

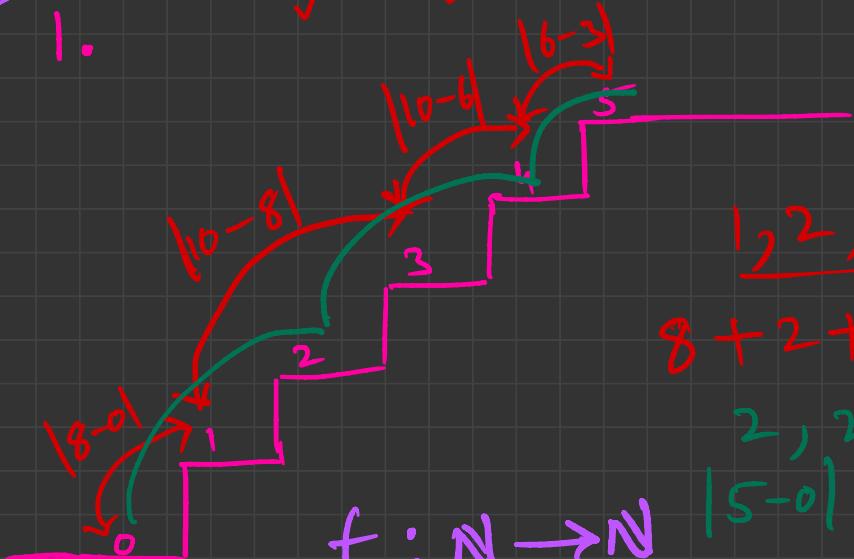


08/08/22

0	1	2	3	4	5
0	8	5	10	6	3

Home Work

1.



$$1, 2, 1, 1$$

$$8 + 2 + 4 + 3 = \underline{\underline{17}}$$

$$2, 2, 1$$

$$(5-0) + (6-5) + (6-2)$$

$$f : \mathbb{N} \rightarrow \mathbb{N}$$

$f \mapsto$ Total no. of ways

$$1\text{way} : 2, 1, 2$$

$$5 + 1 + 3 = \underline{\underline{9}}$$

$$2\text{way} : 1, 1, 2, 1$$

Least

$$3\text{way} : 2, 2, 1$$

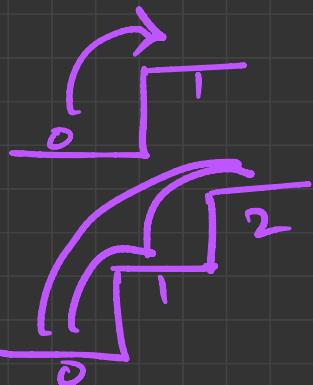
$$4\text{way} : 1, 1, 1, 1, 1$$

:

$f(n) =$ "Total NO. of ways to climb "n" stairs subject to climbing 1 or 2 stairs"

at a time"

$$n=1, f(1) = 1$$



$$n=2, f(2) = 2$$

f(n) = ?

2.

5	1	✓	11	10
6	9	0	3	✓
1	2	✓	8	6
10	-	14	0	✓

R ✓
D ✓

25x30
4x5

RRDRRRDD
DRDRRRDR
RRRRRDDD ✓

{ D,D,D,D,
R,R,R }

mark = ?

$$\frac{7!}{3! \times 4!}$$

X

3.

"set"

$$\underline{\{1, 2, 5, 9, 7\}} \quad \checkmark$$

$$\checkmark S = \{1, 2, 5\} \quad \underline{\text{Sum} = 8}$$

$$\checkmark S = \{7, 1\}$$

• Can we get a subset?

Yes | No

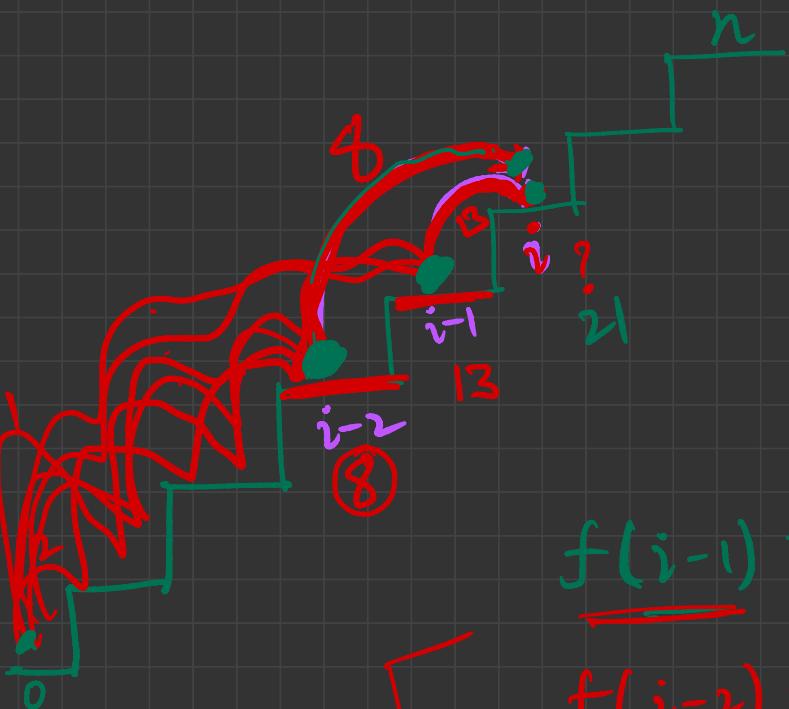
• How many such subsets?

• How many elements in

largest subset?

- How many elements in second largest subsets?
-

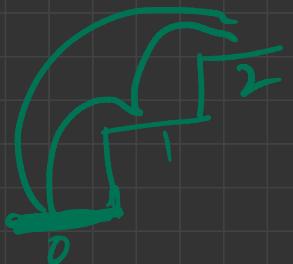
$$f(n) = ?$$



$$\underline{\underline{f(i-1) = 13}}$$

$$\underline{\underline{f(i-2) = 8}}$$

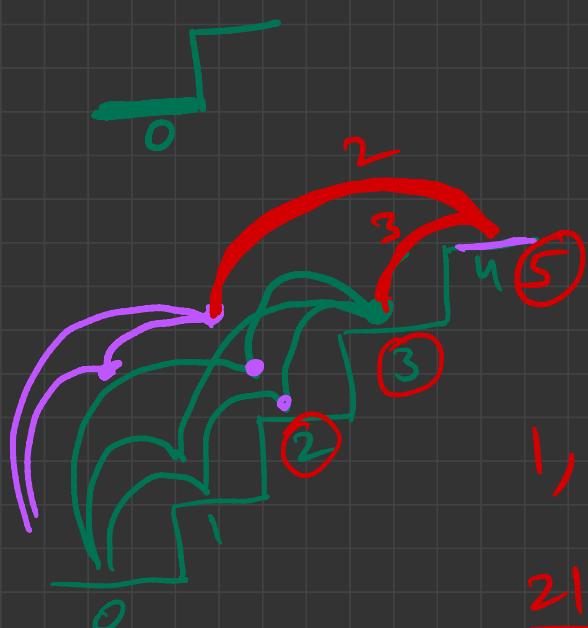
$$f(i) = f(i-1) + f(i-2)$$



$$\begin{aligned}
 f(2) &= f(1) + f(0) \\
 &= 1 + 1 \\
 &= 2
 \end{aligned}$$



$$\begin{aligned}
 f(3) &= f(2) + f(1) \\
 &= 2 + 1 \\
 &= 3
 \end{aligned}$$



$$\begin{aligned}
 f(4) &= f(3) + f(2) \\
 &= 3 + 2 \\
 &= 5
 \end{aligned}$$

$$\begin{aligned}
 1, 1, 2, 3, \underline{5}, 8, \underline{13}, \\
 \underline{21}, \underline{34}, \dots
 \end{aligned}$$

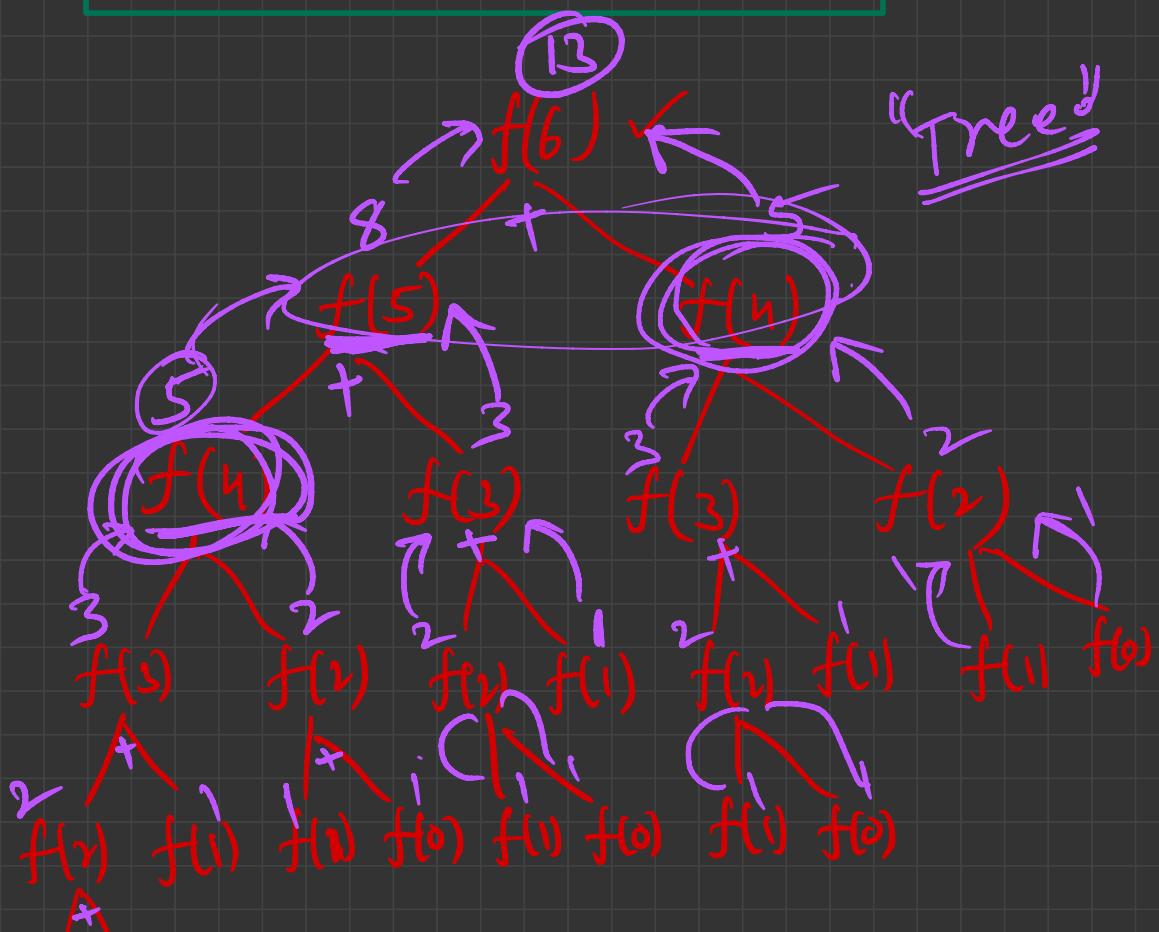
$$\boxed{f(n) = f(n-1) + f(n-2)}$$

Pseudo \approx C++

```
int f(int n){  
    if(n == 0 || n == 1)  
        return 1;  
    return f(n-1) + f(n-2);
```

$$f(0) = 1$$
$$f(1) = 1$$

✓ ✓



$f()$ $f()$

"stack"
"call stack"

