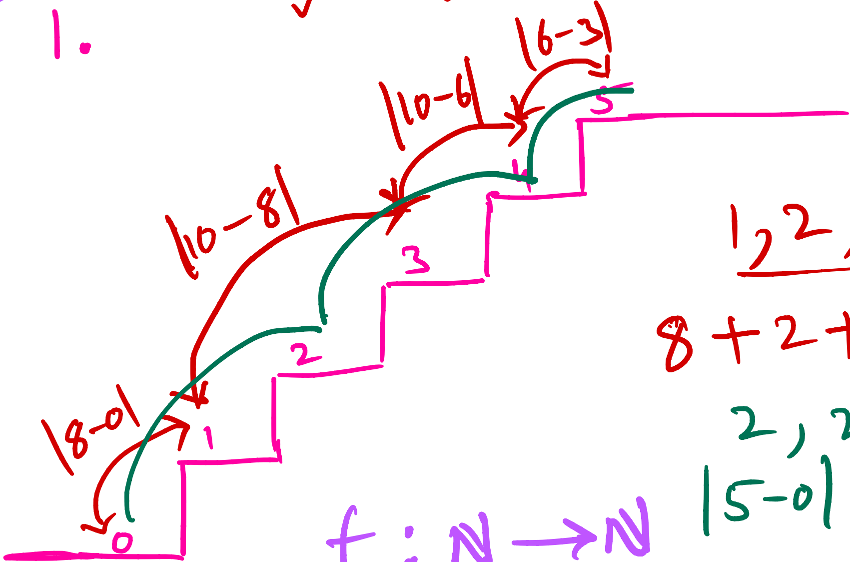
08/08/22

Home Work

1.

0	1	2	3	4	5
0	8	5	10	6	3



1, 2, 1, 1

$8+2+4+3 = \underline{17}$

2, 2, 1

$|5-0| + |6-5| + |6-3|$

$5+1+3 = \underline{9}$

$f : \mathbb{N} \to \mathbb{N}$

$f \mapsto$ Total no. of ways

1 way : 2, 1, 2

2 way : 1, 1, 2, 1

3 way : 2, 2, 1

4 way : 1, 1, 1, 1, 1

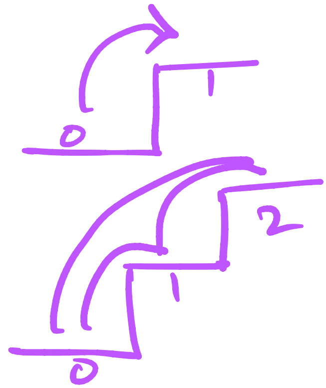
⋮

Least

$f(n)$ = "Total No. of ways to climb "n" stairs subject to climbing 1 or 2 stairs at a time"

$n=1, \; f(1)=1$

$n=2, \; f(2)=2$



$\boxed{f(n) = ?}$

2.

3	1		11	10
6	9	0	3	
1	2		8	6
10	-	14	0	15

R ✓

D ✓

25 X 30

4 x 5

RRDRRDD
DRDRRDR

RRRRDDD ✓

{D, D, D, D,
R, R, R}

max = ?

$\frac{7!}{3! \times 4!}$ ✗

3. "set"

$\{1, 2, 5, 9, 7\}$ ✓

✓ $S = \{1, 2, 5\}$ sum = 8

✓ $S = \{7, 1\}$

- can we get a subset?

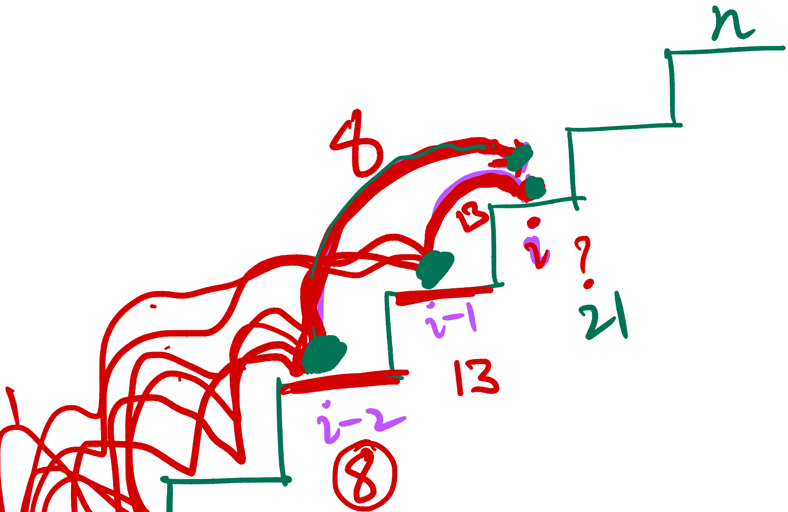
  Yes | No

- How many such subsets?
- How many elements in

largest subset?

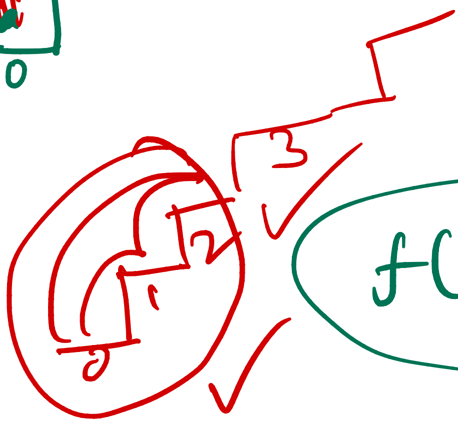
- How many elements in second largest subsets?

