondary. At leading power-factor the phasor diagram is shown in Fig. 1.24.

Approximate regulation at leading power-factor

$$E = \frac{E_2 - V_2}{V_2} = \frac{I_2 R_{e_2} \cos\phi_2 - I_2 X_{e_2} \sin\phi_2}{V_2}$$

1.24 REGULATION AT UNITY POWER FACTOR

For unity power factor

$$\mathbf{I}_2 = I_2 \angle 0^\circ = I_2 + j0$$

$$\mathbf{E}_2 = \mathbf{V}_2 + \mathbf{I}_2 \mathbf{Z}_{e_2}$$

$$= V_2 + j0 + (I_2 + j0)(R_{e_2} + jX_{e_2}) = (V_2 + I_2 R_{e_2}) + jI_2 X_{e_2}$$

$$|\mathbf{E}_2| = \sqrt{[(V_2 + I_2 R_{e_2})^2 + (I_2 X_{e_2})^2]}$$

$$\text{Voltage regulation} = \frac{|\mathbf{E}_2| - |\mathbf{V}_2|}{|\mathbf{V}_2|}$$

The expression for E_2 at unity power can also be obtained from the expression for E_2 at lagging power factor by putting $\phi_2 = 0$. Then $\cos\phi_2 = 1$ and $\sin\phi_2 = 0$.

1.25 PER UNIT RESISTANCE, LEAKAGE REACTANCE AND IMPEDANCE VOLTAGE DROPS

Full-load voltage drops occurring in a transformer can be expressed as a fraction (usually in per unit or per cent) of the full-load terminal voltage.

Let

I_{fl_1} = full-load primary current

I_{fl_2} = full-load secondary current

V_1 = rated primary voltage

V_2 = rated secondary voltage

Per unit resistance voltage drop of a transformer

$$\triangleq \frac{\text{(full-load primary current)} \times \text{(equivalent resistance referred to primary)}}{\text{rated primary voltage}}$$

$$= \frac{\text{(full-load secondary current)} \times \text{(equivalent resistance referred to secondary)}}{\text{rated secondary voltage}}$$

$$= \frac{I_{fl_1} R_{e_1}}{V_1} = \frac{I_{fl_2} R_{e_2}}{V_2} \qquad (1.25.1)$$

The per-unit resistance voltage drop is usually called the **per-unit resistance** Re_{pu}

Thus, per-unit resistance

$$R_{epu} = \frac{I_{fl_1} R_{e_1}}{V_1} = \frac{I_{fl_2} R_{e_2}}{V_2} \tag{1.25.2}$$

$$R_{epu} = \frac{I_{fl_2} R_{e_2}}{V_2}$$

$$R_{epu} = \frac{I_{fl_2}^2 R_{e_2}}{V_2 I_{fl_2}} = \frac{P_{cfl}}{S_{fl_2}} \tag{1.25.3}$$

$$\therefore \quad R_{epu} = \frac{\text{total copper loss at rated current}}{\text{rated voltamperes}} \tag{1.25.4}$$

Similarly, per unit leakage reactance drop of a transformer

$$\triangleq \frac{\text{(full-load primary current)} \times \text{(equivalent leakage reactance refered to primary)}}{\text{rated primary voltage}}$$

$$= \frac{\text{(full-load secondary current)} \times \text{(equivalent leakage reactance refered to secondary)}}{\text{rated secondary voltage}}$$

$$= \frac{I_{fl_1} X_{e_1}}{V_1} = \frac{I_{fl_2} X_{e_2}}{V_2} \tag{1.25.5}$$

The per-unit leakage reactance voltage drop is called the **per-unit leakage reactance** X_{epu}.

$$\therefore \quad X_{epu} = \frac{I_{fl_1} X_{e_1}}{V_1} = \frac{I_{fl_2} X_{e_2}}{V_2} \tag{1.25.6}$$

The *per-unit impedance voltage* drop or simply the *per-unit impedance* Z_{epu} is defined as

$$Z_{epu} = \frac{I_{fl_1} Z_{e_1}}{V_1} = \frac{I_{fl_2} Z_{e_2}}{V_2} \tag{1.25.7}$$

1.26 VOLTAGE REGULATION BY PER-UNIT QUANTITIES

Usually the transformer impedances are given in percent of base impedance. Full-load voltage regulation is most easily calculated by converting these quantities into per unit and per unit values are used throughout the calculations.

From Eq. (1.15.1)

$$\frac{\mathbf{V}_1}{a} = \mathbf{V}_2 + \mathbf{I}_2 \mathbf{Z}_{e_2} \tag{1.26.1}$$

Dividing both the sides of this equation by the secondary base voltage $\mathbf{V}_{2b}$, we get

$$\frac{\mathbf{V}_1}{a\mathbf{V}_{2b}} = \frac{\mathbf{V}_2}{\mathbf{V}_{2b}} + \frac{\mathbf{I}_2 \mathbf{Z}_{e_2}}{\mathbf{V}_{2b}} \tag{1.26.2}$$

But $\quad a\mathbf{V}_{2b} = \mathbf{V}_{1b}$

$$\mathbf{V}_{2b} = \mathbf{I}_{2b} \mathbf{Z}_{2b}$$

Therefore Eq. (1.26.2) may be written as

$$\frac{\mathbf{V}_1}{\mathbf{V}_{1b}} = \frac{\mathbf{V}_2}{\mathbf{V}_{2b}} + \frac{\mathbf{I}_2}{\mathbf{I}_{2b}} \times \frac{\mathbf{Z}_{e2}}{\mathbf{Z}_{2b}}.$$

or

$$\boxed{\mathbf{V}_{1pu} = \mathbf{V}_{2pu} + \mathbf{I}_{2pu}\mathbf{Z}_{epu}} \tag{1.26.3}$$

The relation of Eq. (1.26.3) holds for the voltage regulation of any iron-core transformer at *any* load. But, since voltage regulation is defined at the rated load, at any power factor we have at full load

$$\mathbf{V}_2 = \mathbf{V}_{2b}$$

and

$$|\mathbf{V}_{2pu}| = 1$$

$$|\mathbf{I}_2| = |\mathbf{I}_{2b}|$$

$$|\mathbf{I}_{2pu}| = 1$$

Since

$$\frac{\mathbf{V}_1}{a} = \mathbf{V}_{2b}\angle 0° + \mathbf{I}_{2b}\mathbf{Z}_{e2} \tag{1.26.4}$$

For lagging power factor $\cos\phi_2$

$$\mathbf{V}_{1pu} = 1\angle 0° + (1\angle -\phi_2)\,\mathbf{Z}_{epu} \tag{1.26.5}$$

and for leading power factor $\cos\phi_2$

$$\mathbf{V}_{1pu} = 1\angle 0° + (1\angle +\phi_2)\,\mathbf{Z}_{epu} \tag{1.26.6}$$

Voltage regulation

$$\boxed{= \frac{\left|\dfrac{\mathbf{V}_1}{a}\right| - |\mathbf{V}_{2b}|}{|\mathbf{V}_{2b}|}}$$

If the numerator and denominator are divided by $|\mathbf{V}_{2b}|$, then voltage regulation

$$= \frac{\left|\dfrac{\mathbf{V}_1}{a\mathbf{V}_{2b}}\right| - \left|\dfrac{\mathbf{V}_{2b}}{\mathbf{V}_{2b}}\right|}{\left|\dfrac{\mathbf{V}_{2b}}{\mathbf{V}_{2b}}\right|}$$

$$\boxed{= \left|\frac{\mathbf{V}_1}{\mathbf{V}_{1b}}\right| - 1 = |\mathbf{V}_{1pu}| - 1} \tag{1.26.7}$$

1.27 APPROXIMATE PER-UNIT VOLTAGE REGULATION

Approximate per-unit regulation at lagging power factor $\cos\phi_2$

$$= \frac{I_2 R_{e_2}\cos\phi_2 + I_2 X_{e_2}\sin\phi_2}{V_2}$$

$$= \frac{I_2 R_{e_2}}{V_2}\cos\phi_2 + \frac{I_2 X_{e_2}\sin\phi_2}{V_2}$$

$$\boxed{= R_{epu}\cos\phi_2 + X_{epu}\sin\phi} \tag{1.27.1}$$

Approximate per-unit regulation at leading power factor $\cos\phi_2$

$$= \frac{I_2 R_{e_2} \cos\phi_2 - I_2 X e_2 \sin\phi_2}{V_2} \quad (1.27.2)$$

$$= R_{epu} \cos\phi_2 - X_{epu} \sin\phi_2 \quad (1.27.3)$$

1.28 CONDITION FOR ZERO VOLTAGE REGULATION

Approximate voltage regulation

$$= \frac{I_2 R_{e_2} \cos\phi_2 + I_2 X_{e_2} \sin\phi_2}{V_2}$$

For zero voltage regulation

$$I_2 R_{e_2} \cos\phi_2 + I_2 X_{e_2} \sin\phi_2 = 0$$

$$I_2 X_{e_2} \sin\phi_2 = -I_2 R_{e_2} \cos\phi_2$$

$$\boxed{\tan\phi_2 = -\frac{R_{e_2}}{X_{e_2}}} \quad (1.28.1)$$

$$\boxed{\phi_2 = -\tan^{-1}\left(\frac{R_{e_2}}{X_{e_2}}\right)} \quad (1.28.2)$$

The negative sign indicates that zero voltage regulation occurs when the load is capacitive (that is, the power factor is leading).

1.29 CONDITION FOR MAXIMUM VOLTAGE REGULATION

For maximum voltage regulation, $\frac{d}{d\phi_2}(\text{regulation}) = 0$

$$\frac{d}{d\phi_2} \frac{(I_{e2} R_{es} \cos\phi_2 + I_2 X_{e_2} \sin\phi_2)}{V_2} = 0$$

$$-I_{e_2} R_{e_2} \sin\phi_2 + I_2 X_{e_2} \cos\phi_2 = 0$$

$$\tan\phi_2 = \frac{X_{e_2}}{R_{e_2}} \quad (1.29.1)$$

$$\phi_2 = \tan^{-1}\frac{X_{e_2}}{R_{e_2}} \quad (1.29.2)$$

and

$$\boxed{\cos\phi_2 = \frac{R_{e_2}}{Ze_2}} \quad (1.29.3)$$

Thus, maximum voltage regulation occurs at *lagging power factor* of the load. The lagging power-factor angle of the load is equal to the angle of the equivalent impedance of the transformer.

Maximum value of the voltage regulation

$$= \frac{1}{V_2}\left(I_2 R_{e_2} \frac{R_{e_2}}{Z_{e_2}} + I_2 X_{e_2} \frac{X_{e_2}}{Z_{e_2}} \right) = \frac{I_2}{V_2 Ze_2}(R_{e_2}^2 + X_{e_2}^2)$$

$$= \frac{I_2 Z_{e_2}^2}{V_2 Ze_2} = \frac{I_2 Z_{e_2}}{V_2} = Z_{epu} \qquad (1.29.4)$$

Thus, the magnitude of maximum voltage regulation is equal to the per unit value of equivalent leakage impedance of the transformer.

EXAMPLE 1.11. *A 10 kVA, single-phase transformer for 2000/400 V at no load, has $R_1 = 5.5\ \Omega$, $X_1 = 12\ \Omega$, $R_2 = 0.2\ \Omega$, $X_2 = 0.45\ \Omega$. Determine the approximate value of the secondary voltage at full load, 0.8 power factor (lagging), when the primary applied voltage is 2000 V.*

SOLUTION. $$\frac{T_1}{T_2} = \frac{E_1}{E_2} = \frac{2000}{400} = 5$$

$$R_{e_2} = R_2 + R_1\left(\frac{T_2}{T_1}\right)^2 = 0.2 + 5.5\left(\frac{1}{5}\right)^2 = 0.42\ \Omega$$

$$X_{e_2} = X_2 + X_1\left(\frac{T_2}{T_1}\right)^2 = 0.45 + 12\left(\frac{1}{5}\right)^2 = 0.93\ \Omega$$

$$kVA = \frac{V_2 I_2}{1000}$$

$$I_2 = \frac{1000 \times kVA}{V_2} = \frac{1000 \times 10}{400} = 25\ \text{A}$$

Since $\cos\phi_2 = 0.8$, $\sin\phi_2 = 0.6$

$$E_2 = E_1 \cdot \frac{T_2}{T_1} = V_1 \frac{T_2}{T_1} = 2000 \times \frac{1}{5} = 400\ \text{v}$$

$$E_2 = V_2 + I_2 R_{e_2} \cos\phi_2 + I_2 X_{e_2} \sin\phi_2$$

$$400 = V_2 + 25 \times 0.42 \times 0.8 + 25 \times 0.93 \times 0.6$$

$$V_2 = 400 - 8.4 - 13.95 = 377.65\ \text{V}$$

EXAMPLE 1.12. *A transformer has 2 per cent resistance and 5 per cent reactance. Find its voltage regulation at full load, 0.8 power factor, lagging.*

SOLUTION. Voltage regulation

$$= R_{epu} \cos\phi_2 + X_{epu} \sin\phi_2$$

$$= 0.02 \times 0.8 + 0.05 \times 0.6 = 0.046\ \text{pu} = 4.6\%$$

EXAMPLE 1.13. *Calculate the regulation of a transformer in which ohmic loss is 1% of the output and the reactance drop is 5% of the voltage when the power factor is (a) 0.8 lagging; (b) unity; (c) 0.8 leading.*

SOLUTION. (*a*) Regulation at 0.8 p.f. lagging

$$= R_{e_2} \cos\phi_2 + X_{e_2} \sin\phi_2 = 1 \times 0.8 + 5 \times 0.6 = 3.8\%$$

(*b*) Regulation at unity power factor

$$= R_{e_2} = 1\%$$

(*c*) Regulation at 0.8 p.f. leading

$$= R_{e_2} \cos \phi_2 - X_{e_2} \sin \phi_2 = 1 \times 0.8 - 5 \times 0.6 = -2.2\%$$

EXAMPLE 1.14. *A single-phase, 100 kVA, 2000/200 V, 50 Hz transformer has an impedance drop of 10% and resistance drop of 5%. Calculate the (a) regulation at full load 0.8 power factor lagging; (b) the value of the power factor at which regulation is zero.*

SOLUTION. $$\frac{I_2 Z_{e_2}}{V_2} \times 100 = 10$$

$$I_2 Z_{e_2} = \frac{10\ V_2}{100} = \frac{10 \times 200}{100} = 20 \text{ V}$$

$$\frac{I_2 R_{e_2}}{V_2} \times 100 = 5$$

$$I_2 R_{e_2} = \frac{5\ V_2}{100} = \frac{5 \times 200}{100} = 10 \text{ V}$$

$$I_2 X_{e_2} = \sqrt{(I_2 Z_{e_2})^2 - (I_2 R_{e_2})^2} = \sqrt{20^2 - 10^2} = 17.32 \text{ V}$$

(*a*) Approximate voltage regulation at lagging power factor

$$= \frac{I_2 R_{e_2} \cos \phi_2 + I_2 X_{e_2} \sin \phi_2}{V_2}$$

$$= \frac{10 \times 0.8 + 17.32 \times 0.6}{200} = 0.0919 \text{ pu} = 0.0919 \times 100\% = 9.19\%$$

(*b*) For zero regulation, the power factor must be *leading*

$$\therefore \quad \frac{I_2 R_{e_2} \cos \phi_2 - I_2 X_{e_2} \sin \phi_2}{V_2} = 0$$

or $$\tan \phi_2 = \frac{I_2 R_{e_2}}{I_2 X_{e_2}} = \frac{10}{17.32} = \frac{1}{\sqrt{3}} = \tan 30°$$

$\therefore$ power factor $\cos \phi_2 = \cos 30° = 0.866$ (leading)

EXAMPLE 1.15. *A 100 kVA, single-phase, 1100/220 V, 60 Hz transformer has a high-voltage resistance of 0.1 Ω and a leakage reactance of 0.3 Ω. The low-voltage winding resistance is 0.004 Ω and the leakage reactance is 0.012 Ω. Determine :*

(a) the equivalent winding resistance and reactance referred to the high-voltage side and the low-voltage side;

(b) the equivalent resistance and equivalent reactance drops in volts and in per cent of the rated winding voltages expressed in terms of high-voltage quantities;

(c) the equivalent resistance and equivalent reactance drops in volts and in percent of the rated winding voltages in terms of low-voltage quantities ;

(d) the equivalent leakage impedances of the transformer referred to the high voltage and low-voltage sides.

SOLUTION. $\text{kVA} = \dfrac{V_1 I_1}{1000}$

$$100 = \frac{1100 \times I_1}{1000}, I_1 = 91 \text{ A}$$

Also, $\text{kVA} = \dfrac{V_2 I_2}{1000}$

$$100 = \frac{220\, I_2}{1000}, I_2 = 455 \text{ A}$$

$$R_1 = 0.1\ \Omega, X_1 = 0.3\ \Omega$$

$$R_2 = 0.004\ \Omega, X_2 = 0.012\ \Omega$$

$$a = \frac{T_1}{T_2} = \frac{V_1}{V_2} = \frac{1100}{220} = 5$$

(a) Equivalent resistance referred to high voltage side

$$R_{e_1} = R_1 + R_2\left(\frac{T_1}{T_2}\right)^2 = 0.1 + 0.004\left(\frac{1100}{220}\right)^2 = 0.2\ \Omega$$

Equivalent reactance referred to high voltage side

$$X_{e_1} = X_1 + X_2\left(\frac{T_1}{T_2}\right)^2 = 0.3 + 0.012\left(\frac{1100}{220}\right)^2 = 0.6\ \Omega$$

Equivalent resistance referred to low-voltage side

$$R_{e_2} = R_2 + R_1\left(\frac{T_2}{T_1}\right)^2 = 0.004 + 0.1\left(\frac{1}{5}\right)^2 = 0.008\ \Omega$$

Equivalent reactance referred to low-voltage side

$$X_{e_2} = X_2 + X_1\left(\frac{T_2}{T_1}\right)^2 = 0.012 + 0.3\left(\frac{1}{5}\right)^2 = 0.024\ \Omega$$

(b) Equivalent resistance drop referred to the high-voltage side

$$= I_1 R_{e_1} = 91 \times 0.2 = 18.2 \text{ V}$$

Percent equivalent resistance drop referred to the high-voltage side

$$= \frac{I_1 R_{e_1}}{V_1} \times 100 = \frac{91 \times 0.2}{1100} \times 100 = 1.65\%$$

Equivalent reactance drop referred to the high-voltage side

$$= I_1 X_{e_1} = 91 \times 0.6 = 54.6 \text{ V}$$

Percent equivalent reactance drop referred to the hv side

$$= \frac{I_1 X_{e_1}}{V_1} \times 100 = \frac{91 \times 0.6}{11000} \times 100 = 4.95\%$$

(c) Equivalent resistance drop referred to the low-voltage side

$$= I_2 R_{e_2} = 455 \times 0.008 = 3.64 \text{ V}$$

Percent equivalent resistance drop referred to the low-voltage side

$$= \frac{I_2 R_{e_2}}{V_2} \times 100 = \frac{455 \times 0.008}{220} \times 100 = 1.65\%$$

Equivalent reactance drop referred to *lv* side

$$= I_2 X_{e_2} = 455 \times 0.024 = 10.92 \text{ V}$$

Percent equivalent reactance drop referred to *lv* side

$$= \frac{I_2 X_{e_2}}{V_2} \times 100 = \frac{455 \times 0.024}{220} \times 100 = 4.95\%$$

(d) Equivalent leakage impedance referred to the high voltage side

$$Z_{e_1} = R_{e_1} + jX_{e_1} = 0.2 + j0.6 = 0.634 \angle 71.6° \ \Omega$$

Equivalent leakage impedance referred to the low-voltage side

$$Z_{e_2} = R_{e_2} + jX_{e_2} = 0.008 + j0.024 = 0.0253 \angle 71.6° \ \Omega$$

ALTERNATIVELY

$$Z_{e_2} = Z_{e_1} \left(\frac{T_2}{T_1} \right)^2 = (0.634 \angle 71.6°) \left(\frac{220}{1100} \right)^2 = 0.0253 \angle 71.6° \ \Omega$$

1.30 LOSSES IN TRANSFORMERS

The losses which occur in a transformer are (a) iron loss or core loss P_i
(b) copper loss or I^2R loss P_c

Iron loss or core loss P_i

Iron loss occurs in the magnetic core of the transformer. This loss is the sum of hysteresis loss (P_h) and eddy current loss (P_e).

$$P_i = P_h + P_e$$

The hysteresis and eddy current losses are given by

$$P_h = k_h f B_m^x$$

$$P_e = k_e f^2 B_m^2$$

where k_h = proportionality constant which depends upon the volume and quality of the core material and the units used

k_e = proportionality constant whose value depends upon the volume and resistivity of the core material, thickness of laminations and units used.

B_m = maximum flux density in the core

and f = frequency of the alternating flux

The exponent x is called Steinmetz constant. Its value varies from 1.5 to 2.5 depending upon the magnetic properties of the core material. The total core loss can be written as

$$P_i = P_h + P_e$$

$$P_i = k_h f B_m^x + k_e f^2 B_m^2 \tag{1.30.1}$$

Since the applied voltage is approximately equal to the induced voltage

$$V_1 = E_1 = 4.44\,\phi_m f T_1 = 4.44\,B_m A_i f T_1$$

$$B_m = \frac{V_1}{4.44\,A_i f T_1}$$

and
$$P_h = k_h f B_m^x = k_h f \left(\frac{V_1}{4.44\,A_i f T_1}\right)^x$$

$$= k_h \left(\frac{1}{4.44\,A_i T_1}\right)^x \cdot f \left(\frac{V_1}{f}\right)^x = K_h V_1^x f^{1-x}$$

where
$$K_h = k_h \left(\frac{1}{4.44\,A_i T_1}\right)^x$$

This relation shows that *the hysteresis loss depends upon both the applied voltage and frequency.*

Also,

$$P_e = K_e f^2 B_m^2 = K_e f^2 \left(\frac{V_1}{4.44\,A_i T_1 f}\right)^2 = K_e V_1^2$$

This relation shows that *the eddy current loss is proportional to the square of the applied voltage and is independent of frequency.*

Since, $V_1 = 4.44 B_m A_i f T_1$,

$$\boxed{V_1 \propto B_m f}$$

Therefore for any given voltage if f decreases, B_m increases. Similarly, if f increases, B_m decreases.

The total core loss can be written as

$$P_i = k_h V^x f^{1-x} + k_e V^2 \tag{1.30.2}$$

Copper loss or I^2R loss (P_C)

Copper loss is the I^2R loss which takes place in the primary and secondary windings because of the winding resistances.

Total copper loss in a transformer = primary winding copper loss + secondary winding copper loss

$$P_C = I_1^2 R_1 + I_2^2 R_2$$

Since $\quad I_1 T_1 = I_2 T_2$

$$I_1 = I_2 \frac{T_2}{T_1}$$

$$\therefore \quad P_C = I_2^2 \left(\frac{T_2}{T_1}\right)^2 R_1 + I_2^2 R_2 = I_2^2 \left[R_2 + \left(\frac{T_2}{T_1}\right)^2 R_1\right] = I_2^2 R_{e_2}$$

Thus, copper loss varies as the square of the load current.

Again,

$$P_C = I_1^2 R_1 + I_2^2 R_2 = I_1^2 R_1 + \left(I_1 \frac{T_1}{T_2}\right)^2 R_2 = I_1^2 \left[R_1 + \left(\frac{T_1}{T_2}\right)^2 R_2\right] = I_1^2 R_{e_1}$$

$$\therefore \quad \boxed{P_C = I_1^2 R_{e_1} = I_2^2 R_{e_2}} \tag{1.30.3}$$

Stray loss

Leakage flux in a transformer produces eddy currents in the conductors, tanks etc. These eddy currents produce losses.

Dielectric loss

Dielectric loss occurs in insulating materials, that is, in the transformer oil and the solid insulation of transformers. This loss is significant only in high voltage transformers.

The stray loss and dielectric loss are usually small and negligible.

1.31 SEPARATION OF HYSTERESIS AND EDDY-CURRENT LOSSES

The transformer core loss P_i has two components namely hysteresis loss P_h and eddy-current loss P_e.

$$P_i = P_h + P_e$$

$$P_i = K_h f B_m^n + K_e f^2 B_m^2 \tag{1.31.1}$$

where K_n = a constant whose value depends upon the ferromagnetic material

B_m = maximum value of the flux density

f = supply frequency

The exponent n varies in the range 1.5 to 2.5 depending upon the material. For a given B_m the hysteresis loss varies directly as the frequency and the eddy-current loss varies as the square of the frequency. That is,

constant

$$P_h \propto f \quad \text{or} \quad P_h = af$$

and

$$P_e \propto f^2 \quad \text{or} \quad P_e = bf^2$$

where a and b are constants.

$$\therefore \quad \boxed{P_i = af + bf^2} \tag{1.31.2}$$

For separation of these two losses the no-load test is performed on the transformer. However, the primary of the transformer is connected to a variable frequency and variable sinusoidal supply and the secondary is open circuited.

Now $\quad V \simeq E = 4.44 f \Phi_m T$

or $$\frac{V}{f} = 4.44\, B_m A_i T$$

For any transformer T and A_i are constants. Therefore B_m will remain constant if the test is conducted so that the ratio (V/f) is kept constant.

Dividing Eq. (1.31.2) by f, we get

$$P_i/f = bf + a \tag{1.31.3}$$

During this test, the applied voltage V and frequency f are varied together so that (V/f) is kept constant. The core loss is obtained at different frequencies. A graph of (P_i/f) versus frequency f is plotted. This graph is a straight line AB of the form $y = mx + c$, as shown in Fig. 1.25.

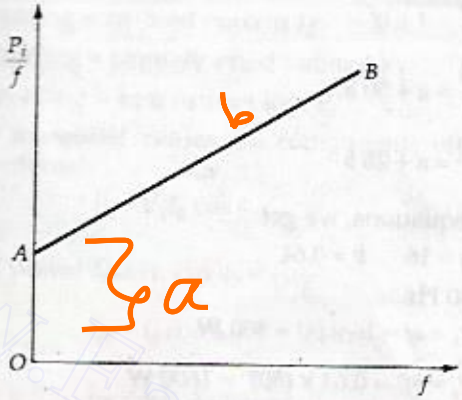


Fig. 1.25. Variation of (P_i/f) with f.

The intercept of the straight line on the vertical axis gives a and the slope of the line AB gives b. Thus, knowing the constants a and b, hysteresis and eddy-current losses can be separated.

EXAMPLE 1.16. *In a transformer, the core loss is 100 W at 40 Hz and 72 W at 30 Hz. Find the hysteresis and eddy current losses at 50 Hz.*

SOLUTION. $$\frac{P_i}{f} = a + bf$$

$$\frac{100}{40} = a + 40\,b$$

and $$\frac{72}{30} = a + 30\,b$$

Solution of these equation gives

$$a = 2.1, \qquad b = 0.01$$

Therefore, hysteresis loss at 50 Hz

$$= af = 2.1 \times 50 = 105 \text{ W}$$

Eddy-current loss at 50 Hz

$$= bf^2 = 0.01 \times (50)^2 = 25 \text{ W}$$

EXAMPLE 1.17. *At 400 V and 50 Hz the total core loss of a transformer was found to be 2400 W. When the transformer is supplied at 200 V, and 25 Hz, the core loss is 800 W. Calculate the hysteresis and eddy current loss at 400 V and 50 Hz.*

SOLUTION. $$\frac{V_1}{f_1} = \frac{400}{50} = 8$$

$$\frac{V_2}{f_2} = \frac{200}{25} = 8$$

Since $$\frac{V_1}{f_1} = \frac{V_2}{f_2} = 8$$

the flux density B_m remains constant. Hence

$$\frac{P_i}{f} = a + bf$$

$$\therefore \quad \frac{2400}{50} = a + 50\,b$$

and $$\frac{800}{5} = a + 25\,b$$

Solving these equations, we get

$$a = 16 \qquad b = 0.64$$

Therefore, at 50 Hz

$$P_h = af = 16 \times 50 = 800 \text{ W}$$

$$P_e = bf^2 = 0.64 \times (50)^2 = 1600 \text{ W}$$

1.32 OPEN-CIRCUIT AND SHORT-CIRCUIT TESTS

Open-circuit and short-circuit tests are performed to determine the circuit constants, efficiency and regulation without actually loading the transformer. These tests give more accurate results than those obtained by taking measurements on fully loaded transformers. Also, the power consumption in these tests is very small compared with the full-load output of the transformer.

1.33 OPEN-CIRCUIT TEST

Figure 1.26 shows the connection diagram for the open circuit test. The high voltage (hv) side is left open.

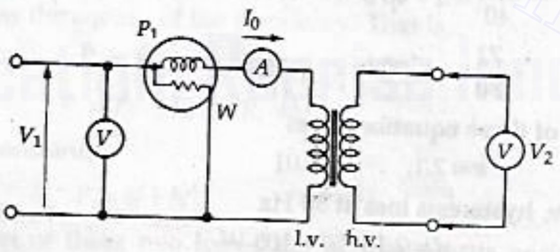


Fig. 1.26. *Open-circuit test on a transformer.*

A voltmeter V, an ammeter A, and a wattmeter W are connected in the low-voltage (lv) side (primary in our case) which is supplied at rated voltage and frequency. Thus, the voltmeter V reads the rated voltage V_1 of the primary. Since the secondary is open-circuited, a very small current I_0, called the no-load current, flows in the primary. The ammeter A, therefore, reads the no-load current I_0. The power loss in the transformer is due to core loss and a very small I^2R loss in the primary. There is no I^2R loss in the secondary since it is open and $I_2 = 0$. Since the no-load current I_0 is very small (usually 2 to 5 percent of the full-load primary current), the I^2R loss in the primary winding can be neglected. The core loss depends upon the flux. Since the rated voltage V_1 is applied, the flux set up by it will have a normal value so that normal core losses will occur. This core loss is the same at all loads. Therefore the wattmeter which is connected to measure input power reads the core loss (iron loss) P_i only. The readings of the instruments in an open-circuit test are as follows :

Ammeter reading = no-load current I_0

Voltmeter reading = primary rated voltage V_1

Wattmeter reading = iron or core loss P_i

From these measured values the components of the no-load equivalent circuit can be determined.

$$P_i = V_1 I_0 \cos \phi_0$$

The no-load power factor, $\cos \phi_0 = \dfrac{P_i}{V_1 I_0}$

$$I_W = I_0 \cos \phi_0, \; I_\mu = I_0 \sin \phi_0$$

$$R_0 = \frac{V_1}{I_W}, \; X_0 = \frac{V_1}{I_\mu}$$

1.34 SHORT-CIRCUIT TEST

In the short circuit (SC) test (Fig. 1.27), usually the low-voltage side is short-circuited by a thick conductor (or through an ammeter which may serve an additional purpose of indicating rated load current). An ammeter, a voltmeter and

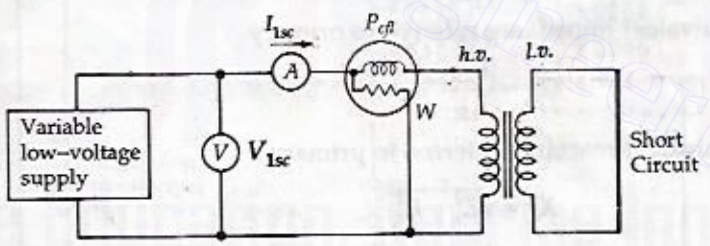


Fig. 1.27. *Short-circuit test on a transformer.*

a wattmeter are connected on the high-voltage side. The reasons for short-circuiting the lv side and taking measurements on the hv side are as follows :

(1) The rated current on hv side is lower than that on lv side This current can be safely measured with the available laboratory ammeters.

(2) Since the applied voltage is less than 5 percent of the rated voltage of the winding, greater accuracy in the reading of the voltmeter is possible when the hv side is used as the primary.

The high voltage winding is supplied at the reduced voltage from a variable voltage supply. The supply voltage is gradually increased until full-load primary current flows. When the rated full-load current flows in the primary winding rated full-load current will flow in the secondary winding by transformer action.

Readings of the ammeter, voltmeter and wattmeter are noted. The ammeter reading I_{1SC} gives the full-load primary current. The voltmeter reading V_{1SC} gives the value of the primary applied voltage when full-load currents are flowing in the primary and secondary. Since the applied voltage is low (usually about 5 to 10 percent of the normal rated supply voltage), the flux Φ produced is low. Also, since the core loss is *nearly* proportional to the square of the flux, the core loss is so small that it can be neglected. However, the windings are carrying normal full-load currents and therefore the input is supplying the normal full-load copper losses. Thus the wattmeter gives the full-load copper losses P_{cfl}. The output voltage V_2 is zero because of the short circuit. Consequently, whole of the primary voltage is used in supplying the voltage drop in the total impedance Z_{1e} referred to the primary

$$\boxed{V_{1SC} = I_{1SC} Z_{1e}}$$

If $\cos \phi_{SC}$ = *power factor at short circuit then* $P_{cfl} = V_{1SC} I_{1SC} \cos \phi_{SC}$

The readings of the instruments in a short-circuit test are as follows :

Ammeter reading = full-load primary current, I_{1SC}

Voltmeter reading = short circuit voltage V_{1SC}

Wattmeter reading = full load copper loss of the transformer P_{cfl}

From the readings of the instruments on short-circuit test, the following calculations can be made :

Equivalent resistance of the transformer referred to primary

$$R_{e_1} = \frac{P_{cfl}}{I_{1SC}^2}$$

Equivalent impedance referred to primary

$$Z_{e_1} = \frac{V_{1SC}}{I_{1SC}}$$

Equivalent reactance referred to primary

$$X_{e_1} = \sqrt{Z_{e_1}^2 - R_{e_1}^2}$$

$$\cos \phi_{sc} = \frac{R_{e_1}}{Ze_1}$$

With short-circuit test performed only on one side the equivalent circuit constants referred to other side can also be calculated as follows :

$$Z_{e_2} = Z_{e_1}\left(\frac{T_2}{T_1}\right)^2 = \frac{Z_{e_1}}{a^2}$$

$$R_{e_2} = R_{e_1}\left(\frac{T_2}{T_1}\right)^2 = \frac{R_{e_1}}{a^2}$$

$$X_{e_2} = X_{e_1}\left(\frac{T_2}{T_1}\right)^2 = \frac{X_{e_1}}{a^2}$$

1.25 BACK-TO-BACK TEST (SUMPNER'S TEST OR REGENERATIVE TEST)

In order to determine the maximum temperature rise it is necessary to conduct a full-load test on a transformer. For small transformers full-load test is conveniently possible, but for large transformers full-load test is very difficult. A suitable load to absorb full-load power of a large transformer may not be easily available. It will also be very expensive as a large amount of energy will be wasted in the load during the test. Large transformers can be tested for determining the maximum temperature rise by the back-to-back test. This test is also called the *Regenerative test* or *Sumpner's test.*

The back-to-back test on single-phase transformers requires two identical transformers. Figure 1.28, shows the circuit diagram for the back-to-back test on two identical single-phase transformers T_{r_1} and T_{r_2}. The primary windings of the two transformers are connected in parallel and supplied at rated voltage and rated frequency. A voltmeter, an ammeter and a wattmeter are connected to the input side as shown in Fig. 1.28.

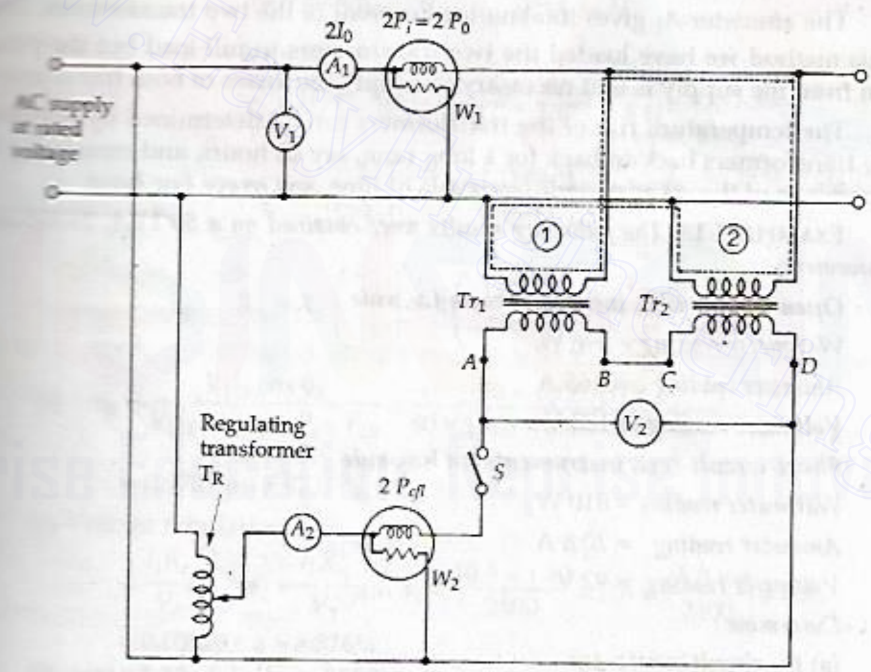


Fig. 1.28. *Back-to-back test on two identical single-phase transformers*

The secondaries are connected in series with their polarities in phase opposition, which can be checked by the voltmeter V_2. The range of this voltmeter should be double the rated voltage of either transformer secondary. In order to check that the secondaries are connected in series opposition, any two terminals (say *B* and *C*) are joined together and the voltage is measured between the remaining terminals *A* and *D*. If the voltmeter V_2 reads zero, the two secondaries are in series opposition and terminals A and D are used for test. If the voltmeter reads a value approximately equal to twice the rated secondary voltage of either transformer, then the secondaries are acting in the same direction. Then terminals A and C are joined and the terminals B and D used for the test.

If the primary circuit is now closed, the total voltage across the two secondaries in series will be zero. There will be no current in the secondary windings. The transformers will behave as if their secondary windings are open circuited. Hence, the reading of wattmeter W_1 gives the iron losses of *both* the transformers.

A small voltage is injected in the secondary circuit by a regulating transformer T_R excited by the main supply. The magnitude of the injected voltage is adjusted till the ammeter A_2 reads full-load secondary current. The secondary current produces full-load current to flow through the primary windings. This current will follow a circulatory path through the main busbars as shown dotted in Fig. 1.28. The reading of wattmeter W_1 will not be affected by this current. Thus, wattmeter W_2 gives the *full-load copper losses* of the *two* transformers.

The ammeter A_1 gives total no-load current of the two transformers. Thus, in this method we have loaded the two transformers to full load but the power taken from the supply is that necessary to supply the losses of both transformers.

The temperature rise of the transformers can be determined by operating these transformers back-to-back for a long time, say 48 hours, and measuring the temperature of the oil at periodic intervals of time, say every one hour.

EXAMPLE 1.18. *The following results were obtained on a 50 kVA, 2400/120 V transformer :*

Open-circuit test, instruments on l.v. side

Wattmeter reading = 396 W

Ammeter reading = 9.65 A

Voltmeter reading = 120 V

Short-circuit test, instruments on h.v. side

Wattmeter reading = 810 W

Ammeter reading = 20.8 A

Voltmeter reading = 92 V

Determine :

(*a*) *the circuit constants;*

(*b*) *the efficiency at full load, 0.8 power factor lagging;*

(*c*) *the approximate voltage regulation.*

SOLUTION. (*a*) *Open-circuit test*

$$P_i = 396\ \text{W},\ I_0 = 9.65\ \text{A},\ V_1 = 120\ \text{V}$$

$$P_i = V_1 I_0 \cos \phi_0 = V_1 I_W$$

$$396 = 120\, I_W,\ I_W = \frac{396}{120} = 3.3\ \text{A}$$

$$I_0^2 = I_W^2 + I_\mu^2$$

$$I_\mu = \sqrt{I_0^2 - I_W^2} = \sqrt{9.65^2 - 3.3^2} = 9.06 \text{ A}$$

$$I_W R_0 = V_1,\ R_0 = \frac{V_1}{I_W} = \frac{120}{3.3} = 36.3\ \Omega$$

$$I_\mu X_0 = V_1,\ X_0 = \frac{V_1}{I_\mu} = \frac{120}{9.06} = 13.2\ \Omega$$

Short-circuit test

$$P_{cfl} = 810\text{W},\ I_{1fl} = 20.8 \text{ A},\ V_{SC} = 92 \text{ V}$$

$$I_{1fl}^2 R_{e_1} = P_{cfl}$$

$$(20.8)^2 R_{e_1} = 810$$

$$R_{e_1} = \frac{810}{(20.8)^2} = 1.87\ \Omega$$

$$V_{sc} = I_{1fl} Z_{e_1}$$

$$92 = 20.8\, Z_{e_1},\quad Z_{e_1} = \frac{92}{20.8} = 4.42\ \Omega$$

$$R_{e_1}^2 + X_{e_1}^2 = Z_{e_1}^2$$

$$X_{e_1} = \sqrt{Z_{e_1}^2 - R_{e_1}^2} = \sqrt{4.42^2 - 1.87^2} = 4\ \Omega$$

$$R_{e_2} = R_{e_1}\left(\frac{T_2}{T_1}\right)^2 = 1.87\left(\frac{120}{2400}\right)^2 = 0.0047\ \Omega$$

$$X_{e_2} = X_{e_1}\left(\frac{T_2}{T_1}\right)^2 = 4\left(\frac{120}{2400}\right)^2 = 0.01\ \Omega$$

(b) $$\eta_{fl} = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + P_{cfl}} = \frac{50 \times 1000 \times 0.8}{50 \times 1000 \times 0.8 + 396 + 810}$$

$$= 0.9707 \text{ pu} = 97.07\%$$

(c) Voltage regulation

$$= \frac{I_1 R_{e_1}}{V_1} \cos\phi_2 + \frac{I_1 X_{e_1}}{V_1} \sin\phi_2 = \frac{20.8 \times 1.87}{2400} \times 0.8 + \frac{20.8 \times 4}{2400} \times 0.6$$

$$= 0.03376 \text{ pu} = 3.376\%$$

Example 1.19. *A 1-phase, 250/500 V transformer gave the following results :*

Open-circuit test *250 V, 1 A, 80 W on l.v. side*

Short-circuit test *20 V, 12 A , 100 W on h.v. side.*

Calculate the circuit constants and show them on an equivalent circuit.

Solution. *Open-circuit test*

$$V_1 = 250 \text{ V},\ I_0 = 1 \text{ A},\ P_i = 80 \text{ W}$$

$$P_i = V_1 I_0 \cos\phi_0$$

$$\cos \phi_0 = \frac{P_i}{V_1 I_0} = \frac{80}{250 \times 1} = 0.32$$

$$I_W = I_0 \cos \phi_0 = 1 \times 0.32 = 0.32 \text{ A}$$

$$I_\mu = I_0 \sin \phi_0 = \sqrt{I_0^2 - I_W^2} = \sqrt{1^2 - (0.32)^2} = 0.947 \text{ A}$$

$$R_0 = \frac{V_1}{I_W} = \frac{250}{0.32} = 781.25\ \Omega$$

$$X_0 = \frac{V_1}{I_\mu} = \frac{250}{0.947} = 264\ \Omega$$

Short-Circuit test

As the primary is short-circuited, therefore all the values refer to the secondary winding. The results of the short-circuit test are given in terms of the h.v. side, while the results of the open-circuit test are in terms of low-voltage side. The results obtained in short-circuit test are, therefore, converted in terms of the l.v. side.

$$V_{2sc} = 20 \text{ V}, P_{cfl} = 100 \text{ W}, I_{2sc} = 12 \text{ A}$$

$$\frac{T_1}{T_2} = \frac{V_1}{V_2} = \frac{250}{500} = 0.5$$

Voltage applied on the l.v. side

$$V_{1sc} = V_{2sc} \frac{T_1}{T_2} = 20 \times \frac{250}{500} = 10 \text{ V}$$

Primary (l.v.) full-load current

$$I_{1sc} = I_{2sc} \frac{T_2}{T_1} = 12 \times \frac{500}{250} = 24 \text{ A}$$

$$P_{cfl} = I_{1sc}^2 R_{e_1}$$

$$R_{e_1} = \frac{P_{cfl}}{I_{ISC}^2} = \frac{100}{(24)^2} = 0.1736\ \Omega$$

$$Z_{e_1} = \frac{V_{1sc}}{I_{1sc}} = \frac{10}{24} = 0.417\ \Omega$$

$$X_{e_1} = \sqrt{Z_{e_1}^2 - R_{e_1}^2} = \sqrt{(0.417)^2 - (0.1736)^2} = 0.379\ \Omega$$

The equivalent circuit is shown in Fig. 1.29.

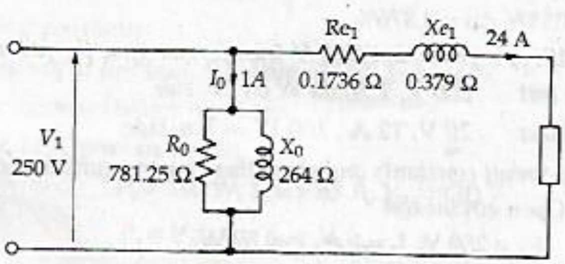


Fig. 1.29. *Equivalent circuit of Example 1.19.*

EXAMPLE 1.20. *A 50-Hz, 1-phase transformer has a turn-ratio of 6. The resistances are 0.90 Ω and 0.03 Ω, and the reactances 5 Ω and 0.13 Ω for high-voltage and low-voltage windings respectively. Find (a) the voltage to be applied to the high-voltage side to obtain full-load current of 200 A in the low-voltage winding on short-circuit, (b) the power factor on short circuit.*

SOLUTION. $\frac{T_1}{T_2} = 6, R_1 = 0.9\ \Omega, R_2 = 0.03\ \Omega$

$$X_1 = 5\ \Omega, X_2 = 0.13\ \Omega$$

$$R_{e_1} = R_1 + R_2\left(\frac{T_1}{T_2}\right)^2 = 0.9 + 0.03(6)^2 = 1.98\ \Omega$$

$$X_{e_1} = X_1 + X_2\left(\frac{T_1}{T_2}\right)^2 = 5 + 0.13(6)^2 = 9.68\ \Omega$$

$$Z_{e_1} = \sqrt{R_{e_1}^2 + X_{e_1}^2} = \sqrt{(1.98)^2 + (9.68)^2} = 9.88\ \Omega$$

$$\cos\phi_{sc} = \frac{R_{e_1}}{Z_{e_1}} = \frac{1.98}{9.88} = 0.2$$

$$I_{sc_2}T_2 = I_{sc_1}T_1$$

$$I_{sc_1} = I_{sc_2}\frac{T_2}{T_1} = 200 \times \frac{1}{6} = 33.33\ \text{A}$$

$$V_{sc} = I_{sc_1}Z_{e_1} = \frac{200}{6} \times 9.88 = 329.3\ \text{A}$$

1.26 TRANSFORMER EFFICIENCY

The ratio of the output power to the input power in a transformer is known as transformer efficiency (η).

$$\eta \triangleq \frac{\text{output power}}{\text{input power}}$$

$$= \frac{\text{output power}}{\text{output power} + \text{copper loss} + \text{iron loss}} \quad \text{pu}$$

Thus, the per unit efficiency at load current I_2 and power factor $\cos\phi_2$ is

$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + I_2^2 R_{e_2} + P_i} \quad \text{pu}$$

The per-unit efficiency at full load is

$$\eta_{fl} = \frac{V_2 I_{2fl} \cos\phi_2}{V_2 I_{2fl} \cos\phi_2 + I_{2fl}^2 R_{e_2} + P_i}$$

If $\quad S_{2fl} = (VA)_{2fl} = V_2 I_{2fl} = \text{full-load VA} = \text{rated VA}$

then $\quad \eta_{fl} = \dfrac{S_2\cos\phi_2}{S_2\cos\phi_2 + P_{cfl} + P_i}$

Since the transformer is a static device, there are no rotational losses such as windage and frictional losses in a rotating machine. In a well-designed transformer the efficiency can be as high as 99%.

1.37 CONDITION FOR MAXIMUM EFFICIENCY

The per-unit (pu) efficiency at load current I_2 is

$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + I_2^2 R_{e_2} + P_i} \tag{1.37.1}$$

$$= \frac{V_2 \cos\phi_2}{V_2 \cos\phi_2 + I_2 R_{e_2} + (P_i/I_2)} \tag{1.37.2}$$

Equation (1.37.2) shows that the efficiency varies with the load. The plot of efficiency η versus load (or load current) is shown in Fig. 1.30.

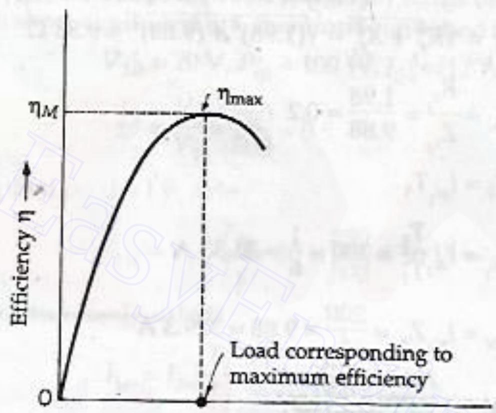


Fig. 1.30. *Efficiency versus load curve.*

It is seen that the efficiency is low at small loads and reaches a maximum value for a certain load. The efficiency then decreases as the load is increased. Sometimes it is desired to know under what conditions and at which value of the load, the transformer will operate at its maximum efficiency.

At maximum efficiency

$$\frac{d\eta}{dI_2} = 0 \text{ and } \frac{d^2\eta}{dI_2^2} < 0$$

Since V_2 and $\cos\phi_2$ are constants for a given load, the efficiency will be a maximum when the denominator $Dr(= V_2\cos\phi_2 + I_2 R_{e_2} + \frac{Pi}{I_2})$ is a minimum.

For a minimum value of the denominator Dr

$$\frac{dDr}{dI_2} = 0 \text{ and } \frac{d^2Dr}{dI_2^2} > 0$$

$$\frac{d}{dI_2} Dr = \frac{d}{dI_2}(V_2\cos\phi_2 + I_2 R_{e_2} + \frac{Pi}{I_2}) = 0 + R_{e_2} - \frac{P_i}{I_2^2}$$

For a minimum Dr,

$$R_{e_2} - \frac{P_i}{I_2^2} = 0$$

$$I_2^2 R_{e_2} = P_i \qquad (1.37.3)$$

Also,

$$\frac{d^2 Dr}{dI_2^2} = \frac{d}{dI_2}\left(R_{e_2} - \frac{P_i}{I_2^2}\right) = 0 + \frac{2Pi}{I_2^3} > 0$$

Since $\frac{d^2 Dr}{dI_2^2}$ is positive, the expression given by Eq. (1.37.3) is a condition for the minimum value of Dr, and therefore the condition for maximum value of efficiency.

Equation (1.37.3) shows that *the efficiency of a transformer for a given power factor is a maximum when the variable copper loss is equal to the constant iron (core) loss.*

1.38 CURRENT AND kVA AT MAXIMUM EFFICIENCY

Let I_{2fl} = full-load secondary current

I_{2M} = secondary current at maximum efficiency

S_{fl} = full load VA = rated VA = $V_2 I_{2fl}$

S_M = VA at maximum efficiency = $V_2 I_M$

For maximum efficiency, variable copper loss = constant iron loss

$$I_{2M}^2 R_{e_2} = P_i$$

$$I_{2M}^2 = \frac{P_i}{R_{e_2}} = \frac{I_{2fl}^2 P_i}{I_{2fl}^2 R_{e_2}}$$

$$I_{2M} = I_{2fl}\sqrt{\frac{P_i}{I_{2fl}^2 R_{e_2}}} \qquad (1.38.1)$$

$$\boxed{\text{Current at maximum efficiency} = (\text{full-load current}) \times \sqrt{\frac{\text{constant iron loss}}{\text{full-load copper loss}}}}$$

Equation (1.38.1) gives the value of current at maximum efficiency.

Multiplying both the sides of Eq. (1.38.1) by V_2, we get

$$V_2 I_{2M} = V_2 I_{2fl}\sqrt{\frac{P_i}{P_{cfl}}}$$

$$\boxed{S_M = S_{fl}\sqrt{\frac{P_i}{P_{cfl}}}} \qquad (1.38.2)$$

Equation (1.38.2) gives the value of VA at maximum efficiency.

Maximum efficiency

$$\eta_M = \frac{V_2 I_{2M} \cos\phi_2}{V_2 I_{2M} \cos\phi_2 + I_{2M}^2 R_{e_2} + P_i}$$

At maximum efficiency,

$$I_{2M}^2 R_{e_2} = P_i\,, \quad I_{2M} = m\, I_{2fl}\,; \quad m = \sqrt{\frac{P_i}{P_{cfl}}}$$

$$\eta_M = \frac{m V_2 I_{2fl} \cos\phi_2}{m V_2 I_{2fl} \cos\phi_2 + P_i + P_i}$$

$$\boxed{\eta_M = \frac{m(\text{rated VA})\cos\phi_2}{m\,(\text{rated VA})\cos\phi_2 + 2P_i}} \qquad (1.38.3)$$

1.39 EFFICIENCY CURVES OF A TRANSFORMER

We have seen that maximum efficiency of a transformer occurs at the load point where variable copper loss is equal to the fixed core loss P_i. That is

$$I_2^2 R_{e_2} = P_i \qquad (1.39.1)$$

The load current at which maximum efficiency occurs is

$$I_M = I_{2fl}\sqrt{\frac{P_i}{I_{2fl}^2 R_{e_2}}} = I_{2fl}\sqrt{\frac{Pi}{P_{cfl}}} \qquad (1.39.2)$$

Equations (1.39.1) and (1.39.2) enable us to predict the shape of the efficiency curves under various load and power factor conditions. Equation (1.39.2) shows that regardless of the power factor of the load, maximum efficiency occurs at the same load (current) value I_M as shown in Fig. 1.31. The maximum efficiency for any power factor occurs at the same load and the highest possible efficiency occurs at unity power factor.

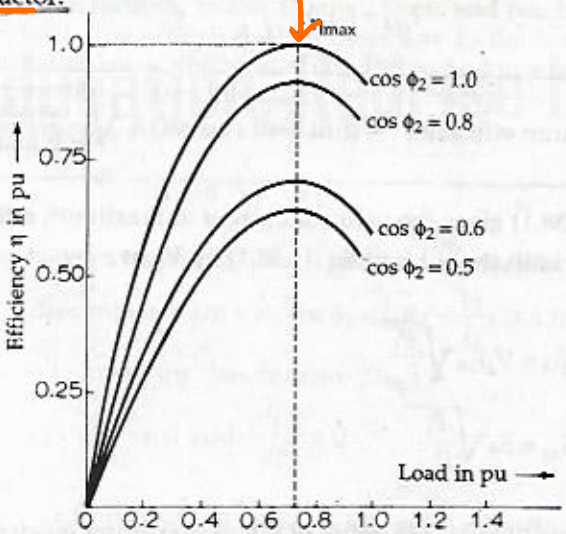


Fig. 1.31. *Effect of power factor on efficiency.*

1.40 PER-UNIT TRANSFORMER VALUES

A 2-winding transformer has three essential ratings :

Rated primary voltage $= V_{1b}$

Rated secondary voltage $= V_{2b}$

Rated voltamperes $= S_b$

Primary rated voltamperes = secondary rated voltamperes

$= S_b = (VA)_b$

Primary base current

$$I_{1b} \overset{\Delta}{=} \frac{S_b}{V_{1b}} \tag{1.40.1}$$

Secondary base current

$$I_{2b} \overset{\Delta}{=} \frac{S_b}{V_{2b}} \tag{1.40.2}$$

Primary base impedance

$$Z_{1b} \overset{\Delta}{=} \frac{V_{1b}}{I_{1b}} \tag{1.40.3}$$

Secondary base impedance

$$Z_{2b} \overset{\Delta}{=} \frac{V_{2b}}{I_{2b}} \tag{1.40.4}$$

Relationship between base quantities

By convention $\dfrac{V_{1b}}{V_{2b}} = a$ (1.40.5)

By Eqs. (1.40.1) and (1.40.2)

$$\frac{I_{1b}}{I_{2b}} = \frac{1}{a} \tag{1.40.6}$$

By Eqs. (1.40.3) and (1.40.4)

$$\frac{Z_{1b}}{Z_{2b}} = \frac{V_{1b}}{V_{2b}} \times \frac{I_{2b}}{I_{1b}} = a \times a$$

$$\therefore \quad \frac{Z_{1b}}{Z_{2b}} = a^2 \tag{1.40.7}$$

$$Z_{1b} = a^2 Z_{2b} \tag{1.40.8}$$

Similarly, the equivalent impedance reflected to the primary

$$Z_{1e} = a^2 Z_{2e} \tag{1.40.9}$$

The per-unit equivalent primary impedance may be defined as

$$\boxed{Z_{1e\,pu} = \frac{Z_{1e}}{Z_{1b}} = \frac{Z_{1e}}{a^2 Z_{2b}} = \frac{Z_{2e}}{Z_{2b}} = Z_{2e\,pu}} \tag{1.40.10}$$

Thus *the per unit equivalent impedance of a 2-winding transformer is the sa whether referred to the primary or the secondary.*

It can also be proved that *the per unit impedance referred to either side of 3-phase transformer is the same irrespective of the 3-phase connections whether they a delta/delta, star/star, delta/star or star/delta.*

In a transformer, when voltages or currents of either side are expressed i a per-unit system, they have the same per-unit values.

$$I_{1\,pu} = \frac{I_1}{I_{1b}} = \frac{I_2/a}{I_{2b}/a} = \frac{I_2}{I_{2b}} = I_{2\,pu} \quad (1.40$$

$$V_{1\,pu} = \frac{V_1}{V_{1b}} = \frac{a\,V_2}{a\,V_{2b}} = \frac{V_2}{V_{1b}} = V_{2\,pu} \quad (1.40$$

1.41 FULL-LOAD COPPER LOSS IN PER UNIT

Copper loss in per unit

$$= \frac{\text{copper loss}}{\text{base VA}}$$

$$P_{c\,pu} = \frac{I_2^2 R_{e_2}}{S_{2b}} = \frac{I_2^2 R_{e_2}}{V_{2b}\, I_{2b}}$$

At full load, $I_2 = I_{2b}$

Therefore, full-load copper loss in per unit

$$P_{c_{fl\,pu}} = \frac{I_{2b}^2 R_{e_2}}{V_{2b}\, I_{2b}} = \frac{R_{e_2}}{(V_{2b}/I_{2b})}$$

$$= \frac{R_{e_2}}{Z_{2b}} = R_{e_{2\,pu}}$$

Again, $\boxed{P_{c_{fl\,pu}} = \frac{R_{e_2}}{Z_{2b}}}$

$$= \frac{a^2 R_{e_2}}{a^2 Z_{2b}} = \frac{R_{e_1}}{Z_{1b}} = R_{e_{1\,pu}}$$

$$\therefore \quad \boxed{R_{e_{1\,pu}} = R_{e_{2\,pu}} = \text{full load copper loss in per unit}} \quad (1.41$$

Hence *the equivalent resistance in per unit is equal to the full-load copper loss i per unit.* The per unit value of the resistance is, therefore, more useful than i ohmic value to determine the transformer performance.

1.42 PER-UNIT EQUIVALENT LEAKAGE REACTANCE

Per-unit equivalent leakage reactance

$$X_{e\,pu} = \frac{X_{e_1}}{Z_{1b}} = \frac{a^2 X_{e_2}}{a^2 Z_{2b}} = \frac{X_{e_2}}{Z_{2b}} = \frac{I_{2b}\, X_{e_2}}{V_{2b}}$$

$$= \sqrt{Z_{e\,pu}^2 - R_{e\,pu}^{\;2}}$$

It is to be noted that all ohmic values (impedance, resistance, reactance) are divided by the *base impedance* to determine their per-unit values.

1.43 TRANSFORMER EFFICIENCY BY PER-UNIT QUANTITIES

The transformer efficiency at any load and any power factor $\cos\phi_2$ is given by

$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + P_i + I_2^2 R_{e_2}}$$

$$= \frac{S_2 \cos\phi_2}{S_2 \cos\phi_2 + P_i + I_2^2 R_{e_2}}$$

Dividing the numerator and denominator by the voltampere rating S_b

$$\eta = \frac{\left(\dfrac{S_2}{S_b}\right)\cos\phi_2}{\left(\dfrac{S_2}{S_b}\right)\cos\phi_2 + \dfrac{P_i}{S_b} + \left(\dfrac{I_2^2 R_{e_2}}{S_b}\right)}$$

rated values= full load values

At full load $S_b = S_2$, therefore,

$$\text{per unit efficiency at full load} = \frac{\cos\phi_2}{\cos\phi_2 + P_{i\,pu} + R_{e\,pu}} \qquad (1.43.1)$$

Maximum Transformer Efficiency

Maximum transformer efficiency occurs when

$$P_i = P_c$$

$$\therefore \quad \frac{P_i}{S_b} = \frac{P_c}{S_b}$$

$$\boxed{P_{i\,pu} = \left(\frac{S}{S_b}\right)^2 R_{e\,pu}}$$

$$\frac{S}{S_b} = \sqrt{\frac{P_{i\,pu}}{R_{e\,pu}}}$$

The ratio (S/S_b) is called the **load factor (L.F.)**.

$$\text{Load factor} \overset{\Delta}{=} \frac{S}{S_b}$$

$$\text{At full load } \frac{S}{S_b} = 1$$

$$\text{At half load } \frac{S}{S_b} = \frac{1}{2}$$

$$\therefore \quad \eta = \frac{(\text{L.F.})\cos\phi_2}{(\text{L.F.})\cos\phi_2 + P_{i\,pu} + (\text{L.F.})^2 R_{e\,pu}} \qquad (1.43.2)$$

EXAMPLE 1.21. *In a transformer if the load current is kept constant, find the power factor at which the maximum efficiency occurs.*

SOLUTION.
$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + I_2^2 R_{e_2} + P_i}$$

If the load current I_2 is constant, $I_2^2 R_{e_2}$ is constant.

$$\therefore \quad I_2^2 R_{e_2} + P_i = \text{a constant (say } k)$$

and
$$\eta = \frac{V_2 I_2 \cos\phi_2}{V_2 I_2 \cos\phi_2 + k}$$

$$\eta = \frac{1}{1 + \dfrac{k}{V_2 I_2 \cos\phi_2}}$$

The efficiency η is a maximum if the denominator $1 + \dfrac{k}{V_2 I_2 \cos\phi_2}$ is a minimum. Since $V_2 I_2$ is also a constant, the minimum value of the denominator occurs when $\cos\phi_2$ is a maximum. The maximum value of $\cos\phi_2 = 1$. Hence *for a constant load current, maximum efficiency occurs when the load power factor is unity* (that is resistive load).

EXAMPLE 1.22. *A transformer is rated at 100 kVA. At full load its copper loss is 1200 W and its iron loss is 960 W. Calculate*

(*a*) *the efficiency at full load, unity power factor,*

(*b*) *the efficiency at half load, 0.8 power factor,*

(*c*) *the efficiency at 75% full load, 0.7 power factor,*

(*d*) *the load kVA at which maximum efficiency will occur,*

(*e*) *the maximum efficiency at 0.85 power factor.*

SOLUTION.
$$S = 100 \text{ kVA} = 100 \times 10^3 \text{ VA}$$
$$P_{cfl} = 1200 \text{ W}, Pi = 960 \text{ W}$$
$$\eta = \frac{mS\cos\phi_2}{mS\cos\phi_2 + P_i + m^2 P_{cfl}}$$

where
$$m = \frac{\text{given load}}{\text{full load}}$$

(*a*) At full load $m = 1$, $\cos\phi_2 = 1$

$$\therefore \quad \eta = \frac{1 \times 100 \times 10^3 \times 1}{1 \times 100 \times 10^3 \times 1 + 960 + (1)^2 \times 1200}$$
$$= 0.9788 \text{ pu or } 97.88\%$$

(*b*) At half load $m = \frac{1}{2}$, $\cos\phi_2 = 0.8$

$$\eta = \frac{\frac{1}{2} \times 100 \times 10^3 \times 0.8}{\frac{1}{2} \times 100 \times 10^3 \times 0.8 + 960 + \frac{1}{(2)^2} \times 1200}$$
$$= 0.9694 \text{ pu} = 96.94\%$$

(ii) $\cos\phi_2 = 0.7$

At 75% full load $m = \frac{75}{100} = 0.75$

$$\eta = \frac{0.75 \times 100 \times 10^3 \times 0.7}{0.75 \times 100 \times 10^3 \times 0.7 + 960 + (0.75)^2 \times 1200}$$

$= 0.9698$ pu or 96.98%

(d) $S_M = S_{fl}\sqrt{\frac{Pi}{P_{cfl}}}$

$$= 100\sqrt{\frac{960}{1200}} = 89.44 \text{ kVA}$$

(e) Maximum efficiency

$$\eta_M = \frac{S_M \cos\phi_2}{S_M \cos\phi_2 + 2Pi} = \frac{89.44 \times 10^3 \times 0.85}{89.44 \times 10^3 \times 0.85 + 2 \times 960}$$

$= 0.9753$ pu or 97.53%

EXAMPLE 1.23. *A 100 kVA, 50 Hz, 440/11000 V, 1-phase transformer has an efficiency of 98.5% when supplying full-load current at 0.8 power factor lagging, and an efficiency of 99% when supplying half full-load current at unity power factor. Find the core losses and the copper losses corresponding to full-load current. At what value of load current will the maximum efficiency be attained ?*

SOLUTION. $S = 100 \text{ kVA} = 100 \times 10^3 \text{ VA}$

$$\eta = \frac{mS\cos\phi_2}{mS\cos\phi_2 + P_i + m^2 P_{cfl}}$$

At full load, $m = 1, \cos\phi_2 = 0.8$

$$\eta_{fl} = \frac{1 \times 100 \times 10^3 \times 0.8}{1 \times 100 \times 10^3 \times 0.8 + P_i + P_{cfl}} = 0.985$$

$$\frac{100 \times 10^3 \times 0.8}{0.985} = 100 \times 10^3 \times 0.8 + P_i + P_{cfl}$$

$$P_i + P_{cfl} = 100 \times 10^3 \times 0.8\left(\frac{1}{0.985} - 1\right)$$

$$P_i + P_{cfl} = 1218 \qquad \text{(E1.23.1)}$$

At half full load, $m = \frac{1}{2}, \cos\phi_2 = 1$

$$\eta_{1/2fl} = \frac{(\frac{1}{2}) \times 100 \times 10^3 \times 1}{\frac{1}{2} \times 100 \times 10^3 + P_i + (\frac{1}{2})^2 P_{cfl}} = 0.99$$

$$P_i + \frac{1}{4}P_{cfl} = 50 \times 10^3\left(\frac{1}{0.99} - 1\right) = 505 \qquad \text{(E1.23.2)}$$

Subtracting (E1.23.2) from (E1.23.1), we get

$$\frac{3}{4} P_{cfl} = 713, \quad P_{cfl} = 950.7 \text{ W}$$

$$\therefore \quad P_i = 1218 - 950.7 = 267.3 \text{ W}$$

Full load current on the secondary side

$$= \frac{100 \times 1000}{11000} = 9.09 \text{ A}$$

At maximum efficiency,

$$I_{2M} = I_{2fl} \sqrt{\frac{P_i}{P_c fl}} = 9.09 \times \sqrt{\frac{267.3}{950.7}} = 4.82 \text{ A}$$

EXAMPLE 1.24. *A single-phase transformer working at unity power factor has an efficiency of 90% at both half load and at the full-load of 500 W. Determine the efficiency at 75% full load and the maximum efficiency.*

SOLUTION.
$$\eta_{fl} = \frac{V_2 I_2 \cos\phi}{V_2 I_2 \cos\phi + P_i + P_{cfl}}$$

$$0.9 = \frac{500}{500 + Pi + P_{cfl}} \qquad \text{(E1.24.1)}$$

$$\eta_{1/2fl} = \frac{V_2\left(\frac{I_2}{2}\right)\cos\phi}{V_2\left(\frac{I^2}{2}\right)\cos\phi + P_i + \left(\frac{1}{2}\right)^2 P_{cfl}}$$

$$0.9 = \frac{250}{250 + P_i + \frac{1}{4} P_c fl} \qquad \text{(E1.24.2)}$$

From Eqs. (E1.24.1) and (E1.24.2)

$$500 + P_i + P_{cfl} = \frac{500}{0.9}$$

$$250 + P_i + \frac{1}{4} P_{cfl} = \frac{250}{0.9}$$

Solution of these equations gives

$$P_{cfl} = 37.04 \text{ W}$$

$$P_i = \frac{1}{2} P_{cfl} = \frac{1}{2} \times 37.04 = 18.52 \text{ W}$$

Efficiency at 75% full load

$$= \frac{500 \times 3/4}{500 \times \frac{3}{4} + P_i + \left(\frac{3}{4}\right)^2 P_{cfl}}$$

$$= \frac{375}{375 + 1.52 + \frac{9}{16} \times 37.04} = 0.905 \text{ pu}$$

Output at maximum efficiency $= 500\sqrt{\dfrac{18.52}{37.04}} = 353.55$ W

At maximum efficiency $P_C = P_i$

$\therefore$ maximum efficiency $= \dfrac{353.55}{353.55 + 18.52 + 18.52} = 0.9051$ pu

EXAMPLE 1.25. *When a 100 kVA, single-phase transformer was tested, the following results were obtained :*

On open circuit the power consumed was 1300 W and on short circuit at full-load current the power consumed was 1200 W. Calculate the efficiency of transformer on full load and half-full load when working at unity power factor.

SOLUTION. $P_i = 1300$ W, $P_{cfl} = 1200$ W, $\cos\phi = 1$

$$\eta_{fl} = \frac{\text{output}}{\text{output} + \text{losses}} = \frac{V_2 I_2 \cos\phi}{V_2 I_2 \cos\phi + P_i + P_{cfl}}$$

$$= \frac{100 \times 1000 \times 1}{100 \times 1000 \times 1 + 1300 + 1200} = 0.9756 \text{ pu}$$

$$\eta_{1/2fl} = \frac{V_2\left(\dfrac{I_2}{2}\right)\cos\phi}{V_2\left(\dfrac{I_2}{2}\right)\cos\phi + P_i + \left(\dfrac{1}{2}\right)^2 P_{cfl}}$$

$$= \frac{50 \times 1000}{50 \times 1000 + 1300 + \dfrac{1}{4} \times 1200} = 0.969 \text{ pu}$$

EXAMPLE 1.26. *The maximum efficiency of a 100 kVA, 1-phase transformer is 98% and occurs at 80% of full load at 0.8 power factor lagging. If the leakage impedance of the transformer is 5%, find the voltage regulation at full load.*

SOLUTION.
$$\eta_{max} = \frac{mS\cos\phi_2}{mS\cos\phi_2 + 2P_i}$$

$$0.98 = \frac{0.8 \times 100 \times 1000 \times 0.8}{0.8 \times 100 \times 1000 \times 0.8 + 2P_i}$$

$$0.98 = \frac{64000}{64000 + 2P_i}$$

$$64000 + 2P_i = \frac{64000}{0.98}$$

$$P_i = \frac{1}{2} \times 64000\left(\frac{1}{0.98} - 1\right) = 653 \text{ W}$$

At maximum efficiency

$$m = \sqrt{\frac{P_i}{P_{cfl}}}$$

$$P_{cfl} = \frac{P_i}{m^2} = \frac{653}{(0.8)^2} = 1020 \text{ W}$$

$$R_{epu} = \frac{P_{cfl}}{S} = \frac{1020}{100 \times 1000} = 0.0102$$

$$Z_{epu} = \frac{5}{100} = 0.05$$

$$Z_{epu}^2 = R_{epu}^2 + X_{epu}^2$$

$$X_{epu} = \sqrt{Z_{epu}^2 - R_{epu}^2}$$

$$= \sqrt{(0.5)^2 - (0.0102)^2} = 0.04895$$

Voltage regulation for lagging p.f.

$$= v_r \cos \phi_r + v_x \sin \phi_r = R_{epu} \cos \phi_r + X_{epu} \sin \phi_r$$

$$= 0.0102 \times 0.8 + 0.04895 \times 0.6$$

$$= 0.03753 \text{ pu} \quad \text{or} \quad 3.753\%$$

Example 1.27. *A 20 kVA, single-phase transformer of 1100/220 V has an iron loss of 175 W ; the resistance of the primary winding is 0.25 Ω and that of secondary is 0.012 Ω. The corresponding leakage reactances are 1.1 Ω and 0.055 Ω respectively. Calculate the percentage resistance and reactance drops at full load and power factor 0.8 lagging. At what percentage of full load will the efficiency be maximum ?*

Solution. $R_1 = 0.25\ \Omega, \quad R_2 = 0.012\ \Omega, \quad X_1 = 1.1\ \Omega, \quad X_2 = 0.055\ \Omega$

$$\frac{T_1}{T_2} = \frac{V_1}{V_2} = \frac{1100}{220} = 5$$

$$R_{e_2} = R_2 + R_1 \left(\frac{T_2}{T_1}\right)^2 = 0.012 + 0.25 \left(\frac{1}{5}\right)^2 = 0.022\ \Omega$$

$$X_{e_2} = X_2 + X_1 \left(\frac{T_2}{T_1}\right)^2 = 0.055 + 1.1 \left(\frac{1}{5}\right)^2 = 0.099\ \Omega$$

Secondary full-load current

$$I_2 = \frac{20 \times 1000}{220} = 90.91\ \Omega$$

Percentage resistive drop

$$v_r = \frac{I_2 R_{e_2}}{V_2} \times 100 = \frac{90.91 \times 0.022}{220} \times 100 = 0.91\ \%$$

Percentage reactance drop

$$v_x = \frac{I_2 X_{e_2}}{V_2} \times 100 = \frac{90.91 \times 0.099}{220} \times 100 = 4.09\%$$

Full-load copper loss $= I_2^2 R_{e_2} = (90.91)^2 \times 0.022 = 181.8$ W

At maximum efficiency $\dfrac{I_M}{I_{2fl}} = \sqrt{\dfrac{P_i}{P_{cfl}}} = \sqrt{\dfrac{175}{181.8}} = 0.981 = 98.1\%$

Therefore at 98.1 percent of full load the efficiency will be maximum.

EXAMPLE 1.28. *In a 25 kVA, 2000/200 V transformer the iron and copper losses are 350 W and 400 W respectively. Calculate the values of iron and copper losses which will give maximum efficiency and also calculate the value of maximum efficiency.*

SOLUTION. Let $\cos\phi_2 = 1$

At maximum efficiency,

constant iron loss = variable copper loss

$$P_i = P_c\,, \quad P_i + P_c = 2\,P_i\,, \quad P_i = 350$$

$$\eta_{max} = \frac{V_2\, I_2 \cos\phi_2}{V_2\, I_2 \cos\phi_2 + P_i + P_i}$$

$$= \frac{25 \times 1000 \times 1}{25 \times 1000 + 2 \times 350} = 0.973 \text{ pu} = 97.3\ \%$$

EXAMPLE 1.29. *The maximum efficiency of a 500 kVA, 3300/500 V, 50 Hz, 1-phase transformer is 0.97 pu and occurs at 75% full load and unity power factor. If the leakage impedance is 10%, calculate the voltage regulation at full load, power factor 0.8 lagging.*

SOLUTION. $V_2 I_2 = 500 \times 10^3$, $\quad m = 0.75\,,\quad \cos\phi_2 = 1.0$

At maximum efficiency $P_i = P_c$

$$\therefore \qquad \eta_{\max} = \frac{m V_2 I_2 \cos\phi_2}{m V_2 I_2 \cos\phi_2 + 2P_i}$$

$$0.97 = \frac{0.75 \times 500 \times 10^3 \times 1}{0.75 \times 500 \times 10^3 \times 1 + 2P_i}$$

$$P_i = 5799 \text{ W}$$

At maximum efficiency

$$m = \sqrt{\frac{P_i}{P_{cfl}}}$$

$$0.75 = \sqrt{\frac{5799}{P_{cfl}}}\,, P_{cfl} = 10309 \text{ W}$$

$$R_{epu} = \frac{P_{cfl}}{V_2 I_2} = \frac{10309}{50 \times 1000} = 0.02061$$

$$Z_{epu} = \frac{10}{100} = 0.1$$

$$Z_{epu}^2 = R_{epu}^2 + X_{epu}^2$$

$$X_{epu} = \sqrt{Z_{epu}^2 - R_{epu}^2} = \sqrt{(0.1)^2 - (0.02061)^2} = 0.09785$$

Per-unit voltage regulation

$$= R_{epu} \cos\phi_2 + X_{epu} \sin\phi_2 = 0.02061 \times 0.8 + 0.09785 \times 0.6$$

$$= 0.0752$$

Percentage voltage regulation $= 0.0752 \times 100 = 7.52\%$

EXAMPLE 1.30. *Open-circuit and short-circuit tests on a 5 kVA, 220/400 V, 50 Hz single-phase transformer gave the following results :*

O.C. test 220 V, 2 A, 100 W (l.v. side)

S.C. test 40 V, 11.4 A, 200 W (h.v. side)

Determine the efficiency and approximate regulation of the transformer at full load 0.9 power factor lagging.

SOLUTION. From O.C. test, core loss $P_i = 100$ W

From S.C. test on h.v. side

$$I_{2sc} = 11.4 \text{ A}$$

Copper loss at short-circuit current

$$P_{2sc} = I_{2SC}^2 R_{e_2} = 200 \text{ W}$$

$$\therefore \quad R_{e_2} = \frac{P_{2Sc}}{I_{2sc}^2} = \frac{200}{(11.4)^2} = 1.54\ \Omega$$

$$kVA = \frac{V_2 I_{2_{fl}}}{1000}$$

$$5 = \frac{400 \times I_{2fl}}{1000}$$

$$I_{2fl} = \frac{5 \times 1000}{400} = 12.5 \text{ A}$$

Copper loss at full load

$$P_{cfl} = I_{2fl}^2 R_{e_2} = (12.5)^2 \times 1.54 = 240.6 \text{ W}$$

Efficiency at full load

$$= \frac{\text{full-load output}}{\text{full-load output + f.l. copper loss + iron loss}}$$

$$= \frac{V_2 I_2 \cos\phi}{V_2 I_2 \cos\phi + P_{cfl} + P_i} = \frac{5 \times 1000 \times 0.9}{5 \times 1000 \times 0.9 + 240.6 + 100}$$

$$= 0.9296 \text{ pu} \quad \text{or} \quad 92.96\%$$

From short-circuit test, $V_{2sc} = 40$ V

$$V_{2sc} = I_{2sc} Z_{e_2}$$

$$Z_{e_2} = \frac{V_{2sc}}{I_{2sc}} = \frac{40}{11.4} = 3.5\ \Omega$$

$$R_{e_2}^2 + X_{e_2}^2 = Z_{e_2}^2$$

$$X_{e_2} = \sqrt{Z_{e_2}^2 - R_{e_2}^2} = \sqrt{3.5^2 - 1.54^2} = 3.15\ \Omega$$

$$\cos\phi_2 = 0.9, \ \sin\phi_2 = 0.4359$$

Approximate voltage regulation

$$= \frac{I_2}{V_2} [R_{e_2} \cos\phi + X_{e_2} \sin\phi] = \frac{12.5}{400} (1.54 \times 0.9 + 3.15 \times 0.4359)$$

$$= 0.0862 \text{ pu} \quad \text{or} \quad 8.62\ \%$$

Example 1.31. *Two similar 250 kVA single-phase transformers gave the following results when tested by back-to-back method :*

Mains wattmeter, $W_1 = 5.0$ *kW*

Primary series circuit wattmeter, $W_2 = 7.5$ *kW (at full-load current).*

Find out the individual transformer efficiency at 75% full load and 0.8 p.f. lead.

Solution. Iron losses for both transformers

$$= \text{reading of wattmeter } W_1 = 5.0 \text{ kW}$$

Iron loss for one transformer $P_i = \frac{5}{2} = 2.5$ kW

Full-load copper losses for both transformers

$$= \text{reading of primary series circuit wattmeter } W_2 = 7.5 \text{ kW}$$

Full-load copper per loss for one transformer $P_{c_{fl}} = \frac{7.5}{2} = 3.75$ kW

Copper loss of one transformer at 75% full load

$$= (0.75)^2 P_{c_{fl}}$$

$$= (0.75)^2 \times 3.75 = 2.109 \text{ kW}$$

Output of each transformer at 75% full load and 0.8 power factor

$$= 75\% \text{ of kVA at full load} \times \text{power factor}$$

$$= \left(\frac{75}{100} \times 250\right) \times 0.8 = 150 \text{ kW}$$

Efficiency at 75% full load

$$= \frac{\text{output at 75\% full load}}{\text{output at 75\% full load} + P_i + (0.75)^2 P_{c_{fl}}}$$

$$= \frac{150}{150 + 2.5 + 2.109} = 0.9702 \, pu = 97.02\%$$

ALL-DAY (OR ENERGY) EFFICIENCY

The primary of a distribution transformer is connected to the line for 24 hours a day. Thus the core losses occur for the whole 24 hours whereas copper losses occur only when the transformer is on load. Distribution transformers operate well below the rated power output for most of the time. It is therefore necessary to design a distribution transformer for maximum efficiency occurring at the average output power. The performance of a distribution transformer is more appropriately represented by all-day or energy efficiency. **Energy efficiency** *of a transformer is defined as the ratio of total energy output for a certain period to the total energy input for the same period.* The energy efficiency can be calculated for any specified period. When the energy efficiency is calculated for a day of 24 hours it is called the all-day efficiency. **All-day efficiency** *is defined as the ratio of the energy output to the energy input taken over a 24-hour period.*

$$\eta_{AD} = \frac{\text{energy output over 24 hours}}{\text{energy input over 24 hours}}$$

$$= \frac{\text{energy output over 24 hours}}{\text{energy output over 24 hours} + \text{energy losses over 24 hours}}.$$

If the load cycle of the transformer is known, the all-day efficiency ... determined.

EXAMPLE 1.32. *A 2300/230 V, 500 kVA, 50 Hz distribution transformer ... loss of 1600 W at rated voltage and copper loss 7.5 kW at full load. During the ... loaded as follows :*

% load	0%	20%	50%	80%	100%	[illegible]
Power factor		0.7 lag	0.8 lag	0.9 lag	1	[illegible]
Hours	2	4	4	5	7	[illegible]

Determine the all-day efficiency of the transformer.

SOLUTION. Energy output = kVA × cos φ × hours kWh. Total energy ... over the 24 hour period is given in the following table :

% rated load	*p.f.*	*kVA cos φ*	*kW*	*hours*	*output energy ...*
20	0.7	$0.2 \times 500 \times 0.7$	70	4	[illegible]
50	0.8	$0.5 \times 500 \times 0.8$	200	4	[illegible]
80	0.9	$0.8 \times 500 \times 0.9$	360	5	[illegible]
100	1.0	$1.0 \times 500 \times 1.0$	500	7	[illegible]
125	0.85	$1.25 \times 500 \times 0.85$	531.25	2	[illegible]
					[illegible]

∴ total energy output over 24 hour period (excluding 2 hours at no load)

$$W_{out} = 7442.5 \text{ kWh}$$

Total energy loss in the core for 24 hours including 2 hours at no load

$$W_i = P_i \times t = \frac{1600}{1000} \times 24 = 38.4 \text{ kWh}$$

Copper loss at rated load = 7.5 kW

Copper loss at any other load = m^2 × copper loss at rated load.

where $m = \dfrac{\text{given load}}{\text{full load}}$

The various energy losses in the winding of the transformer can be ... lated as given in the following table :

% rated load	*m*	*Copper loss $m^2 P_{cfl}$*	*hours (h)*	*Energy loss in the winding ($m^2 P_{cfl}$ ...)*
20	0.2	$(0.2)^2 \times 7.5$	4	1.2
50	0.5	$(0.5)^2 \times 7.5$	4	7.5
80	0.8	$(0.8)^2 \times 7.5$	5	24.0
100	1.0	$(1.0)^2 \times 7.5$	7	52.5
125	1.25	$(1.25)^2 \times 7.5$	2	23.44
				108.64 kWh

Total energy loss in the transformer winding for 24 hours (excluding 2 hours at no load)

$$W_c = 108.64 \text{ kWh}$$

Total energy loss in 24 hours $= W_i + W_C = 38.4 + 108.64 = 147.04$ kWh

Total energy output in 24 hours, $W_{out} = 7442.5$ kWh

All-day efficiency, $\eta_{AD} = \dfrac{W_{out}}{W_{out} + W_i + W_c} = \dfrac{7442.5}{7442.5 + 147.04} = 0.9806$ pu

$= 98.06$ %

1.45 DISTRIBUTION TRANSFORMERS

Transformers used to step down the distribution voltage to a standard service voltage or from transmission voltage to distribution voltage are known as distribution transformers. They are kept in operation all the 24 hours a day whether they are carrying any load or not. They have a good voltage regulation and are designed for a small value of leakage reactance.

1.46 POWER TRANSFORMERS

Power transformers are used in generating stations or substations at each end of a power transmission line for stepping up or stepping down the voltage. They are put in operation during load periods and are disconnected during light periods. They are designed to have maximum efficiency at or near full load. Power transformers are designed to have considerably greater leakage reactance. For power transformers the voltage regulation is less important than the current limiting effect of higher leakage reactance.

1.47 APPLICATION OF TRANSFORMERS

Transformers are used in a number of applications :

(a) To change the level of voltage and current in electric power systems.

(b) As impedance-matching device for maximum power transfer in low-power electronic and control circuits.

(c) As a coupling device.

(d) To isolate one circuit from another, since primary and secondary are not interconnected.

(e) To measure voltage and currents ; these are known as *instrument transformers*.

(f) Converting hvac to hvdc in combined ac/dc power systems.

Transformers are extensively used in ac power systems because of the following reasons :

(1) Electric energy can be generated at the most economic level (11 kV – 33 kV).

(2) Stepping of the generated voltage to high voltage, extra high voltage EHV (voltage above 230 kV), or to even ultra high voltage UHV (750 kV and above) to suit the power transmission requirement to minimise losses and increase transmission capacity of lines.

(3) The transmission voltage is stepped down in many stages for distribution and utilisation of domestic, commercial and industrial consumers.

EXERCISES

1.1 Describe the operation of a single-phase transformer, explaining clearly the functions of the different parts. Why are the cores laminated ?

1.2 Explain briefly the action of a transformer and show that the voltage ratio of the primary and secondary windings is the same as their turns ratio.

1.3 Derive an expression for the induced e.m.f. of a transformer. A 3000/200 V, 50 Hz, single-phase transformer is built on a core having an effective cross-sectional area of 150 cm^2 and has 80 turns in the low-voltage winding. Calculate (*a*) the value of the maximum flux density in the core and (*b*) the number of turns in the high-voltage winding. [(*a*) 0.75 T, (*b*) 1200]

1.4 A 3300/230 V, 50 Hz, single-phase transformer is to be worked at a maximum flux density of 1.2 T in the core. The effective cross-sectional area of the core is 150 cm^2. Calculate the suitable values of primary and secondary turns. [830, 58]

1.5 A 100 kVA, 6600/440 V, 50 Hz single-phase transformer has 80 turns on the low-voltage winding. Calculate (*a*) the maximum flux in the core, (*b*) the number of turns on the high-voltage winding, (*c*) the current in each winding. [24.8 mWb, 1200 turns, 15.1 A, 227 A]

1.6 A 30 kVA single-phase transformer has 500 primary turns and 30 secondary turns. The primary is connected to a 3300 V, 50 Hz supply. Calculate (*a*) the maximum flux in the core, (*b*) the secondary e.m.f., (*c*) the primary and secondary currents. [29.7 mWb, 198 V ; 9.09 A, 151.5 A]

1.7 A single-phase transformer has 550 primary turns and 40 secondary turns. The primary is connected to a 3300 V a.c. supply. Neglecting losses, calculate (*a*) the secondary voltage, and (*b*) the primary current when the secondary current is 200 A. [240 V, 14.6 A]

1.8 A single-phase 240/20 V, 50 Hz transformer has the secondary full-load current of 180 A. It has 45 turns on its secondary. Calculate (*a*) the voltage per turn, (*b*) the number of primary turns, (*c*) the full-load primary current, and (*d*) the kVA output of the transformer. [(*a*) 0.444, (*b*) 540, (*c*) 15 A, (*d*) 3.6 kVA]

1.9 A transformer has its primary winding connected to mains whose voltage varies according to a sine law, the frequency being 50 Hz. The secondary coil has 50 turns and gives 100 V on open circuit. The cross-section of the core is 125 cm^2. Determine the maximum value of the flux density in the core. [0.7207 T]

1.10 A 6600/440 V, 50 Hz, single-phase transformer is built on a core having an effective cross-sectional area of 320 cm^2 and has 80 turns in the low-voltage winding. Calculate : (*a*) the value of the maximum flux density in the core ; (*b*) the number of turns in the high-voltage winding. [(*a*) 0.7742 T ; (*b*) 1200]

1.11 A single-phase, 50 Hz, core type transformer has square core of 20 cm side. The permissible maximum flux density is 1 T. Calculate the number of turns per limb on the high-voltage and low-voltage sides for a 3000/220 V ratio. [191, 14]

1.12 A single-phase transformer has a no-load voltage ratio of 400/33300 V. The low-voltage winding has 80 turns and the net cross-sectional area of the core is 200 cm^2. The frequency of the applied voltage is 50 Hz. Calculate the maximum value of the flux density and the number of turns on the secondary. [1.126 T, 6660]

1.13 A single-phase transformer has 400 primary and 1000 secondary turns. The net cross-sectional area of the core is 60 cm^2. If the primary winding be connected to a 50 Hz supply at 500 V, calculate

(*a*) the peak value of the flux density in the core, and (*b*) the voltage induced in the secondary winding. [(*a*) 0.94 T ; (*b*) 1250 V] [illegible] Derive expressions for the r.m.s. values of the induced voltages in the two windings of a single-phase transformer connected to a sinusoidal supply.

A 500/250 V, 50 Hz, single-phase transformer is to be worked at a maximum flux density of 1.2 T in the core. The effective cross-sectional area of the core is 90 cm^2. Calculate the suitable values of the primary and secondary turns. [208, 104]

[illegible] The required no-load ratio in a single-phase, 50 Hz, core-type transformer is [illegible]/250 V. Find the number of turns in each winding if the flux is to be about [illegible] Wb. [480, 20]

[illegible] The primary winding of a step-down transformer takes a current of 22 A at 3300 V when working at full load. If the transformation ratio is 15 : 1, calculate the secondary voltage and current. [220 V, 330 A]

[illegible] A 500 kVA transformer has a total loss of 4.5 kW on short circuit and a total loss of 2.5 kW on open circuit. Determine the efficiency at 0.7 power factor. [99.8%]

[illegible] A 150 kVA transformer has an iron loss of 700 W and a full-load copper loss of 1800 W. Calculate the efficiency at full load, 0.8 power factor lagging. [97.9%]

[illegible] The efficiency of a 20 kVA, 2500/250 V, single-phase transformer at unity power factor is 98% at rated load and also at half rated load. Determine : (*a*) transformer iron loss ; (*b*) full-load copper loss ; (*c*) per-unit value of the equivalent resistance of the transformer. [(*a*) 136 W ; (*b*) 272 W ; (*c*) 0.0136]

[illegible] A 100 kVA, 2000/200 V, 50 Hz distribution transformer has core loss of 500 W at rated voltage and copper loss of 1200 W at full load. It has the following load cycle :

% load	0%	50%	75%	100%	110%
Power factor	–	1	0.8 lag	0.85 lag	1.0
Hours	3	6	8	5	2

Determine the all-day efficiency of the transformer. [0.9806 p.u.]

[illegible] A 40 kVA single-phase transformer has iron losses of 800 W and copper loss of 1140 W when supplying its full load at unity power factor. Calculate the efficiency of the transformer at unity power factor at full load and half load. [0.9537, 0.9485]

[illegible] The full load copper and iron losses of a 15 kVA single-phase transformer are 320 W and 200 W respectively. Calculate the efficiency of the transformer on (*a*) full load, (*b*) half load, when the load power factor is 0.8 lagging in each case. [(*a*) 0.9585 (*b*) 0.9554]

[illegible] A single-phase transformer working at unity power-factor has an efficiency of 0.9 p.u. both at half load and at the full load of 500 W. Determine the efficiency at 75 per cent of full load. [0.905 p.u.]

[illegible] A 150 kVA transformer has an iron loss of 700 W and a full-load copper loss of 1800 W. Calculate the efficiency at full load at 0.8 p.f. [0.98 p.u.]

[illegible] Calculate the voltage regulation at 0.8 lagging power factor for a transformer which has an equivalent resistance of 2 per cent and an equivalent resistance of 2 per cent and an equivalent leakage reactance of 4 per cent. [4%]

[illegible] Describe the back-to-back test for determining the regulation and efficiency of a pair of similar transformers, giving the circuit diagram, and indicating what readings will be necessary. What are the limitations of this test ?

[illegible] Two similar 200 kVA, 1-phase transformers gave the following results when tested by back-to-back method : W_1 in the supply line, 4 kW ; W_2 in the primary series circuit, when full-load current circulated through the secondaries, 6 kW. Calculate the efficiency of each transformer. [97.56%]

1.28 An 800 kVA transformer at normal voltage and frequency requires an input of 7.5 kW on open circuit. At reduced voltage and full-load current it requires 1.42 kW input when the secondary is short circuited. Calculate the all-day efficiency if the transformer operates on the following duty cycle :

6 hours	500 kW	0.8 p.f.
4 hours	700 kW	0.9 p.f.
4 hours	300 kW	0.95 p.f.
10 hours	no load	

[95.86%]

1.29 What are the two general types of transformers ? Why is the low-voltage winding placed near the core ? What will be the output of transformer if it is operated on dc supply ?

1.30 Why the primary of the transformer draws current from the mains when the secondary is not carrying any load (open circuit) ?

1.31 Develop the exact equivalent circuit of a 1-phase transformer. From this derive the approximate and simplified equivalent circuits of the transformer. State the various assumptions made.

1.32 Draw the approximate model of a transformer. Also draw phasor diagram on load.

1.33 In what way is the approximate equivalent circuit of a transformer different from the exact equivalent circuit ? How does the approximate circuit simplify the calculations ?

1.34 Develop the phasor diagram of a single-phase transformer under load condition. Assume lagging power factor load.

1.35 Define voltage regulation of a transformer. For which type of load the voltage regulation is negative ? Derive the expression using the equivalent circuit.

1.36 Define voltage regulation of a transformer and derive conditions for (*a*) zero regulation, (*b*) maximum regulation.

Also draw the curve of variation of voltage regulation with power factor.

1.37 Describe the various losses in a transformer. Explain how each loss varies with the load current, supply voltage and frequency.

1.38 Why is the short-circuit test is performed on the hv side of a transformer ? Why is the core loss almost negligible in this test ?

1.39 Explain regenerative test on transformer. How can it be used for measurement of efficiency ?

1.40 Describe the tests on a 1-phase transformer that gives its ohmic losses and core losses. Give the determination of the equivalent circuit parameters which can be determined from these tests.

1.41 Derive the condition for maximum efficiency for a single-phase transformer.

1.42 State and prove the condition for maximum efficiency of a transformer.

1.43 In a transformer if the load current is kept constant, find the power-factor at which the maximum efficiency occurs.

1.44 Derive an expression for computing per-unit voltage regulation of a transformer for lagging power-factor load.

1.45 Explain Sumpner's test for testing two single-phase transformers. Also explain why this is beneficial for finding efficiency of transformers.

1.46 Define power efficiency and all-day efficiency of a transformer. Obtain the condition for maximum power efficiency of a single-phase transformer.

1.47 Distinguish between distribution and power transformers.

2

Transformer — II

2.1 SINGLE-PHASE AUTO TRANSFORMER

A *single-phase autotransformer* is a one-winding transformer in which a part of the winding is common to both high-voltage and low-voltage sides.

Consider a single winding *abc* of Fig. 2.1. The terminals *a* and *c* are the high-voltage terminals. The low-voltage terminals are *b* and *c* where *b* is a suitable tapping point. The portion *bc* of the full winding *abc* is common to both high-voltage and low-voltage sides. The winding *bc* is called the ***common winding*** and the smaller winding *ab* is called the ***series winding*** because it is connected in series with the common winding.

A *step-down autotransformer* is one in which the primary voltage is greater than the secondary voltage. The source voltage V_H is applied to the full winding *abc* and the load is connected across the secondary terminals *bc*. This arrangement is called the *step-down autotransformer* as shown in Fig. 2.1.

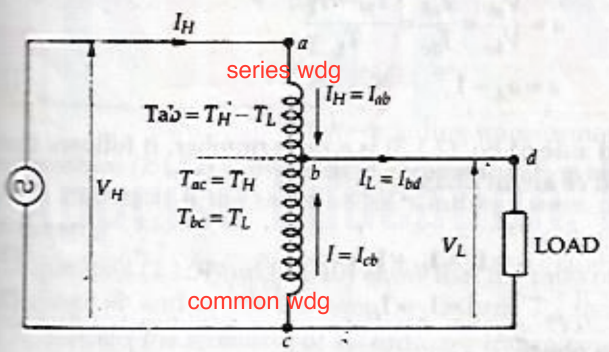


Fig. 2.1. *Stepdown autotransformer.*

Since the transformer windings are physically connected, a different terminology is used for the autotransformer than for other types of transformers. Let

$T_H = T_{ac} =$ number of turns of full winding *abc*

$=$ number of turns of *hv* side

$T_L = T_{bc} =$ number of turns of the common winding *bc*

$=$ number of turns of the *lv* side

$T_{ab} = T_H - T_L =$ number of turns of the series winding *ab*

V_H = input voltage on the *hv* side

V_L = output voltage on the *lv* side

I_H = input current in the *hv* side

I_L = output current in the *lv* side

Current in the series winding $= I_{ab} = I_H$

Current in the common winding $bc = I_{cb} = I$

In an autotransformer there are two voltage ratios namely circuit voltage ratio and winding voltage ratio. The ***circuit-voltage ratio***

$$\boxed{\frac{V_H}{V_L} = \frac{T_H}{T_L} = a_A} \tag{2.1.1}$$

The quantity a_A is called the ***transformation ratio*** of the autotransformer. It is seen from Eq. (2.1.1) that a_A is always greater than 1.

When the load is connected across the secondary terminals a current I flows in the common winding *bc*. It has a tendency to reduce the main flux but the primary current I_H increases to such a value that the mmf in winding *ab* neutralizes the mmf in winding *bc*. That is,

$$\boxed{\mathrm{I}_{ab}\, T_{ab} = \mathrm{I}_{bc}\, T_{bc}}$$

or

$$\mathrm{I}_H (T_H - T_L) = \mathrm{I}\, T_L \tag{2.1.2}$$

$$\frac{\mathrm{I}}{\mathrm{I}_H} = \frac{T_H - T_L}{T_L} = \frac{T_H}{T_L} - 1 = a_A - 1 \tag{2.1.3}$$

The winding-voltage ratio is

$$\boxed{a = \frac{V_{ab}}{V_{bc}}} = \frac{T_{ab}}{T_{bc}} = \frac{T_H - T_L}{T_L}$$

or

$$\boxed{a = a_A - 1} \tag{2.1.4}$$

Since the right-hand side of Eq. (2.1.3) is a pure number, it follows that the current in windings *ab* and *cb* are in phase.

By KCL at point *b*

$$\mathrm{I}_L = \mathrm{I}_H + \mathrm{I} \tag{2.1.5}$$

$\therefore$

$$\mathrm{I} = \mathrm{I}_L - \mathrm{I}_H \tag{2.1.6}$$

Since I_H and I are in phase

$$\frac{\mathrm{I}}{\mathrm{I}_H} = \frac{I}{I_H} \tag{2.1.7}$$

Therefore Eq. (2.1.3) becomes

$$\frac{I}{I_H} = a_A - 1$$

$$\frac{I_L - I_H}{I_H} = a_A - 1$$

$$\frac{I_L}{I_H} = a_A \tag{2.1.8}$$

and

$$I_H = \frac{I_L}{a_A} \tag{2.1.9}$$

From Eqs. (2.1.3) and (2.1.9)

$$\frac{I}{I_L} = \frac{a_A - 1}{a_A} \tag{2.1.10}$$

The induced voltages in windings *ab* and *bc* are in time phase because they are induced by the same flux.

Let

$$E_1 = E_{ac} = E_H$$

$$E_2 = E_{bc} = E_L$$

$$E_{ab} = E_{ac} - E_{bc}$$

$$\frac{E_{ab}}{E_{bc}} = \frac{E_{ac} - E_{bc}}{E_{bc}} = \frac{E_{ab}}{E_{bc}} - 1$$

$$= \frac{E_H}{E_L} - 1$$

$$\therefore \quad \frac{E_{ab}}{E_{bc}} = a_A - 1 \tag{2.1.11}$$

and

$$a_A = \frac{E_{ab}}{E_{bc}} + 1$$

or

$$a_A = a + 1 \tag{2.1.12}$$

where

$$a = \frac{E_{ab}}{E_{bc}}$$

= two–winding transformation ratio

Equation (2.1.12) shows that the transformation ratio of an autotransformer is greater than that if the same set of windings were connected as a 2-winding transformer.

Equations (2.1.3) and (2.1.10) show that the ratio of voltages and currents in the windings *ab* and *bc* are the same as if turns T_{ab} formed the *primary* and the turns T_{bc} formed the *secondary* of an ordinary transformer having a ratio of transformation of $(a_A - 1)$. Thus, an autotransformer may be considered as an ordinary transformer, treating winding *ab* as the *primary* and the winding *bc* as the *secondary*. In other words, the transformer action present in the autotransformer takes place in the windings *ab* and *bc*.

2.2 VOLTAMPERE RELATIONS

A 2-winding transformer transfers electrical power from primary to secondary by induction. Unlike 2-winding transformer an autotransformer transfers electrical power between primary and secondary circuits partly through a magnetic link (induction) and partly by direct electrical connection (conduction). Thus an autotransformer has two types of voltamperes namely, the transformed voltamperes and the conducted voltamperes. Since the transformer action in the autotransformer takes place in the windings ab and bc (Fig. 2.1),

voltamperes in winding ab = voltamperes in winding bc

$$E_{ab} I_{ab} = E_{bc} I_{bc} = S_{trans} \tag{2.21}$$

where S_{trans} is called the ***transformed voltamperes***.

If the winding resistances and leakage reactances are negligible then transformed VA,

$$\boxed{S_{trans} = V_{ab} I_{ab} = V_{bc} I_{bc}} \tag{2.22}$$

The input voltamperes (VA) to the autotransformer

$$S_{in} = V_{ac} I_{ab}$$

$$= (V_{ab} + V_{bc}) I_{ab}$$

$$= V_{ab} I_{ab} + V_{bc} I_{ab}$$

or $$\boxed{S_{in} = S_{trans} + V_{bc} I_{ab}} \tag{2.23}$$

The quantity $V_{bc} I_{ab}$ is called the **conducted voltamperes** $S_{cond.}$.

$\therefore$ $$S_{cond.} = V_{bc} I_{ab} \tag{2.24}$$

and $$\boxed{S_{in} = S_{trans} + S_{cond.}} \tag{2.25}$$

The conducted VA, $S_{cond.}$ has been transmitted from the line input to the line output by direct conduction. The main advantage of using an autotransformer is due to conducted VA. In Fig. 2.1,

transformed VA, $$\boxed{S_{trans} = V_{bc} I_{bc} = V_L I} \tag{2.26}$$

conducted VA, $$\boxed{S_{cond.} = V_L I_H} \tag{2.27}$$

An autotransformer is rated on the basis of output VA rather than the transformer's VA.

The output VA of an autotransformer can be compared to the output of an equivalent 2-winding transformer (assuming that the same core and coils are used).

If the autotransformer of Fig. 2.1 is used as a 2-winding transformer, its VA rating is

$$(VA)_{TW} + (V_H - V_L) I_H = V_L I = (I_L - I_H) V_L$$

When used as an autotransformer, its rating is

$$\boxed{(VA)_{auto} = V_H I_H = V_L I_L}$$

$$\therefore \quad \frac{\text{output VA of autotransformer}}{\text{output VA of equivalent 2-winding conventional transformer}}$$

$$\boxed{\frac{(VA)_{auto}}{(VA)_{TW}} = \frac{V_L I_L}{V_L I} = \frac{I_L}{I}} \tag{2.28}$$

But from Eq. (2.1.10)

$$\frac{I}{I_L} = \frac{a_A - 1}{a_A}$$

$$\frac{(VA)_{auto}}{(VA)_{TW}} = \frac{a_A}{a_A - 1} \quad (2.2.9)$$

Equation (2.2.9) shows that

$$(VA)_{auto} > (VA)_{TW} \quad (2.2.10)$$

This result shows that *two windings connected as an autotransformer will have greater VA rating than when connected as a two-winding transformer.*

For example, if $a_A = 2$, then from Eq. (2.2.9) VA of autotransformer

$$= 2 \times \text{VArating of 2-winding transformer.}$$

Therefore, we can use a 500-kVA auto-transformer instead of using a 1000 kVA 2-winding transformer. It is to be noted that as a_A approaches 1, which means that the voltage ratios approach 1, such as 11.8 kV/11 kV, then the savings, in terms of core and coil sizes of autotransformer, increases. When the voltage ratio (or the turns ratio) is 1, there is maximum saving, but then there is no need for any transformer since the high and low voltages are equal.

2.3 STEP-UP AUTOTRANSFORMER

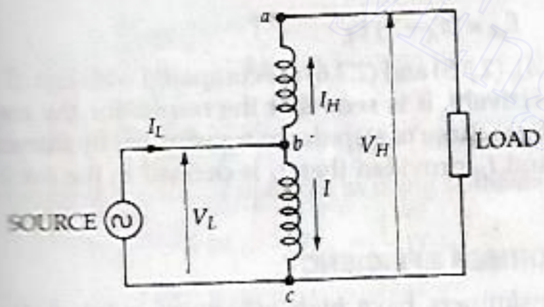


Fig. 2.2. *Step-up auto-transformer.*

Fig. 2.2 shows a step-up autotransformer. Here the source is connected to the winding *bc* and the load is connected across the full windings *abc*. The step-up auto transformer may be analysed in a manner similar to that used for step-down autotransformer. The relationships between input, output and common winding currents can be derived as follows :

Applying the balancing mmf condition in windings *ab* and *bc*

$$\boxed{I_H T_{ab} = I T_{bc}} \quad (2.3.1)$$

Let the transformation for the step-up autotransformer be defined as follows :

$$a_A = \frac{T_{ac}}{T_{bc}} = \frac{V_H}{V_L} \quad (2.3.2)$$

Here $a_A > 1$.

From Eq. (2.3.1)

$$\frac{I}{I_H} = \frac{T_{ab}}{T_{bc}} = \frac{T_{ac} - T_{bc}}{T_{bc}} = \frac{T_{ac}}{T_{bc}} - 1$$

$$\therefore \quad \boxed{\frac{I}{I_H} = a_A - 1} \tag{2.3.3}$$

$$\frac{I + I_H}{I_H} = a_A$$

But $$I + I_H = I_L \tag{2.3.4}$$

or $$\boxed{\frac{I_L}{I_H} = a_A} \tag{2.3.5}$$

Dividing Eq. (2.3.3) by Eq. (2.3.5), we get

$$\boxed{\frac{I}{I_L} = \frac{a_A - 1}{a_A}} \tag{2.3.6}$$

By KVL in the high-voltage side

$$E_{ab} + E_{bc} = E_{ac}$$

$$\therefore \quad E_{ab} = E_{ac} - E_{bc}$$

$$= a_A E_{bc} - E_{bc}$$

$$= a_A E_L - E_L$$

or $$\boxed{E_{ab} = (a_A - 1) E_L} \tag{2.3.7}$$

If Eqs. (2.3.3), (2.3.4), (2.3.5) and (2.3.6) are compared with Eqs. (2.1.3), (2.1.5), (2.1.8) and (2.1.10) respectively, it is seen that the results for the step-up transformer can be obtained from those of step-down transformer by interchanging the roles of the currents I_L and I_H provided that a_A is defined in the same manner in both cases.

2.4 AUTOTRANSFORMER EFFICIENCY

Conventional transformers have high efficiency, autotransformers have even higher efficiencies due to following reasons :

1. In an ordinary transformer the total electrical power is transferred from primary to secondary by *transformation*. Power transformation results in power loss.
2. In an autotransformer electrical power is transferred from primary to secondary partly by the process of transformation and partly by direct electrical connection. Power conductively transferred produces no transformer losses.

2.5 SAVING IN CONDUCTOR MATERIAL

The cross-section of a conductor is proportional to the current through it and the length of the conductor in a winding is proportional to the number of turns. Hence the weight of conductor material in a winding is proportional to the product of current and number of turns.

For a two-winding transformer, weight of conductor material in primary $\alpha\, I_H T_H$ weight of conductor material in secondary $\alpha\, I_L T_L$ total weight of conductor material $\alpha\, (I_H T_H + I_L T_L)$.

For the autotransformer (Fig. 2.1), the portion ab has $(T_H - T_L)$ turns and the current through it is I_H. Therefore the weight of conductor material in section ab

$$\alpha\, I_H (T_H - T_L).$$

The portion bc has T_L turns and the current through it is $I\,(= I_L - I_H)$. Therefore the weight of conductor material in section bc

$$\alpha\, (I_L - I_H)\, T_L$$

Total weight of conductor material

$$\alpha\, [I_H (T_H - T_L) + (I_L - I_H)\, T_L]$$

$$\frac{\text{Weight of conductor material in autotransformer}}{\text{Weight of conductor material in two–winding transformer}}$$

$$\frac{W_{auto}}{W_{2W}} = \frac{I_H (T_H - T_L) + (I_L - I_H)\, T_L}{I_H T_H + I_L T_L} = \frac{(I_H T_H + I_L T_L) - 2\, I_H T_L}{I_H T_H + I_L T_L}$$

$$= \frac{2\, I_H T_H - 2\, I_H T_L}{2\, I_H T_H} \qquad [\because\ I_L T_L = I_H T_H]$$

$$= 1 - \frac{T_L}{T_H} = 1 - \frac{1}{a_A}$$

$$\therefore \qquad \boxed{\frac{W_{auto}}{W_{2W}} = 1 - \frac{1}{a_A}}$$

$$1 - \frac{W_{auto}}{W_{2W}} = \frac{1}{a_A}\ \text{pu}$$

Saving of conductor material in using autotransformer

$$= W_{2W} - W_{auto} = \frac{1}{a_A} W_{2W}$$

If $\frac{1}{a_A} = 0.1$, saving of conductor material is 10 percent. If $\frac{1}{a_A} = 0.9$, saving is 90 percent. Hence the use of autotransformer is more economical when $\frac{1}{a_A}\left(= \frac{T_L}{T_H}\right)$ is close to unity. The ratios of transformations used in autotransformers are 3 : 1 or 4 : 1.

$$\text{If} \quad \frac{V_H}{V_L} = 3, \quad \frac{W_{auto}}{W_{2W}} = 1 - \frac{1}{3} = \frac{2}{3}$$

$$\text{If} \quad \frac{V_H}{V_l} = 2, \quad \frac{W_{auto}}{W_{2W}} = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\text{If} \quad \frac{V_H}{V_L} = 4, \quad \frac{W_{auto}}{W_{2W}} = 1 - \frac{1}{4} = \frac{3}{4}$$

If the ratio of transformation is greater than 4 the advantage in the reduction of conductor material is not much.

2.6 CONVERSION OF A TWO-WINDING TRANSFORMER TO AN AUTOTRANSFORMER

Fig. 2.3(*a*) shows a conventional two-winding transformer with its polarity markings. It can be converted to a step-up autotransformer by connecting the two

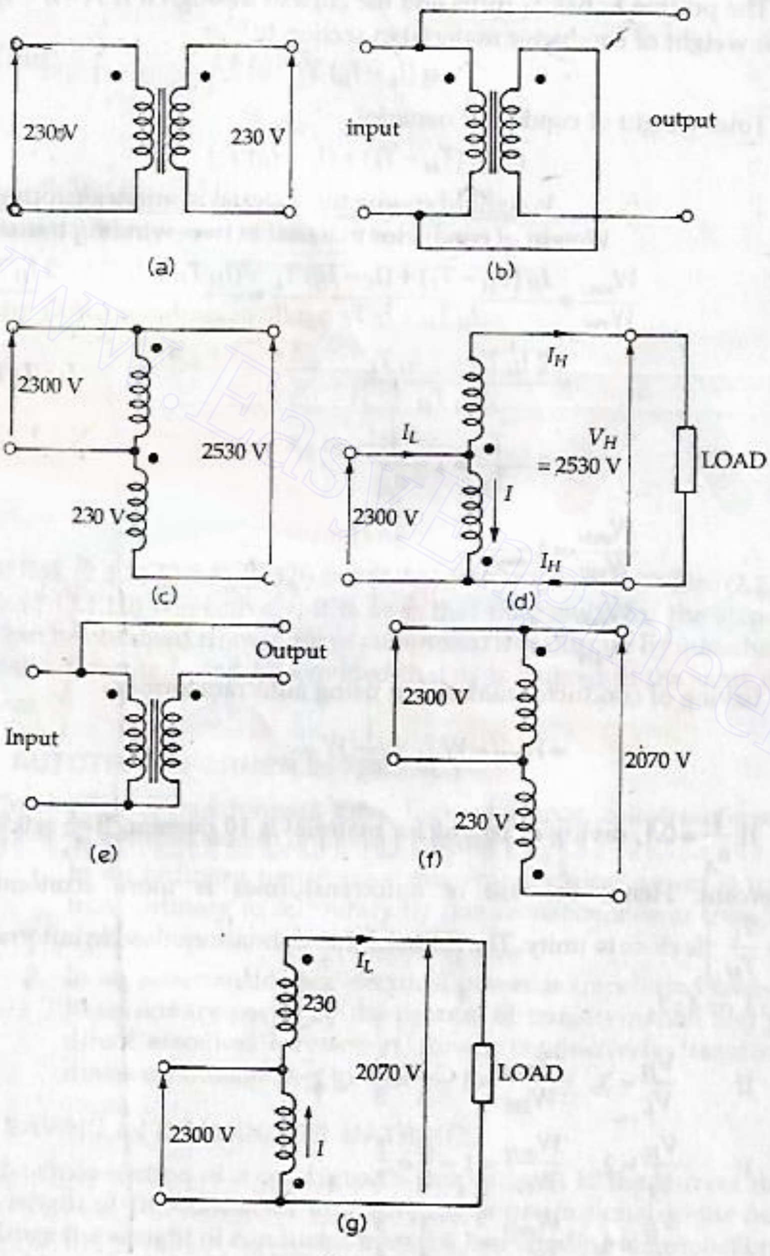


Fig. 2.3.

windings electrically in series with additive or subtractive polarities. With additive polarity between the high-voltage and low-voltage sides, a step-up transformer is obtained. With subtractive polarity a step-down transformer is obtained.

Let us consider a conventional 24 kVA, 2300/230 V transformer to be connected in autotransformer configuration.

Additive Polarity

Fig. 2.3(*b*) shows the series connections of the windings with additive polarity. The circuit is redrawn in Fig. 2.3(*c*) showing common terminal of the autotransformer at the top. Fig. 2.3(*d*) shows the same circuit with common terminal at the bottom. Since the polarity is additive, $V_H = 2300 + 230 = 2530$ V and $V_L = 2300$ V, the transformer acts as a step-up autotransformer.

Subtractive Polarity

Fig. 2.3(*e*) shows the series connections of the windings with subtractive polarity. The circuit is redrawn in Fig. 2.3(*f*) with common terminal at the top. Fig. 2.3(*g*) shows the same circuit with common terminal at the bottom. Since the polarity is subtractive $V_H = 2300$ V and $V_L = 2300 - 230 = 2070$ V, the transformer acts as a step-down autotransformer.

2.7 ADVANTAGES OF AUTOTRANSFORMERS

1. An autotransformer uses less winding material than a 2-winding transformer. The saving is large if the transformation ratio is small.
2. An autotransformer is smaller in size and cheaper than the two-winding transformer of the same output.
3. Since there is a reduction in conductor and core materials the ohmic losses in conductor and the core losses are smaller, an autotransformer has higher efficiency than the equivalent 2-winding transformer.
4. Since one winding has been completely eliminated, the resistance and leakage flux of this winding are zero. Hence the voltage regulation of the autotransformer is superior because of reduced voltage drops in the resistance and reactance.
5. An autotransformer has variable output voltage when a sliding contact is used for the secondary.

2.8 DISADVANTAGES OF AUTOTRANSFORMERS

1. There is a direct connection between the high-voltage and low-voltage sides. In case of an open circuit in the common winding *bc* (Fig. 2.1), the full primary voltage would be applied to the load on the secondary. This high voltage may burn out or seriously damage the equipment connected to the secondary side.
2. The effective per-unit impedance of an autotransformer is smaller compared to a 2-winding transformer. The reduced internal impedance results in a larger short-circuit (fault) current.
3. In an autotransformer there is a loss of isolation between input and output circuits. This is particularly important in three-phase transformers where one may wish to use a different winding and earthing arrangement on each side of the transformer.

2.9 APPLICATIONS OF AUTOTRANSFORMERS

1. Interconnection of power systems of different voltage levels, for example, 132 kV and 230 kV.
2. Boosting of supply voltage by a small amount in distribution systems to compensate voltage drop.
3. Autotransformers with a number of tappings are used for starting induction motors and synchronous motors.
4. Autotransformer is used as variac (variable a.c.) in laboratory and other situations that require continuously variable voltage over broad ranges.

EXAMPLE 2.1. *An 11500/2300 V transformer is rated at 100 kVA as a 2-winding transformer. If the windings are connected in series to form an autotransformer, what will be the possible voltage ratios and output ? Also calculate the saving in conductor material.*

SOLUTION. For the two-winding transformer rated current for 11500 V winding

$$= \frac{100 \times 1000}{11500} = 8.69 \text{ A}$$

and rated current for 2300 V winding

$$= \frac{100 \times 1000}{2300} = 43.48 \text{ A}$$

It is to be noted that if the windings of the 2-winding transformer are connected in series to form an autotransformer, the rated currents are not exceeded.

There are two possible connections of autotransformers.

(a) First Configuration

The winding AB is for 2300 V and winding BC for 11500 V as shown in Fig. 2.4(*a*).

Here $V_{AB} = 2300$ V, $V_{BC} = 11500$ V

$$\therefore \quad V_H = V_{AB} + V_{BC} = 2300 + 11500 = 13800 \text{ V}$$

$$V_L = V_{BC} = 11500 \text{ V}$$

Therefore, the voltage ratio for the autotransformer of Fig. 2.4(*a*) is

$$a_L = \frac{V_H}{V_L} = 13800/11500 \text{ V}$$

By KCL at point B

$$I_L = I_{AB} + I_{CB} = 43.48 + 8.69 = 52.17 \text{ A}$$

The current distribution is shown in Fig. 2.4(*a*).

kVA of the autotransformer of 13800/11500 ratio :

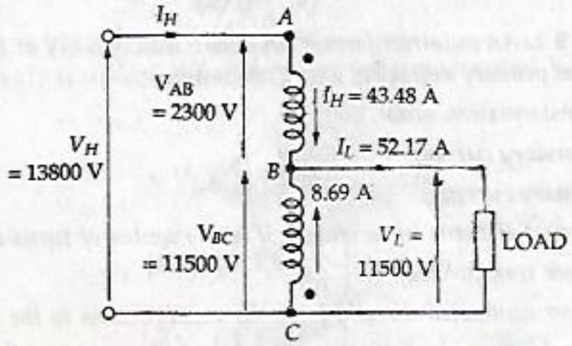
$$= \frac{V_L I_L}{1000} = \frac{11500 \times 52.17}{1000} = 600 \text{ kVA}$$

$$= \frac{V_H I_H}{100} = \frac{13800 \times 43.48}{1000} = 600 \text{ kVA}$$

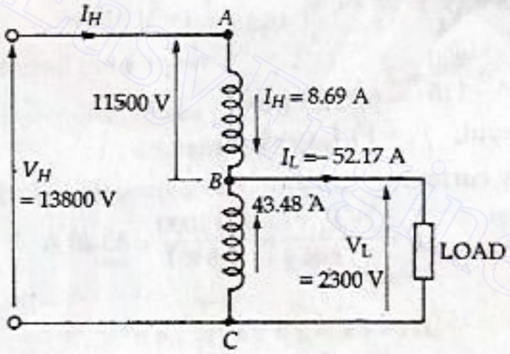
Saving in conductor material

$$= \frac{1}{a_L} = \frac{11500}{13800} = 0.833 \text{ pu}$$

$$= 83.3 \text{ per cent}$$



(a)



(b)

Fig. 2.4.

Second Configuration

Here the winding AB is for 11500 V and BC for 2300 V, as shown in Fig. 2.4(b). Therefore $V_{AB} = 11500$ V, $V_{BC} = 2300$ V

$$V_H = V_{AB} + V_{BC} = 11500 + 2300 = 13800 \text{ V}$$

$$V_L = V_{BC} = 2300 \text{ V}$$

$$a_L = \frac{V_H}{V_L} = 13800/2300 \text{ V}$$

By KCL at point B,

$$I_L = I_{AB} + I_{CB} = 8.69 + 43.48 = 52.17 \text{ A}$$

The current distribution is shown in Fig. 2.4(b).

kVA of the autotransformer of 13800/2300 V ratio

$$= \frac{V_L I_L}{1000} = \frac{2300 \times 52.17}{1000} = 120 \text{ kVA}$$

$$= \frac{V_H I_H}{1000} = \frac{13800 \times 8.69}{1000} = 120 \text{ kVA}$$

Saving in conductor material $= \frac{1}{a_L} = \frac{2300}{13800} = 0.166 \text{ pu} = 16.6$ per cent

EXAMPLE 2.2. *An autotransformer supplies a load of 5 kW at 115 V and at unity power factor. If the primary voltage is 230 V, determine :*

(*a*) *transformation ratio*
(*b*) *secondary current*
(*c*) *primary current*
(*d*) *number of turns in secondary if total number of turns is 400*
(*e*) *power transformed*
(*f*) *power conducted directly from the supply mains to the load.*

SOLUTION. (*a*) Transformation ratio of the autotransformer

$$a_A = \frac{V_H}{V_L} = \frac{T_H}{T_L}$$

$$\therefore \quad a_A = \frac{230}{115} = 2$$

(*b*) Power output, $P_L = V_L I_L \cos \phi$

$\therefore$ secondary current

$$I_L = \frac{P_L}{V_L \cos \phi} = \frac{5 \times 1000}{115 \times 1} = 43.48 \text{ A}$$

(*c*) $\frac{I_L}{I_H} = a_A$

$\therefore$ primary current, $I_H = \frac{I_L}{a_A} = \frac{43.48}{2} = 21.74$ A

(*d*) $\frac{T_H}{T_L} = a_A$

$\therefore$ secondary turns, $T_L = \frac{T_H}{a_A} = \frac{400}{2} = 200$

(*e*) Power transformed

$$P_{trans} = \left(1 - \frac{1}{a_A}\right) \times \text{power output} = \left(1 - \frac{1}{2}\right) \times 5 = 2.5 \text{ kW}$$

(*f*) Power conducted

$$P_{cond.} = \frac{1}{a_A} \times \text{power output} = \frac{1}{2} \times 5 = 2.5 \text{ kW}$$

EXAMPLE 2.3. *An autotransformer supplies a load of 5 kW at 110 V at unity power factor. If the applied primary voltage is 220 V, calculate the power transferred to the load (a) inductively, (b) conductively.*

SOLUTION. Input voltamperes of the autotransformer

$$S_{in} = \frac{\text{input power}}{\text{power factor}} = \frac{5}{1} = 5 \text{ kVA}$$

$$\frac{T_H}{T_L} = \frac{V_H}{V_L} = \frac{220}{110} = 2$$

(*a*) Inductively transferred voltamperes (or transformed VA)

$$S_{trans} = V_L I = V_L (I_L - I_H)$$

$$= V_L I_L \left(1 - \frac{I_H}{I_L}\right)$$

$$= V_L I_L \left(1 - \frac{V_L}{V_H}\right)$$

$$= V_L I_L \left(1 - \frac{T_L}{T_H}\right)$$

$$= S_{in}\left(1 - \frac{110}{220}\right) = 5 \times \frac{1}{2} = 2.5 \text{ kVA}$$

Power transferred inductively

$$= S_{trans} \times \text{power factor}$$

$$= 2.5 \times 1 = 2.5 \text{ kW}$$

(*b*) Conductively transferred voltamperes

$$S_{cond.} = V_L I_H = V_L \cdot \frac{I_L T_L}{T_H}$$

$$= S_{in} \frac{T_L}{T_H} = 5 \times \frac{1}{2} = 2.5 \text{ kVA}$$

Power transferred inductively

$$P_{cond} = S_{cond} \times \text{p.f.} = 2.5 \times 1 = 2.5 \text{ kW}$$

ALTERNATIVELY

$$P_{in} = P_{trans} + P_{cond}$$

$$P_{cond} = P_{in} - P_{trans} = 5 - 2.5 = 2.5 \text{ kW}$$

EXAMPLE 2.4. *A 400/100 V, 5 kVA, two-winding transformer is to be used as an autotransformer to supply power at 400 V from 500 V source. Draw the connection diagram and determine the kVA output of the autotransformer.*

SOLUTION. For a two-winding transformer

$$S_{in} = S_{out} = V_H I_H = V_L I_L$$

$$5 \times 1000 = 400\, I_H, \quad I_H = \frac{5000}{400} = 12.5 \text{ A}$$

and $$5 \times 1000 = 100\, I_L,\ I_L = \frac{5000}{100} = 50 \text{ A}$$

Fig. 2.5 shows the use of 2-winding transformer as an autotransformer to supply power at 400 V from a 500 V source.

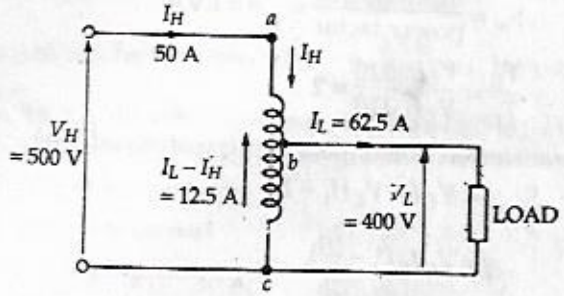


Fig. 2.5.

Here $V_H = 500$ V, $V_L = 400$ V

Transformation ratio

$$a_A = \frac{V_H}{V_L} = \frac{500}{400} = 1.25$$

$$I_H = \frac{I_L}{a_A} = \frac{I_L}{1.25}$$

Current through 400 V winding

$$I_{cb} = I_L - I_H = I_L - \frac{I_L}{1.25} = 0.2\, I_L.$$

Since the current rating of 400 V winding is 12.5 A

$$0.2\, I_L = 12.5,\ I_L = \frac{12.5}{0.2} = 62.5 \text{ A}$$

The kVA output of the autotransformer

$$= \frac{V_L I_L}{1000} = \frac{400 \times 62.5}{1000} = 25$$

Example 2.5. *A 25 kVA, 2000/200 V, 2-winding transformer is to be used as an autotransformer with constant source voltage of 2000 V. At full load of unity power factor, calculate the power output, power transformed and power conducted. If the efficiency of the 2-winding transformer at 0.8 power factor is 95 per cent, find the efficiency of the autotransformer.*

Solution. Rated current of 2000 V winding

$$= \frac{25 \times 1000}{2000} = 12.5 \text{ A}$$

Rated current of 200 V winding

$$= \frac{25 \times 1000}{200} = 125 \text{ A}$$

With the polarities shown in Fig. 2.6, the output voltage,

$$= 2000 + 200 = 2200 \text{ V}.$$

By KCL at point B, the input line current $= 125 + 12.5 = 137.5$ A.

The current distribution is shown in Fig. 2.6.

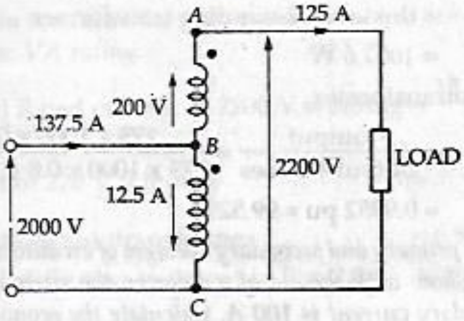


Fig. 2.6.

Output current of autotransformer, $I_L = 125$ A.

kVA rating of autotransformer $= \dfrac{2200 \times 125}{1000} = 275$ kVA

ALTERNATIVELY

kVA rating of autotransformer

$$= \frac{2000 \times 137.5}{1000} = 275 \text{ kVA}$$

Power output at full load of unity power factor

$$= \text{kVA} \cos\phi = 275 \times 1 = 275 \text{ kW}$$

Here winding BC acts as the primary and the winding AB as the secondary.

kVA transformed $= \dfrac{V_{AB} I_{AB}}{1000} = \dfrac{200 \times 125}{1000} = 25$ kVA

Also, kVA transformed $= \dfrac{V_{BC} I_{BC}}{1000} = \dfrac{2000 \times 12.5}{1000} = 25$ kVA

Power transformed $= \text{kVA} \cos\phi = 25 \times 1 = 25$ kW.

kVA conducted $= \dfrac{V_{BC} I_{AB}}{1000} = \dfrac{2000 \times 125}{1000} = 250$ kVA

Check

kVA conducted = input kVA − transformed kVA

$$= 275 - 25 = 250 \text{ kVA}$$

Power conducted $= \text{kVA} \cos\phi = 250 \times 1 = 250$ kW.

Calculation of efficiency

Efficiency, $\eta = \dfrac{\text{output}}{\text{output} + \text{losses}}$

$\therefore$ losses $= \left(\dfrac{1}{\eta} - 1\right) \times \text{output}$

Losses in a 2-winding transformer

$$= \left(\frac{1}{0.95} - 1\right) \times (25000 \times 0.8) = 1052.6 \text{ W}$$

Since autotransformer operates at rated voltages and rated currents, the loss in autotransformer

$$= \text{losses in 2-winding transformer}$$
$$= 1052.6 \text{ W}$$

Efficiency of autotransformer

$$= \frac{\text{output}}{\text{output} + \text{losses}} = \frac{275 \times 1000 \times 0.8}{275 \times 1000 \times 0.8 + 1052.6}$$
$$= 0.9952 \text{ pu} = 99.52\%$$

EXAMPLE 2.6. *The primary and secondary voltages of an autotransformer are 500 V and 400 V respectively. Show with the aid of a diagram the current distribution in the windings when the secondary current is 100 A. Calculate the economy in the conductor material.*

SOLUTION. Transformation ratio

$$a_A = \frac{V_H}{V_L} = \frac{500}{400} = 1.25$$

$$\frac{I_L}{I_H} = a_A$$

$$I_H = \frac{I_L}{a_A} = \frac{100}{1.25} = 80 \text{ A}$$

The current distribution in the windings is shown in Fig. 2.7.

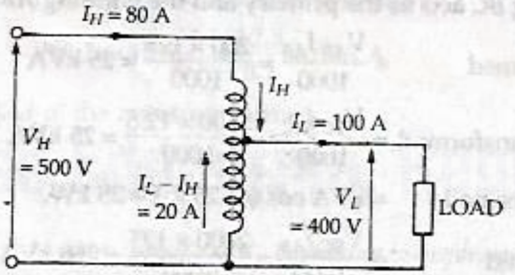


Fig. 2.7.

Saving in conductor material in using autotransformer

$$= \frac{1}{a_A} \text{ pu}$$

$$= \frac{1}{1.25} = 0.8 \text{ pu or } 80\%$$

EXAMPLE 2.7. *A 2200/220 V transformer is rated at 15 kVA as a 2-winding transformer. It is connected as an autotransformer with low-voltage winding connected additively in series with high-voltage winding as shown in Fig. 2.8. The autotransformer*

...from a 2420 V source. The autotransformer is loaded so that the rated currents of the windings are not exceeded. Find

(a) the current distribution in the windings,
(b) kVA output,
(c) kVA transferred conductively and inductively from input to output,
(d) saving in conductor material as compared to a two-winding transformer of the same VA rating.

SOLUTION. (a) Rated current of 2200 V winding $= \dfrac{15 \times 1000}{2200} = 6.82$ A

Rated current of 220 V winding $= \dfrac{15 \times 1000}{220} = 68.2$ A

Output current of autotransformer $= 6.82 + 68.2 = 75.02$ A

The current distribution is shown in Fig. 2.8.

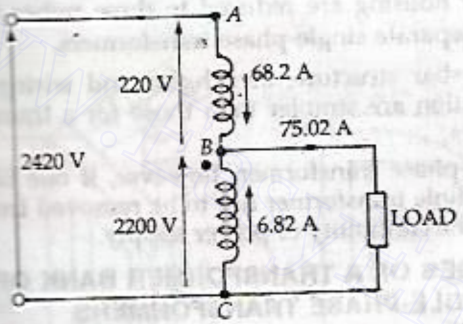


Fig. 2.8.

Output voltage = 2200 V

(b) kVA output $= \dfrac{2200 \times 75.02}{1000} = 165$ kVA

(c) kVA transferred conductively $= \dfrac{2200 \times 68.2}{1000} = 150$ kVA

kVA transferred inductively $= \dfrac{2200 \times 6.82}{1000} = 15$ kVA

(d) Saving in conductor material $= \dfrac{1}{a_A}$ pu $= \dfrac{V_L}{V_H} = \dfrac{2200}{2420}$

$= 0.909$ pu or 90.9%

2.13 THREE-PHASE TRANSFORMERS

A three-phase system is used to generate and transmit large amount of power. Three-phase transformers are required to step up or step down voltages at various stages of a power system network.

Transformers for 3-phase circuits can be constructed in one of the following ways :

1. Three separate single-phase transformers are suitably connected for 3-phase operation. Such an arrangement is called a 3-phase ***bank of transformers***.
2. A single three-phase transformer in which the cores and windings for all the three phases are combined in a single structure.

2.11 ADVANTAGES OF A 3-PHASE UNIT TRANSFORMER

A 3-phase unit transformer has following advantages over three single-phase transformer bank of the same kVA rating :

1. It takes less space.
2. It is lighter, smaller and cheaper.
3. It is slightly more efficient.
4. The costly high voltage terminals to be brought out of the transformer housing are reduced to three rather than six necessary for three separate single-phase transformers.

Thus, the busbar structure, switchgear and wiring for a single 3-phase transformer installation are simpler than those for a transformer bank of three 1-phase transformers.

In a single 3-phase transformer, however, if one of the phase windings breaks down, the whole transformer has to be removed from service for repairs, thereby disturbing the continuity of power supply.

2.12 ADVANTAGES OF A TRANSFORMER BANK OF THREE SINGLE-PHASE TRANSFORMERS

A transformer bank of three 1-phase transformers has the following advantages over a unit three-phase transformer of the same kVA rating :

(*a*) One single-phase transformer in a bank may be provided with a higher kVA rating than the others to supply an unbalanced load.

(*b*) When one single-phase transformer of a bank is damaged and removed from the service, the remaining two units may be used in open-delta or $V-V$ at reduced capacity.

(*c*) Where single units are concerned only one spare single-phase transformer is needed as a standby instead of a complete spare 3-phase transformer. The provision of a spare standby 3-phase transformer is more costly than provision of a single-phase spare transformer for 3-phase transformer bank. Thus standby requirement is lesser.

(*d*) The transport of 1-phase transformers is more convenient.

It is a common practice to use single-unit 3-phase transformers. However, 3-phase banks are also used depending upon the requirements.

2.13 THREE-PHASE TRANSFORMER CONSTRUCTION

Construction of the magnetic core of a 3-phase core-type transformer may be understood by considering three single-phase core-type transformers positioned at 120° to each other as shown in Fig. 2.9(*a*). For simplicity, only the primary windings are shown. If balanced 3-phase sinusoidal voltages are applied to the windings, the fluxes Φ_a, Φ_b and Φ_c will also be sinusoidal and balanced. If the three legs carrying these fluxes are merged, the total flux in the merged leg is zero. This leg can therefore be removed because it carries no flux, as shown in Fig. 2.9(*b*) This structure is not convenient to build. Usually the structure of Fig. 2.9(*c*) with the three limbs in the same plane is used. This structure can be obtained from Fig. 2.9(*b*) by eliminating the yokes of section *b* and fitting the remainder between sections *a* and *c*. It can be built using stacked laminations. Each leg carries both low voltage and high voltage windings. Since it is easier to insulate the *lv* windings from the core than the *hv* windings, the *lv* windings are placed next to the core with suitable insulation between the core and the *lv* windings. The *hv* windings are placed over the *lv* windings. The *hv* windings are placed over the *lv* windings with suitable insulation between them.

Since the magnetic paths of legs *a* and *c* are greater than that of leg *b*, the construction is not symmetrical, but the resultant imbalance in magnetizing currents is not significant.

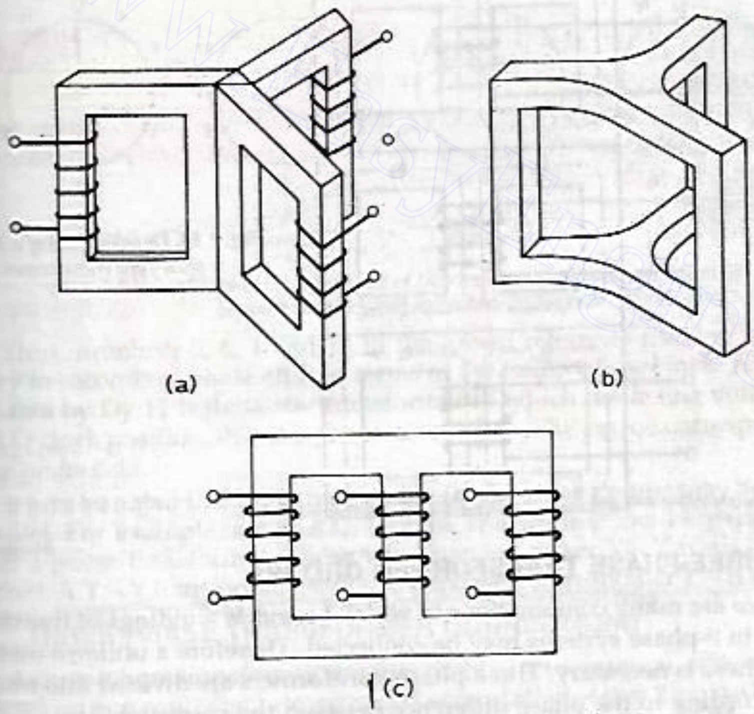


Fig. 2.9. *Development of a 3-phase core-type transformer.*
(a) Three single-phase cores in contact with another.
(b) The same, with central limb removed because it carries no flux.
(c) Usual construction, with the three limbs in the same plane.
(d) Core structure using stacked laminations.

The shell-type 3-phase transformer can be constructed by stacking three single-phase shell transformers as shown in Fig. 2.10. The winding direction of the central unit *b* is made opposite to that of units *a* and *c*. If the system is balanced with phase sequence *a–b–c*, the fluxes will also be balanced. That is,

$$\Phi_a = \alpha\,\Phi_b = \alpha^2\,\Phi_c$$

where
$$\alpha = 1\,\angle 120°,\ \alpha^2 = 1\,\angle 240°$$

The adjacent yoke sections of units a and b carry a combined flux of

$$\frac{1}{2}\Phi_a + \frac{1}{2}\Phi_b = \frac{1}{2}\Phi_a\,(1\,\angle 0° + 1\,\angle 240°) = \frac{1}{2}\Phi_a\,\angle -60°$$

Thus, the magnitude of this combined flux is equal to the magnitude of each of its components. In this way the cross-sectional area of the combined yoke sections may be reduced to the same value as that used in the outer legs and in the top and bottom yokes. The slight imbalance in the magnetic paths among the three phases has very little effect on the performance of the 3-phase shell-type transformer. Its behavior is essentially the same as that of a bank of three single phase transformers. The windings of either core or shell-type, 3-phase transformers may be connected in Y or Δ as desired.

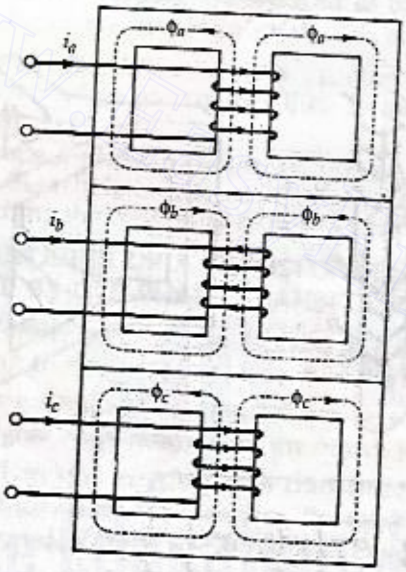


Fig. 2.10. *Development of a 3-phase shell-type transformer.*

2.14 THREE-PHASE TRANSFORMER GROUPS

There are many combinations in which *hv* and *lv* windings of transformers employed in 3-phase systems may be connected. Therefore a uniform method of grouping these is necessary. Three-phase transformers are divided into four main groups according to the phase difference between the corresponding LINE voltages on the *hv* and *lv* sides. The phase difference is the angle by which the *lv* line voltage lags the *hv* line voltage, and is measured in units of 30° in *clockwise* direction. These groups are :

Group number 1 — no phase displacement
Group number 2 — 180° phase displacement
Group number 3 — (– 30°) phase diplacement
Group number 4 — (+ 30°) phase displacement

Thus, a connection $\boxed{Y\,d\,11}$ gives the following information :

Y indicates that *hv* is connected in star *d* indicates that *lv* is connected in delta and 11 indicates that the *lv* line voltage lags high voltage line voltage by $11 \times 30° = 330°$ measured from *hv* phasor in a *clockwise* direction.

Instead of expressing the phase difference between the voltages in degrees, it is more convenient to use clock method of angle designation. For this purpose, the phasor of the line voltage on *hv* side is considered as the minute hand and set at 12 O'clock. The phasor of the line voltage on the *lv* side is considered as the hour hand and is set on the dial of the clock according to its position in relation to the phasor of the line voltage on *hv* side. Here the angle of 30° is the angle between two adjacent figures on the clock dial and is taken as the unit of dial shift. When the hour hand of the clock is at 12 O'clock position, the phase displacement is zero. When the hour hand is at 1 O'clock position, the phase shift is $-30°$ (anticlockwise direction of angle is positive). At 6 O'clock position, the phase shift is $6 \times 30° = 180°$. Similarly, when the hour hand is at the 11 O'clock position, the phase shift is $11 \times 30° = 330$ in the clockwise direction (or $+30°$). This is shown in Fig. 2.11.

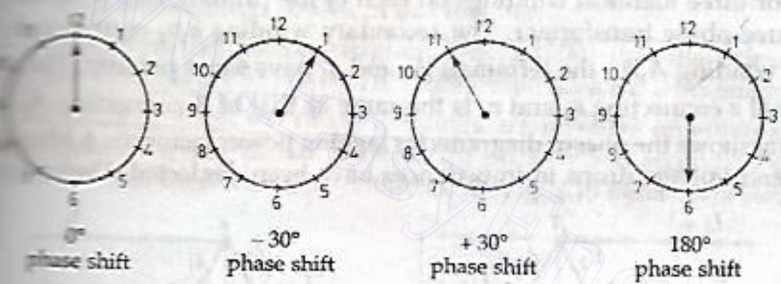


Fig. 2.11. *Positions of the hour hand of the clock to represent the phase shift between primary and secondary voltages.*

imp for parallel connection of tx

Thus, numbers 0, 6, 1 and 11 in the group reference number indicate the primary to secondary phase shift in terms of the hours of the clock. A connection designated by Dy 11 is delta-star transformer in which the *lv* line voltage phasor is at 11 O'clock position, that is a phase advance of $+30°$ on the corresponding line voltage on *hv* side.

It is to be noted that only transformers in the same groups may be connected in parallel. For example, a star-star, 3-phase transformer can be paralleled with another 3-phase transformer whose windings are either $Y-Y$ connected or $\Delta-\Delta$ connected. A $Y-Y$ transformer cannot be paralleled with another $Y-\Delta$ transformer.

2.16 THREE-PHASE TRANSFORMER CONNECTIONS

A three-phase transformer consists of three transformers, either separate or combined on one core. The primaries and secondaries of any 3-phase transformer can be independently connected in either a star (Y) or delta (Δ). Thus, there are four possible connections for a 3-phase transformer bank :

1. $\Delta-\Delta$ (Delta primary-Delta secondary)
2. $Y-Y$ (Star primary-Star secondary)
3. $\Delta-Y$ (Delta primary - Star secondary)
4. $Y-\Delta$ (Star primary - Delta secondary)

Here it is assumed that all transformers in the bank have same kVA rating.

2.16 FACTORS AFFECTING THE CHOICE OF CONNECTIONS

Some of the factors governing the choice of connections are as follows :

1. Availability of a neutral connection for grounding, protection, or load connections.
2. Insulation to ground and voltage stress.
3. Availability of a path for the flow of third harmonic (exciting) currents and zero-sequence (fault) currents.
4. Need for partial capacity with one circuit out of service.
5. Parallel operation with other transformers.
6. Operation under fault conditions.
7. Economic considerations.

2.17 DELTA-DELTA ($\Delta - \Delta$) CONNECTION

Fig. 2.12(a) shows the $\Delta - \Delta$ connection of three identical single-phase transformers or three identical windings on each of the primary and secondary sides of the three-phase transformer. The secondary winding a_1a_2 corresponds to the primary winding A_1A_2, the terminals A_1 and a_1 have same polarity. The polarity of terminal a connecting a_1 and c_2 is the same as that of A connecting A_1 and C_2. Fig. 2.12(b) shows the phasor diagrams for lagging power factor cos ϕ. Magnetizing current and voltage drops in impedances have been neglected. Under balanced

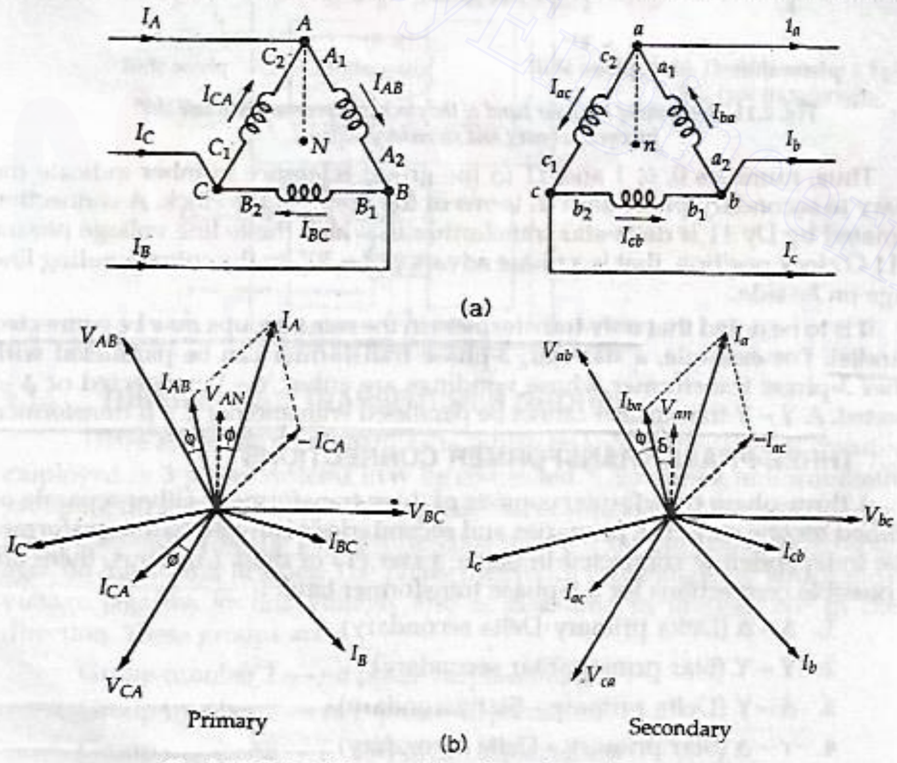


Fig. 2.12. *Delta-delta connection of transformer (0° phase shift).*

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The current ratio

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Fig. 2.13.

conditions, the line currents are $\sqrt{3}$ times the phase (winding) currents and displaced behind the phase currents. In the $\Delta - \Delta$ configuration the corresponding line and phase voltages are identical in magnitude on both primary and secondary sides.

The secondary line-to-line voltages V_{ab}, V_{bc} and V_{ca} are in phase with primary line-to-line voltages V_{AB}, V_{BC} and V_{CA} with voltage ratios equal to the turns ratio.

$$\frac{V_{AB}}{V_{ab}} = \frac{V_{BC}}{V_{bc}} = \frac{V_{CA}}{V_{ca}} = a$$

The current ratios when the magnetizing current is neglected are

$$\frac{I_{AB}}{I_{ab}} = \frac{I_{BC}}{I_{bc}} = \frac{I_{CA}}{I_{ca}} = \frac{I_A}{I_a} = \frac{I_B}{I_b} = \frac{I_C}{I_c} = \frac{1}{a}$$

It is to be noted that in Fig. 2.12 each winding is drawn along the line of the phasor of its induced voltage. The voltage and current phasors are determined very easily from the windings drawn in this manner.

It is seen from the phasor diagram [Fig. 2.12(*b*)] that the primary and secondary line voltages are in phase. This connection is called *0°–connection.*

If the connections of the phase windings are reversed on either side, we obtain the phase difference of 180° between the primary and secondary systems. Such a connection is known as *180°–connection.* In Fig. 2.13 delta-delta connection

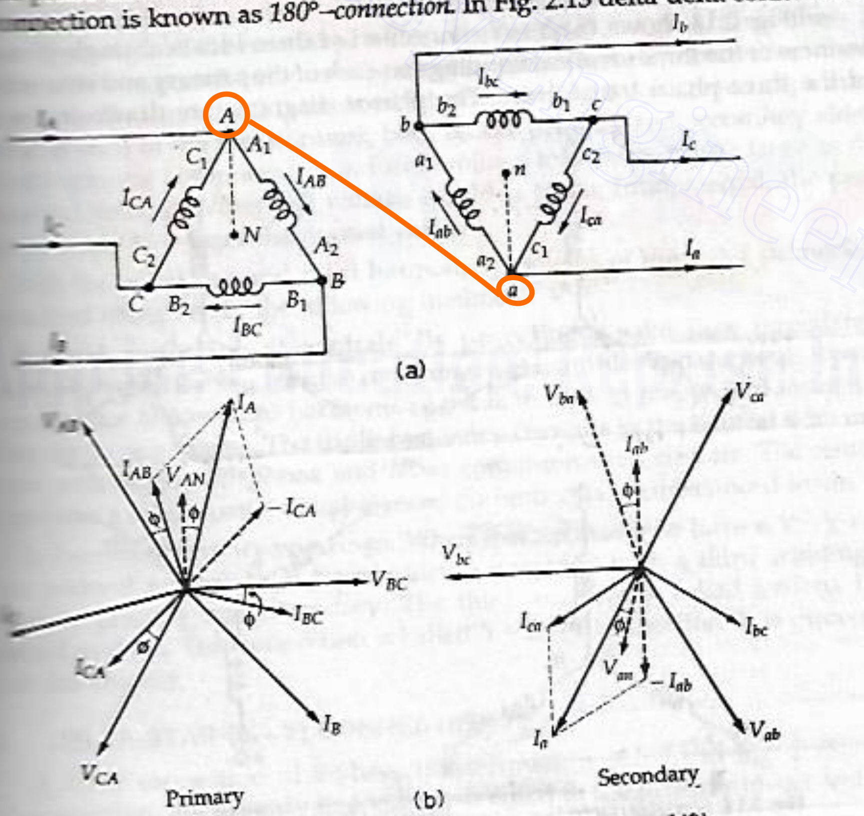


Fig. 2.13. Delta-delta connection of transformer (180° phase shift).

with 180° phase shift is shown. Here $b_1 c_2$, $c_2 a_2$ and $a_1 b_2$ are connected to form delta on secondary side as shown in Fig. 2.13(*a*). Fig. 2.13(*b*) shows the phasor diagrams. It is seen that the secondary voltages are in phase opposition to the primary voltages.

The $\Delta - \Delta$ transformer has no phase shift associated with it, and no problems with unbalanced loads or harmonics.

2.17.1 Advantages of $\Delta - \Delta$ Transformation

1. The $\Delta - \Delta$ connection is satisfactory for both balanced and unbalanced loading.
2. If a third harmonic is present, it circulates in the closed path and therefore does not appear in the output voltage wave.
3. If one transformer fails, the remaining two transformers will continue to supply three-phase power. This is called open-delta (or $V - V$) *connection*. The operation of $V - V$ connection is discussed later in this chapter.

However, the $\Delta - \Delta$ connection has the disadvantage that there is no star point (neutral point) available. A $\Delta - \Delta$ connection is useful when neither primary nor secondary requires a neutral and the voltages are low and moderate.

2.18 STAR-STAR (Y – Y) CONNECTION

Fig. 2.14 shows the Y – Y connection of three identical single-phase transformers or the three identical windings on each of the primary and secondary sides of the three-phase transformer. The phasor diagrams are drawn in the similar

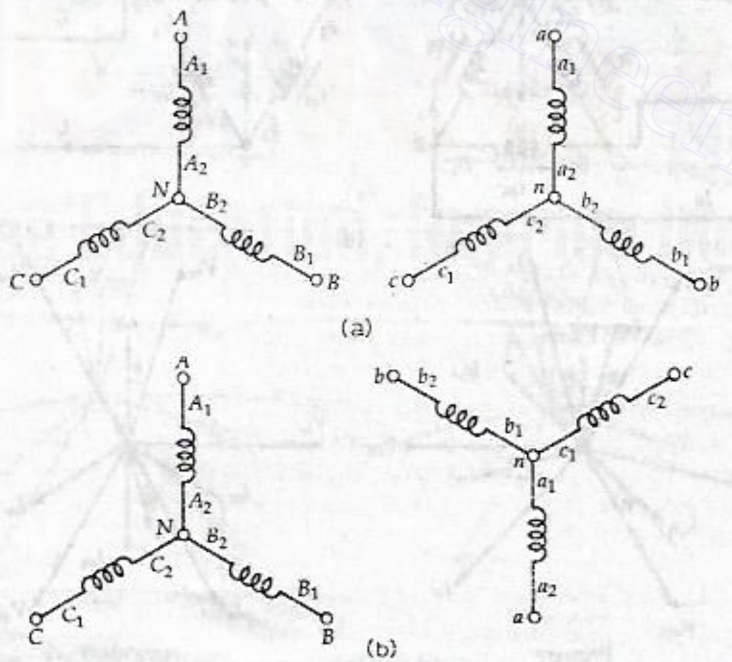


Fig. 2.14. *Star-star connection of transformer (a) 0° phase shift (b) 180° phase shift.*

manner as done in $\Delta - \Delta$ connection. The phase current is equal to the line current and they are in phase. The line voltage is $\sqrt{3}$ times the phase voltage. There is a phase separation of 30° between line and phase voltages. Fig. 2.14 *a* shows the Y – Y connection for 0° phase shift and in Fig. 2.14(*b*) there is a phase shift of 180° between primary and secondary systems.

For ideal transformers the voltage ratios are

$$\frac{V_{AN}}{V_{an}} = \frac{V_{BN}}{V_{bn}} = \frac{V_{CN}}{V_{cn}} = a$$

and current ratios are

$$\frac{I_A}{I_a} = \frac{I_B}{I_b} = \frac{I_C}{I_c} = \frac{1}{a}$$

The Y – Y connection has two very serious problems :

1. If the neutral is not provided, the phase voltages tend to become severely unbalanced when the load is unbalanced. Therefore, the Y – Y connection is not satisfactory for unbalanced loading in absence of a neutral connection.

2. The magnetising current of any transformer is very nonsinusoidal and contains a very large third harmonic, which is necessary to overcome saturation in order to produce a sinusoidal flux. In a balanced three-phase system, the third harmonic components in the magnetising currents of three primary windings are equal in magnitude and in phase with each other. Therefore they will be directly additive. Their sum at the neutral of a star connection is not zero. Since there is no path for these current components in an ungrounded star connection these components will distort the flux wave which will produce a voltage having a third harmonic in each of the transformers, both on the primary and secondary sides. The third harmonic component of induced voltage may be nearly as large as the fundamental voltage. When this voltage is added to the fundamental, the peak voltage is nearly two times the normal value.

Both the unbalance and third harmonic problems of the Y – Y connection can be solved using one of the following methods :

1. **Solid grounding of neutrals.** By providing a solid (low impedance) connection between the star point of the primary transformer and the neutral point of the alternator allows third harmonic currents to flow in the neutral instead of building up large voltages. The triple-frequency currents in the neutral wire may interfere with nearby telephone and other communication circuits. The neutral also provides a return path for unbalanced currents due to unbalanced loads.

2. **Providing tertiary windings.** When it is necessary to have a Y – Y connection without neutral, each transformer is provided with a third winding in addition to primary and secondary. The third winding is called *tertiary*. It is connected in delta. This connection is called Y – Δ – Y connection. It is discussed later in this chapter.

2.18 DELTA-STAR (Δ – Y) CONNECTION

A Δ – Y connection of 3-phase transformers is shown in Fig. 2.15(*a*). In Δ – Y connection, the primary line voltage is equal to the primary phase voltage

($V_{LP} = V_{pP}$). The relationship between secondary voltages is $V_{LS} = \sqrt{3}\, V_{pS}$. Therefore, the line-to-line voltage ratio of this connection is

$$\frac{V_{LP}}{V_{LS}} = \frac{V_{pP}}{\sqrt{3}\, V_{pS}}$$

But $$\frac{V_{pP}}{V_{pS}} = a$$

∴ $$\frac{V_{LP}}{V_{LS}} = \frac{a}{\sqrt{3}}$$

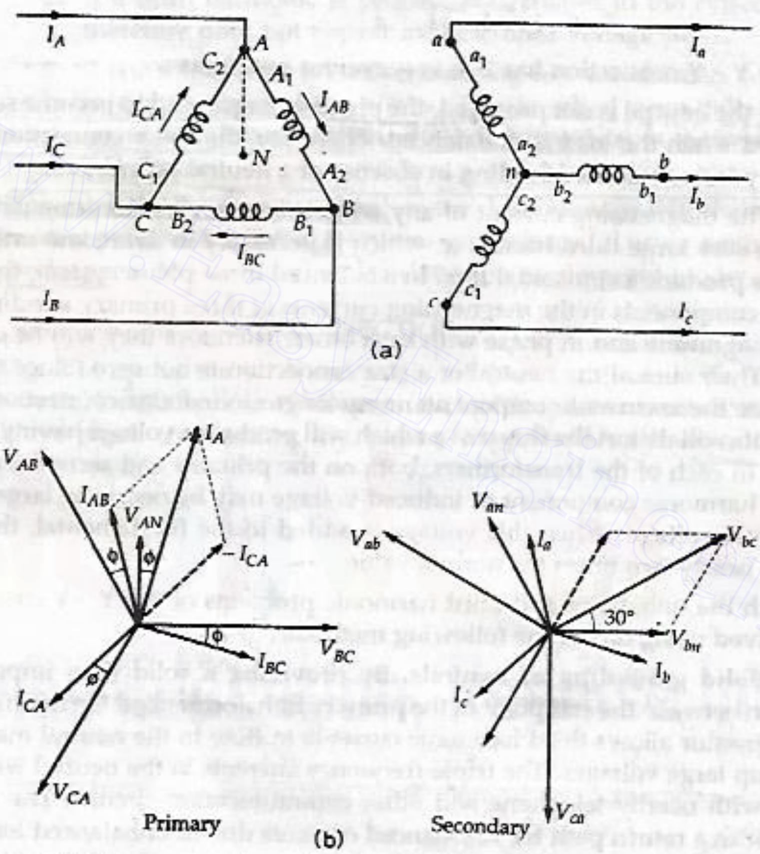


Fig. 2.15. *(a) Delta-star connections of transformer (phase shift 30° lead), (b) Phasor diagrams.*

Fig. 2.15(*b*) shows the phasor diagrams for the Δ – Y connection supplying a balanced load at power factor cos ϕ lagging. It is seen from the phasor diagram that the secondary phase voltage V_{an} leads the primary phase voltage V_{AN} by 30°. Similarly, V_{bn} leads V_{BN} by 30° and V_{cn} leads V_{CN} by 30°. This is also the phase relationship between the respective line-to-line voltages. This connection is called + 30° connection.

By reversing the connections on either side, the secondary system voltage can be made to lag the primary system by 30° as shown in Fig. 2.16. This connection is called – 30° connection.

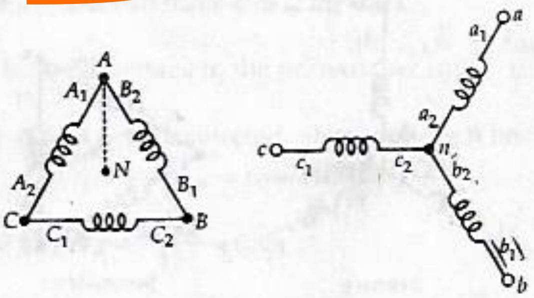


Fig. 2.16. *Delta-star connection of transformer (phase shift 30° lag).*

2.20 STAR-DELTA (Y – Δ) CONNECTION

The Y – Δ connection of three-phase transformers is shown in Fig. 2.17. In this connection, the primary line voltage is equal to $\sqrt{3}$ times the primary phase voltage ($V_{LP} = \sqrt{3}\ V_{pP}$). The secondary line voltage is equal to the secondary phase voltage ($V_{LS} = V_{pS}$). The voltage ratio of each phase is

$$\frac{V_{pP}}{V_{pS}} = a$$

Therefore line-to-line voltage ratio of a Y – Δ connection is

$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}\ V_{pP}}{V_{pS}} = \sqrt{3}\ a.$$

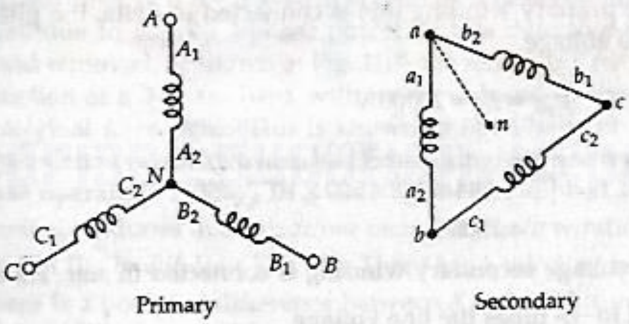


Fig. 2.17. *Y – Δ connections of transformer (Phase shift of 30° lead)*

The phasor diagrams can be drawn with the help of winding diagrams. There is a phase shift of 30° lead between respective line-to-line voltages. Similarly, a phase shift of 30° lead exists between respective phase voltages. This connection is called + 30° connection.

Fig. 2.18 shows the star-delta connection of transformer for a phase shift of 30° lag. This connection is known as – 30° *connection.*

It is to be noted that a $Y-\Delta$ connection is simply obtained by interchanging the primary and secondary roles in $\Delta-Y$ connection.

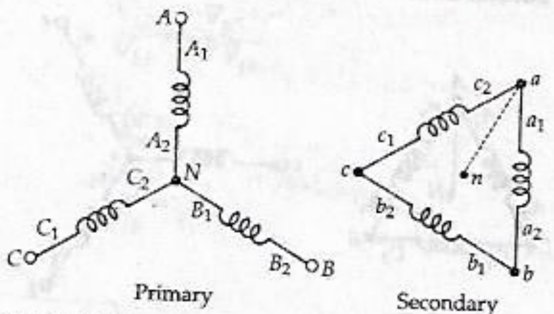


Fig. 2.18. *Star-delta connection of a transformer (Phase shift of 30° lag).*

The $\Delta-Y$ connection or $Y-\Delta$ connection has no problem with unbalanced loads and third harmonics. The delta connection assures balanced phase voltages on the Y side and provides a path for the circulation of the third harmonics and their multiples without the use of a neutral wire.

EXAMPLE 2.8. *Determine the number of turns per phase in each winding of a 2-phase transformer with a ratio of 20,000/2000 V at 50 Hz. The high-voltage winding is delta connected and the low voltage winding is star connected. Each core has a gross section of 500 cm². Assume a flux density of about 1.2 Wb/m².*

SOLUTION. $E_{ph_1} = 4.44\, B_m\, A f\, T_{ph_1}$

$$T_{ph_1} = \frac{E_{ph_1}}{4.44\, B_m\, A\, f}$$

Since the primary winding (*hv*) is connected in delta, the phase voltage is equal to the line voltage.

$$\therefore \quad E_{ph_1} = E_{l_1} = 20000 \text{ V}$$

and
$$T_{ph_1} = \frac{20000}{4.44 \times 1.2 \times 500 \times 10^{-4} \times 50} = 1500$$

The low voltage secondary winding is connected in star. Hence the phase voltage is equal to $\frac{1}{\sqrt{3}}$ times the line voltage.

$$\therefore \quad E_{ph_2} = \frac{1}{\sqrt{3}} El_2 = \frac{1}{\sqrt{3}} \times 2000 = 1154.7 \text{ V}$$

$$\frac{E_{ph_2}}{E_{ph_1}} = \frac{T_{ph_2}}{T_{ph_1}}$$

$$T_{ph_2} = T_{ph_1} \times \frac{E_{ph_2}}{E_{ph_1}} = 1500 \times \frac{1154.7}{20000} = 86$$

EXAMPLE 2.9. *An 11000/440 V, 50 Hz, 3-phase transformer is delta connected on the hv side and the lv windings are star connected. There are to be 12 V per turn and the flux density is not to exceed 1.2 Wb/m². Calculate the number of turns per phase on each winding and the net iron cross-sectional area of the core.*

SOLUTION. Induced voltage in the primary per turn $\frac{E_{ph_1}}{T_{p_1}} = 12$

Since the *hv* side is delta connected, phase voltage = line voltage

$$E_{ph_1} = E_{l_1} = 11000$$

$$\therefore \quad \frac{11000}{T_{p_1}} = 12, \quad T_{p_1} = \frac{11000}{12} = 917.$$

$$\text{Also,} \quad \frac{E_{ph_2}}{T_{p_2}} = 12, \quad E_{ph_2} = \frac{1}{\sqrt{3}} \times 440$$

$$T_{ph_2} = \frac{E_{ph_2}}{12} = \frac{440/\sqrt{3}}{12} = 21$$

$$\frac{E_{ph_2}}{T_{ph_2}} = 4.44\, B_m Af$$

$$12 = 4.44 \times 1.2 \times A \times 50$$

$$A = \frac{12}{4.44 \times 1.2 \times 50} = 0.0450 \text{ m}^2$$

$$= 0.0450 \times 10^4 \text{ cm}^2 = 450 \text{ cm}^2$$

OPEN-DELTA OR V-V CONNECTION

If one transformer of a $\Delta - \Delta$ system is damaged or accidently opened, the system will continue to supply 3-phase power. If this defective transformer is disconnected and removed, as shown in Fig. 2.19, the remaining two transformers continue to function as a 3-phase bank with rating reduced to about 58 per cent of that of the original $\Delta - \Delta$ bank. This is known as open-delta or $V - V$ system. Thus, in the open-delta system, two instead of three single-phase transformers are used for 3-phase operation. Let V_{AB}, V_{BC} and V_{CA} be the applied voltages of the primary. The voltage induced in transformer secondary or *lv* winding *I* is $\mathbf{V}_{ab}$. The voltage induced in the *lv* winding II is $\mathbf{V}_{bc}$. There is no winding between points *a* and *c*, but there is a potential difference between *a* and *c*. This voltage may be found by applying KVL around closed path made up of points *a*, *b* and *c*. Thus,

$$\mathbf{V}_{ab} + \mathbf{V}_{bc} + \mathbf{V}_{ca} = 0 \tag{2.21.1}$$

$$\therefore \quad \mathbf{V}_{ca} = -\mathbf{V}_{ab} - \mathbf{V}_{bc} \tag{2.21.2}$$

Let $\quad \mathbf{V}_{AB} = V_P \angle 0°$

$\mathbf{V}_{BC} = V_P \angle -120°$

and $\quad \mathbf{V}_{CA} = V_P \angle +120°$

where V_p is the magnitude of the line voltage on the primary side.

If the leakage impedances of the transformers are negligible, then

$$\mathbf{V}_{ab} = V_s \angle 0^\circ$$

$$\mathbf{V}_{bc} = V_s \angle -120^\circ$$

where V_s is the magnitude of the secondary voltage.

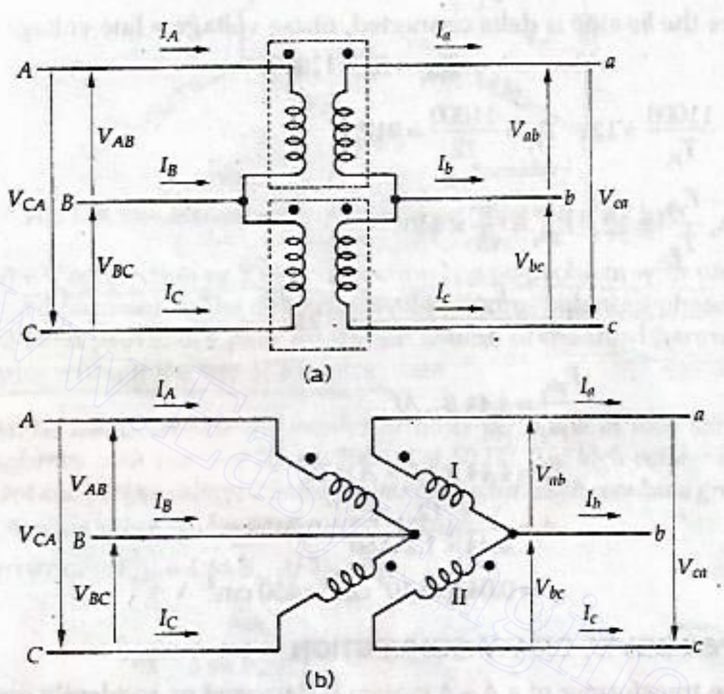


Fig. 2.19. *Open delta (or V – V connection) (a) Common physical arrangement (b) Schematic diagram.*

Substituting the values of V_{ab} and V_{bc} in Eq. (2.21.2)

$$V_{ca} = -V_s \angle 0^\circ - V_s \angle -120^\circ$$

$$= -\mathbf{V}_s - (-0.5\, V_s - j\, 0.866\, V_s)$$

$$= -0.5\, V_s + j\, 0.866\, V_s$$

$$\boxed{= V_s \angle +120^\circ}$$

It is seen that V_{ca} is equal in magnitude to the secondary transformer voltage and 120° apart in time from both of them. Thus balanced 3-phase line voltages applied to the $V-V$ primaries produce balanced 3–phase voltages on the secondary side if leakage impedances are negligible.

It appears that removal of one transformer would permit the remaining two transformers to carry two-thirds (66.7%) of the load kVA. This, however, is not the case. If V_{2B} and I_{2B} are the rated secondary voltage and rated secondary current of the transformers, the line current to the load of a closed delta system is $\sqrt{3}\, I_{2B}$.

∴ Closed delta load VA

$$S_{\Delta-\Delta} = \sqrt{3} \times \text{ line voltage} \times \text{line current}$$

$$= \sqrt{3}\ V_{2B}\ (\sqrt{3}\ I_{2B}) = 3\ V_{2B} I_{2B}$$

When one transformer is removed, the $\Delta - \Delta$ transformer becomes open-delta ($V - V$) connected transformer. The line is in series with the windings of the transformers and therefore secondary line current I_{2B} is equal to the rated secondary current I_{2B} of the transformer. The VA load that can be carried by the open-delta bank without exceeding the ratings of the transformers is $S_{V-V} = \sqrt{3}\ V_{2B}\ I_{2B}$

$$\therefore \quad \frac{S_{V-V}}{S_{\Delta-\Delta}} = \frac{\sqrt{3}\ V_{2B}\ I_{2B}}{3\ V_{2B}\ I_{2B}} = \frac{1}{\sqrt{3}} = 0.577$$

Thus, is seen that the load that can be carried by the open-delta bank *without exceeding the ratings of the transformers* is 57.7 per cent of the original load carried by the $\Delta - \Delta$ bank.

$$\boxed{S_{V-V} = \frac{1}{\sqrt{3}} S_{\Delta-\Delta} = 57.7\% \text{ of } S_{\Delta-\Delta}.}$$

Also in open-delta system

$$\frac{\text{VA per transformer}}{\text{total } 3\ \phi \text{ VA}} = \frac{V_{2B}\ I_{2B}}{\sqrt{3}\ V_{2B}\ I_{2B}} = \frac{1}{\sqrt{3}} = 0.577$$

Thus, the VA supplied by each transformer in a $V - V$ system is not half (50%) of the total VA but it is 57.7 per cent.

If three transformers in $\Delta - \Delta$ are supplying rated load and as soon as it becomes a $V - V$ transformer, the current in each phase winding is increased by $\sqrt{3}$ times. That is, full line current flows in each of the two phase windings of the transformers. Thus each transformer in the $V - V$ system is *overloaded* by 73.2 per cent.

Therefore an important precaution is that the load should be reduced by $\sqrt{3}$ times in case of an open-delta connected transformer. Otherwise, serious overheating and further breakdown of the remaining two transformers may take place.

Summary

1. Load in VA per transformer in $V - V$ bank

$$= \frac{1}{\sqrt{3}} \times \text{ original load in } \Delta - \Delta \text{ bank.}$$

2. Per cent rated VA load carried by each transformer in $V - V$ bank,

$$= \frac{\text{load in VA per transformer in } V - V}{\text{VA rating per transformer}} \times 100$$

3. Total VA rating of $V - V$ bank

$$= \sqrt{3} \times \text{VA rating per transformer in } V - V \text{ bank}$$

4. Ratio of VA ratings $\dfrac{S_{V-V}}{S_{\Delta-\Delta}} = \dfrac{1}{\sqrt{3}} = 0.577$

5. Per cent increase in VA load on each transformer when one transformer is removed

$$= \frac{[(\text{VA load per transformer in } V - V) - (\text{original VA load per transformer in } \Delta - \Delta)] \times 100}{\text{original VA load per transformer in } \Delta - \Delta} = 73.2\ \%$$

Power Supplied by Open-Delta System

When a $V-V$ bank of two transformers supplies a balanced 3-phase load operating at a power factor $\cos\phi$, the angle between the line voltage and line current in one transformer is $(30° + \phi)$ while the angle between the line voltage and line current in the other transformer is $(30° - \phi)$. Therefore one transformer operates at a power factor of $\cos(30° + \phi)$ and the other at $\cos(30° - \phi)$ and the powers supplied by the two transformers are

$$P_1 = V_L I_L \cos(30° + \phi)$$
$$P_2 = V_L I_L \cos(30° - \phi).$$

The total power supplied by the transformers

$$P = P_1 + P_2$$
$$= V_L I_L \cos(30° + \phi) + V_L I_L \cos(30° - \phi)$$
$$= V_L I_L [\cos 30° \cos\phi - \sin 30° \sin\phi + \cos 30° \cos\phi + \sin 30° \sin\phi]$$
$$= 2\, V_L I_L \cos 30° \cos\phi$$
$$\boxed{P = \sqrt{3}\, V_L I_L \cos\phi}$$

At unity power factor of the load, $\cos\phi = 1$, $\phi = 0°$.

Therefore the power supplied by each transformer is

$$P_1 = P_2 = V_L I_L \cos 30° = \frac{\sqrt{3}}{2} V_L I_L$$

2.22 APPLICATIONS OF OPEN-DELTA SYSTEM

The open-delta system is used in one of the following circumstances :

1. As a temporary measure when one transformer of a $\Delta - \Delta$ system is damaged and removed for repair and maintenance.

2. To provide service in a new development area where the full growth of load may require several years. In such cases a $V-V$ system is installed in the initial stage. This reduces the initial cost. Whenever the need arises at a future date to accommodate the growth in the power demand, a third transformer is added for $\Delta - \Delta$ operation. The addition of one transformer increases the capacity of the total bank by 73.2 per cent.

3. To supply a combination of large single-phase and smaller 3-phase loads.

Example 2.10. *A 400 kVA load at 0.7 pf lagging is supplied by three $1-\phi$ transformers connected in $\Delta - \Delta$. Each of the $\Delta - \Delta$ transformers is rated at 200 kVA, 2300/230 V. If one defective transformer is removed from service, calculate for the $V-V$ connection :*

(a) *the kVA load carried by each transformer.*

(b) *percent rated load carried by each transformer.*

(c) *total kVA ratings of the transformer bank in $V-V$.*

(d) *ratio of $V-V$ bank to $\Delta - \Delta$ bank transformer ratings.*

(e) *percent increase in load on each transformer when one transformer is removed.*

SOLUTION. (*a*) Load in kVA per transformer in $V-V$ bank

$$= \frac{1}{\sqrt{3}} \times \text{ original load in } \Delta - \Delta \text{ bank}$$

$$= \frac{1}{\sqrt{3}} \times 400 = 230.9 \text{ kVA}$$

(*b*) Per cent rated load carried by each transformer

$$= \frac{\text{load in kVA per transformer in } V-V}{\text{kVA rating per transformer in } V-V} \times 100$$

$$= \frac{230.9}{200} \times 100 = 115.5 \text{ per cent}$$

(*c*) Total kVA rating of the $V-V$ bank

$$= \sqrt{3} \times \text{kVA rating per transformer in } \Delta - \Delta$$

$$= \sqrt{3} \times 200 = 346.4 \text{ kVA}$$

(*d*) $\dfrac{V-V \text{ bank rating}}{\Delta - \Delta \text{ bank rating}}$

$$\frac{S_{V-V}}{S_{\Delta-\Delta}} = \frac{346.4}{3 \times 200} = 0.577 \text{ pu} = 57.7\%$$

(*e*) Original load kVA per transformer in $\Delta - \Delta$

$$= \frac{1}{3} \times \text{total load kVA}$$

$$= \frac{1}{3} \times 400 = 133.3 \text{ kVA}$$

Percent increase in load on each transformer when one transformer is removed

$$= \frac{\text{increase in kVA load per transformer in } V-V \text{ bank}}{\text{original kVA load per transformer in } \Delta - \Delta \text{ bank}}$$

$$= \frac{(\text{VA load per transformer in V-V} - \text{original VA load per transformer in } \Delta - \Delta)}{\text{original VA load per transformer in } \Delta - \Delta} \times 100$$

$$= \frac{230.9 - 133.3}{133.3} \times 100 = 73.2\%$$

EXAMPLE 2.11. *A 900-kVA load is supplied by three transformers connected in delta-delta. The primaries are connected to a 2300-V supply line, while the secondaries are connected to a 230-V load. If one transformer is removed for repair, what load can the remaining two transformers supply without overloading ? What are the currents in the high-and low voltage sides of the transformer windings when connected in open delta ?*

SOLUTION. Delta-delta operation

If V_{L_1} and I_{L_1} are the line voltage and line current respectively on *hv* side,

$$S_{\Delta-\Delta} = \sqrt{3}\, V_{L_1} I_{L_1}$$

$$900 \times 10^3 = \sqrt{3} \times 2300 \times I_{L_1}$$

$$\therefore \quad I_{L_1} = \frac{900 \times 10^3}{\sqrt{3} \times 2300} = 225.9 \text{ A}$$

Transformer current per phase on *hv* side

$$I_{p1} = \frac{1}{\sqrt{3}} \times \text{line current} = \frac{1}{\sqrt{3}} \times 225.9 = 130.4 \text{ A}$$

Transformer current per phase on *lv* side

$$I_{p2} = 130.4 \times \frac{2300}{230} = 1304 \text{ A}$$

Open-delta operation

When one transformer is removed from chosed delta, the currents through the phase windings of the transformers should not exceed the rated currents to avoid overloading. Therefore the permitted phase winding currents on *hv* and *lv* sides are 130.4 A and 1304 A respectively. For open delta the line is in series with the windings of the transformer and therefore, the secondary line current is equal to the rated secondary current of the transformer. Therefore the VA load that can be carried by the open-delta bank without overloading is

$$S_{V-V} = \sqrt{3}\, V_{2B}\, I_{2B}$$
$$= \sqrt{3} \times V_{L_2} \times I_{p2}$$
$$= \sqrt{3} \times 230 \times 1304 \text{ VA} = 519.5 \text{ kVA}$$

ALTERNATIVELY

$$S_{V-V} = \sqrt{3}\, V_{L_2} \times \frac{I_{L_2}}{\sqrt{3}} = V_{L_2} I_{L_2} = V_{L_1} I_{L_1}$$
$$= \frac{S_{\Delta-\Delta}}{\sqrt{3}} = \frac{900}{\sqrt{3}} = 519.6 \text{ kVA}$$

It is seen that the kVA supplied by the $V-V$ bank is $\frac{1}{\sqrt{3}}$ times the original closed-delta kVA.

EXAMPLE 2.12. *Two 40-kVA single-phase transformers are connected in open-delta to supply a 230-V balanced 3-phase load.*

(*a*) *What is the total load that can be supplied without overloading either transformer ?*

(*b*) *When the delta is closed by the addition of a third 40-kVA transformer, what total load can now be supplied ?*

(*c*) *Percent increase in load.*

SOLUTION. (*a*) The rated secondary transformer current

$$I_{2B} = \frac{40 \times 10^3}{230} = 173.9 \text{ A}$$

This is also the load line current. Therefore the load kVA is

$$= \sqrt{3}\, V_{2B}\, I_{2B} \times 10^{-3}$$
$$= \sqrt{3} \times 230 \times 173.9 \times 10^{-3} = 69.28 \text{ kVA}$$

$$\frac{\text{load kVA}}{\text{total capacity of two individual transformers}} = \frac{69.28}{2 \times 40} = 0.866$$

(b) When the delta is closed by the addition of a third 40-kVA transformer, the Δ – Δ bank will operate at full capacity of the individual transformers. Therefore the load kVA supplied by the Δ – Δ bank is

$$S_{\Delta-\Delta} = 3 \times 40 = 120 \text{ kVA}$$

(c) Percent increase in load kVA $= \dfrac{120 - 69.28}{69.28} \times 100 = 73.2$

2.23 SCOTT THREE-PHASE/TWO-PHASE CONNECTION

The Scott connection is the most common method of connecting two single-phase transformers to perform the 3-phase to two-phase conversion and vice-versa. The two transformers are connected electrically but not magnetically. One transformer is called **main transformer** and the other is known as **auxiliary** or **teaser transformer**. Fig. 2.20 shows the Scott transformer connection. The main transformer is centre-tapped at D and is connected across the lines B and C of the 3-phase side. It has primary BC and secondary $a_1 a_2$. The teaser transformer is connected between the line terminal A and the centre tapping D. It has primary AD and secondary $b_1 b_2$.

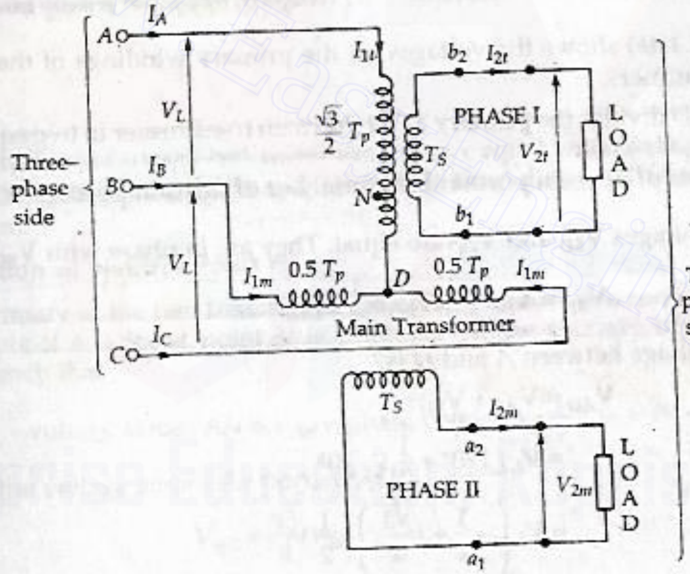


Fig. 2.20. *Scott connection of transformers.*

Frequently identical interchangeable transformers are used for the Scott connection, in which each transformer has a primary winding of T_p turns and is provided with tappings at 0.289 T_p, 0.5 T_p and 0.866 T_p.

Phasor Diagram

The line voltages of the 3-phase system V_{AB}, V_{BC} and V_{CA}, which are balanced, are shown in Fig. 2.21(a). The same voltages are shown as a closed equilateral triangle in Fig. 2.21(b).

$$|\mathbf{V}_{AB}| = |\mathbf{V}_{BC}| = |\mathbf{V}_{CA}| = V_L \qquad \text{(say)}$$

Let $\mathbf{V}_{BC}$ be taken as reference voltage so that

$$V_{BC} = V_L \angle 0°$$
$$V_{CA} = V_L \angle -120°$$
$$V_{AB} = V_L \angle +120°$$

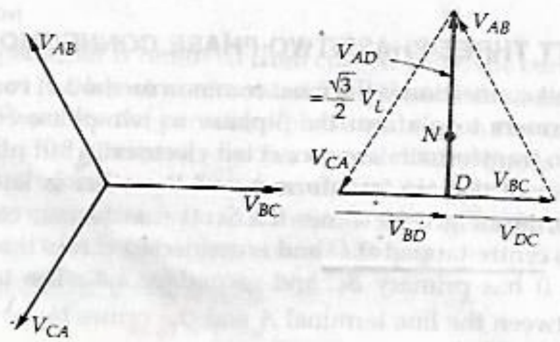


Fig. 2.21 *(a) Three-phase input voltages, (b) Voltages on transformer primary windings*

Fig. 2.21(*b*) shows the voltages on the primary windings of the main and teaser transformers.

Since *D* divides the primary *BC* of the main transformer in two equal halves,

number of turns in portion BD = number of turns in portion $DC = \frac{T_P}{2}$.

The voltages $\mathbf{V}_{BD}$ and $\mathbf{V}_{DC}$ are equal. They are in phase with $\mathbf{V}_{BC}$.

$$\therefore \qquad \mathbf{V}_{BD} = \mathbf{V}_{DC} = \frac{1}{2}\mathbf{V}_{BC} = \frac{1}{2} V_L \angle 0°$$

The voltage between *A* and *D* is

$$\mathbf{V}_{AD} = \mathbf{V}_{AB} + \mathbf{V}_{BD}$$
$$= V_L \angle 120° + \frac{1}{2} V_L \angle 0°$$
$$= V_L\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) + \frac{1}{2} V_L$$
$$= j\frac{\sqrt{3}}{2} V_L = 0.866\, V_L \angle 90°$$

Thus, the voltage $\mathbf{V}_{AD}$ in the primary of the teaser transformer is 0.866 of that in main transformer and is 90° from it in time. In other words, the teaser transformer has a primary voltage rating that is $\frac{\sqrt{3}}{2}$ or 0.866 (or 86.6 per cent) of the voltage rating of the main transformer.

Voltage V_{AD} is applied to the primary of the teaser transformer and therefore, the secondary voltage V_{2t} of the teaser transformer will lead the secondary terminal voltage V_{2m} of the main transformer by 90° as shown in Fig. 2.22.

For the same flux in each transformer, the voltage per turn should be the same. In order to keep voltage per turn same in the primary of the main transformer and primary of the teaser transformer, the number of turns in the primary of the teaser transformer, that is, in portion AD, should be equal to $\frac{\sqrt{3}}{2} T_p$.

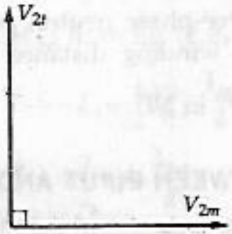


Fig. 2.22.

Then,
$$\frac{V_{s_1}}{V_{AD}} = \frac{T_s}{T_{AD}}$$

$$V_{2t} = \frac{T_s}{T_{AD}} V_{AD} = \frac{T_s}{\frac{\sqrt{3}}{2} T_p} \times \frac{\sqrt{3}\, V_L}{2}$$

$$= \frac{T_s}{T_p} V_L = V_{2m}.$$

Thus, the secondaries of *both* transformers have equal voltage ratings. Since V_{2t} and V_{2m} are equal in magnitude and 90° apart in time, they result in a balanced 2-phase system.

2.23.1 Position of Neutral Point N

The primary of the two transformers may have a four-wire connection to a 3-phase supply if a tapping point N is provided on the primary of the teaser transformer such that

$$\text{voltage across } AN = V_{AN} = \text{phase voltage} = \frac{V_L}{\sqrt{3}}$$

Since the voltage across the portion AD

$$V_{AD} = \frac{\sqrt{3}}{2} V_L,$$

voltage across the portion ND

$$V_{ND} = V_{AD} - V_{AN} = \frac{\sqrt{3}}{2} V_L - \frac{V_L}{\sqrt{3}} = \frac{V_L}{2\sqrt{3}}.$$

In order to keep the same voltage per turn,

turns in portion AN, $T_{AN} = \frac{T_p}{\sqrt{3}} = 0.577\, T_p$

turns in portion ND, $T_{ND} = \frac{1}{2\sqrt{3}} T_p = 0.288\, T_p$

turns in portion AD, $T_{AD} = \frac{\sqrt{3}}{2} T_p = 0.866\, T_p$

$$\therefore \quad \frac{T_{AN}}{T_{ND}} = \frac{T_p}{\sqrt{3}} \div \left(\frac{T_p}{2\sqrt{3}}\right) = 2$$

Thus, it is observed that the neutral point N divides the primary of the teaser transformer in a ratio

$$AN : ND = 2 : 1$$

In other words, the three-phase neutral point N is located on the teaser primary at one third of the winding distance from D where the voltage is $\frac{V_L}{\sqrt{3}}$ (= 0.577) in AN and 0.289 V_L in ND.

2.24 RELATIONSHIP BETWEEN INPUT AND OUTPUT CURRENTS

Let I_{1m} = current in the primary of the main transformer

I_{2m} = current in the secondary of the main transformer

I_{1t} = current in the primary of the teaser transformer

I_{2t} = current in the secondary of the teaser transformers

I_A, I_B and I_C are the line currents on 3-phase side.

From Fig. 2.20, $I_{1t} = I_A$.

Since the two secondaries are identical

$$|\mathbf{I}_{2m}| = |\mathbf{I}_{2t}| = I_2 \text{ (say)}$$

Let the magnetizing current be neglected. For the mmf balance of the *teaser* transformer

$$I_{1t} T_{AD} = I_{2t} T_s$$

$$\mathbf{I}_A \left(\frac{\sqrt{3}}{2} T_p\right) = \mathbf{I}_{2t} T_s$$

$$\therefore \quad \mathbf{I}_A = \mathbf{I}_{1t} = \frac{2}{\sqrt{3}} \frac{T_s}{T_p} \mathbf{I}_{2t} = \frac{2}{\sqrt{3}} \frac{\mathbf{I}_{2t}}{a} = 1.15 \frac{\mathbf{I}_{2t}}{a} \qquad (2.24.1)$$

and

$$\mathbf{I}_{2t} = \frac{\sqrt{3}}{2} \frac{T_p}{T_s} \mathbf{I}_A = \frac{\sqrt{3}}{2} a\, \mathbf{I}_A \qquad (2.24.2)$$

For the mmf balance of the *main* transformer

$$\mathbf{I}_B \frac{T_p}{2} - \mathbf{I}_c \frac{T_p}{2} = \mathbf{I}_{2m} T_s$$

$$\therefore \quad \mathbf{I}_B - \mathbf{I}_C = 2 \frac{T_s}{T_p} \mathbf{I}_{2m} = 2 \frac{\mathbf{I}_{2m}}{a} \qquad (2.24.3)$$

and

$$\mathbf{I}_{2m} = \frac{1}{2} \frac{T_p}{T_s} (\mathbf{I}_B - \mathbf{I}_C) = \frac{a}{2} (\mathbf{I}_B - \mathbf{I}_C) \qquad (2.24.4)$$

For a 3-wire system

$$\mathbf{I}_A + \mathbf{I}_B + \mathbf{I}_C = 0 \qquad (2.24.5)$$

$$\therefore \quad \mathbf{I}_C = -\mathbf{I}_A - \mathbf{I}_B \qquad (2.24.6)$$

Substituting the value of I_C form Eq. (2.24.6) in Eq. (2.24.3)

$$\mathbf{I}_B + (\mathbf{I}_A + \mathbf{I}_B) = \frac{2\,\mathbf{I}_{2m}}{a}$$

$$\mathbf{I}_B = -\frac{1}{2}\mathbf{I}_A + \frac{\mathbf{I}_{2m}}{a} \tag{2.24.7}$$

Substituting the value of $\mathbf{I}_B$ from Eq. (2.24.7) in Eq. (2.24.6)

$$\mathbf{I}_C = -\mathbf{I}_A + \frac{1}{2}\mathbf{I}_A - \frac{\mathbf{I}_{2m}}{a}$$

$$\mathbf{I}_C = -\frac{1}{2}\mathbf{I}_A - \frac{\mathbf{I}_{2m}}{a} \tag{2.24.8}$$

In the above treatment it has not been mentioned whether the transformer is converting from 3-phase to 2-phase or vice-versa. Both the modes are possible. Equations (2.24.2) and (2.24.4) can be solved for 2-phase currents when the three-phase currents are known. Similarly, equations (2.24.1) (2.24.7) and (2.24.8) can be solved for 3-phase currents when 2-phase currents are known. These equations are valid for balanced as well as unbalanced loads.

Balanced load

If the 2-phase currents are balanced, the three-phase currents are also balanced. This may be proved as follows :

Since the two secondaries are identical with T_s turns

$$|\mathbf{I}_{2m}| = |\mathbf{I}_{2t}| = I_2 \qquad \text{(say)}$$

Let $\mathbf{I}_{2t}$ be taken as the reference phasor, so that

$$\mathbf{I}_{2t} = \mathbf{I}_2 \angle 0^\circ \text{ and } \mathbf{I}_{2m} = I_2 \angle -90^\circ = -j\,I_2$$

From Eq. (2.24.1)

$$\mathbf{I}_A = \frac{2}{\sqrt{3}}\frac{\mathbf{I}_{2t}}{a} = \frac{2}{\sqrt{3}}\frac{I_2}{a}\angle 0^\circ \tag{2.24.9}$$

From Eq. (2.24.7)

$$\mathbf{I}_B = -\frac{1}{2}\mathbf{I}_A + \frac{\mathbf{I}_{2m}}{a} = -\frac{1}{\sqrt{3}}\frac{I_2}{a}\angle 0^\circ - j\frac{I_2}{a}$$

$$= \frac{2}{\sqrt{3}}\frac{I_2}{a}\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right)$$

or

$$\mathbf{I}_B = \frac{2}{\sqrt{3}}\frac{I_2}{a}\angle -120^\circ \tag{2.24.10}$$

From Eq. (2.24.8), $\mathbf{I}_C = -\frac{1}{2}\mathbf{I}_A - \frac{\mathbf{I}_{2m}}{a}$

$$= \frac{2}{\sqrt{3}}\frac{I_2}{a}\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right)$$

or

$$\mathbf{I}_C = \frac{2}{\sqrt{3}}\frac{I_2}{a}\angle +120^\circ \tag{2.24.11}$$

From Eqs. (2.24.9), (2.24.10) and (2.24.11) it is seen that the three line currents I_A, I_B and I_C are equal in magnitude and 120° apart in phase and so they form a balanced 3-phase system. In other words, if the 2-phase load is balanced, the three-phase input is also balanced and vice versa.

For a 2-phase balanced load

$$|\mathbf{I}_{2t}| = |\mathbf{I}_{2m}| = I_2$$

The phase difference ϕ between $\mathbf{V}_{2t}$ and $\mathbf{I}_{2t}$ is the same as the phase difference between $\mathbf{V}_{2m}$ and $\mathbf{I}_{2m}$. For lagging power factor cos ϕ, the phasor diagrams are shown in Fig. 2.23.

