Multiclass Support Vector Machine exercise

Complete and hand in this completed worksheet (including its outputs and any supporting code outside of the worksheet) with your assignment submission. For more details see the <u>assignments page (https://compsci682-fa19.github.io/assignments2019/assignment1/)</u> on the course website.

In this exercise you will:

- implement a fully-vectorized loss function for the SVM
- implement the fully-vectorized expression for its analytic gradient
- check your implementation using numerical gradient
- use a validation set to tune the learning rate and regularization strength
- optimize the loss function with SGD
- · visualize the final learned weights

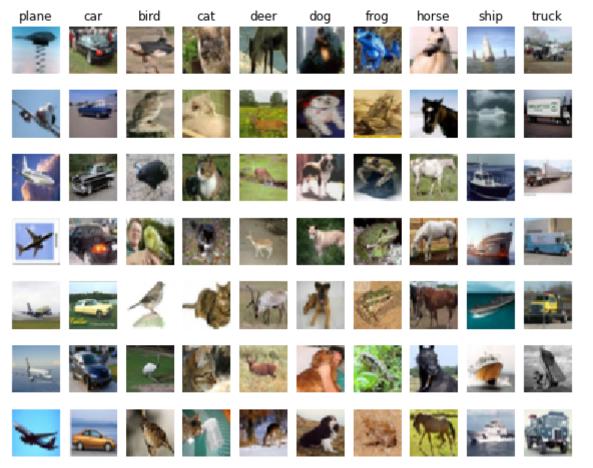
```
In [1]: # Run some setup code for this notebook.
        from future import print function
        import random
        import numpy as np
        from cs682.data_utils import load_CIFAR10
        import matplotlib.pyplot as plt
        # This is a bit of magic to make matplotlib figures appear inline in the
        # notebook rather than in a new window.
        %matplotlib inline
        plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
        plt.rcParams['image.interpolation'] = 'nearest'
        plt.rcParams['image.cmap'] = 'gray'
        # Some more magic so that the notebook will reload external python modul
        # see http://stackoverflow.com/questions/1907993/autoreload-of-modules-i
        n-ipython
        %load ext autoreload
        %autoreload 2
```

CIFAR-10 Data Loading and Preprocessing

```
In [2]: # Load the raw CIFAR-10 data.
        cifar10 dir = 'cs682/datasets/cifar-10-batches-py'
        # Cleaning up variables to prevent loading data multiple times (which ma
        y cause memory issue)
        try:
           del X_train, y_train
           del X test, y test
           print('Clear previously loaded data.')
        except:
           pass
        X train, y train, X test, y test = load CIFAR10(cifar10 dir)
        # As a sanity check, we print out the size of the training and test dat
        a.
        print('Training data shape: ', X_train.shape)
        print('Training labels shape: ', y_train.shape)
        print('Test data shape: ', X_test.shape)
        print('Test labels shape: ', y_test.shape)
```