$f(n, A) \{$

$A = [-1, -1, -1, -1]$

if ($n == 0 \text{ || } n == 1$)

 return 1;

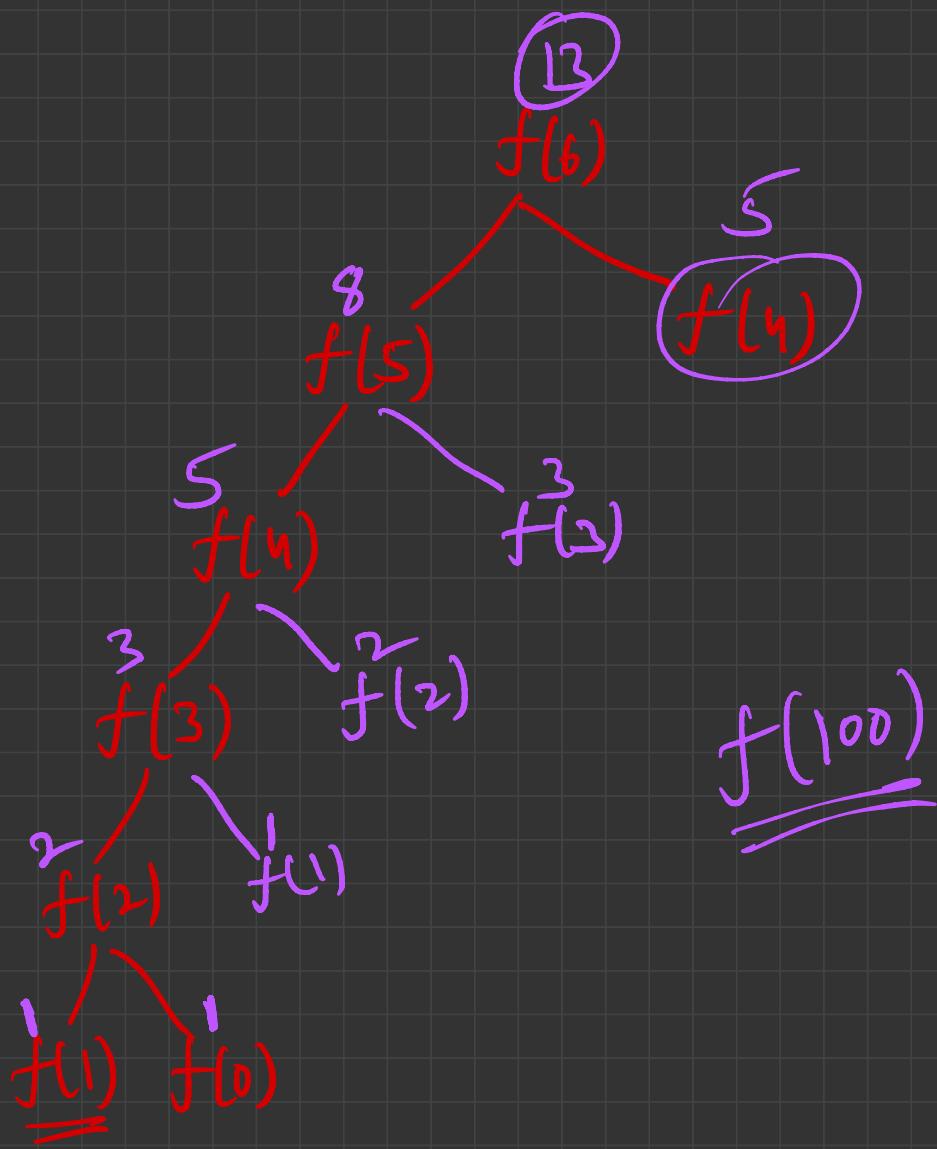
if ($A[n] \neq -1$)

 return $A[n]$;

 return $A[n] = f(n-1) + f(n-2)$;

}

"memoization"



	0	1	2	3	4
0	•				
1					•
2				•	

• No. of ways:

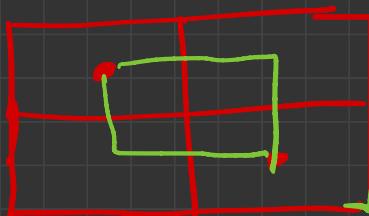
(2,4)



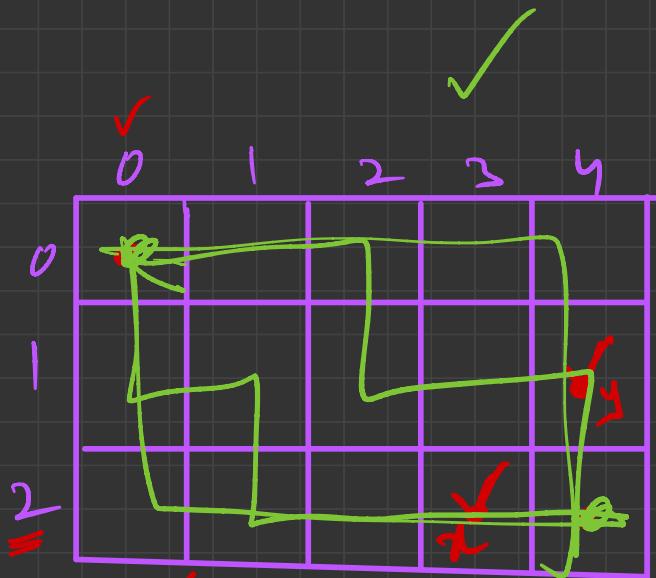
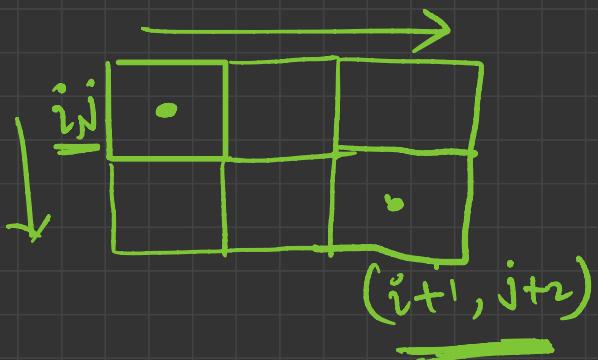
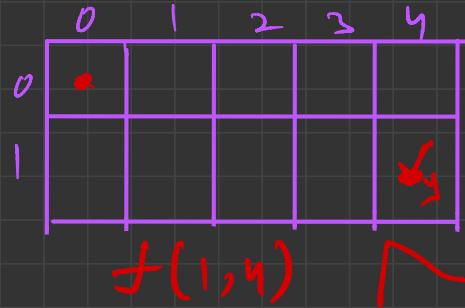
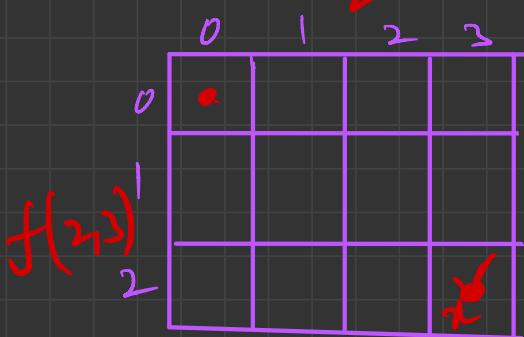
1 way