$f(n) = ?$



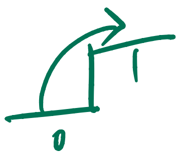
$f(i-1) = 13$

$f(i-2) = 8$

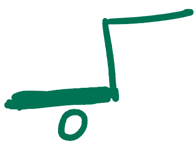


$$f(i) = f(i-1) + f(i-2)$$

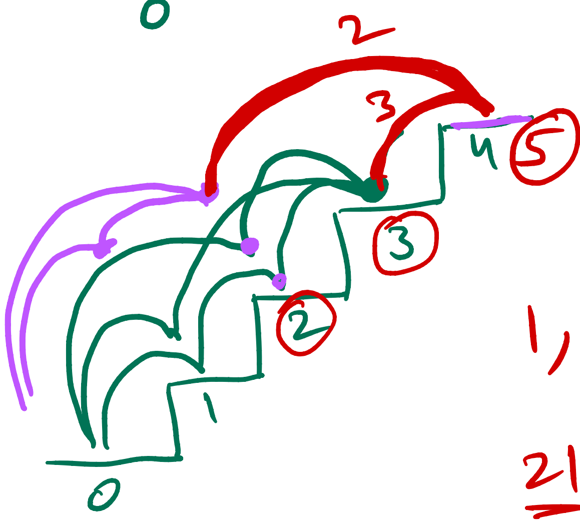
$$f(2) = f(1) + f(0)$$
$$= 1 + 1$$
$$= 2$$



$$f(3) = f(2) + f(1)$$
$$= 2 + 1$$
$$= 3$$



$$f(4) = f(3) + f(2)$$
$$= 3 + 2$$
$$= 5$$



$$1, 1, 2, 3, 5, 8, \overset{6}{13},$$

$$21, 34, \ldots$$

$$\boxed{f(n) = f(n-1) + f(n-2)}$$

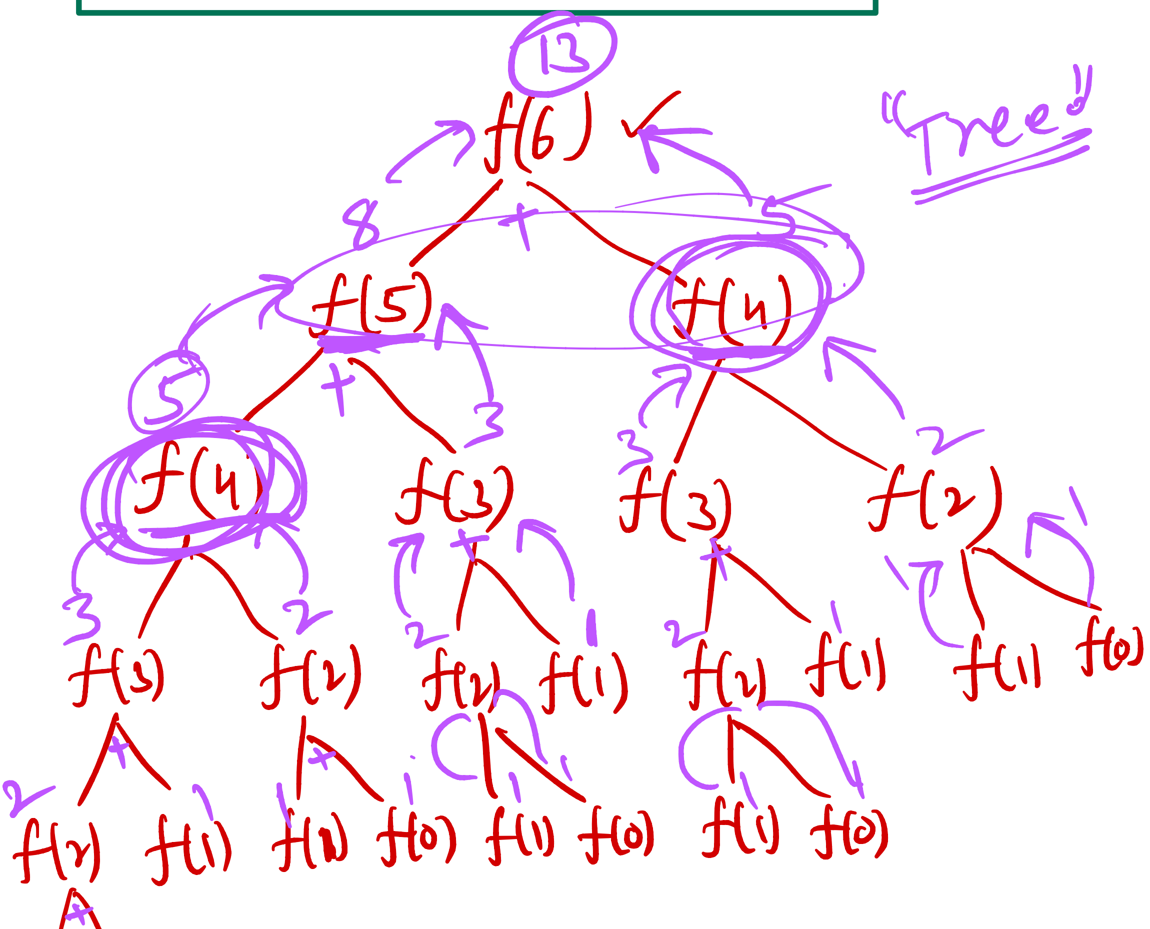
pseudo ≈ C++

```
int f(int n){
    if(n==0 || n==1)
        return 1;
    return f(n-1)+f(n-2);
```