Training data shape: (50000, 32, 32, 3)
Training labels shape: (50000,)
Test data shape: (10000, 32, 32, 3)
Test labels shape: (10000,)

```
In [3]: # Visualize some examples from the dataset.
        # We show a few examples of training images from each class.
        classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse'
        , 'ship', 'truck']
        num_classes = len(classes)
        samples per class = 7
        for y, cls in enumerate(classes):
            idxs = np.flatnonzero(y train == y)
            idxs = np.random.choice(idxs, samples_per_class, replace=False)
            for i, idx in enumerate(idxs):
                plt_idx = i * num_classes + y + 1
                plt.subplot(samples_per_class, num_classes, plt_idx)
                plt.imshow(X_train[idx].astype('uint8'))
                plt.axis('off')
                if i == 0:
                    plt.title(cls)
        plt.show()
```



```
In [4]: # Split the data into train, val, and test sets. In addition we will
        # create a small development set as a subset of the training data;
        # we can use this for development so our code runs faster.
        num_training = 49000
        num validation = 1000
        num test = 1000
        num dev = 500
        # Our validation set will be num validation points from the original
        # training set.
        mask = range(num training, num training + num validation)
        X val = X train[mask]
        y_val = y_train[mask]
        # Our training set will be the first num train points from the original
        # training set.
        mask = range(num training)
        X train = X train[mask]
        y_train = y_train[mask]
        # We will also make a development set, which is a small subset of
        # the training set.
        mask = np.random.choice(num_training, num_dev, replace=False)
        X dev = X train[mask]
        y dev = y train[mask]
        # We use the first num test points of the original test set as our
        # test set.
        mask = range(num test)
        X test = X test[mask]
        y_test = y_test[mask]
        print('Train data shape: ', X train.shape)
        print('Train labels shape: ', y_train.shape)
        print('Validation data shape: ', X_val.shape)
        print('Validation labels shape: ', y_val.shape)
        print('Test data shape: ', X test.shape)
        print('Test labels shape: ', y_test.shape)
        Train data shape: (49000, 32, 32, 3)
```

```
Train data shape: (49000, 32, 32, 3)
Train labels shape: (49000,)
Validation data shape: (1000, 32, 32, 3)
Validation labels shape: (1000,)
Test data shape: (1000, 32, 32, 3)
Test labels shape: (1000,)
```

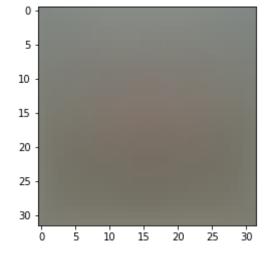
```
In [5]: # Preprocessing: reshape the image data into rows
    X_train = np.reshape(X_train, (X_train.shape[0], -1))
    X_val = np.reshape(X_val, (X_val.shape[0], -1))
    X_test = np.reshape(X_test, (X_test.shape[0], -1))
    X_dev = np.reshape(X_dev, (X_dev.shape[0], -1))

# As a sanity check, print out the shapes of the data
    print('Training data shape: ', X_train.shape)
    print('Validation data shape: ', X_val.shape)
    print('Test data shape: ', X_test.shape)
    print('dev data shape: ', X_dev.shape)
```

Training data shape: (49000, 3072) Validation data shape: (1000, 3072) Test data shape: (1000, 3072) dev data shape: (500, 3072)

In [6]: # Preprocessing: subtract the mean image
 # first: compute the image mean based on the training data
 mean_image = np.mean(X_train, axis=0)
 print(mean_image[:10]) # print a few of the elements
 plt.figure(figsize=(4,4))
 plt.imshow(mean_image.reshape((32,32,3)).astype('uint8')) # visualize the emean image
 plt.show()

[130.64189796 135.98173469 132.47391837 130.05569388 135.34804082 131.75402041 130.96055102 136.14328571 132.47636735 131.48467347]



In [7]: # second: subtract the mean image from train and test data
X_train -= mean_image
X_val -= mean_image
X_test -= mean_image
X_dev -= mean_image

```
In [8]: # third: append the bias dimension of ones (i.e. bias trick) so that our
    SVM
# only has to worry about optimizing a single weight matrix W.
    X_train = np.hstack([X_train, np.ones((X_train.shape[0], 1))])
    X_val = np.hstack([X_val, np.ones((X_val.shape[0], 1))])
    X_test = np.hstack([X_test, np.ones((X_test.shape[0], 1))])
    X_dev = np.hstack([X_dev, np.ones((X_dev.shape[0], 1))])
    print(X_train.shape, X_val.shape, X_test.shape, X_dev.shape)
(49000, 3073) (1000, 3073) (1000, 3073) (500, 3073)
```

SVM Classifier

Your code for this section will all be written inside cs682/classifiers/linear_svm.py.

As you can see, we have prefilled the function svm_loss_naive which uses for loops to evaluate the multiclass SVM loss function.

```
In [9]: # Evaluate the naive implementation of the loss we provided for you:
    from cs682.classifiers.linear_svm import svm_loss_naive
    import time

# generate a random SVM weight matrix of small numbers
W = np.random.randn(3073, 10) * 0.0001

loss, grad = svm_loss_naive(W, X_dev, y_dev, 0.000005)
    print('loss: %f' % (loss, ))
```

loss: 8.798825

The <code>grad</code> returned from the function above is right now all zero. Derive and implement the gradient for the SVM cost function and implement it inline inside the function <code>svm_loss_naive</code>. You will find it helpful to interleave your new code inside the existing function.