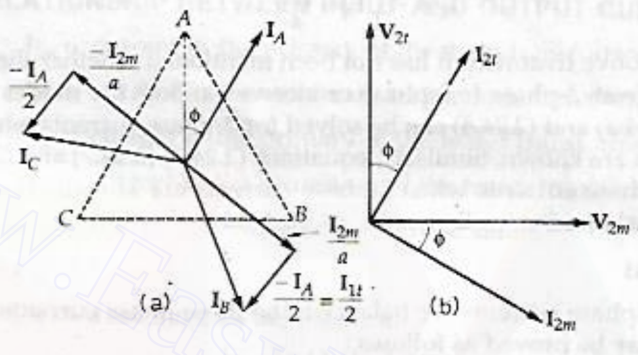


Fig. 2.23. *Phasor diagrams (a) 3-phase side (b) 2 phase side.*

From the phasor diagram of Fig. 2.23,

$$|\mathbf{I}_B| = |\mathbf{I}_C| = \sqrt{\left(\frac{I_{2m}}{a}\right)^2 + \left(\frac{1}{2}I_{1t}\right)^2}$$

$$|\mathbf{I}_A| = |\mathbf{I}_{1t}|$$

Unbalanced load

The phasor diagram for an unbalanced 2-phase load is shown in Fig. 2.24. The power factors of currents of 2-phase sides are different and their magnitudes are also different. The procedure of drawing phasor diagram is similar to that as explained earlier.

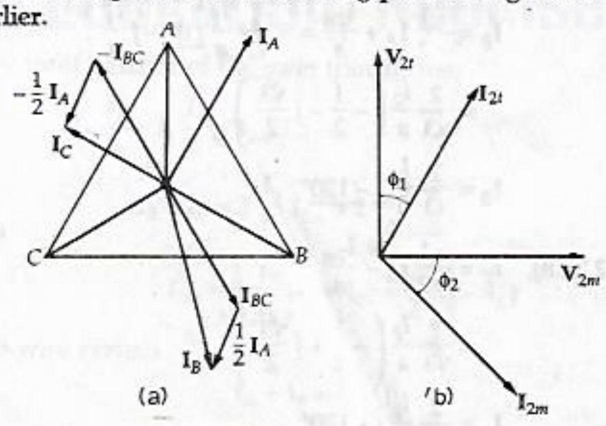


Fig. 2.24. *Phasor diagrams for Scott connection (unbalanced load). (a) 3-phase side (b) 2-phase side.*

2.25 APPLICATIONS OF SCOTT CONNECTION

1. Electric furnace installations where it is desired to operate two single-phase furnaces together and draw a balanced load from the 3-phase supply.
2. To supply single-phase loads such as electric trains which are so scheduled as to keep the load on the 3-phase system as nearly balanced as possible.
3. To link a 3-phase system with a 2-phase system with flow of power in either direction.

The Scott connection permits conversion of a 3-phase system to a 2-phase system and vice versa. But since 2-phase generators are not available, the conversion from 2-phase to 3-phase is not used in practice.

EXAMPLE 2.13. *A Scott-connected transformer supplies two single-phase furnaces at 100 V, each taking 200 kW. The load on the leading phase is at unity power factor and that on the other phase is 0.8 lagging power factor. The 3-phase input line voltage is 11000 V. Calculate the line currents on the primary side. Neglect the magnetizing current and leakage impedance. Draw the phasor diagrams.*

SOLUTION. Main transformers turns ratio

$$a = \frac{11000}{100} = 110$$

For teaser transformer

$$P_{2t} = V_{2t} I_{2t} \cos\phi_{2t}$$

$$I_{2t} = \frac{P_2}{V_{2t}\cos\phi_{2t}} = \frac{200\times10^3}{100\times1} = 2000 \text{ A}$$

For main transformer

$$P_{2m} = V_{2m} I_{2m} \cos\phi_{2m}$$

$$I_{2m} = \frac{P_{2m}}{V_{2m}\cos\phi_{2m}} = \frac{200\times10^3}{100\times0.8} = 2500 \text{ A}$$

Let $\mathbf{V}_{2m}$ be taken as reference phasor.

$$\therefore \quad \mathbf{I}_{2t} = 2000 \angle 90^\circ = j\,2000 \text{ A}$$

$$\mathbf{I}_{2m} = 2500 \angle -\cos^{-1} 0.8$$

$$= 2500\,(0.8 - j\,0.6) = (2000 - j\,1500) \text{ A}$$

$$\therefore \quad \mathbf{I}_A = \frac{2}{\sqrt{3}}\frac{\mathbf{I}_{2t}}{a}$$

$$= \frac{2}{\sqrt{3}} \times \frac{1}{110}(j\,2000) = j\,21 \text{ A}$$

$$|\mathbf{I}_A| = 21 \text{ A}$$

$$\mathbf{I}_B = -\frac{1}{2}\mathbf{I}_A + \frac{\mathbf{I}_{2m}}{a}$$

$$= -j\frac{21}{2} + \frac{1}{110}(2000 - j\,1500)$$

$$= (18.18 - j\,24.1)\text{ A}$$

$$|\mathbf{I}_B| = \sqrt{(18.18)^2 + (24.1)^2} = 30.1\text{ A}$$

$$\mathbf{I}_C = -\frac{1}{2}\mathbf{I}_A - \frac{\mathbf{I}_{2m}}{a}$$

$$= -j\frac{21}{2} - \frac{1}{110}(2000 - j\,1500)$$

$$= (-18.18 + j\,3.13)\text{ A}$$

$$|\mathbf{I}_C| = \sqrt{(18.18)^2 + (3.13)^2} = 18.44\text{ A}$$

Thus, the numerical values of line currents are 21 A, 30.1 A, and 18.44 A respectively.

The phasor diagrams are shown in Fig. 2.25.

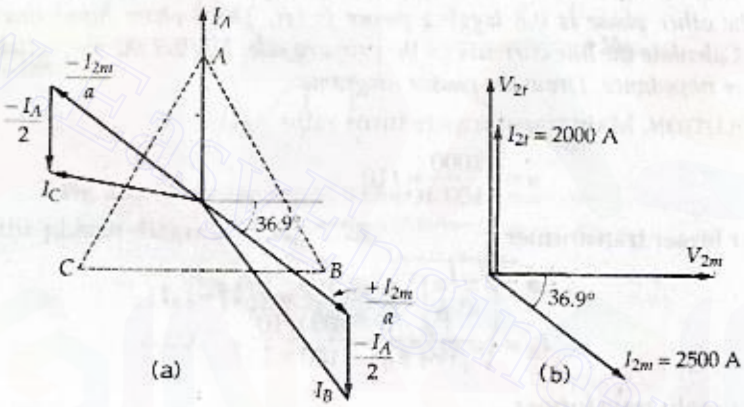


Fig. 2.25. *Phasor diagrams.*

EXAMPLE 2.14. *Two 100 V, 1-phase furnaces take loads of 600 kW and 900 kW respectively at a power factor of 0.707 lagging and are supplied from 6600 V, 3-phase supply through a Scott connected transformer. Calculate the currents in the 3-phase lines. Neglect transformer losses.*

SOLUTION. $a = \frac{660}{100} = 66$

For main transformer

$$P_{2m} = V_{2m} I_{2m} \cos\phi_{2m}$$

$$I_{2m} = \frac{P_{2m}}{V_{2m}\cos\phi_{2m}} = \frac{900 \times 10^3}{100 \times 0.707} = 12730\text{ A}$$

For teaser transformer

$$I_{2t} = \frac{P_{2t}}{V_{2t}\cos\phi_{2t}} = \frac{600 \times 10^3}{100 \times 0.707} = 8486\text{ A}$$

$$I_{1t} = \frac{2}{\sqrt{3}} \frac{I_{2t}}{a} = \frac{2}{\sqrt{3}} \times \frac{8486}{66} = 148.5 \text{ A}$$

$$I_{1m} = \frac{I_{2m}}{a} = \frac{12730}{66} = 192.9 \text{ A}$$

$$\therefore \quad I_A = I_{1t} = 148.5 \text{ A}$$

$$I_B = I_C = \sqrt{I_{1m}^2 + \left(\frac{1}{2} I_{1t}\right)^2}$$

$$= \sqrt{(192.9)^2 + \left(\frac{148.5}{2}\right)^2} = 206.7 \text{ A}$$

EXAMPLE 2.15. *Two single-phase furnaces working at 100 V are connected to a 3300-V, 3-phase supply through Scott-connected transformers. Determine the currents in the 3-phase lines when the power taken by each furnace is 500 kW at a power factor of 0.8 lagging. Neglect transformer losses.*

SOLUTION. $a = \frac{3300}{100} = 33$

For teaser transformer

$$P_{2t} = V_{2t} I_{2t} \cos \phi_{2t}$$

$$I_{2t} = \frac{P_{2t}}{V_{2t} \cos \phi_{2t}} = \frac{500 \times 10^3}{100 \times 0.8} = 6250 \text{ A}$$

Since the 2-phase load is balanced, the 3-phase side is also balanced. For the mmf balance of the teaser transformer

$$I_A \times \frac{\sqrt{3}}{2} T_p = I_{2t} T_s$$

$$I_A = \frac{2}{\sqrt{3}} \frac{T_s}{T_p} I_{2t} = \frac{2}{\sqrt{3}} \times \frac{6250}{33} = 218.7 \text{ A}$$

$$\therefore \quad |\mathbf{I}_A| = |\mathbf{I}_B| = |\mathbf{I}_C| = 218.7 \text{ A}$$

EXAMPLE 2.16. *A Scott-connected transformer is fed from 6600 V, 2-phase network and supplies 3-phase power at 500 V between lines on a 4-wire system. If there are 500 turns per phase on the 3-phase side, find the number of turns in the low-voltage windings and the position of the tapping of the neutral wire.*

SOLUTION. $\frac{V_{hv}}{T_{hv}} = \frac{V_{lv}}{T_{lv}}$

$$\frac{6600}{500} = \frac{500}{T_{lv}}$$

$$T_{lv} = \frac{500 \times 500}{6600} = 38$$

$$T_{AD} = \frac{\sqrt{3}}{2} \times 38 = 33$$

$$T_{AN} = \frac{2}{3} T_{AD} = \frac{2}{3} \times 33 = 22$$

Therefore the neutral point is located at 22nd turn from point A.

EXAMPLE 2.17. *Two electric furnaces are supplied with single-phase current at 80 V from a 3-phase, 11000 V system by means of two single-phase Scott-conne transformers, with similar secondary windings. When the load on the main transfor is 800 kW and on the teaser transformer is 500 kW, determine the currents in three-phase lines (a) at unity power factor, (b) at 0.5 power factor lagging. Draw the pha diagrams.*

SOLUTION. (*a*) *Unity power factor* $a = \dfrac{11000}{80}$

$$I_{2t} = \frac{500 \times 10^3}{80 \times 1} = 6250 \text{ A}$$

$$I_{1t} = I_A = \frac{2}{\sqrt{3}} \frac{I_{2t}}{a} = \frac{2}{\sqrt{3}} \times \frac{6250 \times 80}{11000} = 52.48 \text{ A}$$

$$I_{2m} = \frac{800 \times 10^3}{80 \times 1} = 10{,}000 \text{ A}$$

$$\frac{I_{2m}}{a} = \frac{80}{11000} \times 10000 = 72.72 \text{ A}$$

$$|\mathbf{I}_B| = |\mathbf{I}_C| = \sqrt{\left(\frac{I_{2m}}{a}\right)^2 + \left(\frac{1}{2} I_{1t}\right)^2}$$

$$= \sqrt{(72.72)^2 + \left(\frac{52.48}{2}\right)^2} = 77.3 \text{ A}$$

(*b*) *0.5 power factor lagging*

$$I_{2t} = \frac{500 \times 10^3}{80 \times 0.5} = 12500 \text{ A}$$

$$I_A = I_{1t} = \frac{2}{\sqrt{3}} \frac{I_{2t}}{a} = \frac{2}{\sqrt{3}} \times \frac{12500 \times 80}{11000} = 104.97 \text{ A}$$

$$I_{2m} = \frac{800 \times 10^3}{80 \times 0.5} = 20000 \text{ A}$$

$$\frac{I_{2m}}{a} = \frac{2000 \times 80}{11000} = 145.45 \text{ A}$$

$$|\mathbf{I}_B| = |\mathbf{I}_C| = \sqrt{\left(\frac{I_{2m}}{a}\right)^2 + \left(\frac{1}{2} I_{1t}\right)^2}$$

$$= \sqrt{(145.45)^2 + \left(\frac{104.97}{2}\right)^2} = 154.6 \text{ A}$$

ALTERNATIVELY

At 0.5 p.f., each component of current is doubled. Hence the resultant is al doubled.

$$\therefore \quad I_A = 2 \times 52.48 = 104.96 \text{ A}$$

$$|\mathbf{I}_B| = |\mathbf{I}_C| = 2 \times 77.3 = 154.6 \text{ A}$$

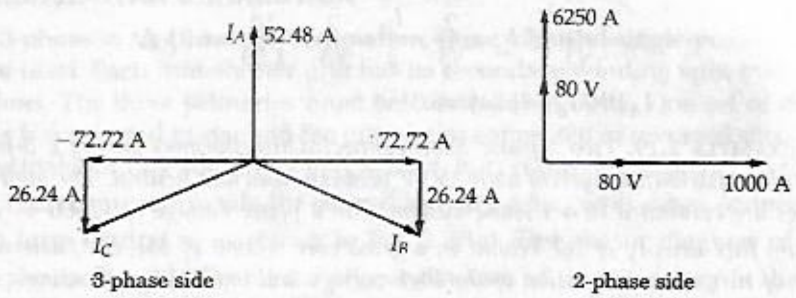


Fig. 2.26. *Phasor diagram at unity power factor.*

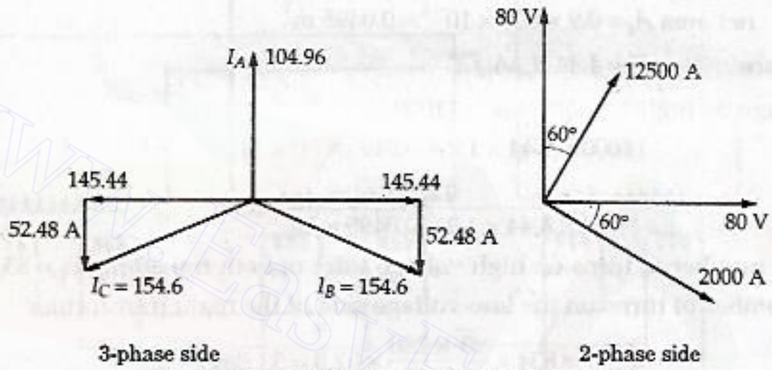


Fig. 2.27. *Phasor diagram at 0.5 power factor lagging.*

The phasor diagrams are shown in Fig. 2.26 and 2.27.

EXAMPLE 2.18. *A 2-phase 240 V supply is to be obtained from a 3-phase 3-wire 440 V supply by means of a pair of Scott-connected single-phase transformers. Determine the turns ratio of the main and teaser transformers. Find the input current in each of the three-phase lines when each of the 2-phase currents is 10 A lagging behind the respective phase voltage by 36.9°.*

SOLUTION. *Main transformer*

$$V_{P_1} = 440\text{ V}, \quad V_{2m} = 240\text{ V}$$

$$\frac{\text{primary turns}}{\text{secondary turns}} = a = \frac{V_{p1}}{V_{2m}} = \frac{440}{240} = 1.83$$

Teaser transformer

Primary voltage $\quad V_{p2} = \frac{\sqrt{3}}{2} V_{p1} = \frac{\sqrt{3}}{2} \times 440 = 381\text{ V}$

Secondary voltage $\quad V_{2t} = 240\text{ V}$

$$\frac{\text{primary turns}}{\text{secondary turns}} = \frac{381}{240} = 1.588$$

Since the 2-phase load is balanced, the 3-phase side is also balanced. For the mmf balance of the teaser transformer

$$I_A \times \frac{\sqrt{3}}{2} T_p = I_{2t} T_s$$

$$I_A = \frac{2}{\sqrt{3}} \frac{T_s}{T_p} I_{2t} = \frac{2}{\sqrt{3}} \times \frac{I_{2t}}{a} = \frac{2}{\sqrt{3}} \times \frac{10}{1.83} = 6.31 \text{ A}$$

$$\therefore \qquad |\mathbf{I}_A| = |\mathbf{I}_B| = |\mathbf{I}_C| = 6.31 \text{ A}$$

EXAMPLE 2.19. *Two 1-phase Scott-connected transformers supply a 3-phase, 4-wire, 50 Hz distribution system with 250 V between lines and neutral. The high-voltage windings are connected to a 2-phase system with a phase voltage of 11000 V. Allow a maximum flux density of 1.2 Wb/m^2 in a gross core section of 550 cm^2, determine the number of turns in each section of the high-voltage and low-voltage windings, and the position of the neutral point.*

SOLUTION. Let net area = 0.9 × gross area

$\therefore$ net area $A_i = 0.9 \times 550 \times 10^{-4} = 0.0495 \text{ m}^2$

Since $\qquad E = 4.44\, B_m A_i f T$

and voltage on high-voltage side = 11000 V

$$11000 = 4.44 \times 1.2 \times 0.0495 \times 50 \times T_s$$

$$T_s = \frac{11000}{4.44 \times 1.2 \times 0.0495 \times 50} = 834$$

$\therefore$ number of turns on high voltage sides of both transformers = 834.

Number of turns on the low-voltage side of the main transformer

$$= 834 \times \frac{\sqrt{3} \times 250}{11000} \times 32.8 = 33 \ (say)$$

Number of turns on the low-voltage side of the teaser transformer

$$= \frac{\sqrt{3}}{2} \times 33 = 28.6 = 30 \text{ say}$$

$$T_{AN} = \frac{2}{3} \times 30 = 20.$$

Therefore the neutral point is located at 20th turn from point A.

2.26 THREE-TO-SIX PHASE TRANSFORMATION

Rectifiers convert ac power to dc power. A smoother waveform is obtained on the dc side as the number of phases is increased. Objectionable harmonics in alternating currents are also reduced with a greater number of phases. The efficiency of conversion from ac to dc by rectifier and thyristor circuits increases with the increase in number of phases. Six phase is, therefore, preferable to 3-phase for rectification. Since 12-phase units are more complex and costlier than 6-phase units, 12-phase is used in larger installation units.

The following schemes of connections are commonly used for 3-phase to six-phase transformation :

1. Double star
2. Double delta
3. Six-phase star
4. Diametrical

2.27 DOUBLE-STAR CONNECTION

For 3-phase to six-phase transformation, three identical single-phase transformers are used. Each transformer unit has its secondary winding split into two equal sections. The three primaries must be connected in delta. One set of three secondaries is connected in star and the other set is connected in reversed star. The output terminals for one star are $a_1\, b_1\, c_1$ with $a_2\, b_2\, c_2$ connected together to form neutral n. The output terminals for second star are $a_4 b_4 c_4$ with $a_3 b_3 c_3$ connected together to form neutral n_1 as shown in Fig. 2.28(a). The phasor diagram of the primary is shown Fig. 2.28(b) and the phasor diagram of the secondary is shown in Fig. 2.28(c). Terminals $a_1\, b_1\, c_1$ and $a_4\, b_4\, c_4$ are connected to 6-phase load terminals

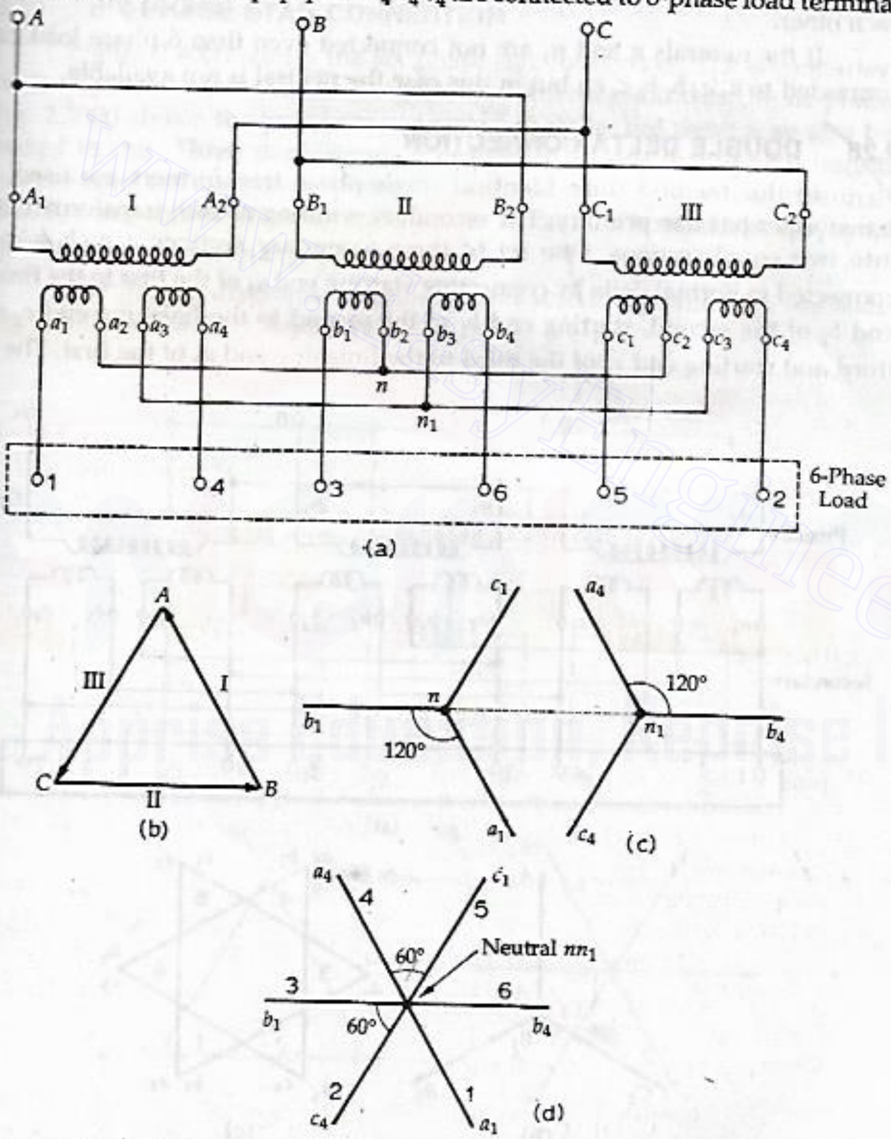


Fig. 2.28. *Three-to-six phase transformation : (a) Double star connection, (b) Phasor diagram of primary, (c) Phasor diagram of secondary, (d) Six-phase star connection.*

1, 3, 5, 4, 6, 2 respectively. The two neutral points n and n_1 may be conne together as shown in Fig. 2.28c. This neutral point will serve as the neutral p of dc supply from the rectifier. Thus, a true 6-phase star system with a neutr obtained.

In order to determine the connections of the transformer terminals to l terminals, the transformer terminal is marked 1 as shown in Fig. 2.28(d). The ot terminals are marked 2, 3, 4, 5, 6 in the clockwise direction as shown in Fig. 2.2 This Fig. shows that c_4 should be connected to load terminal 2, b_1 to load term 3, a_4 to load terminal 4 and so on.

In this system six voltages are obtained which are displaced by 60° fr each other.

If the neutrals n and n_1 are not connected even then 6-phase load ca connected to $a_1\ a_4\ b_1\ b_4\ c_1\ c_4$, but in this case the neutral is not available.

2.28 DOUBLE DELTA CONNECTION

In this method three identical single-phase transformers are used. E transformer has one primary. The secondary winding of each transformer is s into two equal sections. One set of three secondary sections $a_1\ a_2$, $b_1\ b_2$, $c_1\ c$ connected in normal delta by connecting starting end a_1 of the first to the finish end b_2 of the second, starting end b_1 of the second to the finishing end c_2 of t third and starting end c_1 of the third to the finishing end a_2 of the first. The ot

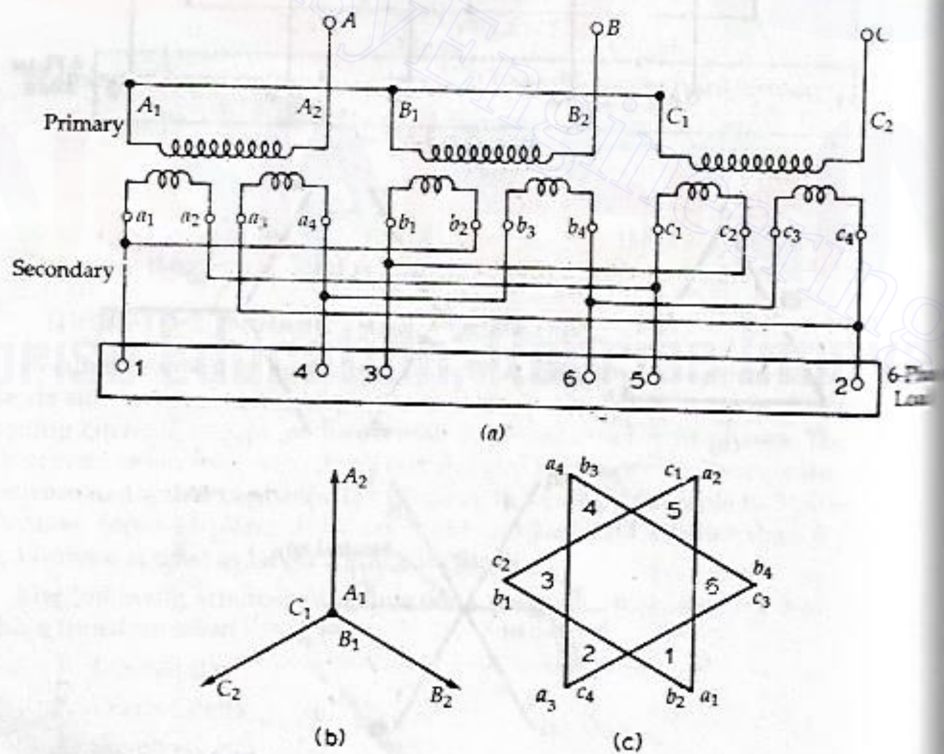


Fig. 2.29. *Three-to-six phase transformation (a) Double-delta connection (b) Phasor diagram of primary (c) Phasor diagram of secondary.*

set $a_3 a_4$, $b_3 b_4$, $c_3 c_4$ of the secondary winding is connected in reverse delta by connecting finishing end a_4 of the first to the starting end b_3 of the second and so on as shown in Fig. 2.29. The double-delta connection has the advantage of good harmonic elimination, but since most rectifier circuits require a secondary neutral, this connection is not suitable for rectifier circuits.

A true 6-phase supply is only obtained when the six terminals are connected to a suitable 6-phase load.

The primary windings may be connected in star because the two secondary deltas provide the required path for third harmonic currents.

2.29 SIX-PHASE STAR CONNECTION

Figure 2.30(*a*) shows the six phase star connection. The secondaries have centre taps which are connected together to form the neutral on the six-phase side. Fig. 2.30(*a*) shows the primaries connected in delta, but these may also be connected in star. Three single-phase transformers or one three-phase transformer may be used for three-to-six phase transformation. Six-phase load terminals 1, 2, 3, 4, 5, 6 are connected to six transformer secondary terminals $a_1\, c_2\, b_1\, a_2\, c_1\, b_2$ respectively as shown in Fig. 2.30(*a*).

The phasor diagrams of primary and secondary windings are shown in Fig. 2.30(*b*) and 2.30(*c*) respectively.

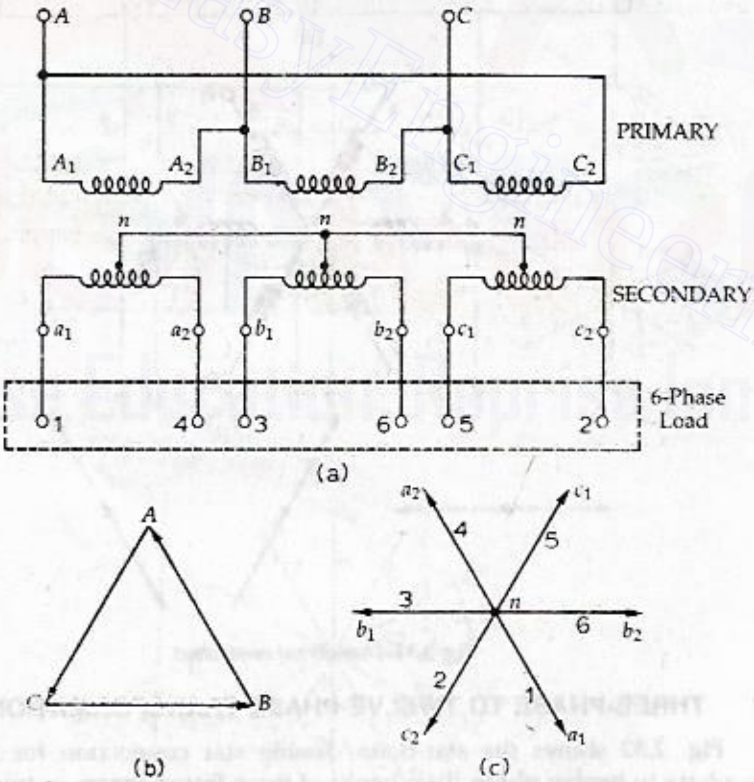


Fig. 2.30. *Three-to-six phase transformation (a) Six-phase star connection (b) Phasor diagram of primary (c) Phasor diagram of secondary*

2.30 DIAMETRICAL CONNECTION

This connection is also known as **diametral connection.** Fig. 2.31 shows another method in which six phase star may be split up. Here no centre tapping or neutral connections are required. In order to fix the phase relations of the voltages and produce a true six-phase supply, it is necessary that the six secondary terminals should be connected to six-phase load as shown in Fig. 2.31. Here transformer terminals a_1 a_2 b_1 b_2 c_1 c_2 are be connected to load terminals 1 4 3 6 5 2 respectively. Since the neutral is not available, this method cannot be used for rectifier circuits.

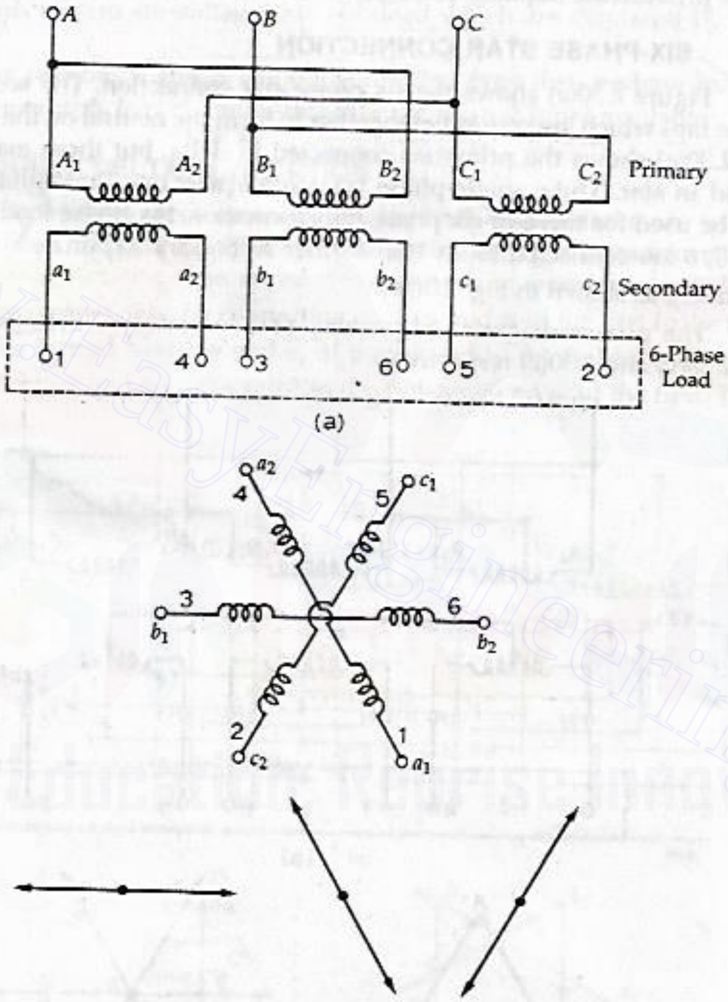


Fig. 2.31. *Diametrical connection.*

2.31 THREE-PHASE TO TWELVE-PHASE TRANSFORMATION

Fig. 2.32 shows the star-delta/double star connection for transforming three-phase to twelve phase. Two banks of three transformers or two three-phase transformers are required. The secondary windings are arranged in the form of pair of double-star connections. The primary of one set of transformers or of one of three-phase transformers is connected in star (Fig. 2.32*a*) while the primary of the other is connected in delta (Fig. 2.32*a*). There is a phase shift of 30° between the secondary star voltages of the two six-phase systems leading to a balanced

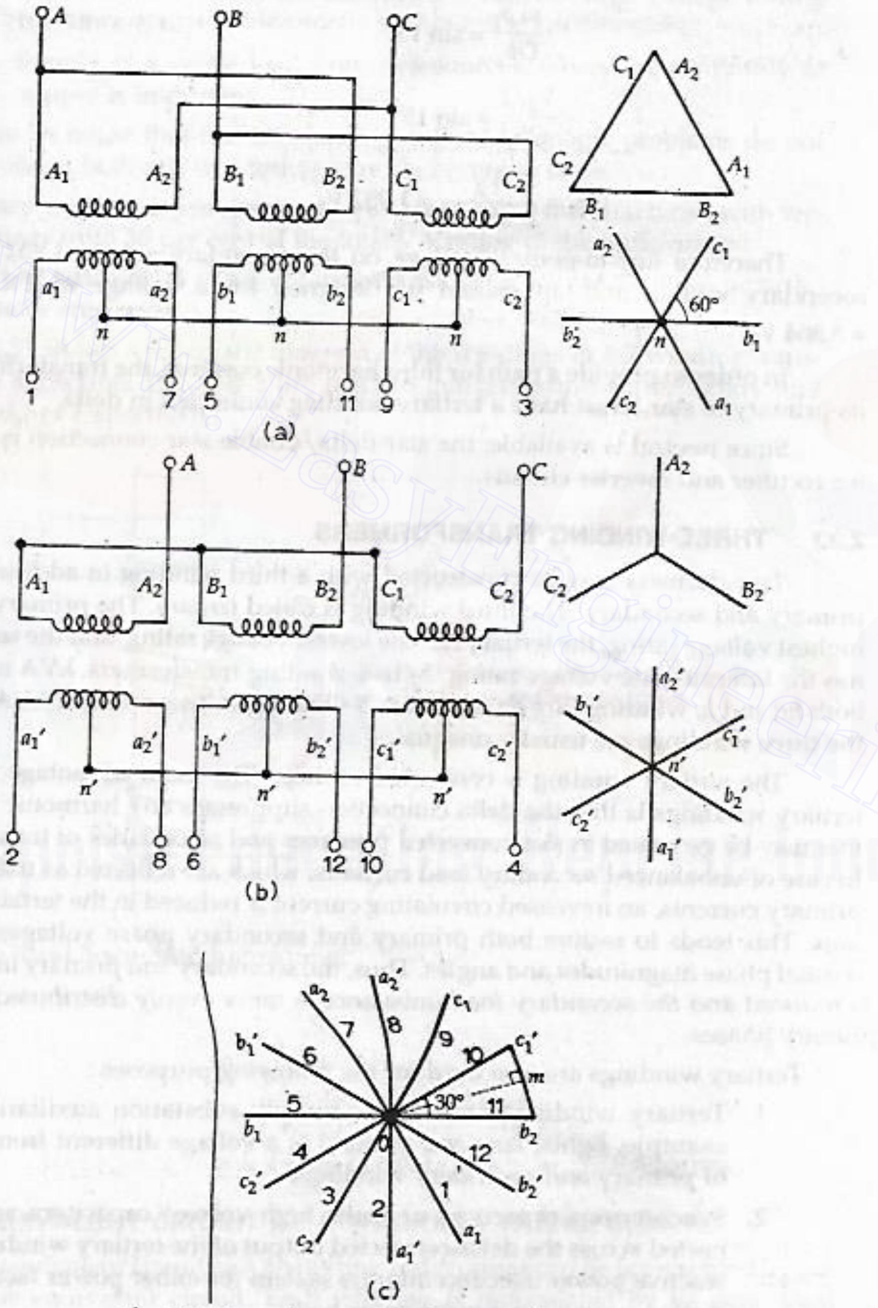


Fig. 2.32. *Three-phase to twelve-phase transformation.*

12- phase system. The marking of secondary terminals is done with the help of Fig. 2.32(c). If V_1 is the input line voltage, each phase winding of star-connected primary is designed for $(V_1/\sqrt{3})$ volts, and each phase winding of delta connected primary is designed for V_1 volts. If V_2 is the line-to-line voltage of the 12-phase circuit that is V_2 = length of phasor $b_2 c_1'$ or $c_1' c$ etc., then from $\Delta\, Om\, b_2$

$$\frac{m\,b_2}{Ob_2} = \sin 15^\circ$$

$$\frac{V_2/2}{Ob_2} = \sin 15^\circ$$

$$Ob_2 = \frac{V_2}{2 \sin 15^\circ} = 1.932\, V_2$$

Therefore line-to-neutral voltage on the secondary side is $1.932\, V_2$. The secondary with a mid-tap should be designed for a voltage of $2 \times 1.932\, V_2 = 3.864\, V_2$.

In order to provide a path for third harmonic currents, the transformer, with its primary in star, must have a tertiary winding connected in delta.

Since neutral is available, the star-delta/double star connection is suitable for rectifier and inverter circuits.

2.32 THREE-WINDING TRANSFORMERS

Transformers may be constructed with a third winding in addition to the primary and secondary. The third winding is called *tertiary*. The primary has the highest voltage rating, the tertiary has the lowest voltage rating, and the secondary has the intermediate voltage rating. In two-winding transformers, kVA ratings of both *hv* and *lv* windings are equal, but in 3-winding transformers, kVA ratings of the three windings are usually unequal.

The tertiary winding is connected in delta. The main advantage of using tertiary windings is that the delta connection suppresses any harmonic voltages that may be generated in star-connected primaries and secondaries of transformers. In case of unbalanced secondary load currents, which are reflected as unbalanced primary currents, an increased circulating current is reduced in the tertiary windings. This tends to restore both primary and secondary phase voltages to their normal phase magnitudes and angles. Thus, the secondary and primary imbalance is reduced and the secondary load imbalance is more evenly distributed among primary phases.

Tertiary windings are also used for the following purposes :

1. Tertiary windings are used to supply substation auxiliaries (for example, lights, fans and pumps) at a voltage different from those of primary and secondary windings.
2. Synchronous capacitors or static high-voltage capacitors are connected across the delta-connected output of the tertiary windings for reactive power injection into the system for either power factor correction or voltage regulation or both.

3. Tertiary windings are used to interconnect three supply systems operating at different voltages.
4. A delta-connected tertiary reduces the impedance offered to the zero-sequence currents to allow sufficient earth fault current for proper operation of protective devices.
5. Tertiary can be used for measuring voltage of high voltage testing transformers.
6. Supply of a single load from two sources, where the continuity of supply is important.

It is to be noted that the unbalance and third harmonic problems do not arise when one or both sets of windings are connected in delta.

Tertiary winding transformers are currently being manufactured with tertiary VA ratings upto 35 per cent of the total VA rating of the transformer.

The chief advantages of a 3-winding transformer are economy of construction and greater efficiency.

Fig. 2.33 shows a schematic diagram of the windings in a 3-winding transformer. The subscripts 1, 2 and 3 are used to indicate primary, secondary and tertiary windings respectively.

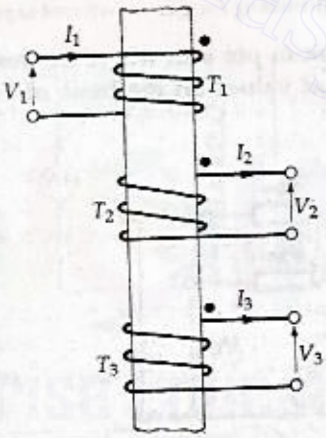


Fig. 2.33. Schematic diagram of the windings of a 3-winding transformer.

For an ideal 3-winding transformer

$$\frac{V_2}{V_1} = \frac{T_2}{T_1}$$

$$\frac{V_3}{V_1} = \frac{T_3}{T_1}$$

and $$I_1 T_1 = I_2 T_2 + I_3 T_3$$

2.33 EQUIVALENT CIRCUIT OF A 3-WINDING TRANSFORMER

The equivalent circuit of a 3-winding transformer can be represented by the single-phase equivalent circuit. Each winding is represented by its equivalent resistance and reactance. Fig. 2.34 shows the equivalent circuit referred to primary. Here the terminals 1, 2, and 3 indicate primary, secondary and tertiary terminals respectively. R_1, R_2 and R_3 are the resistances and X_1, X_2, X_3 are the leakage reactances of primary, secondary and tertiary windings respectively. If the exciting current is considered, then R_0 and X_0 can be connected as shown in Fig. 2.34. Three external circuits are connected between terminals 1, 2, 3 and common terminal 0. Let V_1, V_2, V_3 be the voltages and I_1, I_2, I_3 be the currents of primary, secondary and tertiary windings.

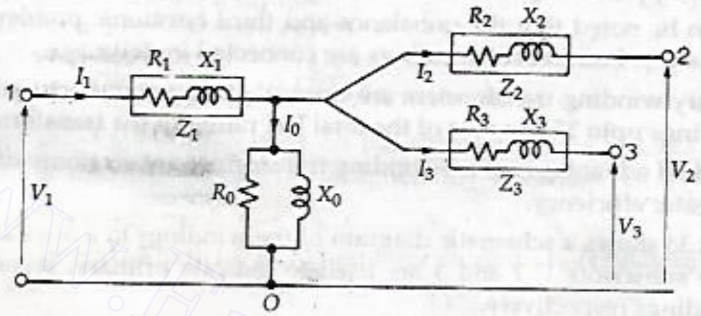


Fig. 2.34. *Single-phase equivalent circuit of a 3-winding transformer referred to primary.*

Fig. 2.35 shows the equivalent circuit in per unit where the resistance and reactances have been converted to per-unit values on the basis of common VA rating base and respective base voltages.

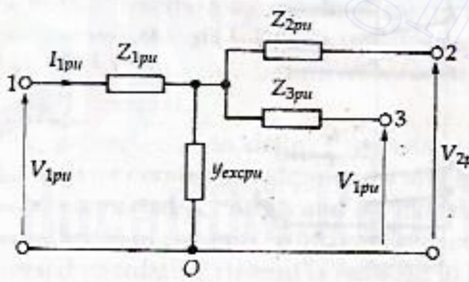


Fig. 2.35. *Single-phase equivalent circuit of a 3-winding transformer in per unit.*

The equivalent circuit neglecting magnetising admittance y_{exc} is shown in Fig. 2.36.

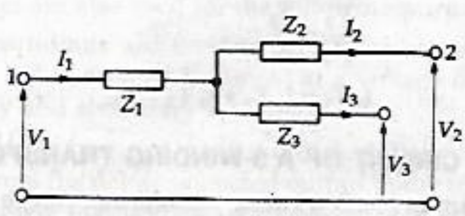


Fig. 2.36. *Equivalent circuit neglecting magnetising admittance.*

2.34 DETERMINATION OF PARAMETERS OF THREE-WINDING TRANSFORMERS

The parameters of the equivalent circuit can be determined from open-circuit and three short-circuit tests.

(*a*) **Short-circuit tests.** The equivalent leakage impedances Z_1, Z_2 and Z_3 referred to a common base can be determined by performing three short-circuit tests as follows :

In the first test (Fig. 2.37*a*) winding 2 is short circuited, winding 3 is kept open circuited and a low voltage is applied to winding 1 so that full-load current flows in winding 2. The voltage, current and power input to winding 1 are measured. Let V_1, I_1 and P_1 be the voltmeter, ammeter and wattmeter readings respectively. If $\mathbf{Z}_{12}$ indicates the short-circuit impedance of windings 1 and 2 with windings 3 open, then $Z_{12} = \frac{V_1}{I_1}$. Equivalent resistance $R_{12} = \frac{P_1}{I_1^2}$, and equivalent leakage reactance $X_{12} = \sqrt{Z_{12}^2 - R_{12}^2}$. It is seen from the equivalent circuit of Fig. 2.37(*b*) that $\mathbf{Z}_{12}$ is a series combination of $\mathbf{Z}_1$ and $\mathbf{Z}_2$.

$$\therefore \quad \mathbf{Z}_{12} = R_{12} + j\,X_{12} = \mathbf{Z}_1 + \mathbf{Z}_2 \tag{2.34.1}$$

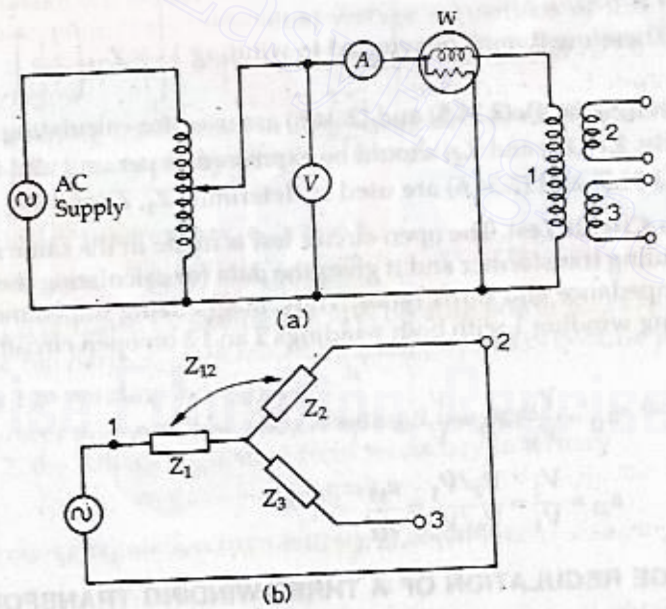


Fig. 2.37. Short-circuit test on a 3-winding transformer (a) Connection diagram (b) Equivalent circuit.

In the second short-circuit test, winding 3 is short circuited and winding 2 is kept open. A low voltage is applied to winding 1 to circulate full-load current in winding 3. If $\mathbf{Z}_{13}$ represents the short-circuit impedance of windings 1 and 3 with winding 2 left open

$$\mathbf{Z}_{13} = \mathbf{Z}_1 + \mathbf{Z}_3 \tag{2.34.2}$$

In the third short-circuit test winding 3 is short circuited and winding 1 is kept open. *A low voltage is applied to winding 2 to circulate full-load current in the short-circuited winding 3. If Z_{23} represents the short-circuit impedance of* windings 2 and 3 with winding 1 open

$$\mathbf{Z}_{23} = \mathbf{Z}_2 + \mathbf{Z}_3 \tag{2.34.3}$$

All the impedances are referred to a common base.

Solving Eqs. (2.34.1), (2.34.2), and (2.34.3) we get the leakage impedances $\mathbf{Z}_1$, $\mathbf{Z}_2$ and $\mathbf{Z}_3$ all referred to primary as

$$\mathbf{Z}_1 = \frac{1}{2}(\mathbf{Z}_{12} + \mathbf{Z}_{13} - \mathbf{Z}_{23}) \tag{2.34.4}$$

$$\mathbf{Z}_2 = \frac{1}{2}(\mathbf{Z}_{23} + \mathbf{Z}_{12} - \mathbf{Z}_{13}) \tag{2.34.5}$$

$$\mathbf{Z}_3 = \frac{1}{2}(\mathbf{Z}_{13} + \mathbf{Z}_{23} - \mathbf{Z}_{12}) \tag{2.34.6}$$

where $\quad \mathbf{Z}_1 = R_1 + j\,X_1,\ \mathbf{Z}_2 = R_2 + j\,X_2,\ \mathbf{Z}_3 = R_3 + j\,X_3$

It is to be noted that impedances $\mathbf{Z}_{12}$ and $\mathbf{Z}_{13}$ are referred to winding 1, because the instruments are connected in winding 1. The impedance $\mathbf{Z}_{23}$ is referred to winding 2. Therefore it must be referred to winding 1 $\left[= \mathbf{Z}_{23}\left(\frac{T_1}{T_2}\right)^2 \right]$ and only then the equations (2.34.4), (2.34.5) and (2.34.6) are used for calculating $\mathbf{Z}_1$, $\mathbf{Z}_2$ and $\mathbf{Z}_3$. Alternatively, $\mathbf{Z}_{12}$, $\mathbf{Z}_{13}$ and $\mathbf{Z}_{23}$ should be expressed in per unit and then equations (2.34.4), (2.34.5) and (2.34.6) are used to determine $\mathbf{Z}_1$, $\mathbf{Z}_2$ and $\mathbf{Z}_3$.

(*b*) **Open-Circuit Test.** The open-circuit test is made in the same manner as that for a 2-winding transformer and it gives the data for calculating the core loss, magnetizing impedance and turns ratios. Thus, magnetizing impedance may be found by exciting winding 1 with both windings 2 and 3 on open circuit. Then we have

$$a_{12} = \frac{V_1}{V_2},\ a_{13} = \frac{V_1}{V_3}$$

$$a_{23} = \frac{V_2}{V_3} = \frac{V_2/V_1}{V_3/V_1} = \frac{a_{13}}{a_{12}}$$

2.35 VOLTAGE REGULATION OF A THREE-WINDING TRANSFORMER

The voltage regulation of a 3-winding transformer can be determined as follows :

1. Determine the kVA in each winding for the given load. Determine k for each winding. Here k is the ratio of the magnitude of the actual kVA loading of the winding to the base kVA used in determining the network parameters.

Thus, $\quad k_1 = \dfrac{\text{primary kVA loading}}{\text{base kVA}}$

$$k_2 = \frac{\text{secondary kVA loading}}{\text{base kVA}}$$

and $$k_3 = \frac{\text{tertiary kVA loading}}{\text{base kVA}}$$

2. Calculate the voltage regulation for each winding at its operating power factor.

Let $\cos\phi_1$, $\cos\phi_2$, $\cos\phi_3$ be the operating power factors of windings 1, 2, 3 respectively. If $\varepsilon_{r_1}, \varepsilon_{r_2}, \varepsilon_{r_3}$ are the per unit resistance drops for windings 1, 2, 3 respectively and $\varepsilon_{x_1}, \varepsilon_{x_2}, \varepsilon_{x_3}$ are the per unit leakage reactance drops for winding 1, 2, 3 respectively.

Then, for primary winding *alone* the per unit voltage regulation is given by

$$\varepsilon_1 = k_1 (\varepsilon_{r_1} \cos\phi_1 + \varepsilon_{x_1} \sin\phi_1)$$

For secondary winding *alone* the pu voltage regulation is

$$\varepsilon_2 = k_2 (\varepsilon_{r_2} \cos\phi_2 + \varepsilon_{x_2} \sin\phi_2)$$

For tertiary winding *alone* the pu voltage regulation is

$$\varepsilon_3 = k_3 (\varepsilon_{r_3} \cos\phi_3 + \varepsilon_{x_3} \sin\phi_3)$$

3. The voltage regulation for any pair of windings is obtained by the algebraic (not phasor) sum of the individual voltage regulations of this pair under consideration, if power flows from one to another. Otherwise a negative sign is used as shown below.

For a 3-winding transformer with primary energised from ac source and with both secondary and tertiary windings connected to loads, the voltage regulations are,

from primary to secondary, $\varepsilon_{12} = \varepsilon_1 + \varepsilon_2$

and from primary to tertiary, $\varepsilon_{13} = \varepsilon_1 + \varepsilon_3$.

Here ε_{12} is obtained by adding ε_1 and ε_2 because power flows from winding 1 to winding 2. Similarly, ε_{13} is obtained by adding ε_1 and ε_3 because power flows from winding 1 to winding 3.

Since power does not flow from winding 2 to winding 3, or from winding 3 to winding 2, the voltage regulation from secondary to tertiary

$$\varepsilon_{23} = \varepsilon_2 - \varepsilon_3$$

Also, voltage regulation from tertiary to secondary

$$\varepsilon_{32} = \varepsilon_3 - \varepsilon_2$$

Here ε_{23} is obtained by subtracting ε_3 from ε_2 and ε_{32} is obtained by subtracting ε_2 from ε_3.

Voltage regulation from secondary to primary

$$\varepsilon_{21} = -(\varepsilon_1 + \varepsilon_2)$$

Here negative sign is used before $(\varepsilon_1 + \varepsilon_2)$ because voltage regulation is determined from secondary to primary, whereas power flows from primary to secondary. Similarly, the voltage regulation from tertiary to primary $\varepsilon_{31} = -(\varepsilon_1 + \varepsilon_3)$

EXAMPLE 2.20. *A single-phase 3-winding transformer gave the following results from three short-circuit tests :*

Secondary shorted, primary excited : 125 V, 25 A, 700 W

Tertiary shorted, primary excited : 130 V, 25 A, 800 W

Tertiary shorted, secondary excited : 30 V, 120 A, 830 W

The ratings of the windings are as follows :

Primary 100 kVA, 3300 V

Secondary 50 kVA, 1100 V

Tertiary 50 kVA, 400 V

Find the resistances and leakage reactances of star equivalent circuit. Also calculate their values for each winding.

SOLUTION.
$$R_{12} = \frac{P_1}{I_1^2} = \frac{700}{(25)^2} = 1.12\ \Omega$$

$$R_{13} = \frac{800}{(25)^2} = 1.28\ \Omega$$

$$R_{23} = \frac{820}{(120)^2} = 0.0569\ \Omega$$

The computed resistances R_{12} and R_{13} are referred to primary winding because the instruments are connected in the primary winding. The computed resistance R_{23} is measured on the secondary side. Therefore it should be referred to primary winding. Therefore R_{23} when referred to primary winding is given by

$$R_{23}' = R_{23}\left(\frac{T_1}{T_2}\right)^2$$

$$= 0.0569\left(\frac{3300}{1100}\right)^2 = 0.5121\ \Omega$$

The equivalent circuit resistances are given by

$$R_1 = \frac{1}{2}(R_{12} + R_{13} - R_{23}')$$

$$= \frac{1}{2}(1.12 + 1.28 - 0.5121) = 0.944\ \Omega$$

$$R_2 = \frac{1}{2}(R_{12} + R_{23}' - R_{13})$$

$$= \frac{1}{2}(1.12 + 0.5121 - 1.28) = 0.176\ \Omega$$

$$R_3 = \frac{1}{2}(R_{13} + R_{23}' - R_{12})$$

$$= \frac{1}{2}(1.28 + 0.5121 - 1.12) = 0.336\ \Omega$$

The short-circuit impedances are

$$Z_{12} = \frac{125}{25} = 5\ \Omega$$

$$Z_{13} = \frac{130}{25} = 5.2\ \Omega$$

$$Z_{23} = \frac{30}{120} = 0.25\ \Omega$$

The computed impedance Z_{23} is measured on the secondary side. Its value should be referred to primary. Therefore the Z_{23} when referred to primary is given by

$$Z_{23}' = Z_{23}\left(\frac{T_1}{T_2}\right)^2$$

$$= 0.25\left(\frac{3300}{1100}\right)^2 = 2.25\ \Omega$$

The leakage impedances referred to the primary side are

$$Z_1 = \frac{1}{2}(Z_{12} + Z_{13} - Z_{23'})$$

$$= \frac{1}{2}(5 + 5.2 - 2.25) = 3.975\ \Omega$$

$$Z_2 = \frac{1}{2}(Z_{12} + Z_{23}' - Z_{13})$$

$$= \frac{1}{2}(5 + 2.25 - 5.2) = 1.025\ \Omega$$

$$Z_3 = \frac{1}{2}(Z_{13} + Z_{23}' - Z_{12})$$

$$= \frac{1}{2}(5.2 + 2.25 - 5) = 1.225\ \Omega$$

Leakage reactance of primary winding

$$X_1 = \sqrt{Z_1^2 - R_1^2}$$

$$= \sqrt{(3.975)^2 - (0.944)^2} = 3.849\ \Omega$$

Leakage reactance of secondary winding referred to primary

$$X_2 = \sqrt{Z_2^2 - R_2^2}$$

$$= \sqrt{(1.025)^2 - (0.176)^2} = 1.0098\ \Omega$$

Leakage reactance of tertiary winding referred to primary

$$X_3 = \sqrt{Z_3^2 - R_3^2}$$

$$= \sqrt{(1.225)^2 - (0.336)^2} = 1.178\ \Omega$$

Leakage impedance of primary winding

$$\mathbf{Z}_1 = R_1 + j\,X_1 = (0.944 + j\,3.849)\ \Omega$$

Leakage impedance of secondary winding

$$\mathbf{Z}_2 = (R_2 + j\,X_2)\left(\frac{1100}{3300}\right)^2$$

$$= (0.176 + j\,1.0098)\left(\frac{1}{9}\right) = (0.0196 + j\,0.1122)\ \Omega$$

Leakage impedance of tertiary winding

$$\mathbf{Z}_3 = (R_3 + j\,X_3)\left(\frac{400}{3300}\right)^2$$

$$= (0.336 + j\,1.178)\left(\frac{4}{33}\right)^2 = (0.00493 + j\,0.0173)\ \Omega$$

EXAMPLE 2.21. *A three-winding transformer in star/star/delta has the following ratings :*

Primary 1 : 10 MVA, 33 kV

Secondary 2 : 5 MVA, 11 kV

Tertiary 3 : 5 MVA, 3.3 kV

Three short-circuit tests on this transformer gave the following results :

Secondary shorted, primary excited : 3000 V, 160 A, 100 kW

Tertiary shorted, primary excited : 200 V, 12 A, 1.25 kW

Tertiary shorted, secondary excited : 100 V, 40 A, 1.5 kW.

Calculate the resistances and reactances in ohms of the star equivalent circuit of the three-winding transformer. Also determine the per unit values of leakage impedances.

SOLUTION. In the first test, the instruments are connected in the primary. Since the primary is connected in star, the phase current is equal to the line current. $I_{p1} = I_{l1} = 160$ A

$$3\,I_{p1}^2\,R_{12} = 100 \times 10^3$$

$$R_{12} = \frac{100 \times 10^3}{3 \times (160)^2} = 1.302\ \Omega$$

In the second test also the instruments are connected in the primary

$$\therefore \qquad R_{13} = \frac{1.25 \times 10^3}{3\,(12)^2} = 2.894\ \Omega$$

$$R_{23} = \frac{1.5 \times 10^3}{3\,(40)^2} = 0.3125\ \Omega$$

The computed resistances R_{12} and R_{13} are referred to primary because the instruments are connected in this winding. In the third tests, the instruments are connected in the secondary and therefore R_{23} calculated above should be referred to the primary.

R_{23} referred to primary is

$$R_{23}' = R_{23}\left(\frac{33}{11}\right)^2 = 0.3125 \times 9 = 2.8125\ \Omega$$

$$\therefore \quad R_1 = \frac{1}{2}(R_{12} + R_{13} - R_{23}')$$

$$= \frac{1}{2}(1.302 + 2.894 - 2.8125) = 0.692\ \Omega$$

$$R_2 = \frac{1}{2}(R_{12} + R_{23}' - R_{13})$$

$$= \frac{1}{2}(1.302 + 2.8125 - 2.894) = 0.610\ \Omega$$

$$R_3 = \frac{1}{2}(R_{13} + R_{23}' - R_{12}) = \frac{1}{2}(2.894 + 2.8125 - 1.302)$$

$$= 2.202\ \Omega$$

Since the primary is in star, primary phase voltage $= \frac{1}{\sqrt{3}} \times$ line voltage

$$= \frac{3000}{\sqrt{3}}\ \text{V}$$

Primary phase current = primary line current

$$= 160\ \text{A}$$

$$\therefore \quad Z_{12} = \frac{\text{primary phase voltage}}{\text{primary phase current}}$$

$$= \frac{3000/\sqrt{3}}{160} = 10.826\ \Omega$$

Similarly $\quad Z_{13} = \frac{200/\sqrt{3}}{12} = 9.62\ \Omega$

$$Z_{23} = \frac{100/\sqrt{3}}{40}$$

Z_{23} when referred to primary is

$$Z_{23}' = \frac{100}{40\sqrt{3}}\left(\frac{33}{11}\right)^2 = 12.99\ \Omega$$

$$\therefore \quad Z_1 = \frac{1}{2}(Z_{12} + Z_{13} - Z_{23}')$$

$$= \frac{1}{2}(10.826 + 9.62 - 12.99) = 3.728\ \Omega$$

$$Z_2 = \frac{1}{2}(Z_{12} + Z_{23}' - Z_{13})$$

$$= \frac{1}{2}(10.826 + 12.99 - 9.62) = 7.098\ \Omega$$

$$Z_3 = \frac{1}{2}(Z_{13} + Z_{23}' - Z_{12})$$

$$= \frac{1}{2}(9.62 + 12.99 - 10.826) = 5.892\ \Omega$$

$$X_1 = \sqrt{Z_1^2 - R_1^2} = \sqrt{(3.728)^2 - (0.692)^2} = 3.663\ \Omega$$
$$X_2 = \sqrt{Z_2^2 - R_2^2} = \sqrt{(7.098)^2 - (0.61)^2} = 7.071\ \Omega$$
$$X_3 = \sqrt{Z_3^2 - R_3^2} = \sqrt{(5.892)^2 - (2.202)^2} = 5.465\ \Omega$$

The leakage impedances Z_1, Z_2 and Z_3 determined above are in ohms and are referred to primary. In order to calculate their per unit values, these should be divided by the base impedance of the primary.

Per phase base voltage for primary

$$V_{b1} = \frac{33 \times 10^3}{\sqrt{3}}\ \text{V} = 19053\ \text{V}$$

Per phase base current for primary

$$I_{b1} = \frac{10 \times 10^6}{\sqrt{3} \times 33 \times 10^3} = 174.95\ \text{A}$$

Primary base impedance

$$Z_{b1} = \frac{V_{b1}}{I_{b1}} = \frac{19053}{174.95} = 108.9\ \Omega$$

$$Z_{1pu} = \frac{Z_1}{Z_{b1}} = \frac{3.728}{108.9} = 0.0342\ \text{pu}$$

$$Z_{2pu} = \frac{Z_2}{Z_{b1}} = \frac{7.098}{108.9} = 0.0652\ \text{pu}$$

$$Z_{3pu} = \frac{Z_3}{Z_{b1}} = \frac{5.892}{108.9} = 0.0541\ \text{pu.}$$

EXAMPLE 2.22. *A 3-phase, 3-winding delta/delta/star, 33000/11000/400-V, 200 kVA transformer has a secondary load of 150 kVA at 0.8 p.f. lagging, and a tertiary load of 50 kVA at 0.9 p.f. lagging. The magnetising current is 4% of rated load, the iron loss being 1 kW. Calculate the value of the primary current when the other two windings are delivering the above loads.*

SOLUTION. Secondary load kVA per phase $= \frac{150}{3} = 50$ kVA

Secondary voltage per phase

$$= 1100\ \text{V (In } \Delta,\ V_p = V_l) = 1.1\ \text{kV}$$

∴ Secondary current per phase

$$= \frac{\text{load kVA per phase}}{\text{voltage per phase in kV}} = \frac{50}{1.1}\ \text{A}$$

Phasor secondary current per phase referred to primary

$$I_2' = \left(\frac{50}{1.1} \times \frac{1100}{33000}\right) \angle -\cos^{-1} 0.8°$$
$$= 1.515\ (0.8 - j\,0.6)$$
$$= (1.212 - j\,0.909)\ \text{A}$$

Tertiary load kVA per phase

$$= \frac{50}{3} \text{ kVA}$$

Tertiary voltage per phase

$$= \frac{400}{\sqrt{3}} = 231 \text{ V} = 0.231 \text{ kV}$$

Tertiary current per phase

$$= \frac{\text{load kVA per phase}}{\text{voltage per phase in kV}}$$

$$= \frac{50/3}{0.231} \text{ A}$$

Phasor tertiary current per phase referred to primary

$$\mathbf{I}_3' = \left(\frac{50/3}{0.231}\right)\left(\frac{231}{3300}\right) \underline{/-\cos^{-1} 0.9^\circ}$$

$$= 0.505\,(0.9 = j\,0.436)$$

$$= (0.4545 - j\,0.220) \text{ A}$$

Magnetising current

$$I_\mu = 4\% \text{ of rated current}$$

$$= \frac{4}{100} \times \frac{(200 \times 10^3)/3}{33000} = 0.0808 \text{ A}$$

Core loss component of no-load current

$$I_W = \frac{(1 \times 1000)/3}{33000} = 0.0101 \text{ A}$$

∴ Primary no-load current

$$\mathbf{I}_0 = I_w - j\,I_\mu = (0.0101 - j\,0.0808) \text{ A}$$

Total primary current

$$\mathbf{I}_1 = \mathbf{I}_2' + \mathbf{I}_3' + \mathbf{I}_0$$

$$= 1.212 - j\,0.909 + 0.4545 - j\,0.220 + 0.0101 - j\,0.0808$$

$$= 1.6766 - j\,1.2908 = 2.116 \underline{/-37.6^\circ}$$

$$= 2.116 \text{ A}$$

at a lagging power factor of cos 37.6° that is 0.7923.

Example 2.23. *A 3-winding 132/33/6.6-kV, 3-phase star-star-delta transformer having negligible resistance has the following measured reactances between the windings :*

hv to lv 0.15 pu ; hv to mv 0.09 pu ; mv to lv 0.08 pu

all are referred to 30 MVA base. The lv winding supplies a balanced load of 2000 A at 0.8 lagging power factor. The mv winding supplies a star-connected inductive reactor of $(0 + j\,50)\,\Omega$ *per phase. Determine the voltage required at the hv terminals to maintain 6.6 kV at the l.v. terminals.*

SOLUTION. Here $X_{13} = 0.15$ pu, $X_{12} = 0.09$ pu, $X_{23} = 0.08$ pu

$$\therefore \quad X_1 = \frac{1}{2}(X_{12} + X_{13} - X_{23})$$

$$= \frac{1}{2}(0.09 + 0.15 - 0.08) = 0.08 \text{ pu}$$

$$X_2 = \frac{1}{2}(X_{12} + X_{23} - X_{13})$$

$$= \frac{1}{2}(0.09 + 0.08 - 0.15) = 0.01 \text{ pu}$$

$$X_3 = \frac{1}{2}(X_{13} + X_{23} = X_{12})$$

$$= \frac{1}{2}(0.15 + 0.08 - 0.09) = 0.07 \text{ pu}$$

The primary winding is star connected. Primary base voltage per phase

$$V_{b1} = \frac{132}{\sqrt{3}} \text{ kV}$$

Base kVA = 30 MVA = 30×10^3 kVA.

Primary base current

$$I_{b1} = \frac{\text{primary base kVA/phase}}{\text{primary base voltage/phase in kVA}}$$

$$= \frac{(30/3) \times 10^3}{(132/\sqrt{3}\,)} \text{ A}$$

For secondary, base kV = 33

$$\text{base current } I_{b2} = \frac{10 \times 10^3}{(33/\sqrt{3})}$$

The tertiary is connected in delta.

Base kV = 6.6

Base current in tertiary

$$I_{b3} = \frac{10 \times 10^3}{6.6} \text{ A}$$

Load current in secondary

$$\mathbf{I}_2 = \frac{V_{2p}}{Z_{2p}} = \frac{(33 \times 10^3)/\sqrt{3}}{j\,50}$$

Per-unit load current in secondary

$$= \frac{\text{load current in secondary in amperes}}{\text{base current in secondary in amperes}}$$

$$I_{2\text{pu}} = \frac{I_2}{I_{b2}} = \frac{33 \times 10^3}{50\sqrt{3}} \times \frac{33}{10 \times 10^3 \sqrt{3}} = 1.2575 \text{ pu}$$

at zero power factor lagging

$$\mathbf{I}_{2\text{pu}} = 1.2575 \,\underline{/-90^\circ} = -j\,1.2575$$

Per phase load current in tertiary $= \dfrac{2000}{\sqrt{3}}$ A

Per unit load current in tertiary

$$\mathbf{I}_{3pu} = \frac{I_2}{I_{b2}} = \frac{2000}{\sqrt{3}} \times \frac{6.6}{10 \times 10^3}$$
$$= 0.7621 \text{ pu at } 0.8 \text{ p.f. lagging}$$
$$= 0.7621 \underline{/ -\cos^{-1} 0.8°}$$
$$= 0.7621 \underline{/ -36.87°}$$
$$= 0.60968 - j\,0.045726.$$

Per phase primary current

$$\mathbf{I}_1 = \mathbf{I}_2 + \mathbf{I}_3$$
$$= -j\,1.2575 + 0.60968 - j\,0.45726$$
$$= (0.60968 - j\,1.7147) \text{ pu}$$

From the star equivalent circuit the voltage required at the hv terminals is iven by

$$\mathbf{V}_1 = \mathbf{I}_1\,\mathbf{Z}_1 + \mathbf{I}_3\,\mathbf{Z}_3 + \mathbf{V}_3$$
$$= (0.60968 - j\,1.7147)\,(j\,0.08) + 0.60968 - j\,0.45726)\,(j\,0.07) + 1$$
$$= j\,0.09145 + 0.1372 + 0.032 + 1$$
$$= 1.1692 + j\,0.09145$$
$$= 1.1728 \text{ pu}$$

Voltage required at the hv terminals in kV

$$= 1.1728 \times 132 = 154.8 \text{ kV}$$

Example 2.24. *A 3-phase bank consisting of three single-phase, 3-winding transformers connected in star-delta-star is used to step down the voltage of a 3-phase, 220 kV transmission line. The data pertaining to one of the transformers is as follows :*

Ratings

Primary 1 : 25 MVA, 220 kV

Secondary 2 : 12.5 MVA, 33 kV

Tertiary 3 : 12.5 MVA, 11 kV

Short-circuit reactances on 12.5 MVA base :

$$X_{12} = 0.2 \text{ pu},\ X_{23} = 0.15,\ X_{13} = 0.3 \text{ pu}.$$

Transformer resistances are neglected. The delta-connected secondaries supply their rated current to a balanced load at 0.8 p.f. lagging. The tertiaries deliver the rated current to a balanced load at unity power factor.

(a) *Calculate the primary line-to-line voltage to maintain rated voltage at the secondary terminals.*

(b) *For the condition in part (a) find the line voltage at the tertiary terminals.*

(c) *If the primary voltage found in part (a) is held fixed, to what value the tertiary voltage rises if the secondary load is reduced to zero ?*

Solution. The leakage reactances referred to common base of 12.5 MVA are

$$X_1 = \frac{1}{2}(X_{12} + X_{13} - X_{23})$$
$$= \frac{1}{2}(0.2 + 0.3 - 0.15) = 0.175 \text{ pu}$$
$$X_2 = \frac{1}{2}(X_{12} + X_{23} - X_{13})$$
$$= \frac{1}{2}(0.2 + 0.15 - 0.3) = 0.025 \text{ pu}$$
$$X_3 = \frac{1}{2}(X_{13} + X_{23} - X_{12})$$
$$= \frac{1}{2}(0.3 + 0.15 - 0.2) = 0.125 \text{ pu}$$

Both the secondary and tertiary windings operate at rated currents. Let us assume that secondary and tertiary terminal voltages $\mathbf{V}_2$ and $\mathbf{V}_3$ are in phase. Then with $\mathbf{V}_2$ and $\mathbf{V}_3$ as reference phasors,

secondary current $\quad \mathbf{I}_2 = 1 \underline{/-\cos^{-1} 0.8°} = (0.8 - j\,0.6)$ pu

tertiary current $\quad \mathbf{I}_3 = 1 \underline{/0°} = 1 + j\,0$ pu

primary current $\quad \mathbf{I}_1 = \mathbf{I}_2 + \mathbf{I}_3 = 0.8 - j\,0.6 + 1 = (1.8 - j\,0.6)$ pu

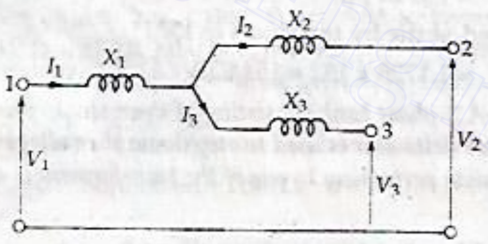


Fig. 2.38.

(a) From Fig. 2.38,

$$\mathbf{V}_1 = \mathbf{V}_2 + \mathbf{I}_2\,\mathbf{Z}_2 + \mathbf{I}_1\,\mathbf{Z}_1$$

Since the secondary voltage is equal to the rated voltage $V_2 = 1.0$

$$\mathbf{V}_2 = 1 \underline{/0°} = 1 + j\,0$$
$$\mathbf{I}_2 = 0.8 - j\,0.6 \text{ pu}$$
$$\mathbf{I}_1 = 1.8 - j\,0.6 \text{ pu}$$
$$\mathbf{Z}_1 = j\,X_1 = j\,0.175 \text{ pu}$$
$$\mathbf{Z}_2 = j\,X_2 = j\,0.025 \text{ pu}$$
$$\therefore \quad \mathbf{V}_1 = 1 + j\,0 + (0.8 - j\,0.6)(j\,0.025) + (1.8 - j\,0.6)(j\,0.175)$$
$$= 1.12 + j\,0.335 = 1.169 \underline{/16.7°} \text{ pu}$$

$\therefore$ primary line-to-line voltage

$$= 1.169 \times 220 = 257.18 \text{ kV}.$$

(*b*) Again from Fig. 2.38, by KVL in secondary and tertiary circuits,

$$\mathbf{V}_2 + \mathbf{I}_2 \mathbf{Z}_2 - \mathbf{I}_3 \mathbf{Z}_3 - \mathbf{V}_3 = 0$$

$$\mathbf{V}_3 = \mathbf{V}_2 + \mathbf{I}_2 \mathbf{Z}_2 - \mathbf{I}_3 \mathbf{Z}_3$$

$$= 1 + j\,0 - (0.8 - j\,0.6)\,(j\,0.025) - (1 + j\,0)\,(j\,0.125)$$

$$= 1.015 - j\,0.105 - 1.0204 \,\underline{/-5.9°} \text{ pu}$$

$$\therefore \quad V_3 = 1.0204 \times 11 = 11.22 \text{ kV}$$

(*c*) When the secondary load is reduced to zero, $\mathbf{I}_2 = 0$, and $I_1 = I_3$. By KVL in primary and tertiary circuits

$$\mathbf{V}_1 = \mathbf{I}_1 \mathbf{Z}_1 + \mathbf{I}_3 \mathbf{Z}_3 + \mathbf{V}_3$$

$$\mathbf{V}_1 = \mathbf{I}_3 \mathbf{Z}_1 + \mathbf{I}_3 \mathbf{Z}_3 + \mathbf{V}_3$$

$$1.12 + j\,0.335 = 1\,(j\,0.175) + 1\,(j\,0.125) + \mathbf{V}_3$$

$$\mathbf{V}_3 = 1.12 + j\,0.035 = 1.1205 \,\underline{/1.8°} \text{ pu}$$

$$V_3 = 1.1205 \times 11 = 12.32 \text{ kV}.$$

Therefore with the secondary load reduced to zero, the tertiary voltage rises from 11.22 kV to 12.32 kV.

Example 2.25. *A 3300/400/110 star/star/delta transformer takes a magnetising current of 6 A. It has respective primary, secondary and tertiary per unit resistances of 0.005, 0.006 and 0.008 and per unit reactances of 0.03, 0.025 and 0.035 with 1000 kVA as base kVA. If the secondary and tertiary windings supply balanced loads of 700 kVA at 0.8 p.f. lagging and 250 kVA at 0.6 p.f. leading respectively, determine the primary current, power factor, primary load and various regulations at the given loads.*

Solution. Magnetising current $\mathbf{I}_0 = 0 - j\,6$ A

Secondary current $\quad I_2 = \dfrac{700 \times 1000}{\sqrt{3} \times 400} = 1010.4$ A

Magnitude of the secondary current referred to primary

$$I_2' = \frac{1010.4 \times 400}{3300} = 122.47 \text{ A}$$

$$\mathbf{I}_2' = I_2' \,\underline{/-\cos^{-1} 0.8°} = 122.47\,(0.8 - j\,0.6)$$

$$= 97.98 - j\,43.48 \text{ A}$$

Tertiary load current, $\quad I_3 = \dfrac{250 \times 1000}{\sqrt{3} \times 110} = 1312$ A

Magnitude of tertiary current referred to primary.

$$I_3' = \frac{1312 \times 110}{3300} = 43.74 \text{ A}$$

$$\therefore \quad \mathbf{I}_3' = I'_3 \,\underline{/+\cos^{-1} 0.6°}$$

$$= 43.74\,(0.6 + j\,0.8) = 26.24 + j\,34.99$$

Total primary current

$$I_2 = I_0 + I_2' + I_3'$$
$$= -j\,6 + 97.98 - j\,43.48 + 26.24 + j\,34.99$$
$$= 124.22 - j\,14.49$$
$$= 125\,\underline{/-6.653^\circ}\text{ A}$$

Power factor of primary current

$$= \cos 6.653^\circ = 0.9932 \text{ lagging}$$

The total primary load

$$S_1 = \sqrt{3} \times 3300 \times 125 \times 10^{-3}$$
$$= 714.47 \text{ kVA} \quad \text{at power factor 0.9933 lagging.}$$

Total primary load in kW = kVA cos ϕ

$$= 714.47 \times 0.9933 = 709.6 \text{ kW}$$

This corresponds to $700 \times 0.8 = 560$ kW in the secondary plus $250 \times 0.6 = 150$ kW in the tertiary.

Per unit loading of the primary

$$k_1 = \frac{714.47}{1000} = 0.71447$$

The regulation contributed by the primary

$$\varepsilon_1 = k_1\,(R_{1pu} \cos \phi_1 + X_{1pu} \sin \phi_1)$$
$$= 0.71447\,(0.005 \times 0.9933 + 0.03 \times 0.1155)$$
$$= 0.00602 \text{ pu}$$

Per unit loading of the secondary

$$k_2 = \frac{700}{1000} = 0.7$$

The regulation contributed by the secondary

$$\varepsilon_2 = k_2\,(R_{2pu} \cos \phi_2 + X_{2pu} \sin \phi_2)$$
$$= 0.7\,(0.006 \times 0.8 + 0.025 \times 0.6)$$
$$= 0.01386 \text{ pu}$$

Per unit loading of the tertiary

$$k_3 = \frac{250}{1000} = 0.25$$

Since the load supplied by the tertiary is at leading power factor $\cos \phi_3 = 0.6$, the regulation contributed by the tertiary

$$\varepsilon_3 = K_3\,(R_{3pu} \cos \phi_3 - X_{3pu} \sin \phi_3)$$
$$= 0.25\,(0.008 \times 0.6 - 0.035 \times 0.8)$$
$$= 0.25\,(0.0048 - 0.028)$$
$$= -0.0058 \text{ pu}$$

Therefore, the primary-secondary regulation

$$\varepsilon_{12} = \varepsilon_1 + \varepsilon_2 = 0.00602 + 0.01386 = 0.01988 \text{ pu}$$

The primary-tertiary regulation

$$\varepsilon_{13} = \varepsilon_1 + \varepsilon_3 = 0.00602 - 0.0058 = 0.00022 \text{ pu.}$$

The regulation between secondary and tertiary is

$$\varepsilon_{23} = \varepsilon_2 - \varepsilon_3 = 0.01386 - (-0.0058) = 0.01966 \text{ pu}$$

In the reverse direction regulation is

$$\varepsilon_{32} = -\varepsilon_{23} = -0.01966 \text{ pu}$$

2.36 POLARITY OF TRANSFORMERS

Polarities of a transformer identify the relative directions of induced voltages in the two windings. The polarities result from the relative directions in which the two windings are wound on the core. It is necessary to know the relative polarities for operating transformers in parallel.

2.37 LABELLING OF TRANSFORMER TERMINALS

Terminals on the high-voltage (hv) side of each phase are designated by capital letters *A*, *B*, *C* while the terminals on the low-voltage (lv) side of each phase are labelled as small letters *a*, *b*, *c*. Terminal polarities are indicated by suffixes 1 and 2. Suffix 1 is the neutral end (if any) and the suffix 2 (or higher if there are tappings) is the line end. Fig. 2.39 shows labelling of terminals for phase *a*.

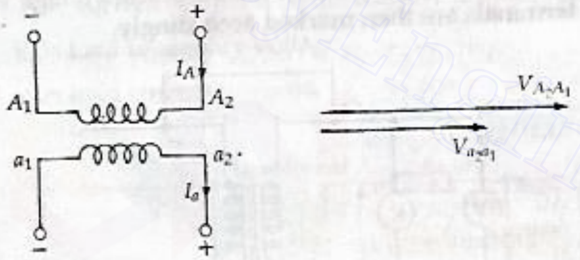


Fig. 2.39. *Labelling of transformer terminals.*

Each winding has two ends designated by subscripts 1, 2 ; or if there are intermediate tappings, these are numbered in order of their separation from end 1. Thus an high-voltage on phase *A* with four tappings would be numbered $A_1, A_2, A_3, ..., A_6$ with A_1 and A_6 forming the phase terminals.

The polarity markings are such that voltages are instantaneously in time phase between terminals with corresponding markings, that is, at instant when the p.d. of A_1 with respect to A_2 is at a positive maximum value, the p.d. of a_1 with respect to a_2 is also at a positive maximum value. If the phase winding is split in sections, the suffix numbering is such that the potential from a_1 to a_2 is in phase with that from a_3 to a_4.

The polarity of a single-phase transformer may be additive or subtractive. With standard markings the voltage from A_1 to A_2 is always in the same direction or in phase with the voltage from a_1 to a_2. In a transformer where A_1 and a_1 terminals are adjacent, as shown in Fig. 2.40(a), the transformer is said to have ***subtractive polarity***. When the terminals A_1 and a_1 are diagonally opposite [Fig. 2.40(b)], the transformer is said to have ***additive polarity***..

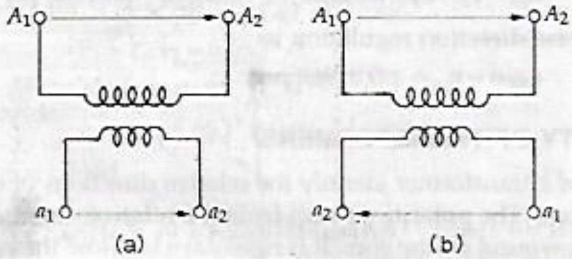


Fig. 2.40. *Polarity of a 1-phase transformer (a) Subtractive polarity (b) Additive polarity.*

2.38 TEST FOR POLARITY

Polarities can be checked by a simple test requiring only voltage measurements with transformer on no load. In this test, rated voltage is applied on one winding, and electrical connection is made between one terminal from one winding and one terminal from the other, as shown in Fig. 2.41. The voltage across the two remaining terminals (one from each winding) is then measured. If this measured voltage V' is greater than the input test voltage V, the polarity is ***additive***. If the measured voltage V' is smaller than the input test voltage V, the polarity is ***subtractive***. The terminals are then marked accordingly.

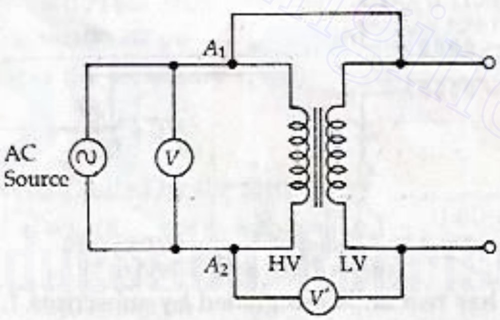


Fig. 2.41. *Polarity test.*

2.39 PARALLEL OPERATION OF TRANSFORMERS

Transformers are said to be connected in parallel when their primary windings are connected to a common voltage supply and their secondary windings are connected to a common load.

2.40 REASONS FOR PARALLEL OPERATION

The main reasons for operating transformers in parallel are as follows :

(a) For large loads it may be impracticable or uneconomical to have a single large transformer.

(b) In substations the total load required may be supplied by an appropriate number of transformers of standard size. This reduces the spare capacity of the substation.

(c) There is a scope of future expansion of a substation to supply a load beyond the capacity of the transformers already installed.

(d) If there is a breakdown of a transformer in a system of transformers connected in parallel, there is no interruption of power supply for essential services. Similarly, when a transformer is taken out of service for its maintenance and inspection, the continuity of supply is maintained.

2.41 SINGLE-PHASE TRANSFORMERS IN PARALLEL

Fig. 2.42 shows the circuit diagram of two transformers A and B in parallel.

Let a_1 = turns ratio of transformer A

a_2 = turns ratio of transformer B

$\mathbf{Z}_A$ = equivalent impedance of transformer A referred to secondary

$\mathbf{Z}_B$ = equivalent impedance of transformer B referred to secondary

$\mathbf{Z}_L$ = load impedance across the secondary

I_A = current supplied to the load by the secondary of transformer A

I_B = current supplied to the load by the secondary of transformer B

V_L = load secondary voltage

I_L = load current

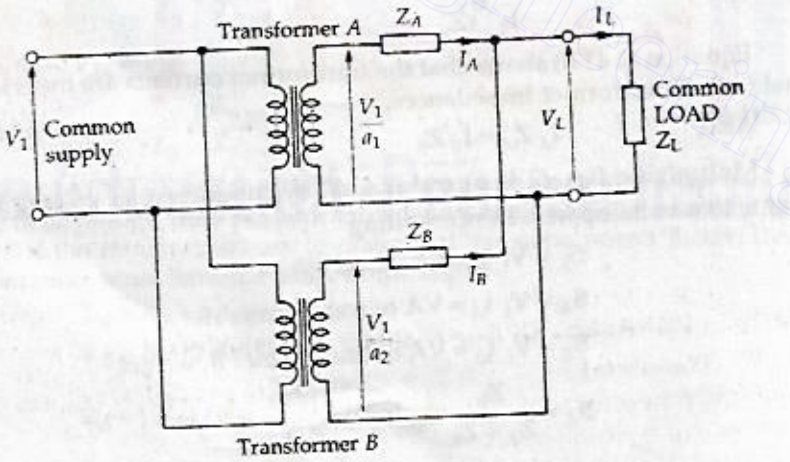


Fig. 2.42. *Two single-phase transformer in parallel.*

By KCL, $$\mathbf{I}_A + \mathbf{I}_B = \mathbf{I}_L \tag{2.41.1}$$

By KVL $$\mathbf{V}_L = \frac{\mathbf{V}_1}{a_1} - \mathbf{I}_A \mathbf{Z}_A \tag{2.41.2}$$

$$\mathbf{V}_L = \frac{\mathbf{V}_1}{a_2} - \mathbf{I}_B\,\mathbf{Z}_B = \frac{\mathbf{V}_1}{a_2} - (\mathbf{I}_L - \mathbf{I}_A)\,\mathbf{Z}_B \tag{2.41.3}$$

Solving Eqs. (2.41.2) and (2.41.3), we get

$$\mathbf{I}_A = \frac{\mathbf{Z}_B\,\mathbf{I}_L}{\mathbf{Z}_A + \mathbf{B}_B} + \frac{\mathbf{V}_1\,(a_2 - a_1)}{a_1 a_2\,(\mathbf{Z}_A + \mathbf{Z}_B)} \tag{2.41.4}$$

$$\mathbf{I}_B = \frac{\mathbf{Z}_A\,\mathbf{I}_L}{\mathbf{Z}_A + \mathbf{Z}_B} - \frac{\mathbf{V}_1\,(a_2 - a_1)}{a_1 a_2\,(\mathbf{Z}_A + \mathbf{Z}_B)} \tag{2.41.5}$$

Each of these currents has two components ; the first component represents the transformer's share of the load current and the second component is a circulating current in the secondary windings. Circulating currents have the following undesirable effects :

(*a*) They increase the copper loss.

(*b*) They overload one transformer and reduce the permmissible load kVA.

Equal Voltage Ratios

In order to eliminate circulating currents, the voltage ratios must be identical. That is, $a_1 = a_2$.

Under this condition,

$$\mathbf{I}_A = \frac{\mathbf{Z}_B\,\mathbf{I}_L}{\mathbf{Z}_A + \mathbf{Z}_B} \tag{2.41.6}$$

$$\mathbf{I}_B = \frac{\mathbf{Z}_A\,\mathbf{I}_L}{\mathbf{Z}_A + \mathbf{Z}_B} \tag{2.41.7}$$

$$\therefore \qquad \frac{\mathbf{I}_A}{\mathbf{I}_B} = \frac{\mathbf{Z}_B}{\mathbf{Z}_A} \tag{2.41.8}$$

Equation (2.41.8) shows that the transformer currents are inversely proportional to the transformer impedances.

Also, $\qquad \mathbf{I}_A\,\mathbf{Z}_A = \mathbf{I}_B\,\mathbf{Z}_B \qquad (2.41.9)$

Multiplying Eqs. (2.41.6) and (2.41.7) by common load voltage V_L changes currents into voltamperes. Hence writing

$\mathbf{S}_L = \mathbf{V}_L\,\mathbf{I}_L$ = total load VA

$\mathbf{S}_A = \mathbf{V}_L\,\mathbf{I}_A$ = VA of transformer *A*

and $\quad \mathbf{S}_B = \mathbf{V}_L\,\mathbf{I}_B$ = VA of transformer *B*, we get

$$\mathbf{S}_A = \frac{\mathbf{Z}_B}{\mathbf{Z}_A + \mathbf{Z}_B}\,\mathbf{S}_L \tag{2.41.10}$$

$$\mathbf{S}_B = \frac{\mathbf{Z}_A}{\mathbf{Z}_A + \mathbf{Z}_B}\,\mathbf{S}_L \tag{2.41.11}$$

From Eqs. (2.41.10) and (2.41.11)

$$\frac{\mathbf{S}_A}{\mathbf{S}_B} = \frac{\mathbf{Z}_B}{\mathbf{Z}_A} \tag{2.41.12}$$

Thus, the voltampere load on each transformer is inversely proportional to its ohmic impedance. Hence to share the load in proportion to their ratings, the transformers should have ohmic impedances which are inversely proportional to their ratings. In terms of per-unit values, the above statement may be expressed as follows :

The transformers should have equal per-unit impedances in order to share the load in proportion to their voltampere ratings.

Equation (2.41.9) shows that for efficient parallel operation of two transformers, the potential differences at full load across the transformers' internal impedances should be equal. This condition ensures that the load sharing between them is according to the rating of each transformer. if the per unit equivalent impedances are not equal, the transformers will not share the load in proportion to their kVA ratings, so that the overall rating of the transformer bank is reduced.

It is often convenient to specify the percentage or per-unit values of resistance and leakage reactance for a transformer. In these circumstances Eqs. (2.41.6) to (2.41.12) do not, in general, apply directly.

We have, per unit impedance $= \dfrac{\text{actual impedance}}{\text{base impedance}}$

$$\mathbf{Z}_{pu} = \frac{\mathbf{Z}}{\mathbf{Z}_b} \tag{2.41.13}$$

Then $\mathbf{Z} = \mathbf{Z}_{pu}\,\mathbf{Z}_b = \mathbf{Z}_{pu}\dfrac{V_r}{I_r} = \mathbf{Z}_{pu}\dfrac{V_r^2}{S_r}$

where V_r is the rated voltage and S_r is the rated VA. Then

$$\frac{\mathbf{Z}_A}{\mathbf{Z}_B} = \frac{\mathbf{Z}_{A\,pu}}{\mathbf{Z}_{B\,pu}}\frac{V_{rA}^2\,S_{rB}}{V_{rB}^2\,S_{rA}} \tag{2.41.14}$$

Since $a_1 = a_2$, $V_{rA} = V_{rB}$

Therefore
$$\frac{\mathbf{Z}_A}{\mathbf{Z}_B} = \frac{\mathbf{Z}_{A\,pu}}{\mathbf{Z}_{B\,pu}}\cdot\frac{S_{rB}}{S_{rA}} \tag{2.41.15}$$

Equation (2.41.15) shows that, if the transformers are to share the load in proportion to their ratings, their per-unit impedances must have the same magnitude and that, if the transformers are to operate at the same power factor, their per-unit impedances must have the same phase angle.

Since
$$\mathbf{Z}_A = R_A + j\,X_A$$
$$\mathbf{Z}_B = R_B + j\,X_B$$
$$\mathbf{Z}_A = |\mathbf{Z}_A|\,\angle \tan^{-1}\frac{X_A}{R_A} = |\mathbf{Z}_A|\,\angle\theta_A$$
$$\mathbf{Z}_B = |\mathbf{Z}_B|\,\angle \tan^{-1}\frac{X_B}{R_B} = |\mathbf{Z}_B|\,\angle\theta_B$$

If the impedance angles of both transformers are equal, that is, $\dfrac{X_A}{R_A} = \dfrac{X_B}{R_B}$ then $\mathbf{I}_A$ and $\mathbf{I}_B$ will be in phase and the load current I_L will be the arithmetic sum of I_A and I_B. That is $I_L = I_A + I_B$

But if $\theta_A \neq \theta_B$, that is, $\dfrac{R_A}{X_A} \neq \dfrac{R_B}{X_B}$, the magnitudes of the currents remain inversely proportional to the magnitudes of the impedances, but $\mathbf{I}_A$ and $\mathbf{I}_B$ will not be in phase. At rated load, the load current $\mathbf{I}_L$ will be the phasor sum of $\mathbf{I}_A$ and $\mathbf{I}_B$. Since the phasor sum of $\mathbf{I}_A$ and $\mathbf{I}_B$ is less than the arithmetic sum of I_A and I_B, the load kVA ($= S_L$) is less than the sum of the transformer kVAs.

Finally both the transformers should have the same polarity while connecting them in parallel.

2.42 CONDITIONS FOR PARALLEL OPERATION OF SINGLE-PHASE TRANSFORMERS

For satisfactory parallel operation of the transformers, *two main* conditions are *necessary.*

Necessary conditions

1. The polarities of the transformers must be the same.
2. The turn ratios of the transformers should be equal.

For efficient operation two further conditions are *desirable.*

Desirable conditions

1. The voltages at full load across transformers' internal impedances *should* be equal.
2. The ratios of their winding resistances to reactances should be equal for both transformers. This condition ensures that both transformers operate the same power factor, thus sharing active power and reactive voltamperes according to their ratings.

EXAMPLE 2.26. *Two single-phase transformers share a load of 400 kVA at power factor 0.8 lagging. Their equivalent impedances referred to secondary windings are $(1 + j\,2.5)\ \Omega$ and $(1.5 + j\,3)\ \Omega$ respectively. Calculate the load shared by each transformer.*

SOLUTION.
$$\mathbf{Z}_A = 1 + j2.5 = 2.693\ \underline{/\,68.2^\circ}\ \Omega$$
$$\mathbf{Z}_B = 1.5 + j3 = 3.354\ \underline{/\,63.4^\circ}\ \Omega$$
$$\mathbf{Z}_A + \mathbf{Z}_B = 1 + j\,2.5 + 1.5 + j\,3$$
$$= 2.5 + j\,5.5 = 6.041\ \underline{/\,65.6^\circ}\ \Omega$$
$$\mathbf{S}_L = S_L\ \underline{/-\phi^\circ} = 400\ \underline{/-\cos^{-1} 0.8^\circ} = 400\ \underline{/-36.9^\circ}\ \text{kVA}$$
$$\mathbf{S}_A = \frac{\mathbf{Z}_B}{\mathbf{Z}_A + \mathbf{Z}_B}\,\mathbf{S}_L = \frac{3.354\ \underline{/\,63.4^\circ}}{6.041\ \underline{/\,65.6^\circ}} \times 400\ \underline{/-36.9^\circ}$$
$$= 222.08\ \underline{/-39.1^\circ}\ \text{kVA}$$
$= 222.08$ kVA at power factor of $\cos 39.1^\circ$ lagging.

$= 222.08$ kVA at power factor of 0.776 lagging.
$$\mathbf{S}_B = \frac{\mathbf{Z}_A}{\mathbf{Z}_A + \mathbf{Z}_B}\,\mathbf{S}_L = \frac{2.693\ \underline{/\,68.2^\circ}}{6.041\ \underline{/\,65.6^\circ}} \times 400\ \underline{/-36.9^\circ}$$
$$= 178.3\ \underline{/-34.3^\circ}\ \text{kVA}$$
$= 178.3$ kVA at power factor of $\cos 34.3^\circ$ ($= 0.8261$) lagging

ALTERNATIVELY

$$\mathbf{S}_A + \mathbf{S}_B = \mathbf{S}_L$$

$$\mathbf{S}_B = \mathbf{S}_L - \mathbf{S}_A = 400\ \underline{/-36.9^\circ} - 222.08\ \underline{/-39.1^\circ}$$

$$= 320 - j\,240 - (172.35 - j\,140) = 147.65 - j\,100$$

$$= 178.33\ \underline{/-34.1^\circ}\ \text{kVA.}$$

EXAMPLE 2.27. *A 500 kVA, 33/3.3 kV single-phase transformer with a resistance voltage drop of 1.5% and a reactance voltage drop of 6% is connected in parallel with a 1000 kVA, 33/3.3 kV single-phase transformer with a resistance voltage drop of 1% and a reactance voltage drop of 6.2%. Find the kVA loading and operating power factor of each transformer when the load is 1200 kVA at power factor 0.8 lagging*

SOLUTION. *Transformer A*, 33/3.3 kV, 500 kVA

$$R_{A\,pu} = \frac{1.5}{100} = 0.015$$

$$X_{A\,pu} = \frac{6}{100} = 0.06$$

$$\mathbf{Z}_{A\,pu} = R_{A\,pu} + j\,X_{A\,pu}$$

$$\therefore \quad \mathbf{Z}_{A\,pu} = 0.015 + j\,0.06 = 0.06185\ \underline{/75.96^\circ}$$

Transformer B, 33/3.3 kV, 1000 kVA

$$R_{B\,pu} = \frac{1}{100} = 0.01$$

$$X_{B\,pu} = \frac{6.2}{100} = 0.062$$

$$\mathbf{Z}_{B\,pu} = 0.01 + j\,0.062 = 0.0628\ \underline{/80.84^\circ}$$

$$\frac{\mathbf{Z}_A}{\mathbf{Z}_B} = \frac{\mathbf{Z}_{A\,pu}}{\mathbf{Z}_{B\,pu}}\frac{S_{Br}}{S_{Ar}} = \frac{0.06185\ \underline{/75.96^\circ}}{0.0628\ \underline{/80.84^\circ}} \times \frac{1000}{500}$$

$$= 1.9697\ \underline{/-4.88^\circ} = 1.9626 - j\,0.1675$$

$$1 + \frac{\mathbf{Z}_A}{\mathbf{Z}_B} = 2.9626 - j\,0.1675 = 2.9673\ \underline{/-3.24^\circ}$$

$$\frac{\mathbf{Z}_B}{\mathbf{Z}_A} = \frac{1}{1.9697\ \underline{/-4.88^\circ}} = 0.5077\ \underline{/+4.88^\circ}$$

$$= 0.5058 + j\,0.0432$$

$$1 + \frac{\mathbf{Z}_B}{\mathbf{Z}_A} = 1.5058 + j\,0.0432 = 1.5064\ \underline{/1.64^\circ}$$

$$\mathbf{S}_L = 1200\ \underline{/-\cos^{-1} 0.8^\circ} = 1200\ \underline{/-36.87^\circ}\ \text{kVA}$$

$$\mathbf{S}_A = \frac{\mathbf{Z}_B}{\mathbf{Z}_A + \mathbf{Z}_B}\,\mathbf{S}_L = \frac{1}{1 + \dfrac{\mathbf{Z}_A}{\mathbf{Z}_B}}\,\mathbf{S}_L$$

$$= \frac{1200\ \underline{/-36.87^\circ}}{2.9673\ \underline{/-3.24^\circ}} = 404.4\ \underline{/-33.63^\circ}\ \text{kVA}$$

$\therefore \qquad S_A = 404.4 \text{ kVA}$

$$\cos \phi_A = \cos(-33.63°) = 0.8326 \text{ (lagging)}$$

$$\mathbf{S}_B = \frac{\mathbf{Z}_A}{\mathbf{Z}_A + \mathbf{Z}_B}\mathbf{S}_L = \frac{1}{1 + \dfrac{\mathbf{Z}_B}{\mathbf{Z}_A}}\mathbf{S}_L$$

$$= \frac{1200 \angle -36.87°}{1.5064 \angle 1.64°} = 796.6 \angle -38.51° \text{ kVA}$$

$\therefore \qquad S_B = 796.6 \text{ kVA}, \cos \phi_B = \cos(-38.51°) = 0.7825 \text{ (lagging)}.$

Example 2.28. *Two single-phase transformers having the same voltage ratio on no-load operate in parallel to supply a load of 1000 kVA at 0.8 power factor lagging. One transformer is rated at 400 kVA and has a per unit equivalent impedance of $0.01 + j\,0.06$; the other is rated at 600 kVA and has a per unit equivalent impedance of $0.01 + j\,0.05$. Determine the load on each transformer in kVA and the operating power factor.*

Solution. Suppose that the per unit impedances of the transformers are referred to a common base of 600 kVA.

$$\therefore \qquad \mathbf{Z}_A = \frac{600}{400}(0.01 + j\,0.06) = 0.015 + j\,0.09$$

$$= 0.0912 \angle 80.5°$$

$$\mathbf{Z}_B = \frac{600}{600}(0.01 + j\,0.05) = 0.01 + j\,0.05$$

$$= 0.0510 \angle 78.7°$$

$$\mathbf{Z}_A + \mathbf{Z}_B = 0.025 + j\,0.14 = 0.142 \angle 79.8°$$

$$\mathbf{S}_L = 1000 \angle -\cos^{-1} 0.8° = 1000 \angle -36.9°$$

$$\therefore \qquad \mathbf{S}_A = \frac{\mathbf{Z}_B}{\mathbf{Z}_A + \mathbf{Z}_B}\mathbf{S}_L$$

$$= \frac{0.051 \angle 78.7°}{0.142 \angle 79.8°} \times 1000 \angle -36.9°$$

$$= 359 \angle -38° \text{ kVA}$$

$\therefore \qquad S_A = 359$ kVA at a lagging power factor of $\cos^{-1} 38°$ or 0.788.

$$\mathbf{S}_B = \frac{\mathbf{Z}_A}{\mathbf{Z}_A + \mathbf{Z}_B}\mathbf{S}_L$$

$$= \frac{0.0912 \angle 80.5°}{0.142 \angle 79.8°} \times 1000 \times \angle -36.9°$$

$$= 642 \angle -36.2° \text{ kVA}$$

$\therefore \qquad S_B = 642$ kVA at a lagging power factor of $\cos^{-1} 36.2$ or 0.8069.

ALTERNATIVELY

$$S_A + S_B = S_L$$

$$S_B = S_L - S_A$$

$$= 1000 \angle -\cos^{-1} 0.8° - 359 \angle -38°$$

$$= 800 - j\,600 - (282.9 - j\,221)$$

$$= 517.1 - j\,379 = 641.1 \angle -36.2° \text{ kVA.}$$

2.43 UNEQUAL VOLTAGE RATIOS

If Z_L = load impedance at secondary terminals.

Let E_{2A}, E_{2B} be the no-load secondary emfs of the two transformers in parallel.

By KVL $\quad V_2 = E_{2A} - I_A Z_A \quad$ (2.43.1)

$V_2 = E_{2B} - I_B Z_B \quad$ (2.43.2)

$V_2 = I_L Z_L \quad$ (2.43.3)

By KCL $\quad I_L = I_A + I_B \quad$ (2.43.4)

$\therefore \quad V_2 = (I_A + I_B) Z_L \quad$ (2.43.5)

Subtracting Eq. (2.43.2) from Eq. (2.43.1)

$$E_{2A} - E_{2B} = I_A Z_A - I_B Z_B$$

$$\therefore \quad I_B = \frac{-(E_{2A} - E_{2B}) + I_A Z_A}{Z_B} \quad (2.43.6)$$

From Eqs. (2.43.1) and (2.43.5)

$$E_{2A} - I_A Z_A = I_A Z_L + I_B Z_L \quad (2.43.7)$$

Substituting the value of I_B from Eq. (2.43.6) in Eq. (2.43.7), we obtain

$$E_{2A} - I_A Z_A = I_A Z_L + \frac{Z_L}{Z_B} [I_A Z_A - (E_{2A} - E_{2B})]$$

$$I_A \left(Z_A + Z_L + \frac{Z_L Z_A}{Z_B} \right) = E_{2A} + (E_{2A} - E_{2B}) \frac{Z_L}{Z_B}$$

$$I_A = \frac{E_{2A}}{Z_A + Z_L + \frac{Z_A Z_L}{Z_B}} + \frac{E_{2A} - E_{2B}}{\left(Z_A + Z_L + \frac{Z_A Z_L}{Z_B} \right)} \cdot \frac{Z_L}{Z_B}$$

$$\therefore \quad I_A = \frac{E_{2A} Z_B + (E_{2A} - E_{2B}) Z_L}{Z_A Z_B + (Z_A + Z_B) Z_L} \quad (2.43.8)$$

$$\text{Similarly, } I_B = \frac{E_{2B} Z_A - (E_{2A} - E_{2B}) Z_L}{Z_A Z_B + (Z_A + Z_B) Z_L} \quad (2.43.9)$$

The expression for I_B can be obtained by interchanging A and B in Eq. (2.43.8).

2.44 CIRCULATING CURRENT

The circulating current may be defined as that current which flows in transformers operating in parallel when they do not supply a load. The unbalanced voltages cause circulating currents between the transformers operating in parallel (transformer bank). These circulating currents also cause a primary current to be drawn from the supply even when the bank as a whole is on no load. This primary current is quite distinct from the normal magnetizing current of the transformers. The **undesirable effects of circulating currents** are as follows :

(*i*) They increase the copper losses.

(*ii*) They reduce the permissible output of the bank.

(*iii*) They overload one transformer.

On load this circulating current will be superimposed on the load current. Equation (2.43.8) can be rearranged as

$$\mathbf{I}_A = \frac{\mathbf{E}_{2A}\,\mathbf{Z}_B}{\mathbf{Z}_A\,\mathbf{Z}_B + (\mathbf{Z}_A + \mathbf{Z}_B)\,\mathbf{Z}_L} + \frac{\mathbf{E}_{2A} - \mathbf{E}_{2B}}{\mathbf{Z}_A + \mathbf{Z}_B + \dfrac{\mathbf{Z}_A\,\mathbf{Z}_B}{\mathbf{Z}_L}}$$

The term $\left(\dfrac{\mathbf{E}_{2A} - \mathbf{E}_{2B}}{\mathbf{Z}_A + \mathbf{Z}_B + \dfrac{\mathbf{Z}_A\,\mathbf{Z}_B}{\mathbf{Z}_L}}\right)$ is sometimes known as a **circulating current** due to the difference voltage. Strictly speaking, it is only a mathematically expressed component of the total current except when $Z_L = \infty$, that is, when the load is open circulated. This true circulating current is then given by

$$\mathbf{I}_C = \frac{\mathbf{E}_{2A} - \mathbf{E}_{2B}}{\mathbf{Z}_A + \mathbf{Z}_B} \tag{2.44.1}$$

This expression can also be deduced directly from the circuit diagram.

2.45 CALCULATION OF LOAD VOLTAGE V_2

$$\mathbf{V}_2 = (\mathbf{I}_A + \mathbf{I}_B)\,\mathbf{Z}_L \tag{2.45.1}$$

$$= \left(\frac{\mathbf{E}_{2A} - \mathbf{V}_2}{\mathbf{Z}_A} + \frac{\mathbf{E}_{2B} - \mathbf{V}_2}{\mathbf{Z}_B}\right)\mathbf{Z}_L \tag{2.45.2}$$

$$\mathbf{V}_2\left(\frac{1}{\mathbf{Z}_L} + \frac{1}{\mathbf{Z}_A} + \frac{1}{\mathbf{Z}_B}\right) = \frac{\mathbf{E}_{2A}}{\mathbf{Z}_A} + \frac{\mathbf{E}_{2B}}{\mathbf{Z}_B} \tag{2.45.3}$$

$$\mathbf{V}_2 = \frac{\dfrac{\mathbf{E}_{2A}}{\mathbf{Z}_A} + \dfrac{\mathbf{E}_{2B}}{\mathbf{Z}_B}}{\dfrac{1}{\mathbf{Z}_L} + \dfrac{1}{\mathbf{Z}_A} + \dfrac{1}{\mathbf{Z}_B}} = \frac{\mathbf{E}_{2A}\,\mathbf{Z}_B + \mathbf{E}_{2B}\,\mathbf{Z}_A}{\mathbf{Z}_A + \mathbf{Z}_B + \dfrac{\mathbf{Z}_A\,\mathbf{Z}_B}{\mathbf{Z}_L}} \tag{2.45.4}$$

Equation (2.45.3) can also be written as

$$\mathbf{V}_2\,(\mathbf{Y}_L + \mathbf{Y}_A + \mathbf{Y}_B) = \mathbf{E}_{2A}\,\mathbf{Y}_A + \mathbf{E}_{2B}\,\mathbf{Y}_B \tag{2.45.5}$$

$$V_2 = \frac{E_{2A} Y_A + E_{2B} Y_B}{Y_A + Y_B + Y_L} \tag{2.45.6}$$

where $\quad Y_A = \frac{1}{Z_A},\ Y_B = \frac{1}{Z_B},\ Y_L = \frac{1}{Z_L}$

If there are a number of transformers in parallel with no-load voltages $E_{2A}, E_{2B}, \ldots E_{2K}$ all being in general different from one another, the secondary load voltage is given by

$$V_2 = \frac{E_{2A} Y_A + E_{2B} Y_B + \ldots + E_{2K} Y_K}{(Y_A + Y_B + \ldots + Y_K) + Y_L}$$

This is **parallel-generator theorem.** Here $Y_K = \frac{1}{Z_K}$ is the p.u. equivalent admittance of transformer K, and Y_L is the load admittance.

EXAMPLE 2.29. *Two transformers A and B are joined in parallel to the same load. Determine the current delivered by each transformer, given : open-circuit emf 6600 V for A and 6400 V for B. Equivalent leakage impedance in terms of the secondary = (0.3 + j 3) Ω for A, and (0.2 + j1) Ω for B. The load impedance is (8 + j 6) Ω.*

SOLUTION. $\quad I_A = \frac{E_{2A} Z_{2B} + (E_{2A} - E_{2B}) Z_L}{Z_A Z_B + (Z_A + Z_B) Z_L}$

$$E_{2A} = 6600 \text{ V},\ E_{2B} = 6400 \text{ V}$$

$$Z_A = 0.3 + j\,3 = 3.015 \angle 84.3^\circ \ \Omega$$

$$Z_B = 0.2 + j\,1 = 1.02 \angle 78.7^\circ \ \Omega$$

$$Z_L = 8 + j\,6 = 10 \angle 36.87^\circ \ \Omega$$

$$Z_A + Z_B = 0.5 + j\,4 = 4.031 \angle 82.87^\circ \ \Omega$$

$$Z_A Z_B = (3.015 \angle 84.3^\circ)(1.02 \angle 78.7^\circ)$$

$$= 3.0753 \angle 163^\circ = -2.941 + j\,0.8991$$

$$Z_L (Z_A + Z_B) = (10 \angle 36.87^\circ)(4.031 \angle 82.87^\circ)$$

$$= 40.31 \angle 119.74^\circ = -19.996 + j\,35$$

$\therefore$

$$I_A = \frac{6600\,(0.2 + j\,1) + (6600 - 6400)\,(8 + j\,6)}{-2.941 + j\,0.8991 - 19.996 + j\,35}$$

$$= \frac{2920 + j\,7800}{-22.937 + j\,35.8991} = \frac{8328.6 \angle 69.5^\circ}{42.6 \angle 122.6^\circ} = 195.5 \angle -53.1^\circ \text{ A}$$

$$I_B = \frac{E_{2B} Z_A - (E_{2A} - E_{2B}) Z_L}{Z_A Z_B + (Z_A + Z_B) Z_L}$$

$$= \frac{6400\,(0.3 + j\,3) - (6600 - 6400)\,(8 + j\,6)}{42.6 \angle 122.6^\circ}$$

$$= \frac{320 + j\,18000}{42.6 \angle 122.6^\circ} = \frac{18002.8 \angle 88.98^\circ}{42.6 \angle 122.6^\circ} = 422.58 \angle -33.62^\circ \text{ A.}$$

Example 2.30. *Two single-phase transformers, one of 100 kVA and the other of 50 kVA are connected in parallel to the same busbars of the primary side, their no-load secondary voltages being 1000 V and 950 V respectively. Their resistances are 1.5% and 2.0% respectively, and their reactances 8% and 6% respectively. Calculate the no-load circulating current in the secondaries.*

Solution. $\text{kVA} = \dfrac{V\,I_{fl}}{1000},\; I_{fl} = \dfrac{1000 \times \text{kVA}}{V}$

Full-load currents of transformer A and B are

$$I_A = \frac{1000 \times 100}{925} = 108.1 \text{ A}$$

$$I_B = \frac{1000 \times 50}{925} = 54.05 \text{ A}$$

It is more convenient to work with ohmic impedances. Therefore percentage impedances are converted into ohmic values. Let us assume that the secondary terminal voltage is 925 V. This arbitrarily chosen value is less than either of the two no-load emfs.

$$I_A R_A = 1.5\% \text{ of V} = \frac{1.5}{100} \times 925$$

$$R_A = \frac{1.5}{100} \times \frac{925}{108.1} = 0.1284\ \Omega.$$

$$I_A X_A = 8\% \text{ of } V = \frac{8}{100} \times 925$$

$$X_A = \frac{8 \times 925}{100 \times 108.1} = 0.6846\ \Omega$$

$$I_B R_B = 2\% \text{ of V} = \frac{2}{100} \times 925$$

$$R_B = \frac{2 \times 925}{100 \times 54.05} = 0.3423\ \Omega$$

$$I_B X_B = 6\% \text{ of V} = \frac{6 \times 925}{100}$$

$$X_B = \frac{6 \times 925}{100 \times 54.05} = 1.0268\ \Omega$$

$$\mathbf{Z}_A = R_A + j\,X_A = 0.1284 + j\,0.6846\ \Omega$$

$$\mathbf{Z}_B = R_B + j\,X_B = 0.3423 + j\,1.0268\ \Omega$$

$$\mathbf{Z}_A + \mathbf{Z}_B = 0.1284 + j\,0.6846 + 0.3423 + j\,1.0268$$
$$= 0.4707 + j\,1.7114 = 1.775\ \underline{/74.62^\circ}\ \Omega$$

Circulating current

$$\mathbf{I}_C = \frac{\mathbf{E}_{2A} - \mathbf{E}_{2B}}{\mathbf{Z}_A + \mathbf{Z}_B} = \frac{1000 - 950}{1.775\ \underline{/74.62^\circ}}$$

$$= 28.17\ \underline{/-74.62^\circ}\ \text{A}$$

EXAMPLE 2.31. *Two single-phase transformers A and B of ratings 500 kVA and 250 kVA are supplying a load of 750 kVA at 0.8 power factor lagging. Their open-circuit voltages are 405 V and 415 V respectively. Transformer A has 1% resistance and 5% reactance and transformer B has 1.5% resistance and 4% reactance. Find (a) circulating current at no load, (b) current supplied by each transformer and (c) kVA shared by each transformer.*

SOLUTION. For convenience let us convert the percentage impedances into ohmic impedances. We shall *arbitrarily* assume that the terminal voltage is 400 V. This assumption is justified since the terminal voltage should be less than either of the two no-load emfs. Such an assumption will not introduce appreciable error.

For transformer A

$$(\text{kVA})_A = \frac{VI_A}{1000}$$

$\therefore$ full-load current of A

$$I_A = \frac{(\text{kVA})_A \times 1000}{V}$$

$$= \frac{500 \times 1000}{400} = 1250 \text{ A}$$

Similarly, full-load current of B

$$I_B = \frac{250 \times 1000}{400} = 625 \text{ A}$$

$$I_A R_A = 1\% \text{ of } 400$$

$$\therefore \quad R_A = \frac{1}{100} \times \frac{400}{1250} = 0.0032\ \Omega$$

$$I_A X_A = 5\% \text{ of } 400$$

$$X_A = \frac{5}{100} \times \frac{400}{1250} = 0.016\ \Omega$$

$$I_B R_B = 1.5\% \text{ of } 400$$

$$R_B = \frac{1.5}{100} \times \frac{400}{625} = 0.0096\ \Omega$$

$$I_B X_B = 4\% \text{ of } 400$$

$$X_B = \frac{4}{100} \times \frac{400}{625} = 0.0256\ \Omega$$

Impedance of transformer A

$$\mathbf{Z}_A = R_A + j\,X_A = 0.0032 + j\,0.016$$

$$= 0.0163 \angle 78.7^\circ\ \Omega$$

Impedance of transformer B

$$\mathbf{Z}_B = R_B + j\,X_B = 0.0096 + j\,0.0256 = 0.02734 \angle 69.4^\circ\ \Omega$$

$$\mathbf{Z}_A + \mathbf{Z}_B = 0.0032 + j\,0.016 + 0.0096 + j\,0.0256$$

$$= 0.0128 + j\,0.0416 = 0.0435 \angle 72.9^\circ\ \Omega$$

(*a*) Circulating current at no load

$$I_C = \frac{E_{2B} - E_{2A}}{Z_A + Z_B} = \frac{415 - 405}{0.0435 \angle 72.9^\circ}$$

$$= 229.9 \angle -72.9^\circ \text{ A}$$

Power factor $= \cos(-72.9^\circ) = 0.294$ (lagging).

Hence the circulating current is 229.9 A at a power factor of 0.294 lagging.

Calculation of load impedance Z_L

Load kVA $\quad S_L = V_2 I_L \times 10^{-3}$

$$= V_2 \times \frac{V_2}{Z_L} \times 10^{-3}$$

$$S_L = \frac{V_2^2}{Z_2} \times 10^{-3}$$

$$750 \angle -\cos^{-1} 0.8^\circ = \frac{(400)^2 \times 10^{-3}}{Z_L}$$

$$Z_L = \frac{(400)^2 \times 10^{-3}}{750 \angle -36.87^\circ}$$

$$= 0.2133 \angle 36.87^\circ \ \Omega$$

$$= 0.1706 + j\,0.128 \ \Omega$$

$$Z_A Z_B = (0.0163 \angle 78.7^\circ)(0.02734 \angle 69.4^\circ)$$

$$= 0.0004456 \angle 148.1^\circ$$

$$= -0.0003783 + j\,0.0002354$$

$$(Z_A + Z_B) Z_L = (0.0435 \angle 72.9^\circ)(0.2133 \angle 36.87^\circ)$$

$$= 0.009278 \angle 109.77^\circ$$

$$= -0.003138 + j\,0.008731$$

$$Z_A Z_B + (Z_A + Z_B) Z_L = -0.0003783 + j\,0.0002354 - 0.003138 + j\,0.0087316$$

$$= -0.003516 + j\,0.008967 = 0.009632 \angle 111.4^\circ$$

$$(E_{2A} - E_{2B}) Z_L = (405 - 415)(0.1706 + j\,0.128)$$

$$= -(1.706 + j\,1.28).$$

$$E_{2A} Z_B = 405\,(0.0096 + j\,0.0256)$$

$$= 3.888 + j\,10.368$$

$$E_{2B} Z_A = 415\,(0.0032 + j\,0.016)$$

$$= 1.328 + j\,6.64$$

$$\therefore \quad I_A = \frac{E_{2A} Z_B + (E_{2A} - E_{2B}) Z_L}{Z_A Z_B + (Z_A + Z_B) Z_L}$$

$$= \frac{3.888 + j\,10.368 - 1.706 - j\,1.28}{0.009632 \angle 111.4^\circ}$$

$$= \frac{2.182 + j\,9.088}{0.009632\,\angle 111.4^\circ} = \frac{9.346\,\angle 76.5^\circ}{0.009632\,\angle 111.4^\circ}$$

$$= 970.3\,\angle -34.9^\circ \text{ A}$$

$$\mathbf{I}_B = \frac{\mathbf{E}_{2B}\,\mathbf{Z}_A - (\mathbf{E}_{2A} - \mathbf{E}_{2B})\,\mathbf{Z}_L}{\mathbf{Z}_A\,\mathbf{Z}_B + (\mathbf{Z}_A + \mathbf{Z}_B)\,\mathbf{Z}_L}$$

$$= \frac{1.328 + j\,6.64 + 1.706 + j\,1.28}{0.009632\,\angle 111.4^\circ}$$

$$= \frac{3.034 + j\,7.92}{0.009632\,\angle 111.4^\circ}$$

$$= \frac{8.4812\,\angle 69^\circ}{0.009632\,\angle 111.4^\circ} = 880\,\angle -42.4^\circ \text{ A}$$

kVA shared by transformer A

$$\mathbf{S}_A = \mathbf{V}\mathbf{I}_A \times 10^{-3} = 400 \times (970.3\,\angle -34.9^\circ) \times 10^{-3} = 388.12\,\angle -34.9^\circ$$

$$\cos\phi_A = \cos(-34.9^\circ) = 0.82 \text{ (lagging)}$$

kVA shared by transformer B

$$\mathbf{S}_B = \mathbf{V}\mathbf{I}_B \times 10^{-3} = 400 \times (880\,\angle -42.4^\circ) \times 10^{-3} = 352\,\angle -42.4^\circ$$

$$\cos\phi_B = \cos(-42.4^\circ) = 0.7385 \quad \text{(lagging)}$$

It is seen that slight differences in open-circuited secondary voltages and percentage impedances has resulted in improper load sharing. Transformer A is under-loaded while transformer B is overloaded.

2.46 THREE-PHASE TRANSFORMERS IN PARALLEL

The conditions for proper parallel operation of *single-phase transformers* are as follows :

1. The polarities of the transformers must be the same.
2. Identical primary and secondary voltage ratings.
3. Impedances inversely proportional to the kVA ratings.
4. Identical X/R ratios in the transformer impedances.

The conditions above apply equally to the paralleling of two or more three-phase transformer banks but with the following additions :

(*a*) The phase sequence must be the same.

(*b*) The phase shift between primary and secondary voltages must be the same for all transformers which are to be connected in parallel. It follows that all transformers in the same main group can be connected in parallel.

Under-balanced loading conditions, the three-phase transformer calculations are made on a *per-phase* basis. It is, however, preferable to perform calculations on a *per-unit* basis, particularly in cases where the primary and secondary connections are different.

EXAMPLE 2.32. *A 500 kVA transformer with 0.01 pu resistance and 0.05 pu reactance is connected in parallel with a 250-kVA transformer with 0.015 pu resistance and 0.04 pu reactance. The secondary voltage of each transformer is 400 V on no load. Find how they share a load of 750 kVA at power factor 0.8 lagging.*

SOLUTION. Here the given per unit values refer to different ratings. They should be converted to the same base kVA say 500 kVA.

$$\mathbf{Z}_A = 0.01 + j\,0.05 = 0.051 \angle 78.7^\circ \text{ pu}$$

$$\mathbf{Z}_B = \frac{500}{250}(0.015 + j\,0.04)$$

$$= 0.03 + j\,0.08 = 0.0854 \angle 69.4^\circ \text{ pu}$$

$$\mathbf{Z}_A + \mathbf{Z}_B = 0.01 + j\,0.05 + 0.03 + j\,0.08$$

$$= 0.04 + j\,0.13 = 0.136 \angle 72.9^\circ \text{ pu}$$

Total load kVA

$$\mathbf{S}_L = S \angle -\phi^\circ = 750 \angle -\cos^{-1} 0.8^\circ = 750 \angle -36.9^\circ$$

$$\mathbf{S}_A = \frac{\mathbf{Z}_B}{\mathbf{Z}_A + \mathbf{Z}_B}\mathbf{S}_L = \frac{0.0854 \angle 69.4^\circ}{0.136 \angle 72.9^\circ} \times 750 \angle -36.9^\circ$$

$$= 470.95 \angle -40.4^\circ \text{ kVA}$$

$$= 470.95 \text{ kVA at p.f. } \cos 40.4^\circ \text{ lagging}$$

$$= 470.95 \text{ kVA at p.f. } 0.7615 \text{ lagging}$$

$$\mathbf{S}_B = \frac{\mathbf{Z}_A \mathbf{S}_L}{\mathbf{Z}_A + \mathbf{Z}_B} = \frac{0.051 \angle 78.7^\circ}{0.136 \angle 72.9^\circ} \times 750 \angle -36.9^\circ$$

$$= 281.25 \angle -31.1^\circ \text{ kVA}$$

$$= 281.25 \text{ kVA at p.f. } \cos 31.1^\circ \text{ lagging}$$

$$= 281.25 \text{ kVA at p.f. } 0.8563 \text{ lagging}$$

EXAMPLE 2.33. *Two three-phase transformers which have the same turns ratio are connected in parallel and supply a total load of 800 kW at 0.8 power factor lagging. Their ratings are as follows :*

Transformer	*Rating*	*Per unit resistance*	*Per unit reactance*
A	*400 kVA*	*0.02*	*0.04*
B	*600 kVA*	*0.01*	*0.05*

Determine the power output and power factor of each transformer.

SOLUTION. On the basis of 1000 kVA

$$R_{A\,\text{pu}} = 0.02 \times \frac{1000}{400} = 0.05$$

$$X_{A\,\text{pu}} = 0.04 \times \frac{1000}{400} = 0.10$$

$$R_{B\,\text{pu}} = 0.01 \times \frac{1000}{600} = 0.0167$$

$$X_{B\,\text{pu}} = 0.05 \times \frac{1000}{600} = 0.0833$$

$$\mathbf{Z}_{A\,\text{pu}} = R_{A\,\text{pu}} + j\,X_{A\,\text{pu}} = 0.05 + j\,0.10 = 0.118\,\underline{/63.43^\circ}$$

$$\mathbf{Z}_{B\,\text{pu}} = R_{B\,\text{pu}} + j\,X_{B\,\text{pu}} = 0.0167 + j\,0.0833 = 0.085\,\underline{/78.6^\circ}$$

$$\mathbf{Z}_{A\,\text{pu}} + \mathbf{Z}_{B\,\text{pu}} = 0.0667 + j\,0.1833 = 0.195\,\underline{/70^\circ}$$

$$P = VI\cos\phi = S\cos\phi$$

$$P_L = S_L\cos\phi$$

$$S_L = \frac{P_L}{\cos\phi} = \frac{800}{0.8} = 1000\text{ kVA}$$

$$\mathbf{S}_L = S_L\,\underline{/-\cos^{-1}0.8^\circ} = 1000\,\underline{/-\cos^{-1}0.8^\circ}$$

$$= 800 - j\,600$$

$$\mathbf{S}_A = \frac{\mathbf{Z}_B}{\mathbf{Z}_A + \mathbf{Z}_B} \times \mathbf{S}_L$$

$$= \frac{0.085\,\underline{/78.6^\circ}}{0.195\,\underline{/70^\circ}} \times 1000\,\underline{/-36.87^\circ}$$

$$= 435.89\,\underline{/-28.7^\circ} = 382.3 - j\,209.3\text{ kVA}$$

$$\mathbf{S}_B = \mathbf{S}_L - \mathbf{S}_A$$

$$= 800 - j\,600 - (382.3 - j\,209.3) = 417.7 - j\,390.7$$

$$= 571.9\,\underline{/-43.08^\circ}\text{ kVA}$$

$\therefore$ $S_A = 435.89$ kVA at a lagging power factor of cos 28.7° (= 0.8771)

$S_B = 571.9$ kVA at a lagging power factor of cos 43.08° (= 0.7304)

2.47 THREE-PHASE AUTOTRANSFORMERS

Three-phase autotransformers are used for small ratios of transformations. Delta connections are avoided and star connections are normally used for three-phase autotransformers. The main application of such transformers is for interconnecting two power systems of different voltages, for example, 66 kV to 132 kV systems, 110 to 220 kV systems, 132 to 220 kV systems, 220 to 400 kV systems etc. A three phase star-connected autotransformer is shown in Fig. 2.43.

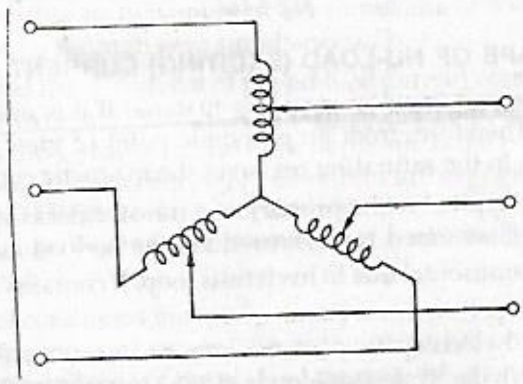


Fig. 2.43. *A three-phase star-connected autotransformer.*

EXAMPLE 2.34. *A 3-phase star-connected autotransformer supplies a balanced 3-phase load of 50 kW at 340 V and at 0.8 p.f. lagging. If the supply voltage is 400 V, determine the currents in the winding as well as in the input and output lines. Neglect exciting current and internal voltage drops.*

SOLUTION. $\sqrt{3}\, V_L I_L \cos\phi = P_{3\phi}$

$$\sqrt{3} \times 340\, I_L \times 0.8 = 50 \times 10^3$$

$\therefore$ load current $I_L = \dfrac{50 \times 10^3}{\sqrt{3} \times 340 \times 0.8} = 106$ A

input voltamperes per phase = output voltamperes per phase

$$\frac{400}{\sqrt{3}} I_H = \frac{340}{\sqrt{3}} I_L$$

$$I_H = \frac{340}{400} \times 106 = 90 \text{ A}$$

Currents flowing from neutral N to tapping points A, B, C are

$$I_{NA} = I_{NB} = I_{NC} = 106 - 90 = 16 \text{ A}$$

The magnitudes and directions of various currents are shown in Fig. 2.44.

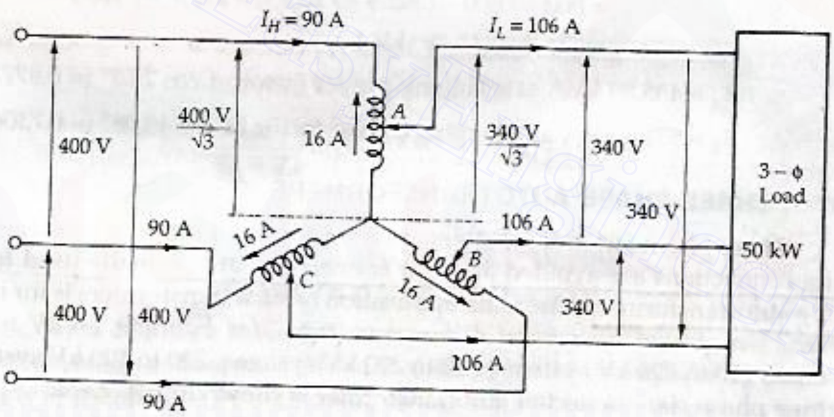


Fig. 2.44.

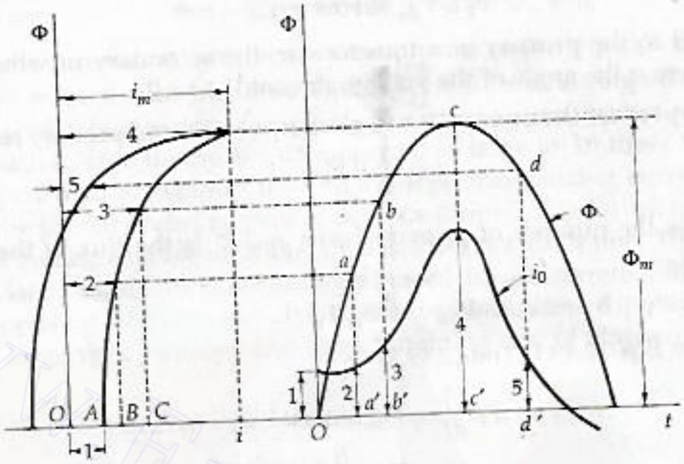
2.48 WAVESHAPE OF NO-LOAD (EXCITING) CURRENT

A transformer requires less magnetic material if it is operated at a higher core flux density. Therefore, from an economic point of view, a transformer is designed to operate in the saturating region of the magnetic core.

If the voltage applied to the primary of a transformer is sinusoidal and the mutual flux set up is assumed to be sinusoidal, the no-load current I_0 (exciting current) will be nonsinusoidal due to hysteresis loop. It contains fundamental and all odd harmonics.

Consider the hysteresis loop of the core as shown in Fig. 2.45(a). Since $\Phi = BA$ and $i = (Hl/N)$, the hysteresis loop is plotted in terms of flux Φ and current

i instead of B and H so that the current required to produce a particular value of flux can be read off directly. The waveform of the no-load current i_0 can be found from the sinusoidal flux waveform and the $\Phi - i$ characteristic of the magnetic core. The graphical procedure is shown in Fig. 2.45.



(a) Upper half of the hysteresis loop *(b) Sinusoidal flux Φ and exciting current waves.*

Fig. 2.45. *Wave shape of the no-load current of a transformer.*

At point O of the flux-time curve, the flux is zero ; this corresponds to a current OA on the hysteresis loop. At point a of the flux-time curve $\Phi = aa'$; this corresponds to a current OB on the hysteresis loop. In brief, the various abscissas of Fig. 2.45(a) are plotted as ordinates to determine the shape of the current wave on Fig. 2.45(b). This procedure is followed round the whole loop until a sufficient number of points is obtained. The current-time curve for the whole loop is plotted. Fig. 2.45(a) shows the waveforms for upper half of the hysteresis loop.

The ascending part of the loop is used for increasing fluxes, and the descending part for decreasing fluxes. The waveform of the current in Fig. 2.45(b) represents the magnetizing component and the hysteresis component of the no-load current. It reaches its maximum at the same time as the flux wave, but the two waves do not go through zero simultaneously.

It is seen that the waveform of the no-load current contains third, fifth and higher-order odd harmonics which increase rapidly if the maximum flux is taken further into saturation. However, the third harmonic is the predominant one. For all practical purposes harmonics higher than third are negligible. At rated voltage the third harmonic in the no-load current is about 5 to 10% of the fundamental. At 150% rated voltage, the third harmonic current can be as high as 30 to 40 percent of the fundamental.

Under load conditions the total primary current is equal to the phasor sum of load current and no-load current. Since the magnitude of the load current is very large as compared that of the no-load current, the primary current is almost sinusoidal under load conditions.

2.49 INRUSH OF MAGNETIZING CURRENT

When a transformer is initially energized, there is a sudden inrush of primary current. The maximum value attained by the flux is over twice the normal flux. The core is driven far into saturation with the result that the magnetizing current has a very high peak value. Let a sinusoidal voltage

$$v_1 = V_{1m} \sin(\omega t + \alpha) \tag{2.49.1}$$

be applied to the primary of a transformer, the secondary of which is an open circuit. Here α the angle of the voltage sinusoid at $t = 0$.

Suppose for the moment we neglect core losses and primary resistance, then

$$v_1 = T_1 \frac{d\Phi}{dt} \tag{2.49.2}$$

where T_1 is the number of primary turns and Φ is the flux in the core. In the steady-state

$$V_{1m} = \omega \Phi_m T_1 \tag{2.49.3}$$

From Eqs. (2.49.1) and (2.49.2)

$$T_1 \frac{d\Phi}{dt} = V_{1m} \sin(\omega t + \alpha)$$

$$\frac{d\Phi}{dt} = \frac{V_{1m}}{T_1} \sin(\omega t + \alpha) \tag{2.49.4}$$

From Eqs. (2.49.3) and (2.49.4)

$$\frac{d\Phi}{dt} = \omega \Phi_m \sin(\omega t + \alpha) \tag{2.49.5}$$

Integration of Eq. (2.49.5) gives

$$\Phi = -\Phi_m \cos(\omega t + \alpha) + \Phi_C \tag{2.49.6}$$

where Φ_C is the constant of integration to be found from initial conditions at $t = 0$. Assume that when the transformer was last disconnected from the supply line, a small residual flux Φ_r remained in the core. Thus at $t = 0$, $\Phi = \Phi_r$.

Substituting these values in Eq. (2.49.6)

$$\Phi_r = -\Phi_m \cos\alpha + \Phi_C$$

$$\therefore \quad \Phi_C = \Phi_r + \Phi_m \cos\alpha \tag{2.49.7}$$

Equation (2.49.6) then becomes

$$\Phi = \underbrace{-\Phi_m \cos(\omega t + \alpha)}_{\text{Steady-state component of flux } \Phi_{ss}} + \underbrace{\Phi_r + \Phi_m \cos\alpha}_{\text{transient component of of flux } \Phi_C} \tag{2.49.8}$$

Equation (2.49.8) shows that the flux consists of two components, the steady-state component Φ_{ss} and the transient component Φ_c. The magnitude of the transient component

$$\Phi_c = \Phi_r + \Phi_m \cos\alpha$$

is a function of α, where α is the instant at which the transformer is switched on to the supply.

If the transformer is switched on at $\alpha = 0$, then $\cos \alpha = 1$

$$\Phi_c = \Phi_r + \Phi_m$$

Under this condition

$$\Phi = -\Phi_m \cos \omega t + \Phi_r + \Phi_m \quad (2.49.9)$$

At $\omega t = \pi$, $\quad \Phi = -\Phi_m \cos \pi + \Phi_r + \Phi_m = 2\,\Phi_m + \Phi_r$

Thus, the core flux attains the maximum value of flux equal to $(2\,\Phi_m + \Phi_r)$, which is over twice the normal flux. This is known as **doubling effect.** Consequently, the core goes into deep saturation. The magnetizing current required for producing such a large flux in the core may be as large as 10 times the normal magnetizing current. Sometimes the rms value of magnetizing current may be larger than the primary rated current of the transformer. This inrush current may produce electromagnetic forces about 25 times the normal value. Therefore the windings of large transformers are strongly braced. Inrush current may also cause improper operation of protective devices like unwarranted tripping of relays, momentary large voltage drops and large humming due to magnetostriction of the core.

To obtain no transient inrush current, Φ_c should be zero :

$$\Phi_c = \Phi_r + \Phi_m \cos \alpha = 0$$

or $$\cos \alpha = \frac{-\Phi_r}{\Phi_m}$$

Since Φ_r is usually very small $\cos \alpha \cong 0$ and $\alpha \cong \dfrac{n\pi}{2}$.

In other words, if the transformer is connected to the supply line near a positive or negative voltage maximum, the current inrush will be minimized. It is usually impractical to attempt to connect a transformer at a predetermined time in the voltage cycle.

Fortunately, inrush currents do not occur as might be thought. The magnitude of the inrush current is also less than the value calculated by purely theoretical considerations. The effects of other transformers in the system, load currents and capacitances all contribute to the reduction.

2.50 HARMONIC PHENOMENA IN THREE-PHASE TRANSFORMERS

The non-sinusoidal nature of magnetizing current necessary to produce sinusoidal flux gives rise to some undesirable phenomena in 3-phase transformers whose phases are magnetically separate, that is, a bank of single-phase transformers, and a 3-phase shell-type transformer.

The phase magnetizing currents should contain third and higher harmonics necessary to produce a sinusoidal flux.

If the phase voltage across each phase is to remain sinusoidal then the phase magnetizing currents must be of the following form :

$$I_{AO} = I_m \sin \omega t + I_{3m} \sin (3\omega t + \phi_3) + I_{5m} \sin (5\,\omega t + \phi_5) + \ldots \quad (2.50.1)$$

$$I_{BO} = I_{1m} \sin (\omega t - 120°) + I_{3m} \sin [3\,(\omega t - 120°) + \phi_3]$$
$$+ I_{5m} \sin [5\,(\omega t - 120°) + \phi_5] + \ldots$$

or $$I_{BO} = I_{1m} \sin(\omega t - 120°) + I_{3m} \sin(3\,\omega t + \phi_3) + I_{5m} \sin(5\,\omega t + 120° + \phi_5) + \ldots \tag{2.50.2}$$

$$I_{CO} = I_{1m} \sin(\omega t - 240°) + I_{3m} \sin[3(\omega t - 240°) + \phi_3] + I_{5m} \sin[5(\omega t - 240°) + \phi_5] + \ldots$$

or $$I_{CO} = I_{1m} \sin(\omega t - 240°) + I_{3m} \sin(3\omega t + \phi_3) + I_{5m} \sin(5\omega t + 240° + \phi_5) + \ldots \tag{2.50.3}$$

It is seen from Eqs. (2.50.1), (2.50.2) and (2.50.3) that the third harmonics in the three currents are cophasal, that is they have the same phase. The fifth harmonics have different phases.

Delta Connection

If I_{AO}, I_{BO} and I_{CO} represent the phase magnetizing currents in a *delta* connection, the line currents can be found by subtracting two phase currents. For example,

$$I_{ABO} = I_{AO} - I_{BO}$$
$$= \sqrt{3}\, I_{1m} \sin(\omega t + 30°) - \sqrt{3}\, I_{5m} \sin(5\,\omega t - 30° + \phi_5) + \ldots \tag{2.50.4}$$

It is seen from Eq. (2.50.4) that the third harmonic present in the phase magnetizing current of a delta-connected three-phase transformer is *not* present in the line current. The third harmonic components, being cophasal, have cancelled out in the line. However, the third harmonic currents flow round the closed loop of delta. A delta connection, therefore, allows a sinusoidal flux and voltage with no third harmonic currents in the supply line. For this reason, majority of 3-phase transformers have a delta-connected winding, and in cases where it is not convenient to have either the primary or secondary connected in delta, a tertiary winding (connected in delta) is provided. The tertiary winding carries the circulating third harmonic current required by the sinusoidal flux in each limb of the core.

Star Connection

If I_{AO}, I_{BO} and I_{CO} represent the phase magnetizing currents in a *star* connection,

$$I_{AO} + I_{BO} + I_{CO} = I_N \tag{2.50.5}$$

where I_N is the current in the neutral wire.

From Eqs. (2.50.1), (2.50.2) and (2.50.3),

$$\therefore \quad I_{AO} + I_{BO} + I_{CO} = 3\, I_{3m} \sin(3\,\omega t + \phi_3) \tag{2.50.6}$$

harmonics above the seventh being neglected. Equation (2.50.6) shows that under balanced conditions, the current in the neutral wire is a third harmonic current having three times the magnitude of each third-harmonic phase current. These third harmonic currents produce inductive interference with communication circuits. If the supply to the star connection is three wire, the neutral current must be zero and therefore

$$3\, I_{3m} \sin(3\,\omega t + \phi_3) = 0$$

or $$I_{3m} = 0.$$

Thus, it is seen that the three-wire star connection suppresses the flow of third harmonic magnetizing currents.

For a four-wire star-connected system, the inphase third-harmonic currents flow in the neutral wire.

A similar treatment is possible for voltages. The three balanced phase voltages containing harmonics can be written

$$V_A = V_{1m} \sin(\omega t + \phi_1') + V_{3m} \sin(3\omega t + \phi_3') + V_{5m} \sin(5\,\omega t + \phi_5') + \ldots \quad (2.50.7)$$

$$V_B = V_{1m} \sin(\omega t - 120° + \phi_1') + V_{3m} \sin(3\omega t + \phi_3') + V_{5m} \sin(5\,\omega t + 120° + \phi_5') + \ldots \quad (2.50.8)$$

$$V_C = V_{1m} \sin(\omega t - 240° + \phi_1') + V_{3m} \sin(3\,\omega\, t + \phi_3') + V_{5m} \sin(5\omega\, t + 240° + \phi_5') + \ldots \quad (2.50.9)$$

Equations (2.50.7), (2.50.8) and (2.50.9) show that the third harmonics in the three phase voltages have the same phase. The line voltage in a star connection can be found by subtracting two phase voltages. For example,

$$V_{AB} = V_A - V_B = \sqrt{3}\, V_{1m} \sin(\omega t + 30° + \phi_1') + \sqrt{3}\, V_{5m} \sin(5\,\omega t - 30° + \phi_5') + \quad (2.50.10)$$

It is seen from Eq. (2.50.10) that the third harmonic is not present in the line-to-line voltage of a star connection. This applies to all triplers harmonics.

In a delta connection, the voltage acting round the closed delta is

$$V_A + V_B + V_C = 3\, V_{3m} \sin(3\,\omega t + \phi_3') \quad (2.50.11)$$

This is a third harmonic voltage, and will circulate a third harmonic current round the closed loop of the delta.

2.51 INSTRUMENT TRANSFORMERS

It is generally not a safe practice to connect instruments, meters, or control apparatus directly to high- voltage (hv) circuits. Instrument transformers (Current transformers and voltage transformers) are universally used to reduce high voltages and currents to safe and practical values which can be measured by conventional instruments (the normal range is 1 A or 5 A for current and 110 V for voltage).

2.51.1 Advantages of Instrument Transformers

The important advantages of instrument transformers are as follows :

1. Ammeters and voltmeters for use with these transformers may be standardized at 1 A or 5 A and 110 V respectively.

2. The single-range instrument may be used to cover a large current or voltage range with instrument transformers. In case of wattmeter or watthour meter it may cover both a large current or voltage range.
3. The measuring instruments may be located away from the high-voltage (hv) circuit for safety to the operator.
4. The current flowing in a busbar or any other conductor can be measured by a current transformer (CT) of split-core type without breaking the current circuit.

2.52 CURRENT TRANSFORMER (CT)

A current transformer is a device for the transformation of current from a higher value to a lower value, or for the transformation of current at a high voltage into a proportionate current at a low voltage with respect to the earth potential. Current transformers (CTs) are used in conjuction with ac instruments, meters or control apparatus where the current to be measured is of such magnitude that the meter or instrument coil cannot conveniently be made of sufficient current-carrying capacity. Current transformers are also used where hv current is to be metered because of the difficulty of providing adequate insulation in meter itself. In meter practice CTs are used when the current to be measured is more than 100 A.

2.53 CONSTRUCTION OF CURRENT TRANSFORMERS

The cores of CTs are usually built up with laminations of silicon steel. A high-permeability nickel steel such as Mumetal or Permalloy is used for cores where a high degree of accuracy is desired. The primary winding carries the current to be measured and is connected to the main circuit. The secondary winding carries a current proportional to the current to be measured and the secondary terminals are connected to the current windings of the meter or the instrument. Both the windings are insulated from the core and from each other. The primary circuit of a CT (Fig. 2.46) is generally a single turn winding B (called a bar primary) and carries full-load current. The secondary winding *S* has a large number of turns.

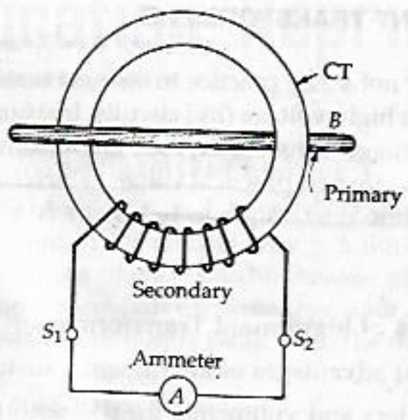


Fig. 2.46. *Current transformer.*

Current transformer ratios are generally specified in terms full-load primary and secondary currents. Usually, the secondary windings are designed for rated values of 5 A, although 1 A and 2 A ranges are also used. For example, 1000/5 A current transformer may be used with a 5-A ammeter to measure currents upto 1000 A. Fig. 2.46 shows the connections of a current transformer.

2.54 DIFFERENCE BETWEEN CURRENT TRANSFORMER AND POWER TRANSFORMER

A current transformer is similar in construction to a power transformer in that it has a magnetic circuit with a primary and a secondary winding. There is a considerable difference in the method of operation. In a power transformer, the primary winding is continuously energized at a substantially constant voltage, and secondary is connected to a load varying in impedance within wide limits. The current in the primary winding is determined by the load connected to the secondary. The magnetic flux in the core is substantially constant at all loads. The current transformer is connected in the line in series with the load. The load determines the current through the primary. The secondary is connected to a load or burden which does not vary, and the primary current is not affected by the load in the secondary. The current in the secondary is determined by the current in the primary. The magnetic flux in the core varies, with the current in the primary. The flux is determined by the connected burden. The flux density in the core is only a very small fraction of that usually used in a power transformer.

2.55 BURDEN OF A CT

The burden of a CT is the value of the load connected across the secondary terminals. It is expressed as the output in voltampere (VA). The rated burden is the value of the burden marked on the nameplate of the CT.

2.56 EFFECT OF OPEN SECONDARY WINDING OF A CT

Under normal operating conditions the secondary winding of a CT is connected to its burden and the secondary is always closed. When the current flows through the primary winding a current also flows through the secondary winding and the ampere turns (mmf) of each winding are substantially equal and opposite. In practice, the secondary ampere turns will be actually 1% to 2% less than the primary ampere turns, the difference being utilized in magnetizing the core. Therefore, if the secondary winding of a CT is opened, while current is flowing through the primary winding, there will be no demagnetizing flux due to the secondary current. Due to the absence of the counter ampereturns of the secondary, the unopposed primary mmf will set up an abnormally high flux in the core. This flux will produce excessive core losses with subsequent heating and a high voltage will be induced across the secondary terminals. This high voltage may be sufficient to cause a danger to life and breakdown of the insulation. Also, loss of accuracy in future may occur, because the excessive mmf leaves residual magnetism in the core. Thus the secondary of a CT should never be open when the primary is carrying current.

2.57 VOLTAGE TRANSFORMER (VT) OR POTENTIAL TRANSFORMER (PT)

A voltage transformer (VT) may be defined as an instrument transformer for the transformation of voltage from a higher value to a lower value. The voltage to be measured or controlled is applied to the primary winding. The terminals of the secondary winding are connected to the meter or instrument. The standard voltage at the terminals of the secondary winding is 110 V.

Potential transformers are made with high quality core operating at very low flux densities so that the magnetizing current is small. The VT should be designed so that the variation of voltage ratio with load is minimum and the phase shift between input and output voltages is also minimum.

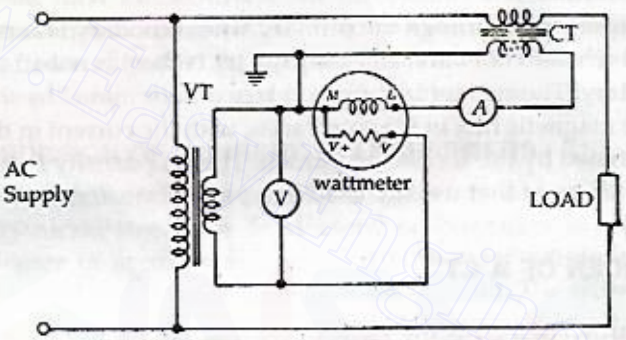


Fig. 2.47.

Fig. 2.47 shows the use of instrument transformers for measurement of current voltage and power in a single-phase circuit.

2.58 TRANSFORMER COOLING

When a transformer is in operation heat is generated due to I^2R losses in the windings and core losses. The removal of heat is called cooling.

The following methods are generally used to cool transformers :

1. ***Air Natural (AN) Cooling.*** In a *dry-type self-cooled transformer*, the natural circulation of surrounding air is used for its cooling. The windings are protected from mechanical injury by a sheet metal enclosure. This type of cooling is satisfactory for low-voltage small transformers upto a few kVA.

2. ***Air Blast (AB) Cooling.*** The dry-type forced air-cooled transformer is similar to that of dry-type self-cooled transformer with the addition that continuous blast of filtered cool air is forced through the core and windings for better cooling. The blast is produced by a fan.

3. ***Oil Natural (ON) Cooling.*** The majority of transformers of medium and large rating have their windings and core immersed in oil which acts both as a cooling medium and an insulating medium. Oil-immersed transformers are enclosed in sheet-steel tank. The heat produced in the cores and windings is passed to the oil. Heated oil becomes lighter and rises to the top and its place is taken by cool oil from the bottom of the tank. The heat of the oil is transferred to the walls of the tank by natural circulation of the oil. The heat is then transferred to the surrounding atmosphere through natural radiation and convection. The oil gets cooler and falls to the bottom. Thus, a continuous natural circulation of oils takes place.

Plain tanks are economical upto a rating of 25 kVA. Above this rating large cooling surface is generally provided by using corrugations, fins, tubes (circular or elliptical) and radiator tanks.

4. ***Oil Blast (OB) Cooling.*** In this type of cooling, forced air is directed over cooling elements of transformer immersed in oil.

5. ***Forced Oil and Forced Air Flow (OFB) Cooling.*** Oil is circulated from top of the transformer tank to a cooling plant. Cool oil is then returned to the bottom of the tank.

6. ***Forced Oil and Water (OFW) Cooling.*** In this type of cooling forced oil flow with water cooling of the oil in the external water heat exchanger takes place. This type of cooling is similar to OFB cooling except that the heat exchanger uses water instead of air for cooling oil. The water is circulated in cooling tubes placed in the heat exchanger.

2.59 CONSERVATORS AND BREATHERS

Oil is not allowed to come in contact with the atmospheric air which may contain moisture. The moisture spoils the insulating properties of oil. Atmospheric air may cause acidity and sludging of oil. A **conservator** is an air- tight metal drum placed above the level of the top of the tank and connected with it by a pipe. It is partially filled with oil.

When the oil expands, or contracts by the change in temperature, there is a displacement of air. When the transformer cools, the oil level goes down and the air is drawn in. This is known as **breathing.** The air coming in is passed through a device called **breather** for the purpose of extracting moisture. The breather consists of a small vessel which contains a drying agent like silica gel crystals impregrated with cobalt chloride. Silica gel is checked regularly and dried and replaced when necessary.

2.60 RATING OF THE TRANSFORMER

It is to be noted that, since the copper loss depends on current and core loss depends on voltage, the total loss in the transformer depends on the volt-ampere product, and not on the phase angle between voltage and current, that is, independent of the load power factor. The rating of the transformer is, therefore, in voltamperes (VA) and not in watts (W). In actual practice, the rating of the transformer is specified in VA, kVA or MVA depending upon its size.

2.61 TRANSFORMER NAME PLATE

According to BIS (Bureau of Indian Standard) 2026 every transformer must be provided with a name plate giving the following information :

- Type (power, auto, booster, etc.)
- Year of manufacture
- Number of phases
- Rated kVA (for multiwinding transformers rated kVA of each winding)
- Rated frequency
- Rated voltage of each winding
- Connection symbol
- Percent impedance voltage at rated current (measured value corrected to 75° C)
- Type of cooling
- Mass and volume of insulating oil

As per BIS 2026, the preferred kVA ratings are

6.3, 10, 16, 25, 40, 63, 100, 160, 250, 400, 630, 1000 kVA, etc.

EXERCISES

2.1 What is an autotransformer ? State its merits and demerits over the two-winding transformer. Give the constructional features and explain the working principle of a single-phase autotransformer.

2.2 Derive an expression for saving in conductor material in an autotransformer over a two-winding transformer of equal rating. State the advantages and disadvantages of autotransformers over two-winding transformers.

2.3 If an autotransformer is made from a 2-winding transformer having a turns ratio $\frac{T_1}{T_2} = a$, show that

$$\frac{\text{magnetizing current as an autotransformer}}{\text{magnetizing current as a 2 - winding transformer}} = \frac{a-1}{a}$$

$$\frac{\text{shortcircuit current as an autotransformer}}{\text{short-circuit current as a 2 - winding transformer}} = \frac{a}{a-1}$$

2.4 An 11500/2300 V transformer is rated at 100 kVA as a 2-winding transformer. If the two windings are connected in series to form an autotransformer what would be the voltage ratio and output ?

[13.8/11.5 kV, 600 kVA or 13.8/2.3 kV, 120 kVA]

2.5 What are the applications of autotransformers ?

2.6 Define an autotransformer. How does the current flow in different parts of its windings ?

2.7 Derive an expression for the saving autotransformer as compared to an equivalent two-winding transformer.

2.8 Give the constructional features of an autotransformer. State the applications of autotransformers.

2.9 What are the reasons of higher efficiency of autotransformers as compared to conventional transformers ?

2.10 What is meant by the terms transformed voltamperes and conducted voltamperes in an autotransformer ? Show that two windings connected as an autotransformer will have greater *VA* rating than when connected as a 2-winding transformer.

2.11 If a transformer having a series impedance Z_e is connected as an autotransformer, show that its series impedance Z_e' as an autotransformer is

$$Z_e' = \frac{T_H - T_L}{T_H} Z_e$$

2.12 Derive an expression for the approximate relative weights of conductors material in an autotransformer and a 2-winding transformer, the primary voltage being V_1 and the secondary voltage V_2. Compare the weights of conductor material when the transformation ratio is 3. Ignore the magnetizing current.

[Ratio $= 1 - (V_2/V_1)$; 2/3]

2.13 A 2-winding 10 kVA 440/110 V transformer is reconnected as a step-down 550/440 V autotransformer. Compare the voltampere rating of the autotransformer with that of original 2-winding transformer. Calculate the power transferred to the load :

(*a*) inductively and

(*b*) conductively.

[Rating of the autotransformer = 50 kVA. Voltampere rating of the 2-winding transformer = 10 kVA (*a*) 10 kVA ; (*b*) 40 kVA]

2.14 What is the difference between a 3-phase transformer bank and a 3-phase transformer unit ? What are the advantages of a three-phase unit transformer over three single-phase transformer bank of the same kVA rating ?

2.15 What are the advantages of a transformer bank of three 1-phase transformers over a unit three-phase transformer of the same kVA rating ?

2.16 What is meant by three-phase transformer groups ? What is the significance of these groups ?

What are the possible connections for a 3-phase transformer bank ?

2.17 What are distinguishing features of Y – Y, Y – Δ, Δ – Y and Δ – Δ 3-phase connections ? Compare their advantages and disadvantages.

2.18 State the factors affecting the choice of 3-phase connections.

2.19 What is an open-delta system ? What are the applications of this system ?

2.20 What schemes of connections are commonly used for 3-phase to six-phase transformation.

2.21 In open-delta transformers, show that the secondary line voltages from a balanced three-phase system of voltages, in case the supply voltages are balanced

2.22 Explain with necessary diagrams how two 3-phase transformers can be used to convert a 3-phase supply to a 2-phase one. If the load is balanced on one side, show that it will be balanced on other side.

2.23 Draw the Scott connection of transformers and mark the terminals and turn ratio. What are the applications of Scott connection ?

2.24 Explain with the help of connection and phasor diagrams, how Scott connections are used to obtain two-phase supply from 3-phase supply mains.

2.25 (*a*) Enumerate the advantages of six or twelve phase power over three-phase power.

(*b*) Describe at least three methods of conversion from three to six phases with suitable circuit and phasor diagrams.

2.26 Two single-phase furnaces are supplied at 250 V from a 6600 V, 3-phase system through a pair of Scott-connected transformers. If the load on the main transformer is 85 kW at 0.9 power factor lagging and that on the teaser transformer is 69 kW at 0.8 power factor lagging, find the values of the line currents on the 3-phase side. Neglect the magnetizing and core-loss currents in the transformers.

[$I_A = 15.1$ A, $I_B = I_C = 14.75$ A]

2.27 Draw and explain the circuit diagram of a transformer arrangement for converting from a 3-phase to a 2-phase supply.

2.28 Two 80 V, single-phase transformers take loads of 480 kW and 720 kW at a power factor of 0.71 lagging and supplied from 6600 V, 3-phase supply through a Scott-connected transformer. Calculate the line currents on the 3-phase side.

[$I_A = 118.3$ A, $I_B = I_C = 165$ A]

2.29 Two 1-phase Scott-connected transformers supply a 3-phase, 4-wire system with 231 V between the lines and the neutral. The *hv* windings are connected to a 2-phase system with a phase voltage of 6600 V. Determine the number of turns in each section of *hv* and *lv* windings and the position of the neutral point if the induced voltage per turn is 8 V.

[*Main transformer*

Number of turns on the *hv* side = 825

Number of turns on the *lv* side = 50

Teaser transformer

Number of turns on the *hv* side = 825

Number of turns on the *lv* side = 44

Tapping point for neutral is 29th turn from point A.]

2.30 Two single-phase furnaces *A* and *B* are supplied at 80 V by means of a Scott-connected transformer combination from a 3-phase, 6600 V system. The voltage of furnace *A* is leading. Calculate the line currents on the 3-phase side when the furnaces take 500 kW and 800 kW respectively

(*a*) at unity power factor ;

(*b*) furnace *A* at unit power factor, furnace *B* at 0.7 power factor lagging. Draw the corresponding phasor diagrams.

[(*a*) 129 A, 129 A, 87.6 A ; (*b*) 207 A, 145 A, 87.6 A]

2.31 (*a*) Describe the standard method of marking the terminals of three-phase transformers.

(*b*) Explain the clock method of angle designation for representing 3-phase transformers.

2.32 Describe the four phasor groups pertaining to 3-phase transformers. Draw the connection schemes and phasor diagrams for each of these four groups.

2.33 Describe the function of the closed delta tertiary winding employed in some three-phase transformers.

2.34 Explain the advantages of using a tertiary winding in a bank of star-star transformers.

2.35 Give the equivalent circuit and applications of 3-winding transformer. Explain how the parameters can be determined experimentally.

2.36 For what purposes are tertiary windings used on 3-phase transformers ? Explain how they can assist in unbalanced loading condition if suitably connected.

2.37 On open circuit, a 3-phase, star/star/delta, 6600/660/220-V transformer takes 50 kVA at 0.15 p.f. What is the primary input kVA and power factor when for balanced loads the secondary delivers 870 A at 0.8 p.f. lagging and tertiary delivers 260 line amperes at unit power factor ? Neglect the leakage impedances.

[1100 kVA, 0.8 p.f.]

2.38 Discuss the essential and desirable conditions to be fulfilled for operating two-single-phase transformers in parallel.

2.39 Discuss briefly the essential and desirable conditions to be fulfilled for operating two three-phase transformers in parallel.

Draw schematically how a 3-phase transformer can be phased in with another 3-phase transformer.

2.40 State the necessary conditions for satisfactory operation of two transformers in parallel. State briefly why all transformers cannot be operated in parallel.

2.41 State the requirements for satisfactory operation of a number of transformers in parallel.

A load of 1600 kW at 0.8 lagging power factor is shared by two 1000 kVA transformers having equal turn ratios and connected in parallel on their primary and secondary sides. The full load resistance drop is 1% and the reactive drop is 6% in one of the transformers, the corresponding values in the other transformer being 1.5% and 5%. Calculate the power and the power factor at which each transformer is operating.

[700 kW, 0.76 kg ; 900 kW, 0.84 lag]

2.42 What are the conditions for satisfactory parallel operation of 1 – ϕ transformer ?

Deduce expressions for the load shared by two transformers in parallel when no-load voltages of these transformers are not equal. What will be the load distribution if the voltage ratio is exactly equal.

2.43 State the conditions necessary before two three-phase transformers may be connected in parallel and the conditions for satisfactory parallel operation on load.

A single-phase, 500-kVA, 3.3 kV transformer having a resistance drop of 2% and a reactance drop of 3% at full load is connected in parallel with a 1000 kVA, 3.3 kV transformer having a resistance drop of 1% and a reactance drop of 5% at full load. Determine the impedance of the two transformers and their voltage drop when the total load is 1500 kVA at 0.8 power factor lagging. Also determine the current and kVA of each transformer. [192 A, 633 kVA ; 271 A, 896 kVA]

2.44 Two single-phase transformers are operating in parallel. Derive an expression for the current drawn by each, sharing a common load, when no-load voltages of these are not equal.

2.45 What is the inrush phenomenon in transformer ? Discuss qualitatively this phenomenon if single-phase transformer is switched on at the instant applied voltage is maximum positive. Show that the doubling effect is not so dangerous as it appears.

2.46 Why is the waveshape of magnetizing current of a transformer non-sinusoidal ? Explain the phenomenon of inrush of magnetizing current. What factors contribute to the magnitude of inrush current ?

2.47 Explain how the exciting (or no load) current of a single-phase transformer contains harmonics even when supply voltage is a sine wave.

2.48 Explain clearly the inrush-current phenomena in transformers and derive an approximate expression for the magnitude of the maximum instantaneous current.

2.49 What are the disadvantages of current and voltage harmonics in transformers ? Explain how these harmonics can be eliminated.

2.50 Explain why it is essential to have one three-phase winding in delta for the transformers used in 3-phase systems.

3

Synchronous Generators (Alternators)

3.1 INTRODUCTION

An alternating voltage is generated in a single conductor or armature coil rotating in a uniform magnetic field with stationary field poles. An alternating voltage will also be generated in stationary armature conductors when the field poles rotate past the conductors. Thus, we see that as long as there is a relative motion between the armature conductors and the field flux there will be a voltage generated in the armature conductors. In both the cases the wave shape of the voltage is a sine curve.

In d.c. generators, the field poles are stationary and the armature conductors rotate. The voltage generated in the armature conductors is of alternating nature. This generated alternating voltage is converted to a *direct* voltage at the brushes with the help of the commutator.

A.c. generators are usually called **alternators.** They are also called **synchronous generators.** Rotating machines that rotate at a speed fixed by the supply frequency and the number of poles are called *synchronous machines*.

A synchronous generator is a machine for converting mechanical power from a prime mover to a.c. electric power at a specific voltage and frequency. A synchronous machine rotates at a constant speed called synchronous speed. A synchronous motor is a synchronous machine that converts electric power to mechanical power. Synchronous generators are usually of 3-phase type because of the several advantages of 3-phase generation, transmission and distribution. Large synchronous generators are used to generate bulk power at thermal, hydro and nuclear power stations. Synchronous generators with power ratings of several hundred MVA are very commonly used in generating stations. The biggest size used in India has a rating of 500 MVA used in super-power thermal power stations. Synchronous generators are the primary sources of world's electric power systems today. For bulk power generation, stator windings of synchronous generators are designed for voltages ranging from 6.6 kV to 33 kV.

3.2 ADVANTAGES OF ROTATING FIELD ALTERNATOR

Most alternators have the rotating field and the stationary armature. The rotating-field type alternator has several advantages over the rotating-armature type alternator :

(1) A stationary armature is more easily insulated for the high voltage for which the alternator is designed. This generated voltage may be as high as 33 kV.

(2) The armature windings can be braced better mechanically against high electromagnetic forces due to large short-circuit currents when the armature windings are in the stator.

(3) The armature windings, being stationary, are not subjected to vibration and centrifugal forces.

(4) The output current can be taken directly from fixed terminals on the stationary armature without using slip rings, brushes, etc.

(5) The rotating field is supplied with direct current. Usually the field voltage is between 100 to 500 volts. Only two slip rings are required to provide direct current for the rotating field, while at least three slip rings would be required for a rotating armature. The insulation of the two relatively low voltage slip rings from the shaft can be provided easily.

(6) The bulk and weight of the armature windings are substantially greater than the windings of the field poles. The size of the machine is, therefore, reduced.

(7) Rotating field is comparatively light and can be constructed for high speed rotation. The armatures of large alternators are forced cooled with circulating gas or liquids.

(8) The stationary armature may be cooled more easily because the armature can be made large to provide a number of cooling ducts.

3.3 SPEED AND FREQUENCY

The frequency of the generated voltage depends upon the number of field poles and on the speed at which the field poles are rotated. One complete cycle of voltage is generated in an armature coil when a pair of field poles (one north and one south pole) passes over the coil.

Let P = total number of field poles

p = pair of field poles

N = speed of the field poles in r.p.m.

n = speed of the field poles in r.p.s.

f = frequency of the generated voltage in Hz

Obviously, $\frac{N}{60} = n$ (3.3.1)

and $\frac{P}{2} = p$ (3.3.2)

In one revolution of the rotor, an armature coil is cut by $\frac{P}{2}$ north poles and $\frac{P}{2}$ south poles. Since one cycle is generated in an armature coil when a pair of field poles passes over the coil, the number of cycles generated in one revolution of the rotor will be equal to the number of pairs of poles. That is,

number of cycles per revolution $= p$

Also, number of revolutions per second $= n$

Now frequency = number of cycles per second

$$= \frac{\text{number of cycles}}{\text{revolutions}} \times \frac{\text{revolutions}}{\text{seconds}}$$

$$f = p \times n \tag{3.3.3}$$

Since $n = N/60$ and $p = P/2$

$$f = \frac{PN}{120} \tag{3.3.4}$$

Equations (3.3.3) and (3.3.4) give the relationship between the number of poles, speed and frequency.

3.4 SYNCHRONOUS SPEED

From Eq. (3.3.4)

$$N_S = \frac{120 f}{p} \tag{3.4.1}$$

Equation (3.4.1) shows that the rotor speed N bears a constant relationship with the field poles and the frequency of the generated voltage in the armature winding. The speed given by Eq. (3.4.1) is called *synchronous speed* N_S. A machine which runs at synchronous speed is called *synchronous machine*. Thus, a synchronous machine is an a.c. machine in which the rotor moves at a speed which bears a constant relationship to the frequency of the generated voltage in the armature winding and the number of poles of the machine. Table 3.1 gives the number of poles and synchronous speeds for a power frequency of 50 Hz.

Table 3.1

Number of poles	*Synchronous speed N_S in r.p.m.*
2	3000
4	1500
6	1000
8	750
10	600
12	500

EXAMPLE 3.1. *Calculate the highest speed at which (a) 50 Hz (b) 60 Hz alternator can be operated.*

SOLUTION. Since it is not possible to have fewer than 2 poles, the minimum value of $P = 2$.

$$f = \frac{PN_S}{120}$$

$$N_S = \frac{120 f}{P}$$

For a minimum value of P the speed N will be a maximum.

(*a*) $f = 50$ Hz, $P = 2$

$$\therefore \quad N_S = \frac{120 \times 50}{2} = 3000 \text{ r.p.m.}$$

(*b*) $f = 60$ Hz, $P = 2$

$$\therefore \quad N_S = \frac{120 \times 60}{2} = 3600 \text{ r.p.m.}$$

3.5 CONSTRUCTION OF THREE-PHASE SYNCHRONOUS MACHINES

Similar to other rotating machines, an alternator consists of two main parts namely, the stator and the rotor. The **stator** is the stationary part of the machine. **It carries the armature winding in which the voltage is generated.** The output of the machine is taken from the stator. The **rotor** is the rotating part of the machine. **The rotor produces the main field flux.**

3.6 STATOR CONSTRUCTION

The various parts of the stator include the frame, stator core, stator windings and cooling arrangement. The frame may be of cast iron for small-size machines and of welded steel type for large size machines. In order to reduce hysteresis and eddy-current losses, the stator core is assembled with high grade silicon content steel laminations. A 3-phase winding is put in the slots cut on the inner periphery of the stator as shown in Fig. 3.1. The winding is star connected. The winding of each phase is distributed over several slots. When current flows in a distributed winding it produces an essentially sinusoidal space distribution of emf.

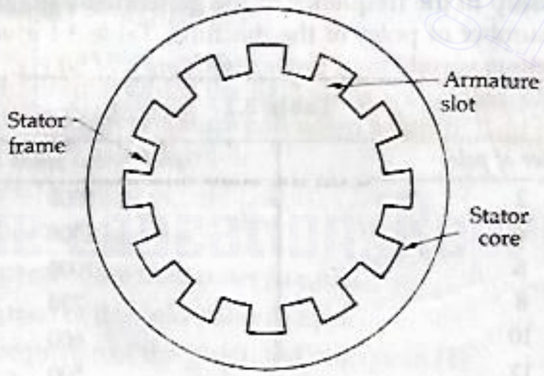


Fig. 3.1. *Alternator stator.*

3.7 ROTOR CONSTRUCTION

There are two types of rotor constructions namely, the salient-pole type and the cylindrical rotor type.

3.7.1 Salient-Pole Rotor

The term **salient** means **'protruding'** or **'projecting'**. Thus, a salient-pole rotor consists of poles projecting out from the surface of the rotor core. Fig. 3.2

shows the end view of a typical 6-pole salient-pole rotor. Salient-pole rotors are normally used for rotors with four or more poles.

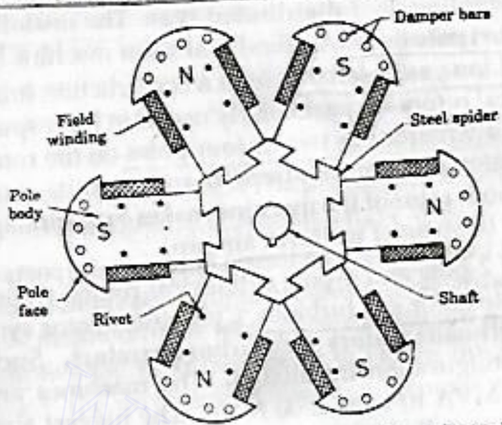


Fig. 3.2. *Six-pole salient-pole rotor.*

Since the rotor is subjected to changing magnetic fields, it is made of this steel laminations to reduce eddy current losses. Poles of identical dimensions are assembled by stacking laminations to the required length and then riveted together. After placing the field coil around each pole body, these poles are fitted by a dove-tail joint to a steel spider keyed to the shaft. Salient-pole rotors have concentrated winding on the poles. Damper bars are usually inserted in the pole faces to damp out the rotor oscillations during sudden change in load conditions. A salient-pole synchronous machine has a non-uniform air gap. The air gap is minimum under the pole centres and it is maximum in between the poles. The pole faces are so shaped that the radial air gap length increases from the pole centre to the pole tips so that the flux distribution in the air gap is sinusoidal. This will help the machine to generate sinusoidal emf.

The individual field-pole windings are connected in series to give alternate north and south polarities. The ends of the field windings are connected to a d.c. source (a d.c. generator or a rectifier) through the brushes on the slip rings. The slip rings are metal rings mounted on the shaft and insulated from it. They are used to carry current to or from the rotating part of the machine (usually a.c. machine) via carbon brushes.

Salient-pole generators have a large number of poles, and operate at lower speeds. A salient-pole generator has comparatively a large diameter and a short axial length. The large diameter accommodates a large number of poles.

Salient-pole alternators driven by water turbines are called **hydro-alternators** or **hydrogenerators.** Hydrogenerators with relatively higher speeds are used with impulse turbines and have horizontal configuration. Hydrogenerators with lower speeds are used with reaction and Kaplan turbines and have vertical configuration.

3.7.2 Cylindrical Rotor

A cylindrical-rotor machine is also called a **non-salient pole rotor machine.** It has its rotor so constructed that it forms a smooth cylinder. The construction is such that there are no physical poles to be seen as in the salient-pole construction. Cylindrical rotors are made from solid forgings of high grade nickel-chrome-molybdenum steel. In about two-third of the rotor periphery, slots are cut at regular intervals and parallel to the shaft. The d.c. field windings are accommodated in these slots. The winding is of **distributed type.** The unslotted portion of the rotor forms two (or four) pole faces. A cylindrical-rotor machine has a comparatively small diameter and long axial length. Such a construction limits the centrifugal forces. Thus, cylindrical rotors are particularly useful in high-speed machines. The cylindrical rotor type alternator has two or four poles on the rotor. Such a construction provides a greater mechanical strength and permits more accurate dynamic balancing. The smooth rotor of the machine makes less windage losses and the operation is less noisy because of **uniform air gap.**

Figure 3.3 shows end views of 2-pole and 4-pole cylindrical rotors. Cylindrical-rotor machines are driven by steam or gas turbines. Cylindrical-rotor synchronous generators are called **turboalternators** or **turbogenerators.** Such machines have always horizontal configuration installation. The machines are built in a number of ratings from 10 MVA to over 1500 MVA. The biggest size used in India has a rating of 500 MVA installed in super thermal power plants.

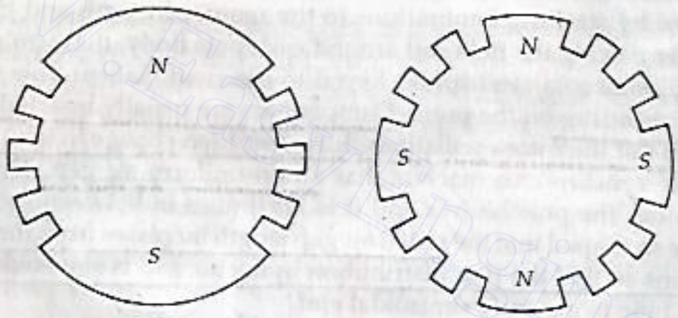


Fig. 3.3. *End view of two-pole and four pole cylindrical rotors.*

3.8 EXCITATION SYSTEMS FOR SYNCHRONOUS MACHINES

Excitation means production of flux by passing current in the field winding.

Direct current is required to excite the field winding on the rotor of the synchronous machines. For *small* machines, dc is supplied to the rotor field by a dc generator called *exciter.* This exciter may be supplied current by a smaller dc generator called *pilot exciter.* The main and pilot exciters are mounted on the main shaft of the synchronous machine (generator or motor). The dc output of the main exciter is given to the field winding of the synchronous machine through brushes and slip rings. In smaller machines, the pilot exciter may be omitted, but this arrangement is not very sensititve or quick acting when changes of the field current are required by the synchronous machine.

For *medium* size machines a.c. exciters are used in place of d.c. exciters. A.c. exciters are three-phase a.c. generators. The output of an a.c. exciter is rectified and supplied through brushes and sliprings to the rotor winding of the main synchronous machine.

For *large* synchronous generators with ratings of the order of few hundred megawatts, the excitation requirements become very large. The problem of conveying such amounts of power through high-speed sliding contacts becomes formidable. At present large synchronous generators and synchronous motors are using *brushless excitation systems*. A brushless exciter is a small direct-coupled a.c. generator with its field circuit on the stator and the armature circuit on the rotor. The three-phase output of the ac exciter generator is rectified by solid-state rectifiers. The rectified output is connected directly to the field winding, thus eliminating the use of brushes and slip rings.

A brushless excitation system requires less maintenance due to absence of brushes and slip rings. The power loss is also reduced.

The d.c. required for the field of the exciter itself is sometimes provided by a small pilot exciter. A *pilot exciter* is a small a.c. generator with *permanent magnets* mounted on the rotor shaft and a three-phase winding on the stator. The permanent magnets of the pilot exciter produce the field current of the exciter. The exciter supplies the field current of the main machine. Thus, the use of a pilot exciter makes the excitation of the main generator completely independent of external supplies.

3.9 VOLTAGE GENERATION

The rotor of the alternator is run at its proper speed by its prime mover. The *prime mover* is a machine which supplies the mechanical energy input to the alternator. The prime movers used for slow and medium speed alternators are water wheels or hydraulic turbines. Steam and gas turbines are used as prime movers in large alternators and run at high speeds. The steam-turbine driven alternators are called *turboalternators* or *turbogenerators*. As the poles of the rotor move under the armature conductors on the stator, the field flux cuts the armature conductors. Therefore voltage is generated in these conductors. This voltage is of alternating nature, since poles of alternate polarity successively pass by a given stator conductor. A 3-phase alternator has a stator with three sets of windings arranged so that there is a mutual phase displacement of 120°. These windings are connected in star to provide a 3-phase output.

3.10 E.M.F. EQUATION OF AN ALTERNATOR

Let

$\Phi =$ useful flux per pole in webers (Wb)

$P =$ total number of poles

$Z_p =$ total number of conductors or coil sides in series per phase

$T_p =$ total number of coils or turns per phase

$n =$ speed of rotation of rotor in revolutions per second (r.p.s.)

$f =$ frequency of generated voltage (Hz)

Since the flux per pole is Φ, each stator conductor cuts a flux $P\Phi$.

The average value of generated voltage per conductor

$$= \frac{\text{flux cut per revolution in Wb}}{\text{time taken for one revolution in seconds}}$$

Since n revolutions are made in one second, one revolution will be made in $1/n$ second. Therefore the time for one revolution of the armature is $1/n$ second. The average voltage generated per conductor

$$E_{av}/\text{conductor} = \frac{P\Phi}{1/n} = nP\,\Phi \quad \text{volts} \tag{3.10.1}$$

We know that

$$f = \frac{PN}{120} = \frac{Pn}{2} \tag{3.10.2}$$

$$Pn = 2f$$

Substituting the value of Pn in Eq. (3.10.1), we get

$$E_{av}/\text{conductor} = 2f\Phi \tag{3.10.3}$$

Since there are Z_p conductors in series per phase, the average voltage generated per phase is given by

$$E_{av}/\text{phase} = 2f\Phi Z_p \tag{3.10.4}$$

Since one turn or coil has two sides, $Z_p = 2T_p$, and the expression for the average generated voltage per phase can be written as

$$E_{av}/\text{phase} = 4f\Phi T_p \tag{3.10.5}$$

For the voltage wave, the form factor is given by

$$k_f = \frac{\text{r.m.s. value}}{\text{average value}}$$

For a sinusoidal voltage, $k_f = 1.11$. Therefore, the r.m.s. value of the generated voltage per phase can be written as

$$E_{r.m.s.}/\text{phase} = k_f \times E_{av}/\text{phase} = 1.11 \times 4\,f\Phi T_p$$
$$= 4.44\,f\Phi T_p$$

The suffix r.m.s. is usually deleted. The r.m.s. value of the generated voltage per phase is given by

$$E_p = 4.44\,f\Phi T_p \tag{3.10.6}$$

Equation (3.10.6) has been derived with the following *assumptions* :

(*a*) Coils have got full pitch.

(*b*) All the conductors are concentrated in one stator slot.

3.11 ARMATURE WINDINGS

The winding through which a current is passed to produce the main flux is called the *field winding*. The winding in which voltage is induced is called the *armature winding*.

Some basic terms related to the armature winding are defined as follows :

A *turn* consists of two conductors connected to one end by an end connector.

A *coil* is formed by connecting several turns in series.

A *winding* is formed by connecting several coils in series.

The turn, coil and winding are shown schematically in Fig. 3.4.

The beginning of the turn, or coil, is identified by the symbol S (Start) and the end of the turn or coil by the symbol F (Finish).

The concept of *electrical degrees* is very useful in the study of machine. If

Θ_{md} = mechanical degrees or angular measure in space

Θ_{ed} = electrical degrees or angular measure in cycles

For a P-pole machine, electrical degree is defined as follows :

$$\boxed{\Theta_{ed} \overset{\Delta}{=} \frac{P}{2}\,\Theta_{md}} \qquad (3.11.1)$$

The advantage of this notation is that expressions written in terms of electrical angles apply to machines having any number of poles.

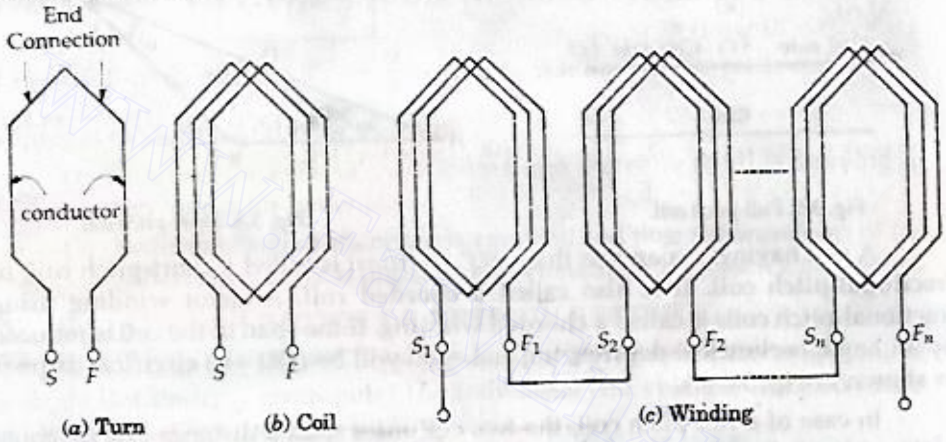


Fig. 3.4. *Turn, coil, and winding.*

The angular distance between the centres of two adjacent poles on a machine is known as **pole pitch or pole span.**

$$\boxed{\text{One pole pitch } = 180^\circ_{ed} = \frac{360^\circ_{md}}{P}} \qquad (3.11.2)$$

Regardless of the number of poles in the machine, a pole-pitch is always 180 electrical degrees.

The two sides of a coil are placed in two slots on the stator surface. The distance between the two sides of a coil is called the **coil-pitch**. If the coil-pitch is one pole pitch, it is called the **full-pitch** coil. If the coil pitch is less than one pole pitch, the coil is called the **short-pitch coil** or **fractional-pitch coil.**

3.12 COIL-SPAN FACTOR OR PITCH FACTOR

The distance between the two sides of a coil is called the *coil span* or coil pitch. The angular distance between the central line of one pole to the central line of the next pole is called *pole pitch*. A pole pitch is always 180 electrical degrees regardless of the number of poles on the machine. *A coil having a span equal to 180° electrical is called a **full-pitch coil*** as shown in Fig. 3.5 (*a*).

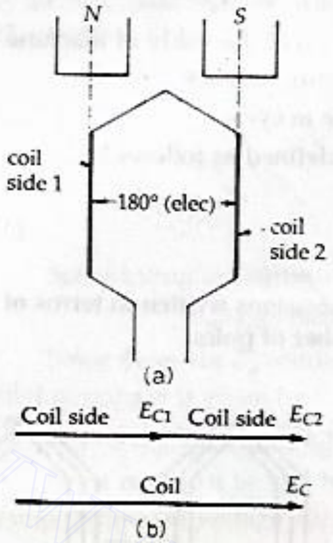


Fig. 3.5. *Full-pitch coil.*

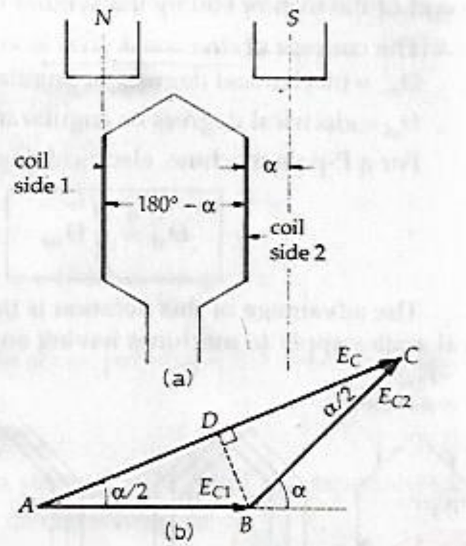


Fig. 3.6. *Short-pitch coil.*

A coil having a span less than 180° electrical is called a **short-pitch coil,** or **fractional-pitch coil.** It is also called a **chorded coil.** A stator winding using fractional-pitch coils is called a **chorded winding.** If the span of the coil is reduced by an angle α electrical degrees, the coil span will be (180 – α) electrical degrees as shown in Fig. 3.6 (*a*).

In case of a full-pitch coil, the two coil sides span a distance exactly equal to the pole pitch of 180 electrical degrees. As a result, the voltage generated in a full-pitch coil is such that the coil-side voltages are in phase as shown in Fig. 3.5 (*b*). Let E_{C_1} and E_{C_2} be the voltages generated in the coil sides and E_C the resultant coil voltage. Then

$$\mathbf{E}_C = \mathbf{E}_{C_1} + \mathbf{E}_{C_2}$$

$$|\mathbf{E}_{C_1}| = |\mathbf{E}_{C_2}| = \mathbf{E}_1 \quad \text{(say)}$$

Since E_{C_1} and E_{C_2} are in phase, the resultant coil voltage E_C is equal to their *arithmetic sum.*

$$\therefore \qquad \mathbf{E}_C = \mathbf{E}_{C_1} + \mathbf{E}_{C_2} = 2\,\mathbf{E}_1$$

If the coil span of a single coil is less than the pole pitch of 180° (elec.), the voltages generated in each coil side are *not* in phase. The resultant coil voltage $\mathbf{E}_C$ is equal to the *phasor sum* of $\mathbf{E}_{C_1}$ and $\mathbf{E}_{C_2}$.

If the coil span is reduced by an angle α electrical degrees, the coil span is (180 – α) electrical degrees. The voltages generated $\mathbf{E}_{C_1}$ and $\mathbf{E}_{C_2}$ in the two coil sides will be out of phase with respect to each other by an angle α electrical degrees as shown in Fig. 3.6 (*b*). The phasor sum of $\mathbf{E}_{C_1}$ and $\mathbf{E}_{C_2}$ is $\mathbf{E}_C$ (= *AC*).

The **coil span-factor** or **pitch-factor** k_C *is defined as the ratio of the voltage generated in the short-pitch coil to the voltage generated in the full-pitch coil.* The coil span-factor is also called the **chording factor.**

$$\boxed{k_C \overset{\Delta}{=} \frac{\text{actual voltage generated in the coil}}{\text{voltage generated in the coil of span } 180^\circ \text{ electrical}}}$$

$$= \frac{\text{phasor sum of the voltages of two coil sides}}{\text{arithmetic sum of the voltages of two coil sides}}$$

$$= \frac{AC}{2AB} = \frac{2AD}{2AB} = \cos\frac{\alpha}{2}$$

$$\therefore \quad \boxed{k_C = \cos\frac{\alpha}{2}} \qquad (3.12.1)$$

For full-pitch coil, $\alpha = 0^\circ$, $\cos\frac{\alpha}{2} = 1$ and $k_C = 1$. For a short pitch coil $k_C < 1$.

Advantages of short pitching or chording

(1) Shortens the ends of the winding and therefore there is a saving in the conductor material.

(2) Reduces effects of distorting harmonics, and thus the waveform of the generated voltage is improved and making it approach a sine wave.

3.13 DISTRIBUTION FACTOR OR BREADTH FACTOR k_d

In a concentrated winding, the coil sides of a given *phase* are concentrated in a single slot under a given pole. The individual coil voltages induced are in phase with each other.

These voltages may be added arithmetically. In order to determine the induced voltage per phase, a given coil voltage is multiplied by the number of series-connected coils per phase. In actual practice, in each phase, coils are not concentrated in a single slot, but are distributed in a number of slots in space to form a polar group under each pole. The voltages induced in coil sides constituting a polar group are not in phase but differ by an angle equal to the angular displacement β of the slots. The total voltage induced in any phase will be the phasor sum of the individual coil voltages.

The **distribution factor or breadth factor** *is defined as the ratio of the actual voltage obtained to the possible voltage if all the coils of a polar group were concentrated in a single slot.*

$$\boxed{k_d \overset{\Delta}{=} \frac{\text{phasor sum of coil voltages per phase}}{\text{arithmetic sum of coil voltages per phase}}} \qquad (3.13.1)$$

Let $m =$ slots per pole per phase, that is slots per phase belt

$$m = \frac{\text{slots}}{\text{poles} \times \text{phases}} \qquad (3.13.2)$$

$\beta =$ angular displacement between adjacent slots in electrical degrees

$$\beta = \frac{180^\circ}{\text{slots/pole}} = \frac{180^\circ \times \text{poles}}{\text{slots}} \qquad (3.13.3)$$

Thus, one phase of the winding consists of coils arranged in m consecutive slots. Voltages E_{C_1}, E_{C_2}, E_{C_3}, ... are the individual coil voltages. Each coil voltage E_C will be out of phase with the next coil voltage by the slot pitch β. Fig. 3.7 shows the voltage polygon of the induced voltages in the four coils of a group ($m = 4$). The voltages E_{C_1}, E_{C_2}, E_{C_3} and E_{C_4} are represented by phasors AB, BC, CD, and DF respectively in Fig. 3.7. Each of these phasors is a chord of a circle with centre O and subtends an angle β at O. The phasor sum AF, representing the resultant winding voltage, subtends an angle $m\beta$ at the centre.

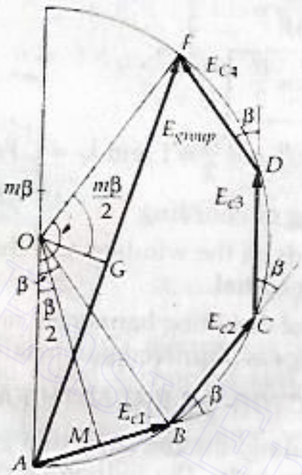


Fig. 3.7.

Arithmetic sum of individual coil voltages

$$= mE_C = mAB = m(2AM)$$

$$= 2m\, OA \sin A\hat{O}M = 2m\, OA \sin \beta/2$$

Phasor sum of individual coil voltages

$$= AF = 2AG = 2\, OA \sin A\hat{O}G = 2OA \sin \frac{m\beta}{2}$$

$$\therefore \quad k_d = \frac{\text{phasor sum of coil voltages per phase}}{\text{arithmetic sum of coil voltages per phase}} = \frac{2OA \sin m\beta/2}{2OA\, m \sin \beta/2}$$

or

$$\boxed{k_d = \frac{\sin m\beta/2}{m \sin \beta/2}} \tag{3.13.4}$$

It is to be noted that the distribution factor k_d for a given number of phases is dependent only on the number of distributed slots under a given pole. It is independent of the type of the winding, lap or wave, or the number of turns per coil, etc. As the number of slots per pole increases, the distribution factor decreases.

3.14 ACTUAL VOLTAGE GENERATED

Taking the coil span factor and distribution factor into account, the actual generated voltage per phase is given by

$$E_p = 4.44\, k_c k_d f \Phi T_p \tag{3.14.1}$$

Equation (3.14.1) is called the *complete emf equation of an alternator.*

The quantity $(k_c k_d T_p)$ is sometimes called effective turns per phase T_{ep}.

$$T_{ep} = k_c k_d T_p \tag{3.14.2}$$

It is smaller than the actual number of turns per phase due to fractional pitch coils and due to distribution of winding over several slots under each pole.

The coil span factor and distribution factor of a winding are sometimes combined into a single *winding* factor k_w which is the product of k_c and k_d. That is,

$$\boxed{k_w \overset{\Delta}{=} k_c k_d} \tag{3.14.3}$$

For a star-connected alternator, the line voltage is $\sqrt{3}$ times the phase voltage.

$$E_L = \sqrt{3}\, E_p$$

Alternative terms for the voltage **E** are

Open-circuit voltage/phase
No-load voltage/phase
Excitation e.m.f./phase
Internal machine voltage/phase
Voltage behind synchronous reactance/phase

The angle between the terminal voltage **V** and the internal voltage **E** is the machine angle or rotor angle δ.

Example 3.2. *A 3-phase, 50 Hz, 8-pole alternator has a star-connected winding with 120 slots and 8 conductors per slot. The flux per pole is 0.05 Wb, sinusoidally distributed. Determine the phase and line voltages.*

Solution. Let us take the full-pitch coil.

