

## Graph Theory; Fourth Set of assignment problems

Thursday, 5 March, 2020

Sane

*Assume that we are dealing with simple graphs.*

1. The graph  $G \vee H$  is obtained from  $G + H$  (disjoint copies) by (additionally) drawing all the edges between  $G$  and  $H$ . Show that  $G$  is  $k$ -connected iff  $G \vee K_r$  is  $(k + r)$ -connected. (2 marks)
2. Let  $G$  be a connected graph with  $n \geq 3$ . Let  $G^*$  be obtained from  $G$  (on the same vertex set and with all the edges of  $G$  retained) by drawing an extra edge  $(xy)$  for all the occurrences of  $d(x, y) = 2$ . Show that  $G^*$  is 2-connected. (2 marks)
3. Find smallest 3-regular connected graph with a cut vertex. (4 marks)