To check that you have correctly implemented the gradient correctly, you can numerically estimate the gradient of the loss function and compare the numeric estimate to the gradient that you computed. We have provided code that does this for you:

```
In [10]: # Once you've implemented the gradient, recompute it with the code below
         # and gradient check it with the function we provided for you
         # Compute the loss and its gradient at W.
         loss, grad = svm loss naive(W, X dev, y dev, 0.0)
         # Numerically compute the gradient along several randomly chosen dimensi
         ons, and
         # compare them with your analytically computed gradient. The numbers sho
         uld match
         # almost exactly along all dimensions.
         from cs682.gradient check import grad check sparse
         f = lambda w: svm loss naive(w, X dev, y dev, 0.0)[0]
         grad numerical = grad check sparse(f, W, grad)
         # do the gradient check once again with regularization turned on
         # you didn't forget the regularization gradient did you?
         loss, grad = svm loss naive(W, X dev, y dev, 5e1)
         f = lambda w: svm_loss_naive(w, X_dev, y_dev, 5e1)[0]
         grad numerical = grad check sparse(f, W, grad)
         numerical: 21.075643 analytic: 21.075643, relative error: 2.869140e-12
         numerical: -23.347696 analytic: -23.347696, relative error: 6.095826e-1
         numerical: 21.065230 analytic: 21.065230, relative error: 1.550595e-12
         numerical: -1.627262 analytic: -1.662916, relative error: 1.083635e-02
         numerical: 18.584493 analytic: 18.584493, relative error: 2.655212e-11
         numerical: 17.996371 analytic: 17.996371, relative error: 1.598412e-11
         numerical: 38.992193 analytic: 38.992193, relative error: 1.951559e-12
         numerical: 5.380711 analytic: 5.380711, relative error: 8.776999e-11
         numerical: 38.182320 analytic: 38.182320, relative error: 1.098502e-12
         numerical: -17.859809 analytic: -17.859809, relative error: 3.995553e-1
         numerical: -30.226370 analytic: -30.226370, relative error: 1.025274e-1
         numerical: -46.757943 analytic: -46.757943, relative error: 2.821629e-1
         numerical: 4.469970 analytic: 4.469970, relative error: 3.751814e-11
         numerical: 0.501495 analytic: 0.501495, relative error: 2.158923e-10
         numerical: -55.380953 analytic: -55.380953, relative error: 6.731372e-1
         numerical: -5.721403 analytic: -5.721403, relative error: 7.292765e-12
         numerical: -4.365053 analytic: -4.365053, relative error: 7.396147e-11
         numerical: 46.799835 analytic: 46.799835, relative error: 1.716847e-12
         numerical: 5.541011 analytic: 5.541011, relative error: 5.879854e-11
```

numerical: 3.597039 analytic: 3.597039, relative error: 4.434875e-11

Inline Question 1:

It is possible that once in a while a dimension in the gradcheck will not match exactly. What could such a discrepancy be caused by? Is it a reason for concern? What is a simple example in one dimension where a gradient check could fail? How would change the margin affect of the frequency of this happening? *Hint: the SVM loss function is not strictly speaking differentiable*

Your Answer: Yes, it is possible that once in a while a dimension in the gradcheck will not match exactly. It could be caused by the discontinuous nature of the SVM function; meaning it is not strictly differentiable. Such an example in one dimension would be x = 0.001. L(x) = max(0, x) = 0. So the analytical gradient would be 0, whereas, the numerical gradient would be non-zero.

```
In [11]: # Next implement the function svm loss vectorized; for now only compute
          the loss;
         # we will implement the gradient in a moment.
         tic = time.time()
         loss naive, grad naive = svm loss naive(W, X dev, y dev, 0.000005)
         toc = time.time()
         print('Naive loss: %e computed in %fs' % (loss_naive, toc - tic))
         from cs682.classifiers.linear svm import svm loss vectorized
         tic = time.time()
         loss_vectorized, _ = svm_loss_vectorized(W, X_dev, y_dev, 0.000005)
         toc = time.time()
         print('Vectorized loss: %e computed in %fs' % (loss vectorized, toc - ti
         C))
         # The losses should match but your vectorized implementation should be m
         uch faster.
         print('difference: %f' % (loss naive - loss vectorized))
```