$f(0)=1$

$f(1)=1$

✓

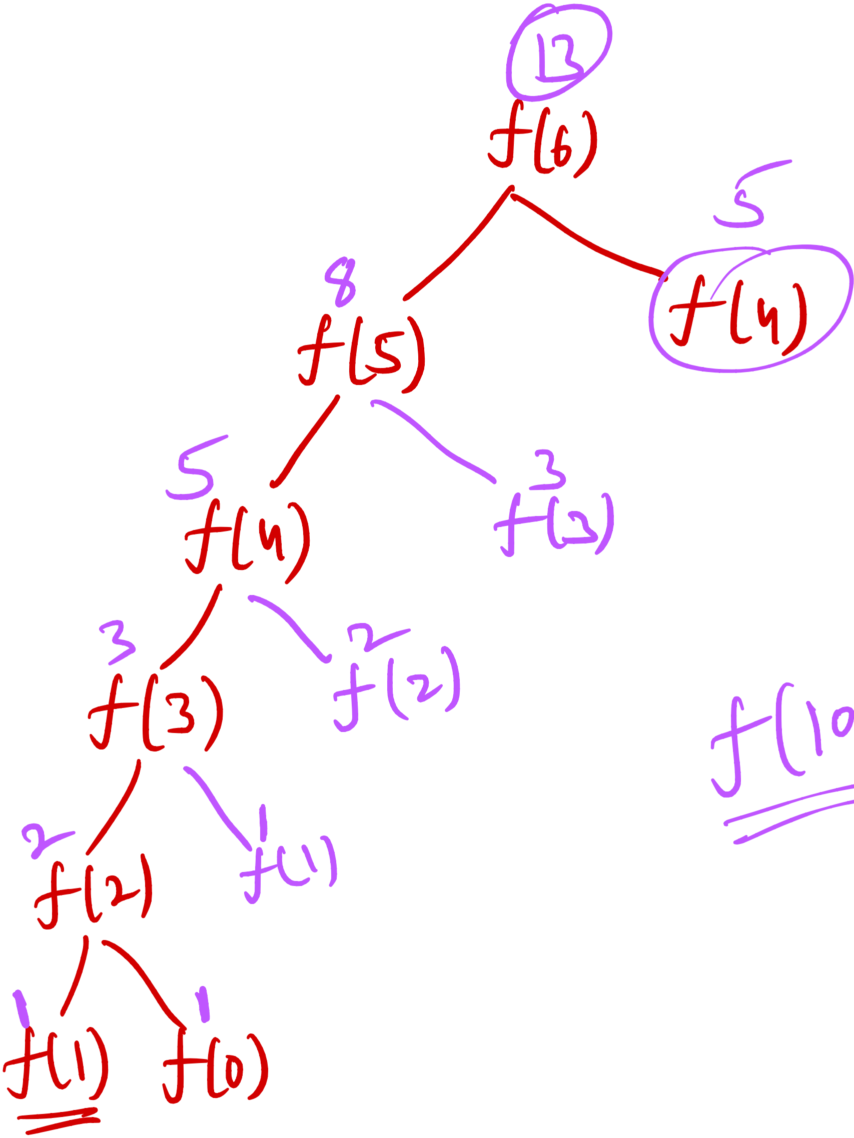


f(1) f(0)

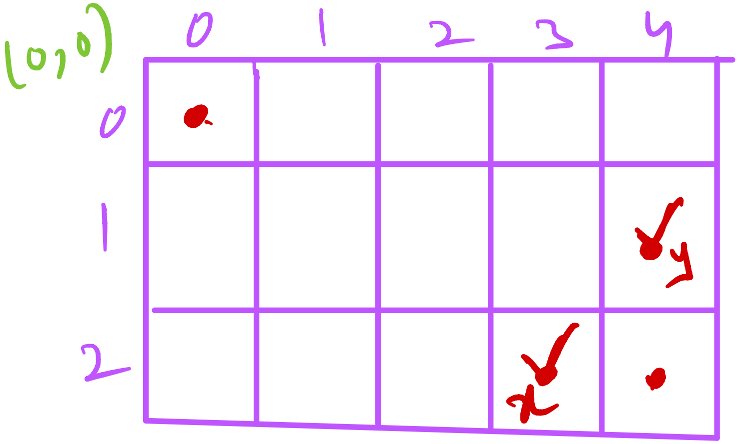
"stack"

"call stack"

```
f(n, A) {                         A = [-1, -1, -1, -1]
    if (n == 0 || n == 1)
        return 1;
    if (A[n] != -1)
        return A[n];
    return A[n] = f(n-1) + f(n-2);
}
```

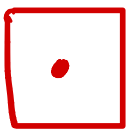


$f(100)$

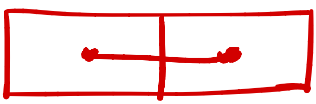


(2,4)