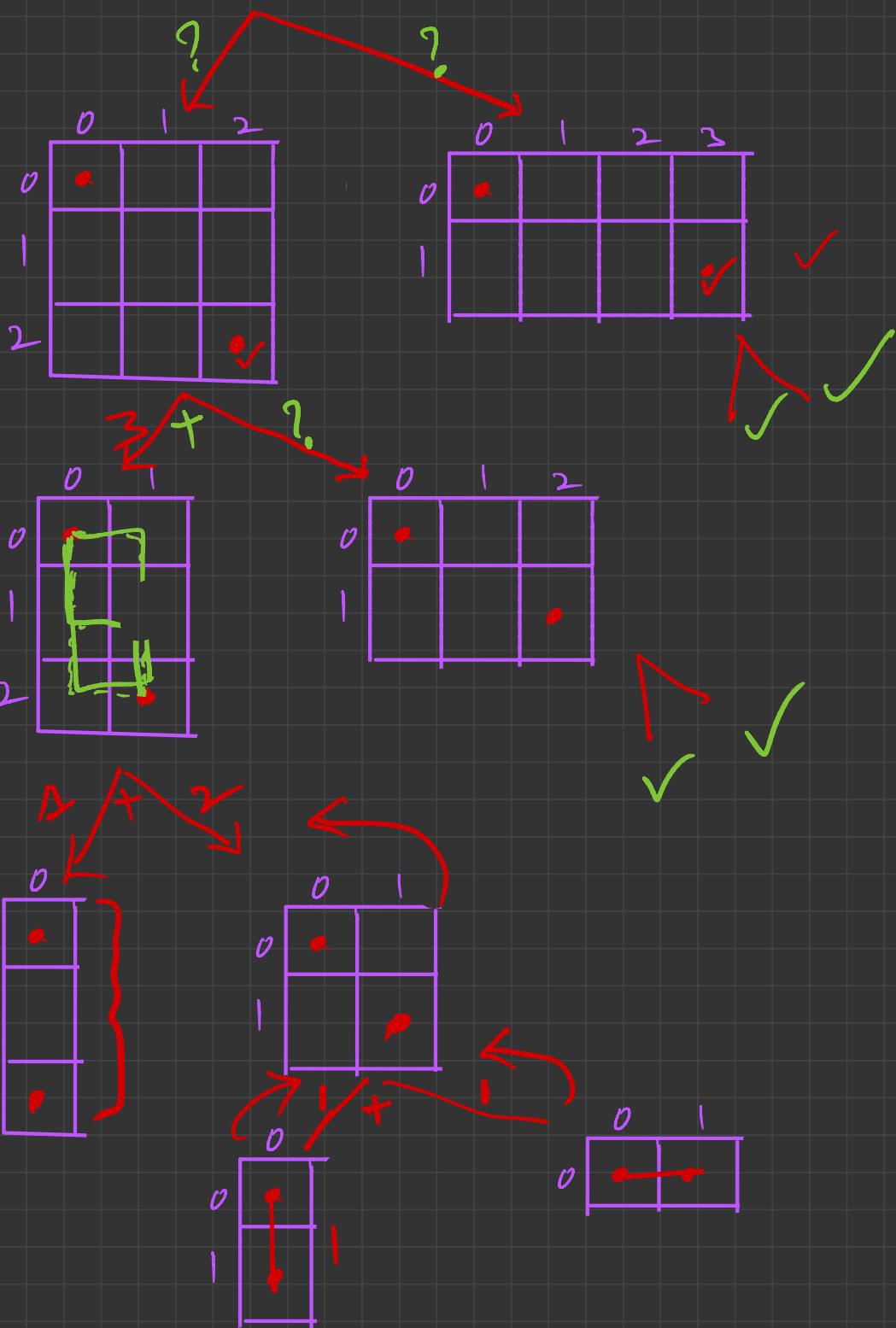


1 way



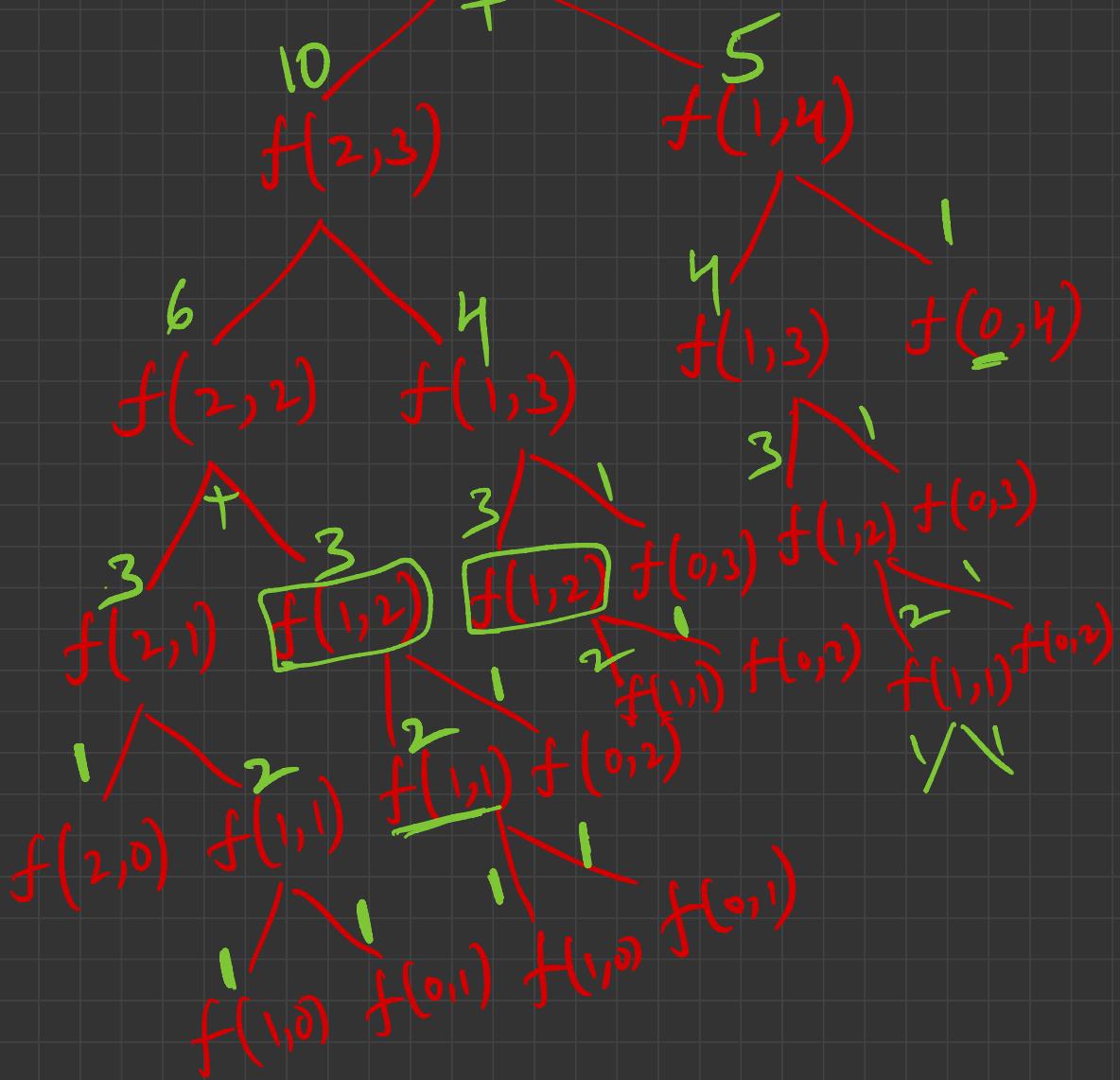
2 ways

$f(i, j)$  $x+y$ 



15

$f(2, 4)$ ✓



$$f(i, j) = f(i-1, j) + f(i, j-1)$$

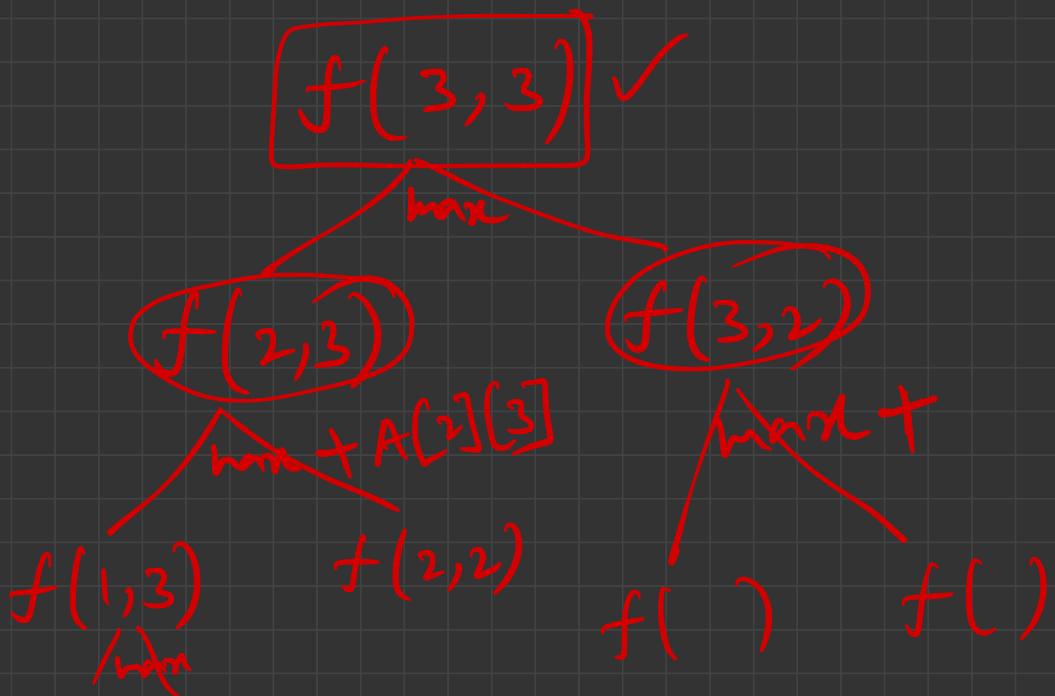
int $f(\text{int } i, \text{ int } j)$ {
 ✓ if ($i == 0 \text{ || } j == 0$)
 ✓ return 1;
 ✓ return $f(i-1, j) + f(i, j-1)$
}

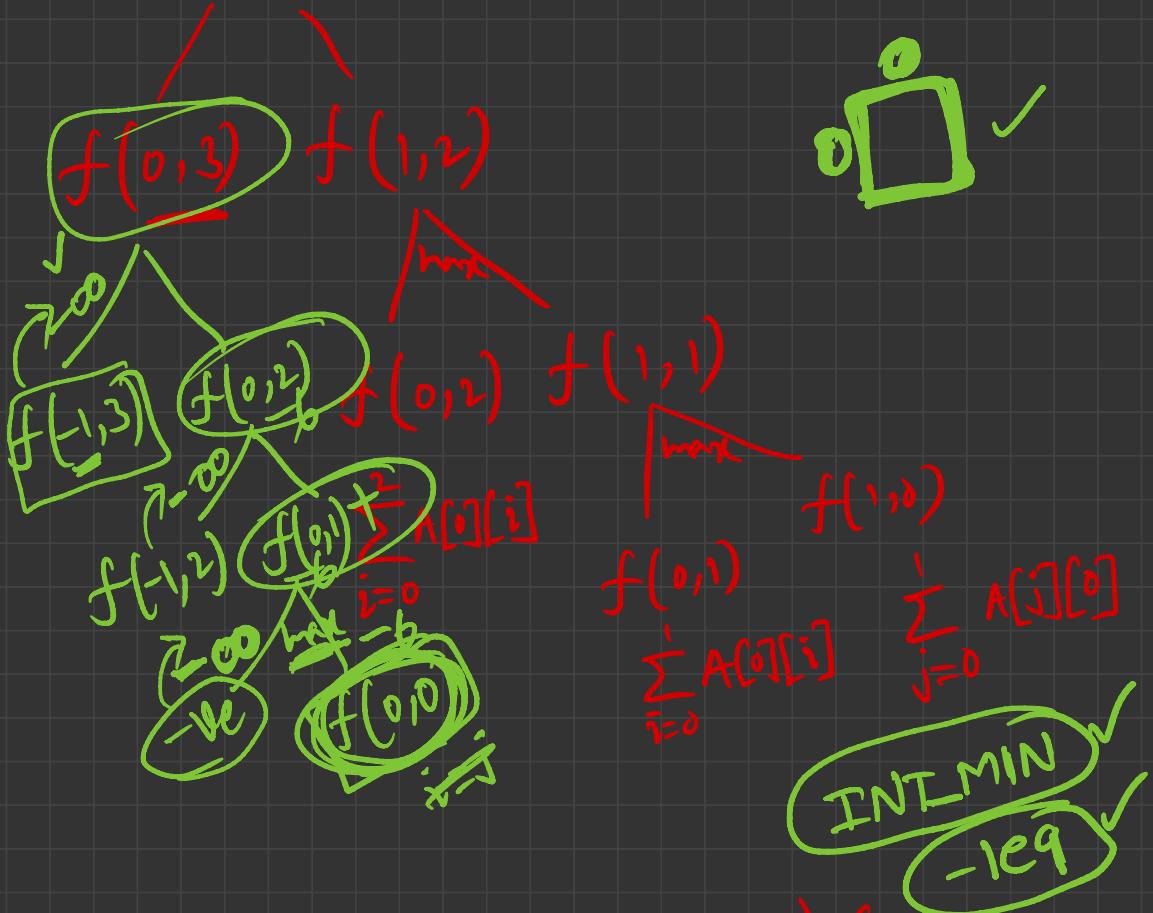


	0	1	2	3	✓
0	-6	5	0	-10	
1	20	-5	12 10 11	-8	m_1
2	3	11 10 11	70	9	
3	5	6	1	18	m_2

tie

m_1, m_2





```
int f(int i, int j){
```

```

    if (i==0) {
        sum=0;
        for (k=0; k<=j; k++)
            sum += A[0][k];
    }
    return sum;
}
```


return $\max(f(i-1, j), f(i, j-1))$
+ $A[i][j]$

✓ $A = \{1, 5, 3, 4\}$ ✓
Distinct

List
Array

✓ $A = \{1, 1, 5, 3, 4, 4\}$ "finite"

✓ $A = \{\underset{\infty}{1}, \underset{\infty}{5}, \underset{\infty}{3}, \underset{\infty}{4}\}$ ✓