Naive loss: 8.798825e+00 computed in 0.079120s Vectorized loss: 8.798825e+00 computed in 0.004615s

difference: 0.000000

```
In [12]: # Complete the implementation of svm loss vectorized, and compute the gr
         adient
         # of the loss function in a vectorized way.
         # The naive implementation and the vectorized implementation should matc
         h, but
         # the vectorized version should still be much faster.
         tic = time.time()
         _, grad_naive = svm_loss_naive(W, X_dev, y_dev, 0.000005)
         toc = time.time()
         print('Naive loss and gradient: computed in %fs' % (toc - tic))
         tic = time.time()
         _, grad_vectorized = svm_loss_vectorized(W, X_dev, y_dev, 0.000005)
         toc = time.time()
         print('Vectorized loss and gradient: computed in %fs' % (toc - tic))
         # The loss is a single number, so it is easy to compare the values compu
         # by the two implementations. The gradient on the other hand is a matri
         X, SO
         # we use the Frobenius norm to compare them.
         difference = np.linalg.norm(grad_naive - grad_vectorized, ord='fro')
         print('difference: %f' % difference)
```

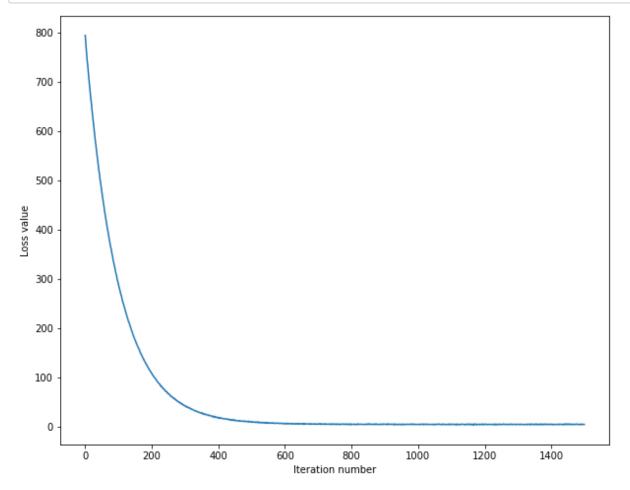
Naive loss and gradient: computed in 0.084399s Vectorized loss and gradient: computed in 0.005446s difference: 0.000000

Stochastic Gradient Descent

We now have vectorized and efficient expressions for the loss, the gradient and our gradient matches the numerical gradient. We are therefore ready to do SGD to minimize the loss.

```
iteration 0 / 1500: loss 794.912996
iteration 100 / 1500: loss 287.932956
iteration 200 / 1500: loss 108.324655
iteration 300 / 1500: loss 42.736626
iteration 400 / 1500: loss 19.395020
iteration 500 / 1500: loss 9.680554
iteration 600 / 1500: loss 7.062760
iteration 700 / 1500: loss 5.893017
iteration 800 / 1500: loss 5.411213
iteration 900 / 1500: loss 5.645085
iteration 1000 / 1500: loss 5.285874
iteration 1100 / 1500: loss 4.944847
iteration 1200 / 1500: loss 5.103733
iteration 1300 / 1500: loss 5.317000
iteration 1400 / 1500: loss 5.734901
That took 3.601004s
```

```
In [14]: # A useful debugging strategy is to plot the loss as a function of
    # iteration number:
    plt.plot(loss_hist)
    plt.xlabel('Iteration number')
    plt.ylabel('Loss value')
    plt.show()
```



```
In [15]: # Write the LinearSVM.predict function and evaluate the performance on b
    oth the
    # training and validation set
    y_train_pred = svm.predict(X_train)
    print('training accuracy: %f' % (np.mean(y_train == y_train_pred), ))
    y_val_pred = svm.predict(X_val)
    print('validation accuracy: %f' % (np.mean(y_val == y_val_pred), ))
```