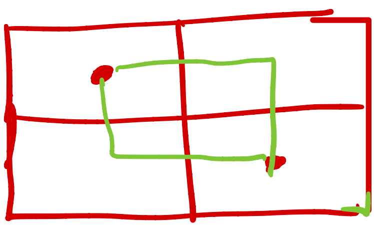
- **No. of ways:**



1 way

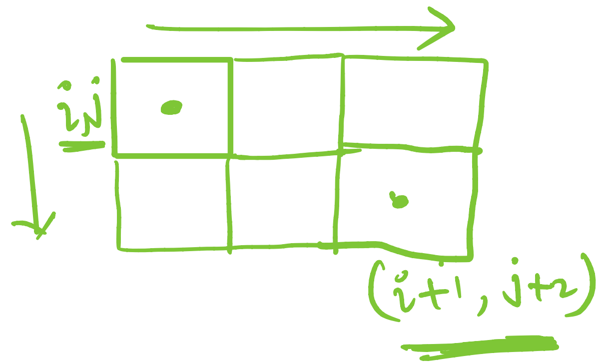


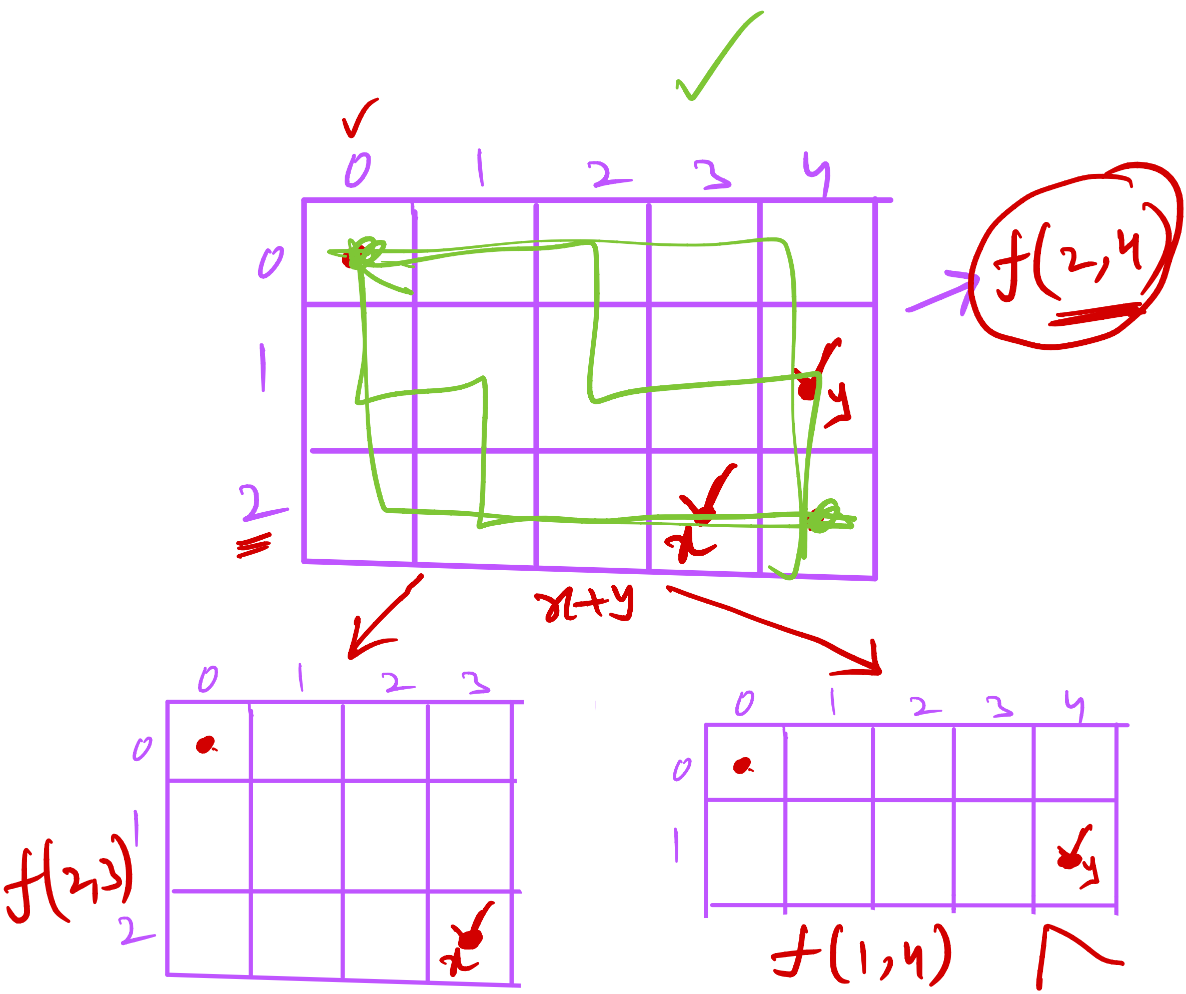
1 way

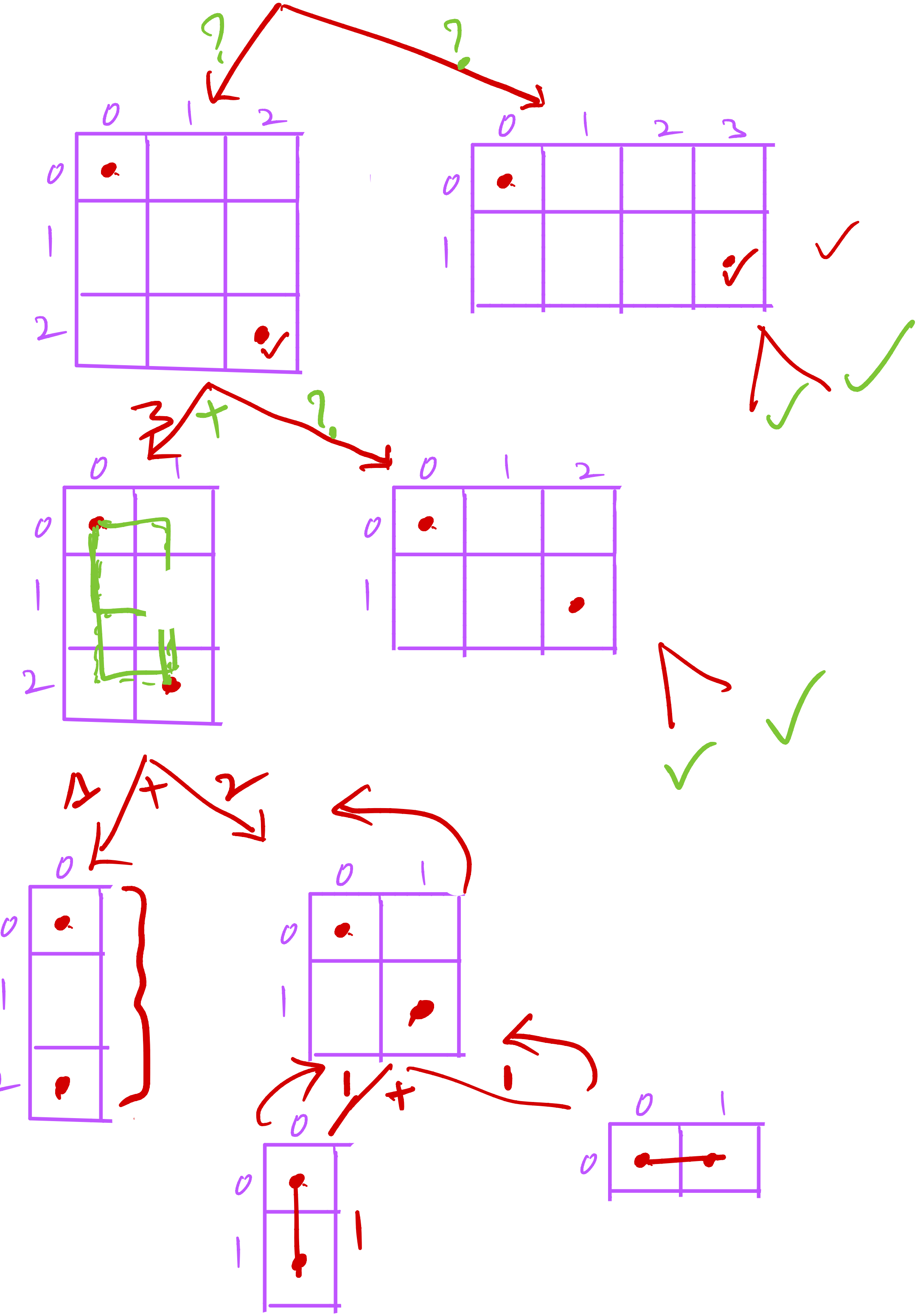


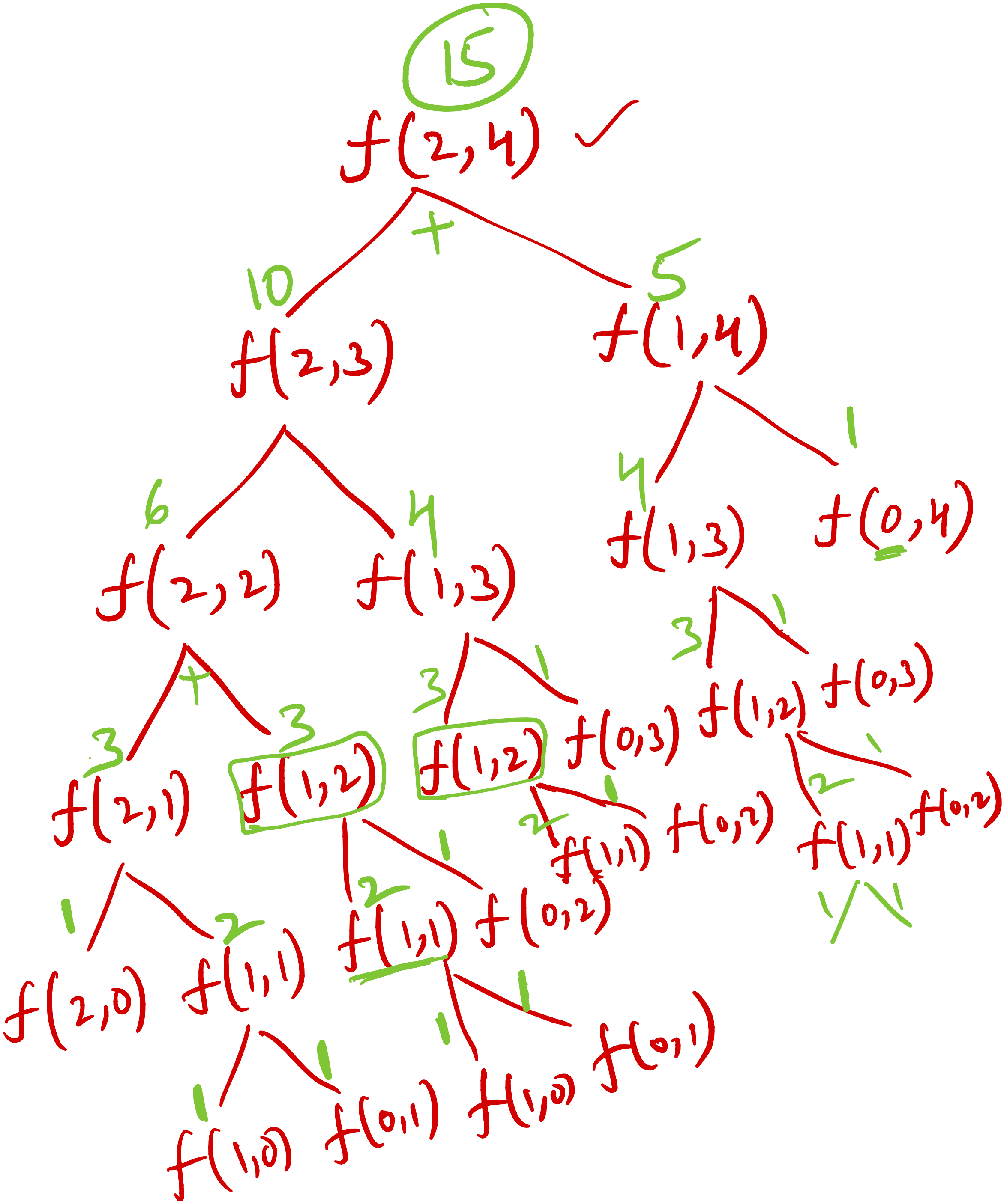
2 ways

$f(i,j)$









15
f(2,4)
+
10
f(2,3)
5
f(1,4)
6
f(2,2)
4
f(1,3)
4
f(1,3)
1
f(0,4)
3
f(2,1)
3
f(1,2)
3
f(1,2)
1
f(0,3)
3
f(1,2)
1
f(0,3)
1
f(2,0)
2
f(1,1)
2
f(1,1)
1
f(0,2)
2
f(1,1)
f(0,2)
2
f(1,1)
f(0,2)
1
f(1,0)
1
f(0,1)
1
f(1,0)
1
f(0,1)

$$f(i,j) = f(i-1,j) + f(i,j-1)$$

```
int f(int i, int j){
    if(i==0 || j==0)
        return 1;
    return f(i-1,j)+f(i,j-1)
}
```