• $A = \{\underset{\infty}{2}, \underset{\infty}{5}, \underset{\infty}{3}\}$, $T = \underline{\underline{11}}$

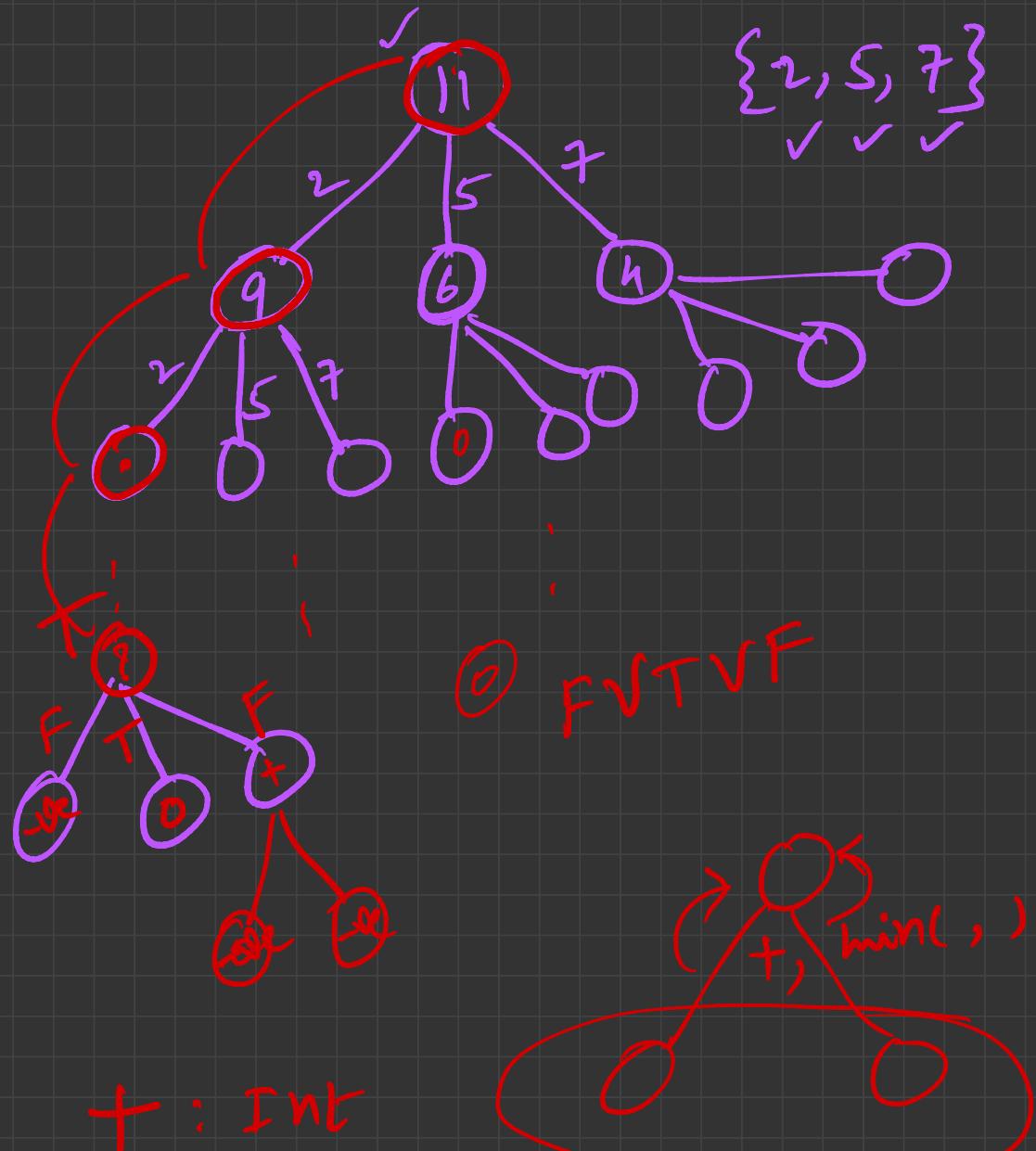
• Is it possible? ✓

T/F

• How many different solutions?

$$f(x) = \begin{cases} 1 & x=0 \\ 0 & x<0 \\ \sqrt{f(x-\alpha)} & 0 < x < \alpha \end{cases}$$

$$\begin{aligned} f(11) &= f(11-2)\sqrt{f(11-5)\sqrt{f(11-7)}} \\ &= f(9)\sqrt{f(6)\sqrt{f(4)}} \\ &\quad \vdots \end{aligned}$$



$+$: Int

$\min(,)$: Intergar

\vee : Boolean

$f(T) \{$

if ($T == 0$)

return True; ✓

if ($T < 0$)

return False; ✓

result = False

for (auto a: A) {

result = result $\vee f(T-a);$

return result;

}

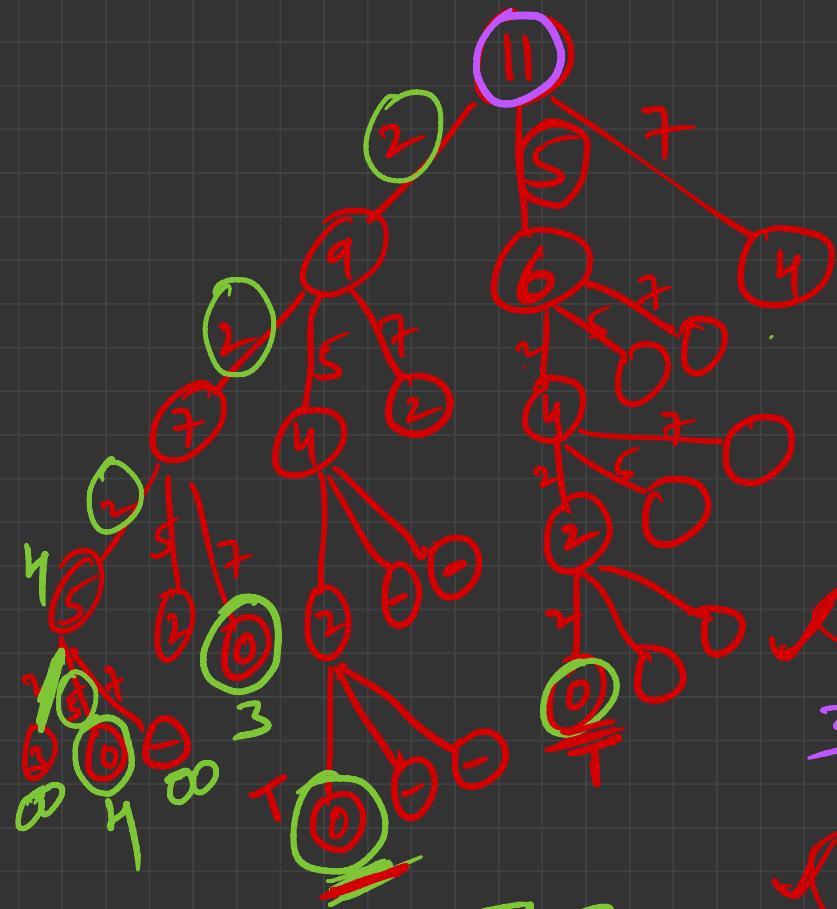
$\vee f(T-a)$
 $a \in A$

result = False
for (_____)
result = result $\vee f()$

$\sum_{n=1}^S n$

Sum = 0 ✓
for (_____)
Sum += n

$\{2, 5, 7\}$



H

$$2+2+2+5$$

$$3(2+2+7)$$

$$\cancel{5+2+2+2}$$

T F

1, 0

min

0
11

$$T \checkmark F = T$$

$$1 + 0 + 1 = 2$$

-ve

11

0
11

DUPNIE

$$1 + 0 + 1 + 1 \quad \checkmark$$

$$f(T) = \begin{cases} 1 & T=0 \\ 0 & T<0 \\ \sum_{a \in A} f(T-a) & \text{otherwise.} \end{cases}$$

$f(T) \{$

if ($T == 0$) return 1

if ($T < 0$) return 0

ans = 0;

for (auto a : A)

ans += f(T-a)

return ans

3

- "minimum number of entries"

$$f(T) = \begin{cases} 0 & T=0 \\ \infty & T < 0 \\ \min_{a \in A} \{ f(T-a) + 1 \} & \text{otherwise} \end{cases}$$

$$\{2, 5, 7\}$$

$$f(9) = \min \left(\underline{f(7)+1}, \underline{f(4)+1}, \underline{\underline{f(2)+1}} \right)$$

$$= \min \left(\min(f(5)+1, f(2)+1, 0+1), \min(f(2)+1, \infty+1, \infty+1) \right)$$

$$\min(0+1, \infty+1, \infty+1)$$
$$\geq \min\left(\min\left(f(5)+1, f(2)+1, 1\right), \min\left(f(2)+1, \infty, \infty\right)\right) + 1$$

$$= \min\left(\underbrace{\min\left(\min\left(f(3)+1, 0+1, \infty\right)\right)}_{\min\left(\min\left(0+1, \infty, \infty\right)+1\right)+1} + 1\right) + 1$$

$$= \min\left(\underbrace{\min\left(\min\left(f(1)+1, \infty, \infty\right)+1\right)+1}_{, 3, 2} + 1\right)$$

$$= \min \{ \infty, 3, 2 \}$$

2

$f(T) \{$

if ($T == 0$) return 0

if ($T < 0$) return INT-MAX

mini = INT-MAX

for (auto a : A) {

mini = min($f(T-a)+1$)

return mini

- Is subset sum possible?
 - Num of ways
 - Least num of entries.
-

$\{2, 5, 7\}$

- ✓ • Is change possible for T ?

$T = 13$

True
False.

- ✓ • How many different ways to produce the coins for T ?

- ✓ • Least num of coins for T ?