training accuracy: 0.372898 validation accuracy: 0.375000

```
In [16]: # Use the validation set to tune hyperparameters (regularization strengt
        h and
        # learning rate). You should experiment with different ranges for the le
        # rates and regularization strengths; if you are careful you should be a
        # get a classification accuracy of about 0.4 on the validation set.
        learning rates = [1e-7, 5e-5]
        regularization strengths = [2.5e4, 5e4]
        # results is dictionary mapping tuples of the form
        # (learning rate, regularization strength) to tuples of the form
        # (training accuracy, validation accuracy). The accuracy is simply the f
        raction
        # of data points that are correctly classified.
        results = {}
        best val = -1 # The highest validation accuracy that we have seen so f
        best svm = None # The LinearSVM object that achieved the highest validat
        ion rate.
        ########
        # TODO:
        # Write code that chooses the best hyperparameters by tuning on the vali
        dation #
        # set. For each combination of hyperparameters, train a linear SVM on th
        # training set, compute its accuracy on the training and validation set
        s, and #
        # store these numbers in the results dictionary. In addition, store the
        # validation accuracy in best val and the LinearSVM object that achieves
        this #
        # accuracy in best svm.
        # Hint: You should use a small value for num iters as you develop your
        # validation code so that the SVMs don't take much time to train; once y
        ou are #
        # confident that your validation code works, you should rerun the valida
        tion
        # code with a larger value for num iters.
        #######
        for learning_rate in learning_rates:
            for regularization strength in regularization strengths:
                svm2 = LinearSVM()
                lost_hist2 = svm2.train(X_train, y_train, learning_rate=learning
        rate, reg=regularization strength,
                             num iters=1500, verbose=True)
                y train pred2 = svm2.predict(X train)
```

```
y_val_pred2 = svm2.predict(X_val)
      validation_accuracy = np.mean(y_val == y_val_pred2)
      training acccuracy = np.mean(y train == y train pred2)
      results[(learning_rate, regularization_strength)] = (training_ac
ccuracy, validation_accuracy)
      if validation_accuracy>best_val:
         best_val = validation_accuracy
         best_svm = svm2
#######
                         END OF YOUR CODE
#
########
# Print out results.
for lr, reg in sorted(results):
   train_accuracy, val_accuracy = results[(lr, reg)]
   print('lr %e reg %e train accuracy: %f val accuracy: %f' % (
             lr, reg, train accuracy, val accuracy))
print('best validation accuracy achieved during cross-validation: %f' %
best_val)
```

```
iteration 0 / 1500: loss 794.525860
iteration 100 / 1500: loss 289.448893
iteration 200 / 1500: loss 107.915433
iteration 300 / 1500: loss 42.616255
iteration 400 / 1500: loss 19.276410
iteration 500 / 1500: loss 10.236430
iteration 600 / 1500: loss 7.015447
iteration 700 / 1500: loss 5.950552
iteration 800 / 1500: loss 5.631508
iteration 900 / 1500: loss 5.585682
iteration 1000 / 1500: loss 5.119921
iteration 1100 / 1500: loss 5.361187
iteration 1200 / 1500: loss 5.215365
iteration 1300 / 1500: loss 5.499823
iteration 1400 / 1500: loss 5.671152
iteration 0 / 1500: loss 1570.540102
iteration 100 / 1500: loss 212.354226
iteration 200 / 1500: loss 33.104767
iteration 300 / 1500: loss 9.943550
iteration 400 / 1500: loss 5.874009
iteration 500 / 1500: loss 5.715917
iteration 600 / 1500: loss 5.589037
iteration 700 / 1500: loss 5.794695
iteration 800 / 1500: loss 5.506864
iteration 900 / 1500: loss 5.919477
iteration 1000 / 1500: loss 5.819220
iteration 1100 / 1500: loss 5.399934
iteration 1200 / 1500: loss 5.977091
iteration 1300 / 1500: loss 6.408548
iteration 1400 / 1500: loss 5.415373
iteration 0 / 1500: loss 799.463349
iteration 100 / 1500: loss 433509728708405248137338445838499708928.0000
0.0
iteration 200 / 1500: loss 71655686210252900593244590173868290850082668
802921243348620202263036559360.000000
iteration 300 / 1500: loss 11844111045812094621393973869415942368696529
271784867037869299840465783138549060030395126052266749057793261568.0000
00
iteration 400 / 1500: loss 19577366973209663426658079951151586942833567
714479524819931898838922559488.000000
iteration 500 / 1500: loss 32359819670826257586542777876592242801516119
77192847377158572997766559083819847746409684906722477484069085437745976
72910915879155207840769933392301873672986833548272318101827092480.00000
iteration 600 / 1500: loss 53488190243425491400860378387082556521331775
14212796018082015039255002202290322285502119228240827816839995984804920\\
77258632651760831787040076801723835951556893686429209669989616692610038
26844697355195777870645428224.000000
iteration 700 / 1500: loss 88411694645386955553244964691986686446482798
59128293019357784275538174255511895788420725800375505043632420236471436
1828427098720713328448820513359827345473842656217793320326266880.000000\\
iteration 800 / 1500: loss 14613745042588961831555877286922697441850823
30994653304524834141501812275305600102452229470137960335423274723783751
22064864471422115344811956579955913391411730492530680666501289026207098
```