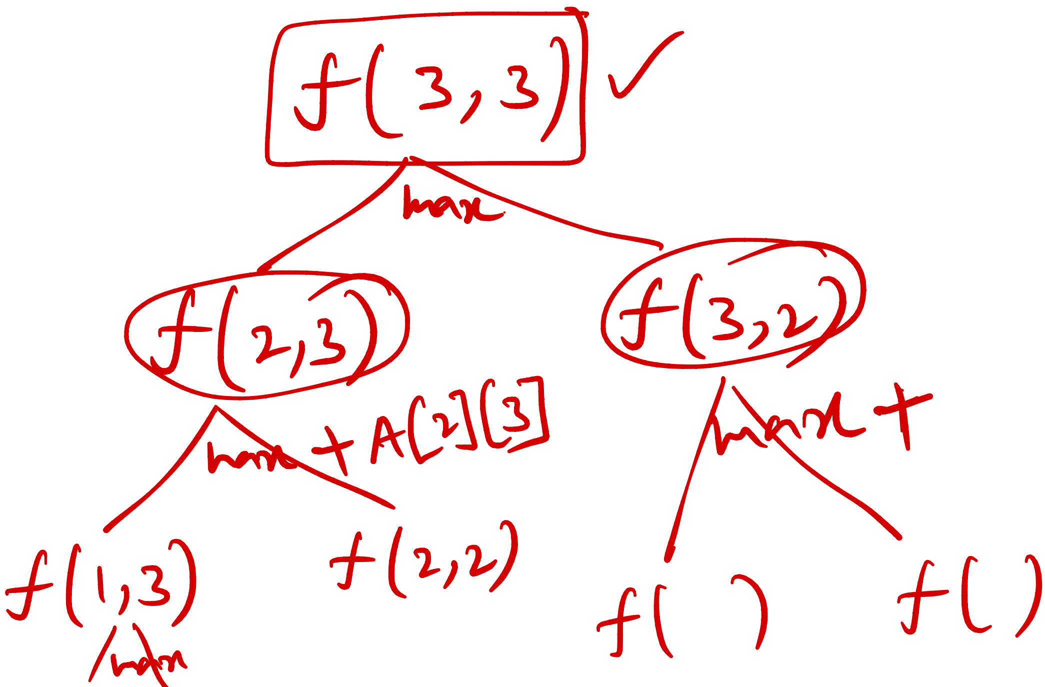
	0	1	2	3
0	-6	5	0	-10
1	20	-5	[illegible]	-8
2	3	[illegible]	70	9
3	5	6	1	18