$\therefore\ \alpha = 0^\circ,\ k_c = \cos \alpha/2 = \cos 0^\circ = 1$

$$m = \frac{\text{slots}}{\text{poles} \times \text{phases}} = \frac{120}{8 \times 3} = 5$$

$$\beta = \frac{180^\circ \times \text{poles}}{\text{slots}} = \frac{180^\circ \times 8}{120} = 12^\circ$$

$$k_d = \frac{\sin \dfrac{m\beta}{2}}{m \sin \dfrac{\beta}{2}} = \frac{\sin \dfrac{5 \times 12^\circ}{2}}{5 \sin \dfrac{12^\circ}{2}} = 0.9567$$

Total number of conductors

$$= \text{conductor per slot} \times \text{number of slots}$$
$$= 8 \times 120 = 960$$

Conductors per phase, $Z_p = \dfrac{960}{3} = 320$

Generated voltage per phase

$$E_p = 2.22\, k_c k_d f \Phi Z_p$$
$$= 2.22 \times 1 \times 0.9567 \times 50 \times 0.05 \times 320 = 1699 \text{ V}$$

Generated line voltage $\quad E_L = \sqrt{3} E_p = \sqrt{3} \times 1699 = 2942.8 \text{ V}$

EXAMPLE 3.3. *A 3-phase, 6-pole, star-connected alternator revolves at 1000 r.p.m. The stator has 90 slots and 8 conductors per slot. The flux per pole is 0.05 Wb (sinusoidally distributed). Calculate the voltage generated by the machine if the winding factor is 0.96.*

SOLUTION. $$f = \frac{PN_s}{120} = \frac{6 \times 1000}{120} = 50 \text{ Hz}$$

Total number of stator conductors

$$= \text{conductors per slot} \times \text{number of slots}$$

$$= 8 \times 90 = 720$$

Stator conductors per phase

$$Z_p = \frac{720}{3} = 240$$

Winding factor $k_w = 0.96$

Generated voltage per phase

$$E_p = 2.22\, k_w f \Phi Z_p$$

$$= 2.22 \times 0.96 \times 50 \times 0.05 \times 240 = 1278.7 \text{ V}$$

Generated line voltage

$$E_L = \sqrt{3}\ E_p = \sqrt{3} \times 1278.7 = 2214.7 \text{ V}$$

EXAMPLE 3.4. *A 3-phase, 16-pole synchronous generator has a resultant air-gap flux of 0.06 Wb per pole. The flux is distributed sinusoidally over the pole. The stator has 2 slots per pole per phase and 4 conductors per slot are accommodated in two layers. The coil span is 150° electrical. Calculate the phase and line induced voltages when the machine runs at 375 r.p.m.*

SOLUTION. $$f = \frac{PN_s}{120} = \frac{16 \times 375}{120} = 50 \text{ Hz}$$

$$\alpha = 180^\circ - 150^\circ = 30^\circ$$

$$k_c = \cos\frac{\alpha}{2} = \cos\frac{30^\circ}{2} = 0.9659$$

$$m = \text{slots per pole per phase}$$

$$= \frac{\text{slots}}{\text{poles} \times \text{phases}}$$

$$\text{slots} = m \times \text{poles} \times \text{phases}$$

$$= 2 \times 16 \times 3 = 96$$

Total number of conductors

$$= \text{slots} \times \text{conductors per slot}$$

$$= 96 \times 4 = 384$$

Number of conductors per phase

$$Z_p = \frac{384}{3} = 128$$

Angular displacement between adjacent slots

$$\beta = \frac{180^\circ \times \text{poles}}{\text{slots}} = \frac{180^\circ \times 16}{96} = 30^\circ$$

$$k_d = \frac{\sin\frac{m\beta}{2}}{m\sin\frac{\beta}{2}} = \frac{\sin 2 \times \frac{30^\circ}{2}}{2\sin\frac{30^\circ}{2}} = 0.9659$$

Since the flux is sinusoidally distributed,

$$k_f = 1.11$$

The generated voltage per phase is given by

$$E_p = 2\, k_f k_c k_d f \Phi Z_p$$

$$= 2 \times 1.11 \times 0.9659 \times 0.9659 \times 50 \times 0.06 \times 128 = 795.3 \text{ V}$$

Generated line voltage

$$E_L = \sqrt{3}\, E_p = \sqrt{3} \times 795.3 = 1377.5 \text{ V}$$

EXAMPLE 3.5. *A 3-phase, 50 Hz, 2-pole, star-connected turboalternator has 54 slots with 4 conductors per slot. The pitch of the coils is 2 slots less than the pole pitch. If the machine gives 3300 V between lines on open circuit with sinusoidal flux distribution, determine the useful flux per pole.*

SOLUTION.

$$m = \frac{\text{slots}}{\text{poles} \times \text{phases}} = \frac{54}{2 \times 3} = 9$$

$$\beta = \frac{180^\circ \times \text{poles}}{\text{slots}} = \frac{180^\circ \times 2}{54} = \frac{20}{3} \text{ degrees}$$

$$\text{Coil pitch} = 25 \text{ slot angles} = 25\,\beta = 25 \times \frac{20}{3} \text{ degrees}$$

$$\alpha = 2\beta = 2 \times \frac{20}{3} = \frac{40^\circ}{3}$$

$$k_c = \cos\frac{\alpha}{2} = \cos\frac{20^\circ}{3} = 0.9932$$

$$k_d = \frac{\sin\frac{m\beta}{2}}{m\sin\frac{\beta}{2}} = \frac{\sin 9 \times \frac{20^\circ}{6}}{9\sin\frac{20^\circ}{6}} = 0.95547$$

Total number of conductors per phase

$$= \frac{\text{slots} \times \text{conductors per slot}}{\text{phases}}$$

$$= \frac{54 \times 4}{3} = 72$$

$$E_p = 2.22\, k_c k_d f \Phi Z_p$$

$$\frac{3300}{\sqrt{3}} = 2.22 \times 0.9932 \times 0.95547 \times 50 \times \Phi \times 72$$

$$\Phi = 0.2512 \text{ Wb}$$

EXAMPLE 3.6. *A 4-pole, 3-phase, 50 Hz, star-connected alternator has 60 slots, with 2 conductors per slot and having armature winding of the two-layer type. Coils are short-pitched in such a way that if one coil side lies in slot number 1, the other lies in slot number 13. Determine the useful flux per pole required to generate a line voltage of 6000 V.*

SOLUTION. Slot angle $\beta = \dfrac{180^\circ \times \text{poles}}{\text{slots}} = \dfrac{180 \times 4}{60} = 12^\circ$

$$\text{Coil span} = 12\,\beta = 12 \times 12 = 144^\circ$$

$$\alpha = 180^\circ - 144^\circ = 36^\circ$$

$$k_c = \cos\frac{\alpha}{2} = \cos\frac{36^\circ}{2} = 0.951$$

$$m = \frac{\text{slots}}{\text{poles} \times \text{phase}} = \frac{60}{4 \times 3} = 5$$

$$k_d = \frac{\sin\dfrac{m\beta}{2}}{m\sin\dfrac{\beta}{2}} = \frac{\sin\dfrac{5\times 12^\circ}{2}}{5\sin\dfrac{12^\circ}{2}} = 0.9567$$

$$Z_p = \frac{60 \times 2}{3} = 40$$

$$E_p = 2.22\,k_c k_d f \Phi Z_p$$

$$\frac{6000}{\sqrt{3}} = 2.22 \times 0.951 \times 0.9567 \times 50 \times \Phi \times 40$$

$$\Phi = 0.8576 \text{ Wb}$$

EXAMPLE 3.7. *A 6-pole alternator rotating at 1000 r.p.m. has a single-phase winding housed in 3 slots per pole, the slots in groups of three being 20° apart. If each slot contains 10 conductors, and the flux per pole is 2×10^{-2} Wb, calculate the voltage generated, assuming the flux distribution to be sinusoidal.*

SOLUTION. $f = \dfrac{PN_s}{120} = \dfrac{6 \times 1000}{1200} = 50 \text{ Hz}$

$$Z_p = 18 \times 10 = 180$$

$$m = 3, \quad \beta = 20^\circ$$

$$k_d = \frac{\sin\dfrac{m\beta}{2}}{m\sin\dfrac{\beta}{2}} = \frac{\sin\dfrac{3\times 20^\circ}{2}}{3\sin\dfrac{20^\circ}{2}} = 0.9598$$

$$E_p = 2.22\,k_c k_d f \Phi Z_p$$

$$= 2.22 \times 1 \times 0.9598 \times 50 \times 2 \times 10^{-2} \times 180 = 383.5 \text{ V}$$

3.15 ARMATURE LEAKAGE REACTANCE

In an a.c. machine, any flux set up by the load current which does not contribute to the useful flux of the machine is a leakage flux. The effect of this leakage flux is to set up a self-induced emf in the armature windings.

The leakage fluxes may be classified as follows :

1. Slot leakage
2. Tooth head leakage
3. Coil-end or overhang leakage.

The voltages induced in the armature windings by the airgap flux are called air gap voltages.

The leakage fluxes also induce voltages in the armature windings. These are taken into account by introduction of leakage reactance drops. Most of the reluctances of the magnetic circuits for armature leakage fluxes are due to air paths. The fluxes are, therefore, nearly proportional to the armature currents producing them and are in phase with these currents. For this reason, the voltages they induce in the armature windings can be taken into account by the use of constant leakage reactances for the phases, which multiplied by the phase currents, give the component voltages induced in the phases by the leakage flux. These voltages are the leakage reactance drops and lead the currents producing them by 90°.

3.16 ARMATURE REACTION IN SYNCHRONOUS MACHINES

When current flows through the armature winding of an alternator, the resulting mmf produces flux. The armature flux reacts with the main pole flux, causing the resultant flux to become either less than or more than the original main field flux. *The effect of armature (stator) flux on the flux produced by the rotor field poles is called* **armature reaction**.

For simplicity, we consider a 3-phase, 2-pole alternator (Fig. 3.8*a*) having a single-layer winding. But this treatment is valid for any number of poles. Also, the winding of each phase is assumed to be concentrated, but the effects of armature reaction will be the same as if a distributed winding were used.

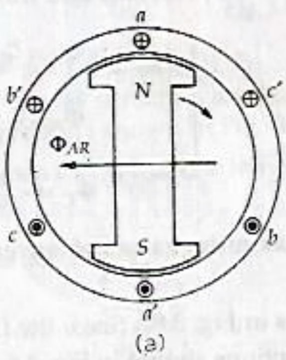


Fig. 3.8(*a*). *Two-pole alternator*

The armature reaction in synchronous machines affects the main-field flux very differently for different power factors. For the sake of simplicity, we shall consider the following three *extreme* conditions namely, unity power factor, zero-power factor lagging, and zero-power factor leading. The results can be generalized for different power factors met in practice.

For the purpose of this discussion, power factor will be defined as the cosine of the angle between the armature phase current and the induced emf in the armature conductor in that phase.

3.17 ARMATURE REACTION : UNITY POWER FACTOR

Suppose that the direction of rotation of the rotor is clockwise. The direction of the induced emf in various conductors can be found by applying the right-hand rule keeping in view that the direction of rotation of the conductors with respect to the rotor poles is anticlockwise.

Suppose that the alternator is supplying current at unity power factor. The phase currents $\mathbf{I}_A$, $\mathbf{I}_B$ and $\mathbf{I}_C$ will be in phase with their respective generated voltage $\mathbf{E}_A$, $\mathbf{E}_B$ and $\mathbf{E}_C$ as shown in Fig. 3.8*b*.

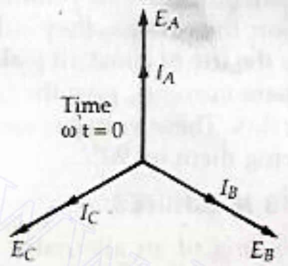


Fig. 3.8(*b*). *Phasor diagram .*

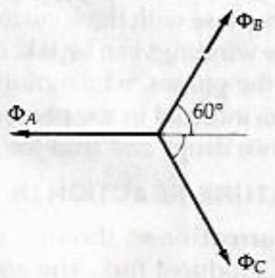


Fig. 3.8(*c*). *Positive directions of fluxes*

The positive directions of fluxes Φ_A, Φ_B and Φ_C are shown in the space diagram of Fig. 3.8*c*.

We know that the projection of a phasor on the vertical axis gives its instantaneous value.

At $t = 0$, the instantaneous values of currents and fluxes are given by

$$i_A = I_m \qquad\qquad \Phi_A = \Phi_m$$

$$i_B = -I_m \cos 60° = -\frac{1}{2} I_m \qquad\qquad \Phi_B = -\frac{1}{2}\Phi_m$$

$$i_C = -I_m \cos 60° = -\frac{1}{2} I_m \qquad\qquad \Phi_C = -\frac{1}{2}\Phi_m$$

where the subscript *m* denotes the maximum values of current and flux. Thus, the flux Φ_A is along *OA* in Fig. 3.8*d*.

This is in the same direction as in Fig. 3.8*c*. Since the fluxes Φ_B and Φ_C are negative they act opposite to the directions shown in Fig. 3.8*c*. They are shown by *OB* and *OC* in Fig. 3.8*d*. The resultant of the fluxes in Fig. 3.8*d* can be found by resolving the fluxes horizontally and vertically.

Resolving along the horizontal direction we get

$$\Phi_h = -\Phi_A - \Phi_B \cos 60° - \Phi_C \cos 60°$$

$$= -\Phi_m - \left(\frac{1}{2}\Phi_m\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\Phi_m\right)\left(\frac{1}{2}\right)$$

or $$\Phi_h = -1.5\,\Phi_m$$

Resolving along the vertical direction we get

$$\Phi_v = -\Phi_B \cos 30° + \Phi_C \cos 30°$$

$$= -\frac{1}{2}\Phi_m \cos 30° + \frac{1}{2}\Phi_m \cos 30° = 0$$

The resultant armature reaction flux is given by

$$\Phi_{AR} = \sqrt{\Phi_h^2 + \Phi_v^2} = \sqrt{(1.5\,\Phi_m)^2 + 0}$$

$$= 1.5\,\Phi_m$$

The direction of this resultant flux is along OA in Fig. 3.8(d). It is seen that the resultant armature reaction flux Φ_{AR} is constant in magnitude equal to $1.5\,\Phi_m$.

Also, Φ_{AR} lags behind 90° with the main field flux.

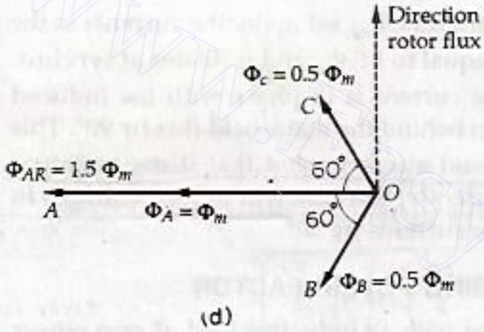


Fig. 3.8(d).

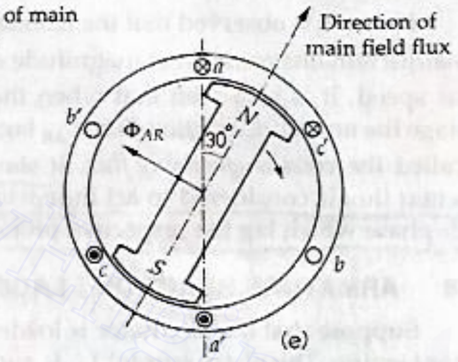


Fig. 3.8(e). Rotor position at ωt = 30°

Let us now consider the case at an instant when the rotor has rotated through 30° in the clockwise direction as shown in Fig. 3.8e. The corresponding phasor diagram of currents is shown in Fig. 3.8f.

At the instant when $\omega t = 30°$, the instantaneous values of currents and fluxes are given by

$$i_A = I_m \cos 30° = \frac{\sqrt{3}}{2} I_m \qquad \Phi_A = \frac{\sqrt{3}}{2}\Phi_m$$

$$i_B = 0 \qquad \Phi_B = 0$$

$$i_C = -I_m \cos 30° = -\frac{\sqrt{3}}{2} I_m \qquad \Phi_C = -\frac{\sqrt{3}}{2}\Phi_m$$

The space diagram for fluxes at $\omega t = 30°$ is shown in Fig. 3.8g.

Here $\Phi_B = 0$. The resultant armature reaction flux

$$\Phi_{AR} = OD = 2O\,M$$

$$= 2\,OC \cos 30° = 2\left(\frac{\sqrt{3}}{2}\Phi_m\right)\frac{\sqrt{3}}{2} = 1.5\,\Phi_m.$$

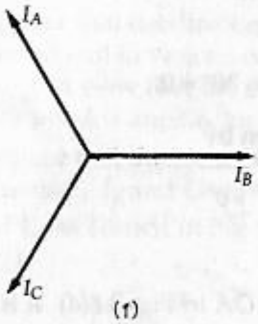


Fig. 3.8(*f*). *Armature currents at* $\omega t = 30°$

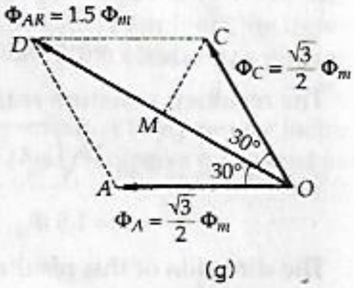


Fig. 3.8(*g*). *Fluxes at* $\omega t = 30°$

The direction of the resultant flux Φ_{AR} is along *OD* which makes an angle with the horizontal in the clockwise direction.

Hence it is observed that the resultant flux Φ_{AR} set up by the currents in the armature remains constant in magnitude equal to 1.5 Φ_m and it rotates at synchronous speed. It is also seen that when the current is in phase with the induced voltage the armature reaction flux Φ_{AR} lags behind the main-field flux by 90°. This is called the *cross-magnetizing flux*. It should also be noted that if the armature reaction flux is considered to act independently, this flux will induce voltages in each phase which lag the respective phase currents by 90°.

3.18 ARMATURE REACTION : LAGGING POWER FACTOR

Suppose that the alternator is loaded with an inductive load of zero power factor lagging. The phase current $\mathbf{I}_A$, $\mathbf{I}_B$ and $\mathbf{I}_C$ will be lagging with their respective phase voltages $\mathbf{E}_A$, $\mathbf{E}_B$ and $\mathbf{E}_C$ by 90°. Figure 3.9*a* shows a 2-pole alternator. The signs within the conductors give the directions of currents, while the signs outside the conductors indicate the direction of induced voltages. Figure 3.9*b* shows the phasor diagram.

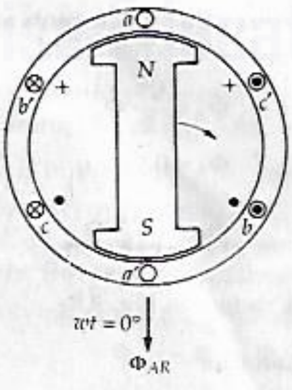


Fig. 3.9 (*a*).

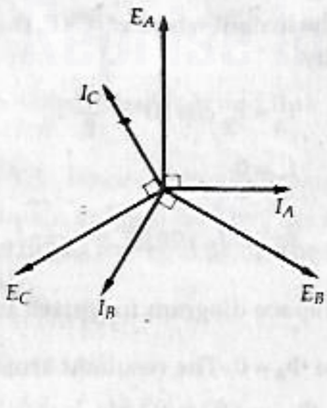


Fig. 3.9 (*b*)

At time $t = 0$, the instanceous values of currents and fluxes are given by

$$i_A = 0 \qquad \Phi_A = 0$$

$$i_B = I_m \sin(-120°) = \frac{-\sqrt{3}}{2} I_m \qquad \Phi_B = \frac{-\sqrt{3}}{2}\Phi_m$$

$$i_C = I_m \sin(+120°) = \frac{\sqrt{3}}{2} I_m \qquad \Phi_C = \frac{\sqrt{3}}{2}\Phi_m$$

The space diagram of magnetic fluxes is shown in Fig. 3.9c.

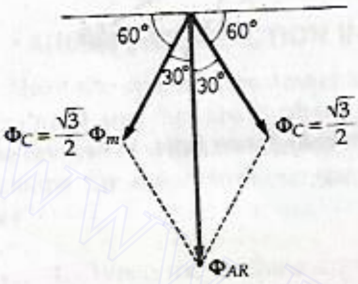


Fig. 3.9(c). *Space diagram of magnetic fluxes.*

The resultant flux Φ_{AR} is given by

$$\Phi_{AR}^2 = \Phi_R^2 + \Phi_C^2 + 2\,\Phi_R\,\Phi_C \cos 60°$$

$$= \left(\frac{\sqrt{3}}{2}\Phi_m\right)^2 + \left(\frac{\sqrt{3}}{2}\Phi_m\right)^2 + 2\left(\frac{\sqrt{3}}{2}\Phi_m\right)\left(\frac{\sqrt{3}}{2}\Phi_m\right)\times\frac{1}{2}$$

$$\therefore \quad \Phi_{AR} = 1.5\,\Phi_m$$

It is seen that the direction of the armature reaction flux is *opposite* to the main field flux. Therefore, it will oppose and weaken the main field flux. It is said to be *demagnetizing*.

Again it can be shown that for successive positions of the rotor, the armature reaction flux remains constant in magnitude and rotates at synchronous speed. Also, *the voltages induced in each phase by the armature reaction flux lag the respective phase currents by* 90°.

3.19 ARMATURE REACTION : LEADING POWER FACTOR

Suppose that the alternator is loaded with a load of zero power factor leading. The phase currents. $\mathbf{I}_A$, $\mathbf{I}_B$ and $\mathbf{I}_C$ will be leading their respective phase voltages $\mathbf{E}_A$, $\mathbf{E}_B$ and $\mathbf{E}_C$ by 90°. Figure 3.10 shows the same stator and poles at $t = 0$. The signs within the conductors give the directions of currents while the signs outside the conductors indicate the directions of induced voltage.

At time $t = 0$, the instantaneous values of currents and fluxes are given by

$$i_A = 0 \qquad \Phi_A = 0$$

$$i_B = I_m \cos 30° = \frac{\sqrt{3}}{2} I_m \qquad \Phi_B = \frac{\sqrt{3}}{2}\Phi_m$$

$$i_C = -I_m \cos 30° = \frac{-3}{2} I_m \qquad \Phi_C = \frac{-\sqrt{3}}{2}\Phi_m$$

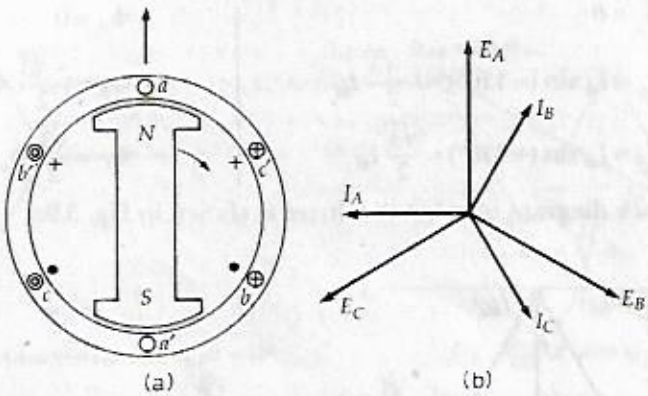


Fig. 3.10. *Armature reaction for zero leading power factor*

The flux directions are shown in Fig. 3.10c.

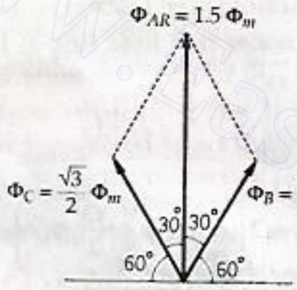


Fig. 3.10(c). *Fluxes for zero leading pf.*

The resultant flux is given by

$$\Phi_{AR}^2 = \Phi_B^2 + \Phi_C^2 + 2\,\Phi_B\,\Phi_C \cos 60°$$

$$= \left(\frac{\sqrt{3}}{2}\,\Phi_m\right)^2 + \left(\frac{\sqrt{3}}{2}\,\Phi_m\right)^2 + 2\left(\frac{\sqrt{3}}{2}\,\Phi_m\right)\left(\frac{\sqrt{3}}{2}\,\Phi_m\right) \times \frac{1}{2}$$

$$\therefore \quad \Phi_{AR} = 1.5\,\Phi_m$$

It is seen that the direction of armature reaction flux is in the *direction* of main field flux. Thus, it will aid it. In other words, it is a *magnetizing flux.*

Again for the successive positions of the rotor, it can be shown that the armature reaction flux remains constant in magnitude and rotates at synchronous speed. Also, *the voltages induced in each phase by the armature reaction lag the respective phase currents by 90°.*

3.20 SUMMARY OF NATURE OF ARMATURE REACTION

From the above discussion, the following more general conclusions regarding the nature of armature reaction can be drawn for a synchronous generator supplying a balanced 3-phase load :

1. The armature reaction flux is constant in magnitude and rotates at synchronous speed.

2. The armature reaction is *cross-magnetizing* when the generator supplies a load at unity power factor.
3. When the generator supplies a load at lagging power, the armature reaction is partly demagnetizing and partly cross-magnetizing.
4. When the generator supplies a load at leading power factor, the armature reaction is partly magnetizing and partly cross magnetizing.
5. In all cases, if the armature-reaction flux is assumed to act independently of the main field flux, it induces voltage in each phase which lags the respective phase currents by 90°.

3.21 ARMATURE REACTION IN A MOTORING MACHINE

Since the synchronous machine operating in motoring mode, the armature reaction mmf and flux are in phase opposition, the nature of armature reaction is just the reverse of what is stated for the synchronous generator. The conesponding conclusions for a synchronous machine operating in a motoring mode are as follows :

1. When the machine draws a lagging power factor current, the armature reaction is partly magnetizing and partly cross-magnetizing.
2. When the machine draws a leading power factor current, the armature reaction is partly demagnetizing and partly cross-magnetizing.

3.22 SYNCHRONOUS IMPEDANCE

The actual generated voltage consists of the summation of two component voltages. One of these component voltages is the voltage that would be generated if there were no armature reaction. It is the voltage that would be generated because of only the field excitation. This component of the generated voltage is called the excitation voltage, E_{exc}.

The other component of the generated voltage is called the *armature reaction voltage*, E_{AR}. This is the voltage that must be added to the excitation voltage to take care of the effect of armature reaction upon the generated voltage.

$$\mathbf{E}_a = \mathbf{E}_{exc} + \mathbf{E}_{AR} \tag{3.22.1}$$

Since armature reaction results, in a voltage effect in a circuit caused by change in flux by current in the same circuit, its effect is of the nature of an inductive reactance. Therefore E_{AR} is equivalent to a voltage of inductive reactance and

$$\mathbf{E}_{AR} = -j\mathbf{X}_{AR}\mathbf{I}_a \tag{3.22.2}$$

The inductive reactance X_{AR} is a **fictitious** reactance which will result in a voltage in the armature circuit to account for the effect of armature reaction upon the voltage relations of the armature circuit. Therefore, armature reaction voltage can be modelled as an inductor in series with the internal generated voltage.

In addition to the effects of armature reaction, the stator winding also has a self-inductance and a resistance.

Let L_a = self-inductance of stator winding

X_a = self-inductive reactance of stator winding

R_a = armature (stator) resistance

The terminal voltage V is given by

$$\mathbf{V} = \mathbf{E}_a - jX_{AR}\mathbf{I}_a - jX_a\mathbf{I}_a - R_a\mathbf{I}_a \tag{3.22.3}$$

where $R_a I_a$ = armature resistance drop

$X_a I_a$ = armature leakage reactance drop

$X_{AR}\, I_a$ = armature reaction voltage

The armature reaction effects and the leakage flux effects in the machine are both represented by inductive reactances. Therefore, it is customary to combine them into a single reactance, called the **synchronous reactance** of the machine, X_S.

$$X_s = X_a + X_{AR} \tag{3.22.4}$$

$$\therefore \quad \mathbf{V} = \mathbf{E}_a - jX_S\mathbf{I}_a - R_a\mathbf{I}_a$$

or

$$\mathbf{V} = \mathbf{E}_a - (R_a + jX_s)\mathbf{I}_a \tag{3.22.5}$$

$$\mathbf{V} = \mathbf{E}_a - \mathbf{Z}_s\mathbf{I}_a \tag{3.22.6}$$

where

$$\boxed{\mathbf{Z}_s = R_a + jX_s} \tag{3.22.7}$$

The impedance $\mathbf{Z}_s$ is called the **synchronous impedance.**

The synchronous reactance X_s is the *fictitious reactance* employed to account for the voltage effects in the armature circuit produced by the actual armature leakage reactance and by the change in air gap flux caused by the armature reaction.

Similarly, the synchronous impedance Z_s is a fictitious impedance employed to account for the voltage effects in the armature circuit produced by the actual armature resistance, the actual armature leakage reactance, and the change in air gap flux produced by armature reaction.

3.23 EQUIVALENT CIRCUIT AND PHASOR DIAGRAMS OF A SYNCHRONOUS GENERATOR

The equivalent circuit of a synchronous generator is shown in Fig. 3.11(*a*). It is redrawn in Fig. 3.11(*b*) by taking $X_s = X_{AR} + X_a$

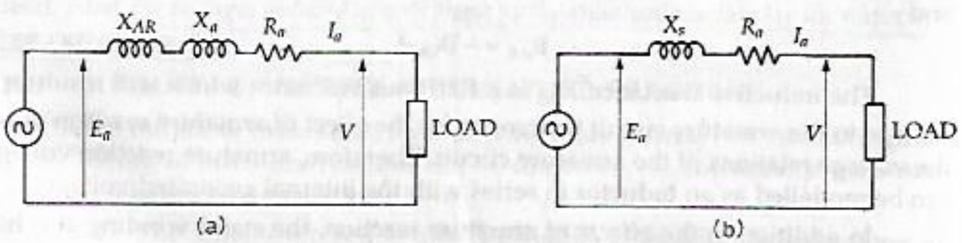


Fig. 3.11. *Equivalent circuit of a synchronous generator.*

(*a*) **Lagging power factor cos ϕ**

Figure 3.12 (*a*) shows the phasor diagram for lagging load. The power factor is cos ϕ lagging. In this diagram the terminal voltage **V** is taken as reference phasor along OA such that $OA = \mathbf{V}$. For lagging power factor cos ϕ, the direction of the armature current $\mathbf{I}_a$ *lags* behind **V** by an angle ϕ along OB, where $OB = \mathbf{I}_a$. The voltage drop in the armature resistance is $\mathbf{I}_a R_a$. It is represented by phasor AC. The voltage drop in the synchronous reactance is $\mathbf{I}_a X_s$. It is represented by CD. It leads the current $\mathbf{I}_a$ by 90° and, therefore, CD is drawn in a direction perpendicular to OB. The total voltage drop in the synchronous impedance is the phasor sum of $\mathbf{I}_a R_a$ and $\mathbf{I}_a X_s$. It is represented by AD. The phasor OD represents $\mathbf{E}_a$.

The magnitude of $\mathbf{E}_a$ can be found from the right-angled triangle *OGD*

$$OD^2 = OG^2 + GD^2 = (OF + FG)^2 + (GC + CD)^2$$

$$E_a^2 = (V\cos\phi + I_a R_a)^2 + (V\sin\phi + I_a X_s)^2$$

$$E_a = \sqrt{(V\cos\phi + I_a R_a)^2 + (V\sin\phi + I_a X_s)^2} \tag{3.23.1}$$

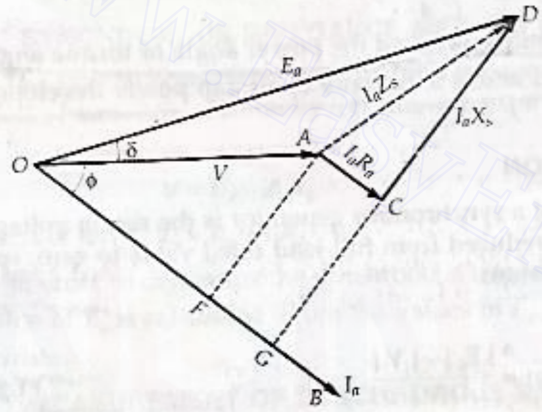


Fig. 3.12 (*a*). *Phasor diagram for lagging power factor cos ϕ*

(*b*) **Unity power factor**

The phasor diagram for unity power factor is shown in Fig. 3.12 (*b*).

From right-angled triangle *OCD*

$$(OD)^2 = (OC)^2 + (CD)^2 = (OA + AC)^2 + (CD)^2$$

$$E_a^2 = (V + I_a R_a)^2 + (I_a X_s)^2$$

$$E_a = \sqrt{(V + I_a R_a)^2 + (I_a X_s)^2} \tag{3.23.2}$$

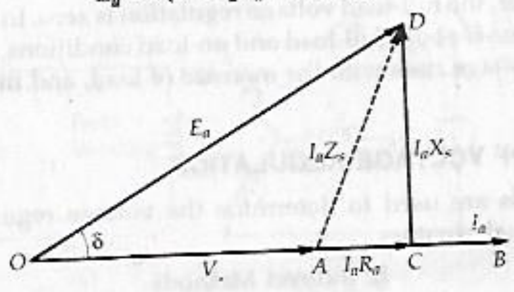


Fig. 3.12 (*b*). *Phasor diagram for unity power factor.*

(c) **Leading power factor cos φ**

The phasor diagram for leading power factor is shown in Fig. 3.12 (c). From right-angled triangle *OGD*

$$OD^2 = OG^2 + GD^2 = (OF + FG)^2 + (GC - CD)^2$$

$$E_a^2 = (V \cos\phi + I_aR_a)^2 + (V \sin\phi - I_aX_s)^2$$

$$E_a = \sqrt{(V\cos\phi + I_aR_a)^2 + (V\sin\phi - I_aX_s)^2} \tag{3.23.3}$$

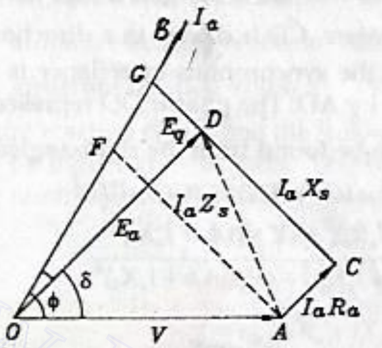


Fig. 3.12 (c). *Phasor diagram for leading power factor cos φ*

The angle δ between $\mathbf{E}_a$ and **V** is called the **power angle** or **torque angle** of the machine. It varies with load and is a measure of air gap power developed in the machine.

3.24 VOLTAGE REGULATION

The **voltage regulation** of a synchronous generator is the rise in voltage at the terminals when the load is reduced from full-load rated value to zero, speed and field current remaining constant.

It can be written as

$$\text{Per unit voltage regulation} \overset{\Delta}{=} \frac{|\mathbf{E}_a| - |\mathbf{V}|}{|\mathbf{V}|} \tag{3.24.1}$$

$$\text{Percent voltage regulation} \overset{\Delta}{=} \frac{|\mathbf{E}_a| - |\mathbf{V}|}{|\mathbf{V}|} \times 100 \tag{3.24.2}$$

where $|\mathbf{E}_a|$ = magnitude of generated voltage per phase

$|\mathbf{V}|$ = magnitude of rated terminal voltage per phase

The voltage regulation depends upon the power factor of the load. For unity and lagging power factors, there is always a voltage drop with the increase of load, but for a certain leading power, the full-load voltage regulation is zero. In this case, the terminal voltage is the same for both full-load and no-load conditions. At lower leading power factors, the voltage rises with the increase of load, and the regulation is negative.

3.25 DETERMINATION OF VOLTAGE REGULATION

The following methods are used to determine the voltage regulation of smooth cylindrical rotor type alternators :

A. Direct load test B. Indirect Methods

(A) **Direct load test**

The alternator is run at synchronous speed and its terminal voltage is adjusted to its rated value V. The load is varied until the ammeter and wattmeter indicate the rated values at the given power factor. Then the load is removed and the speed and field excitation are kept constant. The open-circuit or no-load voltage E_a is recorded. The voltage regulation is found from percentage voltage regulation $= \frac{E_a - V}{V} \times 100\%$. The method of direct loading is suitable only from small alternators of power rating less than 5 kVA.

(B) **Indirect Methods**

For large alternators, the three indirect methods which are used to predetermine the voltage regulation of smooth cylindrical-rotor type alternators are as follows :

1. Synchronous Impedance method or EMF Method.
2. Ampere-turn method or MMF Method.
3. Zero Power Factor method or Potier Method.

3.26 SYNCHRONOUS IMPEDANCE METHOD OR EMF METHOD

The synchronous impedance method is based on the concept of replacing the effect of armature reaction by a fictitious reactance.

For a synchronous generator

$$\mathbf{V} = \mathbf{E}_a - \mathbf{Z}_s \mathbf{I}_a$$

where $\quad \mathbf{Z}_s = R_a + j X_s$

In order to determine the synchronous impedance Z_s is measured and then the value of E_a is calculated. From the values of E_a and V, the voltage regulation is calculated.

3.27 MEASUREMENT OF SYNCHRONOUS IMPEDANCE

The following tests are performed on an alternator to know its performance :

(*a*) D.C. resistance test (*b*) Open-circuit test (*c*) Short-circuit test

3.27.1 D.C. Resistance Test

Assume that the alternator is star connected with d.c. field winding *open* (Fig. 3.13), measure the d.c. resistance between each pair of terminals either by

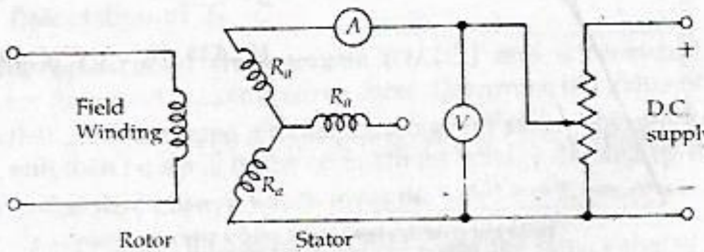


Fig. 3.13. *D.C. resistance test on an alternator.*

using ammeter-voltmeter method or by using Wheatstone's bridge. The average of three sets of resistance values R_t is taken. This value of R_t is divided by 2 to obtain the d.c. resistance (ohmic resistance) per phase. The alternator should be at rest. Since the effective a.c. resistance is larger than d.c. resistance due to skin effect, therefore, the effective a.c. resistance per phase is obtained by multiplying the d.c. resistance by a factor 1.20 to 1.75 depending on the size of the machine. A typical value to use in the calculation would be 1.25.

3.27.2 Open-circuit Test

The alternator is run at rated synchronous speed and the load terminals are kept open (Fig. 3.14). That is, all the loads are disconnected. The field current is set to zero.

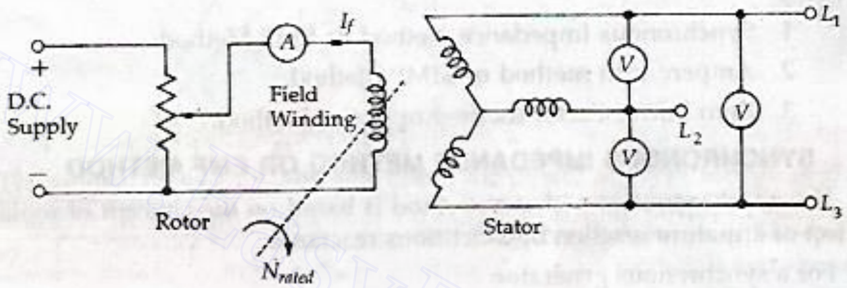


Fig. 3.14. *Open-circuit test on an alternator.*

Then the field current is gradually increased in steps, and the terminal voltage E_t is measured at each step. The excitation current may be increased to get 25% more than rated voltage of the alternator. A graph is plotted between the open-circuit phase voltage $E_p \left(= \frac{Et}{\sqrt{3}}\right)$ and field current I_f. The characteristic curve so obtained is called *open-circuit characteristic* (O.C.C.). It takes the shape of a normal magnetisation curve. The extension of the linear portion of an O.C.C. is called the *air-gap line* of the characteristic. The O.C.C. and the air-gap line are shown in Fig. 3.15.

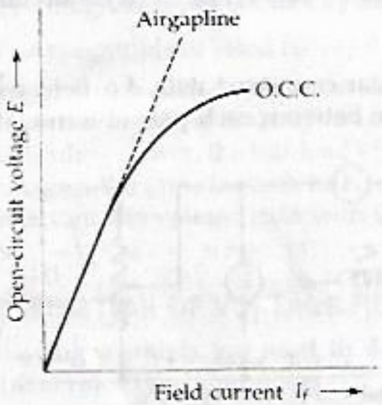


Fig. 3.15. *The O.C.C. of an alternator.*

3.27.3 Short-circuit Test

The armature terminals are shorted through three ammeters (Fig. 3.16). Care should be taken in performing this test, and the field current should first be decreased to zero before starting the alternator. Each ammeter should have a range greater than the rated full-load value. The alternator is then run at synchronous

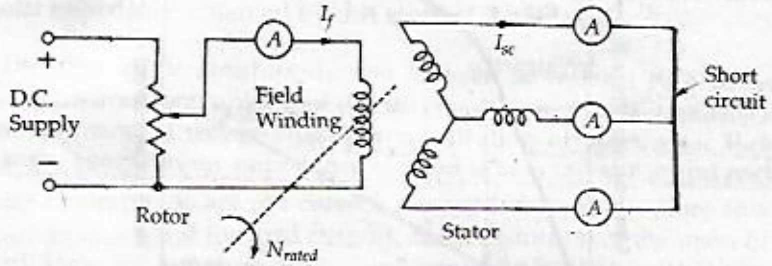


Fig. 3.16. *Short-circuit test on an alternator.*

speed. Then the field current is gradually increased in steps, and the armature current is measured at each step. The field current may be increased to get armature currents upto 150% of the rated value. The field current I_f and the average of three ammeter readings at each step is taken. A graph is plotted between the armature current I_a and the field current I_f. The characteristic so obtained is called *short-circuit characteristic* (SCC). This characteristic is a straight line as shown in Fig. 3.16(*a*).

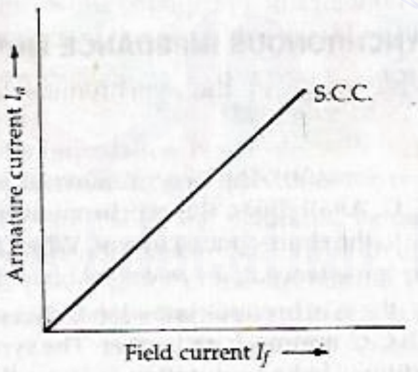


Fig. 3.16 (*a*). *The S.C.C. of an alternator.*

3.27.4 Calculation of Z_s

The open-circuit characteristic (O.C.C.) and short-circuit characteristic (S.C.C) are drawn on the same curve sheet. Determine the value of I_{SC} at the field current that gives the rated alternator voltage *per phase*. The synchronous impedance Z_s will then be equal to the open-circuit voltage divided by the short-circuit current at that field current which gives the rated e.m.f. per phase.

$$Z_s = \frac{\text{open circuit voltage per phase}}{\text{short circuit armature current}} \text{ for the same value of field current.}$$

The synchronous reactance is found as follows :

$$X_s = \sqrt{Z_s^2 - R_a^2}$$

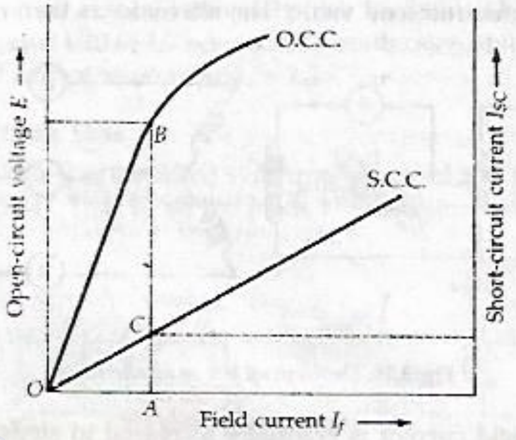


Fig. 3.17.

In Fig. 3.17, consider the field current $I_f = OA$ that produces rated alternator voltage per phase. Corresponding to this field current the open-circuit voltage is AB.

$$\therefore \quad Z_s = \frac{AB \text{ (in volts)}}{AC \text{ (in amperes)}}$$

3.28 ASSUMPTIONS IN THE SYNCHRONOUS IMPEDANCE METHOD

The following assumptions are made in the synchronous impedance method :

1. *The synchronous impedance is constant.* The synchronous impedance is determined from the O.C.C. and S.C.C. At all times, the synchronous impedance is the ratio of the open-circuit voltage to the short-circuit current. When the O.C.C. and S.C.C. are *linear*, the synchronous impedance Z_s is *constant*. Above the knee of the O.C.C. when the saturation starts, the synchronous impedance decreases. This is due to the fact that the O.C.C. and S.C.C. approach each other. The synchronous impedance obtained under test conditions below saturation is *larger* than under normal operating conditions when the magnetic circuit becomes saturated. Thus, we do not take the effect of saturation. This is the greatest source of error in the synchronous impedance method.

2. *The flux under test conditions is the same as that under load conditions.* It is assumed that a given value of field current always produces the same flux. This assumption introduces considerable error. When the armature is short-circuited, the current in the armature lags the generated voltage by almost 90°, and hence armature reaction is almost completely *demagnetizing*. This demagnetizing effect

reduces the degree of saturation still further. The actual resultant flux, and hence the generated voltage is very small. These conditions may be different from those with the actual conditions when the machine is loaded and the field current is equal to that under short-circuit test. Thus, the open-circuit voltage found from the O.C.C. is greater than the short-circuit generated voltage and the value of the synchronous impedance obtained by this method is *too large*.

3. *The effect of the armature reaction flux can be replaced by a voltage drop proportional to the armature current and that the armature reaction voltage drop is added to the armature reactance voltage drop.* The substitution of voltage for flux is the reason that the synchronous-impedance method is also called the **emf method**.

These assumptions are not correct, since the shift of armature flux varies with the power factor and the load current, and a distortion of the main field flux is produced. Thus, the armature reaction voltage is not in phase with the reactance voltage drop.

4. *The magnetic reluctance to the armature flux is constant regardless of the power-factor.* For a cylindrical rotor machine this assumption is substantially true because of the uniform air gap. In salient-pole machines, the position of the armature flux relative to the field poles varies with the power factor. The variation in reluctance and armature flux with the power factor introduces considerable error in salient-pole machines.

Regulation obtained by using synchronous impedance method is higher than that obtained by actual loading. Hence this method is called the **pessimistic method.** The results obtained by this method are more likely on the safer side and the performance of the alternator would be better than the calculations indicate.

At low excitations, Z_s is constant, since the open-circuit characteristic coincides with the air-gap line. This value of Z_s is called the **linear** or **unsaturated synchronous impedance**. However, with increasing excitation the effect of saturation is to decrease Z_s, and the values beyond the linear part of the open-circuit characteristic are called the **saturated values of the synchronous impedance.** These values are not constant but vary with the excitation, that is, with the voltage current and power factor of the machine on load. The value to be used in a given situation is called the **effective synchronous impedance**.

3.29 UNSATURATED SYNCHRONOUS REACTANCE

The unsaturated synchronous reactance (X_{su}) can be obtained from the airgap line voltage and the short-circuit current of the machine for a particular value of the field current. From Fig. 3.18,

$$Z_{su} = \frac{ad}{ab} = R_a + j\,X_{su}$$

If R_a is neglected

$$X_{su} = \frac{ad}{ab}$$

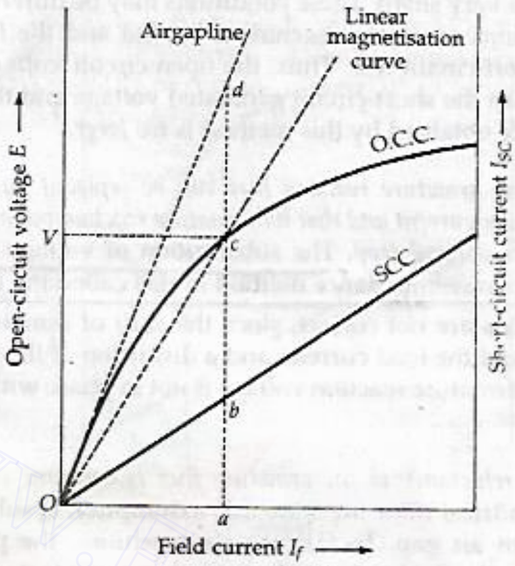


Fig. 3.18.

3.30 SATURATED SYNCHRONOUS REACTANCE

The approximate value of synchronous reactance varies with the degree of saturation of the OCC. Therefore the value of synchronous reactance to be used in a given problem should be one calculated at the approximate load on the machine. If the machine is connected to the infinite bus, its terminal voltage V remains the same at the bus value. If the field current is now changed, the excitation voltage will change not along the OCC, but along line Oc, called the *modified air gap line*. This line represents the same magnetic saturation level as that corresponding to the operating point c.

The *saturated synchronous reactance* at the rated voltage is obtained as follows :

$$Z_{s\,(sat)} = \frac{E_{ca}}{I_{ba}} = R_a + j\,X_{s\,(sat)} \tag{3.30.1}$$

If R_a is neglected

$$X_{s\,(sat)} = \frac{E_{ca}}{I_{ba}} \tag{3.30.2}$$

EXAMPLE 3.8. *A 500 V, 50 kVA single-phase alternator has an effective armature resistance of 0.2 Ω. An excitation current of 10 A produces 200 A armature current on short circuit and an e.m.f. of 450 V on open circuit. Calculate the synchronous reactance.*

SOLUTION. $Z_s = \dfrac{\text{open circuit e.m.f.}}{\text{short circuit current}} = \dfrac{450}{200} = 2.25\ \Omega$

$$X_s = \sqrt{Z_s^2 - R_a^2} = \sqrt{2.25^2 - 0.2^2} = 2.241\ \Omega$$

EXAMPLE 3.9. *A 3-phase, 1500 kVA, star-connected, 50-Hz, 2300 V alternator has a resistance between each pair of terminals as measured by direct current is 0.16 Ω. Assume that the effective resistance is 1.5 times the ohmic resistance. A field current of 70 A produces a short-circuit current equal to full-load current of 376 A in each line. The same field current produces an e.m.f. of 700 V on open circuit. Determine the synchronous reactance of the machine and its full load regulation at 0.8 power factor lagging.*

SOLUTION.
$$Z_s = \frac{\text{open circuit e.m.f. per phase}}{\text{short circuit armature current}}$$

$$= \frac{(700/\sqrt{3})}{376} = 1.075\ \Omega$$

Ohmic resistance per phase $= \dfrac{0.16}{2} = 0.08\ \Omega$

Effective resistance per phase
$$R_a = 1.5 \times 0.08 = 0.12\ \Omega$$

Synchronous reactance
$$X_s = \sqrt{Z_s^2 - R_a^2} = \sqrt{1.075^2 - 0.12^2} = 1.068\ \Omega$$

$$S_{3\Phi} = \sqrt{3}\, V_L I_L$$

$$1500 \times 10^3 = \sqrt{3} \times 2300 I_L,\ I_L = 376\ \text{A}$$

Rated voltage per phase
$$V_p = 2300/\sqrt{3} = 1328\ \text{V}$$

Phase current $\quad I_{ap} = I_L = 376$ A

$$\mathbf{E_p} = \mathbf{V_p} + \mathbf{I_{ap}} \mathbf{Z_s}$$

Let $\mathbf{V_p}$ be taken as reference phasor :

$$\mathbf{V_p} = V_p \angle 0° = 1328 \angle 0°\ \text{V} = 1328 + j0\ \text{V}$$

$$\mathbf{I_{ap}} = I_{ap} \angle -\cos^{-1} 0.8 = 376 \angle -36.87°\ \text{A}$$

$$\mathbf{Z_s} = R_a + jX_s = 0.12 + j1.068 = 1.075 \angle 83.59°\ \Omega$$

$$\mathbf{E_p} = 1328 + j0 + (376 \angle -36.87°)(1.075 \angle 83.59°) = 1328 + 404.2 \angle 46.72°$$

$$= 1328 + 277.1 + j\,294.26 = 1605.1 + j294.26 = 1631 \angle 10.39°\ \text{V}$$

Percentage regulation
$$= \frac{E_p - V_p}{V_p} \times 100$$

$$= \frac{1631 - 1328}{1328} \times 100 = 22.8\%$$

Alternative method of calculating E_p

$$E_p = \sqrt{(V_p \cos\phi + I_a R_a)^2 + (V_p \sin\phi + I_a X_s)^2}$$

$$= \sqrt{(1328 \times 0.8 + 376 \times 0.12)^2 + (1328 \times 0.6 + 376 \times 1.068)^2} = 1631\ \text{V}$$

Example 3.10. *A 3-phase, star-connected alternator is rated at 1600 kVA, 13500 V. The armature effective resistance and synchronous reactance are 1.5 Ω and 30 Ω respectively per phase. Calculate the percentage regulation for a load of 1280 kW at power factors of (a) 0.8 leading; (b) unity; (c) 0.8 lagging.*

Solution. (*a*) $P_{3\phi} = \sqrt{3}\, V_L I_L \cos\phi$

$$1280 \times 10^3 = \sqrt{3} \times 13500 I_L \times 0.8$$

$$I_L = \frac{1280 \times 10^3}{\sqrt{3} \times 13500 \times 0.8} = 68.43 \text{ A} = I_a$$

$\cos\phi = 0.8, \quad \sin\phi = 0.6$

$$R_a = 1.5\ \Omega,\ X_s = 30\ \Omega, \quad V_p = \frac{13500}{\sqrt{3}} = 7794.5 \text{ V}$$

For leading power factor

$$E_p^2 = (V_p \cos\phi + I_a R_a)^2 + (-V_p \sin\phi + I_a X_s)^2$$

$$= (7794.5 \times 0.8 + 68.43 \times 1.5)^2 + (-7794.5 \times 0.6 + 68.43 \times 30)^2$$

$$= (6338)^2 + (-2623.8)^2$$

$$E_p = 6859.6 \text{ V}$$

$$\text{Voltage regulation} = \frac{E_p - V_p}{V_p} \times 100 = \frac{6859.6 - 7794.5}{7794.5} \times 100 = -11.99\%$$

(*b*) *Unity power factor*

$$\cos\phi = 1,\ \sin\phi = 0$$

$$P_{3\phi} = \sqrt{3} V_L I_L \cos\phi$$

$$1280 \times 10^3 = \sqrt{3} \times 13500 I_L \times 1$$

$$I_L = \frac{1280 \times 10^3}{\sqrt{3} \times 13500} = 54.74 A = I_a$$

$$E_p^2 = (V_p + I_a R_a)^2 + (I_a X_s)^2$$

$$= (7794.5 + 54.74 \times 1.5)^2 + (54.74 \times 30)^2 = (7876.6)^2 + (1642.2)^2$$

$$E_p = 8046 \text{ V}$$

Voltage regulation

$$= \frac{E_p - V_p}{V_p} \times 100 = \frac{8046 - 7794.5}{7794.5} \times 100 = 3.227\%$$

(*c*) *Power factor 0.8 lagging*

$$E_p^2 = (V_p \cos\phi + I_a R_a)^2 + (V_p \sin\phi + I_a X_s)^2$$

$$= (7794.5 \times 0.8 + 68.43 \times 1.5)^2 + (7794.5 \times 0.6 + 68.43 \times 30)^2$$

$$= (6338)^2 + (6729.6)^2$$

$$E_p = 9244.4 \text{ V}$$

$$\text{Voltage regulation} = \frac{E_p - V_p}{V_p} \times 100 = \frac{9244.4 - 7794.5}{7794.5} \times 100 = 18.6\%$$

EXAMPLE 3.11. *A straight-line law connects terminal voltage and load of a 3-phase star-connected alternator delivering current at 0.8 power factor lagging. At no load, the terminal voltage is 3500 V and at full load of 2280 kW, it is 3300 V. Calculate the terminal voltage when delivering current to a 3-phase, star-connected load having a resistance of 8 Ω and a reactance of 6 Ω per phase. Assume constant speed and field excitation.*

SOLUTION. $P_{3\Phi} = 3V_pI_p\cos\phi$

$$2280 \times 10^3 = 3 \times \frac{3300}{\sqrt{3}} I_p \times 0.8$$

$$I_p = 498.6 \text{ A}$$

No-load phase voltage $= \frac{3500}{\sqrt{3}} = 2020.7$ V

Full-load phase voltage $= \frac{3300}{\sqrt{3}} = 1905.3$ V

Voltage drop per phase for a current of 498.6 A

$$= 2020.7 - 1905.3 = 115.4 \text{ V}$$

Voltage drop per phase for 1 A current

$$= \frac{115.4}{498.6}$$

Let I be the current supplied by the alternator.

Therefore, the voltage drop per phase for supplying a current I at 0.8 power factor lagging

$$= \frac{115.4}{498.6} \text{I} = 0.2315 \text{ I volts}$$

Terminal voltage per phase for supplying a current I at 0.8 power factor lagging

$$= 2020.7 - 0.2315 \text{ I}$$

Load impedance $Z_L = \sqrt{R_L^2 + X_L^2} = \sqrt{8^2 + 6^2} = 10\ \Omega$

Load terminal voltage

$$= IZ_L = I \times 10 \text{ V}$$

$$10\,I = 2020.7 - 0.2315 \text{ I}$$

$\therefore$

$$I = \frac{2020.7}{10.2315} = 197.5 \text{ A}$$

$\therefore$ terminal voltage per phase

$$= IZ_L = 197.5 \times 10 = 1975 \text{ V}$$

Line value of terminal voltage $= \sqrt{3} \times 1975 = 3420.8$ V

EXAMPLE 3.12. *A 3-phase, star-connected, round-rotor synchronous generator rated at 10 kVA, 230 V has an armature resistance of 0.5 Ω per phase and a synchronous reactance of 1.2 Ω per phase. Calculate the percent voltage regulation at full load at power factors of (a) 0.8 lagging, (b) 0.8 leading, (c) Determine the power factor such that the voltage regulation is zero on full load.*

SOLUTION. $S_{3\phi} = \sqrt{3}\, V_L I_L$

$$10 \times 10^3 = \sqrt{3} \times 230\, I_L$$

$$I_L = \frac{10 \times 10^3}{\sqrt{3} \times 230} = 25.1 \text{ A} = I_{ap}$$

Rated voltage per phase $V_p = \frac{V_L}{\sqrt{3}} = \frac{230}{\sqrt{3}} = 132.8$ V

Let $\mathbf{V}_p$ be taken as reference phasor.

$$\mathbf{V}_p = V_p \angle 0^\circ = 132.8 \angle 0^\circ = 132.8 + j\,0$$

$R_a = 0.5\,\Omega, \quad X_s = 1.2\,\Omega$

$$\mathbf{Z}_s = R_a + j\,X_s = 0.5 + j\,1.2 = 1.3 \angle 67.38^\circ\ \Omega$$

(a) *Power factor 0.8 lagging*

$$\mathbf{I}_{ap} = I_{ap} \angle -\cos^{-1} 0.8 = 25.1 \angle -36.87^\circ \text{ A}$$

$$\mathbf{E}_p = \mathbf{V}_p + \mathbf{I}_{ap} \mathbf{Z}_s$$

$$= (132.8 + j\,0) + (25.1 \angle -36.87^\circ)(1.3 \angle 67.38^\circ)$$

$$= 132.8 + 32.63 \angle 30.51^\circ = 132.8 + 28.1 + j\,16.56$$

$$= 160.9 + j\,16.56 = 161.75 \angle 5.87^\circ \text{ V}$$

Voltage regulation

$$= \frac{E_p - V_p}{V_p} \times 100 = \frac{161.75 - 132.8}{132.8} \times 100 = 21.8\%$$

(b) *Power factor 0.8 leading*

$$\mathbf{I}_{ap} = I_{ap} \angle +\cos^{-1} 0.8 = 25.1 \angle 36.87^\circ \text{ A}$$

$$\mathbf{E}_p = V_p + I_{ap} Z_s$$

$$= 132.8 + (25.1 \angle 36.87^\circ)(1.3 \angle 67.38^\circ)$$

$$= 132.8 + 32.63 \angle 104.25^\circ$$

$$= 132.8 - 8 + j\,31.62 = 124.8 + j\,31.62$$

$$= 128.74 \angle 14.2^\circ \text{ V}$$

Voltage regulation

$$= \frac{E_p - V_p}{V_p} \times 100 = \frac{128.74 - 132.8}{132.8} \times 100 = -3.06\%$$

(c) Let ϕ be the required power-factor angle.

$\therefore$

$$\mathbf{I}_{ap} = I_{ap} \angle \phi = 25.1 \angle \phi \text{ A}$$

$$\mathbf{E}_p = V_p + I_{ap} Z_s$$

$$= 132.8 + (25.1 \angle \phi)(1.3 \angle 67.38^\circ)$$

$$= 132.8 + 32.63 \angle (\phi + 67.38)^\circ$$

$$= 132.8 + 32.63 \cos(\phi + 67.38^\circ) + j\,32.63 \sin(\phi + 67.38^\circ)$$

$$E_p^2 = [132.8 + 32.63 \cos(\phi + 67.38)]^2 + [32.63 \sin(\phi + 67.38)]^2$$

Voltage regulation $= \dfrac{E_p - V_p}{V_p}\, pu$

For zero voltage regulation $E_p = V_p = 132.8$ V

$\therefore \quad (132.8)^2 = [132.8 + 32.63 \cos(\phi + 67.38°)]^2 + [32.63 \sin(\phi + 67.38°)]^2$

or $\quad (132.8)^2 = (132.8)^2 + 2 \times 132.8 \times 32.63 \cos(\phi + 67.38°) + (32.63)^2 \cos^2(\phi + 67.38°) + (32.63)^2 \sin^2(\phi + 67.38°)$

$= (132.8)^2 + 2 \times 132.8 \times 32.63 \cos(\phi + 67.38°) + (32.63)^2$

or $\cos(\phi + 67.38°) = \dfrac{-32.63}{2 \times 132.8} = -0.12285 = \cos 97°$

$\therefore \quad \phi = 97° - 67.38° = +29.62$

and $\quad \cos\phi = 0.8693$ leading

EXAMPLE 3.13. *A 3-phase, 10 kVA, 400 V, 50 Hz star-connected alternator supplies the rated load at 0.8 power factor lagging. If the armature resistance is 0.5 Ω and synchronous reactance is 10 Ω, find the torque angle and voltage regulation.*

SOLUTION. $S_{s\phi} = \sqrt{3}\, V_L I_L$

$10 \times 10^3 = \sqrt{3} \times 400\, I_L$

$I_L = \dfrac{10 \times 10^3}{\sqrt{3} \times 400} = 14.4$ A

$\mathbf{Z}_s = R_a + j X_s = 0.5 + j\,10 = 10.012 \angle 87°\ \Omega$

Phase current $I_{ap} = I_L = 14.4$ A

Rated phase voltage

$V_p = \dfrac{V_L}{\sqrt{3}} = \dfrac{400}{\sqrt{3}} = 230.9$ V

Let $\mathbf{V}_p$ be taken as reference phasor.

$\therefore \quad \mathbf{V}_p = V_p \angle 0° = 230.9 \angle 0° = (230.9 + j\,0)$ V

At a lagging power factor of 0.8

$\mathbf{I}_{ap} = I_{ap} \angle -\cos^{-1} 0.8 = 14.4 \angle -36.87°$ A

$\mathbf{E}_{ap} = \mathbf{V}_p + \mathbf{I}_{ap} \mathbf{Z}_s$

$= 230.9 + j\,0 + (14.4 \angle -36.87°)(10.012 \angle 87°)$

$= 230.9 + 144.2 \angle 50.13° = 230 + 92.4 + j\,110.6$

$= 323.3 + j\,110.6 = 341.7 \angle 18.9°$ V

$\therefore \quad E_{ap} = 341.7$ V, $\delta = 18.9°$

Voltage regulation

$= \dfrac{E_{ap} - V_p}{V_p} = \dfrac{341.7 - 230.9}{230.9} = 0.4798\, pu$

EXAMPLE 3.14. *A 550 V, 55 kVA, single-phase alternator has an effective resistance of 0.2 Ω. A field current of 10 A produces an armature current of 200 A on short circuit and an emf of 450 V on open circuit. Calculate the synchronous reactance and voltage regulation at full load with power factor 0.8 lagging.*

SOLUTION. $S_{1\phi} = V I_a$

$$55 \times 10^3 = 550\, I_a$$

$$I_a = \frac{55 \times 10^3}{550} = 100 \text{ A}$$

$$\cos\phi = 0.8,\ \sin\phi = 0.6$$

Synchronous impedance

$$Z_s = \frac{\text{open-circuit phase voltage}}{\text{short-circuit armature current}}$$

$$= \frac{450}{200} = 2.25\ \Omega$$

Synchronous reactance

$$X_s = \sqrt{Z_s^2 - R_a^2} = \sqrt{(2.25)^2 - (0.2)^2} = 2.24\ \Omega$$

Generated armature voltage per phase for lagging *pf*

$$E_a = \sqrt{(V\cos\phi + I_a R_a)^2 + (V\sin\phi + I_a X_s)^2}$$

$$= \sqrt{(550 \times 0.8 + 100 \times 0.2)^2 + (550 \times 0.6 + 100 \times 2.24)^2}$$

$$= \sqrt{460^2 + 554^2} = 720 \text{ V}$$

Voltage regulation

$$= \frac{E_a - V}{V} \times 100 = \frac{720 - 550}{550} \times 100 = 30.91\%$$

Alternative method of calculating E_a

Let **V** be taken as reference phasor.

$\therefore$ $\mathbf{V} = V\angle 0° = 550\angle 0° = 550 + j0$ V

For lagging *pf* $\cos\phi$

$$\mathbf{I}_a = I_a \angle -\phi° = I_a \angle -\cos^{-1} 0.8° = 100 \angle -36.87° \text{ A}$$

$$\mathbf{Z}_s = R_a + j\,X_s = 0.2 + j\,2.24 = 2.25 \angle 84.9°\ \Omega$$

$$\mathbf{E}_a = \mathbf{V} + \mathbf{I}_a\,\mathbf{Z}_s$$

$$= 550 + j\,0 + (100\angle -36.87°)\,(2.25\angle 84.9°)$$

$$= 550 + 225\angle 48.03°$$

$$= 550 + 150.47 + j\,167.3 = 720 \angle 13.4° \text{ V}$$

EXAMPLE 3.15. *In a 50 kVA, star-connected, 440 V, 3-phase, 50 Hz alternator, the effective armature resistance is 0.25 Ω per phase. The synchronous reactance is 3.2 Ω per phase and the leakage reactance is 0.5 Ω per phase. Determine at rated load and unity power factor : (a) internal emf, (b) no-load emf, (c) percentage voltage regulation at full load, (d) value of the synchronous reactance which replaces armature reaction.*

SOLUTION. $S_{3\phi} = \sqrt{3}\, V_L I_L$

$$50 \times 10^3 = \sqrt{3} \times 440\, I_L$$

$$I_L = \frac{50 \times 10^3}{\sqrt{3} \times 440} = 65.6\ A = I_a$$

Let $\mathbf{V}_p$ be taken as reference phasor.

$$\mathbf{V}_p = V_p \angle 0^\circ = \frac{440}{\sqrt{3}} \angle 0^\circ = 254 \angle 0^\circ \text{ V}$$

At unity power factor

$$\mathbf{I}_a = I_a \angle 0^\circ = 65.6 \angle 0^\circ = 65.6 + j\,0$$

(*a*) Leakage impedance

$$\mathbf{Z}_L = R_a + j\,X_L = 0.25 + j\,0.5 = 0.559 \angle 63.4^\circ\ \Omega$$

Internal emf

$$\mathbf{E}_{p\,exc} = \mathbf{V}_p + \mathbf{I}_a \mathbf{Z}_L$$

$$= 254 \angle 0^\circ + (65.6 \angle 0^\circ)(0.559 \angle 63.4^\circ)$$

$$= 254 + j\,0 + 36.67 \angle 63.4^\circ$$

$$= 254 + 16.42 + j\,32.79 = 272.4 \angle 6.91^\circ \text{ V}$$

Line value of internal emf

$$E_{L\,exc} = \sqrt{3} \times 272.4 = 471.8 \text{ V}$$

(*b*) Synchronous impedance

$$\mathbf{Z}_s = R_a + j\,X_s = 0.25 + j\,3.2 = 3.21 \angle 85.53^\circ\ \Omega$$

No-load emf E_a

$$\mathbf{E}_{ap} = \mathbf{V}_p + \mathbf{I}_a \mathbf{Z}_s$$

$$= 254 \angle 0^\circ + (65.6 \angle 0^\circ)(3.21 \angle 85.53^\circ)$$

$$= 254 + j\,0 + 210.6 \angle 85.53^\circ$$

$$= 254 + 16.4 + j\,210 = 342.37 \angle 37.83^\circ \text{ V}$$

Line value of no-load emf

$$E_{aL} = \sqrt{3}\, E_{ap} = \sqrt{3} \times 342.37 = 593 \text{ V}$$

(*c*) Voltage regulation

$$= \frac{E_{ap} - V_{ap}}{V_{ap}} = \frac{342.37 - 254}{254}$$

$$= 0.3479 \text{ pu} = 34.79\%$$

(*d*) $\qquad X_S = X_L + X_{AR}$

$\therefore \qquad X_{AR} = X_S - X_L = 3.2 - 0.5 = 2.7\ \Omega$

3.31 MAGNETOMOTIVE FORCE (MMF) METHOD

This method is also known as **ampere-turn method.** The synchronous impedance method is based on the concept of replacing the effect of armature reaction by a fictitious reactance. The mmf method replaces the effect of armature leakage reactance by an equivalent additional armature reaction mmf so that this mmf may be combined with the armature reaction mmf F_{ar}.

The following information is required to predict the regulation by the mmf method :

(*a*) The resistance of the stator winding per phase.

(*b*) Open-circuit characteristic at synchronous speed.

(*c*) Short-circuit characteristic.

This method makes use of the phasor diagram of magnetomotive forces. The following procedure is used for drawing the phasor diagram at lagging power factor cos ϕ.

1. The armature terminal voltage per phase (*V*) is taken as the reference phasor along *OA*. (Fig. 3.19)

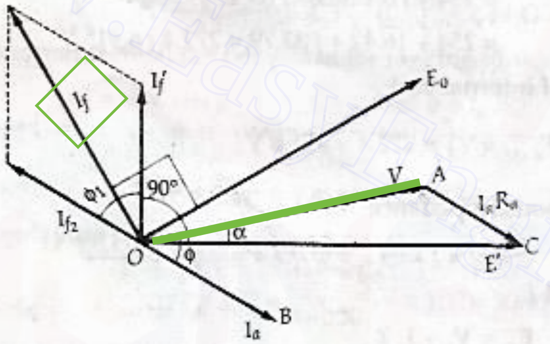


Fig. 3.19.

2. Armature current phasor $\mathbf{I}_a$ is drawn lagging the phasor **V** for lagging power factor angle ϕ for which the regulation is to be calculated.
3. The armature resistance drop phasor $\mathbf{I}_a R_a$ is drawn in phase with $\mathbf{I}_a$ along the line *AC*. Join *O* and *C*. *OC* represents the emf **E′**.
4. From the O.C.C. of Fig. 3.20, the field current I_f' corresponding to voltage **E′** is noted. Draw the field current $\mathbf{I}'_f$ leading the voltage **E′** by 90°. It is assumed that on short circuit all the excitation is opposed by the mmf of armature reaction and armature reactance. Thus, $\mathbf{I}'_f = I'_f \angle 90° - \alpha$.
5. From the S.C.C. of Fig. 3.20 determine the field current $\mathbf{I}_{f_2}$ required to circulate the rated current on short circuit. This is the field current required to overcome the synchronous reactance drop $\mathbf{I}_a X_s$. Draw the field current $\mathbf{I}_{f_2}$ in phase opposition to current $\mathbf{I}_a$. Thus, $\mathbf{I}_{f_2} = I_{f_2} \angle 180° - \phi$.

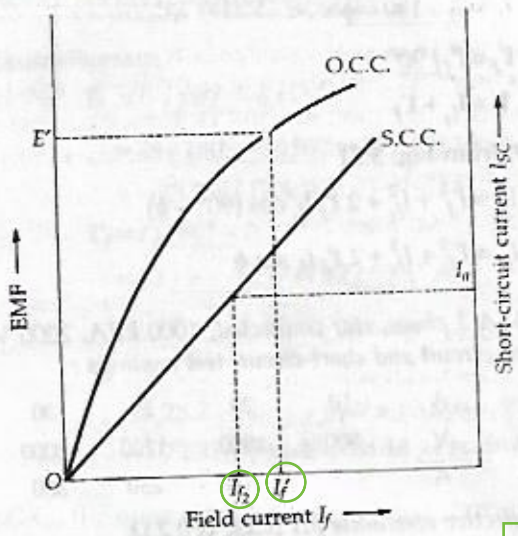


Fig. 3.20.

6. Determine the, phasor sum of field currents $\mathbf{I'}_f$ and $\mathbf{I}_{f_2}$. This gives resultant field current $\mathbf{I}_f$ which would generate a voltage E_0 under no-load conditions of the alternator. The open-circuit emf E_0 corresponding to field current I_f is found from the open-circuit characteristic.
7. The regulation of the alternator is found from the relation,

$$\text{regulation} = \frac{E_0 - V}{V} \times 100\%$$

3.32 AMPERE-TURN METHOD WITH R_a NEGLECTED

The phasor diagram at lagging $pf \cos\phi$, with R_a neglected is shown in Fig. 3.21.

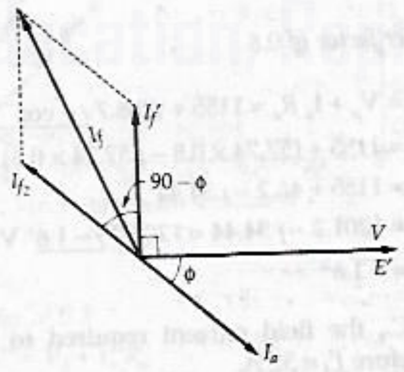


Fig. 3.21. *Phasor diagram at lagging p.f. cos φ with R_a neglected.*

Here $\quad \mathbf{I}_{f_2} = I_{f_2} \angle 180 - \phi$

$$\mathbf{I'}_f = I'_f \angle 90^\circ$$

$$\mathbf{I}_f = \mathbf{I}_{f_2} + \mathbf{I'}_f$$

Alternatively, from Fig. 3.21

$$I_f^2 = I'^2_f + I_{f_2}^2 + 2\, I'_f I_{f_2} \cos(90^\circ - \phi)$$

$$I_f^2 = I'^2_f + I_{f_2}^2 + 2\, I'_f I_{f_2} \sin\phi \qquad (3.32.1)$$

Example 3.16. *A 3-phase, star-connected, 1000 kVA, 2000 V, 50 Hz alternator gave the following open-circuit and short-circuit test readings :*

Field current	A	10	20	25	30	40	50
O.C. voltage	V	800	1500	1760	2000	2350	2600
S.C. armature current	A		200	250	300		

The armature effective resistance per phase is 0.2 Ω.

Draw the characteristic curves and determine the full-load percentage regulation at (a) 0.8 power factor lagging, (b) 0.8 power factor leading.

Solution. The O.C.C. and S.C.C. are shown in Fig. 3.22.

The phase voltage in volts are

$$\frac{800}{\sqrt{3}},\ \frac{1500}{\sqrt{3}},\ \frac{1760}{\sqrt{3}},\ \frac{2000}{\sqrt{3}},\ \frac{2350}{\sqrt{3}},\ \frac{2600}{\sqrt{3}};$$

or $\quad$ 462, 866, 1016, 1155, 1357, 1501.

Full-load phase voltage $V_p = \dfrac{2000}{\sqrt{3}} = 1155$ V

$$\text{kVA} = \frac{\sqrt{3}\, V_L I_{fl}}{1000}$$

$$1000 = \frac{\sqrt{3} \times 2000 \times I_{fl}}{1000},\ I_{fl} = I_a = 288.7 \text{ A}$$

(a) Lagging power factor of 0.8

$$\begin{aligned} \mathbf{E'} = \mathbf{V}_p + \mathbf{I}_a R_a &= 1155 + (288.7 \angle -\cos^{-1} 0.8) \times 0.2 \\ &= 1155 + (57.74 \times 0.8 - j\, 57.74 \times 0.6) \\ &= 1155 + 46.2 - j\, 34.44 \\ &= 1201.2 - j\, 34.44 = 1201.7 \angle -1.6^\circ \text{ V} \end{aligned}$$

Here $\quad \alpha = -1.6^\circ$

From the O.C.C., the field current required to produce the voltage of 1201.7 V is 32 A. Therefore $I'_f = 32$ A.

From the S.C.C., the field current required to produce full-load current of 288.7 is 29 A. Therefore $I_{f_2} = 29$ A. For $\cos\phi = 0.8$, $\phi = 36.87^\circ$

From the phasor diagram

$$\mathbf{I}_{f_2} = I_{f_2} \angle 180° - \phi$$

$$= 29 \angle 180° - 36.87° = 29 \angle 143.13° \text{ A}$$

$$= -23.2 + j\,17.4$$

$$\mathbf{I'}_f = I'_f \angle 90° - \alpha$$

$$= 32 \angle 90° - 1.6° = 32 \angle 88.4° \text{ A}$$

$$= 0.89 + j\,31.98$$

$$\mathbf{I}_f = \mathbf{I}_{f_2} + \mathbf{I'}_f$$

$$= -23.2 + j\,17.4 + 0.89 + j\,31.98$$

$$= -22.31 + j\,44.38 = 54.18 \angle 114.3° \text{ A}$$

From the O.C.C., the open circuit phase voltage corresponding to the field current of 54.18 A is 1559 V.

$\therefore$ percentage voltage regulation

$$= \frac{E_{op} - V_p}{V_p} \times 100 = \frac{1559 - 1155}{1155} \times 100 = 34.97\%$$

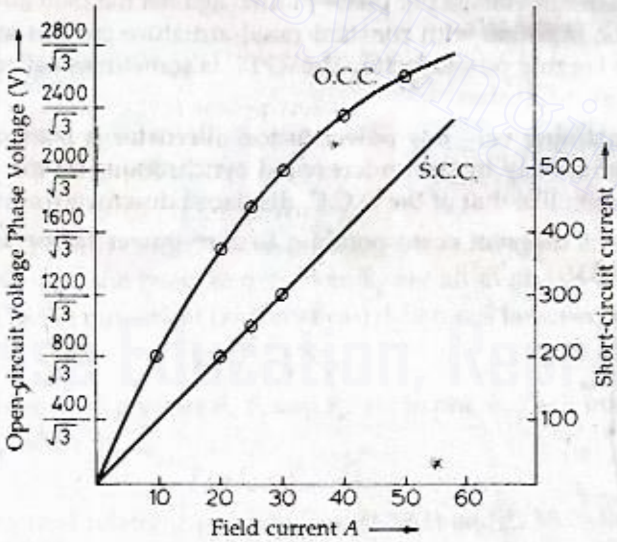


Fig. 3.22.

(b) *Leading power factor of 0.8*

$$\mathbf{E'} = \mathbf{V}_p + \mathbf{I}_a R_a$$

$$= 1155 + (288.7 \angle + \cos^{-1} 0.8) \times 0.2$$

$$= 1155 + 46.2 + j\,34.44$$

$$= 1201.2 + j\,34.44 = 1201.7 \angle +1.6° \text{ V.}$$

From, the phasor diagram

$$\mathbf{I'}_f = I'_f \angle 90^\circ + \alpha = 32 \angle 90^\circ + 1.6 = 32 \angle 91.6^\circ \text{ A}$$
$$= -0.89 + j\,31.98 \text{ A}$$
$$\mathbf{I}_{f_2} = I_{f_2} \angle 180^\circ + \phi$$
$$= 29 \angle 180^\circ + 36.87^{\circ\circ} = 29 \angle 216.87^\circ \text{ A}$$
$$= -23.2 - j\,17.4 \text{ A}$$
$$\mathbf{I}_f = \mathbf{I}_{f_2} + \mathbf{I'}_f = -0.89 + j\,31.98 - 23.2 - j\,17.4$$
$$= -24.09 + j\,14.58 \text{ A}$$
$$= 28.15 \angle 31.18^\circ \text{ A}$$

From the O.C.C., the open-circuit phase voltage corresponding to a field current of 28.15 A is 1098 V.

Percentage voltage regulation

$$= \frac{E_{op} - V_E}{V_p} \times 100 = \frac{1098 - 1155}{1155} \times 100 = -5.02\%$$

3.33 ZERO-POWER FACTOR CHARACTERISTIC (ZPFC)

The zero-power factor characteristic (ZPFC) of an alternator is a curve of the armature terminal voltage per phase plotted against the field current obtained by operating the machine with constant rated armature current at synchronous speed and zero lagging power factor. The ZPFC is sometimes called Potier Characteristic after its originator.

For maintaining very low power factor, alternator is loaded by means of reactors or alternatively by an underexcited synchronous motor. The shape of ZPFC is very much like that of the O.C.C. displaced downwards and to the right.

The phasor diagram corresponding to zero-power factor lagging load is shown in Fig. 3.23.

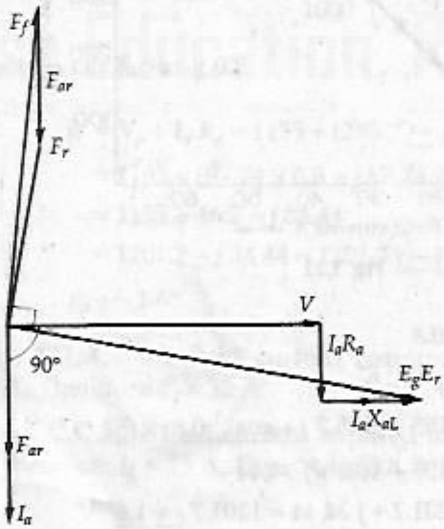


Fig. 3.23. *Phasor diagram at zero power factor lagging.*

In Fig. 3.23., the terminal phase voltage **V** is taken as the reference phasor. At zero power factor lagging, the armature current $\mathbf{I}_a$ lags behind **V** by 90°. Draw $\mathbf{I}_a R_a$ parallel to $\mathbf{I}_a$ and $\mathbf{I}_a X_{aL}$ perpendicular to $\mathbf{I}_a$.

$$\mathbf{V} + \mathbf{I}_a R_a + \mathbf{I}_a X_{aL} = \mathbf{E}_g$$

In Fig. 3.23, $\mathbf{E}_g$ is the generated voltage per phase.

$\mathbf{F}_{ar}$ = armature reaction mmf. It is in phase with $\mathbf{I}_a$

$\mathbf{F}_f$ = mmf of the mainfield winding (field mmf)

$\mathbf{F}_r$ = resultant mmf

The field mmf $\mathbf{F}_f$ is obtained by subtracting $\mathbf{F}_{ar}$ form $\mathbf{F}_r$, so that $\boxed{\mathbf{F}_r = \mathbf{F}_f + \mathbf{F}_{ar}}$

If the armature resistance R_a is neglected, the resulting phasor diagram is shown in Fig. 3.24.

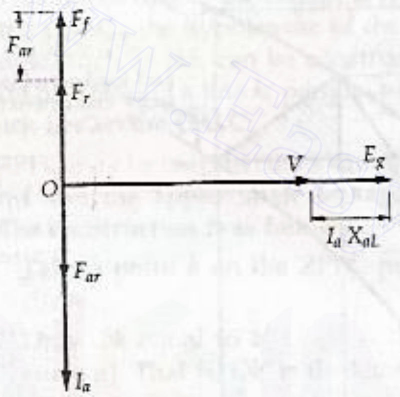


Fig. 3.24. *Phasor diagram at ZPF lagging with R_a neglected.*

From Fig. 3.24, it is seen that the terminal phase voltage **V**, the reactance voltage drop $\mathbf{I}_a X_{aL}$ and the generated voltage $\mathbf{E}_g$ are all in phase. Therefore **V** is practically equal to the numerical (arithmetical) difference between $\mathbf{E}_g$ and $\mathbf{I}_a X_{aL}$.

$$V = E - I_a X_{aL} \qquad (3.33.1)$$

Also the three mmf phasors $\mathbf{F}_f$, $\mathbf{F}_r$ and $\mathbf{F}_{ar}$ are in phase. Their magnitudes are related by the equation

$$F_f = F_r + F_{ar} \qquad (3.33.2)$$

The arithmetical relations given in Eqs. (3.33.1) and (3.33.2) form the basis for the Potier triangle.

Equation (3.33.2) can be converted into its equivalent field-current form by dividing its both sides by T_f, the effective number of turns per pole on the rotor field.

$$\therefore \qquad \frac{F_f}{T_f} = \frac{F_r}{T_f} + \frac{F_{ar}}{T_f}$$

or

$$I_f = I_r + I_{ar} \qquad (3.33.3)$$

3.34 POTIER TRIANGLE

In Fig. 3.25, consider a point b on the ZPFC corresponding to rated terminal voltage V and a field current of $OM = I_f = \dfrac{F_f}{T_f}$. If, for this condition of operation, the armature reaction mmf has a value expressed in equivalent field current of $LM \left(= I_{ar} = \dfrac{F_{ar}}{T_f} \right)$, then the equivalent field current of the resultant mmf would be $OL \left(= I_r = \dfrac{F_r}{T_f} \right)$

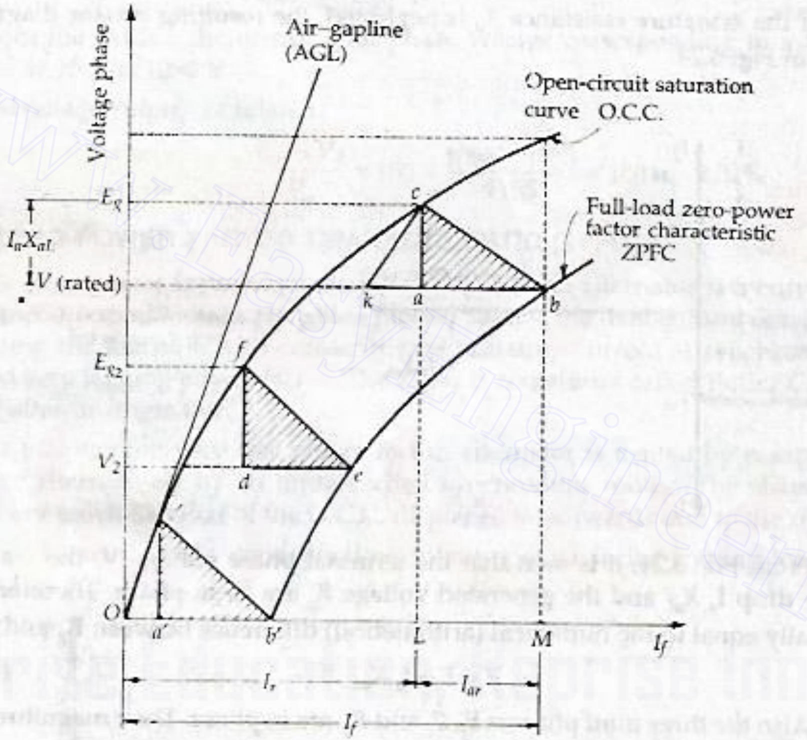


Fig. 3.25. *Potier triangle.*

This field current OL would result in a generated voltage E_g (= Lc) from the no load saturation curve. Since, for lagging zero-power-factor operation

$$\mathbf{E}_g = \mathbf{V} + \mathbf{I}_a \mathbf{X}_{aL}$$

the vertical distance ac must be equal to the leakage-reactance voltage drop $I_a X_{aL}$ where I_a is the rated armature current.

$$\therefore \qquad X_{aL} = \frac{\text{voltage } ac \text{ per phase}}{\text{rated armature current}}$$

The triangle formed by the vertices a, b, c is called the ***Potier triangle.***

For zero-power-factor operation with rated current at any other terminal voltage, such as V_2, since the armature current is of the same value, both the $I_a X_{aL}$ voltage and the armature mmf must be of the same respective values as they were for operation with rated terminal voltage V. Therefore, for all conditions of operation with rated armature current at zero lagging power factor, the same Potier triangle must be located between the terminal voltage V point on the ZPPC and the corresponding E_g point on the O.C.C. Thus, if the Potier triangle *cab* is moved downward so that the side *ab* is kept horizontal and *b* is kept on the ZPFC, the point *c* will move on the O.C.C. When the point *b* reaches the point *e*, the Potier triangle *cab* will be in the position *fde* and the location of point *f* on the O.C.C. will determine the voltage E_{g_2} which will be generated for zero-power-factor operation with terminal voltage V_2. When the point *b* reaches point '*b*'', the Potier triangle will be in the position *c'a'b'*. This is the limiting position which corresponds to short-circuit conditions, because at *b'*, the terminal voltage V is zero.

Since the initial part of the O.C.C. is almost linear, another triangle *Oc'b'* is formed by the O.C.C., the hypotenuse of the Potier triangle and the base line. A similar triangle, such as *ckb*, can be constructed from the Potier triangle in any other location by drawing a line *kc* parallel to *Oc'* through the vertex of the Potier triangle which lies on the O.C.C.

The ZPFC may be used in conjunction with the O.C.C. to find the armature reaction mmf and the approximate leakage reactance voltage of the machine (Fig. 3.25). The construction is as follows :

1. Take a point *b* on the ZPFC preferably well upon the knee of the curve.
2. Draw *bk* equal to *b' O* (*b'* is the point for zero voltage, full-load current). That is, *Ob'* is the short-circuit excitation, F_{SC}.
3. Through *k* draw *kc* parallel to *Oc'* to meet O.C.C. in *c*.
4. Drop the perpendicular *ca* on to *bk*.
5. Then, to scale, *ca* is the leakage reactance drop $I_a X_{aL}$ and *ab* is the armature reaction mmf F_{aR} or field current I_{faR} equivalent to armature reaction mmf at rated current.

The effect of field leakage flux in combination with the armature leakage flux gives rise to an equivalent leakage reactance X_p, known as the **Potier reactance**. It is greater than the armature leakage reactance.

$$\text{Also, Potier reactance } X_p = \frac{\text{voltage drop per phase } (= ac)}{\text{(ZPF rated armature current per phase } I_a)}$$

For cylindrical-rotor machines, Potier reactance X_p is approximately equal to leakage reactance X_{aL}. In salient-pole machines, X_p may be as large as 3 times X_{aL}.

Assumptions

The following assumptions are made in the Potier method :

1. The armature resistance R_a is neglected.

2. The O.C.C. taken on no-load accurately represents the relation between mmf and voltage on load.
3. The leakage reactance voltage $I_a X_{aL}$ is independent of excitation.
4. The armature-reaction mmf is constant.

It is not necessary to plot the entire ZPFC for determining X_{aL} and F_a experimentally. Only two points b and b' in Fig. 3.25 are sufficient. Point b correspondS, to a field current which gives the rated terminal voltage while the ZPF load is adjusted to draw rated current. Point b' corresponds to the short-circuit condition ($V = 0$) on the machine. Thus, $O b'$ is the field current required to circulate the short-circuit current equal to the rated current.

3.35 PROCEDURE TO OBTAIN THE REGULATION BY ZERO-POWER FACTOR METHOD

The following procedure is used to obtain regulation by the zero-power factor method :

The phasor diagram for lagging power cos ϕ is drawn as shown in Fig. 3.26. In the phasor diagram :

$OA = \mathbf{V}$ = terminal phase voltage at full load. It is taken as reference phasor and drawn horizontally.

$OB = \mathbf{I}_a$ = full-load current lagging behind V by an angle ϕ, cos ϕ is the power factor of the load

AC = voltage drop $\mathbf{I}_a R_a$ in the armature resistance (if R_a is given). It is drawn parallel to $\mathbf{I}_a$ (OB)

$CD = \mathbf{I}_a X_{aL}$ = leakage reactance voltage drop. It is perpendicular to AC.

Join OD. It represents the generated e.m.f. E_g

Find the field excitation current I_r corresponding to this generated emf E_g from the O.C.C.

Draw OG (equal to I_r perpendicular to OD).

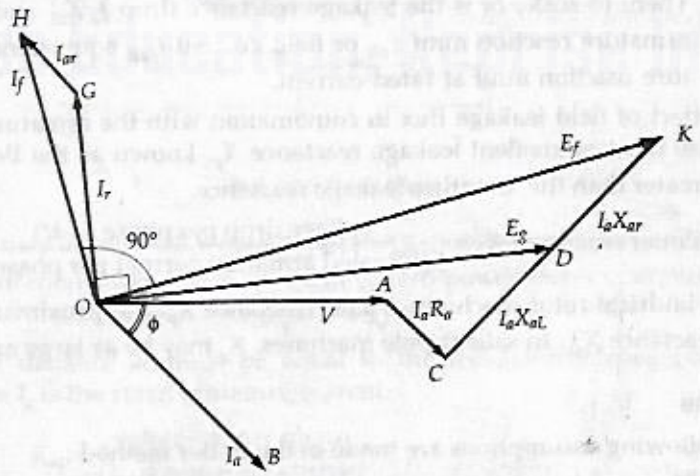


Fig. 3.26.

Draw *GH* parallel to load current *OB* ($= I_a$) to represent excitation (field current) equivalent to full-load armature reaction I_{ar}. *OH* gives the total field current I_f.

If the load is thrown off, then terminal voltage will be equal to generated emf, corresponding to field excitation *OH*.

Determine the emf E_f (= *OK*) corresponding to field excitation *OH* from the O.C.C. Phasor *OK* will lag behind phasor *OH* by 90°. *DK* represents the *voltage drop due to armature reaction.*

Now voltage regulation is obtained from the relation :

$$\text{Percentage voltage regulation} = \frac{E_f - V}{V} \times 100\%$$

EXAMPLE 3.17. *A 5000 kVA, 6600 V, 3-phase, star-connected alternator has a resistance of 0.75 Ω per phase. Estimate by zero power factor method the regulation for a load of 500 A at power factor (a) unity, (b) 0.9 leading, (c) 0.71 lagging, from the following open-circuit and full load, zero power factor curves :*

Field current, A	*Open-circuit terminal voltage, V*	*Saturation curve, zero p.f., V*
32	3100	0
50	4900	1850
75	6600	4250
100	7500	5800
140	8300	7000

SOLUTION. The O.C.C. and the ZPFC are plotted as drawn in Fig. 3.27.

Draw a horizontal line at rated line voltage of 6600 V to meet the ZPFC at *b*. On this line take $bk = Ob' = 32$ A.

Ob' is the field current required to circulate full-load current on S.C.

Draw a line *kc* parallel to Oc' (the initial slope of the O.C.C.) to meet the O.C.C. at *c*. Draw the perpendicular *ca* on the line *kb*. Hence *abc* is the Potier's triangle. In this triangle,

$$ab = \text{field current required to overcome armature reaction on load}$$
$$= I_{ar} = 25 \text{ A}$$

and $$ac = 900 \text{ V (line-to-line)} = \frac{900°}{\sqrt{3}} \text{ V per phase}$$

∴ Leakage impedance voltage drop

$$I_a X_L = \frac{900}{\sqrt{3}} \text{ V per phase}$$
$$= 579.6 \text{ V}, \; I_a = 500 \text{ A}$$

$$X_L = \frac{900}{\sqrt{3} \times 500} = 1.039 \,\Omega$$

Taking $\mathbf{I}_a$ as reference phasor $\mathbf{I}_a = I_a \angle 0° = 500 \angle 0° \text{ A} = 500 + j\,0$

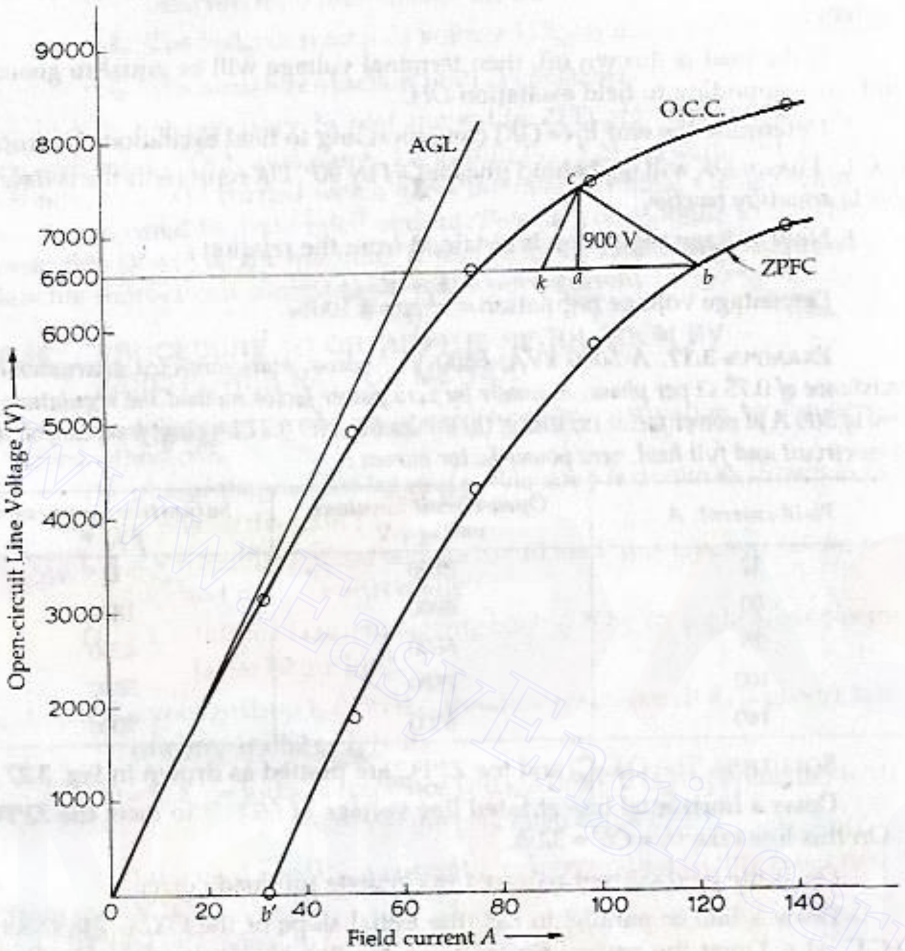


Fig. 3.27.

(a) Unity power factor

$$\mathbf{V}_p = V_p \angle 0° = \frac{6600}{\sqrt{3}} \angle 0° = 3810.6$$

$$\mathbf{E}_{gp} = \mathbf{V}_p + \mathbf{I}_a \mathbf{Z}_L = V_p + I_a (R_a + j X_L) = V_p + I_a R_a + j I_a X_L$$
$$= 3810.6 + 500 \times 0.075 + j\,519.6$$
$$= 3848.1 + j\,519.6 = 3883 \angle 7.69° \text{ V}$$

$$E_{gl} = \sqrt{3} \times 3883 = 6725 \text{ V}$$

From the O.C.C., the field current corresponding to the line voltage of 6725 V is 78 A.

This current leads $\mathbf{E}_g$ by 90°

$$\therefore \quad \mathbf{I}_r = I_r \angle 90° + 7.69° = 78 \angle 97.69° \text{ A}$$

The current $\mathbf{I}_{ar}$ is in phase with $\mathbf{I}_a$.

$\therefore \quad \mathbf{I}_{ar} = I_{ar} \angle 0^\circ = 25 \angle 0^\circ$ A

We have $\quad \mathbf{I}_f + \mathbf{I}_{ar} = \mathbf{I}_r$

$$\begin{aligned} \mathbf{I}_f &= \mathbf{I}_r - \mathbf{I}_{ar} \\ &= 78 \angle 97.69^\circ - 25 \angle 0^\circ \\ &= -10.44 + j\,77.3 - 25 \\ &= -35.44 + j\,77.3 = 85 \angle 114.6^\circ \text{ A} \end{aligned}$$

From the O.C.C., corresponding to a field current of 85 A, the voltage $E_{fl} = 7000$ V (line-line)

$$E_{fp} = \frac{7000}{\sqrt{3}} \text{ V} = 4041.6 \text{ V}$$

$$\therefore \quad \text{voltage regulation } = \frac{E_{fp} - V_p}{V_p} \times 100$$

$$= \frac{4041.6 - 3810.6}{3810.6} \times 100 = 6.06\%$$

(b) 0.9 p.f. leading

$$\cos\phi = 0.9, \qquad \phi = 25.84^\circ, \qquad \mathbf{I}_a = I_a \angle 0^\circ$$

$$\begin{aligned} \mathbf{V}_p &= 3810.6 \angle -25.84^\circ = 3429.6 - j\,1660.9 \\ \mathbf{E}_{gp} &= \mathbf{V}_p + \mathbf{I}_a \mathbf{Z}_L = V_p + I_a (R_a + j\,X_L) \\ &= (3429.6 - j\,1660.9) + (500 \times 0.075 + j\,519.6) \\ &= 3467.1 - j\,1141.3 = 3650 \angle -18.2^\circ \text{ V} \end{aligned}$$

The corresponding line voltage

$$E_{gl} = \sqrt{3}\, E_{gp} = \sqrt{3} \times 3650 = 6321.8 \text{ V}$$

From the O.C.C., the field current corresponding to the line voltage of 6321.8 V is 71 A. This current leads E_g by 90°

$\therefore \quad \mathbf{I}_r = I_r \angle 90^\circ - 18.2^\circ = 71 \angle 71.8^\circ$ A

The current $\mathbf{I}_{ar}$ is in phase with $\mathbf{I}_a$.

$$\mathbf{I}_{ar} = I_{ar} \angle 0^\circ = 25 \angle 0^\circ \text{ A}$$

We have $\quad \mathbf{I}_f + \mathbf{I}_{ar} = \mathbf{I}_r$

$$\begin{aligned} \mathbf{I}_f &= \mathbf{I}_r - \mathbf{I}_{ar} \\ &= 71 \angle 71.8^\circ - 25 \angle 0^\circ \\ &= 22.2 + j\,67.5 - 25 \\ &= -2.8 + j\,67.5 \\ &= 67.6 \angle 92.38^\circ \text{ A} \end{aligned}$$

From the O.C.C., corresponding to a field current of 67.6 A, the voltage

$$E_{fl} = 6000 \text{ V (line to line)}$$

Corresponding phase voltage $E_{fp} = \frac{6000}{\sqrt{3}} = 3464$ V

$\therefore$ voltage regulation $= \frac{E_{fp} - V_p}{V_p} \times 100$

$$= \frac{3464 - 3810.6}{3810.6} \times 100 = -9.1\%$$

(*c*) **0.71 p.f. lagging**

$\cos\phi = 0.71, \quad \phi = 44.77°$

$$\mathbf{I}_a = I_a \angle 0° = 500 \angle 0° \text{ A}$$

$$\mathbf{V}_p = V_p \angle +\phi° = 3810.6 \angle 44.77° = 2705.3 + j\,2683.7$$

$$\mathbf{E}_{gp} = \mathbf{V}_p + \mathbf{I}_a \mathbf{Z}_L = \mathbf{V}_p + I_a (R_a + j\,X_L)$$

$$= \mathbf{V}_p + I_a R_a + j\,I_a X_L$$

$$= (2705.3 + j\,2683.7) + 500 \times 0.075 + j\,519.6$$

$$= 2742.8 + j\,3203.3 = 4217 \angle 49.4° \text{ V}$$

$$E_{gl} = \sqrt{3}\,E_{gp} = \sqrt{3} \times 4217 = 7303.8 \text{ V}$$

From the O.C.C., the field current corresponding to the line voltage of 7307.8 V is 95 A.

$$\therefore \quad \mathbf{I}_r = 95 \angle 90 + 49.4° = 95 \angle 139.4° \text{ A}$$

$$\mathbf{I}_{ar} = 25 \angle 0° \text{ A}$$

$$\mathbf{I}_f = \mathbf{I}_r - \mathbf{I}_{ar} = 95 \angle 139.4° - 25 \angle 0°$$

$$= -72.1 + j\,61.8 - 25 = -97.1 + j\,61.8 = 115 \angle 147.5° \text{ A}$$

From the O.C.C., corresponding to a field current of 115 A, $E_{fl} = 7900$ V

$\therefore$ voltage regulation $= \frac{E_{fl} - V_l}{V_l} \times 100\%$

$$= \frac{7900 - 6600}{6600} \times 100 = 19.7\%$$

EXAMPLE 3.18. *The table gives data for open-circuit and load zero power factor tests on a 6-pole, 440 V, 50 Hz, 3-phase star-connected alternator. The effective ohmic resistance between any two terminals of the armature is 0.3 Ω.*

Field current (A)	2	4	6	7	8	10	12	14	16	18
O.C.terminal voltage (V)	156	288	396	440	474	530	568	592	—	—
S.C.line current (A)	11	22	34	40	46	57	69	80	—	
Zero p.f. terminal voltage (V)	—	—	—	0	80	206	314	398	460	504

Find the regulation at full-load current of 40 A at 0.8 power factor lagging using

(a) synchronous impedance method,

(b) mmf method,

(c) Potier-triangle method

SOLUTION. Armature resistance per phase $= \frac{1}{2} \times 0.3 = 0.15\ \Omega$

Terminal voltage per phase $= \dfrac{440}{\sqrt{3}} = 254$ V

The O.C.C., S.C.C. and ZPFC are plotted as shown in Fig. 3.28.

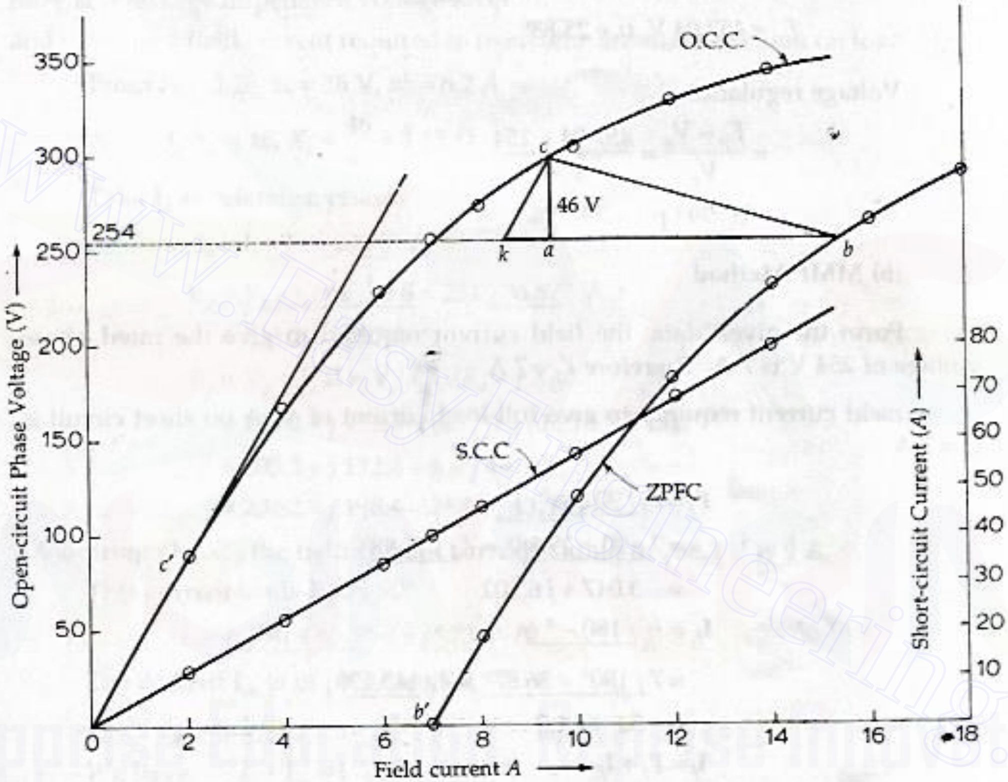


Fig. 3.28.

(a) *Synchronous Impedance Method*

For a field excitation current of 7 A the open-circuit phase voltage is $\dfrac{440}{\sqrt{3}}$ V and the short circuit current is 40 A. Therefore the synchronous impedance is given by

$$Z_s = \frac{\text{O.C. phase voltage for field current of 7 A}}{\text{S.C. current for field current of 7 A}}$$

$$= \frac{440/\sqrt{3}}{40} = 6.35\ \Omega$$

$$X_s = \sqrt{Z_s^2 - R_a^2} = \sqrt{(6.35)^2 - (0.15)^2} = 6.348\ \Omega$$

$$\cos\phi = 0.8 \text{ lagging}, \phi = -36.87°, \mathbf{I}_a = I_a \angle \phi° = 40 \angle -36.87°$$

$$\mathbf{E}_p = \mathbf{V}_p + \mathbf{I}_a \mathbf{Z}_s$$

$$= 254 \angle 0° + (40 \angle -36.87°)\,(0.15 + j\,6.348)$$

$$= 254 + (40 \angle -36.87°)\,(6.349 \angle 88.64°)$$

$$= 254 + 253.96 \angle 51.77°$$

$$= 254 + 157.2 + j\,199.5$$

$$= 411.2 + j\,199.5 = 457.04 \angle 25.88° \text{ V}$$

$$\therefore \quad E_p = 457.04 \text{ V}, \alpha = 25.88°$$

Voltage regulation

$$= \frac{E_p - V_p}{V_p} = \frac{457.04 - 254}{254}$$

$$= 0.7993 \text{ pu} = 79.93\%$$

(b) MMF Method

Form the given data, the field current required to give the rated phase voltage of 254 V is 7 A. Therefore $I'_f = 7$ A.

Field current required to give full-load current of 40 A on short circuit is $I_{f_2} = 7$ A

$$\mathbf{I}'_f = I'_f \angle 90 + \alpha°$$

$$= 7 \angle 90 + 25.88° = 7 \angle 115.88°$$

$$= -3.047 + j\,6.302$$

$$\mathbf{I}_{f_2} = I_{f_2} \angle 180 - °\,\phi$$

$$= 7 \angle 180° - 36.87° = 7 \angle 143.13°$$

$$= -5.6 + j\,4.2$$

$$\therefore \quad \mathbf{I}_f = \mathbf{I}'_f + \mathbf{I}_{f_2}$$

$$= -3.047 + j\,6.302 = -5.6 + j\,4.2$$

$$= -8.647 + j\,10.502$$

$$= 13.6 \angle 129.5° \text{ A}$$

From the O.C.C., the open-circuit phase voltage corresponding to a field current of 13.6 A is 338 V.

$$\text{Voltage regulation} = \frac{E_{op} - V_p}{V_p} \times 100$$

$$= \frac{338 - 254}{254} \times 100 = 33.07\%$$

(c) Zero-Power Factor Method

Draw a horizontal line at rated phase voltage of 254 V to meet the ZPFC at b. On this line take a point k such that

$$bk = Ob' = 7 \text{ A}$$

= field current required to circulate full load current on short circuit.

Through k draw kc parallel to Oc' (the initial slope of the O.C.C.) to meet the O.C.C. at c.

Draw the perpendicular ca on the line kb. From the Potier triangle abc we have ac = leakage impedance voltage drop

and ab = field current required to overcome armature reaction on load = I_{ar}

From Fig. 3.28, $ac = 46$ V, $ab = 6.2$ A

$$\therefore \quad I_a X_L = 46, \; X_L = \frac{46}{40} = 1.15\ \Omega$$

Take $\mathbf{I}_a$ as reference phasor.

$$\mathbf{I}_a = I_a \angle 0° = 40 \angle 0° \text{ A}$$

$$\mathbf{V}_p = V_p \angle +\cos^{-1} 0.8 = 254 \angle 36.87° \text{ V}$$

$$= 203.2 + j\,152.4 \text{ V}$$

$$\mathbf{E}_g = \mathbf{V}_p + \mathbf{I}_a \mathbf{Z} = \mathbf{V}_p + \mathbf{I}_a (R_a + j X_L)$$

$$= 203.2 + j\,152.4 + (40 \angle 0°)(0.15 + j\,1.15)$$

$$= 203.2 + j\,152.4 + 6 + j\,46$$

$$= 209.2 + j\,198.4 = 288.3 \angle 43.5° \text{ V}$$

From O.C.C., the field current corresponding to 288.3 V is 9 A.

This current leads $\mathbf{E}_g$ by 90°

$$\therefore \quad \mathbf{I}_r = I_r \angle 90 + 43.5° = 9 \angle 133.5° \text{ A}$$

The current $\mathbf{I}_{ar}$ is in phase with $\mathbf{I}_a$

$$\mathbf{I}_{ar} = 6.2 \angle 0°$$

We have $\quad \mathbf{I}_f + \mathbf{I}_{ar} = \mathbf{I}_r$

$$\mathbf{I}_f = \mathbf{I}_r - \mathbf{I}_{ar}$$

$$= 9 \angle 133.5° - 6.2 \angle 0°$$

$$= -6.19 + j\,6.53 - 6.2$$

$$= -12.39 + j\,6.53 = 14 \angle 152.2° \text{ A}$$

From O.C.C., corresponding to a field current of $I_f = 14$ A, $E_f = 341.81\%$

$\therefore$ voltage regulation

$$= \frac{E_f - V}{V} \times 100$$

$$= \frac{341.8 - 254}{254} \times 100 = 34.57\%$$

3.36 POWER FLOW TRANSFER EQUATIONS FOR A SYNCHRONOUS GENERATOR

Figure 3.29 shows the circuit model of a phase cylindrical rotor synchronous generator.

Let V = terminal voltage per phase

E_f = excitation voltage per phase

I_a = armature current

δ = phase angle between $\mathbf{E}_f$ and $\mathbf{V}$

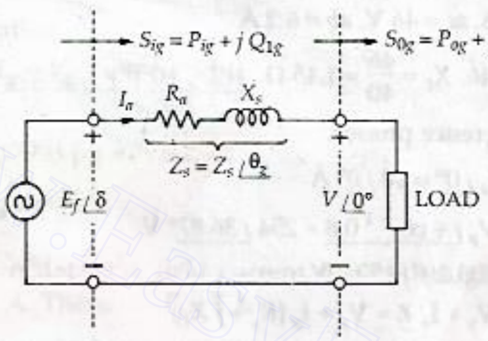


Fig. 3.29 *Circuit model of cylindrical rotor synchronous generator.*

The phasor diagram at lagging power factor is shown in Fig. 3.30.

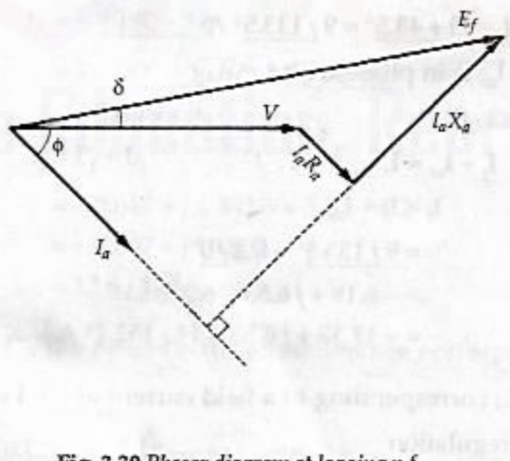


Fig. 3.30 *Phasor diagram at lagging p.f.*

For a synchronous generator $\mathbf{E}_f$ leads $\mathbf{V}$ by angle δ.

$$\mathbf{V} = V \angle 0, \ \mathbf{E}_f = E_f \angle \delta$$

The synchronous impedance is given by

$$\mathbf{Z}_s = R_a + j\,X_s = Z_s \angle \theta_z$$

The impedance triangle is shown in Fig. 3.31. Here

$$\theta_z = \tan^{-1}\frac{X_s}{R_a}$$

$$\alpha_z = 90° - \theta_z = \tan^{-1}\frac{R_a}{X_s}$$

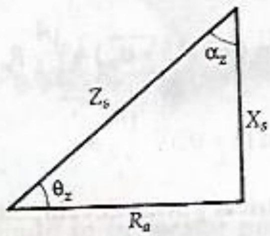


Fig. 3.31 *Impedance triangle.*

Let the subscripts i, o, g and m denote input, output, generator and motor respectively.

By kVL in the network of Fig. 3.29

$$\mathbf{E}_f = \mathbf{V} + \mathbf{Z}_s\,\mathbf{I}_a \tag{3.36.1}$$

$$\mathbf{I}_a = \frac{\mathbf{E}_f - \mathbf{V}}{\mathbf{Z}_s} \tag{3.36.2}$$

3.36.1 Complex Power Output of the Generator Per Phase (S_{0g})

$$S_{og} = P_{og} + j\,Q_{og} = \mathbf{V}\,I_a^* \tag{3.36.3}$$

$$= \mathbf{V}\left(\frac{\mathbf{E}_f - \mathbf{V}}{\mathbf{Z}_s}\right)^*$$

$$= V\angle 0°\left(\frac{E_f\angle\delta - V\angle 0°}{Z_s\angle\theta_z}\right)^*$$

$$= V\angle 0°\left(\frac{E_f}{Z_s}\angle\delta - \theta_z - \frac{V}{Z_s}\angle -\theta_z\right)^*$$

$$= \frac{V\,E_f}{Z_s}\angle\theta_z - \delta - \frac{V^2}{Z_f}\angle\theta_z$$

$$\therefore \quad P_{og} + j\,Q_{og} = \frac{V\,E_f}{Z_s}\cos(\theta_z - \delta) + j\,\frac{V\,E_f}{Z_s}\sin(\theta_z - \delta) - \frac{V^2}{Z_s}(\cos\theta_z + j\sin\theta_z) \tag{3.36.4}$$

Equating real parts of Eq. (3.36.4) we get real power output P_{og} of the generator.

3.36.2 Real Output Power Per Phase of the Generator (P_{og})

$$P_{og} = \frac{V E_f}{Z_s} \cos(\theta_z - \delta) - \frac{V^2}{Z_s} \cos \theta_z \quad (3.36.5)$$

Since $\cos \theta_z = \dfrac{R_a}{Z_s}$

$$P_{og} = \frac{V E_f}{Z_s} \cos(\theta_z - \delta) - \frac{V^2}{Z_s^2} R_a \quad (3.36.5a)$$

But $\theta_z = 90° - \alpha_z$

$$\therefore \quad P_{og} = \frac{V E_f}{Z_s} \cos(90° - \overline{\delta + \alpha_z}) - \frac{V^2}{Z_s^2} R_a$$

or
$$P_{og} = \frac{V E_f}{Z_s} \sin(\delta + \alpha_z) - \frac{V^2}{Z_s^2} R_a \quad (3.36.6)$$

P_{og} is also called the electrical power developed by the generator.

3.36.3 Reactive Output Power Per Phase of the Generator (Q_{og})

Equating imaginary parts of Eq. (3.36.4) we get Q_{og}.

$$Q_{og} = \frac{V E_f}{Z_s} \sin(\theta_z - \delta) - \frac{V_2}{Z_s} \sin \theta_z$$

Since $\sin \theta_z = \dfrac{X_s}{Z_s}$

$$Q_{og} = \frac{V E_f}{Z_s} \sin(\theta_z - \delta) - \frac{V_2}{Z_s^2} X_s \quad (3.36.7)$$

But $\theta_z = 90° - \alpha_z$

$$\therefore \quad Q_{og} = \frac{V E_f}{Z_s} \sin(90° - \overline{\delta + \alpha_z}) - \frac{V^2}{Z_s^2} X_s$$

or
$$Q_{og} = \frac{V E_f}{Z_s} \cos(\delta + \alpha_z) - \frac{V_2}{Z_s^2} \quad (3.36.8)$$

3.36.4 Complex power input to generator per phase (S_{ig})

$$S_{ig} = P_{ig} + j\, Q_{ig} = \mathbf{E}_f \mathbf{I}_a^* \quad (3.36.9)$$

$$= \mathbf{E}_f \angle \delta \left(\frac{E_f}{Z_s} \angle \theta_z - \delta - \frac{V}{Z_s} \angle \theta_z \right)$$

or
$$P_i + j\, Q_{ig} = \frac{E_f^2}{Z_s} \angle \theta_z - \frac{V E_f}{Z_s} \angle \theta_z + \delta \quad (3.36.10)$$

$$\therefore \; P_{ig} + j\, Q_{ig} = \frac{E_f^2}{Z_s} \cos \theta_z + j \frac{E_f^2}{Z_s} \sin \theta_z - \left[\frac{V E_f}{Z_s} \cos(\theta_z + \delta) + j \frac{V E_f}{Z_s} \sin(\theta_z + \delta) \right] \quad (3.36.11)$$

3.36.5 Real power input to generator per phase (P_{ig})

Equating real parts of Eq. (3.36.11) we obtain P_{ig}.

$$P_{ig} = \frac{E_f^2}{Z_s} \cos \theta_z - \frac{V E_f}{Z_s} \cos (\theta_z + \delta)$$

Since $\cos \theta_z = \dfrac{R_a}{Z_s}$

$$P_{ig} = \frac{E_f^2}{Z_s} R_a - \frac{V E_f}{Z_s} \cos (\theta_z + \delta) \tag{3.36.12}$$

But $\theta_z = 90° - \alpha_z$

$$\therefore \quad P_{ig} = \frac{E_f^2}{Z_s} R_a - \frac{V E_f}{Z_s} \cos (90° + \delta - \alpha_z)$$

or
$$P_{ig} = \frac{E_f^2}{Z_s} R_a + \frac{V E_f}{Z_s} \sin (\delta - \alpha_z) \tag{3.36.13}$$

3.36.6 Reactive power input to generator per phase (Q_{ig})

Equating imaginary parts of Eq. (3.36.11) we get Q_{ig}.

$$Q_{ig} = \frac{E_f^2}{Z_s} \sin \theta_z - \frac{V E_f}{Z_s} \sin (\theta_z + \delta)$$

But $\sin \theta_z = \dfrac{X_s}{Z_s}$

$$\therefore \quad Q_{ig} = \frac{E_f^2}{Z_s^2} X_s - \frac{V E_f}{Z_s} \sin (\theta_z + \delta) \tag{3.36.14}$$

Since $\theta_z = 90° - \alpha_z$

$\sin (\theta_z + \delta) = \sin (90° + \delta - \alpha_z) = \cos (\delta - \alpha_z)$

$$\therefore \quad Q_{ig} = \frac{E_f^2}{Z_s^2} X_s - \frac{V E_f}{Z_s} \cos (\delta - \alpha_z) \tag{3.36.15}$$

Mechanical power input to generator = P_{ig} + rotational losses

The rotational losses include friction, windage and core losses.

3.36.7 Maximum power output of the generator per phase $P_{og\,(max)}$

For maximum power output of the generator

$$\frac{d P_{og}}{d \delta} = 0 \quad \text{and} \quad \frac{d^2 P_{og}}{d \delta^2} < 0$$

Differentiating Eq. (3.36.6) w.r.t. δ and equating it to zero we get

$$\frac{d}{d \delta} \left[\frac{V E_f}{Z_s} \sin (\delta + \alpha_z) - \frac{V^2}{Z_s^2} R_a \right] = 0$$

Since V, E_f, Z_s and R_a are constants

$$\frac{V E_f}{Z_s} \cos(\delta + \alpha_z) = 0$$

or

$$\cos(\delta + \alpha_z) = 0$$

$$\delta + \alpha_z = 90°$$

$$\delta = 90° - \alpha_z = \theta_z \tag{3.36.16}$$

Thus for maximum power output of generator

$$\boxed{\text{load angle } \delta = \text{impedance angle } \theta_z}$$

From Eqs. (3.36.6) and (3.36.16), maximum power output of the generator per phase is

$$P_{og\,(\max)} = \frac{V E_f}{Z_s} - \frac{V^2}{Z_s^2} R_a \tag{3.36.17}$$

This occus at $\delta = \theta_z$.

3.36.8 Maximum power input to generator per phase $P_{ig(\max)}$

For maximum power input to the generator $\frac{d P_{ig}}{d\delta} = 0$ and $\frac{d^2 P_{ig}}{d\delta^2} < 0$

Differentiating Eq. (3.36.13) w.r.t. δ and equating it to zero we get

$$\frac{d}{d\delta}\left[\frac{E_f^2}{Z_s^2} R_a + \frac{V E_f}{Z_s} \sin(\delta - \alpha_z)\right] = 0$$

$$\frac{V E_f}{Z_s} \cos(\delta - \alpha_z) = 0$$

$$\delta - \alpha_z = 90°$$

$$\delta = 90° + \alpha_z = 90° + 90° - \theta_z = 180° - \theta_z$$

Thus for maximum power input to the generator

$$\text{load angle } \delta = 180° - \text{impedance angle } \theta_z \tag{3.36.18}$$

From Eqs. (3.36.13) and (3.36.18), maximum power input to the generator per phase is

$$P_{i_{g\,(\max)}} = \frac{E_f^2}{Z_s^2} R_a + \frac{V E_f}{Z_s} \tag{3.36.19}$$

3.36.9 Power flow equations for a generator with armature resistance neglected

In practical polyphase synchronous machines $R_a < X_s$ and R_a can be neglected in the power flow equations.

When armature resistance R_a is neglected $Z_s = X_s$, $\alpha_z = 0$

Equations (3.36.6), (3.36.8), (3.36.13) and (3.36.15) simplify as follows :

$$P_{og} = \frac{V E_f}{X_s} \sin \delta \tag{3.36.20}$$

$$Q_{og} = \frac{V E_f}{X_s} \cos \delta - \frac{V^2}{X_s} \tag{3.36.21}$$

$$P_{ig} = \frac{V E_f}{X_s} \sin \delta = P_{og} \tag{3.36.22}$$

$$Q_{ig} = \frac{E_f^2}{X_s} - \frac{V E_f}{X_s} \cos \delta \tag{3.36.23}$$

Also, $$P_{og\,(\max)} = \frac{V E_f}{X_s} = P_{ig\,(\max)} \tag{3.36.24}$$

3.37 MAGNETIC AXES OF THE ROTOR (Salient Pole)

The axis of symmetry of the north magnetic poles of the rotor is called the ***direct axis*** or ***d*-axis.** *The axis of symmetry of the south magnetic poles is the* ***negative d-axis.*** *The axis of symmetry halfway between adjacent north and south poles is called the* ***'quadrature axis'*** or ***q*-axis.** The q axis lagging the north pole is taken as the positive q axis as shown in Fig. 3.32. The quadrature axis is so named because it is 90 *electrical* degrees (one-quarter cycle) away from the direct axis.

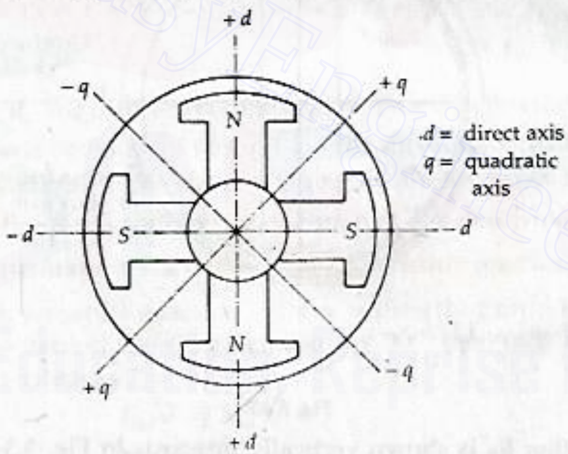


Fig. 3.32.

3.38 TWO-REACTION THEORY

Two-reaction theory was proposed by Andre Blondel. The theory proposes to resolve the given armature mmfs into two mutually perpendicular components, with one located *along the axis* of the rotor salient pole. It is known as the **direct-axis (or *d*-axis) component.** The other component is located *perpendicular* to the axis of the rotor salient pole. It is known as the **quadrature-axis (or *q*-axis) component.** The d-axis component of the armature mmf F_a is denoted by F_d and the q-axis component by F_q. The component F_d is either magnetizing or demagnetizing. The component F_q results in a cross-megnetizing effect.

If ψ is the angle between the armature current $\mathbf{I}_a$ and the excitation voltage $\mathbf{E}_f$ and F_a is the amplitude of the armature mmf, then

$$F_d = F_a \sin \psi$$

and
$$F_q = F_a \cos \psi$$

3.39 SALIENT-POLE SYNCHRONOUS MACHINE–TWO-REACTION MODEL

In the cylindrical-rotor synchronous machine the air gap is uniform. The protruding pole structure of the rotor of a salient-pole machine makes the air gap highly non-uniform. Consider a 2-pole salient-pole rotor rotating in the anticlockwise direction within a 2-pole stator as shown in Fig. 3.33. The axis along the axis of the rotor is called *direct* (or *d*-) *axis* and the axis perpendicular to *d*-axis is called the *quadrature* (or *q*-) *axis*. These axes are shown in Fig. 3.33. It is seen that the direct-axis flux path involves two *small* air gaps and is the path of *minimum* reluctance. The path denoted by ϕ_q in Fig. 3.33 has two *large* air gaps and is the path of maximum reluctance.

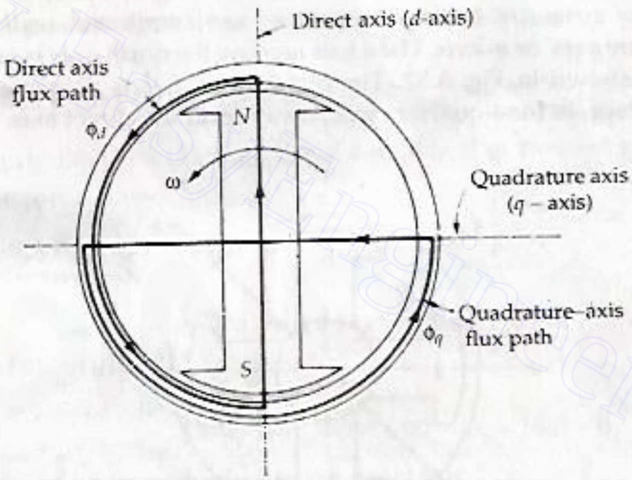


Fig. 3.33.

The rotor flux B_R is shown vertically upwards in Fig. 3.34. The rotor flux induces a voltage $\mathbf{E}_f$ in the stator. If a lagging *p.f.* load is connected to the synchronous generator, a stator current $\mathbf{I}_a$ will flow. The stator current $\mathbf{I}_a$ lags behind the generated voltage $\mathbf{E}_f$ by an angle ψ (Fig. 3.34).

The armature current produces stator magnetomotive force F_s. This mmf lags behind $\mathbf{I}_a$ by 90°. The mmf F_s produces stator magnetic field $\mathbf{B}_s$ along the direction of F_s. The stator mmf F_s is resolved into two components namely the direct-axis component F_d and the quadrature-axis component F_q.

If Φ_d = direct axis flux

Φ_q = quadrature axis flux

R_d = reluctance of direct-axis flux path

$$\therefore \quad \Phi_d = \frac{F_d}{R_d}$$

$$\Phi_q = \frac{F_q}{R_q}$$

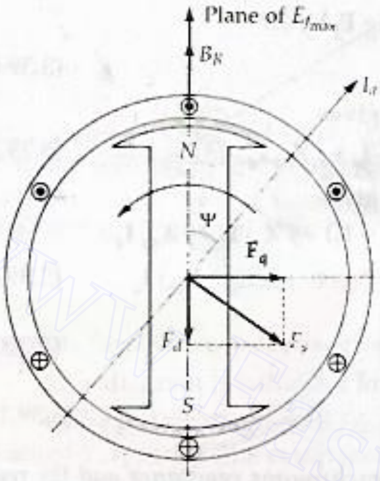


Fig. 3.34.

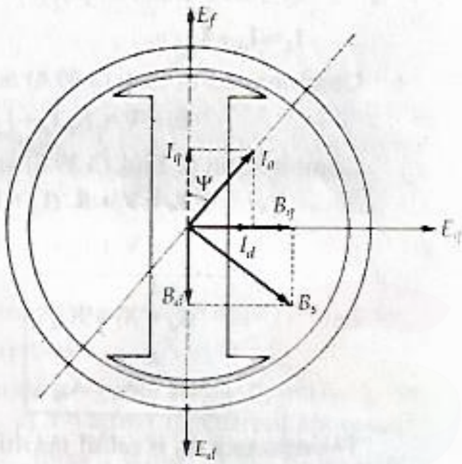


Fig. 3.35.

Since $R_d < R_q$, the direct-axis component of mmf F_d produces more flux than the quadrature-axis component of mmf F_q. The direct and quadrature axis stator fluxes produce voltages in the stator winding by armature reaction.

Let E_{ad} = direct axis component of armature reaction voltage

E_{aq} = quadrature-axis component of armature reaction voltage

Since each armature reaction voltage is directly proportional to its stator current and lags behind the stator current by 90°, therefore armature reaction voltages can be written as

$$\mathbf{E}_{ad} = -j\, X_{ad}\, \mathbf{I}_d \qquad (3.39.1)$$

$$\mathbf{E}_{aq} = -j\, X_{aq}\, \mathbf{I}_q \qquad (3.39.2)$$

where X_{ad} = armature reaction reactance in the direct axis per phase

X_{aq} = armature reaction reactance in the quadrature axis per phase

The value of X_{aq} is always less than X_{ad} since the emf induced by a given mmf acting on the direct axis is smaller than for the quadrature axis due to its higher reluctance.

The total voltage induced in the stator is the sum of emf induced by the field excitation and these two emfs. That is

$$\mathbf{E}' = \mathbf{E}_f + \mathbf{E}_{ad} + \mathbf{E}_{aq} \qquad (3.39.3)$$

or

$$\mathbf{E}' = \mathbf{E}_f - j\, X_{ad}\, \mathbf{I}_d - j\, X_{aq}\, \mathbf{I}_q \qquad (3.39.4)$$

The voltage E′ is equal to the terminal voltage **V** plus the voltage drops in the resistance and leakage reactance of the armature, so that

$$\mathbf{E}' = \mathbf{V} + R_a \mathbf{I}_a + j X_l \mathbf{I}_a \quad (3.39.5)$$

The armature current $\mathbf{I}_a$ is split into two components, one in phase with the excitation voltage $\mathbf{E}_f$ and the other in phase quadrature to it.

If $\mathbf{I}_q$ = the q axis component of $\mathbf{I}_a$ in phase with $\mathbf{E}_f$

$\mathbf{I}_d$ = the d axis component of $\mathbf{I}_a$ lagging $\mathbf{E}_f$ by 90°

$$\therefore \quad \mathbf{I}_a = \mathbf{I}_d + \mathbf{I}_q \quad (3.39.6)$$

Combination of Eqs. (3.39.4) and (3.39.5) gives

$$\mathbf{E}_f = \mathbf{V} + R_a \mathbf{I}_a + j X_l \mathbf{I}_a + j X_{ad} \mathbf{I}_d + j X_{aq} \mathbf{I}_q \quad (3.39.7)$$

Combination of Eqs. (3.39.6) and (3.39.7) gives

$$\mathbf{E}_f = \mathbf{V} + R_a (\mathbf{I}_d + \mathbf{I}_q) + j X_l (\mathbf{I}_d + \mathbf{I}_q) + j X_{ad} \mathbf{I}_d + j X_{aq} \mathbf{I}_q$$

$$= \mathbf{V} + R_a (\mathbf{I}_d + \mathbf{I}_q) + j (X_l + X_{ad}) \mathbf{I}_d + j (X_l + X_{aq}) \mathbf{I}_q \quad (3.39.8)$$

Let

$$\boxed{\begin{aligned} X_d &\overset{\Delta}{=} X_l + X_{ad} \quad (3.39.9) \\ X_q &\overset{\Delta}{=} X_l + X_{aq} \quad (3.39.10) \end{aligned}}$$

*The reactance X_d is called the **direct-axis synchronous reactance** and the reactance X_q is called the **quadrature-axis synchronous reactance**.*

Combination of Eqs. (3.39.8), (3.39.9) and (3.39.10) gives

$$\mathbf{E}_f = \mathbf{V} + R_a \mathbf{I}_d + R_a \mathbf{I}_q + j X_d \mathbf{I}_d + j X_q \mathbf{I}_q \quad (3.39.11)$$

or

$$\mathbf{E}_f = \mathbf{V} + R_a \mathbf{I}_a + j X_d \mathbf{I}_d + j X_q \mathbf{I}_q \quad (3.39.12)$$

Equation (3.39.11) is the final form of the *voltage equation for a salient-pole synchronous generator.*

Phasor diagram

The complete phasor diagram of a salient-pole synchronous generator based on two-axis theory is shown in Fig. 3.36

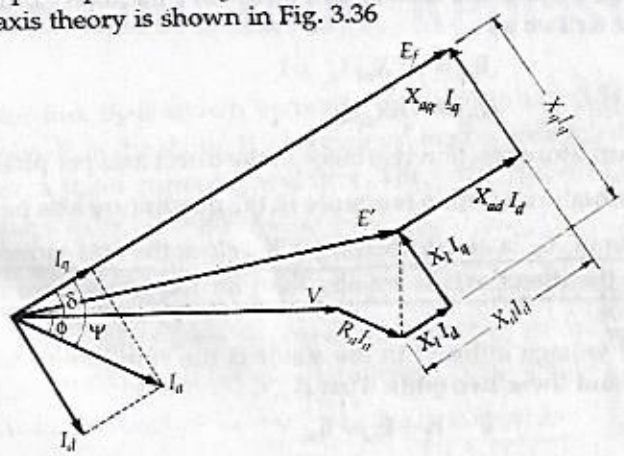


Fig. 3.36. *Phasor diagram of a salient-pole synchronous generator at lagging power factor.*

The simplified phasor diagram based on Eq. (3.39.11) is shown in Fig. 3.37.

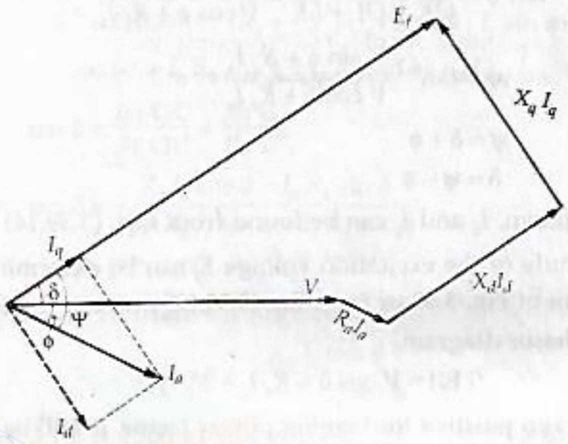


Fig. 3.37. *Simplified phasor diagram of a salient-pole synchronous generator at lagging power factor.*

This diagram is sufficient for many purposes.

In the phasor diagram of Fig. 3.37, the angle $\psi = \phi + \delta$, is not known for given values of V, I_a and ϕ. The components I_d and I_q of the armature current are usually not given. These component currents depend upon δ which is yet to be determined. The following procedure is used to construct the phasor diagram even without prior knowledge of δ.

Substituting $\mathbf{I}_q = \mathbf{I}_a - \mathbf{I}_d$ in Eq. (3.39.12) we get

$$\mathbf{E}_f = \mathbf{V} + R_a \mathbf{I}_a + j X_d \mathbf{I}_d + j X_q (\mathbf{I}_a - \mathbf{I}_d)$$

$$\mathbf{E}_f = \mathbf{V} + R_a \mathbf{I}_a + j X_q \mathbf{I}_a + j (X_d - X_q) \mathbf{I}_d \tag{3.39.13}$$

Figure 3.38 is drawn for the same machine as Fig. 3.37 and for the same operating conditions. In Fig. 3.38, BC is drawn at 90° to $\mathbf{I}_a$ and CD is drawn perpendicular to $\mathbf{E}_f$. In ΔBCD, $\angle BCD = \psi$.

We have
$$I_d = I_a \sin \psi \tag{3.39.14}$$
$$I_q = I_a \cos \psi \tag{3.39.15}$$

In ΔBCD of Fig. 3.38

$$\cos \psi = \frac{CD}{BC} = \frac{X_q I_q}{BC}$$

$$\therefore \quad BC = \frac{X_q I_q}{\cos \psi} = \frac{X_q (I_a \cos \psi)}{\cos \psi} = X_q I_a$$

Thus, the line BC represents the phasor $j X_q \mathbf{I}_a$, and its end point C determines the direction of E_f, that is δ. Now the line BC is extended to point M such that the distance $BM = X_d \mathbf{I}_a$ or $CM = (X_d - X_q) \mathbf{I}_a$. Then a line MN is drawn perpendicular to $\mathbf{E}_f$ making the angle ψ at point M, which makes $CN = (X_d - X_q) I_a \sin \psi = (X_d - X_q) I_d$. The point N is the end point of $\mathbf{E}_f$.

From $\Delta\, OCK$,

$$\tan \psi = \frac{CK}{OK} = \frac{KB + BC}{OL + LK} = \frac{V \sin \phi + X_q I_a}{V \cos \phi + R_a I_a} \tag{3.39.16}$$

$$\therefore \qquad \psi = \tan^{-1} \frac{V \sin \phi + X_q I_a}{V \cos \phi + R_a I_a}$$

Since $\quad \psi = \delta + \phi$

$\delta = \psi - \phi$

Once δ is known, I_d and I_q can be found from Eqs. (3.39.14) and (3.39.15).

The magnitude of the excitation voltage E_f can be determined either from the phasor diagram of Fig. 3.38 or from Eq. (3.39.12) or from Eq. (3.39.13).

From the phasor diagram,

$$|\,\mathbf{E}_f\,| = V \cos \delta + R_a I_q + X_d I_d \tag{3.39.17}$$

Since ϕ is taken positive for lagging power factor, it will be taken negative for leading power factor.

Determination of δ from the phasor diagram

Phasor diagram of Fig. 3.38 can be used to determine δ.

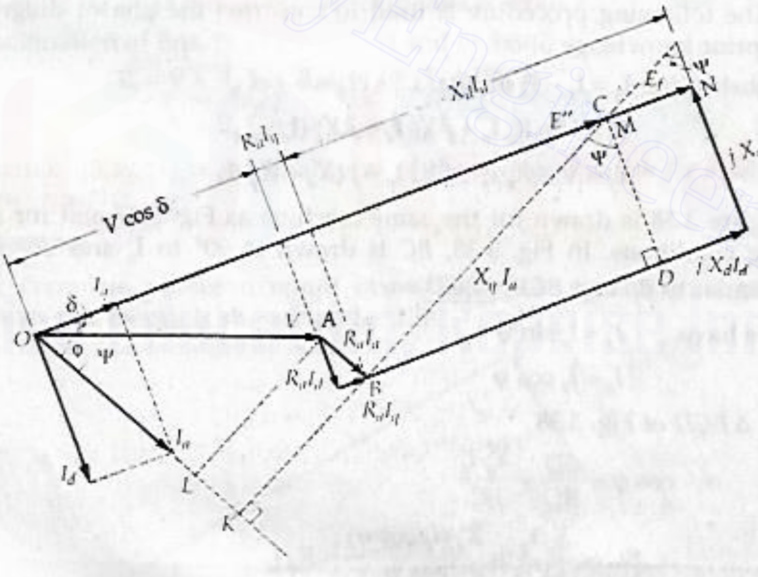


Fig. 3.38.

$$\mathbf{E}'' = \mathbf{OC} = \mathbf{OA} + \mathbf{AB} + \mathbf{BC}$$

$$= V + R_a \mathbf{I}_a + j X_q \mathbf{I}_a$$

$$= V + (R_a + j X_q)\, \mathbf{I}_a$$

For lagging power factor $\cos\phi$,

$$\mathbf{I}_a = I_a \angle -\phi = I_a \cos\phi - j\, I_a \sin\phi$$

$$\mathbf{E}'' = \mathbf{OC} = V + (R_a + j\, X_q)\,(I_a \cos\phi - j\, I_a \sin\phi)$$

$$= (V + I_a R_a \cos\phi + I_a X_q \sin\phi) + j\,(X_q I_a \cos\phi - I_a R_a \sin\phi)$$

$$\tan\delta = \frac{Im\,\mathbf{OC}}{Re\,\mathbf{OC}} = \frac{Im\,\mathbf{E}''}{Re\,\mathbf{E}''}$$

or $$\tan\delta = \frac{X_q I_a \cos\phi - I_a R_a \sin\phi}{V + X_q I_a \sin\phi + I_a R_a \cos\phi}$$

Hence $$\mathbf{E}_f = [E'' + (X_d - X_q)\, I_d] \angle \delta$$

If the armature resistance is neglected

$$\tan\delta = \frac{X_q I_a \cos\phi}{V + X_q I_a \sin\phi}$$

EXAMPLE 3.19. *A 1500 kVA, star-connected, 2300 V, 3-phase, salient-pole synchronous generator has reactances $X_d = 1.95\ \Omega$ and $X_q = 1.40\ \Omega$ per phase. All losses may be neglected. Find the excitation voltage for operation at rated kVA and power factor of 0.85 lagging.*

SOLUTION. $$V_p = \frac{2300}{\sqrt{3}} = 1328 \text{ V}$$

$$(kVA)_{3\phi} = \frac{3\, V_p I_a}{1000}$$

$$1500 = \frac{3 \times 1328\, I_a}{1000},\quad I_a = 376.5 \text{ A}$$

Let $\mathbf{V}_p$ be the reference phasor.

$$\mathbf{V}_p = V_p \angle 0^\circ = 1328 \angle 0^\circ$$

$$\cos\phi = 0.85, \qquad \phi = 31.8^\circ$$

$$\therefore \quad \mathbf{I}_a = I_a \angle -\phi = 376.5 \angle -31.8^\circ \text{ A}$$

$$= 320 - j\,198.4$$

From the phasor diagram of Fig. 3.38,

$$\mathbf{E}'' = \mathbf{OC} = \mathbf{OA} + \mathbf{AB} + \mathbf{BC}$$

$$= \mathbf{V}_p + 0 + j\, X_q I_a$$

$$= (1328 + j\,0) + j\,(1.40)\,(320 - j\,198.4)$$

$$= 1328 + 277.8 + j\,448 = 1605.8 + j\,448 = 1667 \angle 15.6^\circ \text{ V}.$$

The phase difference between $\mathbf{E}''$ and $\mathbf{I}_a$ is angle ψ.

$$\psi = \delta + \phi = 15.6^\circ + 31.8^\circ = 47.4^\circ$$

$$I_d = I_a \sin\psi = 376.5 \sin 47.4^\circ = 277.14 \text{ A}$$

$$(X_d - X_q)\, I_q = (1.95 - 1.40) \times 277.14 = 152.4 \text{ V}$$

Since $\mathbf{E}_f$, $\mathbf{E}''$ and $j\,(X_d - X_q)\, I_d$ are in phase we add the magnitudes.

$$\therefore \quad E_f = E + (X_d - X_q)\, I_d = 1667 + 152.4 = 1819.4 \text{ V}.$$

EXAMPLE 3.20. *An alternator has a direct-axis synchronous reactance of 0.8 per unit and a quadrature-axis synchronous reactance of 0.5 per unit. Determine the per unit open-circuit voltage for full load at a lagging power factor of 0.8. Neglect saturation.*

SOLUTION. Let $\mathbf{V}_p$ be the reference phasor

$\therefore \quad \mathbf{V}_p = 1 \angle 0° = 1 + j\,0$ pu

$I_a = 1$ pu at 0.8 lagging pf

$\therefore \quad \mathbf{I}_a = 1 \angle -\cos^{-1} 0.8 = 1 \angle -36.9°$ pu

$I_a X_d = 0.8$ pu

$I_a X_q = 0.5$ pu

$$\mathbf{E}'' = \mathbf{V}_p + j\,\mathbf{I}_a X_q$$
$$= 1 + j\,0 + j\,1 \angle -36.9° \times 0.5$$
$$= 1 + 0.5 \angle 90° - 36.9° = 1 + 0.5 \angle 53.1°$$
$$= 1 + 0.3 + j\,0.4 = 1.3 + j\,0.4 = 1.36 \angle 17.1° \text{ V}$$

$\therefore \quad \delta = 17.1°$

From the phasor diagram of Fig. 3.38

$$\psi = \phi + \delta = 36.9° + 17.1 = 54°$$
$$I_d = I_a \sin\psi = 1 \times \sin 54° = 0.809$$
$$\therefore \quad E_f = E'' + (X_d - X_q)\, I_d$$
$$= 1.36 + (0.8 - 0.5) \times 0.805 = 1.60 \text{ pu.}$$

EXAMPLE 3.21. *A 400 V, 50 Hz, delta-connected alternator has a direct-axis reactance of 0.1 Ω and a quadrature-axis reactance of 0.07 Ω per phase. The armature resistance is negligible. The alternator is supplying 1000 A at 0.8 lagging pf.*

(a) *Find the excitation emf neglecting saliency and assuming $X_s = X_d$*

(b) *Find the excitation emf taking into account the saliency.*

SOLUTION. $X_d = 0.1\ \Omega$, $X_q = 0.07\ \Omega$

Line voltage $V_L = 400$ V

For delta-connected alternator, phase voltage = line voltage

$\therefore \quad V_p = 400$ V

For delta connection phase current $= \dfrac{1}{\sqrt{3}} \times$ line current

$$I_a = \frac{1}{\sqrt{3}} \times 1000 = 577.4 \text{ A}$$

Taking $\mathbf{V}_p$ as reference phasor.

$$\therefore \quad \mathbf{V}_p = V_p \angle 0° = 400 \angle 0° \text{ V} = 400 + j\,0$$
$$\mathbf{I}_a = 577.4 \angle -\cos^{-1} 0.8$$
$$= 577.4 \angle -36.9° \text{ A}$$

(*a*) *Saliency neglected*

$$\begin{aligned}
\mathbf{E}_{fp} &= \mathbf{V}_p + \mathbf{I}_a \mathbf{Z}_s \\
&= \mathbf{V}_p + \mathbf{I}_a\, j\, X_s = V_p + j\, \mathbf{I}_a X_d \\
&= 400 + (1\,\angle 90^\circ)\,(577.4\,\angle -36.9^\circ) \times 0.1 \\
&= 400 + 57.74\,\angle 90^\circ - 36.9^\circ \\
&= 400 + 57.74\,\angle 53.1^\circ \\
&= 400 + 34.7 + j\,46.2 \\
&= 437.15\,\angle 6.07^\circ \text{ V}
\end{aligned}$$

$$E_{fL} = E_{fp} = 437.15 \text{ V}$$

(*b*) *Saliency taken into account*

Since ϕ is taken positive for lagging pf, $\phi = +\cos^{-1} 0.8 = 36.9^\circ$

From Eq. (3.39.16)

$$\tan\psi = \frac{V_p \sin\phi + X_q I_a}{V_p \cos\phi + R_a I_a}$$

If the armature resistance is neglected, $R_a = 0$.

$$\therefore \qquad \tan\psi = \frac{V_p \sin\phi + X_q I_a}{V_p \cos\phi}$$

$$= \frac{400 \times 0.6 + 0.07 \times 577.4}{400 \times 0.8} = 0.8763$$

$$\begin{aligned}
\psi &= 41.2^\circ \\
\delta &= \psi - \phi = 41.2^\circ - 36.9^\circ = 4.3^\circ \\
I_d &= I_a \sin\psi = 577.4 \sin 41.2^\circ = 380.3 \text{ A} \\
E_{fp} &= V_p \cos\delta + X_d I_d \\
&= 400 \cos 4.3^\circ + 0.1 \times 380.3 = 436.9 \text{ V} \\
E_{fL} &= E_{fp} = 436.9 \text{ V}
\end{aligned}$$

EXAMPLE 3.22. *A 600 kVA, 6600 V, 3-phase, 50-Hz, star connected salient-pole alternator has a resistance of 1.75% and leakage reactance of 10%. The armature reaction for full load on short circuit is equivalent to 40 A field current. The armature cross reaction per armature ampere turn is 50% of the direct reaction. The open-circuit characteristic of the machine if given by :*

Field current	A	21	33	50	66	93
Terminal volts	V	3000	4750	6400	7250	8000

Determine the percentage regulation of the machine on full load and 0.8 power factor lagging.

SOLUTION. The open-circuit characteristic is drawn as shown in Fig. 3.39.

$$\therefore \quad I_q = \frac{V \sin \delta}{X_q} \tag{3.40.3}$$

$$X_d I_d = AC = MD = OD - OM = E_f - V \cos \delta$$

$$\therefore \quad I_d = \frac{E_f - V \cos \delta}{X_d} \tag{3.40.4}$$

Substituting the values of I_q and I_d in Eq. (3.40.2) we get

$$S_{1\phi} = (V \cos \delta - j\, V \sin \delta)\left(\frac{V \sin \delta}{X_q} + j\,\frac{E_f - V \cos \delta}{X_d}\right)$$

$$= \left(\frac{V^2}{X_q} \sin \delta \cos \delta + \frac{V E_f}{X_d} \sin \delta - \frac{V^2}{X_d} \sin \delta \cos \delta\right)$$

$$+ j\left(\frac{V E_f}{X_d} \cos \delta - \frac{V^2}{X_d} \cos^2 \delta - \frac{V^2}{X_q} \sin^2 \delta\right)$$

$$= \left[\frac{V E_f}{X_d} \sin \delta + \frac{V^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right) \sin 2\delta\right]$$

$$+ j\left[\frac{V E_f}{X_d} \cos \delta - \frac{V^2}{2\, X_d}(1 + \cos 2\delta) - \frac{V^2}{2\, X_q}(1 - \cos 2\delta)\right]$$

$$= \left[\frac{V E_f}{X_d} \sin \delta + \frac{V^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right) \sin 2\delta\right]$$

$$+ j\left[\frac{V E_f}{X_d} \cos \delta - \frac{V^2}{2\, X_d X_q}\{(X_d + X_q) - (X_d - X_q) \cos 2\delta\}\right] \tag{3.40.5}$$

Also, $\quad S_{1\phi} = P_{1\phi} + j\, Q_{1\phi}$ (3.40.6)

Therefore the real power per phase in watts is

$$P_{1\phi} = \frac{V E_f}{X_d} \sin \delta + \frac{V^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right) \sin 2\delta$$

Total real power in watts

$$\boxed{P_{3\phi} = 3\, P_{1\phi} = \frac{3\, V E_f}{X_d} \sin \delta + \frac{3\, V^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right) \sin 2\delta} \tag{3.40.7}$$

The reactive power per phase in vars is

$$Q_{1\phi} = \frac{V E_f}{X_d} \cos \delta - \frac{V^2}{2\, X_d X_q}\left[(X_d + X_q) - (X_d - X_q) \cos 2\delta\right]$$

Total reactive power in vars

$$\boxed{Q_{3\phi} = 3\, Q_{1\phi} = \frac{3\, V E_f}{X_d} \cos \delta - \frac{3\, V^2}{2\, X_d X_q}\left[(X_d + X_q) - (X_d - X_q) \cos 2\delta\right]} \tag{3.40.8}$$

The first term of Eq. (3.40.7) is the same as that obtained for the cylindrical-rotor machine. The second term of Eq. (3.40.7) depends on the ***saliency*** defined by the quantity $\left[\frac{1}{X_q} - \frac{1}{X_d}\right]$. The saliency disappears when $X_d = X_q$ (that is, for a cylindrical rotor). Also, this term exists even when there is no field current ($E_f = 0$).

Equations (3.40.7) and (3.40.8) are applicable to both salient-pole synchronous generator and synchronous motor. The torque angle δ is positive for the generator and negative for the motor. P-Q versus δ curves for a salient-pole machine are shown in Fig. 3.42.

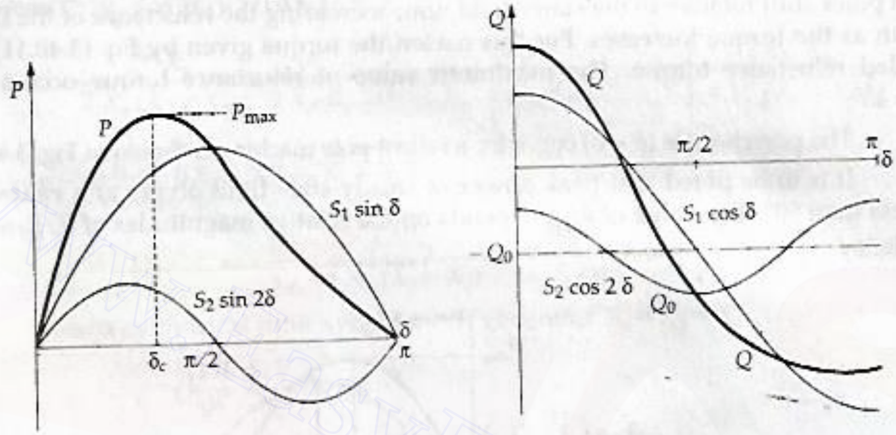


Fig. 3.42 *P–Q versus δ curves for a salient-pole machine.*

The electromagnetic torque or torque developed for a 3-phase synchronous machine is given by

$$\tau_{em} = \frac{3\,P_{1\phi}}{\omega_m} = \frac{3}{2\,\pi\,n_s}\left(\frac{V\,E_f}{X_d}\sin\delta + \frac{X_d - X_q}{2\,X_d\,X_q}\sin 2\delta\right) \tag{3.40.9}$$

It is to be noted that the resulting torque has two components. The first term in Eq. (3.40.9) represents the torque τ_{exc} due to field excitation. Thus,

$$\tau_{exc} = \frac{3V\,E_f}{2\,\pi\,n_s\,X_d}\sin\delta \tag{3.40.10}$$

The second term in Eq. (3.40.9) is known as **reluctance torque,** τ_{rel}.

$$\therefore \quad \boxed{\tau_{rel} = \frac{3}{2\,\pi\,n_s}\left(\frac{X_d - X_q}{2\,X_d\,X_q}\right)\sin 2\delta} \tag{3.40.11}$$

The reluctance torque is independent of excitation and exists only if the machine is connected to a system receiving reactive power from other synchronous machines operating in parallel with the terminal voltage V.

The reluctance torque is due to the saliency of the field poles which tend to align the direct axis with that of the armature mmf.

It is to be noted that if there is no field excitation $E_f = 0$, the first term $\frac{V E_f}{X_d} \sin \delta$ in Eq. (3.40.7) becomes zero, and the machine still has some P generation capability. However, it is impractical to operate a synchronous generator without field excitation on a power system, because it would supply only about 25% or less of its real power rating. Also, it would absorb an excessive amount of reactive power.

If an attempt is made to cause the machine to act as a generator or motor with no field current (V supplied by the bus to which the machine is connected), the poles shift relative to the stator field, thus increasing the reluctance of the flux path as the torque increases. For this reason the torque given by Eq. (3.40.11) is called reluctance torque. The maximum value of reluctance torque occurs at $\delta = 45°$.

The power-angle ($P - \delta$) curve for a salient-pole machine is shown in Fig. 3.43.

It is to be noted that peak power or steady-state limit occurs at a value of δ less than 90°. The value of δ_{max} depends on the relative magnitudes of V, E_f and saliency.

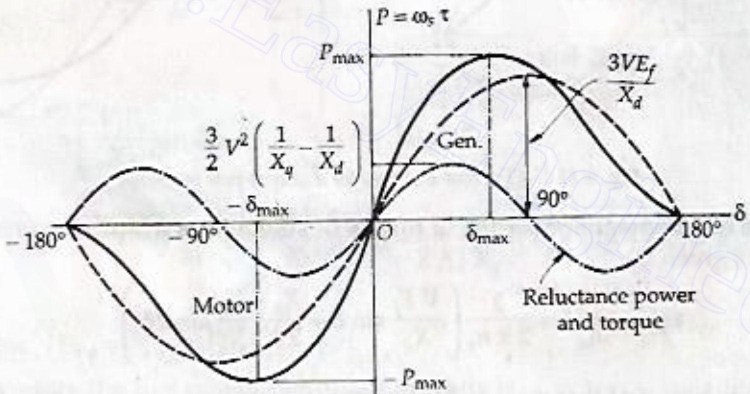


Fig. 3.43 *Power-angle curve of a salient-pole machine.*

For a non-salient pole (cylindrical rotor) machine

$$P_{3\phi} = \frac{3 V E_f}{X_d} \sin \delta \tag{3.40.12}$$

3.41 MAXIMUM REACTIVE POWER FOR A SYNCHRONOUS GENERATOR

For a salient-pole synchronous generator

$$Q_{1\phi} = \frac{V E_f}{X_d} \cos \delta - \frac{V^2}{2 X_d X_q}\left[(X_d + X_q) - (X_d - X_q) \cos 2\delta \right] \tag{3.41.1}$$

For maximum reactive power, $\frac{d Q_{1\phi}}{d \delta} = 0$

$$-\frac{V E_f}{X_d}\sin\delta - \frac{2V^2}{2X_d X_q}(X_d - X_q)\sin 2\delta = 0$$

$$E_f \sin\delta + \frac{V}{X_q}(X_d - X_q)(2\sin\delta\cos\delta) = 0$$

$$\therefore \qquad \cos\delta = -\frac{E_f X_q}{2V(X_d - X_q)} \qquad (3.41.2)$$

Substituting the value of $\cos\delta$ from Eq. (3.41.2) in Eq. (3.41.1) we get

$$Q_{1\phi\,\max} = \frac{V E_f}{X_d}\left[\frac{-E_f X_q}{2V(X_d - X_q)}\right] - \frac{V^2}{2X_d X_q}(X_d + X_q) + \frac{V^2}{2X_d X_q}(X_d - X_q)(2\cos^2\delta - 1)$$

$$= -\frac{E_f^2 X_q}{2X_d(X_d - X_q)} - \frac{V^2}{2X_d X_q}(X_d + X_q) + \frac{V^2}{2X_d X_q}(X_d - X_q)\left[\frac{2E_f^2 X_q^2}{4V^2(X_d - X_q)^2} - 1\right]$$

$$= -\frac{V^2}{2X_d X_q}[(X_d + X_q) - (X_d - X_q)] - \frac{E_f^2 X_q}{2X_d(X_d - X_q)} + \frac{E_f^2 X_q}{4X_d(X_d - X_q)}$$

$$\text{or} \qquad Q_{1\phi\,\max} = -\frac{V^2}{X_d} - \frac{E_f^2 X_q}{4X_d(X_d - X_q)} \qquad (3.41.3)$$

For a cylindrical rotor synchronous generator $X_d = X_q = X_s$

$$\therefore \qquad Q_{1\phi} = \frac{E_f V}{X_s}\cos\delta - \frac{V^2}{X_s}$$

$$\text{or} \qquad Q_{1\phi} = \frac{V}{X_s}(E_f\cos\delta - V) \qquad (3.41.4)$$

Equation (3.41.4) shows that when $E_f \cos\delta = V$, that is, under normal excitation, $Q = 0$, and the generator operates at unity power factor.

When $E_f \cos\delta > V$, that is, the generator is overexcited, Q is positive. Therefore the generator supplies reactive power to the busbars. When $E_f \cos\delta < V$, that is, the alternator is underexcited, Q is negative. Hence, the generator absorbs or consumes reactive power.

In general, an overexcited generator or motor supplies reactive power to the busbars, and an underexcited generator or motor consumes or absorbs reactive power from the busbars.

3.42 DETERMINATION OF X_d AND X_q

The direct and quadrature-axis synchronous reactances of a salient pole synchronous machine can be determined from a simple no-load test known as the **slip test**. In this test, a small voltage at rated frequency, and not more than about 25% of the rated value is applied to the 3-phase stator winding. The field winding is unexcited and left open circuited (Fig. 3.44).

The rotor is driven by an auxiliary motor (preferably a dc motor) at a speed slightly less or slightly more than synchronous speed. The direction of rotation should be the *same* as that of the rotating field produced by the stator. A small voltage reading indicated by the voltmeter across the open field winding terminals shows that the direction of rotation of rotor is proper. Since the rotor is running

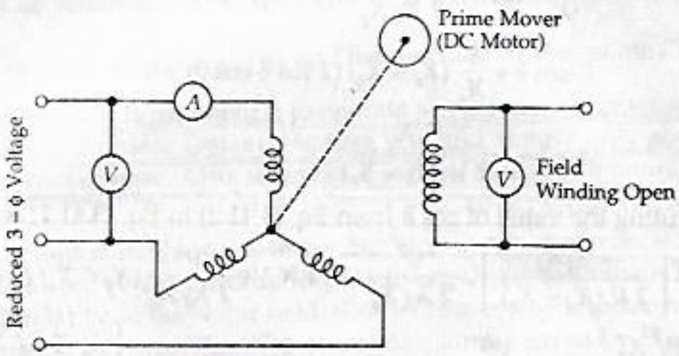


Fig. 3.44. *Slip test connection diagram to determine X_d and X_q*

at a speed n_r close to synchronous speed n_s, there will be a small slip between the rotating magnetic field produced by the armature and the actual salient field poles. The relative speed between the armature mmf and the field poles is equal to the slip speed $(n_s - n_r)$. Since the stator mmf moves slowly past the actual field poles, there will be an instant when the peak of the armature mmf wave is in line with the axis of the actual salient field poles as shown in Fig. 3.45*a*.

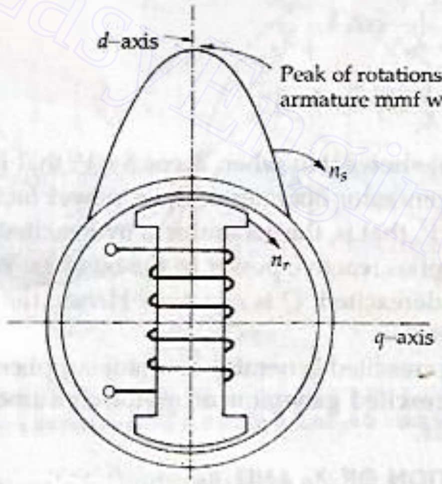


Fig. 3.45. (a)

The axis of the field poles is the direct axis (or *d*-axis). In this position, the reluctance offered by the small air gap is minimum. This results in minimum magnetizing current I_{min} as indicated by the line ammeter *A* in Fig. 3.44. It is to be noted that in this position, the armature flux linkage with the field winding is maximum, and the rate of change of this flux linkage is zero. Therefore, the induced voltage across the field winding is zero. The *d*-axis can, therefore, be located on the oscillogram of Fig. 3.46. From this figure $X_d = (ab/cd)$. Also, the ratio of armature terminal voltage per phase to the corresponding armature current per phase gives X_d.

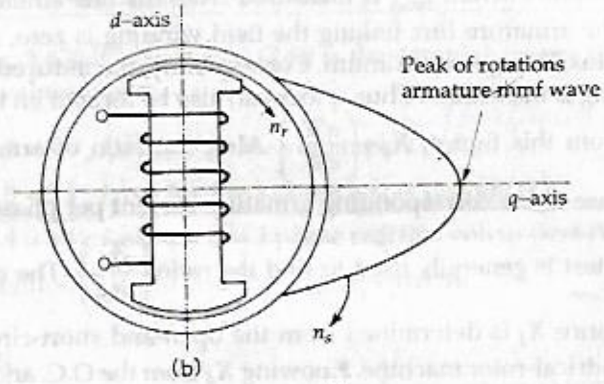


Fig. 3.45 (b).

After one-quarter of slip cycle the peak armature wave is in line with the q axis. In this position, the reluctance offered by long airgap is maximum as shown in Fig. 3.45b. A large magnetizing current is needed to establish the same airgap

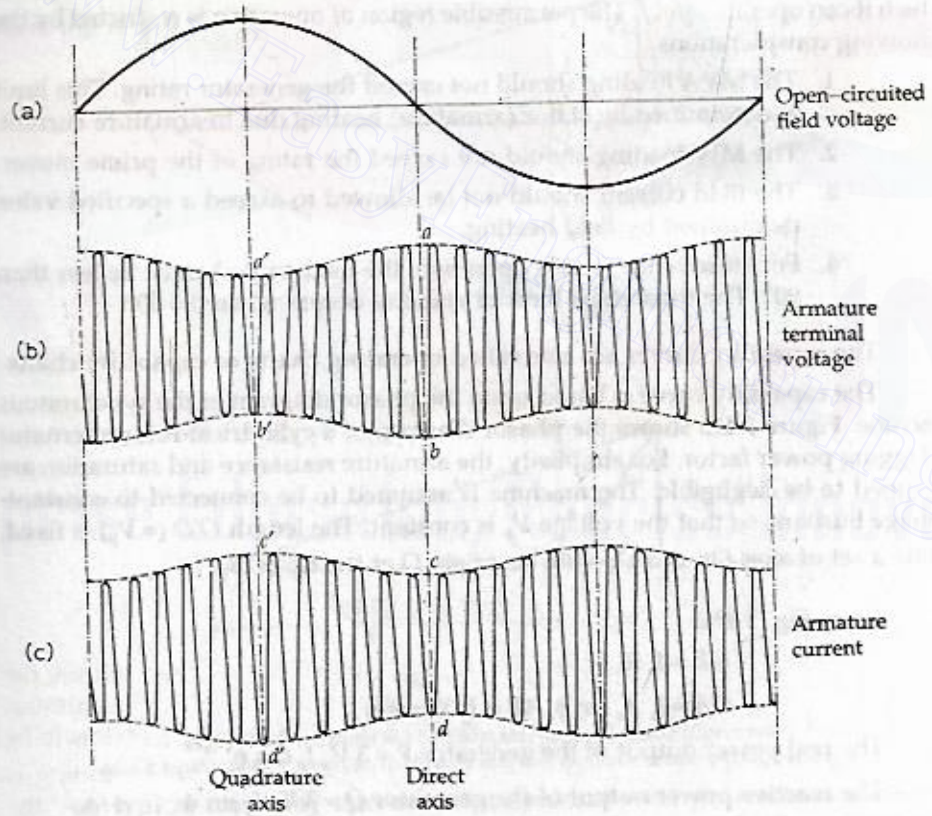


Fig. 3.46. *Typical oscillograms from slip test. (a) Voltage induced in the open-circuited field (b) Applied armature voltage waveform (c) Armature current waveform*

flux. This maximum current I_{max} is measured from the line ammeter A. Also, in this position, the armature flux linking the field winding is zero, and the rate of change of this flux linkage is maximum. Consequently, the induced voltage across the field winding is maximum. Thus, q axis can also be located on the oscillogram of Fig. 3.46. From this figure, $X_q = \left(\frac{a'b'}{c'd'}\right)$. Also, the ratio of armature terminal voltage per phase to the corresponding armature current per phase gives X_q.

The slip test is generally used to find the ratio $\left(\frac{X_d}{X_q}\right)$. The direct-axis synchronous reactance X_d is determined from the open-and short-circuit tests as in the case of cylindrical-rotor machine. Knowing X_d from the O.C. and S.C. tests and the ratio $\left(\frac{X_d}{X_q}\right)$ from the slip test we can calculate X_q.

3.43 SYNCHRONOUS GENERATOR CAPABILITY CURVES

The **capability curve** of a synchronous generator defines the bounds within which it can operate safely. The permissible region of operation is restricted by the following considerations :

1. The MVA loading should not exceed the generator rating. This limit is determined by stator (armature) heating due to armature current.
2. The MW loading should not exceed the rating of the prime mover.
3. The field current should not be allowed to exceed a specified value determined by field heating.
4. For steady-state stable operation, the load angle δ must be less than 90°. The theoretical limit of stability occurs when $\delta = 90°$.

The capability curves are also called **operating charts** or **capability charts.**

The capability curve is based upon the phasor diagram of the synchronous machine. Figure 3.47*a* shows the phasor diagram of a cylindrical-rotor alternator at lagging power factor. For simplicity, the armature resistance and saturation are assumed to be negligible. The machine is assumed to be connected to constant-voltage busbars, so that the voltage V_p is constant. The length $O'O$ ($= V_p$) is fixed. Draw a set of axes Ox and Oy with its origin O at the tip of $\mathbf{V}_p$.

From Fig. 3.47*a*

$$OB = I_a X_s$$

$$OA = I_a X_s \sin\phi, \; AB = I_a X_s \cos\phi$$

The real power output of the generator $P = 3\,V_p I_a \cos\phi$

The reactive power output of the generator $Q = 3\,V_p I_a \sin\phi$

Fig. 3.47*b* shows the phasor diagram with each phasor multiplied by the quantity $\frac{3\,V_p}{X_s}$.

From Fig. 3.47*b* it is seen that *OAB* is the complex power triangle in terms of three phase values where

$$OB = 3\,V_p\,I_a = S = 3 \text{ phase voltampers (VA)}$$
$$AB = 3\,V_p\,I_a \cos\phi = P = 3 \text{ phase active power (W)}$$
$$OA = 3\,V_p\,I_a \sin\phi = Q = 3 \text{ phase reactive voltampers (VAr)}$$

By convention *Q* is positive for lagging current.

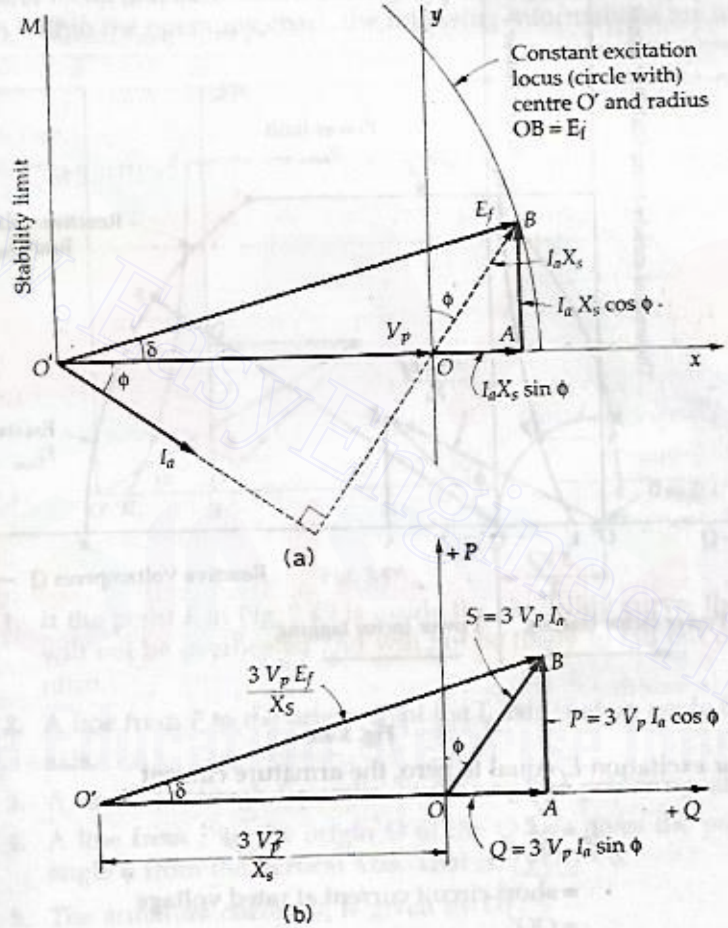


Fig. 3.47. *Derivation fo a synchronous generator capability curve. (a) Generator phasor diagram. (b) Phasor diagram to form a complex power triangle.*

A typical capability curve for a cylindrical-rotor generator is shown in Fig. 3.48. It is plotted on the *S* plane, where *P* is vertical axis and *Q* is the horizontal axis.

For constant current I_a and voltampers $S = $ VA, the locus is a circle with centre at *O* and radius $OB\,(= 3\,V_p\,I_a)$. Constant *P* operation lies on a line parallel to *Q* axis.

The constant excitation locus is a circle with centre O' and radius $O'B\left(=\dfrac{3\,V_p\,E_f}{X_s}\right)$.

Constant power factor lines are radial straight lines from O.

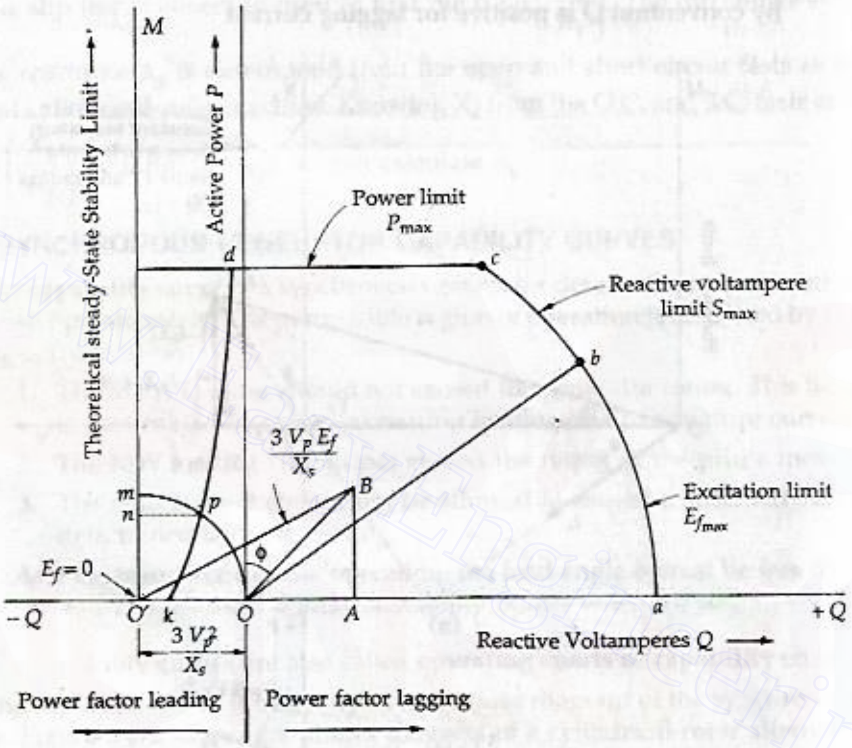


Fig. 3.48.

For excitation E_f equal to zero, the armature current

$$I_a = \frac{V_p}{X_s}$$

$$= \text{short circuit current at rated voltage}$$

$$= OO'$$

The theoretical stability limit is a straight line $O'M$ at right angles to $O'O$ at O'. Here $\delta = 90°$.

Between a and b, the operation of the alternator is limited by the maximum field current, which in Fig. 3.48 is a circle of radius $\left(\dfrac{3\,V\,E_f}{X_s}\right)$ with centre O'.

Between b and c, the operation is limited by the MVA limit, which on this diagram is a circle of radius $3\,V_p\,I_a$ with centre O. Here I_a is the maximum permissible armature current.

Between c and d, the operation is limited by the power of the prime mover.

Between d and e, the operation is limited by the practical stability limit.

The theoretical limit of stability occurs where $\delta = 90°$. But there must be a safety margin between the theoretical limit and that used in practice. The practical limit is taken usually 10% less than the theoretical stability limit.

The complete operating zone of the alternator is *abcdkOa*. Operation of the alternator within this area is safe from the stand points of heating and stability. Once an operating point is located within this area, the desired power P, S, Q, current, power factor and excitation are found. For example, for an operating point F (Fig. 3.49) within the operating chart, the following informations are available :

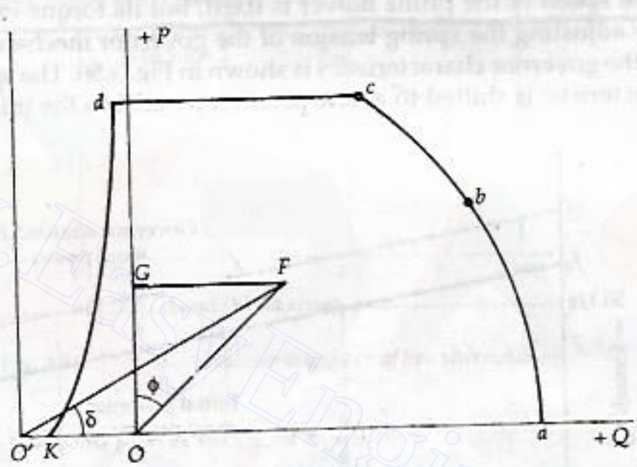


Fig. 3.49.

1. If the point F in Fig. 3.49 is inside the capability curve, the machine will not be overheated and will not be likely to fall out of synchronism.
2. A line from F to the origin O' of the I_f axis is at an angle δ from that axis.
3. A line FG through F parallel to $O'O\ a$ gives power equal to OG.
4. A line from F to the origin O of the Q axis gives the power factor angle ϕ from the vertical axis. That is, $\angle FOG = \phi$.
5. The armature current I_a is given by OF.
6. The VA output is given by ($OF \times$ operating voltage).
7. The VAr output is given by $GF \times$ output voltage.
8. $O'F$ gives the excitation E_f.

3.44 PRIME-MOVER CHARACTERISTICS

In general, for alternators to operate successfully in parallel, the load-speed characteristics of the prime movers should be *drooping*, that is, the speed of the prime mover should decrease slightly with increasing loads. The *speed droop*, also called ***governor droop***, or ***inherent speed regulation***, is usually expressed as a percentage of the full-load speed.

$$\text{Speed droop} = \frac{N_{nl} - N_{fl}}{N_{fl}} \times 100\%$$

where N_{nl} = no-load speed

N_{fl} = full-load speed

The percentage of droop normally varies from 2 to 4 per cent from no-load to full load. Usually the speed-load characteristics are linear.

The amount of power generated by a machine is determined by its prime mover. The speed of the prime mover is fixed, but its torque can be varied. This is done by adjusting the spring tension of the governor mechanism. The effect of changing the governor characteristics is shown in Fig. 3.50. The speed (frequency)-load characteristic is shifted to a new position parallel to the initial position.

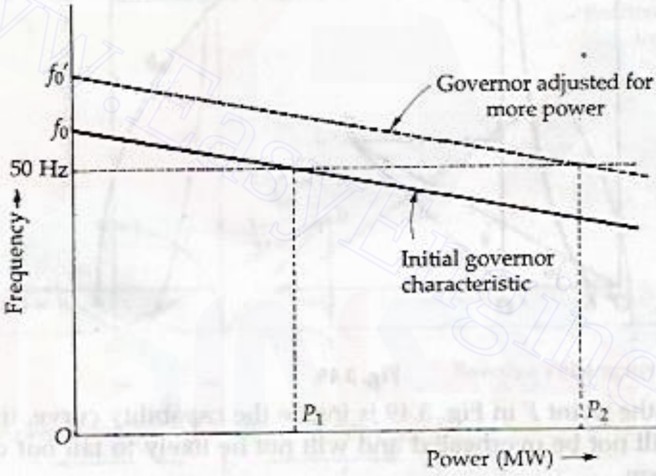


Fig. 3.50. *Shift of speed (frequency) load characteristic.*

When two alternators are operating in parallel, an increase in governor set points on one of them,

1. Increases the system frequency, and
2. Increases the power supplied by that alternator and reduces the power supplied by the other alternator .

When two alternators are operating in parallel and the field current of the second alternator is increased,

(*i*) the system terminal voltage is increased, and

(*ii*) the reactive power Q supplied by that alternator is increased, while the reactive power supplied by the other alternator is decreased.

3.45 EXPRESSIONS FOR POWERS SHARED BY TWO ALTERNATORS

Consider two alternators running in parallel. The frequency-load characteristic of the two machines is shown in Fig. 3.51.

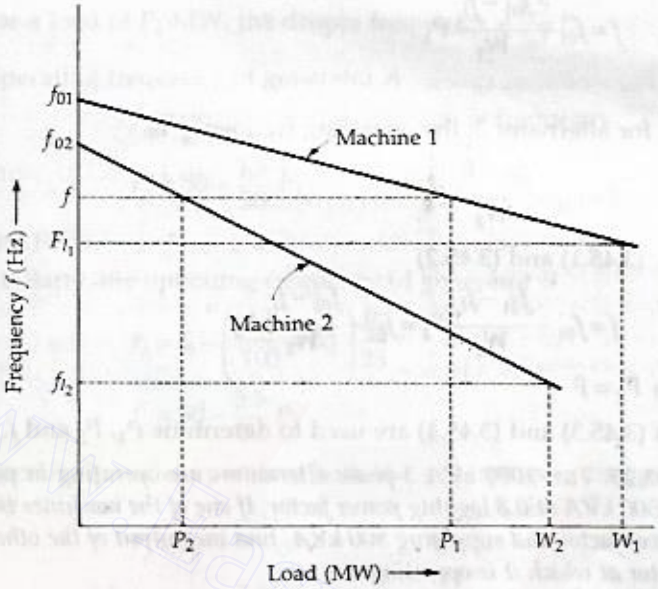


Fig. 3.51. *Frequency-load characteristics of two alternators.*

Let

W_1 = full load power rating of machine 1

W_2 = full load power rating of machine 2

P_1 = power shared by machine 1

P_2 = power shared by machine 2

P = total power supplied by two machines

f_{01} = no load frequency of machine 1

f_{02} = no load frequency of machine 2

f_{l_1} = full load frequency of machine 1

f_{l_2} = full load frequency of machine 2

f = common operating frequency when the two machines are running in parallel

Machine 1

Drop in frequency from no load to full load $= f_{01} - f_{l_1}$

Drop in frequency per unit rating $= \dfrac{f_{01} - f_{l_1}}{W_1}$

Drop in frequency for a load of $P_1 = \dfrac{f_{01} - f_{l_1}}{W_1} P_1$

Operating frequency of machine 1 = (no-load frequency)–(drop in frequency)

$$f = f_{01} - \frac{f_{01} - f_{l_1}}{W_1} P_1 \tag{3.45.1}$$

Machine 2

Similarly, for alternator 2, the operating frequency is

$$f = f_{02} - \frac{f_{02} - f_{l_2}}{W_2} P_2 \tag{3.45.2}$$

From Eqs. (3.45.1) and (3.45.2)

$$f = f_{01} - \frac{f_{01} - f_{l_1}}{W_1} P_1 = f_{02} - \frac{f_{02} - f_{l_2}}{W_2} P_2 \tag{3.45.3}$$

Also, $P_1 + P_2 = P$ (3.45.4)

Equations (3.45.3) and (3.45.4) are used to determine P_1, P_2 and f.

EXAMPLE 3.23. *Two 1000 kVA 3-phase alternators are operating in parallel and supply a load of 1500 kVA at 0.8 lagging power factor. If one of the machines is operating at 0.4 lagging power factor and supplying 800 kVA, find the output of the other machine and the power factor at which it is operating.*

SOLUTION. $\mathbf{S}_{Load} = 1500 \underline{/ -\cos^{-1} 0.8} = 1200 - j\,900$

$$\mathbf{S}_A = 800 \underline{/ -\cos^{-1} 0.9} = 800 \underline{/ -25.84°} = 720 - j\,348.7$$

$$\mathbf{S}_A + \mathbf{S}_B = \mathbf{S}_{Load}$$

$$\mathbf{S}_B = \mathbf{S}_{Load} - \mathbf{S}_A$$

$$= (1200 - j\,900) - (720 - j\,348.7)$$

$$= 480 - j\,551.3 = 730.98 \underline{/ -48.95°} \text{ kVA}$$

$$\cos\phi_B = \cos(-48.95°) = 0.6566 \text{ (lagging)}$$

EXAMPLE 3.24. *Two station generators A and B operate in parallel. Station capacity of A is 50 MW and that of B is 25 MW. Full-load speed regulation of station A is 3% and full-load speed regulation of B is 3.5%. Calculate the load sharing if the connected load is 50 MW. No-load frequency is 50 Hz.*

SOLUTION. Let

P_1 = load taken by generator A in MW

P_2 = load taken by generator B in MW

$$\therefore \quad P_1 + P_2 = 50 \tag{E3.24.1}$$

Original frequency at no load $f_0 = 50$ Hz

Generator A

For a load of 50 MW, the drop in frequency = 3% of f_0

$$= \frac{3}{100} \times 50 = 1.5$$

For a load of 1 MW, the dropin frequency $= \frac{1.5}{50}$

For a load of P_1 MW, the dropin frequency $= \frac{1.5}{50} P_1$

Operating frequency of generator A

$$f_A = \text{original frequency drop in frequency}$$

$$f_A = 50 - \frac{1.5}{50} P_1$$

Generator B

Similarly, the operating frequency of generator B

$$f_B = f_0 - \left(\frac{3.5}{100} \times 50\right)\frac{P_2}{25}$$

$$f_B = 50 - \frac{3.5}{50} P_2$$

Since for parallel operation both the generators must operate at the same frequency

$$f_A = f_B$$

$$50 - \frac{1.5}{50} P_1 = 50 - \frac{3.5}{50} P_2$$

or $$3 P_1 = 7 P_2 \qquad \text{(E3.24.2)}$$

From Eqs. (E3.24.1) and (E3.24.2)

$$\frac{7}{3} P_2 + P_2 = 50$$

$\therefore$ $$P_2 = 15 \text{ MW}$$

and $$P_1 = \frac{7}{3} \times 15 = 35 \text{ MW}$$

EXAMPLE 3.25. *Two three-phase alternators operate in parallel. The rating of one machine is 50 MW and that of the other is 100 MW. Both alternators are fitted with governors having a droop of 4 per cent. How will the machines share a common load of 100 MW?*

SOLUTION. Let the original frequency be f_0.

Droping frequency = 4% of $f_0 = 0.04 f_0$

Let P_1 = load taken by machine 1

P_2 = load taken by machine 2

Machine 1

For a load of 50 MW the drop in frequency $= 0.04 f_0$

For a load of 1 MW the drop in frequency $= \frac{0.04}{50} f_0$

For a load of P_1 MW the drop in frequency $= \frac{0.04}{50} f_0 \times P_1$

Operating frequency of first machine $= f_0 - \frac{0.04}{50} P_1 f_0$

Machine 2

Similarly, the operating frequency of machine $2 = f_0 - \frac{0.04}{100} f_0 P_2$

Since the two alternators are running in parallel, they must operate at the same frequency at steady state

$$\therefore \quad f_0 - \frac{0.04}{50} f_0 P_1 = f_0 - \frac{0.04}{100} f_0 P_2$$

or
$$P_1 = \frac{1}{2} P_2 \qquad \text{(E3.25.1)}$$

The total load to be shared by the two machines is 100 MW.

$$\therefore \quad P_1 + P_2 = 100 \qquad \text{(E3.25.2)}$$

Solving Eqs. (E3.25.1) and (E3.25.2) we get

$$\frac{1}{2} P_2 + P_2 = 100$$

$$\therefore \quad P_2 = \frac{2}{3} \times 100 = 66.66 \text{ MW}$$

$$P_1 = \frac{1}{3} \times 100 = 33.33 \text{ MW}$$

EXAMPLE 3.26. *Two three-phase alternators operate in parallel. The rating of one machine is 200 MW and that of the other is 400 MW. The droop characteristics of their governors are 4% and 5% respectively from no load to full load. Assuming that the governors are operating at 50 Hz at no load, how would a load of 600 MW be shared between them ? What will be the system frequency at this load ? Repeat the problem if both governors have a droop of 4%.*

SOLUTION.

Machine 1

Drop in frequency from no load to full load = 4% of $50 = \frac{4}{100} \times 50 = 2$ Hz

For a load of 200 MW, the drop in frequency = 2 Hz

For a load of P_1 MW, the drop in frequency $= \frac{2}{200} P_1$

Operating frequency of machine $1 = 50 - \frac{2}{200} P_1$

Machine 2

Drop in frequency from no load to full of 400 MW

$$= 5\% \text{ of } 50 = \frac{5}{100} \times 50 = 2.5 \text{ Hz}$$

For a load of P_2 MW, the drop in frequency $= \frac{2.5}{400} P_2$

Operating frequency of machine 2 $= 50 - \frac{2.5}{400} P_2$

Since the two alternators are operating in parallel, they must have the same frequency

$$\therefore \quad 50 - \frac{2}{200} P_1 = 50 - \frac{2.5}{400} P_2$$

or

$$P_1 = \frac{2.5}{4} P_2 \tag{E3.26.1}$$

Since the total load shared by the two machines is 600 MW,

$$P_1 + P_2 = 600 \tag{E3.26.2}$$

From Eqs. (E3.26.1) and (E3.26.2)

$$\frac{2.5}{4} P_2 + P_2 = 600$$

$$\therefore \quad P_2 = 369.23 \text{ MW}$$

and

$$P_1 = \frac{2.5}{4} P_2 = \frac{2.5}{4} \times 369.23$$

$$= 230.77 \text{ MW}$$

Operating frequency of the system

$$= 50 - \frac{2}{200} P_1$$

$$= 50 - \frac{2}{200} \times 230.77 = 47.69 \text{ Hz}$$

It is seen that due to difference in droop characteristics of governors machine 1 is overloaded and machine 2 is underloaded.

If both governors have a droop of 4%, machine 1 will take a load of 200 MW and machine 2 will take 400 MW. That is, the machines are loaded according to their ratings.

EXAMPLE 3.27. *Two identical 2000 kVA alternators operate in parallel. The governor of first machine is such that the frequency drops uniformly from 50 Hz on no load to 48 Hz on full load. The corresponding uniform speed drop of the second machine is 50 Hz to 47.5 Hz. (a) How will the two machines share a load of 3000 kW ? (b) What is the maximum load at unity power factor that can be delivered without overloading either machine ?*

SOLUTION. (*a*) With unity power factor, full-load rating of each machine

$$= 2000 \times 1 = 2000 \text{ kW} = 2 \text{ MW}$$

Let P_1 = load taken by machine 1 (MW)

P_2 = load taken by machine 2 (MW)

Machine 1

Dropin frequency per unit rating $= \dfrac{50-48}{2}$

For a load of P_1 MW the drop in frequency $= \dfrac{50-48}{2} P_1$

Operating frequency of machine 1

= original frequency-drop in frequency

$$f_1 = 50 - \frac{50-48}{2} P_1$$

or $$f_1 = 50 - P_1 \tag{E3.27.1}$$

Machine 2

Similarly, operating frequency of machine 2

$$f_2 = 50 - \frac{50-47.5}{2} P_2 \tag{E3.27.2}$$

Since the two alternators in parallel must operate at the same frequency f

$$f = f_1 = f_2$$

$$50 - P_1 = 50 - \frac{50-47.5}{2} P_2$$

or $$P_1 = 1.25\, P_2 \tag{E3.27.3}$$

Since the two alternators share a load of 3 MW

$$P_1 + P_2 = 3 \tag{E3.27.4}$$

Solving Eqs. (E3.27.3) and (E3.27.4), we get

$$1.25\, P_2 + P_2 = 3$$

$$P_2 = \frac{3}{2.25}\ \text{MW} = 1333\ \text{kW}$$

$$P_1 = 3000 - P_2 = 3000 - 1333 = 1667\ \text{kW}$$

(*b*) For parallel operation, the minimum value of frequency at which the system can operate without overloading either machine is 48 Hz.

Since the full-load frequency of first machine is 48 Hz, the total load taken by it = 2000 kW.

Therefore, for a drop of frequency equal to (50 – 48), the total load taken by second machine $= \dfrac{2000}{2.5} \times 2 = 1600$ kW

Total load taken by the two machines without overloading either machine

$$= 2000 + 1600 = 3600\ \text{kW}$$

Example 3.28. *Two 3-phase, 50 Hz alternators operate in parallel. One is rated at 1000 kW and the other is rated 1500 kW. The first machine has a frequency-load characteristic that varies from 51 Hz at no load to 49.6 Hz at full load. For the other machine the frequency-load characteristic varies from 51.4 Hz to 49.2 Hz. How will the machines* *share a common load of 2000 kW ? If the characteristic of one machine is altered so that the machines share the load in the ratio of their ratings, what is the common frequency at the condition ?*

SOLUTION. Let P_1 = load shared by machine 1

P_2 = load shared by machine 2

$$\therefore \quad P_1 + P_2 = 2000 \tag{E3.28.1}$$

When machine 1 supplies a load of 1000 kW, the drop in frequency from no load to its full load value of 1000 kW = 51 – 49.6 = 1.4 Hz.

That is, the drop in frequency for a load of 1000 kW = 1.4 Hz.

Therefore, the drop in frequency for a load of 1 kW $= \dfrac{1.4}{1000}$ Hz

The drop in frequency for a load of P_1 kW $= \dfrac{1.4}{1000} P_1$

Actual frequency of operation when machine 1 supplies P_1 kW

= original frequency at no load – drop in frequency

$$f_1 = 51 - \frac{1.4}{1000} P_1 \tag{E3.28.2}$$

Similarly, for machine 2, actual frequency of operation when it supplies P_2 kW is

$$f_2 = 51.4 - \frac{51.4 - 49.2}{1500} P_2 \tag{E3.28.3}$$

Since the two machines in parallel run at the same frequency f,

$$f = f_1 = f_2$$

$$51 - \frac{1.4}{1000} P_1 = 51.4 - \frac{51.4 - 49.2}{1500} P_2$$

$$-\frac{1.4}{1000} P_1 + \frac{2.2}{1500} P_2 = 51.4 - 51$$

$$\frac{-4.2\,P_1 + 4.4\,P_2}{3000} = 0.4$$

or $$-4.2\,P_1 + 4.4\,P_2 = 1200 \tag{E3.28.4}$$

Also, $$P_1 + P_2 = 2000 \tag{E3.28.1}$$

Solving Eqs. (E3.28.1) and (E3.28.4) we get

$$P_1 = 883.72 \text{ kW}$$

$$P_2 = 1116.28 \text{ kW}$$

$$f = 51 - \frac{1.4 \times 883.72}{1000} = 49.762 \text{ Hz}$$

(b) $$\frac{P_1}{P_2} = \frac{1000}{1500} = \frac{2}{3}$$

or $$P_2 = 1.5\,P_1 \tag{E3.28.5}$$

Also, $$P_1 + P_2 = 2000 \tag{E3.28.1}$$

From Eqs. (E3.28.1) and (E3.28.5)

$$P_1 + 1.5\, P_1 = 2000$$

$$P_1 = \frac{2000}{2.5} = 800 \text{ kW}$$

and $$P_2 = 2000 - 800 = 1200 \text{ kW}$$

Let f be the new common frequency of operation of the two alternators.

$$\therefore \quad \frac{P_1}{W_1} = \frac{51 - f}{51 - 49.6}$$

or $$\frac{800}{1000} = \frac{51 - f}{51 - 49.6}$$

$$\therefore \quad f = 49.88 \text{ Hz}$$

Let f_{20} be the new value the no-load frequency of machine 2.

$$\therefore \quad \frac{P_2}{W_2} = \frac{f_{20} - 49.88}{51.4 - 51.2}$$

or $$\frac{1200}{1500} = \frac{f_{20} - 49.88}{2.2}$$

and $$f_{20} = 51.64 \text{ Hz}$$

Example 3.29. *The speed regulation of two 500 kW alternators A and B running in parallel are 100% to 104% and 100% to 105% from full load to no load respectively. How will the two alternators share a load of 800 kW and also find the load at which one machine ceases to supply any portion of the load ?*

Solution. The speed-load characteristics of machines A and B are shown in Fig. 3.52

Since the machines are running in parallel the speed of each machine should be the same. When the two machines are fully loaded, the operating point of machines A and B is represented by point C in Fig. 3.52 representing 100% of full-load speed. When the two machines are fully loaded,

total load on machines A and $B = 500 + 500 = 1000$ kW

When the two machines share a load of 800 kW, the speed of each machine will rise to some value x% of full-load speed. The operating point of machine A is now D and the operating point for machine B is E as shown in Fig. 3.52.