98339255360254316316273833124891094497537349146307247949709619428703354 05779815153175703433487843328.000000 /Users/satwikgoyal/Desktop/CS 682/assignment1/cs682/classifiers/linear svm.py:84: RuntimeWarning: overflow encountered in double_scalars loss += reg * np.sum(W * W) /opt/anaconda3/lib/python3.8/site-packages/numpy/core/fromnumeric.py:8 7: RuntimeWarning: overflow encountered in reduce return ufunc.reduce(obj, axis, dtype, out, **passkwargs) /Users/satwikgoyal/Desktop/CS 682/assignment1/cs682/classifiers/linear svm.py:84: RuntimeWarning: overflow encountered in multiply loss += reg * np.sum(W * W) iteration 900 / 1500: loss inf iteration 1000 / 1500: loss inf iteration 1100 / 1500: loss inf iteration 1200 / 1500: loss inf iteration 1300 / 1500: loss inf iteration 1400 / 1500: loss inf iteration 0 / 1500: loss 1572.281940 iteration 100 / 1500: loss 44009254299085813569993141164184559579897168 82416770705616686983438730970659142655293640178815401255049191924344882590646272.000000 iteration 200 / 1500: loss 11364289154851009982558777891966387111999946 89216821240878160709415023508621816701207097528446449522575474948169921 2744152807188085569488917061487326351135281712976394535516394507617590912139647251500538970445149595853182263275112694191879094272.000000 iteration 300 / 1500: loss inf iteration 400 / 1500: loss inf iteration 500 / 1500: loss inf /Users/satwikgoyal/Desktop/CS 682/assignment1/cs682/classifiers/linear svm.py:105: RuntimeWarning: overflow encountered in multiply dW += 2*reg*W/Users/satwikgoyal/Desktop/CS 682/assignment1/cs682/classifiers/linear classifier.py:70: RuntimeWarning: invalid value encountered in subtract self.W -= learning rate*grad iteration 600 / 1500: loss nan iteration 700 / 1500: loss nan iteration 800 / 1500: loss nan iteration 900 / 1500: loss nan iteration 1000 / 1500: loss nan iteration 1100 / 1500: loss nan iteration 1200 / 1500: loss nan iteration 1300 / 1500: loss nan iteration 1400 / 1500: loss nan lr 1.000000e-07 reg 2.500000e+04 train accuracy: 0.367388 val accuracy:

