Assignment 1

AI1110: Probability and Random Variables
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Exemplar, 10.13.3.39:
Question.
A die has its six faces marked 0, 1, 1, 1, 6, 6. Two such dice are thrown together and the total score is recorded.
(i) How many different scores are possible?
(ii) What is the probability of getting a total of 7? **Answer:**

i) 6
ii) $\frac{1}{3}$

Solution:
i) The possible sums are

- **0** (If both the times outcome is zero)
- **1** (If the outcome was 0 and 1 or viceversa)
- **2** (If both times the outcome was 1)
- **6** (If the outcome was 0 and 6 or viceversa
- **7** (If the outcome was 1 and 6 or viceversa
- **12** (If both times the outcome was 6)

$$\therefore 6 \text{ different scores are possible} \tag{1}$$

ii)The sum 7 can be obtained only if
$\boxed{1}\boxed{6}$ or $\boxed{6}\boxed{1}$
The total possible scores are 6 from equation (1)

$$\begin{aligned}\therefore \text{Required probability} &= \frac{2}{6} \\ &= \frac{1}{3}\end{aligned}$$

PMF of the distribution

x Outcome	0	1	6
P(X=x)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$

PMF of sum of numbers on the dice
Let X be the random variable representing the total score when two dice with markings 0, 1, 1, 1, 6, 6 are thrown together. Then the PMF of X can be expressed as:

$$P(X = x) = \begin{cases} \frac{1}{36} & x = 0 \\ \frac{1}{6} & x = 1 \\ \frac{1}{4} & x = 2 \\ \frac{1}{3} & x = 7 \\ \frac{1}{9} & x \in \{6, 12\} \\ 0 & \text{otherwise} \end{cases}$$

where x is the possible score, and the values of the PMF are given in the corresponding cases.

x Score	0	1	2	6	7	12
P(X=x)	$\frac{1}{36}$	$\frac{1}{6}$	$\frac{1}{4}$	$\frac{1}{9}$	$\frac{1}{3}$	$\frac{1}{9}$

Let x[n] be the discrete-time signal representing the probability mass function (PMF) for the total score when two dice with markings 0, 1, 1, 1, 6, 6 are thrown together. Then the Z transform of x[n] is given by:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] \cdot z^{-n}$$

where z is a complex variable and n is a discrete-time index. The Z transform is a powerful tool in digital signal processing, as it allows us to analyze and manipulate signals in the frequency domain. The inverse Z transform can also be used to recover the original signal x[n] from its Z transform X(z). [table]labelformat=empty,labelsep=none

Score	Probability
0	$\frac{1}{36}$
1	$\frac{1}{6}$
2	$\frac{1}{4}$
6	$\frac{1}{9}$
7	$\frac{1}{3}$
12	$\frac{1}{9}$