From Fig. 3.52,

load on machine $A = P_1 = OG = FD$

or load on machine $B = P_2 = OH = FE$

From $\Delta\, KFD$

$$\tan \alpha = \frac{FD}{KF} = \frac{OG}{KF} = \frac{OG}{OK - OF} = \frac{P_1}{104 - x}$$

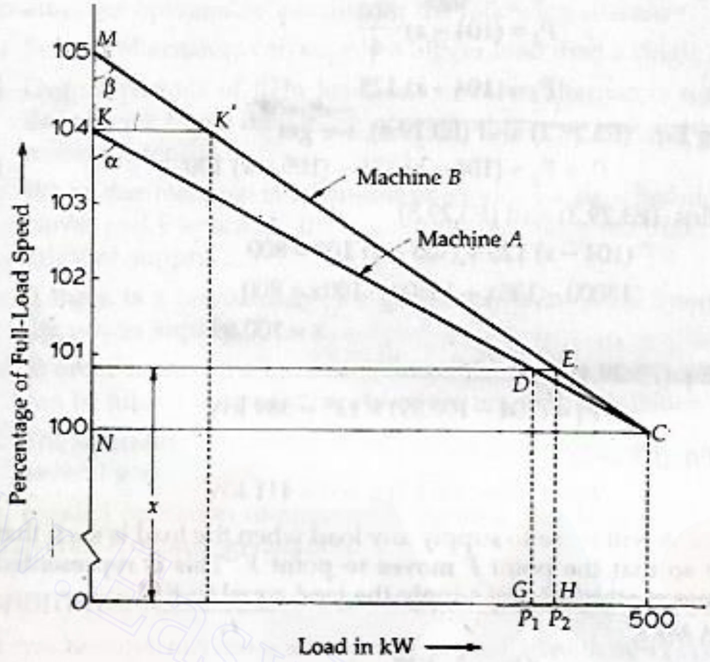


Fig. 3.52 *Speed-load characteristics of machines A and B.*

From $\Delta\, KNC$

$$\tan\alpha = \frac{NC}{KN} = \frac{500}{4}$$

Equating the two values of tan α, we get

$$\frac{P_1}{104 - x} = \frac{500}{4} \tag{E3.29.1}$$

From $\Delta\, MFE$

$$\tan\beta = \frac{FE}{MF} = \frac{OH}{OM - OF} = \frac{P_2}{105 - x}$$

From $\Delta\, MNC$

$$\tan\beta = \frac{NC}{MN} = \frac{OL}{OM - ON} = \frac{500}{105 - 100}$$

Equating the two values of tan β, we get

$$\frac{P_2}{105 - x} = \frac{500}{5}$$

$$P_2 = (105 - x)\,100 \tag{E3.29.2}$$

When the two alternators share a load of 800 kW

$$P_1 + P_2 = 800 \tag{E3.29.3}$$

From Eq. (E3.29.1)

$$P_1 = (104 - x)\frac{500}{4}$$

$$P_1 = (104 - x)\,125 \tag{E3.29.4}$$

Adding Eqs. (E3.29.2) and (E3.29.4), we get

$$P_1 + P_2 = (104 - x)\,125 + (105 - x)\,100 \tag{E3.29.5}$$

From Eqs. (E3.29.3) and (E3.29.5)

$$(104 - x)\,125 + (105 - x)\,100 = 800$$

$$13000 - 125x + 10500 - 100x = 800$$

$$x = 100.89$$

From Eq. (E3.29.4)

$$P_1 = (104 - 100.89) \times 125 = 389 \text{ kW}$$

From Eq. (E3.29.2)

$$P_2 = (105 - 100.89) \times 100 = 411 \text{ kW}$$

Machine *A* will cease to supply any load when the load is such that the line *FDE* rises up so that the point *F* moves to point *K*. This is represented by line *KK′*. In this case machine *B* will supply the load equal to *KK′*.

From $\Delta\, MKK'$

$$\tan\beta = \frac{KK'}{MK} = \frac{KK'}{105 - 104} = KK'$$

From $\Delta\, MNC$

$$\tan\beta = \frac{NC}{MN} = \frac{500}{105 - 100} = 100$$

Equating the two values of $\tan\beta$

$$KK' = 100 \text{ kW}$$

Hence when the load drops from 800 kW to 100 kW, machine *A* will cease to supply any portion of the load.

3.46 PARALLEL OPERATION OF ALTERNATORS

Electric power systems are interconnected for economy and reliable operation. Interconnection of ac power systems requires synchronous generators to operate in parallel with each other. In a generating station two or more generators are connected in parallel. In an interconnected system forming a grid the alternators are located at different places. They are connected in parallel by means of transformers and transmission lines. Under normal operating conditions all the generators and synchronous motors in an interconnected system operate in synchronism with each other.

An arrangement of generators for parallel operation enables a plant engineer to adjust the machines for optimum operating efficiency and greater reliability. As the load increases beyond the generated capacity of the connected units, additional generators are paralleled to carry the load. Similarly, as the load demand falls off, one or more of the machines are generally taken off the line to allow the units to operate at a higher efficiency.

3.47 REASONS OF PARALLEL OPERATION

Alternators are operated in parallel for the following reasons :

1. Several alternators can supply a bigger load than a single alternator.
2. During periods of light load, one or more alternators may be shut down, and those remaining operate at or near full load, and thus more efficiently.
3. When one machine is taken out of service for its scheduled maintenance and inspection, the remaining machines maintain the continuity of supply.
4. If there is a breakdown of a generator, there is no interruption of the power supply.
5. In order to meet the increasing future demand of load more machines can be added without disturbing the original installation.
6. The operating cost and cost of energy generated are reduced when several generators operate in parallel.

Thus, parallel operation of alternators ensures greater security of supply and enables overall economic generation.

3.48 CONDITIONS NECESSARY FOR PARALLELING ALTERNATORS

Most synchronous machines will operate in parallel with other synchronous machines and the process of connecting one machine in parallel with another machine or with an infinite busbar system is known as synchronizing. Those machines already carrying load are known as *running machines*, while the alternator which is to be connected in parallel with the system is known as the *incoming machine*. Before the incoming machine is to be connected to the system, the following conditions should be satisfied :

1. The phase sequence of the busbar voltages and the incoming machine voltage must be the same.
2. The busbar voltages and the incoming machine terminal voltage must be in phase.
3. The terminal voltage of the incoming machine should be equal to that of the alternator with which it is to be run in parallel or with the busbar voltage.
4. The frequency of the generated voltage of the incoming machine must be equal to the frequency of the voltage of the live busbar.

3.49 SYNCHRONIZING PROCEDURE

A stationary alternator must not be connected to live busbars because the induced e.m.f is zero at standstill and a short circuit will result.

The synchronizing procedure and the equipment for checking it are the same whether one alternator is to be connected in parallel with another alternator or an alternator is to be connected to the infinite bus.

The following methods are used for synchronization :

(1) Synchronizing lamps (2) Synchroscope.

3.50 SYNCHRONIZING LAMPS

A set of three synchronizing lamps can be used to check the conditions for paralleling the incoming machine with other machines. The dark lamp method along with a voltmeter used for synchronizing is shown in Fig. 3.53. It is used for synchronizing low-power machines.

The prime mover of the incoming machine is started and brought up to near its rated speed. The field current of the incoming machine is adjusted so that it becomes equal to the bus voltage. The three lamps flicker at a rate equal to the difference in the frequencies of the incoming machine and the bus. If the phases are properly connected, all the lamps will be bright and dark at the same time. If this is not the case, then it means that the phase sequences are not correct. In order to correct the phase sequence, two leads of the line of the incoming machine should be interchanged. The frequency of the incoming machine is adjusted until the lamps flicker at a very slow rate, usually less than one dark period per second. After finally adjusting the incoming voltage, the synchronizing switch is closed in the *middle of their dark period*. Since the voltage across the lamps varies from zero to twice the phase voltage, the lamps of suitable rating (usually two in series) must be used.

Advantages of the dark-lamp method

1. The method is cheap.
2. The proper phase sequence is easily determined.

Disadvantages of the dark-lamp method

1. Since the lamps become dark at about half their rated voltage, it is possible that the synchronizing switch might be closed when there is a considerable phase difference between the machines. This may result in high circulating current to damage the machines.
2. The lamp filaments might burn out.
3. The flicker of the lamps does not indicate which machine has the higher frequency.

3.51 THREE BRIGHT LAMP METHOD

In this method, the lamps are connected across the phases, that is, A_1 is connected to B_2, B_1 is connected to C_2 and C_1 is connected to A_2. If all the three lamps get bright and dark together, then the phase sequences are the same. The correct instant of closing the synchronizing switch in the middle of the *bright* period. The brightest point in the cycle is easier to distinguish than the middle of a dark period and avoids confusing the latter with a lamp filament failure.

3.52 TWO-BRIGHT ONE DARK LAMP METHOD

In this method one lamp is connected between corresponding phases while the two others are cross-connected between the other two phases (Fig. 3.53). That is, A_1 is connected to A_2, B_1 to C_2 and C_1 to B_2. The prime mover of the incoming machine is started and the alternator is brought up to near its rated speed.

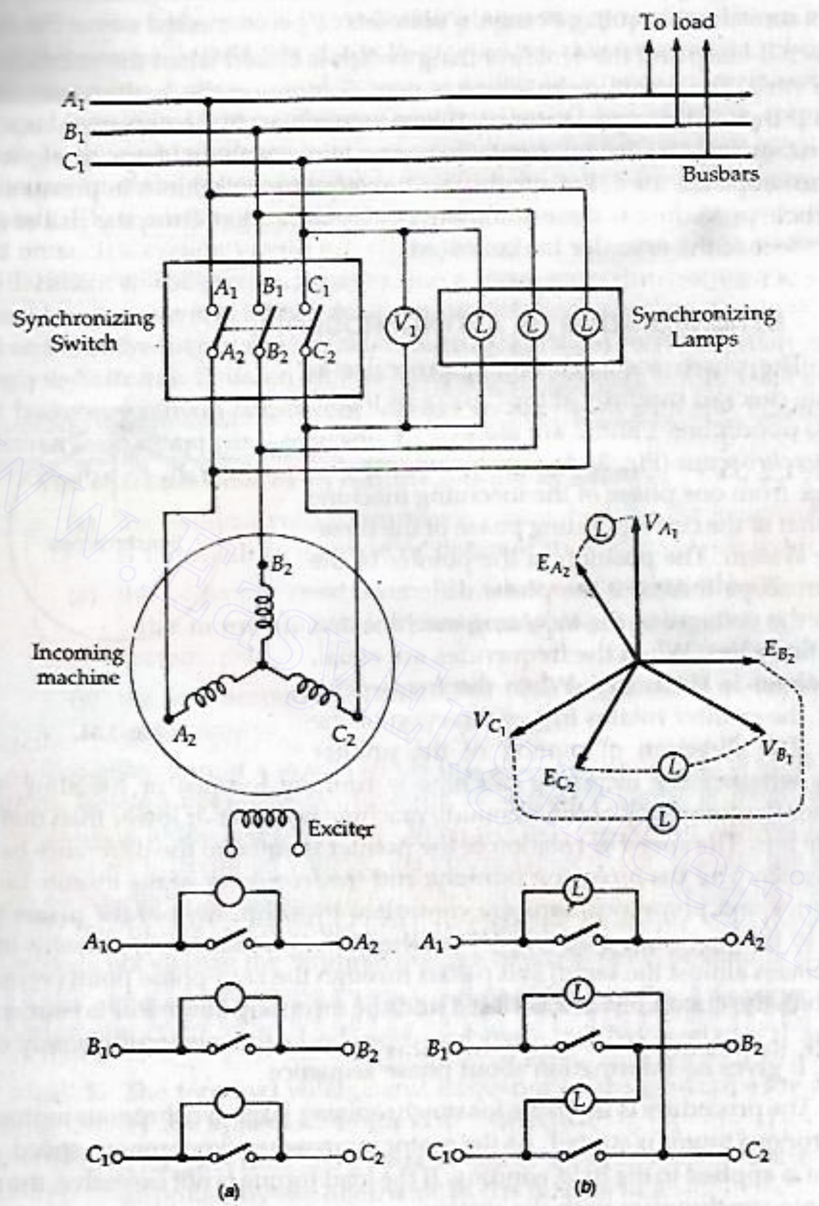


Fig. 3.53. *(a) Straight connection (b) Cross connection.*

The incoming machine excitation is now adjusted until the incoming machine induced voltages E_{A_2}, E_{B_2}, E_{C_2} are equal to the busbar voltages V_{A_1}, V_{B_1} and V_{C_1}. The correct moment to close the switch is obtained at the instant when the straight-connected lamp is dark and the cross-connected lamps are equally bright. If the phase sequence is incorrect no such instant will occur as the cross-connected lamps will, in effect, be straight connected and all the lamps will be dark simultaneously.

In this case the direction of rotation of the incoming machine should be reversed by interchanging two lines of the machine. Since the dark range of a lamp extends over a considerable voltage range, a voltmeter V_1 is connected across the straight-connected lamp and the synchronizing switch is closed when the voltmeter reading is zero. The incoming machine is now floating on the busbars and ready to take up the load as a generator or, if its prime mover is disconnected, as a motor. For paralleling smaller machines in power stations three-lamps along with the synchroscope are used. For synchronizing very large machines in power stations, the whole procedure is done automatically by computer. Thus, the risk of error in judgement of the operator is eliminated.

3.53 SYNCHRONIZING BY A SYNCHROSCOPE

The phase sequence of the generator is usually checked carefully at the time of its installation. Conditions 1 and 2 are assured by means of a *synchroscope* (Fig. 3.54), which compares the voltage from one phase of the incoming machine with that of the corresponding phase of the three-phase system. The position of the pointer of the synchroscope indicates the phase difference between the voltages of the incoming machine and the infinite bus. When the frequencies are equal, the pointer is stationary. When the frequencies differ, the pointer rotates in one direction or the other. The direction of motion of the pointer shows whether the incoming machine is running too fast or too slow, that is whether the frequency of the incoming machine is higher or lower than that of the infinite bus. The speed of rotation of the pointer is equal to the difference between the frequency of the incoming machine and the frequency of the infinite bus. The frequency and phase positions are controlled by adjustment of the prime mover input to the incoming machine. When the indicator moves very slowly (that is, frequencies almost the same) and passes through the zero-phase point (vertical up position), the circuit breaker is closed and the incoming alternator is connected to the bus. It is to be noted that a synchroscope checks the relationships only on one phase. It gives no information about phase sequence.

Fig. 3.54.

The procedure is the same for synchronizing large synchronous motors. The synchronous motor is started. As the motor approaches synchronous speed, direct current is applied to the field winding. If the load torque is not excessive, the motor pulls into synchronism with the system.

3.54 MACHINE FLOATING ON BUSBARS

When synchronized, the generated emf of the incoming machine is just equal to the busbar voltage. The incoming machine is then said to be *floating* on busbars. At this instant, it will neither deliver nor receive any power. The prime mover driving the incoming machine will be supplying the no-load losses only.

3.55 INFINITE BUS

In a power system normally more than one alternators operate in parallel. The machines may be located at different places. A group of machines located at one place may be treated as a single large machine. Also, the machines connected to the same bus but separated by transmission lines of low reactance, may be grouped into one large machine. The operation of one machine connected in parallel with such a large system comprising many machines is of great interest. The capacity of the system is so large that its voltage and frequency may be taken constant. The connection or disconnection of a single small machine or a small load on such a system would not effect the magnitude and phase of the voltage and frequency. The system behaves like a large generator having virtually zero internal impedance and infinite rotational inertia. Such a system of constant voltage and constant frequency regardless of the load is called ***infinite busbar system*** or simply ***infinite bus***. Thus, an infinite bus is a power system so large that its voltage and frequency remain constant regardless of how much real and reactive power is drawn from or supplied to it.

The characteristics of an infinite bus are as follows :

(*a*) the terminal voltage remains constant, because the incoming machine is too small to increase or decrease it,

(*b*) the frequency remains constant, because the rotational inertia is too large to enable the incoming machine to alter the speed of the system, and

(*c*) the synchronous impedance is very small since the system has a large number of alternators in parallel.

The behaviour of a synchronous machine on infinite bus is quite different from its isolated operation because the steady-state speed is fixed at a value corresponding to the line frequency. In an isolated operation, the change of excitation changes its terminal voltage, the power factor depends upon the load only. When an alternator is working in parallel with an infinite bus and its excitation is changed, the power factor of the machine changes. However, the change of excitation does not change the terminal voltage which is held constant in the system.

An alternator connected to an infinite bus has the following operating characteristics :

1. The terminal voltage and frequency of the generator are controlled by the system to which it is connected.
2. The governor set points of the alternator control the real power supplied by the alternator to the infinite bus.
3. The field current (excitation) in the alternator controls the reactive power supplied by the alternator to the infinite bus. Increasing the field current in the alternator operating in parallel with an infinite bus increases the reactive power output of the alternator.

3.56 OBTAINING AN INFINITE BUS

Consider n generators $G_1, G_2, ..., G_n$ connected to an infinite bus (Fig. 3.55).

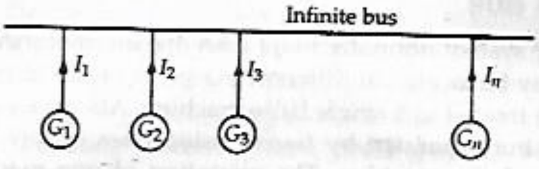


Fig. 3.55.

(a) Proof of voltage remaining constant

Let V = terminal voltage of the bus

E = induced emf of each generator

Z_s = synchronous impedance of each generator

n = number of generators in parallel

$$V = E - I Z_{s\,eq}$$

$$Z_{s\,eq} = \frac{Z_s}{n}$$

When n is very large, $Z_{s\,eq} \longrightarrow 0$

and therefore $I Z_{s\,eq} \longrightarrow 0$

$\therefore$ $V = E$ (constant)

If the number of alternators operating in parallel is infinite only then $Z_s = 0$

(b) Proof of frequency remaining constant

Let J = moment of inertia of each alternator

Total moment of inertia of all n alternators

$$= J + J + J + \ldots + J \text{ (n times)} = nJ$$

Acceleration of alternator

$$= \frac{\text{accelerating torque}}{\text{moment of inertia}}$$

$$= \frac{\tau_a}{\Sigma J} = \frac{\tau_a}{nJ}$$

If n is very large, nJ is very large

$\therefore$ acceleration $\longrightarrow 0$

and speed is constant.

Consequently, frequency is constant.

Therefore, in order to obtain a constant-voltage, constant-frequency of a practical busbar system, the number of alternators connected in parallel should be as large as possible.

EXAMPLE 3.30. *A 3-phase, 11000 V, star-connected turbo-alternator, having synchronous reactance of 6 Ω per phase and negligible resistance has an armature current of 200 A at unity power factor when operating on constant-frequency and constant-voltage busbars. If the steam admission remains the same and the emf is raised by 25%, determine the new values of current and power factor.*

SOLUTION. Phase voltage $V_p = \dfrac{11000}{\sqrt{3}} = 6350$ V

$$I_{a_1} = 200 \text{ A}, \quad X_s = 6\ \Omega$$

At unity power factor

$$E_{a_1p}^2 = V_p^2 + (I_{a_1} X_s)^2$$

$$= 6350^2 + (200 \times 6)^2$$

$$E_{ap} = 6462.4 \text{ V}$$

When the emf is increased by 25%, the new emf

$$E_{a_2p} = 1.25\, E_{a_1p} = 1.25 \times 6462.4 = 8088 \text{ V}$$

Let the new armature current be I_{a_2} and the new power factor be $\cos\phi_2$. Since $E_{a_2} > V_p$, the power factor $\cos\phi_2$ is lagging.

$$\therefore \quad E_{a_2}^2 = (V_{p_2} + I_{a_2} R_{a_2} \cos\phi_2 + I_{a_2} X_s \sin\phi_2)^2 + (I_{a_2} X_s \cos\phi_2 - I_{a_2} R_{a_2} \sin\phi_2)^2$$

At infinite bus $V_{p_2} = V_{p_1} = 6350$ V

Since the steam supply remains the same, the power output will not change

$$V_{p_2} I_{a_2} \cos\phi_2 = V_{p_1} I_{a_1} \cos\phi_1$$

$$I_{a_2} \cos\phi_2 = 200 \times 1$$

At lagging power factor $\cos\phi_2$ with $R_a = 0$,

$$E_{a_2p}^2 = (V_p + I_{a_2} X_s \sin\phi_2)^2 + (I_{a_2} X_s \cos\phi_2)^2$$

$$(8088)^2 = (6350 + 6\, I_{a_2} \sin\phi_2)^2 + (200 \times 6)^2$$

$$6350 + 6\, I_{a_2} \sin\phi_2 = \sqrt{(8088)^2 - (1200)^2}$$

$$I_{a_2} \sin\phi_2 = 274.75 \text{ A}$$

Also $\quad I_{a_2} \cos\phi_2 = 200$

$$I_{a_2} = \sqrt{(I_{a_2} \cos\phi_2)^2 + (I_{a_2} \sin\phi_2)^2}$$

$$= \sqrt{(200)^2 + (274.75)^2} = 339.8 \text{ A}$$

New power factor

$$\cos\phi_2 = \frac{200}{339.8} = 0.5885 \text{ (lagging)}.$$

EXAMPLE 3.31. *A 6600 V, 1200 kVA, 3-phase alternator is delivering full-load at 0.8 power factor lagging. Its reactance is 25% and resistance negligible. By changing the excitation, the emf is increased by 30% at this load. Calculate the new values of current and power factor. The machine is connected to infinite busbars.*

SOLUTION. $V_L = 6600$ V, $\quad V_p = \dfrac{V_L}{\sqrt{3}} = \dfrac{6600}{\sqrt{3}} = 3810.6$ V

$$\frac{\sqrt{3}\, V_L I_a}{1000} = (\text{kVA})_{3\phi}$$

$$\frac{\sqrt{3} \times 6600\, I_a}{1000} = 1200$$

$\therefore$ full load current

$$I_a = \frac{1200 \times 1000}{\sqrt{3} \times 6600} = 105 \text{ A}$$

Percent reactance = 25

$$\frac{I_a X_s}{V_p} \times 100 = 25$$

$$X_s = \frac{25\, V_p}{I_a \times 100} = \frac{25 \times 3810.6}{105 \times 100} = 9.073\ \Omega.$$

With $R_a = 0$, $\quad E_{a,p}^2 = (V_p + I_{a_1} X_s \sin\phi_1)^2 + (I_{a_1} X_s \cos\phi_1)^2$

For $\cos\phi_1 = 0.8$, $\quad \sin\phi_1 = 0.6$

$\therefore \quad E_{a,p}^2 = (3810.6 + 105 \times 9.073 \times 0.6)^2 + (105 \times 9.073 \times 0.8)^2$

$E_{a,p} = 4448$ V

When the emf is increased by 30%, the new emf becomes

$$E_{a,p} = 1.3\, E_{a,p} = 1.3 \times 4448 = 5782.4 \text{ V}$$

Since $E_{a,p} > V_p$, the power factor is lagging. Let the new current be I_{a_2} and new power factor be $\cos\phi_2$.

Since the steam supply or power input is the same, the power output will not change

$\therefore \quad V_{p_1} I_{a_1} \cos\phi_1 = V_{p_2} I_{a_2} \cos\phi_2$

At infinite but $V_{p_1} = V_{p_2} = 3810.6$

$\therefore \quad I_{a_2} \cos\phi_2 = I_{a_1} \cos\phi_1$

$I_{a_2} \cos\phi_2 = 105 \times 0.8 = 84$ A

At lagging power factor $\cos\phi_2$ and $R_a = 0$,

$$E_{a,p}^2 = (V_p + I_{a_2} X_s \sin\phi_2)^2 + (I_{a_2} X_s \cos\phi_2)^2$$

$$5782.4^2 = (3810.6 + 9.073\, I_{a_2} \sin\phi_2)^2 + (84 \times 9.073)^2$$

$I_{a_2} \sin\phi_2 = 211.7$

Also $\quad I_{a_2} \cos\phi_2 = 84$

$\therefore \quad I_{a_2} = \sqrt{(I_{a_2}\cos\phi_2)^2 + (I_{a_2}\sin\phi_2)^2}$

$= \sqrt{84^2 + (211.7)^2} = 227.8$ A

New power factor, $\cos\phi_2 = \dfrac{84}{I_{a_2}} = \dfrac{84}{227.8} = 0.3688$ (lagging).

EXAMPLE 3.32. *A 6600 V, 1200 kVA alternator has a reactance of 25% and is delivering full load at 0.8 p.f. lagging. It is connected to infinite busbars. If the steam supply is gradually increased calculate (a) the output at which the power factor becomes unity, (b) the maximum load which it can supply without losing synchronism and the corresponding power factor.*

SOLUTION. Form Example 3.31

$$V_p = 3810.6 \text{ V}, \quad I_{a_1} = 105 \text{ A}$$

$$X_s = 9.073\ \Omega, \quad E_{a,p} = 4448 \text{ V}$$

$$P = \frac{V E_a}{X_s} \sin\delta = V I_a \cos\phi$$

$$E_a \sin\delta = \frac{X_s}{V} P = kP$$

where $$k = \frac{X_s}{V} = \text{constant}$$

This relation shows that as P increases, $E_a \sin\delta$ and $I_a \cos\phi$ increase. Since excitation is constant, E_a is constant. The locus of E_a is a circle. It is seen that with the increase of P, the quantity $I_a X_s$ goes on increasing so that the relation $\mathbf{E} = V + j I_a X_s$ is satisfied. Thus, the armature current I_a also increases. It is also seen that the power factor angle ϕ also changes.

(a) When the p.f. is unity, $\mathbf{I}_{a_2}$ is along $\mathbf{V}$ and $I_{a_2} X_s$ is perpendicular to $\mathbf{V}$. From $\Delta\, OAC$ of Fig. 3.56.

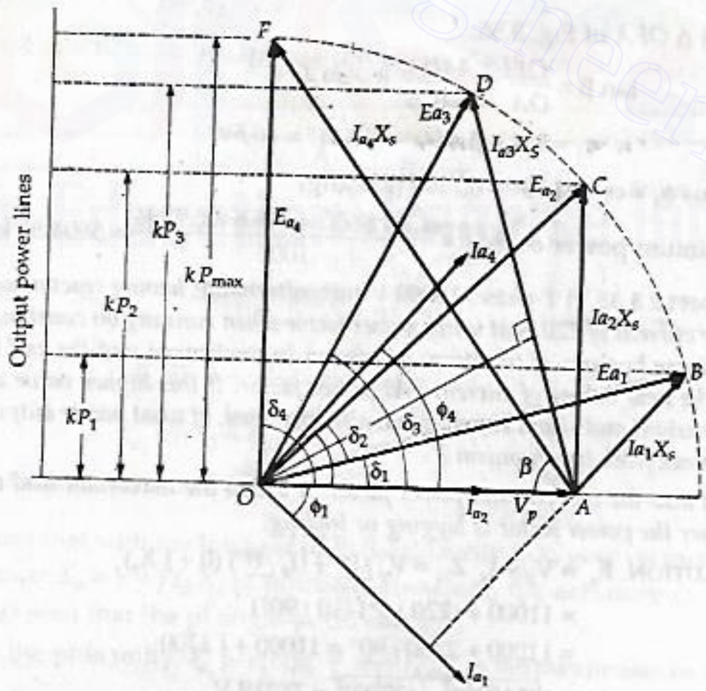


Fig. 3.56.

$$OC^2 = OA^2 + AC^2$$

$$E_{a,p}^2 = V_p^2 + (I_{a_2} X_s)^2$$

$$(I_{a_2} X_s)^2 = E_{a,p}^2 - V_p^2$$

$$= (4488)^2 - (3810.6)^2$$

$$I_{a_2} X_s = 2294.3$$

$$I_{a_2} = \frac{2294.3}{X_s} = \frac{2294.3}{9.073} = 252.88 \text{ A}$$

Power output at unity power factor

$$= \frac{\sqrt{3}\, V_L I_{a_2} \cos\phi}{1000} = \frac{\sqrt{3} \times 6600 \times 252.88 \times 1}{1000} = 2890.7 \text{ kW}$$

(*b*) For maximum output, $\delta = \delta_4 = 90°$.

From $\Delta\, OFA$ of Fig. 3.56,

$$OF^2 + OA^2 = AF^2$$

$$E_{a,p}^2 + V_p^2 = (I_{a_4} X_s)^2$$

$$(4448)^2 + (3810.6)^2 = (I_{a_4} X_s)^2$$

$$I_{a_4} X_s = 5870$$

$$I_{a_4} = \frac{5870}{X_s} = \frac{5870}{9.073} = 645.6 \text{ a}$$

From $\Delta\, OFA$ of Fig. 3.56,

$$\tan\beta = \frac{OF}{OA} = \frac{4448}{3810.6},\ \beta = 49.41°$$

$$\phi_4 = 90° - \beta = 90° - 49.41° = 40.59°$$

$$\cos\phi_4 = \cos 40.59° = 0.7594 \text{ (leading)}$$

$$\text{Maximum power output} = \frac{\sqrt{3} \times 6600 \times 645.6 \times 0.7594}{1000} = 5604.52 \text{ kW}$$

EXAMPLE 3.33. *A 1-phase, 11000 V turboalternator, having reactance of 10 Ω has an armature current of 220 A at unity power factor when running on constant-frequency, constant-voltage busbars. If the steam admission be unchanged and the emf be raised by 25%, find the new values of current and power factor. If this higher value of excitation were kept constant and steam supply gradually increased, at what power output would the alternator break from synchronism ?*

Find also the current and power factor to which the maximum load corresponds. State whether the power factor is lagging or leading.

SOLUTION. $\mathbf{E}_{a_1} = \mathbf{V}_p + \mathbf{I}_{a_1} \mathbf{Z}_{s_1} = V_p \angle 0° + (I_a \angle 0°)\,(0 + j\,X_s)$

$$= 11000 + (220 \angle 0°)\,(10 \angle 90°)$$

$$= 11000 + 2200 \angle 90° = 11000 + j\,2200$$

$$E_{a_1} = \sqrt{(11000)^2 + (2200)^2} = 11218 \text{ V}$$

When the excitation is increased by 25%, the new induced emf $E_{a_2} = 1.25\, E_{a_1} = 1.25 \times 11218 = 14022$ V

Since $E_{a_2} > V_p$, the power factor is lagging.

Let the new armature current be I_{a_2} at power factor $\cos\phi_2$ (lagging).

Since the steam supply is unchanged, the power output will remain constant.

$$\therefore \quad V_{p_1} I_{a_1} \cos\phi_1 = V_{p_2} I_{a_2} \cos\phi_2$$

At infinite bus

$$V_{p_1} = V_{p_2} = 11000 \text{ V}$$

$$\therefore \quad I_a \cos\phi_2 = I_{a_1} \cos\phi_1$$

$$I_{a_2} \cos\phi_2 = 220 \times 1$$

$$\mathbf{E}_{a_2} = \mathbf{V}_{p_2} + \mathbf{I}_{a_2} \mathbf{Z}_{s_2}$$

$$E_{a_2}^2 = (V_{p_2} + I_{a_2} X_s \sin\phi_2)^2 + (I_{a_2} X_s \cos\phi_2)^2$$

$$(14022)^2 = (11000 + 10\, I_{a_2} \sin\phi_2)^2 + (10 \times 220)^2$$

$$\sqrt{(14022)^2 - (2200)^2} = 11000 + 10\, I_{a_2} \sin\phi_2$$

$$I_{a_2} \sin\phi_2 = \frac{13848 - 11000}{10} = 284.83$$

Also, $\quad I_{a_2} \cos\phi_2 = 220$

$$I_{a_2}^2 = (I_{a_2} \cos\phi_2)^2 + (I_{a_2} \sin\phi_2)^2$$

$$= (220)^2 + (284.83)^2$$

$$I_{a_2} = 359.9 \text{ A}$$

$$\cos\phi_2 = \frac{220}{359.9} = 0.6113 \text{ (lagging)}$$

Since $\quad P = \dfrac{VE}{X_s} \sin\delta$

$$E_a \sin\delta = \frac{X_s P}{V} = \text{constant}$$

$$\therefore \quad E_{a_2} \sin\delta_2 = E_{a_1} \sin\delta_1$$

For maximum power, $\delta = 90°$

It is seen that with the increase of P, the quantity $I_a X_s$ goes on increasing so that the relation $E_a = V + j\, I_a X_s$ is satisfied. Therefore, the armature current also increases. It is seen that the pf angle ϕ also changes.

When the pf is unity, $\mathbf{I}_{a_2}$ is along $\mathbf{V}$ and $I_{a_2} X_s$ is perpendicular to V.

In $\Delta\, OAC$ of Fig. 3.56,

$$OC^2 = OA^2 + AC^2$$

$$E_{a_2}^2 = V_p^2 + (I_{a_2} X_s)^2$$

$$(I_{a_2} X_s)^2 = E_{a_2}^2 - V_p^2$$

$$= (14022)^2 - (11000)^2$$

$$I_{a_2} X_s = 8695$$

$$I_{a_2} = \frac{8695}{10} = 869.5 \text{ A}$$

For maximum output $\delta_4 = 90°$

From $\Delta\, OFA$

$$OF^2 + OA^2 = AF^2$$

$$E_a^2 + V_p^2 = (I_a X_s)^2$$

$$\sqrt{14022^2 + 11000^2} = I_a X_s$$

$$17822 = I_a X_s$$

$$I_a = \frac{17822}{10} \Rightarrow 1782.2 \text{ A}$$

From $\Delta\, OFA$, $\quad \tan\beta = \frac{OF}{OA} = \frac{14022}{11000} = \beta = 51.88°$

$$\phi = 90° - \beta = 90 - 51.88 = 38.12°$$

$$\cos\phi = \cos 38.12 = 0.7867 \text{ (leading)}$$

Maximum power output $= V I_a \cos\phi$

$$= 11000 \times 1782.2 \times 0.7867 \text{ W}$$

$$= 15423 \text{ kW.}$$

EXAMPLE 3.34. *A 3-phase, star-connected alternator with $R = 0.4\ \Omega$ and $X = 6\ \Omega$ per phase delivers 300 A at power factor 0.8 to constant frequency 10 kV busbars. If the steam supply is unchanged, find the percentage change in the induced emf necessary to raise the power factor to unity. Ignore the change in losses.*

SOLUTION. $\mathbf{E}_p = \mathbf{V}_p + \mathbf{I}_{a_1} \mathbf{Z}_s$

$$= \frac{10000}{\sqrt{3}} + (300 \angle -\cos^{-1} 0.8)(0.4 + j\,6)$$

$$= 5773.6 + (300 \angle -36.87°)(6.01 \angle 86.18°)$$

$$= 5773.6 + 1803 \angle 49.31°$$

$$= 5773.6 + 1175 + j\,1367$$

$$= 6948.6 + j\,1367 = 7081.8 \angle 11.12° \text{ V}$$

As the output is constant and the current is at unity power factor

$$I_{a_1} \cos\phi_1 = I_{a_2} \cos\phi_2$$

$$300 \times 0.8 = I_{a_2} \times 1, \qquad I_{a_2} = 240 \text{ A}$$

$$\mathbf{E}'_p = \mathbf{V}_p + \mathbf{I}_{a_2} \mathbf{Z}_s$$
$$= 5773.6 + (240 \angle 0°)(6.01 \angle 86.18°)$$
$$= 5773.6 + 1442.4 \angle 86.18°$$
$$= 5773.6 + 96.1 + j\,1439.2$$
$$= 5869.7 + j\,1439.2$$
$$= 6043.6 \angle 13.77° \text{ V}$$

Percentage change in excitation

$$= \frac{E_p - E'_p}{E_p} \times 100$$
$$= \frac{7081.8 - 6043.6}{7081.8} \times 100 = 14.66\%$$

EXAMPLE 3.35. *Two identical, three-phase alternators operating in parallel share equally a load of 1000 kW at 6600 V and 0.8 lagging power factor. The field excitation of the first machine is adjusted so that the armature current is 50 A at lagging power factor. Determine (a) the armature current of the second alternator, and (b) the power factor at which each machine operates.*

SOLUTION. Since the two alternators are identical, the load shared by alternator 1 is

$$P_1 = \frac{1}{2} P_{Load} = \frac{1}{2} \times 1000 = 500 \text{ kW}$$

$$\sqrt{3}\, V_L I_{a_1} \cos\phi_1 = P_1$$
$$\sqrt{3} \times 6600\, I_{a_1} \cos\phi_1 = 500 \times 10^3$$
$$I_{a_1} \cos\phi_1 = \frac{500 \times 10^3}{\sqrt{3} \times 6600} = 43.74 \text{ A}$$

For an armature current of $I_{a_1} = 50$ A, the power factor of alternator 1 is

$$\cos\phi_1 = \frac{43.74}{I_{a_1}} = \frac{43.74}{50}$$
$$= 0.8748 \text{ (lagging)}$$
$$\phi_1 = -28.98°$$
$$\mathbf{I}_{a_1} = I_{a_1} \angle -\phi_1 = 50 \angle -28.98°$$
$$= 43.74 - j\,24.22 \text{ A}$$

Let I be the total load current at power factor 0.8 lagging.

$$\sqrt{3}\, V_L I \cos\phi = P_{load}$$
$$\sqrt{3} \times 6600\, I \times 0.8 = 1000 \times 10^3$$
$$I = \frac{1000 \times 10^3}{\sqrt{3} \times 6600 \times 0.8} = 109.35 \text{ A}$$
$$\mathbf{I} = 109.35 \angle -\cos^{-1} 0.8$$

$$= 109.35 \angle -36.87^\circ \text{ A}$$
$$= 87.48 - j\,65.61 \text{ A}$$
$$\mathbf{I}_{a_1} + \mathbf{I}_{a_2} = \mathbf{I}$$
$$\mathbf{I}_{a_2} = \mathbf{I} - \mathbf{I}_{a_1}$$
$$= (87.48 - j\,65.61) - (43.74 - j\,24.22)$$
$$= 43.74 - j\,41.39$$
$$= 60.22 \angle -43.42^\circ \text{ A}$$

Power factor of the second machine $\cos\phi_2 = \cos 43.42^\circ = 0.7263$ (lagging).

EXAMPLE 3.36. *Two identical three-phase alternators are coupled in parallel to a total load of 1500 kW of 11000 V, power factor 0.8 lagging. The synchronous reactance of each machine is 60 Ω per phase, and resistance 2.8 Ω per phase. The power supplied by each machine being maintained the same, the excitation of first alternator is adjusted so that its armature current is 45 A lagging.*

Calculate :

(a) the armature current of the second alternator,

(b) the power factor at which each alternator operates,

(c) the emf of the first alternator.

SOLUTION. $P_1 + P_2 = P_{load}$

$$P_1 = P_2 = \frac{1}{2} P_{load} = \frac{1}{2} \times 1500 = 750 \text{ kW}$$
$$\sqrt{3}\, V_l I_{a_1} \cos\phi_1 = P_1$$
$$\sqrt{3} \times 11000\, I_{a_1} \cos\phi_1 = 750 \times 10^3$$
$$I_{a_1} \cos\phi_1 = \frac{750 \times 10^3}{\sqrt{3} \times 11000} = 39.36 \text{ A}$$

For $I_{a_1} = 45$ A, $\cos\phi_1 = \dfrac{39.36}{45} = 0.8748$ (lagging)

$$\phi_1 = 28.98^\circ$$
$$\mathbf{I}_{a_1} = I_{a_1} \angle -\phi_1 = 45 \angle -28.98^\circ$$
$$= 39.36 - j\,21.8 \text{ A}$$

Let I be the total load current at power factor 0.8 lagging.

$$\sqrt{3}\, V_l I \cos\phi = P_{load}$$
$$\sqrt{3} \times 11000\, I \times 0.8 = 1500 \times 10^3$$
$$I = \frac{1500 \times 10^3}{\sqrt{3} \times 11000 \times 0.8} = 98.4 \text{ A}$$
$$\mathbf{I} = I \angle -\phi = I \angle -\cos^{-1} 0.8$$
$$= 98.4 \angle -36.87^\circ \text{ A}$$
$$= 78.72 - j\,59.04 \text{ A}$$

$$\mathbf{I}_{a_1} + \mathbf{I}_{a_2} = \mathbf{I}$$

$$\mathbf{I}_{a_2} = \mathbf{I} - \mathbf{I}_{a_1}$$

$$= (78.72 - j\,59.04) - (39.36 - j\,21.8)$$

$$= 39.36 - j\,37.24 = 54.19 \angle -43.4^\circ \text{ A}$$

$$\cos\phi_2 = \cos 43.4^\circ = 0.7264 \text{ (lagging)}$$

$$Z_1 = R_{a_1} + j\,X_{s_1} = 2.8 + j\,60 = 60.06 \angle 87.33^\circ\ \Omega$$

$$\mathbf{E}_{a_1p} = \mathbf{V}_p + \mathbf{I}_{a_1}\mathbf{Z}_1$$

$$= \frac{11000}{\sqrt{3}} + (45 \angle -28.98^\circ)(60.06 \angle 87.33^\circ)$$

$$= 6350.8 + 2702.7 \angle 58.35^\circ$$

$$= 6350.8 + 1418.2 + j\,2300.7$$

$$= 7769 + j\,2300.7 = 8102.5 \angle 16.5^\circ \text{ V}$$

Line value of the emf of the first alternator

$$E_{a_1l} = \sqrt{3}\,E_{a_1p} = \sqrt{3} \times 8102.5 = 14034 \text{ V}$$

Example 3.37. *Two 3-phase, 6.6 kV, star-connected alternators supply a load of 3000 kW at 0.8 power factor lagging. The synchronous impedance per phase of machine A is $0.5 + j\,10\ \Omega$ and of machine B is $0.4 + j\,12\ \Omega$. The excitation of machine A is adjusted so that it delivers 150 A at a lagging power factor, and the governors are so set that the load is shared equally between the machines. Determine the current, power factor, induced e.m.f. and load angle of each machine.*

Solution. For machine 1, $I_{a_A} = 150$ A

$$\sqrt{3} \times 6.6 \times 10^3\, I_{a_1} \cos\phi_A = \frac{1}{2} \times 3000 \times 10^3$$

$$\cos\phi_A = \frac{1500 \times 10^3}{\sqrt{3} \times 6.6 \times 10^3 \times 150} = 0.8748 \text{ (lagging)}$$

$$\phi_A = 28.98^\circ$$

$$\mathbf{I}_{a_A} = I_{a_A} \angle -\phi_1 = 150 \angle -28.98^\circ$$

$$= 131.2 - j\,72.68 \text{ A}$$

Total current

$$I = \frac{P_{3\phi}}{\sqrt{3}\,V_L \cos\phi} = \frac{3000 \times 10^3}{\sqrt{3} \times 6.6 \times 10^3 \times 0.8} = 328 \text{ A}$$

$$\mathbf{I} = I \angle -\phi = 328 \angle -\cos^{-1} 0.8 = 328 \angle -36.87^\circ \text{ A}$$

$$= 262.4 - j\,196.8 \text{ A}$$

$$\mathbf{I}_{a_A} + \mathbf{I}_{a_B} = \mathbf{I}$$

$$\mathbf{I}_{a_B} = \mathbf{I} - \mathbf{I}_{a_A}$$

$$= (262.4 - j\,196.8) - (131.2 - j\,72.68)$$

$$= 131.2 - j\,124.12 = 180.6 \angle -43.14^\circ \text{ A}$$

Power factor of the second machine

$$\cos \phi_B = \cos(-43.14°) = 0.7264 \text{ lagging}$$

$$Z_A = 0.5 + j\,10 = 10.01 \angle 87.14° \ \Omega$$

$$\mathbf{E}_{a_A p} = \mathbf{V}_p + \mathbf{I}_{a_A} \mathbf{Z}_A$$

$$= \frac{6600}{\sqrt{3}} + (150 \angle -28.98°)(10.01 \angle 87.14°)$$

$$= 3810.5 + 1501.5 \angle 58.16°$$

$$= 3810.5 + 792 + j\,1275.6$$

$$= 4602 + j\,1275.6 = 4776 \angle 15.49° \text{ V}$$

Load angle of machine A

$$\delta_A = 15.49°$$

Line value of e.m.f of machine A

$$E_{a_A L} = \sqrt{3}\, E_{a_A p} = \sqrt{3} \times 4776 = 8272 \text{ V}$$

$$Z_B = 0.4 + j\,12 = 12.007 \angle 88.1° \ \Omega$$

$$\mathbf{E}_{a_B p} = \mathbf{V}_p + \mathbf{I}_{a_B} \mathbf{Z}_B$$

$$= 3810.5 + (180.6 \angle -43.14°)(12.007 \angle 88.1°)$$

$$= 3810.5 + 2168.5 \angle 44.96°$$

$$= 3810.5 + 1534.4 + j\,1532.3$$

$$= 5344.9 + j\,1532.3 = 5560.2 \angle 16° \text{ V}$$

$$= E_{a_B p} \angle \delta_B$$

Load angle of machine B, $\delta_B = 16°$

$\therefore \quad E_{a_B p} = 5560.2$ V

Line value of e.m.f. of machine $B = \sqrt{3}\, E_{a_B p} = \sqrt{3} \times 5560.2 = 9631$ V

EXAMPLE 3.38. *Two identical, 3-phase, star-connected generators, operating in parallel, share equally a total load of 750 kW at 6000 V and power factor 0.8. The synchronous reactance and resistance of each machine are respectively 50 Ω and 2.5 Ω per phase. The field of first generator is excited so that the armature current is 40 A (lagging). Find (a) the armature current of the second alternator ; (b) the power factor of each machine ; (c) the electromotive force of each machine ; (d) the load angle of each machine.*

SOLUTION. $\sqrt{3}\, V_L I_{a_1} \cos \phi_1 = P_1$

$$\sqrt{3} \times 6000\, I_{a_1} \cos \phi_1 = \frac{750}{2} \times 10^3$$

$$I_{a_1} \cos \phi_1 = \frac{750 \times 10^3}{2\sqrt{3} \times 6000} = 36.08 \text{ A}$$

$$I_{a_1} = 40 \text{ A}$$

$$\therefore \quad \cos \phi_1 = \frac{36.08}{40} = 0.9021 \text{ (lagging)}$$

$$\phi_1 = 25.56°$$

$$\therefore \quad \mathbf{I}_{a_1} = I_{a_1} \angle -\phi = 40 \angle -25.56^\circ$$

$$= 36.08 - j\,17.26$$

Total current $I = \dfrac{750 \times 10^3}{\sqrt{3} \times 6000 \times 0.8} = 90.21$ A

$$\mathbf{I} = 90.21 \angle -\cos^{-1} 0.8 = 90.21 \angle -36.87^\circ$$

$$= 72.17 - j\,54.13$$

$$\mathbf{I}_{a_1} + \mathbf{I}_{a_2} = \mathbf{I}$$

$$\mathbf{I}_{a_2} = \mathbf{I} - \mathbf{I}_{a_1}$$

$$= (72.17 - j\,54.13) - (36.08 - j\,17.26)$$

$$= 36.09 - j\,36.87 = 51.59 \angle -45.61^\circ$$

$$I_{a_2} = 51.59 \text{ A}, \cos\phi_2 = \cos(-45.61^\circ) = 0.6995 \text{ (lagging)}$$

$$\mathbf{E}_{a_1 p} = \mathbf{V}_p + \mathbf{I}_{a_1} \mathbf{Z}_f$$

$$= \frac{6000}{\sqrt{3}} + (40 \angle -25.56^\circ)(2.5 + j\,50)$$

$$= 3464.2 + (40 \angle -25.56^\circ)(50.06 \angle 87.14^\circ)$$

$$= 3464.2 + 2002.4 \angle 61.98^\circ$$

$$= 3464.2 + 940.7 + j\,1767.7$$

$$= 4404.9 + j\,1767.7$$

$$= 4746.4 \angle 21.86^\circ \text{ V per phase}$$

Line value of emf of machine 1

$$(E_{a_1})_{line} = \sqrt{3}\, E_{a_1 p} = \sqrt{3} \times 4746.4 = 8221 \text{ V}$$

Load angle of machine 1, $\delta_1 = 21.86^\circ$

$$\mathbf{E}_{a_2 p} = \mathbf{V}_p + \mathbf{I}_{a_2} \mathbf{Z}_s$$

$$= 3464.2 + (51.59 \angle -45.61^\circ)(50.06 \angle 87.14^\circ)$$

$$= 3464.2 + 2582.6 \angle 41.53^\circ$$

$$= 3464.2 + 1933.4 + j\,1712.3$$

$$= 5397.6 + j\,1712.3$$

$$= 5662.7 \angle 17.6^\circ \text{ V per phase}$$

Line value of emf of machine 2

$$(E_{a_2})_{line} = \sqrt{3}\, E_{a_2 p} = \sqrt{3} \times 5662.7 = 9807.8 \text{ V}$$

Load angle of machine 2

$$\delta_2 = 17.6^\circ$$

3.57 SYNCHRONIZING POWER AND SYNCHRONIZING TORQUE COEFFICIENTS

A synchronous machine, whether a generator or a motor, when synchronized to infinite busbars has a inherent tendency to remain in synchronism.

Consider a synchronous generator transferring a steady power P_o at a steady load angle δ_o. Suppose that, due to a transient disturbance, the rotor of the generator accelerates, so that the load angle increases by an angle $d\,\delta$. The operating point of the machine shifts to a new constant-power line and the load on the machine increases to $P_o + \delta P$. Since the steady power input remains unchanged, this additional load decreases the speed of the machine and brings it back to synchronism.

Similarly, if due to a transient disturbance, the rotor of the machine retards, so that the load angle decreases. The operating point of the machine shifts to a new constant power line and the load on the machine decreases to $(P_o - \delta P)$. Since the steady power input remains unchanged, the reduction in load accelerates the rotor. Consequently, the machine again comes in synchronism. It is seen that the effectiveness of this correcting action depends on the change in power transfer for a given change in load angle. A measure of effectiveness is given by *synchronizing power coefficient*. It is defined as the rate at which the synchronous power P varies with the load angle δ. It is also called ***stiffness of coupling, rigidity factor***, or ***stability factor*** and is denoted by P_{syn}.

$$P_{syn} \overset{\Delta}{=} \frac{d\,P}{d\,\delta} \tag{3.57.1}$$

Power output per phase of the cylindrical rotor generator

$$P = \frac{V}{Z_s}\left[E_f \cos(\theta_z - \delta) - V\cos\theta_z\right] \tag{3.57.2}$$

$$\therefore \quad P_{syn} = \frac{d\,P}{d\,\delta} = \frac{V\,E_f}{Z_s}\sin(\theta_z - \delta) \tag{3.57.3}$$

The synchronizing torque coefficient

$$\tau_{syn} \overset{\Delta}{=} \frac{d\,\tau}{d\,\delta} = \frac{1}{2\,\pi\,n_s}\,\frac{d\,P}{d\,\delta} \tag{3.57.4}$$

or

$$\tau_{syn} = \frac{V\,E_f}{2\pi\,n_s\,Z_s}\sin(\theta_z - \delta) \tag{3.57.5}$$

In many synchronous machines $X_s >> R$. Therefore, for a cylindrical rotor machine, neglecting saturation and stator resistance Eqs. (3.57.3) and (3.57.5) become

$$\boxed{\begin{aligned} P_{syn} &= \frac{V\,E_f}{X_s}\cos\delta \qquad (3.57.6)\\ \tau_{syn} &= \frac{V\,E_f}{2pi\,n_s\,X_s}\cos\delta \qquad (3.57.7)\end{aligned}}$$

For a salient-pole machine

$$\boxed{P = \frac{V\,E_f}{X_s}\sin\delta + \frac{1}{2}V^2\left(\frac{1}{X_d} - \frac{1}{X_q}\right)\sin 2\delta} \qquad (3.57.8)$$

$$\therefore \quad \boxed{P_{syn} = \frac{V\,E_f}{X_s}\cos\delta + V^2\left(\frac{1}{X_d} - \frac{1}{X_q}\right)\cos 2\delta} \qquad (3.57.9)$$

3.58 UNITS OF SYNCHRONIZING POWER COEFFICIENT P_{syn}

The synchronizing-power coefficient is expressed in watts per electrical radian.

$$\therefore \quad P_{syn_1} = \frac{E_f V}{X_s}\cos\delta \quad \text{W/elec. radian} \qquad (3.58.1)$$

Since π radians $= 180°$

$$1 \text{ radian} = \frac{180}{\pi} \text{ degrees}$$

$$P_{syn_2} = \frac{dP}{d\delta} \quad \text{W}\Big/\left(\frac{180}{\pi}\text{ degrees}\right)$$

or

$$P_{syn_2} = \left(\frac{dP}{d\delta}\right)\frac{\pi}{180} \quad \text{W/elec. degree} \qquad (3.58.2)$$

If P = total number of *pair* of poles of the machine

$$\theta_{electrical} = p\,\theta_{mechanical}$$

Synchronizing power coefficient per mechanical radian is given by

$$P_{syn_3} = p\frac{dP}{d\delta} \quad \text{W} \qquad (3.58.3)$$

Synchronizing power coefficient per mechanical degree

$$P_{syn_4} = \frac{p\,\pi}{180}\frac{dP}{d\delta} \quad \text{W} \qquad (3.58.4)$$

3.59 SYNCHRONIZING TORQUE COEFFICIENT

Synchronizing power coefficient gives rise to synchronizing torque coefficient at synchronous speed. That is, the ***synchronizing torque** is the torque which at synchronous speed gives the synchronizing power.* If τ_{syn} is the synchronizing torque coefficient

$$\tau_{syn} = \frac{1}{\omega_s}\,m\,\frac{dP}{d\delta} \quad \text{Nm/elect. radian} \qquad (3.59.1)$$

or

$$\tau_{syn} = \left(\frac{1}{\omega_s}\,m\,\frac{dP}{d\delta}\right)\frac{\pi p}{180} \quad \text{Nm/mech. degree} \qquad (3.59.2)$$

where m = number of phases of the machine

$\omega_s = 2\pi n_s$

n_s = synchronous speed in r.p.s

$$\tau_{syn} = \frac{P_{syn}}{\omega_s} = \frac{P_{syn}}{2\pi n_s} \qquad (3.59.3)$$

3.60 SIGNIFICANCE OF SYNCHRONIZING POWER COEFFICIENT

The synchronizing power coefficient P_{syn} is a measure of the stiffness of the electromagnetic coupling between the rotor and the stator. A large value of P_{syn} indicates that the coupling is *stiff* or *rigid*. Too rigid a coupling means that the machine will be subjected to shocks with change of load or supply. These shocks may damage the rotor or the windings. We have

$$P_{syn} = \frac{3\,V E_f}{X_s} \cos \delta \qquad (3.60.1)$$

$$\tau_{syn} = \frac{3}{2\pi n_s} \frac{V E_s}{X_s} \cos \delta \qquad (3.60.2)$$

Equations (3.60.1) and (3.60.2) show that P_{syn} is inversely proportional to the synchronous reactance. Machines with large air gaps have relatively small reactances. Therefore a synchronous machine with a larger air gap is more stiff than a machine with smaller air gap. Since P_{syn} is directly proportional to E_f, an overexcited machine is more stiff than an underexcited machine.

Equations (3.60.1) and (3.60.2) also indicate that the restoring action is greatest when $\delta = 0$, that is, at no load. The restoring action is zero when $\delta = \pm 90°$. At these values of δ the machine would be at the steady-state limit of stability and in condition of unstable equilibrium. Therefore it is impossible to run a machine at the steady-state limit of stability since its ability to resist small changes is zero unless the machine is provided with special fast-acting excitation system.

3.61 OSCILLATIONS OF SYNCHRONOUS MACHINES

A machine under steady running conditions has at every instant a driving torque exactly balancing its retarding torque. The retarding torque is developed by phase displacement between the axis of the stator and rotor poles, that is angle δ.

When a mechanical rotary system possesses inertia and restoring torque that tends to restore its position when displaced, the system has a natural frequency of free oscillations. A synchronous machine operating in parallel with other machines or infinite busbars forms such a system. Here the restoring torque is due to the synchronizing torque which depends upon the displacement and opposes displacement. The inertia in this system is due to the moment of inertia of the rotor and the prime mover. A synchronous dead load (lamps, furnaces etc.) has no restoring torque and hence no natural frequency of oscillation.

Let τ_{syn} = synchronizing torque coefficient (Nm per mech. rad)

β = load angle deviation steady state position (mech. rad)

J = moment of inertia of rotating system (kg m^2)

If damping is neglected

$$J \frac{d^2 \beta}{dt^2} = -\tau_{syn}\, \beta \qquad (3.61.1)$$

This represents a single harmonic motion.

The frequency of undamped oscillation is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{\tau_{syn}}{J}} \tag{3.61.2}$$

The period of oscillation is

$$T = \frac{1}{f} = 2\pi\sqrt{\frac{J}{\tau_{syn}}} \tag{3.61.3}$$

Full-load current $= I_a$

Reactance voltage drop $= I_a X_s$

Per-unit reactance voltage drop

$$X_{s\,pu} = \frac{I_a X_a}{V_p}$$

$$\therefore \quad X_s = \frac{V_p}{I_a} X_{s\,pu} \tag{3.61.4}$$

Short-circuit current

$$I_{sc} = \frac{V_p}{X_s} = \frac{I_a}{X_{s\,pu}}$$

$$\therefore \quad \frac{I_{sc}}{I_a} = \frac{1}{X_{s\,pu}} \tag{3.61.5}$$

$$\tau_{syn} = \frac{3\,V_p^2}{2\pi\, n_s\, X_s}\, p \tag{3.61.6}$$

Since

$$p = \frac{f}{n_s} \tag{3.61.7}$$

$$\tau_{syn} = \frac{3\,V_p^2}{2\pi\, n_s\, X_s} \cdot \frac{f}{n_s}$$

Since

$$\frac{V_p}{X_s} = I_{sc}$$

$$\tau_{syn} = \frac{3\,V_p\, I_{sc}\, f}{2\pi\, n_s^2}$$

Time period of oscillation

$$T = 2\pi\sqrt{\frac{J}{\tau_{syn}}} = 2\pi\sqrt{\frac{J \cdot 2\pi\, n_s^2}{3\,V_p\, I_{sc}\, f}}$$

$$T = 9.093\, n_s \sqrt{\frac{J}{V_p\, I_{sc}\, f}} \tag{3.61.8}$$

Now

$$(\text{kVA})_{3\phi} = \frac{3\,V_p\, I_a}{1000}$$

$$V_p = \frac{1000\,(\text{kVA})_{3\phi}}{3\,I_a}$$

$$\therefore \quad T = 9.093\, n_s \sqrt{\frac{J}{\left(\frac{1000}{3}\right)(\text{kVA})_{3\phi}\left(\frac{I_{sc}}{I_u}\right) f}}$$

$$= 0.498\, n_s \sqrt{\frac{J}{(\text{kVA})_{3\phi}\left(\frac{I_{sc}}{I_u}\right) f}}$$

$$T = 0.498\, n_s \sqrt{\frac{J X_{s\,pu}}{(\text{kVA})_{3\phi} f}} \quad (3.61)$$

EXAMPLE 3.39. *A 2-pole, 50 Hz, 3-phase, turbo-alternator is excited to generate the busbar voltage of 11 kV on no load. The machine is star-connected and the short circuit current for this excitation is 1000 A. Calculate the synchronizing power per degree of mechanical displacement of the rotor and the conesponding synchronizing torque.*

SOLUTION. At no load, $E_f = V_p = \frac{11000}{\sqrt{3}} = 6350$ V and $\delta = 0$.

Synchronous reactance

$$X_s = \frac{V_p}{I_{sc}} = \frac{6350}{1000} = 6.35\ \Omega$$

$$P_{syn} = \left(\frac{3\, V_p E_f}{X_s} \cos\delta\right) \frac{p\,\pi}{180}$$

$$= \frac{3 \times 6350 \times 6350}{6.35} \times 1 \times \frac{1 \times \pi}{180}$$

$$= 332485 \text{ W per mech degree}$$

$$\tau_{syn} = \frac{P_{syn}}{2\pi\, n_s} = \frac{332485}{2\pi \times \frac{3000}{60}} = 1058.33 \text{ Nm}$$

EXAMPLE 3.40. *A 2 MVA, 3 phase, 8-pole alternator is connected to 6000 V, 50 Hz busbars and has a synchronous reactance of 4 Ω per phase. Calculate the synchronizing power and the synchronizing torque per mechanical degree of rotor displacement at no load. Assume normal excitation.*

SOLUTION. $P = 8, \quad p = \frac{P}{2} = 4$

At no load $\quad \delta = 0°, \quad E_f = V_p$

$$V_L = 6000 \text{ V}, \quad V_p = \frac{6000}{\sqrt{3}}$$

$$\therefore \quad E_f = \frac{6000}{\sqrt{3}}, \quad N_s = \frac{120\, f}{P} = \frac{120 \times 50}{8} = 750 \text{ r.p.m}$$

$$P_{syn} = \left(\frac{3\, V_p E_f}{X_s} \cos\delta\right) \frac{p\,\pi}{180} = \frac{3}{4} \times \frac{6000}{\sqrt{3}} \times \frac{6000}{\sqrt{3}} \times 1 \times \frac{4\pi}{180}$$

$$= 628318 \text{ W/mech degree}$$

$$\tau_{syn} = \frac{P_{syn}}{2\pi\, n_s} = \frac{628318}{2\pi \times \frac{750}{60}} = 8000 \text{ Nm}$$

EXAMPLE 3.41. *A 3 MVA, 6-pole alternator runs at 1000 r.p.m. in parallel with other machines on 3.3 kV busbars. The synchronous reactance is 25 percent. Calculate the synchronizing power per mechanical degree of rotor displacement at no load and the corresponding synchronizing torque.*

SOLUTION. On no-load $E_f = V$

$$S = \sqrt{3}\, V_L\, I_a$$

$$I_a = \frac{S}{\sqrt{3}\, V_L} = \frac{3 \times 10^6}{\sqrt{3} \times 3.3 \times 10^3} = 524.86 \text{ A}$$

$$V_p = \frac{3.3 \times 10^3}{\sqrt{3}} = 1905 \text{ V}$$

$$X_{s\,pu} = \frac{I_a X_{s\,\Omega}}{V_p}, \quad X_{s\,pu} = \frac{25}{100} = 0.25 \text{ pu}$$

$$X_{s\,\Omega} = X_{s\,pu} \times \frac{V_p}{I_a} = \frac{0.25 \times 1905}{524.86} = 0.9075\ \Omega$$

Synchronizing power per mechanical degree

$$P_{sy} = \left(\frac{3\, V_p E_f}{X_s} \cos\delta\right) \frac{p\,\pi}{180}$$

At no load, $\delta = 0$, $E_f = V_p$

Also $P = 6$, $p = \frac{P}{2} = \frac{6}{2} = 3$

$$\therefore \quad P_{sy} = \frac{3\, V_p^2}{X_s} \times 1 \times \frac{p\,\pi}{180} = \frac{3 \times 1905^2}{0.9075} \times \frac{3\pi}{180}$$

$$= 628149 \text{ W} = 628.149 \text{ kW}$$

$$\tau_{syn} = \frac{P_{syn}}{2\pi\, n_s} = \frac{628149}{2\pi \times \frac{1000}{60}} = 5998.4 \text{ Nm}$$

EXAMPLE 3.42. *A 10000 kVA, 4-pole, 6600 V, 50 Hz, 3-phase, star-connected alternator has a synchronous reactance of 25% and operates on constant-voltage, constant-frequency busbars. If the natural period of oscillation while operating at full load and unity power factor is to be limiied to 1.5 seconds, calculate the moment of inertia of the rotating system.*

SOLUTION. $T = 0.498\, n_s \sqrt{\dfrac{J X_{s\,pu}}{(\text{kVA})_{3\phi} f}}$

$$n_s = \frac{f}{p} = \frac{50}{2} = 25 \text{ r.p.s.}$$

$$\therefore \quad 1.5 = 0.498 \times 25 \times \sqrt{\frac{J \times 0.25}{10000 \times 50}}$$

$$J = \left(\frac{1.5}{0.498 \times 25}\right)^2 \times \frac{1000 \times 50}{0.25} = 29032 \text{ kg m}^2.$$

EXAMPLE 3.43. *A 5000 kVA, 3-phase, 10000 V, 50 Hz alternator runs at 1500 r.p.m, connected to constant-voltage busbars. If the moment of inertia of the rotating system is 1.5×10^4 kg m^2 and the steady-state short-circuit current is five times the normal full-load current, find the natural time period of oscillation.*

SOLUTION. $X_{s\,pu} = \frac{I_a}{I_{sc}} = \frac{1}{5} = 0.2$

$$T = 0.498\, n_s \sqrt{\frac{J X_{s\,pu}}{(\text{kVA})_{3\phi} f}}$$

$$= 0.498 \times \frac{1500}{60} \times \sqrt{\frac{1.5 \times 10^4 \times 0.2}{5000 \times 50}}$$

$$= 1.3638 \text{ s.}$$

EXAMPLE 3.44. *A 10 MVA, 10 kV, 3-phase, 50 Hz, 1500 r.p.m. alternator is paralleled with others of much greater capacity. The moment of inertia of the rotor is 2×10^5 kg m^2 and the synchronous reactance of the machine is 40%. Calculate the frequency of oscillation of the rotor.*

SOLUTION. $n_s = \frac{1500}{60} = 25$

$$T = 0.498\, n_s \sqrt{\frac{J X_{s\,pu}}{(\text{kVA})_{3\phi} f}}$$

$$= 0.498 \times 25 \times \sqrt{\frac{2 \times 10^5 \times 0.4}{(10 \times 10^3) \times 50}}$$

$$= 4.98 \text{ s}$$

Undamped frequency of oscillation $= \frac{1}{T} = \frac{1}{4.98} = 0.2$ Hz

EXAMPLE 3.45. *A 2-pole, 50 Hz, 3-phase, 100 MVA, 33 kV turboalternator connected to the infinite bus has a moment of inertia of 10^6 kg m^2 in its rotating parts. It has a synchronous reactance of 0.5 pu. Calculate the natural frequency of oscillation.*

SOLUTION. $p\, n_s = f, \quad n_s = \frac{f}{p} = \frac{50}{1} = 50$ rps

$$T = 0.498\, n_s \sqrt{\frac{J X_{s\,pu}}{(\text{kVA})_{3\phi} f}}$$

$$= 0.498 \times 50 \times \sqrt{\frac{10^6 \times 0.5}{(100 \times 10^3) \times 50}} = 7.874 \text{ s}$$

EXAMPLE 3.46. *A 5000 kVA, 10000 V, 1500 r.p.m., 50 Hz alternator runs in parallel with other machines. Its synchronous reactance is 20%. Find for (a) no load, (b) full load at power factor 0.8 lagging, synchronizing power per unit mechanical angle of phase displacement and calculate the synchronizing torque if mechanical displacement is 0.5°.*

SOLUTION. Voltage per phase $V_p = \frac{V_L}{\sqrt{3}} = \frac{10000}{\sqrt{3}} = 5774$ V

$$S = \sqrt{3}\, V_L\, I_a$$

$$I_a = \frac{S}{\sqrt{3}\, V_L} = \frac{5000 \times 10^3}{\sqrt{3} \times 10000} = 288.7 \text{ A}$$

$$X_{s\,\Omega} = X_{s\,pu}\frac{V_p}{I_a} = \frac{20}{100} \times \frac{5774}{288.7} = 4\,\Omega$$

$$P = \frac{120\,f}{N_s} = \frac{120 \times 50}{1500} = 4,\ p = \frac{P}{2} = 2$$

(*a*) *At no load* $\quad E_f = V_p = 5774$ *and* $\delta = 0°$.

$$P_{syn} = \left(\frac{3\,V_p\,E_f}{X_s}\cos\delta\right)\frac{p\,\pi}{180}$$

$$= \frac{3 \times 5774 \times 5774}{4} \times 1 \times \frac{2\pi}{180} = 872815\text{ W}$$

$$\tau_{syn} = \frac{P_{syn}}{2\pi\,n_s} = \frac{872815}{2\pi \times \dfrac{1500}{60}} = 5556\text{ Nm/mech degree}$$

For $\quad S = 0.5°, \quad \tau'_{syn} = 0.5\,\tau_{syn}$

$$\tau'_{syn} = 5556 \times 0.5 = 2778\text{ Nm}$$

(*b*) *Full load, 0.8 p.f. lagging*

$$\mathbf{E}_f = \mathbf{V}_p + \mathbf{I}_a\,\mathbf{Z}_s$$

$$= V_p + (I_a \angle -\phi)\,(0 + j\,X_s)$$

$$= V_p + (I_a \angle -\phi)\,X_s \angle 90°$$

$$= V_p + I_a\,X_s \angle 90° - \phi$$

$$= V_p + I_a\,X_s\,[\cos(90° - \phi) + j\sin(90° - \phi)]$$

$$= (V_p + I_a\,X_s\sin\phi) + j\,I_a\,X_s\cos\phi$$

$$V_p = 5774\text{ V}, \quad I_a = 288.7\text{ A}, \quad X_s = 4\,\Omega$$

$$\cos\phi = 0.8,\ \sin\phi = 0.6$$

$$\therefore \quad E_f = (5774 + 288.7 \times 4 \times 0.6) + j\,288.7 \times 4 \times 0.8$$

$$= 6466.88 + j\,923.84 = 6532.5 \angle 8.13°\text{ V}$$

$$\therefore \quad E_f = 6532.5\text{ V}, \quad \delta = 8.13°$$

$$P_{syn} = \left(\frac{3\,V_p\,E_f}{X_s}\cos\delta\right)\frac{p\,\pi}{180}$$

$$= \frac{3 \times 5774 \times 6532.5}{4}\,(\cos 8.13°) \times \frac{2\,\pi}{180}$$

$$= 977548\text{ W}$$

$$\tau_{syn} = \frac{P_{syn}}{2\pi\,n_s} = \frac{977548}{2\pi \times \dfrac{1500}{60}} = 6223\text{ Nm per deg mech}$$

Synchronizing torque for 0.5 degree mechanical displacement

$$\tau'_{syn} = 0.5\,\tau_{syn} = 0.5 \times 6223 = 3111.5\text{ Nm}$$

EXAMPLE 3.47. *A 1500 kVA, 3-phase, star-connected 6.6 kV, 8-pole, 50 Hz synchronous generator has a reactance of 0.6 pu and negligible resistance. Calculate (a) the synchronizing power per mechanical degree at full load and 0.8 power factor lagging.*

SOLUTION. $S = \sqrt{3}\, V_l I_a$

$$I_a = \frac{1500 \times 10^3}{\sqrt{3} \times 6.6 \times 10^3} = 131.2 \text{ A}$$

$$V_p = \frac{V_l}{\sqrt{3}} = \frac{6.6 \times 10^3}{\sqrt{3}} = 3810.5 \text{ V}$$

$$\therefore \quad X_{s\,pu} = \frac{I_a X_{s\,\Omega}}{V_p} \quad X_{s\,\Omega} = X_{s\,pu} \frac{V_p}{I_a} = 0.6 \times \frac{3810.5}{131.2} = 17.42\ \Omega$$

$$\begin{aligned}
\mathbf{E}_f &= \mathbf{V}_p + \mathbf{I}_a \mathbf{Z}_s \\
&= V_p + (I_a \angle{-\phi})(X_s \angle 90^\circ) \\
&= V_p + I_a X_s \angle 90 - \phi^\circ = V_p + I_a X_s[\cos(90-\phi) + j\sin(90-\phi)] \\
&= V_p + I_a X_s(\sin\phi + j\cos\phi) \\
&= (V_p + I_a X_s \sin\phi) + j I_a X_s \cos\phi \\
&= (3810.5 + 131.2 \times 17.42 \times 0.6) + j\,131.2 \times 17.42 \times 0.8 \\
&= 5638.9 + j\,1828.40 = 5928 \angle 17.96^\circ \text{ V}
\end{aligned}$$

$$\therefore \quad E_f = 5928 \text{ V}, \delta = 17.96^\circ$$

Synchronizing power per mechanical degree

$$\begin{aligned}
P_{syn} &= \left(\frac{dP}{d\delta}\right) p \frac{\pi}{180} \\
&= \left(\frac{3 V_p E_f}{X_s} \cos\delta\right) p \frac{\pi}{180} \\
&= \frac{3 \times 3810.5 \times 5928}{17.42} \cos 17.96^\circ \times \frac{4\pi}{180} \\
&= 258340.6 \text{ W} = 258.3406 \text{ kW}
\end{aligned}$$

Synchronizing torque

$$\tau_{syn} = \frac{P_{syn}}{2\pi n_s} = \frac{258340.6}{2\pi \times (50/4)} = 3289 \text{ Nm}$$

EXAMPLE 3.48. *A 2-MVA, 3-phase, 8-pole alternator runs at 750 r.p.m. in parallel with other machines on 6000 V busbars. Find the synchronizing power on full load at power factor 0.8 lagging per mechanical degree of displacement and the corresponding synchronizing torque. The synchronous reactance of the machine is 6 Ω per phase.*

SOLUTION. $I_a = \dfrac{S}{\sqrt{3}\, V_L} = \dfrac{2 \times 10^6}{\sqrt{3} \times 6000} = 192.45 \text{ A}$

$$\begin{aligned}
\mathbf{E}_f &= \mathbf{V}_p + \mathbf{I}_a \mathbf{Z}_s = V_p + I_a \angle{-\phi}\, X_s \angle 90^\circ \\
&= (V_p + I_a X_s \sin\phi) + j I_a X_s \cos\phi
\end{aligned}$$

$$= (3464 + 192.45 \times 6 \times 0.6) + j\,(192.45 \times 6 \times 0.8)$$

$$= 4156.8 + j\,923.76 = 4258.2 \angle 12.53° \text{ V}$$

$$E_f = 4258.2 \text{ V}, \quad \delta = 12.53°$$

$$P_{syn} = \left(\frac{3\,V_p\,E_f}{X_s} \cos \delta \right) \frac{p\,\pi}{180}$$

$$= \frac{3 \times 3464 \times 4258.2 \times 4\,\pi}{6 \times 180} \cos 12.53°$$

$$= 502625 \text{ W} = 502.625 \text{ kW}$$

$$\tau_{syn} = \frac{P_{syn}}{2\pi\,n_s} = \frac{502625}{2\pi \times \frac{750}{60}} = 6399.6 \text{ Nm}$$

EXAMPLE 3.49. *A synchronous generator has a synchronous reactance of 1.2 pu. It is running overexcited with an excitation voltage of 1.5 pu and supplies a synchronous power of 0.6 pu to the bus. If the prime mover torque is increased by 1%, by how much will the synchronous power P and reactive power Q change ?*

SOLUTION. For a cylindrical-rotor alternator synchronous power

$$P = \frac{E_f\,V}{X_s} \sin \delta$$

$$0.6 = \frac{1.5 \times 1}{1.2} \sin \delta$$

$$\sin \delta = \frac{1.2 \times 0.6}{1.5} = 0.48, \; \delta = 28.68°$$

If the prime mover torque is increased by 1%, there will be 1% increase in real power.

$$\therefore \qquad d\,P = 1\% \text{ of } P$$

$$= \frac{1}{100} \times 0.6 = 0.006 \text{ pu}$$

For a clindrical rotor machine, the reactive power Q is given by

$$Q = \frac{E_f\,V}{X_s} \cos \delta - \frac{V^2}{X_s}$$

$$\therefore \qquad \frac{d\,Q}{d\,\delta} = -\frac{E_f\,V}{X_s} \sin \delta$$

$$\text{Also,} \qquad \frac{d\,P}{d\,\delta} = \frac{E_f\,V}{X_s} \cos \delta$$

$$\therefore \qquad \frac{d\,Q}{d\,P} = -\tan \delta = -\tan 28.68° = -0.547$$

$$d\,Q = -0.547\,(1\%) = -0.547\%$$

Hence if the prime-mover torque is increased by 1% there will be 1% increase in real power and 0.547% decrease in reactive power.

EXAMPLE 3.50. *A synchronous machine has been synchronized with an infinite bus. Now, without changing the field current, the machine is made to deliver real power to the bus. Will it, at the same time, generate or consume reactive power ?*

SOLUTION. $Q = \frac{E_f V}{X_s} - \frac{V^2}{X_s}$

At synchronism, $E_f = V$

$$\therefore \quad Q = \frac{V^2}{X_s}(\cos\delta - 1)$$

Since the machine is supplying real power to the bus, δ cannot be zero. That is, $\cos\delta < 1$. Consequently, Q is negative. Hence the synchronous generator is consuming reactive power under the condition.

3.62 TRANSIENT CONDITIONS OF ALTERNATORS

A synchronous machine may be subjected to various disturbances. Any cause of disturbance will produce electrical and mechanical transients. These transients may result from switching ; from sudden changes of load ; from sudden short circuits between line and ground, between line and line or between all the three lines. These short circuits produce large mechanical stresses which may damage the machine. The machine may also lose synchronism.

It is necessary to calculate short-circuit currents under all fault conditions. The fault analysis enables us to select appropriate protective schemes, relays and circuit breakers in order to save the system from abnormal conditions within minimum time.

3.63 CONSTANT-FLUX LINKAGE THEOREM

The constant-flux linkage concept is of considerable importance in studying alternator transients. This concept is stated as follows :

The flux linkage after sudden disturbance in a closed circuit having zero resistance and zero capacitance remain constant at their predisturbed value.

There is no capacitance in the armature and field windings of an alternator. Their resistances are negligibly small in comparison with their inductances. Thus armature and field windings may be assumed to be purely inductive, and the flux linkages in the armature and field circuits cannot be changed suddenly by the application of the short circuit to the armature winding. Therefore, any abrupt change of current in one winding must be accompanied by a change of current in the other to keep the flux linkages constant.

3.64 PROOF OF CONSTANT FLUX-LINKAGE THEOREM

The mesh voltage equations for any circuit may be written in the form

$$\Sigma e = \Sigma i R + \Sigma N \frac{d\Phi}{dt} + \Sigma \frac{q}{c}$$

Using the symbol Ψ for the flux linkage ($N\Phi$), the equations may be written

$$\Sigma e - \Sigma i R - \Sigma \frac{q}{c} = \Sigma \frac{d\Psi}{dt} = \frac{d}{dt}\Sigma\Psi \quad \text{or} \quad \frac{d}{dt}\Sigma\Psi = e_1$$

where e_1 is the resultant voltage, which will, in general, be some function of time

Integrating this equation, the change in flux linkage from some arbitrarily chosen zero of time will be

$$\Delta \ (\Sigma \Psi) = \int_0^{\Delta t} e_1 \, dt$$

where Δt is a small interval of time. As Δt tends to zero, so will be integral ; hence

$$\Sigma \Psi = 0$$

i.e., the instantaneous change of flux linkage is zero.

3.65 SYMMETRICAL SHORT-CIRCUIT TRANSIENT

In order to get an idea about the transient phenomenon in a synchronous machine, it is useful to analyze the transient following a sudden three-phase short circuit at the armature terminals. This is the most severe transient condition that can occur in a synchronous generator. The machine is assumed to be initially unloaded and to continue operating at synchronous speed after the short circuit

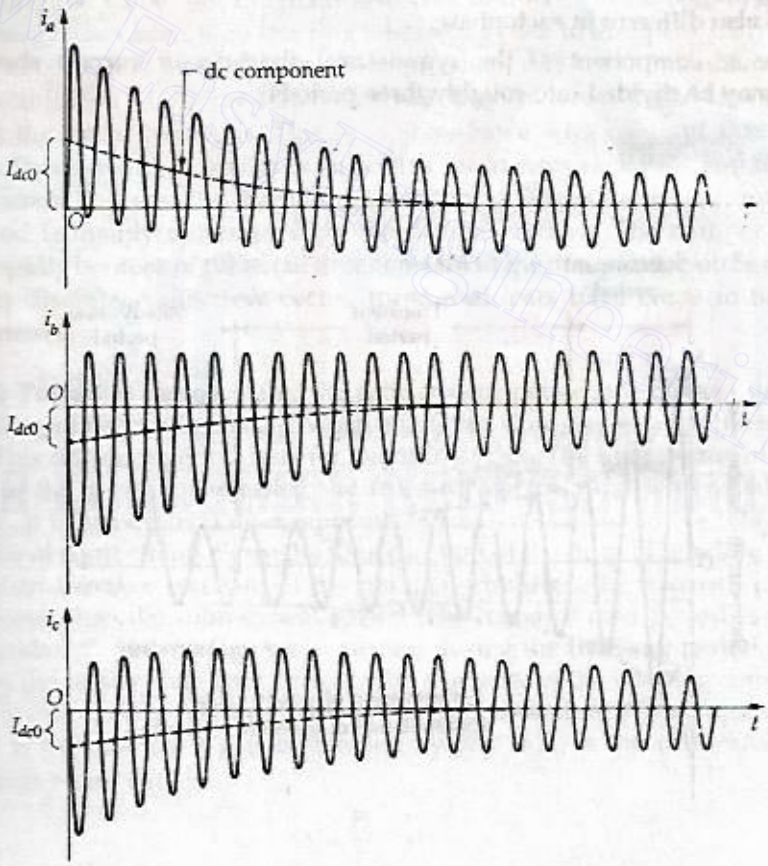


Fig. 3.57 *Transient armature currents in an alternator after a sudden 3-phase short circuit*

occurs. The machine is developing normal voltage under no-load condition such that the instantaneous values of 3-phase voltage are given by

$$e_A = E_m \sin \omega t$$

$$e_B = E_m \sin (\omega t - \frac{2\pi}{3})$$

$$e_C = E_m \sin (\omega t + \frac{2\pi}{3})$$

Since the machine is initially unloaded, the only predisturbance current in the machine is the field current. Each armature phase sees a resultant time-varying flux linkage as the rotor rotates.

If the stator terminals are now shorted, a large transient current will flow through them. According to constant-flux linkage theorem, currents flowing through armature windings maintain their flux linkages constant at the values which existed at the time of short circuit. The current in each phase consists of an *ac* component and a *dc* component as shown in Fig. 3.57. Since the voltages of the three phases are displaced by 120°, the short circuit occurs at different points on the voltage waves of each phase. Consequently, the *dc* component of armature current is also different in each phase.

The *ac* component of the symmetrical short-circuit current shown in Fig. 3.58 may be divided into roughly three periods.

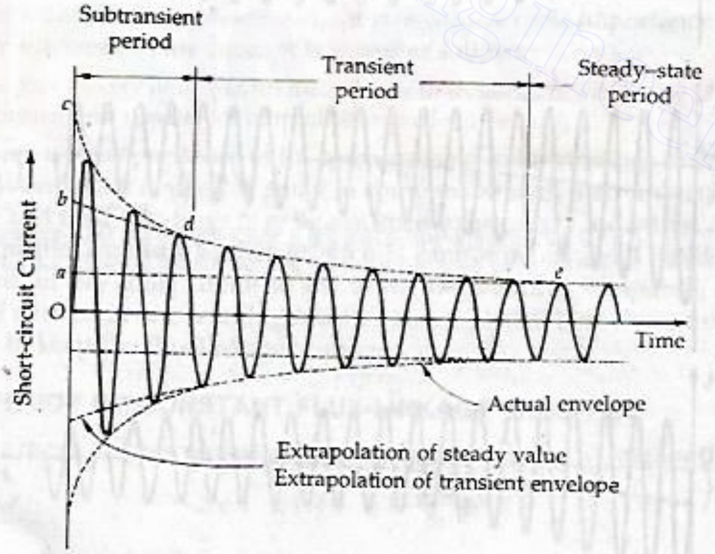


Fig. 3.58 *AC component of the symmetrical short-circuit current in an alternator*

(*a*) **Subtransient period.** This period lasts for only about 2 cycles after the fault occurs. The current is very large and during this period the current decays very rapidly. The r.m.s. value of initial current (that is, the current at the instant of short circuit) is called the ***subtransient current***. It is denoted by the symbol I''. The time constant of the subtransient current is given by the symbol τ'' and it can be determined from the slope of the subtransient current. The reactance of the winding corresponding to I'' is called the ***direct-axis subtransient reactance*** X''_d or simply the ***subtransient reactance***. This reactance is essentially due to the presence of damper windings.

If E_o is the r.m.s. value of the open-circuit phase voltage $X''_d = \dfrac{E_o}{I''}$

where I'' is the r.m.s. value of subtransient current without *dc* offset. This current is about 5 to 10 times the rated current.

During the subtransient period there is a large initial current in the armature. This current lags by almost 90° behind the voltage. It produces a large demagnetizing mmf in the direct axis tending to reduce the main field pole mmf from its original value. But the main field flux cannot decrease suddenly as the stored energy associated with this flux takes some time to dissipate. Currents are induced in both field and damper windings which will try to maintain the flux linkage conditions in the machine exactly as they were at the instant of short circuit at the stator terminals. This is in accordance with constant flux-linkage theorem. This in effect, is similar to large increase in rotor excitation and therefore a large current flows during the subtransient period. The armature current during this period is mainly determined by the damper current. The damper current decays rapidly because of the small time constant of the damper circuit. Since these transients disappear after few cycles, these transients have come to be called **subtransients.**

(*b*) **Transient period.** After the subtransient period, the current decreases slowly. The period of time during which it falls at a slow rate is called the transient period. This transient period lasts for about 30 cycles. The r.m.s. value of current flowing in the generator is called the ***transient current***, and is denoted by the symbol I'. It is caused by a dc component of current induced in the *field winding* at the time of short circuit. Since the time constant of the dc field winding is much greater than the time constant of the damper windings, the transient period is much greater than the subtransient period. The transient time period is denoted by the symbol τ'. The average r.m.s. current during the transient period is about five times the steady-state fault current. The reactance of the winding corresponding to I' is called the **direct-axis transient reactance** or simply the **transient reactance**. It is denoted by X'_d. It is denoted by X'_d. If E_o is the rms value of the open-circuit phase voltage

$$X'_d = \frac{E_o}{I'}$$

where I' is the rms value of the transient current without dc offset.

(*c*) **Steady-state period.** After the transient period, the fault current reaches its steady-state value. The steady-state current during a fault is denoted by the symbol I_{ss}. The corresponding reactance is called direct-axis *synchronous reactance* X_d.

$$X_d = \frac{E_o}{I_{ss}}$$

where I_{ss} = r.m.s. value of steady-state short-circuit current

E_o = r.m.s. value of the open circuit phase voltage that is, no-load line-to neutral r.m.s. voltage

The rms magnitude alternating fault current varies continuously with time. At any instant after a fault occurs at the terminal, of the generator, its magnitude is given by

$$I_{sc}(t) = (I'' - I')\, e^{-t/\tau''} + (I' - I_{ss})\, e^{-t/\tau'} + I_{ss}$$

where all quantities are in rms values and are equal but displaced 120 electrical degrees in the three phases.

3.66 THREE-PHASE SHORT CIRCUIT ON LOADED SYNCHRONOUS GENERATOR

Short circuits at the terminals of unloaded synchronous generators are very rare. They generally occur due to insulation failure or accidental damage on some part of the power system supplied by the generator. It is, therefore, important to deal with the case of a three-phase generator delivering power to a load or to an infinite bus. Short-circuit analysis of a loaded synchronous machine is quite complex and is beyond the scope of this book. The following description gives the simple method to determine the subtransient and transient currents without proof. However, the results are reasonably accurate.

If a short circuit is applied across the stator terminals, the short circuit armature current will pass through a subtransient period, and a transient period and finally will settle down to a steady-state condition. On the application of a short circuit, the machine reactance changes from X_d to X''_d. In order to satisfy the initial conditions of constant flux linkage, the excitation voltages must also change. The equivalent circuits (circuit models) during the three periods of the short circuit are shown in Fig. 3.59.

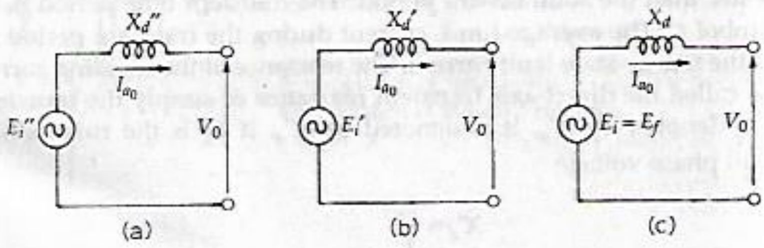


Fig. 3.59. *Circuit models for computing (a) subtransient current (b) transient current (c) steady-state current.*

Here the voltages E_i, E'_i and E''_i are the internal voltages.

They are found from the prefault conditions as follows :

Voltage behind subtransient reactance before fault

$$\mathbf{E}''_i = \mathbf{V}_o + j\,\mathbf{I}_{a_o}\,\mathbf{X}''_d \tag{3.66.1}$$

Voltage behind transient reactance before fault

$$\mathbf{E}'_i = \mathbf{V}_o + j\,I_{a_o}\,X'_d \tag{3.66.2}$$

Voltage behind synchronous reactance before fault

$$\mathbf{E}_i = \mathbf{E}_f = \mathbf{V}_o + j\,I_{a_o}\,X_d \tag{3.66.3}$$

where V_o is the machine terminal voltage and I_{a_o} is the prefault steady-state current.

The subtransient current during short circuit is

$$I'' = \frac{E''_i}{X''_d} \tag{3.66.4}$$

The transient current during short circuit is

$$I' = \frac{E'_i}{X'_d} \tag{3.66.5}$$

The steady-state short-circuit current is

$$I = \frac{E_f}{X_d} \tag{3.66.6}$$

The short-circuit current is given by

$$i_{sc} = \sqrt{2}\left[\frac{E_i}{X_d} - \left(\frac{E'_i}{X'_d} - \frac{E_i}{X_d}\right)e^{-t/\tau'_d} + \left(\frac{E''_i}{X''_d} - \frac{E'_i}{X'_d}\right)e^{-t/\tau''_d}\right]\sin\omega t + I_{dc0}\,e^{-t/\tau_a} \tag{3.66.7}$$

where $$\tau'_d = \frac{X'_d}{X_d}\tau'_{d_o} \tag{3.66.8}$$

$$\tau''_d = \frac{X''_d}{X'_d}\tau''_{d_o} \tag{3.66.9}$$

3.67 SHORT-CIRCUIT RATIO (SCR)

The **short-circuit ratio (SCR)** *of a synchronous machine is defined as the ratio of the field current required to generate rated voltage on open circuit to the field current required to circulate rated armature current on short circuit.*

The short-circuit ratio (SCR) can be calculated from the open-circuit characteristic (O.C.C.) at rated speed and short-circuit characteristic (S.C.C.) of a three-phase synchronous machine as shown in Fig. 3.60.

From Fig. 3.60,

$$\text{SCR} = \frac{I_f \text{ for rated O.C. voltage}}{I_f \text{ for rated S.C. current}} = \frac{Oa}{Od} \tag{3.67.1}$$

Since triangles Oab and Ode are similar,

$$\text{SCR} = \frac{Oa}{Od} = \frac{ab}{de} \tag{3.67.2}$$

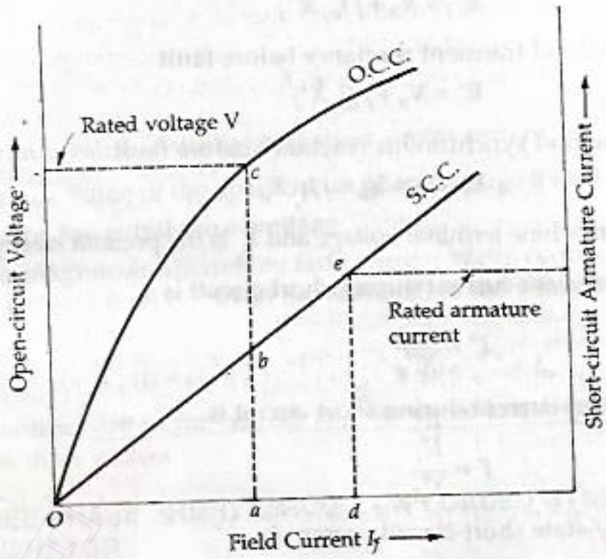


Fig. 3.60 *Determination of SCR*

The direct-axis synchronous reactance X_d is defined as the ratio of open-circuit voltage for a given field current to the armature short-circuit current for the same field current.

From Fig. 3.60 for a field current equal to Oa, the direct-axis synchronous reactance in ohms is given by

$$X_{d\,\Omega} = \frac{ac}{ab} \tag{3.67.3}$$

The per-unit value of X_d is given by

$$X_{d\,pu} = \frac{X_{d\,\Omega}}{\text{base impedance}} \tag{3.67.4}$$

But base impedance

$$= \frac{\text{per phase rated voltage}}{\text{per phase rated armature current}}$$

$$= \frac{V_{rated}}{I_{a\,rated}} = \frac{ac}{de}\,\Omega \tag{3.67.5}$$

$$\therefore \quad X_{d\,pu} = \frac{ac}{ab} \cdot \frac{de}{ac} = \frac{de}{ab} \tag{3.67.6}$$

From Eqs. (3.67.2) and (3.67.6)

$$\text{SCR} = \frac{ab}{de} = \frac{1}{(de/ab)} = \frac{1}{X_{d\,pu}} \tag{3.67.7}$$

Equation (3.67.7) shows that *the short-circuit ratio (SCR) is equal to the reciprocal of the per-unit value of the direct axis synchronous reactance.*

In a saturated magnetic circuit, the value of X_d depends upon the degree of saturation. It is to be noted that SCR is single valued because it pertains to the rated voltage on O.C.C. and rated armature current on S.C.C.

Significance of S.C.R.

The S.C.R. is an important factor for the synchronous machine. It affects the operating characteristics, physical size and cost of the machine. With a low value of the S.C.R. a synchronous generator has a large variation in terminal voltage with a change in load. That is, the machine is very sensitive to load variations. In order to keep the terminal voltage constant, field current is to be varied over a wide range. The synchronizing power is small if the S.C.R. is small. Since the synchronizing power keeps the machine in synchronism, a low value of the S.C.R. has a low stability limit. In other words, a machine with a low S.C.R. is less stable when operating in parallel with other generators. But the armature current under short-circuit conditions is small for a low S.C.R.

A synchronous machine with high value of S.C.R. has a better voltage regulation and improved steady-state stability limit but the short-circuit fault current in the armature is high.

The size and cost of the machine are also affected by the S.C.R. For a synchronous machine the excitation voltage E_f is given by

$$E_f = 4.44\, k_w f \Phi\, T_{ph}$$

For the same T_{ph},

$$E_f \propto \text{field flux per pole}$$

or

$$E_f \propto \frac{\text{field mmf per pole}}{\text{reluctance of air gap}}$$

Also, synchronous inductance

$$L_s \propto \frac{1}{\text{reluctance of air gap}}$$

$$\therefore \quad \text{SCR} \propto \frac{1}{L_s}$$

$$\propto \text{air gap reluctance or air gap length}$$

It follows that the S.C.R. may be increased by increasing the length of the air gap. With increased air-gap length, the field mmf is to be increased for the same E_f. In order to increase the field mmf either field current I_f or the number of field turns T_f is to be increased. This requires greater height of field poles. Consequently, the overall diameter of the machine increases. Thus, a large S.C.R. will increase the size, weight and cost of the machine.

Typical values of S.C.R. are as follows :

Cylindrical rotor machines 0.5 to 0.9

Salient-pole machines 1.0 to 1.5

Synchronous compensators 0.4

3.68 WINDING FACTORS FOR HARMONIC WAVEFORMS

In equation $E_p = 4.44\, k_w f \Phi\, T_p$ (3.68.1)

it is assumed that the induced voltage is sinusoidal. However, if the flux density distribution is non-sinusoidal, the induced voltage in the winding will be non-sinusoidal. The pitch factor, distribution factor and winding factor will be different for each harmonic voltage.

From the emf equation, the fundamental emf per phase is

$$E_{p_1} = 4.44\, k_{w_1} f \Phi_1 T_p \quad (3.68.2)$$

For third harmonic, emf per phase is

$$E_{p_3} = 4.44\, k_{w_3} (3f)\, \Phi_3 T_p \quad (3.68.3)$$

In general for n^{th} harmonic, emf per phase is

$$E_{p_n} = 4.44\, k_{wn} (nf)\, \Phi_n T_p \quad (3.68.4)$$

Here subscripts 1, 3 and n denote fundamental, third and n^{th} harmonics respectively.

$$\therefore \quad \frac{E_{p_n}}{E_{p_1}} = \frac{k_{w_n}}{k_{w_1}} \cdot \frac{n\,\Phi_n}{\Phi_1} \quad (3.68.5)$$

Φ_1 = total fundamental flux per pole

= (average flux density) × area under one pole

$$= \left(\frac{\text{peak flux density}}{\pi/2}\right) \times (\text{area under one pole})$$

$$= \left(\frac{B_{m_1}}{\pi/2}\right)\left(\frac{\pi DL}{P}\right)$$

$$\therefore \quad \Phi_1 = \frac{2DL}{P} B_{m_1} \quad (3.68.6)$$

where B_{m_1} = peak value of fundamental component of flux density wave

D = diameter of armature

= mean air gap diameter

L = axial length of armature

= active coil side length

Similarly, for n^{th} harmonic

$$\text{pole pitch} = \frac{\pi D}{P_n} \quad (3.68.7)$$

$$\Phi_n = \frac{2DL}{nP} B_{nm} \quad (3.68.8)$$

where B_{nm} = peak value of n^{th} harmonic flux density

$$\therefore \quad \frac{E_{p_n}}{E_{p_1}} = \frac{k_{wn}\, B_{mn}}{k_{w1}\, B_{m_1}} \quad (3.68.9)$$

3.69 COIL SPAN FACTOR FOR n^{th} HARMONIC

As the electrical angle is directly proportional to the number of poles $\left(\theta_e = \frac{P}{2}\theta_m\right)$ and the angle between the adjacent slots, the short pitch angle (chording angle) increases with an increase in the order of harmonics, n.

In a short pitch coil the chording angle is $\alpha°$ electrical for fundamental flux wave. For the n^{th} space field harmonic, the chording angle becomes $n\,\alpha°$ electrical. Therefore, the pitch factor for the n^{th} harmonic is

$$k_{cn} = \cos\frac{n\,\alpha}{2} \tag{3.69.1}$$

The harmonic voltages decrease in a short pitch coil, thereby improving the waveform of the induced voltage in the winding. Actually, a certain harmonic can be completely eliminated from the winding voltage by choosing a pitch for the coils that makes the pitch factor zero for that harmonic. In order to eliminate the n^{th} harmonic voltage

$$\cos\frac{n\,\alpha}{2} = 0$$

$$\cos\frac{n\,\alpha}{2} = \cos 90°$$

$$\frac{n\,\alpha}{2} = 90°$$

$$\alpha = \frac{180°}{n} \tag{3.69.2}$$

Thus, to eliminate the third harmonic, the coils should be shorted by $\alpha = \frac{180°}{3} = 60°$

3.70 DISTRIBUTION FACTOR FOR n^{th} HARMONIC

The phase difference between the n^{th} harmonic voltages of adjacent coils is $n\,\beta$. Therefore, the distribution factor for the n^{th} harmonic is

$$k_{dn} = \frac{\sin\frac{mn\,\beta}{2}}{m\sin\frac{n\,\beta}{2}} \tag{3.70.1}$$

3.71 WINDING FACTOR FOR n^{th} HARMONIC

The winding factor corresponding to the n^{th} harmonic voltage is

$$k_{wn} = k_{cn}\,k_{dn} \tag{3.71.1}$$

where k_{cn} and k_{dn} are given by Eqs. (3.69.1) and (3.70.1).

Therefore n^{th} order harmonic induced emf per phase

$$E_p = 4.44\,k_{cn}\,k_{dn}\,(nf)\,\Phi_n\,T_p \tag{3.71.2}$$

where $$\Phi_n = \frac{2\,DL}{nP}B_{mn} \tag{3.71.3}$$

In addition to the fundamental flux sine wave, the flux wave may consist of space-field harmonics also, which give rise to corresponding time harmonics in the generated emf wave. In other words, the induced voltage in a winding will contain harmonics because of nonsinusoidal space flux density distribution.

Since the positive and negative halves of flux density wave are identical, only odd harmonics can be present and even harmonics are absent. Therefore, phase voltage may contain third, fifth, seventh and higher order harmonics.

Three-phase alternators are invariably star connected. The third harmonic voltages of all the phases are equal in magnitude and phase, as the phase difference between any two phases is equal to $3 \times 120° = 360°$, that is, 0°. Since the phases of a star-connected machine are so connected that the voltage across any two lines is the phasor difference in voltages of corresponding phases, there cannot be any third harmonic in the line-to-line of a star-connected synchronous machine. For the same reason, all harmonics which are multiples of third, do not appear in the line-to-line voltage. Thus, triplens (that is, third and its multiples) are absent in the line voltage of star-connected synchronous machines. Since the strength of harmonic components of voltage decreases with increasing frequency, only fifth and seventh harmonics are important. These are known as ***belt harmonics***.

Thus, rms voltage of the induced voltage across lines of a 3-phase, star-connected machine is given by

$$E_{line} = \sqrt{3} \times \sqrt{E_1^2 + E_5^2 + E_7^2 + E_{11}^2 + ...} \quad (3.71 A)$$

where subscripts 1, 5, 7, 11, ... denote fundamental, fifth, seventh, eleventh harmonics respectively.

EXAMPLE 3.51. *A 4-pole ac machine has a 3-phase winding wound in 60 slots. The coils are short pitched in such a way that if one coil side lies in slot number 1, the other side of the same coil lies in slot number 13. Calculate the winding factor for (a) fundamental, (b) third harmonic and (c) fifth harmonic frequency waveforms.*

SOLUTION. m = slots per pole per phase

$$= \frac{\text{slots}}{\text{poles} \times \text{phases}} = \frac{60}{4 \times 3} = 5$$

$$\beta = \text{slot angle}$$

$$= \frac{180° \times \text{poles}}{\text{slots}} = \frac{180° \times 4}{60} = 12°$$

$$\text{slots per pole} = \frac{\text{slots}}{\text{poles}} = \frac{60}{4} = 15$$

For full-pitch coil, the coil span is 15 slots. For the given coil the coil span is $13 - 1 = 12$ slot angles $= 12\,\beta$

$$\therefore \quad \alpha = (15 - 12) \text{ slot angles} = 3\,\beta = 3 \times 12° = 36°$$

(a) Fundamental-frequency waveform

$$\text{Coil-span factor } k_{c_1} = \cos\frac{\alpha}{2} = \cos\frac{36}{2} = 0.951$$

Distribution factor

$$k_{d_1} = \frac{\sin (m\,\beta/2)}{m \sin (\beta/2)} = \frac{\sin (5 \times 12/2)}{5 \sin (12/2)} = 0.957$$

Winding factor

$$k_{w_1} = k_{c_1} k_{d_1} = 0.951 \times 0.957 = 0.91$$

(b) *Third-harmonic frequency waveform*

$$k_{c_3} = \cos (3\,\alpha/2) = \cos (3 \times 36/2) = 0.588$$

$$k_{d_3} = \frac{\sin (m \times 3\,\beta/2)}{m \sin (3\,\beta/2)} = \frac{\sin (5 \times 3 \times 12/2)}{5 \sin (3 \times 12/2)} = 0.647$$

$$k_{w_3} = k_{c_3} \times k_{d_3} = 0.588 \times 0.647 = 0.38$$

(c) *Fifth-harmonic frequency waveform*

$$k_{c_5} = \cos (5\,\alpha/2) = \cos (5 \times 36/2) = 0$$

$$k_{d_5} = \frac{\sin (m \times 5\,\beta/2)}{m \sin (5\,\beta/2)} = \frac{\sin (5 \times 5 \times 12/2)}{5 \sin (5 \times 12/2)} = 0.2$$

$$k_{w_5} = k_{c_5} \times k_{d_5} = 0 \times 0.2 = 0$$

3.72 COOLING OF SYNCHRONOUS GENERATORS

Natural cooling is not adequate to dissipate great amount of heat produced in alternators. In the ***forced air-cooling system***, air is forced into the alternator so that greater quantity of air is passed over the surface and greater heat is removed. For still better cooling ***closed-circuit ventilation system*** is used. In this totally enclosed air system, clean hot air from the alternator is cooled by water-cooled heat exchanger and forced through the alternator by fans.

For increasing the surface area in contact with the cooling air, ducts are provided in the stator and rotor cores and sometimes in field coils of machines. These ducts can either be radial or axial depending upon the direction of air flow in them.

In the ***radial flow ventilation system*** the cooling air enters the ducts through stator by way of air gap and passes radially to the back of the stator from where it is removed.

Advantages of radial ventilation

1. Minimum energy losses for ventilation and sufficiently uniform temperature rise of the machine in axial direction.
2. This system is applicable both to small and large machines.

Limitations of radial ventilation

1. It makes the machine less compact since ventilating ducts occupy about 20 per cent of the armature length.
2. The heat dissipation is less than that in other systems and the system in certain cases is unstable to quantity of cooling air flowing through the machine.

3.73 AXIAL FLOW VENTILATING SYSTEM

In the method air is forced in the axial direction through passages formed by the holes in stator and rotor.

It is highly effective except for machines with considerable axial length. The *disadvantage* of axial ventilation is nonuniform heat transfer. The air outlet part of machine is cooled less because air in passing through the axial ducts has time to become heated.

Combined radial and axial ventilation systems also find application.

3.74 CIRCUMFERENTIAL VENTILATION

In this method air is supplied at one or more points on the outer periphery of the stator core and forced circumferentially through the ducts between the laminations to suitable outlets. But in this method the duct area can be increased.

In some cases this method is combined with the radial flow system but the resultant interference in the two streams of air have to be avoided. For this alternating radial ducts are closed at the outer surface.

3.75 REQUIREMENTS OF COOLING AIR

The cooling air should be free from dust and soot specially in industrial surroundings. These will clog the ducts to reduce area which results in reducing heat transfer by conduction. Air filters are used. Cheese cloth filters are generally used which can be renewed frequently. Sometimes air may have to be washed in a spray chamber. In most cases air is cooled by water coolers and used again.

3.76 LIMITATIONS OF AIR COOLING

1. For large-capacity machines, the sizes of fans required for circulation of air increase and require considerable power and corresponding expensive auxiliary equipment.
2. There is an ultimate rating of machine beyond which air cooling will not be able to keep the temperature within safe limits.

3.77 HYDROGEN COOLING

Hydrogen gas is used as a cooling medium in the generator casing because of its superior cooling properties.

Advantages of hydrogen cooling over air cooling

1. *Cooling*. Hydrogen gas has a higher thermal conductivity and 1.5 times heat transfer compared with air. Therefore cooling with hydrogen gas is faster.
2. *Windage, efficiency and noise*. The density of hydrogen is about (1/14) times the density of air at the same temperature and pressure. Since the revolving parts rotate in low-density hydrogen, windage loss and noise produced in the machine are reduced. The efficiency of the machine also increases.
3. *Corona*. When air is used as a cooling medium in generators, corona discharge may take place to produce ozone, oxides of nitrogen, nitric acid etc., which damage the insulation. With hydrogen, cooling corona does not occur and therefore the life of insulation is increased.

Certain mixtures of hydrogen and air are explosive. Explosion may take place with a range of 6 per cent hydrogen and 94 per cent air upto 71 per cent hydrogen and 29 per cent air. When there is more than 71 per cent hydrogen, the mixture is neither combustible nor supporter of combustion. In practice 9 : 1 ratio of hydrogen to air is used in very large turboalternators.

To prevent an explosive mixture of hydrogen and air from occurring in the generator, the hydrogen gas is maintained at a pressure above atmosphere to prevent inward seepage of contaminating air. Hydrogen cooling at 1, 2 and 3 times the atmospheric pressure can raise the rating of the generator by 15, 30 and 40 per cent respectively above its air-cooled rating.

Hydrogen cooling requires completely seated circulating system. Special oil-seated glands are used between shaft and casing. Since oil absorbs both hydrogen leaking out and air leaking in, it is purified periodically.

The hydrogen gas is circulated by blowers and fans through rotor and stator and then it is passed over cooling coils inside the seated casing. The coils carry oil or water to extract heat from the circulating hydrogen.

Hydrogen cooling increases the overall full-load efficiency of generator by about 1 per cent, but increases the generator capacity by about 25 per cent of the generator of the same physical size using air. The latter is the main reason that justifies the use of hydrogen cooling.

Limitations of hydrogen cooling

1. The frame of hydrogen-cooled alternator is more costly because of necessity to provide explosion-proof construction and gas-tight shaft seals.
2. Means are necessary to admit hydrogen to the alternator without creating explosion. It is done by either (*a*) scouring the air with CO_2 and then admitting hydrogen or (*b*) by vacuum pumping the unit to (1/5) atmosphere and admitting hydrogen.
3. Cooling coils carrying oil or water inside the casing are to be provided to extract heat from hydrogen.

3.78 DIRECT WATER COOLING

Hydrogen cooling is not sufficient to extract heat generated in large turboalternators of sizes of 500 MW or more. For such large machines, the volume of hydrogen gas required may be so large that its use may become uneconomical. Moreover, the middle portion of longer rotors may not be cooled effectively with hydrogen.

In such cases direct water cooling is also used. In very large turbogenerators, rotors are direct hydrogen cooled and stator windings are direct demineralised water cooled. Water is circulated by an ac motor centrifugal pump. Cartridge filters are used to filter water. These filters are designed to prevent metallic corrosive particles generated in winding and piping from entering into winding hollow conductors.

Advantages of using water cooling over hydrogen cooling

1. Thermal conductivity of water is higher than that of hydrogen. Therefore, water cooling is faster and more efficient.
2. The duct area of water is smaller to allow more space for conductor in the slot.

Disadvantages of water cooling

1. Water should be highly purified so that the conductivity of water does not increase.
2. Water cooling is costlier than hydrogen cooling.

EXERCISES

3.1 Why is a rotating field system used in preference to a stationary field ?

A 6-pole alternator rotates at 1000 r.p.m. What is the frequency of the generated voltage ? [50 Hz]

3.2 Explain the essential difference between cylindrical (smooth) and salient-pole rotors used in large alternators. What type of rotor would you expect to find in (i) a 2-pole machine, (ii) a 12-pole machine ?

At what speed would each of the machines be driven in order to produce a frequency of 50 Hz ? [3000 r.p.m.; 500 r.p.m]

3.3 Deduce the expression showing the relationship between speed, frequency and number of poles of a synchronous machine.

What frequency is generated by a 6-pole alternator that rotates at 1200 r.p.m ? [60 Hz]

3.4 A waterwheel alternator has 20 poles. Calculate the speed for a frequency of 50 Hz. [300 r.p.m.]

3.5 By means of a neat diagram, describe the main parts of an alternator with their functions.

3.6 Describe the difference in construction of rotors of alternators used in hydroelectric plants and steam plants.

Draw neat sketches of the two types of rotor.

3.7 Explain the different methods of excitation system of alternators.

3.8 Derive e.m.f. equation for an alternator. Explain clearly the meaning of (a) distribution factor and (b) coil-span factor. Give expressions for them.

3.9 Explain the effect of distribution of winding and use of short-pitch coil on the magnitude of the generated voltage of an alternator.

3.10 Explain how rotating magnetic fields are produced by (a) two-phase currents, (b) three-phase currents.

3.11 Write short notes on (a) two-phase rotating field, (b) three-phase rotating filed.

State how the direction of rotation of a rotating magnetic field may be changed.

3.12 Explain the terms coil-span factor and distribution factor in connection with alternator armature windings and deduce the e.m.f. equation of an alternator incorporating the effects of these factors.

3.13 The stator of a 3-phase, 8-pole, 750 r.p.m. alternator has 72 slots, each of which contains 10 conductors. Calculate the r.m.s. value of the e.m.f. per phase if the flux per pole is 0.1 Wb sinusoidally distributed. Assume full-pitch coils and a winding distribution factor of 0.96. [2557 V]

3.14 A 3-phase alternator has windings distributed in 36 slots around stator circumference. Each winding is made up of full-pitched coils formed from 40 conductors accommodated in each slot. A 4-pole rotor is driven at 25 r.p.s and the resultant air gap flux is sinusoidally distributed. Total flux per pole is 0.2 Wb. Calculate the breadth factor and the voltage generated in each phase. [0.979; 10.44 kV]

3.15 A 3-phase, star-connected alternator has the following data :
Voltage generated on open circuit = 4000 V ; speed = 500 r.p.m ; frequency = 50 Hz; stator slots per pole per phase = 3, conductors per slot = 12. Calculate : (*a*) the number of poles ; (*b*) the useful flux per pole. Assume all conductors per phase to be connected in series and coil to be full pitch. [(*a*) 12; (*b*) 0.0502 Wb]

3.16 A 10 MVA, 1 kV, 50 Hz, 3-phase, star-connected synchronous generator is driven at 300 r.p.m. The armature winding is housed in 360 slots with 6 conductors per slot. The coil span is five-sixth of a pole pitch. Calculate the flux per pole required to give 11 kV line voltage on open circuit. [0.086 Wb]

3.17 A star-connected, 3-phase, 4-pole, 50 Hz alternator has a single layer winding in 24 stator slots. There are 50 turns in each coil and the flux per pole is 0.05 Wb. Find the open-circuit voltage. [3715 V]

3.18 A 400 V, 50 kVA, 50 Hz, 3-phase, star-connected alternator has the armature effective resistance of 0.1 Ω per phase. An excitation of 2.5 A produces on open circuit on e.m.f. of 130 V (line). The same excitation produces a current of 90 A on short circuit.
Calculate : (*a*) the synchronous impedance and reactance; (*b*) the full-load regulation of the alternator for (*i*) 0.866 lagging power factor; (*ii*) unity power factor.
[(*a*) $Z_s = 0.834\ \Omega$, $X_s = 0.828\ \Omega$; (*b*) (*i*) 17.3% (*ii*) 5.1 %]

3.19 A 1500 kVA, 6600 V, 3-phase, star-connected alternator with a resistance of 0.4 Ω and a reactance of 6 Ω per phase, delivers full load current at power factor 0.8 lagging, and normal rated voltage. Estimate the terminal voltage for the same excitation and load current at 0.8 power factor leading. [8220 V]

3.20 A 3-phase, 8-pole, 60-Hz, star-connected, salient-pole synchronous generator has 96 slots, with 4 conductors per slot connected in series in each phase. The coil pitch is 10 slots. If the maximum value of the airgap flux is 60 mWb and the flux-density distribution in the airgap is sinusoidal, determine (*a*) the rms phase voltage and (*b*) the rms line voltage (*c*) If each phase is capable of carrying 650 A current, what is the kVA rating of the machine ?
[(*a*) 946.7 V (*b*) 1639.7 V, (*c*) 1846 kVA]

3.21 A 3-phase, 60-Hz, star-connected armature winding of a generator has 6 slots per pole per phase. The pole pitch is 10 slots and the coil pitch is 9 slots. The winding is double layer and has 30 turns per phase. If the airgap flux is sinusoidally distributed, what must be its maximum value to give 600 V across the lines ?
[45.89 mWb]

3.22 In a 60 kVA, 220 V, 50 Hz, single-phase alternator, the effective armature resistance and leakage reactance are 0.016 Ω and 0.07 Ω respectively. Calculate the voltage induced in the armature when the alternator is delivering rated current at a load power factor of (*a*) unit, (*b*) 0.7 lagging and (*c*) 0.7 leading.
[(*a*) 225.2 V, (*b*) 236.9 V, (*c*) 210 V]

3.23 The effective resistance of a 2200 V, 440 kVA, single-phase alternator is 0.5 Ω. On short circuit, a field current of 40 A gives the full-load current. The emf on open circuit for the same field current is 1160 V. Calculate the synchronous impedance and reactance. [5.8 Ω, 5.778 Ω]

3.24 From the following test results, determine the voltage regulation of a 2000 V 1 – ϕ alternator delivering a current of 100 A at (*a*) A unit pf, (*b*) 0.8 leading pf and (*c*) 0.71 lagging pf.

Test results. Full-load current of 100 A is produced on short circuit by a field excitation of 2.5 A. An emf of 500 V is produced on open circuit by the same excitation. The armature resistance is 0.8 Ω. [(*a*) 7%, (*b*) – 9%, (*c*) 21

3.25 A 3-phase, 11 kV star-connected alternator supplies a load of 10 MW at pf of lagging. Calculate the generated voltage if the armature resistance is 0.1 Ω per phase and the synchronous reactance is 0.66 Ω per phase is 0.66 Ω. [11.475

3.26 Explain the phenomena of armature reaction when an alternator is delivering load current at (*a*) purely lagging pf, (*b*) unity pf and (*c*) purely leading pf.

3.27 Explain the concept of replacing the armature reaction by a reactance.

3.28 What do you mean by synchronous reactance ? Explain the term synchronous impedance of an alternator.

3.29 Define voltage regulation of an alternator. Explain the various factors which affect the regulation of an alternator.

3.30 Draw the phasor diagram of a loaded alternator for the following conditions (*a*) lagging power factor, (*b*) leading power factor, and (*c*) unity power factor.

3.31 What is armature reaction ? Explain the effect of armature reaction on the terminal voltage of an alternator at (*i*) unity power factor load, (*ii*) zero lagging pf load and (*iii*) zero leading pf load. Draw the relevant phasor diagrams.

3.32 Name and explain the factors responsible for making terminal voltage of an alternator less than the induced voltage.

3.33 Sketch and explain the open-circuit and short-circuit characteristics of a synchronous machine. How voltage regulation can be calculated by the use of their results

3.34 Define the terms synchronous impedance and voltage regulation of an alternator. Explain the synchronous impedance method of determining regulation of an alternator. State the assumptions made in the synchronous impedance method.

3.35 Explain the terms unsaturated synchronous reactance and saturated synchronous reactance.

3.36 Explain how open-circuit and short-circuit tests are conducted on a synchronous machine. What is an air-gap line ?

3.37 In an alternator, explain why short-circuit characteristic is a straight line whereas open-circuit characteristic is a curve.

3.38 Explain why synchronous-impedance method of computing the voltage regulation leads to a pessimistic value at lagging power factor loads.

3.39 What is synchronous impedance ? How can it be measured in laboratory ?

3.40 Explain the synchronous impedance method of determining the voltage regulation of an alternator. Comment on the merits and limitations of this method. Why this method is considered as pessimistic method ?

3.41 Explain the MMF method of determining the voltage regulation of alternator.

3.42 Explain the Potier-triangle method of determining the voltage regulation of an alternator.

3.43 Define the terms synchronous reactance and voltage regulation of an alternator.

3.44 Derive the phasor diagram of a cylindrical rotor alternator. What is the effect of armature reaction and how is it included in the phasor diagram ? Draw phasor diagrams for lagging, unity and leading power factors.

3.45 Compare synchronous impedance method and ampere-turn method of predetermining regulation of alternators.

3.46 A 6600-V, star-connected, three-phase non-salient pole synchronous generator has the following open-circuit characteristic :

Phase voltage (V)	2600	3500	4130	4600	5000	550
Field current (A)	100	150	200	250	300	400

Full-load current on short circuit is obtained with an excitation of 175 A. Using the ampere-turn method, determine the full-load regulation when the power factor is 0.9 lagging. The resistance drop is negligible and the reactive drop is 10 per cent on full load. [31.9%]

3.47 An 11-kV, three-phase cylindrical-rotor type alternator has the following open-circuit characteristic at rated speed :

Line Voltage (V)	. . . .	7300	10300	12400	14000
Field Current (A)	. . . .	40	60	80	100

The excitation to produce full-load current on short circuit is 34 A, and when the machine supplies full-load output at 11 kV and zero power factor, the excitation is 106 A.

Determine :

(*a*) The percentage synchronous-reactance drop.

(*b*) The percentage leakage-reactance drop.

(*c*) The armature reaction in equivalent field amperes at full load. Neglect the armature resistance. [57%, 12.7%, 26 A]

3.48 An 11-kV, 1000-kVA, three-phase, star-connected alternator has a resistance of 2 Ω per phase. The open-circuit curve and the characteristic with rated full-load current at zero power factor are given in the following table. Find the voltage regulation of the alternator for full-load current at power factor of 0.8 lagging.

Field Current (A)	. . .	40	50	110	140	180
Lime Volts	. . .	5800	7000	12500	13750	15000
Lime volts zero p.f.	. . .	0	1500	8500	10550	12500

[21.4%]

3.49 A 6000 V, star-connected, three-phase cylindrical-rotor-type alternator has the following open-circuit characteristic at rated speed :

Voltage (lime-to-star point) (V)	1500	2600	3500	4150
Field Current (A)	25	50	75	100

Full-load current on steady-state short circuit is obtained with an excitation of 87.5 A.

Assuming the resistance drop to be negligible, and the leakage reactance 20%, find the excitation to produce full-load current at rated voltage at a power factor of 0.9 lagging. [158 A]

3.50 Explain the terms direct-axis synchronous reactance and quadrature-axis synchronous of a salient-pole alternator. Upon what factors do these values depend ?

3.51 Describe the slip test method for the measurement of X_d and X_q of synchronous machines.

3.52 Explain the two reaction theory applicable to salient-pole synchronous mach

3.53 Derive an expression for finding regulation of salient-pole alternator using reaction theory. Draw its phasor diagram.

3.54 Discuss Blondel's two-reaction theory of salient-pole synchronous machines.

3.55 Draw and explain the phasor diagram of a salient-pole synchronous gen supplying a lagging power factor load.

3.56 Draw and explain the phasor diagram of a salient-pole synchronous gen supplying full-load lagging current. Show that the power output per pha given by

$$P = \frac{VE_f}{X_d}\sin\delta + \frac{V^2}{2}\left[\frac{1}{X_q} - \frac{1}{X_d}\right]\sin 2\delta$$

3.57 For a salient-pole synchronous machine, neglecting the effect of armature tance, derive an expression for power developed as a function of load angle

3.58 What is the capability curve of a synchronous generator ? What information available from this curve ?

3.59 Show that for alternators running in parallel, the division of load between is governed mainly by the speed load characteristics of their prime movers.

3.60 Explain why prime movers driving alternators operating in parallel should drooping speed-load characteristics.

3.61 What is the necessity of parallel operation of alternators ?

3.62 State the conditions necessary for paralleling alternators.

3.63 What do you mean by synchronizing of alternators ? Describe any one meth synchronizing.

3.64 What are the various methods of synchronizing alternators ?

3.65 Explain the following :

(*a*) Why bright lamp of synchronizing is preferred over dark lamp meth

(*b*) How do synchronizing lamps indicate the phase variation of the inco machine and the running machine ?

3.66 The governors on the prime movers of two 1000 kW alternators running in par are so adjusted that the frequency of one of the alternators drops from 51 H 48.5 Hz, an that of other drops from 51 Hz to 49 Hz. Calculate (*a*) the load each machine when the total load is 1250 kW and (*b*) the frequency at this lo
[(*a*) 695 kW, 555 kW (*b*) 49.61 H

3.67 Two 50 MVA, 3-phase alternators operate in parallel. The settings of gover are such that the rise in speed from full load to no load is 2 per cent in one mach and 3 per cent in one machine and 3 per cent in the other, the speed-load chara teristics being straight lines in both cases. If each machine is fully loaded, wh the total load is 100 MW, what would be the load on each machine when the t load is 60 MW ? [26 MW, 34 M

3.68 Two 1000 kVA, 3-phase alternators are running in parallel. The setting of th governors is such that rise of speed from full load to no load of machine A i per cent and that of machine B is 3 per cent, the speed-load characteristics be straight lines in both cases.

(*a*) If both machines are fully loaded when the total load is 2000 kVA, find th load on each machine when the total load is 1166.6 kVA.

(*b*) Also find the load at which one machine ceases to supply any load.

[(*a*) $S_A = 500$ kVA, $S_B = 666.6$ kVA ; (*b*) $S_A' = 333.3$ k

3.69 Two exactly similar 3000 kVA synchronous generators operate in parallel. The governor of the first machine is such that the frequency drops uniformly from 50 Hz on load to 48 Hz on full load. The corresponding uniform speed drop of second machine is from 50 Hz to 47.5 Hz. (*a*) How will the two machines share a load of 5000 kW ? (*b*) What is the maximum load at unity power factor that can be delivered without overloading either machine ? [(*a*) 2777 kW, 2223 kW (*b*) 5400 kW]

3.70 What is an infinite bus ? State the characteristics of an infinite bus. What are the operating characteristics of an alternator connected to an infinite bus ?

3.71 Show that the behaviour of a synchronous machine on infinite bus is quite different from its isolated operation.

3.72 Show that in order to obtain a constant-voltage, constant-frequency of a practical bus bar system, the number of alternators connected in parallel should be as large as possible.

3.73 What conditions must be fulfilled before an alternator can be connected to an infinite bus ?

3.74 Define synchronizing power coefficient. State its units. What is its significance ?

3.75 A synchronous generator operates on constant-voltage constant frequency busbars. Explain the effect of variation of (*a*) excitation and (*b*) steam supply on power output, power factor, armature current and load angle of the machine.

An 11 kV 3 phase star-connected synchronous generator delivers 4000 kVA at unit power factor when running on constant voltage constant frequency busbars. If the excitation is raised by 20%, determine the kVA and power factor at which the machine now works. The steam supply is constant and the synchronous reactance is 30 Ω/ phase. Neglect power losses and assume the magnetic circuit to be unsaturated. [4280 kVA, 0.935 lagging]

3.76 Show that the maximum power that a synchronous generator can supply when connected to constant voltage constant frequency busbars increases with the excitation.

An 11 kV 3 phase star-connected turbo-alternator delivers 240 A at unity power factor when running on constant voltage and frequency busbars. If the excitation is increased so that the delivered current rises to 300 A, find the power factor at which the machine now works and the percentage increase in the induced emf assuming a constant steam supply and unchanged efficiency. The armature resistance is 0.5 Ω per phase and the synchronous reactance 10 Ω per phase.

[0.802 lagging ; 24 per cent]

3.77 An 11 kV 300 MVA 3-phase alternator has a steady short-circuit current equal to half its rated value. Determine graphically or otherwise the maximum load the machine can deliver when connected to 11 kV constant voltage constant-frequency busbars with its field excited to give an open circuit voltage of 12.7 kV per phase. Find also the armature current and power factor corresponding to this load. Ignore armature resistance. [300 MW ; 17.4 kVA ; 0.895 leading]

3.78 An alternator having a synchronous impedance of $(R + jX)$ ohms per phase is supplying constant voltage and frequency busbars. Describe, with the aid of phasor diagrams, the changes in current and power factor when the excitation is varied over a wide range, the steam supply remaining unchanged. The phasor diagrams should show the bars of the induced emf.

3.79 A star-connected alternator supplies 300 A at unity power factor to 6600 V constant voltage and frequency busbars. If the induced e.m.f. is now reduced by 20 per cent, the steam supply remaining unchanged, determine the new values of the current and power factor. Assume the synchronous reactance is 5 Ω/phase, the resistance is negligible and the efficiency constant. [350 A ; 0.85 leading]

3.80 Deduce an expression for the synchronizing torque on no load of a 3-phase synchronous machine in terms of the line voltage V, the short-circuit line current I_{SC}, the electrical angle of displacement θ, and the speed n in revolution per sec

[$\sqrt{3}\ VI_{Sc}\ (\theta/2\pi n)$ Nm

3.81 Calculate the synchronizing power in kilowatts per degree of mechanical displacement at full load for a 1000 kVA, 6600 V, 0.8 power factor, 50 Hz, 8-pole, star-connected alternator having a negligible resistance and a synchronous reactance of 60%. [158 kW per mechanical degree

3.82 Calculate the value of synchronizing power in kilowatts for 1 mechanical degree of displacement at full load, 0.8 power factor (lagging) for a 3-phase, 1000 kVA, 3300 V, 50 Hz, 500 r.p.m. machine having a synchronous reactance of 20% and negligible resistance. [585 kW per degree

3.83. A 40 MVA 50 Hz 3000 r.p.m. turbine driven alternator has an equivalent moment of inertia of 1310 kgm^2, and the machine has a steady short-circuit current of five times its normal full-load current

Deduce any formula used, estimate the frequency at which hunting may take place when the alternator is connected to an infinite grid system. [31.4 Hz

3.84. A 500 kVA, 3-Phase, 6-pole, 11 kV star-connected alternator is running in parallel with other synchronous machines on 11000 V bus. The synchronous reactance of the machines is 5 Ω per phase. Calculate the synchronizing power per mechanical degree at full load and 0.8 power factor lagging. [1423.8 kW ; 13596 Nm

3.85 A 10 MVA, 3-phase alternator has an equivalent short-circuit reactance of 20%. Calculate the synchronizing power of the armature per mechanical degree of phase displacement when running in parallel with 10000 V, 50 Hz busbars at 1500 r.p.m. [1745.3 kW

3.86 State and prove constant-flux-linkage theorem.

3.87 Discuss the phenomenon of sudden 3-phase short-circuit at the armature terminals of an alternator. Draw a typical waveshape of current and mark the different regions. Write an expression for the current.

3.88 What is short-circuit ratio ? Why are modern alternators designed with high short-circuit ratio ?

3.89 Show that the short-circuit ratio of a synchronous generator is the reciprocal of per-unit value of synchronous reactance adjusted to saturation at rated voltage.

3.90 Describe different methods of cooling alternators. What are the advantages of hydrogen as a cooling medium as compared to air ?

What special precaution should be taken for hydrogen cooled alternators ?

4

Three-Phase Induction Motors

4.1 INTRODUCTION

Three-phase induction motor is the most popular type of a.c. motor. It is very commonly used for industrial drives since it is cheap, robust, efficient and reliable. It has good speed regulation and high starting torque. It requires little maintenance. It has a reasonable overload capacity.

4.2 CONSTRUCTION

A three-phase induction motor essentially consists of two parts : the *stator* and the *rotor*. The stator is the stationary part and the rotor is the rotating part. The stator is built up of high-grade alloy steel laminations to reduce eddy-current losses. The laminations are slotted on the inner periphery and are insulated from each other. These laminations are supported in a stator frame of cast iron or fabricated steel plate. The insulated stator conductors are placed in these slots. The stator conductors are connected to form a three-phase winding. The phase winding may be either star or delta-connected [Fig. 4.1(*a*)].

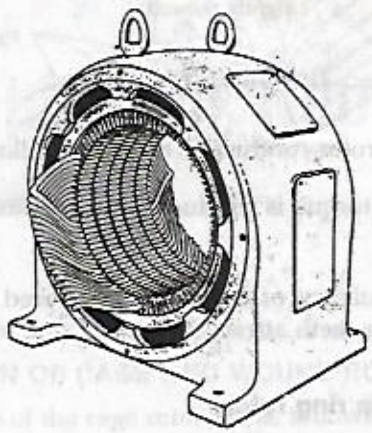


Fig. 4.1(a). *Induction Motor Stator with double-layer winding partly wound.*

The **rotor** is also built up of thin laminations of the same material as stator. The laminated cylindrical core is mounted directly on the shaft or a spider carried by the shaft. These laminations are slotted on their outer periphery to receive the rotor conductors. There are two types of induction motor rotors :

(*a*) Squirrel-cage rotor or simply cage rotor.

(*b*) Phase wound or wound rotor. Motors using this type of rotor are also called slip-ring motors.

4.2.1 Cage rotor

It consists of a cylindrical laminated core with slots nearly parallel to the shaft axis, or *skewed*. Each slot contains an uninsulated bar conductor of aluminium or copper. At each end of the rotor, the rotor bar conductors are short-circuited by heavy end rings of the same material. The conductors and the end rings form a cage of the tyre which was once commonly used for keeping squirrels ; hence its name. A cage rotor is shown in Fig. 4.1(*b*).

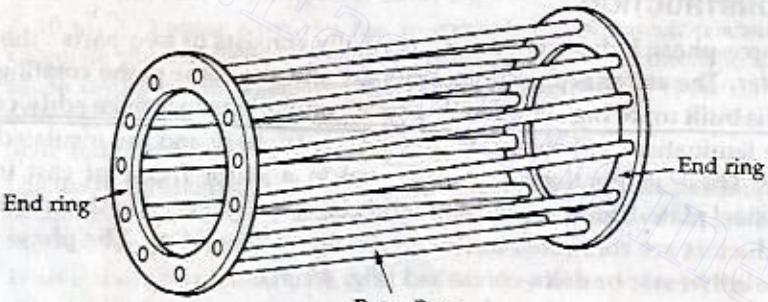


Fig. 4.1(b). *Cage rotor.*

The skewing of cage rotor conductors offers the following advantages :

1. More uniform torque is produced and the noise is reduced during operation.

2. The *locking* tendency of the rotor is reduced. During locking, the rotor and stator teeth attract each other due to magnetic action.

4.2.2 Wound rotor or slip ring rotor

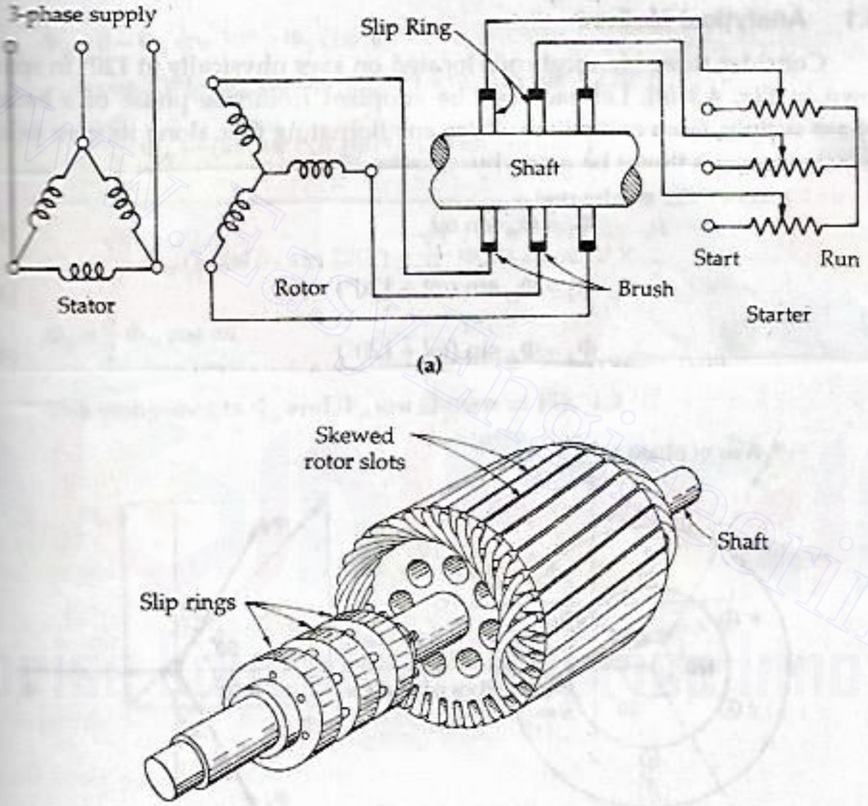
The wound rotor consists of a slotted armature. Insulated conductors are put in the slots and connected to form a three-phase double layer distributed winding similar to the stator winding. The rotor windings are connected in star

The open ends of the star circuit are brought outside the rotor and connected to three insulated slip rings. The slip rings are mounted on the shaft with brushes resting on them. The brushes are connected to three variable resistors connected in star. The purpose of slip rings and brushes is to provide a means for connecting external resistors in the rotor circuit. The resistors enable the variation of each rotor phase resistance to serve two purposes :

(*a*) to increase the starting torque and decrease the starting current from the supply.

(*b*) to control the speed of the motor.

A slip ring induction motor is shown in Fig. 4.2(*a*) &(*b*).



(a)

(b)

Fig. 4.2. *Slip ring induction motor.*

4.3 COMPARISON OF CAGE AND WOUND ROTORS

The advantages of the cage rotor are as follows :

(*a*) Robust construction and cheaper

(*b*) The absence of brushes reduces the risk of sparking.

(*c*) Lesser maintenance.

(*d*) Higher efficiency and higher power factor.

The wound rotors have the following merits :

(a) High starting torque and low starting current.

(b) Additional resistance can be connected in the rotor circuit to control speed.

4.4 PRODUCTION OF ROTATING FIELD

When 3-phase windings displaced in space by 120° are supplied by 3-phase currents displaced in time by 120°, a magnetic flux is produced which rotates in space.

4.4.1 Analytical Method

Consider three identical coils located on axes physically at 120° in space as shown in Fig. 4.3 (a). Let each coil be supplied from one phase of a balanced 3-phase supply. Each coil will produce an alternating flux along its own axis. Let the instantaneous fluxes be given by

$$\Phi_1 = \Phi_m \sin \omega t$$

$$\Phi_2 = \Phi_m \sin (\omega t - 120°)$$

$$\Phi_3 = \Phi_m \sin (\omega t + 120°)$$

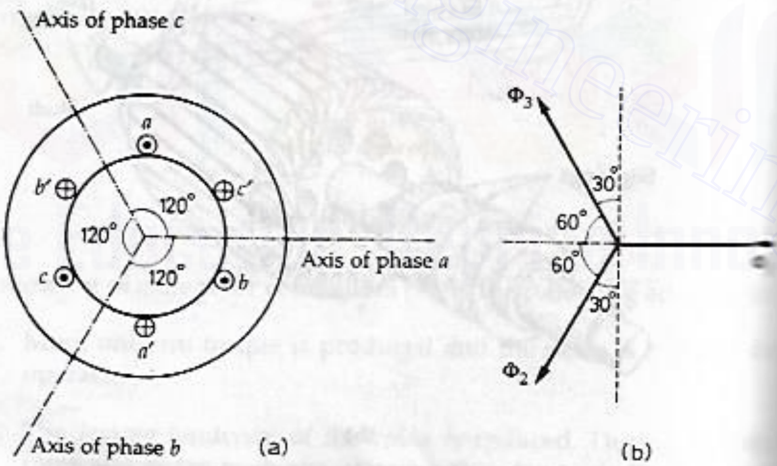


Fig. 4.3. (a) & (b)

The resultant flux produced by this system may be determined by resolving the components with respect to the physical axes as shown in Fig. 4.3 (b).

The resultant horizontal component of flux is given by

$$\Phi_h = \Phi_1 - \Phi_2 \cos 60° - \Phi_3 \cos 60° = \Phi_1 - (\Phi_2 + \Phi_3) \cos 60°$$

$$= \Phi_1 - \frac{1}{2}(\Phi_2 + \Phi_3) = \Phi_m \sin \omega t - \frac{1}{2}[\Phi_m \sin(\omega t - 120°) + \Phi_m \sin(\omega t + 120°)]$$

$$= \Phi_m \sin \omega t - \frac{\Phi_m}{2}(\sin \omega t \cos 120° - \cos \omega t \sin 120° + \sin \omega t \cos 120° + \cos \omega t \sin 120°)$$

$$= \Phi_m \sin \omega t - \frac{\Phi_m}{2} \times (2 \sin \omega t)(-\tfrac{1}{2})$$

or $$\Phi_h = \frac{3}{2}\Phi_m \sin \omega t \qquad (4.4.4)$$

The resultant vertical component of flux is given by

$$\Phi_v = 0 - \Phi_2 \cos 30° + \Phi_3 \cos 30°$$

$$= \cos 30° [-\Phi_m \sin(\omega t - 120°) + \Phi_m \sin(\omega t + 120°)]$$

$$= \frac{\sqrt{3}}{2}\Phi_m [-(\sin \omega t \cos 120° - \cos \omega t \sin 120°) + (\sin \omega t \cos 120° + \cos \omega t \sin 120°)]$$

$$= \frac{\sqrt{3}}{2}\Phi_m (2 \cos \omega t \sin 120°) = \frac{\sqrt{3}}{2}\Phi_m \times 2 \cos \omega t \times \frac{\sqrt{3}}{2}$$

or $$\Phi_v = \frac{3}{2}\Phi_m \cos \omega t \qquad (4.4.5)$$

The components Φ_h and Φ_v are shown in Fig. 4.3 (*c*).

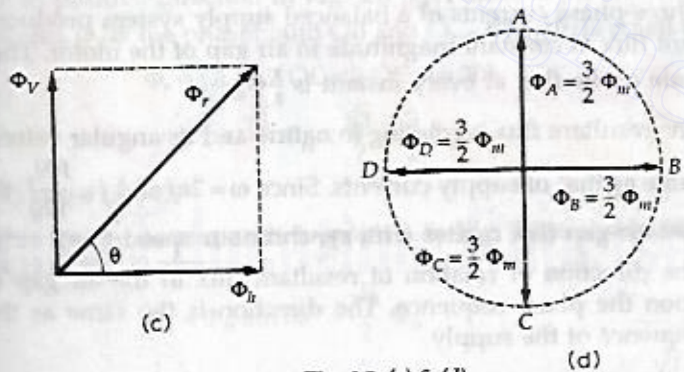


Fig. 4.3. *(c) & (d)*

Resultant flux

$$\Phi_r = \sqrt{\Phi_h^2 + \Phi_v^2} = \sqrt{\left(\frac{3}{2}\Phi_m \sin \omega t\right)^2 + \left(\frac{3}{2}\Phi_m \cos \omega t\right)^2}$$

$$= \frac{3}{2}\Phi_m \sqrt{(\sin^2 \omega t + \cos^2 \omega t)}$$

or $$\Phi_r = \frac{3}{2}\Phi_m \qquad (4.4.6)$$

Also,

$$\tan\theta = \frac{\Phi_v}{\Phi_h} = \left(\frac{3}{2}\Phi_m \cos\omega t\right) \div \left(\frac{3}{2}\Phi_m \sin\omega t\right)$$

$$= \cot\omega t = \tan\left(\frac{\pi}{2} - \omega t\right)$$

$$\therefore \quad \theta = \frac{\pi}{2} - \omega t \tag{4.4.7}$$

Equation (4.4.6) shows that the resultant flux Φ_r is independent of time. It is a constant flux of magnitude equal to $\frac{3}{2}$ times the maximum flux per phase.

Equation (4.4.7) shows that angle θ is dependent on time.

From Eq. (4.4.7), $\theta = 90° - \omega t$,

(*a*) At $\omega t = 0°$, $\theta = 90°$ corresponding to position A in Fig. 4.3 (*d*).

(*b*) At $\omega t = 90°$, $\theta = 0°$ corresponding to position B.

(*c*) At $\omega t = 180°$, $\theta = -90°$ corresponding to position C.

(*d*) At $\omega t = 270°$, $\theta = -180°$ corresponding to position D.

It is seen that the resultant flux rotates in space in the clockwise direction with angular velocity of ω radians per second.

Since $\omega = 2\pi f$ and $f = \dfrac{PN_s}{120}$, the resultant flux rotates with synchronous speed.

The following *conclusions* are drawn from the above discussion :

1. Three-phase currents of a balanced supply system produce a resultant flux of constant magnitude in air gap of the motor. The magnitude of the flux at every instant is $\frac{3}{2}\Phi_m$.
2. The resultant flux is rotating in nature and its angular velocity is the same as that of supply currents. Since $\omega = 2\pi f$ and $f = \dfrac{PN_s}{120}$, the resultant air-gap flux rotates with synchronous speed.
3. The direction of rotation of resultant flux in the air gap depends upon the phase sequence. The direction is the same as the phase sequence of the supply.

4.4.2 Graphical Method

Figure 4.4 shows the waveforms of the fluxes produced by the three coils. The maximum value of the flux due to any one of the three phases is Φ_m. The positive directions of flux phasors for each phase are shown in Fig. 4.3 (*a*). The resultant flux Φ_r, at any instant, is equal to the phasor sum of the fluxes due to three phases. The magnitude of the phasors is proportional to the ordinates of the waveforms in each case, and the direction is taken from Fig. 4.3 (*a*). We shall determine the values of Φ_r at four instants 60° apart corresponding to points 0, 1, 2 and 3 in Fig. 4.4.

(*i*) *When* $\omega t = 0$

This instant corresponds to point 0 in Fig. 4.4. Putting $\omega t = 0$ in Eqs. (4.4.1), (4.4.2) and (4.4.3) gives

$$\Phi_1 = \Phi_m \sin \omega t = \Phi_m \sin 0° = 0$$

$$\Phi_2 = \Phi_m \sin (\omega t - 120°) = \Phi_m \sin (-120°) = -\frac{\sqrt{3}}{2} \Phi_m$$

$$\Phi_3 = \Phi \sin (\omega t + 120°) = \Phi_m \sin 120° = \frac{\sqrt{3}}{2} \Phi_m$$

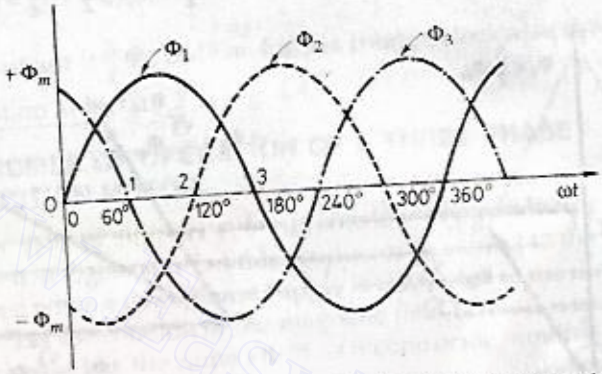


Fig. 4.4. *Waveforms of fluxes produced by three-phase currents.*

The phasor for Φ_2 in Fig. 4.5 (*a*) is shown along OB which is in a direction opposite to positive direction in Fig. 4.3 (*a*). Phasor Φ_3 is shown along OC. The resultant flux Φ_r is the phasor sum OB and OC. Thus Φ_r is equal to phasor OD.

$$\Phi_r = OD = 2\,OE = 2\,OC \cos 30°$$

$$= 2 \times \frac{\sqrt{3}}{2} \Phi_m \times \frac{\sqrt{3}}{2} = \frac{3}{2} \Phi_m$$

(*ii*) *When* $\omega t = 60°$

This instant corresponds to point 1 in Fig. 4.4. Substitution of $\omega t = 60°$ in Eqs. (4.4.1) to (4.4.3) gives

$$\Phi_1 = \Phi_m \sin 60° = \frac{\sqrt{3}}{2} \Phi_m$$

$$\Phi_2 = \Phi_m \sin (60° - 120°) = \Phi_m \sin (-60°) = -\frac{\sqrt{3}}{2} \Phi_m$$

$$\Phi_3 = \Phi_m \sin (60° + 120°) = \Phi_m \sin 180° = 0$$

Phasors Φ_1, Φ_2 and their resultant Φ_r is shown in Fig. 4.5 (*b*).

$$\Phi_r = OD = 2\,DA \cos 30° = 2 \times \frac{\sqrt{3}}{2} \Phi_m \times \frac{\sqrt{3}}{2} = \frac{3}{2} \Phi_m$$

It is seen that the resultant flux Φ_r is again $\frac{3}{2} \Phi_m$ but has rotated clockwise through an angle of 60° from position at instant 0.

(iii) When $\omega t = 120°$

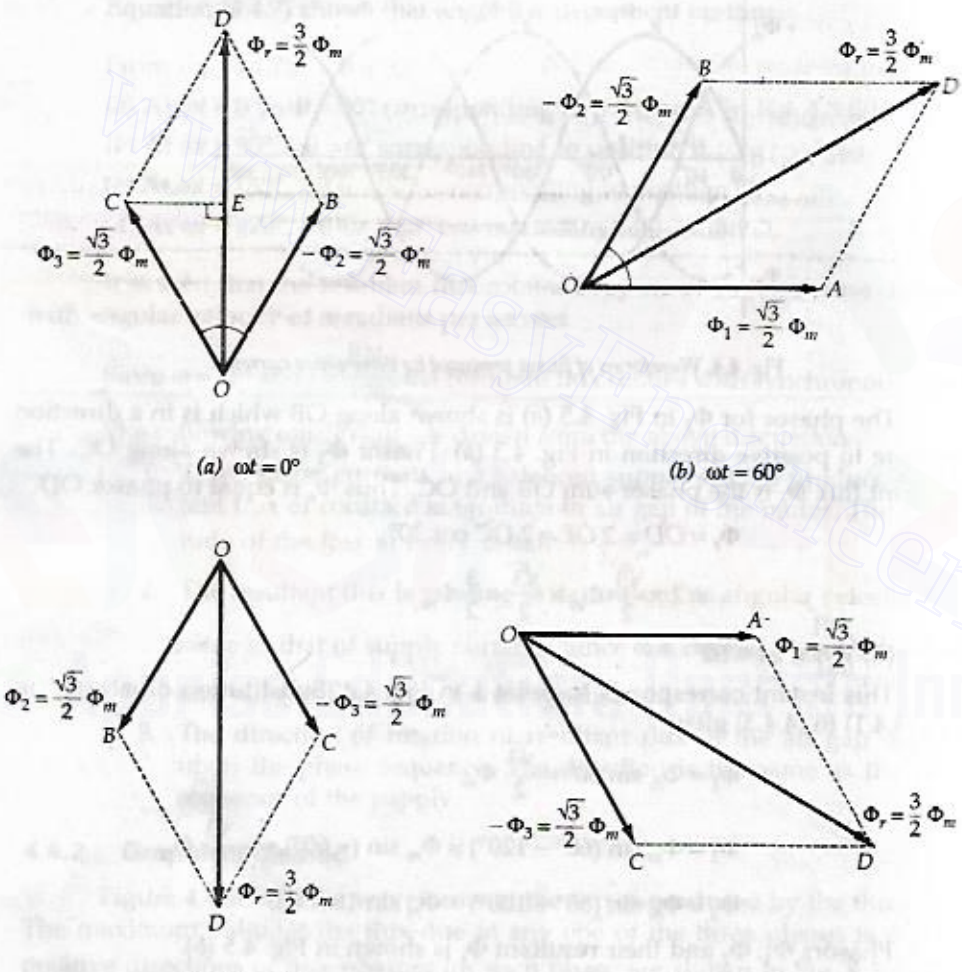
This instant corresponds to position 2 in Fig. 4.4. Here

$$\Phi_1 = \Phi_m \sin 120° = \frac{\sqrt{3}}{2}\Phi_m \text{ along OA in Fig. 4.5 } (c)$$

$$\Phi_2 = \Phi_m \sin(120° - 120°) = 0$$

$$\Phi_3 = \Phi_m \sin(120° + 120°) = -\frac{\sqrt{3}}{2}\Phi_m \text{ along OC in Fig. 4.5 } (c)$$

$$\Phi_r = OD = 2\,OA \cos 30° = 2 \times \frac{\sqrt{3}}{2}\Phi_m \times \frac{\sqrt{3}}{2} = \frac{3}{2}\Phi_m$$



(a) $\omega t = 0°$ *(b)* $\omega t = 60°$

(d) $\omega t = 180°$ *(c)* $\omega t = 120°$

Fig. 4.5.

Hence the resultant is $\frac{3}{2}\Phi_m$ but has further rotated clockwise through an angle of 60° from position at instant 1 in Fig. 4.4.

(*iv*) *When* $\omega t = 180°$

This instant corresponds to position 3 in Fig. 4.4. Here

$$\Phi_1 = \Phi_m \sin 180° = 0$$

$$\Phi_2 = \Phi_m \sin (180° - 120°) = \frac{\sqrt{3}}{2}\Phi_m \text{ along OB in Fig. 4.5 } (d)$$

$$\Phi_3 = \Phi_m \sin (180° + 120°) = -\frac{\sqrt{3}}{2}\Phi_m \text{ along OC in Fig. 4.5 } (d)$$

$$\Phi_r = OD = 2\,OB \cos 30° = 2 \times \frac{\sqrt{3}}{2}\Phi_m \times \frac{\sqrt{3}}{2} = \frac{3}{2}\Phi_m$$

The resultant is $\frac{3}{2}\Phi_m$ but has further rotated clockwise through an angle of 60° from position at instant 2 in Fig. 4.4.

4.5 PRINCIPLE OF OPERATION OF A THREE-PHASE INDUCTION MOTOR

For the sake of simplicity, let us consider one conductor on the stationary rotor as shown in Fig. 4.6 (*a*). Let this conductor be subject to the rotating magnetic field produced when a three-phase supply is connected to the three-phase winding of the stator. Let the rotation of the magnetic field be clockwise. A magnetic field moving clockwise has the same effect as a conductor moving anticlockwise in a

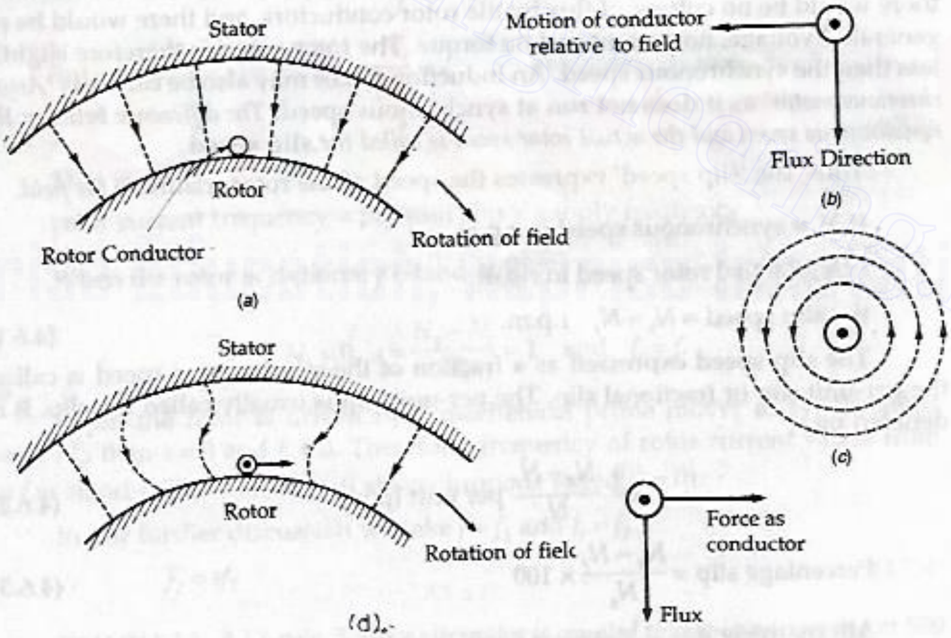


Fig. 4.6. *Production of torque*

stationary field. By Faraday's law of electromagnetic induction, a voltage will be induced in the conductor. Since the rotor circuit is complete, either through the end rings or an external resistance the induced voltage causes a current to flow in the rotor conductor. By right-hand rule we can determine the direction of induced current in the conductor. Since the magnetic field is rotating clockwise, and the conductor is stationary we can *assume* that the conductor is in motion in the anticlockwise direction with respect to the magnetic field. By right hand rule the direction of the induced current is outwards (shown by dot) as given in Fig. 4.6 (*b*). The current in the rotor conductor produces its own magnetic field [Fig. 4.6 (*c*)].

We know that when a conductor carrying current is put in a magnetic field a force is produced on it. Thus, a force is produced on the rotor conductor. The direction of this force can be found by left-hand rule [Fig. 4.6 (*d*)]. It is seen that the force acting on the conductor is in the same direction as the direction of the rotating magnetic field. Since the rotor conductor is in a slot on the circumference of the rotor, this force acts in a tangential direction to the rotor and develops a torque on the rotor. Similar torques are produced on all the rotor conductors. Since the rotor is free to move, it starts rotating in the *same* direction as the rotating magnetic field. Thus, a three-phase induction motor is self-starting. Since the operation of this motor depends upon the induced voltage in its rotor conductors, it is called an **induction motor.**

4.6 SPEED AND SLIP

An induction motor cannot run at synchronous speed. Let us consider for a moment that is rotor is rotating at synchronous speed. Under this condition, there would be no cutting of flux by the rotor conductors, and there would be no generated voltage, no current and no torque. The rotor speed is therefore slightly less than the synchronous speed. An induction motor may also be called as *'Asynchronous motor'* as it does not run at synchronous speed. *The difference between the synchronous speed and the actual rotor speed is called the* **slip speed.**

Thus, the 'slip speed' expresses the speed of the rotor *relative to the field.*

If N_s = synchronous speed in r.p.m.

N_r = actual rotor speed in r.p.m.

the slip speed = $N_s - N_r$ r.p.m. (4.6.1)

The slip speed expressed as a fraction of the synchronous speed is called the per-unit slip or fractional slip. The per-unit slip is usually called the slip. It is denoted by s.

$$s \overset{\Delta}{=} \frac{N_s - N_r}{N_s} \text{ per unit (p.u.)} \tag{4.6.2}$$

$$\text{Percentage slip} = \frac{N_s - N_r}{N_s} \times 100 \tag{4.6.3}$$

Alternatively if

n_s = synchronous speed in r.p.s.

n_r = actual rotor speed in r.p.s.

Then
$$s \overset{\Delta}{=} \frac{n_s - n_r}{n_s} \text{ p.u.} \tag{4.6.4}$$

and percentage slip $= \dfrac{n_s - n_r}{n_s} \times 100$ (4.6.5)

Also,
$$s = \frac{\omega_s - \omega_r}{\omega_s} \tag{4.6.6}$$

The slip at full load varies from about 5 per cent for small motors to about 2 percent for large motors.

4.7 FREQUENCY OF ROTOR VOLTAGE AND CURRENT

The frequency of current and voltage in the stator must be the same as the supply frequency given by

$$f = \frac{PN_s}{120} \tag{4.7.1}$$

The frequency in the rotor winding is variable and depends on the difference between the synchronous speed and the rotor speed. Hence the rotor frequency depends upon the slip. The rotor frequency is given by

$$f_r = \frac{P(N_s - N_r)}{120} \tag{4.7.2}$$

Division of Eq. (4.7.2) by Eq. (4.7.1) gives

$$\frac{f_r}{f} = \frac{N_s - N_r}{N_s}$$

But
$$\frac{N_s - N_r}{N_s} = s$$

$\therefore$
$$f_r = sf \tag{4.7.3}$$

That is,

rotor current frequency = per unit slip × supply frequency

When the rotor is stationary (stand-still)

$$N_r = 0,\ s = \frac{N_s - N_r}{N_s} = 1 \quad \text{and} \quad f_r = f.$$

When the rotor is driven by a mechanical prime mover at synchronous speed N_s, then $s = 0$ and $f_r = 0$. Therefore, frequency of rotor current varies from $f_r = f$ at stand-still ($s = 1$) to $f_r = 0$ at synchronous speed ($s = 0$).

In our further discussion we take $f = f_1$ and $f_r = f_2$.

$\therefore$
$$f_2 = sf_1 \tag{4.7.4}$$

EXAMPLE 4.1. *A 12-pole, 3-phase alternator is coupled to an engine running at 500 r.p.m. It supplies an induction motor which has a full-load speed of 1440 r.p.m. Find the slip and the number of poles of the motor.*

Solution. $f_1 = \frac{PN_s}{120} = \frac{12 \times 500}{120} = 50$ Hz

The speed of the induction motor is 1440 r.p.m. Under normal operating conditions, an induction motor operates at a speed slightly less than its synchronous speed. The supply frequency for induction motor is 50 Hz. The possible synchronous speeds for 50 Hz are 3000, 1500, 1000 r.p.m. etc., so the closest synchronous speed corresponding to the actual speed of 1440 r.p.m. is 1500 r.p.m. that is, $N_{sm} = 1500$ r.p.m.

Slip of the motor

$$s = \frac{N_{sm} - N_r}{N_{sm}} = \frac{1500 - 1440}{1500} = 0.04 = 4\%$$

Number of poles of the motor

$$P_m = \frac{120 f}{N_{sm}} = \frac{120 \times 50}{1500} = 4$$

Example 4.2. *The frequency of the e.m.f. in the stator of a 4 pole induction motor is 50 Hz, and that in the rotor is 1.5 Hz. What is the slip, and at what speed is the motor running ?*

Solution. $f_2 = sf_1$

$$s = \frac{f_2}{f_1} = \frac{1.5}{50} = 0.03 \text{ p.u.} = 3\%$$

$$f_1 = \frac{PN_s}{120}$$

$$N_s = \frac{120 f_1}{P} = \frac{120 \times 50}{4} = 1500 \text{ r.p.m.}$$

Speed of the motor

$$N = (1 - s) N_s = (1 - 0.03) \times 1500 = 1455 \text{ r.p.m.}$$

Example 4.3. *A 3-phase, 6-pole, 50 Hz induction motor has a slip of 1% at no load, and 3% at full load. Determine : (a) synchronous speed ; (b) no-load speed ; (c) full-load speed ; (d) frequency of rotor current at standstill ; (e) frequency of rotor current at full load.*

Solution.

(a) $N_s = \frac{120 f_1}{P} = \frac{120 \times 50}{6} = 1000$ r.p.m.

(b) $N_0 = (1 - s_0)\, N_s = (1 - 0.01) \times 1000 = 990$ r.p.m

(c) $N_{fl} = (1 - s_{fl})\, N_s = (1 - 0.03) \times 1000 = 970$ r.p.m.

(d) Frequency of rotor current at standstill

$$f_2 = sf_1 = 1 \times 50 = 50 \text{ Hz}$$

(e) Frequency of rotor current at full-load

$$f_2 = s_{fl} f_1 = 0.03 \times 50 = 1.5 \text{ Hz}$$

4.8 ROTOR CURRENT

(a) *Standstill conditions*

Let E_{20} = e.m.f. induced per phase of the rotor at standstill.

R_2 = resistance per phase of the rotor

X_{20} = reactance per phase of the rotor at standstill

$= 2\pi f_1 L_2$

Z_{20} = rotor impedance per phase at standstill

I_{20} = rotor current per phase at standstill

$$\mathbf{Z}_{20} = R_2 + jX_{20} \tag{4.8.1}$$

$$\mathbf{I}_{20} = \frac{\mathbf{E}_{20}}{\mathbf{Z}_{20}} \tag{4.8.2}$$

Power factor at standstill

$$\cos \Phi_{20} = \frac{R_2}{Z_{20}} = \frac{R_{20}}{\sqrt{R_2^2 + X_{20}^2}} \tag{4.8.3}$$

(b) *Rotor current at slip s*

Induced emf per phase in the rotor winding at slip s is

$$E_{2s} = sE_{20} \tag{4.8.4}$$

Rotor winding resistance per phase = R_2

Rotor winding reactance per phase at slip s is

$$X_{2s} = 2\pi f_2 L = 2\pi\,(sf_1)\,L = sX_{20} \tag{4.8.5}$$

Rotor winding impedance per phase at slip s is

$$\mathbf{Z}_{2s} = R_2 + jX_{2s} = R_2 + jsX_{20} \tag{4.8.6}$$

Rotor current at slip s is

$$\mathbf{I}_{2s} = \frac{\mathbf{E}_{2s}}{\mathbf{Z}_{2s}} \tag{4.8.7}$$

Power factor at slip s is

$$\cos \Phi_{2s} = \frac{R_2}{Z_{2s}} \tag{4.8.8}$$

EXAMPLE 4.4. *A 3-phase, 50 Hz, 4-pole induction motor has a slip of 4%. Calculate : (a) speed of the motor ; (b) frequency of rotor emf.*

If the rotor has a resistance of 1 Ω and standstill reactance of 4 Ω, calculate the power factor (i) at standstill, and (ii) at a speed of 1400 r.p.m.

SOLUTION. $N_s = \frac{120 f_1}{P} = \frac{120 \times 50}{4} = 1500$ r.p.m.

$s = 4\% = 0.04$ p.u.

(a) Speed of the motor

$$N = (1 - s)\,N_s = (1 - 0.04) \times 1500 = 1440 \text{ r.p.m.}$$

(*b*) Frequency of rotor emf

$$f_2 = sf_1 = 0.04 \times 50 = 2 \text{ Hz}$$

(*i*) $R_2 = 1\ \Omega,\ X_{20} = 4\ \Omega$

Rotor impedance at standstill

$$Z_{20} = R_2 + jX_{20} = 1 + j4 = 4.123\ \underline{/\ 75.96^\circ}\ \Omega$$

Power factor at standstill

$$\cos \Phi_{20} = \cos 75.96^\circ = 0.2425 \text{ (lag)}$$

(*ii*) The slip at a speed of 1400 r.p.m. is

$$s_1 = \frac{N_s - N}{N_s} = \frac{1500 - 1400}{1500} = \frac{1}{15}$$

Rotor impedance at slip s_1 is

$$Z_{2s_1} = R_2 + js_1 X_{20}$$

$$= 1 + j \times \frac{1}{15} \times 4 = 1 + j0.2667 = 1.03495\ \underline{/\ 14.93^\circ}\ \Omega$$

Power factor at 1400 r.p.m. is

$$\cos \Phi_{2s_1} = \cos 14.93^\circ = 0.9662 \text{ (lag)}$$

EXAMPLE 4.5. *A 3-phase slip-ring induction motor gives a reading of 60 V across slip rings when at rest with normal stator voltage applied. The rotor is star connected and has an impedance of (0.8 + j6) Ω per phase. Find the rotor current when the machine is (a) at standstill with the slip-rings joined to a star-connected starter with a phase impedance of (4 + j3) Ω and (b) running normally with a 5% slip.*

SOLUTION. E_{20} = e.m.f. induced per phase of the rotor at standstill

$$= \frac{60}{\sqrt{3}} = 34.64 \text{ V}$$

Total impedance of the rotor at standstill = impedance of rotor + impedance of starter

$$= (0.8 + j6) + (4 + j3) = 4.8 + j9\ \Omega$$

(*a*) Current at standstill

$$\mathbf{I}_{20} = \frac{\mathbf{E}_{20}}{\mathbf{Z}_{20}} = \frac{34.64\ \angle 0^\circ}{4.8 + j9}$$

$$= \frac{34.64\ \angle 0^\circ}{10.2\ \angle 61.93^\circ} = 3.396\ \angle -61.93^\circ \text{ A}$$

(*b*) $s = 5\% = 0.05$ p.u.

During normal running, the starting resistances are cut off :

$$\mathbf{I}_{2s} = \frac{\mathbf{E}_{2s}}{\mathbf{Z}_{2s}} = \frac{sE_{20}}{R_2 + jsX_{20}}$$

$$= \frac{0.05 \times 34.64}{0.8 + j0.05 \times 6} = \frac{1.732}{0.8 + j0.3} = \frac{1.732}{0.8544\ \angle 20.56^\circ}$$

$$= 2.027\ \underline{/ -20.56^\circ}\ \text{A}$$

4.9 RELATIONSHIP BETWEEN ROTOR COPPER LOSS AND ROTOR INPUT

Let τ_d = developed torque = torque exerted on the rotor by rotating flux

n_s = synchronous speed (r.p.s.)

n_r = rotor speed (r.p.s.)

Power transferred from stator to rotor = air-gap power P_g

$$P_g = \omega_s \tau_d = 2\pi n_s \tau_d = \text{input power to rotor} \tag{4.9.1}$$

Total mechanical power developed by the rotor

$$P_{md} = \omega_r \tau_d = 2\pi n_r \tau_d \quad \text{W} \tag{4.9.2}$$

Total I^2R loss in rotor = (power transferred from stator to rotor) − (total mechanical power developed by rotor)

$$P_{rc} = P_g - P_{md} = 2\pi (n_s - n_r) \tau_d \tag{4.9.3}$$

$$\therefore \quad \frac{\text{total } I^2R \text{ loss in rotor}}{\text{input power to rotor}} = \frac{2\pi (n_s - n_r) \tau_d}{2\pi n_s \tau_d} = s \tag{4.9.4}$$

$\therefore$ rotor copper loss $= s \times$ rotor input

$$p_{rc} = s P_g = s P_{ir} \tag{4.9.5}$$

Thus, the rotor copper loss is equal to slip times the rotor input (air-gap power). The term sP_g is known as *slip power*, because it is proportional to the slip for a given value of P_g. It is the portion of the air-gap power which is not converted into mechanical power.

Also, rotor input = mechanical power developed + rotor copper loss

$$P_{ir} = P_{md} + p_{rc} \tag{4.9.6}$$

$$p_{rc} = s (P_{md} + p_{rc})$$

$$p_{rc} (1 - s) = s P_{md}$$

$$p_{rc} = \frac{s}{1-s} P_{md} \tag{4.9.7}$$

That is, rotor copper loss $= \frac{s}{1-s} \times$ mechanical power developed by the rotor

$$P_g : p_{rc} : P_{md} = 1 : s : (1 - s) \tag{4.9.8}$$

From the above discussion, it is seen that once the air-gap power P_g is determined, three quantities may be found from the slip and synchronous speed.

$$p_{rc} = s P_g \tag{4.9.9}$$

$$P_{md} = (1 - s) P_g \tag{4.9.10}$$

$$\tau_d = \frac{P_g}{\omega_s} \tag{4.9.11}$$

4.10 DEVELOPED TORQUE τ_d

The ***developed torque*** or ***induced torque*** in a machine is defined as the torque generated by the internal electric-to-mechanical power conversion. The torque is also called the ***electromagnetic torque***. This torque differs from the torque actually available at the terminals of the motor by an amount equal to the friction and windage torques in the machine. The developed torque is given by

$$\tau_d = \frac{\text{mechanical power developed}}{\text{mechanical angular velocity of the rotor}} = \frac{P_{md}}{\omega_r} \qquad (4.10.1)$$

Since $\quad P_{md} = (1-s)\,P_g$

and $\quad \omega_r = (1-s)\,\omega_s$

$$\tau_d = \frac{(1-s)\,P_g}{(1-s)\,\omega_s} = \frac{P_g}{\omega_s} \qquad (4.10.2)$$

Equation (4.10.2) is specially useful because it expresses developed torque directly in terms of air-gap power P_g and synchronous speed ω_s. Since ω_s is constant and independent of load conditions, τ_d is found directly if P_g is known. Equation (4.10.2) is applicable to the starting condition when $s = 1$ and the torque cannot be calculated directly from Eq. (4.10.1) which becomes an indeterminate form.

Since the developed torque is given by Eq. (4.10.2), the air-gap power P_g is often called *"the torque in synchronous watts"*.

Synchronous watt is the torque that develops power of 1 watt when the machine is running at synchronous speed.

$$\text{Output power } P_0 = \omega_r\,\tau_{load} \qquad (4.10.3)$$

$$\tau_{load} = \frac{P_0}{\omega_r} = \frac{P_{md} - P_{rot}}{\omega_r} \qquad (4.10.4)$$

EXAMPLE 4.6. *The power input to a 3 phase induction motor is 60 kW. The stator loses total 1 kW. Find the total mechanical power developed and the rotor copper loss per phase if the motor is running with a slip of 3%.*

SOLUTION. Stator input $P_{is} = 60$ kW, $\quad s = 3\% = \frac{3}{100} = 0.03$ pu

Stator losses = 1 kW

Stator output = 60 − 1 = 59 kW

Rotor input = stator output = 59 kW

Total rotor copper loss = s × rotor input = 0.03 × 59 = 1.77 kW

Rotor copper loss per phase = $\frac{1}{3} \times 1.77 = 0.59$ kW

Mechanical power developed = rotor input − rotor copper loss
= 59 − 1.77 = 57.23 kW

EXAMPLE 4.7. *A 6-pole, 50 Hz, 3-Φ induction motor running on full load develops a useful torque of 150 Nm at a rotor frequency of 1.5 Hz. Calculate the shaft power output. If the mechanical torque lost in friction be 10 Nm, determine (a) rotor copper loss, (b) the input to the motor, and (c) the efficiency.*

The total stator loss in 700 W.

SOLUTION. $N_s = \dfrac{120 f_1}{p} = \dfrac{120 \times 50}{6} = 1000$ r.p.m.

$$s = \frac{f_2}{f_1} = \frac{1.5}{50} = 0.03 \quad \text{or } 3\%$$

$$N_r = (1 - s)\, N_s = (1 - 0.003) \times 1000 = 970 \text{ r.p.m.}$$

$$\omega_r = 2\pi\, n_r = \frac{2\pi \times 970}{60} = 101.58 \text{ rad/s}$$

Shaft power output, $P_0 = \tau_0\, \omega_r$

$$= 150 \times 101.58 = 15236 \text{ W} = 15.236 \text{ kW}$$

Mechanical power developed

$$P_{md} = (150 + 10) \times 101.58 = 16252 \text{ W} = 16.252 \text{ kW}$$

(*a*) Rotor copper loss $p_{rc} = \left(\dfrac{s}{1-s}\right) P_{md}$

$$= \frac{0.03}{1 - 0.03} \times 16252 = 502.6 \text{ W} = 0.5026 \text{ kW}$$

(*b*) Input to motor, $P_i = P_{md} + p_{rc} + p_{sc}$

$$= 16.252 + 0.5026 + 0.700 = 17.4546 \text{ kW}$$

(c) Efficiency $= \dfrac{P_0}{P_i} = \dfrac{15.236}{17.4546} = 0.8729 \text{ pu} = 87.29\%.$

EXAMPLE 4.8. *The power input to the rotor of 440 V, 50 Hz, 6-pole, 3-phase induction motor is 80 kW. The rotor emf is observed to make 100 complete alternations per min. Calculate (a) the slip ; (b) the rotor speed ; (c) the mechanical power developed ; (d) the rotor copper loss per phase ; (e) the rotor resistance per phase if the rotor current is 65 A.*

SOLUTION. $f_1 = 50$ Hz, $f_2 = \dfrac{100}{60}$ Hz

$$f_2 = s f_1, \quad s = \frac{f_2}{f_1} = \frac{100}{60 \times 50} = 0.033 \text{ pu}$$

(*a*) $s = \dfrac{N_s - N_r}{N_s}, \quad N_s = \dfrac{120 f}{P} = \dfrac{120 \times 50}{6} = 1000$ rpm

(*b*) $N_r = (1 - s)\, N_s = \left(1 - \dfrac{2}{60}\right) \times 1000 = 1000 - 33.3 = 966.7$ rpm (*c*) Mechanical power developed = rotor input − rotor copper loss

Also, rotor copper loss = s × rotor input

$$= \frac{1}{30} \times 80 \times 1000 \text{ W}$$

$$\text{Mechanical power developed} = 80 \times 1000 - 2667 = 77333 \text{ W}$$

$$= \frac{77333}{746} \text{ hp} = 103.66 \text{ hp}$$

(*d*) Rotor copper loss per phase $= \dfrac{80 \times 1000}{30 \times 3} = 889$ W

(*e*) Rotor resistance per phase $R_2 = \dfrac{\text{rotor per loss/phase}}{I^2} = \dfrac{889}{65^2} = 0.2104\ \Omega$

EXAMPLE 4.9. *A 25 h.p., 6-pole, 50-Hz, 3-phase slip-ring induction motor runs at 960 revolutions per minute on full load with a rotor current per phase of 35 A. Allowing 250 W for the copper loss in the short-circuiting gear, and 1000 W for mechanical losses, find the resistance per phase of the three-phase rotor winding.*

SOLUTION. $f_1 = \dfrac{PN_s}{120}$

$$50 = \frac{6 \times N_s}{120}, \quad N_s = \frac{120 \times 50}{6} = 1000 \text{ r.p.m.}$$

$$s = \frac{N_s - N_r}{N_s} = \frac{1000 - 960}{1000} = 0.04 \text{ p.u.}$$

Rotor copper loss

$$= \frac{s}{1-s} \times \text{mechanical power developed}$$

$$3 I_2^2 R_2 + 250 = \frac{0.04}{1 - 0.04}(25 \times 746 + 1000)$$

$$3 \times 35^2 R_2 = 818.75 - 250$$

$$R_2 = \frac{568.75}{3 \times 35^2} = 0.15476\ \Omega$$

EXAMPLE 4.10. *A 500 V, 6-pole, 50 Hz, 3-Φ induction motor develops 20 kW inclusive of mechanical losses when running at 995 r.p.m., the p.f. being 0.87. Calculate (a) the slip, (b) the rotor I^2R loss, (c) the total input if the stator loss is 1500 W, (d) line current, (e) the rotor current frequency.*

SOLUTION. $N_s = \dfrac{120 f_1}{P} = \dfrac{120 \times 50}{6} = 1000$ r.p.m.

$$s = \frac{N_s - N_r}{N_s} = \frac{1000 - 995}{1000} = 0.005 \text{ pu}$$

Rotor on loss = slip × rotor power input

$$= s\,(\text{mech power developed} + \text{rotor Cu loss})$$

$$p_{rc} = s\,(P_m + p_{rc})$$

$$p_{rc}\,(1-s) = s\,P_m$$

$$p_{rc} = \frac{s\,P_m}{(1-s)} = \frac{0.005}{1-0.005} \times 20 \times 1000 = 100.5 \text{ W}$$

Total input to stator = rotor power input + stator loss

$$\text{Rotor input} = \frac{1}{s} \times \text{rotor Cu loss}$$

$$= \frac{1}{0.005} \times 100.5 = 20100 \text{ W} = 20.1 \text{ kW}$$

Hence total input $= 20.1 + 1.5 = 21.6$ kW

(*d*) Line current $= \dfrac{21600}{\sqrt{3} \times 500 \times 0.87} = 28.7$ A

(*e*) $f_r = sf_1 = 0.005 \times 50 = 0.25$ Hz

EXAMPLE 4.11. *The stator loss of a 3–Φ induction motor is 2 kW. When the power input is 90 kW, what will be the rotor mechanical power developed and the rotor copper loss if the motor is running with a slip of 4%.*

SOLUTION. $s = 0.04$, stator input = 90 kW, stator loss = 2 kW

Rotor input P_{ri} = stator ouput

= stator input − stator loss = 90 − 2 = 88 kW

Rotor Cu loss $p_{cr} = s \times \text{rotor input} = 0.04 \times 88 = 3.52$ kW

Mechanical power developed

$$P_m = P_{ri} - p_{cr} = 88 - 3.52 = 84.48 \text{ kW}$$

EXAMPLE 4.12. *A 3-phase induction motor with star-connected rotor has an induced emf of 60 V between slip rings at standstill on open circuit with normal voltage applied to the stator. The resistance and standstill reactance of each rotor phase are 0.6 Ω and 4 Ω respectively. Calculate the current per phase in the rotor (a) when at standstill and connected to a star-connected rheostat of resistance 5 Ω and reactance 2 Ω per phase, (b) when running short-circuited with 4% slip.*

SOLUTION. (*a*) $E_{20} = \dfrac{60}{\sqrt{3}}$ V per phase

Total rotor resistance $= 0.6 + 5 = 5.6\ \Omega$

Total rotor reactance $= 4 + 2 = 6\ \Omega$

$$I_{20} = \frac{E_{20}}{Z_{20}} = \frac{60/\sqrt{3}}{\sqrt{R_2^2 + X_{20}^2}} = \frac{60/\sqrt{3}}{\sqrt{5.6^2 + 6^2}} = 4.22 \text{ A}$$

(*b*) $E_{2s} = sE_{20} = 0.04 \times \dfrac{60}{\sqrt{3}} = 1.3856$ V

$$Z_{2s} = \sqrt{R_2^2 \times (sX_{20})^2} = \sqrt{0.6^2 + (0.04 \times 4)^2} = 0.62\ \Omega$$

$$I_{2s} = \frac{E_{2s}}{Z_{2s}} = \frac{1.3856}{0.62} = 2.235 \text{ A}$$

4.11 TORQUE OF AN INDUCTION MOTOR

Electrical power generated in rotor

$$= 3\, E_{2s}\, I_{2s} \cos \Phi_{2s} \quad \text{W}$$

$$= 3\, E_{2s} \cdot \frac{E_{2s}}{Z_{2s}} \cdot \frac{R_2}{Z_{2s}} = \frac{3\, E_{2s}^2\, R_2}{Z_{2s}^2}$$

$$= \frac{3\, s^2\, E_{20}^2\, R_2}{R_2^2 + (sX_{20})^2} \qquad (4.11.1)$$

All this power is dissipated as $I^2 R$ loss (copper loss) in the rotor circuit.

Input power to rotor $= 2\pi\, n_s\, \tau_d$

$s \times$ rotor input = rotor copper loss

$$s \times 2\pi\, n_s\, \tau_d = \frac{3\, s^2\, E_{20}^2\, R_2}{R_2^2 + s^2\, X_{20}^2}$$

$$\tau_d = \frac{3\, E_{20}^2}{2\pi\, n_s} \cdot \frac{sR_2}{R_2^2 + s^2 X_{20}^2} \qquad (4.11.2)$$

$$\tau_d = \frac{k\, s\, E_{20}^2\, R_2}{R_2^2 + s^2 X_{20}^2} \qquad (4.11.3)$$

where $k = \dfrac{3}{2\pi\, n_s} = \dfrac{3}{\omega_s}$ = a constant (4.11.4)

Starting Torque

At start, $s = 1$. Therefore, starting torque may be obtained by putting $s = 1$ in Eq. (4.11.2)

$$\tau_{dst} = \frac{3\, E_{20}^2\, R_2}{2\pi\, n_s\, (R_2^2 + X_{20}^2)} \qquad (4.11.5)$$

The starting torque is also known as ***standstill torque***.

Torque at Synchronous Speed

At synchronous speed, $s = 0$, and therefore $\tau_d = 0$. That is, at synchronous speed, developed torque is zero.

$$\frac{E_{20}}{E_1} = \frac{T_{c_2}}{T_{c_1}} \qquad (4.11.6)$$

$$E_{20} = \frac{T_{c_2}}{T_{c_1}} E_1$$

$$\therefore \quad \tau_d = \frac{3}{2\pi\, n_s} \left(\frac{T_{c_2}}{T_{c_1}} \right)^2 E_1^2 \frac{s\, R_2}{R_2^2 + s^2 X_{20}^2} \qquad (4.11.7)$$

Let $$\frac{3}{2\pi n_s}\left(\frac{T_{e_2}}{T_{e_1}}\right)^2 = k \text{ (a constant)} \quad (4.11.8)$$

$$\tau_d = \frac{kE_1^2 sR_2}{R_2^2 + s^2 X_{20}^2} \quad (4.11.9)$$

Since E_1 is nearly equal to V_1

$$\tau_d = \frac{kV_1^2 sR_2}{R_2^2 + s^2 X_{20}^2} \quad (4.11.10)$$

Starting torque is obtained by putting $s = 1$ in the above expression.

$$\tau_{st} = \frac{k\, V_1^2\, R_2}{R_2^2 + X_{20}^2} \quad (4.11.11)$$

$$\therefore \quad \tau_{st} \propto V_1^2 \quad (4.11.12)$$

That is, the starting torque is proportional to the square of the stator applied voltage.

4.12 CONDITION FOR MAXIMUM TORQUE

The value of torque when motor is running is given by

$$\tau_d = \frac{ks\, R_2\, E_{20}^2}{R_2^2 + s^2 X_{20}^2} \quad (4.12.1)$$

If the impedance of the stator winding is assumed to be negligible, then for the given supply voltage V_1, E_{20} remains constant.

Let $k\, E_{20}^2 = k_1$ (a constant)

$$\therefore \quad \tau_d = \frac{k_1 s\, R_2}{R_2^2 + s^2 X_{20}^2} \quad (4.12.2)$$

$$\tau_d = \frac{k_1 R_2}{\dfrac{R_2^2}{s} + s X_{20}^2} \quad (4.12.3)$$

$$= \frac{k_1 R_2}{\left(\dfrac{R_2}{\sqrt{s}} - X_{20}\sqrt{s}\right)^2 + 2\, R_2 X_{20}} \quad (4.12.4)$$

The developed torque τ_d will be maximum when the right-hand side of Eq. (4.12.4) is a maximum which is possible when

$$\frac{R_2}{\sqrt{s}} - X_{20}\sqrt{s} = 0$$

$$R_2 = sX_{20} \quad (4.12.5)$$

or $$R_2 = X_{2s} \quad (4.12.6)$$

Hence, the developed torque is a maximum when the rotor resistance per phase is equal to the rotor reactance per phase under running conditions.

The maximum torque is obtained by putting $sX_{20} = R_2$ in the expression for torque in Eq. (4.12.1),

$$\tau_{dmax} = \frac{k\,s\,R_2\,E_{20}^2}{R_2^2 + R_2^2} = \frac{k\,s\,E_{20}^2}{2\,R_2} = \frac{ks\,E_{20}^2}{2sX_{20}}$$

$$\tau_{dmax} = \frac{k\,E_{20}^2}{2X_{20}} \quad (4.12.7)$$

This relation shows that the maximum torque is independent of rotor resistance.

If s_M = value of slip corresponding to maximum torque then from Eq. (4.12.5),

$$s_M = \frac{R_2}{X_{20}} \quad (4.12.8)$$

We have

$$N = N_s\,(1 - s)$$

Therefore, the speed of the rotor at maximum torque is

$$N_M = N_S\,(1 - s_M) \quad (4.12.9)$$

From Eq. (4.12.7) for maximum torque the following conclusions can be drawn :

(*a*) Maximum torque is independent of rotor circuit resistance.

(*b*) Maximum torque varies inversely as standstill reactance of the rotor. Hence, for maximum torque, X_{20} and, therefore, the inductance of the rotor should be kept as small as possible.

(*c*) The slip at which the maximum torque depends upon the rotor resistance ($s_M = R_2/X_{20}$). Therefore, by varying the resistance in the rotor circuit, maximum torque can be obtained at any desired slip or motor speed. It is to be noted that the resistance in the rotor circuit can only be varied in slipring rotors. In order to develop maximum torque at standstill, the rotor resistance must be high (and equal to X_{20}), but to develop maximum torque under running conditions the rotor resistance must be low.

Maximum Torque at Starting

To determine the condition for maximum torque at starting, put $s = 1$ in Eq. (4.12.5). Therefore, the starting torque will be a maximum when

$$\frac{R_2}{X_{20}} = s = 1$$

or

$$R_2 = X_{20} \quad (4.12.10)$$

The rotor resistance is not more than 1 or 2 percent of its leakage reactance for higher efficiency. In order to increase the starting torque, extra resistance should be added to the rotor circuit at start and cut out gradually as motor speeds up.

4.13 TORQUE-SLIP AND TORQUE-SPEED CHARACTERISTICS

We have

$$\tau = \frac{ks\,R_2\,E_{20}^2}{R_2^2 + (sX_{20})^2} \tag{4.13.1}$$

It is seen that if R_2 and X_{20} are kept constant, the torque τ depends upon the slip s. The torque-slip characteristic curve can be divided roughly into three regions :

(*a*) low-slip region

(*b*) medium-slip region

(*c*) high-slip region

(*a*) ***Low-slip region***

At synchronous speed $s = 0$, therefore, the torque is zero. When the speed is very near to synchronous speed, the slip is very low and $(sX_{20})^2$ is negligible in comparison with R_2. Therefore,

$$\tau = \frac{k_1 s}{R_2}$$

If R_2 is constant,

$$\tau = k_2 s \tag{4.13.2}$$

when $k_2 = k_1/R_2$.

Relation (4.13.2) shows that the torque is proportional to the slip. Hence, when the slip is small (which is the normal working region of the motor), the *torque-slip curve* is a *straight line*.

(*b*) ***Medium-slip region***

As slip increases (that is, as the speed decreases with the increase in load), the term $(sX_{20})^2$ becomes large, so that R_2^2 may be neglected in comparison with $(sX_{20})^2$ and

$$\tau = \frac{k_3 R_2}{sX_{20}^2} \tag{4.13.3}$$

Thus, the torque is inversely proportional to slip towards standstill conditions. The torque-slip characteristic is represented by a rectangular hyperbola. For intermediate values of the slip, the graph changes from one form to another. In doing so, it passes through the point of maximum torque when $R_2 = sX_{20}$. The maximum torque developed in an induction motor is called the **pull-out torque or breakdown torque.** This torque is a measure of the short-time overloading capability of the motor.

(*c*) ***High-slip region***

The torque decreases beyond the point of maximum torque. The result is that the motor slows down and eventually stops. At this stage, the overload protection must immediately disconnect the motor from the supply to prevent damage due to overheating.

The motor operates for the values of the slip between $s = 0$ and $s = s_M$, where s_M is the value of the slip corresponding to maximum torque. For a typical induction motor, the pull-out torque is 2 to 3 times the rated full-load torque. Thus, the motor can handle short-time overload, without stalling. The starting torque is about 1.5 times the rated full-load torque.

Fig. 4.7 shows the torque-slip curves for various values of rotor resistance. The torque-speed curves are shown in Fig. 4.8.

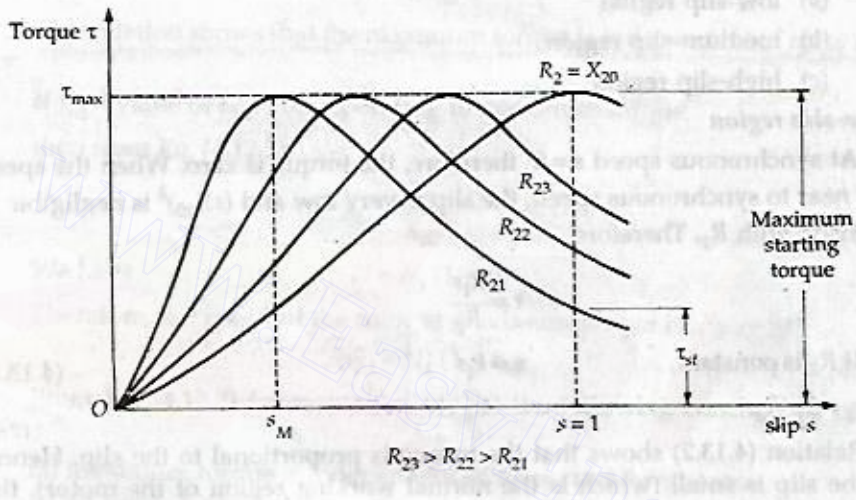


Fig. 4.7. *Torque-slip curves.*

It is seen that although the maximum torque is independent of rotor resistance, yet the exact location of τ_{max} is dependent on it. Greater the value of R_2, greater is the value of slip at which maximum torque occurs. It is also seen that as the rotor resistance is increased, the pull-out speed of the motor decreases, but the maximum torque remains constant.

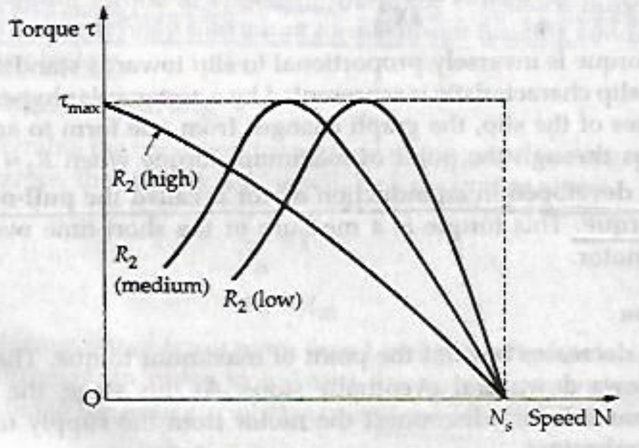


Fig. 4.8. *Torque-speed curves.*

EXAMPLE 4.13. *The rotor resistance and standstill reactance per phase of a 3–Φ slip-ring induction motor are 0.05 Ω and 0.2 Ω respectively. What should be the value of external resistance per phase to be inserted in the rotor circuit to give maximum torque at starting ?*

SOLUTION. Let external resistance per phase added to the rotor circuit be r ohms.

Rotor resistance per phase, $R_2 = (0.05 + r)$

The starting torque will be maximum when

$$R_2 = X_{20}$$

$0.05 + r = 0.1,\ \ r = 0.05\ \Omega$

EXAMPLE 4.14. *A 6-pole, 3 Φ, 50 Hz induction motor runs on full load with a slip of 4 per cent. Given the rotor standstill impedance per phase as (0.01 + j0.05) Ω, calculate the available maximum torque in terms of full-load torque. Also determine the speed at which the maximum torque occurs.*

SOLUTION. $s = 4\% = 0.04$

$$s_M = \frac{R_2}{X_{20}} = \frac{0.01}{0.05} = 0.2$$

$$N_M = (1 - s_M)\,N_s = (1 - 0.2) \times 1000 = 800 \text{ r.p.m.}$$

$$\frac{\tau_{max}}{\tau_{fl}} = \frac{s^2 + s_M^2}{2s\,s_M} = \frac{(0.04)^2 + (0.2)^2}{2 \times 0.04 \times 0.2} = 2.6$$

$\therefore \qquad \tau_{max} = 2.6\,\tau_{fl}\,.$

EXAMPLE 4.15. *A 6-pole, 3-phase, 50 Hz induction motor develops a maximum torque of 30 Nm at 960 r.p.m. Determine the torque exerted by the motor at 5% slip. The rotor resistance per phase is 0.6 Ω.*

SOLUTION. $N_s = \dfrac{120\,f}{P} = \dfrac{120 \times 50}{6} = 1000 \text{ r.p.m.}$

Speed at maximum torque, $N_M = 960$ r.p.m.

Slip at maximum speed

$$s_M = \frac{N_s - N_M}{N_s} = \frac{1000 - 960}{1000} = 0.04$$

Also, $\qquad s_M = \dfrac{R_2}{X_{20}}$

$$X_{20} = \frac{R_2}{s_M} = \frac{0.6}{0.04} = 15\ \Omega$$

If τ_s is the torque at slip s

$$\frac{\tau_s}{\tau_{max}} = \frac{2s\,s_M}{s^2 + s_M^2}$$

Here $s = 0.05,\ \ \tau_{max} = 30$ Nm

$$\tau_s = \frac{2 \times 0.05 \times 0.04}{(0.05)^2 + (0.04)^2} \times 30 = 29.27 \text{ Nm}$$

EXAMPLE 4.16. *A 4-pole, 50 Hz, 3-phase induction motor develops a maximum torque of 110 Nm at 1360 r.p.m. The resistance of the star-connected rotor is 0.25 Ω/phase. Calculate the value of resistance that must be inserted in series, with each rotor phase to produce a starting torque equal to half the maximum torque.*

SOLUTION. Synchronous speed, $N_s = \dfrac{120 f}{P} = \dfrac{120 \times 50}{4} = 1500$ r.p.m.

Speed at maximum torque $N_M = 1360$ r.p.m.

Slip at maximum torque
$$s_M = \frac{N_s - N_M}{N_s}$$
$$= \frac{1500 - 1360}{1500} = 0.0933$$

Also,
$$s_M = \frac{R_2}{X_{20}}$$
$$X_{20} = \frac{R_2}{s_M} = \frac{0.25}{0.0933} = 2.68\ \Omega$$
$$\tau_{max} = \frac{kE_{20}^2}{2X_{20}} = \frac{K}{2X_{20}} = \frac{K}{2 \times 2.68} = 0.1866\ \text{K}$$

where $K = k E_{20}^2$

Let r be the external resistance inserted per phase in the rotor circuit, then starting torque
$$\tau_{st} = \frac{k E_{20}^2 (R_2 + r)}{(R_2 + r)^2 + X_{20}^2} = \frac{K(0.25 + r)}{(0.25 + r)^2 + (2.68)^2}$$

Let $0.25 + r = R_T$

$$\therefore \quad \tau_{st} = \frac{KR_T}{R_T^2 + (2.68)^2}$$

It is given that
$$\tau_{st} = \frac{1}{2}\tau_{max}$$
$$\frac{K R_T}{R_T^2 + (2.68)^2} = \frac{1}{2} \times 0.1866\ \text{K}$$
$$R_T^2 + (2.68)^2 = \frac{2 R_T}{0.1866}$$
$$R_T^2 - 10.718\ R_T + 7.1824 = 0$$
$$R_T = \frac{10.718 \pm \sqrt{10.718^2 - 4 \times 7.1824}}{2}$$
$$= \frac{1}{2}(10.718 \pm 9.28) = 9.999\ \Omega \text{ or } 0.719\ \Omega$$
$$R_T = 0.25 + r$$

$$r = R_T - 0.25 = 9.999 - 0.25 = 9.749\ \Omega$$

or

$$r = 0.719 - 0.25 = 0.469\ \Omega\ .$$

We reject the value $r = 9.749\ \Omega$ as it corresponds to τ_{max} lying in the region where $s > 1$.

$$\therefore \quad r = 0.469\ \Omega$$

EXAMPLE 4.17. *A 746 kW, 3-phase, 50 Hz, 16-pole induction motor has a rotor impedance of* $(0.02 + j0.15)\ \Omega$ *at standstill. Full load torque is obtained at 360 r.p.m. Calculate (a) the speed at which maximum torque occurs ; (b) the ratio of maximum to full-load torque ; (c) the external resistance per phase to be inserted in the rotor circuit to get maximum torque at starting.*

SOLUTION. $N_s = \dfrac{120 f}{P} = \dfrac{120 \times 50}{16} = 375$ r.p.m.