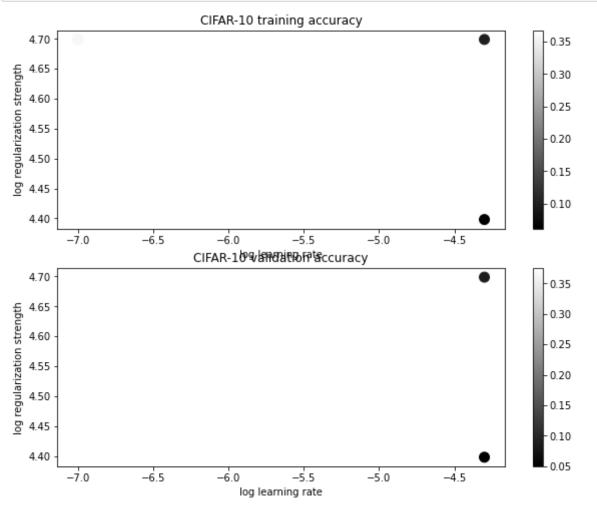
0.375000 lr 1.000000e-07 reg 5.000000e+04 train accuracy: 0.357408 val accuracy: 0.375000

1r 5.000000e-05 reg 2.500000e+04 train accuracy: 0.061184 val accuracy: 0.050000

1r 5.000000e-05 reg 5.000000e+04 train accuracy: 0.100265 val accuracy:

best validation accuracy achieved during cross-validation: 0.375000

```
# Visualize the cross-validation results
In [17]:
         import math
         x_scatter = [math.log10(x[0]) for x in results]
         y_scatter = [math.log10(x[1]) for x in results]
         # plot training accuracy
         marker_size = 100
         colors = [results[x][0] for x in results]
         plt.subplot(2, 1, 1)
         plt.scatter(x_scatter, y_scatter, marker_size, c=colors)
         plt.colorbar()
         plt.xlabel('log learning rate')
         plt.ylabel('log regularization strength')
         plt.title('CIFAR-10 training accuracy')
         # plot validation accuracy
         colors = [results[x][1] for x in results] # default size of markers is 2
         plt.subplot(2, 1, 2)
         plt.scatter(x_scatter, y_scatter, marker_size, c=colors)
         plt.colorbar()
         plt.xlabel('log learning rate')
         plt.ylabel('log regularization strength')
         plt.title('CIFAR-10 validation accuracy')
         plt.show()
```



```
In [18]: # Evaluate the best svm on test set
    y_test_pred = best_svm.predict(X_test)
    test_accuracy = np.mean(y_test == y_test_pred)
    print('linear SVM on raw pixels final test set accuracy: %f' % test_accuracy)
```

linear SVM on raw pixels final test set accuracy: 0.360000

```
In [19]: # Visualize the learned weights for each class.
         # Depending on your choice of learning rate and regularization strength,
         these may
         # or may not be nice to look at.
         w = best_svm.W[:-1,:] # strip out the bias
         w = w.reshape(32, 32, 3, 10)
         w_min, w_max = np.min(w), np.max(w)
         classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse'
         , 'ship', 'truck']
         for i in range(10):
             plt.subplot(2, 5, i + 1)
             # Rescale the weights to be between 0 and 255
             wimg = 255.0 * (w[:,:,:,i].squeeze() - w_min) / (w_max - w_min)
             plt.imshow(wimg.astype('uint8'))
             plt.axis('off')
             plt.title(classes[i])
```





Inline question 2:

Describe what your visualized SVM weights look like, and offer a brief explanation for why they look they way that they do.

Your answer: Each visualized SVM weight is the looks like an average over all the images in a class. It is because each SVM weight is computed using the training data in each class, and so images are mapped over each other to give the correct class' maximum weight. For example, the horse image looks like an image with 2 horse heads probably because there were many pictures in the class with the horse's head facing right and many pictures with the horse's head facing the left. Siminaly, in the car class we can sort of see a car's pink/red front with a blueish windshield. Similarly, for the deer class we can see a brown coloured deer in the center on the image with green background which is most probably the greenery around it.

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