$f(T) \{$

$T < 0$

if $T = 0$:

return True

if $T < 0$:

return False

Ans = False

for (auto coin: A):

Ans = Ans \vee f($T - \underline{\text{coin}}$)

return Ans

$f(T) \{$

if ($T == 0$)

return 1

if ($T < 0$)

return 0

$sum = 0$

for (auto coin : A) {

$sum = sum + f(T - coin)$

return sum

Duplicate

{ 1, 2, 3 }

$$\begin{aligned}\checkmark 5 &= 2 + 3 \\ \checkmark 5 &= 3 + 2\end{aligned}$$

$f(T) \{$

if $T == 0 :$

 return 0

if $T < 0 :$

 return ∞

 mini = ∞

 for (auto coin : A) {

 mini = min(mini,

 f(T - coin) + 1)

 return mini

✓ • Draw Tree diagram

✓ • Look at Nature of subproblems

✓ • Decide base cases

✓ • Pseudo code

✓ • Implement

$$A = \{ \overset{\infty}{2}, \overset{\infty}{3}, \overset{\infty}{5} \}$$

$$A = \{ \overset{\checkmark}{2}, \overset{\checkmark}{3}, \overset{\checkmark}{5} \}$$

✓

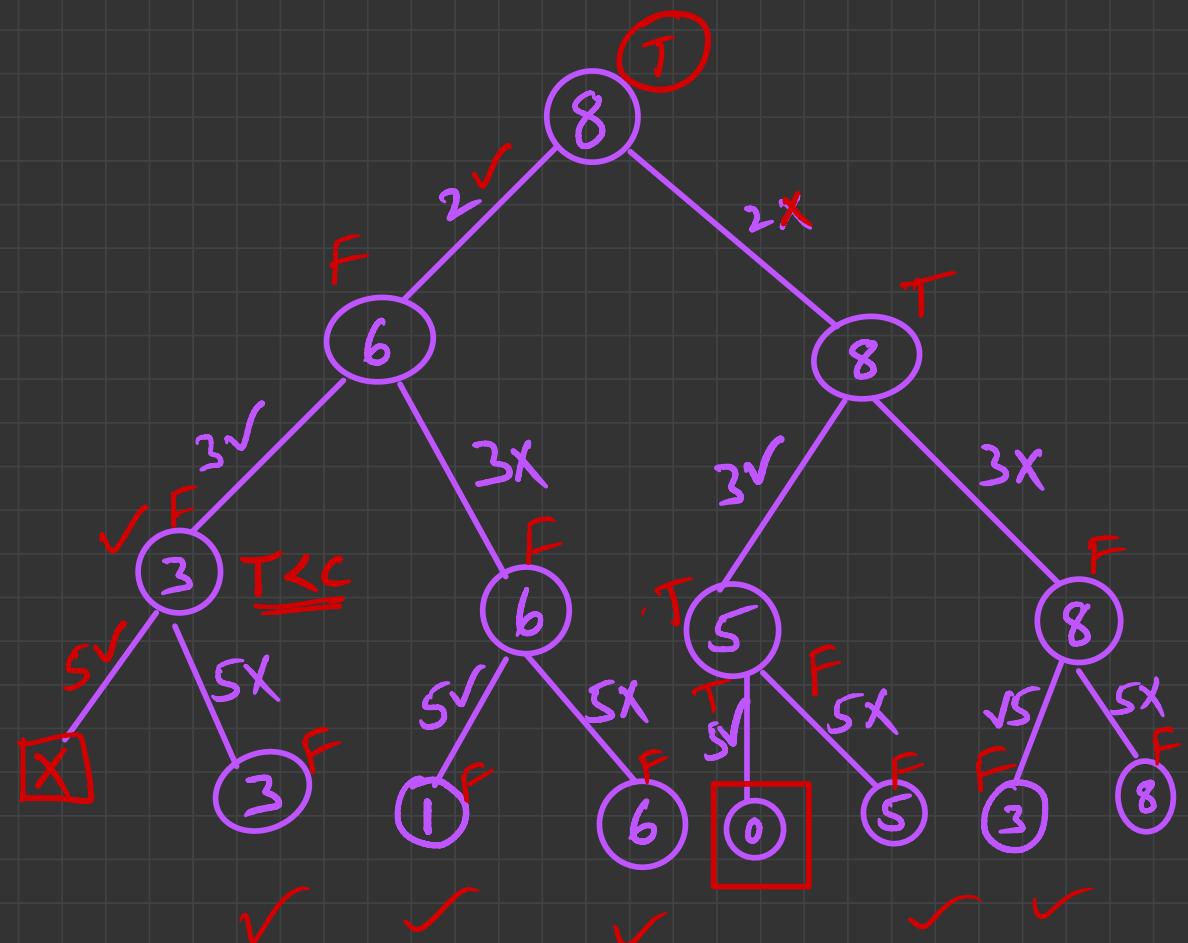
- Is subset sum Possible

$$T \checkmark \Rightarrow \{ \underbrace{\quad}_{S} \} \subseteq A$$

$$\sum S = T \checkmark$$

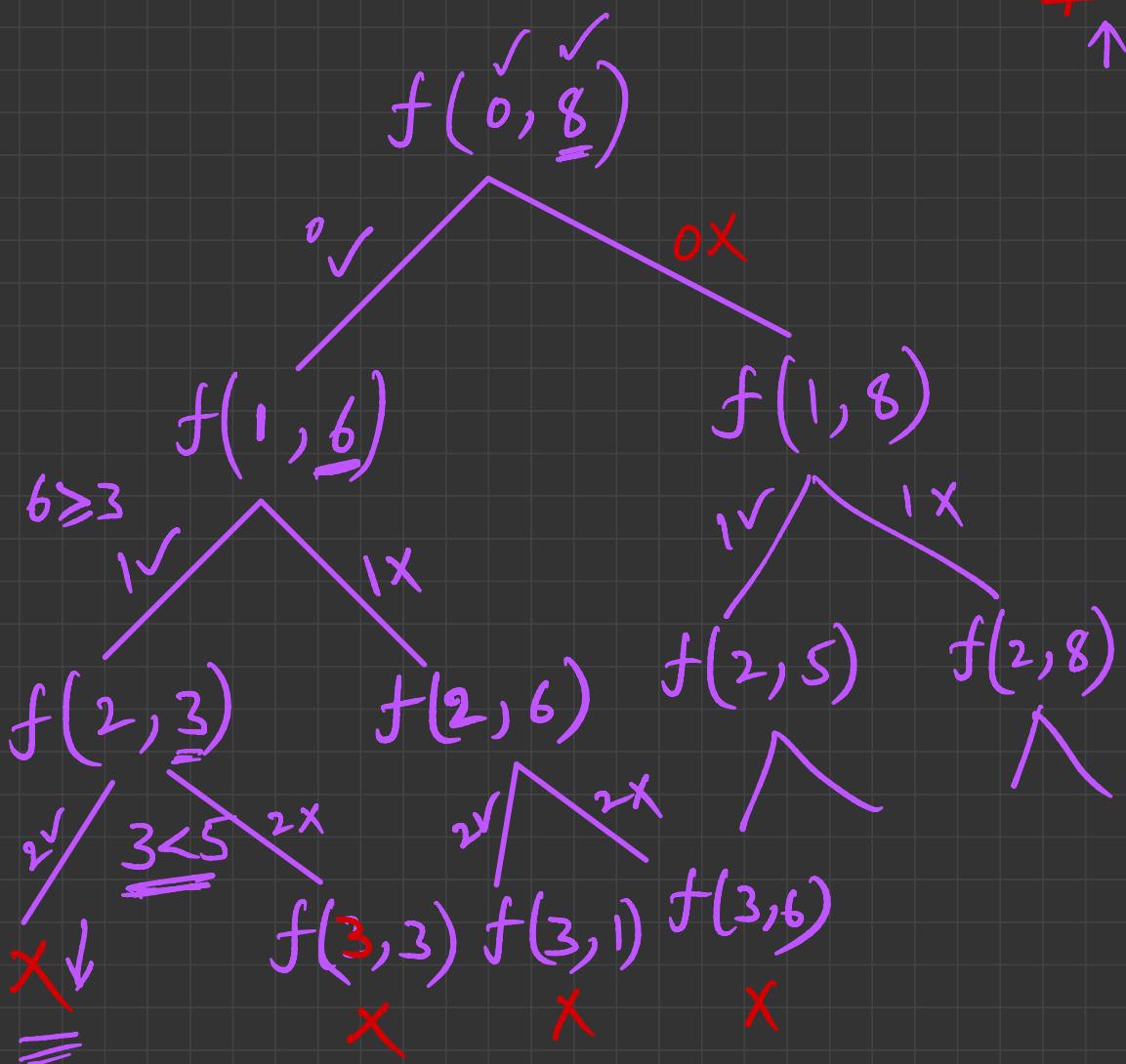
$$A = \{ \overset{\checkmark}{2}, \overset{\checkmark}{3}, \overset{\checkmark}{5} \}$$

$$T = 8$$



$f(i, T, A)$

$$A = \{ \begin{matrix} 0 & 1 & 2 \\ 2, 3, 5 \end{matrix} \}$$



$f(i, T, A) \{$

if ($T == 0$) : return True

if ($i >= \text{size}(A)$) return False.

if ($T < 0$) : return False

return $f(i+1, T - A[i], A)$ \vee
 $f(i+1, T, A)$

$f(i, T, A) \{$

if ($T == 0$) : return True

if ($i >= \text{size}(A)$) : return False.

if ($T \geq A[i]$)

Take = $f(i+1, T - A[i], A)$

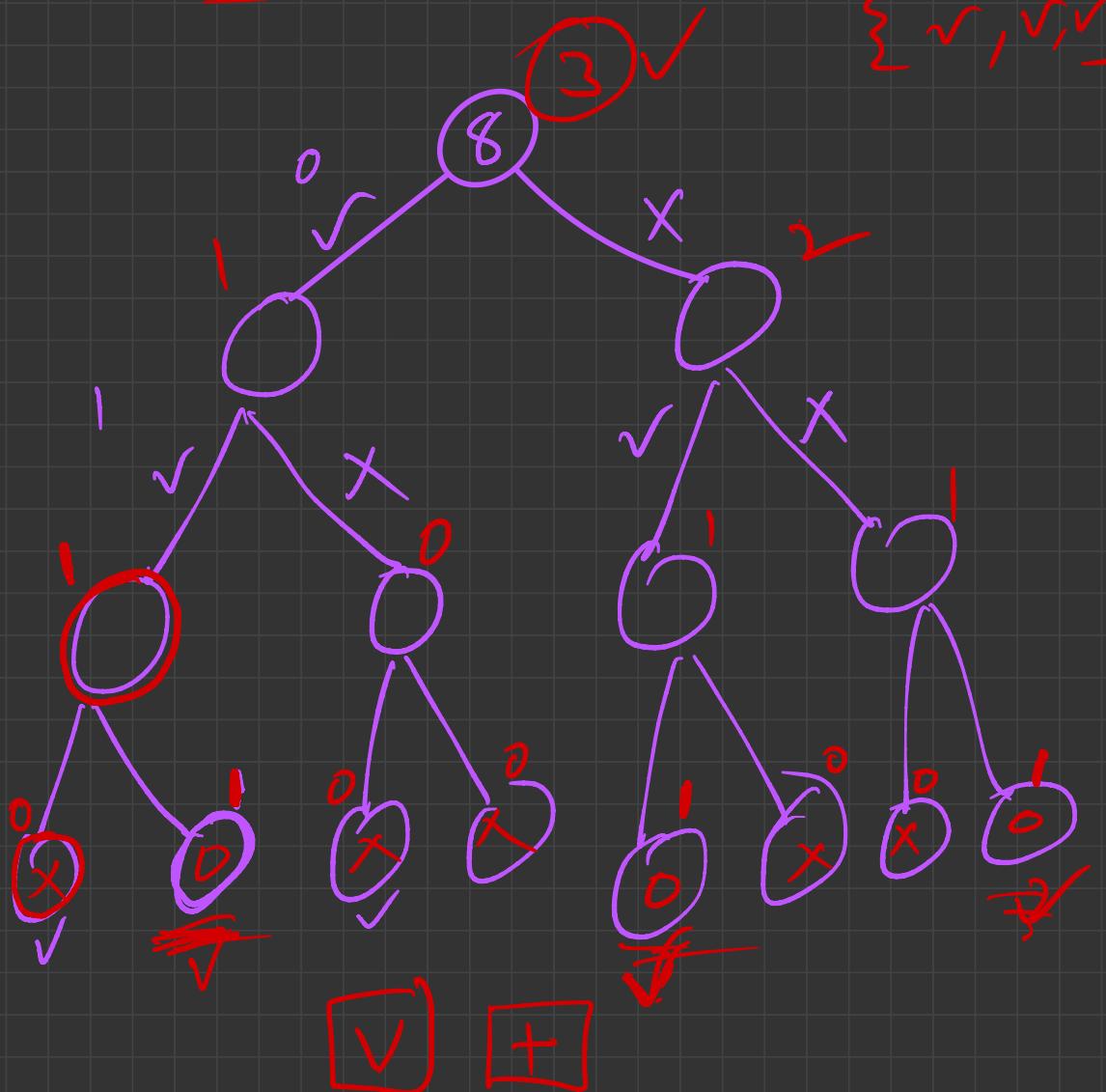
NotTake = $f(i+1, T, A)$

return Take \vee NotTake

✓ • No of subsets to produce T ✓

✓ No of elements in smallest subset
for sum T.