Speed at full load = 360 r.p.m.

Slip at full load $\quad s_{fl} = \dfrac{N_s - N_r}{N_r} = \dfrac{375 - 360}{375} = 0.04$

Slip at maximum torque

$$s_M = \frac{R_2}{X_{20}} = \frac{0.02}{0.15} = \frac{2}{15}$$

(*a*) Speed at which maximum torque occurs

$$N_M = (1 - s_M)\, N_s = \left(1 - \frac{2}{15}\right) \times 375 = 325 \text{ r.p.m.}$$

(*b*)
$$\frac{\tau_{max}}{\tau_{fl}} = \frac{s_{fl}^2 + s_M^2}{2\, s_{fl}\, s_M} = \frac{(0.04)^2 + \left(\dfrac{2}{15}\right)^2}{2 \times 0.04 \times \dfrac{2}{15}} = 1.8167$$

(*c*) Let the external resistance per phase added to the rotor circuit be r ohms.

$\therefore$ rotor resistance per phase, $R_2 = (0.02 + r)$

The starting torque will be maximum when

$$R_2 = X_{20}$$

$$0.02 + r = 0.15, \quad r = 0.15 - 0.02 = 0.13\ \Omega$$

EXAMPLE 4.18. *A 3-phase, 50 Hz, 6-pole induction motor runs at 940 r.p.m. and delivers 7 kW output. What starting torque will the motor develop when switched directly on to the supply, if maximum torque is developed at 800 r.p.m. ? The friction and windage losses total 840 W.*

SOLUTION. $s = \dfrac{N_s - N}{N_s} = \dfrac{1000 - 940}{1000} = 0.06$

$$s_M = \frac{N_s - N_M}{N_s} = \frac{1000 - 800}{1000} = 0.2$$

$$P_{md} = 7000 + 840 = 7840 \text{ W}$$

$$P_{md} = 2\pi n \, \tau_d$$

$$7840 = 2\pi \times \frac{940}{60} \tau_{dfl}$$

$$\tau_{dfl} = \frac{7840 \times 60}{2\pi \times 940} = 79.645 \text{ Nm}$$

$$\tau_{st} = \frac{s^2 + s_M^2}{s\,(1 + s_M^2)} \tau_{dfl} = \frac{(0.06)^2 + (0.2)^2}{0.06\,(1 + 0.2)^2} \times 79.645 = 55.65 \text{ Nm}$$

EXAMPLE 4.19. *A 3-phase induction motor has a 4-pole, star-connected stator winding. The motor runs on a 50 Hz supply with 200 V between lines. The rotor resistance and standstill rotor reactance per phase are 0.1 Ω and 0.9 Ω respectively. The ratio of rotor to stator turns is 0.67. Calculate : (a) total torque at 4% slip ; (b) maximum torque ; (c) speed at maximum torque ; (d) maximum mechanical power. Neglect stator impedance.*

SOLUTION. $E_{20} = E_1 \left(\frac{T_{e_2}}{T_{e_1}} \right) = \frac{200}{\sqrt{3}} \times 0.67 = 77.37 \text{ V}$

(*a*)
$$\tau_d = \frac{3\,s\,E_{20}^2 R_2}{2\pi\, n_s\,[R_2^2 + (sX_{20})^2]}$$

$$= \frac{3 \times 0.04 \times (77.37)^2 \times 0.1}{2\pi \left(\frac{1500}{60} \right) [(0.1)^2 + (0.04 \times 0.9)^2]} = 40.48 \text{ Nm}$$

(*b*)
$$\tau_{max} = \frac{k\,E_{20}^2}{2\,X_{20}} = \frac{3\,E_{20}^2}{2\pi\, n_s \times 2X_{20}}$$

$$= \frac{3 \times (77.37)^2}{2\pi \times \frac{1500}{60} \times 2 \times 0.9} = 63.51 \text{ Nm}$$

(*c*) Slip at maximum torque

$$s_M = \frac{R_2}{X_{20}} = \frac{0.1}{0.9} = \frac{1}{9}$$

Speed at maximum torque

$$N_M = (1 - s_M)\, N_s = \left(1 - \frac{1}{9} \right) \times 1500 = 1333.3 \text{ r.p.m.}$$

(*d*)
$$(P_{md})_{max} = \omega_M\, \tau_{max} = \frac{2\pi\, N_M}{60}\, \tau_{max} = \frac{2\pi \times 1333.3}{60} \times 63.51 = 8867.5 \text{ Nm}$$

4.14 FULL-LOAD TORQUE AND MAXIMUM TORQUE

Let s = full–load slip of the motor

τ_{fl} = full–load torque

τ_{st} = starting torque

$$\tau_{fl} = \frac{k\, s\, R_2\, E_{20}^2}{R_2^2 + (sX_{20})^2} \tag{4.14.1}$$

$$\tau_{max} = \frac{k\, E_{20}^2}{2X_{20}} \tag{4.14.2}$$

At starting, $s = 1$

$$\tau_{st} = \frac{k\, R_2\, E_{20}^2}{R_2^2 + X_{20}^2} \tag{4.14.3}$$

$$\therefore \qquad \frac{\tau_{fl}}{\tau_{max}} = \frac{ks\, R_2\, E_{20}^2}{R_2^2 + (sX_{20})^2} \div \frac{k\, E_{20}^2}{2\, X_{20}} = \frac{2\, s\, R_2 X_{20}}{R_2^2 + (sX_{20}^2)}$$

But $R_2 = s_M X_{20}$ (4.14.4)

$$\therefore \qquad \frac{\tau_{fl}}{\tau_{max}} = \frac{2s \,.\, s_M X_{20}^2}{s_M^2\, X_{20}^2 + s^2 X_{20}^2}$$

$$\frac{\tau_{fl}}{\tau_{max}} = \frac{2\, s\, s_M}{s^2 + s_M^2} \tag{4.14.5}$$

Also,
$$\frac{\tau_{st}}{\tau_{max}} = \frac{k\, R_2\, E_{20}^2}{R_2^2 + X_{20}^2} \div \frac{k\, E_{20}^2}{2\, X_{20}}$$

$$= \frac{2\, R_2 X_{20}}{R_2^2 + X_{20}^2} = \frac{2\,(s_M X_{20})\, X_{20}}{(s_M X_{20})^2 + X_{20}^2}$$

$$\therefore \qquad \frac{\tau_{st}}{\tau_{max}} = \frac{2\, s_M}{1 + s_M^2} \tag{4.14.6}$$

Equation (4.14.6) can also be obtained from Eq. (4.14.5) by putting $s = 1$ in it.

$$\frac{\tau_{st}}{\tau_{max}} = \frac{2 \times 1 \times s_M}{1 + s_M^2} = \frac{2\, s_M}{1 + s_M^2}$$

Also,
$$\frac{\tau_{st}}{\tau_{fl}} = \frac{s^2 + s_M^2}{s\,(1 + s_M^2)} \tag{4.14.7}$$

EXAMPLE 4.20. *A 3-phase induction motor has a starting torque of 100% and a maximum torque of 200% of the full-load torque. Determine :*

(*a*) *slip at which maximum torque occurs ;*

(*b*) *full-load slip ;*

(*c*) *rotor current at starting in per unit of full-load rotor current.*

SOLUTION. (*a*) $\dfrac{\tau_{st}}{\tau_{max}} = \dfrac{\tau_{fl}}{2\,\tau_{fl}} = \dfrac{1}{2}$

Also, $\dfrac{\tau_{st}}{\tau_{max}} = \dfrac{2\, s_M}{1 + s_M^2} = \dfrac{1}{2}$

$$\therefore \quad s_M^2 - 4\,s_M + 1 = 0$$

$$s_M = \frac{4 \pm \sqrt{16-4}}{2} = 3.732,\ 0.268$$

Neglecting the higher value

$$s_M = 0.268$$

(b) $$\frac{\tau_{fl}}{\tau_{max}} = \frac{2s\,s_M}{s^2 + s_M^2}$$

$$\frac{1}{2} = \frac{2s\,s_M}{s^2 + s_M^2}$$

$$s^2 - 4s\,s_M + s_M^2 = 0$$

$$s = \frac{4s_M \pm \sqrt{16\,s_M^2 - 4\,s_M^2}}{2}$$

Neglecting the higher value

$$s = 0.268\,s_M = 0.268 \times 0.268 = 0.0718$$

(c) Let the slip at full load be s.

$$I_{2fl} = \frac{E_{2s}}{Z_{2s}} = \frac{s\,E_{20}}{\sqrt{R_2^2 + (sX_{20})^2}}$$

At starting $s = 1$

$$I_{2st} = \frac{E_{20}}{\sqrt{R_2^2 + X_{20}^2}}$$

$$\therefore \quad \frac{I_{2st}}{I_{2fl}} = \frac{E_{20}}{\sqrt{R_2^2 + X_{20}^2}} \div \frac{s\,E_{20}}{\sqrt{R_2^2 + (s\,X_{20})^2}}$$

$$\left(\frac{I_{2st}}{I_{2fl}}\right)^2 = \frac{R_2^2 + (sX_{20})^2}{s^2\,(R_2^2 + X_{20}^2)} = \frac{X_{20}^2\left(\dfrac{R_2^2}{X_{20}^2} + s^2\right)}{X_{20}^2\left(\dfrac{R_2^2}{X_{20}^2} + 1\right)s^2}$$

Since, $$s_M = \frac{R_2}{X_{20}}$$

$$\left(\frac{I_{2st}}{I_{2fl}}\right)^2 = \frac{s_M^2 + s^2}{s^2\,(1 + s_M^2)} = \frac{(0.268)^2 + (0.0718)^2}{(0.0718)^2\,(1 + 0.268^2)} = 13.9316$$

$$\therefore \quad \frac{I_{2st}}{I_{2fl}} = \sqrt{13.9316} = 3.7325$$

EXAMPLE 4.21. *A 3-phase induction motor with rotor resistance per phase equal to the standstill rotor reactance, has a starting torque of 25 Nm. For negligible stator impedance and no-load current, determine the starting torque in case the rotor circuit resistance per phase is (a) doubled, (b) halved.*

SOLUTION. $R_2 = X_{20}$

$$\tau_{st} = \frac{k\,R_2}{R_2^2 + X_{20}^2}$$

$$25 = \frac{kR_2}{R_2^2 + R_2^2}$$

$$\therefore \quad k = 50\,R_2$$

(a) New rotor resistance $= 2\,R_2$

$$\therefore \quad \tau_{st} = \frac{k\,(2\,R_2)}{(2R_2)^2 + R_2^2} = \frac{50\,R_2 \times 2R_2}{5\,R_2^2} = 20 \text{ Nm}$$

(b) Rotor resistance $= \frac{1}{2}\,R_2$

$$\tau_{st} = \frac{K(\frac{1}{2}R_2)}{(\frac{1}{2}\,R_2)^2 + R_2^2} = \frac{50\,R_2 \times \frac{1}{2}\,R_2}{\frac{R_2^2}{4} + R_2^2} = 20 \text{ Nm}$$

EXAMPLE 4.22. *The rotor resistance and reactance of a 4-pole, 50 Hz, 3-phase slip ring induction motor are 0.4 and 4 Ω/phase respectively. Calculate the speed at maximum torque and the ratio (max torque)/(starting torque). What value should the resistance per phase have so that the starting torque is half of maximum torque ?*

SOLUTION. $N_s = \frac{120\,f_1}{P} = \frac{120 \times 50}{4} = 1500$ rpm

$$s_M = \frac{R_2}{X_{20}} = \frac{0.4}{4} = 0.1$$

$$N_M = (1 - s_M)\,N_s = (1 - 0.1) \times 1500 = 1350 \text{ rpm}$$

$$\frac{\tau_{max}}{\tau_{st}} = \frac{1 + s_M^2}{2\,s_M} = \frac{1 + (0.1)^2}{2 \times 0.1} = 5.05$$

Let the additional resistance required per phase be r Ω.

$$\tau_{st} = \frac{K\,(R_2 + r)}{(R_2 + r)^2 + X_{20}^2}$$

$$\tau_{max} = \frac{K}{2\,X_{20}}$$

$$\frac{\tau_{st}}{\tau_{max}} = \frac{1}{2}$$

$$\frac{K\,(R_2 + r)}{(R_2 + r)^2 + X_{20}^2} \div \frac{K}{2\,X_{20}} = \frac{1}{2}$$

$$\frac{2\,X_{20}\,(R_2 + r)}{(R_2 + r)^2 + (X_{20})^2} = \frac{1}{2}$$

$$2 \times 2 \times 4\,(0.4 + r) = (0.4 + r)^2 + (4)^2$$

$$6.4 + 16r = 0.16 + 0.8r + r^2 + 16$$

$$r^2 - 15.2r + 9.76 = 0$$

$$r = \frac{15.2 \pm \sqrt{(15.2)^2 - 4 \times 9.76}}{2}$$

$$= \frac{1}{2}(15.2 \pm 13.856) = 14.52 \text{ or } 0.807$$

The lower value r is taken otherwise R would be greater than X_{20} and the maximum torque would never be reached. For example, if $r = 14.52$, then for maximum torque

$$14.52 = sX_{20}$$

$$s = \frac{14.52}{4} > 1$$

which is not possible

$$\therefore \qquad r = 0.807\ \Omega$$

EXAMPLE 4.23. *A 440 V, 50 Hz squirrel cage induction motor has a ratio of standstill reactance to resistance of rotor per phase of 3 to 1 and a maximum torque which is 4 times the normal full load torque. Calculate (a) full-load slip (b) ratio of starting torque to full-load torque (c) minimum voltage required to develop the normal full load torque at starting.*

SOLUTION. $s_M = \dfrac{R_2}{X_{20}} = \dfrac{1}{3}$

$$\frac{\tau_{max}}{\tau_{fl}} = \frac{s_M^2 + s^2}{2s\, s_M}$$

$$4 = \frac{\left(\frac{1}{3}\right)^2 + s^2}{2s \times \frac{1}{3}}$$

$$4 = \frac{1 + 9s^2}{6s}$$

$$9s^2 - 24s + 1 = 0$$

$$s = \frac{24 \pm \sqrt{24^2 - 4 \times 9 \times 1}}{2 \times 9} = 0.0423 \text{ or } 2.624$$

$$\therefore \qquad s = 0.0423$$

(b) $$\frac{\tau_{st}}{\tau_{fl}} = \frac{s^2 + s_M^2}{s\,(1 + s_M^2)} = \frac{(0.0423)^2 + \left(\frac{1}{3}\right)^2}{0.0423\left(1 + \frac{1}{9}\right)} = 2.402$$

(c) $\tau_{st} = \tau_{fl}$

$$\frac{K V_{11}^2 R_2}{R_2^2 + X_{20}^2} = \frac{K V_1^2 s R_2}{R_2^2 + (sX_{20})^2}$$

$$V_{11}^2 = \frac{s\,V_1^2\,(R_2^2 + X_{20}^2)}{R_2^2 + (sX_{20})^2}$$

$$= \frac{0.04\left(\frac{440}{\sqrt{3}}\right)^2 (R_2^2 + 9\,R_2^2)}{R_2^2 + (0.04 \times 9\,R_2)^2} = \frac{0.04 \times (440)^2\,(1+9)}{3\,(1+0.1276)} = 22853$$

$$V_{11} = 151.17 \text{ V}$$

Line voltage $= \sqrt{3} \times 151.17 = 261.8$ V

4.15 WINDING E.M.F.s

Let suffixes 1 and 2 be used for stator and rotor quantities respectively.

V_1 = stator applied voltage per phase

T_1 = number of stator winding turns in series per phase

T_2 = number of rotor winding turns in series per phase

Φ = flux per pole produced by the stator mmf
= resultant air gap flux

E_1 = stator induced emf per phase

E_{20} = emf induced in the rotor per phase when the rotor is at standstill

E_{2s} = emf induced in the rotor per phase when the rotor is rotating at a slip s

R_1 = resistance of stator winding per phase

R_2 = resistance of rotor winding per phase

L_{20} = rotor inductance per phase at standstill due to leakage flux

X_{20} = leakage reactance of the rotor winding per phase when the rotor is at standstill

f_1 = stator emf frequency (supply frequency)

f_2 = frequency of the induced emf in the rotor at a slip s

X_{2s} = leakage reactance of rotor winding per phase when the rotor is rotating at a slip s

k_{d_1} = distribution factor of stator winding

k_{d_2} = distribution factor of rotor winding

k_{c_1} = pitch factor or coil span factor of stator winding

k_{c_2} = pitch factor or coil span factor of rotor winding

Stator induced emf per phase

$$E_1 = 4.44\,k_c\,k_{d_1} f_1\,\Phi\,T_1 \tag{4.15.1}$$

Induced emf per phase in the rotor when the rotor is at standstill

$$E_{20} = 4.44\,k_{c_2}\,k_{d_2} f_1\,\Phi\,T_2 \tag{4.15.2}$$

Induced emf per phase in the rotor when the rotor is rotating at a slip s

$$E_{2s} = s\, E_{20} \tag{4.15.3}$$

$$\therefore \quad E_{2s} = 4.44\, k_{c_2}\, k_{d_2}\, sf\, \Phi\, T_2 \tag{4.15.4}$$

Let $k_{c_1}\, k_{d_1} = k_{w_1}$ = winding factor of stator (4.15.5)

and $k_{c_2}\, k_{d_2} = k_{w_2}$ = winding factor of rotor (4.15.6)

$$\therefore \quad E_1 = 4.44\, k_{w_1} f_1\, \Phi\, T_1 \tag{4.15.7}$$

$$E_{2s} = 4.44\, k_{w_2}\, s f_1\, \Phi\, T_2 \tag{4.15.8}$$

Let us define

$$T_{e_1} \stackrel{\Delta}{=} k_{w_1}\, T_1 \tag{4.15.9}$$

$$T_{e_2} \stackrel{\Delta}{=} k_{w_2}\, T_2 \tag{4.15.10}$$

$$\therefore \quad \frac{E_1}{E_{20}} = \frac{k_{w_1}\, T_1}{k_{w_2}\, T_2} = \frac{T_{e_1}}{T_{e_2}} = a_{eff} \tag{4.15.11}$$

where T_{e_1} and T_{e_2} are called effective stator and rotor turns per phase respectively.

a_{eff} = effective turns ratio of an induction motor

$$\frac{I'_2}{I_2} = \frac{T_{e_2}}{T_{e_1}} = \frac{1}{a_{eff}} \tag{4.15.12}$$

Equation (4.15.11) shows that the ratio between stator and rotor emfs is constant at standstill. This ratio depends on the turns ratio modified by distribution and pitch factors of the windings. The induction motor, therefore, behaves like a transformer. It is to be noted that the factors for stator and rotor windings are not the same, because the number of slots in them may be different.

4.16 DEVELOPMENT OF CIRCUIT MODEL (EQUIVALENT CIRCUIT) OF AN INDUCTION MOTOR

An induction motor is based for its operation on the induction of voltages and currents in its rotor circuit from the stator circuit. Because the induction of voltages and currents in the rotor circuit of an induction motor is essentially a transformer operation, the equivalent circuit of an induction motor is very similar to the equivalent circuit of a transformer.

Equivalent circuit enables the performance characteristics of the induction motor to be evaluated for steady-state conditions by simple network calculations. The equivalent circuit of an induction motor is drawn only for one phase.

4.17 THE STATOR CIRCUIT MODEL

The stator model of the induction motor is shown in Fig. 4.9.

It consists of a stator phase winding resistance R_1, a stator phase winding leakage reactance X_1. These two components appear right at the input to the machine model.

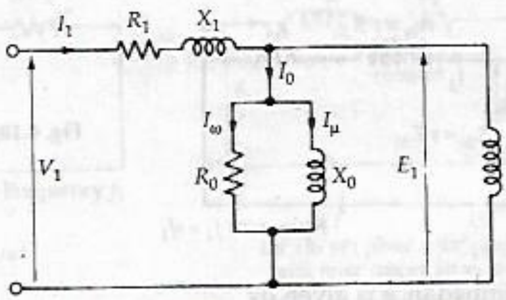


Fig. 4.9. *Stator model of an induction motor.*

The no-load current I_0 is simulated by a pure inductive reactor X_0 taking the magnetizing component I_μ and a non-inductive resistor R_0 carrying the core-loss current I_ω. Thus

$$\mathbf{I_0} = \mathbf{I_\mu} + \mathbf{I_\omega} \tag{4.17.1}$$

It is to be noted that the total magnetizing current I_0 is considerably larger in the case of the induction motor as compared to a transformer. This is due to the higher reluctance caused by the air gap of the induction motor. The magnetizing reactance X_0 in an induction motor will have a much smaller value. In a transformer, I_0 is about 2 to 5% of the rated current while in an induction motor it is approximately 25 to 40% of the rated current depending upon the size of the motor.

4.18 ROTOR CIRCUIT MODEL

In an induction motor, when a 3 ϕ supply is applied to the stator windings, a voltage is induced in the rotor windings of the machine. In general, the greater the relative motion of the rotor and the stator magnetic fields, the greater the resulting rotor voltage. The largest relative motion occurs when the rotor is stationary. This condition is called the *standstill* condition. This is also known as the *locked-rotor* or *blocked-rotor* condition. If the induced rotor voltage at this condition is E_{20} then the induced voltage at any slip is given by

$$\mathbf{E_{2s}} = s\mathbf{E_{20}} \tag{4.18.1}$$

The rotor resistance R_2 is a constant (except for the skin effect). It is independent of slip.

The reactance of the induction motor rotor depends upon the inductance of the rotor and the frequency of the voltage and current in the rotor.

If L_2 = inductance of rotor, the rotor reactance is given by

$$X_2 = 2\pi f_2 L_2$$

But $$f_2 = sf_1$$

$\therefore$ $$X_2 = 2\pi sf_1 L_2 = s\,(2\pi f_1 L_2)$$

or $$X_2 = sX_{20} \tag{4.18.2}$$

where X_{20} is the standstill reactance of the rotor.

The rotor circuit is shown in Fig. 4.10.

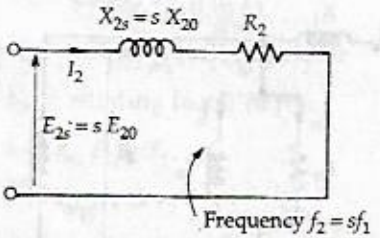


Fig. 4.10. *Rotor Circuit Model.*

The rotor impedance is given by

$$Z_{2s} = R_2 + jX_{2s}$$

or

$$Z_{2s} = R_2 + jsX_{20} \tag{4.18.3}$$

The rotor current per phase may be expressed as

$$\mathbf{I}_{2s} = \frac{\mathbf{E}_{2s}}{Z_{2s}}$$

$$\mathbf{I}_{2s} = \frac{s\mathbf{E}_{20}}{R_2 + jsX_{20}} \tag{4.18.4}$$

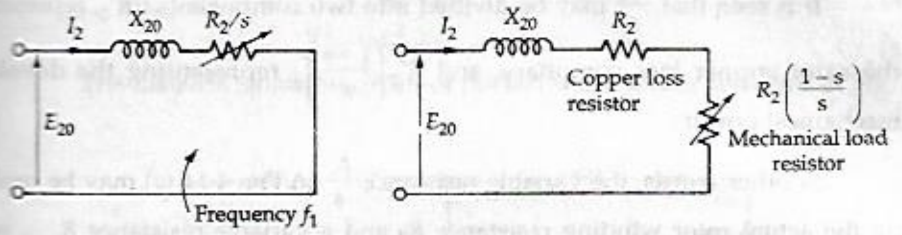
The circuit interpretation of Eq. (4.18.4) is shown in Fig. 4.10. It shows that I_2 is a slip-frequency current produced by a slip-frequency induced voltage sE_{20} acting in the rotor circuit having an impedance per phase of $(R_2 + jsX_{20})$.

By dividing both the numerator and the denominator of Eq. (4.18.4) by the slip s, we get

$$\mathbf{I}_{2s} = \frac{\mathbf{E}_{20}}{\dfrac{R_2}{s} + jX_{20}} \tag{4.18.5}$$

The circuit interpretation of Eq. (4.18.5) is shown in Fig. 4.11. It is to be noted that the magnitude and phase angle of $\mathbf{I}_{2s}$ remain the same by this operation. However, there is a significant difference between Eqs. (4.18.4) and (4.18.5). In Eq. (4.18.5), $\mathbf{I}_{2s}$ considered to be produced by a constant line-frequency voltage $\mathbf{E}_{20}$ acting in a rotor circuit having an impedance per phase of $\left(\dfrac{R_2}{s} + jX_{20}\right)$. Hence the $\mathbf{I}_{2s}$ of Eq. (4.18.5) is a *line-frequency current*, while $\mathbf{I}_{2s}$ of Eq. (4.18.4) is a *slip-frequency current*.

It is also to be noted that the rotor circuit model of Fig. 4.10 has a constant resistance R_2 and a variable leakage reactance sX_{20}. Similarly, the rotor circuit model of Fig. 4.11 has a constant leakage reactance X_{20} and a variable resistance $\dfrac{R_2}{s}$.



(a) *(b) The per phase rotor equivalent circuit with rotor copper losses and P_{md} separated.*

Fig. 4.11.

The significance of Eq. (4.18.5) should be understood clearly. This equation describes the secondary circuit of a fictitious transformer, one with a constant voltage ratio and with the *same* frequency of both sides. This *fictitious stationary rotor* carries the same current as the actual rotating rotor, and, thus produces the same mmf wave. This concept of fictitious stationary rotor makes it possible to transfer the secondary (rotor) impedance to the primary (stator) side.

It should be noted that when rotor currents and voltages are reflected into the stator, their frequency is also changed to stator frequency.

4.19 SEPARATION OF MECHANICAL LOAD FROM ROTOR COPPER LOSS IN THE CIRCUIT MODEL

In the circuit model shown in Fig. 4.12, the resistor R'_2/s consumes the total rotor input (air-gap power). Therefore, the air-gap power is given by

$$P_g = 3 I'^2_2 \frac{R'_2}{s} \tag{4.19.1}$$

The actual resistive losses (copper losses) in the rotor circuit are given by

$$p_{RCL} = 3 I'^2_2 R'_2 \tag{4.19.2}$$

Developed mechanical power

$$P_{md} = P_g - p_{RCL} = 3 I'^2_2 \frac{R'_2}{s} - 3 I'^2_2 R'_2$$

$$P_{md} = 3 I'^2_2 R'_2 \left(\frac{1}{s} - 1\right) \tag{4.19.3}$$

$$P_g = p_{RCL} + P_{md} = 3 I'^2_2 R'_2 + 3 I'^2_2 R'_2 \left(\frac{1}{s} - 1\right)$$

$$P_g = 3 I'^2_2 \left[R'_2 + R'_2 \left(\frac{1-s}{s}\right)\right] \tag{4.19.4}$$

Also,

$$R'_2 + R'_2\left(\frac{1-s}{s}\right) = R'_2 + \frac{R'_2}{s} - R'_2 = \frac{R'_2}{s}$$

It is seen that $\frac{R'_2}{s}$ may be divided into two components : R'_2, representing the rotor copper loss per phase, and $R'_2\left(\frac{1-s}{s}\right)$, representing the developed mechanical power.

In other words, the variable resistance $\frac{R_2}{s}$ in Fig. 4.11 (*a*) may be replaced by the actual rotor winding resistance R_2 and a variable resistance R_{mech} which represents the mechanical shaft load. That is,

$$R_{mech} = \frac{R_2}{s}(1-s) \qquad (4.19.5)$$

This expression is useful in analysis because it allows any mechanical load to be represented in the equivalent circuit by a resistor. The modified per phase rotor equivalent circuit is shown in Fig. 4.11 (*b*).

4.20 THE COMPLETE CIRCUIT MODEL (EQUIVALENT CIRCUIT) REFERRED TO STATOR

In order to obtain the complete per-phase equivalent circuit for an induction motor, it is necessary to refer the rotor part of the model over to the stator circuit frequency and voltage level.

In an ordinary transformer, the voltage, currents, and impedances on the secondary side can be transferred to the primary side by means of the turns ratio '*a*' of the transformer.

$$\mathbf{E}_1 = \mathbf{E}'_2 = a\,\mathbf{E}_2 \qquad (4.20.1)$$

$$\mathbf{I}_1 = \mathbf{I}'_2 = \frac{\mathbf{I}_2}{a} \qquad (4.20.2)$$

and

$$\mathbf{Z}'_2 = a^2\,\mathbf{Z}_2 \qquad (4.20.3)$$

where the prime refers to the reflected values of voltage, current and impedance.

Similar transformation can be done for the induction motor's rotor circuit.

If a_{eff} = effective turns ratio of the induction motor

R'_2 = resistance of the rotor winding per phase referred to the stator side

X'_{20} = standstill rotor reactance per phase referred to the stator side

$$\frac{\mathbf{E}_2}{T_{e_2}} = \frac{\mathbf{E}'_2}{T_{e_1}} \qquad (4.20.4)$$

$$\mathbf{E}'_2 = \frac{T_{e_1}}{T_{e_2}}\mathbf{E}_2 = a_{eff}\,.\,\mathbf{E}_2 = \mathbf{E}_1 \qquad (4.20.5)$$

Similarly,

$$\mathbf{I}'_2 = \frac{\mathbf{I}_2}{a_{eff}} \qquad (4.20.6)$$

$$\mathbf{Z}'_{20} = a_{eff}^2\left(\frac{R_2}{s} + jX_{20}\right) \qquad (4.20.7)$$

$$R'_2 = a_{eff}^2 R_2 \tag{4.20.8}$$

$$X'_{20} = a_{eff}^2 X_{20} \tag{4.20.9}$$

The *complete equivalent circuit* of the induction motor is shown in Fig. 4.12.

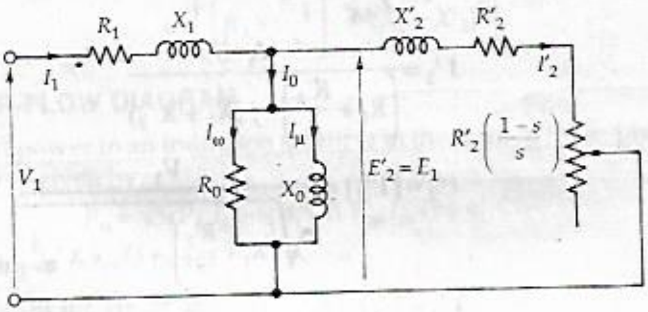


Fig. 4.12. *Per phase complete equivalent circuit of the induction motor referred to the stator.*

It is to be noted that the form of this circuit is identical with that of the 2-winding transformer.

4.21 APPROXIMATE EQUIVALENT CIRCUIT

It is usual to simplify this equivalent circuit still further by shifting the shunt impedance branches R_0 and X_0 to the input terminals as shown in Fig. 4.13. This approximation is based on the assumption that $\mathbf{V_1 = E_1 = E'_2}$. The circuit shown in Fig. 4.13 is called the "approximate equivalent circuit per phase of the induction motor". In this circuit, the only component that depends on slip s is the resistance representing the developed mechanical power by the rotor. All other quantities are constant, and reactances correspond to those at the fixed stator frequency f_1. This approximate equivalent circuit model has become the standard for all performance calculation of an induction motor.

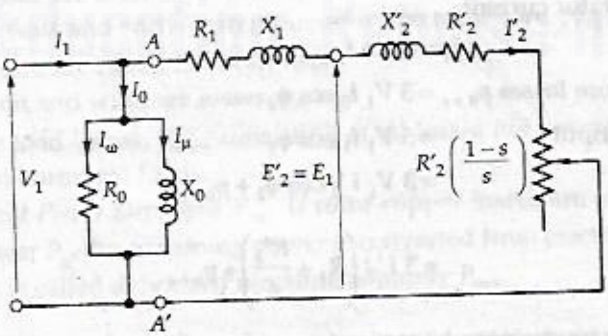


Fig. 4.13. *Approximate equivalent circuit.*

Referring to Fig. 4.13, the following equations can be written down for one phase at any given slip s :

Impedance beyond AA'

$$\mathbf{Z}_{AA'} = \left(R_1 + \frac{R'_2}{s}\right) + j(X_1 + X'_2) \tag{4.21.1}$$

$$\mathbf{I}'_2 = \frac{\mathbf{V}_1}{\mathbf{Z}_{AA'}} \tag{4.21.2}$$

or

$$\mathbf{I}'_2 = \frac{\mathbf{V}_1}{\left(R_1 + \frac{R'_2}{s}\right) + j(X_1 + X'_2)} \tag{4.21.3}$$

$$\therefore \quad I'_2 = |\mathbf{I}'_2| = \frac{V_1}{\sqrt{\left(R_1 + \frac{R'_2}{s}\right)^2 + (X_1 + X'_2)^2}} \tag{4.21.4}$$

$$\therefore \quad \mathbf{I}'_2 = I'_2 \angle{-\phi_2} \tag{4.21.5}$$

$$= I'_2 \cos\phi_2 - j\, I'_2 \sin\phi_2 \tag{4.21.6}$$

where $\quad \tan\phi_2 = \dfrac{X_1 + X'_2}{R_1 + \dfrac{R'_2}{s}}$ (4.21.7)

$$\cos\phi_2 = \frac{R_1 + (R'_2/s)}{|Z_{AA'}|} \tag{4.21.8}$$

No-load current, $\mathbf{I}_0 = \mathbf{I}_w + \mathbf{I}_\mu$

$$\therefore \quad \mathbf{I}_0 = \frac{\mathbf{V}_1}{R_0} + \frac{\mathbf{V}_1}{jX_0}$$

$$\mathbf{I}_0 = \mathbf{V}_1\left(\frac{1}{R_0} - j\frac{1}{X_0}\right) \tag{4.21.9}$$

Total stator current

$$\mathbf{I}_1 = \mathbf{I}'_2 + \mathbf{I}_0$$

Total core losses $p_{h+e} = 3\, V_1 I_0 \cos\phi_0$ (4.21.10)

Stator input

$$= 3\, V_1 I_1 \cos\phi_1$$

$$= 3\, V_1 I'_2 \cos\phi_2 + p_{h+e}$$

$$= 3\, I'^2_2 \left(R_1 + \frac{R'_2}{s}\right) + p_{h+e}$$

Air-gap power per phase

$$P_g = V_1 I'_2 \cos\phi_2 = I'^2_2 \frac{R'_2}{s} = \frac{V_1^2 (R'_2/s)}{\left(R_1 + \frac{R'_2}{s}\right)^2 + (X_1 + X'_2)^2}$$

Developed torque $\tau_d = \dfrac{P_g}{\omega_s}$

or
$$\tau_d = \frac{V_1^2 (R'_2/s)}{\omega_s \left[\left(R_1 + \dfrac{R'_2}{s} \right)^2 + (X_1 + X'_2)^2 \right]}$$

4.22 POWER-FLOW DIAGRAM

The input power to an induction motor is in the form of three-phase voltage and currents. It is given by

$$P_{is} = \sqrt{3}\ V_L I_L \cos\phi_i = 3\ V_{sp} I_{sp} \cos\phi_i$$

where $\cos\phi_i$ = input power factor

The losses in the stator are

(*a*) I^2R losses in the stator winding resistances, $p_{SCL} = 3 I_{sp}^2 R_{sp}$

These losses are known as *stator copper losses*.

(*b*) Hysteresis and eddy-current losses in the stator core, $p_{s(h+e)}$.

These losses are called ***stator-core losses***.

The power output of the stator $P_{os} = P_{is} - p_{sc} - p_{s(h+e)}$

This power P_{os} is transferred to the rotor of the machine across the air gap between the stator and rotor. It is called the ***air-gap power*** P_g of the machine.

Thus,

power output of the stator = air-gap power = input power to rotor

or
$$P_{os} = P_g = P_{ir}$$

Rotor Losses

(*a*) I^2R losses in the rotor resistance $p_{rc} = 3 I_2^2 R_2$

These losses are known as *rotor-copper losses*.

(*b*) Hysteresis and eddy-current losses in the rotor core, $p_{r(h+e)}$.

These losses are called *rotor-core losses*.

(*c*) Friction and windage losses, p_{fw}

(*d*) Stray load losses, p_{misc}, consisting of all losses not covered above, such as losses due to harmonic fields.

Mechanical Power Developed P_{md}. If rotor copper losses are subtracted from rotor input power P_g, the remaining power is converted from electrical to mechanical form. This is called ***developed mechanical power*** P_{md}.

Developed mechanical power = rotor input − rotor copper loss

$$P_{md} = P_{ir} - p_{rc}$$

or
$$P_{md} = P_g - p_{rc}$$

$$P_{md} = P_g - 3 I_2^2 R_2$$

The output of the motor is given by

$$P_0 = P_{md} - P_{fw} - P_{misc}$$

P_0 is also called ***shaft power*** or ***useful power***.

ROTATIONAL LOSSES

At starting and during acceleration, the rotor core losses are high. With the increase in speed these losses decrease. The friction and windage losses are zero at start. With increase in speed these losses increase. As a result, the sum of friction, windage, and core losses is roughly constant with changing speed. Therefore these categories of losses are sometimes lumped together and called *rotational losses*. *Rotational losses* are defined as follows :

$$p_{rot} \overset{\Delta}{=} p_{fw} + p_{h+e} + p_{misc}$$

Then $$P_0 = P_{md} - p_{rot} = P_{md} - p_{fw} - p_{h+e} - p_{misc}$$

The relationship between the input electric power and the output mechanical power of an induction motor is shown in the power-flow diagram in Fig. 4.14.

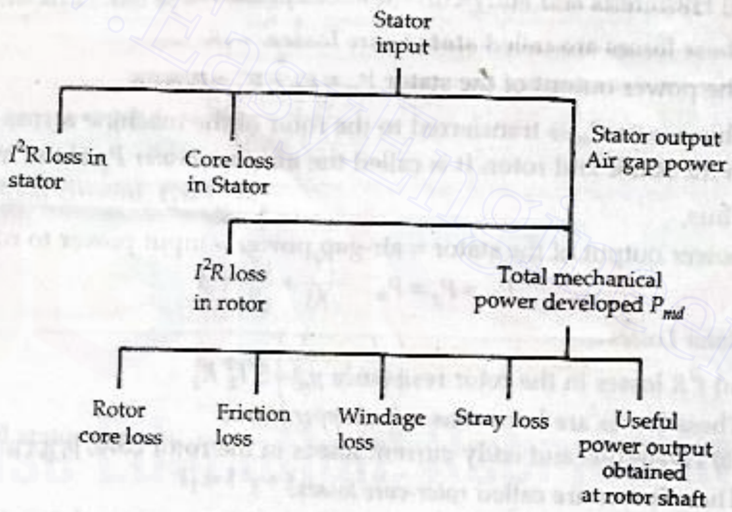


Fig. 4.14. *The power-flow diagram of an induction motor.*

Being a purely mechanical quantity, the rotational losses are not represented by any element of the equivalent circuit.

4.23 REPRESENTATION OF CORE LOSSES IN THE CIRCUIT MODEL

There is no general agreement as to how to treat core losses in the circuit model. The core losses of an induction motor consist of stator-core losses and rotor core losses. The rotor core losses vary with rotor frequency and hence with the slip. Under normal running conditions the slip is of the order of 0.03 (3%) and therefore the rotor frequency is about 1.5 Hz for stator frequency of 50 Hz. For this reason, the rotor-core losses are negligible, and all the core losses are lumped together in the stator of the circuit model. These losses are represented in the induction motor equivalent circuit by the resistor R_0.

4.24 STARTING INDUCTION MOTORS

When the supply is connected to the stator of a three-phase induction motor, a rotating magnetic field is produced and the rotor starts rotating. Thus, a three-phase induction motor is self-starting. At the time of starting the motor slip is unity and the starting current is very large. The purpose of a starter is not to start the motor as the name implies. The starter of the motor performs two functions :

1. To reduce the heavy starting current.
2. To provide *overload* and *under-voltage protection*.

function of a starter

In general, three-phase induction motors may be started either by connecting the motor directly to the full voltage of the supply or by applying a reduced voltage to the motor during starting period. The torque of an induction motor is proportional to the square of the applied voltage. Thus, a greater torque is exerted by a motor when it is started on full voltage than when it is started on reduced voltage.

4.25 STARTING OF CAGE MOTORS

The following are the commonly used starters for cage motors :

1. Direct on-line starter.
2. Star-delta starter.
3. Autotransformer starter.

4.26 DIRECT-ON-LINE STARTER

In the direct-on-line method of starting cage motors, the motor is connected by means of a starter across the *full* supply voltage. Figure 4.15 shows the connections

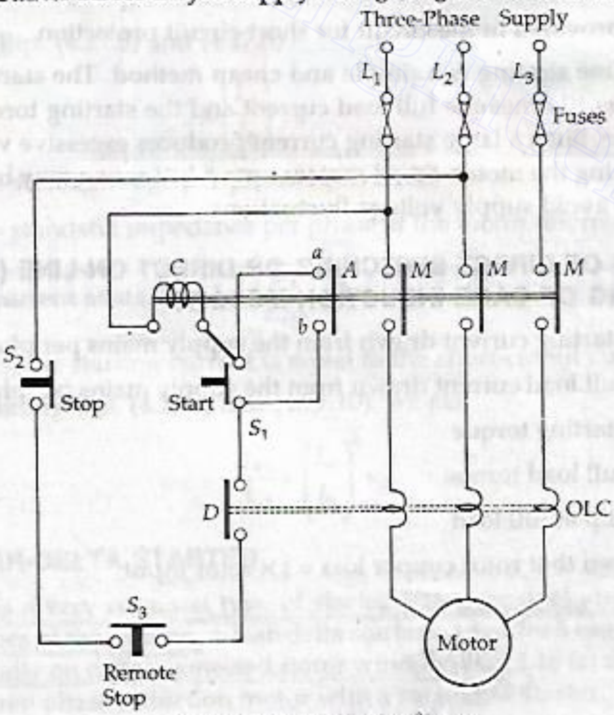


Fig. 4.15. *Direct-on-line starter.*

for one type of the direct-on-line (D.O.L.) starter. It consists of a coil-operated contactor C controlled by start and stop push buttons which may be installed at convenient places remote from the starter. On pressing the START push button S_1 (which is normally held open by a spring) the contactor coil C is energised from two line conductors L_1 and L_2. The three main contacts M and the auxiliary contact A close and the terminals a and b are short-circuited. The motor is thus connected to the supply. When the pressure on S_1 is released, it moves back under spring action. Even then the coil C remains energised through ab. Thus, the main contacts M remain closed and the motor continues to get supply. For this reason, contact A is called *hold-on-contact*.

When the STOP push button S_2 (which is normally held closed by spring) is pressed, the supply through the contactor coil C is disconnected. Since the coil C is de-energised, the main contacts M and auxiliary contact A are opened. The supply to motor is disconnected and the motor stops.

Undervoltage protection

When the voltage falls below a certain value, or in the event of failure of supply during motor operation, the coil C is de-energised. The motor is then disconnected from the supply.

Overload protection

In case of an overload on the motor, one or all the overload coils (O.L.C.) are energised. The normally closed contact D is opened and the contactor coil C is de-energised to disconnect the supply to the motor.

Fuses are provided in the circuit for short-circuit protection.

Direct-on-line starting is a simple and cheap method. The starting current may be as large as 10 times the full load current and the starting torque is equal to full-load torque. Such a large starting current produces excessive voltage drop in the line supplying the motor. Small motors upto 5 kW rating may be started by D.O.L. starters to avoid supply voltage fluctuations.

4.27 THEORY OF DIRECT SWITCHING OR DIRECT ON-LINE (DOL) STARTING OF CAGE INDUCTION MOTORS

Let I_{st} = starting current drawn from the supply mains per phase

I_{fl} = full-load current drawn from the supply mains per phase

τ_{est} = starting torque

τ_{efl} = full-load torque

s_{fl} = slip at full load

We have seen that rotor copper loss = $s \times$ rotor input

$$3 I_2^2 R_2 = s \times 2\pi n_s \tau_e \tag{4.27.1}$$

$$\therefore \quad \tau_e = \frac{3 I_2^2 R_2}{2\pi n_s s} \tag{4.27.2}$$

At starting, $s = 1$, $I_2 = I_{2st}$, $\tau_e = \tau_{est}$

$$\therefore \quad \tau_{est} = \frac{3 I_{2st}^2 R_2}{2\pi n_s \times 1} \tag{4.27.3}$$

At full-load

$$s = s_{fl}, \; I_2 = I_{2fl}, \; \tau_e = \tau_{efl}$$

$$\tau_{efl} = \frac{3 I_{2fl}^2 R_2}{2\pi n_s \times s_{fl}} \tag{4.27.4}$$

$$\therefore \quad \frac{\tau_{est}}{\tau_{efl}} = \left(\frac{3 I_{2st}^2 R_2}{2\pi n_s \times 1} \right) \div \left(\frac{3 I_{2fl}^2 R_2}{2\pi n_s \times s_{fl}} \right)$$

$$\text{or} \quad \frac{\tau_{est}}{\tau_{efl}} = \left(\frac{I_{2st}}{I_{2fl}} \right)^2 \times s_{fl} \tag{4.27.5}$$

If the no-load current is neglected

$$I_{st} \times \text{effective stator turns} = I_{2st} \times \text{effective rotor turns} \tag{4.27.6}$$

Also

$$I_{fl} \times \text{effective stator turns} = I_{2fl} \times \text{effective rotor turns} \tag{4.27.7}$$

From Eqs. (4.27.6) and (4.27.7)

$$\therefore \quad \frac{I_{st}}{I_{fl}} = \frac{I_{2st}}{I_{2fl}} \tag{4.27.8}$$

From Eqs. (4.27.5) and (4.27.8)

$$\frac{\tau_{est}}{\tau_{efl}} = \left(\frac{I_{st}}{I_{fl}} \right)^2 s_{fl} \tag{4.27.9}$$

If V_1 = stator voltage per phase equivalent

Z_{e10} = standstill impedance per phase of the motor referred to stator

then current at starting $I_{st} = \dfrac{V_1}{Z_{e10}} = I_{sc}$ (4.27.10)

That is, the starting current is equal to the short-circuit current.

Combining Eqs. (4.27.9) and (4.27.10), we get

$$\frac{\tau_{est}}{\tau_{efl}} = \left(\frac{I_{sc}}{I_{fl}} \right)^2 s_{fl} \tag{4.27.11}$$

4.28 STAR-DELTA STARTER

This is a very common type of starter and extensively used, compared to the other types of the starters. A star-delta starter is used for a cage motor designed to run normally on delta-connected stator winding. Fig. 4.16 (*a*) shows the connections of a three-phase induction motor with a star-delta starter. When the switch

S is in the START position, the stator windings are connected in STAR [Fig. 4.16 (*b*)]. When the motor picks up speed, say 80 per cent of its rated value, the changeover switch S is thrown quickly to the RUN position which connects the stator windings in DELTA [Fig. 4.16 (*c*)]. By connecting the stator windings, first in star and then in delta, the line current drawn by the motor at starting is reduced to one-third as compared to starting current with the windings connected in delta. At the time of starting when the stator windings are star connected, each stator phase gets a voltage $V_L/\sqrt{3}$, where V_L is the line voltage. Since the torque developed by an induction motor is proportional to the square of the applied voltage, star-delta starting reduces the starting torque to one-third that obtainable by direct-delta starting.

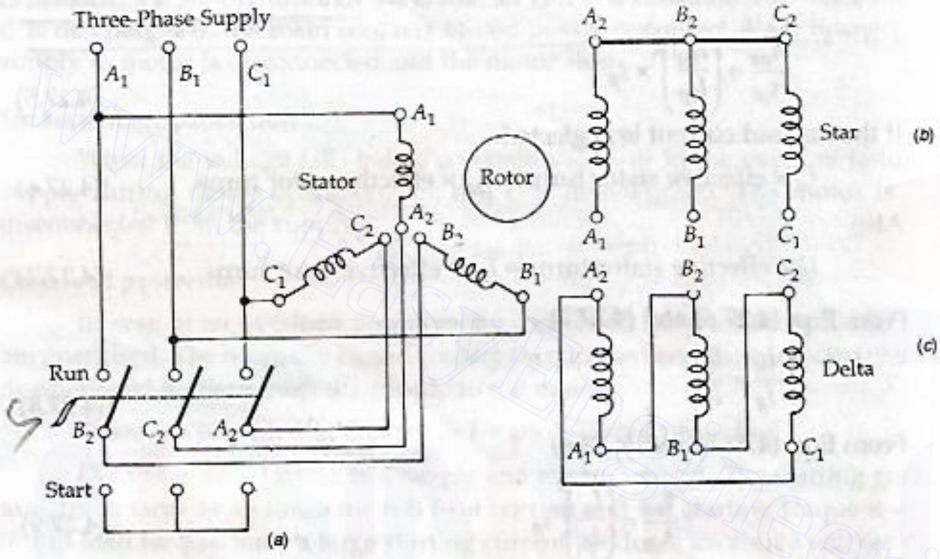


Fig. 4.16. *Star-delta starter.*

4.29 THEORY OF STAR-DELTA STARTING

At starting, the stator windings are connected in star and therefore voltage across each phase winding is equal to $\frac{1}{\sqrt{3}}$ times the line voltage.

Let

V_L = line voltage

I_{stYp} = starting current per phase with stator windings connected in star

I_{stYl} = starting line current with stator winding in star

For star connection, line current = phase current

$\therefore \quad I_{stYl} = I_{stYp}$

If $\quad V_1$ = phase voltage

V_L = line voltage

$I_{st\Delta p}$ = starting current per phase by direct switching with stator windings connected in delta

$I_{st\Delta l}$ = starting line current by direct switching with stator windings in delta

$I_{sc\Delta p}$ = short-circuit phase current by direct switching with stator windings in delta

Z_{e10} = standstill equivalent impedance per phase of the motor referred to stator

$$I_{stYp} = \frac{V_1}{Z_{e10}} = \frac{V_L}{\sqrt{3}\, Z_{e10}}$$

$$I_{st\Delta p} = \frac{V_L}{Z_{e10}}$$

For delta connection,

$$\text{line current} = \sqrt{3} \times \text{phase current}$$

$$\therefore \quad I_{st\Delta l} = \sqrt{3}\, I_{st\Delta p} = \frac{\sqrt{3}\, V_L}{Z_{e10}}$$

$$\frac{\text{starting line current with star-delta starting}}{\text{starting line current with direct switching in delta}} = \frac{I_{stYp}}{I_{st\Delta l}} = \frac{(V_L/\sqrt{3}\, Z_{e10})}{\sqrt{3}\,(V_L/Z_{e10})} = \frac{1}{3} \tag{4.29.1}$$

Thus, with star-delta starter, the starting current from the main supply is one-third of that with direct switching in delta.

Also,

$$\frac{\text{starting torque with star-delta starting}}{\text{starting torque with direct switching in delta}} = \frac{(V_L/\sqrt{3})^2}{V_L^2} = \frac{1}{3} \tag{4.29.2}$$

Hence, with star-delta starting, the starting torque is reduced to one-third of the starting torque obtained with direct switching in delta.

$$\frac{\text{starting torque with star-delta starting}}{\text{full-load torque with stator winding in delta}} = \left[\frac{(I_{stYp})^2 \cdot \frac{R_2}{1}}{2\pi\, n_s}\right] \div \left[\frac{I_{fl\Delta p}^2}{2\pi\, n_s} \cdot \frac{R_2}{s_{fl}}\right]$$

$$= \left(\frac{I_{stYp}}{I_{fl\Delta p}}\right)^2 s_{fl} \tag{4.29.3}$$

where $I_{fl\Delta p}$ = full-load phase current with winding in delta

But $\quad I_{stYp} = \dfrac{V_L/\sqrt{3}}{Z_{e10}}$

$$I_{st\Delta p} = \frac{V_L}{Z_{e10}}$$

$$\therefore \quad I_{stYp} = \frac{1}{\sqrt{3}} I_{st\Delta p}$$

and $\quad I_{stYp}^2 = \dfrac{1}{3} I_{st\Delta p}^2$

$$\therefore \frac{\text{starting torque with star delta starting}}{\text{full load torque with stator winding in delta}} = \left(\frac{I_{stYp}}{I_{fl\Delta p}}\right)^2 s_{fl}$$

$$= \frac{1}{3}\left(\frac{I_{st\Delta p}}{I_{fl\Delta p}}\right)^2 s_{fl} \quad (4.2$$

4.30 AUTO-TRANSFORMER STARTER

An auto-transformer starter is suitable for both star- and delta-conne motors. In this method, the starting current is limited by using a three-ph auto-transformer to reduce the initial stator applied voltage. Fig. 4.17 shows th motor with the auto-transformer starter. The auto-transformer is provided wi number of tappings.

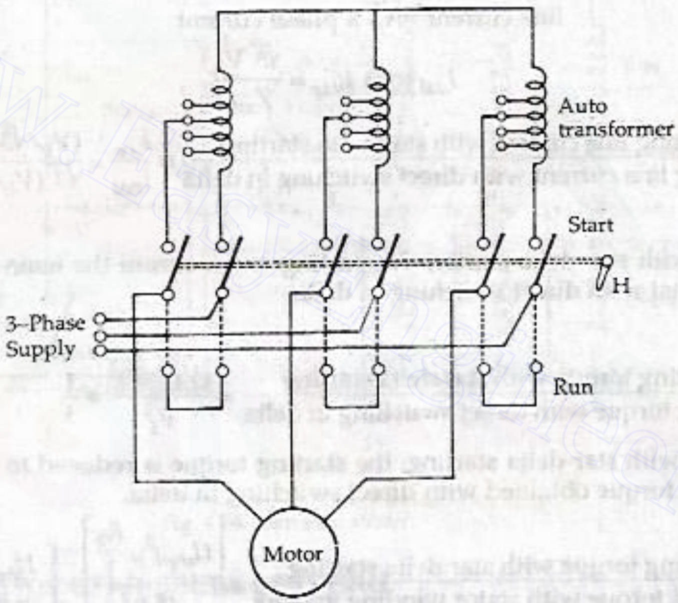


Fig. 4.17. *Auto-transformer starter.*

In practice, the starter is connected to one particular tapping to obtain th most suitable starting voltage. A double throw switch S is used to connect th auto-transformer in the circuit for starting. When the handle H of the switch S i the START position, the primary of the auto-transformer is connected to th supply line and the motor is connected to the secondary of the auto-transforme When the motor picks up the speed, say to about 80 per cent of its rated value, th handle H is quickly moved to the RUN position. The autotransformer is disco nected from the circuit and the motor is directly connected to the line and gets i full rated voltage. The handle is held in the RUN position by the under-volta relay. In case the supply voltage fails or falls below a certain value, the handle i released and returns to the OFF position. Overload protection is provided b thermal overload relays.

4.30.1 Theory of Auto-Transformer Starting

Fig. 4.18 (*a*) shows the condition when the motor is directly switched on to lines.

Let Z_{e10} = equivalent standstill impedance per phase of the motor referred to stator side

V_1 = supply voltage per phase

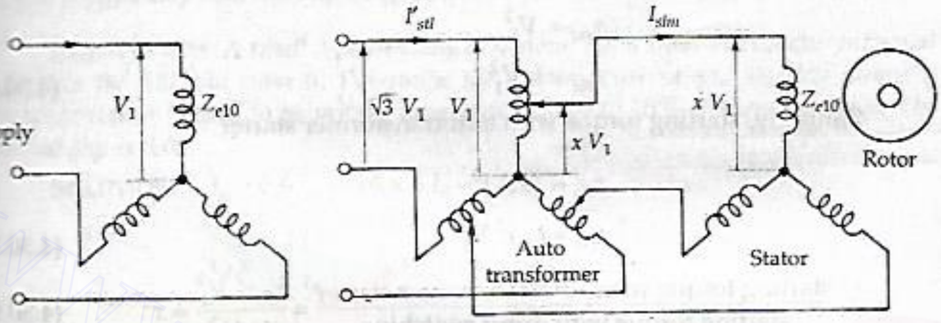


Fig. 4.18. *(a) Direct switching of the motor, (b) Auto-transformer starting of the motor.*

When full voltage V_1/phase is applied with direct switching, the starting current drawn from the supply is given by

$$I_{stl} = \frac{V_1}{Z_{e10}} \tag{4.30.1}$$

With auto-transformer starting, if a tapping of transformation ratio x is used, then the voltage per phase across the motor is xV_1. Therefore, the motor current at starting is given by

$$I_{stm} = \frac{xV_1}{Z_{e10}} \tag{4.30.2}$$

In a transformer, the ratio of currents is inversely proportional to the voltage ratio provided that the no-load current is neglected. That is,

$$\frac{I_1}{I_2} = \frac{V_2}{V_1}$$

or $\quad V_1 I_1 = V_2 I_2$

If $\quad I'_{stl}$ = current taken from the supply by the autotransformer

then $\quad V_1 I'_{stl} = (xV_1)\, I_{stm}$

$$I'_{stl} = x\, I_{stm} \tag{4.30.3}$$

Substituting the value of I_{stm} from Eq. (4.30.2) in Eq. (4.30.3),

$$I'_{stl} = x \cdot \left(\frac{xV_1}{Z_{e10}}\right),$$

$$= \frac{x^2 V_1}{Z_{e10}} \tag{4.30.4}$$

Therefore, $\dfrac{\text{starting current with autotransformer}}{\text{starting current with direct switching}} = \dfrac{I'_{stl}}{I_{stl}}$

$$= \frac{(x^2V_1/Z_{e10})}{(V_1/Z_{e10})} = x^2 \quad (4.30.5)$$

Since the torque developed is proportional to the square of the applied voltage, we have starting torque with direct switching

$$\tau_{std} \propto V_1^2$$

$$\tau_{std} = k_2 V_1^2 \quad (4.30.6)$$

Similarly, starting torque with autotransformer starter

$$\tau_{sta} \propto (xV_1)^2$$

$$\tau_{sta} = k_2\, x^2\, V_1^2 \quad (4.30.7)$$

$$\therefore \frac{\text{starting torque with autotransformer starter}}{\text{starting torque with direct switching}} = \frac{k_2\, x^2\, V_1^2}{k_2\, V_1^2} = x^2 \quad (4.30.8)$$

Also, with auto-transformer, the motor current at starting

$$I_{stm} = \frac{xV_1}{Z_{e10}} = x\, I_{sc} \quad (4.30.9)$$

From Eqs. (4.30.3) and (4.30.9)

$$I'_{stl} = x^2 I_{sc} \quad (4.30.10)$$

From Eq. (4.27.5)

$$\frac{\tau_{est}}{\tau_{efl}} = \left(\frac{I_{stm}}{I_{fl}}\right)^2 s_{fl} = \left(\frac{x\, I_{sc}}{I_{fl}}\right)^2 s_{fl}$$

$$= x^2 \left(\frac{I_{sc}}{I_{fl}}\right)^2 s_{fl} \quad (4.30.11)$$

Equations (4.30.5) and (4.30.8) show that with an autotransformer, the starting current I'_{stl} from the main supply and the starting torque τ_{sta} are reduced to x^2 times their corresponding values with direct on-line starting.

Comparison of Eqs. (4.29.4) and (4.30.11) show that

$$x^2 = \frac{1}{3}$$

or

$$x = \frac{1}{\sqrt{3}} = 0.58$$

Thus, the star-delta starter is equivalent to an auto-transformer starter of ratio $x = 0.58$. But a star-delta starter is much cheaper than an auto-transformer starter, and is commonly used for both small and medium size motors.

4.31 SLIP RING INDUCTION MOTOR STARTER

Figure 4.2 shows the connection of a 3-phase slip ring induction motor with a starter. Full supply voltage is connected across the stator. Full starting resistances are connected, and thus the supply current to the stator is reduced. The rotor begins to rotate and the rotor resistances are gradually cut out as the motor speeds up. When the motor is running at its rated full speed, the starting resistances are cut out completely and the slip rings are short-circuited.

EXAMPLE 4.24. *A small 3-phase induction motor has a short-circuit current equal to 5 times the full-load current. Determine the starting current and starting torque if resistance starter is used to reduce the impressed voltage to 60% of normal voltage. The full-load slip is 0.05.*

SOLUTION. $I_{st} = 0.6\, I_{sc} = 0.6 \times 5\, I_{fl} = 3\, I_{fl}$

$$\tau_{st} = \tau_{fl} \left(\frac{I_{st}}{I_{fl}}\right)^2 \times s_{fl} = \tau_{fl}\,(3)^2 \times 0.05 = 0.45\, \tau_{fl}$$

EXAMPLE 4.25. *Find the ratio of starting to full load current for a 10 kW, 400 V three-phase induction motor with star-delta starter, given that the full-load efficiency is 0.86, the full-load power factor is 0.8 and the short-circuit current is 30 A at 100 V.*

SOLUTION. Full-load line current of the delta- connected motor

$$= \frac{\text{output}}{\sqrt{3}\, V_l \cos\phi \times \eta}$$

$$= \frac{10 \times 10^3}{\sqrt{3} \times 400 \times 0.8 \times 0.86} = 20.98 \text{ A}$$

The line value of short-circuit current with 100 V is 30 A. Therefore, the line value of short-circuit current with normal supply voltage of 400 V is $30 \times \frac{400}{100} = 120$ A. Phase value of short-circuit current of the delta-connected motor $= \frac{120}{\sqrt{3}}$ A. The starting current per phase taken by the motor when connected in star during starting is equal to $\frac{1}{\sqrt{3}}$ times the phase value of short-circuit current that is $\frac{1}{\sqrt{3}} \times \frac{120}{\sqrt{3}} = 40$ A

At start the motor is connected in star.

$\therefore$ line value of starting current = phase value of starting current = 40 A

$$\therefore \frac{\text{line value of starting current}}{\text{line value of full load current}} = \frac{40}{20.98} = 1.9$$

EXAMPLE 4.26. *Calculate the reduction in starting current and starting torque when the supply voltage to a cage motor is 75 per cent instead of 100 per cent.*

SOLUTION. Starting current with normal voltage $= I_{sc}$

Starting current with 75 per cent of normal supply voltage $= 0.75\, I_{sc}$

Reduction in starting current $= I_{sc} - 0.75\, I_{sc}$

Percentage reduction in starting current $= \dfrac{I_{sc} - 0.75\, I_{sc}}{I_{sc}} \times 100 = 25$ perce

Starting torque with normal supply voltage

$$\tau_{st} = \tau_{fl}\left(\frac{I_{sc}}{I_{fl}}\right)^2 s_f$$

Starting torque with 75% normal supply voltage applied to the stator

$$\tau'_{st} = \tau_{fl}\left(\frac{0.75\, I_{sc}}{I_{fl}}\right)^2 s_f = 0.5625\, \tau_{st}$$

Percentage reduction in torque

$$= \frac{-\tau'_{st} + \tau_{st}}{\tau_{st}} \times 100$$

$$= \left(1 - \frac{\tau'_{st}}{\tau_{st}}\right) \times 100 = (1 - 0.5625) \times 100 = 43.75$$

Example 4.27. *A three-phase delta- connected cage-type induction motor connected directly to 400 V, 50 Hz supply takes a starting current of 100 A in each phase. Calculate*

(*i*) *the line current for 'direct-on-line' starting*

(*ii*) *line and phase starting currents for star-delta starting, and*

(*iii*) *line and phase starting currents for a 70 per cent tapping on auto-transformer starting.*

Solution.

(*i*) Direct on-line starting current $= \sqrt{3} \times 100 = 173.2$ A

(*ii*) In *star-delta starting*, phase voltage on starting $= \dfrac{V_L}{\sqrt{3}} = \dfrac{400}{\sqrt{3}} = 230.9$ V

Since 400 V produce 100 A in phase winding, $\dfrac{400}{\sqrt{3}}$ V will produ

$\dfrac{100}{\sqrt{3}} = 57.7$ A

$\therefore$ starting phase current = 57.7 A

In star connection, line current = phase current

$\therefore$ starting line current = 57.7 A

(*iii*) *Auto-transformer starting*

With 70 per cent tapping on auto-transformer, the line voltage across the delta connected motor $= 0.7 \times 400$ V. For delta connection

phase voltage = line voltage $= 0.7 \times 400$ V

Since 400 V produce 100 A in phase winding, (0.7×400) V produ $0.7 \times 100 = 70$ A

Hence motor phase current = 70 A

Motor line current $= \sqrt{3} \times 70 = 121.2$ A

But $\dfrac{\text{supply line current}}{\text{motor line current}} = \dfrac{\text{motor applied voltage}}{\text{supply voltage}} = 0.7$

$\therefore$ supply line current $= 0.7 \times 121.2 = 84.8$ A

The distribution of voltages and currents in the three cases is shown in Fig. 4.19.

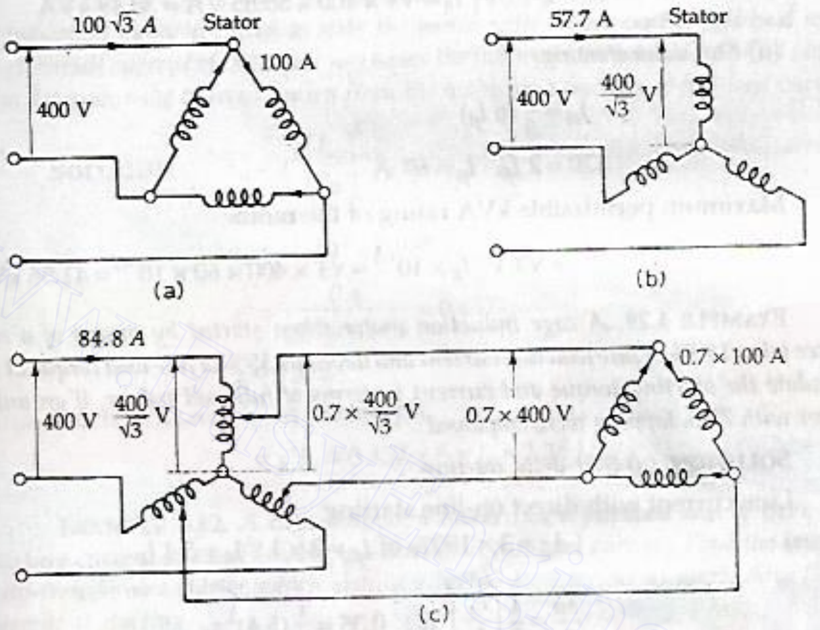


Fig. 4.19. *(a) Direct on-line starting (b) Star-delta starting (c) Auto-transformer starting.*

EXAMPLE 4.28. *It is desired to instal a 3-phase cage induction motor restricting the maximum line current drawn from a 400 V 3-phase supply to 120 A. If the starting current is 6 times full load current, what is the maximum permissible full load kVA of the motor when*

(i) it is directly connected to the mains

(ii) it is connected through an auto-transformer with a tapping of 60%

(iii) it is designed for use with star-delta starter.

SOLUTION. (i) *Direct-on-line starting*

Maximum line current, $I_L = 120$ A

Starting current $I_{st} = 6 \times$ full load current $= 6\, I_{fl}$

Since the maximum line current drawn from the supply is 120 A

$$6\, I_{fl} = 120, \quad I_f = \frac{120}{6} = 20 \text{ A}$$

Maximum permissible rating of the motor

$$= \sqrt{3}\, V_L\, I_{fl} = \sqrt{3} \times 20 \times 400 = 13856 \text{ VA} = 13.856 \text{ kVA}$$

(ii) *Auto-transformer starting*

$$I_{st} = x^2 I_{sc} = x^2 (6 I_{fl})$$

$$120 = (0.6)^2 (6 I_{fl})$$

$$I_{fl} = \frac{120}{6 \times (0.6)^2} = 55.55 \text{ A}$$

Maximum permissible rating of the motor

$$= \sqrt{3}\, V_L I_{fl} = \sqrt{3} \times 400 \times 55.55 \text{ VA} = 38.49 \text{ kVA}$$

(ii) *Star-delta starting*

$$I_{st} = \frac{1}{3} (6 I_{fl})$$

$$120 = 2 I_{fl}, \quad I_{fl} = 60 \text{ A}$$

Maximum permissible kVA rating of the motor

$$= \sqrt{3}\, V_L I_{fl} \times 10^{-3} = \sqrt{3} \times 400 \times 60 \times 10^{-3} = 41.56 \text{ kVA}$$

Example 4.29. *A cage induction motor when started by means of a star-delta starter takes 180% of full-load line current and develops 35% of full-load torque at starting. Calculate the starting torque and current in terms of full-load values, if an auto-transformer with 75% tapping were employed.*

Solution. (*a*) *Star-delta starting*

Line current with direct on-line starting

$$I_{sc} = 3 \times 180\% \text{ of } I_{fl} = 3 \times 1.8\, I_{fl} = 5.4\, I_{fl}$$

$$\frac{\tau_{st}}{\tau_{fl}} = \frac{1}{3} \left(\frac{I_{sc}}{I_{fl}}\right)^2 s_{fl}\, ; \quad 0.35 = \frac{1}{3} (5.4)^2 s_{fl}$$

(*b*) *Auto-transformer starting*

$$I_{st} = x^2 I_{sc} = (0.75)^2 \times 5.4\, I_{fl} = 3.0375\, I_{fl}$$

$$\frac{\tau_{st}}{\tau_{fl}} = x^2 \left(\frac{I_{sc}}{I_{fl}}\right)^2 s_{fl} = (0.75)^2 \times (5.4)^2 s_{fl} = (0.75)^2 \times 3 \times 0.3$$

or $$\tau_{st} = 59\% \text{ of full-load torque}$$

Example 4.30. *A cage induction motor has a short-circuit current of four times the full-load value and has a full-load slip of 0.05. Determine a suitable auto-transformer ratio if the supply current is not to exceed twice the full-load current. Determine the starting torque in terms of the full-load torque.*

Solution. Line current taken from the supply

$$I_{st} = x^2 I_{sc}$$

The supply line current at start $= 2\, I_{fl}$

Short-circuit current, $I_{sc} = 4\, I_{fl}$

$$\therefore \quad 2\,I_{fl} = x^2 \times 4\,I_{fl}$$

$$x^2 = \frac{1}{2},\; x = \frac{1}{\sqrt{2}} = 0.707$$

Also, $$\frac{\tau_{st}}{\tau_{fl}} = x^2 \left(\frac{I_{sc}}{I_{fl}}\right)^2 s_{fl} = \frac{1}{2}\,(4)^2 \times 0.05 = 0.4$$

Example 4.31. *Determine the suitable tapping on an auto-transformer starter for an induction motor required to start the motor with 40 per cent of full-load torque. The short-circuit current of the motor is 5 times the full load current and full-load slip is 0.035. Also determine the current drawn from the mains as a fraction of full-load current.*

Solution. $$\tau_{st} = x^2 \left(\frac{I_{sc}}{I_{fl}}\right)^2 s_{fl}\,\tau_{fl}$$

$$0.4\,\tau_{fl} = x^2\,(5)^2 \times 0.035\,\tau_{fl}$$

$$x^2 = \frac{0.4}{(5)^2 \times 0.035} = 0.457$$

$$x = 0.676$$

Current drawn from the supply

$$= x^2 I_{sc} = 0.457 \times 5 \times I_{fl} = 2.28\,I_{fl}$$

Example 4.32. *A cage induction motor has a full-load slip of 0.05. The motor starting current at rated voltage is 5.5 times it full-load current. Find the tapping on the auto-transformer starter which should give full-load torque at start. Also find the line current at starting.*

Solution. $$\frac{\tau_{est}}{\tau_{efl}} = x^2 \left(\frac{I_{sc}}{I_{fl}}\right)^2 s_{fl}$$

$$1 = x^2 \left(\frac{5.5\,I_{fl}}{I_{fl}}\right)^2 \times 0.05$$

$$x = 0.813 \text{ or } 81.3\% \text{ tapping}$$

Starting current $I_{st} = x^2\,I_{sc} = (0.813)^2\,(5\,I_{fl}) = 3.3\,I_{fl}$

Example 4.33. *A 3-phase cage induction motor has a short-circuit current equal to 5 times the full-load current. Find the starting torque as a percentage of full-load torque if the motor is started by (i) direct switching to the supply, (ii) star-delta starter, (iii) an auto-transformer (iv) a resistance in the stator circuit. The starting current in (iii) and (iv) is limited to 2.5 times the full-load current and the full-load slip is 4%.*

Solution. (*i*) Starting torque with direct switching

$$\tau_{st} = \left(\frac{I_{sc}}{I_{fl}}\right)^2 s_{fl}\,\tau_{fl} = (5)^2 \times 0.04\,\tau_{fl} = \tau_{fl}$$

(*ii*) Starting torque with star-delta starter

$$\tau_{st} = \frac{1}{3}\left(\frac{I_{sc}}{I_{fl}}\right)^2 s_{fl}\,\tau_{fl} = \frac{1}{3}(5)^2 \times 0.04\,\tau_{fl} = \frac{1}{3}\tau_{fl} = \frac{100}{3}\tau_{fl}\,\% = 33.3\% \text{ of } \tau_{fl}$$

(*iii*) Current taken from the supply by the auto-transformer

$$I'_{stl} = x^2 I_{sc}$$

Also $\quad I'_{stl} = 2.5\, I_{fl}$

$\therefore \quad x^2 I_{sc} = 2.5\, I_{fl}$

But $\quad I_{sc} = 5\, I_{fl}$ (given)

$\therefore \quad x^2 \times 5\, I_{fl} = 2.5\, I_{fl}$ and $x^2 = 0.5$

Starting torque with auto-transformer starter

$$\tau_{st} = x^2\left(\frac{I_{sc}}{I_{fl}}\right)^2 s_{fl}\,\tau_{fl} = \frac{1}{2}(5)^2 \times 0.04\,\tau_{fl} = 0.5\,\tau_{fl}$$

$\therefore \quad \tau_{st} = 50\%$ of full-load torque.

(*iv*) Starting torque with a resistance in the stator circuit

$$\tau_{st} = \left(\frac{I_{st}}{I_{fl}}\right)^2 s_{fl}\,\tau_{fl} = (2.5)^2 \times 0.04\,\tau_{fl} = 0.25\,\tau_{fl}$$

$\therefore \quad \tau_{st} = 25\%$ of full-load torque.

4.32 DETERMINATION OF EFFICIENCY

The efficiency of small motors can be determined by directly loading them and by measuring their input and output powers. For larger motors, it may be difficult to arrange loads for them. Moreover, the power loss will be large with direct loading tests. Therefore, indirect methods are used to determine the efficiency of 3-phase induction motors. The following tests are performed on the motor : (*a*) No-load test (*b*) Blocked-rotor test

These tests also enable us to determine the circuit parameters of the equivalent circuit of a 3-phase induction motor.

4.33 NO-LOAD TEST OR OPEN-CIRCUIT TEST

This test is similar to the open-circuit test on a transformer. The motor is uncoupled from its load and the rated voltage at the rated frequency is applied to the stator to run the motor without load. The input power is measured by the 2-wattmeter method. An ammeter and a voltmeter are connected as shown in Fig. 4.20 (*a*). The ammeter measures the no-load current and the voltmeter gives the normal rated supply voltage. Since the no-load current is 20 – 30% of the full-load current, the I^2R losses in the primary may be neglected as they vary with the square of the current. Since the motor is running at no load, total input power is equal to constant iron loss, friction and windage losses of the motor.

$$P_{constant} = P_i = P_1 + P_2 = \text{sum of the two wattmeter readings}$$

Since the power factor of the induction motor under no-load condition is generally less than 0.5, one wattmeter will show negative reading. It is, therefore, necessary to reverse the direction of current-coil terminals to take the reading.

As in the case of a transformer, the constants R_0 and X_0 can be calculated from the readings obtained in the no-load test.

If V_{inl} = input line voltage

P_{inl} = total 3 phase input power at no load

I_0 = input line current

V_{ip} = input phase voltage

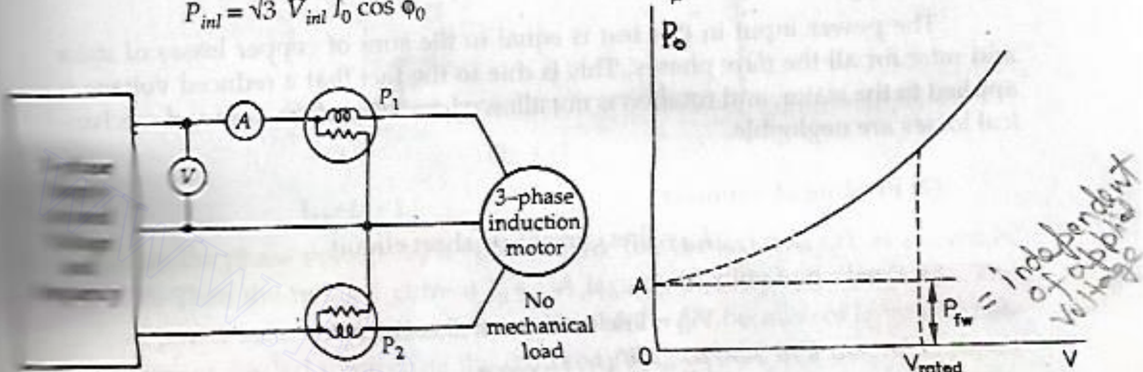
$$P_{inl} = \sqrt{3}\, V_{inl}\, I_0 \cos \phi_0$$

$$I_\mu = I_0 \sin \phi_0$$

$$I_\omega = I_0 \cos \phi_0$$

$$R_0 = \frac{V_{ip}}{I_\omega}$$

$$X_0 = \frac{V_{ip}}{I_\mu}$$



(a) Circuit diagram for no-load test on 3-phase induction motor.

(b) Separation of friction and windage losses.

Fig. 4.20.

SEPARATION OF LOSSES

Friction and windage losses can be separated from the no-load loss P_0. A number of readings of P_0 at no-load is taken at different stator applied voltages from rated to breakdown value at rated frequency. A curve P_0 versus V is plotted as shown in Fig. 4.20(*b*). The curve is nearly parabolic at voltages near normal, since iron losses are almost proportional to the square of the flux density and, therefore, the applied voltage. The curve is extended to the left to cut the vertical axis at A. At the vertical axis $V = 0$ and hence the intercept OA represents the voltage independent loss, that is, the loss due to friction and windage P_{fw}.

4.34 BLOCKED ROTOR OR SHORT-CIRCUIT TEST

This test is analogous to the short-circuit test of a transformer. In this test, the shaft of the motor is clamped (locked) so that it cannot move and rotor winding

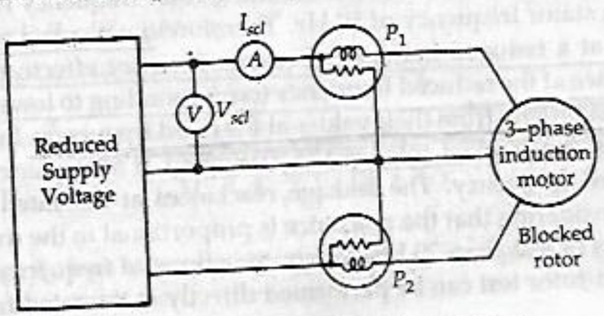


Fig. 4.21. *Circuit diagram for blocked rotor test.*

is short-circuited. In a slip-ring motor, the rotor winding is short-circuited through sliprings and in cage motors, the rotor bars are permanently short-circuited. This test is also called the *locked-rotor test*. The circuit diagram for blocked rotor test is shown in Fig. 4.21.

A reduced voltage at reduced frequency is applied to the stator through a 3-phase auto-transformer so that full-load rated current flows in the stator. The following three readings are obtained :

(1) Total power input on short-circuit P_{sc} = *algebraic sum* of the two wattmeter readings

The power input in this test is equal to the sum of copper losses of *stator* and *rotor* for all the *three* phases. This is due to the fact that a reduced voltage is applied to the stator, and rotation is not allowed and, therefore, core and mechanical losses are negligible.

(2) Reading of ammeter

$$I_{scl} = \text{line current on short circuit}$$

(3) Reading of voltmeter

$$V_{scl} = \text{line voltage on short circuit}$$

$$\therefore \quad P_{sc} = \sqrt{3}\, V_{scl}\, I_{scl} \cos\phi_{sc}$$

where $\cos\phi_{sc}$ = power factor on short circuit

Equivalent resistance of the motor referred to stator

$$R_{e_1} = \frac{P_{scp}}{I_{scp}^2}$$

Equivalent impedance of the motor referred to stator

$$Z_{e_1} = \frac{V_{scp}}{I_{scp}}$$

Equivalent reactance of the motor referred to stator

$$X_{e_1} = \sqrt{Z_{e_1}^2 - R_{e_1}^2}$$

It is to be noted that the blocked-rotor test should be performed under the same conditions of rotor current and frequency that will exist under normal operating conditions. At normal operating conditions, the slip of most induction motors is only 2 to 4 per cent, and the resulting rotor frequency is in the range of 1 to 2 Hz for a stator frequency of 50 Hz. Therefore the blocked rotor test should be performed at a reduced frequency because the rotor effective resistance and leakage reactance at the reduced frequency (corresponding to lower values of slip) may differ considerably from their values at the rated frequency. In order to obtain accurate results, the blocked-rotor test is performed at a frequency 25 percent or less of the rated frequency. The leakage reactances at the rated frequency are obtained by considering that the reactance is proportional to the frequency. However, for motors of less than 20 kW rating, the effects of frequency are negligible and the blocked-rotor test can be performed directly at the rated frequency.

4.35 CIRCLE DIAGRAM

The circle diagram of an induction motor is very useful to study its performance under all operating conditions. Its construction is based on the *approximate equivalent circuit* shown in Fig. 4.22.

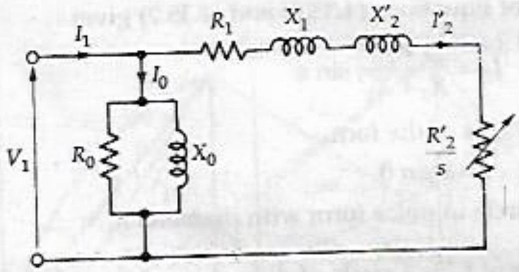


Fig. 4.22.

By KCL, $\mathbf{I}_1 = \mathbf{I}_0 + \mathbf{I}_2'$

Let the phase voltage $\mathbf{V}_1$ be taken along the vertical axis OY as shown in Fig. 4.23. Then the no-load current $\mathbf{I}_0 = OA$ lags behind $\mathbf{V}_1$ by an angle ϕ_0. The no-load power factor angle ϕ_0 is of the order of 60° – 80° because of large magnetizing current needed to produce the required flux per pole in a magnetic circuit containing air gaps.

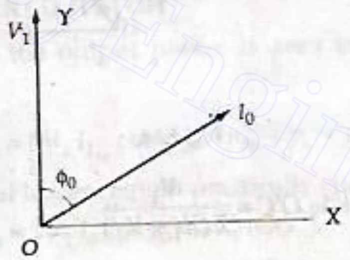


Fig. 4.23.

At no load, $s = 0$ and $\frac{R_2}{s}$ is infinite. In other words, $\frac{R_2}{s}$ is an open circuit at no load, and

$$\mathbf{I}_0 = \frac{\mathbf{V}_1}{\mathbf{Z}_{nl}} \quad \text{where} \quad \mathbf{Z}_{nl} = (R_0 \,||\, j X_0)$$

It is to be noted that all rotational losses are taken into account by R_0.

No-load loss, $P_0 = V_1 I_0 \cos \phi_0$

The rotor current referred to stator is given by

$$I_2' = \frac{V_1}{\sqrt{\left(R_1 + \frac{R_2'}{s}\right)^2 + (X_1 + X_2')^2}} \qquad (4.35.1)$$

The current $\mathbf{I}_2'$ lags behind $\mathbf{V}_1$ by the impedance angle ϕ (Fig. 4.24*b*) where $$\sin\phi = \frac{X_1 + X_2'}{\sqrt{\left(R_1 + \frac{R_2'}{s}\right)^2 + (X_1 + X_2')^2}} \tag{4.35.2}$$

Combination of Equations (4.35.1) and (4.35.2) gives

$$I_2' = \frac{V_1}{X_1 + X_2'} \sin\phi \tag{4.35.3}$$

Equation (4.35.3) is of the form

$$r = a \sin\theta$$

which represents a circle in polar form with diameter a.

Thus, the locus of $\mathbf{I}_2'$ is a circle of diameter $\dfrac{V_1}{X_1 + X_2'}$ as shown in Fig. 4.24b.

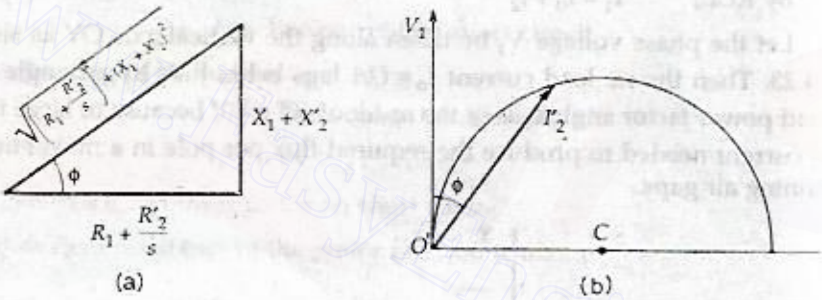


Fig. 4.24.

The radius of the circle $O'C = \dfrac{V_1}{2(X_1 + X_2')}$

The centre C has the coordinates

$$\left[\frac{V_1}{2(X_1 + X_2)}, 0\right]$$

Since, $\mathbf{I}_1 = \mathbf{I}_0 + \mathbf{I}_2'$, the stator current I_1 is found by combining the results shown in Fig. 4.23 and 4.24. The resulting diagram is shown in Fig. 4.25. It is seen that the tip of the phasor $\mathbf{I}_1$ coincides with that of phasor $\mathbf{I}_2'$. Thus, the locus of both $\mathbf{I}_1$ and $\mathbf{I}_2'$ is the upper semicircle. It is to be noted that $\mathbf{I}_1$ radiates from the origin 0 while $\mathbf{I}_2'$ radiates from the origin 0′.

Consider the instant when the motor is started ($s = 1$) with the rated voltage. The tips of $\mathbf{I}_1$ and $\mathbf{I}_2'$ will be at some point F of the circle. As the motor, accelerates, the tips of $\mathbf{I}_1$ and $\mathbf{I}_2$ move around the circle in an anticlockwise direction until the output torque matches the load torque. If there is no shaft load, the motor accelerates to synchronous speed. At this point $\mathbf{I}_2' = 0$ and $\mathbf{I}_1 = \mathbf{I}_0$.

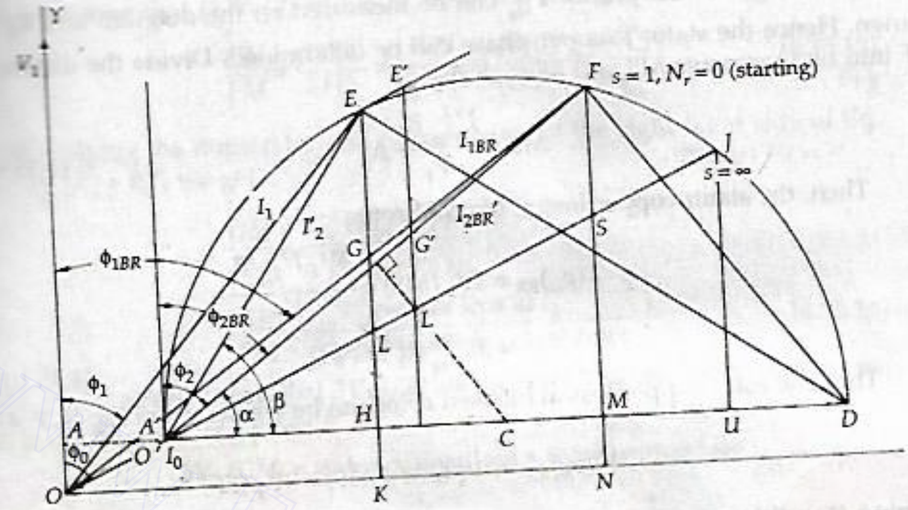


Fig. 4.25.

CONDITIONS AT BLOCKED ROTOR

At blocked rotor, the output power is zero and the electrical power is consumed as losses :

$$P_{in\,BR} = 3V_1 I_{1_{BR}} \cos \phi_{BR} = p_{rot} + p_{sc} + p_{rc} \tag{4.35.4}$$

Since the rotational losses remain practically constant

$$p_{rot} = 3V_1 I_0 \cos \phi_0 = 3V_1 (OO')$$

From Fig. 4.25,

$$I_{1_{BR}} \cos \phi_{1_{BR}} = FN = FM + MN = FM + OO' = I'_{2_{BR}} \cos \phi_{2_{BR}} + I_0 \cos \phi_0 \tag{4.35.5}$$

I_2' at blocked rotor is $I'_{2_{BR}}$.

Multiplying both sides of Eq. (4.35.5) by $3V_1$ we get

$$3V_1 I_{1_{BR}} \cos \phi_{1_{BR}} = 3V_1 I_2' \cos \phi_{2_{BR}} + 3V_1 I_0 \cos \phi_0$$

$$3V_1 (FN) = 3V_1 (FM) + 3V_1 (OO')$$

$$p_{rot} + p_{sc} + p_{rc} = 3V_1 (FM) + 3V_1 (OO') \tag{4.35.6}$$

$$\therefore \quad 3V_1 (FM) = p_{sc_{BR}} + p_{rc_{BR}} = 3I'^{2}_{2_{BR}} (R_1 + R_2) \tag{4.35.7}$$

The distance FM can be divided into two parts to represent the two copper losses individually. For this purpose, measure the length of the phasor $I'_{2_{BR}}$ on the diagram.

The stator copper loss per phase with rotor blocked is given by

$$\frac{1}{3} p_{sc} = I'^2_{2_{BR}} R_1 \tag{4.35.8}$$

The length of the phasor $I'_{2_{BR}}$ can be measured on the diagram and R_1 is known. Hence the stator loss per phase can be determined. Divide the distance MF into two segments MS and SF such that

$$MS = \frac{I'^2_{2_{BR}} R_1}{V_1} \text{ A} \tag{4.35.9}$$

Then, the stator copper loss at blocked rotor

$$(p_{sc})_{BR} = 3V_1 (MS) = \frac{3V_1 I'^2_{2_{BR}} R_1}{V_1}$$

$$= 3I'^2_{2_{BR}} R_1 \text{ W} \tag{4.35.10}$$

Then the rotor copper loss at blocked rotor can be written from Eq. (4.35.4) as

$$(p_{rc})_{BR} = 3V_1 (FM - MS) = 3V_1 (SF) \text{ W}$$

Running Conditions

Consider the performance of the motor when it operates at some point on the current locus, such as E in Fig. 4.25. The input power for the operating point E is

$$P_i = 3V_1 I_1 \cos\phi_1$$

and $$\cos\phi_1 = \frac{KE}{OE} = \text{power factor}$$

The rotational losses are $3V_1 (KH)$ in this case also. Thus,

$$3V_1 (HE) = p_{sc} + p_{rc} + \text{output power.}$$

Join $O'S$ and extend it to meet the current locus at J.

From triangles $O'GH$ and $O'FM$,

$$\tan\alpha = \frac{GH}{O'H} = \frac{FM}{O'M} \tag{4.35.11}$$

From triangles $O'EH$ and $O'ED$

$$\cos\beta = \frac{O'H}{O'E} = \frac{O'E}{O'D}$$

$$\therefore \quad O'H = \frac{(O'E)^2}{O'D} \tag{4.35.12}$$

From triangles $O'FM$ and $O'FD$

$$\cos\alpha = \frac{O'M}{O'F} = \frac{O'F}{O'D}$$

$$\therefore \quad O'M = \frac{(O'F)^2}{O'D} \tag{4.35.13}$$

From Eq. (4.35.11)

$$\frac{GH}{FM} = \frac{O'H}{O'M} \tag{4.35.14}$$

Substituting Eqs. (4.35.12) and (4.35.13) in Eq. (4.35.14) we obtain

$$\frac{GH}{FM} = \frac{(O'E)^2}{O'D} \times \frac{O'D}{(O'F)^2} = \frac{(O'E)^2}{(O'F)^2} = \frac{I'^2_2}{I'^2_{2_{BR}}} \tag{4.35.15}$$

Multiplying the numerator and denominator of the right-hand side of Eq. (4.35.15) by $(R_1 + R_2')$ we get

$$\frac{GM}{FM} = \frac{I'^2_2 (R_1 + R_2')}{I'^2_2 (R_1 + R_2')}$$

or

$$\frac{GH}{FM} = \frac{\text{copper loss at } I_2'}{\text{copper loss at blocked rotor}} \tag{4.35.16}$$

We have already seen that $3V_1 (FM)$ is the total blocked-rotor copper loss. Therefore at the operating point E.

$$3V_1 (GH) = \text{stator copper loss} + \text{rotor copper loss}$$

By similar triangles,

$$\frac{HL}{HG} = \frac{MS}{MF} = \frac{R_1}{R_1 + R_2'}$$

Therefore,

$$\text{stator core loss} = 3V_1 (HL) \quad \text{W}$$

and

$$\text{rotor copper loss} = 3V_1 (HG - HL) = 3V_1 (GL) \quad \text{W}$$

The output power is given by

$$P_0 = p_i - losses$$
$$= 3V_1 (KE) - 3V_1 (KH + HL + LG)$$

or

$$P_0 = 3V_1 (GE) \quad \text{W}$$

The efficiency of the motor is given by

$$\eta = \frac{P_0}{P_i} = \frac{GE}{KE}$$

Air-gap power = rotor input. P_g

$$P_g = P_0 + \text{rotor copper loss}$$
$$= 3V_1 (LE) \quad \text{W}$$

Since rotor copper loss $= s P_g$

$$\therefore \quad s = \frac{\text{rotor copper loss}}{P_g} = \frac{LG}{LE}$$

The speed of the motor is given by

$$\omega = \omega_s (1 - s) = \omega_s \frac{GE}{LE}$$

The torque is

$$\tau_{out} = \frac{P_g}{\omega_s} = \frac{3V_1\,(LE)}{\omega_s}\ Nm$$

The maximum value of torque corresponds to maximum value of LE. It is found by drawing a tangent to the circle parallel to the line $O'S$, locating the point E'. Then the slip for maximum torque is

$$s_M = \frac{L'G'}{L'E'}$$

The maximum torque is

$$\tau_{max} = \frac{3V_1\,(L'E')}{\omega_s}\ Nm$$

If $O'S$ is produced to meet the circle at J, then

JU = stator copper loss, while corresponding rotor copper loss is zero

This is only possible when

$$\frac{R_2'}{s} = 0 \text{ or } s = \infty$$

Thus, the point J corresponds to infinite slip.

4.36 CONSTRUCTION OF THE CIRCLE DIAGRAM

The following data are required for constructing the circle diagram :

(i) Stator phase voltage, $V_1 = \frac{V_L}{\sqrt{3}}$

(ii) No-load current I_0

(iii) No-load power factor $\cos \phi_0$

(iv) Blocked rotor current and power factor

(v) Stator phase resistance R_1.

Procedure

1. Take the voltage phasor $\mathbf{V}_1$ along y axis.
2. Choose a convenient current sole. With O as origin, draw a line $OO' = \mathbf{I}_0$ at an angle ϕ_0 with $\mathbf{V}_1$.
3. Draw a line OKN perpendicular to $\mathbf{V}_1$. Similarly, draw a line $O'T$ perpendicular to $\mathbf{V}_1$.
4. From O draw the line of equal to the blocked rotor current $I_{1_{BR}}$ to the same scale as $\mathbf{I}_0$. This line lags behind $\mathbf{V}_1$ by the blocked-rotor power-factor angle $\phi_{1_{BR}}$.
5. Join $O'F$ and measure its magnitude in amperes. The line $O'F$ represents $I'_{2_{BR}}$.

6. From the point F, draw a line FMN parallel to V_1. This line is perpendicular to $O'D$ and ON.

7. Calculate $MS = \dfrac{I'^2_{2_{BR}} R_1}{V_1}$ and locate point S. Join $O'S$ and extend it to meet the circle at J. It is to be noted that $s = \infty$ is at J.

8. Draw the perpendicular bisector of the chord $O'F$. This bisector will pass through the centre of the circle at C. With radius CD' or CD, draw the circle.

4.37 RESULTS OBTAINABLE FROM THE CIRCLE DIAGRAM

Let us assume that the line current I_1 is known. With centre at O, draw an arc with radius I_1. This arc intersects the circle at the operating point E. Draw the line EK, and locate the points H, L, G. Then the following results may be obtained from the circle diagram :

1. Input power $= 3V_1 (KE)$ watts.
2. Rotational loss $= 3V_1 (KH)$ watts
3. Stator copper loss $= 3V_1 (HL)$ watts
4. Rotor copper loss $= 3V_1 (LG)$ watts
5. Output power $= 3V_1 (GE)$ watts
6. Output torque $= 3V_1 \dfrac{(LE)}{\omega_s}$ Nm
7. Starting torque $= 3V_1 \dfrac{(SF)}{\omega_s}$ Nm
8. Slip $= \dfrac{LG}{LE}$
9. Speed $= \dfrac{GE}{LE} n_s$
10. Efficiency $= \dfrac{GE}{KE}$
11. Power factor $= \dfrac{KE}{OE}$

4.38 SIGNIFICANCE OF SOME LINES IN THE CIRCLE DIAGRAM

Input line ON. The vertical distance between any point on the circle and line ON represents the input power. Therefore, line ON is called the *input line*.

Output line $O'F$. The vertical distance between any point on the circle and line $O'G$ represents the output power. Hence line $O'F$ is called the *output line*.

Air-gap power line or torque line $O'J$. Since EL represents the air-gap power P_g, line $O'LJ$ is called the air-gap power line. Since $\tau_d = \dfrac{P_g}{\omega_s}$, this line is also called *torque line*.

EXAMPLE 4.34. *A 50 kW, 6 pole, 50 Hz, 450 V 3 – φ slip ring induction furnished the following test figures :*

No-load test : 450 V, 20 A, p.f. = 0.15

Blocked rotor test : 200 V, 150 A, p.f. = 0.3

The ratio of stator to rotor copper losses on short circuit was 5 : 4. Draw the diagram and determine from it

(a) the full-load current and power factor,

(b) the maximum torque and the maximum power input,

(c) slip at full load,

(d) efficiency at full load.

SOLUTION. Voltage applied, $V = 450$ V

No-load current, $I_0 = 20$ A

No-load power factor, $\cos \phi_0 = 0.15$

No-load phase angle, $\phi_0 = \cos^{-1}(0.15) = 81.37°$

Short-circuit voltage applied, $V_s = 200$ V

Short-circuit current, $I_s = 150$ A

Short-circuit power factor, $\cos \phi_s = 0.3$

Short-circuit phase angle, $\phi_s = \cos^{-1}(0.3) = 72.54°$

Short-circuit current at normal voltage

$$I_{SC} = I_s \frac{V}{V_s}$$

$$= 150 \times \frac{450}{200} = 337.5 \text{ A}$$

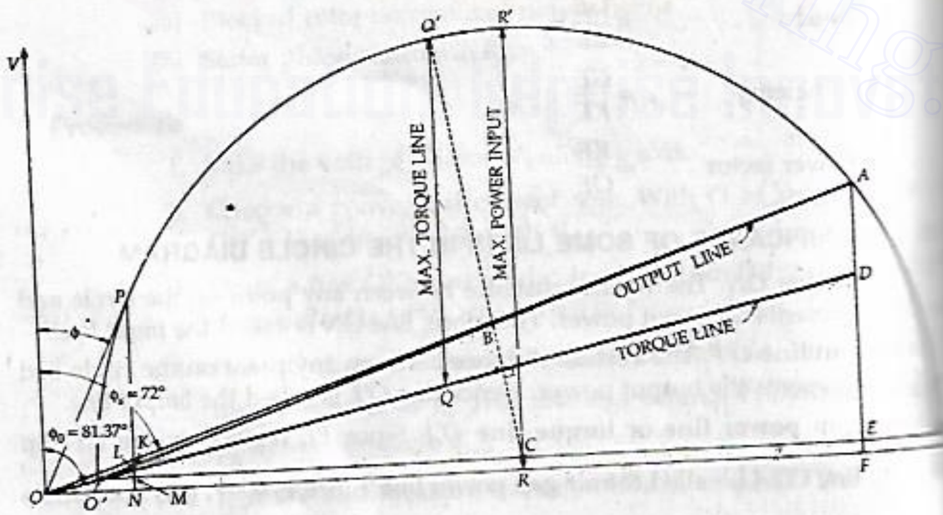


Fig. 4.26.

Short-circuit power input with this current,

$$P_{SC} = \sqrt{3}\, V\, I_{SC} \cos \phi_s$$

$$= \sqrt{3} \times 450 \times 337.5 \times 0.3 = 78916 \text{ W}$$

Let the current scale be 15 A/cm.

The circle diagram shown in Fig. 4.26 is constructed as follows :

Step I. No-load current phasor OO' represents 20 A and measures $\frac{20}{15} = 1.33$ cm. It is drawn at an angle of 81.37° with V-axis.

Step II. Phasor OA represents 337.5 A. It measures $\frac{337.5}{15} = 22.5$ cm. It is drawn at an angle of 72.54° with V-axis.

Step III. $O'G$ is drawn parallel to OX (that is, x-axis). BC is right bisector of $O'A$.

Step IV. With C as centre and $O'C$ as radius, a semi-circle is drawn.

Step V. AF represents power input on short circuit with normal voltage applied. It measures 6.7 cms and represents 78916 W. (as calculated above).

Hence power scale becomes

$$1 \text{ cm} = \frac{78916}{6.7} \text{ W} = 11778 \text{ W}$$

(*a*) Full-load motor output = 50 kW

According to the above power scale the intercept between the semicircle and output line $O'A$ should measure $\frac{50000}{11778} = 4.25$ cm.

Hence, vertical line PN is found which measures 4.25 cms. Point P represents the full-load operating point.

(*b*) Full-load current *i.e.*, line current

$$= OP = 4.7 \text{ cm} = 4.7 \times 15 \text{ A} = 70.5 \text{ A}$$

$$\phi = 27° \text{ (from circle diagram)}$$

$$\therefore \quad \text{Power factor} = \cos \phi = \cos 27° = 0.891$$

Step VI. Since stator copper loss $= \frac{5}{4}$ rotor copper loss = 1.25 rotor copper loss

$\therefore$ total copper loss = stator copper loss + rotor copper loss = 2.25 rotor copper loss

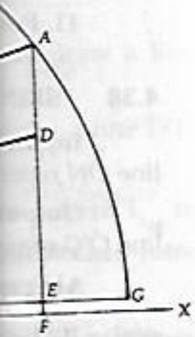
From circle diagram,

total copper loss $= AE = 6.5$ cm

and rotor copper loss $= AD$

stator copper loss $= DE$

$$\therefore \quad AE = 2.25 \times AD$$



or $$AD = \frac{6.5}{2.25} = 2.9 \text{ cm}$$

and $$DE = AE - AD = 6.5 - 2.9 = 3.6 \text{ cm}.$$

where point D separates stator and rotor copper losses. The length AD (= 2.9 cm) represents rotor copper loss and DE (= 3.6 cm) represents stator copper loss. The line $O'D$ represents the TORQUE LINE.

For finding maximum torque, draw a line CQ' perpendicular to $O'D$ and dropping vertical line from Q' intersecting torque line $O'D$ at point Q. Now QQ' represents maximum torque.

(*c*) $\therefore$ maximum torque $= QQ' \times$ power scale

$$= 9.4 \times 11778 = 110713 \text{ Syn. watts}$$

Step VII. For maximum power input, draw perpendicular RR' from centre C of the semi-circle. RR' represents maximum power input.

$\therefore$ maximum power input

$$= RR' \times \text{power scale}$$

$$= 11.4 \times 11778 = 134269 \text{ W} = 134.269 \text{ kW}$$

(*d*) Slip at full load

$$= \frac{KL}{PL} = \frac{0.1}{3.9} = 0.0256 \text{ pu.} = 2.56\%$$

(*e*) Efficiency at full load

$$= \frac{PK}{PN} = \frac{3.8}{4.25} = 0.894 \text{ pu} = 89.4\%$$

EXAMPLE 4.35. *Draw the circle diagram for a 3 – φ, 6-pole, 50 Hz, 400 V, star connected induction motor from the following data (line values)*

No-load test : 400 V, 10 A, 1400 W

Short-circuit test : 200 V, 55 A, 7000 W.

The stator loss at standstill is 60 % of the total copper losses and full-load current is 30 A. From the circle diagram determine :

(*i*) *power factor, slip, output, efficiency speed, and torque at full load,*
(*ii*) *maximum power factor,*
(*iii*) *starting torque,*
(*iv*) *maximum power output,*
(*v*) *maximum power input,*
(*vi*) *maximum torque in synchronous watts and slip for maximum torque.*

SOLUTION.

Voltage applied, $V = 400$ V

No-load current, $I_0 = 10$ A

No-load input, $W_0 = 1400$ W

No load power factor, $\cos\phi_0 = \dfrac{W_0}{\sqrt{3}\,V\,I_0}$

$$= \frac{1400}{\sqrt{3}\times 400\times 10}$$

$$= 0.2021$$

No load phase angle, $\phi_0 = \cos^{-1}(0.2021) = 78.34°$

Short-circuit voltage applied, $V_s = 200$ V

Short-circuit current, $I_s = 55$ A

Short-circuit power input, $W_s = 7000$ W

Short-circuit current with normal voltage of 400 V applied 70 the stator

$$I_{SC} = I_s\left(\frac{V}{V_s}\right) = 55\times\frac{400}{200} = 110 \text{ A}$$

Short-circuit power input with normal voltage,

$$P_{SC} = \left(\frac{I_{SC}}{I_s}\right)^2 W_s = \left(\frac{110}{55}\right)^2 \times 7000 = 28000 \text{ W}$$

Short-circuit power factor,

$$\cos\phi_s = \frac{P_{SC}}{\sqrt{3}\,V\,I_{SC}} = \frac{28000}{\sqrt{3}\times 400\times 110} = 0.3674$$

Short-circuit phase angle

$$\phi_s = \cos^{-1} 0.3674$$

$$= 68.44°$$

Let the current scale be 5 A/cm

The circle diagram shown in Fig. 4.27 is constructed as follows :

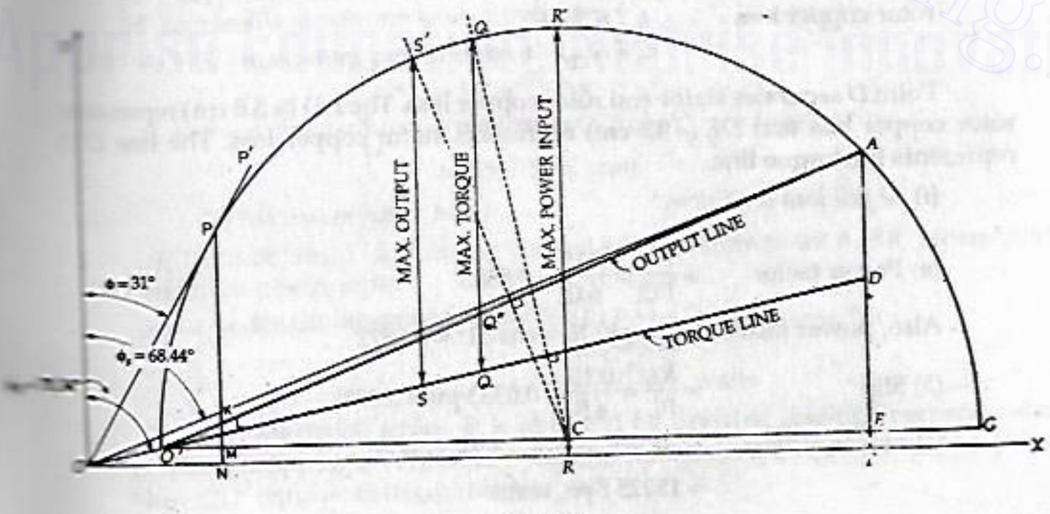


Fig. 4.27.

Step I. No-load current phasor OO'. It represents 10 A and meas
$\frac{10}{5} = 2$ cm.

It is drawn at an angle of 78.34° with V-axis.

Step II. Phasor OA represents 110 A and measures $\frac{110}{5} = 22$ cm. It is dra
at an angle of 68.44° with V-axis.

Step III. Join O' and A. $O'G$ is drawn parallel to OX (X-axis) BC is rig
bisector of $O'A$.

Step IV. With C as centre and $O'C$ as radius, a semi-circle is drawn.

Step V. AF represents power input on short circuit with normal volta
applied. It measures 8 cm and represents 28000 W (as calculated above).

Hence power scale becomes

$$1 \text{ cm} = \frac{28000}{8} \text{ W} = 3500 \text{ W}$$

Step VI. Full-load current is 30 A. It is represented by the len
$\frac{30}{5} = 6$ cm. With O' as centre and 6 cm as radius, an arc is drawn interse
semicircle at point P. This is representing full-load condition. Join points O an
P. Draw perpendicular PN from point P on the X-axis. This line intersects the lin
$O'A$ (output line), $O'D$ (torque line), $O'E$ and OX at points K, L, M and N resp
tively.

Since stator copper loss = 60 % of total copper loss

From circle diagram,

total copper loss $= AE = 7.6$ cm

$\therefore$ stator copper loss $= 0.6 \times AE$

$= 0.6 \times 7.6$

$= 4.6$ cm

rotor copper loss $= 7.6 - 4.6$

$= 3.0$ cm

Point D separates stator and rotor copper loss. The AD (= 3.0 cm) repres
rotor copper loss and DE (= 4.6 cm) represents stator copper loss. The line
represents the torque line.

(*i*) *At full load conditions*

(*a*) Power factor $= \frac{PN}{PO} = \frac{5.15}{6.0} = 0.8583$

Also, power factor $= \cos \angle VOP = \cos 31° = 0.8572$

(*b*) Slip $= \frac{KL}{PL} = \frac{0.15}{4.5} = 0.0333 \text{ pu} = 3.33\%$

(*c*) Power output $= PK = 4.35 \text{ cm} = 4.35 \times 3500$

$= 15225$ Syn. watts

(*d*) Efficiency $= \dfrac{PK}{PN} = \dfrac{4.35}{5.15} = 0.845$ pu

$= 84.5\%$

(*e*) Speed at full load

$$= N_s(1-s) = \frac{120f}{P}(1-s)$$

$$= \frac{120 \times 50 \times (1-0.0333)}{6}$$

$$= 966.7 \text{ rpm}$$

(*f*) Torque at full load

$$\tau_{fl} = LP = 4.5 \text{ cm} = 4.5 \times 3500$$

$$= 15750 \text{ Syn. watts}$$

(*ii*) *Maximum power factor cos* ϕ_m

It is obtained by drawing a line from point 0 tangential to semicircle at point P'

$$\therefore \quad \phi_m = \angle VOP' = 31°$$

Maximum power factor

$$\cos\phi_m = \cos 31°$$

$$= 0.8572$$

(*iii*) *Starting torque*

$$\tau_{st} = AD = 3.0 \text{ cm}$$

$$= 3 \times 3500$$

$$= 10500 \text{ Syn. watts}$$

(*iv*) *Maximum power output*

It is obtained by producing BC intersecting semicircle at S' and then dropping perpendicular from point S' on horizontal line meeting output line at S. Now SS' represents maximum power output.

$\therefore$ maximum power output

$$= SS' = 7.3 \text{ cm}$$

$$= 7.3 \times 7500$$

$$= 25550 \text{ Syn. watts}$$

(*v*) *Maximum power input*

It is obtained by drawing vertical line RR' from point R. RR' represents the maximum power input.

$\therefore$ maximum power input $= RR' = 11.2$ cm

$$= 11.2 \times 3500$$

$$= 39200 \text{ Syn. watts}$$

(*vi*) *Maximum torque.* It is obtained by drawing line CQ' perpendicular to $O'D$ and dropping a vertical line from Q' intersecting torque line $O'D$ at point Q. Now QQ' represents maximum torque.

$\therefore$ maximum torque $= QQ' = 8.5$ cm

$= 8.5 \times 3500$

$= 29750$ Syn. watts

Slip for maximum torque

$$= \frac{QQ''}{QQ'} = \frac{1.35}{8.5}$$

$= 0.159$ pu

$= 15.9\%$.

EXAMPLE 4.36. *The following test results refer to a 3-phase, 20 h.p (metric), 440 V, delta-connected, 50 Hz, 4 pole induction motor.*

Running light test : 440 V, 10 A (line) 1.5 kW (input)

Locked rotor test : 120 V, 30 A (line) 2.25 kW (input).

Draw the circle diagram of this induction motor and determine therefrom

(a) *full-load current and power factor,*

(b) *maximum possible power output,*

(c) *the best possible operating power factor.*

SOLUTION.

Applied voltage, $V = 440$ V

no-load current, $I_0 = 10$ A

no-load input power, $P_0 = 1.5$ kW

No-load power factor, $\cos\phi_0 = \dfrac{P_0}{\sqrt{3}\, V I_0}$

$$= \frac{1500}{\sqrt{3} \times 440 \times 10}$$

$= 0.1968$

$\therefore$ No-load phase angle $\phi_0 = \cos^{-1} 0.1968$

$= 78.649°$

On short circuit voltage applied $V_s = 120$ V

Short-circuit current $I_s = 30$ A

Short-circuit input power $P_s = 2250$ W

Short-circuit current at normal voltage

$$I_{SC} = I_s \frac{V}{V_s}$$

$$= 30 \times \frac{440}{120} = 110 \text{ A}$$

Short-circuit input power at normal voltage

$$P_{SC} = P_s \times \left(\frac{I_{SC}}{I_s}\right)^2$$

$$P_{SC} = 2250 \times \left(\frac{110}{30}\right)^2$$

$$= 30250 \text{ W} = 30.25 \text{ kW}$$

Short-circuit power factor,

$$\cos \phi_s = \frac{P_{SC}}{\sqrt{3}\, V\, I_{SC}}$$

$$= \frac{30250}{\sqrt{3} \times 440 \times 110} = 0.3608$$

Short-circuit phase angle,

$$\phi_s = 68.85°$$

The circle diagram shown in Fig. 4.28 is constructed as described in Example 4.34.

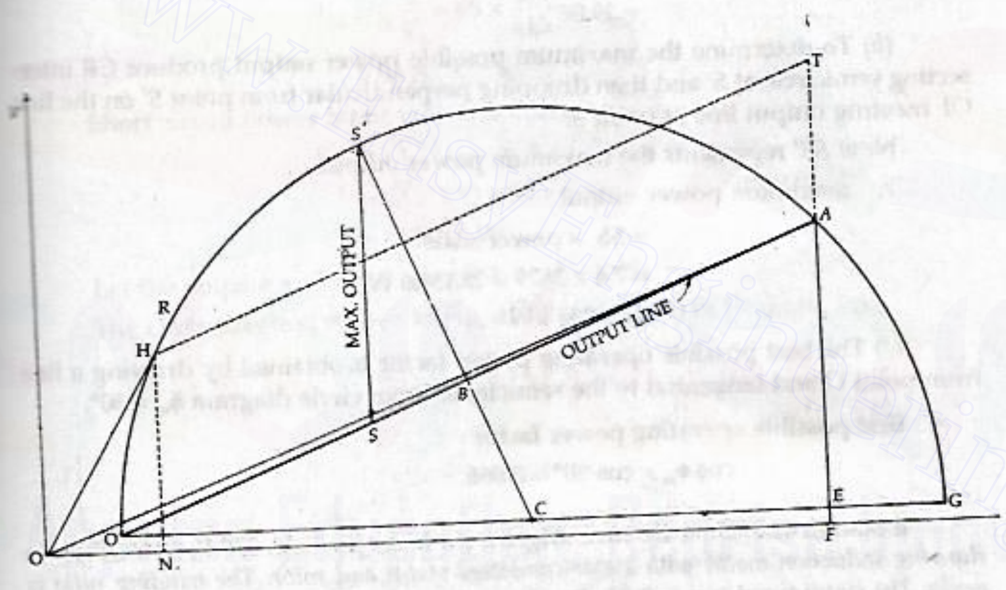


Fig. 4.28.

Let the current scale be 5.0 A/cm.

To determine the power scale a perpendicular AF is drawn. AF represents total input on short circuit with normal voltage applied *i.e.*, 30250 watts.

Since $AF = 7.9$ cm

$\therefore$ power scale, $1 \text{ cm} = \dfrac{30250}{7.9}$

$= 3829$ W

Now motor output, $= 20\ h.p = 20 \times 735.5$

$= 14710$ W

which will be represented by $\frac{14710}{3829} = 3.84$ cm. on the circle diagram. The line FA is extended to point T, so that $AT = 3.84$ cm. From point T line TH is drawn parallel to output line $O'A$ intersecting the circle at H. Point H is joined to the origin O and perpendicular HN is drawn.

Now from circle drawn.

(*a*) Full-load current $= OH$

$$= 5.65 \text{ cm}$$

$$= 5.65 \times 10 = 56.5 \text{ A}$$

Full-load power factor

$$\cos \phi_{fl} = \frac{NH}{OH} = \frac{4.9}{5.65} = 0.8673$$

Full-load phase angle

$$\phi_f = \cos^{-1}(0.8673)$$

$$= 29.86°$$

(*b*) To determine the maximum possible power output produce CB intersecting semicircle at S' and then dropping perpendicular from point S' on the line OF meeting output line at point S.

Now SS' represents the maximum power output.

∴ maximum power output

$$= SS' \times \text{power scale}$$

$$= 7.4 \times 3829 = 28334.6 \text{ W}$$

$$= 28.335 \text{ kW}$$

(*c*) The best possible operating power factor is obtained by drawing a line from point O and tangential to the semicircle. From circle diagram $\phi_m = 30°$.

Best possible operating power factor

$$\cos \phi_m = \cos 30° = 0.866.$$

Example 4.37. *Draw the circle diagram of a 15 h.p (British), 230 V, 50 Hz, 3-phase slip ring induction motor with a star-connected stator and rotor. The winding ratio is unity. The stator resistance is 0.42 ohm/phase and the rotor resistance is 0.3 ohm/phase. The following are the test readings :*

No-load test : 230 V, 9A, $\cos \phi_0 = 0.2143$

Short-circuit test : 115 V, 45 A, $\cos \phi_s = 0.454$.

Find

(*a*) *starting torque,*

(*b*) *maximum torque,*

(*c*) *maximum power factor,*

(*d*) *slip for maximum torque,*

(*e*) *maximum output.*

SOLUTION.

No-load voltage applie' $V = 230$ V

No-load current, $I_0 = 9$ A

No-load power factor, $\cos\phi_0 = 0.2143$

No-load phase angle, $\phi_0 = \cos^{-1} 0.2143$

$= 77.63°$

Short-circuit voltage applied, $V_s = 115$ V

Short-circuit current $I_s = 45$ A

Short-circuit power factor, $\cos\phi_s = 0.454$

Short-circuit phase angle $\phi_s = \cos^{-1} 0.454$

$= 63°$

Short-circuit current at normal voltage

$$I_{SC} = I_s \frac{V}{V_s} = 45 \times \frac{230}{115}$$

$$= 90 \text{ A}$$

Short circuit power input with this current,

$$P_{SC} = \sqrt{3}\, V\, I_{SC} \cos\phi_s$$

$$= \sqrt{3} \times 230 \times 90 \times 0.454$$

$$= 16277.5 \text{ W}$$

Let the ampere scale be 1 cm = 5 A.

The circle diagram shown in Fig. 4.29 is drawn as in Example 4.34.

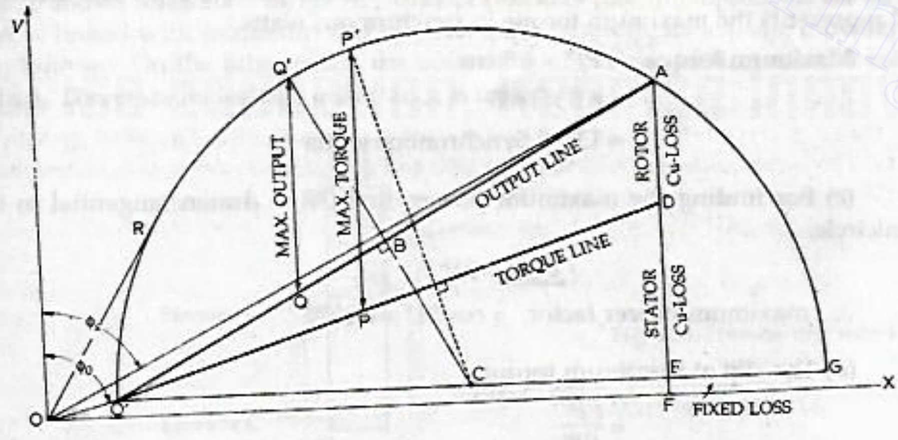


Fig. 4.29.

Phasor OO' measures $\frac{9}{5} = 1.8$ cm and represents the no-lead current of 9A.

Similarly, phasor OA represents I_{SC} that is, short-circuit current with normal voltage and measures $\frac{90}{5} = 18$ cm and is drawn at an angle of 63° with voltage axis.

The vertical line AF measures the power input on short circuit with normal voltage and is equal to 16277.5 W.

From circle diagram, AF measures 8.2 cms, then the power scale is

$$1 \text{ cm} = \frac{16277.5}{8.2}$$

$$= 1985 \text{ W}$$

Point D is such that

$$\frac{AD}{AE} = \frac{\text{Rotor copper loss}}{\text{Total copper loss}}$$

$$= \frac{\text{Rotor resistance}}{\text{Rotor + stator resistance}} \qquad (\because \text{ winding ratio} = 1)$$

$$= \frac{0.30}{0.30 + 0.42} = 0.417$$

Since $AE = 7.8$ cm

$\therefore$ $AD = 7.8 \times 0.417$

$= 3.25$ cm

and $DE = AE - AD$

$= 7.8 - 3.25$

$= 4.55$ cm.

(*a*) Starting torque $= AD = 3.25 \text{ cm} = 3.25 \times 1985$

$= 6451$ Synchronous watts

(*b*) Line CP' is drawn perpendicular to the torque line $O'D$. The intercept PP' represents the maximum torque in synchronous watts.

Maximum torque $= PP' = 6.8$ cm

$= 6.8 \times 1985$

$= 13498$ Synchronous watts

(*c*) For finding the maximum power, line OR is drawn tangential to the semicircle.

$$\angle VOR = 31°$$

$\therefore$ maximum power factor, $= \cos 31° = 0.8572$

(*d*) The slip at maximum torque

$$= \frac{PP''}{PP'}$$

$$= \frac{1.45}{6.8} = 0.213 = 21.3\%$$

(e) Line CQ' is drawn perpendicular to the output line $O'A$. From Q' is drawn the vertical line QQ'. It measures 5.6 cm and represents the maximum output.

$\therefore$ maximum output $= 5.6 \times 1985 = 11116$ Synchronous watts.

4.39 HIGH-TORQUE CAGE MOTORS

Conventional squirrel-cage motors suffer from the disadvantage of low starting torque because of low rotor resistance. The starting torque can be increased by using bar material of higher resistivity. A higher rotor resistance gives a higher starting torque and lower starting line current at a higher power factor. However, higher rotor resistance reduces the full-load speed, increases rotor ohmic losses and lower efficiency. A low rotor resistance is required for normal operation, when running, so that the slip is low and the efficiency is high. Therefore for good starting performances, the rotor resistance should be high, and under normal operating speeds, the rotor resistance should be low.

In wound-rotor induction motors these conditions are fulfilled by connecting external resistances in the rotor circuit at the time of starting. As the motor speeds up, the external resistance is cut out in steps. At normal running speed the entire external resistance is cut out and the rotor windings are short circuited through the slip rings.

In order to obtain high rotor resistance at starting and low rotor resistance at running, two types of rotor connections are used in cage motors :

1. Deep bar rotor.
2. Double-cage rotor.

4.40 DEEP-BAR CAGE MOTORS

Figure 4.30 shows a cage rotor with deep and narrow bars. A bar may be assumed to be made up of number of narrow layers connected in parallel. Figure 4.30 shows three such layers A, B and C. It is seen that the topmost layer element A is linked with minimum leakage flux and, therefore, its leakage inductance is minimum. On the other hand, the bottom layer C links with maximum leakage flux. Therefore its leakage inductance is maximum.

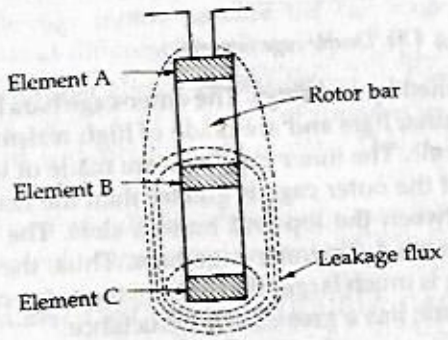


Fig. 4.30. *Deep-bar cage rotor bar.*

At starting the rotor-frequency is equal to the supply frequency. The bottom layer element C offers more impedance to the flow of current than the top layer element A. Therefore maximum current flows through the top layer and minimum through the bottom layer. Because of the unequal current distribution of current, the effective rotor resistance increases and the leakage reactance decreases. With a high rotor resistance at starting conditions, the starting torque is relatively higher and the starting current is relatively lower.

Under normal operating conditions, the slip and the rotor frequency are very small. The reactances of all the layers of the bar are small compared to their resistances. The impedances of all layers of the bar are nearly equal, so current flows through all parts of the bar equally. The resulting large cross-reactional area makes the rotor resistance quite small, resulting in a good efficiency at low slips.

4.41 DOUBLE-CAGE INDUCTION MOTORS

An induction motor with two rotor windings or cages is used for obtaining high starting torque at low starting current. The stator of a double-cage rotor induction motor is similar to that of an ordinary induction motor. In the double-cage rotor there are two layers of bars as shown in Fig. 4.31.

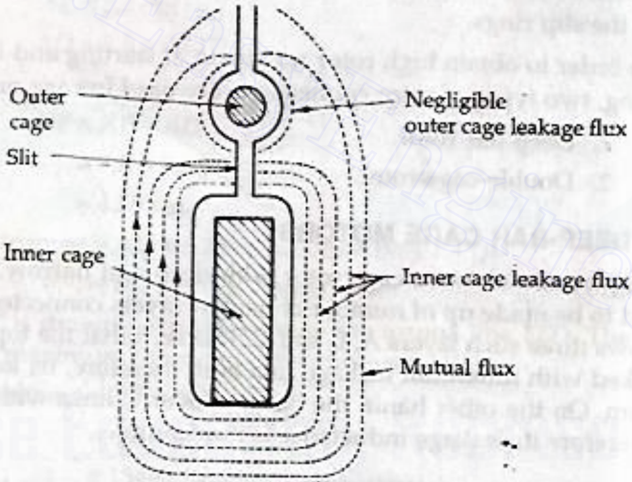


Fig. 4.31. *Double-cage rotor slot.*

Each layer is short circuited by end rings. The outer-cage bars have a smaller cross-sectional area than the inner bars and are made of high resistivity materials like brass, aluminium, bronze etc. The inner-cage bars are made of low-resistance copper. Thus, the resistance of the outer cage is greater than the resistance of the inner cage. There is a slit between the top and bottom slots. The slit increases permeance for leakage flux around the inner-cage bars. Thus, the leakage flux linking the inner-cage winding is much larger than that of the outer-cage winding, and the inner winding, therefore, has a greater self inductance.

At starting, the voltage induced in the rotor is same as the supply frequency ($f_2 = f_1$). Hence, the leakage reactance of the inner-cage winding ($= 2\pi fL$) is much larger than that of the outer-cage winding. Therefore most of the starting current is flowing in the outer-cage winding which offers low-impedance to the flow of current. The high-resistance outer cage winding, therefore, develops a high starting torque.

As the rotor speed increases, the frequency of the rotor emf ($f_r = sf$) decreases. At normal operating speed, the leakage reactances ($= 2\pi\, sf\, L$) of both the windings become negligibly small. The rotor current division between the two cages is governed mainly by their resistances. Since the resistance of the outer cage is about 5 to 6 times that of the inner cage, most of the rotor current flows through the inner cage. Hence under normal operating speed, torque is developed mainly by the low-resistance inner cage.

It is to be noted that for low-starting torque requirements an ordinary cage motor is generally used. For higher torque requirements a deep-bar cage motor is used. A double-cage motor is used for still higher torques. For large-size motors with very large starting torques and exceptionally long starting periods, slip-ring construction is used.

4.42 COMPARISON BETWEEN SINGLE-CAGE AND DOUBLE-CAGE MOTORS

A single-cage motor and a double-cage motor of the same rating can be compared as follows :

1. A double-cage rotor has low starting current and high starting torque. Therefore, it is more suitable for direct-on-line starting.
2. Since effective rotor resistance of a double-cage motor is higher, there is a larger rotor heating at the time of starting as compared to that of a single-cage rotor.
3. The high resistance of the outer cage increases the effective resistance of a double-cage motor. Therefore, full-load copper losses are increased and the efficiency of the double-cage motor is decreased.
4. A double-cage induction motor has higher effective leakage reactance due to additional reactance of the inner cage. Therefore, the full-load power factor is reduced.
5. The pull-out torque a double-cage motor is smaller than that of a single-cage motor because the two cages produce the maximum torque at different speeds.
6. By a proper choice of resistances and reactances of the outer and inner cages, a wide range of torque-slip characteristics can be obtained with double-cage motors. This is not possible with a single-cage motor.
7. The cost of a double-cage motor is about 20 to 30 % higher tan that of a single-cage motor of the same rating.

4.43 EQUIVALENT CIRCUIT OF A DOUBLE-CAGE INDUCTION MOTOR

Let R_1 = resistance per phase of stator

X_1 = reactance per phase of stator

R_{2o}' = resistance per phase of *outer cage* referred to stator

X_{2o}' = stand still leakage reactance per phase of *outer cage* referred to stator

R_{2i}' = resistance per phase of the *inner cage* referred to stator

X_{2i}' = stand still leakage reactance per phase of the *inner cage* referred to stator

s = fractional slip

If it is assumed that the main flux completely links both the cages, the impedances of the two cages can be considered in parallel.

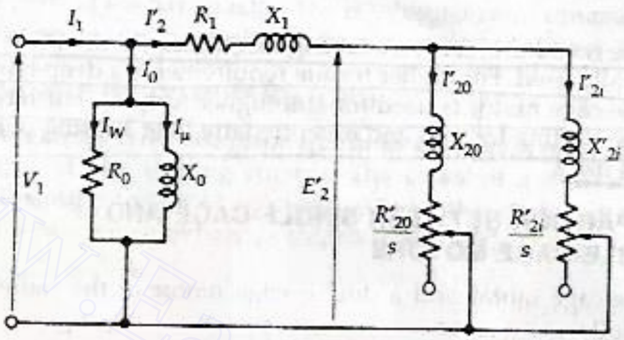


Fig. 4.32. *Equivalent circuit of a double-cage induction motor*

The equivalent circuit of the double-cage induction motor at slip s is shown in Fig. 4.32. If the shunt branches containing R_o and X_o are neglected, the equivalent circuit is simplified to that shown in Fig. 4.33.

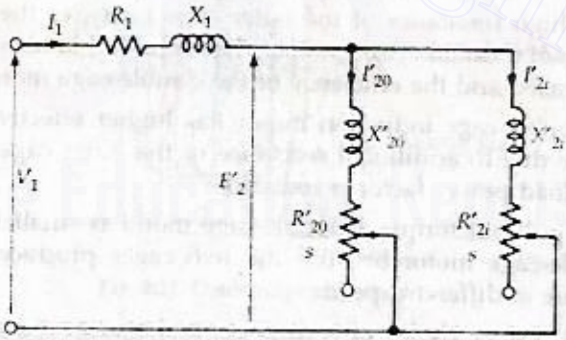


Fig. 4.33. *Approximate equivalent circuit of a double-cage induction motor with magnetising current neglected.*

At slip s, the outer-cage impedance, $\mathbf{Z}_{2o}' = \dfrac{R_{2o}'}{s} + j\,X_{2o}'$

At slip s, the inner-cage impedance, $\mathbf{Z}_{2i} = \dfrac{R_{2i}'}{s} + j\,X_{2i}'$

The impedance of the stator, $\mathbf{Z}_1 = R_1 + j\,X_1$

Equivalent impedance per phase of the motor referred to stator

$$Z_{e_1} = Z_1 + (Z_{2o}' \parallel Z_{2i}')$$

$$\therefore \quad Z_{e_1} = R_1 + j X_1 + \frac{1}{\dfrac{1}{Z_{2o}'} + \dfrac{1}{Z_{2i}'}} = R_1 + j X_1 + \frac{Z_{2o}' Z_{2i}'}{Z_{2o}' + Z_{2i}'} \tag{4.43.1}$$

Current through the outer cage

$$I_{2o}' = \frac{E_2}{Z_{2o}'} \tag{4.43.2}$$

Current through the inner cage

$$I_{2i}' = \frac{E_2}{Z_{2i}} \tag{4.43.3}$$

The rotor current (referred to the stator) is equal to the phasor sum of the currents through the outer and inner cages.

$$I_2' = I_{2o}' + I_{2i} \tag{4.43.4}$$

4.44 TORQUE-SLIP CHARACTERISTICS OF A DOUBLE-CAGE INDUCTION MOTOR

It is assumed that the two cages develop two separate torques. The total torque of the motor is equal to the sum of the two cage torques. The torque-slip characteristics of the two cages are shown in Fig. 4.33. The total torque of the motor is also shown in Fig. 4.34.

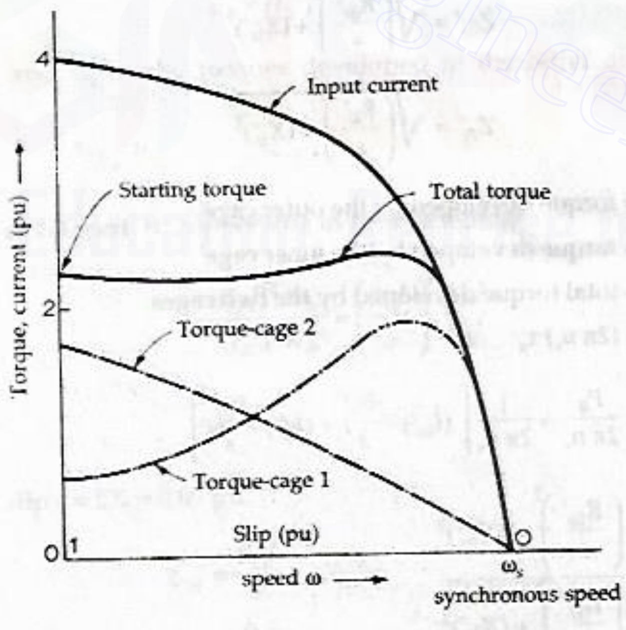


Fig. 4.34. *Torque and current characteristics of a double-cage induction motor.*

The resultant torque-speed characteristic can be modified according to the requirement. This is done by modifying the individual cage resistances and leakage reactances. The resistances can be changed by changing the area of cross-section of bars. The leakage reactances can be changed by changing the width of the slot openings and the depth of the inner cage.

4.45 COMPARISON OF CAGE TORQUES

Power developed per phase by the outer cage

$$P_{do} = I_{2o}'^{2} \frac{R_{2o}'}{s}$$

Power developed per phase by the inner cage

$$P_{di} = I_{2i}'^{2} \frac{R_{2i}'}{s}$$

Power developed per phase by both the cages

$$P_d = P_{do} + P_{di} = (I_{2o}')^2 \frac{R_{2o}'}{s} + (I_{2i}')^2 \frac{R_{2i}'}{s}$$

From the equivalent circuit of the double-cage motor

$$I_{2o}' = \frac{E_2'}{Z_{2o}'}$$

$$I_{2i}' = \frac{E_2'}{Z_{2i}'}$$

$$Z_{2o}' = \sqrt{\left(\frac{R_{2o}'}{s}\right)^2 + (X_{2o}')^2}$$

$$Z_{2i}' = \sqrt{\left(\frac{R_{2i}'}{s}\right)^2 + (X_{2i}')^2}$$

If τ_{do} = torque developed by the outer cage

τ_{di} = torque developed by the inner cage

τ_d = total torque developed by the two cages

$P_d = (2\pi n_s)\, \tau_d$

$$\tau_d = \frac{P_d}{2\pi n_s} = \frac{1}{2\pi n_s}\left[(I_{2o}')^2 \frac{R_{2o}'}{s} + (I_{2i}')^2 \frac{R_{2i}'}{s}\right]$$

$$\frac{\tau_{do}}{\tau_{di}} = \frac{\left(\frac{R_{2o}'}{s}\right)^2 + (X_{2o}')^2}{\left(\frac{R_{2i}'}{s}\right)^2 + (X_{2i}')^2}$$

EXAMPLE 4.38. *The standstill impedance of the outer cage of a double-cage induction motor is $(0.3 + j\,0.4)\ \Omega$ and that of the inner cage is $(0.1 + j\,1.5)\ \Omega$. Compare the relative currents and torques of the two cages (a) at standstill, (b) at a slip of 5 %. Neglect stator impedance.*

SOLUTION. (*a*) At standstill, $s = 1$

Outer-cage impedance, $\mathbf{Z}_{2o}{'} = R_{2o}{'} + j\,X_{2o}{'}$

$$\mathbf{Z}_{2o}{'} = 0.3 + j\,0.4 = 0.5\ \underline{/53.13^\circ}\ \Omega$$

Inner-cage impedance, $\mathbf{Z}_{2i} = R_{2i}{'} + j\,X_{2i}$

$$\mathbf{Z}_{2i}{'} = 0.1 + j\,1.5 = 1.503\ \underline{/86.18^\circ}\ \Omega$$

Current through the outer cage

$$I_{2o}{'} = \frac{E_2{'}}{Z_{2o}{'}} = \frac{E_2{'}}{0.5} = 2E_2$$

Current through the inner cage

$$I_{2i}{'} = \frac{E_2{'}}{Z_{2i}{'}} = \frac{E_2{'}}{1.503}$$

$$\therefore \quad \frac{I_{2o}{'}}{I_{2i}{'}} = \frac{Z_{2i}{'}}{Z_{2o}{'}} = \frac{1.503}{0.5} = 3.006$$

Copper loss in the outer cage

$$p_{2o} = (I_{2o}{'})^2 R_{2o}{'}$$

Copper loss in the inner cage

$$p_{2i} = (I_{2i}{'})^2 R_{2i}{'}$$

Let τ_{2o} and τ_{2i} be the torques developed in the outer and inner cages respectively.

Since $\quad \dfrac{\tau_{2o}}{\tau_{2i}} = \dfrac{p_{2o}}{p_{2i}}$

torque developed α copper loss in rotor windings

$$= \frac{(I_{2o}{'})^2 R_{2o}{'}}{(I_{2i}{'})^2 R_{2i}{'}} = \left(\frac{I_{2o}{'}}{I_{2i}{'}}\right)^2 \frac{R_{2o}{'}}{R_{2i}{'}}$$

$$= (3.006)^2 \times \frac{0.3}{0.1} = 27.1$$

(*b*) At slip $s = 5\% = 0.05$ pu

$$\mathbf{Z}_{2o}{'} = \frac{R_{2o}{'}}{s} + j\,X_{2o}{'}$$

$$= \frac{0.3}{0.05} + j\,0.4 = 6.013\ \underline{/38.1^\circ}\ \Omega$$

$$\mathbf{Z}_{2i}' = \frac{R_{2i}'}{s} + j\,X_{2i}'$$

$$= \frac{0.1}{0.05} + j\,1.5 = 2.5\,\underline{/\,36.87°}\;\Omega$$

$$\frac{I_{2o}}{I_{2i}} = \frac{Z_{2i}'}{Z_{2o}} = \frac{2.5}{6.013} = 0.4158$$

$$\frac{\tau_{2o}}{\tau_{2i}} = \left(\frac{I_{2o}'}{I_{2i}'}\right)^2 \times \frac{R_{2o}'}{R_{2i}'} = (0.4158)^2 \times \frac{0.3}{0.1} = 0.518$$

EXAMPLE 4.39. *The standstill impedances of outer and inner cages of a double-cage induction motor are $(2 + j\,1.2)\ \Omega$ and $(0.5 + j\,3.5)\ \Omega$ respectively. Determine the slip at which the two cages develop equal torques.*

SOLUTION. Standstill impedance of the outer cage

$$\mathbf{Z}_{2o}' = (2 + j\,1.2)\ \Omega = R_{2o}' + j\,X_{2o}'$$

Standstill impedances of the inner cage

$$\mathbf{Z}_{2i}' = (0.5 + j\,3.5)\ \Omega = R_{2i}' + j\,X_{2i}'$$

Let s be the slip at which two cages develop equal torques.

The impedance of the outer cage at slip s

$$\mathbf{Z}_{2o}' = \frac{R_{2o}'}{s} + j\,X_{2o}' = \frac{2}{s} + j\,1.2$$

$$Z_{2o}' = \sqrt{\left(\frac{2}{s}\right)^2 + (1.2)^2}$$

The impedance of the inner cage at slip s

$$\mathbf{Z}_{2i}' = \frac{R_{2i}'}{s} + j\,X_{2i}' = \frac{0.5}{s} + j\,3.5$$

$$Z_{2i}' = \sqrt{\left(\frac{0.5}{s}\right)^2 + (3.5)^2}$$

Current through the outer cage

$$I_{2o}' = \frac{E_2'}{Z_{2o}'}$$

Current through the inner cage

$$I_{2i}' = \frac{E_2'}{Z_{2i}'}$$

$$\therefore \qquad \frac{I_{2o}'}{I_{2i}'} = \frac{Z_{2i}'}{Z_{2o}'}$$

Copper loss in the outer cage

$$p_{2o} = (I_{2o}')^2\,R_{2o}$$

Copper loss in the inner cage

$$p_{2i} = (I_{2i}')^2\,R_{2i}'$$

Since torque developed is proportional to the copper loss in the rotor winding,

$$\frac{\text{torque developed in outer cage}}{\text{torque developed in inner cage}} = \frac{\text{copper loss in the outer cage}}{\text{copper loss in the inner cage}}$$

$$\frac{\tau_{do}}{\tau_{di}} = \frac{p_{2o}}{p_{2i}}$$

$$= \frac{(I_{2o}')^2 R_{2o}'}{(I_{2i}')^2 R_{2i}'}$$

$$= \frac{(Z_{2i}')^2}{(Z_{2o}')^2} \times \frac{R_{2o}'}{R_{2i}'} = \frac{\left(\frac{0.5}{s}\right)^2 + (3.5)^2}{\left(\frac{2}{s}\right)^2 + (1.2)^2} \times \frac{2}{0.5}$$

Since $\tau_{do} = \tau_{di}$

$$\left[\left(\frac{0.5}{s}\right)^2 + (3.5)^2\right] 4 = \left(\frac{2}{s}\right)^2 + (1.2)^2 \frac{1}{s^2} + 49 = \frac{4}{s^2} + 1.44$$

$\therefore$ $s = 0.251$ pu or 25.1%.

Example 4.40. *The resistance and reactance (equivalent) values of a double-cage induction motor for stator, outer and inner cage are 0.25, 1.0 and 0.15 ohm resistance and 3.5, zero and 3.0 ohm reactance respectively. Find the starting torque if the phase voltage is 250 V and the synchronous speed is 1000 rpm.*

Solution. $R_1 = 0.25\ \Omega$, $X_1 = 3.5\ \Omega$

$R_{2o}' = 1.0\ \Omega$, $X_{2o}' = 0$

$R_{2i}' = 0.15\ \Omega$, $X_{2i}' = 3\Omega$

At starting $s = 1$

Impedance of the outer cage at starting

$$\mathbf{Z}_{2o}' = 1 + j\,0 = 1 \angle 0^\circ\ \Omega$$

Impedance of the inner cage at starting

$$\mathbf{Z}_{2i}' = 0.15 + j\,3 = 3.004 \angle 87.1^\circ\ \Omega$$

Since the two impedances $\mathbf{Z}_{2o}'$ and $\mathbf{Z}_{2i}'$ are in parallel, therefore their equivalent impedance is given by

$$\mathbf{Z}_{2l}' = \frac{\mathbf{Z}_{2o}'\,\mathbf{Z}_{2i}'}{\mathbf{Z}_{2o}' + \mathbf{Z}_{2i}'}$$

$$= \frac{(1\angle 0^\circ)(3.004\angle 87.1^\circ)}{1 + j\,0 + 0.15 + j\,3} = \frac{3.004\angle 87.1^\circ}{3.213\angle 69^\circ}$$

$$= 0.935\angle 18.1^\circ = (0.889 + j\,0.290)\ \Omega$$

Impedance of the stator

$$\mathbf{Z}_1 = R_1 + j\,X_1 = 0.25 + j\,3.5$$

Equivalent impedance per phase of the motor referred to stator at starting

$$\mathbf{Z}_{e1} = \mathbf{Z}_1 + \mathbf{Z}_{e2}{}'$$

$$= 0.25 + j\,3.5 + 0.889 + j\,0.29$$

$$= 1.139 + j\,3.79 = 3.96\,\underline{/\,73.2^\circ}\ \Omega$$

Stator starting current

$$I_1 = \frac{\text{phase voltage}}{\text{total phase impedance}}$$

$$= \frac{250}{3.96\,\underline{/\,73.2^\circ}} = 63.13 \text{ A}$$

Starting torque per phase

$$= \frac{I_1^2}{\omega_s} \times (\text{equivalent rotor resistance})$$

$$= \frac{(63.13)^2 \times 0.889}{2\pi \times (1000/60)} = 33.833 \text{ Nm}$$

Total starting torque $= 3 \times 33.833 = 101.5$ Nm

EXAMPLE 4.41. *A double-cage induction motor has the following equivalent circuit parameters all of which are phase values referred to the primary :*

Primary	$R_1 = 1.0\ \Omega$	$X_1 = 2.8\ \Omega$
Outer cage	$R_{2o}{}' = 3.0\ \Omega$	$X_{2o}{}' = 1.0\ \Omega$
Inner cage	$R_{2i}{}' = 0.5\ \Omega$	$X_{2i}{}' = 5.0\ \Omega$

The primary is delta connected and supplied from 440 V. Calculate the starting torque when running at a slip of 4 %. The magnetizing branch can be assumed connected across the primary terminals.

SOLUTION. Since the torque and speed are not dependent upon the magnetizing impedance Z_o, the magnetizing branch may be neglected.

(*a*) At $s = 1$

Equivalent impedance per phase of the motor referred to stator

$$\mathbf{Z}_{e1} = \mathbf{Z}_1 + (\mathbf{Z}_{2o}{}' \parallel \mathbf{Z}_{2i}{}')$$

$$= R_1 + j\,X_1 + \frac{\mathbf{Z}_{2o}{}'\,\mathbf{Z}_{2i}{}'}{\mathbf{Z}_{2o}{}' + \mathbf{Z}_{2i}{}'}$$

$$= 1 + j\,2.8 + \frac{(3 + j\,1)\,(0.5 + j\,5)}{3 + j\,1 + 0.5 + j\,5}$$

$$= 1 + j\,2.8 + \frac{(3.16\,\underline{/\,18.4^\circ})\,(5.025\,\underline{/\,84.3^\circ})}{3.5 + j\,6}$$

$$= 1 + j\,2.8 + \frac{15.88\,\underline{/\,102.7^\circ}}{6.95\,\underline{/\,59.7^\circ}} = (1 + j\,2.8) + 2.285\,\underline{/\,43^\circ}$$

$$= 1 + j\,2.8 + 1.67 + j\,1.56 = (2.67 + j\,4.36)\ \Omega$$

$$= 5.1\,\underline{/\,58.5^\circ}\ \Omega$$

Since the stator is delta connected, phase voltage = line voltage = 440 V

Rotor current referred to stator

$$I_2' = \frac{V_1}{Z_{e1}} = \frac{440}{5.1\,\angle 58.5^\circ} = 86.27\,\angle -58.5^\circ \text{ A}$$

Combined resistance $R_2 = 1.67\ \Omega$

Starting torque per phase

$$= I_2^{2'} R_2 = (86.27)^2 \times 1.67$$

$$= 12429 \text{ synchronous watts}$$

(*b*) At slip $s = 4\% = 0.04$ pu

$$Z_{2o}' = \frac{R_{2o}'}{s} + j\,X_{2o}'$$

$$= \frac{3}{0.04} + j\,1 = (75 + j\,1)\ \Omega$$

$$Z_{2i}' = \frac{R_{2i}'}{s} + j\,X_{2i}'$$

$$= \frac{0.5}{0.04} + j\,5 = (12.5 + j\,5)\ \Omega$$

$$Z_{2o}' \parallel Z_{2i}' = \frac{Z_{2o}'\,Z_{2i}'}{Z_{2o}' + Z_{2i}'}$$

$$= \frac{(75 + j\,1)\,(12.5 + j\,5)}{75 + j\,1 + 12.5 + j\,5} = \frac{(75\,\angle 0.76^\circ)\,(13.46\,\angle 21.8^\circ)}{87.5 + j\,6}$$

$$= \frac{1009.5\,\angle 22.56^\circ}{87.7\,\angle 3.92^\circ} = 11.51\,\angle 18.64^\circ\ \Omega$$

$$= (10.9 + j\,3.68)\ \Omega$$

$$Z_{e1} = Z_1 + \frac{Z_{2o}'\,Z_{2i}'}{Z_{2o}' + Z_{2i}'} = 1 + j\,2.8 + 10.9 + j\,3.68$$

$$= 11.9 + j\,6.48 = 13.55\,\angle 28.57^\circ\ \Omega$$

$$I_2' = \frac{V_1}{Z_{e1}} = \frac{440}{13.55\,\angle 28.57^\circ} = 32.47\,\angle -28.57^\circ \text{ A}$$

Combined resistance $= 10.9\ \Omega$

Full load torque per phase $= I_2^{2'} R_{2o} = (32.47)^2 \times 10.9$

$$= 11492 \text{ synchronous watts}$$

4.46 EFFECT OF SPACE HARMONICS ON THREE-PHASE INDUCTION MOTOR PERFORMANCE

The air gap flux set up by the three-phase stator windings carrying sinusoidal currents is of nonsinusoidal wave shape. According to Fourier series analysis, any nonsinusoidal flux is equivalent to the combination of a number of sinusoidal fluxes of fundamental and higher order harmonics. Since the flux waveshapes have half-wave symmetry, all even harmonics (2, 4, 6, ...) are absent in Fourier series. Therefore, a nonsinusoidal flux can be resolved into fluxes of fundamental and higher-order odd harmonics (3rd, 5th, 7th, 11th, 13th etc.). The third harmonic flux waves produced by each of the three phases neutralize one another. Therefore, the resultant air gap flux is free from triplen (that is, third and its multiples–3, 9, etc.), harmonics. This is due to the fact that the third harmonics in the flux wave of all the three phases are in space phase, but differ in time phase by 120°. Space harmonic fluxes are produced by windings, slotting, magnetic saturation, inequalities in the air gap length etc. These harmonic fluxes induce voltages and circulate harmonic currents in the rotor windings. These harmonic currents in the rotor interact with the harmonic fluxes to produce harmonic torques, vibrations and noise.

Harmonic Induction Torques

A 3-phase winding carrying sinusoidal currents produces space harmonics of the order

$$h = 6k \pm 1$$

where k is a positive integer (1, 2, 3, ...). The synchronous speed of the hth harmonic is $(1/h)$ times the speed of the fundamental wave. The space harmonic waves rotate in the same direction as the fundamental wave if $h = 6k + 1$, and in the opposite direction if $h = 6k - 1$.

A space harmonic wave of order h is equivalent to a machine with number of poles equal to ($h \times$ number of poles of the stator). Therefore, the synchronous speed of the hth space harmonic wave is

$$n_{s(h)} = \frac{n_s}{h} = \frac{120\,f}{h \times P}$$

where f = supply frequency

P = number of poles of the stator

Thus, for $k = 1$, a 3-phase winding will produce predominant backward rotating fifth harmonic rotating at a speed of (1/5) of synchronous speed and forward rotating seventh harmonic rotating at a speed of (1/7) of synchronous speed. These harmonics *alone* will have little effect on the operation of the motor. The torque-speed characteristics for the fundamental flux and fifth and seventh space harmonic flux are shown in Fig. 4.34. The fifth and seventh harmonic torques have the same general shape as that of the fundamental.

Since fifth harmonic flux rotates in the direction opposite to the rotation of the rotor, the fifth harmonic torque opposes the fundamental component torque. In other words, the fifth harmonic flux produces a braking torque. The seventh harmonic flux rotates in the same direction as the fundamental flux. Therefore, the seventh harmonic induction torque aids the fundamental component torque. The resultant torque speed-characteristic will be the combination of the fundamental, fifth and seventh harmonic characteristics as shown in Fig. 4.34. The resultant torque-speed characteristic has two dips, one near (1/5) of synchronous speed and the other near (1/7) of synchronous speed. The dip near (1/5) of synchronous speed occurs in the negative direction of the motor rotation. The dip near (1/7) of synchronous speed is more important. Figure 4.34 also shows load-torque speed characteristic. If the motor torque is developed due to the fundamental flux alone, the motor will accelerate to the point L which is the intersection of the load torque characteristic and the motor torque-speed curve. Due to the presence of seventh harmonic flux torque, the load torque curve intersects the motor torque-speed characteristic at point A. Since the seventh harmonic flux torque curve has a negative slope at point A stable running condition over the torque range between the maximum and minimum points results. The motor torque falls below the load torque. At this stage the motor will not accelerate upto its normal speed, but will remain running at a speed which is nearly (1/7) of its normal speed and the operating point would be A. This tendency of the motor to run at a stable speed as low as one-seventh of the normal speed N_S and being unable to pick up its normal speed is known as **crawling** of the motor.

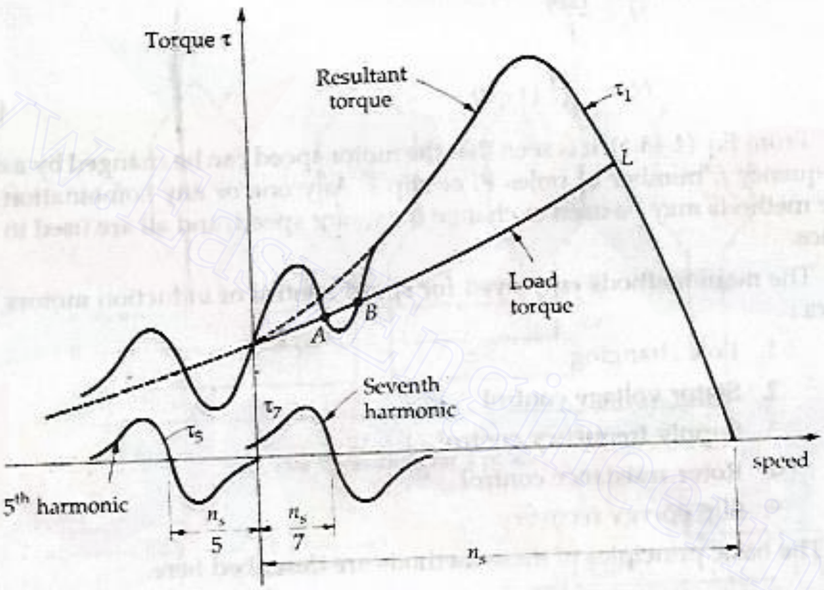


Fig. 4.34. *Torque-speed characteristics of a 3-phase induction motor showing the effect of space harmonic asynchronous (harmonic) torques.*

Crawling can be reduced by reducing fifth and seventh harmonics. This can be done by using a chorded (or short pitched) winding.

4.47 COGGING OR MAGNETIC LOCKING

Sometimes, even when full voltage is applied to the stator winding, the rotor of a 3-phase cage induction motor fails to start. This happens when the number of stator and rotor slots are either equal or have an integral ratio. With the number of stator slots equal to or an integral multiple of rotor slots, strong alignment forces are produced between stator and rotor at the instant of starting. These forces may create an alignment torque greater than the accelerating torque with consequent failure of the motor to start. This phenomenon of magnetic locking between stator and rotor teeth is called **cogging** or **teeth locking**.

The reluctance of the magnetic path is minimum when the stator and rotor teeth face each other. Under this condition there is a magnetic locking between stator and rotor teeth.

In order to reduce or eliminate cogging the number of stator slots are never made equal to or have an integral ratio. Cogging can also be reduced by using skewed rotor.

Cogging and crawling are much less prominent in wound rotor motors because of their higher starting torques.

4.48 SPEED CONTROL OF INDUCTION MOTORS

The rotor speed of an induction motor is given by

$$N_r = (1 - s)\, N_s$$

and

$$N_s = \frac{120f}{P}$$

$$\therefore \quad N_r = \frac{120f}{P}(1 - s) \tag{4.48.1}$$

From Eq. (4.48.1), it is seen that the motor speed can be changed by a change in frequency f, number of poles P, or slip s. Any one or any combination of the above methods may be used to change the motor speed, and all are used in actual practice.

The main methods employed for speed control of induction motors are as follows :

1. Pole changing
2. Stator voltage control
3. Supply frequency control
4. Rotor resistance control
5. Slip energy recovery.

The basic principles of these methods are described here.

4.49 POLE-CHANGING METHODS

The number of stator poles can be changed by (*a*) multiple stator windings, (*b*) method of consequent poles, and (*c*) pole-amplitude modulation (PAM). The methods of speed control by pole changing are suitable for cage motors only because the cage rotor automatically develops number of poles equal to the poles of the stator winding.

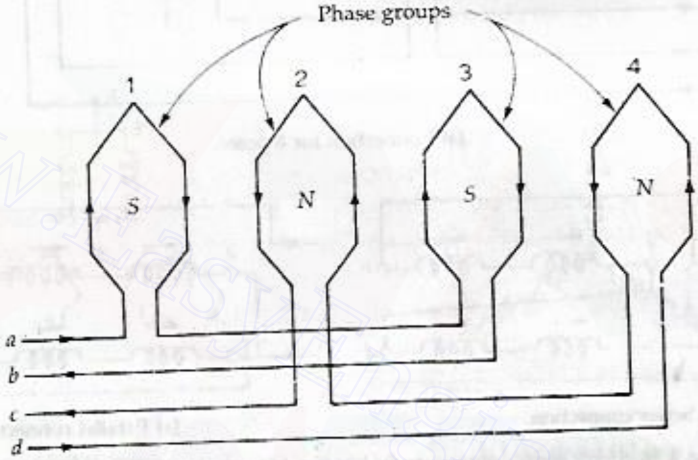
4.49.1 Multiple Stator Winding

In this method the stator is provided with two separate windings which are wound for two different pole numbers. One winding is energized at a time. Suppose that a motor has two windings for 6 and 4 poles. For 50 Hz supply the synchronous speeds will be 1000 and 1500 rpm respectively. If the full-load slip is 5% in each case, the operating speeds will be 950 rpm and 1425 rpm respectively. This method is less efficient and more costly, and therefore, used only when absolutely necessary.

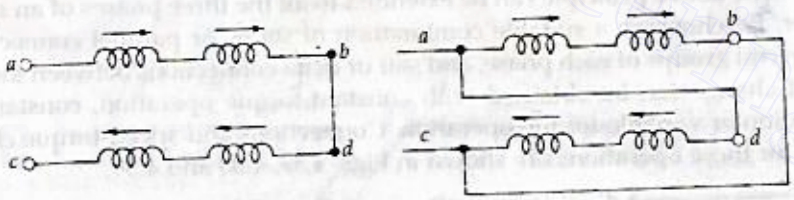
4.49.2 Method of Consequent Poles

The method of consequent poles was originally developed in 1897. In this method a single stator winding is divided into few coil groups. The terminals of all these groups are brought out. The number of poles can be changed with only simple changes in coil connections. In practice, the stator winding is divided only in two coil groups. The number of poles can be changed in the ratio of 2 : 1.

Fig. 4.35 shows one phase of a stator winding consisting of 4 coils divided into two groups $a-b$ and $c-d$. Group $a-b$ consists of odd-numbered coils (1, 3) and connected in series. Group $c-d$ has even numbered coils (2, 4) connected in series. The terminals a, b, c, d are taken out as shown. The coils can be made to carry current in the given directions by connecting coil groups either in series or parallel shown in Fig. 4.35(*b*) and Fig. 4.35(*c*) respectively.



(*a*) Connection for 4 poles.



(*b*) Series connection

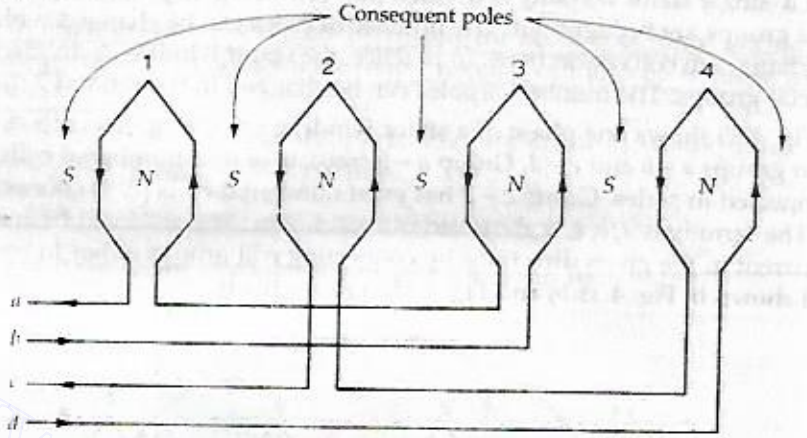
(*c*) Parallel connection

Fig. 4.35. *Stator phase connections for high speed (4 poles).*

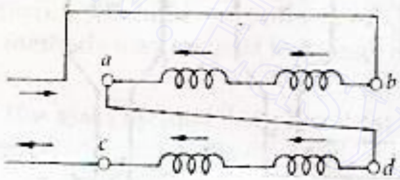
With this connection, there will be a total of 4 poles giving a synchronous speed of 1500 rpm for a 50 Hz system. If the current through the coils of group $a-b$ is reversed (Fig. 4.36(*a*)), then all the coils will produce north (N) poles.

In order to complete the magnetic path, the flux of the pole groups must pass through the spaces between the groups, thus inducing magnetic poles of opposite polarity (S poles) in the inter-pole spaces. These induced poles are called **consequent poles**. Thus, machine has twice as many poles as before (that is, 8 poles) and the synchronous speed is half of the previous speed (that is 750 rpm).

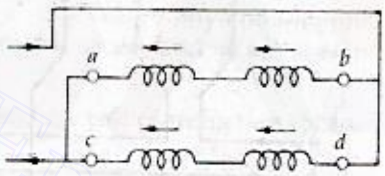
It is to be noted that two sets of coil groups $a - b$ and $c - d$ can be connected either in series for one speed, or in parallel for the other speed as shown in Fig. 4.36 (*b*) and (*c*).



(*a*) Connection for 8 poles.



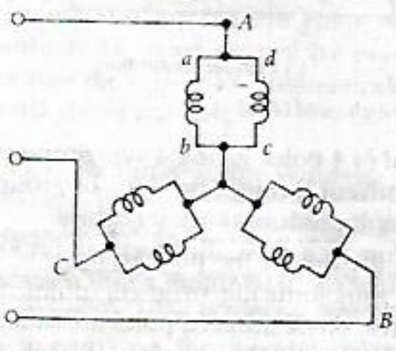
(*b*) Series connection



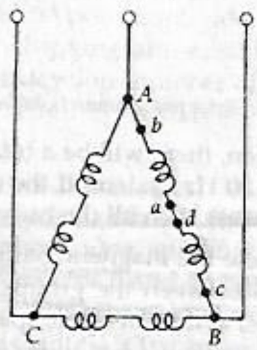
(*c*) Parallel connection

Fig. 4.36. *Stator phase connections for low speed (4 poles) with 4 consequent poles.*

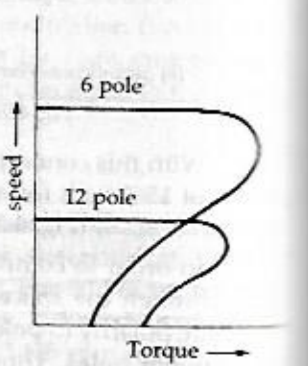
The above principle can be extended to all the three phases of an induction motor. By choosing a suitable combination of series or parallel connections between coil groups of each phase, and star or delta connections between the phases, speed change can be obtained with constant-torque operation, constant-power operation or variable-torque operation. Connections and speed-torque characteristics for these operations are shown in Figs. 4.37, 4.38 and 4.39.



(*a*) High speed (6-pole)

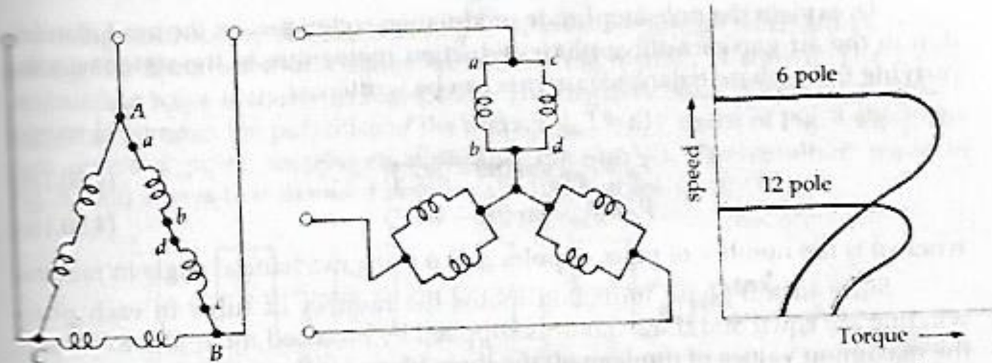


(*b*) Low speed (12-pole)



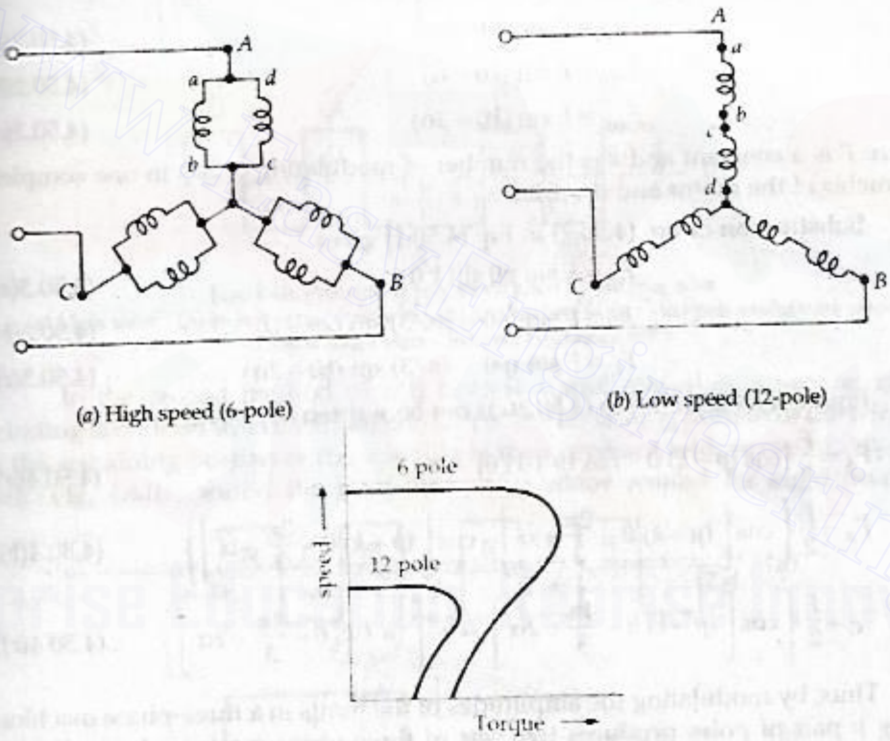
(*c*) Speed-torque characteristics

Fig. 4.37. *Constant-torque control.*



(a) High speed (6-pole) (b) Low speed (12-pole) (c) Speed-torque characteristics

Fig. 4.38. *Constant-power control.*



(a) High speed (6-pole) (b) Low speed (12-pole)

(c) Speed-torque characteristics

Fig. 4.39. *Variable-torque control.*

4.50 POLE AMPLITUDE MODULATION (PAM) TECHNIQUE

Pole amplitude modulation (PAM) technique is a flexible method of pole changing which can be used in applications where speed ratios other than 2 : 1 are required. The motors designed of speed changing based on poled amplitude modulation scheme are known as PAM motors.

To explain the pole amplitude modulation technique, let the mmf distribution in the air gap of a three-phase induction motor due to the stator winding carrying three-phase balanced currents can be written as

$$F_A = F_{mA} \sin p\theta \quad (4.50.1(a))$$

$$F_B = F_{mB} \sin (p\theta - 2\pi/3) \quad (4.50.1(b))$$

$$F_C = F_{mC} \sin (p\theta - 4\pi/3) \quad (4.50.1(c))$$

where p is the number of pairs of poles and θ is the mechanical angle in radians.

Since in a 3-phase induction motor the number of turns in each phase winding are equal and if the motor is supplied by balanced three-phase currents, the maximum values of mmfs in all the three phases is the same.

If three modulating mmf waves of amplitude F but displaced from each other by $2\pi/3$ radians are used to modulate the mmf waves of Eq. (4.50.1), then it is possible to write F_{mA}, F_{mB} and F_{mC} as follows :

$$F_{mA} = F \sin k\theta \quad (4.50.2(a))$$

$$F_{mB} = F \sin (k\theta - \alpha) \quad (4.50.2(b))$$

$$F_{mC} = F \sin (k\theta - 2\alpha) \quad (4.50.2(c))$$

where F is a constant and k is the number of modulating cycles in one complete perimeter of the motor and $\alpha = \pm 2\pi/3$.

Substitution of Eq. (4.50.2) in Eq. (4.50.1) gives

$$F_A = F \sin p\theta \sin k\theta \quad (4.50.3(a))$$

$$F_B = F \sin (p\theta - 2\pi/3) \sin (k\theta - \alpha) \quad (4.50.3(b))$$

$$F_C = F \sin (p\theta - 4\pi/3) \sin (k\theta - 2\alpha) \quad (4.50.3(c))$$

Equations (4.50.3(*a*)) to (4.50.3(*c*)) can be written as

$$F_A = \frac{F}{2} [\cos (p-k)\theta - \cos (p+k)\theta] \quad (4.50.4(a))$$

$$F_B = \frac{F}{2}\left\{ \cos\left[(p-k)\theta - \frac{2\pi}{3} + \alpha\right] - \cos\left[(p+k)\theta - \frac{2\pi}{3} - \alpha\right]\right\} \quad (4.50.4(b))$$

$$F_C = \frac{F}{2}\left\{ \cos\left[(p-k)\theta - \frac{4\pi}{3} + 2\alpha\right] - \cos\left[(p+k)\theta - \frac{4\pi}{3} - 2\alpha\right]\right\} \quad ...(4.50.4(c))$$

Thus, by modulating the amplitudes of the mmfs in a three-phase machine having p pair of poles produces two sets of three-phase mmfs with $(p-k)$ and $(p+k)$ poles. These two sets of poles will produce torques in opposite directions. To obtain steady torque in one direction only, one of these pole pairs must be suppressed and the other pair should be retained. A rectangular space mmf wave of unit amplitude and of period equal to the length of the stator periphery is used for modulation. Two methods of connections are used to obtaine the desired modulation. The first method is known as **Coil inversion** and the other is the **coil inversion and omission.** In both the methods the windings of each phase are divided in two parts. In the method of coil inversion, the current through the latter half of winding in each phase is reversed.

mmf distribu-
tator winding

(4.50.1(a))
(4.50.1(b))
(4.50.1(c))

gle in radians.
in each phase
phase currents,

aced from each
(4.50.1), then it

(4.50.2(a))
(4.50.2(b))
(4.50.2(c))

n one complete

(4.50.3(a))
(4.50.3(b))
(4.50.3(c))

(4.50.4(a))

(4.50.4(b))

...(4.50.4(c))

e-phase machine
with $(p-k)$ and
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le pairs must be
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periphery is used
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e other is the coil
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Fig. 4.40 shows the basic principle of pole amplitude modulation. In Fig. 4.40(a), the mmf wave of a stator wound for eight poles is shown. The 2-pole modulating wave is shown in Fig. 4.40(b). The negative half cycle of the modulating wave reverses the polarities of the main poles 5, 6, 7 and 8 of Fig. 4.40(a). The sign reversed poles are shown dotted in Fig. 4.40(c). The resultant wave in Fig. 4.40(c) shows that the modulated wave has 6 poles.

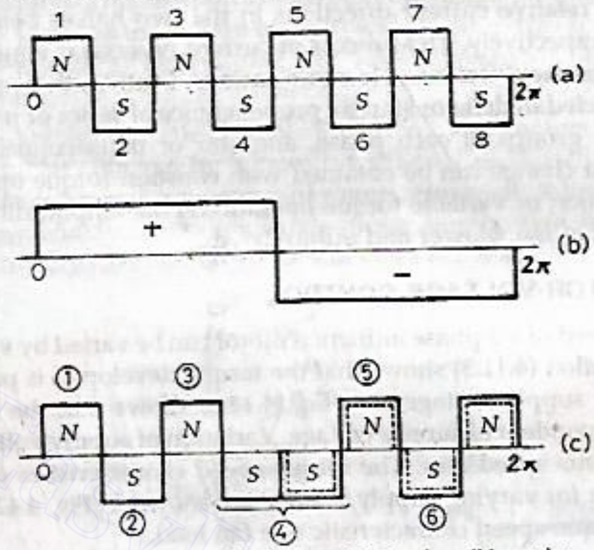


Fig. 4.40. *Principle of pole modulation by coil inversion*
(a) Main wave : Eight-pole stator mmf wave, (b) Modulating wave : Two-pole modulating wave, (c) Modulated wave : Six-pole modulated wave.

In the second method of coil inversion and omission, a section of the winding is omitted from each half and half of the remaining portion of the winding is the remaining portion of the winding is then reversed with respect to the first half. Fig. 4.41(a) shows the mmf wave of a stator wound for eight poles. In

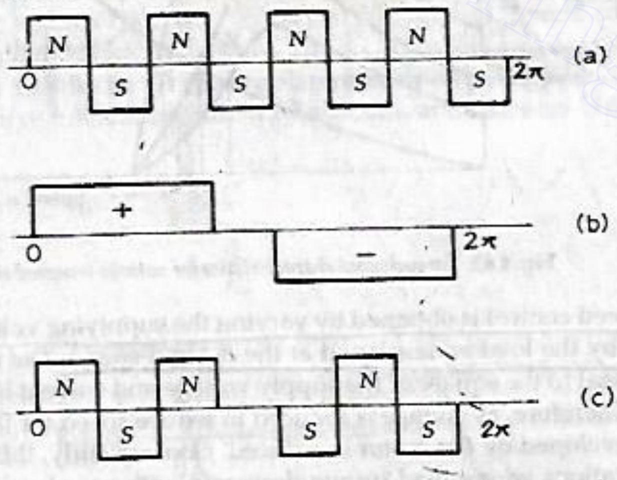


Fig. 4.41. *Principle of pole amplitude modulation by coil inversion and omission.*
(a) Main wave : Eight-pole stator mmf wave, (b) Modulating wave ; Two-pole modulating wave, (c) Modulated wave : Six-pole modulated wave.

Fig. 4.41(*b*) fourth and eighth coil are omitted and fifth, sixth and seventh coils a
then reversed with respect to the first three. This results in six poles as show
Fig. 4.41(*c*). Thus, the motor can run corresponding to 8 (original) and 6 (mo
lated) poles.

The basic feature of PAM winding is the layout, which is generally irreg The winding is in two parts. The two parts are either connected in series o parallel, the relative current directions in the two halves being the same o opposition respectively. The process of current reversal is equivalent to modu tion and gives the different pole combinations. Further, three phases of mach can be connected in delta or star. By proper choice of series or parallel connect between coil groups of each phase, and star or delta connection between phases, speed change can be obtained with constant-torque operation, const power operation or variable torque operation. Pole amplitude modulation tec nique is used in fan, blower and pump drives.

4.51 STATOR VOLTAGE CONTROL

The speed of a 3-phase induction motor can be varied by varying the sup voltage. Equation (4.11.3) shows that the torque developed is proportional to th square of the supply voltage, and Eq. (4.12.8) shows that the slip at maxim torque is independent of supply voltage. Variation of supply voltage does not al the synchronous speed also. The torque-speed characteristics of three-phase in duction motor for varying supply voltage are shown in Fig. 4.42. This figure als shows the torque-speed characteristic of a fan load.

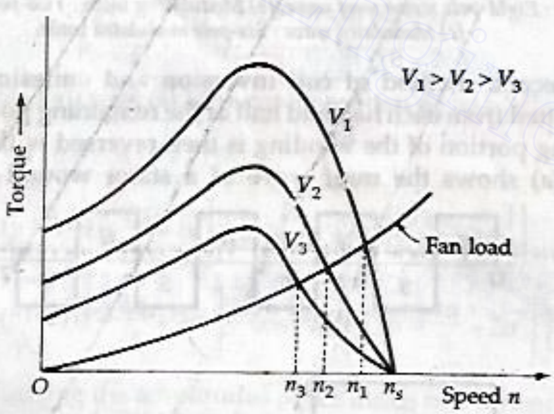


Fig. 4.42. *Torque-speed characteristics for various terminal voltages.*

Speed control is obtained by varying the supplying voltage until the torq required by the load is developed at the desired speed. The torque develope proportional to the square of the supply voltage and current is proportional to th voltage. Therefore, as voltage is reduced to reduce speed for the same current, th torque developed by the motor is reduced. Consequently, this method is suita for applications where load torque decreases with speed, as in the case of a fa load.

From Fig. 4.42, it is seen that for a given load, the speed of the motor can be varied within a small range by this method.

Since the operation at voltages higher than the rated voltage is not permissible, this method allows speed control only below the normal rated speed.

The stator voltage control is more suitable where intermittent operation of the drive is required. This method is also suitable for fan or pump drives where the load torque varies as the square of the speed. These drives require low torque at low speeds and this can be obtained with lower applied voltage without excessive motor current.

Variable voltage for speed control of small size motors, particularly for single-phase, can be obtained by connecting external resistance or inductance in the stator circuit or by using autotransoformers. However, thyristor voltage controllers are now widely used. For single-phase supply, two thyristors in anti-parallel (back-to-back) are connected as shown in Fig. 4.43.

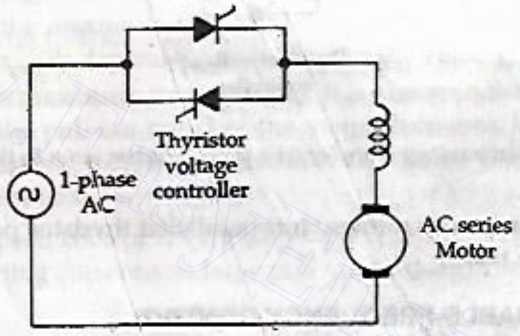


Fig. 4.43. *Single-phase variable voltage supply using thyristor voltage controller for speed control of ac series motor.*

Domestic fan motors, which are always single phase, are controlled by a single-phase triac voltage controller as shown in Fig. 4.44. Speed control is obtained by varying firing angle of the triac. These controllers are commonly known as solid-state fan regulators. These regulators are preferred over conventional variable resistance regulators because they are compact and more efficient.

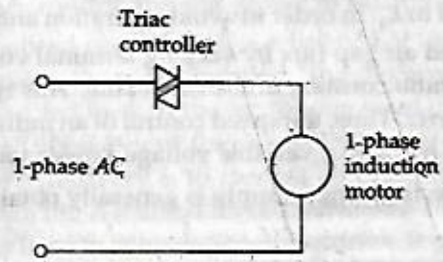


Fig. 4.44. *Stator voltage control by triac controller.*

For a three-phase induction motor three pairs of back-to-back conn thyristors are required, one pair in each phase (Fig. 4.45). Each pair of thyr controls the voltage of the phase to which it is connected. Speed control is ob by varying conduction period of thyristors.

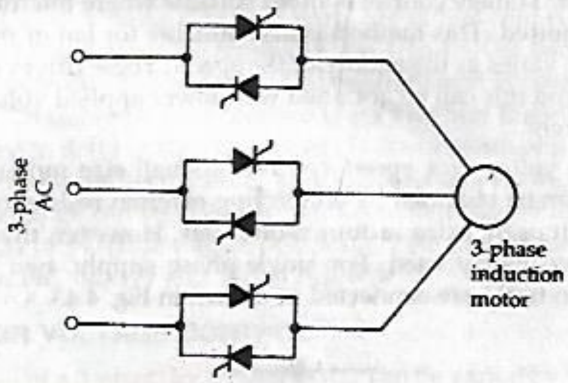


Fig. 4.45. *Stator voltage control of three-phase induction motor by thyristor voltage contr*

For low-power ratings, anti-paralleled thyristor pair in each phase replaced by a triac.

4.52 VARIABLE-FREQUENCY CONTROL

The synchronous speed of an induction motor is given by $N_S = \frac{120f}{P}$

The synchronous speed and, therefore, the speed of the motor can be trolled by varying the supply frequency.

The emf induced in the stator of the induction motor is given by

$$E_1 = 4.44\, k_{w_1} f \phi\, T_1$$

Therefore, if the supply frequency is changed, E_1 will also change tain the same air gap flux. If the stator voltage drop is neglected the voltage V_1 is equal to E_1. In order to avoid saturation and to minimize losses is operated at rated air gap flux by varying terminal voltage with frequen to maintain (V/f) ratio constant at the rate value. This type of control is kn *constant volts per hertz*. Thus, the speed control of an induction motor using frequency supply requires a variable voltage power source.

The variable frequency supply is generally obtained by the followi verters :

1. Voltage source inverter
2. Current source inverter
3. Cycloconverter.

An inverter converts a fixed voltage dc to a fixed (or variable) voltage ac with variable frequency.

A cycloconverter converts a fixed voltage and fixed frequency ac to a variable voltage and variable (lower) frequency ac.

The variable frequency control allows good running and transient performance to be obtained from a cage induction motor.

Cycloconverter controlled induction motor drive is suitable only for large power drives and to get low speeds.

4.53 ROTOR RESISTANCE CONTROL

The speed of wound induction motor can be controlled by connecting external resistance in the rotor circuit through slip rings, as shown in Fig. 4.2. This method is not applicable to cage motors. Fig. 4.7 shows the torque-slip curves for various values of rotor resistance. The torque-speed curves are shown in Fig. 4.8. It is seen that although the maximum torque is independent of rotor resistance, yet the exact location of τ_{max} is dependent on it. Greater the value of R_2, greater is the value of slip at which maximum torque occurs. It is also seen that as the rotor resistance is increased, the pull-out speed of the motor decreases, but the maximum torque remains constant. Therefore, by this method, control is provided from the rated speed to lower speeds.

This method of speed control is very simple. It is possible to have a large starting torque, low starting current and large pull-out torques at small values of slip.

The major disadvantage of the rotor resistance control method is that the efficiency is low due to additional losses in resistors connected in the rotor circuit. The efficiency is greatly reduced at low speeds because of higher slips. Because of low cost and high torque capability at low speeds, this method is used in cranes, Ward-Leonard Ilgener drives and other intermittent load applications. This method can also be used in fan or pump drives, where speed variation over a small range near the top speed is required.

4.54 SLIP-ENERGY RECOVERY

In the rotor resistance control, the slip power in the rotor circuit is wasted as I^2R loss during the low speed operation. The efficiency of the drive system by this method of speed control is, therefore, reduced. The slip power from the rotor circuit can be recovered and fed back to the a.c. source so as to utilize it outside the motor. Thus, the overall efficiency of the drive system can be increased. The basic principle of slip power recovery is to connect an external source of emf of slip frequency to the rotor circuit. A method for recovering the slip power is shown in Fig. 4.46. This method is known as static Scherbius drive. It provides the speed control of a slipring induction motor below synchronous speed. A portion of rotor a.c. power (slip power) is converted into d.c. by a diode bridge. The rectified current is smoothed by the smoothing reactor. The output of the rectifier is then connected to the d.c. terminals of the inverter, which inverts this d.c. power to a.c. power and feeds it back to the a.c. source. The inverter is a controlled rectifier operated in the inversion mode.

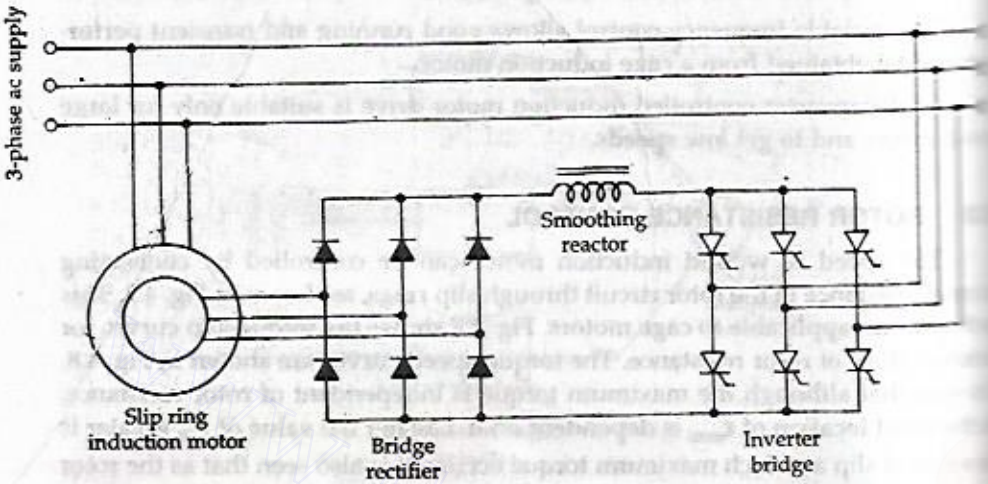


Fig. 4.46. *Static Scherbius drive for speed control of slip ring induction motor.*

This method of speed control is used in large power applications where variation of speed over a wide range involves a large amount of slip power.

4.55 APPLICATIONS OF POLYPHASE WOUND-ROTOR INDUCTION MOTORS

Wound-rotor motors are suitable for loads requiring high starting torque and for applications where the starting current is low. They are also used for loads having high inertia, which results in extremely large rotor energy losses during acceleration. Wound-rotor motors are also used for loads which require a gradual buildup of torque of soft start and for loads that require some speed control.

The maximum torque is usually above 200% of full-load value while the full-load slip may be as low as 3%, which makes for a high full-load efficiency of about 90%.

Typical applications are conveyors, crushers, plunger pumps, hoists, cranes, elevators, and compressors.

4.56 APPLICATIONS OF POLYPHASE CAGE INDUCTION MOTORS

To meet the various starting and running requirements of variety of industrial applications, several standard designs of squirrel-cage motors are available in the market. The torque-speed characteristics of most common designs are shown in Fig. 4.47. The most significant design variable in these motors is the effective resistance of the rotor cage circuits.

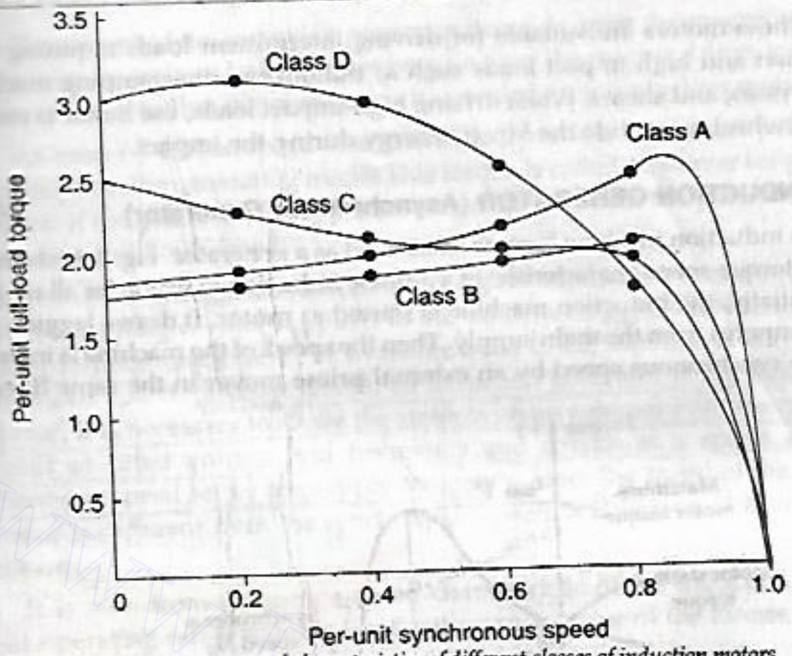


Fig. 4.47. *Typical torque-speed characteristics of different classes of induction motors*

Class A Motors

Class A motors have normal starting torque, high starting current and low operating slip (0.005 – 0.015). This design usually has a low-resistance single-cage rotor. The full-load efficiency is high.

Examples of loads are blowers, fans, machine tools and centrifugal pumps.

Class B Motors

Class B motors are characterized by normal starting torque, low starting current and low operating slip. The starting current is reduced by designing for relatively high leakage reactance, and the starting torque is maintained by use of a double-cage or deep-bar rotor.

Design B motors are most popular and used for full-voltage starting. They have about the same starting torque as design A, with only about 75% of the starting current. Their applications are the same as those for the design A.

Class C Motors

Class C motors have high starting torque and low starting current. Such motors are of the double-cage and deep-bar construction with higher rotor resistance than class B motors. The application is for practically constant-speed loads requiring fairly high torque with low starting current. Typical loads are compressors, crushers, conveyors, and reciprocating pumps.

Class D Motors

Class D motors have the highest starting torque of all squirrel-cage induction motors. The rotor cage bars are made of high resistance material such as brass instead of copper. These motors have low starting current and high operating slip. The full-load operating slip is 8 to 15 percent and, therefore, the running efficiency is low. These motors are suitable for driving intermittent loads requiring rap acceleration and high impact loads such as bulldozers, die-stamping machine punch presses, and shears. When driving high-impact loads, the motor is coupl with a flywheel to provide the kinetic energy during the impact.

4.57 INDUCTION GENERATOR (Asynchronous Generator)

An induction machine is sometimes used as a generator. Fig. 4.48 shows the complete torque-speed characteristic of a 3-phase induction machine for all range of speed. Initially, the induction machine is started as motor. It draws lagging rea tive voltamperes from the main supply. Then the speed of the machine is increas above the synchronous speed by an external prime mover in the same directio

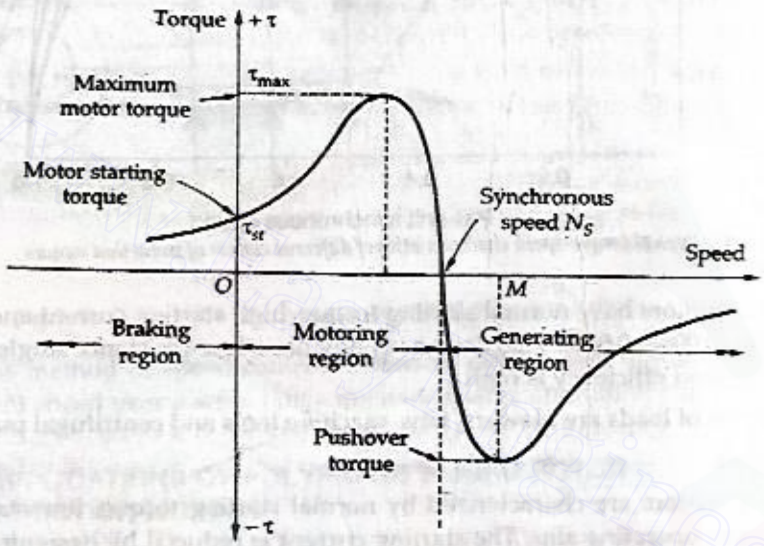


Fig. 4.48. *Torque-speed characteristic of an induction machine.*

as the rotating field produced by the stator windings. Then the induction machin will operate as an induction generator, and will produce a generating torque. Th generating torque is opposite to the rotation of the rotor (or opposite to the rotati field produced by the rotor). Under these circumstances, the slip is negative an the induction generator delivers electrical energy to the supply mains.

In the equivalent circuit of an induction motor of Fig. 4.2, the mechani shaft load has been replaced by a resistor of value given by

$$R_{mech} = \frac{R_2'}{s}(1-s)$$

In an induction generator, the slip s is negative and, therefore, the loa resistance R_{mech} is also negative. This shows that load resistance no longer abso power, but acts as a source of power. In other words, the induction genera supplies electrical energy to the supply mains to which it is connected.

The output of the induction generator depends upon the magnitude of the negative slip, or on how fast *above synchronous speed* the rotor is driven in the *same direction* or rotation that occurred when it operated as an induction motor.

As seen by torque-speed characteristic, there is a maximum possible induced torque in the generating mode. This torque is called **pushover torque** of the generator. If the prime mover applies a torque greater than the pushover torque, the generator will overspeed.

The rotating magnetic field in the polyphase induction motor is produced due to the exciting current supplied to the stator winding from the supply line. The supply must continue to be available even if the machine is driven above synchronous speed. In other words, an induction generator is not a self-excited generator. It is necessary to excite the stator with an external polyphase source at all times at rated voltage and frequency and driven at a speed above the synchronous speed set by the supply frequency. Since the speed of the induction generator is different from the synchronous speed, it is known as **asynchronous generator.**

It is seen from the torque-speed characteristic of the induction generator that its operating range is limited to the maximum value of the torque (pushover torque) corresponding to a slip at a speed OM in Fig. 4.48.

4.58 ISOLATED INDUCTION GENERATOR

An induction machine can work as a generator even without an external supply system. A three-phase delta-connected capacitor bank (Fig. 4.49(*a*)) is connected across the terminals of the machine to provide necessary excitation. The presence of residual flux is necessary to provide the initial excitation. In case there is no residual flux, the machine must be momentarily run as an induction motor to create residual flux. The motor is run slightly above synchronous speed at no

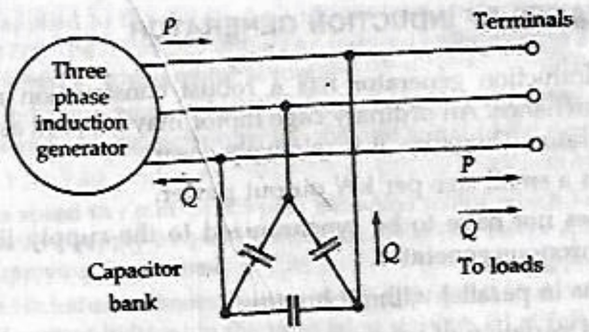


Fig. 4.49. *(a) An induction generator operating along with a capacitor bank to supply reactive power.*

load by a prime mover. A small emf is induced in the stator at a frequency proportional to the rotor speed. This voltage appears across the 3-phase capacitor bank giving rise to a leading current drawn by the capacitor bank. This is equivalent to the lagging current supplied back to the generator. The flux set up by this current assists the initial residual flux causing an increase in the net flux, which in turn causes a net increase in voltage. This increase in voltage causes further increase in exciting current causing further increase in the terminal voltage. This voltage build-up continues upto a point where the magnetization characteristic of the machine and the voltage-current characteristic ($V \sim I_C$) of the capacitor bank (Fig. 4.49. (*b*)) intersect each other. At this point the reactive voltamperes demanded by the generator is equal to the reactive voltamperes supplied by the capacitor bank. The operating frequency depends upon the rotor speed and is affected by the load. The voltage is mainly governed by the capacitive reactance at the operating frequency. For a lagging power factor load, the voltage collapses very rapidly. This is a serious disadvantage of an induction generator.