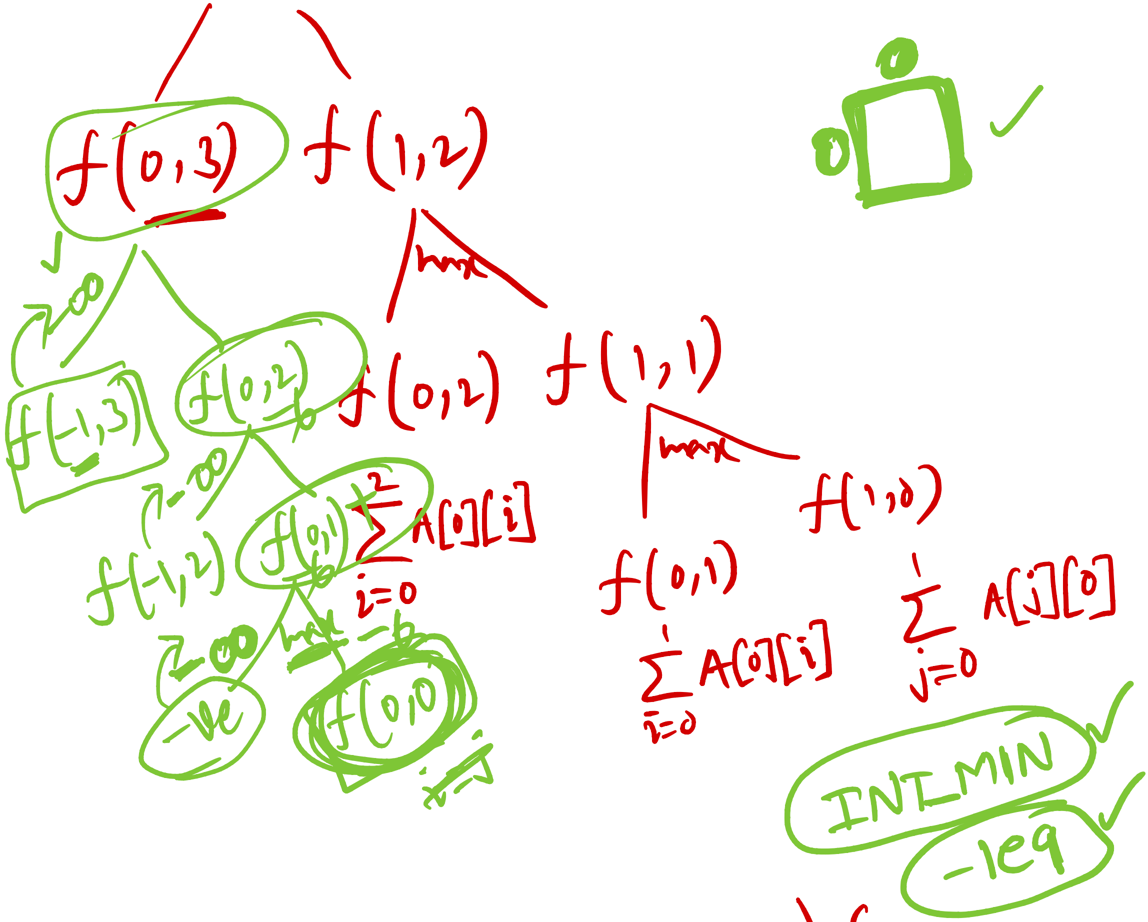
m_1

m_2

+ve

m_1, m_2






```
int f(int i, int j){
    if(i==0){
        sum=0;
        for(k=0; k<=j; k++)
            sum += A[0][k];
        return sum;
    }
```

```
if(j==0){
    sum=0;
    for(k=0; k<=i; k++)
        sum += A[k][0];
    return sum;
}

return max(f(i-1,j), f(i,j-1))
                     + A[i][j]
```

```
int f(int i, int j){
    if(i==0 && j==0){
        return A[0][0];
    if(i<0 || j<0}
        return INT_MIN;
```

return $\max(f(i-1,j), f(i,j-1)) + A[i][j]$

✓ $A = \{1, 5, 3, 4\}$ ✓

Distinct

List

Array

✓ $A = \{1, 1, 5, 3, 4, 4\}$ "finite"

✓ $A = \{\underbrace{1}_{\infty}, \underbrace{5}_{\infty}, \underbrace{3}_{\infty}, \underbrace{4}_{\infty}\}$ ✓

- $A = \{\underbrace{2}_{\infty}, \underbrace{5}_{\infty}, \underbrace{3}_{\infty}\}$, $T = 11$

- Is it possible? ✓

T/F

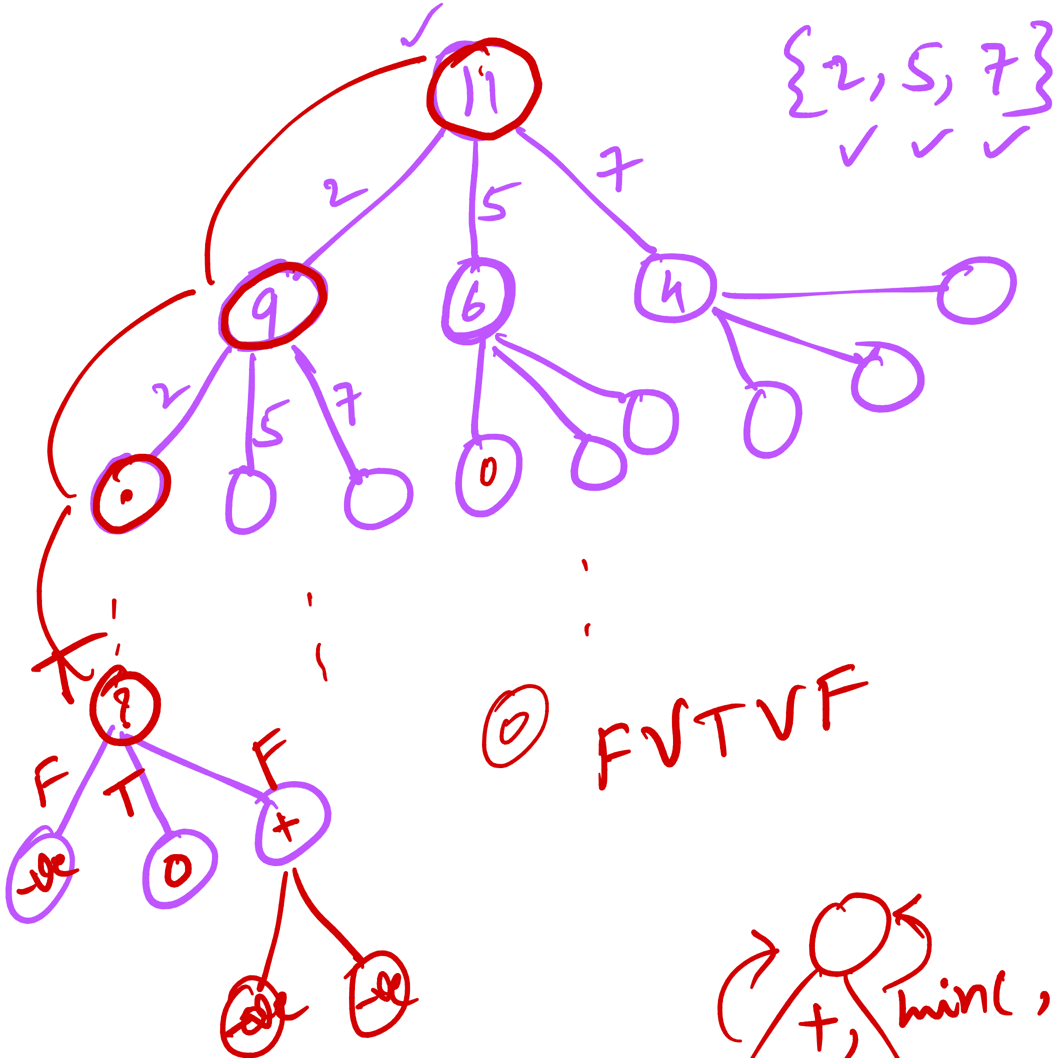
- How many different solutions?