{ ✓, ✓, ✓ }



$f(i, T, A) \{$

if ($T == 0$) return 1

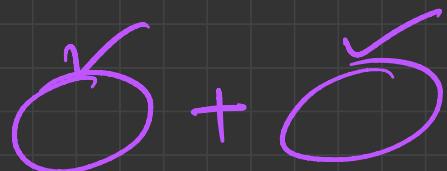
if ($i \geq \text{size}(A)$) return 0

if ($T \geq A[i]$) {

Take = $f(i+1, T-A[i], A)$

NotTake = $f(i+1, T, A)$

return Take + NotTake



$f(i, T, A) \{$

if ($T \geq A[i]$) ↓ ↓

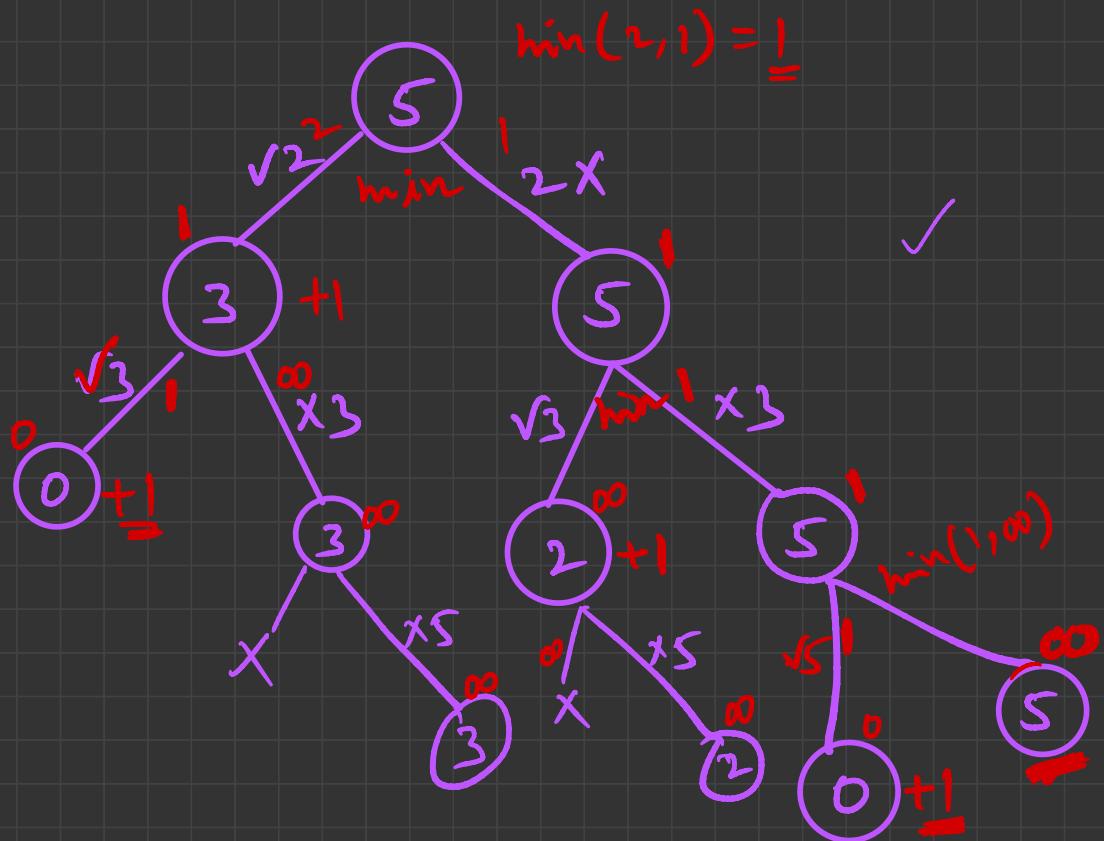
Take = $f(\underline{i+1}, \underline{T-A[i]}, A) + 1$

NotTake = $f(\underline{i+1}, \underline{T}, A)$

1D, 2D

$\{2, 3, 5\}$

$T = S$



$f(i, T, A) \{$

if $T == 0$: return 0

if $i \geq \text{size}(A)$: return ∞

if ($T \leq A[i]$)

Take = $f(i+1, T-A[i], A)$

NotTake = $f(i+1, T, A)$

return min(Take, NotTake)

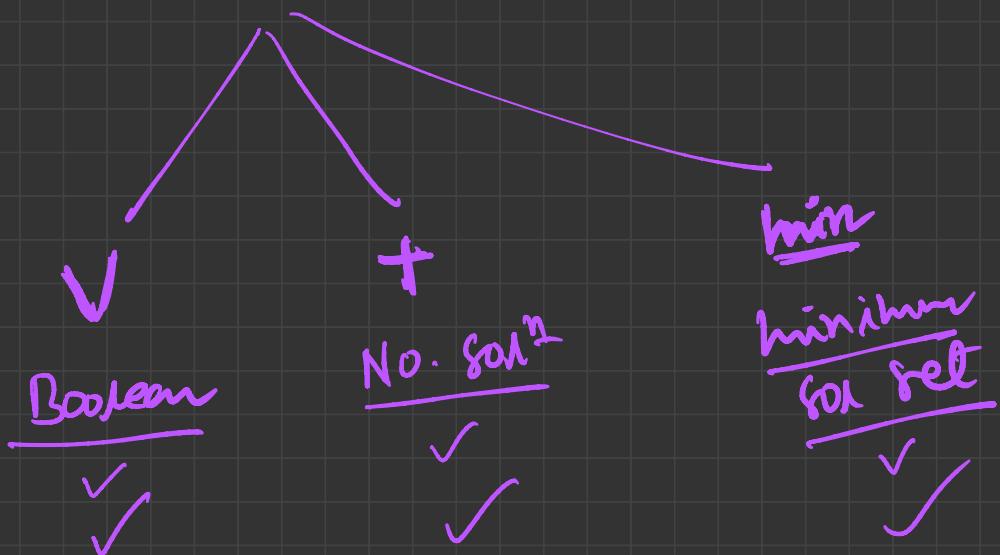
A, T

- no. of elements in largest subset.
- Is it possible to produce a subset with K-no. of elements
- no. of elements in second "smallest"

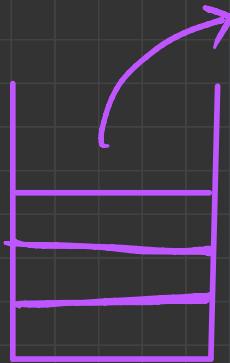
① $\{ \underset{\infty}{2}, \underset{\infty}{3}, \underset{\infty}{5} \} \checkmark$

② $\{ \underset{1}{2}, \underset{1}{3}, \underset{1}{5} \} \checkmark$

③ $\{ \underset{2}{2}, \underset{2}{3}, \underset{5}{5}, \underset{5}{5}, \underset{5}{5} \} ??$



"STACK:"



"LIFO"

$$\{2, 3, 5\} \rightarrow T=5$$

"solution" "subset" ✓

→ T/F ✓ ✓

→ int ✓ ✓

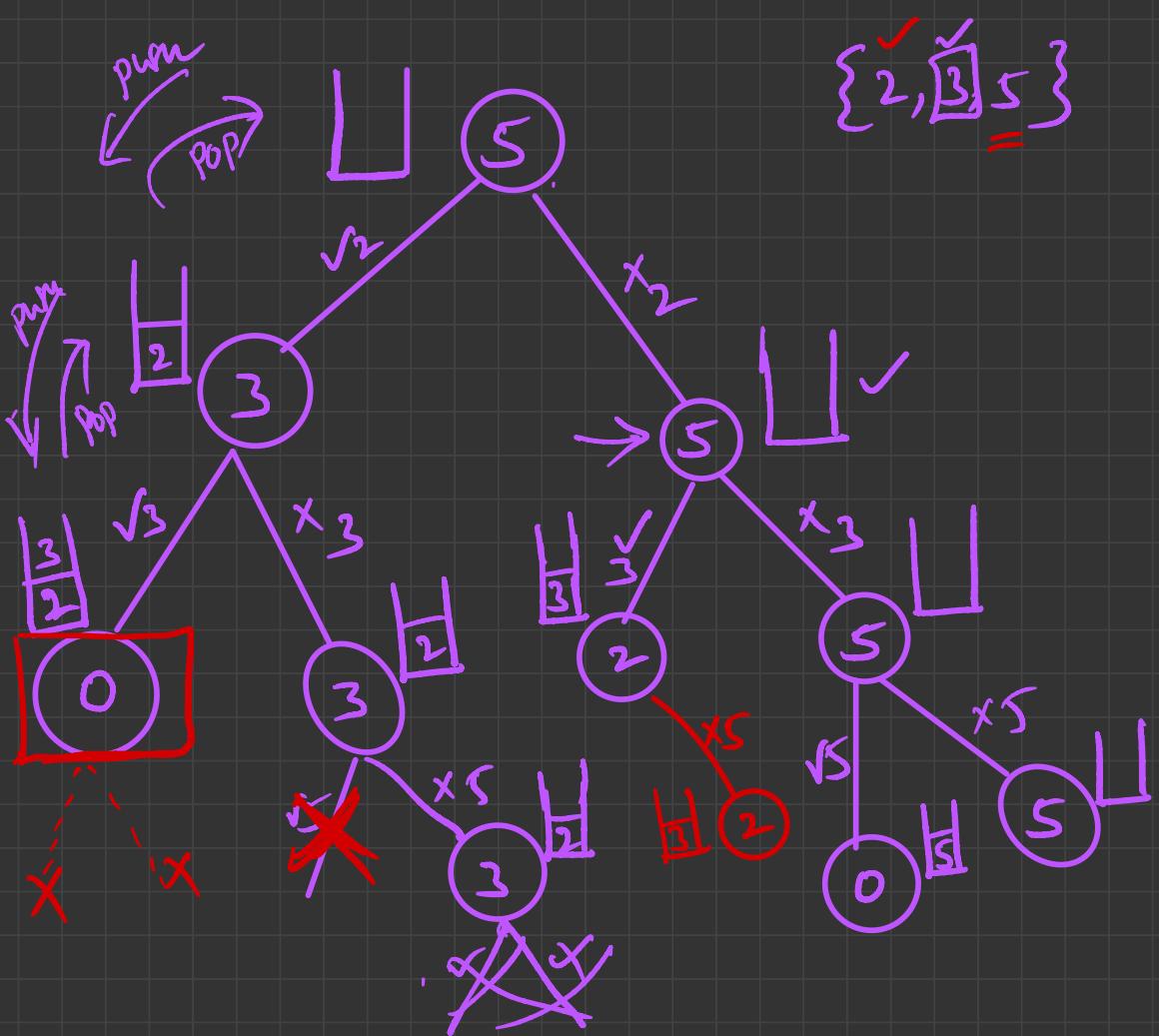
→ int ✓ ✓

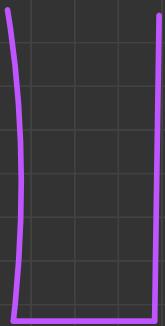
→ $\{\underbrace{\{2, 3\}}, \underbrace{\{5\}}\}$ ✓