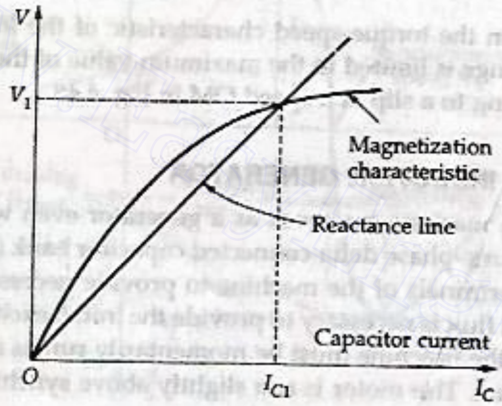


Fig. 4.49. *(b) Magnetization curve and V–I_C characteristic.*

4.59 ADVANTAGES OF INDUCTION GENERATOR

1. An induction generator has a robust construction requiring less maintenance. An ordinary cage motor may be used as an induction generator. Therefore, it is relatively cheap.
2. It has a small size per kW output power.
3. It does not have to be synchronized to the supply line as does a synchronous generator.
4. It runs in parallel without hunting.
5. Speed variation of prime mover is less important.
6. An induction generator needs little auxiliary equipment.
7. It has a self-protective feature. If a short-circuit fault occurs on its terminals, the excitation fails and the machine stops generation.

4.60 LIMITATIONS OF INDUCTION GENERATOR

An induction generator cannot generate reactive voltamperes. Actually, it requires reactive voltamperes from the supply line to furnish its excitation, since it has no means for establishing air gap flux with the stator open-circuited. Operation of an induction generator requires synchronous machines, whether generators or motors, on the line to supply the induction generator with its needed reactive voltamperes. It is this limitation of reactive voltampere requirements which restricts the use of induction generators to a few rather unusual applications.

4.61 INDUCTION GENERATOR APPLICATIONS

Induction generators have been used since early in the twentieth century, but by the 1960s and 1970s they had largely disappeared from use. However, with the increasing oil prices since 1973, its use started again. Induction generators are used principally with alternative energy sources, such as wind mills, or with energy recovery systems in industrial processes. They are often used to supply additional power to a load in a remote area that is being supplied by a weak transmission line.

EXERCISES

4.1 Describe with neat sketches the construction of a 3-phase cage-type induction motor.

4.2 Describe with neat sketches the construction of a 3-phase wound induction motor.

4.3 Compare cage and wound 3-phase induction motor with reference to construction, performance and applications.

4.4 Explain the principle of operation of a 3-phase induction motor.

4.5 What is meant by slip in an induction motor ? Why must slip be present for motor action ?

4.6 Define slip. Why cannot an induction motor run at synchronous speed ?

4.7 Deduce an expression for the frequency of rotor current in an induction motor.

4.8 The voltage applied to the stator of a 3-phase, 4-pole induction motor has a frequency of 50 Hz. The frequency of the emf induced in the rotor is 2 Hz. Calculate the slip and speed at which motor is running. [0.04, 1440 r.p.m.]

4.9 If an 8-pole induction motor running from a supply of 50 Hz has an emf in the rotor of frequency 1.5 Hz, determine the slip and speed of the motor. [0.03, 727.5 r.p.m.]

4.10 Calculate the speed in r.p.m. of a 6-pole induction motor which has a slip of 6% at full load with a supply frequency of 50 Hz. What will be the speed of a 4-pole alternator supplying the motor ? [940 r.p.m. ; 1500 r.p.m.]

4.11 A 4-pole, 50 Hz induction motor runs with 4% slip at full load. What will be the frequency of current induced in the rotor (*a*) at starting, (*b*) at full load ? [(*a*) 50 Hz, (*b*) 2 Hz]

4.12 Two three-phase induction motors when connected across a 400 V, 50 Hz supply and running at 1440 and 940 r.p.m. respectively. Determine which of the two motors is running at higher slip. [Motor running at 940 r.p.m.]

4.13 A 4-pole, 50 Hz induction motor runs with a slip of 0.01 p.u. on full load. Calculate the frequency of the rotor current (*a*) at standstill and (*b*) on full load. [(*a*) 50 Hz ; (*b*) 0.5 Hz]

4.14 A 4-pole induction motor is fed from 50 Hz supply, and has a rotor speed of 1425 r.p.m.
Find (*a*) slip speed, (*b*) per unit slip, (*c*) per cent slip. [(*a*) 75 r.p.m., (*b*) 0.05 ; (*c*) 5%]

4.15 A 12-pole, 3-phase alternator driven at a speed of 500 r.p.m. supplies power to an 8-pole, 3-phase induction motor. If the slip of the motor at full load is 0.03 p.u., calculate the full-load speed of the motor. [727.5 r.p.m.]

4.16 A 6-pole, 50 Hz induction motor runs with 5 per cent slip. What is its speed ? What is the frequency of the rotor current ? [950 r.p.m. ; 2.5 Hz]

4.17 A 4-pole, 3300 V, 50 Hz induction motor runs at the rated frequency and voltage. The frequency of the rotor currents is 2.5 Hz. Find the per unit slip and the running speed. [0.05, 1425 r.p.m.]

4.18 A 3-phase, 6-pole, 400 V, 50 Hz induction motor has a speed of 950 r.p.m. on full load. Calculate the slip. How many complete alternations will the rotor voltage make per minute ? [0.05 p.u., 150 cycles per minute]

4.19 A 4-pole, 3-phase induction motor operates from a supply whose frequency is 50 Hz. Calculate

(*a*) the speed at which the magnetic field of the stator is rotating ;

(*b*) the speed of the rotor when the slip is 0.04 ;

(*c*) the frequency of the rotor current when the slip is 0.03 ;

(*d*) the frequency of the rotor current at standstill.

[(*a*) 1500 r.p.m. ; (*b*) 1440 r.p.m. ; (*c*) 1.5 Hz ; (*d*) 50 Hz]

4.20 Why starters are necessary for starting induction motors ? Name different starting methods for 3-phase induction motors.

4.21 Describe with construction diagrams the working of the following starters :

(*a*) Direct on-line starter

(*b*) Auto-transformer starter

(*c*) Star-delta starter

(*d*) Slip-ring motor starter

4.22 A 4-pole, 50 Hz, 3-phase induction motor has a rotor resistance of 0.02 Ω per phase and standstill reactance of 0.5 Ω per phase. Determine the speed at which the maximum torque is developed. [1440 r.p.m.]

4.23 3-phase induction motor has a synchronous speed of 250 r.p.m. and 4% slip at full load. The rotor has a resistance of 0.02 Ω per phase and a standstill leakage reactance of 0.15 Ω per phase. Calculate :

(*a*) the speed at which maximum torque is developed ;

(*b*) the ratio of maximum to full-load torque ;

(*c*) the ratio of maximum to starting torque ;

(*d*) What value should the resistance per phase have so that the starting torque is half the maximum torque ?

[(*a*) 217 r.p.m. ; (*b*) 1.82 ; (*c*) 3.82 (*d*) 0.04 Ω]

4.24 A small 3-phase induction motor has a short-circuit current equal to 3.5 times the full-load current. Determine the starting torque as a fraction of full-load torque if the slip at full load is 0.03 p.u. [0.3675]

4.25 An induction motor is to be started directly from the mains. If the starting torque is equal to the full-load torque, find the starting current in terms of full-load current if the slip of the motor at full load is 4%. [$I_{st} = 5\ I_{fl}$]

4.26 A cage induction motor has a short-circuit current of 5 times the full-load value and has a full-load slip of 3%. Determine a suitable auto-transformer ratio for starting this motor when the supply current is not to exceed 2.5 times the full-load current. Also find the starting torque in terms of the full-load torque. [0.707, $\tau_{st} = 0.375\ \tau_{fl}$]

4.27 Determine the suitable auto-transfer ratio for starting a 3-phase induction motor with line current not exceeding three times the full-load current. The short-circuit current is 5 times the full-load current and the full-load slip is 5%. Determine also the starting torque in terms of the full-load torque. [0.775, $\tau_{st} = 0.75\ \tau_{fl}$]

4.28 The full-load slip of a 400 V, 3-phase cage induction motor is 3.5 per cent, and with locked rotor, full-load current is circulated when 90 V is applied between lines. Find the necessary tapping on an auto-transformer to limit the starting current to twice the full-load current of the motor. Determine also the starting torque in terms of the full-load torque. [0.67, $\tau_{st} = 0.311\ \tau fl$]

4.29 A 4-pole, 50 Hz, 3-phase induction motor has rotor resistance and standstill rotor reactance of 0.04 Ω and 0.16 Ω per phase respectively. Calculate the value of the external rotor resistance per phase to be inserted to obtain 70% of maximum torque at starting. [0.02533 Ω]

4.30 A 4-pole, 50 Hz, 3-phase induction motor has a rotor resistance and standstill rotor reactance of 0.025 Ω and 0.1 Ω per phase respectively. Calculate : (*a*) the speed at which maximum torque occurs ; (*b*) the value of external rotor resistance per phase to be inserted to obtain 80% of maximum torque at starting. [(*a*) 1125 r.p.m. ; (*b*) 0.025 Ω]

4.31 Show that in a 3-phase induction motor

$$\frac{\tau_{dm}}{\tau_d} = \frac{1}{2}\left[(s/s_M) + (s_M/s)\right]$$

where τ_{dm} = breakdown torque

s_M = slip for τ_{dm}

τ_d = torque at slip s

4.32 Starting from the first principles develop the equivalent circuit of a 3-phase induction motor. Draw and explain the phasor diagram.

4.33 Derive the relationship for torque developed by a 3-phase induction motor. Draw a typical torque-slip characteristic and deduce the condition for maximum torque.

4.34 Develop the equivalent circuit for a 3-phase induction motor and explain how the mechanical power developed is taken care in the equivalent circuit.

4.35 Show that in a 3-phase induction motor

$$\frac{\tau_{max}}{\tau_{fl}} = \frac{1}{2}\,\frac{\beta^2 + s_{fl}^2}{\beta s_{fl}} \quad \text{where } \beta = \frac{R_2}{X_2}$$

4.36 Sketch the torque-slip characteristic of a 3-phase induction motor indicating therein the starting torque, maximum torque and the operating region. How do starting and maximum torques vary with the rotor resistance ?

4.37 In a 3-phase induction motor show that $P_g : p_{rc} : P_{md} = 1 : s : (1-s)$ where the symbols have their usual meanings.

4.38 Explain the procedure of drawing the circle diagram of an induction motor. What information can be drawn from the circle diagram ?

4.39 Explain the procedure of no-load and blocked rotor tests on a 3-phase induction motor. How are the parameters of equivalent circuit determined from test results ?

4.40 A 3-phase induction motor has full-load output of 18.65 kW at 220 V, 720 r.p.m. The full-load power factor is 0.83 and efficiency is 85%. When running light, the motor takes 5 A at 0.2 power factor. Draw the circle diagram and use it to determine the maximum torque which the motor can exert (*a*) in Nm (*b*) in terms of full-load torque and (*c*) in terms of the starting torque.

[(*a*) 268.7 Nm (*b*) 1.08 (*c*) 7.2 approx.

4.41 Draw the circle diagram of a 10 hp, 200 V, 50 Hz, 3-phase slip ring induction motor with a star-connected stator and rotor, a winding ratio of unity, a stator resistance of 0.38 Ω / phase and a rotor resistance of 0.24 Ω / phase. The following are test readings :

No load	200 V	7.7 A	p.f. 0.195
Short Circuit	100 V	47.6 A	p.f. 0.454

Find

(*a*) the starting torque in synchronous watts,

(*b*) maximum torque in synchronous watts

(*c*) the maximum power factor

(*d*) the slip for maximum torque

(*e*) the maximum output.

[(*a*) 5,600 synchronous watts, (*b*) 12500 synchronous watts, (*c*) 0.879, (*d*) 0.195, (*e*) 10.4 kW]

4.42 A 20 h.p. (14.92 kW), 50 Hz, 440 V, 3-phase starting induction motor furnished the following test figures (line values) :

No-load :	440 V	10 A	p.f. 0.2
s.c. test :	200 V	50 A	p.f. 0.4

The ratio of stator to rotor copper losses on short circuit was unity. Draw the circle diagram and find from it (*a*) the full-load current and power factor (*b*) the maximum power developed (*c*) the starting torque.

[(*a*) 28.1 A at 0.844 *p.f.* (*b*) 27.75 kW (*c*) 11.6 synchronous kW per phase]

A 40 h.p. (29.84 kW), 440 V, 50 Hz, 3-phase induction motor gave the following test results :

No-load :	440 V	16 A	p.f. 0.15
s.c. test :	100 V	55 A	p.f. 0.225

Ratio of rotor to stator losses on short circuit is 0.9. Find the full-load current and power factor, the pull-out torque and the maximum output power developed.

[49 A at 0.88 *p.f.* ; 78.5 synchronous kW or 2.575 times full-load torque, 70.2 kW]

4.43 What is the purpose of using deep-bar cage rotors ? Explain the construction and working of a deep-bar cage motor.

4.44 Describe the construction of a double-cage induction motor. Explain its working.

4.45 Compare a single-cage motor with a double-cage induction motor of the same rating.

4.46 Draw the equivalent circuit of a double-cage induction motor sketch torque and current characteristics of a double-cage induction motor.

4.47 Derive the relationship between the torques developed by outer and inner cages of a double cage induction motor.

4.48 What are the effects of space harmonics on 3-phase induction motor performance ?

4.49 Explain the phenomenon of crawling in a 3-phase induction.

4.50 Explain the phenomenon of cogging in a 3-phase induction motor.

4.51 Distinguish between harmonic induction torque and harmonic synchronous torque developed in a 3-phase induction motor. What are their effects ?

4.52 Explain how improved starting performance of three-phase squirrel-cage motors may be obtained by means of a double-cage rotor winding. Sketch typical slots and a speed-torque characteristic, and compare the latter with the speed-torque characteristic of a normal squirrel-cage rotor.

4.53 The impedances at standstill of the inner and outer windings of a double-cage rotor are $(0.01 + j0.5)\ \Omega$ and $(0.05 + j0.1)$ respectively. Calculate the ratio of torques due to the two windings

(*i*) at starting, (*ii*) when running with a slip of 5%.

[1 : 100, 1 : 1.44]

4.54 A six-pole, 400 V double-cage induction motor has a delta-connected primary winding of impedance $(1 + j2)\ \Omega$ per phase. The corresponding referred impedances of the cages are $(2 + j1)\ \Omega$ and $(1 + j4)\ \Omega$ per phase. The full-load slip is 5%. Determine the ratio of starting torque to full-load torque for direct-on-line starting.

[1.36 : 1]

4.55 Describe, with sketches, the construction of a double-cage induction motor and point out its advantages compared with a single-cage motor.

If the outer cage has an equivalent impedance of $(0.5 + j0.5)\ \Omega$ and the inner cage an equivalent impedance of $(0.1 + j0.9)\ \Omega$, both at supply frequency, calculate the current in amperes and the torque in synchronous watts for the two cages at standstill and at 6% slip.

The effective standstill e.m.f of each cage is 100 V.

[238 A, 11.25 kW ; 63.6 A, 5.82 kW]

4.56 If the outer cage has an equivalent impedance of $(0.6 + j0.6)\ \Omega$ and the inner cage an equivalent impedance of $(0.1 + j0.8)\Omega$ both at supply frequency, calculate the current and torque in synchronous watts for the two cages at standstill and at 10% slip. The effective standstill e.m.f. of each cage is 200 V.

[453 A, 39.6 kW ; 185 A, 31.1 kW]

4.57 Discuss briefly the various methods of speed control of 3-phase induction motors.

4.58 Discuss the pole-changing methods of speed control of 3-phase induction motors.

4.59 Explain the pole amplitude modulation technique of speed control of 3-phase induction motors.

4.60 Explain the stator voltage control of 3-phase induction motors.

4.61 Explain the method of speed control of 3-phase induction motor by varying the supply frequency.

4.62 Explain the method of speed control of 3-phase induction motor by varying the rotor resistance.

4.63 What is meant by slip-energy recovery ? How this principle is used to control the speed of 3-phase induction motors ?

4.64 Explain the principle of operation of an induction generator. What are its limiations ?

4.65 Draw and explain the complete torque-speed characteristic of a three-phase induction machine for all ranges of speed.

4.66 Why an induction generator is not a self-excited generator ? How does an isolated induction generator work ?

4.67 Explain the voltage build-up of an isolated induction generator.

5

Three-Phase Synchronous Motors

5.1 INTRODUCTION

Like most rotating machines a synchronous machine can also operate as both a generator and a motor. A synchronous motor is a machine that converts ac electric power to mechanical power at a constant speed called synchronous speed. A synchronous motor is a **doubly-excited machine.** Its rotor poles are excited by direct current (dc) and its stator windings are connected to the ac supply. The air gap flux is, therefore, the resultant of the fluxes due to both rotor current and stator current. An important feature of a synchronous motor is that it can draw either lagging or leading reactive current from the ac supply system.

5.2 CONSTRUCTION

The construction of a 3-phase synchronous motor is essentially the same as that of a synchronous generator. The three-phase armature winding is on the stator

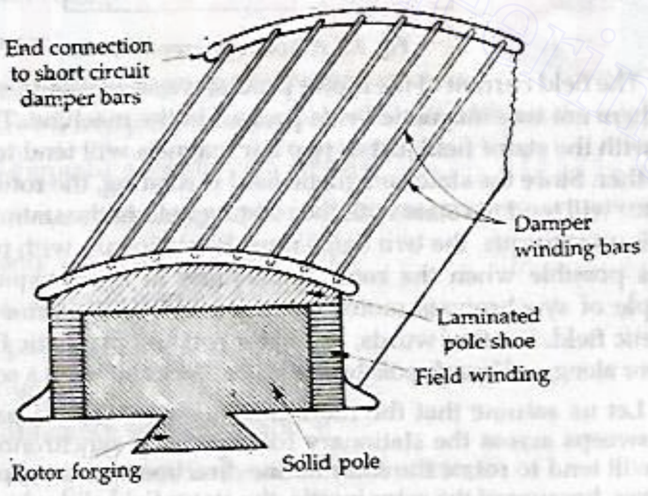


Fig. 5.1. *Pole of a synchronous motor showing damper windings.*

and is wound for the same number of poles as the rotor. The rotor of a synchronous motor can be of the salient-pole or cylindrical-pole type of construction. Generally it is of salient-pole type, except for exceedingly high speed machines. An additional set of windings, called the damper winding, is mounted on the rotor. This winding is placed in slots located in the pole faces and parallel to the shaft as shown in Fig. 5.1. The ends of the copper bars are short-circuited in the same manner as the cage rotor of an induction motor. Damper windings provide a means of starting the synchronous motor. They also serve to increase the stability of the motor during load transients.

A synchronous motor is a doubly excited machine, its armature winding is energized from an a.c. source and its field winding from a d.c. source.

5.3 PRINCIPLE OF OPERATION

Consider the 2-pole synchronous motor shown in Fig. 5.2. When a three phase a.c. voltage is applied to the stator winding, a rotating magnetic field is produced in the air gap. The stator field rotates at synchronous speed.

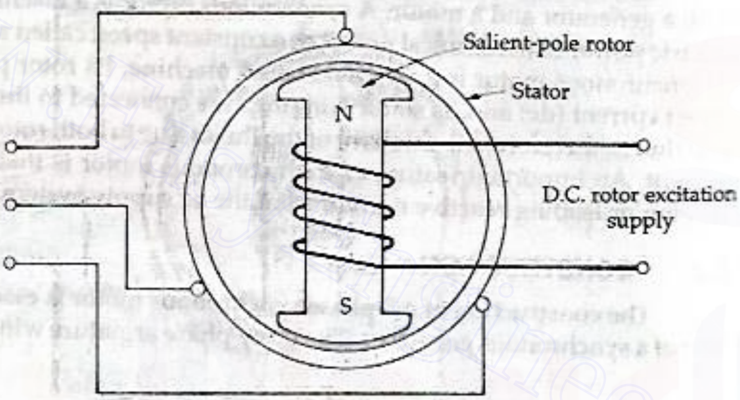


Fig. 5.2. *A 2-pole synchronous motor.*

The field current of the motor produces a steady-state magnetic field. Therefore, there are two magnetic fields present in the machine. The rotor will tend to align with the stator field just as two bar magnets will tend to align if placed near each other. Since the stator magnetic field is rotating, the rotor magnetic field and the rotor will tend to rotate with the rotating field of the stator. In order to develop a continuous torque, the two fields must be stationary with respect to each other. This is possible when the rotor also rotates at synchronous speed. The basic principle of synchronous motor operation is that the rotor "chases" the stator magnetic field. In other words, the stator rotating magnetic field tends to "drag" the rotor along, as if north pole on the stator "locks in" with a south pole of the rotor.

Let us assume that the rotor is stationary. When a pair of rotating stator poles sweeps across the stationary rotor poles at synchronous speed, the stator poles will tend to rotate the rotor in one direction and then in the other direction. However, because of the rotor inertia, the stator field slides by so fast that the rotor cannot follow it. Consequently, the rotor does not move and we say that the starting torque is zero. In other words, a synchronous motor is not self-starting.

Let us now assume that the rotor is also rotating at synchronous speed.

5.4 MAIN FEATURES OF SYNCHRONOUS MOTOR

Some *characteristic features* of synchronous motor are as follows :

1. It runs either at synchronous speed or not at all. That is, while running it maintains a constant speed. The speed is independent of load.
2. It is not inherently self-starting. It has to be run upto synchronous speed by some means before it can be synchronized to the supply.
3. It can be operated under wide range of power factors both lagging and leading.
4. It will stall if, while running, the counter torque is increased beyond the maximum torque that the machine can develop.

5.5 EQUIVALENT CIRCUIT AND PHASOR DIAGRAMS OF A CYLINDRICAL ROTOR SYNCHRONOUS MOTOR

A synchronous motor is the same in all respects as a synchronous generator except that the direction of power flow is reversed. The equivalent circuit of a 3-phase synchronous motor is shown in Fig. 5.3.

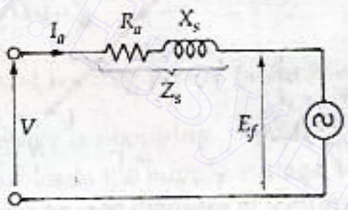


Fig. 5.3. *Equivalent circuit of a 3-phase synchronous motor.*

Let

E_f = excitation voltage

I_f = field current

V = terminal phase voltage applied to the armature

I_a = armature current per phase drawn by the motor from the supply

R_a = effective armature resistance per phase

X_s = synchronous reactance per phase of the motor stator armature winding

Z_s = impedance per phase of the armature

ϕ = phase angle between $\mathbf{V}$ and $\mathbf{I}_a$

$\cos\phi$ = power factor

δ = torque angle

= phase difference between $\mathbf{E}_f$ and $\mathbf{V}$

I_aR_a = voltage drop per phase in the armature resistance

I_aX_s = reactive voltage drop per phase due to armature reactance and armature reaction effects

$$\mathbf{Z}_s = R_a + jX_s \tag{5.5.1}$$

For a synchronous motor

$$\mathbf{V} = \mathbf{E}_f + \mathbf{I}_a \mathbf{Z}_s$$

$$\mathbf{V} = \mathbf{E}_f + \mathbf{I}_a (R_a + jX_s)$$

or $$\mathbf{E}_f = \mathbf{V} - \mathbf{I}_a R_a - j\,\mathbf{I}_a X_s$$

Phasor Diagrams

Phasor diagrams of a 3-phase cylindrical rotor synchronous motor op- ing at different power factors can be drawn with the help of Eq. (5.5.3).

(a) Phasor diagram at lagging power factor cos φ

Suppose that the synchronous motor is taking a lagging current fro supply. Equation (5.5.3a) is used to draw the phasor diagram. The supply v

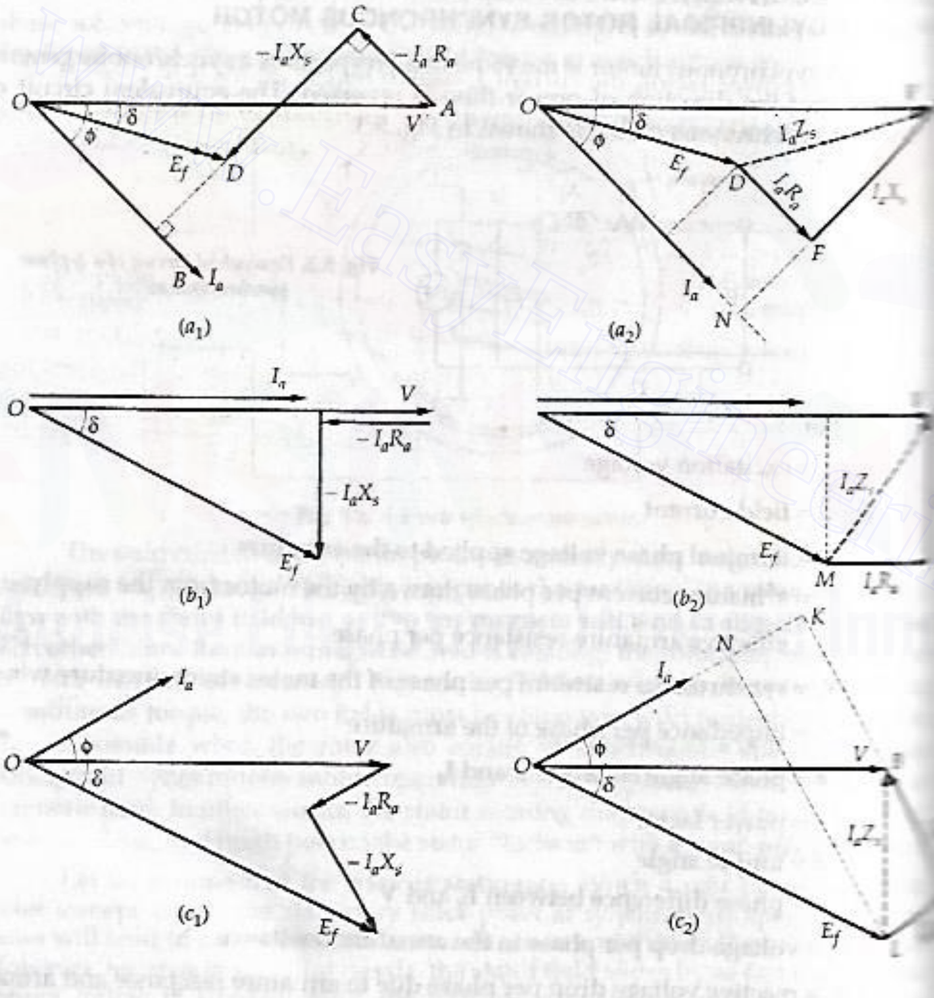


Fig. 5.4. *Phasor diagrams for a cylindrical rotor synchronous motor. (a_1), (a_2) Lagging power factor (b_1), (b_2) Unity power factor (c_1), (c_2) Leading power factor*

V is taken as reference phasor along OA such that $OA = \mathbf{V}$. For lagging power factor $\cos\phi$, the direction of armature current $\mathbf{I}_a$ lags behind **V** by an angle ϕ along OB, where $OB = I_a$. The voltage drop per phase in the armature resistance is $\mathbf{I}_a R_a$. The phasor $(-\mathbf{I}_a R_a)$ is represented by AC. It is in a direction opposite to that of $\mathbf{I}_a$. The voltage drop per phase in the synchronous reactance is $\mathbf{I}_a X_s$. The phasor $(-j\mathbf{I}_a X_s)$ is represented by CD. It is in a direction perpendicular to the phasor $(-\mathbf{I}_a R_a)$. The phasor $\mathbf{E}_f$ is equal to the phasor sum of **V**, $(-\mathbf{I}_a R_a)$ and $(-j\mathbf{I}_a X_s)$. It is represented by OD. The angle δ between **V** and $\mathbf{E}_f$ is the power angle (or torque angle). It plays an important role in the power transfer and in the stability of the synchronous motor operation. Fig. 5.4(a_1), shows the phasor diagram of the synchronous motor operating at a lagging power factor $\cos\phi$. The phasor diagram at lagging power factor $\cos\phi$ can also be drawn as shown in Fig. 5.4(a_2).

(b) Phasor diagram at unity power factor

At unity power factor, the current $\mathbf{I}_a$ drawn by the motor is in phase with supply voltage **V**. The procedure for drawing the phasor diagram at unity power factor is the same as that for lagging power factor. It is shown in Fig. 5.4(b_1) and alternatively in Fig. 5.4(b_2).

(c) Phasor diagram at leading power factor cos φ

When the motor is operating at leading power factor $\cos\phi$, the current $\mathbf{I}_a$ drawn by the motor leads the supply voltage **V** by the phase angle ϕ. The procedure for drawing the phasor diagram at leading power factor $\cos\phi$ is the same as that given for lagging power factor. Fig. 5.4(c_1) shows the phasor diagram of a synchronous motor at leading power factor $\cos\phi$. This diagram can also be drawn as shown in Fig. 5.4(c_2).

Calculation of $\mathbf{E}_f$

The excitation voltage $\mathbf{E}_f$ can be found for different power factors either by using complex algebra or phasor diagrams.

Determination of $\mathbf{E}_f$ by using complex algebra

Let **V** be taken as reference phasor.

$$\therefore \qquad \mathbf{V} = V\angle 0^\circ = V + j\,0$$

For lagging power factor $\cos\phi$

$$\mathbf{I}_a = I_a \angle -\phi = I_a\cos\phi - j\,I_a\sin\phi$$

For unity power factor

$$\mathbf{I}_a = I_a \angle 0^\circ = I_a + j0$$

For leading power factor

$$\mathbf{I}_a = I_a \angle +\phi = I_a\cos\phi + j\,I_a\sin\phi$$

The synchronous impedance is given by

$$\mathbf{Z} = R_a + jX_s$$

The excitation voltage is given by

$$\mathbf{E}_f = \mathbf{V} - \mathbf{I}_a \mathbf{Z}_s$$

For lagging power factor cos φ

$$E_f \underline{/\delta} = V \underline{/0^\circ} - (I_a \underline{/-\phi})\,(R_a + j\,X_s)$$

$$= V + j0 - (I_a \cos\phi - j\,I_a \sin\phi)\,(R_a + j\,X_s)$$

$$= (V - I_a R_a \cos\phi - I_a X_s \sin\phi) - j\,(I_a X_s \cos\phi - I_a R_a \sin\phi)$$

$$E_f = \sqrt{(V - I_a R_a \cos\phi - I_a X_s \sin\phi)^2 + (I_a X_s \cos\phi - I_a R_a \sin\phi)^2} \qquad (5.5.4)$$

$$\delta = \tan^{-1} \frac{I_a R_a \sin\phi - I_a X_s \cos\phi}{V - I_a R_a \cos\phi - I_a X_s \sin\phi} \qquad (5.5.5)$$

Similarly, *for leading power factor cos φ*

$$E_f = \sqrt{(V - I_a R_a \cos\phi + I_a X_s \sin\phi)^2 + (I_a X_s \cos\phi + I_a R_a \sin\phi)^2} \qquad (5.5.6)$$

and

$$\delta = -\tan^{-1}\left(\frac{I_a X_s \cos\phi + I_a R_a \sin\phi}{V - I_a R_a \cos\phi + I_a X_s \sin\phi}\right) \qquad (5.5.7)$$

For unity power factor (cos φ = 1)

$$E_f = \sqrt{(V - I_a R_a)^2 + (I_a X_s)^2} \qquad (5.5.8)$$

$$\delta = -\tan^{-1}\left(\frac{I_a X_s}{V - I_a R_a}\right) \qquad (5.5.9)$$

Determination of E_f by phasor diagram

For lagging power factor we use Fig. 5.4(a_2).

From triangle *ODM*

$$OD^2 = OM^2 + MD^2 = OM^2 + NF^2$$

$$= (ON - MN)^2 + (NA - FA)^2$$

$$E_f^2 = (V \cos\phi - I_a R_a)^2 + (V \sin\phi - I_a X_s)^2 \qquad (5.5.10)$$

For unity power factor Fig. 5.4(b_2)

$$OM^2 = OL^2 + LM^2$$

$$E_f^2 = (V - I_a R_a)^2 + (I_a X_s)^2 \qquad (5.5.11)$$

For leading power factor Fig. 5.4(c_2)

$$OL^2 = ON^2 + NL^2 = (OK - NK)^2 + (KB + BM)^2$$

$$E_f^2 = (V \cos\phi - I_a R_a)^2 + (V \sin\phi + I_a X_s)^2 \qquad (5.5.12)$$

5.6 DIFFERENT TORQUES IN A SYNCHRONOUS MOTOR

The following torques are considered in the selection of a synchronous motor for a particular application :

1. Locked-rotor torque
2. Running torque
3. Pull-in torque
4. Pull-out torque

Locked Rotor Torque

It is the minimum torque at any angular rotor position that is developed with the rotor locked (that is, stationary) and rated voltage at rated frequency is applied to the terminals. This torque is provided by the stator windings.

Running Torque

It is the torque developed by the motor under running conditions. It is determined by the power rating and speed of the driven machine.

Pull-in torque

A synchronous motor is started as induction motor till it runs 2 to 5 per cent below the synchronous speed. The d.c. excitation is then applied and the rotor pulls into step with the synchronously rotating stator field. The pull-in torque is the maximum constant torque at rated voltage and frequency under which a motor will pull a connected load into synchronism when the d.c. motor excitation is applied.

Pull-out torque

It is the maximum value of torque which a synchronous motor can develop at rated voltage and frequency without losing synchronism.

5.7 POWER FLOW EQUATIONS FOR A SYNCHRONOUS MOTOR

Figure 5.3 shows the circuit model of a cylindrical rotor synchronous motor. The phasor diagram at lagging power factor is shown in Fig. 5.4a_2.

Here $\mathbf{E}_f$ lags $\mathbf{V}$ by angle δ so that

$$\mathbf{V} = V\angle 0^\circ, \qquad \mathbf{E}_f = E_f\angle -\delta$$

By KVL in Fig. 5.3

$$\mathbf{V} = \mathbf{E}_f + \mathbf{Z}_s\,\mathbf{I}_a \tag{5.7.1}$$

$$\mathbf{I}_a = \frac{\mathbf{V} - \mathbf{E}_f}{\mathbf{Z}_s} \tag{5.7.2}$$

$$= \frac{V\angle 0^\circ}{Z_s\angle\theta_z} - \frac{E_f\angle -\delta}{Z_s\angle\theta_z}$$

$$= \frac{V}{Z_s} \angle -\theta_z - \frac{E_f}{Z_s} \angle -(\delta + \theta_z)$$

$$\mathbf{I}_a^* = \frac{V}{Z_s} \angle \theta_z - \frac{E_f}{Z_s} \angle \delta + \theta_z \qquad (5.7.3)$$

Complex power input to motor per phase (S_{im})

$$\mathbf{S}_{im} = P_{im} + j\,Q_{im} = \mathbf{V}\mathbf{I}_a^* \qquad (5.7.4)$$

$$= \frac{V^2}{Z_s} \angle \theta_z - \frac{V E_f}{Z_s} \angle \delta + \theta_z$$

$$\therefore \quad P_{im} + j\,Q_{im} = \left(\frac{V^2}{Z_s} \cos \theta_z + j \frac{V^2}{Z_s} \sin \theta_z \right)$$

$$- \left[\frac{V E_f}{Z_s} \cos (\delta + \theta_z) + j \frac{V E_f}{Z_s} \sin (\delta + \theta_z) \right] \qquad (5.7.5)$$

Real input power per phase to the motor (P_{im})

Equating real parts of Eq. (5.7.5), we get P_{im}.

$$P_{im} = \frac{V^2}{Z_s} \cos \theta_z - \frac{V E_f}{Z_s} \cos (\delta + \theta_z)$$

or
$$P_{im} = \frac{V^2}{Z_s^2} R_a - \frac{V E_f}{Z_s} \cos (\delta + \theta_z) \qquad (5.7.6)$$

But $\theta_z = 90° - \alpha_z$

$$\cos (\delta + \theta_z) = \cos (90° + \overline{\delta - \alpha_z}) = -\sin (\delta - \alpha_z)$$

$$\therefore \quad P_{im} = \frac{V^2}{Z_s^2} R_a + \frac{V E_f}{Z_s} \sin (\delta - \alpha_z) \qquad (5.7.7)$$

Reactive input power per phase to the motor (Q_{im})

Equating imaginary parts of Eq. (5.7.5), we get Q_{im}.

$$Q_{im} = \frac{V^2}{Z_s} \sin \theta_z - \frac{V E_f}{Z_s} \sin (\delta + \theta_z)$$

or
$$Q_{im} = \frac{V^2}{Z_s^2} X_s - \frac{V E_f}{Z_s} \sin (\delta + \theta_z) \qquad (5.7.8)$$

But $\theta_z = 90° - \alpha_z$, $\sin (\delta + \theta_z) = \sin (90 + \overline{\delta - \alpha_z}) = \cos (\delta - \alpha_z)$

$$\therefore \quad Q_{im} = \frac{V^2}{Z_s^2} X_s - \frac{V E_f}{Z_s} \cos (\delta - \alpha_z) \qquad (5.7.9)$$

Complex power output per phase of the motor (S_{om})

$$\mathbf{S}_{om} = P_{om} + j\,Q_{om} = \mathbf{E}_f \mathbf{I}_a^* \qquad (5.7.10)$$

$$= E_f \angle -\delta \left(\frac{V}{Z_s} \angle \theta_z - \frac{E_f}{Z_s} \angle \delta + \theta_z \right)$$

$$= \frac{V E_f}{Z_s} \angle \theta_z - \delta - \frac{E_f^2}{Z_s} \angle \theta_z$$

$$\therefore \quad P_{om} + j\, Q_{om} = \frac{V E_f}{Z_s} \cos(\theta_z - \delta) + j \frac{V E_f}{Z_s} \sin(\theta_z - \delta)$$

$$- \left(\frac{E_f^2}{Z_s} \cos \theta_z + j \frac{E_f^2}{Z_s} \sin \theta_z \right) \qquad (5.7.11)$$

$\frac{V E_f}{Z_s} \cos \theta_z$

Real power output per phase of the motor (P_{om})

Equating real parts of Eq. (5.7.11), we get P_{om}.

$$P_{om} = \frac{V E_f}{Z_s} \cos(\theta_z - \delta) - \frac{E_f^2}{Z_s} \cos \theta_z$$

or

$$P_{om} = \frac{V E_f}{Z_s} \cos(\theta_z - \delta) - \frac{E_f^2}{Z_s^2} R_a \qquad (5.7.12)$$

But $\quad \theta_z = 90° - \alpha_z$

$$\cos(\theta_z - \delta) = \cos(90° - \overline{\delta + \alpha_z}) = \sin(\delta + \alpha_z)$$

$$\therefore \quad P_{om} = \frac{V E_f}{Z_s} \sin(\delta + \alpha_z) - \frac{E_f^2}{Z_s^2} R_a \qquad (5.7.13)$$

Reactive power output per phase of the motor (Q_{om})

Equating imaginary parts of Eq. (5.7.11), we get Q_{om}.

$$Q_{om} = \frac{V E_f}{Z_s} \sin(\theta_z - \delta) - \frac{E_f^2}{Z_s} \sin \theta_z$$

or

$$Q_{om} = \frac{V E_f}{Z_s} \sin(\theta_z - \delta) - \frac{E_f^2}{Z_s^2} X_s \qquad (5.7.14)$$

But $\quad \theta_z = 90° - \alpha_z$

$$\sin(\theta_z - \delta) = \sin(90° - \overline{\delta + \alpha_z}) = \cos(\delta + \alpha_z)$$

$$\therefore \quad Q_{om} = \frac{V E_f}{Z_s} \cos(\delta + \alpha_z) - \frac{E_f^2}{Z_s^2} X_s \qquad (5.7.15)$$

For a synchronous motor, power at the shaft

$$= P_{om} - \text{rotational losses}$$

P_{om} **is mechanical power developed or gross power developed**

Rotation losses include friction, windage and core losses.

Maximum power output of the motor

For maximum power output of the motor

$$\frac{d P_{om}}{d \delta} = 0 \quad \text{and} \quad \frac{d^2 P_{om}}{d \delta^2} < 0$$

Differenciating Eq. (5.7.13) w.r.t. δ and equating it to zero

$$\frac{d}{d\delta}\left[\frac{V E_f}{Z_s} \sin(\delta + \alpha_z) - \frac{E_f^2}{Z_s^2} R_a\right] = 0$$

$$\frac{V E_f}{Z_s} \cos(\delta + \alpha_z) = 0$$

$$\therefore \quad \delta + \alpha_z = 90°$$

$$\delta = 90 - \alpha_z = \theta_z$$

The maximum power output of the motor is given by

$$P_{om\,(\max)} = \frac{V E_f}{Z_s} - \frac{E_f^2}{Z_s^2} R_a$$

This occurs at $\delta = \theta_z$ which defines the limit of steady-state stability.

P_{om} (max) is also called the **maximum power developed.**

5.8 PHASOR DIAGRAMS OF A SALIENT-POLE SYNCHRONOUS MOTOR

The voltage equation for a salient-pole synchronous motor is

$$\mathbf{V} = \mathbf{E}_f + R_a \mathbf{I}_a + j \mathbf{I}_d X_d + j \mathbf{I}_q X_q$$

where the symbols have their usual meanings.

(a) Lagging power factor cos ϕ

The phasor diagram at lagging pf cos ϕ is shown in Fig. 5.5.

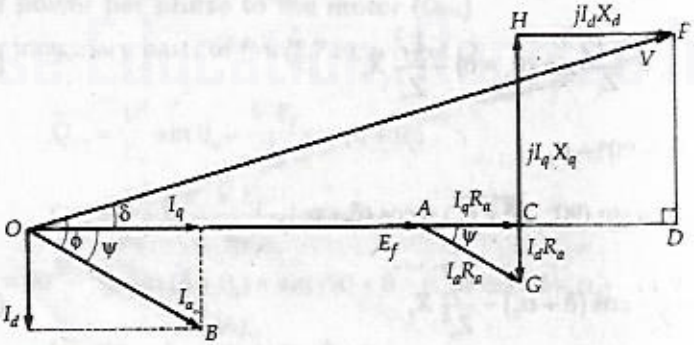


Fig. 5.5. Phasor diagram of a salient-pole synchronous motor at lagging pf cos ϕ

Here $OA = E_f$, $AG = I_a R_a$, $GH = I_q X_q$, $HF = I_d X_d$, $OF = V$

$$OD = OA + AC + CD$$

$$V \cos\delta = E_f + I_q R_a + I_d X_d$$

$$GH = GC + CH$$

$$I_q X_q = I_d R_a + V \sin \delta \tag{5.8.2}$$

$$\psi = \phi - \delta$$

$$I_d = I_a \sin \psi = I_a \sin (\phi - \delta) \tag{5.8.3}$$

$$I_q = I_a \cos \psi = I_a \cos (\phi - \delta) \tag{5.8.4}$$

Combination of Eqs. (5.8.2), (5.8.3) and (5.8.4) gives

$$I_a X_q \cos (\phi - \delta) = I_a R_a \sin (\phi - \delta) + V \sin \delta$$

$$I_a X_q (\cos \phi \cos \delta + \sin \phi \sin \delta)$$
$$= I_a R_a (\sin \phi \cos \delta - \cos \phi \sin \delta) + V \sin \delta$$

$$(V - I_a R_a \cos \phi - I_a X_q \sin \phi) \sin \delta$$
$$= (I_a X_q \cos \phi - I_a R_a \sin \phi) \cos \delta$$

$$\therefore \qquad \tan \delta = \frac{I_a X_q \cos \phi - I_a R_a \sin \phi}{V - I_a X_q \sin \phi - I_a R_a \cos \phi} \tag{5.8.5}$$

(c) Leading power factor cos ϕ

The phasor diagram at leading pf cos ϕ is shown in Fig. 5.6.

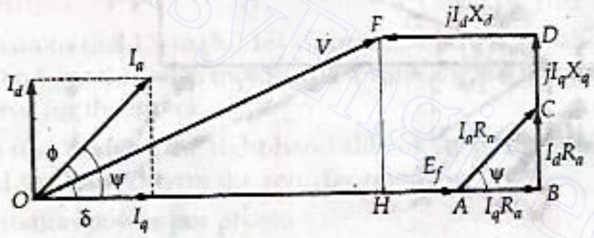


Fig. 5.6. *Phasor diagram of a salient-pole synchronous motor at leading pf cos ϕ.*

In Fig. 5.6,

$$OA = E_f,\ \ AC = I_a R_a,\ \ CD = I_q X_q,\ \ DF = I_d X_d$$

$$OF = V, AB = AC \cos \psi = I_a R_a \cos \psi = I_q R_a$$

$$BC = AC \sin \psi = I_a R_a \sin \psi = I_d R_a$$

Also, $\quad OA = OH + HA = OH + HB - AB = OH + FD = AB$

$$E_f = V \cos \delta + I_d X_d - I_q R_a \tag{5.8.6}$$

$$HF = BD = BC + CD$$

$$V \sin \delta = I_d R_a + I_q X_q \tag{5.8.7}$$

$$\psi = \phi + \delta \tag{5.8.8}$$

$$I_d = I_a \sin \psi = I_a \sin (\phi + \delta) \tag{5.8.9}$$

$$I_q = I_a \cos \psi = I_a \cos (\phi + \delta) \tag{5.8.10}$$

Substituting the values of I_d and I_q from Eqs. (5.8.9) and (5.8.10) in Eq. (5.8.7), we get

$$V \sin \delta = I_a R_a \sin (\phi + \delta) + I_a X_q \cos (\phi + \delta)$$

$$= I_a R_a (\sin \phi \cos \delta + \cos \phi \sin \delta) + I_a X_q (\cos \phi \cos \delta - \sin \phi \sin \delta)$$

Collecting the terms containing sin δ and cos δ, we get

$$(V + I_a X_q \sin \phi - I_a R_a \cos \phi) \sin \delta = (I_a X_q \cos \phi + I_a R_a \sin \phi) \cos \delta$$

$$\therefore \quad \tan \delta = \frac{I_a X_q \cos \phi + I_a R_a \sin \phi}{V + I_a X_q \sin \phi - I_a R_a \cos \phi} \tag{5.8.11}$$

(c) Unity power factor

$$\cos \phi = 1, \quad \phi = 0°, \quad \sin \phi = 0$$

$$\therefore \quad \tan \delta = \frac{I_a X_a}{V - I_a R_a} \tag{5.8.12}$$

The phasor diagrams of a salient-pole synchronous motor neglecting the armature resistance R_a are shown in Fig. 5.7.

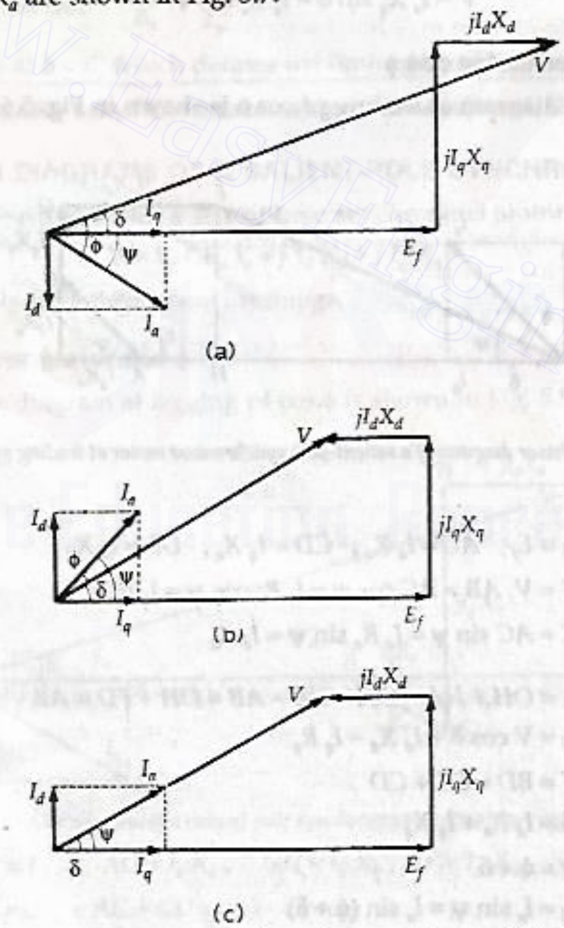


Fig. 5.7. *Phasor diagram of a salient-pole synchronous motor, neglecting R_a (a) Lagging pf cos φ (b) Leading pf cos φ (c) Unity pf*

THREE-PHASE SYNCHRON

Power Develop

The expressions f generator derived in cha

Real power per ph

$P_{1\phi} = \frac{V E_f}{X_d}$ s

Total real power f

$P_{3\phi} = 3 P_{1\phi}$

The reactive powe

$Q_{1\phi} = \frac{V E_f}{X_d}$ c

Total reactive pov

$Q_{3\phi} = 3 Q_{\phi}$ =

Equations (5.8.13) generator and synchron and negative for the mo

The first term on power and the second t

Excitation power

Reluctance powe

EFFECT OF V

The effect of fiel explained with the help R_a is neglected and sync constants.

The power per p

5.8.1 Power Developed By a Salient-pole Synchronous Motor

The expressions for the power developed by a salient-pole synchronous generator derived in chapter 3 also apply to a salient-pole synchronous motor.

Real power per phase in watts is

$$P_{1\phi} = \frac{V E_f}{X_d} \sin \delta + \frac{V^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right) \sin 2\delta \tag{5.8.13}$$

Total real power for three phases in watts is

$$P_{3\phi} = 3 P_{1\phi} = \frac{3V E_f}{X_d} \sin \delta + \frac{3V^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right) \sin 2\delta \tag{5.8.14}$$

The reactive power phase in vars is

$$Q_{1\phi} = \frac{V E_f}{X_d} \cos \delta - \frac{V^2}{2 X_d X_d}\left[(X_d + X_q) - (X_d - X_q) \cos 2\delta \right] \tag{5.8.15}$$

Total reactive power for three phases in vars is

$$Q_{3\phi} = 3 Q_{\phi} = \frac{3V E_f}{X_d} \cos \delta - \frac{3 V^2}{2 X_d X_q}\left[(X_d + X_q) - (X_d - X_q) \cos 2\delta \right] \tag{5.8.16}$$

Equations (5.8.13) to (5.8.16) are applicable to both salient-pole synchronous generator and synchronous motor. The torque angle δ is positive for the generator and negative for the motor.

The first term on the right-hand side of Eq. (5.18.13) is called the **excitation power** and the second term the **reluctance power**.

Excitation power per phase

$$= \frac{V E_f}{X_d} \sin \delta \tag{5.8.17}$$

Reluctance power per phase

$$= \frac{V^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right) \sin 2\delta \tag{5.8.18}$$

5.9 EFFECT OF VARYING FIELD CURRENT

The effect of field current I_f on synchronous motor power factor can be explained with the help of its phasor diagram. For simplicity, armature resistance R_a is neglected and synchronous reactance X_s and terminal voltage V are assumed constants.

The power per phase is given by

$$P = \frac{E_f V}{X_s} \sin \delta = V I_a \cos \phi \tag{5.9.1}$$

Since V and X_s are constants, therefore, for constant power output $E_f \sin \delta$ and $I_a \cos \phi$ should remain constant. That is,

$$E_f \sin \delta = \text{constant} \quad (5.9.2)$$

$$I_a \cos \phi = \text{constant} \quad (5.9.3)$$

Also, $$\mathbf{E}_f + j\mathbf{I}_a X_s = \mathbf{V} \quad (5.9.4)$$

When the field current increases, the magnitude of $\mathbf{E}_f$ increases, but the component of $\mathbf{E}_f$ normal to $\mathbf{V}$, that is, $E_f \sin \delta$ must remain constant. From Fig. 5.8 it is seen that, as $\mathbf{E}_f$ varies, $\mathbf{I}_a X_s$ and therefore the armature current $\mathbf{I}_a$ also varies subject to the condition that $I_a \cos \phi$ remains constant.

Equations (5.9.1) and (5.9.2) allow us to draw power loci for the phasors $\mathbf{E}_f$ and $\mathbf{I}_a$ on the phasor diagram in Fig. 5.8. When I_f is varied slowly enough to avoid hunting, $\mathbf{E}_f$ varies in magnitude and the tip of $\mathbf{E}_f$ phasor moves along the constant power locus CD so that $E_f \sin \delta$ remains constant.

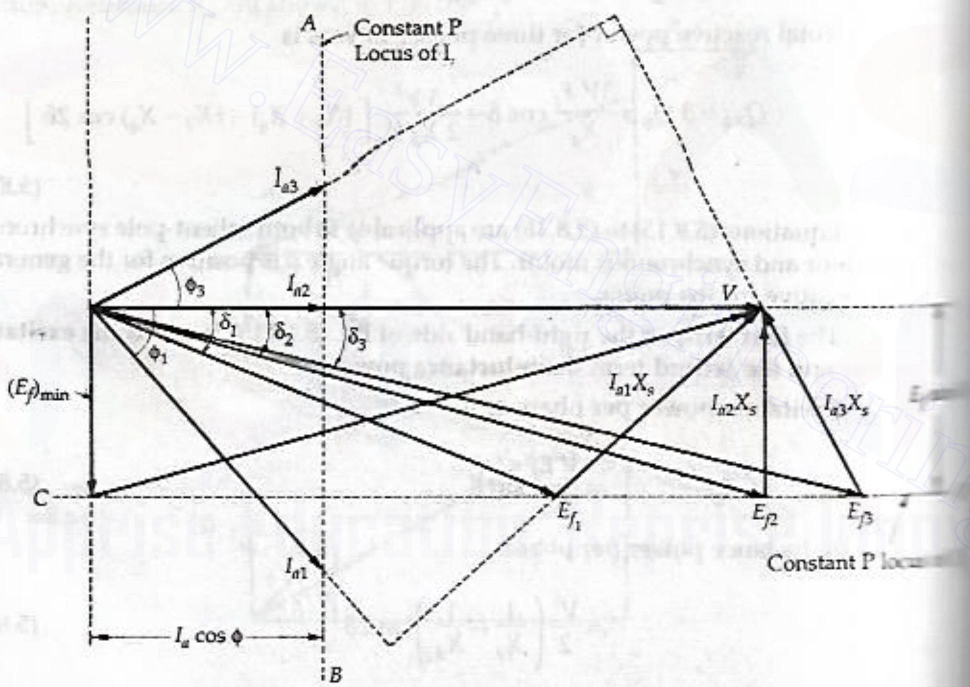


Fig. 5.8. *Effect of field current on power factor and line current.*

Since $$\mathbf{I}_a (jX_s) = \mathbf{V} - \mathbf{E}_f$$

and $$\mathbf{I}_a = \frac{\mathbf{V} - \mathbf{E}_f}{jX_s}$$

Equation (5.9.6) shows that with the restricted variation of $\mathbf{E}_f$ (that is, $E_f \sin \delta = a$ constant), the armature current $\mathbf{I}_a$ also varies with the constraint that $I_a \cos \phi$ remains constant. The tip of the $\mathbf{I}_a$ phasor moves along the constant power locus AB so that $I_a \cos\phi$ remains constant. Also, the phasor $\mathbf{I}_a$ must always remain perpendicular to $-jI_aX_s$ drop as the tip of the current phasor $\mathbf{I}_a$ moves along its locus. These three constraints

$$\left.\begin{array}{r} E_f \sin\delta = \text{a constant} \\ I_a \cos\phi = \text{a constant} \\ \text{and } I_a \text{ is perpendicular to } -jI_aX_s \end{array}\right\}$$

enable us to draw the phasor diagram of a synchronous motor for varying field currents. Fig. 5.8 shows the phasor diagram for lagging *pf*, unity *pf* and leading *pf*.

When excitation voltage is $\mathbf{E}_{f_1}$, the motor is underexcited and the armature current $\mathbf{I}_{a_1}$ lags behind V by *pf* angle ϕ_1 so that

$$\mathbf{E}_{f_1} + j\mathbf{I}_{a_1}X_s = \mathbf{V}$$

When the excitation voltage is increased to $\mathbf{E}_{f_2}$ by increasing the field current, the torque angle decreases from δ_1 to δ_2 so that $E_{f_1} \sin\delta_1 = E_{f_2} \sin\delta_2$.

Since $\mathbf{E}_f + j\mathbf{I}_aX_s = \mathbf{V}$ is to be satisfied, therefore

$$\mathbf{E}_{f_2} + j\mathbf{I}_{a_2}X_s = \mathbf{V}$$

and the armature current changes to $\mathbf{I}_{a_2}$. Since in Fig. 5.8, $\mathbf{I}_{a_2}$ is in phase with $\mathbf{V}$, the motor operates at unity power factor.

Suppose that the excitation voltage is now increased to $\mathbf{E}_{f_3}$. The torque angle decreases from δ_2 to δ_3 so that $E_{f_3} \sin\delta_3 = E_{f_2} \sin\delta_2 = E_{f_1} \sin\delta_1$.

In order to satisfy the voltage relation $\mathbf{E}_f + j\mathbf{I}_aX_s = \mathbf{V}$ again, the armature current $\mathbf{I}_{a_3}$ leads the voltage $\mathbf{V}$ and the motor operates at a leading power factor as shown in Fig. 5.8.

It is to be noted that the active components of armature currents are equal, that is,

$$I_{a_1} \cos\phi_1 = I_{a_2} \cos\phi_2 = I_{a_3} \cos\phi_3$$

It is seen from Fig. 5.8 that as the value of E_f increases, the magnitude of the armature current first decreases and then increases again. The armature current is minimum at unity *pf* and more at leading or lagging power factors.

When E_f is small the armature current is lagging and the motor is an inductive load. It acts like an inductor-resistor combination, consuming reactive power Q. As the field current is increased, the armature current $\mathbf{I}_a$ becomes in phase with the terminal voltage $\mathbf{V}$ and the motor becomes a purely resistor load. As the field current is increased further, the armature current $\mathbf{I}_a$ becomes leading and the motor becomes a capacitive load. It acts like a capacitor-resistor combination, *consuming* negative reactive power $-Q$, or, alternatively, *supplying* reactive power $+Q$ to the system. Therefore, by controlling the field current of a synchronous motor, the reactive power supplied to or consumed from the power system can be controlled. When $E_f \cos \delta < V$, the synchronous motor has a lagging *consumes Q*. Since the field current is small in this case, the motor is *underexcited*.

If $E_f \cos \delta > V$, the synchronous motor has a leading current and to the system. Since the field current is large in this case, the motor is *overexcited*. The motor diagrams of the underexcited and overexcited shown in Fig. 5.9.

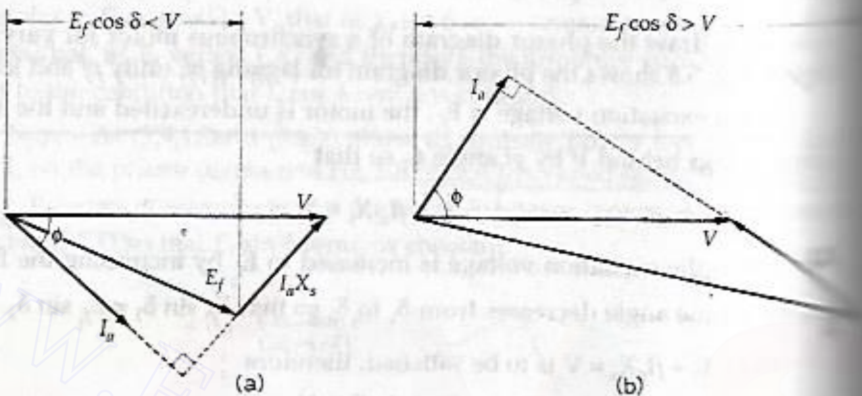


Fig. 5.9. *(a) Phasor diagram of an underexcited motor. (b) Phasor diagram of overe*

When $E_f \cos \delta = V$, the motor is said to be *normally* excited. Here is, the motor is neither delivering nor absorbing reactive power.

The excitation corresponding to unity power factor is known excitation. Here $\cos \phi = 1$ and $\phi = 90°$.

5.10 EFFECT OF LOAD CHANGES ON A SYNCHRONOUS MOT

A synchronous motor runs at absolutely constant synchronous gardless of the load. Let us examine the effect of load change on the

Consider a synchronous motor operating initially with a lead factor. The phasor diagram for leading power factor is shown in Fig. 5.

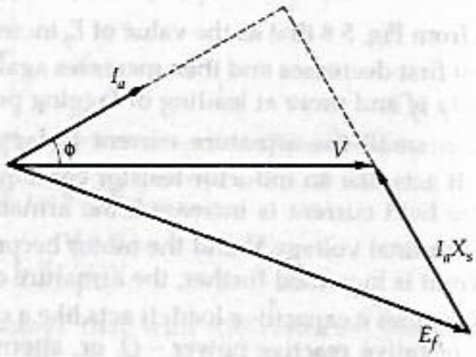


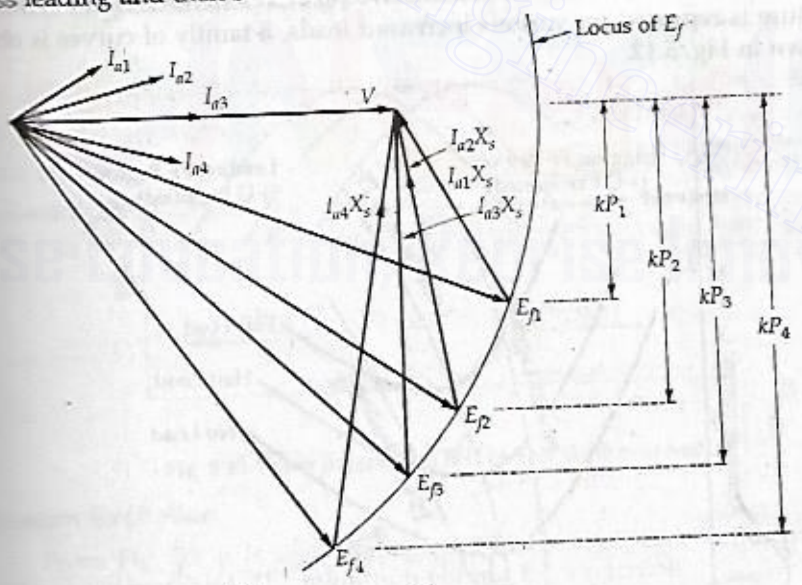
Fig. 5.10. *Phasor diagram for leading power factor.*

that the load on the shaft is increased. The rotor slows down momentarily since it takes some time for the motor to take increased power from the line. In other words, although still rotating at synchronous speed, the rotor slips back in space as a result of increased loading. In this process the torque angle δ becomes larger and therefore the induced torque $\left(\tau_{ind} = \dfrac{V\,E_f \sin\delta}{\omega\,X_s}\right)$ increases. The increased torque increases the rotor speed and the motor again picks up the synchronous speed but with a larger torque angle δ. Since the excitation voltage E_f is proportional to $\Phi\omega$, it only depends upon the field current and the speed of the motor. Since the motor is moving with a constant synchronous speed, and since the field circuit is also untouched, the field current remains constant. Therefore the magnitude of excitation voltage $|\mathbf{E}_f|$ remains constant with the change in load on the shaft.

We have, $$P = \frac{V\,E_f \sin\delta}{X_s} = VI_a\cos\phi,$$

$$\therefore \quad E_f \sin\delta = \frac{X_s}{V}P = KP \quad \text{where} \quad K = \frac{X_s}{V} = \text{a constant.}$$

These relations show that the increase in P increases $E_f \sin\delta$ and $I_a \cos\phi$. The locus of $\mathbf{E}_f$ is shown in Fig. 5.11. It is seen from Fig. 5.11 that with the increase of the load, the quantity $j\mathbf{I}_a X_s$ goes on increasing so that the relation $\mathbf{V} = \mathbf{E}_f + j\mathbf{I}_a X_s$ is satisfied and therefore the armature current $\mathbf{I}_a$ also increases. It is also seen from Fig. 5.11 that the power factor angle ϕ also changes. It becomes less and less leading and then becomes more and more lagging.



$$P = \frac{VE_f}{X_s}\sin\delta,\; E_f \sin\delta = \frac{X_s}{V}P = KP$$

Fig. 5.11. *Effect of increase in load on the operation of a synchronous motor.*

Thus, when the load on a synchronous motor is increased,

(*i*) the motor continues to run at synchronous speed.

(*ii*) the torque angle δ increases.

(*iii*) the excitation voltage E_f remains constant.

(*iv*) the armature current $\mathbf{I}_a$ drawn from the supply increases.

(*v*) the phase angle ϕ increases in the lagging direction.

There is a limit to the mechanical load that can be applied to a synch... motor. As the load is increased, the torque angle δ also increases till a st... reached when the rotor is pulled out of synchronism and the motor is st... The maximum value of torque which a synchronous motor can develop at ra... voltage and frequency without losing synchronism is called the **pull-out tor...** Its value varies from 1.5 to 3.5 times the full-load torque.

5.11 SYNCHRONOUS MOTOR V CURVES

We have seen that the power factor of a synchronous motor can be con... trolled by variation of field current I_f. It has also been observed that the arma... current I_a changes with the change in field current I_f. Let us assume that the m... is operating at no load. If the field current is increased from this small value, the armature current I_a decreases until the armature current becomes minimum. At this minimum armature current the motor is operating at unity power factor. Up to this point the motor was operating at a lagging power factor. If the field current is increased further, the armature current increases again and the motor starts to operate at a leading power factor. If a graph is plotted between armature current I_a and field current I_f at no load the lowest curve in Fig. 5.12 is obtained. If the procedure is repeated for various increased loads, a family of curves is obtained as shown in Fig. 5.12.

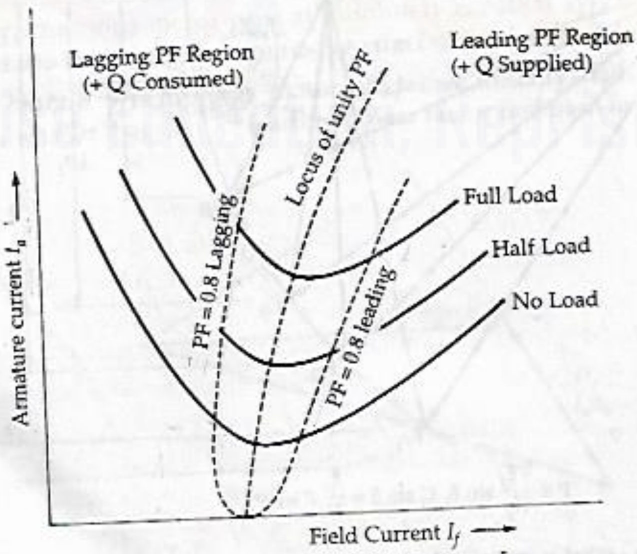


Fig. 5.12. *V-curves of a synchronous motor.*

Since the shape of these curves resembles the letter "V", these curves are commonly known as *V curves* of a synchronous motor. Thus, *V* curves are plots of stator current versus field current for different constant loads. The point at which unity power factor occurs is at the point where armature current is minimum. The curve connecting the lowest points of all *V* curves for various power levels is called the unity power factor *compounding curve*. Similarly, compounding curves for 0.8 power factor lag and 0.8 power factor lead are shown by dotted curves in Fig. 5.12. The compounding curves for other power factors can be drawn. Thus, the loci of constant power factor points on the *V* curves are called *compounding curves*. The compounding curves show the manner in which the field current should be varied in order to maintain constant power factor under changing loads. Points to the right of the unity power factor compounding curve correspond to overexcitation and leading current input ; points to the left correspond to underexcitation and lagging current input.

The *V* curves are useful in adjusting the field current. Increasing the field current I_f beyond the level for minimum armature current I_a results in leading power factor. Similarly, decreasing the field current I_f below that for minimum armature current I_a results in lagging power factor. Therefore, by controlling the field current of a synchronous motor, the reactive power supplied to or consumed from the power system can be controlled.

A family of curves is obtained by plotting the power factor versus field current. These are inverted *V* curves as shown in Fig. 5.13. The highest point on each of these curves indicates unity power factor. It is to be noted that the field current for unity power factor at full load is more than the field current for unity power factor at no load. Figure. 5.13 also shows that if the synchronous motor at full load is operating at unity power factor then removal of the shaft load causes the motor to operate at a leading power factor.

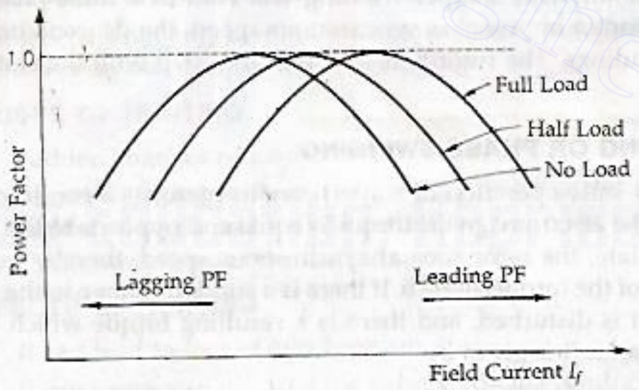


Fig. 5.13. *Power factor versus field current at different loads.*

Minimum Excitation

From Fig. 5.8 it is seen that as excitation is reduced, the torque angle δ continuously increases. The minimum permissible excitation, $E_{f\,(min)}$, corresponds to the stability limit, that is, $\delta = 90°$.

Therefore, $$E_{f\,(min)} = \frac{PX_s}{V}$$

5.12 STARTING OF SYNCHRONOUS MOTORS

A synchronous motor is not self-starting. It can be started by the following two methods :

1. Starting with the help of an external prime mover.
2. Starting with the help of damper windings.

5.12.1 Motor Starting With An External Prime Mover

In this method an external motor drives the synchronous motor and brings it to synchronous speed. The synchronous machine is then synchronized with the bus-bar as a synchronous generator. The prime mover is then disconnected. Once in parallel, the synchronous machine will work as a motor. Now the load can be connected to the synchronous motor. Since load is not connected to the synchronous motor before synchronising, the starting motor has to overcome the inertia of the synchronous motor at no load. Therefore the rating of the starting motor is much smaller than the rating of the synchronous motor.

At present most large synchronous motors are provided with brushless excitation systems mounted on their shafts. These exciters are used as starting motors.

5.12.2 Motor Starting With Damper Windings

Today the most widely used method of starting a synchronous motor is to use damper windings. A damper winding consists of heavy copper bars inserted in slots of the pole faces of the rotor as shown in Fig. 5.1. These bars are short-circuited by end rings at both ends of the rotor. Thus, these short-circuited bars form a squirrel-cage winding. When a three-phase supply is connected to the stator, the synchronous motor with damper winding will start as a three-phase induction motor. As the motor approaches synchronous speed, the dc excitation is applied to the field windings. The rotor will then pull into step with the stator magnetic field.

5.13 HUNTING OR PHASE SWINGING

A steady-state operation of a synchronous motor is a condition of *equilibrium* in which the electromagnetic torque is equal and opposite to the load torque. In the steady state, the rotor runs at synchronous speed, thereby maintaining a constant value of the torque angle δ. If there is a sudden change in the load torque, the equilibrium is disturbed, and there is a resulting torque which changes the speed of the motor. It is given by

$$\tau_e - \tau_{load} = J\frac{d\omega_M}{dt} \qquad (5.13.1)$$

where J = moment of inertia

ω_M = angular velocity of the rotor in mechanical units

When there is a sudden increase in the load torque, the motor slows down temporarily and the torque angle δ is sufficiently increased to restore the torque equilibrium and the synchronous speed.

The electromagnetic torque is given by

$$\tau_e = \frac{3\,VE_f}{\omega_s\,X}\sin\delta \qquad (5.13.2)$$

Since δ is increased, the electromagnetic torque increases. Consequently, the motor is accelerated. When the rotor reaches synchronous speed, the torque angle δ is larger than the required value δ_1 for the new state of equilibrium. Hence, the motor speed continues to increase beyond the synchronous speed. As a result of motor acceleration above synchronous speed, the torque angle δ decreases. At the point where motor torque becomes equal to the load torque, the equilibrium is not restored, because now the speed of the rotor is greater than the synchronous speed. Therefore the rotor continues to swing backwards. The torque angle goes on decreasing. When the load angle δ becomes less than the required value δ_1, the mechanical load becomes greater than the developed power. Therefore, the motor starts to slow down. The load angle is increased again. Thus, the rotor swings or oscillates around synchronous speed and the required value δ_1 of the torque angle before reaching the new steady state.

Similarly, the motor responds to a decreasing load torque by a temporary increase in speed, and thereby, a reduction of the torque angle δ. The rotor swings or oscillates around synchronous speed and the new required value δ_2 of the torque angle before reaching the new equilibrium position (steady state).

This phenomenon of oscillation of the rotor about its final equilibrium position is called **hunting**. Since during rotor oscillations, the phase of the phasor $\mathbf{E}_f$ changes relative to phasor **V**, hunting is also known as **phase swinging**. The term hunting is used to signify that after sudden application of load, the rotor attempts to search for or hunt for its new equilibrium space position.

Hunting occurs not only in the synchronous motors but also in the synchronous generators upon the abrupt change in loading.

5.14 CAUSES OF HUNTING

1. Sudden changes of load
2. Faults occurring in the system which the generator supplies
3. Sudden changes in the field current
4. Cyclic variations of the load torque.

5.15 EFFECTS OF HUNTING

1. It can lead to loss of synchronism.
2. It can cause variations of the supply voltage producing undesirable lamp flicker.
3. It increases the possibility of resonance. If the frequency of the torque component becomes equal to that of the transient oscillations of the synchronous machine, resonance may take place.
4. Large mechanical stresses may develop in the rotor shaft.
5. The machine losses increase and the temperature of the machine rises.

Of these effects, the first is the most important phenomenon to be avoided.

5.16 REDUCTION OF HUNTING

The following are some of the techniques used to reduce hunting :

(a) *Damper windings*

(b) *Use of flywheels*

The prime mover is provided with a large and heavy flywheel. This increases the inertia of the prime mover and helps in maintaining the rotor speed constant.

(c) By designing synchronous machines with suitable synchronizing power coefficients.

5.17 COMPARISON BETWEEN THREE-PHASE SYNCHRONOUS AND INDUCTION MOTORS

S. No.	*Synchronous motor*	*Induction motor*
1.	A synchronous motor is a doubly excited mahine. Its armature winding is energised from an ac source, and its field winding from a dc source.	An induction motor is a singly-excited machine. Its stator winding is energised from an ac source.
2.	It always runs at synchronous speed. The speed is independent of load.	Its speed falls with the increase in load and is always less than the synchronous speed.
3.	It is not self-starting. It has to be run upto synchronous speed by some means before it can be synchronised to ac supply.	An induction motor has got self-starting torque.
4.	A synchronous motor can be operated under wide range of power factors, both lagging and leading by changing its excitation.	An induction motor operates at only lagging power factor, which becomes very poor at high loads.
5.	It can be used for power factor correction in addition to supplying torque to drive mechanical loads.	An induction motor is used for driving mechanical loads only.
6.	It is more efficient than induction motor of the same output and voltage rating.	Its efficiency is lesser than that of a synchronous motor of the same output and voltage rating.
7.	A synchronous motor is costlier than an induction motor of the same output and voltage rating.	An induction motor is cheaper than a synchronous motor of the same output and voltage rating.

5.18 SYNCHRONOUS COMPENSATOR (SYNCHRONOUS CONDENSER)

When the motor power factor is unity, the dc excitation is said to be normal. Overexcitation causes the motor to operate at a leading power factor. Underexcitation causes it to operate at a lagging power factor. When the motor is operated at no load with overexcitation, it takes a current that leads the voltage by nearly 90°. In this way it behaves like a capacitor and under such operating conditions, the synchronous motor is called a *synchronous capacitor*. It is also known as *synchronous compensator* or *synchronous phase modifier*. A synchronous compensator is therefore, a synchronous motor running without a mechanical load. It can generate or absorb reactive voltamperes (VAr) by varying the excitation of its field winding. It can be made to take a leading current with over-excitation of the field winding. In such a case it delivers inductive (or absorbs capacitive) VAr. If it is under-excited, it takes a lagging current and, therefore, supplies capacitive (or absorbs inductive) VAr. Thus, the current drawn by a synchronous capacitor can be varied from lagging to leading smoothly by varying its excitation.

5.19 APPLICATIONS OF SYNCHRONOUS MOTORS

Synchronous motors were mainly used in constant speed applications. The development of semiconductor variable frequency sources, such as inverters and cycloconverters, has allowed their use in variable speed applications such as high power and high speed compressors, blowers, induced and forced draft fans, main-line traction, servo drives, etc.

Since a synchronous condenser behaves like a variables inductor or a variable capacitor, it is used in power transmission systems to regulate line voltage. In industry, synchronous motors are used with induction motors and operated with over excitation to draw leading current from the supply. Thus, they compensate the lagging current drawn by the induction motors to improve the overall power factor of the plant.

EXAMPLE 5.1. *A 3000 V, 3 phase synchronous motor running at 1500 r.p.m. has its excitation kept constant corresponding to no-load terminal voltage of 3000 V. Determine the power input, power factor and torque developed for an armature current of 250 A if the synchronous reactance is 5 Ω per phase and armature resistance is neglected.*

SOLUTION. Supply voltage per phase

$$V = \frac{3000}{\sqrt{3}} = 1732 \text{ V}$$

Induced e.m.f. per phase

$$E_f = \frac{3000}{\sqrt{3}} = 1732 \text{ V}$$

$$\mathbf{Z}_s = R_a + jX_s = 0 + j5 = 5 \angle 90° \ \Omega$$

$$\mathbf{E}_f = \mathbf{V} - \mathbf{I}_a \mathbf{Z}_s$$

If **V** is taken as reference phasor, then for lagging power factor,

$$\mathbf{I}_a = I_a \angle -\phi$$

$$\mathbf{E}_f = V - (I_a \angle -\phi)(5 \angle 90°) = V - 5\, I_a \angle 90° - \phi$$

$$= V - 5 \times 250 \angle 90° - \phi$$

$$= V - 1250\,[\cos(90° - \phi) + j \sin(90° - \phi)]$$

$$= V - 1250\,(\sin\phi + j\cos\phi)$$

$$= (V - 1250 \sin\phi) - j\,1250\cos\phi$$

$$E_f^2 = (V - 1250\sin\phi)^2 + (1250\cos\phi)^2$$

$$= V^2 - 2\,V \times 1250 \sin\phi + (1250\sin\phi)^2 + (1250\cos\phi)^2$$

$$1732^2 = 1732^2 - 2 \times 1732 \times 1250 \sin\phi + (1250)^2$$

$$2 \times 1732 \times 1250 \sin\phi = (1250)^2$$

$$\sin\phi = \frac{1250}{2 \times 1732} = 0.3608$$

$$\cos\phi = 0.9326 \text{ (lagging)}$$

Input power

$$P_i = \sqrt{3}\, V_L I_a \cos\phi = \sqrt{3} \times 3000 \times 250 \times 0.9326$$

$$= 1211483 \text{ W} = 1211.483 \text{ kW}$$

Also, $$P_i = 2\pi n_s \tau = 2\pi \frac{N_s}{60}\tau$$

$\therefore$ Torque $$\tau = \frac{P_i \times 60}{2\pi N_s}$$

$$= \frac{1211483 \times 60}{2\pi \times 1500} = 7712.5 \text{ Nm}$$

EXAMPLE 5.2. *A 1000 kVA, 11000 V, 3-phase star-connected synchronous motor has an armature resistance and reactance per phase of 3.5 Ω and 40 Ω respectively. Determine the induced e.m.f. and angular retardation of the rotor when fully loaded at (a) unity power factor, (b) 0.8 power factor lagging, (c) 0.8 power factor leading.*

SOLUTION. $$V = \frac{11000}{\sqrt{3}} = 6351 \text{ V}$$

$$R_a = 3.5\ \Omega,\ X_s = 40\ \Omega$$

$$(\text{kVA})_{3\phi} = \frac{\sqrt{3}\, V_L I_a}{1000}$$

$$1000 = \frac{\sqrt{3} \times 11000\, I_a}{1000},\ I_a = 52.49 \text{ A}$$

(a) Unity power factor

$$\cos\phi = 1.0,\quad \phi = 0°,\quad \mathbf{I}_a = 52.49 \angle 0° \text{ A}$$

$$\mathbf{E}_f = \mathbf{V} - \mathbf{I}_a \mathbf{Z}_s = \mathbf{V} - \mathbf{I}_a (R_a + jX_s)$$

$$= 6351 - (52.49 \angle 0°)(3.5 + j40) = 6351 - (183.7 + j2099.6)$$

$$E_f \angle \delta = 6167.3 - j2099.6 = 6515 \underline{/-18.8°} \text{ V}$$

$\therefore$ $$E_f = 6515 \text{ V per phase}$$

$$\delta = -18.8°$$

(b) 0.8 power factor lagging

$$\cos\phi = 0.8,\quad \sin\phi = 0.6$$

$$\mathbf{I}_a = I_a \angle -\phi$$

$$\mathbf{E}_f = \mathbf{V} - \mathbf{I}_a \mathbf{Z}_s$$

$$= V - (I_a \underline{/-\phi})(R_a + jX_s) = V - (I_a \cos\phi - j I_a \sin\phi)(R_a + jX_s)$$

$$= (V - I_a R_a \cos\phi - I_a X_s \sin\phi) - j(I_a X_s \cos\phi - I_a R_a \sin\phi)$$

$$= (6351 - 52.49 \times 3.5 \times 0.8 - 52.49 \times 40 \times 0.6)$$
$$- j\,(52.49 \times 40 \times 0.8 - 52.49 \times 3.5 \times 0.6)$$
$$E_f \angle \delta = 4944 - j\,1569.5 = 5187 \,\underline{/-17.6^\circ}\ \text{V}$$
$$\therefore \quad E_f = 5187 \text{ per phase}, \quad \delta = -17.6^\circ$$

Induced line voltage

$$= \sqrt{3} \times 5187 = 8984 \text{ V}$$

(c) *0.8 power factor leading*

$$\mathbf{I}_a = I_a \angle + \phi$$
$$\mathbf{E}_f = \mathbf{V} - \mathbf{I}_a \mathbf{Z}_s = V - (I_a \angle + \phi)(R_a + jX_s)$$
$$= (V - I_a R_a \cos\phi + I_a X_s \sin\phi) - j\,(I_a X_s \cos\phi + I_a R_a \sin\phi)$$
$$= (6351 - 52.49 \times 3.5 \times 0.8 + 52.49 \times 40 \times 0.6)$$
$$- j\,(52.49 \times 40 \times 0.8 + 52.49 \times 3.5 \times 0.6)$$
$$E_f \angle \delta = 7463.8 - j1790 = 7675 \,\underline{/-13.48^\circ}\ \text{V}$$
$$E_f = 7675 \text{ V per phase}$$
$$\delta = -13.48^\circ$$

Induced line voltage $= \sqrt{3} \times 7675 = 13293$ V

EXAMPLE 5.3. *A 2000 V, 3-phase, star-connected synchronous motor has an effective resistance and synchronous reactance of 0.2 Ω and 2.2 Ω per phase respectively. The input is 800 kW at normal voltage and the induced line e.m.f. is 2500 V. Calculate the line current and power factor.*

SOLUTION. Supply voltage per phase

$$V = \frac{2000}{\sqrt{3}} = 1154.7 \text{ V}$$

Induced e.m.f. per phase

$$E_f = \frac{2500}{\sqrt{3}} = 1443.4 \text{ V}$$

Since the induced e.m.f. is greater than the supply voltage, the motor is operating with a leading power factor $\cos\phi$.

If **V** is taken as reference phasor.

$$\therefore \quad \mathbf{V} = V \angle 0^\circ \text{ and } \mathbf{I}_a = I_a \angle + \phi = I_a \cos\phi + j\,I_a \sin\phi$$

For a star-connected system line current = phase current

$$I_L = I_a$$

Power input $= \sqrt{3}\, V_L I_L \cos\phi$

$$800 \times 10^3 = \sqrt{3} \times 2000\, I_a \cos\phi$$
$$I_a \cos\phi = \frac{800 \times 10^3}{\sqrt{3} \times 2000} = 231$$
$$R_a = 0.2\,\Omega, \quad X_s = 2.2\,\Omega$$
$$\mathbf{E}_f = \mathbf{V} - \mathbf{I}_a \mathbf{Z}$$

$$= V - [(I_a \cos\phi + jI_a \sin\phi)\,(R_a + jX_s)]$$

$$= V - [(I_aR_a \cos\phi - I_aX_s \sin\phi) + j\,(I_aX_s \cos\phi + I_aR_a \sin\phi)]$$

$$= (V - I_aR_a \cos\phi + I_aX_s \sin\phi) - j\,(I_aX_s \cos\phi + I_aR_a \sin\phi)$$

$$E_f^2 = (V - I_aR_a \cos\phi + I_aX_s \sin\phi)^2 + (I_aX_s \cos\phi + I_aR_a \sin\phi)^2$$

$$1443.4^2 = (1154.7 - 0.2 \times 231 + 2.2\, I_a \sin\phi)^2 + (231 \times 2.2 + 0.2\, I_a \sin\phi)^2$$

$$= (1108.5 + 2.2\, I_a \sin\phi)^2 + (508.2 + 0.2\, I_a \sin\phi)^2$$

$$2083404 = 1228772 + 4877.4\, I_a \sin\phi + 4.84\, I_a^2 \sin^2\phi + 258267$$

$$+ 203.3\, I_a \sin\phi + 0.04\, I_a^2 \sin^2\phi$$

$$4.88\, I_a^2 \sin^2\phi + 5080.7\, I_a \sin\phi - 596365 = 0$$

$$I_a^2 \sin^2\phi + 1041\, I_a \sin\phi - 122206 = 0$$

$$I_a \sin\phi = \frac{-1041 \pm \sqrt{1041^2 + 4 \times 122206}}{2} = \frac{1}{2}(-1041 + 1254) = 106.5$$

$\therefore$ $$\mathbf{I}_a = I_a \cos\phi + j\, I_a \sin\phi$$

$$= 231 + j\,106.5 = 254.4 \angle 24.75^\circ \text{ A}$$

$\therefore$ Line current $I_L = I_a = 254.4$ A

Power factor $= \cos 24.75^\circ = 0.9081$ (leading)

EXAMPLE 5.4. *A 3-phase synchronous motor of 8000 W at 1100 V has synchronous reactance of 8 Ω per phase. Find the minimum current and the corresponding induced e.m.f. for full-load condition. The efficiency of the machine is 0.8. Neglect armature resistance.*

SOLUTION. The current in the motor is minimum when the power factor is unity, that is, $\cos\phi = 1$.

$$\text{Motor input} = \frac{\text{motor output}}{\text{efficiency}}$$

$$P_i = \frac{8000}{0.8} = 10000 \text{ W} = 10 \text{ kW}$$

$$P_i = \sqrt{3}\, V_L I_L \cos\phi$$

$$I_L = \frac{P_i}{\sqrt{3}\, V_L \cos\phi} = \frac{10 \times 10^3}{\sqrt{3} \times 1100 \times 1} = 5.249 \text{ A}$$

For unity power factor

$$E_f^2 = V^2 + (I_aX_s)^2 = \left(\frac{1100}{\sqrt{3}}\right)^2 + (5.249 \times 8)^2$$

$\therefore$ $$E_f = 636.49 \text{ V per phase}$$

EXAMPLE 5.5. *A 3-phase, 400 V synchronous motor takes 52.5 A at a power factor of 0.8 leading. Determine the induced e.m.f. and the power supplied. The motor impedance per phase is (0.25 + j 3.2) Ω.*

SOLUTION. For leading power factor

$$E_f^2 = (V\cos\phi - I_a R_a)^2 + (V\sin\phi + I_a X_s)^2$$

$$= \left(\frac{400}{\sqrt{3}} \times 0.8 - 52.2 \times 0.25\right)^2 + \left(\frac{400}{\sqrt{3}} \times 0.6 + 52.5 \times 3.2\right)^2$$

$$= (171.6)^2 + (306.57)^2$$

$$E_f = 351.3 \text{ V}$$

Line e.m.f. $= \sqrt{3} \times 351.3 = 608.5$ V

Power supplied $P_i = \sqrt{3}\, V_L I_a \cos\phi$

$$= \sqrt{3} \times 400 \times 52.5 \times 0.8 = 29098 \text{ W}$$

EXAMPLE 5.6. *A 660 V, 3-phase, star-connected synchronous motor draws 50 kW at power factor 0.8 lagging. Find the new current and power factor when the back e.m.f. increases by 50%. The machine has synchronous reactance of 3 Ω and effective resistance is negligible.*

SOLUTION. $P_{3\Phi} = \sqrt{3}\, V_L I_a \cos\phi$

$$50 \times 10^3 = \sqrt{3} \times 660\, I_a \times 0.8$$

$$I_a = \frac{50 \times 10^3}{\sqrt{3} \times 660 \times 0.8} = 54.67 \text{ A}$$

$$V_p = \frac{V_L}{\sqrt{3}} = \frac{660}{\sqrt{3}} = 381 \text{ V}$$

$$\mathbf{Z}_s = R_a + jX_s = 0 + j3 = 3\,\angle 90° \ \Omega$$

Let V_p be taken as reference phasor.

$$\mathbf{V}_p = V_p \angle 0° = 381 \angle 0° = 381 + j0 \text{ V}$$

$$\mathbf{I}_a = I_a \angle -\phi = I_a \angle -\cos^{-1} 0.8 = 54.67\,\angle -36.87° \text{ A}$$

$$\mathbf{E}_{fp} = \mathbf{V}_p - \mathbf{I}_a \mathbf{Z}_s$$

$$= 381 + j0 - (54.67\,\angle -36.87°)(3\,\angle 90°)$$

$$= 381 - (54.67 \times 3)\,\angle 90° - 36.87°$$

$$= 381 - 164.01\,\angle 53.13° = 381 - (98.4 + j131.2) = 282.6 - j131.2$$

$$= 311.57\,\angle -24.9° \text{ V}$$

When the e.m.f. is increased by 50%, the new value of e.m.f. will be

$$E_{f_{p1}} = 1.5 \times 311.57 = 467.36 \text{ V}$$

Since $E_{f_{p1}} > V_p$, the power factor is leading.

$$P_{3\phi} = \sqrt{3}\, V_L I_{a_1} \cos\phi_1$$

$$50 \times 10^3 = \sqrt{3} \times 660\, I_{a_1} \cos\phi_1$$

$$I_{a_1} \cos\varphi_1 = \frac{50 \times 10^3}{\sqrt{3} \times 660} = 43.74 \text{ A}$$

For leading power factor, when $R_a = 0$,

$$E_{f_{p1}}^2 = (V_p + I_{a_1} X_s \sin \phi_1)^2 + (I_{a_1} X_s \cos \phi_1)^2$$

$$E_{f_{p1}}^2 - (I_{a_1} X_s \cos \phi_1)^2 = (V_p + I_{a_1} X_s \sin \phi_1)^2$$

$$I_a \sin \phi_1 = \frac{\sqrt{E_{f_1}{}^2 - (I_{a_1} \cos \phi_1)^2 X_s^2} - V_p}{X_s}$$

$$= \frac{1}{3}\left[\sqrt{467.36^2 - (3 \times 43.74)^2} - 381\right] = 22.52 \text{ A}$$

For leading power factor

$$\mathbf{I}_{a_1} = I_{a_1} \angle \phi_1 = I_{a_1} \cos \phi_1 + j\, I_{a_1} \sin \phi_1$$

$$= 43.74 + j22.52 = 49.2 \underline{/27.242°} \text{ A}$$

New power factor $\cos \phi_1 = \cos 27.242° = 0.8890$ (leading)

New current $I_{a_1} = 49.2$ A

EXAMPLE 5.7. *The efficiency of a 3-phase, 400 V, star-connected synchronous motor is 95% and it takes 24 A at full load and unity power factor. What will be the induced e.m.f. and total mechanical power developed at full load and 0.9 p.f. leading ? The synchronous impedance per phase is* $(0.2 + j2)\ \Omega$.

SOLUTION. $V_p = \dfrac{400}{\sqrt{3}} = 231$ V

$\cos \phi = 0.9$, $\sin \phi = 0.4359$

$R_a = 0.2\ \Omega$, $X_s = 2\ \Omega$

Current at 0.9 power factor

$$I_a = \frac{24}{0.9} = 26.66 \text{ A}$$

For leading power factor

$$\mathbf{E}_{fp} = (V_p - I_a R_a \cos \phi + I_a X_s \sin \phi) - j\,(I_a X_s \cos \phi + I_a R_a \sin \phi)$$

$$= (231 - 26.66 \times 0.2 \times 0.9 + 26.66 \times 2 \times 0.4359)$$

$$- j\,(26.66 \times 2 \times 0.9 + 26.66 \times 0.2 \times 0.4359)$$

$$= 249.44 - j50.31 = 254.46 \underline{/-11.4°} \text{ V}$$

Induced line e.m.f. $= \sqrt{3} \times 254.46 = 440.7$ V

Total copper loss $= 3 I_a^2 R_a = 3 \times \left(\dfrac{24}{0.9}\right)^2 \times 0.2 = 426.6\dot{7}$ W

Input power $P_i = \sqrt{3}\, V_L I_a \cos \phi = \sqrt{3} \times 400 \times \dfrac{24}{0.9} \times 0.9 = 16627.68$ W

Mechanical power developed

$= $ input power $-$ copper loss

$= 16627.68 - 426.67 = 16201$ W $= 16.201$ kW

EXAMPLE 5.8. *A 6600 V, 3-phase, star-connected synchronous motor draws a full-load current of 80 A at 0.8 p.f. leading. The armature resistance is 2.2 Ω and synchronous reactance 22 Ω per phase. If the stray losses of the machine are 3200 W, determine : (a) the e.m.f. induced ; (b) the output power ; (c) the efficiency.*

SOLUTION. $V_p = \dfrac{6600}{\sqrt{3}} = 3810.6$ V

$$I_a = 80 \text{ A}, \quad \cos\phi = 0.8, \quad \sin\phi = 0.6$$

$$R_a = 2.2\ \Omega, \qquad X_s = 22\ \Omega$$

For leading power factor

$$\mathbf{E}_{fp} = (V_p - I_a R_a \cos\phi + I_a X_s \sin\phi) - j\,(I_a X_s \cos\phi + I_a R_a \sin\phi)$$

$$= (3810.6 - 80 \times 2.2 \times 0.8 + 80 \times 22 \times 0.6) - j\,(80 \times 22 \times 0.8 + 80 \times 2.2 \times 0.6)$$

$$= 4725.8 - j\,1513.6 = 4962 \angle{-17.76^\circ} \text{ V}$$

Induced line e.m.f. $= \sqrt{3} \times 4962 = 8594$ V

Power input $= \sqrt{3}\, V_L I_a \cos\phi = \sqrt{3} \times 6600 \times 80 \times 0.8 = 731618$ W

Total copper loss $= 3\, I_a^2 R_a = 3 \times 80^2 \times 2.2 = 42240$ W

Stray loss $= 3200$ W

Power output = power input − copper losses − stray loss

$$= 731618 - 42240 - 3200 = 686178 \text{ W}$$

Efficiency $= \dfrac{\text{output}}{\text{input}} = \dfrac{686178}{731618} = 0.9379$ p.u.

EXAMPLE 5.9. *A 6-pole, 2200 V, 50 Hz, 3-phase, star-connected synchronous motor has armature resistance of 0.4 Ω per phase and synchronous reactance of 4 Ω per phase. While running on no load, the excitation has been adjusted so as to make the e.m.f. numerically equal to and antiphase with the terminal voltage. With a certain load torque applied, if the rotor gets retarded by 3 mechanical degrees, calculate the armature current and power factor of the motor.*

SOLUTION. $V = \dfrac{2200}{\sqrt{3}} = 1270.2$ V, $\quad E_f = V = 1270.2$ volts per phase

Load angle $\delta = 3^\circ$ mechanical

$$= 3 \times \frac{P}{2} = 3 \times \frac{6}{2} = 9^\circ \text{ electrical}$$

$$\mathbf{V} = V \angle 0^\circ = 1270.2 \angle 0^\circ$$

$$\mathbf{E}_f = E_f \angle -\delta = 1270.2 \angle -9^\circ$$

$$\mathbf{E}_f = \mathbf{V} - \mathbf{I}_a \mathbf{Z}_s$$

$$\mathbf{I}_a = \frac{\mathbf{V} - \mathbf{E}_f}{\mathbf{Z}_s} = \frac{1270.2 \angle 0^\circ - 1270.2 \angle -9^\circ}{0.4 + j4}$$

$$= \frac{1270.2\,[(1 - (\cos 9^\circ - j \sin 9^\circ)]}{0.4 + j4} = \frac{1270.2\,(1 - 0.9877 + j0.1564)}{4.02 \angle 84.289^\circ}$$

$$= \frac{1270.2 \times 0.15688 \angle 85.50^\circ}{4.02 \angle 84.289^\circ} = 49.57 \angle 1.211^\circ \text{ A}$$

$\therefore$ Armature current $I_a = 49.57$ A

Power factor $= \cos\phi = \cos 1.211° = 0.9998$ (leading)

EXAMPLE 5.10. *The synchronous reactance per phase of a $3-\phi$, star-conn 6600 V synchronous motor is 20 Ω. For a certain load the input is 900 kW at norm voltage and the induced line emf is 8500 V. Determine the line current and power fa*

SOLUTION. $V_l = 6600\text{ V},\ V = \dfrac{6600}{\sqrt{3}} = 3810.5\text{ V}$

$$X_s = 20\,\Omega,\ R_a = 0,\ \mathbf{Z}_s = jX_s$$

Input $\quad P_i = \sqrt{3}\, V_l\, I_a \cos\phi$

$$900 \times 10^3 = \sqrt{3} \times 6600\, I_a \cos\phi$$

$$I_a \cos\phi = \frac{900 \times 10^3}{\sqrt{3} \times 6600} = 78.73\text{ A}$$

Line induced emf $\quad E_{fl} = 8500$ V

Phase induced emf $\quad E_f = \dfrac{8500}{\sqrt{3}} = 4907.5$ V

Since $E_{fl} > V_l$, the power factor is leading

$$\mathbf{E}_f = \mathbf{V}_p - \mathbf{I}_a \mathbf{Z}_s$$

$$= (V + j0) - (I_a \angle \phi)(20 \angle 90°) = V - 20\, I_a \angle 90° + \phi$$

$$= V - 20\, I_a\, [\cos(90° + \phi) + j \sin(90° + \phi)]$$

$$= V - 20\, I_a\, [-\sin\phi + j\cos\phi]$$

$$\mathbf{E}_f = (V + 20\, I_a \sin\phi) - j\,(20\, I_a \cos\phi)$$

$$E_f^2 = (V + 20\, I_a \sin\phi)^2 + (20\, I_a \cos\phi)^2$$

$$(4907.5)^2 = (3810.5 + 20\, I_a \sin\phi)^2 + (20 \times 78.73)^2$$

$$(3810.5 + 20\, I_a \sin\phi) = \sqrt{(4907.5)^2 - (20 \times 78.73)^2} = 4648$$

$$I_a \sin\phi = \frac{4648 - 3810.5}{20} = 41.876\text{ A}$$

$$\therefore \quad \mathbf{I}_a = I_a \cos\phi + jI_a \sin\phi$$

$$= 78.73 + j41.876 = 89.17 \angle 28°\text{ A}$$

$$\cos\phi = \cos 28° = 0.8829\text{ (lead)}$$

$$I_a = 89.17\text{ A}$$

EXAMPLE 5.11. *A 400 V, 6-pole, 3-phase, 50 Hz, star-connected synchronous moto has a resistance and synchronous impedance of 0.5 Ω and 4 Ω per phase respectively. It takes a current of 15 A at unity power factor when operating with a certain field curren If the load torque is increased until the line current is increased to 60 A, the field curren remaining unchanged, calculate the gross torque developed and the new power factor.*

SOLUTION. $V = \dfrac{400}{\sqrt{3}} = 231$ V

$$R_a = 0.5\,\Omega,\ \ Z_s = 4\,\Omega,\ \ X_s = \sqrt{4^2 - 0.5^2} = 3.968\,\Omega$$

At unity p.f., *i.e.*, $\cos\phi = 1$, $\sin\phi = 0$

$$E_f = V - I_a R_a - jI_a X_s$$

$$= 231 - 15 \times 0.5 - j15 \times 3.968 = 223.5 - j59.52 = 231.29 \angle -14.9^\circ \text{ V}$$

With the increased load torque, the field current remains the same and therefore, E_f remains the same.

For lagging power factor $\cos\phi$,

$$E_f^2 = (V\cos\phi + I_a R_a)^2 + (V\sin\phi + I_a X_s)^2$$

$$= V^2 + (I_a Z_s)^2 - 2\,V I_a Z_s \cos(\theta - \phi)$$

$$(231.29)^2 = 231^2 + (60 \times 4)^2 - 2 \times 231 \times 60 \times 4 \cos(\theta - \phi)$$

$$\cos(\theta - \phi) = \frac{231^2 + (60 \times 4)^2 - (231.29)^2}{2 \times 231 \times 60 \times 4} = 0.51827 = \cos 58.78^\circ$$

$$\theta - \phi = 58.78^\circ$$

$$\tan\theta = \frac{X_s}{R_a} = \frac{3.968}{0.5},\ \theta = 82.82^\circ$$

$$\therefore \quad \phi = \theta - 58.78^\circ = 82.82^\circ - 58.78^\circ = 24.04^\circ$$

New power factor $\cos\phi = \cos 24.04^\circ = 0.9133$ (lag)

Motor input $P_i = \sqrt{3}\,V_L I_a \cos\phi = \sqrt{3} \times 400 \times 60 \times 0.9133 = 37965$ W

Total armature copper loss

$$= 3\,I_a^2 R_a = 3 \times (60)^2 \times 0.5 = 5400 \text{ W}$$

Electrical power converted into mechanical power, $P_m = P_i - 3\,I_a^2 R_a$

$$= 37965 - 5400 = 32565 \text{ W}$$

Synchronous speed

$$N_s = \frac{120\,f}{P} = \frac{120 \times 50}{6} = 1000 \text{ r.p.m.}$$

$$n_s = \frac{N_s}{60} = \frac{1000}{60}$$

$$\therefore \quad 2\pi\, n_s T = P_m$$

$$T = \frac{P_m}{2\pi\, n_s} = \frac{32565 \times 60}{2\pi \times 1000} = 310.97 \text{ Nm}$$

Example 5.12. *A 2200 V, 3-phase, star-connected, 8-pole synchronous motor has impedance per phase equal to $(0.4 + j6)\ \Omega$. When the motor runs at no load, the field excitation is adjusted so that E_f is made equal to V. When the motor is loaded, the rotor is retarded by 3° mechanical. Calculate the armature current, power factor and power of the motor. What is the maximum power the motor can supply without falling out of step ?*

Solution. $E_f = V = \dfrac{2200}{\sqrt{3}} = 1270.2$ V

$$\delta = 3^\circ \text{ mechanical} = 3 \times \frac{P}{2} = \frac{3 \times 8}{2} = 12^\circ \text{ (elec.)}$$

$$\mathbf{I}_a = \frac{\mathbf{V} - \mathbf{E}_f}{\mathbf{Z}_s} = \frac{V\angle 0^\circ - E_f\angle -\delta}{R_a + jX_s}$$

$$= \frac{1270.2\angle 0^\circ - 1270.2\angle -12^\circ}{0.4 + j6} = \frac{1270.2\,[1 - (\cos 12^\circ - j\sin 12^\circ)]}{6.0133\,\angle 86.18^\circ}$$

$$= \frac{1270.2 \times [0.02185 + j0.2079]}{6.0133\,\angle 86.18^\circ} = \frac{1270.2 \times 0.2090\,\angle 84^\circ}{6.0133\,\angle 86.18^\circ}$$

$$= 44.15\,\angle -2.18^\circ \text{ A}$$

Armature current $I_a = 44.15$ A

Power factor, $\cos\phi = \cos 2.18^\circ = 0.9993$ (lag)

Total power input $= \sqrt{3}\, V_L\, I_a \cos\phi$

$$= \sqrt{3} \times 2200 \times 44.15 \times 0.9993 = 168116 \text{ W} = 168.116 \text{ kW}$$

Total copper loss

$$= 3\, I_a^2\, R_a = 3 \times (44.15)^2 \times 0.4 = 2339 \text{ W}$$

Power developed by the motor

$$= \text{motor input} - \text{copper losses}$$

$$= 168116 - 2339 = 165777 \text{ W} = 165.777 \text{ kW}$$

Maximum power

$$(P_m)_{max} = \frac{E_f V}{Z_s} - \frac{E_f^2}{Z_s^2} R_a = \frac{1270.2 \times 1270.2}{6.0133} - \frac{(1270.2)^2}{(6.0133)^2} \times 0.4$$

$$= 250459 \text{W} = 250.459 \text{ kW}$$

Example 5.13. *The 400 V, 50 kVA, 0.8 power factor leading delta conn synchronous motor is supplying a 12 kW load with initial power factor of 0.86 lagging. The windage and friction losses are 2.0 kW and the core losses are 1.5 kW*

(*a*) *Determine the line current, armature current and excitation voltage*

(*b*) *If the flux of the motor is increased by 30 per cent determine the excitation voltage, armature current and the new power factor.*

Solution. $P_{in} = P_{out} + p_{mech} + p_{core} + p_{elec} = 12 + 2 + 1.5 + 0 = 15.5$ kW

$$3V_p\, I_{a_1} \cos\phi = P_{in}$$

$$I_{a_1} = \frac{P_{in}}{3\, V_p \cos\phi} = \frac{15.5 \times 10^3}{3 \times 400 \times 0.86} = 15 \text{ A}$$

Since the power factor of the motor is 0.86 lagging, the phasor armature current is given by

$$\mathbf{I}_{a_1} = I_{a_1}\angle -\cos^{-1} 0.86 = 15\,\angle -30.68^\circ \text{ A}$$

Line current $\quad I_L = \sqrt{3}\, I_{a_1} = \sqrt{3} \times 15 = 25.98$ A

The excitation voltage E_f is given by

$$\mathbf{E}_{f_1} = \mathbf{V}_p - \mathbf{Z}_s\, \mathbf{I}_{a_1}$$

$$= \mathbf{V}_p - jX_s \mathbf{I}_{a_1}$$

$$= 400 \angle 0° - j3 (15 \angle -30.69°)$$

$$= 400 \angle 0° - 45 \angle 90° - 30.68° = 400 - 45 \angle 59.32°$$

$$= 400 - (22.96 + j38.7) = 377.04 - j38.7 = 379 \angle -5.86° \text{ V}$$

(*b*) If the flux is increased by 30%, then $E_f (= k \Phi \omega)$ will also increase by 30%.

$$\therefore \quad E_{f_2} = 1.3 E_{f_1} = 1.3 \times 379 = 492.7 \text{ V}$$

With the increase of flux by 30%, the power supplied to the load remains constant and

$$E_{f_1} \sin \delta_1 = E_{f_2} \sin \delta_2$$

$$\sin \delta_2 = \frac{E_{f_1}}{E_{f_2}} \sin \delta_1 = \frac{1}{1.3} \sin (-5.86°)$$

$$\delta_2 = -4.5°$$

The new armature current is given by

$$\mathbf{I}_{a_1} = \frac{\mathbf{V}_p - \mathbf{E}_{f_2}}{jX_s} = \frac{400 \angle 0° - 492.7 \angle -4.5°}{j3} = \frac{1}{j3} [400 - (491.2 - j38.66)]$$

$$= \frac{1}{j3} (-91.2 + j38.66) = \frac{99 \angle 157°}{3 \angle 90°} = 33 \angle 67° \text{ A}$$

New power factor of the motor

$$\cos \phi_2 = \cos 67° = 0.3902 \text{ leading}$$

Example 5.14. *A 400 V, 50 kVA, 0.8 power factor leading, delta-connected 50 Hz synchronous machine has a synchronous reactance of 3 Ω and negligible armature resistance. Its friction and windage losses are 2 kW and its core losses are 1.5 kW. Initially, the shaft load is 10 kW and the power factor of the motor is 0.8 leading.*

(*a*) *Determine the line current, armature current and excitation voltage.*

(*b*) *If the shaft load is increased to 20 kW determine the new values of line current, armature current and the motor power factor.*

Solution. (*a*) Power input to motor

$$P_{in} = P_{out} + p_{mech} + p_{core} + p_{elec}$$

$$= 10 + 2 + 1.5 + 0 = 13.5 \text{ kW}$$

$$\sqrt{3}\, V_L I_L \cos \phi = P_{in}$$

$$I_L = \frac{P_{in}}{\sqrt{3}\, V_L \cos \phi} = \frac{13.5 \times 10^3}{\sqrt{3} \times 400 \times 0.8} = 24.36 \text{ A}$$

Armature current

$$I_a = \frac{1}{\sqrt{3}} \times \text{line current}$$

$$= \frac{1}{\sqrt{3}} \times 24.36 = 14.06 \text{ A}$$

Since the power factor is 0.8 leading, the phasor armature current is gi

$$\mathbf{I}_a = I_a \angle \cos^{-1} 0.8 = 14.06 \underline{/36.87^\circ} \text{ A}$$

The excitation voltage E_f is given by

$$\mathbf{E}_f = \mathbf{V}_p - Z\mathbf{I}_a = \mathbf{V}_p - jX_s\,\mathbf{I}_a$$

$$= 400 \angle 0^\circ - j3\,(14.06 \underline{/36.87^\circ}) = 400 \angle 0^\circ - 3 \times 14.06 \underline{/90^\circ + 36.8}$$

$$= 400 \angle 0^\circ - 42.18 \underline{/126.87^\circ} = 400 + j0 - (-25.3 + j33.74)$$

$$= 425.3 - j33.74 = 426.63 \underline{/-4.536^\circ} \text{ V}$$

(*b*) If the shaft load is increased to 20 kW, the shaft slows down mome and the excitation voltage remains constant but the torque angle δ is incre

The new power input

$$P_{in} = P_{out} + p_{mech} + p_{core} + p_{elec}$$

$$= 20 + 2 + 1.5 + 0 = 23.5 \text{ kW}$$

Also, $$P_{in} = \frac{3\,V_p\,E_f \sin\delta}{X_s}$$

$$\sin\delta = \frac{P_{in}\,X_s}{3\,V_p E_f}$$

$$= \frac{23.5 \times 10^3 \times 3}{3 \times 400 \times 426.63} = 0.1377$$

$$\delta = 7.92^\circ$$

$$\mathbf{E}_f = 426.63 \underline{/-7.92^\circ} \text{ V}$$

$$\mathbf{I}_a = \frac{\mathbf{V}_p - \mathbf{E}_f}{jX_s} = \frac{400 \angle 0^\circ - 426.63 \underline{/-7.92^\circ}}{j3}$$

$$= \frac{400 - (422.56 - j58.78)}{3 \underline{/90^\circ}} = \frac{-22.56 + j58.78}{3 \underline{/90^\circ}}$$

$$= \frac{62.96 \underline{/111^\circ}}{3 \underline{/90^\circ}} = 20.98 \underline{/21^\circ} \text{ A}$$

Line current $I_L = \sqrt{3}\,I_a = \sqrt{3} \times 20.98 = 36.34$ A

New power factor of the motor

$$= \cos 21^\circ = 0.9336 \text{ (leading)}.$$

EXAMPLE 5.15. *A 3-phase, 11-kV, star-connected synchronous motor take input current. The effective resistance and synchronous reactance per phase are 1 Ω a 30 Ω respectively. Calculate the induced emf for a power factor of (a) 0.8 lagging leading and (c) the power supplied to the motor.*

SOLUTION. (*a*) *Lagging power factor*

$$\cos\phi = 0.8, \qquad \phi = 36.86^\circ$$

$$\mathbf{Z}_s = R_a + j\,X_s = 1 + j\,30 = 30.01 \underline{/88.09^\circ}\ \Omega$$

$$V_p = \frac{11000}{\sqrt{3}} = 6350.8 \text{ V}, I_a = 50 \text{ A}$$

$$\mathbf{E}_{fp} = \mathbf{V}_p - \mathbf{I}_a\,\mathbf{Z}_s$$
$$= 6350.8 - (50\,\underline{/-36.86^\circ})\,(30.01\,\underline{/88.09^\circ})$$
$$= 6350.8 - 1500.5\,\underline{/51.23^\circ}$$
$$= 6350.8 - (939.6 + j\,1169.8)$$
$$= 5411.2 - j\,1169.8 = 5536.2\,\underline{/-12.2^\circ}\text{V}$$

(*b*) *Leading power factor*

$$\mathbf{E}_{fp} = \mathbf{V}_p - \mathbf{I}_a\,\mathbf{Z}_s$$
$$= 6350.8 - (50\,\underline{/+36.86^\circ})\,(30.01\,\underline{/88.09^\circ})$$
$$= 6350.8 - 1500.5\,\underline{/124.95^\circ}$$
$$= 6350.8 - (-859.6 + j\,1229.9)$$
$$= 7210.4 - j\,1229.9 = 7314.5\,\underline{/-9.7^\circ}\text{V}$$

(c) $P_{3\phi} = \sqrt{3}\,V_L\,I_a \cos\phi = \sqrt{3} \times 11000 \times 50 \times 0.8 = 762102$ W

$= 762.102$ kW

EXAMPLE 5.16. *A 3-phase, 5000 kVA, 11 kV, 50 Hz, 1000 r.p.m, star-connected synchronous motor operates at full load at a power factor of 0.8 leading. The synchronous reactance is 60% and the resistance may be neglected. Calculate the synchronizing power per mechanical degree of angular displacement. What is the ratio of maximum to full-load torque and the value of the maximum torque ?*

SOLUTION. $S = \sqrt{3}\,V_L\,I_a$

$$I_a = \frac{S}{\sqrt{3}\,V_L} = \frac{5000 \times 10^3}{\sqrt{3} \times 11 \times 10^3} = 262.4 \text{ A}$$

$$V_p = \frac{V_L}{\sqrt{3}} = \frac{11 \times 10^3}{\sqrt{3}} = 6351 \text{ V}$$

$$X_{spu} = \frac{X_s \text{ in ohms}}{V_p/I_a}$$

$$X_{s\Omega} = X_{spu} \cdot \frac{V_p}{I_a} = \frac{60}{100} \times \frac{6351}{262.4} = 14.52\ \Omega$$

$$\mathbf{E}_f = \mathbf{V}_p - \mathbf{I}_a\,\mathbf{Z}_s$$
$$= V_p - (I_a\,\underline{/\phi})\,(X_s\,\underline{/90^\circ})$$
$$= V_p - I_a\,X_s\,\underline{/90^\circ + \phi}$$
$$= (V_p + I_a\,X_s \sin\phi) - j\,I_a\,X_s \cos\phi$$
$$= (6351 + 262.4 \times 14.52 \times 0.6) - j\,262.4 \times 14.52 \times 0.8$$
$$= (6351 + 2286) - j\,3048 = 8637 - j\,3048$$
$$= 9159\,\underline{/-19.44^\circ}\text{V}$$

$\therefore\quad E_f = 9159$ V, $\delta = -19.44^\circ$

$$f = \frac{P\,N_s}{120},\ 50 = \frac{P \times 1000}{120},\ P = 6,\ p = 3$$

$$P_{syn} = \left(\frac{3V_p E_f}{X_s} \cos \delta \right) p \frac{\pi}{180}$$

$$= \frac{3 \times 6351 \times 9159}{14.52} (\cos 19.44°) \times \frac{3\pi}{180}$$

$$= 593404 \text{ W}$$

$$\tau_{syn} = \frac{P_{syn}}{2\pi n_s} = \frac{593404}{2\pi \times \frac{1000}{60}} = 5666.6 \text{ Nm per mech degree}$$

Full-load torque = (maximum torque) . sin δ

$$\frac{\text{maximum torque}}{\text{full-load torque}} = \frac{1}{\sin \delta} = \frac{1}{\sin 19.44°} = 3$$

$$\text{Maximum power} = \frac{3 V_p E_f}{X_s}$$

$$P_{max} = \frac{3 \times 6351 \times 9159}{14.52} = 12018349$$

$$\text{Maximum torque} = \frac{P_{max}}{2\pi n_s}$$

$$= \frac{12018349}{2\pi \times \frac{1000}{60}} = 114767 \text{ Nm}$$

EXAMPLE 5.17. *A 3-phase, star-connected synchronous motor takes 20 kW at 400 V from the mains. The synchronous reactance is 4 Ω and the effective resistance is negligible. If the exciting current is so adjusted that the back emf is 550 V, calculate the line current and the power factor of the motor.*

SOLUTION. $V_p = \frac{400}{\sqrt{3}}$ V, $E_{fp} = \frac{550}{\sqrt{3}}$ V

$R_a = 0,\quad X_s = 4\ \Omega$

Since $E_{fp} > V_p$, the p.f. is leading.

Input power $P = 3\, V_p\, I_a \cos \phi$

$$20 \times 10^3 = 3 \times \frac{400}{\sqrt{3}} I_a \cos \phi$$

$$I_a \cos \phi = \frac{20 \times 10^3}{\sqrt{3} \times 400} = 28.86$$

For leading p.f.

$$E_{fp}^2 = (V_p + I_a X_s \sin \phi)^2 + (I_a X_s \cos \phi)^2$$

$$V_p + I_a X_s \sin \phi = \sqrt{E_{fp}^2 - (I_a X_s \cos \phi)^2}$$

$$I_a \sin \phi = \frac{1}{X_s} \left[\sqrt{E_{fp}^2 - (X_s I_a \cos \phi)^2} - V_p \right]$$

$$= \frac{1}{4}\left[\sqrt{\left(\frac{550}{\sqrt{3}}\right)^2 - (4 \times 28.86)^2} - \frac{400}{\sqrt{3}}\right] = 16.21$$

$$\mathbf{I}_a = I_a \angle \phi = I_a \cos\phi + j\, I_a \sin\phi$$

$$= 28.86 + j\,16.21 = 33.1 \angle 29.32° \text{ A}$$

Power factor $\cos\phi = \cos 29.32° = 0.8719$ (leading)

EXAMPLE 5.18. *A 3-phase, 100 hp, 440 V, star-connected synchronous motor has a synchronous impedance per phase of $0.1 + j\,1\Omega$. The excitation and torque losses are 4 kW and may be assumed constant. Calculate the current, power factor and efficiency when operating at full load with an excitation equivalent to 400 line volts.*

SOLUTION. $\mathbf{Z}_s = 0.1 + j\,1 = 1.005 \angle 84.28°\ \Omega,\ \theta_z = 84.28°$

Gross output $= 100 \times 746 + 4000 = 78600$ W

$$P_{om} = \frac{3\,V\,E_f}{Z_s}\cos(\theta_z - \delta) - \frac{3\,E_f^2}{Z_s^2}R_a$$

$$78600 = \frac{3\,(440/\sqrt{3})\,(400/\sqrt{3})}{1.005}\cos(84.28° - \delta) - \frac{3\,(400/\sqrt{3})^2 \times 0.1}{1.005}$$

$$78600 + 15920 = 175124 \cos(84.28° - \delta)$$

$$\cos(84.28° - \delta) = \frac{94520}{175125} = 0.5397 = \cos 57.33°$$

$$84.28° - \delta = 57.33°, \quad \delta = 84.28° - 57.33° = 26.95°$$

$$\mathbf{E}_f = E_f \angle -\delta = \frac{400}{\sqrt{3}} \angle -26.95°° = 205.9 - j\,104.7$$

$$V_p = \frac{440}{\sqrt{3}} = 254,\ \mathbf{V}_p = V_p \angle 0° = 254 + j\,0$$

$$\mathbf{I}_a = \frac{\mathbf{V} - \mathbf{E}_f}{\mathbf{Z}_s} = \frac{254 + j\,0 - 205.9 + j\,104.7}{1.005 \angle 84.28°}$$

$$= \frac{48.1 + j\,104.7}{1.005 \angle 84.28°} = \frac{115.22 \angle 65.32°}{1.005 \angle 84.28°}$$

$$\mathbf{I}_a = 114.64 \angle -18.94° \text{ A}$$

Power factor $\cos\phi = \cos(-18.94°) = 0.9459$

Efficiency $= \dfrac{746 \times 100}{\sqrt{3} \times 440 \times 114.64 \times 0.9459}$

$= 0.9027$ per $= 90.27\%$

EXAMPLE 5.19. *A 3-phase, 50 MVA, 11 kV, 50 Hz, salient pole synchronous motor has reactances $X_d = 0.8$ pu and $X_q = 0.4$ pu. It draws rated current at a supply power factor of 0.8 lagging. Rotational losses are 0.15 pu and armature resistance losses are neglected.*

(a) *Determine the excitation voltage.*

(b) *Determine the power due to field excitation and that due to saliency of the machine.*

(c) *If the field current is reduced to zero, will the machine stay in synchronism ?*

(d) *If the shaft load is removed before the field current is reduced to zero, determine the resultant supply current in pu and the supply power factor. Draw the phasor diagram for the machine in this condition.*

SOLUTION. (*a*) Let V be taken as reference phasor.

$\therefore \quad V = 1 \angle 0^\circ$ pu

$\cos\phi = 0.8$ lagging, $\quad \phi = -36.9^\circ$

$I_a = 1 \angle -36.9^\circ$ A

From Eq. (5.8.5)

$$\tan\delta = \frac{I_a X_q \cos\phi}{V - I_a X_q \sin\phi}$$

$$= \frac{1 \times 0.4 \times 0.8}{1 - 1 \times 0.4 \times 0.6} = 0.421$$

$$\delta = 22.83^\circ$$

From Fig. 5.5, $\quad \psi = \phi - \delta = 36.9^\circ - 22.83^\circ = 14.07^\circ$

$$I_d = I_a \sin\psi = 1 \times \sin 14.07^\circ = 0.243 \text{ pu}$$

$$I_q = I_a \cos\psi = 1 \times \cos 14.07^\circ = 0.97 \text{ pu}$$

From Eq. (5.8.1), $\quad E_f = V\cos\delta - I_d X_d$

$$= 1 \times \cos 22.83^\circ - 0.243 \times 0.8$$

$$= 0.727 \text{ pu}$$

(*b*) From Eq. (3.40.7), power due to field excitation

$$P_f = \frac{VE}{X_d}\sin\delta$$

$$= \frac{1 \times 0.727}{0.8}\sin 22.83^\circ = 0.3526 \text{ pu}$$

Power due to saliency of the machine

$$P_{reluctance} = \frac{V^2}{2 X_d X_q}(X_d - X_q)\sin 2\delta$$

$$= \frac{1^2 \times (0.8 - 0.4)}{2 \times 0.8 \times 0.4}\sin 2 \times 22.83^\circ$$

$$= 0.447 \text{ pu}$$

(*c*) Power output

$$P_0 = VI_a\cos\phi = 1 \times 1 \times 0.8 = 0.8 \text{ pu}$$

Power due to saliency of the machine

$$P_{rel} = \frac{V^2 (X_d - X_q)}{2 X_d X_q}\sin 2\delta$$

P_{rel} is maximum when $\sin 2\delta = 1$.

$$\therefore \quad (P_{rel})_{\max} = \frac{V^2 (X_d - X_q)}{2\, X_d\, X_q}$$

$$= \frac{1^2\,(0.8 - 0.4)}{2 \times 0.8 \times 0.4} = 0.625 \text{ pu}$$

Since the output power is greater than the power the machine can develop, the machine will lose synchronism.

(*d*) No-load power = 0.15 pu

$$0.15 = 0.625 \sin 2\delta, \; \delta = 6.94°$$

With $E_f = 0$ and $R_a = 0$, $V \cos \delta = I_d X_d$ and $V \sin \delta = I_q X_q$

The phasor diagram is shown in Fig. 5.14.

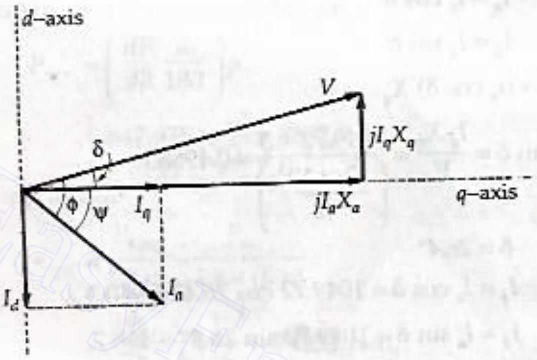


Fig. 5.14.

With V as reference phasor, the q-axis is $\delta = 6.94°$ behind it.

$$I_d = \frac{V}{X_d} \cos \delta = \frac{1}{0.8} \cos 6.94° = 1.24 \text{ pu}$$

$$I_q = \frac{V}{X_q} \sin \delta = \frac{1}{0.4} \sin 6.94° = 0.302 \text{ pu}$$

$$I_a = \sqrt{I_d^2 + I_q^2} = \sqrt{1.24^2 + 0.302^2} = 1.276 \text{ pu}$$

$$\tan \psi = \frac{I_d}{I_q} = \frac{1.24}{0.302}, \; \psi = \tan^{-1} \frac{1.24}{0.302} = 76.31°$$

$$\phi = \psi + \delta$$
$$= 76.31° + 6.94°$$
$$= 83.25°$$

Power factor $= \cos \phi = \cos 83.25°$
$= 0.1175$ lagging

EXAMPLE 5.20. *A 20 MVA, 3-phase, star-connected, 11-kV, 12-pole, 50-Hz salient-pole synchronous motor has reactances of $X_d = 5\ \Omega$, $X_q = 3\ \Omega$. At full-load, unity power factor and rated voltage determine.*

(*a*) *the excitation voltage,*

(b) *active power,*

(c) *synchronizing power per electrical degree and the correspon*

(d) *synchronizing power per mechanical degree and the correspon*

(e) *maximum value of the power angle and the corresponding pow*

SOLUTION. $S = \sqrt{3}\, V_L I_a$

$$20 \times 10^6 = \sqrt{3} \times 11 \times 10^3 I_a$$

$$I_a = \frac{20 \times 10^6}{\sqrt{3} \times 11 \times 10^3} = 1049.72 \text{ A}$$

From the phasor diagram at unity power factor

$$V \sin \delta = I_q X_q$$

$$I_q = I_a \cos \delta$$

$$I_d = I_a \sin \delta$$

$$\therefore \quad V \sin \delta = (I_a \cos \delta) X_q$$

$$\tan \delta = \frac{I_a X_q}{V} = \frac{1049.72 \times 3}{\left(\frac{11 \times 10^3}{\sqrt{3}}\right)} = 0.49585$$

$$\delta = 26.4°$$

$$I_q = I_a \cos \delta = 1049.72 \cos 26.4° = 940.3$$

$$I_d = I_a \sin \delta = 1049.72 \sin 26.4° = 466.7.$$

(a) Excitation voltage per phase

$$E = V \cos \delta + I_d X_d$$

$$= \frac{11 \times 10^3}{\sqrt{3}} \cos 26.4° + 466.7 \times 5$$

$$= 5688 + 2333.5 = 8021.5 \text{ V}.$$

(b) Active power for 3 phases

$$P = \frac{3VE}{X_d} \sin \delta + \frac{3V^2}{2} \frac{(X_d - X_q)}{X_d X_q} \sin 2\delta$$

$$= \frac{3 \times 11 \times 10^3 \times 8021.5}{\sqrt{3} \times 5} \sin 26.4° + \frac{3}{2}\left(\frac{11 \times 10^3}{\sqrt{3}}\right)^2 \left(\frac{5-3}{5 \times 3}\right) \sin 52.8°$$

$$= 13591127 + 6423615$$

$$= 200147.42 \text{ W} = 2001.47 \text{ kW}.$$

(c) Synchronizing power per electrical degree

$$P_{syn_2} = \frac{dP}{d\delta} \frac{\pi}{180} \text{ watts}$$

$$= \left[\frac{3EV}{X_d} \cos \delta + 3V^2 \left(\frac{X_d - X_q}{X_d X_q}\right) \cos 2\delta\right] \frac{\pi}{180}$$

$$= \left[\frac{3 \times 8021.5 \times 6350}{5} \cos 26.4° + 3 \times (6350)^2 \left(\frac{5-3}{5 \times 3} \right) \cos 52.8° \right] \frac{\pi}{180} \text{ W}$$

$$= 647.975 \text{ kW/elec degree}$$

Synchronizing torque

$$\tau_{syn_2} = \frac{\text{synchronizing power per elec. degree}}{2\pi\, n_s}$$

$$\tau_{syn_2} = \frac{P_{syn_2}}{2\pi\, n_s} = \frac{P_{syn_2}}{2\pi\,(f/p)}$$

$$= \frac{647.975 \times 10^3}{2\pi\,(50/6)} = 12375 \text{ Nm.}$$

(*d*) Synchronizing per mechanical degree

$$P_{syn_4} = \left(\frac{dP}{d\delta} \frac{\pi}{180} \right) p$$

$$= 647.975 \times \frac{12}{2} = 3887.8 \text{ kW}$$

Corresponding torque

$$\tau_{syn_4} = \frac{P_{syn_4}}{2\pi\, n_s} = \frac{P_{syn_4}}{2\pi \times (f/p)}$$

$$= \frac{3887.8 \times 10^3}{2\pi \times 50/6} = 74251 \text{ Nm.}$$

EXAMPLE 5.21. *A 20 MVA, 3-phase, star-connected 11 kV, 12-pole, 50 Hz salient-pole synchronous motor has a direct-axis reactance of 5 Ω and a quadrature-axis reactance of 3 Ω per phase, the armature resistance being negligible. At rated load, unity power factor and rated voltage determine*

(*a*) *excitation voltage,*

(*b*) *synchronizing power per electrical degree and corresponding torque,*

(*c*) *synchronizing power per mechanical degree and corresponding torque and*

(*d*) *the maximum load on the motor before it would lose synchronism.*

SOLUTION. $S = \sqrt{3}\, V_L\, I_a$

$$I_a = \frac{S}{\sqrt{3}\, V_L} = \frac{20 \times 10^6}{\sqrt{3} \times 11 \times 10^3} = 1049.7 \text{ A}$$

From the phasor diagram of Fig. 5.7c,

$$V \sin\delta = I_q X_q$$

$$I_q = I_a \cos\delta$$

$$I_d = I_a \sin\delta$$

$$\therefore \quad V \sin\delta = I_a \cos\delta\, X_q$$

$$\tan\delta = \frac{I_a X_q}{V}$$

$$= \frac{1049.72 \times 3}{\left(\dfrac{11 \times 10^3}{\sqrt{3}}\right)} = 0.49585$$

$$\delta = 26.4°$$

$$I_q = I_a \cos\delta = 1049.72 \times \cos 26.4° = 940.45 \text{ A}$$

$$I_d = I_a \sin\delta = 1049.72 \sin 26.4° = 466.32 \text{ A}$$

(a) $E_f = V\cos\delta + I_d X_d$

$$= 6350 \cos 26.4° + 466.32 \times 5 = 8021 \text{ V}$$

(b) $P_{syn_1} = \left(\dfrac{dP}{d\delta}\right) \cdot \dfrac{\pi}{180}$ W/elec. degree

$$= \left[\frac{3V E_f}{X_d}\cos\delta + 3V^2\left(\frac{X_d - X_q}{X_d X_q}\right)\cos 2\delta\right]\frac{\pi}{180}$$

$$= \left[\frac{3 \times 6350 \times 8021}{5}\cos 26.4° + 3 \times 6350^2\left(\frac{5-3}{5 \times 3}\right)\cos 52.8\right]\frac{\pi}{180}$$

$$= 648292 \text{ W/elec. degree} = 648.292 \text{ kW/elec. deg.}$$

$$p\, n_s = f$$

$$n_s = \frac{f}{p} = \frac{50}{6} = \frac{25}{3} \text{ rps}$$

Synchronizing torque

$$\tau_{syn} = \frac{P_{syn_1}}{2\pi n_s} = \frac{648292 \times 3}{2\pi \times 25} = 12381.5 \text{ Nm}$$

(c) Synchronizing power per mechanical degree

$$P_{syn_2} = \frac{dP}{d\delta} \cdot \frac{\pi}{180} \times p = 648.292 \times 6 = 3889.752 \text{ kW}$$

Synchronizing torque

$$\tau_{syn} = \frac{P_{syn_2}}{2\pi n_s} = \frac{38889752 \times 3}{2\pi \times 25} = 74288.79 \text{ Nm}$$

(d) $P_{3\phi} = \dfrac{3V E_f}{X_d}\sin\delta + \dfrac{3V^2}{2}\left(\dfrac{X_d - X_q}{X_d X_q}\right)\sin 2\delta$

For maximum power,

$$\frac{d P_{3\phi}}{d\delta} = 0$$

$$\therefore \quad \frac{3V E_f}{X_d}\cos\delta + 3V^2\left(\frac{X_d - X_q}{X_d X_q}\right)\cos 2\delta = 0$$

$$\frac{E_f}{X_d}\cos\delta + \frac{V}{X_d X_q}(X_d - X_q)\cos 2\delta = 0$$

$$\frac{8020.8}{5}\cos\delta+\frac{6350}{5\times 3}(5-3)\cos 2\delta=0$$

$$1604\cos\delta+846.666\cos 2\delta=0$$

$$\cos 2\delta+\frac{1604}{846.66}\cos\delta=0$$

$$2\cos^2\delta+1.89448\cos\delta-1=0$$

$$\cos\delta=\frac{-1.89448\pm\sqrt{(1.89448)^2+8}}{4}$$

Taking the positive sign only

$$\cos\delta=0.337448$$
$$\delta=67.824°$$

This, the maximum value of power the torque angle is 67.824°.

$$\therefore \quad P_{max}=\frac{3\times 6350\times 8020.8}{5}\sin 67.824°+\frac{3\times(6350)^2}{2}\left(\frac{5-3}{5\times 3}\right)\sin 135.648°$$

$$=33936296 \text{ W}=33936.296 \text{ kW}$$

EXAMPLE 5.22. *A 6.6 kV, 3-phase, star-connected synchronous motor is running in parallel with an infinite bus. Its direct-and quadrature-axis synchronous reactances are 10 Ω and 5 Ω respectively. If the field current is reduced to zero, find the maximum load that can be put on the synchronous motor. Also calculate the armature current and the maximum power. Neglect armature resistance.*

SOLUTION. $P=\frac{E_f V}{X_d}\sin\delta+\frac{V^2}{2}\left(\frac{1}{X_q}-\frac{1}{X_d}\right)\sin 2\delta$

When the field current becomes zero, $E_f=0$

$$\therefore \quad P=\frac{V^2}{2}\left(\frac{1}{X_q}-\frac{1}{X_d}\right)\sin 2\delta$$

For maximum reluctance power,

$$\sin 2\delta=1, \qquad \delta=45°$$

$$\therefore \quad P_{max}=\frac{V^2}{2}\left(\frac{1}{X_q}-\frac{1}{X_d}\right)$$

$$=\frac{1}{2}\left(\frac{6.6\times 10^3}{\sqrt{3}}\right)^2\left(\frac{1}{5}-\frac{1}{10}\right)=726\times 10^3 \text{ W per phase}$$

Total maximum power for all the three phases

$$=3\times 726\times 10^3 \text{ W}=2178 \text{ kW}$$

For maximum power, $\delta=45°$ and

$$I_d=\frac{V\cos\delta}{X_d}=\frac{6.6\times 10^3}{\sqrt{3}}\times\frac{\cos 45°}{10}=269.45 \text{ A}$$

$$I_q=\frac{V\sin\delta}{X_q}=\frac{6.6\times 10^3}{\sqrt{3}}\times\frac{\sin 45°}{10}=538.90 \text{ A}$$

Armature current at maximum power

$$I_a = \sqrt{I_d^2 + I_q^2} = \sqrt{(269.45)^2 + (538.90)^2} = 602.5 \text{ A}$$

EXAMPLE 5.23. *A 3-phase, 11-kV, 50-Hz, 10-pole, 200-kW star-... pole synchronous motor has $X_d = 1.2$ pu and $X_q = 0.8$ pu. It operates at rated ... power factor leading. Determine*

(a) *the internal emf and the load angle,*

(b) *the maximum mechanical torque.*

SOLUTION. $V_p = \dfrac{11000}{\sqrt{3}} = 6350 \text{ V}$

$$3V_p I_a \cos\phi = 200 \times 10^3$$

$$I_a = \frac{200 \times 10^3}{3 \times 6350 \times 0.98} = 10.7 \text{ A}$$

For a synchronous motor operating at leading p.f. $\cos\phi$

$$\tan\delta = \frac{I_a X_q \cos\phi + I_a R_a \sin\phi}{V_p + I_a X_q \sin\phi - I_a R_a \cos\phi}$$

Here $R_a = 0$,

$$\therefore \quad \tan\delta = \frac{I_a X_q \cos\phi}{V_p + I_a X_q \sin\phi}$$

$$X_{pu} = \frac{X_\Omega}{(V_p/I_a)}$$

$$X_\Omega = X_{pu} \frac{V_p}{I_a}$$

$$\therefore \quad \tan\delta = \frac{(I_a \cos\phi)\, X_{q\,pu}\, (V_p/I_a)}{V_p + (I_a \sin\phi)\, X_{q\,pu}\, (V_p/I_a)}$$

$$= \frac{X_{q\,pu} \cos\phi}{1 + X_{q\,pu} \sin\phi}$$

$$= \frac{0.8 \times 0.98}{1 + 0.8 \times 0.199}$$

$$\therefore \quad \delta = 34°$$

$$V_p \sin\delta = I_q X_q$$

$$= I_q \,.\, X_{q\,pu} \frac{V_p}{I_a}$$

$$\therefore \quad I_q = \frac{I_a \sin\delta}{X_{q\,pu}} = \frac{10.7 \sin 34°}{0.8} = 7.479 \text{ A}$$

$$I_d = \sqrt{I_a^2 - I_q^2} = \sqrt{(10.7)^2 - (7.479)^2} = 7.65 \text{ A}$$

$$E_{fp} = V_p \cos\delta + I_d X_d$$

$= V_p \cos\delta + I_a X_{d\,pu} \cdot$ ...

$= 6350 \cos 34° + 7.65$ ...

$E_f = \sqrt{3}\, E_{fp} = \sqrt{3} \times 1071$ ...

$X_d = X_{d\,pu} \frac{V_p}{I_a} = 1.2 \times \frac{63...}{10...}$

$X_q = X_{q\,pu} \frac{V_p}{I_a} = 0.8 \times \frac{635...}{10....}$

$\frac{1}{X_q} - \frac{1}{X_d} = \frac{1}{474.766} - \frac{1}{712.14...}$

... power developed pe...

$P_{mp} = \frac{E_{fp} V_p}{X_d} \sin\delta + \frac{V...}{2...}$

$= \frac{10712 \times 6350}{712.149}$ si...

$= 95515.4 \sin\delta +$ ...

$= 66534 \text{ W} = 66.5$...

...phase mechanical powe...

$P_{m3\phi} = 3\, P_{mp} = 3 \times 66.5$...

...maximum power develope...

...5515.4 $\cos\delta + 14153.2$ ...

...$\delta - 1) \times 2 \times 14153.2 + 955$...

$2\cos^2\delta + \frac{95515.4}{2 \times 14153.2}$ ...

$2\cos^2\delta + 3.3743$...

$\cos\delta = \frac{1}{4}[-3.37434 \pm \sqrt{(3}$...

$\cos\delta = 0.2571, \delta = 75.1°$

$P_{m(max)} = 95515.4 \sin 75.1°$ ...

$= 99337 \text{ W} = 99.337$...

Maximum 3-phase power

$= 3\, P_{m\,(max)} = 3 \times 99$...

...is neglected

$P_{mp_{(max)}} = 95515$...

$P_{m\,3\phi_{(max)}} = 3 \times 95$...

$$= V_p \cos\delta + I_a X_{d\,pu} \cdot \frac{V_p}{I_a}$$

$$= 6350 \cos 34° + 7.65 \times 1.2 \times \frac{6350}{10.7} = 10712 \text{ V}$$

$$E_{fl} = \sqrt{3}\, E_{fp} = \sqrt{3} \times 10712 = 18553 \text{ V}$$

$$X_d = X_{d\,pu} \frac{V_p}{I_a} = 1.2 \times \frac{6350}{10.7} = 712.149\ \Omega$$

$$X_q = X_{q\,p} \frac{V_p}{I_a} = 0.8 \times \frac{6350}{10.7} = 474.766\ \Omega$$

$$\frac{1}{X_q} - \frac{1}{X_d} = \frac{1}{474.766} - \frac{1}{712.149} = 7.02 \times 10^{-4}$$

Mechanical power developed per phase

$$P_{mp} = \frac{E_{fp} V_p}{X_d} \sin\delta + \frac{V_p^2}{2}\left(\frac{1}{X_q} - \frac{1}{X_d}\right) \sin 2\delta$$

$$= \frac{10712 \times 6350}{712.149} \sin 34° + \frac{(6350)^2}{2} (7.02 \times 10^{-4}) \times \sin 68°$$

$$= 95515.4 \sin\delta + 14153.2 \sin 2\delta$$

$$= 66534 \text{ W} = 66.534 \text{ kW}$$

Total 3-phase mechanical power

$$P_{m\,3\phi} = 3\,P_{mp} = 3 \times 66.534 = 199.6 \text{ kW}$$

For maximum power developed $\dfrac{d\,P_{mp}}{d\,\delta} = 0$

or
$$95515.4 \cos\delta + 14153.2 \times 2 \cos 2\delta = 0$$

$$(2\cos^2\delta - 1) \times 2 \times 14153.2 + 95515.4 \cos\delta = 0$$

$$2\cos^2\delta + \frac{95515.4}{2 \times 14153.2} \cos\delta - 1 = 0$$

$$2\cos^2\delta + 3.37434 \cos\delta - 1 = 0$$

$$\cos\delta = \frac{1}{4}[-3.37434 \pm \sqrt{(3.37434)^2 - 8}]$$

$$\cos\delta = 0.2571,\ \delta = 75.1°$$

$$\therefore\ P_{mp\,(\max)} = 95515.4 \sin 75.1° + 14153.2 \sin 2 \times 75.1°$$

$$= 99337 \text{ W} = 99.337 \text{ kW}$$

Maximum 3-phase power

$$= 3\,P_{m\,(\max)} = 3 \times 99.337 = 298 \text{ kW}$$

If saliency is neglected

$$P_{mp_{(\max)}} = 95515.4 \text{ W power phase}$$

$$P_{m\,3\phi_{(\max)}} = 3 \times 95515 \text{ W} = 286.5 \text{ kW}$$

EXAMPLE 5.24. *A 125 MVA, 3-phase, star-connected 11 kV, 4-pole, 50 Hz synchronous motor has a reactance of 0.15 pu and negligible armature resistance. Calculate the synchronizing power per mechanical degree when it supplies full load at 11 kV and 0.8 power factor leading.*

SOLUTION. $S = \sqrt{3}\, V_L I_a$

$$I_a = \frac{125 \times 10^6}{\sqrt{3} \times 11 \times 10^3} = 6561 \text{ A}$$

$$V_p = \frac{V_l}{\sqrt{3}} = \frac{11 \times 10^3}{\sqrt{3}} = 6350 \text{ V}$$

$$X_{s\,pu} = \frac{X_s \text{ in ohms}}{V_p / I_a}$$

$$X_{s\,\Omega} = X_{s\,pu} \frac{V_p}{I_a} = 0.15 \times \frac{6350}{6561} = 0.14518\ \Omega$$

$$\mathbf{E}_f = \mathbf{V}_p - \mathbf{I}_a \mathbf{Z}_s$$
$$= V_p - (I_a \angle \phi)(X_s \angle 90^\circ)$$
$$= V_p - I_a X_s \angle 90^\circ + \phi$$
$$= V_p - I_a X_s [\cos(90 + \phi) + j \sin(90 + \phi)]$$
$$= (V_p + I_a X_s \sin\phi) - j\, I_a X_s \cos\phi$$
$$= 6350 + 6561 \times 0.14518 \times 0.6 - j\, 6561 \times 0.14518 \times 0.8$$
$$= 6921.5 - j\, 762 = 6963.4 \angle 6.2826^\circ$$

$E_f = 6963.4$ V, $\quad \delta = 6.2826^\circ$

Synchronizing power per mechanical degree

$$P_{syn} = \left(\frac{dP}{d\delta}\right) p \frac{\pi}{180}$$

since
$$P = \frac{3 V_p E_f}{X_s} \sin\delta$$

$$P_{syn} = \left(\frac{3 V_p E_f}{X_s} \cos\delta\right) p \frac{\pi}{180}$$

$$= \left(\frac{3 \times 6350 \times 6963.4}{0.14518} \cos 6.2826^\circ\right) \times \frac{6\pi}{180}$$

$$= 95109087 \text{ W} = 95.10 \text{ MW}$$

EXAMPLE 5.25. *A 3-phase, 3.3 kV, 2-pole, 3000 r.p.m. 934 kW synchronous motor has an efficiency of 0.95 pu and delivers full-load torque with its excitation adjusted so that the input power factor is unity. The moment of inertia of the motor and its load is 30 kg m^2 and its synchronous impedance is $(0 + j\, 11.1)\ \Omega$. Determine the period of undamped oscillation on full load for small changes in load angle.*

SYNCHRONOUS MOT

SOLUTION. Power input =

$934 \times 10^3 =$

$I =$

Phase voltage $V_p = \frac{3.3 \times 1}{\sqrt{3}}$

Taking the phase voltage

$\mathbf{E}_f = \mathbf{V}_p - \mathbf{I}$

$= 1905 \angle$

$= 1905 -$

$= 1905 -$

The synchronizing torque

$\tau_{syn} = \frac{3}{2\pi n_s}$

$= \frac{3}{2\pi \times}$

$= 3.125$

The undamped frequency

$f = \frac{1}{2\pi}\sqrt{}$

The period of oscillation

$T = \frac{1}{f} = 2\pi$

$= 2\pi\sqrt{}$

EXAMPLE 5.26. *A synchron... lagging to 0.95 laggin... the leading kVAr supplie... power factor at which the motor op...*

SOLUTION. Load, $P_1 =$

Motor load, $P_2' = 100$ kW

Power factor of the given

Power factor of combine

Total load, $P = P_1 + P_2 =$

In Fig. 5.15, Δ OAB is the

$OA =$

SOLUTION. Power input $= (\sqrt{3}\, V_L\, I \cos\phi) \times \text{efficiency}$

$$934 \times 10^2 = \sqrt{3}\,(3.3 \times 10^3 \times I \times 1) \times 0.95$$

$$I = \frac{934 \times 10^3}{\sqrt{3} \times 3.3 \times 10^3 \times 0.95} = 172 \text{ A}$$

Phase voltage $V_p = \dfrac{3.3 \times 10^3}{\sqrt{3}} = 1905$ V

Taking the phase voltage $\mathbf{V}_p$ as reference

$$\begin{aligned}\mathbf{E}_f &= \mathbf{V}_p - \mathbf{I}\, X_s \\ &= 1905 \angle 0^\circ - (172 \angle 0^\circ) \times 11.1 \angle 90^\circ \\ &= 1905 - 1909 \angle 90^\circ \\ &= 1905 - j\,1909 = 2697 \angle -45^\circ \text{ V}\end{aligned}$$

The synchronizing torque coefficient is

$$\tau_{syn} = \frac{3}{2\pi n_s} \times \frac{V_p E_f}{X_s} \cos\delta$$

$$= \frac{3}{2\pi \times \dfrac{3000}{60}} \times \frac{3.3 \times 10^3}{\sqrt{3}} \times \frac{2697}{11.1} \cos 45^\circ$$

$$= 3.125 \times 10^3 \text{ Nm/rad}$$

The undamped frequency of oscillation is

$$f = \frac{1}{2\pi}\sqrt{\frac{\tau_{syn}}{J}}$$

The period of oscillation is

$$T = \frac{1}{f} = 2\pi \sqrt{\frac{J}{\tau_{syn}}}$$

$$= 2\pi \sqrt{\frac{30}{3.125 \times 10^3}} = 0.616 \text{ s}$$

EXAMPLE 5.26. *A synchronous motor improves the power factor of a load of 500 kW from 0.707 lagging to 0.95 lagging. Simultaneously the motor carries a load of 100 kW. Find (i) the leading kVAr supplied by the motor, (ii) kVA rating of the motor, and (iii) power factor at which the motor operates.*

SOLUTION. Load, $P_1 = 500$ kW

Motor load, $P_2 = 100$ kW

Power factor of the given load, $\cos\phi_1 = 0.707$ (lag)

Power factor of combined load, $\cos\phi_2 = 0.95$ (*lag*)

Total load, $P = P_1 + P_2 = 500 + 100 = 600$ kW

In Fig. 5.15, Δ OAB is the power triangle for the given load. Here,

$$OA = P_1 = 500 \text{ kW}, \quad \angle AOB = \phi_1$$

Δ OCD is the power triangle for the combined load. Here,

$$OD = OA + AD = 500 + 600 \text{ kW}, \quad \angle COD = \phi_2.$$

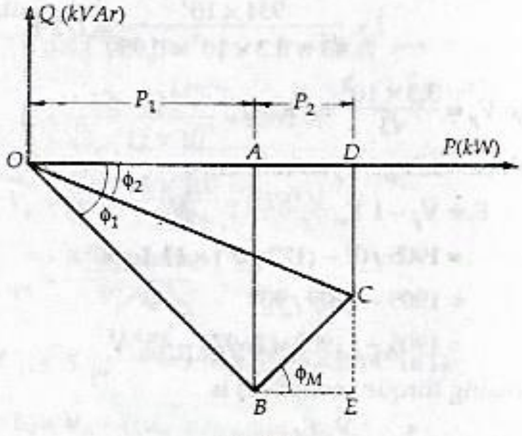


Fig. 5.15.

The power triangle for the synchronous motor is Δ BEC. In this triangle

$BE = P_2 = 100$ kW

EC = leading kVAr supplied by the motor

BC = kVA rating of the synchronous motor

$\angle CBE = \phi_M$ = power factor angle at which the motor operates.

(*i*) Leading kVAr supplied by the motor

$$= EC = DE - DC = AB - DC$$

$$= P_1 \tan \phi_1 - (P_1 + P_2) \tan \phi_2$$

$$= 500 \tan (\cos^{-1} 0.707) - 600 \tan (\cos^{-1} 0.95)$$

$$= 500 \times 1 - 600 \times 0.32868 = 302.79 \text{ kVAr}$$

(*ii*) kVA rating of the motor

$$= BC = \sqrt{(BE)^2 + (EC)^2} = \sqrt{100^2 + (302.79)^2} = 318.875 \text{ kVA}$$

(*iii*) Power factor of the motor,

$$\cos \phi_M = \frac{\text{motor kW}}{\text{motor kVA}} = \frac{BE}{BC} = \frac{100}{318.875} = 0.3136 \text{ (leading)}$$

Alternative method

Let S_1 = kVA of the load

S_M = kVA of the synchronous motor

S_T = total kVA supplied.

$$S_1 = P_1 - j\,Q = P_1 - j\,P_1 \tan \phi_1$$

$$S_M = P_M + j\,Q_M = P_2 + j\,P_2 \tan \phi_M$$

$$S_T = P_T - j\,Q_T = (P_1 + P_2) - j\,(P_1 + P_2) \tan \phi_2$$

It is to be noted that the j term in the expression for S_M is positive because the synchronous motor supplies leading kVAr.

Now $S_T = S_1 + S_M$

$$(P_1 + P_2) - j(P_1 + P_2)\tan\phi_2 = P_1 - j P_1 \tan\phi_1 + P_2 + j P_2 \tan\phi_M$$

$$(P_1 + P_2)\tan\phi_2 = P_1 \tan\phi_1 - P_2 \tan\phi_M$$

$$\tan\phi_M = \frac{1}{P_2}[P_1 \tan\phi_1 - (P_1 + P_2)\tan\phi_2]$$

The p.f. at which the motor operates is $\cos\phi_M$ (leading).

Leading kVAr supplied by the motor, $Q_M = P_M \tan\phi_M$

In this problem

$$\tan\phi_M = \frac{1}{100}[500\tan(\cos^{-1} 0.707) - (500 + 100)\tan(\cos^{-1} 0.95)]$$

$$= \frac{1}{100}(500 \times 1 - 600 \times 0.32868) = 3.0279$$

Power factor of the motor, $\cos\phi_M = 0.3136$ leading.

Leading kVAr supplied by the motor,

$$Q_M = P_M \tan\phi_M = 100 \times 3.0279 = 302.79 \text{ kVAr}$$

$$S_M = P_M + jQ_M = P_M + jP_M \tan\phi_M = 100 + j\,302.79$$

$$= \sqrt{(100)^2 + (302.79)^2} = 318.875 \text{ kVA.}$$

EXAMPLE 5.27. *A 400 V, 3 ϕ installation takes a current of 36 A at 0.8 p.f. (lagging). A synchronous motor improves the overall p.f. to 0.92 (lagging). Simultaneously the synchronous motor drives a 15 hp (metric) load at an efficiency of 0.85. Determine (a) the power factor of the synchronous motor, (b) the leading kVAr supplied by the motor, and (c) the kVA rating of the motor.*

SOLUTION. Active power requirement of the installation

$$P_1 = \sqrt{3}\, V_L I_L \cos\phi_1 = \sqrt{3} \times 400 \times 36 \times 0.8 = 19953 \text{ W} = 19.953 \text{ kW}$$

Input to the synchronous motor

$$P_2 = \frac{\text{output}}{\text{efficiency}} = \frac{15 \times 735.5}{0.85} = 129.79 \text{ W} = 12.979 \text{ kW}$$

(*a*) As in Example 5.26

$$\tan\phi_M = \frac{1}{P_2}[P_1 \tan\phi_1 - (P_1 + P_2)\tan\phi_2]$$

$$= \frac{1}{12.979}[19.953\tan(\cos^{-1} 0.8) - (19.953 + 12.979)\tan(\cos^{-1} 0.92)]$$

$$= \frac{1}{12.979}[19.953 \times 0.75 - 32.932 \times 0.462] = \frac{1}{12.979}(14.9647 - 14.0289)$$

$$= 0.0721$$

$\therefore \cos\phi_M = 0.9974$ leading (*b*) Leading kVAr supplied by the motor

$$Q_M = P_M \tan \phi_M = 12.979 \times 0.0721 = 0.9358 \text{ kVAr}$$

(*c*) kVA rating of the motor

$$S_M = P_M + j\,Q_M = 12.979 + j\,0.9358$$

$$S_M = \sqrt{(12.979)^2 + (0.9358)^2} = 13.0127 \text{ kVA}$$

EXERCISES

5.1 Explain the principle of operation of a 3-phase synchronous motor.

5.2 Describe briefly the effect of varying excitation upon armature current and power factor of a synchronous motor when input power to the motor is maintained constant.

5.3 Explain the operation of a synchronous motor under (*a*) constant load, varying excitation, (*b*) constant excitation, varying load. Discuss how a synchronous motor can function as a synchronous capacitor.

5.4 State the applications of synchronous motors. Compare synchronous motor and induction motor drives.

5.5 Explain hunting of a synchronous machine. What is the purpose of damper windings in a synchronous machine ?

5.6 Why is synchronous motor not self-starting ? What methods are generally used to start the synchronous motors ?

5.7 What are V-curves of a synchronous motor ? What are the main characteristics of a synchronous motor ?

5.8 A 3-phase, 11000 V, star-connected synchronous motor takes a load current of 100 A. The effective reactance and resistance per phase are 30 Ω and 0.8 Ω respectively. Find the power supplied to the motor and the induced e.m.f. for a (*a*) power factor lagging, (*b*) 0.8 power factor leading.

[(*a*) 1524.205 kW, 8774 V ; (*b*) 1524.205 kW, ...

5.9 A 3-phase, 6600 V, 50 Hz star-connected synchronous motor takes 50 A current. The resistance and synchronous reactance per phase are 1 Ω and 20 Ω respectively. Find the power supplied to the motor and the induced e.m.f. for a power factor of (*a*) 0.8 lagging, (*b*) 0.8 leading. [(*a*) 457261 W, 5649 V ; (*b*) 457261 W, ...

5.10 A 10 h.p. (metric), 400 V, 3-phase, star-connected synchronous motor has a synchronous impedance per phase of $(0.35 + j2.8)$ Ω. Find the angle of retard and the voltage to which the motor must be excited to give a full-load output at ... leading power factor. Assume an efficiency of 88%. [431 ...

5.11 A 3-phase, 40 kW, 400 V, 50 Hz star-connected synchronous motor has a full-load efficiency of 90%. The synchronous impedance of the motor is $(0.25 + j1...)$ Ω per phase. If the excitation of the motor is adjusted to give leading p.f. of 0.8, calculate the induced e.m.f. and total mechanical power developed at full load.

[545.5 V, 41.3...

5.12 A 15 kW, 3-phase, 400 V, star-connected synchronous motor operating on full load from infinite busbars, has its excitation so adjusted that the power factor is 0.8 lagging. Load being constant, excitation is now increased by 25%. Synchronous reactance is 1.0 per unit. Find the new power factor. [0.965 lagging...

5.13 The synchronous reactance per phase of a 3-phase star-connected 6600 V synchronous motor is 10 Ω. For a certain load, the input is 900 kW and the induced line e.m.f. is 8900 V. Determine the line current. Neglect resistance. [149.4 A]

5.14 A 2200 V, 373 kW, 3-phase, star-connected synchronous motor has a resistance of 0.3 Ω and a synchronous reactance of 3 Ω per phase respectively. Determine the induced e.m.f. per phase if the motor works on full load with an efficiency of 94% and a power factor of 0.8 leading. [1510 $\angle 12.7°$ V]

5.15 A 100 h.p., 440 V, 1000 r.p.m., 50 Hz, 3 phase synchronous motor has a star-connected stator and is designed to operate at unity power factor at full-load. The rated line current is 106 A. The armature resistance is 0.09 Ω per phase and the synchronous reactance is 2.25 Ω per phase.

(*a*) Find the generated voltage per phase, the torque angle, the power developed at the rated conditions.

(*b*) Repeat for 0.8 lagging power factor and rated current.

[(*a*) 341.56 $\angle -44.3°$ V, 77.72 kW ; (*b*) 212 $\angle -60.84°$ V, 61.57 kW]

5.16 State the characteristic features of a three-phase synchronous motor.

5.17 Derive an expression for the power developed in a 3-phase synchronous motor.

5.18 A synchronous motor develops torque only at the synchronous speed whereas an induction motor develops torque at all speeds except at synchronous speed. Mention the reasons.

5.19 Explain how a synchronous motor operate, and synchronous capacitor (condenser) and mention its applications.

5.20 A 3-phase salient-pole synchronous, motor has a direct-axis synchronous reactance of 0.95 pu and a quadrature axis, synchronous reactance of 0.6 pu. Draw a phasor diagram for the motor when operating on full load at 0.8 p.f. leading and determine the load angle. [40°]

5.21 Explain with neat sketches the principle of operation of a 3-phase synchronous motor. Also explain why it will not run at other than synchronous speed.

5.22 Explain two important functions served by damper windings in a synchronous motor. State the various applications of synchronous motors.

5.23 Explain why a three-phase synchronous motor develops torque only at the synchronous speed, whereas a three-phase induction motor develops torque at all speeds except the synchronous speed.

5.24 Explain the effect of varying excitation on armature current and power factor in a synchronous motor. Draw *V*-curves and state their significance.

5.25 What are *V*-curves and inverted *V*-curves of a 3-phase synchronous motor ?

5.26 What is a synchronous condenser ? Explain with the help of phasor diagram its operation. What are its applications ?

6

Direct-Current Generators

6.1 BASIC STRUCTURE OF ELECTRIC MACHINES

A rotating electric machine has two main parts, stator and rotor, separated by the air gap.

The **stator** of the machine does not move and normally is the outer frame of the machine.

The **rotor** is free to move and normally is the inner part of the machine. Both stator and rotor are made of ferromagnetic materials. Slots are cut on the inner periphery of the stator and the outer periphery of the rotor. Conductors are placed in the slots of the stator or rotor. They are interconnected to form windings. The winding in which voltage is induced is called the **armature winding**.

The winding through which a current is passed to produce the main flux is called the **field winding**.

Permanent magnets are used in some machines to provide the main flux of the machine.

There are two types of d.c. machines, the d.c. generator and the d.c. motor. The d.c. generator converts mechanical energy into electrical energy. The d.c. motor converts electrical energy into mechanical energy. The d.c. generator is based on the principle that when a conductor is rotated in a d.c. magnetic field, a voltage will be generated in the conductor.

6.2 D.C. GENERATOR CONSTRUCTION

A d.c. generator consists of three main parts [Fig. 6.1(*a*)].

1. Magnetic-field system 2. Armature 3. Commutator and brushgear

6.2.1 Magnetic-field system

The magnetic-field system is the stationary (fixed) part of the machine.

It produces the main magnetic flux. The outer frame or **yoke** is a hollow cylinder of cast steel or rolled steel. An even number of **pole cores** are bolted to the yoke. The yoke serves the following two purposes :

(*a*) It supports the pole cores and acts as protecting cover to the machine.

(*b*) It forms a part of the magnetic circuit.

...the poles project inw...
...pole shoe having a curved...
(i) It supports the field...
(ii) It increases the cross...
its reluctance.

The pole cores are made o...
...and riveted together. T...

Each pole core has one o...
...magnetic field. The fi...
...one another such that wh...
...and south poles are produ...

... Armature

The rotating part of the a...
...of a shaft upon which...
... The armature core ha...
...are insulated from each o...
...the laminations are keye...
...on a spider. The purpo...

The insulated conducto...
...conductors are wedged and b...
...them flying under ce...
... This connected arrang...
...types of windings are use...

... Commutator and bru...

Alternating voltage is ...
...direct current in the ex...
..., which rotates with the a...

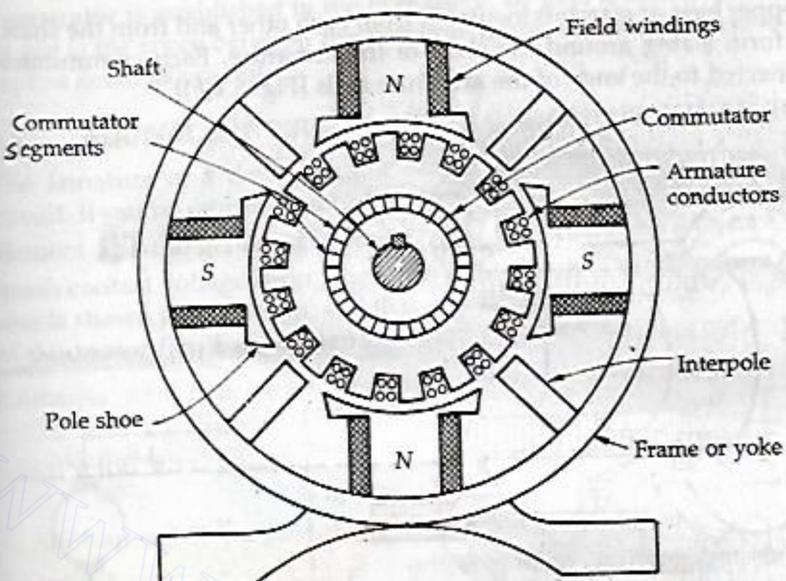


Fig. 6.1(a). *Main parts of a 4-pole d.c. machine.*

Since the poles project inwards they are called **salient poles.** Each pole core has a **pole shoe** having a curved surface. The pole shoe serves two purposes :

(*i*) It supports the field coils.

(*ii*) It increases the cross-sectional area of the magnetic circuit and reduces its reluctance.

The pole cores are made of sheet steel laminations that are insulated from each other and riveted together. The poles are laminated to reduce eddy-current loss.

Each pole core has one or more **field coils** (windings) placed over it to produce a magnetic field. The field coils (or exciting coils) are connected in series with one another such that when the current flows through the coils, alternate north and south poles are produced in the direction of rotation.

6.2.2 Armature

The rotating part of the d.c. machine is called the **armature.** The armature consists of a shaft upon which a laminated cylinder, called **armature core,** is mounted. The armature core has grooves or slots on its outer surface. The laminations are insulated from each other and tightly clamped together. In small machines the laminations are keyed directly to the shaft. In large machines they are mounted on a spider. The purpose of using laminations is to reduce eddy-current loss.

The insulated conductors are put in the slots of the armature core. The conductors are wedged and bands of steel wire are fastened round the core to prevent them flying under centrifugal forces. The conductors are suitably connected. This connected arrangement of conductors is called **armature winding.** Two types of windings are used–*wave* and *lap*.

6.2.3 Commutator and brushgear

Alternating voltage is produced in a coil rotating in a magnetic field. To obtain direct current in the external circuit a commutator is needed. The **commutator,** which rotates with the armature, is made from a number of wedge-shaped hard-drawn copper bars or *segments* insulated from each other and from the ... The segments form a ring around the shaft of the armature. Each ... segment is connected to the ends of the armature coils [Fig. 6.1(*b*)].

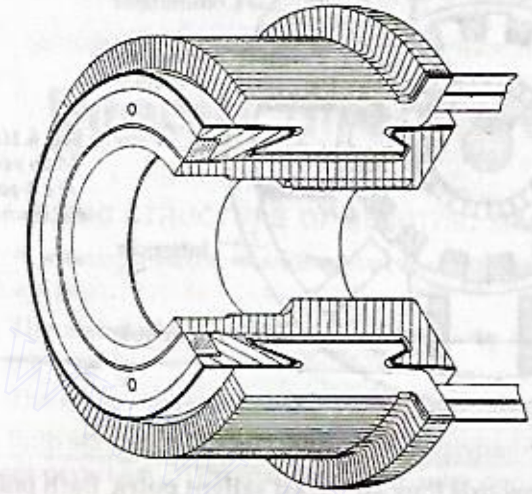


Fig. 6.1(*b*). Comm...

Current is collected from the armature winding by means of two or more carbon **brushes** mounted on the commutator. Each brush is supported in a ... box called a *brush box* or *brush holder*. The pressure exerted by the brushes on the commutator can be adjusted and is maintained at a constant value by means of springs. Current produced in the armature winding is passed on to the commuta-tor and then to the external circuit by means of brushes.

6.3 MAGNETIC CIRCUIT OF A D.C. GENERATOR

The magnetic circuit of a four-pole d.c. generator is shown in Fig. 6.2. The broken lines indicate the main flux paths. Flux produced by the field winding ...

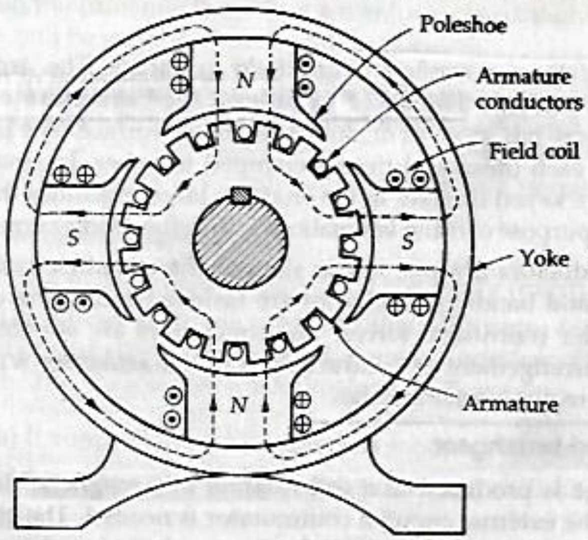


Fig. 6.2. *Magnetic circuit of a 4-pole d.c. generator*

a generator is established in the pole cores, air gap, armature core and yoke. The air gap is the space between the armature surface and the pole face. This space is kept as small as possible.

6.4 EQUIVALENT CIRCUIT OF A D.C. MACHINE ARMATURE

The armature of a d.c. generator can be represented by an equivalent electric circuit. It can be represented by three series-connected elements E, R_a and V_b. The element E is the generated voltage, R_a is the armature resistance, and V_b is the brush contact voltage drop. The equivalent circuit of the armature of a d.c. generator is shown in Fig. 6.3 (*a*), and that of a d.c. motor is shown in Fig. 6.3 (*b*). In case of d.c. motor E is the back e.m.f.

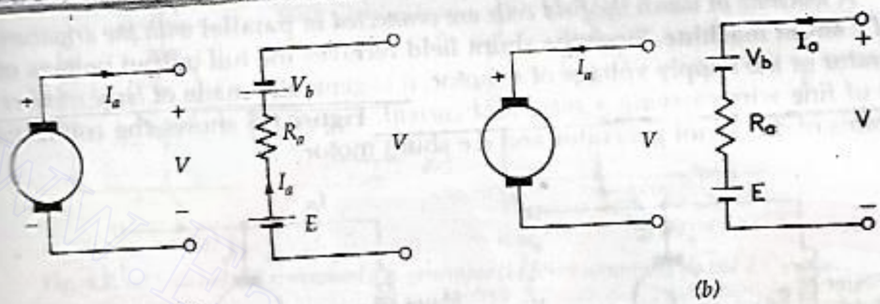


Fig. 6.3. *Equivalent circuits of the armature of (a) d.c. generator (b) d.c. motor.*

6.5 TYPES OF D.C. MACHINE

The magnetic flux in a d.c. machine is produced by field coils carrying current. *The production of magnetic flux in the machine by circulating current in the field winding is called* **excitation.**

There are two methods of excitation, namely **separate excitation** and **self-excitation.** In separate excitation the field coils are energised by a separate d.c. source. In self-excitation the current flowing through the field winding is supplied by the machine itself.

Direct current machines are named according to the connection of the field winding with the armature. The principal types of d.c. machine are :

1. Separately excited d.c. machine.
2. Shunt wound or shunt machine.
3. Series wound or series machine.
4. Compound wound or compound machine.

The four types of machines given above could be either generators or motors.

6.5.1 Separately excited d.c. machine

As the name implies, the field coils are energised by a separate d.c. source. The connections showing the separately excited d.c. machines are given in Fig. 6.4.

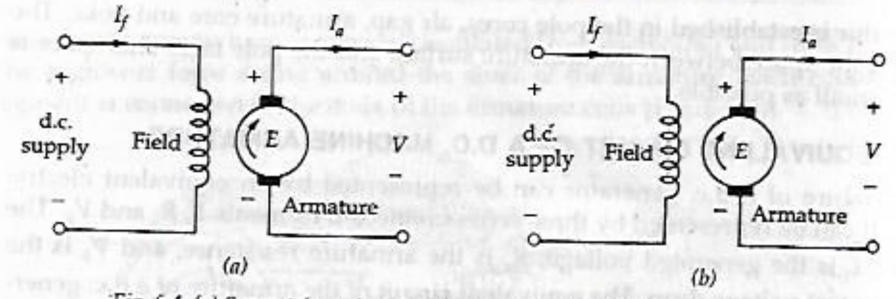


Fig. 6.4. *(a) Separately excited d.c. generator, (b) Separately excited d.c. motor.*

6.5.2 Shunt wound d.c. machine

A machine in which the field coils are connected in **parallel** *with the armature is called a* **shunt machine.** Since the shunt field receives the full output voltage of a generator or the supply voltage of a motor, it is generally made of *large number of turns* of fine wire carrying a *small field current.* Figure 6.5 shows the connection diagrams of d.c. shunt generator and d.c shunt motor.

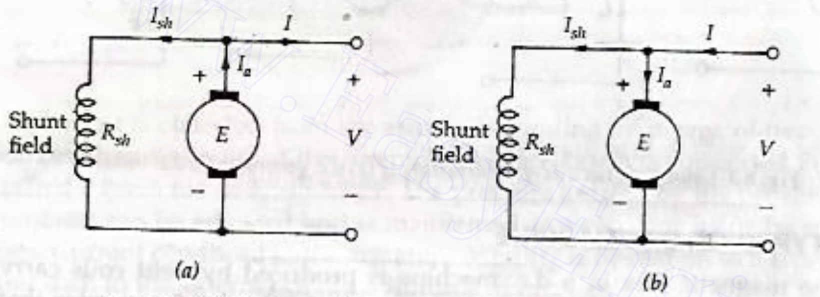


Fig. 6.5. *(a) Shunt wound d.c. generator, (b) Shunt wound d.c. motor.*

6.5.3 Series wound d.c. machine

A d.c. machine in which the field coils are connected in **series** *with the armature is called a* **series machine.** The series field winding carries the armature current and since the armature current is large, the series field winding consists of *few turns* of wire of large cross-sectional area. Figure 6.6 shows the connections of d.c. series generator and d.c. series motor.

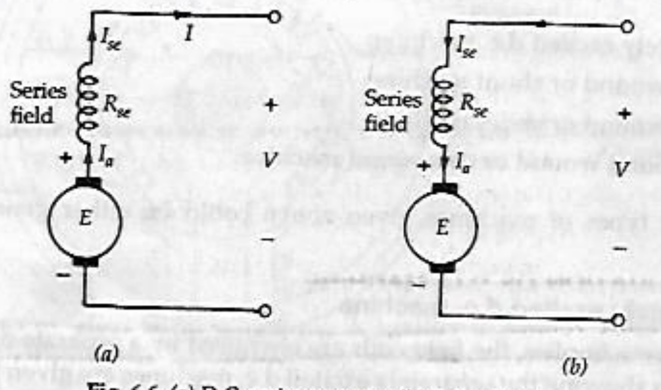


Fig. 6.6. *(a) D.C. series generator (b) D.C. series motor.*

DIRECT-CURRENT GENERATORS

6.5.4 Compound wound d.c

A d.c. machine having both
each field pole of the machine c
turns of fine wire and the seri
area.

The compound machine
is connected in parallel with th
shunt compound machine. Su

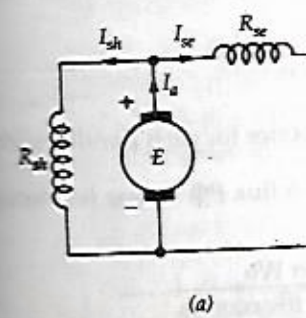


Fig. 6.7. *(a) Short-shunt com*

If the shunt field is in p
the machine is called the long

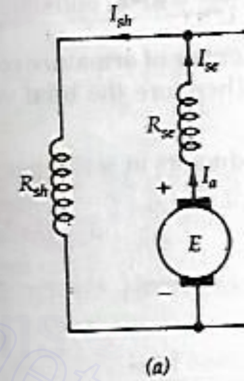


Fig. 6.8. *(a) Lon*

If the magnetic flux p
produced by the shunt fiel
compounded. If the series f
said to be differentially co
shunt connected.

6.6 E.M.F. EQUATION

As the armature rot
generator, the e.m.f. of rota
$E_a = E_g$

6.5.4 Compound wound d.c. machine

A d.c. machine having both shunt and series fields is called a compound machine. Each field pole of the machine carries two windings. The shunt winding has many turns of fine wire and the series winding has few turns of large cross-sectional area.

The compound machine may be connected in two ways. If the shunt field is connected in parallel with the armature *alone* the machine is called the **short-shunt compound machine.** Such a machine is shown in Fig. 6.7.

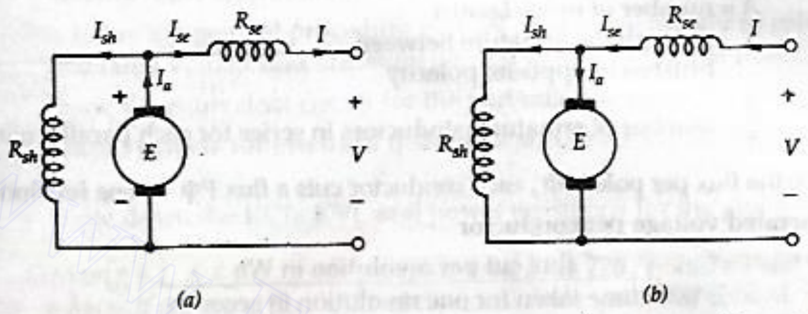


Fig. 6.7. *(a) Short-shunt compound d.c. generator (b) Short-shunt compound d.c. motor.*

If the shunt field is in parallel with both armature and series field (Fig. 6.8), the machine is called the **long-shunt compound machine.**

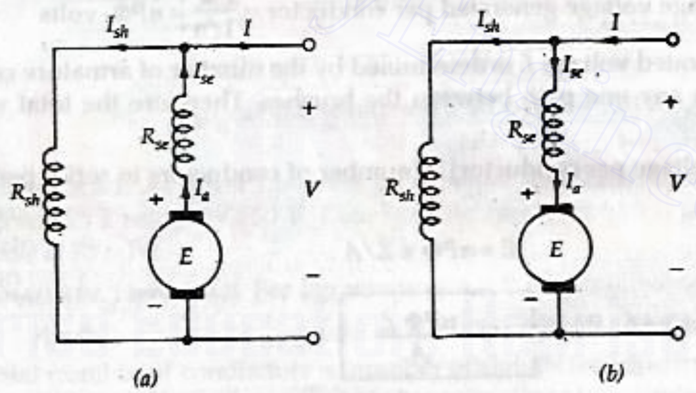


Fig. 6.8. *(a) Long-shunt d.c. generator (b) Long-shunt d.c. motor.*

If the magnetic flux produced by the series winding *assists* (*aids*) the flux produced by the shunt field winding, the machine is said to be **cumulatively compounded.** If the series field flux opposes the shunt field flux, the machine is said to be **differentially compounded.** Either type may be long-shunt or short-shunt connected.

6.6 E.M.F. EQUATION OF D.C. MACHINE

As the armature rotates, a voltage is generated in its coils. In case of a generator, the e.m.f. of rotation is called the *generated emf* (or armature e.m.f.) and $E_r = E_g$.

In case of a motor, the e.m.f. of rotation is known as *back e.m.f.* (or counter e.m.f.), and $E_r = E_b$. The expression, however, is the same for both conditions of operation.

Let Φ = useful flux per pole in webers(Wb)

P = total number of poles

Z = total number of *conductors* in the armature

n = speed of rotation of armature in revolutions per second (*r.p.s.*)

A = number of *parallel* paths through the armature between brushes of opposite polarity

$\therefore \quad \dfrac{Z}{A}$ = number of armature conductors in series for each parallel path

Since the flux per pole is Φ, each conductor cuts a flux $P\Phi$ in one revolution.

Generated voltage per conductor

$$= \frac{\text{flux cut per revolution in Wb}}{\text{time taken for one revolution in seconds}}$$

Since n revolutions are made in one second, one revolution will be made in $1/n$ second. Therefore the time for one revolution of the armature is $1/n$ second.

The average voltage generated per conductor $= \dfrac{P\Phi}{1/n} = nP\Phi$ volts

The generated voltage E is determined by the number of armature conductors in series in any one path between the brushes. Therefore the total voltage generated

E = (average voltage per conductor) × (number of conductors in series per path)

that is,

$$E = nP\Phi \times Z/A$$

$$\boxed{E = \frac{nP\Phi Z}{A}} \qquad (6.6.1)$$

Equation (6.6.1) is called the **e.m.f. equation of a d.c. machine.**

6.7 LAP AND WAVE WINDINGS

Armature coils can be connected to the commutator to form either **Lap** or **Wave** windings.

6.7.1 Lap Winding

The ends of each armature coil are connected to adjacent segments on the commutator so that the total number of parallel paths is equal to the total number of poles. That is, for **LAP** winding **A = P**. This may be remembered by the letters A and P in LAP.

Wave Winding

...each armature c... so that only two p... brushes. That is, for... general, the lap winding... winding is used in...

GENERAL PROCEDUR... GENERATED VOLTA...

The following general p... generated voltage and...

1. Draw the equivalent...

2. Mark symbols for el...

3. Write down the KCI...

EXAMPLE 6.1. *A 4-pole, ... rev/min. If the useful flu...*

SOLUTION. Here $P = 4$ ...

EXAMPLE 6.2. *An 8-pole ... generates a voltage of 5... pole is 50 mWb.*

SOLUTION. Here $P = 8$...

$\therefore A = 8$

Total number of condu...

$Z = 40 \times 1$...

$E = 500$ vo...

Since $N = 60 \times n$ ...

EXAMPLE 6.3. *A d.c. ge... pole is 20 mWb and the... same flux and a speed of 10... r.p.m.*

6.7.2 Wave Winding

The ends of each armature coil are connected to commutator segments some distance apart, so that only two parallel paths are provided between the positive and negative brushes. That is, for WAVE winding $A = 2$.

In general, the lap winding is used in low-voltage, high-current machines, and the wave winding is used in high-voltage, low-current machines.

6.8 GENERAL PROCEDURE FOR SOLVING PROBLEMS ON GENERATED VOLTAGE AND ARMATURE CURRENT

The following general procedure may be used conveniently to solve problems on generated voltage and armature current in a d.c. machine :

1. Draw the equivalent circuit for the particular machine.
2. Mark symbols for electrical quantities on the circuit diagram at proper places.
3. Write down the KCL, KVL and power equations for the given machine.

EXAMPLE 6.1. *A 4-pole, wave-wound armature has 720 conductors and is rotated at 1000 rev/min. If the useful flux is 20 mWb, calculate the generated voltage.*

SOLUTION. Here $P = 4, \quad A = 2, \quad Z = 720$

$$N = 1000 \text{ r.p.m.}$$

$$\therefore \quad n = \frac{N}{60} = \frac{1000}{60} \text{ r.p.s.}$$

$$\Phi = 20 \text{ mWb} = 20 \times 10^{-3} \text{ Wb}$$

$$E = \frac{n P \Phi Z}{A} = \frac{1000 \times 4 \times 20 \times 10^{-3} \times 720}{60 \times 2} = 480 \text{ V}$$

EXAMPLE 6.2. *An 8-pole lap-connected armature has 40 slots with 12 conductors per slot generates a voltage of 500 V. Determine the speed at which it is running if the flux per pole is 50 mWb.*

SOLUTION. Here $P = 8$. For lap winding $A = P$

$\therefore \quad A = 8$

Total number of conductors = (number of slots) × (conductors per slot)

$$Z = 40 \times 12 = 480$$

$$E = 500 \text{ volts}, \ \Phi = 50 \text{ mWb} = 50 \times 10^{-3} \text{ Wb}$$

$$E = \frac{n P \Phi Z}{A}, \ n = \frac{EA}{P \Phi Z} = \frac{500 \times 8}{8 \times 50 \times 10^{-3} \times 480} = \frac{125}{6} \text{ r.p.s}$$

Since $\quad N = 60 \times n$

$$N = 60 \times \frac{125}{6} = 1250 \text{ r.p.m.}$$

EXAMPLE 6.3. *A d.c. generator has an armature e.m.f. of 100 V when the useful flux per pole is 20 mWb and the speed is 800 r.p.m. Calculate the generated e.m.f. (a) with the same flux and a speed of 1000 r.p.m., (b) with a flux per pole of 24 mWb and a speed of 900 r.p.m.*

SOLUTION. $E = \dfrac{NP\Phi Z}{60\,A} = \left(\dfrac{PZ}{60A}\right)N\Phi$

Since P, Z and A are constants for a given machine, $\left(\dfrac{PZ}{60\,A}\right)$ is also a constant say k.

The generated voltage can therefore be written as

$$E = kN\Phi$$

If the subscripts 1 and 2 denote the initial and final values

$$E_1 = kN_1\Phi_1, \quad E_2 = kN_2\Phi_2$$

$$\frac{E_2}{E_1} = \frac{kN_2\Phi_2}{kN_1\Phi_1} = \frac{N_2\Phi_2}{N_1\Phi_1}$$

(*a*) $\Phi_2 = \Phi_1 = 20 \text{ mWb} = 20 \times 10^{-3} \text{ Wb}$

$N_1 = 800$ r.p.m., $N_2 = 1000$ r.p.m.

$$E_2 = E_1 \frac{N_2\Phi_2}{N_1\Phi_1} = \frac{100 \times 1000 \times 20 \times 10^{-3}}{800 \times 20 \times 10^{-3}} = 125 \text{ V}$$

(*b*) $\Phi_2 = 24 \text{ mWb} = 24 \times 10^{-3} \text{ Wb}$

$N_2 = 900$ r.p.m.

$$E_2 = E_1 \frac{N_2\Phi_2}{N_1\Phi_1} = \frac{100 \times 900 \times 24 \times 10^{-3}}{800 \times 20 \times 10^{-3}} = 135 \text{ V}$$

EXAMPLE 6.4. *An 8-pole generator has 500 armature conductors and has a useful flux per pole of 0.065 Wb. What will be the e.m.f. generated if it is lap connected and runs at 1000 r.p.m. ? What must be the speed at which it is to be driven to produce the same e.m.f. if it is wave wound ?*

SOLUTION. $E_1 = \dfrac{N_1 P\,\Phi\,Z}{60\,A} = \dfrac{1000 \times 0.065 \times 500}{60} \times \dfrac{P}{A}$

For lap connection $A = P$

$$\therefore \quad E_1 = \frac{1000 \times 0.065 \times 500}{60} = 541.67 \text{ V}$$

$$E_2 = \frac{N_2 \times 8\,\Phi\,Z}{60 \times 2}$$

Since $\quad E_2 = E_1$

$$\frac{8\,N_2\,\Phi\,Z}{60 \times 2} = \frac{8\,N_1 \times \Phi\,Z}{60 \times 8}$$

$$N_2 = \frac{N_1 \times 2}{8} = \frac{1000 \times 2}{8} = 250 \text{ r.p.m.}$$

EXAMPLE 6.5. *A lap-wound d.c. shunt generator having 80 slots with 10 conductors per slot generates at no load an e.m.f. of 400 V when running at 1000 r.p.m. At what speed should it be rotated to generate a voltage of 220 V on open circuit ?*

SOLUTION. Total number of conductors

$$= \text{slots} \times \text{number of conductors per slot} = 80 \times 10 = 800$$

For lap winding $A = P$

$$E = \frac{N P \Phi Z}{60 A}$$

$$400 = \frac{1000\,\Phi \times 800}{60} \times \frac{P}{A}$$

$$\Phi = \frac{400 \times 60}{1000 \times 800} = 0.03 \text{ Wb}$$

Let N_1 be the speed of rotation to generate 220 V on open circuit

$$\therefore \quad 220 = \frac{N_1 \times 0.03 \times 800}{60} \times \frac{P}{A}$$

$$N_1 = \frac{220 \times 60}{0.03 \times 800} = 550 \text{ r.p.m.}$$

Alternative method

$$E_1 = \frac{N_1 P \Phi Z}{60 A}, \quad E_2 = \frac{N_2 P \Phi Z}{60 A}$$

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

$$N_2 = \frac{E_2}{E_1} \times N_1 = \frac{220}{400} \times 1000 = 550 \text{ r.p.m.}$$

EXAMPLE 6.6. *A 4-pole d.c. generator has 1200 armature conductors and generates 250 V on open circuit when running at a speed of 500 r.p.m. The diameter of the pole-shoe circle is 0.35 m and the ratio of pole arc to pole pitch is 0.7 while the length of the shoes is 0.2 m. Find the mean flux density in the air gap. Assume lap-connected armature winding.*

SOLUTION. Pole pitch = distance between two adjacent poles

$$= \frac{\text{periphery of the armature}}{\text{number of poles of the generator}} = \frac{\pi D}{P} = \frac{\pi \times 0.35}{4} \text{ m}$$

$$\frac{\text{Pole arc}}{\text{pole pitch}} = 0.7$$

$$\text{Pole arc} = 0.7 \times \text{pole pitch} = \frac{0.7 \times \pi \times 0.35}{4}$$

$$\text{Area of pole face} = \text{pole arc} \times \text{axial length}$$

$$= \frac{0.7\,\pi \times 0.35}{4} \times 0.2 = 0.03848 \text{ m}^2$$

$$E = \frac{N P \Phi Z}{60 A}$$

$$250 = \frac{500 \times 4\,\Phi \times 1200}{60 \times 4}$$

$$\Phi = \frac{250 \times 60 \times 4}{500 \times 4 \times 1200} = 0.025 \text{ Wb}$$

Flux density in the air gap

$$B = \frac{\text{flux per pole}}{\text{area of pole shoe}} = \frac{0.025}{0.03848} = 0.65 \text{ T}$$

EXAMPLE 6.7. *A short-shunt compound d.c. generator delivers 100 A to a load at 250 V. The generator has shunt field, series field and armature resistance of 130 Ω, 0.1 Ω and 0.1 Ω respectively. Calculate the voltage generated in armature winding. Assume 1V drop per brush.*

SOLUTION. The circuit diagram of a short-shunt compound d.c. generator is shown in Fig. 6.9.

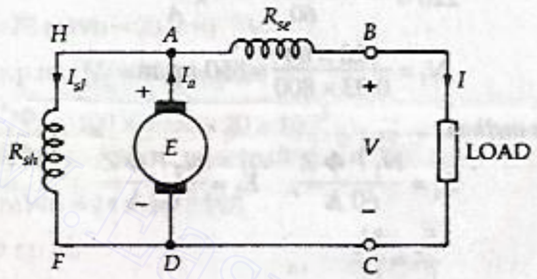


Fig. 6.9. *Short-shunt compound generator.*

Here $I = 100$ A, $V = 250$ V, $R_{sh} = 130\ \Omega$

$R_{se} = 0.1\ \Omega$, $R_a = 0.1\ \Omega$

Brush voltage drop $V_b = 2 \times 1 = 2$ V

By KVL in the outer loop FHABCDF

$$V_{FH} + V_{HA} + V_{AB} + V_{BC} + V_{CD} + V_{DF} = 0$$

$$+ I_{sh} R_{sh} + 0 - I_{se} R_{se} - V + 0 + 0 = 0$$

$$I_{sh} \times 130 = I_{se} R_{se} + V = I R_{se} + V = 100 \times 0.1 + 250 = 260 \text{ V}$$

$$\therefore \qquad I_{sh} = \frac{260}{130} = 2 \text{ A}$$

By KCL at node A,

$$I_a = I_{se} + I_{sh} = I + I_{sh} = 100 + 2 = 102 \text{ A}$$

By KVL in mesh DABCD

$$E - I_a R_a - V_b - I_{se} R_{se} - V = 0$$

$$E = I_a R_a + V_b + I R_{se} + V = 102 \times 0.1 + 2 + 100 \times 0.1 + 250 = 272.2 \text{ V}$$

EXAMPLE 6.8. *A long-shunt compound generator delivers a load current of 50 A at 500 V, and the resistances of armature, series field and shunt fields are 0.05 Ω, 0.03 Ω, and 250 Ω respectively. Calculate the generated e.m.f. and the armature current. Allow 1.0 V per brush for contact drop.*

SOLUTION. The circuit diagram of a long-shunt compound d.c. generator is shown in Fig. 6.10.

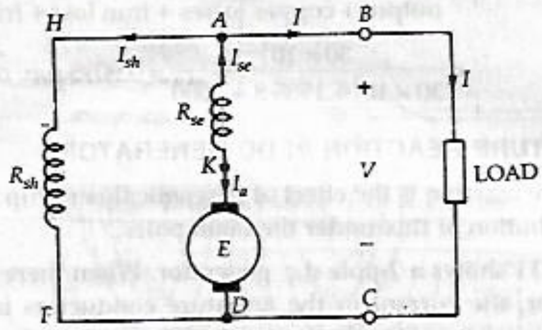


Fig. 6.10. *Long-shunt compound generator.*

Here $I = 50$ A, $V = 500$ V, $R_a = 0.05$, $R_{se} = 0.03\ \Omega$, $R_{sh} = 250\ \Omega$.

Brush contact drop $V_b = 2 \times 1 = 2$ V

$$I_{sh} = \frac{V_{sh}}{R_{sh}} = \frac{V}{R_{sh}} = \frac{500}{250} = 2 \text{ A}$$

By KCL at node K, $I_a = I_{se}$

By KCL at node A,

$$I_{se} = I_a = I_{sh} + I = 2 + 50 = 52 \text{ A}$$

By KVL in mesh DKABCD

$$+E - I_a R_a - V_b - I_{se} R_{se} - V = 0$$

$$E = I_a R_a + V_b + I_{se} R_{se} + V$$

$$= 52 \times 0.05 + 2 + 52 \times 0.03 + 500 = 506.16 \text{ V}$$

EXAMPLE 6.9. *A shunt generator gives full-load output of 30 kW at a terminal voltage of 200 V. The armature and shunt field resistances are 0.05 Ω and 50 Ω respectively. The iron and friction losses are 1000 W. Calculate : (a) generated e.m.f. ; (b) copper losses ; (c) efficiency.*

SOLUTION. $V = 200$ V, $P = 30 \times 10^3$ W

$R_a = 0.05\ \Omega$, $R_{sh} = 50\ \Omega$, $p_{i+f} = 1000$ W

$$I = \frac{P}{V} = \frac{30 \times 10^3}{200} = 150 \text{ A}$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{200}{50} = 4 \text{ A}$$

$$I_a = I + I_{sh} = 150 + 4 = 154 \text{ A}$$

(a) $E = V + I_a R_a = 200 + 154 \times 0.05 = 207.7$ V

(b) Copper losses $= I_a^2 R_a + I_{sh}^2 R_{sh}$

$$= (154)^2 \times 0.05 + 4^2 \times 50 = 1985.8 \text{ W}$$

(c) Efficiency

$$\eta = \frac{\text{output}}{\text{output} + \text{copper losses} + \text{iron loss} + \text{friction loss}}$$

$$= \frac{30 \times 10^3}{30 \times 10^3 + 1985.8 + 1000} = 0.9095 \text{ p.u. or } 90.95\%$$

6.9 ARMATURE REACTION IN DC GENERATORS

Armature reaction is the effect of magnetic flux set up by armature current upon the distribution of flux under the main poles.

Figure 6.11 shows a 2-pole d.c. generator. When there is *no load* connected to the generator, the current in the armature conductors is zero. Under these conditions there exists in it only the mmf of the main poles which produce the *mainflux* ϕ. This flux is distributed symmetrically with respect to the polar axis, that is, the centre line of the north and the south poles. The direction of Φ_m is shown by an arrow. The **magnetic neutral axis or plane** (MNA) *is a plane perpendicular to the axis of the flux.*

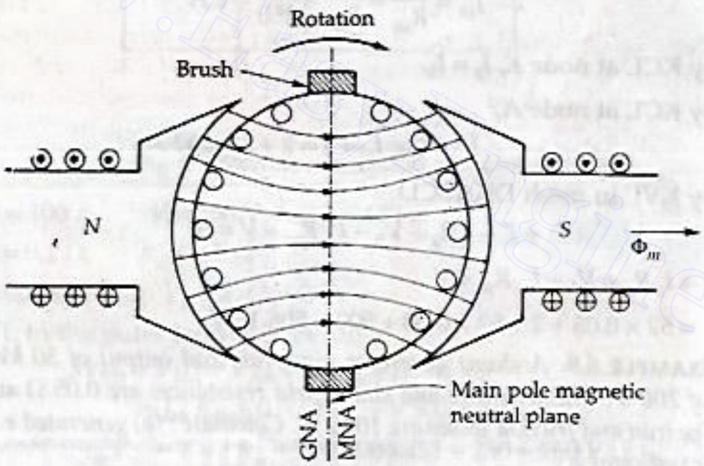


Fig. 6.11. *Main pole magnetic flux distribution.*

The MNA coincides with the geometrical neutral axis or plane (GNA). *Brushes are always placed along MNA. Hence MNA is also called the axis of commutation.*

Figure 6.12 shows armature conductors carrying current with no current in field coils. The direction of current in the armature conductors may be determined by Flemings right-hand rule. The current flows in the same direction in all the conductors lying under one pole. The direction of flux produced by armature conductors may be determined by cork-screw rule. The conductors on the left-hand side of the armature carry current in the direction into the paper. The flux produced by current in these armature conductors in shown in Fig. 6.12.

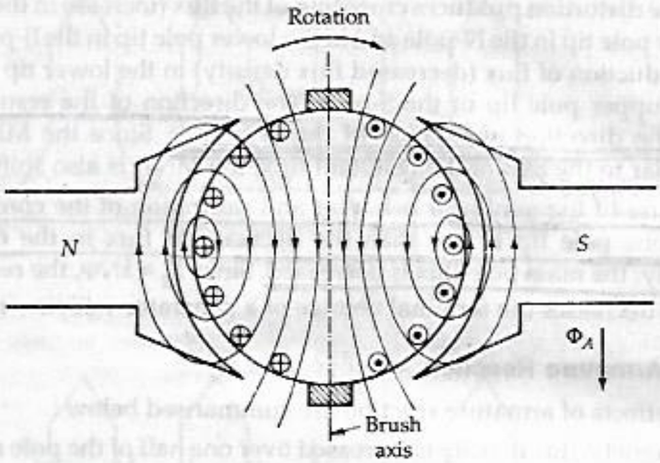


Fig. 6.12. *Armature flux distribution.*

These conductors combine their mmfs to produce a flux through the armature in the downward direction. Similarly, the conductors on the right-hand side of the armature carry current in the direction out of the paper. These conductors also combine their mmfs to produce a flux through the armature in the downward direction. Thus, the conductors on both sides of the armature combine their mmfs in such a manner as to send a flux through the armature in the downward direction. This flux Φ_A is represented by an arrow as shown in Fig. 6.12. The armature flux produced is analogous to that produced in the equivalent iron-cored solenoid with its axis along the brush axis.