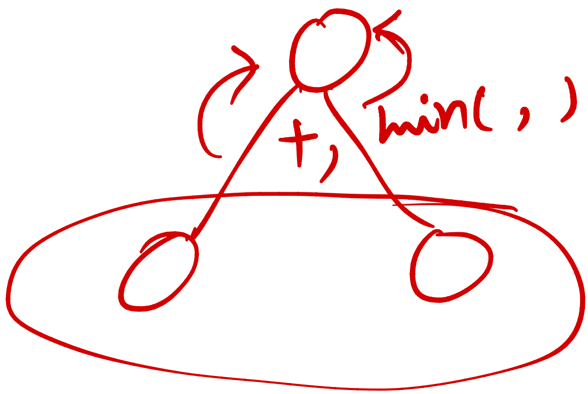
$$f(x) = \begin{cases} 1 & x = 0 \\ 0 & x < 0 \\ \vee f(x-a) & \text{o.w} \checkmark \end{cases}$$

$$f(11) = f(11-2) \vee f(11-5) \vee f(11-7)$$

$$= f(9) \vee f(6) \vee f(4)$$

$$\vdots$$





+ : Int

min(,) : Integer

V : Boolean

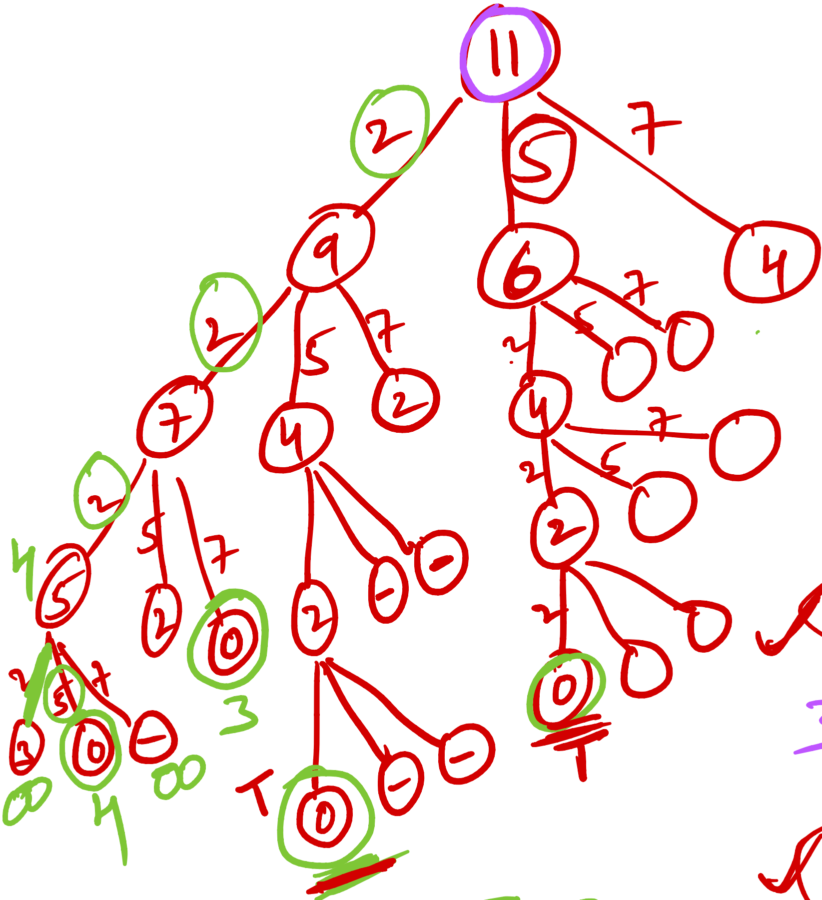
```
f(T){
  if (T==0)
     return True; ✓
  if (T<0)
     return False; ✓
  result = False
  for (auto a : A){
     result = result ∨ f(T-a);
  return result;
}
```

$$\bigvee_{a \in A} f(T-a) \qquad \sum_{n=1}^{5} n$$

```
result = False
for(______)
  result = result ∨ f( )
```

```
sum=0 ✓
for(______)
  sum += n
```

{2, 5, 7}



4
2+2+2+5

3 2+2+7

5+2+2+2

T, F
1, 0

T $\boxed{V}$ F = T
1+0+1 = 2

Dupric

min

0	-ve	0	0
=	=	=	=
1	+ 0	+ 1	+ 1

$$f(T) = \begin{cases} 1 & T = 0 \\ 0 & T < 0 \\ \sum_{a \in A} f(T-a) & \text{otherwise.} \end{cases}$$

```
f(T) {
    if (T==0) return 1
    if (T<0)  return 0
    ans =0;
    for (auto a : A)
        ans += f(T-a)
    return ans
}
```

- "minimum number of entries"

$$f(T) = \begin{cases} 0 & T = 0 \\ \infty & T < 0 \\ \min\limits_{a \in A} \{ f(T-a) + 1 \} \end{cases}$$

$\{ \underline{2}, 5, \underline{\underline{7}} \}$

$$f(9) = \min\left(\underline{f(7)}+1, \underline{f(4)}+1, f(2)+1 \right)$$

$$= \min\Big(\min(f(5)+1, f(2)+1, 0+1),$$

$$\min(f(2)+1, \infty+1, \infty+1),$$

$$\min(0+1,\ \infty+1,\ \infty+1))\}$$

$$= \min(\min(f(5)+\underline{1}, f(2)+1,\ 1),\ \min(f(2)+1,\ \infty,\ \infty),\ \underline{1}) + \underline{1}$$

$$= \min(\underline{\min}(\underline{\min(f(3)+1,\ 0+1,\ \infty)+1},\ \underline{\min(\min(0+1,\infty,\infty)+1)+1},\ 1)+1$$

$$= \min(\min(\min(f(1)+1,\ \infty,\infty)+1)+1)+1,\ 3,\ 2)$$

$= \min\{\infty, 3, 2\}$

$\underline{\underline{2}}$

```
f(T) {
    if (T == 0) return 0
    if (T < 0) return INT_MAX
     mini = INT_MAX
    for (auto a : A) {
        mini = min(f(T-a)+1)
return mini
```