AWS

B

B





S

S.push(5)

S.push(6)

S.pop() ←

S.pop(8)

S.pop() ←

S.pop()

$f(i, T, A, B, \underline{\text{Ans}}) \{$

if ($T == 0$):

$\text{Ans.push_back}(B)$

 return -

 if ($i \geq \underline{\text{size}(A)}$) return

 if ($T \geq A[i]$) ✓

$B.\text{push_back}(A[i]); \checkmark$

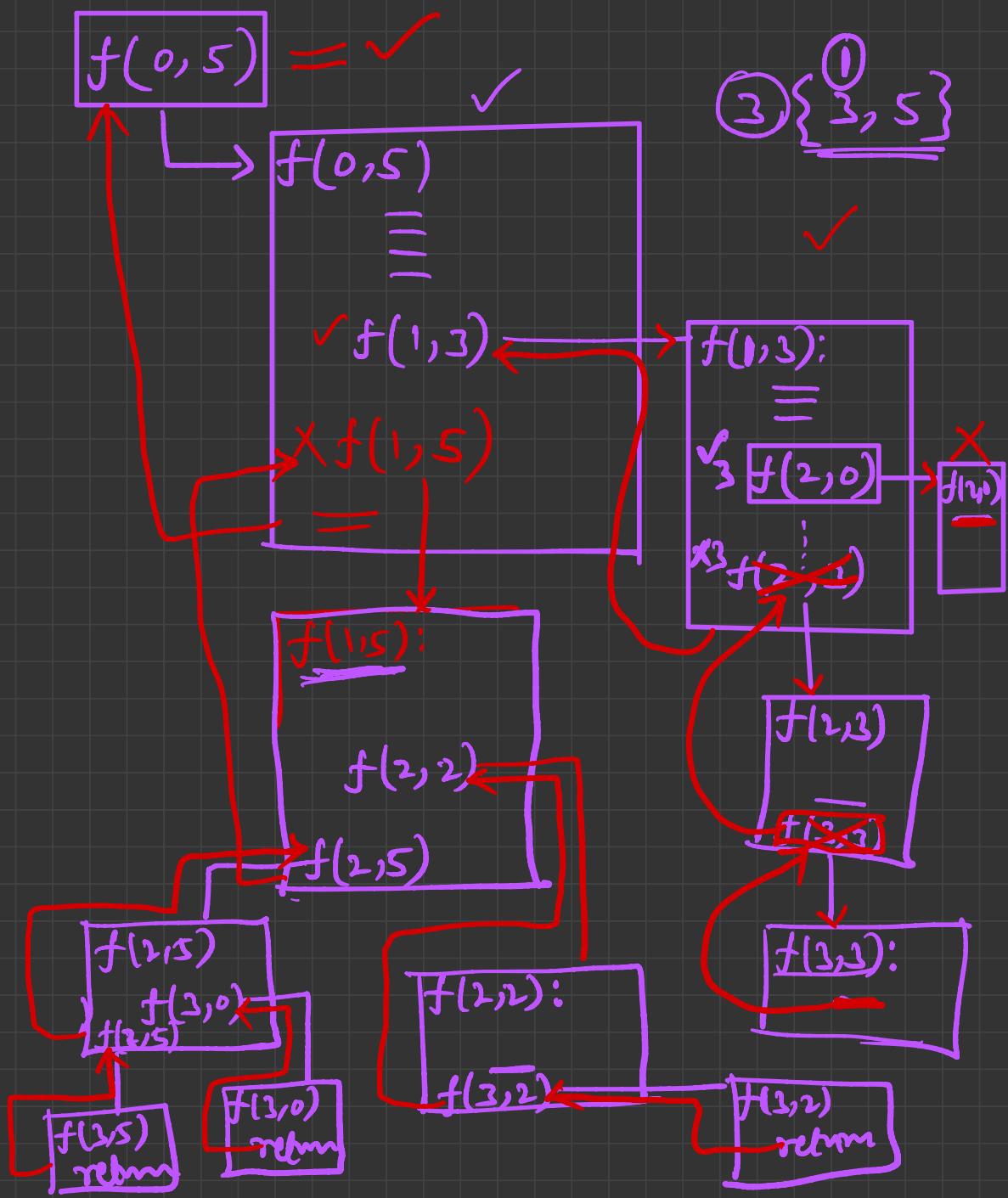
✓ $\rightarrow f(i+1, T-A[i], A, B, \underline{\text{Ans}});$

$B.\text{pop}(); \checkmark$

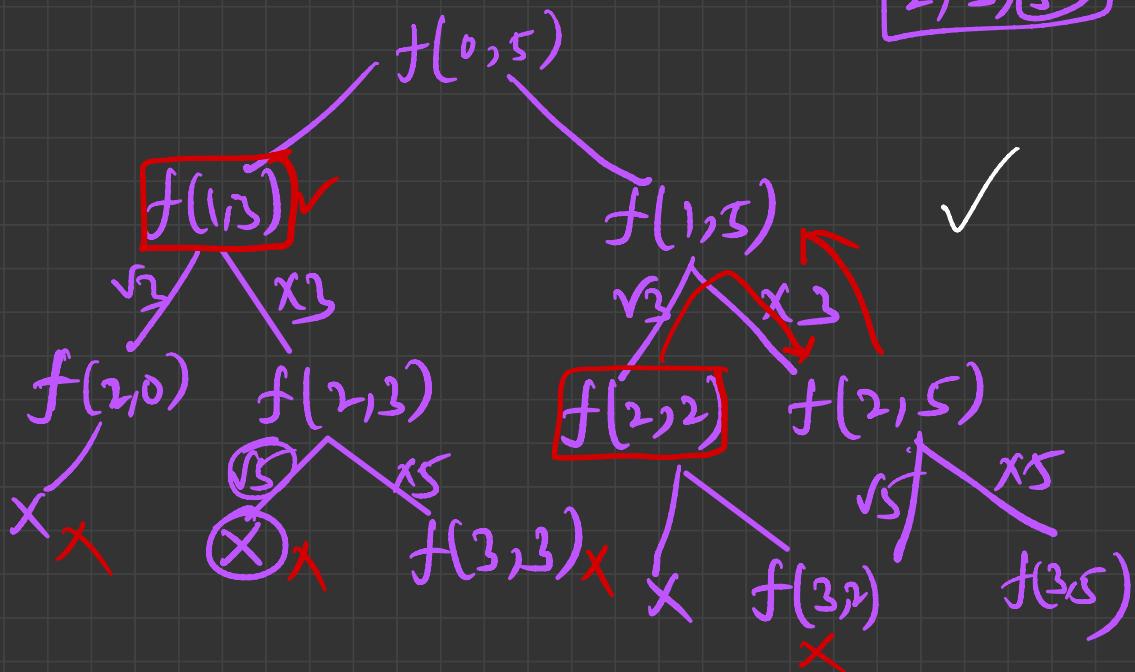
✗ $f(i+1, T, A, B, \underline{\text{Ans}})$ ✓

Tree \rightarrow "call stack"

$\downarrow \downarrow \downarrow$
 $\{2, 3, 5\}$



2, 3, 1
3



~~DAY-4~~ prime numbers!

$$1 \leq n \leq 1000 ; 0 \leq k \leq n$$

$$n = 15, k = 2$$

$$15 = \left(\frac{1}{p_1} + \frac{1}{p_2} \right) \quad \underline{\underline{k=2}}$$

$$\left(\frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3} \right) \text{ has}$$

$$k=2; 15 = \underline{2} + \underline{13}$$

$$\boxed{f(15, 2) = 1}, f(24, 2)$$

$$\underline{2, 3, 5, 7, 11, 13, 17, 19, 23}$$

$$f(24, 2) = n \{ 11 + 13, 19 + 5, 7 + 17 \}$$

$$f(24, 2) = 3$$

$$f(29, 3) = 2$$

$$1 \leq n \leq 1000$$

$$1 \leq k \leq n$$

$$f(\underline{7}, 1) = 1$$

$$f(\underline{7}, 2) = 1$$

$$f(\underline{7}, 3) = 0$$

$$n <$$

$$k <$$

$$\text{Ans: } f(n, k) = \underline{\underline{?}}$$

1) • we have already solved this?

2) • We haven't solved?

$$S = \{ \checkmark^2 \checkmark^3 \checkmark^5 \checkmark \checkmark \checkmark \checkmark \checkmark \checkmark \checkmark \checkmark \checkmark \}$$

k = "length"

target sum $\rightarrow n$

$$f(i, n, k)$$

$$n=7$$
$$A = \{2, 3, 5, 7\}$$

0 1 2 3

$$K=2$$

1 (3, 7, 2) $\boxed{H(7, 2)} = 1$

$$\sqrt{7} \quad + \quad \times \sqrt{7}$$

$^0 (2, 0, 1)$

$$5^{<0} \quad X \quad \times 5$$

$^1 (2, 7, 2)$

$$\sqrt{5} \quad + \quad \times 5$$

$^1 (1, 2, 1) \quad ^0 (1, 7, 2)$

$$3 \quad + \quad \times 3$$

X

$(0, 2, 1)$

$$\sqrt{3} \quad + \quad \times 3$$

$^0 (0, 4, 1) \quad ^0 (0, 7, 2)$

$\sqrt{2} \quad + \quad \times 2$

$^0 (-1, 2, 0) \quad ^0 (-1, 7, 1)$

$\sqrt{2} \quad + \quad \times 2$

$(-1, 0, 0) \quad ^0 (-1, 2, 1)$

$k=0 \text{ and } n=0$

"Tree Diagram"

$f(i, n, k) :$

if ($n == 0 \& k == 0$) return 1

if ($n == 0 \text{ or } i < 0$) return 0

if ($k > i + 1$) return 0

if ($A[i] \leq n$):

Take = $f(i - 1, n - A[i], k - 1)$;

NotTake = $f(i - 1, n, k)$

return Take + NotTake;

$f(i, n, k)$:

if ($n == 0 \wedge k == 0$) return

if ($n == 0 \vee i < 0$) return 0

if ($k > i + 1$) return 0

if ($DP[i][n][k] != -1$)

return $DP[i][n][k]$

if ($A[i] \leq n$):