Figure 6.13 shows the condition when the field current and armature current are acting simultaneously. This occurs when the generator is on load. Now there are two fluxes inside the machine, one produced by the main field poles of the generator and the other by the current in the armature conductors. These two fluxes now combine to form a resultant flux Φ_R as shown in Fig. 6.13.

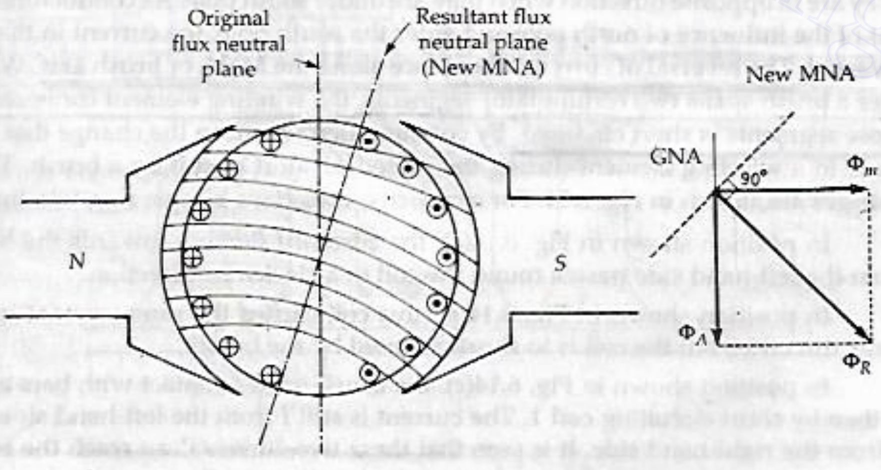


Fig. 6.13. *Resultant flux distribution.*

It is seen that the field flux entering the armature is not only shifted but also *distorted.* The distortion produces crowding of the flux (increase in the flux density) in the upper pole tip in the N-pole and in the lower pole tip in the S-pole. Similarly, there is a reduction of flux (decreased flux density) in the lower tip of the N-pole and in the upper pole tip of the S-pole. The direction of the resultant flux is shifted in the direction of rotation of the generator. Since the MNA is always perpendicular to the axis of the resultant flux, the MNA is also shifted.

Because of the nonlinear behavior and saturation of the core, the increase in flux in one pole tip is less than the decrease in flux in the other pole tip; consequently, the main pole flux is decreased. Since $E_g = kN\Phi$, the reduction in the field flux Φ decreases the terminal voltage of a generator with increased load.

Effects of Armature Reaction

The effects of armature reaction are summarised below :

1. Magnetic flux density is increased over one half of the pole and decreased over the other half. But the total flux produced by each pole is slightly reduced and, therefore, the terminal voltage is slightly reduced. The effect of total flux reduction by armature reaction is known as **demagnetizing effect.**

2. The flux wave is distorted and there is shift in the position of the magnetic neutral axis (MNA) in the direction of rotation for the generator and against the direction of rotation for the motor.

3. Armature reaction establishes a flux in the neutral zone (or commutating zone). Armature reaction flux in the neutral zone will induce conductor voltage that aggravates the commutation problem.

6.10 COMMUTATION

The currents induced in the armature conductors of a d.c. generator are alternating in nature. The commutation process involves the change from a generated alternating current to an externally applied direct current. These induced currents flow in one direction when the armature conductors are under north pole. They are in opposite direction when they are under south pole. As conductors pass out of the influence of north pole and enter the south pole, the current in them is reversed. The reversal of current takes place along the MNA or brush axis. Whenever a brush spans two commutator segments, the winding element connected to those segments is short circuited. By commutation we mean the change that takes place in a winding element during the period of short circuit by a brush. These changes are shown in Fig. 6.14. For simplicity, consider a simple ring winding.

In position shown in Fig. 6.14(*a*), the current I flowing towards the brush from the left-hand side passes round the coil in a clockwise direction.

In position shown in Fig. 6.14(*b*), this coil carries the same current in the same direction, but the coil is to short circuited by the brush.

In position shown in Fig. 6.14(*c*), the brush makes contact with bars *a* and *b*, thereby short circuiting coil 1. The current is still I from the left-hand side and I from the right-hand side. It is seen that these two currents can reach the brush without passing through coil 1.

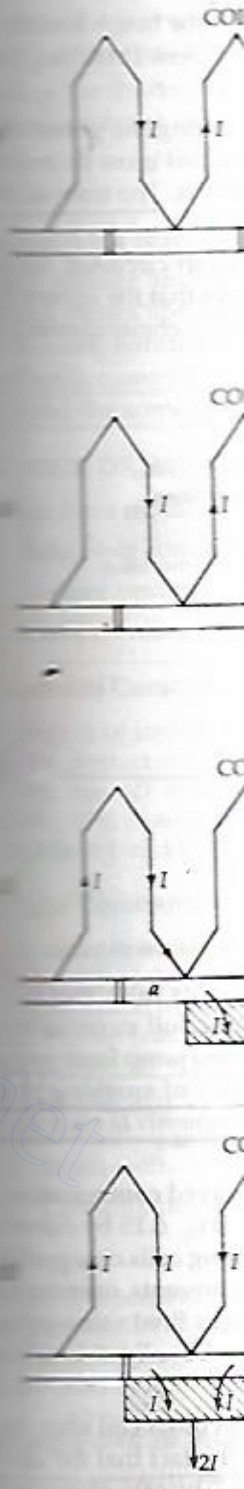


Fig. 6.14. C

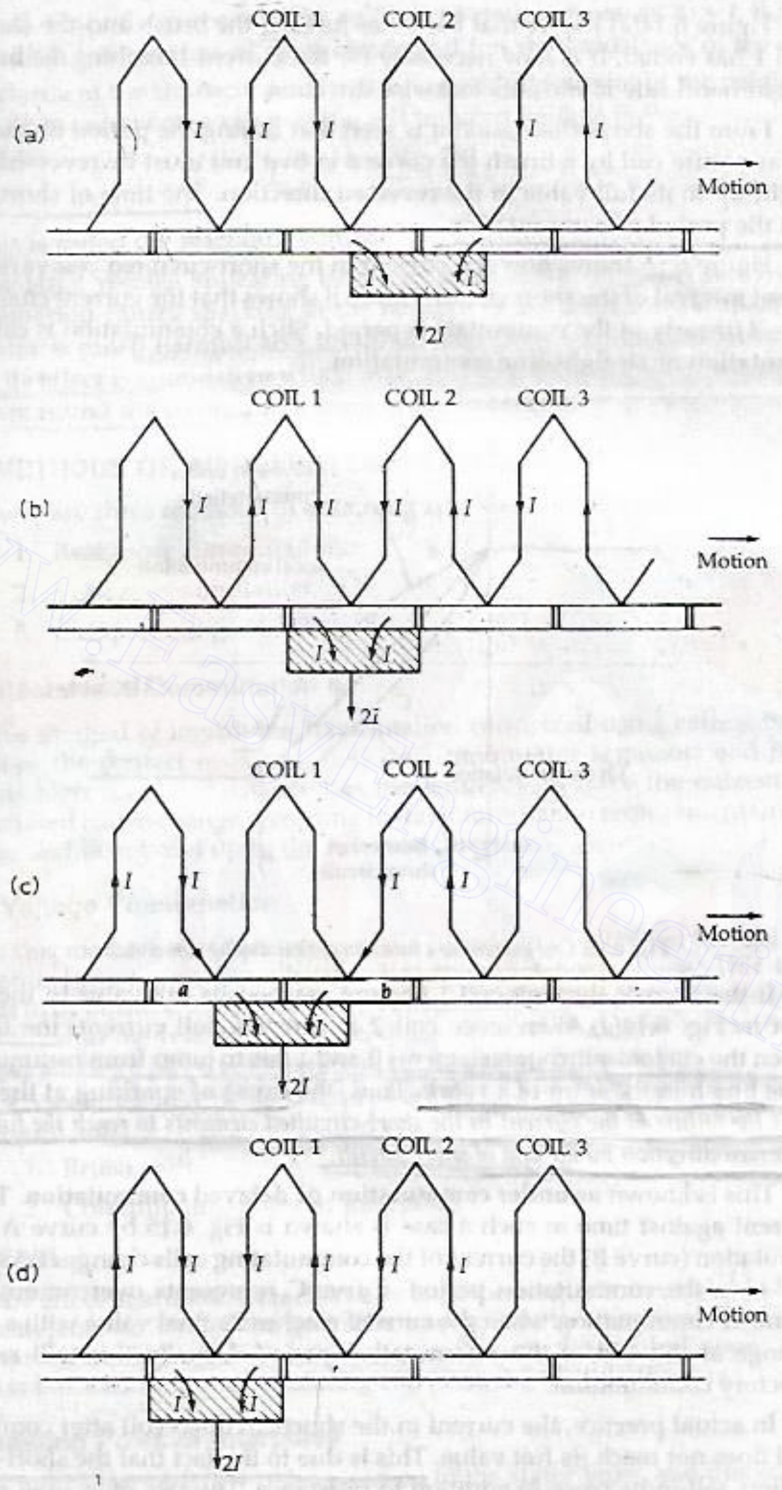


Fig. 6.14. *Current collection at the commutator.*

Figure 6.14(*d*) shows that bar *b* has just left the brush and the short c of coil 1 has ended. It is now necessary for the current *I* reaching the brus the right-hand side in the anticlockwise direction.

From the above discussion it is seen that during the period of short c of an armature coil by a brush the current in that coil must be reversed an brought up to its full value in the reversed direction. The time of short c called the **period of commutation.**

Figure 6.15 shows how the current in the short-circuited coil varies d the brief interval of the short circuit. Curve B shows that the current change $+I$ to $-I$ linearly in the commutation period. Such a commutation is called **commutation or straight-line commutation.**

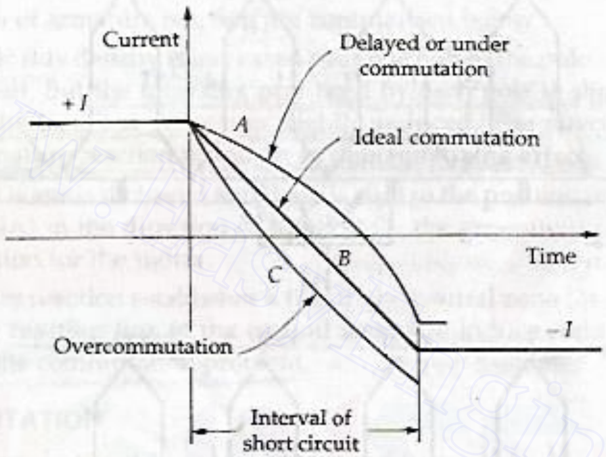


Fig. 6.15. *Coil current as a function of time during commutation.*

If the current through coil 1 has not reached its full value in the pos shown in Fig. 6.14(*d*), then since coil 2 is carrying full current, the diffe between the currents, through elements 2 and 1 has to jump from commutat *b* to the brush in the form of a spark. Thus, the **cause of sparking at the com tator** *is the failure of the current in the short-circuited elements to reach the full v the reversed direction by the end of short circuit.*

This is known as **under commutation or delayed commutation.** The cu of current against time in such a case is shown is Fig. 6.15 by curve A. In commutation (curve B) the current of the commutating coils changes linearly fr $+I$ to $-I$ in the commutation period. Curve C represents **overcommutatio** accelerated commutation when the current reaches its final value with a zero of change at the end of the commutation period. Usually this will result satisfactory commutation.

In actual practice, the current in the short-circuited coil after commutat period does not reach its full value. This is due to the fact that the short-circu coil offers self-inductance in addition to resistance. The rate of change of cu is so great that the self-inductance of the coil sets up a back emf which oppo

Since the current in
If t_c is the time of sh
of the short-circui
then the average value

L

is called the **reactanc**

The large voltage appeari
ected causes sparkin
is much harmful ar
effect is cumulative
round the commuta

METHODS OF IMPRO

There are three methods

1. Resistance commu
2. Voltage commutat
3. Compensating wi

Resistance Commuta

This method of improvi
the contact resista
high contact resista
coil to change ac
and then build up in

Voltage Commutatio

In this method, arrange
commutation, which w
opposition to the r
equal to the reactance
will take place and
Two methods may be u
voltage :

1. Brush shift.
2. Commutating p

Brush Shift

The effect of armature
direction of rotation for th
Armature reactio
induced in the com

Commutating Poles or Inte

Interpoles are narrow
between the main p

the reversal. Since the current in the coil has to change from $+I$ to $-I$, the total change is $2I$. If t_c is the time of short circuit and L is the inductance of the coil (= self-inductance of the short-circuited coil + mutual inductances of the neighbouring coils), then the average value of the self induced voltage is

$$L\frac{di}{dt} = \frac{L \times 2I}{t_c} = \frac{2LI}{t_c}$$

This is called the **reactance voltage.**

The large voltage appearing between commutator segments to which the coil is connected causes sparking at the brushes of the machine the sparking of commutator is much harmful and it will damage both commutator surface and brushes. Its effect is cumulative which may lead to a short circuit of the machine with an arc round the commutator from brush to brush.

6.11 METHODS OF IMPROVING COMMUTATION

There are three methods of obtaining sparkless commutation :

1. Resistance commutation.
2. Voltage commutation.
3. Compensating windings.

6.11.1 Resistance Commutation

This method of improving commutation consists of using carbon brushes. This makes the contact resistance between commutator segments and brushes high. This high contact resistance has the tendency to force the current in the short-circuited coil to change according to the commutation requirements, namely, to reverse and then build up in the reversed direction.

6.11.2 Voltage Commutation

In this method, arrangement is made to induce a voltage in the coil undergoing commutation, which will neutralize the reactance voltage. This injected voltage is in opposition to the reactance voltage. If the value of the injected voltage is made equal to the reactance voltage, quick reversal of current in the short-circuited coil will take place and there will be sparkless commutation.

Two methods may be used to produce the injected voltage in opposition to the reactance voltage :

1. Brush shift.
2. Commutating poles or interpoles.

Brush Shift

The effect of armature reaction is to shift the magnetic neutral axis MNA in the direction of rotation for the generator and against the direction of rotation for the motor. Armature reaction establishes a flux on the neutral zone. A small voltage is induced in the commutating coil since it is cutting the flux.

Commutating Poles or Interpoles

Interpoles are narrow poles attached to the stator yoke, and placed exactly midway between the main poles. Interpoles are also called commutating poles or compoles. The interpole windings are connected in series with the arm because the interpoles must produce fluxes that are directly proportional to the armature current. The armature and interpole mmfs are affected simultan by the same armature current. Consequently, the armature flux in the comm ing zone which tends to shift the magnetic neutral axis, is neutralized by appropriate component of interpole flux. The neutral plane is, therefore, fixed position regardless of the load.

The interpoles must induce a voltage in the conductors undergoing com mutation that is *opposite* to the voltage caused by the neutral-plane shift and reactance voltage. For a generator, the neutral plane shifts in the direction of rotation. Thus, the conductors undergoing commutation have the same polarity of the voltage as the pole they just left. To oppose this voltage, the interpoles must have the opposite flux, which is the flux of the main pole ahead according to the direction of rotation.

For a motor, the neutral plane shifts opposite to the direction of rotation and the conductors undergoing commutation have the same flux as the main pole then are approaching. For opposing this voltage, the interpoles must have the same polarity as the previous main pole. Thus we have the following rules for the polarity of the interpoles :

1. For a *generator*, the polarity of the interpole must be the same as that of the next main pole further ahead in the direction of rotation.

2. For a *motor*, the polarity of an interpole is opposite to that of the next main pole in the direction of rotation.

The polarity of interpoles is shown in Fig. 6.16.

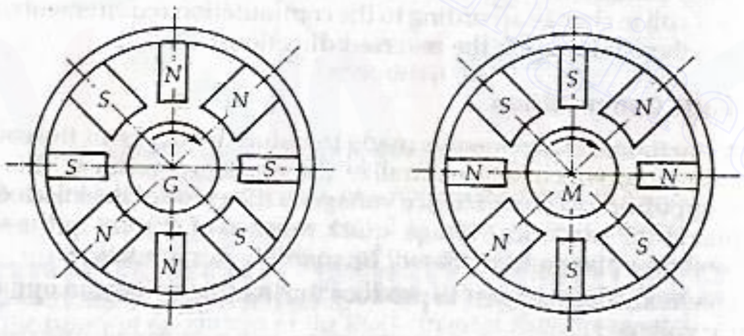


Fig. 6.16. *Polarity of interpoles.*

It is to be noted that the interpoles serve only to provide sufficient flux to assure good commutation. They do not overcome the distortion of the flux result ing from cross-magnetizing mmf of the armature.

The use of interpoles is very common to nearly all dc machines of more than 1 hp.

During severe overloads or rapidly changing loads the voltage between adjacent commutator segments may become very high. This may ionize the air around the commutator to the extent that it becomes sufficiently conductive. An arc is established from brush to brush. This phenomenon is known as flashover. This arc is sufficiently hot to melt the commutator segments. It should be extin guished quickly. In order to prevent flashover compensating windings are used.

6.11.3 Compensating Windings

Compensating windings are the most effective means for eliminating the problems of armature reaction and flashover by balancing the armature mmf. Compensating windings are placed in slots provided in pole faces parallel to the rotor (armature) conductors. These windings are connected in series with the armature windings. The direction of currents in the compensating winding must be opposite to that in the armature winding just below the pole faces. Thus, compensating winding produces an mmf that is equal and opposite to the armature mmf. In effect the compensating winding demagnetizes or neutralizes the armature flux produced by the armature conductors lying just under the pole faces. The flux per pole is then undisturbed by the armature flux regardless of the load conditions.

The major drawback with the compensating windings is that they are very costly. Their use can only be justified in the following special cases :

1. In large machines subject to heavy overloads or plugging.
2. In small motors subject to sudden reversal and high acceleration.

6.12 DEMAGNETIZING AND CROSS MAGNETIZING AMPERE TURNS

We have seen that the generating voltage in the commutating coils should be made to oppose the reactance voltage for smooth commutation. This can be made possible of the commutating coils cut a flux in the direction as that in the post-commutation period for a dc generator and that in the precommutation period for dc motor. For this purpose, the brushes may be shifted from the GNP through an angle $\beta°$ electrical in the direction of rotation for a generator. For a motor the brushes are given a backward shift.

The nature of demagnetizing and cross-magnetizing ampere turns can be calculated by considering the Fig. 6.17.

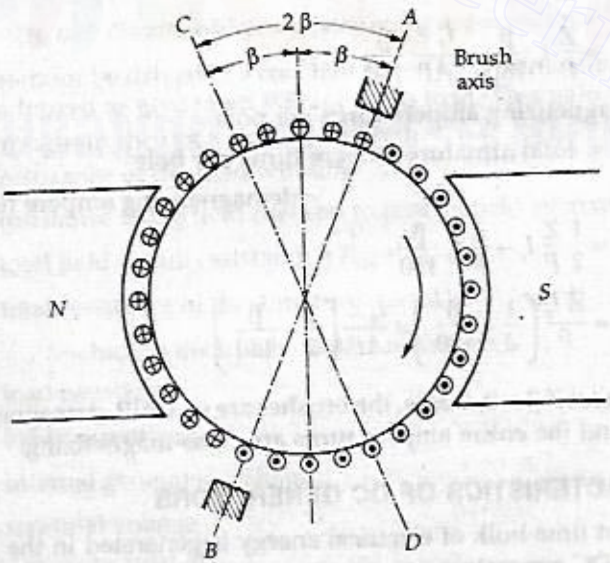


Fig. 6.17. *Demagnetizing and cross-magenitizing components of armature reaction.*

If the brush shift $\beta°$ electrical, then the direction of currents in the ... conductors between the lines AB and CD in the interpolar zones spread ... at the top and bottom of the armature, is such as to produce a flux opp... main pole flux. Hence these conductors are called *demagnetizing armature conductors*. The rest of the conductors (that is lying in an angle $180° - 2\beta°$) carry... produce only cross-magnetizing effect.

Let Z = total number of conductors in the armature

P = total number of poles

$\beta°$ = brush shift in electrical degrees

A = parallel paths

I_a = armature current.

Total number of conductor per pole $= \dfrac{Z}{P}$

Since one turn consists of two conductors, the number of turns per...

$$= \frac{1}{2}\frac{Z}{P}$$

If I_c is the current in each armature conductor, the total ampere turns ... pole $= \dfrac{1}{2}\dfrac{Z}{P} I_c$

These ampere turns are spread over one pole pitch (= 180° electrical)

$\therefore$ the armature ampere turns per degree electrical $= \dfrac{1}{2}\dfrac{Z}{P}\dfrac{I_c}{180}$

The demagnetizing ampere turn pole

$$= \text{armature ampere turns per degree} \times 2\beta$$

$$= \frac{ZI_c \times 2\beta}{2P \times 180°}$$

$$= \frac{Z}{P} I_c \frac{\beta}{180} = \frac{I_a Z}{AP}\frac{\beta}{180} \qquad \left(\because I_c = \ldots\right.$$

Cross-magnetizing ampere turns per pole

$$= \text{total armature ampere turns per pole} - \text{demagnetizing ampere turns per pole}$$

$$= \frac{1}{2}\frac{Z}{P} I_c - \frac{Z}{P} I_c \frac{\beta}{180}$$

$$= \frac{Z I_c}{P}\left(\frac{1}{2} - \frac{\beta}{180}\right) = \frac{I_a Z}{AP}\left(\frac{1}{2} - \frac{\beta}{180}\right)$$

It is seen that if $\beta = 0$, that is, the brushes are on GNP, demagnetizing ampere turns are zero and the entire ampere turns are cross-magnetizing.

6.13 CHARACTERISTICS OF DC GENERATORS

At present time bulk of electrical energy is generated in the form of alternating current. DC generators are no more used in modern power systems. F...

the sake of continuity, the characteristics of dc generators are briefly given here. Characteristic is the graph between two dependent quantities.

Following are the three important characteristics of a dc generator :

1. Magnetization Characteristic. Magnetization characteristic gives the variation of generated voltage (or no-load voltage) with field current at a constant speed. It is also called no-load or open-circuit characteristic (O.C.C.).

2. Internal Characteristic. It is the plot between the generated voltage and load current.

3. External Characteristic or Load Characteristic

It is a graph between the terminal voltage and load current at a constant speed.

6.14 SEPARATELY EXCITED DC GENERATOR

In the separately excited dc generator, the field winding is connected to a separate source of dc power. This source may be another dc generator, a battery, a diode rectifier, or a controlled rectifier.

The circuit for a separately excited dc generator on load is shown in Fig. 6.18.

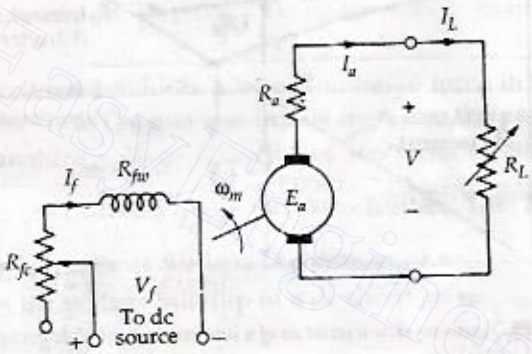


Fig. 6.18. *Circuit model of a separately excited dc generator.*

Let the generator be driven at a constant speed by a prime mover. The field excitation (I_f) is adjusted to give rated voltage at no load. This value of voltage is kept constant throughout the operation considered.

Let R_{fw} = resistance of the field winding

R_{fc} = resistance of the field rheostat to control field current

R_f = total field circuit resistance = $R_{fw} + R_{fc}$

R_a = total resistance of the armature circuit

(including the brush-contact resistance)

R_L = load resistance

I_L = load current

E_a = internal generated voltage

V = terminal voltage

I_a = armature current

The defining equations for the separately excited dc generator are as follows :

$$V_f = R_f I_f$$

$$E_a = V + I_a R_a$$

$$E_a = K_a \Phi \omega_m$$

$$V = I_L R_L$$

$$I_a = I_L$$

If there were no armature reaction, the generated voltage V_0 would be constant as shown by a straight line in Fig. 6.19. Because of the demagnetizing effect of armature reaction there is a voltage drop ΔV_{AR}. The internal characteristic $(E_a \sim I_L)$ is shown in Fig. 6.19.

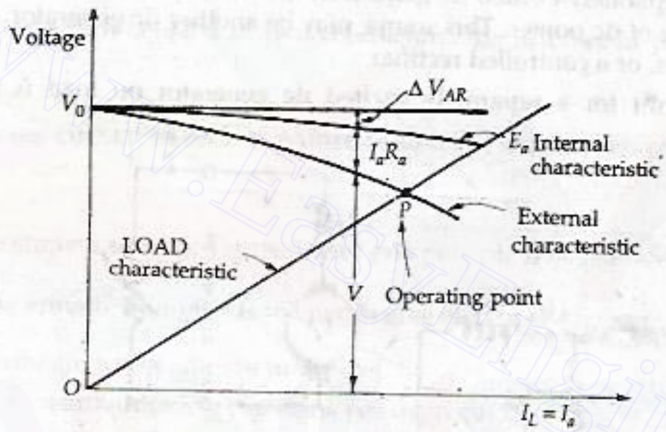


Fig. 6.19. *Terminal characteristics of a separately excited dc generator.*

There is a voltage drop $I_a R_a$ across R_a. The generator external characteristic $(V \sim I_L)$ defined by the relation

$$V = E_a - I_a R_a$$

is shown in Fig. 6.19. The point of intersection between the generator external characteristic and the load characteristic given by the relation $V = I_L R_L$ determines the operating point P. The operating point gives the operating values of terminal voltage V and terminal (load) current I_L.

6.15 VOLTAGE BUILDUP IN SELF-EXCITED GENERATORS

A self-excited dc generator supplies its own field excitation. A self-excited generator shown in Fig. 6.20 is known as a shunt generator because its field winding is connected in parallel with the armature. Thus, the armature voltage supplies the field current.

This generator will build up a desired terminal voltage. Assume that the generator in Fig. 6.20 has no load connected to it and the armature is driven at a certain speed by a prime mover. We shall study the conditions under which the voltage buildup takes place. The voltage buildup in a dc generator depends upon the presence of a *residual flux* in the field poles of the generator. A small voltage E_{ar} will be generated. It is given by

$$E_{ar} = K\,\Phi_{res}\,\omega \tag{6.15.1}$$

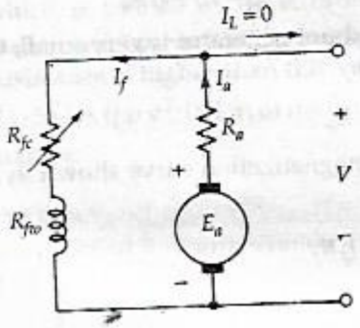


Fig. 6.20. *Equivalent circuit of a shunt dc generator.*

This voltage is of the order of 1V or 2V. it causes a current I_f to flow in the field winding of the generator. The field current is given by

$$I_f = \frac{V}{R_f} \tag{6.15.2}$$

This field current produces a magnetomotive force in the field winding, which increases the flux. The increase in flux increases the generated voltage E_a. The increased armature voltage E_a increases the terminal voltage V. With the increase in V, the field current $I_f = \dfrac{V}{R_f}$ increases further. This in turn increases Φ and consequently E_a increases further. The process of voltage buildup continues. Figure 6.21 shows the voltage buildup of a dc shunt generator.

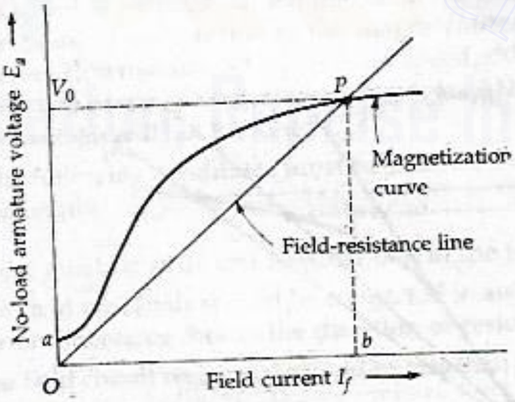


Fig. 6.21. *Voltage build up of a dc shunt generator.*

The effect of magnetic saturation in the pole faces limits the terminal voltage of the generator to a steady-state value.

We have assumed that the generator is on no load during the buildup process. The following equations describe the steady-state operation :

$$I_a = I_f \tag{6.15.3}$$

$$V = E_a - I_a R_a = E_a' - I_f R_a$$

Since the field current I_f in a shunt generator is very small, the voltage drop $I_f R_a$ can be neglected,

and
$$V = E_a. \tag{6.15.4}$$

The E_a versus I_f curve is the magnetization curve shown in Fig. 6.21.

For the field circuit

$$V = I_f R_f \tag{6.15.5}$$

The straight line given by

$$V = I_f R_f$$

is called the **field-resistance line.**

The field-resistance line is a plot of the voltage $I_f R_f$ across the field circuit versus the field current I_f. The slope of this line is equal to the resistance of the field circuit.

The solution of Equations (6.15.4) and (6.15.5) gives the no-load terminal voltage V_0 of the generator. Thus, the intersection point P of the magnetization curve and the field-resistance line gives the no-load terminal voltage V_0 (= bP) and the corresponding field current (Ob). Normally, in the shunt generator the voltage builds up to the value given by the point P. At this point $E_a = I_f R_f = V_0$.

If the field current corresponding to point P is increased further, there is no further increase in the terminal voltage.

Figure 6.22 shows the voltage buildup in the dc shunt generator for various field circuit resistances.

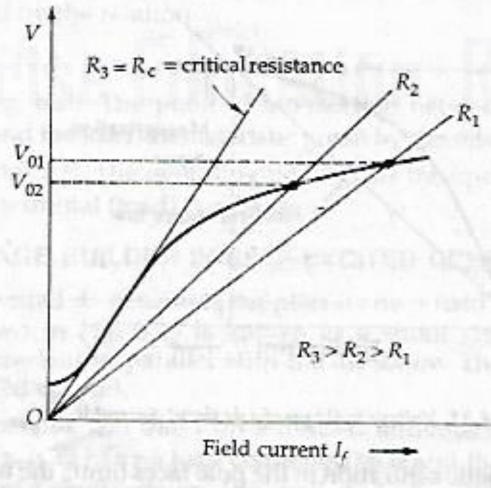


Fig. 6.22. *Effect of field resistance on no-load voltage.*

A decrease in the resistance of the field circuit reduces the slope of the field-resistance line resulting in a higher voltage. If the speed remains constant. An increase in the resistance of the field circuit increases the slope of the field resistance line, resulting in a lower voltage. If the field circuit resistance is increased to R_c which is termed as the critical resistance of the field, the field resistance line becomes a tangent to the initial part of the magnetization curve. when the field resistance is higher than this value, the generator fails to excite.

Figure 6.23 shows the variation of no-load voltage with fixed R_f and variable speed of the armature.

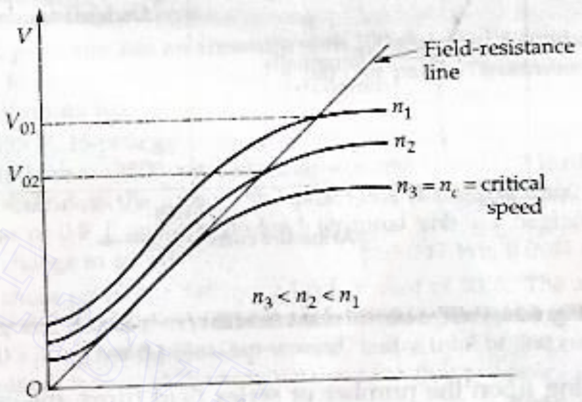


Fig. 6.23. *Variation of no-load voltage with speed.*

The magnetization curve varies with the speed and its ordinate for any field current is proportional to the speed of the generator. If the field resistance is kept constant and the speed is reduced, all the points on the magnetization curve are lowered, and the point of intersection of the magnetization curve and the field resistance line moves downwards. At a particular speed, called the critical speed, the field-resistance line becomes tangential to the magnetization curve. Below the critical speed the voltage will not build up.

In brief, the following conditions must be satisfied for voltage buildup in a self-excited dc generator.

1. There must be sufficient residual flux in the field poles.
2. The field terminals should be connected in such a way that the field current increases flux in the direction of residual flux.
3. The field circuit resistance should be less than the critical field circuit resistance.

If there is no residual flux in the field poles, disconnect the field from the armature circuit and apply a dc voltage to the field winding. This process is called flashing the field. It will induce some residual flux in the field poles.

6.16 CHARACTERISTICS OF COMPOUND DC GENERATORS

The voltage-current characteristics of compound generators are sho Fig. 6.24.

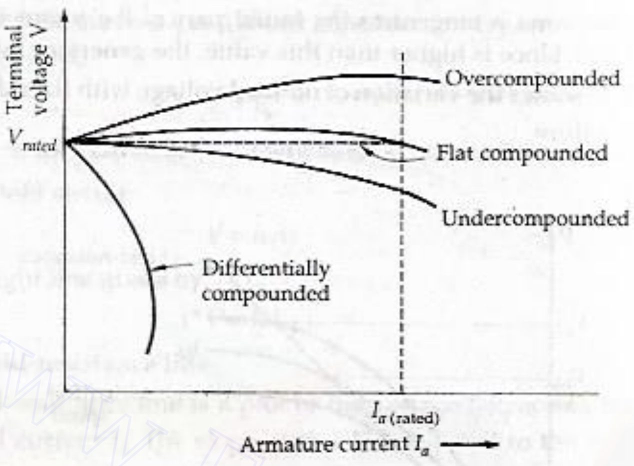


Fig. 6.24. *Voltage-current characteristics of compound dc generators.*

Depending upon the number of series field turns, the cumulatively com pounded generators may be overcompounded, flat compounded, and un dercompounded for an overcompounded generator full-load terminal voltage is higher than the no-load terminal voltage. For a flat compounded (or level com pounded) generator the terminal voltage at full load is equal to the no-load termi nal voltage. In an undercompounded generator the terminal voltage at full load is less than the no-load terminal voltage.

In differential compounded generators, the terminal voltage drops very quickly with increasing armature current.

EXERCISES

6.1 Draw a neat sketch of a d.c. generator. State the functions of each part. Derive the e.m.f. of equation of a d.c. generator.

6.2 An 8-pole lap-wound d.c. generator armature has 960 conductors, a flux of 4 mWb and a speed of 400 r.p.m. Calculate the e.m.f. generated on open circuit. If the same armature is wave wound, at what speed must it be driven to generate 400 V ? [256 V ; 156.25 r.p.m.]

6.3 A 4-pole generator with wave wound armature has 51 slots each having 48 con ductors. The flux per pole is 7.5 mWb. At what speed must the armature be driven to give an induced e.m.f. of 440 V. [719 r.p.m.]

6.4 A 6-pole d.c. generator runs at 850 r.p.m. and each pole has a flux of 12 mWb. If there are 150 conductors in series between each pair of brushes, what is the value of generated e.m.f. ? [153 V]

6.5 A 2-circuit armature of a 4-pole generator has 51 slots, each slot containing 20 conductors. What will be the voltage generated in the machine when driven at 1500 r.p.m., assuming the useful flux per pole to be 0.7×10^{-2} Wb ? [357 V]

6.6 A 4-pole, wave-wound d.c. armature has a bore diameter of 0.7 m. It has 520 conductors and the ratio of pole arc to pole pitch is 0.62. The armature is running at 720 r.p.m. and the flux density in the air gap is 1.1 T. Calculate the e.m.f. generated in the armature if the effective length of the armature conductors is 0.2 m. [936 V]

6.7 A 20 kW, 4-pole shunt generator has a terminal voltage of 250 V when running at 400 r.p.m. The armature has a resistance of 0.16 Ω and consists of 652 conductors which are lap wound. The diameter of the pole shoe circle is 0.38 m. The poles are 0.2 m long and subtend an angle of 60°. Calculate the flux density in the air gap. Neglect shunt field current. [1.519 T]

6.8 An 8-pole d.c. generator has an armature with 100 slots and 8 conductors per slot. The winding is so connected to have 8 parallel paths. Determine the speed to generate 240 V on no load, if the flux per pole is 30 mWb. [600 r.p.m.]

6.9 A 1500 kW, 550 V, 16-pole generator runs at 150 r.p.m. What must be the useful flux per pole if there are 2500 conductors lap-connected and full-load copper losses are 25 kW ? Calculate the area of the pole shoe if the gap flux density has a uniform value of 0.9 T and the no-load terminal voltage, neglecting armature reaction and change in speed. [0.08947 Wb, 0.0994 m^2 ; 559.16 V]

6.10 A 110 V d.c. shunt generator delivers a load current of 50 A. The armature resistance is 0.2 Ω, and the field circuit resistance is 55 Ω. The generator, rotating at a speed of 1800 r.p.m., has 6 poles, lap-wound, and a total of 360 conductors. Calculate the no-load voltage at the armature and the flux per pole. [120.4 V, 11.15 mWb]

6.11 A 4-pole, 250 V d.c. long-shunt compound generator supplies a load of 10 kW at the rated voltage. The armature, series field, and shunt field resistances are 0.1 Ω, 0.15 Ω, and 250 Ω respectively. The armature is lap wound with 50 slots, each slot containing 6 conductors. If the flux per pole is 50 mWb, calculate the speed of the generator. [1041 r.p.m.]

6.12 A 440 V, d.c. compound generator has an armature, series field, and shunt field resistances of 0.5 Ω, 1.0 Ω and 200 Ω respectively. Calculate the generated voltage while delivering 40 A to external circuit for both long-shunt and short-shunt connections. [503.3 V, 501.2 V]

6.13 A short-shunt compound generator supplies a current of 100 A at a voltage of 220 V. The resistances of shunt field, series field and armature are 50 Ω, 0.025 Ω, and 0.05 Ω respectively. The total brush drop is 2 V and the total iron and friction losses are 1000 V. Determine : (*a*) the generated voltage ; (*b*) the copper losses ; (*c*) the output of the prime mover driving the generator ; (*d*) generator efficiency. [(*a*) 229.72 V ; (*b*) 1785.6 W ; (*c*) 24785.6 W ; (*d*) 88.76%]

6.14 What is armature reaction ? Describe the effects of armature reaction on the operation of d.c. machines. How the armature reaction is minimized ?

6.15 What do you understand by demagnetizing and cross magnetizing effects of armature reaction in a d.c. machine ?

6.16 Define commutation. Explain the process of commutation in d.c. generators with neat sketches.

6.17 Explain the process of commutation in a d.c. machine and describe the methods to improve it.

6.18 Explain (i) period of commutation, (ii) reactance voltage during com (iii) emf commutation and (iv) resistance commutation.

6.19 What do you understand by linear commutation under commutation an commutation in a d.c. machine ?

6.20 Explain clearly the functions of a following in d.c. machines :
(a) interpoles (b) compensating windings

6.21 What are commutating poles ? Why are they used ?

6.22 Explain the methods of improving commutation with relevant figures.

6.23 What are the different types of d.c. generators according to the ways fields are excited ? Show the connection diagram of each type.

6.24 Explain the process of building up of voltage in a d.c. shunt generator an the conditions to be satisfied for voltage buildup.

6.25 What is the critical field resistance of a d.c. shunt generator ? What is cance ?

6.26 Distinguish between self-excited and separately excited d.c. generators. Ho self-excited d.c. generators classified ? Give their circuit diagrams.

6.27 Mention the reasons for compounding d.c. generator. Neatly sketch and the external characteristics of a d.c. compound generator.

6.28 State the principle of operation of a dc generator and derive the expression emf generated.

6.29 Describe with relevant diagrams the different methods of excitation machines.

6.30 Explain why the external characteristic of a dc shunt generator is more dr than that of a separately excited generator.

6.31 What are various possible causes for dc shunt generator not building up volta

Direct-C

INTRODUCTION

motor is a machine that con
is very similar to a d.c. g
in a more protected locatio
type. On the other hand
exposed to dust, moisture
protective enclosures for e
requirements.

MOTOR PRINCIPLE

When a conductor carryin
on it. The effect of plac
shown in Fig. 7.1. Let us
and suppose that it is ac
motor. By applying left-ha
to the left-hand side. Sin
the force F_C acts in a
effect) is developed on t
conductors. Since the rotor
direction.

Fig. 7.1. A current-

7

Direct-Current Motors

7.1 INTRODUCTION

A motor is a machine that converts electrical energy into mechanical energy. The d.c. motor is very similar to a d.c. generator in construction. Generators are usually operated in more protected locations and therefore their construction is generally of the *open* type. On the other hand, motors are generally used in locations where they are exposed to dust, moisture, fumes and mechanical damage. Thus, motors require protective enclosures for example, drip-proof, fire-proof etc., according to the requirements.

7.2 MOTOR PRINCIPLE

When a conductor carrying current is put in a magnetic field, a force is produced on it. The effect of placing a current-carrying conductor in a magnetic field is shown in Fig. 7.1. Let us consider one such conductor placed in a slot of armature and suppose that it is acted upon by the magnetic field from a north pole of the motor. By applying left-hand rule it is found the conductor has a tendency to move to the left-hand side. Since the conductor is in a slot on the circumference of the rotor, the force F_C acts in a tangential direction to the rotor. Thus, a torque (turning effect) is developed on the rotor. Similar torques are produced on all the rotor conductors. Since the rotor is free to move, it starts rotating in the anticlockwise direction.

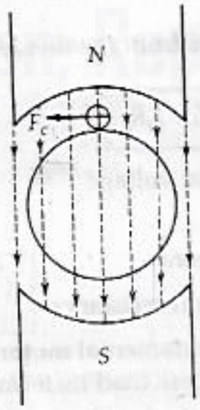


Fig. 7.1. *A current-carrying conductor placed in a magnetic field.*

The torque produced on the rotor is transferred to the shaft of the
can be utilized to drive a mechanical load.

7.3 BACK E.M.F.

When the motor armature rotates, its conductors cut the magn
Therefore, the e.m.f. of rotation E, is induced in them. In case of a motor, th
of rotation is known as **back e.m.f.** or **counter e.m.f.** The back e.m.f. op
applied voltage. Since the back e.m.f. is induced due to generator action its
nitude is, therefore, given by the same expression as that for the gener
in a d.c. generator. That is,

$$\boxed{E = \frac{NP\Phi Z}{60\,A}}$$

where the symbols have their usual meanings.

7.4 EQUIVALENT CIRCUIT OF A D.C. MOTOR ARMATURE

The armature of a d.c. motor can be represented by an equivalent
It can be represented by three series-connected elements E, R_a and V_b. The
E is the back e.m.f., the element R_a is the armature resistance and V_b is the
contact voltage drop. The equivalent circuit of the armature of a d.c. m
shown in Fig. 7.2.

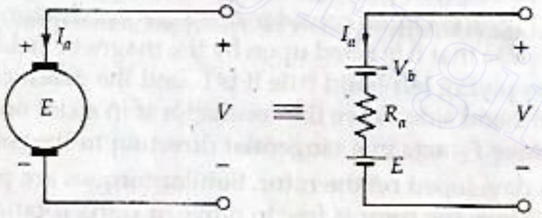


Fig. 7.2. *Equivalent circuit of the armature of a d.c. motor.*

In a motor, current flows from the line *into* the armature against the
ated voltage. By KVL

$$\boxed{V = E + I_a R_a}$$

where V = motor terminal voltage

E = back e.m.f.

I_a = armature current

R_a = armature circuit resistance

Equation (7.4.1) is the fundamental **motor equation.** It is seen that the
e.m.f. E of the motor is always less than its terminal voltage V.

If the voltage drop V_b in the brushes is also considered, then by KVL

$$V = E + I_a R_a + V_b$$

7.5 TORQUE OF A DC MACHINE

When a dc machine is loaded either as a motor or as a generator, the rotor conductors carry current. These conductors lie in the magnetic field of the air gap. Thus each conductor experiences a force. The conductors lie near the surface of the rotor at a common radius from its centre. Hence a torque is produced around the circumference of the rotor and the rotor starts rotating.

When the machine operates as a generator at constant speed, this torque is equal and opposite to that provided by the prime-mover. When the machine is operating as a motor the torque is transferred to the shaft of the rotor and drives the mechanical load.

The expression for the torque is the same for the generator and the motor. It can be deduced as follows :

The voltage equation of a d.c. motor is

$$V = E + I_a R_a \tag{7.5.1}$$

Multiplying both the sides of Eq. (7.5.1) by I_a we obtain

$$VI_a = EI_a + I_a^2 R_a \tag{7.5.2}$$

But VI_a = electrical power input to the armature

$I_a^2 R_a$ = copper loss in the armature

We also know that input = output + losses (7.5.3)

Comparison of Eqs. (7.5.2) and (7.5.3) shows that

EI_a = electrical equivalent of gross mechanical power developed by the armature (electromagnetic power)

Let τ_{av} = average electromagnetic torque developed by the armature in newton metres (Nm)

At this value of torque the electromechanical power conversion takes place.

Mechanical power developed by the armature,

$$P_m = \omega\tau_{av} = 2\pi n\tau_{av}$$

Therefore

$$P_m = EI_a = \omega\tau_{av} = 2\pi n\tau_{av} \tag{7.5.4}$$

But

$$E = \frac{nP\Phi Z}{A}$$

Therefore

$$\frac{nP\Phi Z}{A} I_a = 2\pi n\tau_{av}$$

and

$$\boxed{\tau_{av} = \frac{PZ}{2\pi A}\Phi I_a} \tag{7.5.5}$$

Equation (7.5.5) is called the **torque equation** of d.c. motor.

For a given d.c. machine, P, Z and A are constant, therefore $\left(\frac{PZ}{2\pi A}\right)$ is also a constant.

Let $\dfrac{PZ}{2\pi A} = k$

$\therefore$
$$\boxed{\begin{aligned} \tau_{av} &= k\Phi I_a \\ \tau_{av} &\propto \Phi I_a \end{aligned}}$$

Hence *the torque developed by a d.c. motor is directly proportional* ... *pole and armature current.*

Alternative Proof

Consider a turn $a\,a'\,b'\,b$ whose two conductors aa' and bb' are pl... two adjacent poles as shown in Fig. 7.3.

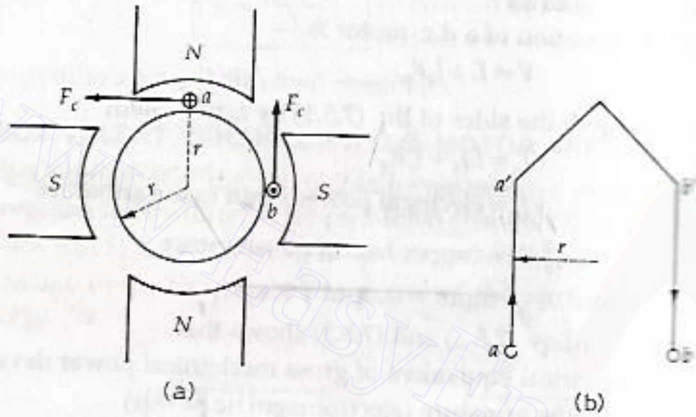


Fig. 7.3.

The force on a conductor of length l placed on the periphery of the ... tor lying in a field of flux density B is

$$F_c = BI_c\, l = B\left(\frac{I_a}{A}\right) l$$

where I_c = current in the *conductor* of the armature winding

I_a = armature terminal current

A = number of parallel paths

In a motor the flux density B is not constant for all conductors. The ave... flux density is taken as B_{av} and l is taken as the length of the armature co...

The average torque developed by a conductor a

$$\tau_c = F_c r = B_{av}\left(\frac{I_a}{A}\right) lr$$

But, $B_{av} = \dfrac{\text{flux per pole}}{\text{area per pole}} = \dfrac{\varphi}{(2\pi r l/P)} = \dfrac{\Phi P}{2\pi r l}$

$$\therefore \quad \tau_{av} = \frac{\Phi P}{2\pi r l}\,\frac{I_a}{A}\, lr = \frac{\Phi P I_a}{2\pi A}$$

...nductors in the ar
... the average torque
... That is, the total e

$$\tau_e = Z\tau_{av}$$

$$\boxed{\tau_e = \frac{PZ}{2\pi A}\,\Phi I_a}$$

... D.C. MOTORS

... motors are nam... armature. There are
... or shunt moto...
... or series moto...
... wound or comp...

...
... common type ...
... the armature, as sho...

Fig. 7.4

... current, voltage and pow...

... equation

... at junction A of Fig. ...
... the incoming current
= sum of the outgoin...

$I = I_a + I_{sh}$

... I = input line current ...

I_{sh} = shunt field curren...

... equations

... equations are w...
... circuit

$$V = I_{sh} R_{sh}$$

...armature-winding circu...

$$V = E + I_a R_a$$

Since all the conductors in the armature winding develop torque in the same direction, therefore, the average torque developed by the armature will be the sum of all these torques. That is, the total electromagnetic torque developed is :

$$\tau_e = Z\tau_{av}$$

or
$$\boxed{\tau_e = \frac{PZ}{2\pi A}\Phi I_a} \tag{7.5.9}$$

7.6 TYPES OF D.C. MOTORS

Direct current motors are named according to the connection of the field winding with the armature. There are three types of d.c. motors.

1. Shunt wound or shunt motor.
2. Series wound or series motor.
3. Compound wound or compound motor.

7.6.1 Shunt motor

This is the most common type of d.c. motors. The field winding is connected in parallel with the armature, as shown in Fig. 7.4.

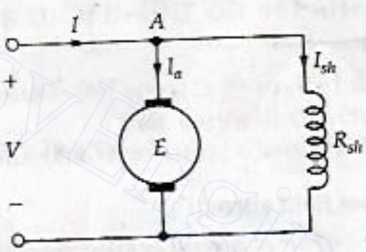


Fig. 7.4. *D.C. Shunt motor.*

The current, voltage and power equations for a shunt motor are written as follows :

Current equation

By KCL at junction A of Fig. 7.4,

Sum of the incoming currents at A

= sum of the outgoing currents at A

$$I = I_a + I_{sh} \tag{7.6.1}$$

where I = input line current ; I_a = armature current

I_{sh} = shunt field current

Voltage equations

The voltage equations are written by using Kirchhoff's voltage law (KVL).

For field-winding circuit

$$V = I_{sh} R_{sh} \tag{7.6.2}$$

For armature-winding circuit

$$V = E + I_a R_a \tag{7.6.3}$$

Power equations

Power input = mechanical power developed + losses in the armature + loss in the field

$$VI = P_m + I_a^2 R_a + I_{sh}^2 R_{sh}$$

$$= P_m + I_a^2 R_a + VI_{sh}$$

$$P_m = VI - VI_{sh} - I_a^2 Ra = V(I - I_{sh}) - I_a^2 R_a$$

$$= VI_a - I_a^2 R_a = (V - I_a R_a) I_a$$

$$\therefore \quad P_m = EI_a$$

Multiplying Eq. (7.6.3) by I_a we get

$$VI_a = EI_a + I_a^2 R_a$$

$$VI_a = P_m + I_a^2 R_a$$

where VI_a = electrical power supplied to the armature of the ...

7.6.2 Series motor

In the series motor (Fig. 7.5), the field winding is connected in se... the armature.

Current equation

By KCL in Fig. 7.5

$$I = I_{se} = I_a \qquad (7.6.8)$$

where I_{se} = series field current

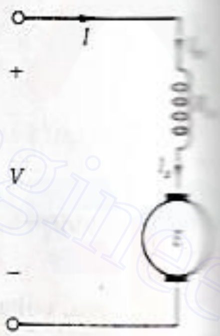


Fig. 7.5. DC series motor

Voltage equation

By KVL in Fig. 7.5

$$V = E + I(R_a + R_{se}) \qquad (7.6.9)$$

Power equations

Multiplying Eq. (7.6.9) by I we get

$$VI = EI + I^2(R_a + R_{se}) \qquad (7.6.10)$$

Power input = mechanical power developed + losses in the armature + losses in the field

$$VI = P_m + I^2 R_a + I^2 R_{se} \qquad (7.6.11)$$

Comparison of Eqs. (7.6.10) and (7.6.11) shows that

$$P_m = EI \qquad (7.6.12)$$

7.6.3 Compound motor

A d.c. motor having both shunt and series field windings is called a compound motor. It may be a short-shunt compound motor or a long-shunt compo...

motor. A d.c. compound motor may be cumulatively compounded or differentially compounded as discussed in Sec. 6.5. The current relationships for a compound motor can be written by using KCL. The voltage relationships are written by using KVL.

7.7 ARMATURE REACTION IN A D.C. MOTOR AND INTERPOLES

The armature reaction is the effect of armature flux on the main flux. In case of a d.c. motor the resultant flux is *strengthened* at the *leading pole tips* and *weakened* at the *trailing pole tips*.

Armature reaction causes sparking at the brushes due to delay in commutation. Interpoles are placed in between the main poles in order to neutralize the effects of armature reaction in brush region and minimize sparking at brushes. Interpoles generate voltage necessary to neutralize the e.m.f. of self-induction in the armature coils undergoing commutation. Motor interpoles have a polarity opposite to that of the following main pole in the direction of rotation of armature. Since the interpoles are connected in series with the armature, the change in direction of current in the armature changes the polarity of the interpole. Thus, a d.c. machine that has correct interpole polarity when used as a generator will have the correct interpole polarity when used as a motor.

7.8 CHARACTERISTICS OF A SHUNT OR SEPARATELY EXCITED D.C. MOTOR

In both the cases of shunt and separately excited d.c. motors, the field is supplied from a constant voltage so that the field current is constant. The two motors, therefore, have similar characteristics. Characteristic is a graph between two dependent quantities.

7.8.1 Speed-armature current characteristics

In a shunt motor, $I_{sh} = V/R_{sh}$. If V is constant I_{sh} will also be a constant. Hence the flux is constant at no load. The flux decreases slightly due to armature reaction. If the effect of armature reaction is neglected, the flux Φ will remain constant. The motor speed is given by

$$N \propto \frac{V - I_a R_a}{\Phi} \tag{7.8.1}$$

$V - I_a R_a \downarrow$ → practically a constant speed motor

$\Phi \downarrow$

If Φ is constant the speed can be written as

$$N \propto V - I_a R_a \tag{7.8.2}$$

Equation (7.8.2) is the equation of a straight line with a negative slope. That is, the speed N of the motor decreases linearly with the increase in armature current as shown in Fig. 7.6.

Since $I_a R_a$ at full load is very small compared to V, the drop in speed from no load to full load is very small. The decrease in N is partially neutralized by a reduction in Φ due to armature reaction. Hence for all practical purposes the shunt motor may be taken as a constant-speed motor.

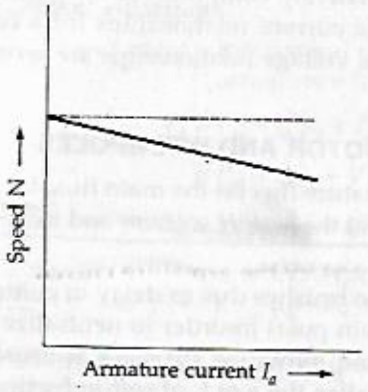


Fig. 7.6. *Speed-armature current (N/I_a) characteristic of a shunt or separately excited d.c. motor.*

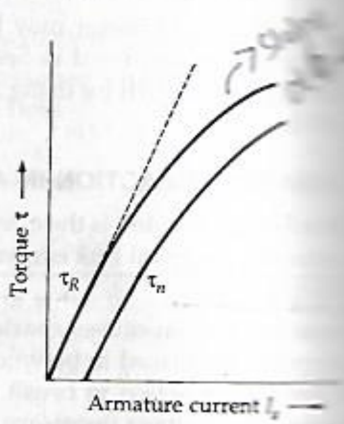


Fig. 7.7. *Torque/armature current characteristic of a shunt or separately excited d.c. motor*

7.8.2 Torque/armature current characteristic

From Eq. (7.5.8)

$$\boxed{\tau_g \propto \Phi I_a}$$

If the effect of armature reaction is neglected, Φ is nearly constant and

$$\tau_g \propto I_a$$ → if Φ is constant

Equation (7.8.4) shows that the graph between τ_g and I_a is a straight line passing through the origin (Fig. 7.7).

If the effect of armature reaction is taken into account, the value of Φ decreases slightly with the increase in armature current. Hence at higher values of I_a the gross or total torque τ_g decreases slightly.

The relation between various torques is given by the relation

$$\tau_n = \tau_g - (\tau_f + \tau_w)$$

where τ_n = net torque or useful torque or load torque at the output shaft

τ_g = gross or total torque

τ_f = frictional torque

τ_w = windage torque

The graph showing the relationship between the net torque and the armature current is a curve parallel to the corresponding gross torque curve but slightly below it.

7.9 CHARACTERISTICS OF A D.C. SERIES MOTOR

7.9.1 Speed/armature current characteristic

The motor speed N is given by

$$N \propto \frac{V - I_a(R_a + R_{se})}{\Phi}$$

At low values of I_a, the voltage drop $I_a(R_a + R_{se})$ is negligibly small in comparison with V. Therefore

$$N \propto \frac{V}{\Phi} \tag{7.9.2}$$

Since V is constant

$$N \propto \frac{1}{\Phi} \tag{7.9.3}$$

In a series motor, the flux Φ is produced by the armature current flowing in the field winding so that $\Phi \propto I_a$. Hence the series motor is a variable flux machine. Equation (7.9.3) now becomes

$$\boxed{N \propto \frac{1}{I_a}} \tag{7.9.4}$$

Thus, for the series motor, the speed is inversely proportional to the armature (load) current. The speed-load characteristic is a rectangular hyperbola as shown in Fig. 7.8.

Equation (7.9.4) shows that when the load current is small, the speed will be very large. Therefore at no load or at light loads there is a possibility of dangerously high speeds, which may damage the motor due to large centrifugal forces. Hence *a series motor must never run unloaded.* It should always be coupled to a mechanical load either directly or through gearing. It should never be coupled by belt, which may break at any time. With the increase in armature current (which is also the field current) the flux also increases and therefore the speed is reduced.

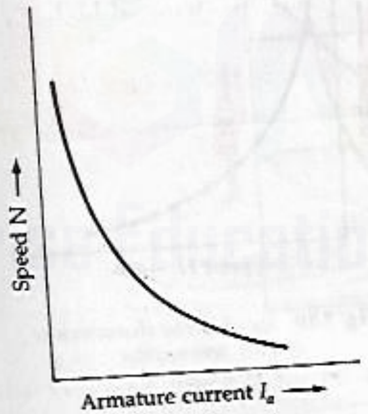


Fig. 7.8. *Speed-armature current characteristic of a d.c. series motor.*

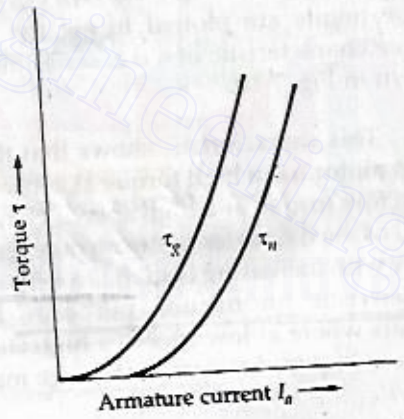


Fig. 7.9. *Torque/armature current characteristic of a d.c. series motor.*

7.9.2 Torque/Armature Current Characteristic

From Eq. (7.5.8)

$$\tau_g \propto \Phi I_a \tag{7.9.5}$$

Before saturation, $\Phi \propto I_a$ and hence at light loads

$$\tau_g \propto I_a^2 \tag{7.9.6}$$

Equation (7.9.6) shows that the torque/armatue current (τ/I_a) curve will be parabolic. When the iron becomes magnetically saturated, Φ becomes almost constant, so that at heavy loads

$$\tau_g \propto I_a \tag{7.9.7}$$

Equation (7.9.7) shows that the τ/I_a characteristic is a straight line. Thus, the torque/current characteristic of a d.c. series motor is initially parabolic and finally becomes linear when the load current becomes large. The characteristic changes smoothly from one curve to another. This characteristic is shown in Fig. 7.9.

The characteristic relating the net torque or useful torque τ_n to the armature current is parallel to the T_g/I_a characteristic, but is slightly below it (Fig. 7.9). The difference between the two curves is due to friction and windage losses.

7.9.3 Speed/torque characteristic

The speed/torque characteristic of a series motor can be derived from its speed-armature current (N/I_a) and torque-armature current (τ/I_a) characteristics as follows :

For a given value of I_a find τ from τ/I_a curve and N from N/I_a curve. This gives one point (τ, N) on speed-torque (N/τ) curve. Repeat this procedure for a number of values of I_a and find the corresponding values of speed and torque (τ_1, N_1), (τ_2, N_2) etc. These points are plotted to get the speed-torque characteristic of a d.c. series motor as shown in Fig. 7.10.

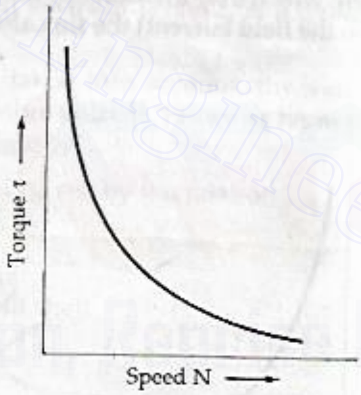


Fig. 7.10. *Speed-torque characteristic of a d.c. series motor.*

This characteristic shows that the d.c. series motor has a high torque at a low speed and a low torque at a high speed. Hence the speed of the d.c. series motor changes considerably with increasing load. It is a very useful characteristic for traction purposes, hoists and lifts where at low speeds a high starting torque is required to accelerate large masses.

7.10 CHARACTERISTICS OF A COMPOUND MOTOR

A compound motor has both shunt and series field windings, so its characteristics are intermediate between the shunt and series motors. The cumulative compound motor is generally used in practice. The speed-armature current characteristics are shown in Fig. 7.11.

The torque-armature current characteristics are shown in Fig. 7.12.

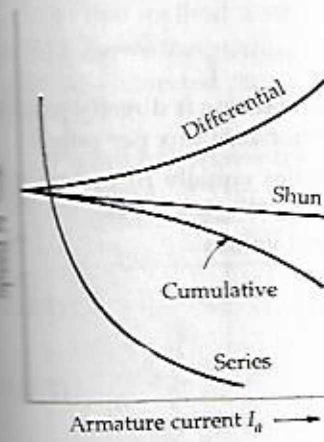


Fig. 7.11. *Speed-armature current characteristic of a d.c. motor*

Figure 7.13 shows th
tor. It is found that a com
no-load speed. These f
s such as lifts, hoists etc.

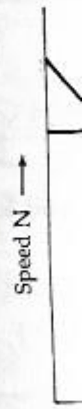


Fig. 7.13. Sp

SPEED OF A D.C.

e.m.f. equation of a d.c

Solving for N gives

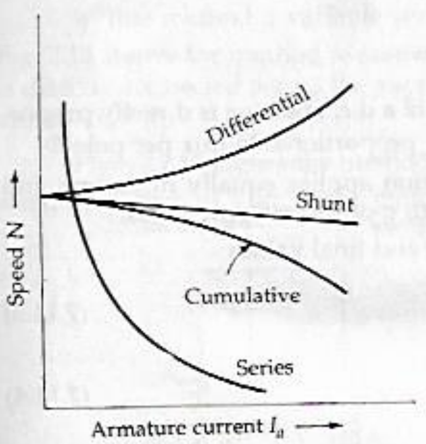


Fig. 7.11. *Speed-armature current characteristic of a d.c. motor.*

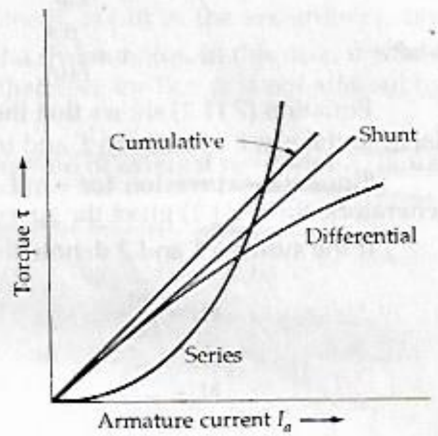


Fig. 7.12. *Torque/armature current characteristic of a d.c. motor.*

Figure 7.13 shows the speed-torque (N/τ) characteristic of a compound motor. It is found that a compound motor has a high starting torque together with a safe no-load speed. These factors make it suitable for use with heavy intermittent loads such as lifts, hoists etc.

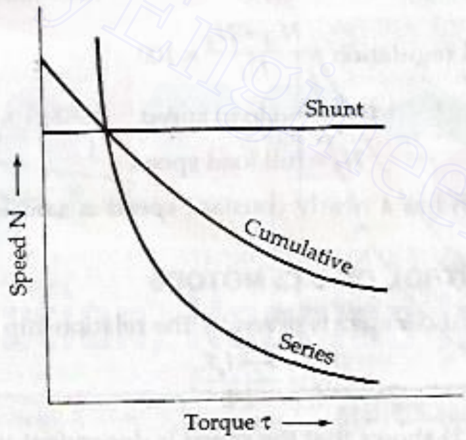


Fig. 7.13. *Speed-torque characteristic of a compound motor.*

7.11 SPEED OF A D.C. MACHINE

The e.m.f. equation of a d.c. machine is given by

$$E = \frac{NP\Phi Z}{60\,A}$$

Solving for N gives

$$N = \frac{60A}{PZ}\frac{E}{\Phi} \tag{7.11.1}$$

$$\therefore \qquad N = \frac{E}{k\Phi}$$

where $$k = \frac{PZ}{60A}.$$

Equation (7.11.2) shows that the speed of a d.c. machine is directly proportional to the e.m.f. of rotation E and inversely proportional to flux per pole Φ.

Since the expression for e.m.f. of rotation applies equally to motors and generators, Eq. (7.11.1) gives the speed for both motors and generators.

If the suffixes 1 and 2 denote the initial and final values

$$N_1 = \frac{E_1}{k\Phi_1}$$

$$N_2 = \frac{E_2}{k\Phi_2}$$

$$\frac{N_2}{N_1} = \frac{E_2}{E_1} \times \frac{\Phi_1}{\Phi_2}$$

7.11.1 Speed regulation

The **speed regulation** *is defined as the change in speed from no load to full load expressed as a fraction or a percentage of the full load speed.* It can be written as

$$\text{Per unit speed regulation} = \frac{N_{nl} - N_{fl}}{N_{fl}}$$

$$\text{Per cent speed regulation} = \frac{N_{nl} - N_{fl}}{N_{fl}} \times 100$$

where N_{nl} = no load speed

N_{fl} = full load speed

A motor which has a nearly constant speed is said to have a good speed regulation.

7.12 SPEED CONTROL OF D.C. MOTORS

The speed of a d.c. motor is given by the relationship

$$N = \frac{V - I_a R_a}{k\Phi}$$

Equation (7.12.1) shows that the speed is dependent upon the supply voltage V, the armature circuit resistance R_a, and the field flux Φ, which is produced by the field current. In practice, the variation of these three factors is used for speed control. Thus, there are three general methods of speed control of d.c. motors.

1. Variation of resistance in the armature circuit.

   This method is called **armature resistance control.** (Rheostatic control)
2. Variation of field flux Φ

   This method is called **field flux control.**
3. Variation of applied voltage. (Armature Voltage Control)

7.12.1 Armature resistance control (Rheostatic Control)

In this method a variable series resistor R_e is put in the armature circuit. Fig. 7.14 shows the method of connection for a shunt motor. In this case, the field is directly connected across the supply and therefore the flux Φ is not affected by variation of R_e.

Figure 7.15 shows the method of connection of external resistance R_e in the armature circuit of a d.c. series motor.

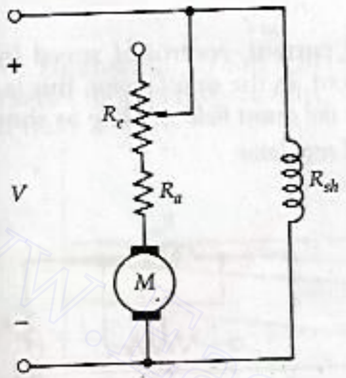


Fig. 7.14. Speed control of a d.c. shunt motor by armature resistance control.

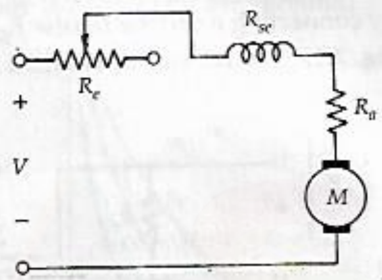


Fig. 7.15. Speed control of a d.c. series motor by armature resistance control.

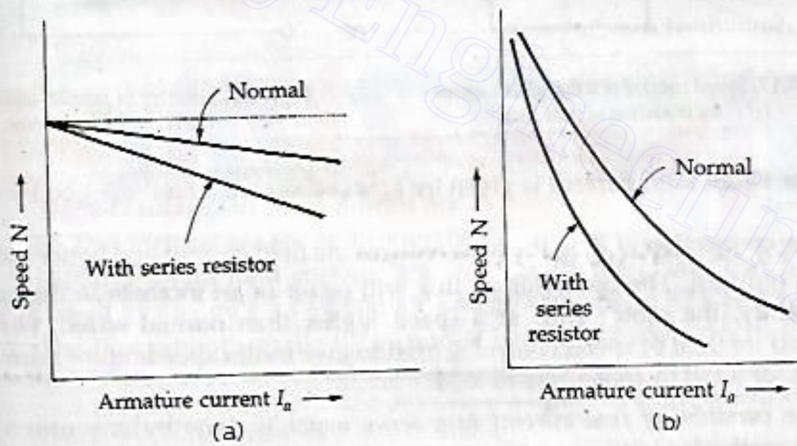


Fig. 7.16. Speed/current characteristics (a) shunt motor (b) series motor.

In this case the current and hence the flux are affected by the variation of the armature circuit resistance. The voltage drop in R_e reduces the voltage applied to the armature and therefore the speed is reduced. Figures 7.16 (*a*) and 7.16 (*b*) show typical speed/current characteristics for shunt and series motors respectively. In both the cases the motor runs at a lower speed as the value of R_e is increased. Since R_e carries full armature current, it must be designed to carry continuously the full armature current.

This method suffers from the following *drawbacks* :

(*i*) A large amount of power is wasted in the external resistance R_e.

(*ii*) Control is limited to give speeds below normal and increase of speed cannot be obtained by this method.

(*iii*) For a given value of R_e the speed reduction is not constant but varies with the motor load.

This method is only used for small motors.

7.12.2 Variation of field flux Φ (Field Flux Control)

Since the flux is produced by the field current, control of speed by this method is obtained by control of the field current. In the *shunt motor*, this is done by connecting a *variable resistor* R_C *in series with the shunt field winding* as shown in Fig. 7.17. The resistor R_c is called the *shunt field regulator.*

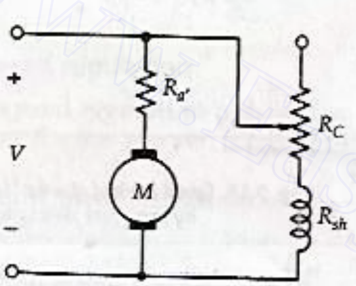


Fig. 7.17. *Speed control of a d.c. shunt motor by variation of field flux.*

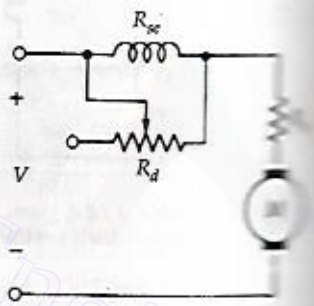


Fig. 7.18. *Diverter in parallel with the series of d.c. motor*

The shunt field current is given by $I_{sh} = \dfrac{V}{R_{sh} + R_c}$

The connection of R_c in the field reduces the field current and hence the flux Φ is also reduced. The reduction in flux will result in an increase in the speed. Consequently, the motor runs at a speed higher than normal speed. For this reason, this method of speed control is used to give motor speeds *above* normal or to correct for a fall in speed due to load.

The *variation of field current in a series motor* is done by any one of the following methods :

(*a*) A variable resistance R_d is connected in parallel with the series field winding as shown in Fig. 7.18. The parallel resistor is called the **diverter**. A portion of the main current is diverted through R_d.

Thus, the diverter reduces the current flowing through the field winding. This reduces the flux and *increases* the speed.

(*b*) The second method uses a **tapped field control** as shown in Fig. 7.1

Here the ampere-turns are varied by varying the number of field turns. This arrangement is used in electric traction.

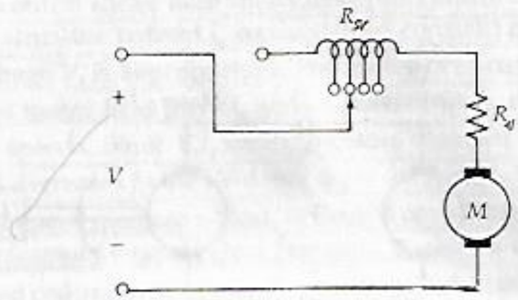


Fig. 7.19. *Tapped series field on d.c. motor.*

Figures 7.20 (*a*) and 7.20 (*b*) show the typical speed/torque curves for shunt and series motors respectively, whose speeds are controlled by the variation of the field flux.

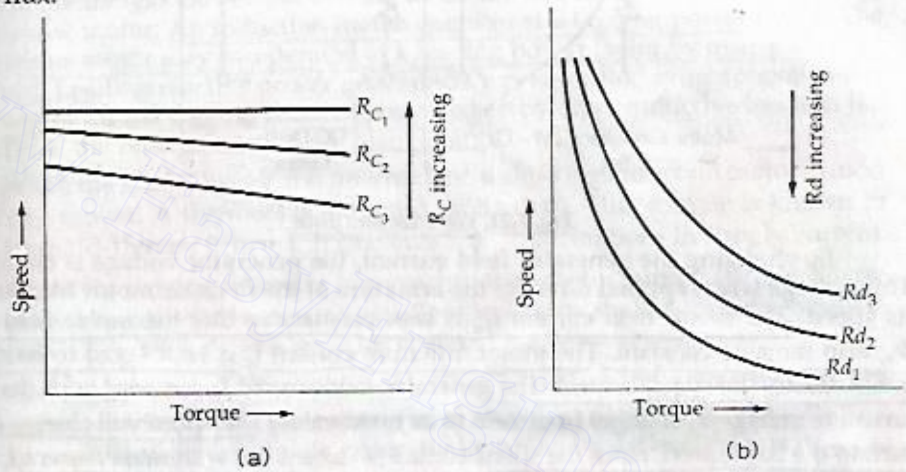


Fig. 7.20. *Typical speed/torque curves (a) shunt motor (b) series motor.*

The **advantages of field control** are as follows :

(*i*) This method is easy and convenient.

(*ii*) Since shunt field current I_{sh} is very small, the power loss in the shunt field is small.

The flux cannot usually be increased beyond its normal value because of saturation of the iron, so speed control by flux is limited to weakening, which gives an *increase* in speed. It is applicable over only a limited range, because if the field is weakened too much there is a loss of stability.

7.12.3 Armature Voltage Control

Speed control of dc motors can also be obtained by varying the applied voltage to the armature. **Ward-Leonard System** of speed control is based on this principle. This method was introduced in 1891. The schematic diagram of the Ward-Leonard method of speed control of a dc shunt motor is shown in Fig. 7.21. In this system M is the main dc motor whose speed is to be controlled, and G is a separately excited dc generator. The generator G is driven by a 3-phase driving motor which may be an induction motor or a synchronous motor. The combination of ac driving motor and the dc generator is called the motor-generator ($M-G$) set.

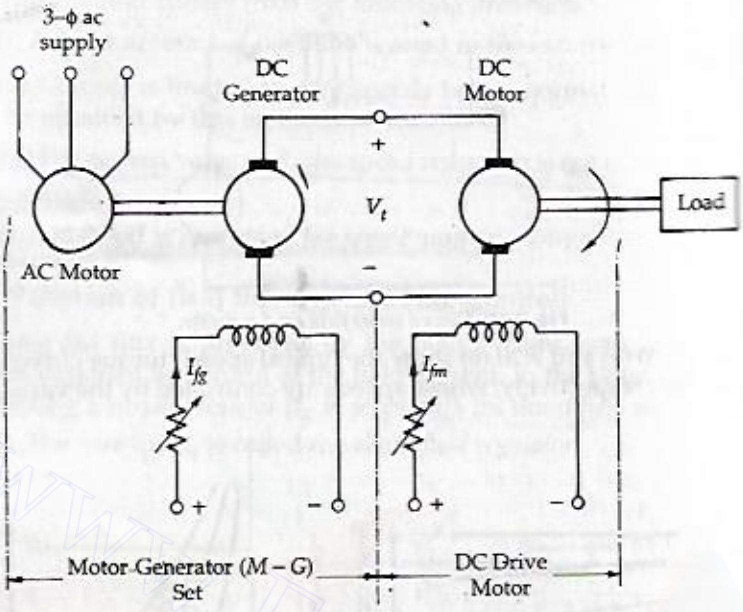


Fig. 7.21. *Ward-Leonard drive.*

By changing the generator field current, the generator voltage is changed. This voltage when applied direct to the armature of the main dc motor M changes its speed. The motor field current I_{fm} is kept constant so that the motor field flux Φ_m also remains constant. The motor armature current I_a is kept equal to its rated value during the speed control. The generator field current I_{fg} is varied such that the armature voltage V_t changes from zero to its rated value. The speed will change from zero to the base speed. Since the speed control is carried out with rated current I_a and with constant motor field flux Φ_m, a constant torque ($\propto \Phi_m I_a$) upto base (rated) speed is obtained. Since the power P (= torque × speed) is proportional to speed, it increases with speed. *Hence with armature voltage control method constant torque and variable power drive is obtained from speed below the base speed.* This is shown in Fig. 7.22.

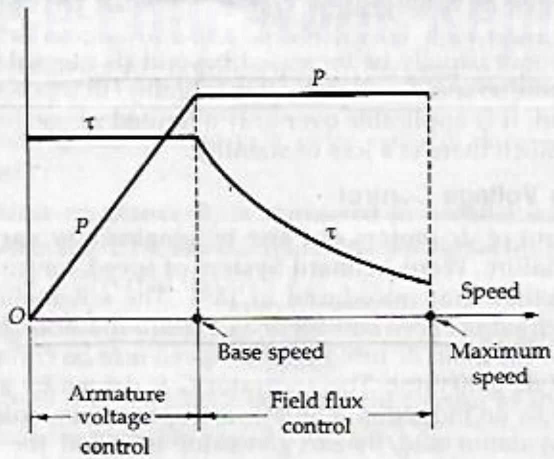


Fig. 7.22. *Torque and power characteristics is combined armature voltage and field control.*

For speed control above base speed field flux control is used. In this mode of operation, the armature current I_a is maintained constant at its rated value and the generator voltage V_t is kept constant. The motor field current I_{fm} is decreased and, therefore, the motor field flux Φ_m is decreased. That is, the field is weakened to obtain higher speeds. Since V_tI_a or EI_a remains constant, the electromagnetic torque τ ($\propto \Phi_m I_a$) decreases as the field flux Φ_m is decreased. Therefore, the torque τ decreases, as the speed increases. Thus, *in the field control mode, constant power and variable torque is obtained for speeds above base speed* as shown in Fig. 7.22.

When speed control over a wide range is required, combination of armature voltage control and field flux control is used. This combination permits the ratio of maximum to minimum available speeds to be 20 to 40. With closed loop control, this range can be extended upto 200.

As mentioned earlier the driving ac motor can be an induction motor or synchronous motor. An induction motor operates at a lagging power factor. The synchronous motor may be operated at a leading power factor by over-excitation of its field. Leading reactive power generated by over-excited synchronous motor compensates for the lagging reactive power taken by other inductive loads in the plant. Thus, the power factor of the plant is improved.

When the load is heavy and intermittent, a slip-ring induction motor is used as a prime mover. A flywheel is mounted on its shaft. This scheme is known as **Ward-Leonard-Ilgener Scheme.** It prevents heavy fluctuations in supply current.

When the driving ac motor is a synchronous motor supply current fluctuations cannot be reduced by mounting a flywheel on its shaft, because a synchronous motor operates only at a constant speed.

In another form of Ward-Leonard drive, non-electrical prime movers can also be used to drive the dc generator. For example, in diesel electric locomotive and ship populsion drives, the dc generator is driven by a diesel engine or a gas turbine. In this system regenerative braking is not possible because energy cannot flow in the reverse direction in the prime mover.

Advantages of Ward-Leonard Drives

The main advantages of the Ward-Leonard drive are as follows :

1. Smooth speed control of dc motors over a wide range in both directions is possible.
2. It has inherent regenerative braking capacity.
3. By using an overexcited synchronous motor as the drive for dc generator, the lagging reactive voltamperes of the plant are compensated. Therefore the overall power factor of the plant improves.
4. When the load is intermittent as in rolling mills, the drive motor used is an induction motor with a flywheel mounted on its shaft to smooth out the intermittent loading to a low value.

Drawbacks of Classical Ward-Leonard System

The classical Ward-Leonard system with rotating machines ($M-G$ set) suffers from the following drawbacks :

1. Higher initial cost due to use of two additional machines ($M-G$ set) of the same rating as the main dc motor.

2. Larger size and weight.
3. Requires more floor area and costly foundation.
4. Frequent maintenance is needed.
5. Lower efficiency due to higher losses.
6. The drive produces more noise.

7.13 SOLID-STATE CONTROL

Static Ward-Leonard drives are being used these days because of the dra- backs of the classical method. Rotating motor-generator sets have been rep by solid-state converters to control the speed of dc motors. The converters are controlled rectifiers or choppers.

In case of ac supply, controlled-rectifiers are used to convert fixed ac su voltage into a variable ac supply voltage.

When the supply is dc, choppers are used to obtain variable dc voltage the fixed-voltage dc supply.

Drawbacks of Static Ward-Leonard Drives

The main drawbacks of static Ward-Leonard drives are as follows :

1. They are not suitable for intermittent loads because load fluctu produce large fluctuations of supply voltage and current. Ther no provision of load equalisation in static Ward-Leonard system
2. Harmonics are generated in the system which affect the qualit supply.
3. Such a system operates at low power factor particularly at lo speeds.

In general static Ward-Leonard drives are used in most applications. How- ever, conventional Ward-Leonard drives are used in large-size intermittent loa In case of nonelectrical prime movers conventional Ward-Leonard system can be used.

Applications of Ward-Leonard Drives

Ward-Leonard drives are used where a smooth speed control of dc mot over a wide range in both directions is required as in rolling mills, elevat cranes, paper mills, diesel-electric locomotives, mine hoists etc.

7.14 STARTING D.C. MOTORS

A **starter** is a device to start and accelerate a motor. A **controller** is a dev to start, control speed, reverse, stop and protect the motor.

7.14.1 Need for starters

The armature current of a motor is given by

$$I_a = \frac{V - E}{R_a} \qquad (7.14$$

Thus, I_a depends upon E and R_a if V is kept constant. When a motor is first switched on, the armature is stationary so the back e.m.f. E is zero. The initial starting armature current I_{as} is given by

$$I_{as} = \frac{V-0}{R_a} = \frac{V}{R_a} \tag{7.14.2}$$

Since the armature resistance of a motor is very small, generally less than one ohm; therefore the starting armature current I_{as} would be very large. For example, if a motor with armature resistance of 0.5 ohm is connected directly to a 230-V supply, then

$$I_{as} = \frac{V}{R_a} = \frac{230}{0.5} = 460 \text{ A}$$

This large current would damage the brushes, commutator, or windings.

As the motor speed increases, the back e.m.f. increases and the difference $(V - E)$ goes on decreasing. This results in the gradual decrease of I_a until the motor attains its stable speed and the corresponding back e.m.f. Under this condition the armature current reaches its desired value. Thus, it is found that the back e.m.f. helps the armature resistance in limiting the current through the armature.

Since the starting current is very large, at the time of starting of all d.c. motors (except very small motors) an extra resistance must be connected in series with the armature. This would limit the initial current to a safe value until the motor has built up the stable speed and back e.m.f. E. The series resistance is divided into sections which are cut out one by one as the speed of the motor rises and the back e.m.f. builds up. When the speed of the motor builds up to its normal value, the extra resistance is completely cut out.

7.15 THREE-POINT D.C. SHUNT MOTOR STARTER

Figure 7.23 shows a three-point d.c. shunt motor starter. It consists of a graded resistance R to limit the starting current. Prior to starting, the handle H is kept in the OFF position by a spring S. For starting the motor, the handle H is moved manually and when it makes contact with the resistance stud 1 it is in the START position. In this position the field winding receives the full supply voltage, but the armature current is limited by the graded resistance $R(= R_1 + R_2 + R_3 + R_4)$. The starter handle is then gradually moved from stud to stud, allowing the speed of the motor to build up until it reaches the RUN position. In this position (*a*) the motor attains full speed, (*b*) the supply is directly across both the windings of the motor, and (*c*) the resistance R is completely cut out. The handle H is held in RUN position by an electromagnet energized by a no-volt trip coil NVC. The no-volt trip coil is connected in series with the field winding of the motor. In the event of switching off, or when the supply voltage falls below a predetermined value, or the complete failure of supply while the motor is running, NVC is deenergized. This results in release of the handle, which is then pulled back to OFF position by the action of the spring. The current to the motor is cut off, and the motor is not restarted without resistance R in the armature circuit. The NVC also provides protection against an open-circuit in the field winding. The NVC is called **no-volt** or **undervoltage protection** of the motor. Without this protection, the supply voltage might be restored with the handle in the RUN position. Consequently, full line voltage may be applied directly to the armature resulting in a very large current.

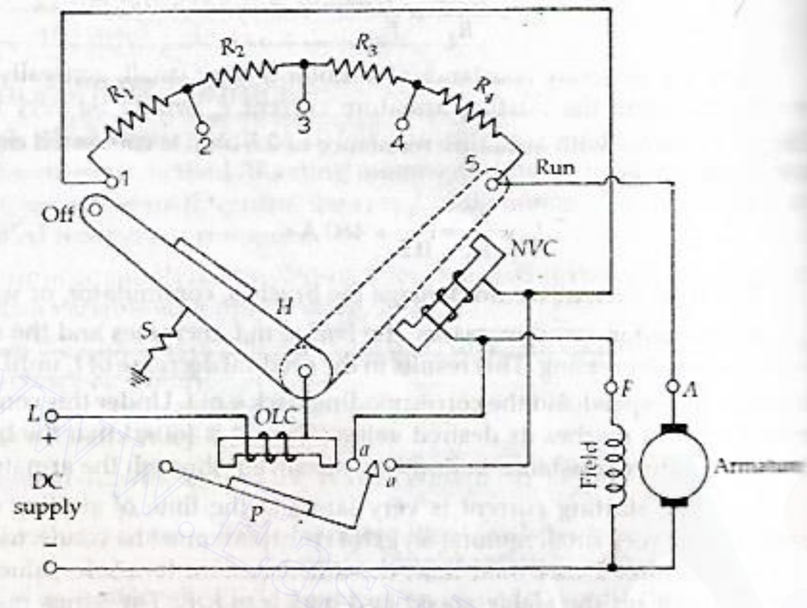


Fig. 7.23. *Three-point D.C. shunt motor starter.*

The other protective device incorporated in the starter is the **overload protection.** Overload protection is provided by the overload trip coil OLC and the NVC. The overload coil is a small electromagnet. It carries the armature current and for normal values of armature current the magnetic pull of OLC is insufficient to attract the strip *P*. When the armature current exceeds the normal rated value (that is, when the motor is overloaded), *P* is attracted by the electromagnet of OLC and closes the contacts *aa*. Thus, NVC is short-circuited. This results in the release of the hundle *H*, which returns to the OFF position and the motor supply is cut off.

If the motor is to be stopped the main switch should be opened. To stop the motor, the starter handle should never be pulled back as this would result in burning the starter contacts.

7.16 DRAWBACKS OF A THREE-POINT STARTER

The three-point starter suffers from a serious drawback for motors with large variation of speed by adjustment of the field rheostat. To increase the speed of the motor the field resistance should be increased. Therefore the current through the shunt field is reduced. The field current may become very low because of the addition of high resistance to obtain a high speed. A very low field current will make the holding electromagnet too weak to overcome the force exerted by the spring. The holding magnet may release the arm of the starter during the normal operation of the motor and thus disconnect the motor from the line. This is not desirable. A four-point starter is used to overcome this difficulty.

7.17 FOUR-POINT STARTER

The schematic connection diagram of four-point starter is shown in Fig. 7.24. The basic difference in the circuit of a 4-point starter as compared to a 3-point starter is that, the holding coil is removed from the shunt field circuit and is connected directly across the line with a current limiting resistance R in series. Such an arrangement forms three parallel circuits :

1. Armature, starting resistance and overload release.
2. A variable resistance and shunt field winding.
3. Holding coil and current limiting resistance.

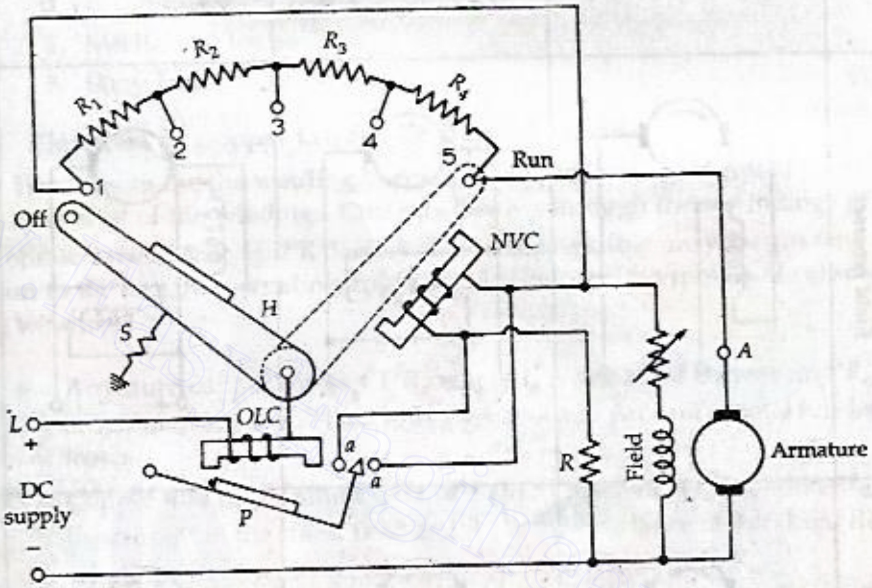


Fig. 7.24. *Four-point D.C. shunt motor starter.*

With this arrangement, a change in field current for variation of speed of the motor, does not affect the current through the holding coil, because the two circuits are independent of each other.

Nowadays automatic push button starters are also used. In such starters the ON push button is pressed to connect the current-limiting starting resistors in series with the armature circuit. These resistors are gradually disconnected by an automatic controlling arrangement until full line voltage is available to the armature circuit. With pressing the OFF button, the circuit is disconnected. Automatic starter circuits have been developed using electromagnetic contactors and time-delay relays. The automatic starters enable even an inexperienced operator to start and stop the motor without any difficulty.

7.18 REVERSAL OF ROTATION

The direction of rotation of a dc motor can be reversed by reversing the connections of *either* the field winding *or* the armature *but not both*. Figure 7.25 shows the alterations in connections required for the reversed rotation of series, shunt and compound motors.

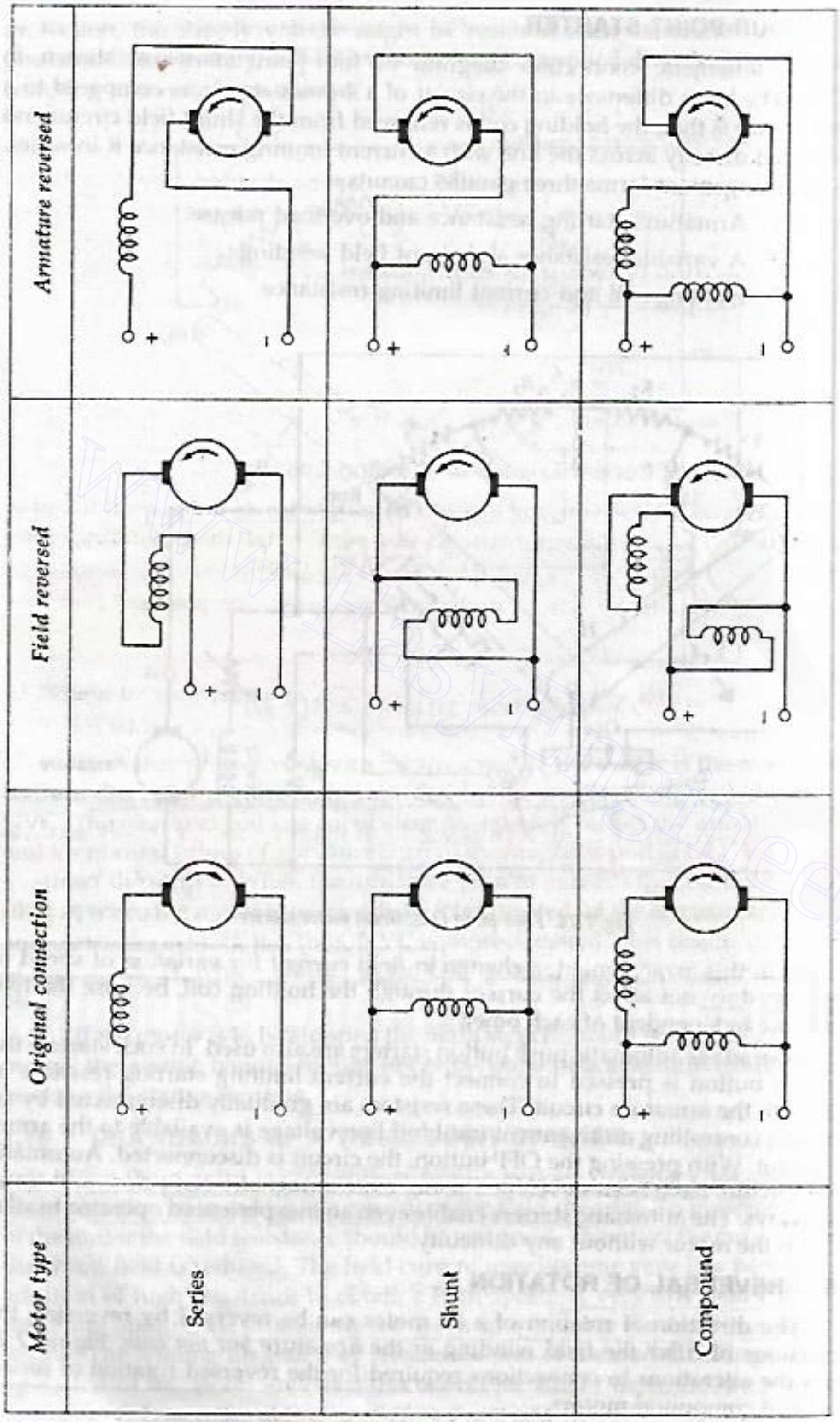


Fig. 7.28 Reversing direction of rotation of d.c. motors

It is to be noted that … compound motor the revers… same windings.

LOSSES IN DC MA…

The losses that occu… categories :

1. Electrical or co…
2. Core losses or …
3. Brush losses
4. Mechanical los…
5. Stray-load loss…

Electrical or Copp…

These losses are the … the resistance of the win… these ohmic losses (that i… addition to the armature w… rating windings.

- Armature coppe… armature resista… losses.
- Copper loss in … the current in th… winding. The sh…
- Copper loss in … the current thro… the series field …
- In a compound … losses are abou…
- Copper loss in … interpole wind…
- Copper loss in … resistance of co…

Magnetic Losse…

The core losses a… machines are usually ope… losses are almost constan…

It is to be noted that in order to reverse the direction of rotation of a compound motor the reversal of the field connections involves both shunt and series windings.

7.19 LOSSES IN DC MACHINES

The losses that occur in dc machines can be divided into five basic categories :

1. Electrical or copper losses (I^2R losses)
2. Core losses or iron losses
3. Brush losses
4. Mechanical losses
5. Stray-load losses

7.19.1 Electrical or Copper Losses or Winding Losses

These losses are the winding losses. The copper losses are present because of the resistance of the windings. Currents flowing through these windings produce ohmic losses (that is, I^2R losses). The windings that may be present in addition to the armature winding are the field windings, interpole and compensating windings.

- Armature copper losses $= I_a^2 R_a$ where I_a is armature current and R_a is armature resistance. There losses are about 30 per cent of total full-load losses.
- Copper loss in the shunt field of a shunt machine $= I_{sh}^2 R_{sh}$ where I_{sh} is the current in the shunt field and R_{sh} is the resistance of the shunt field winding. The shunt regulating resistance is included in R_{sh}.
- Copper loss in the series field of a series machine $= I_{se}^2 R_{se}$ where I_{se} is the current through the series field winding and R_{se} is the resistance of the series field winding.
- In a compound machine, both shunt and series field losses occur. These losses are about 20% of full load losses.
- Copper loss in interpole windings $= I_a^2 R_i$ where R_i is the resistance of interpole windings.
- Copper loss in compensating winding if any $= I_a^2 R_c$ where R_c is the resistance of compensating winding.

7.19.2 Magnetic Losses or Core Losses or Iron Losses

The core losses are the hysteresis losses and eddy-current losses. Since machines are usually operated at constant flux density and constant speed, these losses are almost constant. These losses are about 20 per cent of full-load losses.

7.19.3 Brush Losses

There is a power loss at the brush contacts between the copper commutator and the carbon brushes. In practice, thin loss depends upon the brush contact voltage drop and the armature current I_a. It is given by

$$P_{BD} = V_{BD}\, I_a$$

The voltage drop across a set of brushes is approximately constant over a large range of armature currents. Unless stated otherwise, the brush voltage drop is usually assumed to be about 2 V. The brush droploss is, therefore, taken as $2I_a$.

7.19.4 Mechanical Losses

The losses associated with mechanical effects are called mechanical losses. They consist of *bearing friction loss* and *windage loss*. Windage losses are those associated with overcoming are friction between the moving parts of the machine and the air inside the machine for cooling purposes. These losses are usually very small.

7.19.5 Stray-Load Losses

Stray-load loss consists of all losses, not covered above. These are the miscellaneous losses that result from such factors as (*i*) the distortion of flux because of armature reaction, (*ii*) short circuit currents in the coil, undergoing commutation, etc. These losses are very difficult to determine. The indeterminate nature of the stray-load loss makes it necessary to assign it a reasonable value. For most machines stray losses are taken by convention to be one percent of the full-load output power. The term stray powerless should not be confused with stray load loss.

7.20 POWER-FLOW DIAGRAM

Power-flow diagram is used for determining the generator and motor efficiencies. A power-flow diagram for a d.c. generator is shown in Fig. 7.26.

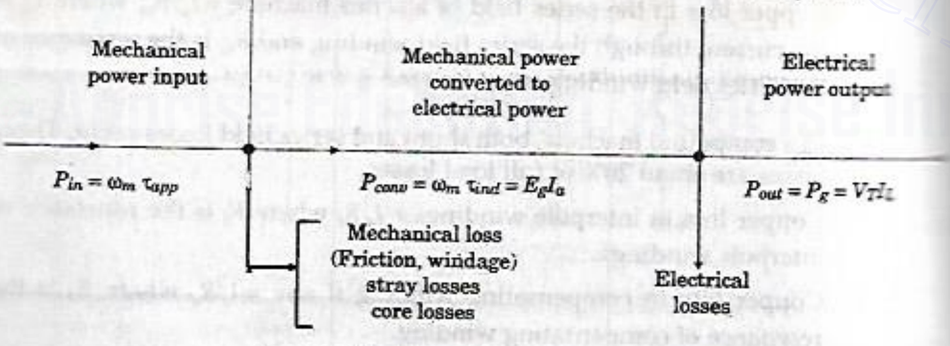


Fig. 7.26. *Power-flow diagram of a d.c. generator.*

In a d.c. generator, the input is the mechanical power input given by

$$P_{in} = \omega_m\, \tau_{app} \tag{7.20.1}$$

where ω_m = angular speed of armature in rad/s

τ_{app} = applied torque in Nm

The sum of stray losses, mechanical losses and core losses are subtracted from P_{in} to get the net mechanical power converted to electrical power by electro-mechanical conversion.

$$P_{conv} = P_i - \text{stray loss} - \text{mechanical loss} - \text{core losses}$$

$$= \omega_m \tau_{ind} = \omega_m \tau_e \tag{7.20.2}$$

where τ_e is the electromagnetic torque. The resulting electric power produced is given by

$$P_{conv} = E_g I_a \tag{7.20.3}$$

The net electrical power output is obtained by subtracting electrical I^2R losses and brush losses from P_{conv}.

$$P_{out} = P_{conv} - \text{electrical } I^2 R \text{ loss} - \text{brush losses} \tag{7.20.4}$$

$$P_{out} = V_T I_L \tag{7.20.5}$$

where V_T is the terminal voltage and I_L is the current delivered to the load.

The power-flow diagram for a dc motor motor is shown in Fig. 7.27.

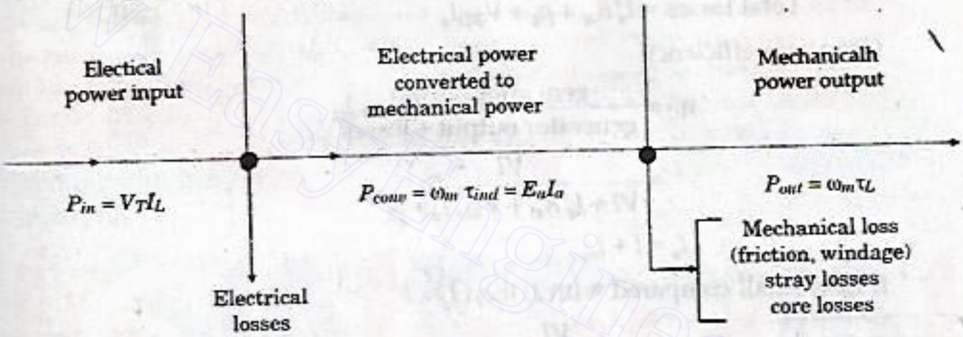


Fig. 7.27. *Power-flow diagram of a dc motor.*

In a dc motor, the input electrical power P_{in} is given by

$$P_{in} = V_T I_L \tag{7.20.6}$$

$$P_{conv.} = P_i - \text{copper losses} \tag{7.20.7}$$

Power output

$$P_{out} = \omega_m \tau_L \tag{7.20.8}$$

Also, $P_{out} = P_{conv.} - \text{core losses} - \text{mechanical losses} - \text{stray losses}$ (7.20.9)

where τ_L = load torque in N_m.

7.21 EFFICIENCY OF A D.C. MACHINE

(a) Generator

Let R = total resistance of the armature circuit (including the brush-contact resistance, at series winding resistance, interpole winding resistance, and compensating winding resistance, if any)

I = output current

I_{sh} = current through the shunt field

I_a = armature current = $I + I_{sh}$

V = terminal voltage

Total copper loss in the armature circuit = $I_a^2 R_{at}$

Power loss in the shunt circuit

= VI_{sh} (this includes the loss in the shunt regulating resistan

Mechanical losses = friction loss at bearings + friction loss at comm

+ windag

Core losses = hysteresis loss + eddy-current loss

Stray loss = mechanical loss + core loss

The sum of the shunt field copper loss and stray losses may be consi as a combined *fixed* (*constant*) loss that does not vary with the load current I

∴ constant losses (in shunt and compound generators) = stray loss

+ shunt field copper los

$$\text{Total losses} = I_a^2 R_{at} + p_k + V_{BD} I_a$$

Generator efficiency

$$\eta_G = \frac{\text{generator output}}{\text{generator output + losses}}$$

$$= \frac{VI}{VI + I_a^2 R_{at} + V_{BD} I_a + p_k}$$

$$I_a = I + I_{sh}$$

If I_{sh} is small compared with I, then $I_a = I$

$$\therefore \quad \eta_G = \frac{VI}{VI + I^2 R_{at} + V_{BD} I + p_k}$$

$$= \frac{1}{1 + \frac{I R_{at}}{V} + \frac{V_{BD}}{V} + \frac{p_K}{VI}}$$

The efficiency η_G will be a maximum when the denominator D_r is a mini mum, where $D_r = 1 + \frac{IR_{at}}{V} + \frac{V_{BD}}{V} + \frac{p_k}{VI}$

D_r is a minimum when

$$\frac{dD_r}{dI} = 0 \quad \text{and} \quad \frac{d^2 D_r}{dI^2} > 0$$

$$\frac{dD_r}{dI} = \frac{d}{dI}\left(1 + \frac{IR_{at}}{V} + \frac{V_{BD}}{V} + \frac{p_k}{VI}\right)$$

$$0 = 0 + \frac{R_{at}}{V} + \frac{p_k}{V}\left(-\frac{1}{I^2}\right)$$

or

$$\boxed{I^2 R_{at} = p_k} \qquad (7.21.1)$$

Also, $$\frac{d^2 D_r}{dI^2} = \frac{d}{dI}\left(\frac{R_{at}}{V} - \frac{p_k}{VI^2}\right) = \frac{2p_k}{VI^3} > 0$$

Since $\frac{d^2 D_r}{dI^2}$ is positive, the expression given by Eq. (7.21.1) is a condition for the minimum value of D_r, and therefore the condition for maximum value of efficiency.

Equation (7.21.1) shows that *the efficiency of a dc generator is a maximum when those losses proportional to the square of the load current are equal to the constant losses of the dc generator.* This relation applies equally well to *all* rotating machines, *regardless* of type.

This relationship is sometimes *incorrectly* stated as "maximum efficiency occurs when the variable losses are equal to the constant losses".

Load Corresponding to Maximum Efficiency

Let I_{fl} = full-load current

I_M = current at maximum efficiency

For maximum efficiency

$$I_M^2 R_{at} = p_k$$

$$I_M^2 = \frac{p_k}{R_{at}}$$

$$= \frac{I_{fl}^2 p_k}{I_{fl}^2 R_{at}}$$

$$I_M = I_{fl}\sqrt{\frac{p_k}{I_{fl}^2 R_{at}}}$$

$$\therefore \text{ Current at maximum efficiency} = \text{full-load current} \times \sqrt{\frac{\text{constant loss}}{\text{f.l. copper loss}}} \tag{7.21.2}$$

7.22 TESTING OF DC MACHINES

Machines are tested for finding out losses, efficiency and temperature rise. Direct-loading tests may be performed on small machines. For large shunt machines, indirect methods are used. Swinburne's test and Hopkinson's test are mostly used in practice.

7.23 SWINBURNE'S TEST

It is an ***indirect method*** of testing dc machines. In this method the losses are measured separately, and the efficiency at any desired load is predetermined.

The machine is run as a motor at rated voltage and speed. Figure 7.28 shows the connection diagram for the test for dc shunt machine.

Let V = supply voltage

I_0 = no-load current

I_{sh} = shunt field current

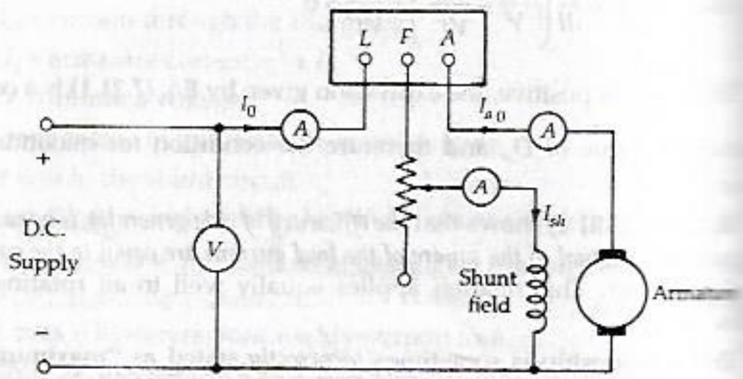


Fig. 7.28. *Swinburne's test.*

$\therefore$ no-load armature current

$$I_{a0} = I_0 - I_{sh}$$

No-load input $= VI_0$.

The no-load power input to the machine supplies the following :

(*i*) iron loss in the core,

(*ii*) friction losses at bearings and commutator,

(*iii*) windage loss

(*iv*) armature copper loss at no load.

When the machine is loaded, the temperature of the armature windi... field winding increases due to I^2R losses. In calculating the I^2R losses hot ... ances should be used. A stationary measurement of resistances at room tempera... ture of, say, $t°$ C is made by passing current through armature and then field ... a low voltage dc supply. Then the hot resistance, allowing a temperature ... say 50° C, is found as follows :

$$R_{t_1} = R_0(1 + \alpha_0 t_1)$$

$$R_{t_1+50°} = R_0[1 + \alpha_0(t_1 + 50°)]$$

where $\alpha_0 =$ temperature coefficient of resistance at 0° C

$$\therefore \quad R_{t_1+50°} = R_{t_1}\frac{1 + \alpha_0(t_1 + 50°)}{1 + \alpha_0 t_1}$$

Stray loss = iron loss + friction loss + windage loss = input at no load – field copper loss – no load armature copper loss

$$= VI_0 - p_f - p_{a0} = p_s \text{ (say)}$$

Also, constant losses

$$p_c = \text{no-load input} - \text{(no-load armature copper loss)}$$

$$p_c = p_s + p_f$$

By knowing the constant losses of the machine, its efficiency at any other load can be determined as follows :

Let I be the load current at which efficiency is required.

Efficiency when running as motor

Motor input $= VI$

Armature copper loss $= I_a^2 R_a = (I - I_{sh})^2 R_a$

Constant losses $= p_c$ (found above)

$\therefore$ total losses $= (I - I_{sh})^2 R_a + p_c$

Motor efficiency

$$\eta_m = \frac{\text{input} - \text{losses}}{\text{input}}$$

$$= \frac{VI - (I - I_{sh})^2 R_a - p_c}{VI}$$

Efficiency when running as generator

Armature current $I_a = I + I_{sh}$

Generator output $= VI$

Armature copper loss $= (I + I_{sh})^2 R_a$

Constant losses $= p_c$ (found above)

$\therefore$ total losses $= (I + I_{sh})^2 R_a + p_c$

Efficiency of generator

$$\eta_g = \frac{\text{Output}}{\text{Output} + \text{losses}} = \frac{VI}{VI + (I + I_{sh})^2 R_a + p_c}$$

Advantages of Swimburne's Test

The main advantages of Swinburne's test are :

1. It is a convenient and economical method of testing dc machines since power required to test a large machine is small.
2. The efficiency can be predetermined at any load because constant losses are known.

Main Disadvantages

1. No account is taken of the change in iron loss from no load to full load. At full load, due to armature reaction, flux is distorted which increases the iron losses.
2. As the test is on no load, it does not indicate whether the commutation on full load is satisfactory and whether the temperature rise would be within specified limits.

Limitations

1. Swinburne's test is applicable to those machines in which the flux is practically constant, that is shunt machines and level compound generators.
2. Series machines cannot be tested by this method as they cannot be run on light loads and secondly flux and speed vary greatly.

7.24 HOPKINSON'S TEST

This test is also called

(*a*) **regenerative test**

(*b*) **back-to-back test**

(*c*) **heat-run test.**

The test requires two *identical* shunt machines which are coupled mechanically and also connected electrically in parallel. One of them acts as a motor and the other as the generator. The motor takes its input from supply. The mechanical output of motor drives the generator and the electrical output of the generator is used in supplying the input to motor. Thus, output of each machine is fed as input to the other. When both machines are run on full load, the input from supply will be equal to the total losses of both the machines. Hence the power input from the supply is very small.

The circuit diagram for Hopkinson's test is shown in Fig. 7.29 machine M is started from the supply as motor with the help of a starter (not shown). The switch S is kept open. The field current of M is adjusted with the help of field rheostat R_M to enable the motor to run at rated speed. Machine G acts as a generator. Since G is mechanically coupled to M, it runs at the rated speed of M. The excitation of the generator G is so adjusted with the help of its field rheostat R_G that the voltage across the armature of G is slightly higher than the supply voltage. In actual practice, the terminal voltage of the generator is kept 1 or 2 V higher than the supply busbar voltage. When this is achieved, that is, the voltage of the generator being equal and of the same polarity as the busbar voltage, the main switch S is closed and the generator is connected to the busbars. Thus, both the machines are now in parallel across the supply. Under this condition, the generator is said to float. That is, it is neither taking any current from nor giving any current to the supply. Any required load can now be thrown on the machines by adjusting the excitation of the machines with the help of field rheostats.

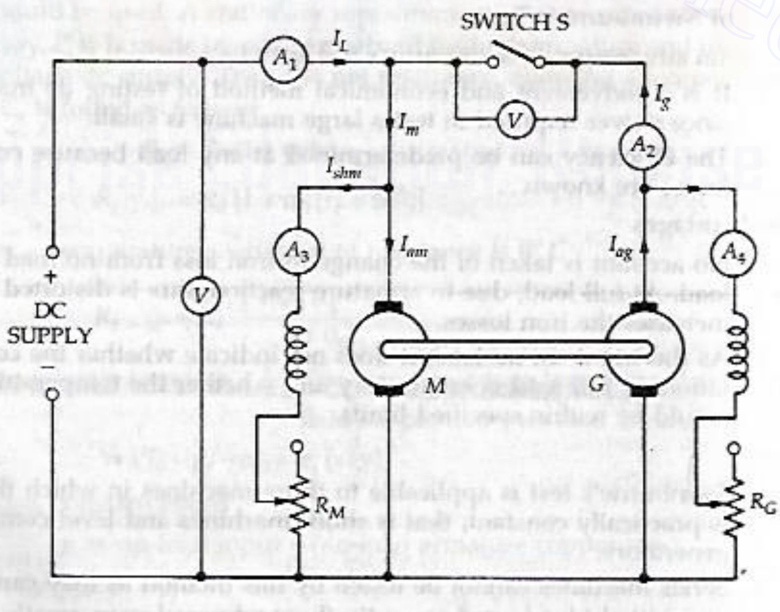


Fig. 7.29. *Hopkinson's test on two similar dc shunt machines.*

V = supply voltage

I_L = line current

I_m = input current t

I_g = output current

I_{am} = motor armatur

I_{ag} = generator arma

I_{shm} = motor shunt fi

I_{shg} = generator shur

R_a = armature resis

R_{shm} = motor shunt fi

R_{shg} = generator shur

E_g = generator indu

E_m = motor induced

We have

$E_g = V + I_{ag} R_a$

$E_m = V - I_{am} R_a$

$E_g > E_m$

But $E_g \propto \Phi_g N$

$E_m \propto \Phi_m N$

$\Phi_g > \Phi_m$

Since $\Phi \propto I_f$

$I_{shg} > I_{shm}$

Thus, the excitation of t

... That is, the machine wit

The load on the machin

... efficiencies of the two ma

Power input from the s

Armature copper loss c

Field copper loss of the

Armature copper loss c

Field copper loss of the

For identical machines

... are assumed to be equ

Constant losses of both

– arm

$P_c = VI_L - (I_{am}^2 R_a$

Let V = supply voltage

I_L = line current

I_m = input current to motor

I_g = *output current from the generator*

I_{am} = motor armature current

I_{ag} = generator armature current

I_{shm} = motor shunt field current

I_{shg} = generator shunt field current

R_a = armature resistance of each machine

R_{shm} = motor shunt field resistance

R_{shg} = generator shunt field resistance

E_g = generator induced voltage

E_m = motor induced voltage (back emf)

We have

$$E_g = V + I_{ag} R_a$$

$$E_m = V - I_{am} R_a$$

$$\therefore \quad E_g > E_m$$

But $E_g \propto \Phi_g N$

$$E_m \propto \Phi_m N$$

$$\therefore \quad \Phi_g > \Phi_m$$

Since $\Phi \propto I_f$ (I_f is field current and Φ is field flux)

$$I_{shg} > I_{shm}$$

Thus, the excitation of the generator shall always be greater than that of the motor. That is, *the machine with smaller excitation acts a a motor.*

The load on the machines can be adjusted as desired and readings taken. The efficiencies of the two machines can be determined as follows :

Power input from the supply = VI_L = total losses of both the machines

Armature copper loss of the motor $= I_{am}^2 R_a$

Field copper loss of the motor $= I_{shm}^2 R_{shm}$

Armature copper loss of the generator $= I_{ag}^2 R_a$

Field copper loss of the generator $= I_{shg}^2 R_{shg}$

For identical machines the constant losses P_c (iron, friction and windage losses) are assumed to be equal.

Constant losses of both the machines = power drawn from the supply − armature and shunt copper losses of both the machines

$$P_c = VI_L - (I_{am}^2 R_a + I_{shm}^2 R_{shm} + I_{ag}^2 R_a + I_{shg}^2 R_{shg})$$

Assuming that the constant losses (stray losses) are equal divided between the two machines,

total stray loss per machine $= \frac{1}{2} P_c$

The efficiencies of the two machines can be determined as follows :

Generator

$$\text{Output} = VI_{ag}$$

$$\text{Constant losses for generator} = \frac{P_c}{2}$$

$$\text{Armature copper loss} = I_{ag}^2 R_a$$

$$\text{Field copper loss} = I_{shg}^2 R_{shg}$$

Efficiency of the generator

$$\eta_g = \frac{\text{Output}}{\text{Output + Losses}}$$

$$= \frac{VI_{ag}}{VI_{ag} + I_{ag}^2 R_a + I_{shg}^2 R_{shg} + \frac{1}{2} P_c}$$

Motor

$$\text{Input} = VI_m = V(I_{am} + I_{shm})$$

$$\text{Constant losses for motor} = \frac{P_c}{2}$$

$$\text{Armature copper loss} = I_{am}^2 R_a$$

$$\text{Field copper loss} = I_{shm}^2 R_{shm}$$

Efficiency of the motor

$$\eta_m = \frac{\text{Output}}{\text{input}} = \frac{\text{input} - \text{losses}}{\text{input}}$$

$$= \frac{V(I_{am} + I_{shm}) - \left(\frac{P_c}{2} + I_{am}^2 R_a + I_{shm}^2 R_{shm}\right)}{V(I_{am} + I_{shm})}$$

Merits of Hopkinson's Test

The main *advantages* of using Hopkinson's test for determination of efficiency are :

1. The total power taken from the supply is very low. Therefore this method is very economical.
2. The temperature rise and the commutation conditions can be checked under rated load conditions.
3. Stray losses are considered, as both the machines are operated under rated load conditions.

Large machines can be tested at rated load without consuming much power from the supply.

5. Efficiency at different loads can be determined.

Disadvantages

The main *disadvantage* of this method is the necessity of two practically identical machine to be available. Consequently, this test is suitable for manufacturers of large dc machines.

7.25 ELECTRIC BRAKING OF DC MOTORS

Electric braking is usually employed in applications to stop a unit driven by motors in an exact position or to have the speed of the driven unit suitably controlled during its deceleration. In applications requiring frequent, quick, accurate or rapid emergency stops, electric braking is used. For example, in suburban electric trains quick stops are required. Electric braking allows smooth stops without any inconvenience to passengers.

When a loaded hoist is lowered, electric braking keeps the speed within safe limits, otherwise, the drive speed will reach dangerous values.

When a train goes down a steep gradient, electric braking is employed to hold the train speed within safe limits. Similarly, in applications involving other active loads, electric braking is very commonly used.

The braking force can also be obtained by using mechanical brakes.

7.26 DISADVANTAGES OF MECHANICAL BRAKING

The following are the main disadvantages of mechanical braking :

1. It requires frequent maintenance and replacement of brake shoes.
2. Braking power is wasted as heat.

In spite of the disadvantages, mechanical braking is used along with electric braking to ensure reliable operation of the drive. Mechanical brakes are also used to hold the drive at standstill because many braking methods do not produce torque at standstill.

7.27 TYPES OF ELECTRIC BRAKING

There are three types of braking a dc motor :

1. Regenerative braking
2. Dynamic braking or rheostatic braking
3. Plugging or reverse current braking.

7.28 REGENERATIVE BRAKING

It is a form of braking in which the kinetic energy of the motor and its driven machinery is returned to the power supply system. This type of braking is possible when the driven load forces the motor to run at a speed higher than its no-load speed with a constant excitation. Under this condition, the motor back emf E_b is greater than the supply voltage V, which reverses the direction of motor ar... current. The machine now begins to operate as a generator and the energy ge... ated is supplied to the source.

Regenerative braking can also be carried out upto very low speeds if the motor is connected as a separately excited generator and excitation is increas... the speed is reduced, so that $E_b = \frac{nP\,\Phi\,Z}{A}$ and $V = E_b - I_a R_a$ are satisfied an... motor does not enter into saturation on increasing excitation.

Regeneration is possible with a shunt and separately excited moto... with compound motors with weak series compounding. Regenerative braki... used specially where more frequent braking or slowing of drives is require... most useful in holding a descending load of high potential energy at a con... speed. For example, regenerative braking is used to control the speed of m... driving loads such as electric locomotives, elevators, cranes and hoists. W... descending, the load in this operation acts as the prime mover by virtu... potential energy. The motor acts as a generator. The generated power is... returned to the supply. The returned power is available for other devices ope... from the same source of supply. Regenerative braking cannot be used for sto... the motor. It is used for controlling the speed above the no-load speed of the m... driving the descending loads.

The necessary condition for regeneration is that the back emf E_b shoul... greater than the supply voltage so that the armature current is reversed an... mode of operation changes from motoring to generating.

7.28.1 Regenerative Braking in dc Shunt Motors

Under normal operating conditions the armature current is given by

$$I_a = \frac{V - E_b}{R_a}$$

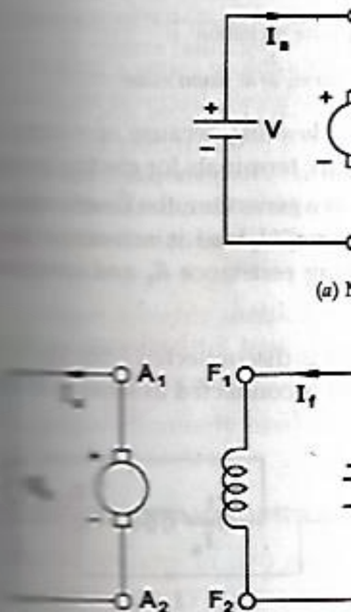
When the load (such as lowering of load by a crane, hoist or lift) cause... motor speed to be greater than the no-load speed, the back emf E_b becomes gre... than the supply voltage V. Consequently, armature current I_a becomes neg... The machine now begins to operate as a generator.

7.28.2 Regenerative Braking in dc Series Motors

In case of a dc series motor an increase in speed is followed by a decr... in the armature current and field flux. The back emf E_b cannot be greater tha... supply voltage. Regeneration is not possible in a plain dc series motor sinc... field current cannot be made greater than armature current. However, in appl... tions such as traction, elevators, hoists etc., where dc series motors are u... extensively, regeneration may be required. For example, in an electroloco... moving down a gradient, a constant speed may be necessary, and in hoist d... the speed is to be limited whenever it becomes dangerously high. One comm... used method of regenerative braking of the dc series motor is to connect it a... shunt motor. Since the resistance of the field winding is low, a series resista... connected in the field circuit to limit the current within the safe value.

...C BRAKING OR RHE...

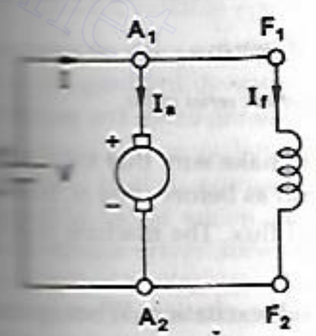
...braking, the dc mot... R_b is immediately conn... ...generator, producing the br... ...braking operation, the s... ...as a separately excited ... connected as a self-excited ... the armature. Fig. 7.30 s... ... Fig. 7.31 shows the d...



(a) ...

...with separate excitation

Fig. 7.30. Dynamic brakin...



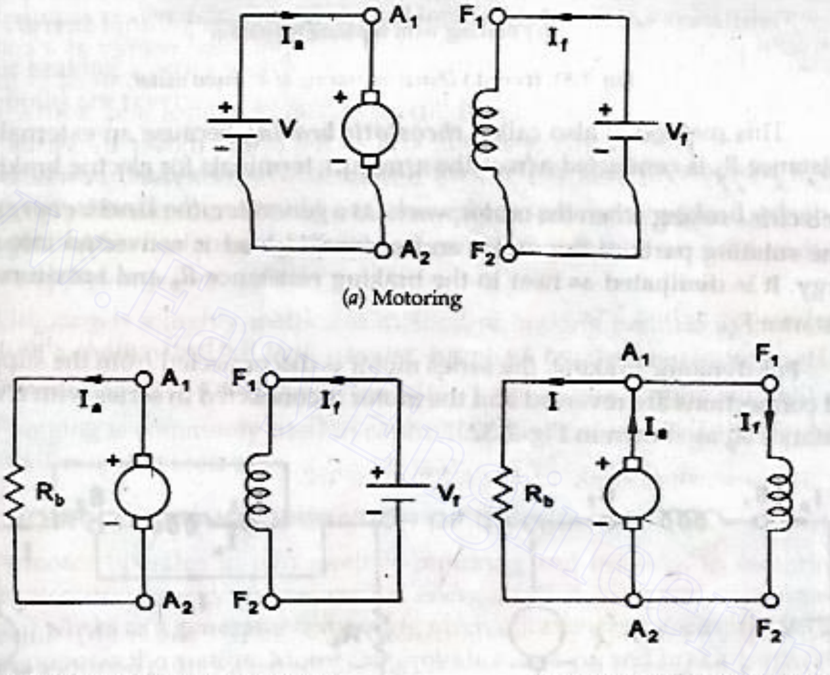
...motoring

Fig. 7.31. Dynamic ...

7.29 DYNAMIC BRAKING OR RHEOSTATIC BRAKING

In dynamic braking, the dc motor is disconnected from the supply and a braking resistor R_b is immediately connected across the armature. The motor now works as a generator, producing the braking torque.

For the braking operation, the separately excited (or shunt) motor can be connected either as a separately excited generator, where the flux is kept constant, or it can be connected as a self-excited shunt generator, with the field winding in parallel with the armature. Fig. 7.30 shows the dynamic braking of separately excited dc motor. Fig. 7.31 shows the dynamic braking of a dc shunt motor.

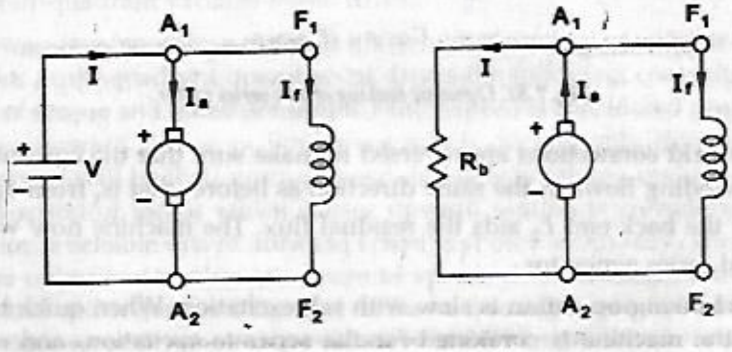


(*a*) Motoring

(*b*) Braking with separate excitation

(*c*) Braking with self excitation

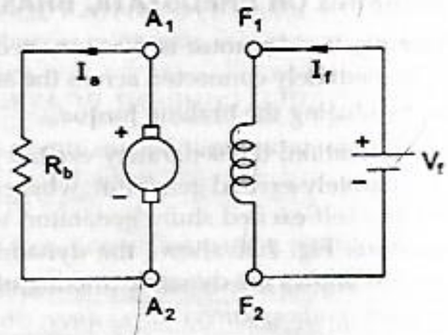
Fig. 7.30. *Dynamic braking of separately excited dc motor.*



(*a*) Motoring

(*b*) Braking with self excitation

Fig. 7.31. *Dynamic braking of dc shunt motor.*

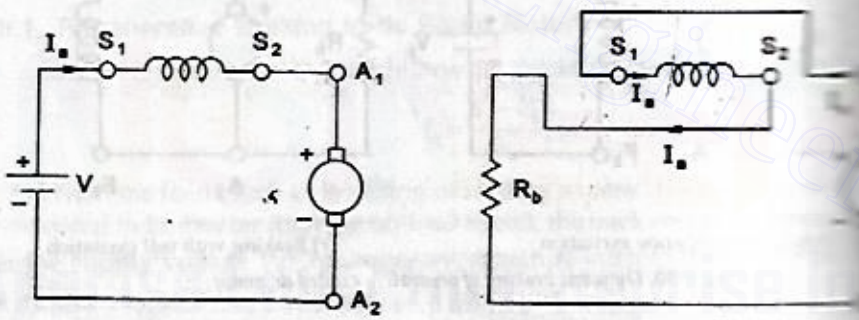


(*c*) Braking with separate excitation

Fig. 7.31. (contd.) *Dynamic braking of dc shunt motor.*

This method is also called ***rheostatic braking*** because an exte... resistance R_b is connected across the armature terminals for electric braki... ing electric braking when the motor works as a generator, the kinetic ener... in the rotating parts of the motor and connected load is converted ... energy. It is dissipated as heat in the braking resistance R_b and arma... resistance R_a.

For dynamic braking, the series motor is disconnected from the su... field connections are reversed and the motor is connected in series wi... resistance R_b as shown in Fig. 7.32.



(*a*) Motoring (*b*) Braking with self excitation

Fig. 7.32. *Dynamic braking of a dc series motor.*

The field connections are reversed to make sure that the curr... the field winding flows in the same direction as before (that is, from S_1 ... order that the back emf E_b aids the residual flux. The machine now w... self-excited series generator.

The braking operation is slow with self excitation. When quick b... required, the machine is connected for the separate excitation, and a ... resistance is connected in series with the field to limit the current ... value.

Dynamic braking is an inefficient method of braking, because all the generated energy is dissipated as heat in resistances.

7.30 PLUGGING OR REVERSE CURRENT BRAKING

In this method the armature terminals (or supply polarity) of a separately excited (or shunt) motor when running are reversed. Therefore, the supply voltage V and the induced voltage E_b (back emf) will act in the same direction. Thus during braking the effective voltage across the armature will be $(V + E_b)$ which is almost twice the supply voltage. The armature current is reversed and a high braking torque is produced. In order to limit the armature current to a safe value, an external current-limiting resistor is connected in series with the armature.

For braking a series motor either the armature terminals or field terminals (but not both) are reversed. Reversing of both gives only normal working operation.

The braking torque is not zero at zero speed. When used for stopping a load, the motor must be disconnected from the supply at or near zero speed, otherwise, it will speed up in the reverse direction. Centrifugal switches are used to disconnect the supply.

Plugging is a highly inefficient method of braking because in addition to the power supplied by the load, power supplied by the source is wasted in resistances.

Plugging is commonly used in controlling elevators, rolling mills, printing presses and machine tools etc.

7.31 FOUR-QUADRANT OPERATION OF DRIVES

A motor operates in two modes—motoring and braking. In motoring, it converts electrical energy to mechanical energy, which supports its motion. In braking, it works as a generator converting mechanical energy to electrical energy, and thus opposes the motion. Motor can provide motoring and braking operations for both forward and reverse directions. A motor drive capable of operating in both directions of rotation and of producing both motoring and regeneration is called a four-quadrant variable-speed drive.

Power developed by a motor is given by the product of angular speed and torque. For multi-quadrant operation of drives the following conventions about the signs of torque and speed are useful: Motor speed is considered positive when rotating in forward direction. For drives which operate only in one direction, forward speed will be their normal speed. In loads involving up-and-down motions, the speed of motor which causes upward motion is considered forward motion. For reversible drives, forward speed is chosen arbitrarily. Then the rotation in the opposite direction gives reverse speed which is assigned the negative sign. Positive motor torque is defined as the torque which produces acceleration or the positive rate of change of speed in forward direction. Motor torque is taken negative if it produces retardation. Load torque is opposite in direction to the positive motor torque.

Fig. 7.33 shows the four-quadrant operation of drives. In qua... oped power is positive, hence, the machine works as a motor supp... ical energy. Operation in quadrant I is, therefore, called forward m...

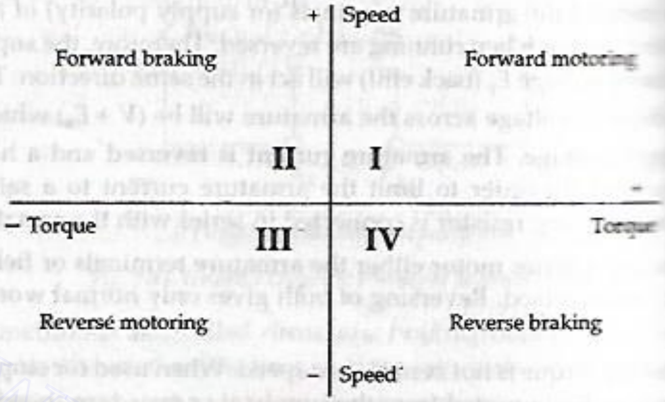


Fig. 7.33. *Four-quadrant operation*

Operation in the quadrant II represents braking, because in this ... torque-speed plane the direction of rotation is positive and the torque ... The machine operates as a generator developing a negative torque which ... motion. The kinetic energy of the rotating parts is available as electr... which may be supplied back to the mains, or in dynamic braking, dis... some resistance.

In the third quadrant which corresponds to the motor action in the ... direction both speed and torque have negative values while the power is ... Operation in quadrant III is similar to that in the first quadrant with ... rotation reversed.

In the fourth quadrant, the torque is positive and the speed is neg... quadrant corresponds to braking in reverse motoring.

The four-quadrant operation and its relationship to speed, torq... power output are summarized in Table 7.1.

Table 7.1. Four-quadrant dc motor drive characteristics

Function	*Quadrant*	*Speed*	*Torque*	*Powe...*
Forward motoring	I	+	+	[illegible]
Forward braking	II	+	–	[illegible]
Reverse motoring	III	–	–	[illegible]
Reverse braking	IV	–	+	[illegible]

Compressor, pump and fan type loads require operation in the first quadrant only, since their operation is unidirectional. They are one-quadrant drive systems.

Transportation drives require operation in both directions. The method of braking depends upon the conditions of availability of power supply. If regeneration is necessary, application in all four quadrants may be required. If not, operation is restricted to quadrants I and III dynamic braking or mechanical braking may be required. In hoist drives, a four-quadrant operation is needed.

7.32 PRESENT-DAY USES OF D.C. MACHINES

At present time bulk of electrical energy is generated in the form of alternating current. Hence the use of d.c. generators is very limited. They are mainly used in supplying excitation of small and medium range alternators. For industrial applications of d.c. like electrolytic processes, welding processes and variable speed motor drives, the present trend is to generate a.c. and then to convert a.c. into d.c. by rectifiers. Thus, dc generators have generally been superseded by rectified ac supplies for many applications.

Direct current motors are very commonly used as variable-speed drives and in applications where severe torque variations occur.

The main applications of the three types of d.c. motors are given below :

Series motors

These motors are used where high starting torque is required and speed can vary, for example, traction, cranes, etc.

Shunt motors

These motors are used where constant speed is required and starting conditions are not severe, for example, lathes, centrifugal pumps, fans, blowers, conveyors, lifts etc.

Compound motors

These motors are used where high starting torque and fairly constant speed is required, for example, presses, shears, conveyors, elevators, rolling mills, heavy planers etc.

Small d.c. machines (in fractional kilowatt ratings) are used primarily as control devices such as techogenerators for speed sensing and servomotors for positioning and tracking.

EXAMPLE 7.1. *A d.c. shunt machine, connected to 250 V supply, has an armature resistance (including brushes) of 0.12 Ω and the resistance of the field circuit is 100 Ω. Find the ratio of the speed as a generator to the speed as a motor, the line current in each case being 80 A.*

SOLUTION. $V = 250\text{ V}, \quad I_L = 80\text{ A}, \quad R_a = 0.12\ \Omega$

$$R_{sh} = 100\ \Omega, \quad I_{sh} = \frac{V}{R_{sh}} = \frac{250}{100} = 2.5\text{ A}$$

Let suffixes 1 and 2 be used for generator and motor respectively.

Generator

$$I_{a_1} = I_L + I_{Sh} = 80 + 2.5 = 82.5 \text{ A}$$

$$E_1 = V + I_{a_1} R_a = 250 + 82.5 \times 0.12 = 259.9 \text{ V}$$

Motor

$$I_{a_2} = I_L - I_{Sh} = 80 - 2.5 = 77.5 \text{ A}$$

$$E_2 = V - I_{a_2} R_a = 250 - 77.5 \times 0.12 = 240.7$$

$$\frac{E_1}{E_2} = \frac{N_1 \Phi_1}{N_2 \Phi_2}$$

Since the field current is the same, $\Phi_2 = \Phi_1$

$$\therefore \quad \frac{N_1}{N_2} = \frac{E_1}{E_2} = \frac{259.9}{240.7} = 1.0797$$

Example 7.2. *The armature resistance of a 200 V shunt motor is 0.4 Ω and no-load current is 2 A. When loaded and taking an armature current of 50 A, the speed is 1200 r.p.m. Find approximately the no-load speed.*

Solution. $E_0 = V - I_{a_0} R_a = 200 - 2 \times 0.4 = 199.2 \text{ V}$

$$E_1 = V - I_{a_1} R_a = 200 - 50 \times 0.4 = 180 \text{ V}$$

$$\frac{N_1}{N_0} = \frac{E_1}{E_0} \times \frac{\Phi_0}{\Phi_1}$$

For a shunt motor, $\Phi_1 = \Phi_0$

$$\therefore \quad N_0 = \frac{E_0}{E_1} \times N_1 = \frac{199.2 \times 1200}{18}$$

$$= 1328 \text{ r.p.m.}$$

Example 7.3. *A 250 V shunt motor on no load runs at 1000 r.p.m. and takes 5 A. The total armature and shunt field resistance are respectively 0.2 Ω and 250 Ω. Calculate the speed when loaded and taking a current of 50 A, if the armature reaction weakens the field by 3%.*

Solution. $I_L = 5 \text{ A}, \quad I_{sh} = \frac{V}{R_{sh}} = \frac{250}{250} = 1 \text{ A}, \quad I_{a_1} = I_{L_1} - I_{sh} = 5 - 1 = 4 \text{ A}$

$$E_1 = V - I_{a_1} R_a = 250 - 4 \times 0.2 = 249.2 \text{ V}$$

Armature current on load

$$I_{a_2} = I_{L_2} - I_{sh} = 50 - 1 = 49 \text{ A}$$

$$E_2 = V - I_{a_2} R_a = 250 - 49 \times 0.2 = 240.2 \text{ V}$$

$$\Phi_2 = 0.97\,\Phi_1$$

$$\frac{N_2}{N_1} = \frac{E_2 \Phi_1}{E_1 \Phi_2} = \frac{240.2\,\Phi_1}{249.2 \times 0.97\,\Phi_1}$$

$$N_2 = \frac{240.2 \times 1000}{249.2 \times 0.97} = 993.69 \text{ r.p.m.}$$

EXAMPLE 7.4. *A shunt generator delivers 50 kW at 250 V when running at 400 r.p.m. The armature and field resistance are 0.02 Ω and 50 Ω respectively. Calculate the speed of the machine when running as a shunt motor and taking 50 kW input at 250 V. Allow 1 V per brush for contact drop.*

SOLUTION. *As generator*

Load current $$I_L = \frac{50 \times 10^3}{250} = 200 \text{ A}$$

Shunt field current $$I_{sh} = \frac{V}{R_{sh}} = \frac{250}{50} = 5 \text{ A}$$

Armature current $$I_{a_1} = I_{L_1} + I_{sh} = 200 + 5 = 205 \text{ A}$$

Generated emf

$$E_1 = V + Ia_1 R_a + \text{voltage drop in the brushes}$$

$$= 250 + (205 \times 0.02) + 2 \times 1 = 256.1 \text{ V}$$

Speed of the generator $N_1 = 400$ r.p.m.

As motor

Input line current $$I_{L_2} = \frac{P_i}{V} = \frac{50 \times 10^3}{250} = 200 \text{ A}$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{250}{50} = 5 \text{ A}$$

Armature current $$I_{a_2} = I_{L_2} - I_{sh} = 200 - 5 = 195 \text{ A}$$

$$E_2 = V - I_{a_2} R_a - \text{brush drop}$$

$$= 250 - 195 \times 0.02 - 2 \times 1 = 244.1 \text{ V}$$

$$\therefore \quad \frac{N_2}{N_1} = \frac{E_2}{E_1} \times \frac{\Phi_1}{\Phi_2}$$

Since the field current is constant, $\Phi_2 = \Phi_1$

$$\therefore \quad N_2 = \frac{E_2}{E_1} N_1 = \frac{244.1}{256.1} \times 400 = 381.3 \text{ r.p.m.}$$

EXAMPLE 7.5. *A 4-pole, 250 V, wave-connected shunt motor gives 10 kW when running at 1000 r.p.m. and drawing armature and field currents of 60 A and 1 A respectively. It has 560 conductors. Its armature resistance is 0.2 Ω. Assuming a drop of 1 V per brush, determine : (a) total torque ; (b) useful torque ; (c) useful flux per pole ; (d) rotational losses ; (e) efficiency.*

SOLUTION. $E = V - I_a R_a - \text{brush drop}$

$$= 250 - 60 \times 0.2 - 2 \times 1 = 236 \text{ V}$$

(*a*) $$\tau \times \frac{2\pi N}{60} = E\, I_a$$

$$\tau = \frac{60\, E\, I_a}{2\pi N} = \frac{60 \times 236 \times 60}{2\pi \times 1000} = 135.22 \text{ Nm}$$

(*b*) $\tau_{useful} = \tau_{shaft}$, $P_{out} = 10 \text{ kW} = 10 \times 10^3 \text{ W}$

$$\tau_{useful} \times \frac{2\pi N}{60} = P_{out}$$

$$\tau_{useful} = \frac{60 \times 10 \times 10^3}{2\pi \times 1000} = 95.49 \text{ Nm}$$

(*c*) $E = \dfrac{N P \Phi Z}{60 A}$

$$\Phi = \frac{60 E A}{N P Z} = \frac{60 \times 236 \times 2}{1000 \times 4 \times 560}$$

$$= 0.0126 \text{ Wb}$$

(*d*) Armature input $= V I_a = 250 \times 60 = 15000 \text{ W}$

Armature copper loss $= I_a^2 R_a = (60)^2 \times 0.2 = 720 \text{ W}$

Brush contact loss $= V_b I_a = 2 \times 60 = 120 \text{ W}$

Power developed $= V I_a - I_a^2 R_a - V_b I_a$

$$= 15000 - 720 - 120 = 14160 \text{ W}$$

Total power output + rotational losses = power developed

∴ rotational losses = power developed − total power output

$$= 14160 - 10000 = 4160 \text{ W}$$

(*e*) Total input to motor

$$= VI = V(I_a + I_{sh})$$

$$= 250 \times (60 + 1) = 15250 \text{ W}$$

Motor efficiency

$$= \frac{\text{motor output}}{\text{motor input}} \times 100\%$$

$$= \frac{10 \times 10^3}{15250} \times 100 = 65.57\%$$

EXAMPLE 7.6. *A 460 V series motor runs at 500 r.p.m. taking a current of 4... Calculate the speed and percentage change in torque if the load is reduced so that the m... is taking 30 A. Total resistance of the armature and field circuits is 0.8 Ω. Assume ... and field current to be proportional.*

SOLUTION. $E_1 = V - I_{a_1} R_a = 460 - 40 \times 0.8 = 428 \text{ A}$

$$E_2 = V - I_{a_2} R_a = 460 - 30 \times 0.8 = 436 \text{ V}$$

Since $\Phi \propto I_a$, $\dfrac{\Phi_1}{\Phi_2} = \dfrac{I_{a_1}}{I_{a_2}}$

$$N_2 = \frac{E_2}{E_1} \times \frac{\Phi_1}{\Phi_2} N_1$$

$$= \frac{E_2}{E_1} \times \frac{I_{a_1}}{I_{a_2}} N_1 = \frac{436}{428} \times \frac{40}{30} \times 500 = 679 \text{ r.p.m.}$$

$$\tau \propto \Phi I_a$$

$$\tau \propto I_a^2, \quad \tau_1 = k I_{a_1}^2, \ \tau_2 = k I_{a_2}^2$$

$$\frac{\tau_2}{\tau_1} = \frac{k I_{a_2}^2}{k I_{a_1}^2} = \frac{I_{a_2}^2}{I_{a_1}^2} = \frac{(30)^2}{(40)^2} = \frac{9}{16}$$

Percentage change in torque

$$= \frac{\tau_1 - \tau_2}{\tau_1} \times 100$$

$$= \frac{\tau_1 - \frac{9}{16}\tau_1}{\tau_1} \times 100 = \frac{7}{16} \times 100 = 43.75\%$$

EXAMPLE 7.7. *A 220 V, d.c. series motor is running at a speed of 800 r.p.m. and draws 100 A. Calculate at what speed the motor will run when developing half the torque. Total resistance of the armature and field is 0.1 Ω. Assume that the magnetic circuit is unsaturated.*

SOLUTION. For a series motor $\Phi \propto I_a$

Torque $\tau \propto \Phi I_a \propto I_a^2$

$$\tau = k I_a^2$$

$$\tau_1 = k I_{a_1}^2, \quad \tau_2 = k I_{a_2}^2$$

$$\frac{\tau_2}{\tau_1} = \frac{I_{a_2}^2}{I_{a_1}^2}$$

$$\frac{1}{2} = \frac{I_{a_2}^2}{(100)^2}, \ I_{a_2} = \frac{100}{\sqrt{2}} = 70.7 \text{ A}$$

$$E_1 = V - I_{a_1} R_a = 220 - 100 \times 0.1 = 210 \text{ V}$$

$$E_2 = V - I_{a_2} R_a = 220 - 70.7 \times 0.1 = 212.93 \text{ V}$$

$$\frac{N_2}{N_1} = \frac{E_2}{E_1} \times \frac{\Phi_1}{\Phi_2} = \frac{E_2}{E_1} \times \frac{I_{a_1}}{I_{a_2}}$$

$$\frac{N_2}{800} = \frac{212.93}{210} \times \frac{100}{70.7}$$

$$N_2 = 1147.3 \text{ r.p.m.}$$

EXAMPLE 7.8. *A 110 kW belt-driven shunt generator running at 400 r.p.m. on 220 V busbars continues to run as a motor when the belt breaks. As a motor it takes 11 kW. Find the speed at which it will run as a motor if the resistance of the armature and field are 0.025 Ω and 55 Ω respectively. Brush contact drop is 2 V.*

SOLUTION. *As generator*

Load current $I_{L_1} = \frac{110 \times 1000}{220} = 500$ A

$$I_{sh} = \frac{V}{R_{sh}} = \frac{220}{55} = 4 \text{ A}$$

Armature current

$$I_{a_1} = I_{L_1} + I_{sh} = 500 + 4 = 504 \text{ A}$$

Generated voltage

$$E_1 = V + I_{a_1} R_a + \text{brush contact drop}$$

$$= 220 + 504 \times 0.025 + 2 = 234.6 \text{ V}$$

As motor

When the belt breaks and the machine terminals of the generator ... connected across the brushes, the machine continues to run as a motor. The direction of the armature current is reversed compared to the direction of current in the armature while running as generator.

However, the direction of rotation remains the same.

The line input current to the motor

$$I_{L_2} = \frac{11 \times 1000}{220} = 50 \text{ A}$$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{220}{55} = 4 \text{ A}$$

Armature current

$$I_{a_2} = I_{L_2} - I_{sh} = 50 - 4 = 46 \text{ A}$$

Back e.m.f. of the motor

$$E_2 = V - I_{a_2} R_a - \text{brush drop}$$

$$= 220 - 46 \times 0.025 - 2 = 216.85 \text{ V}$$

$$E = k N \Phi, \quad N = \frac{E}{k\Phi}$$

Speed of the motor

$$N_2 = \frac{E_2 \Phi_1}{E_1 \times \Phi_2} N_1$$

For the same machine, $\Phi_2 = \Phi_1$

$$\therefore \quad N_2 = \frac{E_2}{E_1} \times N_1 = \frac{216.85}{234.6} \times 400 = 369.7 \text{ r.p.m.}$$

EXAMPLE 7.9. *A 250 V d.c. shunt motor having an armature resistance of 0.25 Ω carries an armature current of 50 A and runs at 750 r.p.m. If the flux is reduced by 10%, find the speed. Assume that the load torque remains the same.*

SOLUTION. *Initial conditions*

$$V = 250 \text{ V}, \quad I_{a_1} = 50 \text{ A}, \quad R_a = 0.25\ \Omega, \quad N_1 = 750 \text{ r.p.m.}$$

$$E_1 = V - I_{a_1} R_a = 250 - 50 \times 0.25 = 237.5 \text{ V}$$

Conditions after reducing the flux

$$\Phi_2 = 0.9\,\Phi_1$$

Load torque $\quad \tau \propto \Phi\, I_a$

Since the load torque remains the same

$$\tau_2 = \tau_1$$

$$\Phi_2 I_{a_2} = \Phi_1 I_{a_1}$$

$$I_{a_2} = \frac{\Phi_1}{\Phi_2} I_{a_1} = \frac{50}{0.9} = 55.56 \text{ A}$$

$$E_2 = V - I_{a_2} R_a = 250 - 55.6 \times 0.25 = 236.1 \text{ V}$$

$$\frac{N_2}{N_1} = \frac{E_2\,\Phi_1}{E_1\,\Phi_2}$$

$$N_2 = \frac{E_2\,\Phi_1}{E_1\,\Phi_2} N_1 = \frac{236.1 \times 750}{237.5 \times 0.9} = 828.5 \text{ r.p.m.}$$

EXAMPLE 7.10. *A 120 V d.c. shunt motor having an armature circuit resistance of 0.2 Ω and field circuit resistance of 60 Ω, draws a line current of 40 A at full load. The brush voltage drop is 3 V and rated full-load speed is 1800 r.p.m. Calculate : (a) the speed at half load ; (b) the speed at 125 per cent full load.*

SOLUTION. $V = 120$ V, $\quad R_a = 0.2\,\Omega, \quad R_{sh} = 60\,\Omega$

$$I_{sh} = \frac{V}{R_{sh}} = \frac{120}{60} = 2 \text{ A}, \qquad I_L = 40 \text{ A}$$

$$I_{a_1} = I_L - I_{sh} = 40 - 2 = 38 \text{ A}$$

$$E_1 = V - I_a R_a - \text{brush drop}$$

$$= 120 - 38 \times 0.2 - 3 = 109.4 \text{ V}$$

At rated speed of 1800 r.p.m.,

$$E_1 = 109.4 \text{ V} \quad \text{and} \quad I_{a_1} = 38 \text{ A (full load)}$$

(a) At half load

Line current $\quad I_{L_2} = \dfrac{40}{2} = 20$ A

$$I_{a_2} = I_{L_2} - I_{sh} = 20 - 2 = 18 \text{ A}$$

$$E_2 = V - I_a R_a - \text{brush drop}$$

$$= 120 - 18 \times 0.2 - 3 = 113.4 \text{ V}$$

If N_2 is the speed at half load,

$$N_2 = \frac{E_2}{E_1} \times N_1 = \frac{113.4}{109.4} \times 1800 = 1865.8 \text{ r.p.m.}$$

(b) At 125 percent full load

Line current $\quad I_{L_3} = 40 \times 1.25 = 50$ A

Armature current $I_{a_3} = I_{L_3} - I_{sh} = 50 - 2 = 48$ A

EXAMPLE 7.11. *A shunt wound motor has an armature resistance of 0.1 Ω connected across 220 V supply. The armature current taken by the motor is 20 A and motor runs at 800 r.p.m. Calculate the additional resistance to be inserted in series with the armature to reduce the speed to 520 r.p.m. Assume that there is no change in armature current.*

SOLUTION. $E_1 = V - I_{a_1} R_{a_1} = 220 - 20 \times 0.1 = 218 \text{ V}$

$$E_2 = \frac{N_2 \Phi_2}{N_1 \Phi_1} E_1$$

Since $I_{sh} = \dfrac{V}{R_{sh}}$, the shunt field current I_{sh} remains constant, and, therefore

$$\Phi_2 = \Phi_1$$

$$E_2 = \frac{N_2}{N_1} E_1 = \frac{520}{800} \times 218 = 141.7 \text{ V}$$

If R_A is the additional resistance inserted in the armature circuit

$$E_2 = V - I_{a_2}(R_{a_1} + R_A)$$

$$141.7 = 220 - 20\,(0.1 + R_A)$$

$$R_A = 3.815\ \Omega$$

EXAMPLE 7.12. *A 240 V dc series motor takes 40 A when giving its rated output at 1500 r.p.m. Its resistance is 0.3 Ω. Calculate the value of resistance that must be added to obtain the rated torque (a) at starting, (b) at 1000 r.p.m.*

SOLUTION. Rated voltage $V = 240$ V

Rated current $I = I_a = 40$ A

$$N_1 = 1500 \text{ r.p.m.}, \quad R_a = 0.3\ \Omega$$

$$E = V - I_a R_a = 240 - 40 \times 0.3 = 228 \text{ V}$$

(*a*) *At starting*, back e.m.f. is zero. In order to obtain rated torque at rated current, an additional resistance R_1 is connected in series with the armature.

$$E_1 = V - I_a (R_a + R_1)$$

$$0 = 240 - 40\,(0.3 + R_1)$$

$$R_1 = \frac{240 - 12}{40} = 5.7\ \Omega$$

(*b*) Let R_2 be the resistance connected in series with the armature to obtain the rated torque at a speed of 1000 r.p.m.

$$E_2 = V - I_a (R_a + R_2)$$

$$E_2 = 240 - 40\,(0.3 + R_2) = 228 - 40\,R_2$$

$$\frac{N_2 \Phi_2}{N \Phi} = \frac{E_2}{E}$$

$$\frac{N_2 I_{a_2}}{N I_a} = \frac{E_2}{E}$$

Since $I_{a_2} = I_a$

$$\frac{N_2}{N} = \frac{E_2}{E}$$

$$\frac{1000}{1500} = \frac{228 - 40\,R_2}{228}, \quad R_2 = 1.9\,\Omega$$

EXAMPLE 7.13. *A 250 V shunt motor takes a current of 41 A and runs at 800 r.p.m. on full-load. Armature and field resistances are 0.2 Ω and 250 Ω respectively. If a resistance of 2 Ω is placed in series with the armature, determine :*

(a) the speed at full-load torque ;

(b) the speed at double full-load torque ;

(c) the stalling torque in terms of the full-load torque.

Assume that the flux remains constant throughout.

SOLUTION. (*a*) *Full-load conditions*

Armature current $I_{a_1} = I - I_{sh} = 41 - \frac{250}{250} = 40 \text{ A}$

Back e.m.f. on full-load

$$E_1 = V - I_{a_1} R_a = 250 - 40 \times 0.2 = 24.2 \text{ V}$$

When a resistance of 2 Ω is placed in series with the armature, the back e.m.f. is

$$E_2 = V - I_{a_1}\,(0.2 + 2) = 250 - 40\,(2.2) = 162 \text{ V}$$

Since the flux remains constant

$$N_2 = \frac{E_2}{E_1} N_1 = \frac{162}{242} \times 800 = 535.54 \text{ r.p.m.}$$

(*b*) *Double full-load conditions*

At double full-load torque, the armature current

$$I_{a_2} = 40 \times 2 = 80 \text{ A}$$

With 2 Ω resistance in the armature circuit, the back e.m.f. at double full-load torque

$$E_3 = V - I_{a_2}\,(0.2 + 2) = 250 - 80\,(2.2) = 74 \text{ V}$$

$$N_3 = \frac{E_3}{E_1} N_1 = \frac{74}{242} \times 800 = 244.6 \text{ r.p.m.}$$

(*c*) *Stalling torque*

Under stalling conditions, the speed is zero and therefore the back e.m.f. is zero.

Let I_{a_0} be the armature current taken by the motor under stalling conditions.

$$\therefore \quad E_{b_0} = V - I_{a_0}\,(0.2 + 2)$$

$$0 = 250 - 2.2\,I_{a_0}$$

$$I_{a_s} = \frac{250}{2.2} = 113.64 \text{ A}$$

$$\tau \propto I_a$$

Stalling torque ∝ stalling current

Full-load torque ∝ full-load armature current

$$\therefore \quad \frac{\text{stalling torque}}{\text{full-load torque}} = \frac{\text{stalling current}}{\text{full-load armature current}}$$

$$= \frac{113.64}{40} = 2.84$$

∴ stalling torque = 2.84 × full-load torque.

EXAMPLE 7.14. *A 200 V d.c. series motor runs at 1000 r.p.m. and ... Combined resistance of armature and field is 0.4 Ω. Calculate the resistance ... in series so as to reduce the speed to 800 r.p.m., assuming torque to vary as ... speed and linear magnetization curve.*

SOLUTION. $I_L = I_{a_1} = 20$ A, $N_1 = 1000$ r.p.m., $N_2 = 800$ r.p.m.

$$E_1 = V - I_{a_1} R_{a_1} = 200 - 20 \times 0.4 = 192 \text{ V}$$

$$\frac{\tau_2}{\tau_1} = \frac{\Phi_2 I_{a_2}}{\Phi_1 I_{a_1}} = \left(\frac{I_{a_2}}{I_{a_1}}\right)^2 = \left(\frac{N_2}{N_1}\right)^2 = \left(\frac{800}{1000}\right)^2$$

$$\therefore \quad I_{a_2} = 0.8\, I_{a_1} = 0.8 \times 20 = 16 \text{ A}$$

$$E_2 = V - I_{a_2}(0.4 + R) = 200 - 16\,(0.4 + R)$$

$$E_2 = 193.6 - 16\,R$$

$$\frac{E_2}{E_1} = \frac{N_2\,\Phi_2}{N_1\,\Phi_1} = \frac{N_2\,I_{a_2}}{N_1\,I_{a_1}} = \frac{800 \times 16}{1000 \times 20} = 0.64$$

$$\frac{193.6 - 16\,R}{192} = 0.64$$

$$R = \frac{193.6 - 192 \times 0.64}{16} = 4.42\ \Omega$$

EXAMPLE 7.15. *A series motor, with an unsaturated magnetic circuit and ... total resistance, when running at a certain speed takes 60 A at 500 V. If the load ... varies as the cube of the speed, calculate the resistance required to reduce the speed ...*

SOLUTION. $E_1 = V - I_a R_a = 500 - 60 \times 0.5 = 470$ V

$$N_2 = 0.75\, N_1$$

$$\tau \propto N^3$$

$$\frac{\tau_2}{\tau_1} = \left(\frac{N_2}{N_1}\right)^3 = (0.75)^3$$

$$\tau \propto \Phi\, I_a$$

$$\frac{\tau_2}{\tau_1} = \frac{\Phi_2 I_{a_2}}{\Phi_1 I_{a_1}} = \frac{I_{a_2}^2}{I_{a_1}^2}$$

$$\therefore \quad \frac{I_{a_2}^2}{I_{a_1}^2} = \left(\frac{N_2}{N_1}\right)^3$$

$$\frac{I_{a_2}^2}{(60)^2} = (0.75)^3, \; I_{a_2} = 60\sqrt{(0.75)^3} = 38.97 \text{ A}$$

Let R be the additional resistance to be connected in series with the armature.

$$E_2 = V - I_{a_2}(R_a + R) = 500 - 38.97\,(0.5 + R) \;;\; E_2 = 480.5 - 38.97\,R$$

$$\frac{E_2}{E_1} = \frac{N_2 \Phi_2}{N_1 \Phi_1} = \frac{N_2 I_{a_2}}{N_1 I_{a_1}}$$

$$\frac{480.5 - 38.97\,R}{470} = \frac{0.75 \times 38.97}{60} \quad \Rightarrow \quad R = 6.455\ \Omega$$

EXAMPLE 7.16. *A 500 V shunt motor takes 4 A on no load. The armature resistance including that of brushes is 0.2 Ω and the field current is 1 A. Estimate the output and the efficiency when the input current is (a) 20 A, (b) 100 A.*

SOLUTION. Constant loss

= no-load power input – no load copper losses

$$p_c = VI_0 - (I_0 - I_{sh})^2 R_a = 500 \times 4 - (4-1)^2 \times 0.2 = 1998.2 \text{ W}.$$

(a) For 20 A input current

Power input $= VI = 500 \times 20 = 10000$ W

Armature copper loss $= (I - I_{sh})^2 R_a$

$= (20-1)^2 \times 0.2 = 72.2$ W.

Total losses $= p_c$ + armature copper loss

$= 1998.2 + 72.2 = 2070.4$ W.

Efficiency of motor

$$\eta_m = \frac{\text{input} - \text{losses}}{\text{input}} = \frac{10000 - 2070.4}{10000} = 0.793 \text{ pu} = 79.3\%.$$

(b) For 100 A input current

Power input $= VI = 500 \times 100 = 50000$ W

Armature copper loss $= (I - I_{sh})^2 R_a = (100-1)^2 \times 0.2 = 1960$ W.

Total losses $= p_c$ + armature copper loss

$= 1998.2 + 1960 = 3958.2$ W.

$$\eta_m = \frac{\text{input} - \text{losses}}{\text{input}} = \frac{50000 - 3958.2}{50000} = 0.921 \text{ pu} = 92.1\%$$

EXAMPLE 7.17. *The Hopkinson test on two shunt machines gave the following results for full load :*

Line voltage, 250 V ; line current excluding field currents, 50 A ; motor armature current, 380 A ; field currents, 5 A and 4.2 A. Calculate the efficiency of each machine. Armature resistance of each machine 0.02 Ω.

SOLUTION. In the problem it is mentioned that line current is 5... field currents. This indicates that the fields are separately excited... supplied will only be the armature copper losses and stray losses... fields which are separately excited.

The generator armature current $I_{ga} = I_{ma} - I_L = 380 - 50 = 330$ A

The machine with smaller excitation acts as a motor.

Input from the supply = total losses in the set = $250 \times 50 = 12500$ W

Copper loss in the motor armature $= I_{ma}^2 R_{am} = (380)^2 \times 0.02 = 2888$ W

Copper loss in the generator armature $= I_{ga}^2 R_{ag} = (330)^2 \times 0.02 = 2178$ W

Total armature copper loss of the set $= 2888 + 2178 = 5066$ W

$\therefore$ total stray loss of the set = input – total losses in the armature

$= 12500 - 5066 = 7434$ W.

Stray loss per machine $= \dfrac{7434}{2} = 3717$ W

Motor efficiency

Input $= 250\,(380 + 4.2) = 96050$ W

Armature copper loss $= 2888$ W

Field copper loss $= 250 \times 4.2 = 1050$ W

Stray loss $= 3717$ W

Total losses $= 2888 + 1050 + 3717 = 7655$ W

The efficiency of motor

$$= \frac{\text{input} - \text{losses}}{\text{input}} \times 100 = \frac{96050 - 7655}{96050} \times 100 = \ldots$$

Generator efficiency

Output $= 250 \times 330 = 82500$ W

Armature copper loss $= 2178$ W

Field loss $= 250 \times 5 = 1250$ W

Stray loss $= 3717$ W

Total losses $= 2178 + 1250 + 3717 = 7145$ W

The efficiency of generator

$$= \frac{\text{output}}{\text{output} + \text{losses}} = \frac{82500}{82500 + 7145} \times 100 = 92.03\%.$$

EXERCISES

7.1 Explain what is meant by back e.m.f. Explain the principle of ... in a d.c. motor.

7.2 Derive the torque equation of a d.c. motor.

7.3 What is the necessity of a starter for a d.c. motor. Explain, with a ... working of a 3-point d.c. shunt motor starter, bringing out the ... incorporated in it.

7.4 Discuss different methods of speed control of a d.c. motor.

7.5 Sketch the speed-load characteristics of a d.c. (*a*) shunt motor, ... (*c*) cumulatively compounded motor. Account for the shape of the ... teristic curves.

7.6 Why is the starting current very high in a d.c. motor ? How does the starter reduce the starting current to a safe value ?

7.7 A 50 kW, 250 V d.c. shunt generator runs at 1200 r.p.m. If this machine is run as a motor taking 30 kW at 250 V, what will be its speed ? The armature and shunt field resistances are 0.1 Ω and 125 Ω respectively. Brush drop is 2 V. [1782.6 r.p.m.]

7.8 A 4-pole, 500 V d.c. shunt motor has 700 wave-connected conductors on its armature. The full-load armature current is 60 A and the flux per pole is 30 mWb. Calculate the full-load speed if the motor armature resistance is 0.2 Ω and the brush drop is 1 V per brush. [694.3 r.p.m.]

7.9 A 4-pole d.c. shunt motor working on 250 V takes a current of 2 A when running on 1000 r.p.m. What will be its back e.m.f., speed and percentage speed drop if the motor takes 51 A at a certain load ? Armature and shunt field resistances are 0.2 Ω and 250 Ω respectively. [240 V, 960.77 r.p.m., 3.92%]

7.10 A-4 pole, d.c. shunt motor has a flux per pole of 0.04 Wb and the armature is lap wound with 720 conductors. The shunt field resistance is 240 Ω and the armature resistance is 0.2 Ω. Brush contact drop is 1 V per brush. Determine the speed of the machine when running (*a*) as a motor taking 60 A and (*b*) as a generator supplying 120 A. The terminal voltage in each case is 480 V. [972 r.p.m., 1055 r.p.m.]

7.11 A 440 V d.c. motor takes an armature current of 60 A when its speed is 750 r.p.m. If the armature resistance is 0.25 Ω, calculate the torque produced. [324.68 Nm]

7.12 A 10 h.p., 230 V shunt motor takes an armature current of 6 A from the 230 V lime at no load and runs at 1200 r.p.m. The armature resistance is 0.25 Ω. Determine the speed and torque when the armature takes 36 A with the same flux. [1160.6 r.p.m. ; 65.46 Nm]

7.13 The armature of a 4-pole d.c. shunt motor has a lap winding accommodated in 60 slots, each containing 20 conductors. If the useful flux per pole is 23 mWb, calculate the total torque developed when the armature current is 50 A. [219.63 Nm]

7.14 Explain the speed-current, torque-current and speed-torque characteristics of d.c. series motor.

7.15 Explain the speed-current torque-current and speed-torque characteristics of d.c. shunt motor.

7.16 Explain why a d.c. series motor should never run unloaded.

7.17 Define the term speed regulation of a d.c. motor. What is meant by good speed regulation ?

7.18 Why a d.c. series motor should not be started at no load ?

7.19 Neatly sketch the speed-load, torque-load and speed-torque characteristics of a d.c. compound motor.

7.20 What are the general methods of speed control of d.c. motors ?

7.21 Explain the necessity of starter in a d.c. motor and describe three-point starter with a neat sketch.

7.22 What are the drawbacks of three-point starter ? Describe a four-point starter with a neat sketch.

7.23 What are the losses that occur in d.c. machines ? Derive the condition for maximum efficiency of a d.c. generator.

7.24 Draw the power flow diagrams of a d.c. generator and a d.c. motor.

7.25 Describe Swinburne's test with the help of a neat diagram to find out the efficiency of a d.c. machine. What are the main advantages and disadvantages of this test ?

7.26 Explain briefly Hopkinson's test for determination of efficiency of d.c. shunt machines. What are the main advantages and limitations of this test ?

7.27 When running on no load, a 400-V shunt motor takes 5 A. Armature 0.5 Ω and field resistance 200 Ω. Find the output of the motor and running on full load and taking a current of 50 A. Also, find the pe in speed from no load to full load. [16852.5 W,

7.28 A 200-V, shunt motor develops an output of 17.158 kW when taking field resistance is 50 Ω and an armature resistance 0.06 Ω. What is and power input when the output is 7.46 kW ?

7.29 Two identical d.c. machines when tested by Hopkinson's method ing test results : Field currents are 2.5 A and 2 A. Line voltage current including both the field currents is 10 A. Motor armature The armature resistance of each machine is 0.05 Ω. Calculate the the machines. [$\eta_g = 92.64\%$;

7.30 A 200-V, 14.92 kW d.c. shunt motor when tested by Swinburne's following results :

Running light : Armature current was 6.5 A and field current 2.2 A.

With armature locked : The current was 70 A when a potential was applied to the brushes.

Estimate the efficiency of the motor when working under full-load

7.31 The Hopkinson's test on two shunt machines gave the following load :

Line voltage 250 V ; Line current excluding field currents 50 A

Motor armature current 380 A ; Field currents 5 A and 4.2 A

Assuming resistance of each machine as 0.02 Ω, determine the machine. [$\eta_g = 91.9\%$;

7.32 State some present-day uses of d.c. machines.

7.33 Derive torque and emf equations for a dc motor.

7.34 Derive an expression for the electromagnetic torque developed in a by using BIL concept.

7.35 Draw and explain the characteristics of a dc series motor.

7.36 Draw the speed-torque characteristics of a dc shunt, series, and compou in one figure and compare them. Which characteristic is more suitable purpose and why?

7.37 Explain what would happen if a dc motor is directly switched on to the without any starter.

7.38 What are the losses taking place in dc machine and how they vary current and derive the condition for maximum efficiency ?

7.39 Name various methods of electric braking of dc motors and describe any them.

7.40 Explain the principle of regenerative braking of dc motors.

7.41 Explain why regenerative braking of dc series motor is not as simple as dc shunt motor.

7.42 Explain the principle of dynamic braking of dc motors.

7.43 Explain the principles of plugging and rhesostatic braking of a dc drive

7.44 Compare various methods of electric braking of dc motors.

7.45 Explain how a dc series motor is stopped by : (*a*) plugging, (*b*) rheostatic

7.46 Describe a four-quadrant operation of a dc motor.

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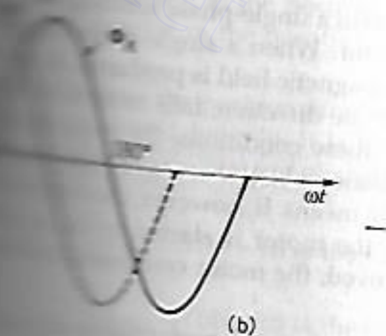
$\Phi_A = \Phi$

$\Phi_B = \Phi$



(b)

Fig. 8.1. Production of a u

8

Single-Phase Motors

8.1 INTRODUCTION

The most common type of electric motor is the single-phase type, which finds wide domestic, commercial and industrial applications. Single-phase motors are small-size motors of fractional-kilowatt ratings. Domestic appliances like fans, hair driers, washing machines, vacuum cleaners, mixers, refrigerators, food processors and other kitchen equipment employ these motors. These motors also find applications in air-conditioning fans, blowers, office machinery, small power tools, dairy machinery, small farming equipment etc.

Single-phase motors are classified as follows :

1. Induction motors
2. Commutator motors
3. Synchronous motors

8.2 PRODUCTION OF ROTATING FIELD

Consider two windings A and B so displaced that they produce magnetic fields 90° apart in *space*, as shown in Fig. 8.1 (*a*). Suppose that these windings produce magnetic fields equal in magnitude and 90° apart in time given by

$$\Phi_A = \Phi_m \sin \omega t$$

$$\Phi_B = \Phi_m \sin (\omega t + 90°)$$

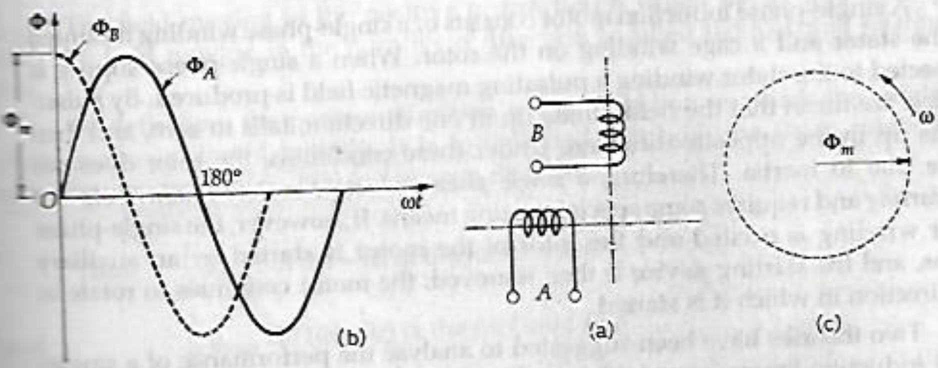


Fig. 8.1. *Production of a uniform magnetic field.*

The waveforms of these fields are shown in Fig. 8.1 (*b*). The resultant of these two fields is a rotating magnetic field of constant magnitude Φ_m. This rotating magnetic field may be represented by a phasor of constant magnitude Φ_m as shown in Fig. 8.1 (*c*). This phasor describes a circle in each revolution (Fig. 8.1 (*c*)). Each revolution of the phasor corresponds to one cycle of the supply frequency.

Suppose that the two windings A and B are displaced 90° in *space* but produce fields that are either not equal or not 90° apart in time as shown in Fig. 8.2. The resultant of these two fields is again a rotating field but this field is variable in magnitude throughout each revolution. Similarly, the resultant of two fields displaced both in time and space by some angle other than 90 degrees is a nonuniform rotating field. A nonuniform magnetic field produces a nonuniform torque which makes the operation of the motor noisy. The other effect of nonuniform field is upon the starting torque. A motor having a more uniform rotating field has the larger starting torque in comparison to a motor of the same rating having a nonuniform rotating field.

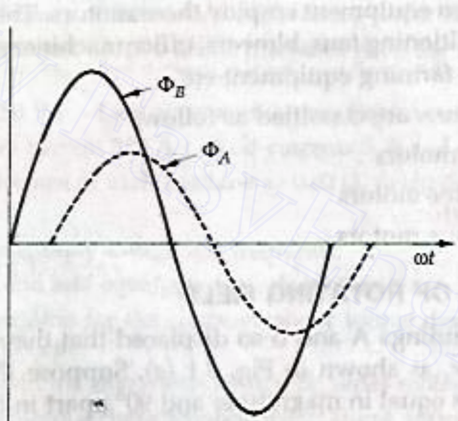


Fig. 8.2.

8.3 SINGLE-PHASE INDUCTION MOTOR PRINCIPLE

A single-phase induction motor consists of a single-phase winding mounted on the stator and a cage winding on the rotor. When a single-phase supply is connected to the stator winding a pulsating magnetic field is produced. By *pulsating field* we mean that the field builds up in one direction, falls to zero, and then builds up in the opposite direction. Under these conditions, the rotor does not rotate due to inertia. Therefore, *a single phase induction motor is inherently not self-starting* and requires some special starting means. If, however, the single-phase stator winding is excited and the rotor of the motor is started by an auxiliary means, and the starting device is then removed, the motor continues to rotate in the direction in which it is started.

Two theories have been suggested to analyse the performance of a single-phase induction motor, namely the ***double-revolving-field theory*** and the ***cross-field theory***. Both the theories are fairly complicated, and neither has any advantage over the other in numerical calculations. Almost similar results are *obtained with both the theories. These two theories explain why a torque is produced in the rotor once it is turning. Here we shall discuss the double revolving* field theory.

8.4 DOUBLE-REVOLVING-FIELD THEORY OF SINGLE-PHASE INDUCTION MOTORS

The double-revolving-field theory of single-phase induction motors basically states that a stationary pulsating magnetic field can be resolved into two rotating magnetic fields, each of equal magnitude but rotating in opposite directions. The induction motor responds to each magnetic field separately, and the net torque in the motor is equal to the sum of the torques due to each of the two magnetic fields.

The equation for an alternating magnetic field whose axis is fixed in space is given by

$$b(\alpha) = \beta_{max} \sin \omega t \cos \alpha \tag{8.3.1}$$

where β_{max} is the maximum value of the sinusoidally distributed air-gap flux density produced by a properly distributed stator winding carrying an alternating current of frequency ω and α is space-displacement angle measured from the axis of the stator winding.

Since $\quad \sin A \cos B = \frac{1}{2} \sin (A - B) + \frac{1}{2} \sin (A + B)$

Eq. (8.3.1) can be written as

$$b(\alpha) = \frac{1}{2} \beta_{max} \sin (\omega t - \alpha) + \frac{1}{2} \beta_{max} \sin (\omega t + \alpha) \tag{8.3.2}$$

The first term on the right-hand side of Eq. (8.3.2) represents the equation of a revolving field moving in the positive α direction. It has a maximum value equal to $\frac{1}{2}\beta_{max}$. The second term on the right-hand side of Eq. (8.3.2) represents the equation of a revolving field moving in the negative α direction. Its amplitude is also equal to $\frac{1}{2}\beta_{max}$.

The field moving in the positive α direction is called the *forward rotating* field. The field moving in the negative α direction is called the *backward rotating* field.

By definition, the positive direction is that direction in which the single-phase motor is started initially. It is to be noted that both the fields rotate at synchronous speed ω_s $(= 2\pi f)$ in opposite directions.

Thus, $\quad \frac{1}{2} \beta_{max} \sin (\omega t - \alpha)$ is the *forward field*

and $\quad \frac{1}{2} \beta_{max} \sin (\omega t + \alpha)$ is the *backward field*

It is therefore concluded that a stationary pulsating magnetic field can be resolved into two *rotating* magnetic fields, both of equal magnitude and moving at synchronous speed in opposite directions at the same frequency as the ... ary magnetic field alternates. The theory based on such a resolution of an alt... ing field into two counter rotating fields is called the **double-revolving... theory** of single-phase induction motors.

When the rotor is stationary (that is, at standstill), the induced vol... equal and opposite. Consequently, the two torques are also equal and op... Hence, at standstill the net torque is zero. In other words, *a single-phase ind... motor with single-stator winding inherently has no starting torque.*

However, if the rotor is given an initial rotation by auxiliary means in ... directions the torque due to the rotating field acting in the direction of ... rotation will be more than the torque due to the other rotating field. Hence ... motor will develop a net positive torque in the same direction as the ... rotation. The motor will, therefore, keep running in the direction of initial ro...

8.5 ROTOR SLIP WITH RESPECT TO TWO ROTATING FIELDS

If the rotor is started by auxiliary means, it will develop torque and ... to run in the same direction as one of the fields. By definition, the dire... which the rotor is started initially will be called the ***forward field***.

Let $n_s =$ synchronous speed, $n =$ rotor speed

The slip of the rotor with respect to the forward rotating field is

$$s_f = s = \frac{n_s - n}{n_s} = 1 - \frac{n}{n_s}$$

Since the **backward rotating flux** rotates opposite to the stator, the ... n must be changed in Eq. (8.5.1) to obtain the backward slip. Thus, the slip ... rotor with respect to the backward rotating field is

$$s_b = \frac{n_s - (-n)}{n_s} = \frac{n_s + n}{n_s} = 1 + \frac{n}{n_s}$$

Adding Eqs. (8.5.1) and (8.5.2), we get

$$s + s_b = \left(1 - \frac{n}{n_s}\right) + \left(1 + \frac{n}{n_s}\right)$$

$$\therefore \quad s_b = 2 - s$$

Thus, the rotor slips with respect to the two rotating fields are differe... are given by Eqs. (8.5.1) and (8.5.3).

In order to make clear the influence of the two rotating fluxes on the ... it will be assumed that $n < n_s$. Equation (8.5.1) corresponds to a motor op... and Eq. (8.5.3) denotes the braking region. Thus, the two torques have an op... influence on the rotor.

The rotor equivalent circuits for the forward and backward rotating fi... are shown in Fig. 8.3.

At standstill, the impedances are equal and, therefore, the current I_{2f} ... I_{2b} are equal. These currents produce mmfs which oppose the stator mmfs ... Therefore, the rotating forward and backward fluxes in the air gap are eq...

magnitude, and no torque is developed. However, when the rotor rotates, the impedances of the rotor circuits (Fig. 8.3) are unequal and the rotor current I_{2b} is greater than the rotor current I_{2f}. Their mmfs, which oppose the stator mmfs, will

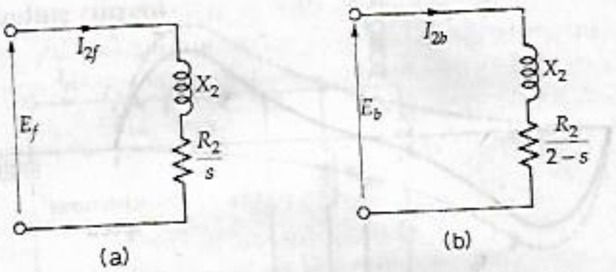


Fig. 8.3. *Rotor equivalent circuits (a) For forward-rotating flux wave (b) For backward-rotating flux wave.*

result in the reduction of the backward rotating flux. Consequently, as the speed increases, the forward flux increases while the backward flux decreases. However, the resultant flux remains essentially constant. This resultant flux induces voltage in the stator winding. Both flux waves induce voltages in the rotor and produce torques in the rotor. These two torques are in opposite directions. The net induced torque in the motor is equal to the *difference* between these torques. Figure 8.4 shows torques produced by the two revolving fields and also the resultant torque produced by the motor.

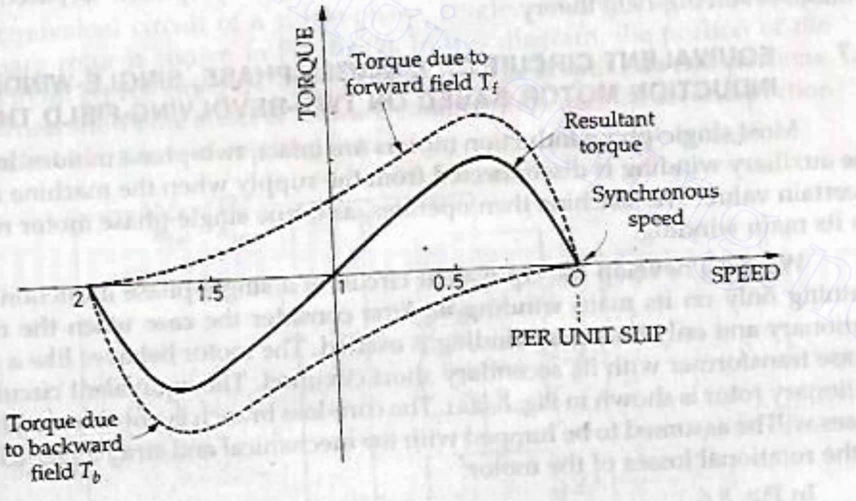


Fig. 8.4. *Torque-speed characteristic of a single-phase induction motor based on constant forward and backward flux waves.*

If the voltage drops across the winding resistance and leakage reactance are neglected the induced voltage is almost equal to the applied voltage. As the speed increases, forward torque increases and reverse torque decreases compared with those shown in Fig. 8.4. The actual torque-speed characteristics are app those shown in Fig. 8.5.

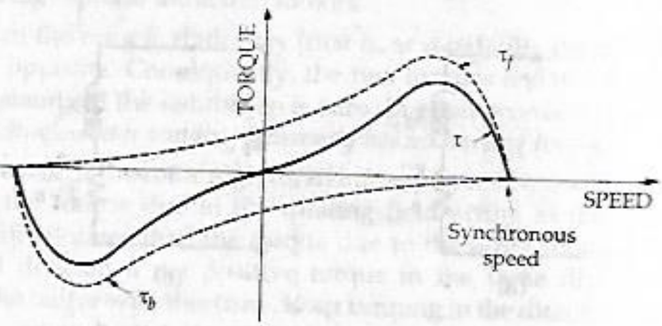


Fig. 8.5. *Actual torque-speed characteristic of a single-phase induction motor taking into account changes in forward and backward flux waves.*

8.6 EQUIVALENT CIRCUIT (CIRCUIT MODEL) OF A SINGLE-PHASE SINGLE-WINDING INDUCTION MOTOR

The induced torque in a single-phase induction motor can be under by either the double revolving-field theory or the cross-field theory of single-pha motors. The equivalent circuit of the motor can be obtained by either me Similarly, the torque-speed characteristic can be derived through either appr First, we shall develop an equivalent circuit of a single-phase induction mo when only its main winding is energized. This development is based on the double-revolving-field theory.

8.7 EQUIVALENT CIRCUIT OF A SINGLE-PHASE, SINGLE-WINDING INDUCTION MOTOR BASED ON TWO-REVOLVING-FIELD THEORY

Most single-phase induction motors are infact, two-phase motors, in wh the auxiliary winding is disconnected from the supply when the machine rea a certain value. The machine then operates, as a true single-phase motor run on its main winding.

We shall develop the equivalent circuit of a single-phase induction mo running only on its main winding m. First consider the case when the rot stationary and only the main winding is excited. The motor behaves like a sing phase transformer with its secondary short circuited. The equivalent circuit wi stationary rotor is shown in Fig. 8.6(*a*). The core-loss branch is not shown. The losses will be assumed to be lumped with the mechanical and stray losses as pa of the rotational losses of the motor.

In Fig. 8.6,

R_{1m} = resistance of the main stator winding

X_{1m} = leakage reactance of the main stator winding

X_M = magnetizing reactance

R_2' = standstill rotor resistance referred to the main stator winding

X_2' = standstill rotor leakage reactance referred to the main stator winding.

V_m = applied voltage

I_m = main winding current

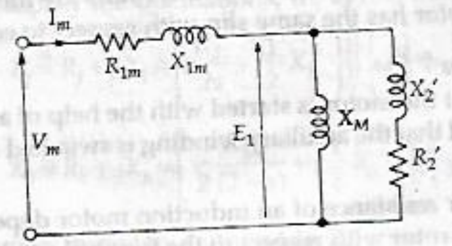


Fig. 8.6. *Equivalent circuit of a single-phase induction motor with only its main winding energized (a) At standstill.*

According to the double-revolving-field theory, the pulsating air-gap flux in the motor at standstill can be resolved into two equal and opposite fluxes with the motor. Since the magnitude of each rotating flux is one-half of the alternating flux, it is convenient to assume that the two rotating fluxes are acting on two separate rotors. Thus, a single-phase induction motor may be considered as consisting of two motors having a common stator winding and two *imaginary* rotors, which rotate in opposite directions. The standstill impedance of each rotor referred to the main stator winding is $\left(\dfrac{R_2'}{2} + j\dfrac{X_2'}{2}\right)$.

The equivalent circuit of a single-phase, single-winding induction motor with stationary rotor is shown in Fig. 8.6(*b*). In this diagram, the portion of the equivalent circuit representing the effects of air-gap flux is split into two portions. The first portion shows the effect of forward rotating flux, and the second portion shows the effect of backward rotating flux.

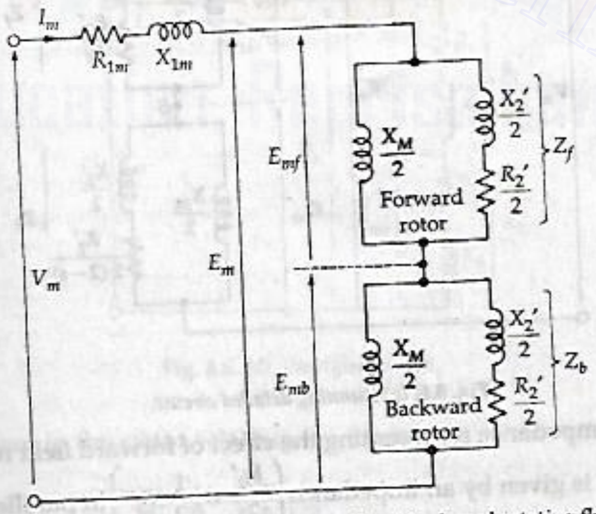


Fig. 8.6. *(b) At standstill with the effects of forward and backward rotating fluxes separated.*

The forward flux induces a voltage E_{mf} in the main stator winding ... the backward rotating flux induces a voltage E_{mb} in the main stator winding ... resultant induced voltage in the main stator winding is E_m, where

$$\mathbf{E}_m = \mathbf{E}_{mf} + \mathbf{E}_{mb}$$

The circuits of the forward and backward rotors are identical under ... still conditions as the rotor has the same slip with respect to each rotating ...

At standstill, $\quad \mathbf{E}_{mf} = \mathbf{E}_{mb}$

Now suppose that the motor is started with the help of an auxiliary winding. It is further assumed that the auxiliary winding is switched out after ... gains it normal speed.

The effective rotor resistance of an induction motor depends on ... the rotor. The slip of the rotor with respect to the forward rotating flux ... the effective rotor resistance in the portion of the circuit associated with ... forward rotating flux is $\frac{R_2'}{2s}$. The slip of the rotor with respect to the backward rotating flux is $(2 - s)$. Therefore, the effective rotor resistance (referred ... in the portion of the circuit associated with the backward rotating ... $\frac{R_2'}{2(2-s)}$.

When the forward and backward slips are taken into account, the ... the equivalent circuit shown in Fig. 8.6(*c*) which represents the motor ... the main winding alone.

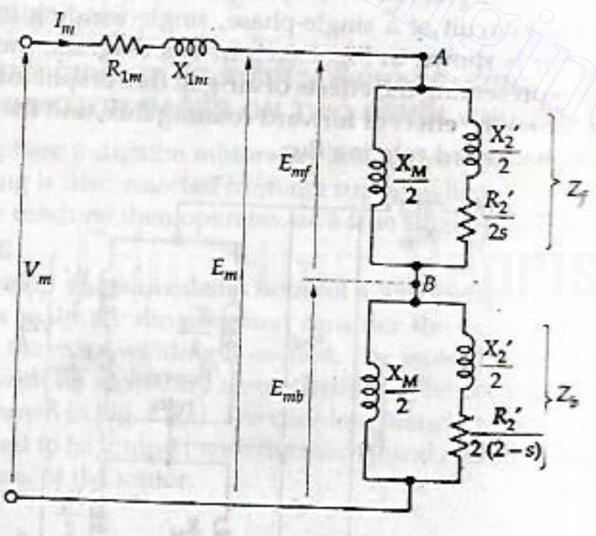


Fig. 8.6. (*c*) *Running detailed circuit.*

The rotor impedance representing the effect of forward field referred ... stator winding m is given by an impedance $\left(\frac{R_2'}{2s} + j\frac{1}{2}X_2'\right)$ in parallel with ...

Similarly, the rotor impedance representing the effect of backward field referred to the stator winding m is given by an impedance $\left[\frac{R_2'}{2(2-s)}+j\frac{1}{2}X_2'\right]$ in parallel with $j\frac{1}{2}X_M$.

In order to simplify the calculations, we define the following impedances :

$$Z_f \triangleq R_f + jX_f = \left(\frac{R_2'}{2s}+j\frac{1}{2}X_2'\right) \| \left(j\frac{1}{2}X_M\right)$$

$$Z_b \triangleq R_b + jX_b = \left[\frac{R_2'}{2(2-s)}+j\frac{1}{2}X_2'\right] \| \left(j\frac{1}{2}X_M\right)$$

Here Z_f = rotor impedance offered to the forward field

Z_b = rotor impedance offered to the backward field

$$\therefore \quad Z_f = \frac{\left(\frac{R_2'}{2s}+j\frac{1}{2}X_2'\right)\left(j\frac{1}{2}X_M\right)}{\frac{R_2'}{2s}+j\frac{1}{2}X_2'+j\frac{1}{2}X_M} \tag{8.7.1}$$

and

$$Z_b = \frac{\left[\frac{R_2'}{2(2-s)}+j\frac{1}{2}X_2'\right]\left(j\frac{1}{2}X_M\right)}{\frac{R_2'}{2(2-s)}+j\frac{1}{2}X_2'+j\frac{1}{2}X_M} \tag{8.7.2}$$

The simplified equivalent circuit of a single-phase induction motor with only its main winding energized is shown in Fig. 8.6*d*.

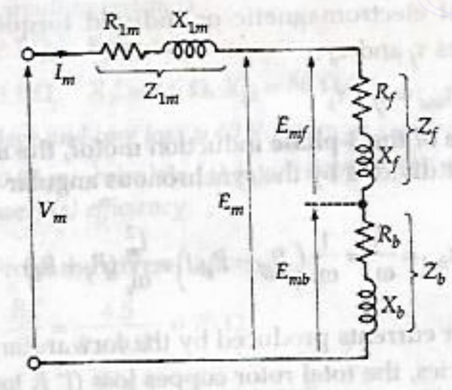


Fig. 8.6. *(d) Simplified circuit.*

The current in the stator winding is

$$I_m = \frac{V_m}{Z_{1m}+Z_f+Z_b} \tag{8.7.3}$$

8.8 PERFORMANCE CALCULATIONS OF A SINGLE-PHASE, SINGLE-WINDING INDUCTION MOTOR

The performance calculations of a single-phase, single-windi motor can be done with the help of the equivalent circuit shown in Fig circuit is similar to that of the three-phase induction motor with the e there are both forward and backward components of power and t in it. The relationships for power and torque obtained for 3-phase motors can also be applied for either the forward or the backward co the single-phase induction motor.

The torque of the backward field is in the opposite direction t forward field, and therefore the total air-gap power in a single-phase motor is

$$P_g = P_{gf} - P_{gb}$$

where P_{gf} = air-gap power for forward field

or
$$P_{gf} = I_m^2 R_f$$

P_{gb} = air-gap power for backward field $= I_m^2 R_b$

$$\therefore \quad P_g = I_m^2 R_f - I_m^2 R_b = I_m^2 (R_f - R_b)$$

The torque produced by the forward field

$$\tau_f = \frac{1}{\omega_s} P_{gf} = \frac{P_{gf}}{2\pi n_s}$$

The torque produced by the backward field

$$\tau_b = \frac{1}{\omega_s} P_{gb} = \frac{P_{gb}}{2\pi n_s}$$

where ω_s = synchronous speed in rad/s.

The resultant electromagnetic or induced torque τ_{ind} is the between the torques τ_f and τ_b :

$$\tau_{ind} = \tau_f - \tau_b$$

As in the case of the 3-phase induction motor, the induced torq to the air-gap power divided by the synchronous angular velocity.

$$\tau_{ind} = \frac{P_g}{\omega_s} = \frac{1}{\omega_s}\left(P_{gf} - P_{gb}\right) = \frac{I_m^2}{\omega_s}(R_f - R_b)$$

Since the rotor currents produced by the forward and backwar of different frequencies, the total rotor copper loss ($I^2 R$ loss) is the sum copper loss due to the forward field and the rotor copper loss due to the field.

$$p_{rc} = p_{rcf} + p_{rcb}$$

The rotor copper loss in a 3-phase induction motor

= slip × airgap power

Similarly, the rotor copper loss due to the forward field of a single-phase induction motor is given by

$$p_{rcf} = s\, P_{gf} \tag{8.8.10}$$

and the rotor copper loss due to backward field of a single-phase induction motor is given by

$$p_{rcb} = (2 - s)\, P_{gb} \tag{8.8.11}$$

Total rotor copper loss

$$p_{cr} = sP_{gf} + (2 - s)\, P_{gb} \tag{8.8.12}$$

The power converted from electrical to mechanical form in a single-phase induction motor is given by

$$P_{mech} = P_{conv} = \omega\, \tau_{ind} \tag{8.8.13}$$

where $\omega =$ angular velocity of the rotor in rad/s.

Since, $\omega = (1 - s)\, \omega_s$ (8.8.14)

$$P_{mech} = P_{conv} = (1 - s)\, \omega_s\, \tau_{ind} \tag{8.8.15}$$

$$= (1 - s)\, P_g = (1 - s)\, (P_{gf} - P_{gb})$$

or $$P_{mech} = I_m^2\, (R_f - R_b)\, (1 - s) \tag{8.8.16}$$

Shaft output power

$$P_{out} = P_{conv} - \text{core loss} - \text{mechanical losses} - \text{stray losses} \tag{8.8.17}$$

or $$P_{out} = P_{mech} - p_{rot} \tag{8.8.18}$$

where $p_{rot} =$ rotational losses

$$= \text{friction loss} + \text{windage loss} + \text{core loss} \tag{8.8.19}$$

Example 8.1. *A 230 V, 50 Hz, 4-pole single-phase induction motor has the following equivalent circuit impedances :*

$$R_{1m} = 2.2\ \Omega, \quad R_2' = 4.5\ \Omega$$

$$X_{1m} = 3.1\ \Omega, \quad X_2' = 2.6\ \Omega, X_M = 80\ \Omega.$$

Friction, windage and core loss = 40 W

For a slip of 0.03 pu, calculate (a) input current, (b) power factor, (c) developed power, (d) output power, (e) efficiency.

Solution. From the given data

$$\frac{R_2'}{2s} = \frac{4.5}{2 \times 0.03} = 75\ \Omega$$

$$\frac{R_2'}{2\,(2 - s)} = \frac{4.5}{2\,(2 - 0.03)} = 1.142\ \Omega$$

$$\frac{1}{2} X_2' = \frac{1}{2} \times 2.6 = 1.3\ \Omega$$

$$\frac{1}{2} X_M = \frac{1}{2} \times 80 = 40\ \Omega$$

For the forward field circuit

$$Z_f = R_f + jX_f = \frac{\left(\dfrac{R_2'}{2s} + j\dfrac{X_2'}{2}\right)\left(j\dfrac{X_M}{2}\right)}{\dfrac{R_2'}{2s} + j\dfrac{X_2'}{2} + j\dfrac{X_M}{2}}$$

$$= \frac{(75 + j\,1.3)\,(j\,40)}{75 + j\,1.3 + j\,40} = \frac{(75.011\,\angle 0.993°)\,(40\,\angle 90°)}{85.619\,\angle 28.84°}$$

$$= 35.04\,\angle 62.15°\ \Omega = 16.37 + j\,30.98\ \Omega$$

For the backward field

$$Z_b = R_b + jX_b = \frac{\left[\dfrac{R_2'}{2\,(2-s)} + j\dfrac{X_2'}{2}\right]\left(j\dfrac{X_M}{2}\right)}{\dfrac{R_2'}{2\,(2-s)} + j\dfrac{X_2'}{2} + j\dfrac{X_M}{2}}$$

$$= \frac{(1.142 + j\,1.3)\,(j\,40)}{1.142 + j\,1.3 + j\,40} = \frac{(1.73\,\angle 48.7°)\,(40\,\angle 90°)}{41.316\,\angle 88.4°}$$

$$= 1.675\,\angle 50.3° = 1.07 + j\,1.29\ \Omega$$

$$\mathbf{Z}_{1m} = R_{1m} + j\,X_{1m} = 2.2 + j\,3.1$$

The total series impedance

$$\mathbf{Z}_e = \mathbf{Z}_{1m} + \mathbf{Z}_f + \mathbf{Z}_b$$

$$= 2.2 + j\,3.1 + 16.37 + j\,30.98 + 1.07 + j\,1.29$$

$$= 19.64 + j\,35.37 = 40.457\,\angle 60.96°\ \Omega$$

(*a*) Input current

$$\mathbf{I}_m = \frac{\mathbf{V}_m}{\mathbf{Z}_e} = \frac{230\,\angle 0°}{40.457\,\angle 60.96°} = 5.685\,\angle -60.96°\ \text{A.}$$

(*b*) Power factor $= \cos(-60.95°) = 0.4856$ lagging.

(*c*) Developed power

$$P_{conv} = P_d = I_m^2\,(R_f - R_b)\,(1 - s)$$

$$= (5.685)^2\,(16.37 - 1.07)\,(1 - 0.03) = 479.65\ \text{W}$$

(*d*) Output power $= P_d - P_{rot} = 479.65 - 40 = 439.65$ W

Input power $= VI_m \cos\phi = 230 \times 5.685 \times 0.4856 = 634.9$ W

(*e*) Efficiency $= \dfrac{\text{output}}{\text{input}} = \dfrac{439.65}{634.9} = 0.692$ pu.

8.9 DETERMINATION OF EQUIVALENT CIRCUIT PARAMETERS

The parameters of the equivalent circuit of a single-phase induction motor can be determined from the blocked-rotor and no-load tests. These tests are similar to those made on 3-phase induction motor. However, except for the capacitor motor, these tests are performed with auxiliary winding kept open.

8.9.1 Blocked-rotor test

In this test the rotor is at rest (blocked). A low voltage is applied to the stator so that rated current flows in the main winding. The voltage, current and power input are measured. Let V_{sc}, I_{sc} and P_{sc} denote the voltage, current and power respectively under these conditions. With the rotor blocked, $s = 1$ the impedance $\frac{X_M}{2}$ in the equivalent circuit of Fig. 8.6(*c*) is so large compared with $\left(\frac{R_2'}{2} + j\frac{X_2'}{2}\right)$ that it may be neglected from the equivalent circuit. Therefore, the equivalent circuit of Fig. 8.6(*c*) at $s = 1$ reduces to that shown in Fig. 8.7(*a*).

The equivalent impedance referred to stator is given by

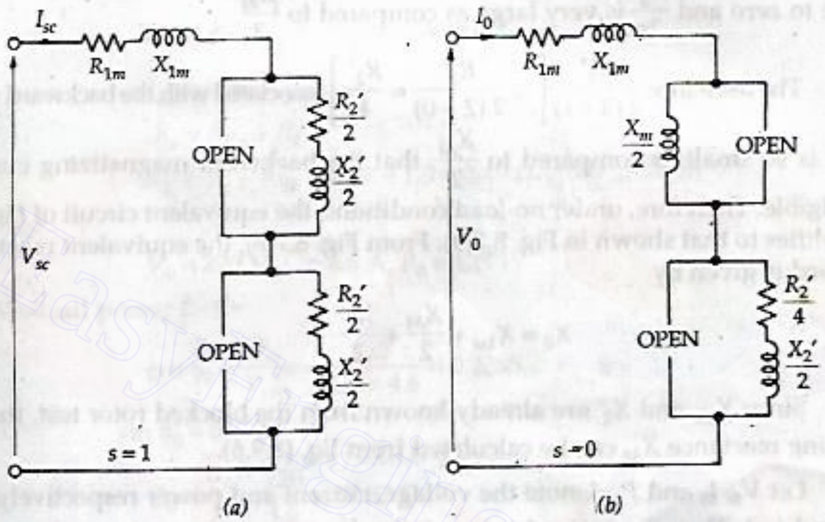


Fig. 8.7. *Simplified equivalent circuit of a single-phase induction motor (a) with locked rotor, (b) at no load.*

$$Z_e = \frac{V_{sc}}{I_{sc}} \tag{8.9.1}$$

From Fig. 8.7(*a*), the equivalent series resistance R_e of the motor is

$$R_e = R_{1m} + \frac{R_2'}{2} + \frac{R_2'}{2} = R_{1m} + R_2' = \frac{P_{sc}}{I_{sc}^2} \tag{8.9.2}$$

Since the resistance of the main stator winding R_{1m} is already measured, the effective rotor resistance at line frequency is given by

$$R_2' = R_e - R_{1m} = \frac{P_{sc}}{I_{sc}^2} - R_{1m} \tag{8.9.3}$$

From Fig. 8.7(*a*), the equivalent reactance X_e is given by

$$X_e = X_{1m} + \frac{X_2'}{2} + \frac{X_2'}{2} = X_{1m} + X_2' \tag{8.9.4}$$

Since the leakage reactances X_{1m} and X_2' cannot be separated ... a simplifying assumption that $X_{1m} = X_2'$.

$$\therefore \quad X_{1m} = X_2' = \frac{1}{2} X_e = \frac{1}{2} \times \sqrt{Z_e^2 - R_e^2}$$

Thus, from blocked-rotor test, the parameters R_2', X_{1m}, X_2' can be ... R_{1m} is known.

8.9.2 No-load test

The motor is run without load at rated voltage and rated freq... voltage, current and input power are measured. At no load, the slip s is ... close to zero and $\frac{R_2'}{2s}$ is very large as compared to $\frac{X_M}{2}$.

The resistance $\frac{R_2'}{2(2-s)} \left[\approx \frac{R_2'}{2(2-0)} = \frac{R_2'}{4} \right]$ associated with the backward ... field is so small as compared to $\frac{X_M}{2}$, that the backward magnetizing ... negligible. Therefore, under no-load conditions, the equivalent circuit of Fig. ... simplifies to that shown in Fig. 8.7(*b*). From Fig. 8.7(*b*), the equivalent ... no load is given by

$$X_0 = X_{1m} + \frac{X_M}{2} + \frac{X_2'}{2}$$

Since X_{1m} and X_2' are already known from the blocked rotor test, the ... netizing reactance X_M can be calculated from Eq. (8.9.6).

Let V_0, I_0 and P_0 denote the voltage, current and power respectively ... no-load test. Then the no-load power factor is

$$\cos \phi_0 = \frac{P_0}{V_0 I_0}$$

The no-load equivalent impedance is

$$Z_0 = \frac{V_0}{I_0}$$

The no-load equivalent reactance is

$$X_0 = Z_0 \sin \phi_0 = Z_0 \times \sqrt{1 - \cos^2 \phi_0}$$

Example 8.2. *A 220 V, single-phase induction motor gave the following ... results :*

Blocked-rotor test : 120 V, 9.6 A, 460 W

No-load test : 220 V, 4.6 A, 125 W

The stator winding resistance is 1.5 Ω, and during the blocked-rotor test, the starting winding is open. Determine the equivalent circuit parameters. Also, find the ... friction and windage losses.

SOLUTION.

Blocked-rotor test

$$V_{sc} = 120\text{ V}, I_{sc} = 9.6\text{ A}, P_{sc} = 460\text{ W}$$

$$Z_e = \frac{V_{sc}}{I_{sc}} = \frac{120}{9.6} = 12.5\ \Omega$$

$$R_e = \frac{P_{sc}}{I_{sc}^2} = \frac{460}{(9.6)^2} = 4.99\ \Omega$$

$$\therefore \quad X_e = \sqrt{Z_e^2 - R_e^2} = \sqrt{(12.5)^2 - (4.99)^2}$$

$$= 11.46\ \Omega$$

$$X_{1m} = X_2' = \frac{1}{2} X_e = \frac{1}{2} \times 11.46 = 5.73\ \Omega$$

$$R_{1m} = 1.5\ \Omega$$

$$R_e = R_{1m} + R_2'$$

$$R_2' = R_e - R_{1m} = 4.99 - 1.5 = 3.49\ \Omega.$$

No-load test

$$V_0 = 220\text{ V}, I_0 = 4.6\text{ A}, P_0 = 125\text{ W}$$

No-load power factor

$$\cos\phi_0 = \frac{P_0}{V_0 I_0} = \frac{125}{220 \times 4.6} = 0.1235$$

$$\therefore \quad \sin\phi_0 = 0.9923$$

$$Z_0 = \frac{V_0}{I_0} = \frac{220}{4.6} = 47.83\ \Omega$$

$$\therefore \quad X_0 = Z_0 \sin\phi_0 = 47.83 \times 0.9923 = 47.46\ \Omega$$

Core, friction and windage losses

= power input to motor at no load − no-load copper loss

$$= P_0 - I_0^2 \left(R_{1m} + \frac{R_2'}{4} \right)$$

$$= 125 - (4.6)^2 \left(1.5 + \frac{3.49}{4} \right) = 74.8\text{ W}.$$

8.10 STARTING METHODS AND TYPES OF SINGLE-PHASE INDUCTION MOTORS

We have seen that some means should be used to start the single-phase induction motor. Mechanical methods are impractical and, therefore, the motor is started temporarily converting it into a two-phase motor.

Single-phase induction motors are usually classified according to the auxiliary means used to start the motor. They are classified as follows :

1. Split-phase motor

2. Capacitor-start motor
3. Capacitor-start capacitor-run motor (or two-value capacit
4. Permanent-split capacitor (PSC) motor (or single-value c motor)
5. Shaded-pole motor

All these starting methods depend on two alternating fields di space and phase.

The resultant of the two fields is a rotating field. This rotating fi with the cage rotor to provide the starting torque. One field is produ *main winding* and the other by the *auxiliary winding*. The auxiliary wind called *starting winding*.

8.11 SPLIT-PHASE INDUCTION MOTOR

Figure 8.8 shows a split-phase induction motor. It is also called a *start motor*. It has a single-cage rotor and its stator has two winding winding and a starting (auxiliary) winding. The main field winding starting winding are displaced 90° in space like the windings in a t induction motor. The main winding has very low resistance and hig reactance.

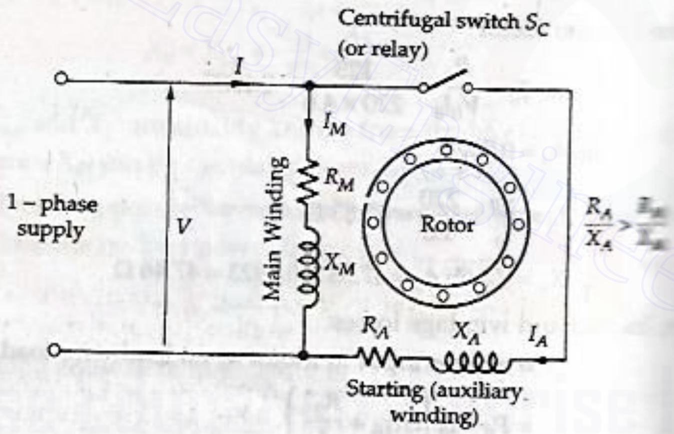


Fig. 8.8. *split-phase induction motor connections.*

Thus, the current I_M in the main winding lags behind the su V by nearly 90° [Fig. 8.9(*a*)]. The auxiliary winding has a resistor co series with it. It has a high resistance and low inductive reacta current I_A in the auxiliary winding is nearly in phase with the line v there is time phase difference between the currents in the two windin phase difference ϕ is not 90° but usually of the order of 30°. This ph is enough to produce a rotating magnetic field. Since the curre windings are not equal, the rotating field is not uniform, and the st is small of the order of 1.5 to 2 times the rated running torque. Th auxiliary windings are connected in parallel during starting. The start is disconnect from the supply automatically when the motor reach

70 to 80 per cent of synchronous speed. For motors rated about 100 W or more, a centrifugally operated switch is used to disconnect the starting winding. For smaller motors a relay is often used. The relay is connected in series with the main winding. At the time of starting, a heavy current flows in the relay coil causing its contacts to close. This brings the starting winding into the circuit. As the motor reaches its predetermined speed of the order of 70 to 80 per cent of synchronous speed, the current through the relay coil decreases. Consequently, the relay opens and disconnects the auxiliary winding from the main supply and the motor then runs only on the main windings. The torque-speed characteristic of this motor is shown in Fig. 8.9 (*b*), which also shows the speed n_0 at which the centrifugal operates.

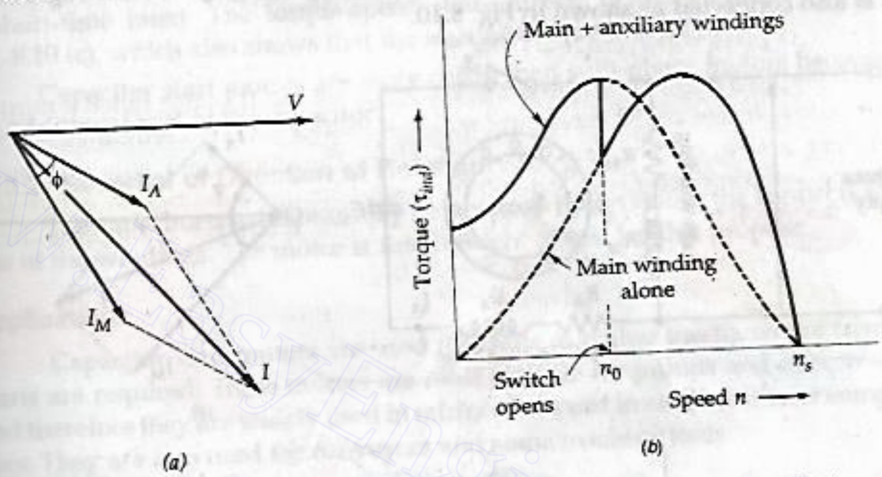


Fig. 8.9 *Resistance split-phase motor (a) Phasor diagram (b) Torque-speed characteristic*

8.11.1 Reversal of direction of Rotation

This motor continues to run in the direction in which it is started. The direction of rotation of the resistance-start induction motor may be reversed by reversing the line connections of either the main winding or the starting winding. The motor must be brought to rest for this purpose. That is, the reversal of rotation can be made only when the motor is standstill but not while running.

8.11.2 Motor Characteristics

The starting torque of a resistance-start induction motor is about 1.5 times full-load torque. The maximum or pull-out torque is about 2.5 times full-load torque at about 75 per cent of synchronous speed. The split-phase motor has a high starting current which is usually 7 to 8 times the full-load value.

8.11.3 Applications

Split-phase motors are cheap and they are most suitable for easily started loads where frequency of starting is limited. The common applications are washing machines, air-conditioning fans, food mixers, grinders, floor polishers, blowers, centrifugal pumps, small drills, lathes, office machinery, dairy machinery, etc. Because of low starting torques, they are seldom used for drives requiring more than 1 kW.

8.12 CAPACITOR MOTORS

Capacitor motors are single-phase induction motors that employ a ca in the auxiliary winding circuit to produce a greater phase difference bet current in the main and auxiliary winnings. There are three types of ca motors.

8.13 CAPACITOR-START MOTOR

Figure 8.10 shows the connections of a capacitor-start motor. It has rotor and its stator has two windings namely, the main winding and the aux winding (starting winding). The two windings are displaced 90° in spa capacitor C_s is connected in series with the starting windings. A centrifug S_c is also connected as shown in Fig. 8.10.

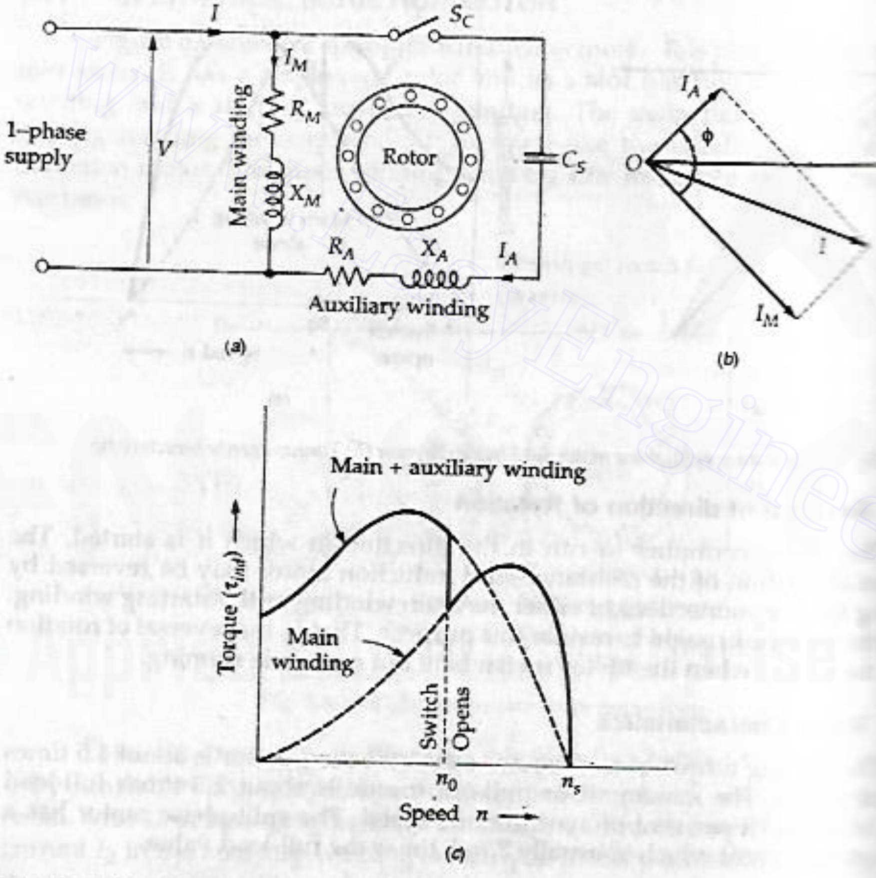


Fig. 8.10. *Capacitor start motor (a) circuit diagram (b) phasor diagram (c) Torque-speed chara*

By choosing a capacitor of the proper rating the current I_M in the ma winding may be made to lag the current I_A in the auxiliary winding by (Fig. 8.10 (b)). Thus, a single-phase supply current is split into two phase applied to the stator windings. Thus the windings are displaced 90° electri their mmf's are equal in magnitude but 90° apart in time phase.

...the motor acts ...rated speed, the ...automatically by t ...named because it u

...Characteristics

...capacitor-start motor ...full-load torque) than ...starting capacitor ...a high starting tor ...electrolytic capacitors ...rated. The torqu ...which also shows

...start motors a ...cost of the capac

...Reversal of Direction

...capacitor-start moto ...windings. The motor

...start motors a ...required. These moto ...they are widely u ...also used for co

...VALUE CAPAC

...8.11 shows the sc ...rotor and its stator ...winding (starting ...motor uses two capa ...starting.

Fig. 8.

Therefore the motor acts like a balanced two-phase motor. As the motor approaches its rated speed, the auxiliary winding and the starting capacitor C_s are disconnected automatically by the centrifugal switch S_c mounted on the shaft. The motor is so named because it uses the capacitor only for the purpose of starting.

8.13.1 Motor Characteristics

The capacitor-start motor develops a much higher starting torque (3.0 to 4.5 times the full-load torque) than does an equally rated resistance-start motor. The value of the starting capacitor must be large and the starting-winding resistance low to obtain a high starting torque. Because of the high VAr rating of the capacitor required, electrolytic capacitors of the order of 250 μF are used. The capacitor C_s is short-time rated. The torque-speed characteristic of the motor is shown in Fig. 8.10 (c), which also shows that the starting torque is high.

Capacitor start motors are more costly than split-phase motors because of the additional cost of the capacitor.

8.13.2 Reversal of Direction of Rotation

The capacitor-start motor may be reversed by reversing the connections of one of the windings. The motor is first brought to rest for this propose.

Applications

Capacitor-start motors are used for loads of higher inertia where frequent starts are required. These motors are most suitable for pumps and compressors, and therefore they are widely used in refrigerators and in air-conditioner compressors. They are also used for conveyors and some machine tools.

8.14 TWO-VALUE CAPACITOR MOTOR

Figure 8.11 shows the schematic diagram of a two-value capacitor motor. It has a cage rotor and its stator has two windings namely the main winding and the auxiliary winding (starting winding). The two windings are displaced 90° in space. The motor uses two capacitors C_S and C_R. The two capacitors are connected in parallel at starting.

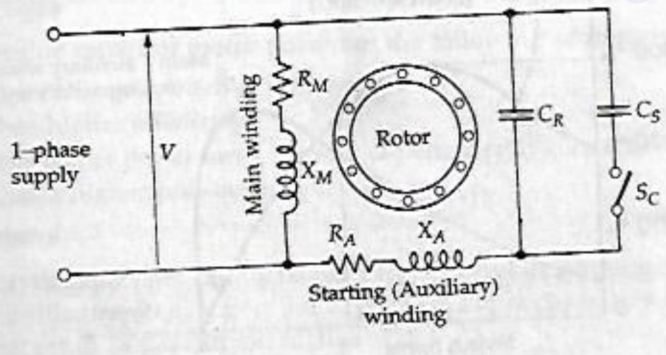


Fig. 8.11. *Two-value capacitor motor.*

The capacitor C_S is called the *starting capacitor*. In order to ... starting torque, a large current is required. For this purpose, the capaci... tance X in the starting winding should be low. Since $X_A = 1/(2\pi f C_A)$, the ... C_S should be *large*. The capacitor C_S is short-time rated and is almost ... electrolytic.

During normal operation, the rated line current is smaller than the ... current. Hence the capacitive reactance should be large. Since $X_R = 1/(2\pi ...$ value of C_R should be *small*. As the motor appropaches synchronous spe... capacitor C_S is disconnected by a centrifugal switch S_C. The capacitor C_R is perma... nently connected in the circuit. It is called the *run-capacitor*. It is long-time ... for continuous running. It is usually of oil-filled paper construction. S... capacitor C_S is used only at starting and the other C_R for continuous running ... motor is also called **capacitor-start capacitor-run motor.**

Figures 8.12 (*a*) and (*b*) show the phasors diagrams of a 2-value capa... motor. At starting both the capacitors are in the circuit and $\phi > 90°$ (Fig. 8.12...). When the capacitor C_s is disconnected ϕ becomes 90° (electrical) as show... Fig. 8.12 (*b*).

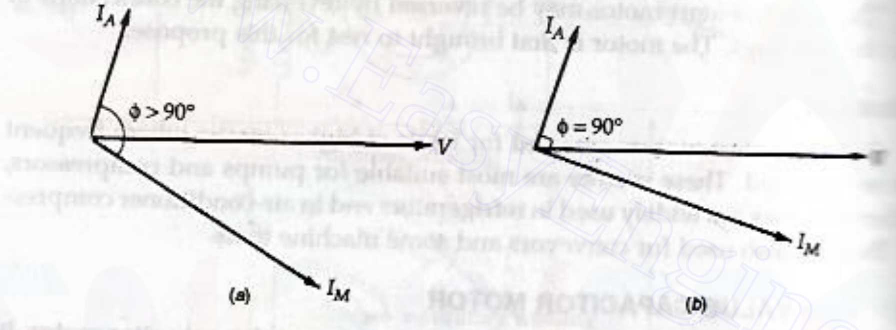


Fig. 8.12. *Phasor diagrams of a 2-value capacitor motor.*

The torque-speed characteristic of a 2-value capacitor motor is shown in Fig. 8.13.

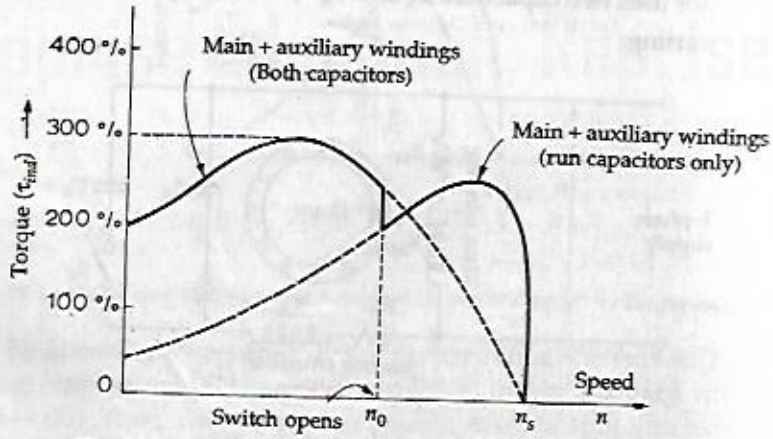


Fig. 8.13. *Torque-speed characteristic of a 2-value capacitor motor*

Two-value capacitor motors are quiet and smooth running. They have a higher efficiency than motors that run on the main windings alone.

Applications

Two-value capacitor motors are used for loads of higher inertia requiring frequent starts where the maximum pull out torque and efficiency required are higher. They are used in pumping equipment, refrigeration, air compressors etc.

8.15 PERMANENT-SPLIT CAPACITOR (PSC) MOTOR

A permanent-split capacitor (PSC) motor is shown in Fig. 8.14. It has a cage rotor and its stator has two windings, namely, the main winding and the auxiliary winding. This single-phase induction motor has only one capacitor C which is connected in series with the starting winding. The capacitor C is permanently connected in the circuit both at starting and running conditions. A permanent-split capacitor motor is also called the **single-value capacitor motor**. Since the capacitor C is always in the circuit, this type of motor has *no starting switch*. The auxiliary winding is always in the circuit, and therefore this motor operates in the same way as a balanced two-phase motor. Consequently, it produces a uniform torque. The motor is therefore less noisy during operations.

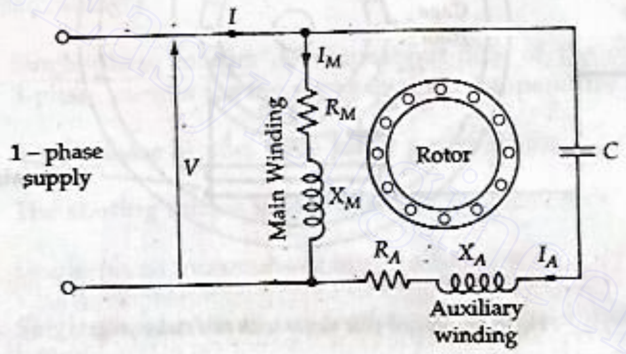


Fig. 8.14. *Permanent-split capacitor motor.*

8.15.1 Advantages

A single-value capacitor motor possesses the following *advantages* :

1. No centrifugal switch is required.
2. It has higher efficiency.
3. It has higher power-factor because of permanently-connected capacitor.
4. It has a higher pull-out torque.

8.15.2 Limitations

1. Electrolytic capacitors cannot be used for continuous running. Therefore paper-spaced oil-filled type capacitors are to be used. Paper capacitors of equivalent rating are larger in size and more costly.

2. A single-value capacitor has a low starting torque usually less than full-load torque.

8.15.3 Applications

Permanent-split capacitor motors are used for fans and blowers in heaters and air conditioners and to drive refrigerator compressors. They are also used to drive office machinery.

8.16 SHADED-POLE MOTORS

A shaded-pole motor is a simple type of self-starting single-phase induction motor. It consists of a stator and a cage-type rotor. The stator is made up of salient poles. Each pole is slotted on side and a copper ring is fitted on the smaller part as shown in Fig. 8.15. This part is called the **shaded pole.** The ring is usually a single-turn coil and is known as **shading coil.**

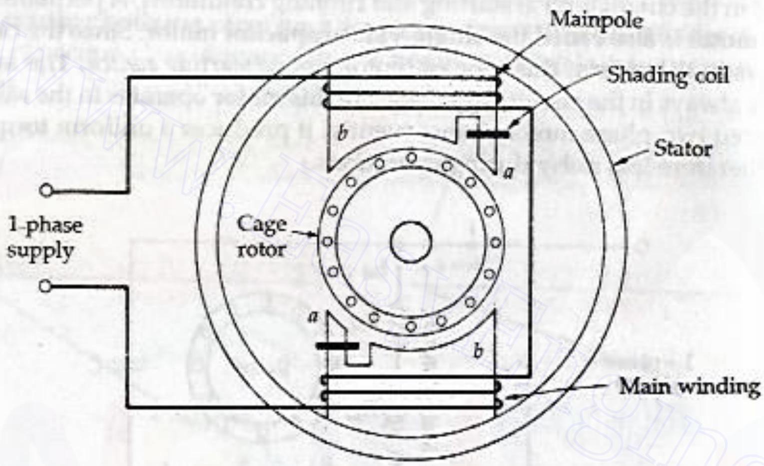


Fig. 8.15. *Shaded-pole motor with two stator poles.*

When alternating current flows in the field winding, an alternating flux is produced in the field core. A portion of this flux links with the shading coil, which behaves as a short-circuited secondary of a transformer. A voltage is induced in the shading coil, and this voltage circulates a current in it. The induced current produces a flux called the induced flux which opposes the main core flux. The shading coil, thus, causes the flux in the shaded portion *a* to lag behind the flux in the unshaded portion *b* of the pole. At the same time, the main flux and the shaded pole flux are displaced in space. This space displacement is less than 90°. Since there is time and space displacement between the two fluxes, the conditions for setting up a rotating magnetic field are produced. Under the action of the rotating flux a starting torque is developed on the cage rotor. The direction of the rotating field (flux) is from the unshaded to the shaded portion of the pole. In Fig. 8.15 the direction of rotation is clockwise. *In a shaded-pole motor the reversal of direction of rotation is not possible.*

Applications

Shaded-pole motors are very cheap. The starting torque developed by a shaded-pole motor is very low. The losses are high and the power factor is low. Consequently, the efficiency is also very low. For this reason, the shaded-pole motors are built only in small sizes of the power rating of the order of 40 W or less. They are used to drive devices which require low starting torque. They are most suitable for small devices like relays, fans of all kinds etc. because of their low initial cost and easy starting. The most common applications are table fans, exhaust fans, hair driers, fans for refrigeration and air-conditioning equipments, electronic equipment, cooling fans etc. They are also used in record players, tape recorders, slide projectors, photo copying machines, in starting electric clocks and other single-phase synchronous timing motors.

8.17 COMPARISON BETWEEN SINGLE-PHASE AND THREE-PHASE INDUCTION MOTORS

Most single-phase induction motors are constructed in fractional kilowatt capacity and are used in places where three phase supply is not readily available. Single-phase motors when compared with 3-phase induction motors have the following disadvantages :

(*i*) Single-phase motors develop about 50% of the output of that of 3-phase motors for the same size and temperature rise.

(*ii*) Single-phase motors have lower power factor.

(*iii*) The starting torque is low in single-phase motors.

(*iv*) Single-phase motors have lower efficiency.

(*v*) Single-phase motors are costlier than 3-phase motors of the same rating.

However, single-phase induction motors are simple robust, reliable and less expensive for small ratings. They are used in low-power drives in small industries and domestic and commercial applications. They are generally available upto 1 kW rating.

8.18 SINGLE-PHASE SERIES (UNIVERSAL) MOTOR

The single-phase series motor is a commutator-type motor. If the polarity of the line terminals of a dc series motor is reversed, the motor will continue to run in the same direction. Thus, it might be expected that a dc series motor would operate on alternating current also. The direction of the torque developed in a dc series motor is determined by both field polarity and the direction of current through the armature ($T \propto \Phi i_a$). Let a dc series motor be connected across a single-phase ac supply. Since the same current flows through the field winding and the armature, it follows that ac reversals from positive to negative, or from negative to positive, will simultaneously affect both the field flux polarity and the current direction through the armature. This means that the direction of the developed torque will remain positive, and rotation will continue in the same direction. The nature of the torque will be pulsating and frequency will be twice the line frequency as shown in Fig. 8.16. Thus, a series motor can run both on dc and ac. Motors that can be used with a single-phase ac source as well as a dc source of supply voltages are called **universal motors.** However, a series motor which is specifically designed for dc operation suffers from the following **drawbacks** when it is used on single-phase ac supply :

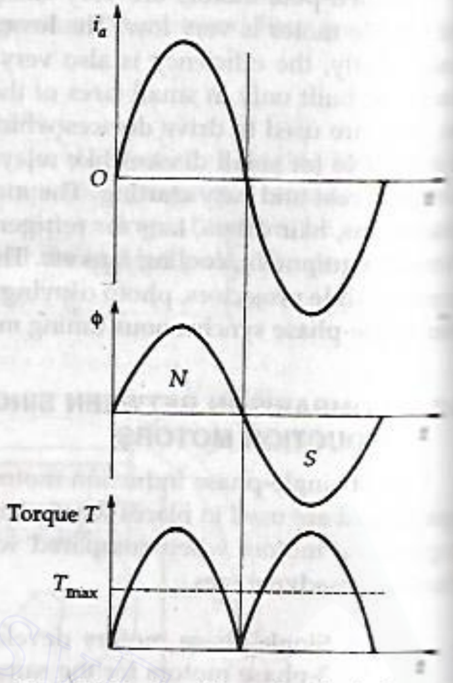


Fig. 8.16. *Developed torque in a single-phase series motor.*

1. Its efficiency is low due to hysteresis and eddy-current losses.

2. The power factor is low due to the large reactance of the field and the armature windings.

3. The sparking at the brushes is excessive.

In order to overcome these difficulties, the following modifications are made in a d.c. series motor that is to operate satisfactorily on alternating current

(*a*) The field core is constructed of a material having low hysteresis loss. It is laminated to reduce eddy-current loss.

(*b*) The field winding is provided with small number of turns. The field pole areas is increased so that the flux density is reduced. This reduces the iron loss and the reactive voltage drop.

(*c*) The number of armature conductors is increased in order to get the required torque with the low flux.

(*d*) In order to reduce the effect of armature reaction, thereby improving commutation and reducing armature reactance, a **compensating winding** is used. This winding is put in the stator slots as shown in Fig. 8.17. The axis of the compensating winding is 90° (electrical) with the main field axis. It may be connected in series with both the armature and field as shown in Fig. 8.18. In such a case the motor is **conductively compensated.**

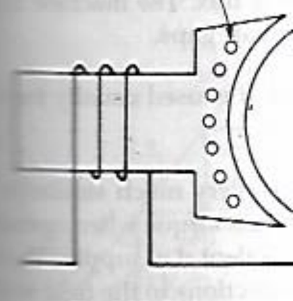
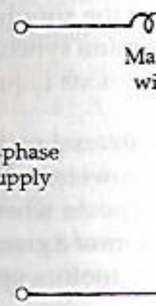


Fig. 8.17. *Series motor w*



The compensating winding ... said to be inductively

Fig. 8.19. *Series motor*

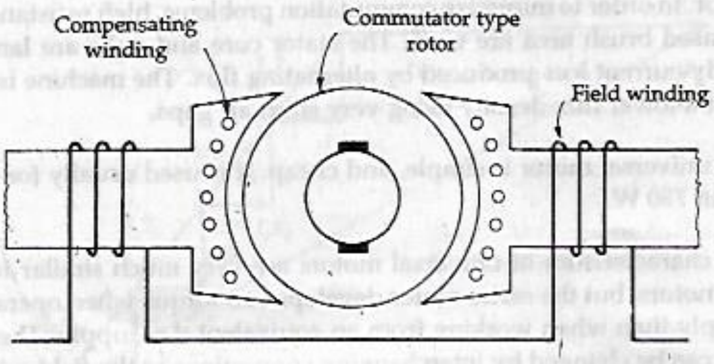


Fig. 8.17. Series motor with conductively compensated winding.

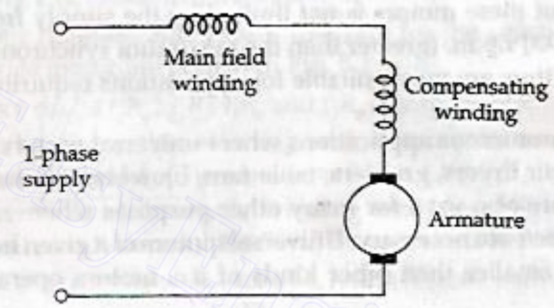


Fig. 8.18.

The compensating winding may be short circuited on itself, in which case the motor is said to be **inductively compensated** (Fig. 8.19).

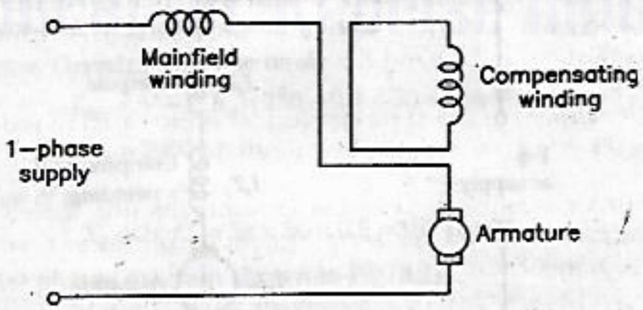


Fig. 8.19. Series motor with inductively compensated winding.

The armature of universal motors is of the same construction as ... series motor. In order to minimize commutation problems, high resistance ... with increased brush area are used. The stator core and yoke are ... reduce eddy-current loss produced by alternating flux. The machine is ... operated at a lower flux density using very short air gaps.

The universal motor is simple, and cheap. It is used usually for ... greater than 750 W.

The characteristics of universal motors are very much similar to ... d.c. series motors, but the series motor develops less torque when operating ... an a.c. supply than when working from an equivalent d.c. supply. The ... of rotation can be changed by interchanging connections to the field ... to the armature as in d.c. series motor.

Speed control of universal motors is best obtained by solid-state ... Since the speed of these motors is not limited by the supply frequency ... be as high as 20,000 r.p.m. (greater than the maximum synchronous speed ... r.p.m. at 50 Hz), they are most suitable for applications requiring high ...

There are numerous applications where universal motors are used, ... portable drills, hair dryers, grinders, table-fans, blowers, polishers, kitchen ... ances etc. They are also used for many other purposes where speed ... high values of speed are necessary. Universal motors of a given horse power ... are significantly smaller than other kinds of a.c. motors operating at the ... frequency.

8.19 PHASOR DIAGRAM OF A.C. SERIES MOTOR

The schematic diagram and phasor diagram for the conductively ... single-phase ac series motor are shown in Fig. 8.20.

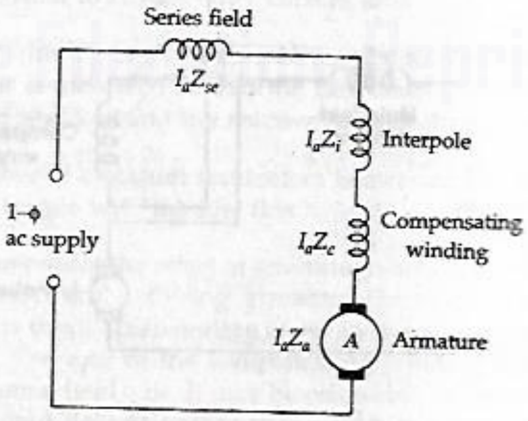


Fig. 8.20. *Conductively coupled ac series motor. (a) Schematic diagram*

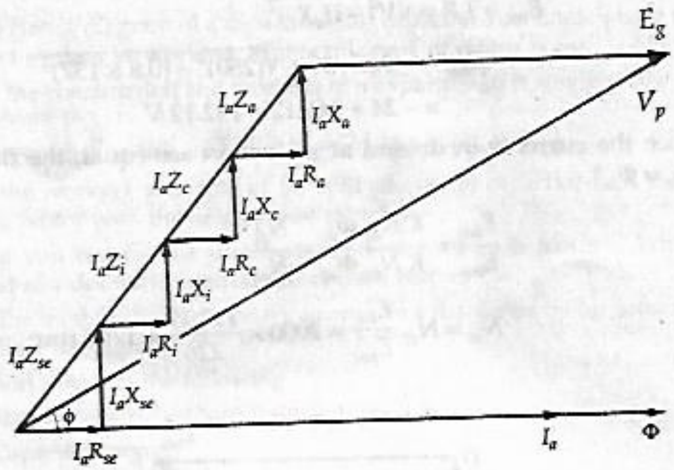


Fig. 8.20. *Conductively coupled ac series motor. (b) Phasor diagram.*

The schematic diagram and phasor diagram for the conductively coupled single-phase ac series motor are shown in Fig. 8.20.

The resistance drops I_aR_{se}, I_aR_i, I_aR_c and I_aR_a due to resistances of series field, interpole winding, compensating winding and of armature respectively are in phase with armature current I_a. The reactance drops I_aX_{se}, I_aX_i, I_aX_c and I_aX_a due to reactances of series field, interpole winding compensating winding and of armature respectively lead current I_a by 90°. The generated armature counter emf is $\mathbf{E}_g$. The terminal phase voltage $\mathbf{V}_p$ is equal to the phasor sum of $\mathbf{E}_g$ and all the impedance drops in series.

$$\mathbf{V}_p = \mathbf{E}_g + I_a\,\mathbf{Z}_{se} + I_aZ_i + I_aZ_c + I_aZ_a$$

The power factor angle between $\mathbf{V}_p$ and $\mathbf{I}_a$ is ϕ.

Example 8.3. *A universal series motor has a resistance of 30 Ω and an inductance of 0.5 H. When connected to a 250 V dc supply and loaded to take 0.8 A it runs at 2000 rpm. Determine the speed, torque and power factor, when connected to a 250 V, 50 Hz ac supply and loaded to take the same current.*

Solution. *Operation of motor on dc*

$$E_{bdc} = V - I_a\,R_a = 250 - 0.8 \times 30 = 226 \text{ V}$$

$$N_{dc} = 2000 \text{ r.p.m.}$$

Operation of motor on ac

$$X_L = 2\pi f L = 2\pi \times 50 \times 0.5 = 157\ \Omega$$

From the phasor diagram shown in Fig. 8.21,

$$AF^2 = AG^2 + GF^2$$

$$V^2 = (AB + BG)^2 + GF^2 = (AB + DF)^2 + GF^2$$

$$= (I_aR_a + E_{bac})^2 + (I_a\,X_L)^2$$

$$E_{bac} + I_a R = \sqrt{V^2 - (I_a X_L)^2}$$

$$E_{bac} = -0.8 \times 30 + \sqrt{(250)^2 - (0.8 \times 157)^2}$$

$$= -24 + 216.12 = 192.12 \text{ V}$$

Since the currents in dc and ac operation are equal, the flux will ... equal $(\Phi_{dc} = \Phi_{ac})$

$$\frac{E_{bdc}}{E_{bac}} = \frac{K N_{dc} \Phi_{dc}}{K N_{ac} \Phi_{ac}} = \frac{N_{dc}}{N_{ac}}$$

$$N_{ac} = N_{dc} \frac{E_{bac}}{E_{bdc}} = 2000 \times \frac{192.12}{226} = 1700 \text{ rpm}$$

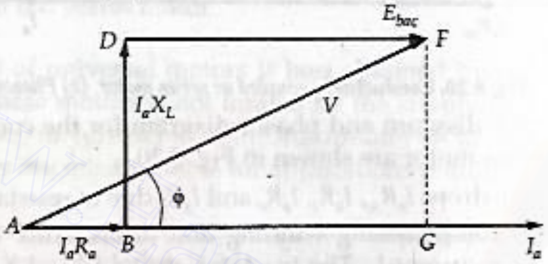


Fig. 8.21.

From Fig. 8.21

Power factor, $\cos\phi = \dfrac{AC}{AF} = \dfrac{E_{bac} + I_a R_a}{V}$

$$= \frac{192.12 + 0.8 \times 30}{250} = 0.8645 \text{ (lagging)}$$

Mechanical power developed

$$P_{mech} = E_{bac} I_a = 192.12 \times 0.8 = 153.7 \text{ W}$$

Torque developed

$$\tau = \frac{P_{mech}}{\omega_m} = \frac{P_{mech}}{2\pi\, n_{ac}}$$

$$= \frac{153.7}{2\pi \times (1700/60)} = 0.8633 \text{ Nm}$$

EXERCISES

8.1 (*a*) Discuss why single-phase induction motors do not have a starting ...
(*b*) Describe with the aid of diagrams of connections and phasor diagrams ... methods of producing starting torque in a single-phase induction motor ...

8.2 Draw the circuit diagram of a capacitor-start capacitor-run single-phase induction motor and explain its working. Where this type of motor is commonly used ?

8.3 Describe the construction and working of a capacitor-start single-phase induction motor.

8.4 Describe the construction and working of a shaded-pole motor.

8.5 Explain the working principle of (*a*) split phase, (*b*) capacitor-start single-phase induction motor with the help of neat sketches.

How can you reverse the direction of rotation of such motor ? What are the industrial and domestic applications of such motors ?

8.6 Discuss the modifications necessary to operate a d.c. series motor satisfactorily on single-phase a.c. supply.

8.7 Write short notes on the following :

(*a*) Starting of single-phase induction motors

(*b*) Capacitor motors

(*c*) Shaded-pole motor

(*d*) Universal motor

8.8 What type of motor would you use in the following applications : washing machine, sewing machine, dishwasher, portable electric drill, food mixer ? State your reasons.

8.9 Explain simply why a universal motor can operate from d.c. as well as a.c. supplies.

What are the chief differences in construction between a.c./d.c. series motors and d.c. series motors ?

8.10 Give details of four methods of starting small single-phase induction motors, and mention typical applications for which these types would be suitable.

8.11 Using double-revolving field theory, explain why a single-phase induction motor is not self starting.

8.12 Explain the double-revolving field theory for single-phase induction motors.

8.13 Draw a torque-speed curve of a single-phase induction motor on the basis of double-revolving-field theory.

8.14 Draw and explain the equivalent circuit of a single-phase induction motor. How can the performance of the motor be analysed ?

8.15 Discuss the procedure for determining the parameters of equivalent circuit of a single-phase induction motor.

8.16 What are the disadvantages of a signle-phase induction motor when compared with a 3-phase induction motor ?

8.17 Draw and explain the phasor diagram of an ac series motor.

8.18 A 230 V, 50 Hz, 4-pole single-phase induction motor has the following equivalent circuit parameters :

$$R_{1m} = R'_2 = 8\ \Omega$$

$$X_{1m} = X'_2 = 12\ \Omega, \quad X_M = 200\ \Omega$$

At a slip of 4%, calculate (*a*) input current (*b*) input power, (*c*) developed power, and developed torque at rated voltage. The motor speed is 140 r.pm.

[(*a*) 2.6 A, (*b*) 384.8 W, (*c*) 293.3 W, (*d*) 1.945 Nm]

8.19 A 200 W, 240 V, 50 Hz, single-phase induction motor runs on ... slip of 0.05 pu. The parameters are :

$$R_{1m} = 11.4\ \Omega, \quad X_{1m} = 14.5$$

$$R'_2 = 13.8\ \Omega, \quad X'_2 = 14.4\ \Omega \quad X_M = 270\ \Omega$$

Core and mechanical loss = 32 W

Calculate (*a*) power factor, (*b*) synchronous power, (*c*) shaft power ... power, and (*e*) efficiency.

[(*a*) 0.6548, (*b*) 210 W, (*c*) 200 W, (*d*) 315 W, ...

8.20 A 230 V, 50 Hz, 4-pole, class A single-phase induction motor has the ... parameters at an operating temperature of 63°C.

$r_{1m} = 2.51\ \Omega$, $r'_2 = 7.81\ \Omega$, $x_m = 150.88\ \Omega$, $x_{1m} = = 4.62\ \Omega$, $x'_2 = 4.62\ \Omega$

Determine stator main winding current and power factor when the motor ... ning at a slip of 0.05 at the specified temperature of 63°C.

[3.737 $\angle -48.24°$ A, ...

8.21 A universal series motor, when operating on 220 V d.c. draws ... 1400 r.p.m. Find the new speed and power factor, when connected to ... supply, the motor current remains the same. The motor has total ... 1 Ω and total inductance of 0.1 H.

Special

...PHASE SYNCHRON...

...phase synchronous ... hundred kilowatts or ... speed machines of sm... used, *reluctance mot*...

... are simple in cons... they use permanent mag...

...CTANCE MOTORS

...phase synchronous relu... type induction motor ... winding and an aux... In general, the stato... reluctance motor is simi... of the single-phase indu... The rotor of a reluctance mo... squirrel cage with some ... at the appropriate ... provide the desired numb... poles. Figure 9.1 shows ... type synchronous motor ... removed in four locati... salient-pole structur... kept intact even in the ... are removed. The tw... these bars as in ...

... the stator is connected ... induction motor. At a ... centrifugal switch disconn... up as a single-phas... is close to the synch... of the rotor to align ... synchronously rotatin...

9

Special Machines

9.1 SINGLE-PHASE SYNCHRONOUS MOTORS

The three-phase synchronous motors are usually large machines of the order of several hundred kilowatts or magawatts. Single-phase synchronous motors are constant-speed machines of small ratings. Two types of small synchronous motors are widely used, *reluctance motors* and *hysteresis motors*.

These motors are simple in construction. They do not require dc field excitation nor do they use permanent magnets.

9.2 RELUCTANCE MOTORS

A single-phase synchronous reluctance motor is basically the same as the single phase cage type induction motor. The stator has the main winding and an auxiliary (starting) winding. In general, the stator of a single-phase reluctance motor is similar to that of any one of the single-phase induction motors. The rotor of a reluctance motor is basically a squirrel cage with some rotor teeth removed at the appropriate places such as to provide the desired number of salient rotor poles. Figure 9.1 shows the 4-pole reluctance type synchronous motor. Here teeth have been removed in four locations to produce a 4-pole salient-pole structure. The rotor bars are kept intact even in the spaces from where teeth are removed. The two end rings short-circuit these bars as in a cage rotor.

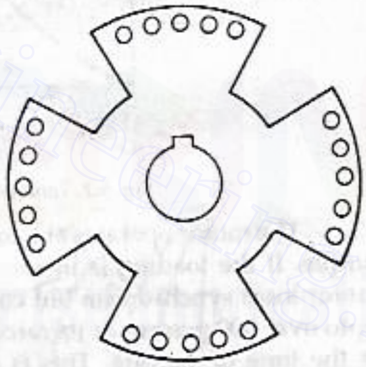


Fig. 9.1. *Reluctance motor rotor.*

When the stator is connected to a single-phase supply, the motor starts as a single-phase induction motor. At a speed, of about 75 per cent of the synchronous speed, a centrifugal switch disconnects the auxiliary winding and the motor continues to speed up as a single-phase motor with the main winding in operation. When the speed is close to the synchronous speed, a reluctance torque is produced due to tendency of the rotor to align itself in the minimum reluctance position with respect to the synchronously rotating flux of the forward field. The rotor pulls into

synchronism. For this to happen effectively, the load inertia must be with
After pulling into synchronism, the induction torque disappears but the
remains in synchronism due to the synchronous reluctance torque alone.

Figure 9.2 shows the typical torque-speed characteristic of the single-
reluctance motor. The starting torque is dependent upon the rotor position
of the salient pole rotor. The value of the starting torque is between 300 to
cent of its full-load torque. At about 75% of the synchronous speed, a cen
switch disconnects the auxiliary winding and the motor continues to run with
main winding only. When the speed is close to synchronous speed, the reluc
torque developed as a synchronous motor pulls the rotor into synchronism
the rotor continues to rotate at synchronous speed.

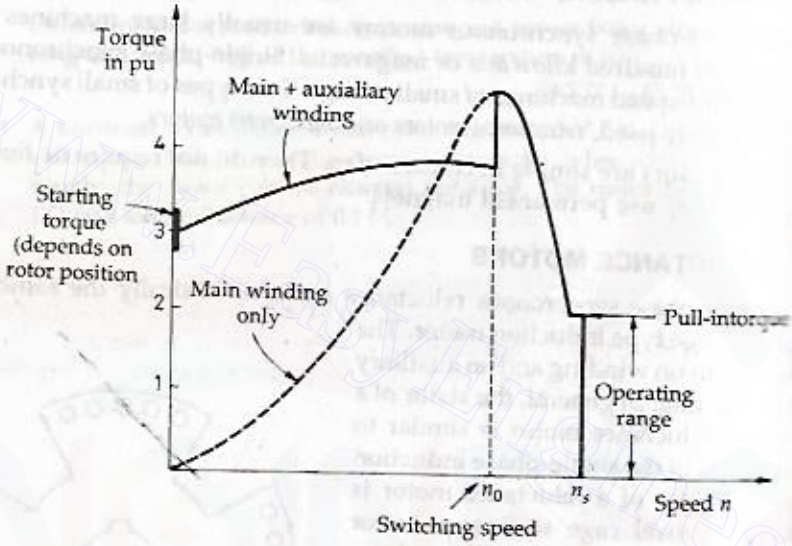


Fig. 9.2. *Torque-speed characteristic of reluctance motor.*

The motor operates at a constant speed upto a little over 200% of its fu
torque. If the loading is increased beyond the value of the pull-out torque
motor loses synchronism but continues to run as a single-phase induction
upto over 500 percent of its rated torque. Reluctance motors are subject to
at the time of starting. This is due to the saliency of the rotor. The cog
minimised by skewing the rotor bars and by having the rotor slots no
multiples of the number of poles.

In reluctance motors, since the rotor is unexcited and has saliency, the
factor is lower than that of the equivalent induction motor. The maximum o
a reluctance motor is greatly reduced due to absence of dc field excitation. The
fore the size of a reluctance motor is larger than that of an equivalent syn
motor. The main advantages of a reluctance motor are its simple constructi
slip rings, no brushes, no dc field winding), low cost and easy maintenan

In spite of its shortcomings, the reluctance motor is widely used fo
constant-speed applications such as electric clocks timers, signalling de
cording instruments and phonographs etc.

9.3 HYSTERESIS MOTORS

A hysteresis motor is basically a synchronous motor with uniform airgap and without d.c. excitation. This motor may operate from single-phase or 3-phase supply. In a hysteresis motor torque is produced due to hysteresis and eddy current induced in the rotor by the action of the rotating flux of the stator windings.

Stator Construction

The stator of a hysteresis motor is similar to that of an induction motor with the basic requirement that it produces a rotating magnetic field. Thus the stator of the motor can be connected to either single-phase supply or 3-phase supply. We know that 3-phase motors produce a more uniform rotating field than single-phase motors. For a single-phase hysteresis motor, the stator winding is of permanent split-capacitor type or of the shaded pole type for very small sizes. In case of the permanent split-capacitor type, the capacitor should be used with an auxiliary winding in order to produce as uniform field as possible.

Rotor Construction

Figure 9.3 shows the rotor of a hysteresis motor. It consists of core of aluminium or some other nonmagnetic material which carries a layer of special magnetic material. The outer layer has a number of thin rings to form the laminated rotor. In smaller motors a solid ring may be used. Thus, the rotor of a hysteresis motor is a smooth cylinder and it does not carry any windings (no rotor bars). The ring is made of special magnetic material such as magnetically hard chrome or cobalt steel having very large hysteresis loop as shown in Fig. 9.4.

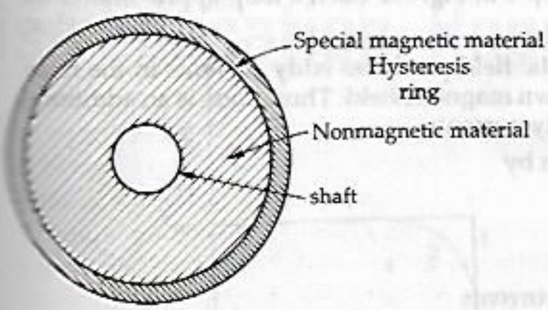


Fig. 9.3. *Rotor of a hysteresis motor.*

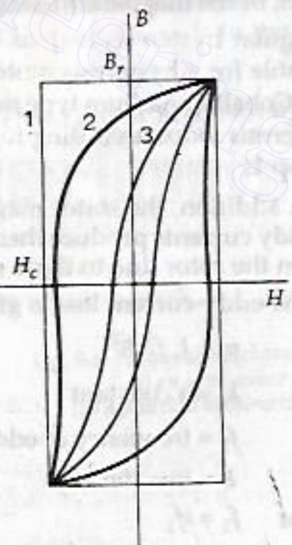


Fig. 9.4. *Various hysteresis loops for different materials.*

Operation

Figure 9.5 shows the basic operation of a hysteresis motor.

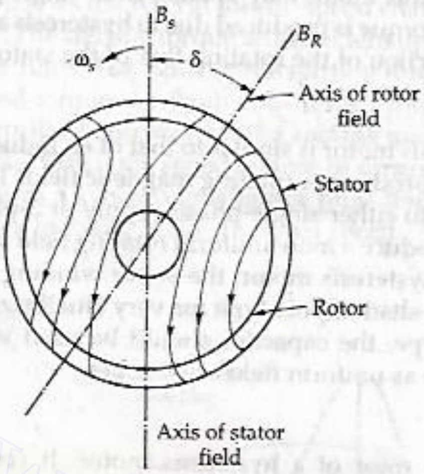


Fig. 9.5. *Magnetic field in hysteresis motor.*

When a 3-phase supply or a single-phase supply is applied to the stator, a rotating magnetic field is produced. This rotating magnetic field magnetises the rotor ring and induces poles within it. A uniform cross-section rotor inherently will match the number of stator poles. The induced rotor flux lags behind the rotating stator flux because of the hysteresis loss in the rotor. The angle δ between the stator magnetic field $\mathbf{B_S}$ and the rotor magnetic field $\mathbf{B_R}$ is responsible for the production of torque. The angle δ depends only on the shape of the hysteresis loop. It does not depend upon the frequency. For this reason, a magnetic material having a wide hysteresis loop should be used. Thus, the coercive force H_c and the residual flux density B_r of the magnetic material should be large. An ideal material would have a rectangular hysteresis loop as shown by loop 1 in Fig. 9.4. Ordinary steels are not suitable for a hysteresis motor since their hysteresis loops resemble loop 3 in Fig. 9.4. Cobalt-vanadium type materials are used in hysteresis motors. They have the hysteresis loops according to loop 2 in Fig. 9.4. Such a loop approximates the ideal loop 1.

In addition, the stator magnetic field produces eddy currents in the rotor. These eddy currents produce their own magnetic field. Thus, there is an additional torque on the rotor due to these eddy currents.

The eddy-current loss is given by

$$p_e = k_e f_2^2 B^2$$

where k_e = a constant

f_2 = frequency of eddy currents

B = flux density

But $f_2 = sf_1$

where s is the slip and f_1 is the stator frequency.

$$\therefore \qquad p_e = k_e s^2 f_1^2 B^2$$

Also, the torque is given by

$$\tau_e = \frac{p_e}{s\,\omega_s}$$

or $$\tau_e = K's \qquad (9.3.2)$$

where $$K' = \frac{k_e f_1^2 B^2}{\omega_s} = \text{a constant}$$

Torque due to hysteresis loss

The hysteresis loss is given by

$$p_h = k_h f_2 B^{1.6}$$
$$= k_h s f_1 B^{1.6} \qquad (9.3.3)$$

The torque due to hysteresis is given by

$$\tau_h = \frac{p_h}{s\,\omega_s}$$

or $$\tau_h = (k_h f_1 B^{1.6})/\omega_s = k'' = \text{a constant} \qquad (9.3.4)$$

It is seen from Eq. (9.3.2) that τ_e is proportional to the slip. Therefore τ_e decreases as the rotor speed increases. When the motor reaches synchronous speed, the slip becomes zero and the torque τ_e becomes zero. The stator current falls off and the rotor acts like a permanent magnet, and the machine runs as a permanent magnet type motor. It is to be noted that the torque τ_e aids in the starting of the motor.

The electromagnetic torque developed by a hysteresis motor due to hysteresis is given by Eq. (9.3.4). This component of torque remains constant at all rotor speeds until the breakdown torque. Since τ_e is zero at synchronous speed, the only torque at synchronous speed is the torque τ_h. Thus, at synchronous speed the induced torque in the motor is proportional to the angle δ between the stator and rotor magnetic fields upto a maximum angle set by the hysteresis in the motor.

Torque-speed Characteristic

An ideal torque-speed curve for the hysteresis motor is shown by curve 1 in Fig. 9.6.

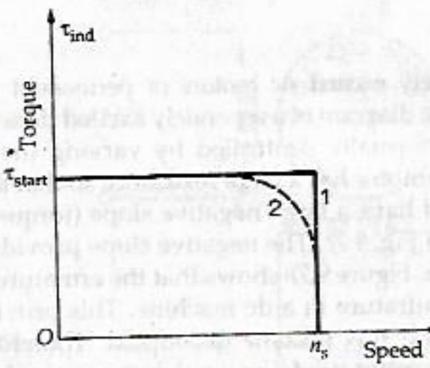


Fig. 9.6. Torque-speed characteristic of : (i) an ideal motor ; (ii) a practical hysteresis motor.

The torque-speed characteristic of a practical hysteresis motor ... curve 2 in Fig. 9.6. The departure from the ideal characteristic 1 is due ... of harmonics in the rotating field and other irregularities. The torque-speed char- acteristic of a hysteresis motor is quite different from that of an induction motor. The torque developed by an induction motor becomes zero at synchronous speed while in an ideal hysteresis motor it is constant at all speeds including synchro- nous speed. Thus, it is seen from the characteristic that locked rotor, ... pullout torques are all equal. This is a valueable property in that such a motor ... pull into synchronism at high inertia loads.

Applications

The hysteresis motor has a very low noise level compared to single-phase induction motor. This is because it operates at one speed (synchronous speed) ... its rotor is smooth (unslotted). With a permanent capacitor stator, the hysteresis motor is the smoothest running, quietest single-phase motor and therefore ... preferred for quality sound reproduction equipment like record players, ... recorders, etc. The most common application of hysteresis motor is for ... clocks and other timing devices. With provisions for pole changing in the ... the motor is multispeed. This motor is made in very small sizes only.

9.4 SERVOMOTORS

Servomotors are also called **control motors**. These motors are used in feed- back control systems as output actuators. Unlike large industrial motors, they are not used for continuous energy conversion. The basic principle of operation of these motors is the same as that of other electromagnetic motors. However, their design, construction and mode of operation are different. Their power ratings vary from a fraction of a watt to a few hundred watts. They have low rotor inertia and, therefore, they have a high speed of response. The rotors of servomotors are designed with relatively long rotor length and smaller diameters. They generally operate at very low speeds and sometimes zero speed. They have larger size than that of conventional motors of similar power rating. Servomotors are widely used in radars, computers, robots, machine tools, tracking and guidance systems, pro- cess controllers etc. Both dc and ac (2-phase and three-phase) servomotors are being used presently.

9.5 DC SERVOMOTORS

DC servomotors are separately excited dc motors or permanent magnet dc motors. Figure 9.7*a* shows a schematic diagram of a separately excited dc servomotor. The speed of dc servomotors is normally controlled by varying the armature voltage. The armature of a dc servomotor has a large resistance so that the torque- speed characteristics are linear and have a large negative slope (torque reducing with increasing speed) as shown in Fig. 9.7*c*. The negative slope provides viscous damping for the servo-drive system. Figure 9.7*b* shows that the armature mmf and the excitation field mmf are in quadrature in a dc machine. This provides a fast torque response because torque and flux become decoupled. Therefore, a small change in the armature voltage or current produces a quick change in the position or speed of the rotor.

The power rating of dc servomotors can vary from a few watts to several hundred watts. In general, most high-power servomotors are dc servomotors.

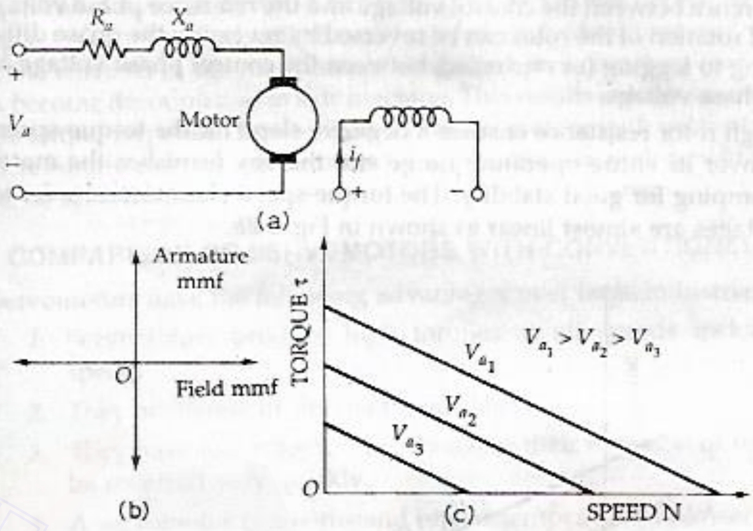


Fig. 9.7. *DC servomotor : (a) Schematic diagram. (b) Armature mmf and field mmf. (c) Torque-speed characteristics.*

9.6 AC SERVOMOTORS

At present, most of the ac servomotors are of the two-phase squirrel cage induction type for low-power applications. Recently, three-phase squirrel-cage induction motors have been modified for application in high-power servo systems.

9.7 TWO-PHASE AC SERVOMOTOR

Figure 9.8*a* shows the schematic diagram of a two-phase ac servomotor. The stator has two distributed windings which are displaced from each other by 90 electrical degrees. One winding, called the *reference* or *fixed phase*, is supplied from a constant voltage source $V_m \angle 0°$. The other winding, called the *control phase*, is

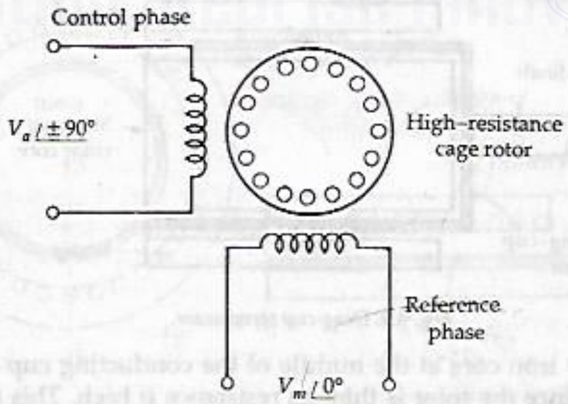


Fig. 9.8. *Two-phase ac servomotor : (a) Schematic diagram.*

supplied with a variable voltage of the same frequency as the reference phase but is phase displaced by 90 electrical degrees. The control phase is usually supplied from a servo amplifier. The speed and torque of the rotor are controlled by the phase difference between the control voltage and the reference phase voltage. The direction of rotation of the rotor can be reversed by reversing the phase difference from leading to lagging (or *vice versa*), between the control phase voltage and the reference phase voltage.

A high rotor resistance ensures a negative slope for the torque-speed characteristics over its entire operating range and thereby furnishes the motor with positive damping for good stability. The torque-speed characteristics for various control voltages are almost linear as shown in Fig. 9.8*b*.

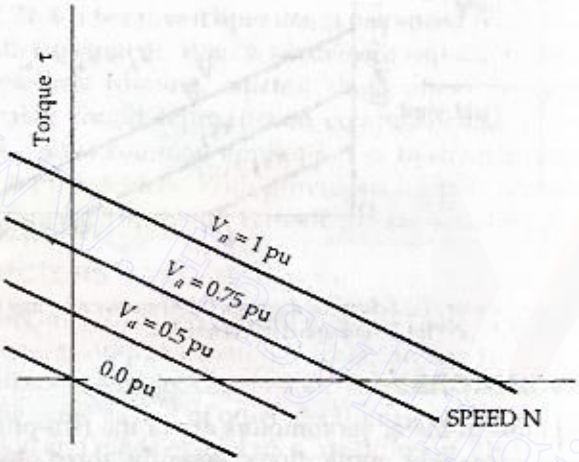


Fig. 9.8. *(b) Torque-speed characteristics.*

The response of a two-phase servomotor to very small control signals is improved by reducing the weight and inertia of the motor in a design known as the **drag-cup servomotor**. A thin cup of nonmagnetic conducting material is used as the rotor, as shown in Fig. 9.9.

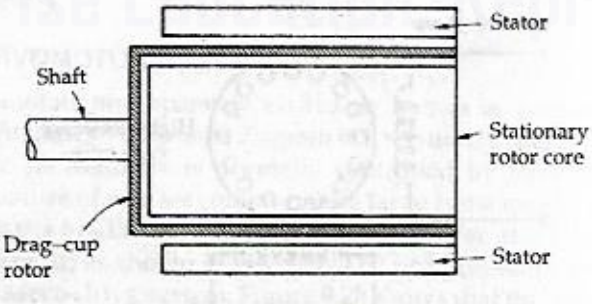


Fig. 9.9. *Drag-cup servomotor.*

A stationary iron core at the middle of the conducting cup completes the magnetic circuit. Since the rotor is thin, its resistance is high. This results in high starting torque.

THREE-PHASE AC SER

... squirrel-cage in ... device. Recently, ... control method call ... the currents in the mach ... decoupled as in ... response. Three ... used as servo ... servomotors.

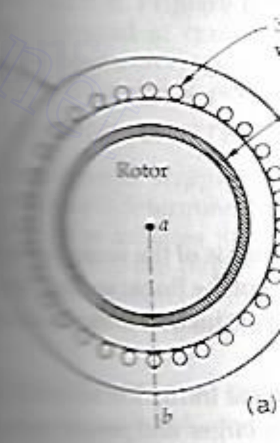
COMPARISON OF SER

... have the foll

1. Servomotors prod speed.
2. They accelerate or
3. They have low rot be reversed very q
4. A servomotor can zero speed. As it c

LINEAR INDUCTION M

... induction motor ... of rotational m ... shows a p ... and spread ... induction motor. ... and secondary ... of a flat alumini



9.8 THREE-PHASE AC SERVOMOTORS

A 3-phase squirrel-cage induction motor is normally a highly nonlinear coupled circuit device. Recently, it has been used as a linear decoupled machine by using a control method called *vector control* or *field-oriented control*. In this method the currents in the machine are controlled in such a way that its torque and flux become decoupled as in a dc machine. This results in high-speed response and high-torque response. Three-phase induction motors with vector control are being increasingly used as servomotors for applications in high-power servo systems like dc servomotors.

9.9 COMPARISON OF SERVOMOTORS WITH CONVENTIONAL MOTORS

Servomotors have the following advantages over large industrial motors :

1. Servomotors produce high torques at all speeds including zero speed.
2. They accelerate or decelerate quickly.
3. They have low rotor inertia, therefore their direction of rotation can be reversed very quickly.
4. A servomotor can withstand higher temperature at lower speeds or zero speed. As it can dissipate heat quickly.

9.10 LINEAR INDUCTION MOTOR (LIM)

A linear induction motor (LIM) is a motor which gives linear or translational motion instead of rotational motion as in the case of a conventional induction motor. Figure 9.10*a* shows a polyphase rotary induction motor. Let the stator by cut along the line *ab* and spread out flat as shown in Fig. 9.10*b*. This forms primary of the linear induction motor. In a linear induction motor,s stator and rotor are called primary and secondary respectively. Secondary of the linear induction motor consists of a flat aluminium conductor with a ferromagnetic core.

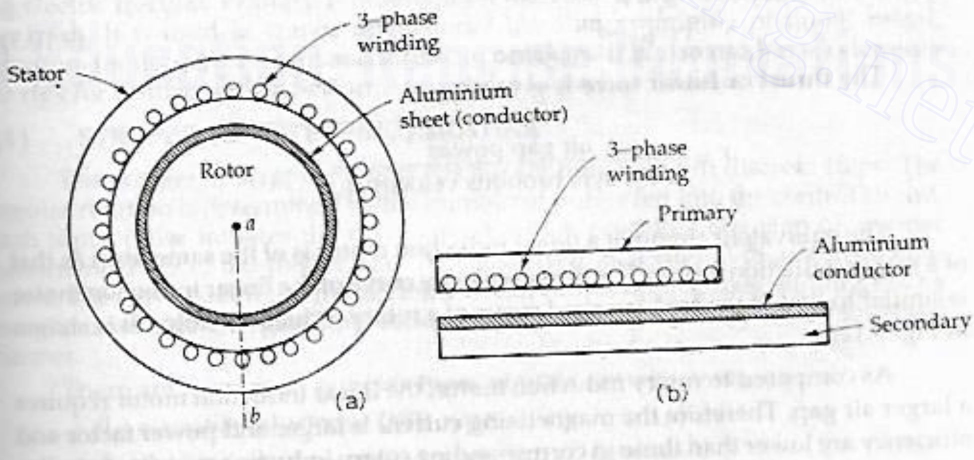


Fig. 9.10. *(a) Rotary induction motor. (b) Linear induction motor.*

If a 3-phase supply is connected to the stator of a conventional rotary induction motor, a rotating flux is produced. This flux rotates at a synchronous speed in the air gap. Similarly, if primary of the linear induction motor is connected to a 3-phase supply, a travelling flux wave is produced that travels along the length of the primary. Current is induced in the aluminium conductor due to the relative motion between the travelling flux wave and aluminium conductor. The induced current interacts with travelling flux wave to produce a linear force (or thrust) F. If secondary is fixed and primary is free to move, the force will move the primary in the direction of the travelling wave. The LIM shown in Fig. 9.10 is called **single-sided linear induction motor (SLIM).** The LIM shown in Fig. 9.11 is known as **double-sided linear induction motor (DLIM).** It has primary on both the sides of the secondary.

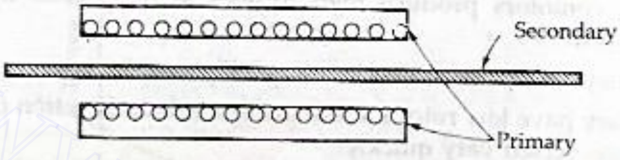


Fig. 9.11. *Double-sided linear induction motor (DLIM).*

Performance of LIM

The **linear synchronous speed** v_s of the travelling wave is given by

$$v_s = 2f\,(\text{pole pitch})\ m/s$$

where f is the supply frequency in Hz.

As in a rotary induction motor the speed of the secondary in LIM is less than the synchronous speed v_s. It is given by

$$v_r = v_s\,(1 - s)$$

where s is the **slip** of the linear induction motor given by

$$s = \frac{v_s - v_r}{v_s}\ pu$$

The **thrust or linear force** is given by

$$F = \frac{\text{air gap power}}{\text{linear synchronous velocity, } v_s}$$

The equivalent circuit of a linear induction motor is of the same form as that of a rotary induction motor. The thrust-velocity curve of the linear induction motor is similar to that of the torque-speed curve of a rotary induction motor. It is shown in Fig. 9.12.

As compared to rotary induction motor, the linear induction motor requires a larger air gap. Therefore the magnetizing current is larger and power factor and efficiency are lower than those in corresponding rotary induction motors of similar rating.

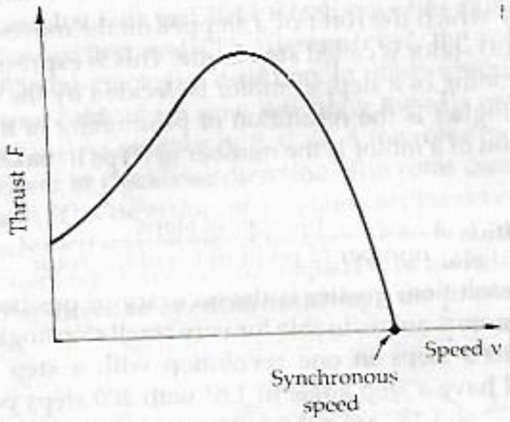


Fig. 9.12. *Thrust-speed curve of a linear induction motor.*

In order to facilitate operational explanations, a linear induction motor has been assumed as a developed version of rotary induction motor where the stator is cut axially and spread out flat. There is, however, a very important difference between the linear development of a rotary induction motor and an actual linear induction motor. In a rotary induction motor, the stator and rotor developments are of the same length (because of the smaller air gap) and after one revolution the rotor and stator are back in the same position with respect to each other. At a constant speed steady-state conditions exist in such a machine. In the linear motor one member will be shorter than the other, and at a steady speed the shorter member will be continuously passing over a new part of the other member. At a steady speed in linear induction motors, the performance is greatly affected by transient conditions which exist at the entry and trailing edges of the shorter member.

Applications

The main application of linear induction motor is in transportation, including electric traction. Primary is mounted on the vehicle and secondary laid along the track. It is used in cranes for material handling, pumping of liquid metal, actuators for door movement and *hv* circuit breakers. It is also used in accelerators for rigs for testing vehicle performance under impact conditions.

9.11 STEPPER (OR STEPPING) MOTORS

The *stepper* or *stepping motor* has a rotor movement in discrete steps. The angular rotation is determined by the number of pulses fed into the control circuit. Each input pulse initiates the drive circuit which produces one step of angular movement. Hence, the device may be considered as a digital-to-analogue converter. The drive circuit has inbuilt logic which causes appropriate windings to be energized and de-energized by solid-state switches in the required sequential manner.

There are three most popular types of rotor arrangements :

1. Variable reluctance (VR) type
2. Permanent magnet (PM) type
3. Hybrid type, a combination of VR and PM.

9.12 STEP ANGLE

The angle by which the rotor of a stepper motor moves when one pulse is applied to the (input) stator is called **step angle.** This is expressed in degrees. The resolution of positioning of a stepper motor is decided by the step angle. Smaller the step angle the higher is the resolution of positioning of the motor. The step **number or resolution** of a motor is the number of steps it makes in one revolution of the rotor.

$$\text{Resolution} = \frac{\text{number of steps}}{\text{number of revolutions of the rotor}}.$$

Higher the resolution, greater is the accuracy of positioning of the motor. Stepper motors are realisable for very small step angles. Some motors can make 1000 steps in one revolution with a step angle of ... standard motor will have a step angle of 1.8° with 200 steps per revolution. step angles of 90°, 45° and 15° are not uncommon in simple motors.

Many different designs of stepping motor have been developed. The number of phases can vary from two to six. Small step angles are obtained by ... of slotted pole pieces to increase the number of effective saliencies (now ... to as teeth) together with multistack stator assemblies.

9.13 VARIABLE RELUCTANCE (VR) STEPPER MOTOR

The principle of operation of a variable reluctance (VR) stepper ... based on the property of flux lines to occupy low reluctance path. The ... rotor therefore get aligned such that the magnetic reluctance is minimum ... variable reluctance (VR) stepper motor can be of single-stack type or the multi... type.

9.13.1 Single-Stack Variable Reluctance Motor

A variable reluctance stepper motor has salient-pole (or tooth) stator ... stator has concentrated windings placed over the stator poles (teeth). The number of phases of the stator depends upon the connection of stator coils. Usually ... or four phase windings are used. The rotor is a slotted structure made ... ferromagnetic material and carries no winding. Both the stator and rotor are ... up of high quality magnetic materials having very high permeability so ... exciting current required is very small. When the stator phases are excited ... proper sequence from dc source with the help of semiconductor switches ... netic field is produced. The ferromagnetic rotor occupies the position which ... ents minimum reluctance to the stator field. That is, the rotor axis aligns ... the stator field axis.

Elementary operation of a variable reluctance motor can be ... through the diagram of Fig. 9.13.

It is a four-phase, 4/2-pole (4 poles in the stator and 2 in rotor), single ... variable reluctance stepper motor. Four phases A, B, C and D are connected ... source with the help of semiconductor switches S_A, S_B, S_C and S_D respectively ... phase windings of the stator are energised in the sequence A, B, C, D, A ... winding A is excited, the rotor aligns with the axis of phase A. The rotor ... in this position and cannot move until phase A is de-energised. Next, ...

... is disconnected. The ... with the resultant air gap ... phase C is excited and ... of 90° in the clockwise ... air gap field which ... excited in the sequence ... transition in clockwise ... steps. The direction ... switching the winding ... depends only on ... of the direction of c...

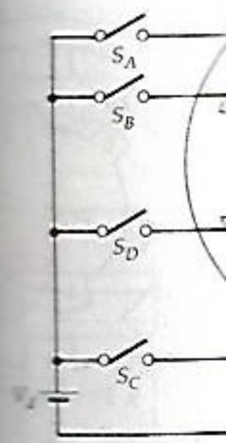


Fig. 9.13. Four-phase ...

... magnitude of step ang...

$$\alpha = \frac{360°}{m_s N_r}$$

α = step angle
m_s = number o...
N_r = number o...

... step angle is also expr...

$$\alpha = \frac{N_s - N_r}{N_s N_r} \times$$

N_s = stator pol...

... step angle can be red... $A, A-B, B, B+C, C, C$... and B are excited tog... poles carrying pha... angle of 45° wit... phase A is excite...

excited and A is disconnected. The rotor moves through 90° in clockwise direction to align with the resultant air gap field which now lies along the axis of phase B. Further, phase C is excited and B is disconnected, the rotor moves through a further step of 90° in the clockwise direction. In this position, the rotor aligns with the resultant air gap field which now lies along the axis of phase C. Thus, as the phases are excited in the sequence A, B, C, D, A the rotor moves through a step of 90° at each transition in clockwise direction. The rotor completes one revolution through four steps. The direction of rotation can be reversed by reversing the sequence of switching the windings, that is, A, D, C, B, A. It is seen that the direction of rotation depends only on the sequence of switching the phases and is independent of the direction of currents through the phases.

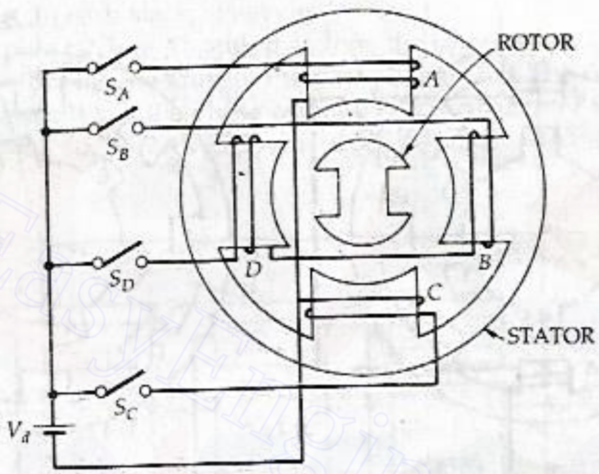


Fig. 9.13. *Four-phase 4/2-pole variable reluctance stepper motor.*

The magnitude of step angle for any PM or VR stepper motor is given by

$$\alpha = \frac{360°}{m_s N_r}$$

where α = step angle

m_s = number of stator phases or stacks

N_r = number of rotor teeth (or rotor poles)

The step angle is also expressed as

$$\alpha = \frac{N_s - N_r}{N_s N_r} \times 360°$$

where N_s = stator poles (or stator teeth)

The step angle can be reduced from 90° to 45° by exciting phases in the sequence $A, A+B, B, B+C, C, C+D, D, D+A, A$. Here $(A+B)$ means that phase windings A and B are excited together and the resultant stator field will be midway between the poles carrying phase windings A and B. That is, the resultant field axis makes an angle of 45° with the axis of pole A in the clockwise direction. Therefore when phase A is excited, the rotor aligns with the axis of phase A. When

phases A and B are excited together, the rotor moves by 45° in the
direction. Thus, it is seen that if the windings are excited in the
$A, A+B, B, B+C, C, C+D, D, D+A, A$ the rotor rotates in steps of 45°
wise direction. The rotor can be rotated in steps of 45° in the anticlock
tion by exciting the phases in the sequence $A, A+D, D, D+C, C$
$B, B+A, A$. This method of gradually shifting excitation from one
other (for example, from A to B with an intermediate step of $A+B$) is
microstepping. It is used to realise smaller steps.

Lower values of step angle can be obtained by using a stepping
more number of poles on stator and teeth on rotor. Consider a four-phase
pole, single stack variable reluctance motor shown in Fig. 9.14.

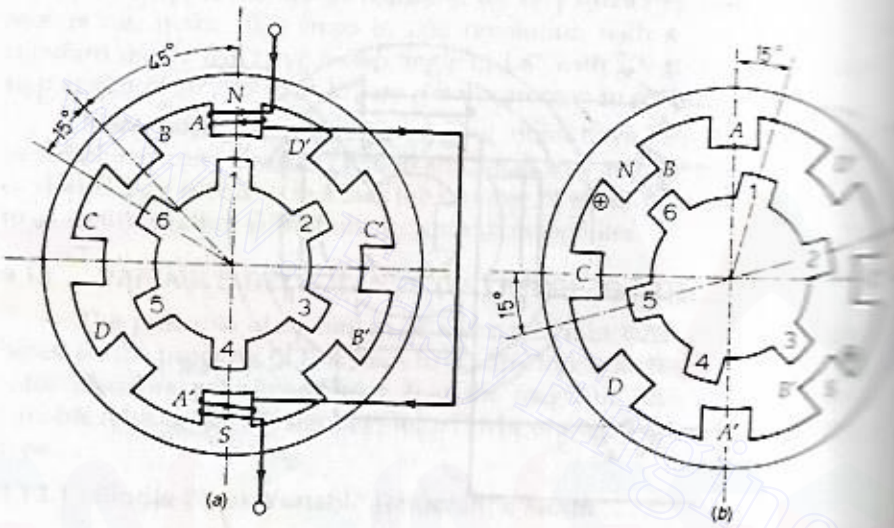


Fig. 9.14. *Four-phase, 8/6-pole variable reluctance stepper motor.*

The coils wound around diametrically opposite poles are
series and four circuits (phases) are formed. These phases are energised
source through electronic switching device. The rotor has six poles (teeth).
sake of simplicity, only phase A winding is shown in Fig. 9.14(a). When
(coil $A-A'$) is excited, rotor teeth numbered 1 and 4 are aligned along the
phase A winding. The rotor occupies the position shown in Fig. 9.14(a). Next
winding A is de-energised and phase winding B is excited. Rotor teeth
3 and 6 get aligned along the axis of phase B and the rotor moves through
angle of 15° in the clockwise direction. Further clockwise rotation of 15° is
by de-energising phase winding B and exciting phase winding C. With
quence A, B, C, D, A, four steps of rotation are completed and the rotor
through 60° in the clockwise direction. For one complete revolution of the
24 steps are required. For anticlockwise rotation of rotor through each
15°, the phase windings are excited in the reverse sequence of A, D, C
Microstepping can also be used in this case to reduce the step size. For
rotation with a step size of 7.5° the sequence $A, A+B$,
$C, C+D, D, D+A, A$ can be used. By choosing different combinations of
of rotor teeth and/or stator exciting coils, any desired step angle can be

9.13.2 Multi-Stack Variable Reluctance Stepper Motor

A multi-stack (or m-stack) variable reluctance stepper motor can be considered to be made up of m identical single-stack variable reluctance motors with their rotors mounted on a single shaft. The stators and rotors have the same number of poles (or teeth) and, therefore, same pole (tooth) pitch. For a m-stack motor, the stator poles (or teeth) in all m stacks are aligned, but the rotor poles (teeth) are displaced by $1/m$ of the pole pitch angle from one another. All the stator pole windings in a given stack are excited simultaneously and, therefore, the stator winding of each stack forms one phase. Thus, the motor has the same number of phases as the number of stacks.

Figure 9.15 shows the cross-section of a three-stack (three-phase) motor parallel to the shaft. In each stack, stators and rotors have 12 poles (teeth). For a 12-pole rotor, the pole pitch is 30, and, therefore, the rotor poles (teeth) are displaced from each other by one-third of the pole pitch or 10°. The stator teeth in each stack are aligned. When the phase winding A is excited rotor teeth of stack A are aligned with the stator teeth as shown in Fig. 9.16(a).

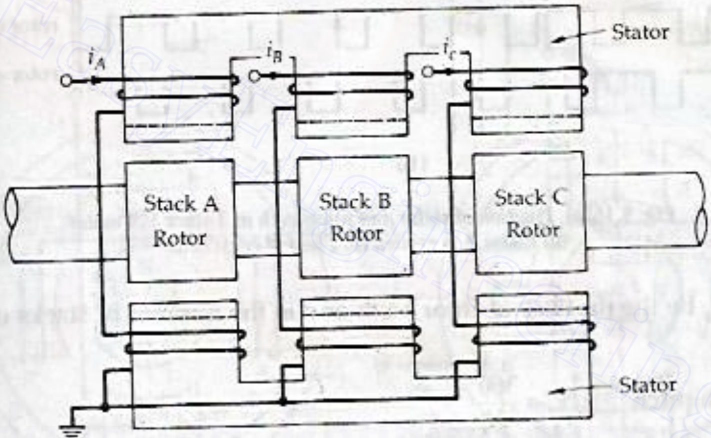


Fig. 9.15. *Cross-section of a 3-stack, VR stepper motor parallel to the shaft.*

When phase A is de-energized and phase B is excited, rotor teeth of stack B are aligned with stator teeth. This new alignment is made by the rotor movement of 10° in the anticlockwise direction. Thus, the motor moves one step (equal to $\frac{1}{3}$ pole pitch) due to change of excitation from stack A to stack B. Next phase B is de-energized and phase C is excited. The rotor moves by another step of one-third of pole pitch in the anticlockwise direction. Another change of excitation from stack C to stack A will once more align the stator and rotor teeth in stack A. However, during this process ($A \longrightarrow B \longrightarrow C \longrightarrow A$) the rotor has moved one rotor tooth pitch.

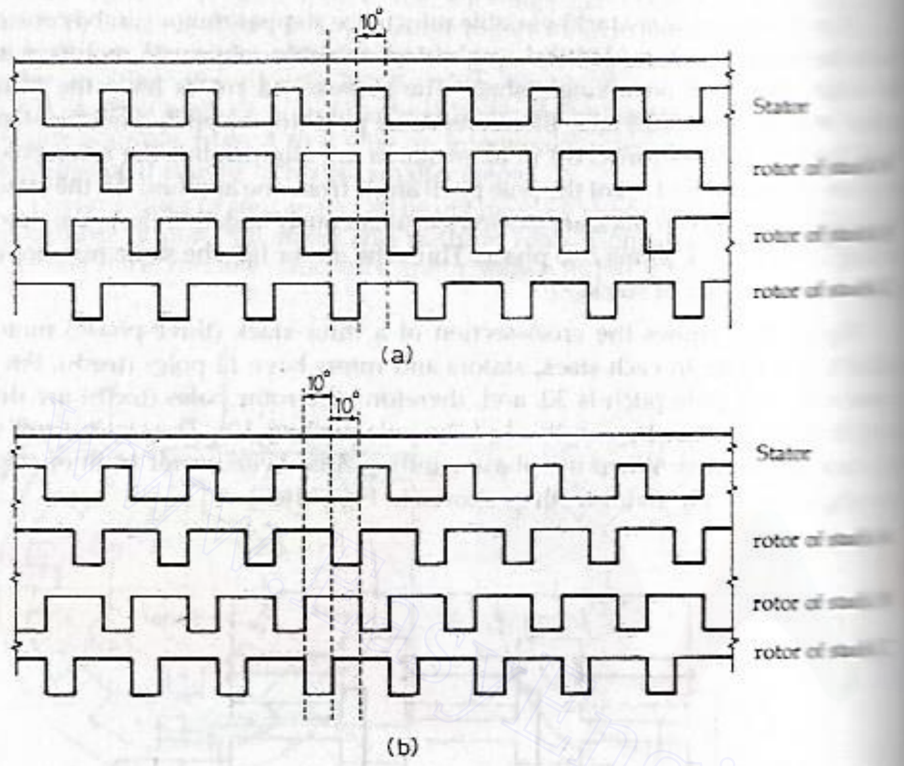


Fig. 9.16(a). *Position of stator and rotor teeth in 3-stack VR motor. (a) Phase A is excited (b) Phase B is excited.*

Let N_r be the number of rotor teeth and m the number of stacks or phases.

Then

Tooth pitch $\quad \tau_p = \dfrac{360°}{N_r}$

Step angle $\quad = \dfrac{360°}{m\,N_r}$

In our case, $\quad \tau_p = \dfrac{360°}{12} = 30°$

Step angle $\quad = \dfrac{360°}{3 \times 12} = 10°$

Multi-stack variable reluctance stepper motors are widely used to obtain smaller step sizes, typically in the range of 2 to 15 degrees.

The variable reluctance motors, both single-and multi-stack types, have high torque to inertia ratio. The reduced inertia enables the VR motor to accelerate the load faster.

PERMANENT MAGNET (

The permanent-magnet (PM ... of the single-stack variable ... permanent-magnet po... 2-pole PM stepper mot... poles are connected in se...

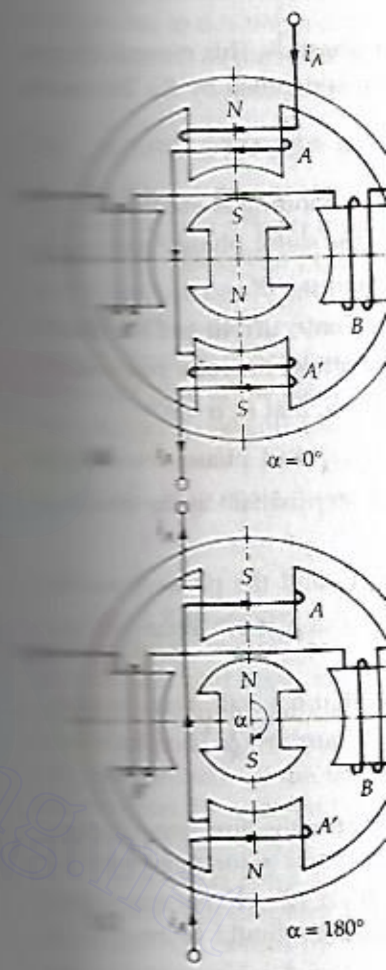


Fig. 9.17. *Tu...*

The rotor poles align wi... of the winding. The t... Similarly, the two coi... windings A and B ca...

(a) Current flows fro... denoted by i_A^+ and...

9.14 PERMANENT MAGNET (PM) STEPPER MOTOR

The permanent-magnet (PM) stepper motor has a stator construction similar to that of the single-stack variable reluctance motor. The rotor is cylindrical and consists of permanent-magnet poles made of high retentivity steel. Figure 9.17 shows a 4/2-pole PM stepper motor. The concentrated windings on diametrically opposite poles are connected in series to form 2-phase winding on the stator.

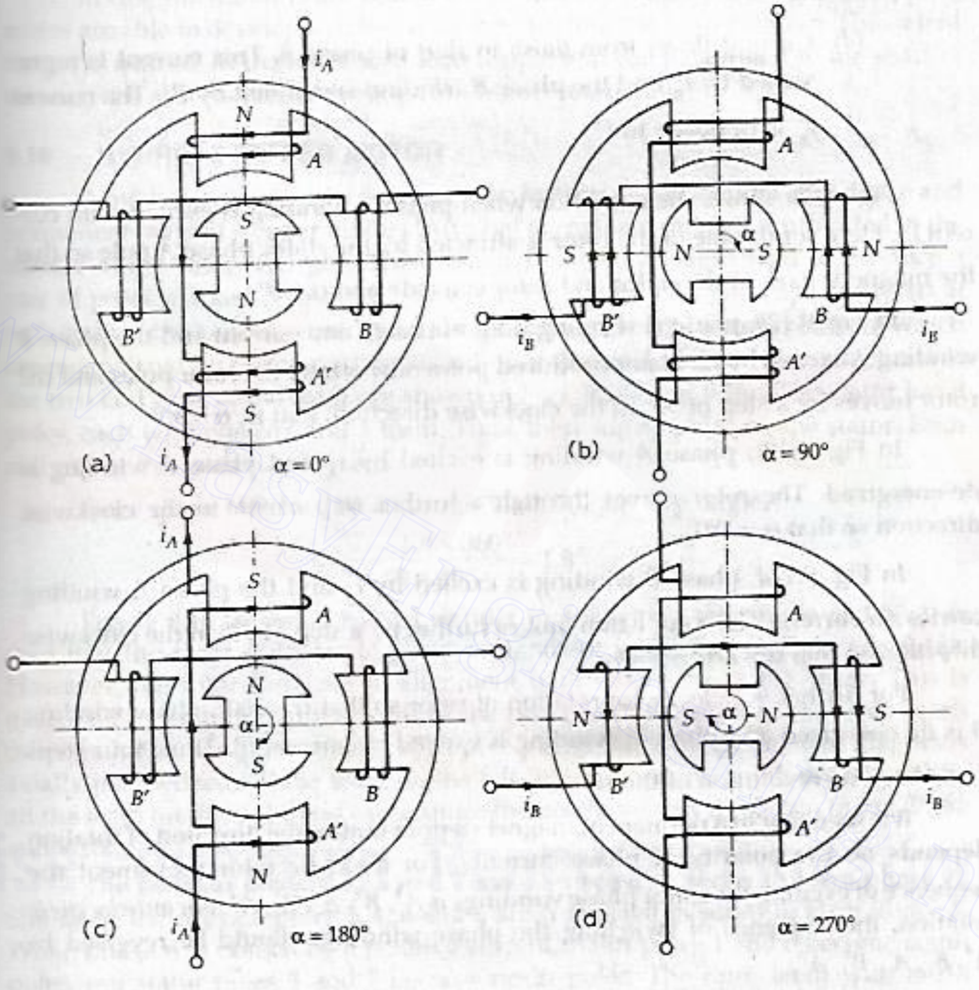


Fig. 9.17. *Two-phase 4/2-pole PM stepper motor.*

The rotor poles align with the stator teeth (or poles) depending on the excitation of the winding. The two coils $A - A'$ connected in series form phase A winding. Similarly, the two coils BB' connected in series form phase B winding. The stator windings A and B can be excited as follows :

(*a*) *Current flows from the start to finish of phase A. This current is* denoted by i_A^+ and the phase A winding is denoted by A^+.

(*b*) Current flows from the *start* to *finish* of phase *B*. This current is denoted by i_B^+ and the phase *B* winding is denoted by B^+.

(*c*) Current flows from the *finish* to *start* of phase *A*. This current is denoted by i_A^- and the phase *A* winding is denoted by A^-. The current i_A^- is opposite to i_A^+.

(*d*) Current flows from *finish* to *start* of phase *B*. This current is represented by i_B^- and the phase *B* winding is denoted by B^-. The current i_B^- is opposite to i_B^+.

Fig. 9.17*a* shows the condition when phase *A* winding is excited with current i_A^+. Here south pole of the rotor is attracted by the stator phase *A* pole so that the magnetic axes of the stator and rotor coincide and $\alpha = 0°$.

In Fig. 9.17*b*, phase *A* winding does not carry any current and the phase *B* winding is excited by i_B^+. Stator produced poles now attract the rotor poles and the rotor moves by a step of 90° in the clockwise direction, that is, $\alpha = 90°$.

In Fig. 9.17*c*, phase *A* winding is excited by i_A^- and phase *B* winding is de-energized. The rotor moves through a further step of 90° in the clockwise direction so that $\alpha = 180°$.

In Fig. 9.17*d*, phase *B* winding is excited by i_B^- and the phase *A* winding carries no current. The rotor again moves further by a step of 90° in the clockwise direction so that $\alpha = 270°$.

For further 90° clockwise rotation of rotor so that $\alpha = 360°$, phase winding *B* is de-energized and phase *A* winding is excited by current i_A^+. Thus, four steps complete one revolution of the rotor.

It is seen that in a permanent magnet stepper motor, the direction of rotation depends on the polarity of phase currents. For *clockwise* rotor movement the sequence of exciting the stator phase windings is A^+, B^+, A^-, B^-, A^+. For anticlockwise rotation, the sequence of switching the phase windings should be reversed, A^+, B^-, A^-, B^+, A^+.

It is difficult to make a small PM rotor with large number of poles and therefore, stepper motors of this type are restricted to larger step sizes in the range of 30° to 90°. However, disc type PM stepper motors are available to give a small step size and low inertia.

Permanent-magnet stepper motors have higher inertia and, therefore, lower acceleration than VR stepper motors. The maximum step rate for PM stepper motors is 300 pulses per second, whereas it can be as high as 1200 pulses per second for VR stepper motors. The PM stepper motor produces more torque per ampere stator current than VR stepper motor.

9.15 DETENT TORQUE OR RESTRAINING TORQUE

The residual magnetism in the permanent-magnet material produces a *detent torque* or *restraining torque* on the rotor when the stator coils are not energized. This torque prevents the rotor from 'drifting' when the machine supply is turned off.

In case the motor is unexcited, the permanent magnet and hybrid stepping motor are able to develop a detent torque restricting the rotor rotation. The detent torque is defined as the maximum load torque that can be applied to the shaft of an *unexcited motor* without causing continuous rotation.

9.16 HYBRID STEPPER MOTOR

A hybrid stepper motor combines the features of the variable reluctance and permanent magnet stepper motors. An axial permanent magnet is provided in the middle of the rotor. The permanent magnet is axially magnetized to produce a pair of poles marked N and S as shown in Fig. 9.18*a*. Two end caps are fitted at both ends of the axial magnet. The end caps consist of equal number of teeth which are magnetized by the respective polarities of the axial magnet. Cross-sections of the two end caps of the rotor are shown in Figs. 9.18*b* and 9.18*c*. The stator has 8 poles, each with one coil and 5 teeth. Thus, there are 40 poles on the stator. Each end cap of the rotor has 50 teeth.

Since stator teeth are 40 and rotor teeth 50, the step angle

$$= \frac{(50-40)\times 360^\circ}{50 \times 40} = 1.8^\circ$$

Figure 9.18 shows a hybrid stepper motor with a step angle of 1.8°. It is seen that the rotor teeth are in perfect alignment with *stator teeth* in Fig. 9.18*b*. However, the rotor teeth are in alignment with *stator slots* in Fig. 9.18*c*. This is described by stating that the teeth of the two end caps are displaced from each other by half a tooth pitch (also called pole pitch). Since the permanent magnet is axially magnetized, all the teeth on the left-end cap acquire south polarity while all the teeth on the right-end cap acquire the north polarity as shown in Fig. 9.18*b* and 9.18*c*.

The coils on poles 1, 3, 5 and 7 are connected in series to form phase A. Similarly, the coils on poles 2, 4, 6 and 8 are connected in series to form phase B. When phase A is excited by a positive current, stator poles 1 and 5 become south poles and stator poles 3 and 7 become north poles. The rotor teeth with north polarity align with the stator poles 1 and 5, while the rotor teeth with south polarity align with the stator poles 3 and 7. When phase A is de-energized and phase B is excited positively, the rotor will turn by a full step of 1.8° in the anticlockwise direction.

Next phase A is energized negatively, there is a further movement of rotor by 1.8° in the same anticlockwise direction. In order to move rotor further by 1.8°, phase B is excited negatively. Thus, for producing the anticlockwise motion of the rotor in steps of 1.8°, the phases are to be energized in the sequence $+A, +B, -A, -B, +B, +A$ .... For producing clockwise rotation the phase sequence should be $+A, -B, +B, +A$ ....

The main advantage of the hybrid stepper motor is that if the motor excita-
tion is removed, the rotor continues to remain locked into the same position
before removal of excitation. This is due to the fact that the rotor is prevented to
move in either direction by the detent torque produced by the permanent magnet.

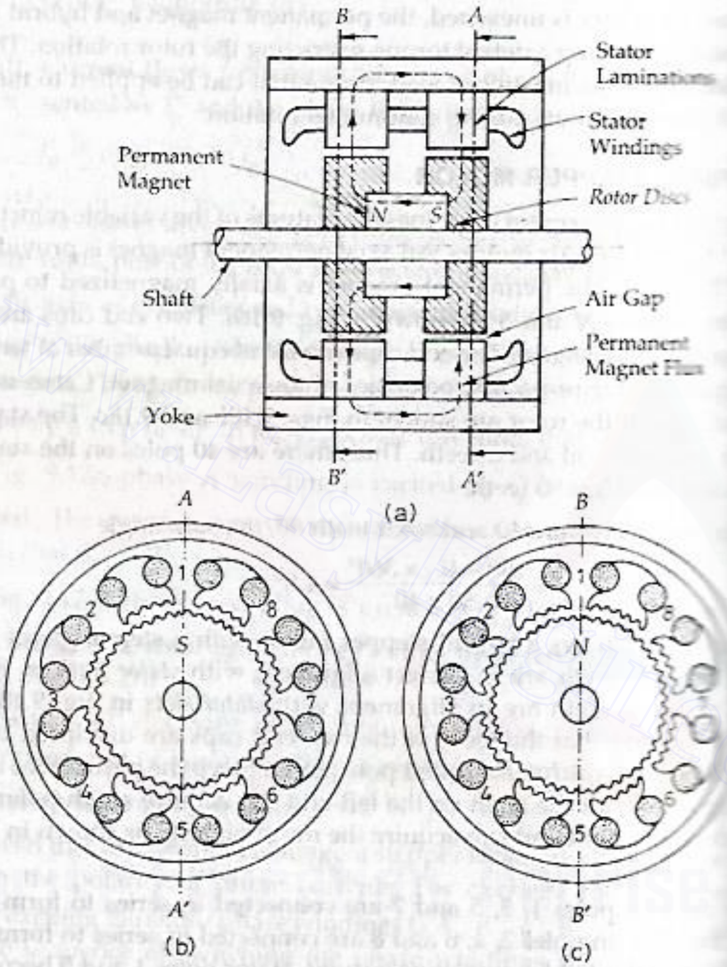


Fig. 9.18. 1.8° *Hybrid stepper motor (a) Axial view (b) Cross-section AA′ (c) Cross section*

Advantages of Hybrid Stepper Motors

The main advantages of hybrid stepper motors compared with variable reluctance stepper motors are as follows :

1. Small step length.
2. Greater torque per unit volume.
3. Provides detent torque with windings de-energized.
4. Less tendency to resonate.
5. High efficiency at lower speeds and lower stepping rates.

Disadvantages of Hybrid Stepper

1. Higher inertia and w
2. Performance affected
3. More costly than var

TORQUE-PULSE RATE

The torque-pulse rate chara
electromagnetic torque as a func
stepper motor is usually describ
Curve 1 is the pull-i
stepping rate at which the motor c
function of load torque. Curve
maximum stepping rate
load torque if already synchro
this rate. For examp
stop or reverse witho
motor has started and s
load torque without r
operation, the stepping
without losing synchroni

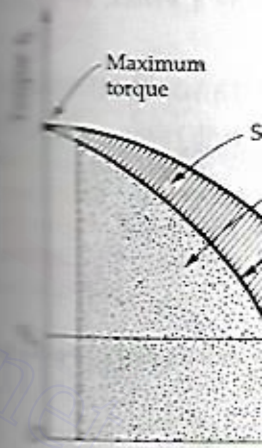


Fig. 9.19. *Torque-puls*

However, if the stepping r
Thus, the area betwe
range of stepping rate
that it has already been s
and the motor is said to
and it is not suit
vertical line and cu

Disadvantages of Hybrid Stepper Motors

1. Higher inertia and weight due to presence of rotor magnet.
2. Performance affected by change in magnetic strength.
3. More costly than variable reluctance stepper motor.

9.17 TORQUE-PULSE RATE CHARACTERISTICS

The torque-pulse rate characteristic of a stepping motor gives the variation of electromagnetic torque as a function of stepping rate in pulses per second (pps). A stepper motor is usually described by two characteristic curves 1 and 2 as shown in Fig. 9.19. Curve 1 is the pull-in torque characteristic. It shows the maximum stepping rate at which the motor can start, synchronise, stop or reverse for different values of load torque. Curve 2 defines the pull-out torque characteristic. It shows the maximum stepping rate at which the motor can run for different values of load torque if already synchronised, but it cannot start, stop or reverse on command at this rate. For example, for the load torque τ_{L_1}, the motor can start, synchronise, stop or reverse without missing a pulse if the pulse rate is less than s_1. Once the rotor has started and synchronized, the stepping rate can be increased for the same load torque without missing a step. For load torque τ_{L_1}, after starting and synchronization, the stepping rate can be increased upto s_2 without missing a step or without losing synchronism.

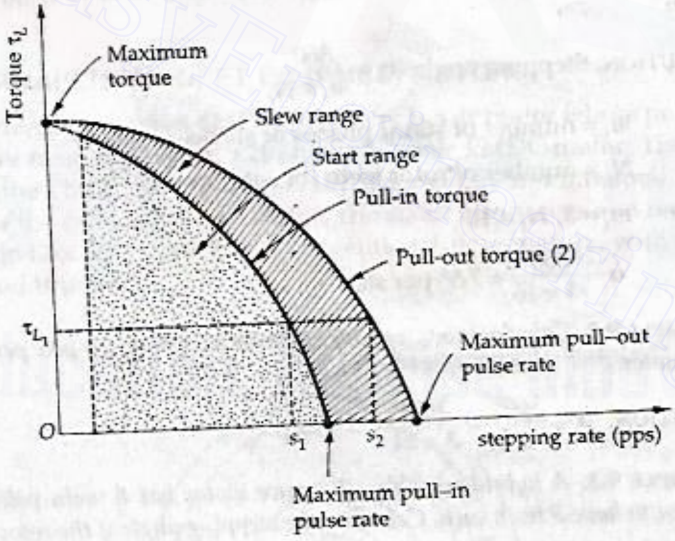


Fig. 9.19. *Torque-pulse rate characteristics of a stepper motor.*

However, if the stepping rate is increased beyond s_2 the motor will lose synchronism. Thus, the area between curves 1 and 2 represents, for various torque values, the range of stepping rate which the motor follows without losing a step provided that it has already been started and synchronized. This area is called the ***slew range*** and the motor is said to operate in a ***slewing mode***. Slew range is useful for speed control and it is not suitable for position control. The area between the zero-pulse rate vertical line and curve 1 is called the ***start range***.

The stepper motor is not used at very low stepping rates (ind... dotted portions of the curves 1 and 2) because of rotor oscillations ... lack of damping at the low pulse rate. Thus, the actual start range ... dotted vertical line indicating the low pulse rate and the pull-in torque ... istic 1.

9.18 APPLICATIONS OF STEPPING MOTORS

Stepper motors have a wide range of applications. Since they ... controlled (using input pulses), they are eminently suitable for use with ... controlled systems. Stepper motors are widely used in numerical cont... chine tools, tape drives, floppy disc drives, printers, $X - Y$ plotters, rob... industry, integrated circuit fabrication, electric watches etc. The other ... of stepper motors are in space crafts launched for scientific exploration... Stepper motors are also used in the production of science fiction mov... motors also find variety of commercial, medical and military applicatio... be increasingly used in future.

Stepper motors are manufactured in sizes ranging from milliwat... dreds of watts, maximum torque values upto 15 Nm and step angles ... to 90 mechanical degrees. Stepper motors of microwatt level are used ... watches, while in machine tools stepper motors with ratings of sever... kilowatts may be found.

EXAMPLE 9.1. *Calculate the stepping angle for a 3-stack, 16-tooth vari... tance motor.*

SOLUTION. Stepping angles $\alpha = \dfrac{360°}{m_s \times N_r}$

where m_s = number of stator phases or stacks

N_r = number of rotor teeth (or rotor poles)

Here $m_s = 3$, $N_r = 16$

$$\therefore \quad \alpha = \frac{360°}{3 \times 16} = 7.5° \text{ per step.}$$

EXAMPLE 9.2. *Calculate the stepping angle for a 3-phase, 24-pole perma... net stepper motor.*

SOLUTION. $\alpha = \dfrac{360°}{m_s P_r} = \dfrac{360°}{3 \times 24} = 5°/\text{step}.$

EXAMPLE 9.3. *A hybrid variable reluctance motor has 8 main poles ... been castleated to have 5 teeth each. Calculate the stepping angle if the rotor has 5...*

SOLUTION. Number of stator teeth $N_s = 8 \times 5 = 40$

Number of rotor teeth $N_r = 50$

$$\text{Step angle } \alpha = \frac{N_r - N_s}{N_r N_s} \times 360°$$

$$= \frac{50 - 40}{50 \times 40} \times 360° = 1.8°$$

EXAMPLE 9.4. *A single-stack, eig... The phases are excited one a... Determine (a) step size, (b) ...*

SOLUTION. $N_s = 8$, $N_r = 6$,

(a) Step size $\alpha = \dfrac{N_s - N_r}{N_s N_r} \times 36...$

$= \dfrac{8 - 6}{8 \times 6} \times 360°$

(b) Steps per revolution $= \dfrac{360}{\alpha}$

(c) Shaft speed $= \dfrac{\alpha f}{360} = \dfrac{15 \times 1...}{360}$

EXAMPLE 9.5. *A three-stack, fo... Determine the step s...*

SOLUTION. Number of stacks ...

Number of rotor teeth, ...

Step angle ...

... PERMANENT-MAGNET D...

... permanent-magnet dc (PM... permanent magnets. Figure 9.20 ... the PMDC motor are ... of the cylindrical steel s... magnetic flux. The rotor has ... and brushes.

Fig. 9.20. Cro...

Example 9.4. *A single-stack, eight-phase (stator) multipole, stepper motor has six rotor teeth. The phases are excited one at a time.*

(a) *Determine (a) step size, (b) steps per revolution, (c) speed, if the excitation frequency is 120 Hz.*

Solution. $N_s = 8, \quad N_r = 6, \quad f = 120$ Hz

(*a*) Step size $\alpha = \dfrac{N_s - N_r}{N_s N_r} \times 360°$

$= \dfrac{8-6}{8 \times 6} \times 360° = 15°$

(*b*) Steps per revolution $= \dfrac{360°}{\alpha} = \dfrac{360°}{15°} = 24$

(*c*) Shaft speed $= \dfrac{\alpha f}{360} = \dfrac{15 \times 120}{360} = 5$ rps

Example 9.5. *A three-stack, four-pole stepper motor has eight teeth on the rotor as well as stator. Determine the step size as excitation is changed from one stack to the next.*

Solution. Number of stacks $m = 3$

Number of rotor teeth, $N_r = 8$

Step angle $\alpha = \dfrac{360°}{m N_r} = \dfrac{360°}{3 \times 8} = 15°$

9.19 PERMANENT-MAGNET DC (PMDC) MOTORS

A permanent-magnet dc (PMDC) motor is a dc motor whose poles are made of permanent magnets. Figure 9.20 shows a 2-pole PMDC motor. The permanent magnets of the PMDC motor are radially magnetized and mounted on the inner periphery of the cylindrical steel stator. The stator also serves as a return path for the magnetic flux. The rotor has a conventional dc armature, with commutator segments and brushes.

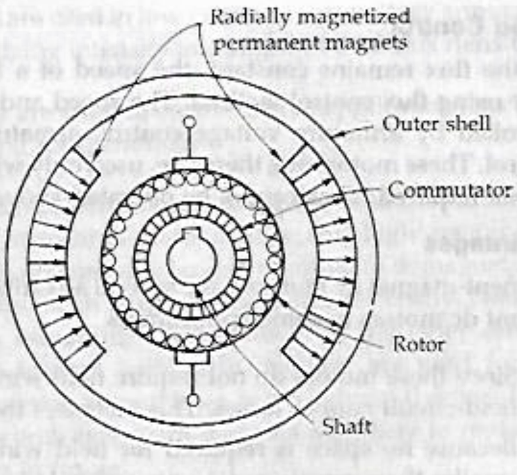


Fig. 9.20. *Cross-sectional view of a PMDC motor.*

Most of the PMDC motors operate on $6V$, $12V$ or $24V$ dc supply from batteries or rectifiers.

The torque is produced by interaction between axial current-carrying rotor conductors and the magnetic flux produced by the permanent magnets.

The circuit model or equivalent circuit of a PMDC motor is shown in Fig. 9.21. Since the field flux in a PMDC motor is produced by permanent magnets, the field winding is not shown in the circuit model.

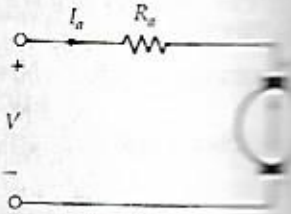


Fig. 9.21. Circuit model of a PMDC motor

In a conventional dc motor, the generated or back emf is given by

$$E = k\,\Phi\,N$$

and the electromagnetic torque is given by

$$\tau_e = k\,\Phi\,I_a$$

In a PMDC motor, flux Φ is constant, Eqs. (9.19.1) and (9.19.2) are written as

$$E = k_1\,N$$

$$\tau_e = k_1\,I_a$$

where $k_1 = k\,\Phi$ is called speed-voltage constant or torque constant. Its value depends upon the number of field poles, armature conductors etc.

From Fig. 9.21 we have

$$V = E + I_a\,R_a$$

$$= k_1\,N + I_a\,R_a$$

$$N = \frac{V - I_a\,R_a}{k_1}$$

9.19.1 Speed Control

Since the flux remains constant, the speed of a PMDC motor cannot be controlled by using flux control method. The speed and torque of PMDC motors can be controlled by armature voltage control, armature rheostat control and chopper control. These motors are, therefore, used only where motor speeds below base speeds are required. They cannot be operated above the base speed.

9.19.2 Advantages

Permanent-magnet dc motors have several advantages compared with conventional shunt dc motors in some applications.

1. Since these motors do not require field windings, they do not have field-circuit copper losses. This increases their efficiency.
2. Because no space is required for field windings, these motors are smaller than corresponding wound-pole motors.

Disadvantages

1. Permanent magne... externally supplie... lower induced torq... shunt motor of the...
2. There is a risk of de... large armature cur... excessive heating, w... a prolonged period...
3. The magnetic field o... the motor is not be... no foreign magnetic... to avoid harm to the...

Applications

PMDC motors are used in... horsepower. They are use... and washers, to ra... air conditioners, in co... industries. Other appl... portable vacuum cleaners... hedge trimmers o... battery sources. PM... in industry.

Types of Permanent Magnets

There are three types of perm...

Alnico. They are used in lo... magnetizing intensity...

Ferrites. They are used in c... and refrigerator...

Rare earths. Rare earth mag... They have high res... earth magnets are l... Such magn... than samarium-cob... the former. Rare... They are used in automob... motors with rare earth... motors in future.

9.19.3 Disadvantages

1. Permanent magnets cannot produce as high a flux density as an externally supplied shunt field. Therefore a PMDC motor has a lower induced torque τ_{ind} per ampere of armature current I_a than a shunt motor of the same rating.

2. There is a risk of demagnetization of poles which may be caused by large armature currents. Demagnetization may also be caused by excessive heating, which can occur if the motor is overloaded over a prolonged period.

3. The magnetic field of PMDC motor is present at all times even when the motor is not being used. The motor is totally enclosed so that no foreign magnetic matter is attracted by the permanent magnets to avoid harm to the motor.

9.19.4 Applications

PMDC motors are used in many applications ranging from fractions to several horsepower. They are used extensibly in automobiles to operate windshield wipers and washers, to raise and lower windows, to drive blowers for heaters and air conditioners, in computer drives etc. Millions of such motors are used in toy industries. Other applications of PMDC motors are in electric tooth brushes, portable vacuum cleaners, food mixers, portable electric tools such as drills, saber saws, hedge trimmers etc. They are often employed in equipment that is supplied from battery sources. PMDC motors have been developed up to about 200 kW for use in industry.

9.19.5 Types of Permanent Magnet Materials

There are three types of permanent magnet materials used for PMDC motors.

(a) Alnicos. They are used in low-current, high-voltage applications because of low coercive magnetizing intensity and high residual flux density.

(b) Ferrites. They are used in cost-sensitive applications such as air conditioners, compressors, and refrigerators.

(c) Rare earths. Rare earth magnets are made of samarium-cobalt, neodymium-iron-boron. They have high residual flux and high coercive magnetizing intensity. The rare earth magnets are largely immune to demagnetizing problems due to armature reaction. Such magnetic materials are costly. Neodymium-iron-boron is cheaper than samarium-cobalt. However, the latter can stand higher temperatures than the former. Rare earth magnets are used for size-sensitive applications. They are used in automobiles, servo industrial drives, large industrial motors. PMDC motors with rare earth magnets are likely to replace most of the conventional dc motors in future.

9.20 PRINTED CIRCUIT BOARD (PCB) MOTORS

The printed circuit board (or disc armature) motor consists of a rot of nonmagnetic and nonconducting (insulating) material. The entire arm winding and the commutator are printed in copper on both the sides of the The brushes are placed around its inner periphery. The disc armature is between two sets of permanent magnets mounted on ferromagnetic end This configuration provides axial flux through the armature. The radial flowing through the disc armature interacts with the axial flux to produce that rotates the rotor. Fig. 9.22 shows the construction of a PCB motor.

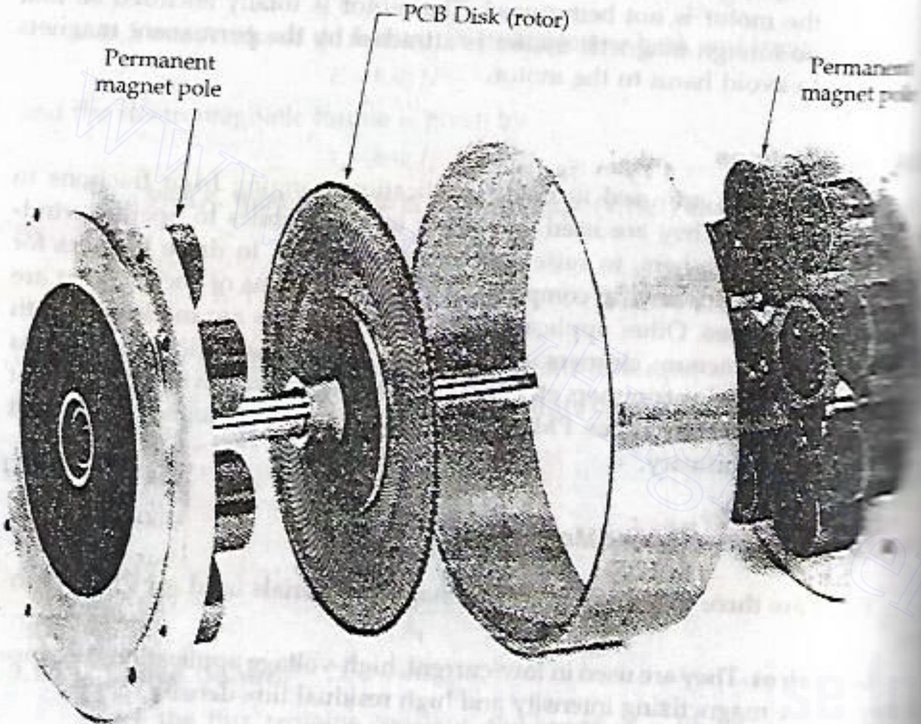


Fig. 9.22. PCB motor assembly.

Advantages of PCB motor

1. It has a very low inertia and, therefore, its (torque/inertia) very high. Therefore it can provide quick acceleration and tion.
2. Since there is no iron in the rotor, the armature inductance The armature time constant is very small due to low arm ductance. Therefore, armature current builds up very quickly than one millisecond, and full torque is developed almost ins

3. The lower inductance brush life.
4. Since the rotor is non
5. A PCB motor has a h
6. There is negligible ar high value of overloa teristic of a PCB moto
7. Since there are a large of three or four hundre to near-zero speed.

The characteristics of PCB mo particularly suitable for c Examples are high-spee robots and other se a competitor to the step area it is also manufact heavy-duty drives such a

Ex

the construction, workin and explain a typical torq a reluctance motor with the construction of a sy a running torque both a

and explain the torque-spee applications of hyste the principle of oper their torque-speed character the advantages of servomoto the construction and wor a two-phase servomotor ? speed characteristics for the constructional feature

the principle of operation o its important applicatio the operating principle of a speed of a rotating in

3. The lower inductance reduces sparking and significantly increases brush life.
4. Since the rotor is nonmagnetic, there is no cogging torque.
5. A PCB motor has a high overload current capacity.
6. There is negligible armature reaction and flux distortion even at a high value of overload current. Therefore the speed-torque characteristic of a PCB motor is linear.
7. Since there are a large number of armature conductors of the order of three or four hundred, the torque of a PCB motor is smooth down to near-zero speed.

Applications

The characteristics of PCB motors (especially the high torque/inertia ratio) make them particularly suitable for control applications. These motors have a wide range of use. Examples are high-speed tape readers, *X-Y* recorders, point-to-point tool positioners, robots and other servo drives. With an in-built optical position encoder, it is a competitor to the stepper motor. Although the main application is in the control area it is also manufactured in sizes upto several kilowatts. It is also suitable for heavy-duty drives such as lawn-mowers, heavy-driven vehicles, etc.

EXERCISES

9.1 Describe the construction, working and uses of a reluctance motor.

9.2 Draw and explain a typical torque-speed characteristic of a reluctance motor. Compare a reluctance motor with an equivalent induction motor.

9.3 Describe the construction of a synchronous hysteresis motor and show that it develops a running torque both at synchronous and asynchronous speed of the motor.

9.4 Draw and explain the torque-speed characteristic of a hysteresis motor. What are the common applications of hysteresis motor ?

9.5 Briefly explain the principle of operation of (*a*) d.c. servomotors (*b*) a.c. servomotors. Draw their torque-speed characteristics.

9.6 State the advantages of servomotors over large industrial motors.

9.7 Describe the construction and working of drag-cup servomotor.

9.8 What is a two-phase servomotor ? Describe its construction and working. Draw its torque-speed characteristics for various control voltages.

9.9 Describe the constructional feature and principle of operation of a linear induction motor.

9.10 Explain the principle of operation of a linear induction motor. Draw its characteristics. State its important applications.

9.11 Explain the operating principle of a linear induction motor. What is equivalent to synchronous speed of a rotating induction motor ? Mention some of its applications.

9.12 Explain the operation of a stepper motor.

9.13 Describe the operation of a variable reluctance type stepper motor. Wh... microstepping ?

9.14 What are the differences in the behaviour of variable reluctance type st... motor and permanent magnet type stepper motor ?

9.15 What are the main features of stepper motor which are responsible for its w... spread use ?

9.16 What are the advantages and disadvantages of stepper motors ?

9.17 Explain the torque versus stepping rate characteristics of a stepper motor. Wh... is the slew range ? What is ramping ?

9.18 What is stepping angle ? Calculate the stepping angle for a 3-phase, 24-pole per... manent magnet type stepper motor.

9.19 Name the most popular types of stepper motors. Describe the operation of a per... manent magnet (PM) type of stepper motor.

9.20 Define detent torque. Describe the construction and operation of a hybrid st... motor. What are the main advantages and disadvantages of hybrid stepper m... compared with variable reluctance stepper motors ?

9.21 State some important applications of stepper motors.

9.22 Describe the construction of a permanent-magnet d.c. motor. What are the adv... tages and disadvantages of permanent-magnet d.c. motors compared with co... ventional shunt d.c. motors ?

9.23 What types of permanent-magnet materials are used for permanent-magn... motors ? State their properties and applications.

9.24 State some important applications of PMDC motors.

10

Principles of Electromechanical Energy Conversion

10.1 INTRODUCTION

An electromechanical energy conversion device is one which converts electrical energy into mechanical energy or, alternatively, mechanical energy into electrical energy. Electromechanical energy conversion takes place via the medium of a magnetic field or an electric field, but most practical converters use magnetic field as the coupling medium between electrical and mechanical systems. This is due to the fact that the energy storing capacity of magnetic field is much greater than that of the electric field. Electromechanical energy converters are either **gross-motion devices** such as electric motors and generators or **incremental motion devices** such as microphones, loudspeakers, electromagnetic relays, and certain electrical measuring instruments, etc.

DC, induction and synchronous machines are used extensively for eletromechanical energy conversion. When the conversion takes place from electrical to mechanical form, the device is called **motor**. When the mechanical energy is converted to electrical energy the device is called a **generator**. In these machines, conversion of energy from electrical to mechanical form or from mechanical to electrical form results from the following two electromagnetic phenomena :

1. When a conductor moves in a magnetic field, voltage is induced in the conductor.
2. When a current-carrying conductor is placed in a magnetic field, the conductor experiences a mechanical force.

These two effects occur simultaneously whenever energy conversion takes place from electrical to mechanical or vice versa. In motoring action, current flows through the conductors placed in the magnetic field. A force is produced on each conductor. The conductors are placed on the rotor which is free to move. An electromagnetic torque is produced on the rotor. It starts rotating at some speed. The torque produced on the rotor is transferred to the shaft of the rotor and can be utilized to drive a mechanical load. Since the conductors rotate in the magnetic field, a voltage is induced in each conductor. In generating action ... driven by a prime mover. A voltage is induced in the rotor ... electrical load is connected to the winding formed by these conductors ... will flow, delivering electric power to the load. Moreover, the current ... through the conductors will interact with the magnetic field to produce ... torque, which will tend to oppose the torque developed by prime mover.

It is to be noted that the same fundamental principles underlie ... tion of both ac and dc machines. They are governed by the same ... computation of developed torque, Amperes' law is used whether the ... ac or dc. The fundamental equations for the developed torque are ... make these expressions according to the constructional features of the ... involved. Similarly, in computation of the generated voltages Faraday ... induced voltage is used whether the machine is ac or dc. Again, the ... are made in the voltage equations according to the constructional features ... machine involved. Thus, the expressions for electromagnetic torque ... ated voltages appear different for ac and dc machines because of their ... mechanical constructions. The expressions for electromagnetic torque ... rated voltages are discussed in detail along with the rotating machines.

10.2 CONSERVATION OF ENERGY

According to the *principle of conservation of energy*, energy can ... created nor destroyed. It is converted from one form to another.

In an energy conversion device, the total input energy is equal ... of the following three components :

energy dissipated, energy stored, and useful output energy.

Thus, with an electromechanical conversion devices the *energy balance equation* can be written as

$$\begin{bmatrix}\text{Electrical}\\ \text{energy}\\ \text{input}\end{bmatrix} = \begin{bmatrix}\text{energy to}\\ \text{electrical}\\ \text{losses}\end{bmatrix} + \begin{bmatrix}\text{energy to field}\\ \text{storage in the}\\ \text{electrical system}\end{bmatrix} + \begin{bmatrix}\text{mechanical}\\ \text{energy}\\ \text{output}\end{bmatrix} \ldots$$

Equation (10.2.1) is written for motor action where electrical energy ... and mechanical energy output are treated as positive terms. For generator ... the energy balance equation is written as

$$\begin{bmatrix}\text{Total mechanical}\\ \text{energy input}\end{bmatrix} = \begin{bmatrix}\text{electrical energy}\\ \text{output}\end{bmatrix} + \begin{bmatrix}\text{total energy}\\ \text{stored}\end{bmatrix} + \begin{bmatrix}\ldots\end{bmatrix}$$

The various forms of energies in Eq. (10.2.1) are as follows :

(*a*) Total electrical energy input from the main supply.

(*b*) All the mechanical energy does not appear as useful output ... A fraction is dissipated in mechanical losses (friction and ...

(*c*) Total energy stored in any electromechanical conversion ...

= (energy stored in magnetic field W_{es})

+ energy stored in mechanical ...

W_{ms}, a potential or kinetic ...

(*d*) Total energy dissipated

= (energy dissipated in electric circuit as ohmic losses)

+ (energy dissipated in magnetic core as hysteresis and eddy-current losses)

+ (energy dissipated in mechanical system as friction and windage losses etc.)

10.3 ENERGY STORED IN A MAGNETIC FIELD

Consider a coil of N turns wound around a magnetic core connected to a voltage source. By KVL, the voltage applied is given by

$$v = iR + e \tag{10.3.1}$$

where e is the voltage induced in the coil and R is the resistance of the coil circuit. The instantaneous power input to the system is given by

$$p = vi = i^2R + ei \tag{10.3.2}$$

Suppose that a dc voltage is applied to the circuit at time $t = 0$ and that at end of T seconds, the current has attained a value of I amperes. The energy input to the system during this interval is

$$W_i = \int_0^T p\,dt = \int_0^T i^2R\,dt + \int_0^T ei\,dt \tag{10.3.3}$$

The expression in Eq. (10.3.3) shows that the input energy consists of two parts. The first part in the energy dissipated as resistance loss in the winding and the second part is the energy stored in the magnetic field.

$$W_f = \int_0^T ei\,dt \tag{10.3.4}$$

By Faraday's law

$$e = \frac{d\psi}{dt} = \frac{d\,(N\Phi)}{dt} \tag{10.3.5}$$

where ψ is the magnetic flux linkage.

$$\therefore \qquad W_f = \int_0^T \frac{d\psi}{dt}\,i\,dt = \int_0^T i\,d\psi \tag{10.3.6}$$

Alternatively,

$$W_f = \int_0^T N\frac{d\Phi}{dt}\,i dt = \int_0^T Ni\,d\Phi = \int_0^{\Phi_T} F\,d\Phi \tag{10.3.7}$$

where $F = Ni$ = mmf.

Equation (10.3.6) shows that the energy stored in the magnetic field is equal to the area between the ψ–*i* curve for the system and the flux linkage (ψ)axis (Fig. 10.1(*a*)). The energy stored in the magnetic field is also equal to the area between Φ–*F* curve and the flux axis (Fig. 10.1(*b*)) as indicated by Eq. (10.3.7).

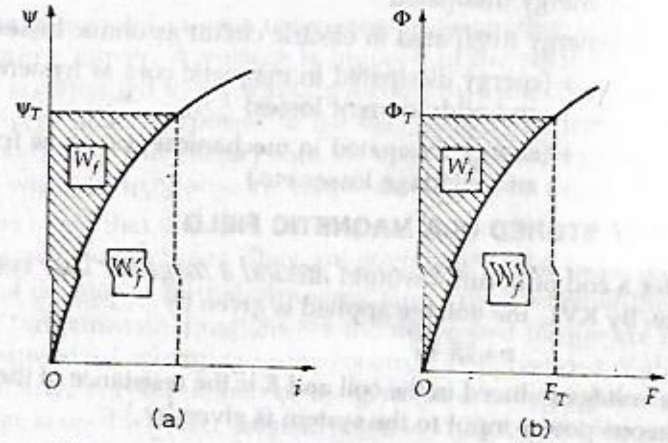


Fig. 10.1. *Graphical representation of energy and coenergy.*

Coenergy

The area between the magnetization curve and the current or mmf axis is called the coenergy. It is denoted by W_f'. Coenergy has no physical significance. However, it is used to derive expressions for force or torque developed in an electromagnetic system. Coenergy has been shown graphically in Fig. 10.1.

10.4 FIELD ENERGY IN A MAGNETICALLY LINEAR SYSTEM

For a linear system, the field energy

$$W_f = \int_0^{\psi} i \, d\psi = \int_0^{\psi} \frac{\psi}{L} \, d\psi = \frac{\psi^2}{2L} \qquad (10.4.1)$$

Since $\psi = Li$

$$W_f = \frac{(Li)^2}{2L} = \frac{1}{2} Li^2 \qquad (10.4.2)$$

The coenergy is given by

$$W_f' = \int_0^{i} \psi \, di = \int_0^{i} Li \, di = \frac{1}{2} Li^2 \qquad (10.4.3)$$

Thus, for a linear magnetic system, the field energy and coenergy are equal.

$$W_f = W_f' = \frac{1}{2} Li^2 = \frac{1}{2} \psi i = \frac{1}{2L} \psi^2 \qquad (10.4.4)$$

10.5 SINGLY-EXCITED SYSTEM

A schematic of singly-excited system is shown in Fig. 10.2. Here a coil is wound around a magnetic core connected to a voltage source. The ferromagnetic rotor experiences a torque urging it towards a region where the magnetic field is stronger. That is, a torque is exerted on the rotor so that it tries to position itself to given minimum reluctance for the magnetic flux. The reluctance is dependent upon the rotor angle. This torque is called the *reluctance* torque or saliency torque due to *saliency* of the rotor.

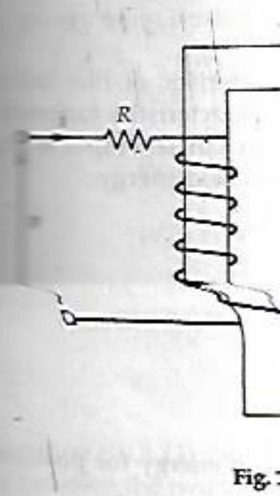


Fig. 1...

The following assumptio...

1. The flux-likage/cu... That is, $\psi = L(\theta)\, i$
2. Hysteresis and edd...
3. The coil has negligi... magnetic path.
4. The magnetic field ...

Let R be the resistance of ...

By KVL, the instantaneou...

$v = Ri$ ...

Multiplying Eq. (10.5.1) b...

$vi = Ri$ ...

Integrating with respect to ... linkages to be initially ...

$\int_0^t vi \, dt =$ ...

That is,

[total electrical input energy] ...

In symbolic form, Eq. (10...

$W_e =$ ...

$\int_0^{\psi} i \, d\psi =$ ...

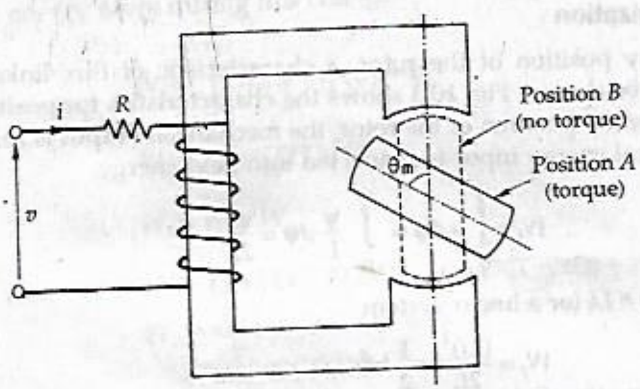


Fig. 10.2. *Singly-excited system.*

The following **assumptions** are made :

1. The flux-likage/current relationship is linear for any rotor position. That is, $\psi = L(\theta)\, i$
2. Hysteresis and eddy-current losses are neglected.
3. The coil has negligible leakage flux and all the flux follows the main magnetic path.
4. The magnetic field predominates and electric field effects are neglected.

Let R be the resistance of the coil.

By KVL, the instantaneous voltage equation for the coil is

$$v = Ri + \frac{d\psi}{dt} \tag{10.5.1}$$

Multiplying Eq. (10.5.1) by i,

$$vi = Ri^2 + i\frac{d\psi}{dt} \tag{10.5.2}$$

Integrating with respect to time t from $t = 0$ to $t = t$ and assuming the current and flux linkages to be initially zero,

$$\int_0^t vi\, dt = \int_0^t Ri^2\, dt + \int_0^\psi i\, d\psi \tag{10.5.3}$$

That is,

$$\begin{bmatrix} \text{total} \\ \text{electrical} \\ \text{input energy} \end{bmatrix} = \begin{bmatrix} \text{energy to} \\ \text{electrical} \\ \text{losses} \end{bmatrix} + \begin{bmatrix} \text{useful} \\ \text{electrical} \\ \text{energy} \end{bmatrix} \tag{10.5.4}$$

In symbolic form, Eq. (10.5.4) can be written from Eq. (10.2.2) as

$$W_e = W_{le} + [W_{fe} + W_{em}] \tag{10.5.5}$$

and

$$\int_0^\psi i\, d\psi = W_{fe} + W_{em} \tag{10.5.6}$$

Static Energization

For any position of the rotor, a characteristic of flux linkage ψ against current i can be drawn. Fig. 10.3 shows the characteristics for positions A and B. For any stationary position of the rotor, the mechanical output is zero, and all the useful electrical energy input is connected into field energy.

$$W_f = \int_0^{\psi} i\, d\psi = \int_0^{\psi} \frac{\psi}{L}\, d\psi = \frac{\psi^2}{2L} \qquad (10.5.7)$$

Since $\psi = Li$ for a linear system

$$W_f = \frac{(Li)^2}{2L} = \frac{1}{2} Li^2 \qquad (10.5.8)$$

The areas OCD and OEF represent the stored energy for positions A and B respectively.

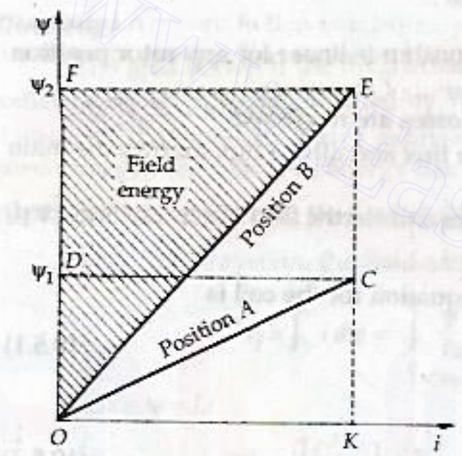


Fig. 10.3.

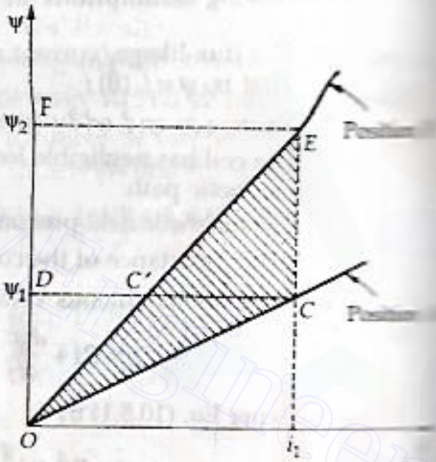


Fig. 10.4.

Dynamic Energization

It is useful to consider the rotor moving either very slowly or instantaneously from position A to position B.

Slow movement

In position A, the current is i_1 and the operating point is C. During the slow movement of the rotor, the counter emf $\frac{d\psi}{dt}$ is very small. Consequently, the exciting current i_1 remains substantially constant. The flux linkage may be assumed to increase from ψ_1 to ψ_2 at constant current i_1. Therefore, the operating point reaches the new operating point E along the vertical line CE as shown in Fig. 10.4.

Change in stored energy of magnetic field

$W_f =$ (magnetic energy stored in position B)

$\qquad$ − (magnetic energy stored in position A)

$$W_f = \text{area } OC'EFDO - \text{area } OCC'DO \qquad (10.5.9)$$

Electric energy input during this change

$$W_e = \int_{\psi_1}^{\psi_2} i_1\, d\psi = i_1(\psi_2 - \psi_1)$$

$$W_e = \text{area } CEFDC'C \tag{10.5.10}$$

But $\quad W_e = W_f + W_m$

$\therefore \quad$ area $CEFDC'C$ = (area $OC'EFDO$ − area $OCC'DO$) + W_m

or
$$W_m = \text{area } OCEFDO - \text{area } OC'EFDO$$
$$= \text{area } OCEC'O \tag{10.5.11}$$

Equation (10.5.11) shows that the mechanical work done is equal to the area enclosed between the two ψ–i characteristics in positions A and B and the vertical ψ–i locus during slow movement of the rotor.

It is also seen that for a linear system considered half of the useful electrical energy input is stored in the magnetic field and the other half appears as mechanical energy output during the slow movement of the rotor.

Electromagnetic torque

$$\tau_e = \lim_{\Delta\theta_m \to 0} \left\{ \frac{\Delta W_{fe}'}{\Delta \theta_m} \right\}_{\text{constant } i} \tag{10.5.12}$$

$$= \left\{ \frac{\partial W_{fe}'}{\partial \theta_m} \right\}_{\text{constant } i}$$

$$\tau_e = \left\{ \frac{\partial}{\partial \theta_m} \left(\frac{1}{2} L i^2 \right) \right\}_{\text{constant } i} \tag{10.5.13}$$

or
$$\tau_e = \frac{i^2}{2} \frac{\partial L}{\partial \theta_m} \tag{10.5.14}$$

For a linear system L is not a function of i

$$\tau_e = \frac{i^2}{2} \frac{dL}{d\theta_m} \tag{10.5.15}$$

Instantaneous movement

In this case, the rotor is assumed to move very fast from position A to position B. That is, the movement is instantaneous. According to the constant-flux-linkage theorem, the flux linkages with an inductive circuit cannot change suddenly. Thus, during the fast movement of the rotor, the flux linkage remains constant at ψ_1. Therefore, the operating point moves horizontally from C to C' (Fig. 10.5).

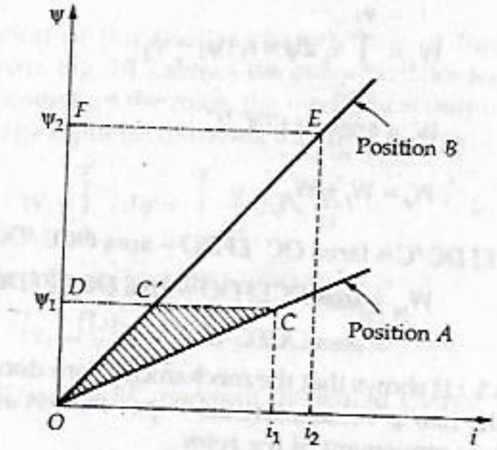


Fig. 10.5. *Instantaneous movement of rotor.*

Both ψ and i gradually increase to their steady-state values. The final operating point is E.

Since
$$W_e = \int_{\psi_1}^{\psi_2} i\, d\psi$$

and $\psi_2 = \psi_1$ during the rapid movement, therefore,

$$W_e = \int_{\psi_1}^{\psi_1} i\, d\psi = 0.$$

Thus, no energy is taken from the supply during the fast movement of the rotor and mechanical energy output is obtained from the stored magnetic field energy which reduces by an equal amount.

For an incremental movement of the rotor angle θ_m, the energy to mechanical work corresponds to the loss of stored magnetic field energy.

Electromagnetic torque

$$\tau_e = \lim_{\Delta\theta_m \to 0}\left\{-\frac{\Delta W_{fe}}{\Delta\theta_m}\right\}_{\text{constant } \psi}$$

$$= \left\{-\frac{\partial W_{fe}}{\partial\theta_m}\right\}_{\text{constant } \psi}$$

$$= \left\{-\frac{\partial}{\partial\theta_m}\frac{\psi^2}{2L}\right\}_{\text{constant } \psi}$$

$$\tau_e = -\frac{\psi^2}{2}\frac{1}{L^2}\frac{\partial L}{\partial\theta_m}$$

$\tau_e = \frac{i^2}{2}$

For a linear system

$\tau_e = \frac{i^2}{2}$

Transient movement

The actual rotor movemen
between the two extreme limits c
change during the mover
along a general path from C to

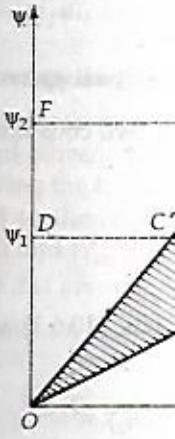


Fig. 10.6.

Here W_e = area CEFDC

W_f = area OEFO – ar

$W_m = W_e - W_f$

= area CEFDC – a

That is, the mechanical e
Since CE is a general m
numerically.

DOUBLY-EXCITED SYS

A doubly-excited magneti
Examples of such systems
loudspeakers, tachome
in Fig. 10.7. Let us consic
assumptions are as for a singly-
The flux linkage equations

$\psi_1 = L_1$

$\psi_2 = L_2$

or
$$\tau_e = \frac{i^2}{2}\frac{\partial L}{\partial \theta_m} \qquad (10.5.18)$$

For a linear system
$$\tau_e = \frac{i^2}{2}\frac{dL}{d\theta_m} \qquad (10.5.19)$$

Transient movement

The actual rotor movement will neither be too slow nor too fast, but will lie between the two extreme limits discussed above. In general, both current and flux linkage change during the movement of the rotor. The corresponding ψ–i relationship is a general path from C to E as shown in Fig. 10.6.

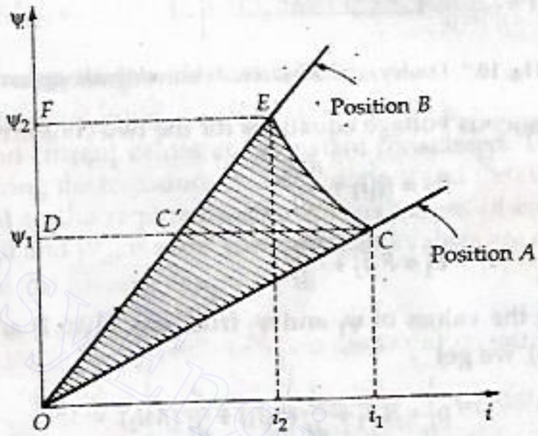


Fig. 10.6. *Transient movement of rotor.*

Here W_e = area $CEFDC$

W_f = area $OEFO$ – area $OCDO$

But $W_m = W_e - W_f$

= area $CEFDC$ – area $OEFO$ + area $OCDO$ = area $OCEO$.

That is, the mechanical energy output is given by the shaded area in Fig. 10.6. Since CE is a general movement, this area is to be computed graphically or numerically.

10.6 DOUBLY-EXCITED SYSTEM

A doubly-excited magnetic system has two independent sources of excitations. Examples of such systems are separately excited dc machines synchronous machine, loudspeakers, tachometers etc. A schematic of doubly excited system is shown in Fig. 10.7. Let us consider that both the stator and rotor have saliency. Assumptions are as for a singly-excited system.

The flux linkage equations for the two windings are
$$\psi_1 = L_1 i_1 + M i_2 \qquad (10.6.1)$$
$$\psi_2 = L_2 i_2 + M i_1 \qquad (10.6.2)$$

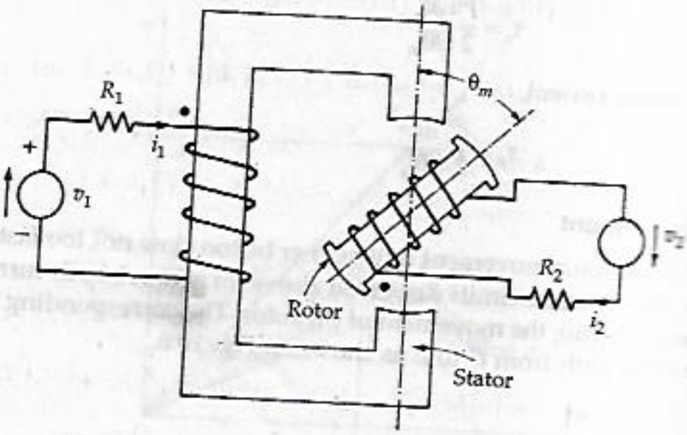


Fig. 10.7. *Doubly excited rotational electromagnetic system.*

The instantaneous voltage equations for the two coils are

$$v_1 = R_1 i_1 + \frac{d\psi_1}{dt}$$

$$v_2 = R_2 i_2 + \frac{d\psi_2}{dt}$$

Substituting the values of ψ_1 and ψ_2 from Eqs. (10.6.1) and (10.6.2) and (10.6.3) and (10.6.4), we get

$$v_1 = R_1 i_1 + \frac{d}{dt}(L_1 i_1) + \frac{d}{dt}(M i_2)$$

$$v_2 = R_2 i_2 + \frac{d}{dt}(L_2 i_2) + \frac{d}{dt}(M i_1)$$

Now the inductances are independent of currents and depend on the ... tion of the rotor angle θ_m which is a function of time. Similarly, currents are ... dependent and are not functions of inductances. Therefore, Eqs. (10.6.5) and (10.6.6) can be written as

$$v_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + i_1 \frac{dL_1}{dt} + M \frac{di_2}{dt} + i_2 \frac{dM}{dt}$$

$$v_2 = R_2 i_2 + L_2 \frac{di_2}{dt} + i_2 \frac{dL_2}{dt} + M \frac{di_1}{dt} + i_1 \frac{dM}{dt}$$

Multiplying Eq. (10.6.7) by i_1 and Eq. (10.6.8) by i_2 we get

$$v_1 i_1 = R_1 i_1^2 + L_1 i_1 \frac{di_1}{dt} + i_1^2 \frac{dL}{dt} + i_1 M \frac{di_2}{dt} + i_1 i_2 \frac{dM}{dt}$$

$$v_2 i_2 = R_2 i_2^2 + L_2 i_2 \frac{di_2}{dt} + i_2^2 \frac{dL_2}{dt} + i_2 M \frac{di_1}{dt} + i_1 i_2 \frac{dM}{dt}$$

Equations (10.6.9) and (10.6.10) are the power equations for the ...

Integrating Eqs. (10.6.9) and (10.6.10) with respect to time and adding, we get

$$\int (v_1 i_1 + v_2 i_2)\, dt = \int (R_1 i_1^2 + R_2 i_2^2)\, dt + \int (L_1 i_1\, di_1 + L_2 i_2\, di_2 + i_1 M\, di_2 + 2\, i_1 i_2\, dM + i_1^2\, dL_1 + i_2^2\, dL_2 + i_2 M\, di_1) \tag{10.6.11}$$

Also,
$$\begin{bmatrix} \text{Useful electrical} \\ \text{energy input} \end{bmatrix} = \int (v_1 i_1 + v_2 i_2)\, dt - \int (R_1 i_1^2 + R_2 i_2^2)\, dt \tag{10.6.12}$$

$$\begin{bmatrix} \text{Energy to field} \\ \text{storage in the} \\ \text{electrical system} \end{bmatrix} + \begin{bmatrix} \text{Electrical to} \\ \text{mechanical} \\ \text{energy} \end{bmatrix}$$

$$= \int (L_1 i_1\, di_1 + L_2 i_2\, di_2 + i_1 M\, di_2 + 2 i_1 i_2\, dM + i_1^2\, dL_1 + i_2^2\, dL_2 + i_2 M\, di_1) \tag{10.6.13}$$

10.6.1 Stored Energy in the Magnetic Field

The instantaneous value of energy stored in the magnetic field depends on the inductance and current values at the instant considered. This energy may be found by considering the transducer to be stationary and the coils to be energized from zero current to the required instantaneous values of current. There is no mechanical output and W_{em} is zero. The inductance values are constant. Therefore terms dL_1, dL_2 and dM become zero.

From Eq. (10.6.13)

$$\int dW_{fe} = \int_0^{i_1} L_1 i_1\, di_1 + \int_0^{i_2} L_2 i_2 di_2 + \int_0^{i_1, i_2} (i_2 M di_1 + i_1 M\, di_2)$$

$$[\text{total } W_{fe}] = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M i_1 i_2 \tag{10.6.14}$$

10.6.2 Electromagnetic Torque

Equation (10.6.14) holds for any transducer position. If the transducer rotates, the rate of change of field energy with respect to time is given by differentiating Eq. (10.6.14)

$$\frac{dW_{fe}}{dt} = \frac{1}{2} L_1 \frac{d}{dt} i_1^2 + \frac{1}{2} i_1^2 \frac{dL_1}{dt} + \frac{1}{2} L_2 \frac{di_2^2}{dt^2} + \frac{1}{2} i_2^2 \frac{dL_2}{dt} + i_1 i_2 \frac{dM}{dt} + i_1 M \frac{di_2}{dt} + i_2 M \frac{di_1}{dt}$$

$$\frac{dW_{fe}}{dt} = L_1 i_1 \frac{di_1}{dt} + \frac{1}{2} i_1^2 \frac{dL_1}{dt} + L_2 i_2 \frac{di_2}{dt} + \frac{1}{2} i_2^2 \frac{dL_2}{dt} + i_1 i_2 \frac{dM}{dt} + i_1 M \frac{di_2}{dt} + i_2 M \frac{di_1}{dt} \tag{10.6.15}$$

Integrating Eq. (10.6.15) with respect to time

$$\int dW_{fe} = W_{fe} = \int (L_1 i_1\, di_1 + \frac{1}{2} i_1^2\, dL_1 + L_2 i_2\, di_2 + \frac{1}{2} i_2^2\, dL_2 + i_1 i_2\, dM + i_1 M\, di_2 + i_2 M\, di_1) \tag{10.6.16}$$

This is a general equation for a moving transducer in which L_1, L_2, M, i_1 and i_2 are all varying with position and time. Comparing Eq. (10.6.16) with Eq. (10.6.13), we get

$$W_{em} = \begin{bmatrix} \text{electrical to} \\ \text{mechanical} \\ \text{energy} \end{bmatrix} = \int \left(\frac{1}{2} i_1^2\, dL_1 + \frac{1}{2} i_2^2\, dL_2 + i_1 i_2\, dM\right) \qquad (10.6.17)$$

Differentiating Eq. (10.6.17) with respect to θ_m,

$$\frac{dW_{em}}{d\theta_m} = \frac{1}{2} i_1^2 \frac{dL_1}{d\theta_m} + \frac{1}{2} i_2^2 \frac{dL_2}{d\theta_m} + i_1 i_2 \frac{dM}{d\theta_m} \qquad (10.6.18)$$

as only L_1, L_2 and M are dependent on θ_m.

Equation (10.6.18) includes the case of singly-excited system when one of the two currents is equal to zero so that the expression for the torque becomes

$$\tau_e = \frac{i^2}{2} \frac{dL}{d\theta_m} \qquad (10.6.19)$$

The first two terms of the torque equation (10.6.18) are **reluctance torques** or **saliency torques**. The last term $i_1 i_2 \frac{dM}{d\theta}$ is called the **co-alignment torque**, that is, two superimposed fields, that try to align.

For machines having uniform air gaps reluctance torque is not produced.

EXAMPLE 10.1. *A rotating transducer shown in Fig. 10.2 has a linear relationship between flux linkage and current. The inductance varies as* $(L_{r1} + L_{r2} \cos 2\theta)$.

(*a*) *Derive a general expression for torque.*

(*b*) *Calculate the average torque when the rotor has a constant angular velocity* ω *radians per-second, that is* $\theta = \omega t$ *and the current varies as* $I_m \cos (\omega t + \delta)$.

SOLUTION. (*a*) $\tau_e = \frac{1}{2} i^2 \frac{dL}{d\theta}$

$$= \frac{1}{2} i^2 \frac{d}{d\theta} (L_{r1} + L_{r2} \cos 2\theta)$$

$$\tau_e = -i^2 L_{r2} \sin 2\theta$$

With steady current, the torque oscillates sinusoidally, but the total torque is zero over one revolution.

(*b*) With a sinusoidally varying current $i = I_m (\cos \omega t + \delta)$

Since $\theta = \omega t$, $i = I_m \cos (\theta + \delta)$

and $\tau_e = -i^2 L_{r2} \sin 2\theta$

$$= -\left[I_m \cos (\omega t + \delta) \right]^2 L_{r2} \sin 2\theta$$

The average torque over a cycle

$$T_e\,(\text{average}) = \frac{1}{2\pi}\int_0^{2\pi} \tau_e\, d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} -\left[I_m \cos(\theta+\delta)\right]^2 L_{r2} \sin 2\theta\, d\theta$$

$$= \frac{-I_m^2}{2\pi}\int_0^{2\pi}\left[\frac{1+\cos 2(\theta+\delta)}{2}\right] \sin 2\,\theta\, d\theta$$

$$= -\frac{I_m^2 L_{r2}}{4\pi}\int_0^{2\pi}\left[\sin 2\theta + \frac{\sin 2\,(2\theta+\delta) - \sin 2\delta}{2}\right] d\theta$$

$$= \frac{I_m^2 L_{r2}}{4\pi}\int_0^{2\pi}\left[-\sin 2\theta - \frac{\sin 2\,(2\,\theta+\delta) - \sin 2\delta}{2}\right] d\theta$$

$$= \frac{I_m^2 L_{r2}}{4\pi}\left[\frac{\cos 2\theta}{2} + \frac{\cos 2\,(2\theta+\delta)}{8} + \frac{\theta \sin 2\delta}{2}\right]_0^{2\pi}$$

$$= \frac{I_m^2 L_{r2}}{4\pi}\cdot\frac{2\pi}{2}\sin 2\delta.$$

$$\tau_e\,(\text{average}) = 0.25\, I_m^2\, L_{r2} \sin 2\delta \qquad \text{Nm.}$$

Useful torque is developed when the angular velocity is equal to the angular frequency of the supply current. Single-phase reluctance motors operate on this principle.

EXAMPLE 10.2. *For the doubly-excited system shown in Fig. 10.7, the inductances are approximated as follows :*

$$L_1 = 11 + 3\cos 2\theta \quad H\,; \qquad L_2 = 7 + 2\cos 2\theta \quad H\,; \qquad M = 11\cos\theta \quad H$$

The coils are energized with direct currents $I_1 = 0.7$ A, $I_2 = 0.8$ A

(*a*) *Find the torque as a function of θ, and its value when θ = – 50.*

(*b*) *Find the energy stored in the system as a function of θ.*

SOLUTION. (*a*) Torque is given by

$$\tau = \frac{1}{2}\, i_1^2 \frac{dL_2}{d\theta} + \frac{1}{2}\, i_2^2 \frac{dL_2}{d\theta} + i_1\, i_2 \frac{dM}{d\theta}$$

$$= \frac{1}{2}(0.7)^2 \frac{d}{d\theta}(11 + 3\cos 2\theta) + \frac{1}{2}(0.8)^2 \frac{d}{d\theta}(7 + 2\cos 2\theta)$$

$$+ (0.7)\,(0.8)\,\frac{d}{d\theta}(11\cos\theta)$$

$$= \frac{1}{2}(0.7)^2(-6\sin 2\theta) + \frac{1}{2}(0.8)^2(-4\sin 2\theta) + (0.7)(0.8)(-11 \sin\theta)$$

$$\tau = -2.75 \sin 2\theta - 6.16 \sin\theta \quad \text{Nm}$$

For $\theta = -50°$

$$\tau = -2.75 \sin(-100°) - 6.16 \sin(-50°)$$

$$= 7.427 \quad \text{Nm}$$

This torque acts counterclockwise on the rotor. If this rotor is allowed to rotate, it will move to the position where $\theta = 0°$ and where the torque is zero. The torque is also zero at $\theta = 180°$, but if the position is away from 180°, the direction of the torque will tend to move the rotor towards 0°).

(*b*) The stored energy is given by

$$W_{fe} = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 + M i_1 i_2$$

$$= \frac{1}{2}(11 + 3\cos 2\theta)(0.7)^2 + \frac{1}{2}(7 + 2\cos 2\theta)(0.8)^2$$

$$+ (11\cos\theta)(0.7)(0.8)$$

$$= 4.935 + 1.375\cos 2\theta + 6.16\cos\theta$$

It is seen that maximum energy is stored when $\theta = 0°$. Energy stored has positive value for any position of the rotor.

EXERCISES

10.1 Define field energy and coenergy. What is the significance of coenergy ?

10.2 Show that the energy stored in a magnetic field is equal to the area between ψ–i curve for the system and the flux-linkage (ψ) axis.

10.3 Show that the energy stored in a magnetic field is equal to the area between Φ–F curve and the flux axis.

10.4 Show that the field energy in a linear magnetic system is given by

$$W_f = \frac{1}{2}L i^2 = \frac{1}{2}\psi i = \frac{1}{2L}\psi^2$$

10.5 Define field energy and coenergy. Prove that field energy and coenergy in a linear magnetic system are given by identical expressions.

10.6 (*a*) Distinguish between singly-excited and doubly-excited systems.

(*b*) For a singly-excited linear magnetic system, derive an expression for electromagnetic torque.

10.7 Why most practical energy conversion devices use magnetic field as the coupling medium between electrical and mechanical systems ?

10.8 Derive an expression for the torque in a doubly-excited system having salient type of stator as well as rotor. State the assumptions made.

10.9 State the electromagnetic phenomena useful for the electromagnetic energy conversion in rotating electric machines.

10.10 Show that in a singly-excited system the mechanical work done is equal to area enclosed between the two ψ–i characteristics in initial and final positions and the vertical ψ–i locus during the slow movement of the rotor.

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INTRODUCTION

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11

Basic Concepts of Rotating Electric Machines

11.1 INTRODUCTION

All electric machines have many similar properties and features. The basic common features in rotating machines are given in the following sections.

The torque production in a machine can be considered in terms of the instantaneous flux pattern. In this concept torque is produced in a machine when the resultant magnetic field has distortion or asymmetry.

The two basic magnetic field effects resulting in the production of mechanical forces are :

(*i*) Alignment of flux lines

(*ii*) Interaction between magnetic fields and current-carrying conductors.

The magnetic fields in practical devices and machines are invariably produced by current-energized coil systems to make a versatile and economic arrangement. With a single-energized coil, the device is called a **singly-excited system. Doubly excited systems** have two coils which are arranged with one on the *stationary part* and one on the *moving part.*

In a singly-excited system a torque is exerted on the magnetic material to align it with the magnetic field or move it into a stronger field. Such a torque is known as *saliency torque*. In a doubly-excited system, a torque is produced by the co-alignment of two magnetic fields.

11.2 BASIC STRUCTURE OF ROTATING ELECTRIC MACHINES

A rotating electric machine has two main parts, stator and rotor, separated by the air gap. The **stator** is the stationary part of the machine. Normally, it is the outer frame of the machine. The **rotor** is the rotating part of the machine. Solid or laminated ferromagnetic materials are used for the stator and rotor to reduce the reluctance of the flux paths. Most rotating machines have windings on the rotating and stationary members. The winding in which voltage is induced is called the **armature winding**. The winding through which a current is passed to produce the main flux is called the **field winding**. Permanent magnets are used in some machines to produce the main flux of the machine.

11.3 DC MACHINE

In the dc machine the field winding is placed on the stator and the armature winding on the rotor. Direct current (dc) is passed through the field winding to produce flux in the machine. Voltage induced in the armature winding is alternating. A mechanical commutator and a brush assembly function as a rectifier or inverter making the armature terminal voltage unidirectional.

11.4 INDUCTION MACHINE

In an induction machine, the stator windings serve as both armature windings and field windings. When the stator windings are connected to an ac supply, flux is produced in the air gap. This flux rotates at a fixed speed called synchronous speed. This rotating flux induces voltages in the stator and rotor windings. If the rotor circuit is closed, current flows through the rotor winding and reacts with the rotating flux and a torque is produced. In the steady state, the rotor rotates at a speed very close to synchronous speed. Two types of induction motor rotors are used :

(*a*) Squirrel-cage rotor or simply cage rotor.

(*b*) Wound rotor or slip-ring rotor.

11.5 SYNCHRONOUS MACHINE

In a synchronous machine, the stator carries the armature winding and rotor carries the field winding. The field winding is excited by direct current to produce flux in the air gap. When the rotor rotates, voltage is induced in the armature winding. The armature current produces a rotating flux in the air gap. The speed of this flux is the same as the speed of the rotor. There are two types of rotor constructions, namely, the salient-pole type and the cylindrical-pole type.

11.6 MMF SPACE WAVE OF A CONCENTRATED COIL

Consider a full-pitch coil on the stator of a 2-pole uniform gap ac machine. The coil consists of N turns and each turn carries a current i as shown in Fig. 11.1(*a*).

The direction of current in the two coil sides is shown by a cross ⊗ and dot ⊙. The magnetic flux set up by this coil current is shown by dotted lines in Fig. 11.1(*a*).

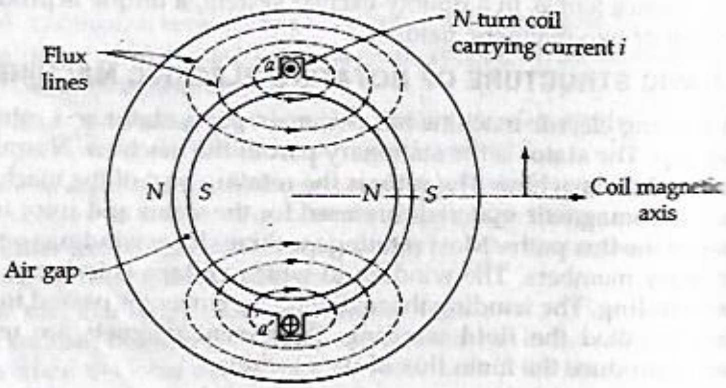

Fig. 11.1. *(a) Full-pitch coil on stator.*

The developed view is shown in Fig. 11.1(*b*). A north and a corresponding south pole are induced in the stator periphery. The magnetic axis of the coil is from the stator north to stator south.

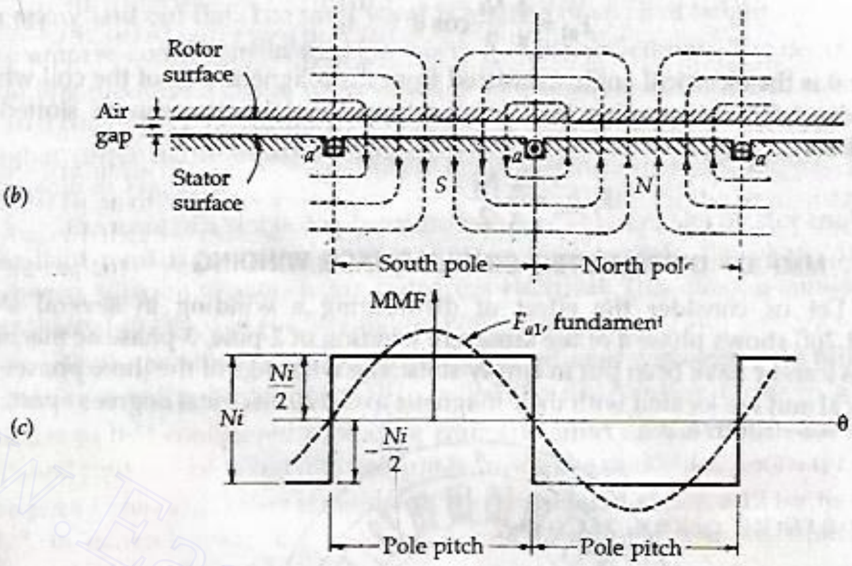

Fig. 11.1. (*b*) *Developed view of Fig. 11.1(a).* (*c*) *MMF distribution along the air gap periphery.*

The following assumptions are made to determine the distribution of coil mmf :

1. The relative permeability of the stator and rotor cores is infinite and the reluctance to the magnetic flux is offered by the air gap alone.
2. The air gap flux is radial and the field distortion in the vicinity of the coil is neglected.
3. The gap length is small with respect to the rotor diameter and the flux density is assumed to be constant along a radius.

Each flux line radially crosses the air gap twice normal to the stator and rotor surfaces. The mmf for any closed path is Ni. Since the reluctance of the ion path is assumed to be negligible, half the mmf $\left(\frac{Ni}{2}\right)$ is used to set up flux from the rotor to the stator in the air gap and the other half is used to establish flux from the stator to the rotor in the air gap. In other words, mmf for each air gap is $\left(\frac{Ni}{2}\right)$. The air gap mmf on the opposite sides of the rotor is equal in magnitude and opposite in direction. Mmf outwards from the rotor to the stator is assumed to be positive and from stator to the rotor is negative. Fig. 11.1(*c*) shows the air-gap mmf distribution. It is rectangular space wave where mmf of $(+ Ni/2)$ in used in setting up flux from the rotor to stator and mmf of $(- Ni/2)$ is consumed in setting up the flux from the stator to the rotor. It has been assumed that the coil sides occupy a narrow space on the stator and the mmf changes abruptly from $(- Ni/2)$ to $(+ Ni/2)$ at one slot and in the reverse direction at the other slot.

The rectangular mmf space wave of a single concentrated full-pitch coil
be resolved into a Fourier series comprising a fundamental component and a
of odd harmonics. By Fourier series, the fundamental component is

$$F_{a1} = \frac{4}{\pi}\frac{Ni}{2}\cos\theta$$

where θ is the electrical angle measured from the magnetic axis of the coil
coincides with the positive peak of the fundamental wave shown
Fig. 11.1(c). It is a sinusoidal space wave of peak-value

$$F_{1\,peak} = \frac{4}{\pi}\frac{Ni}{2}.$$

11.7 MMF OF DISTRIBUTED SINGLE-PHASE WINDING

Let us consider the effect of distributing a winding in several
Fig. 11.2(*a*) shows phase *a* of the armature winding of 2-pole, 3-phase ac
Phases *b* and *c* have been put in empty slots. The windings of the three
identical and are located with their magnetic axes 120 electrical degrees

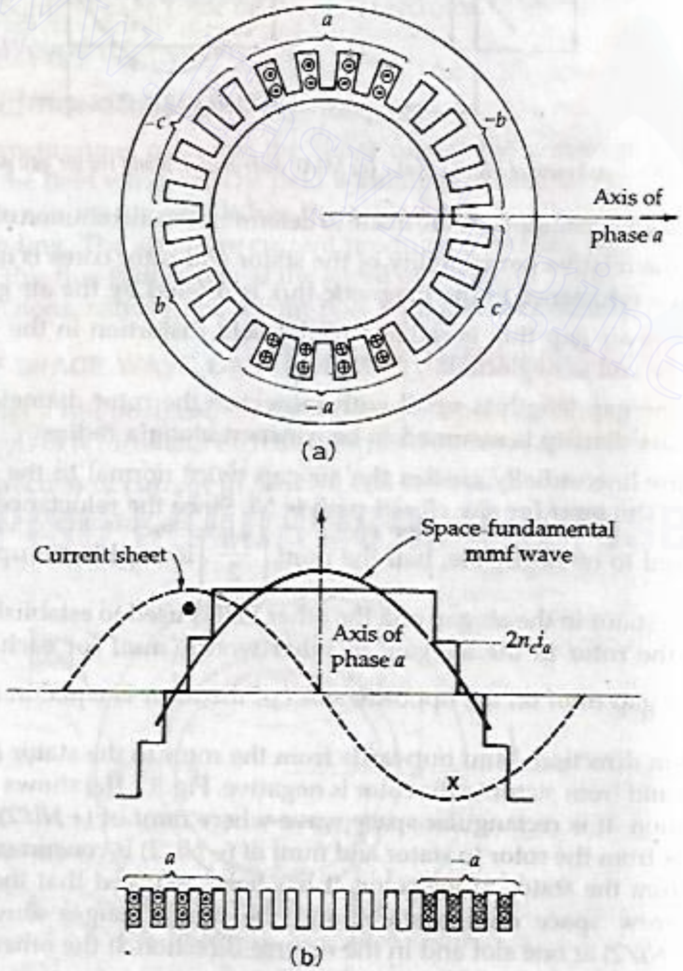

Fig. 11.2. MMF of one phase of a 2 pole, 3-phase winding having full-pitch coils

us consider the mmf of phase a alone. The winding is arranged in two layers, each coil of n_c turns having one side in the top of a slot and the other coil side in the bottom of a slot, nearly one pole pitch away. Fig. 11.2(b) shows one pole of this winding laid out flat. The mmf wave is a series of steps of height $2\pi n_c i_a$ equal to the ampere-conductors in the slot, where i_a is the coil current. The distribution of winding produces a closer approximation to a sinusoid as compared to a concentrated coil (Fig. 11.1). The mmf wave can be split into fundamental component and higher order harmonics. The space-fundamental component is shown by the sinusoid in Fig. 11.2.

The mmf of a slot is displaced from that of adjacent slot by slot angle β. The resultant mmf is found by phasor addition of slot mmfs, the phase difference between adjacent phasors being α degrees electrical. This effect is similar to that considered in Sec. 3.13 for finding out emf.

Thus, the effect of distributed winding on mmf wave can be accounted for by using a multiplication factor k_{dk}, called the **distribution factor** in a manner similar to that considered in finding emf. The effect on the mmf wave of short-pitched coils can be taken into account by using the multiplication factor k_p called the *pitch factor*. This effect is similar to that considered in Sec. 3.12 for finding out emf. In general, when the winding is both distributed and short-pitched, the fundamental space mmf of phase a is given by

$$F_{a1} = \frac{4}{\pi} k_p k_d \frac{N_{ph}}{P} i_a \cos\theta \tag{11.7.1}$$

where k_p = pitch factor

k_d = distribution factor

N_{ph} = number of turns in series per phase

P = number of poles.

Equation (11.7.1) describes the space fundamental component of the mmf wave produced by current in phase a. It is equal to the mmf wave produced by a finely divided sinusoidally distributed current sheet placed on the inner periphery of the stator as shown in Fig. 11.2(b). This component of mmf is a standing wave whose spatial distribution around the periphery is described by $\cos\theta$. Its peak is along the magnetic axis of phase a and its maximum value is proportional to the instantaneous current i_a.

Since $i_a = I_m \cos\omega t$, the maximum value of F_{a1} is

$$F_{max} = \frac{4}{\pi} k_p k_d \frac{N_{ph}}{P} I_m \tag{11.7.2}$$

11.8 MMF OF THREE-PHASE WINDINGS, ROTATING MAGNETIC FIELD

Fig. 11.3 shows the stator of a 2-pole, 3-phase machine. The three-phase windings, represented by aa', bb', and cc', are displaced from each other by 120 electrical degrees in space around the inner circumference of the stator. The concentrated full pitch coils represent the actual distributed windings.

Let us now consider the three phases of an ac winding carrying balanced alternating currents.

$$i_a = I_m \cos \omega t \qquad (11.8.1)$$

$$i_b = I_m \cos (\omega t - 120°) \qquad (11.8.2)$$

$$i_c = I_m \cos (\omega t + 120°) \qquad (11.8.3)$$

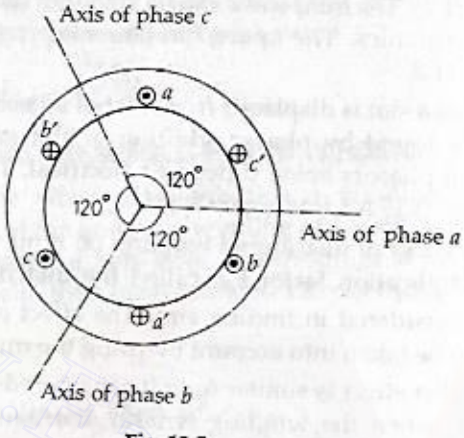

Fig. 11.3.

When these currents flow through the respective phase windings, each produces a sinusoidally distributed mmf wave in space along its axis and having a peak located along the axis. Each mmf wave can be represented by a space phasor along the axis of its phase with magnitude proportional to the instantaneous value of the current. The resultant mmf wave is the net effect of the three component mmf waves. The resultant can be found either analytically or graphically.

11.8.1 Analytical Method

The resultant air gap mmf at any angle θ is due to contribution by all three phases. Let the angle θ be measured from the axis of phase a. The resultant mmf at angle θ is

$$F(\theta) = F_a(\theta) + F_b(\theta) + F_c(\theta) \qquad (11.8.4)$$

At any instant of time, each phase winding produces a sinusoidally distributed mmf wave with its peak along the axis of the phase winding and amplitude proportional to the instantaneous value of the phase current. The contribution of phase a along θ is

$$F_a(\theta) = Ni_a \cos \theta \qquad (11.8.5)$$

where N is the effective number of turns in phase a and i_a is the current in phase a.

Since the phase axes are located 120 electrical degrees apart in space, the contributions of phases b and c are given by

$$F_b(\theta) = Ni_b \cos (\theta - 120°) \qquad (11.8.6)$$

$$F_c(\theta) = Ni_c \cos (\theta + 120°) \qquad (11.8.7)$$

Therefore, the resultant mmf at angle θ is

$$F(\theta) = Ni_a \cos\theta + Ni_b \cos(\theta - 120^\circ) + Ni_c \cos(\theta + 120^\circ) \tag{11.8.8}$$

The currents i_a, i_b and i_c are functions of time.

Substituting the values of i_a, i_b and i_c from Eqs. (11.8.1), (11.8.2) and (11.8.3) in Eq. (11.8.8), we get

$$F(\theta, t) = NI_m \cos\omega t \cos\theta + NI_m \cos(\omega t - 120^\circ)\cos(\theta - 120^\circ) + NI_m \cos(\omega t + 120^\circ)\cos(\theta + 120^\circ) \tag{11.8.9}$$

Using the trigonometric identity

$$\cos A \cos B = \frac{1}{2}\cos(A - B) + \frac{1}{2}\cos(A + B),$$

Eq. (11.8.9) can be written as

$$F(\theta, t) = \underbrace{\frac{1}{2}NI_m \cos(\omega t - \theta) + \frac{1}{2}NI_m \cos(\omega t - \theta) + \frac{1}{2}NI_m \cos(\omega t - \theta)}_{\text{Forward rotating components}} + \underbrace{\frac{1}{2}NI_m \cos(\omega t + \theta) + \frac{1}{2}NI_m \cos(\omega t + \theta - 240^\circ) + \frac{1}{2}NI_m \cos(\omega t + \theta + 240^\circ)}_{\text{Backward rotating components}} \tag{11.8.10}$$

$$= \frac{3}{2}NI_m \cos(\omega t - \theta) + 0$$

$$F(\theta, t) = \frac{3}{2}NI_m \cos(\omega t - \theta) \tag{11.8.11}$$

The expression of Eq. (11.8.11) represents the resultant mmf wave of constant magnitude in the air gap. The term ωt represents rotation of mmf around the gap at a constant angular velocity ω $(= 2\pi f)$. At any time t_1, the wave is distributed sinusoidally around the air gap with its positive peak along $\theta = \omega t_1$. At a later instant t_2, the positive peak of the sinusoidally distributed wave is along $\theta = \omega t_2$, that is, the wave has moved by $\omega(t_2 - t_1)$ around the air gap. At $t = 0$, I_a is maximum and the resultant wave is directed along the axis of phase a. One cycle (that is, 120 electrical degrees) later, the current in phase b is maximum, and the resultant mmf wave is directed along the axis of phase b and so on. The angular velocity of the mmf wave is ω $(= 2\pi f)$ radians per second. For a machine with P poles, the velocity of the rotating mmf is

$$\omega_s = \omega\left(\frac{2}{P}\right) \text{rad/s} \tag{11.8.16}$$

and the synchronous speed is

$$N_s = \frac{120f}{P} \text{rpm} \tag{11.8.17}$$

It can be shown in general that an m phase ($m \geq 3$) distributed winding excited by balanced m phase currents will produce a sinusoidally distributed rotating field of constant amplitude when the phase windings are wound (360/m) electrical degrees apart in space.

The maximum value of mmf for a single-phase winding is given by Eq. (11.7.2). The three-phase mmf has a constant amplitude equal to (3/2) times the maximum value of single-phase mmf. From Eqs. (11.7.2) and (11.8.11) the three-phase mmf is given by

$$F(\theta, t) = \frac{3}{2}\left[\frac{4}{\pi} k_p k_d \frac{N_{ph}}{P} I_m\right] \cos(\omega t - \theta).$$

11.9 GENERATED VOLTAGES IN AC MACHINES

Consider a 3-phase distributed winding. We have seen that when balanced polyphase currents flow through a polyphase distributed winding, a sinusoidally distributed rotating magnetic field is produced in the air gap of the machine. The rotating magnetic field will induce voltages in the phase coils aa', bb', and cc' (Fig. 11.4(a)). Expressions for the induced voltages can be found by using Faraday's law of electromagnetic induction.

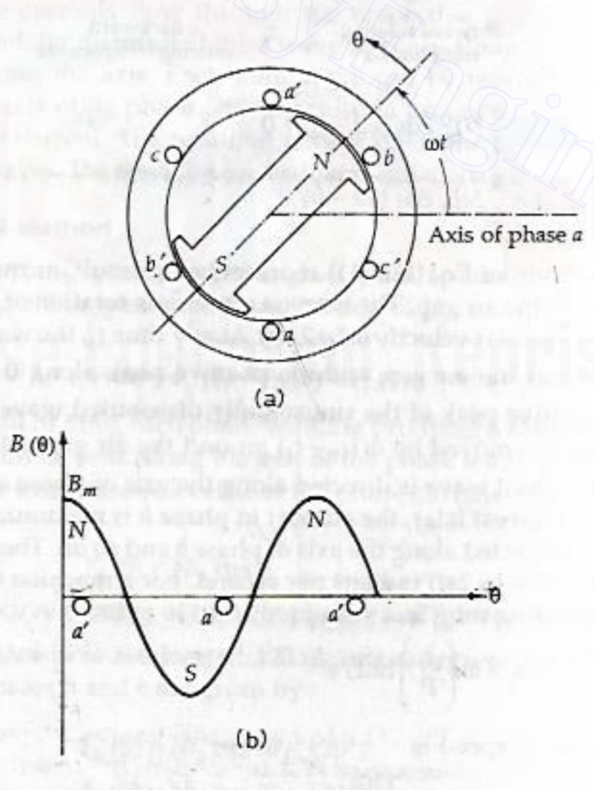

Fig. 11.4.

The flux density distribution in the air gap can be expressed as

$$B(\theta) = B_m \cos\theta \tag{11.9.1}$$

where B_m is the maximum value of the flux density at the rotor pole centre and θ is measured in electrical radians from the rotor pole axis.

For a 2-pole machine if l is the axial length of the stator at the air gap, then the air gap flux per pole, Φ_p, is

$$\Phi_p = \int_{-\pi/2}^{\pi/2} B(\theta)\, lr\, d\theta = 2\, B_m\, lr \tag{11.9.2}$$

If the machine has P poles, then area per pole is $(2/P)$ times for a 2-pole machine. Therefore, the flux per pole for a P–pole machine is

$$\Phi_p = \frac{2}{P}(2\, B_m\, lr) = \frac{4}{P} B_m\, lr \tag{11.9.3}$$

Let us consider that the phase coils are full pitch coils each of T turns. As the rotating field moves, the flux linkage of a coil will vary ; the flux linkage for coil aa' will be maximum $(= \Phi_p T)$ at $\omega t = 0°$ (Fig. 11.4(*b*)) and zero at $\omega t = 90°$. The flux linkage ψ will vary as the cosine of angle ωt.

Hence $\quad \psi = \Phi_p\, T \cos \omega t \qquad (11.9.4)$

By Faraday's law of induction, the voltage induced in the phase coil aa' is

$$e_a = \frac{d\,\psi}{dt} = -\frac{d}{dt}(\Phi_p\, T \cos \omega t)$$

$$e_a = \omega \Phi_p\, T \sin \omega t$$

$$e_c = E_m \sin \omega t \tag{11.9.5}$$

where E_m is the maximum value of the induced emf. The rms value E of induced emf is

$$E = \frac{E_m}{\sqrt{2}} = \frac{\omega\, \Phi_p\, T}{\sqrt{2}} = 2\pi f\, \Phi_p\, T$$

or $\quad E = 4.44\, f\, T\, \Phi_p \qquad (11.9.6)$

The voltages induced in other phase coils are also sinusoidal. They will be the same in magnitude but will be phase shifted from each other by 120 electrical degrees.

Thus,

$$e_b = E_m \sin(\omega t - 120°) \tag{11.9.7}$$

$$e_c = E_m \sin(\omega t + 120°) \tag{11.9.8}$$

Equation (11.9.6) has the same form as that for the induced voltage in transformers. However, Φ_p in Eq. (11.9.6) represents the flux per pole of the machine.

The expression of Eq. (11.9.6) represents the rms value of the induced voltage per phase for concentrated full pitch coils. In an actual machine, each phase winding is distributed in a number of slots and the windings are short-pitched (chorded). Therefore, the above expression must be multiplied by the span factor k_c (pitch factor) and the distribution factor k_d. The actual induced (generated) voltage per phase is given by

$$E_{ph} = 4.44\, k_c\, k_d\, f\, \Phi_p\, T_{ph}$$

The coil span factor and distribution factor are combined into a winding factor k_w which is the product of k_c and k_d. That is,

$$k_w \triangleq k_c\, k_d$$

$$\therefore \qquad E_{ph} = 4.44\, k_w\, f\, \Phi_p\, T_{ph}$$

where T_{ph} is the number of turns in series per phase.

11.10 MACHINE TORQUES

There are essentially two kinds of forces developed in electromechanical devices. The first is due to interaction of the fields produced by the currents in windings which may move relative to each other. This torque is called *electromagnet torque* or *induced torque*. The second, usually called *reluctance* ... *reluctance torque*, is dependent on the current in only one winding and is the ... of variations in the reluctance of the air gap in the magnetic circuit carrying ... flux which links that winding. Both these phenomena are often active simultaneously.

11.10.1 Electromagnetic Torque or Induced Torque in AC Machines

In ac machine under normal operating conditions there are two magnetic fields present-a magnetic field from the rotor circuit and another magnetic ... from the stator circuit. The interaction of these two magnetic fields produces torque in the machine.

11.11 TORQUE IN MACHINES WITH CYLINDRICAL AIR GAPS

Consider a machine with a uniform cylindrical air gap (Fig. 11.5(a)) ... sinusoidally distributed rotor and stator mmfs. Let F_1 and F_2 be the peak ... of the mmfs of the stator and rotor fields. The mmf waves of the stator and ... are sine waves in space with δ_{12} the phase angle between their magnetic ... electrical degrees. They can be represented by the space phasors $\mathbf{F}_1$ and $\mathbf{F}_2$ ... along the magnetic axes of the stator and rotor mmf waves respectively ... in Fig. 11.5(*b*).

The resultant $\mathbf{F}_R$ acting across the air gap is equal to the phasor sum ... and $\mathbf{F}_2$. It is also a sine wave. By parallelogram law of phasors

$$F_R^2 = F_1^2 + F_2^2 + 2F_1F_2\cos\delta_{12}$$

It is to be noted that F_1, F_2 and F_R in Eq. (11.11.1.) are the peak ... mmf waves. The resultant radial H field is a sinusoidal space wave ... value is H_R. It is given by

$$H_R = \frac{F_R}{g}$$

where g is the length of the air gap.

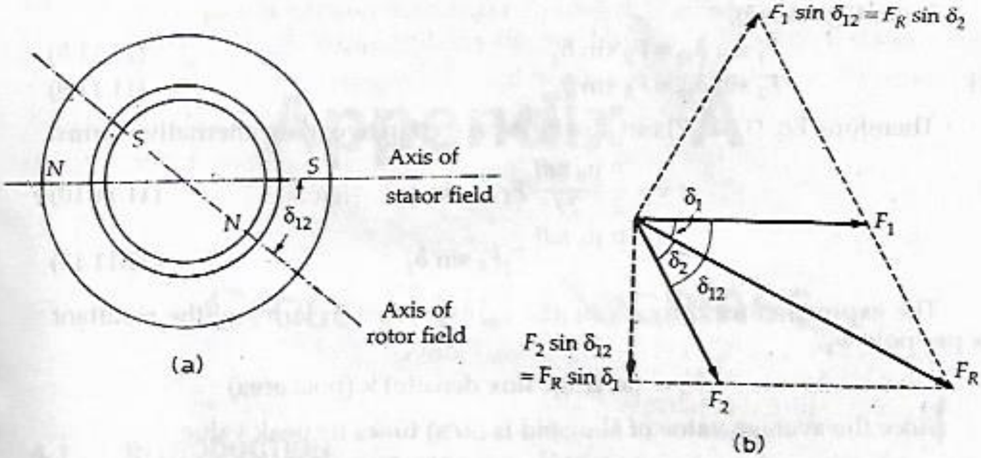

Fig. 11.5. *(a) Machine with a cylindrical airgap. (b) Phasor diagram.*

Total energy in the air gap

$$= \frac{1}{2}\mu_0 H^2 \times (\text{volume of air gap}) \tag{11.11.3}$$

Since H_R is sinusoidally distributed, the average value of $H^2 = \frac{1}{2} H_g^2$

If d and l and the average diameters of the core (at the air gap) and axial length of the core respectively,

$$\text{volume of the air gap} = \pi dlg \tag{11.11.4}$$

Therefore, the total energy in the air gap is

$$W_f = \frac{1}{2}\mu_0\left(\frac{1}{2}Hg^2\right)(\pi dlg) = \frac{1}{4}\mu_0\left(\frac{F_R^2}{g^2}\right)(\pi dlg)$$

$$W_f = \frac{\mu_0 \pi dl}{4g}(F_1^2 + F_2^2 + 2F_1F_2\cos\delta_{12}) \tag{11.11.5}$$

Since $$\tau = +\frac{\partial W_f}{\partial \delta_{12}}$$

$$\tau = -\frac{\mu_0 \pi dl}{2g} F_1 F_2 \sin\delta_{12} \tag{11.11.6}$$

Equation (11.11.6) is the torque per pole pair. Since the total number of pole pairs is $(P/2)$, the torque is

$$\tau = -\left(\frac{P}{2}\right)\left(\frac{\mu_0 \pi dl}{2g}\right) F_1 F_2 \sin\delta_{12} \tag{11.11.7}$$

Equation (11.11.7) shows that the torque is proportional to the peak values of the stator and rotor mmf waves F_1 and F_2 and to the sine of the electrical space-phase angle δ_{12} between them. The negative sign in Eq. (11.11.7) shows that the fields tend to align themselves so as to decrease the angle δ_{12}. Equal and opposite torques are exerted on the stator and rotor. The stator torque is transmitted to the foundation through the frame of the machine.

From Fig. 11.5(*b*),

$$F_1 \sin \delta_{12} = F_R \sin \delta_2 \quad (11.11.8)$$

and

$$F_2 \sin \delta_{12} = F_R \sin \delta_1 \quad (11.11.9)$$

Therefore, Eq. (11.11.7) can also be expressed in two more alternative forms

$$\tau = -\frac{P}{2}\frac{\mu_0 \pi dl}{2g} F_1 F_R \sin \delta_1 \quad (11.11.10)$$

$$\tau = -\frac{P}{2}\frac{\mu_0 \pi dl}{2g} F_2 F_R \sin \delta_2 \quad (11.11.11)$$

The expression for torque can also be expressed in terms of the resultant flux per pole Φ_R.

$$\Phi_R = \text{(average flux density)} \times \text{(pole area)}$$

Since the average value of sinusoid is $(2/\pi)$ times its peak value

$$\Phi_R = \left(\frac{2}{\pi} B_R\right)\left(\frac{\pi dl}{P}\right) = \frac{2 B_R\, dl}{P} \quad (11.11.12)$$

where B_R is the peak value of the corresponding flux density wave.

$$B_e = \mu_0 H_R = \mu_0 \frac{F_R}{g} \quad (11.11.13)$$

Combination of Eqs. (11.11.11), (11.11.12) and (11.11.13) gives

$$\tau = -\frac{\pi}{8} P^2 F_2 \Phi_R \sin \delta_2 \quad (11.11.14)$$

Equation (11.11.14) gives a very useful expression for the torque in machines having cylindrical air gaps.

11.12 RELUCTANCE TORQUE OR ALIGNMENT TORQUE

Reluctance torque is a torque experienced by a ferromagnetic object placed in an external magnetic field, which causes the object to line up with the external magnetic field. This torque occurs because the external magnetic field induces an internal magnetic field in the object, and a torque is produced between the two fields twisting the object around to line up with the external magnetic field. Thus a torque is exerted on the object so that it tries to position itself to give minimum reluctance for the magnetic flux. Reluctance torque is also called the *alignment torque* or *saliency torque*.

A reluctance motor depends on reluctance torque for its operation.

EXERCISES

11.1 Show that a 3-phase distributed winding excited by balanced 3-phase currents will produce a sinusoidally distributed rotating field of constant amplitude when the phase windings are wound 120 electrical degrees apart in space.

11.2 Deduce an expression for the generated voltage per phase of a 3-phase ac machine with distributed windings.

11.3 Define the terms induced torque and reluctance torque.

Derive an expression for electromagnetic torque in an ac machine with cylindrical air gap. State the assumptions made.

11.4 Derive an expression for the mmf of a 3-phase winding. State the assumptions made.

ELECTRIC MACHINES

Second Edition

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