Take = $f(i-1, n-A[i], k-1)$;

NotTake = $f(i-1, n, k)$

return $DP[i][n][k] = Take + NotTake$;

1/0 knapsack

"Bag"

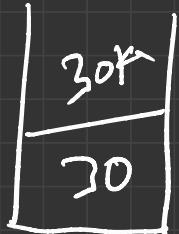
$W =$	50	25	30	✓
-------	----	----	----	---

check

$P =$	270	300	100	✓
-------	-----	-----	-----	---

Optimal price

60 kg



0 1 2

↑

(2, 60)



2

400

✓ 30

max

300

100+ (1, 30)

✓ 25

max

300+ (0, 5)

x 25

0

(0, 30)

(1, 60)

✓ 25

max

x 25

0

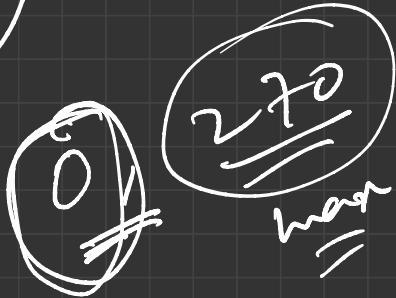
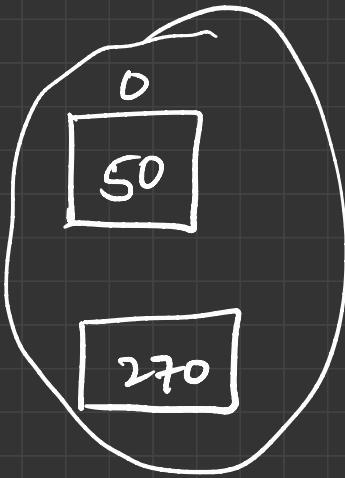
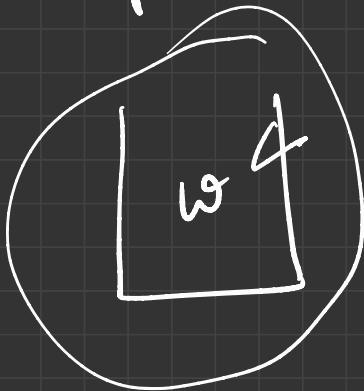
(0, 35)

270

max

3

"Smallest problem"



$f(i, w) :$

if ($i == 0$) :

 if ($w[0] \leq w$) return $p[i]$

 else return 0

take = -INF

if ($w[i] \leq w$) :

\checkmark take = $p[i] + f(i-1, w-w[i])$

notTake = $f(i-1, w)$

return max(take, notTake)

~~$n=30$~~

~~30
2~~

$f(n-1, w)$

$w = \boxed{0 \quad | \quad | \quad | \quad | \quad | \quad n-1}$

$\checkmark w \leftarrow \text{bag capacity.}$

$\checkmark p = \boxed{0 \quad | \quad | \quad | \quad | \quad | \quad n-1}$

$f(i, w) :$

$\text{if } i == 0 :$

$\text{if } w[i] \leq w \text{ return } p[i]$

$\text{else return } 0$

$\text{if } (DP[i][w] != -1) \text{ return } DP[i][w]$

$\text{take} = -\infty$

$\text{if } w[i] \leq w :$

$\text{take} = p[i] + f(i-1, w-w[i])$

$\text{not take} = f(i-1, w)$

$\text{return } DP[i][w] = \max(\text{take}, \text{not take})$

0	1	2	-	-	-	-	-	19
-	-	-	-	-	-	-	-	-

$$W = \frac{100 \text{ kg}}{\checkmark}$$

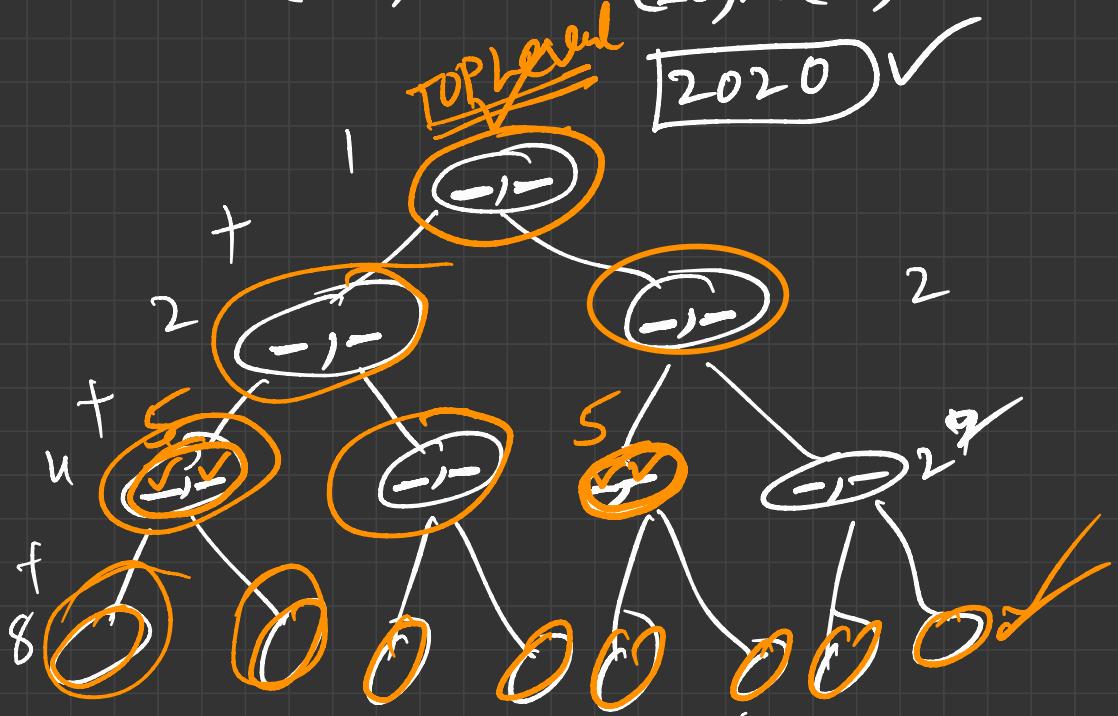
$$(i, w)$$

$\begin{matrix} \swarrow \\ (0-19) \times (0-\underline{\underline{100}}) \end{matrix}$

$$(-, -) \quad (20) \times (101)$$

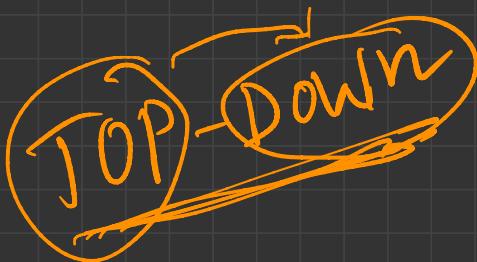
~~TOP level~~

2020 ✓



$$1 + 2 + 2^2 + 2^3 + \dots + 2^{20} = \underline{\underline{2^1 - 1}}$$

$$2020 \leftarrow 2^1 - 1$$



Ex-1

w

0	1	2	3	4	5
60	20	50	25	30	70
100	110	70	200	150	60
3	2	1	4	2	5

TRY!!

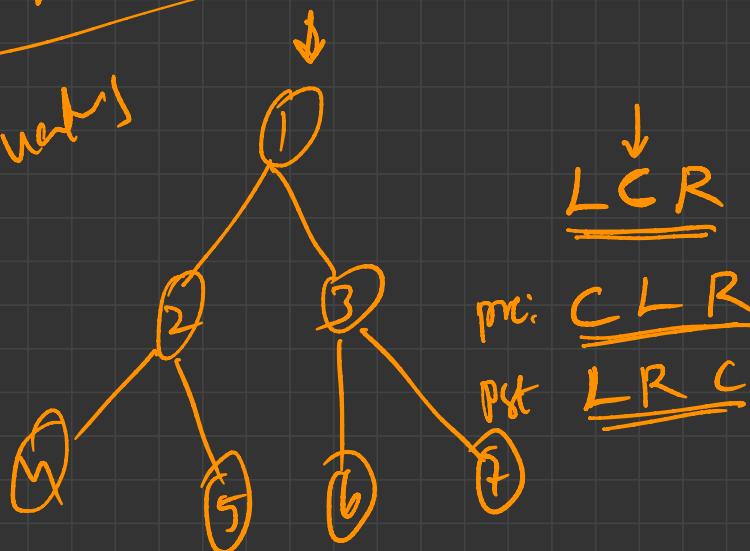
p

$$W = 400$$

"Optimize"

TRY!!!

Tree Traveller
Data Structures
"Tools"



LCR

pre: CLR
post: LR C

Inorder: 4, 2, 5, 1, 6, 3, 7 ✓

"Recursive Thinking"

Practice!!

Ex-2 (1-2) Hours

$$P = \boxed{570 \mid 230 \mid 1500 \mid 2010 \mid 1600}$$

TRY!!

$$f = \boxed{4 \mid 2 \mid 3 \mid 4 \mid 5}$$

$$B = \underline{\underline{3000}}$$

Buy items such that max(f)

$$\underline{\underline{\text{max}(f)}} = ?$$

? > 2000 Discount 200 ✓

~~1800 b 2001~~ $b+200$

$S + P$

$\underline{b+200}$

$b = 1900$

$$\begin{array}{r} 1900 \\ 200 \\ \hline 2 \end{array}$$

1950	30	10	\checkmark	\checkmark	4
5	2	1			

Ans: 3

\checkmark \checkmark S

S

\checkmark \checkmark P_4

$(S + P) \leq b$

$$P_5 > b$$

$$S + P_5 > b$$

$$\underline{S + P_5} \leq 2001$$

$$\underline{S + P_5} \leq b + 200$$

$$\underline{\underline{1900}} + \underline{\underline{200}}$$

$$\underline{\underline{200}}$$

$$\underline{\underline{>2000}}$$

$$S = \underline{\underline{1950}} + \underline{\underline{\checkmark}} + \underline{\underline{\checkmark}} + \underline{\underline{\checkmark}} + \underline{\underline{\checkmark}}$$

$$i \rightarrow \underline{\underline{1}} \quad \underline{\underline{2}} \quad \underline{\underline{3}}$$



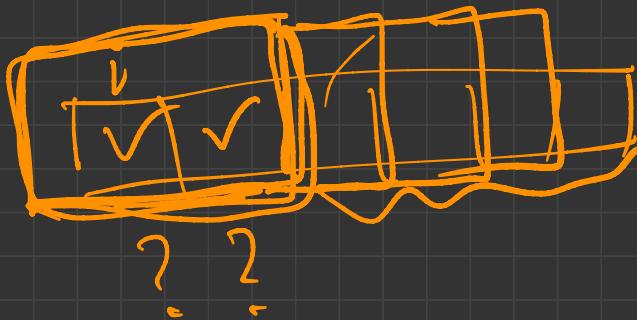
$2000 \downarrow 15^\circ$
5 2 1

$$\underline{\underline{>2000}}$$

$$\underline{\underline{S \leq 2100}}$$

> 2000

$s \leq b + 200$



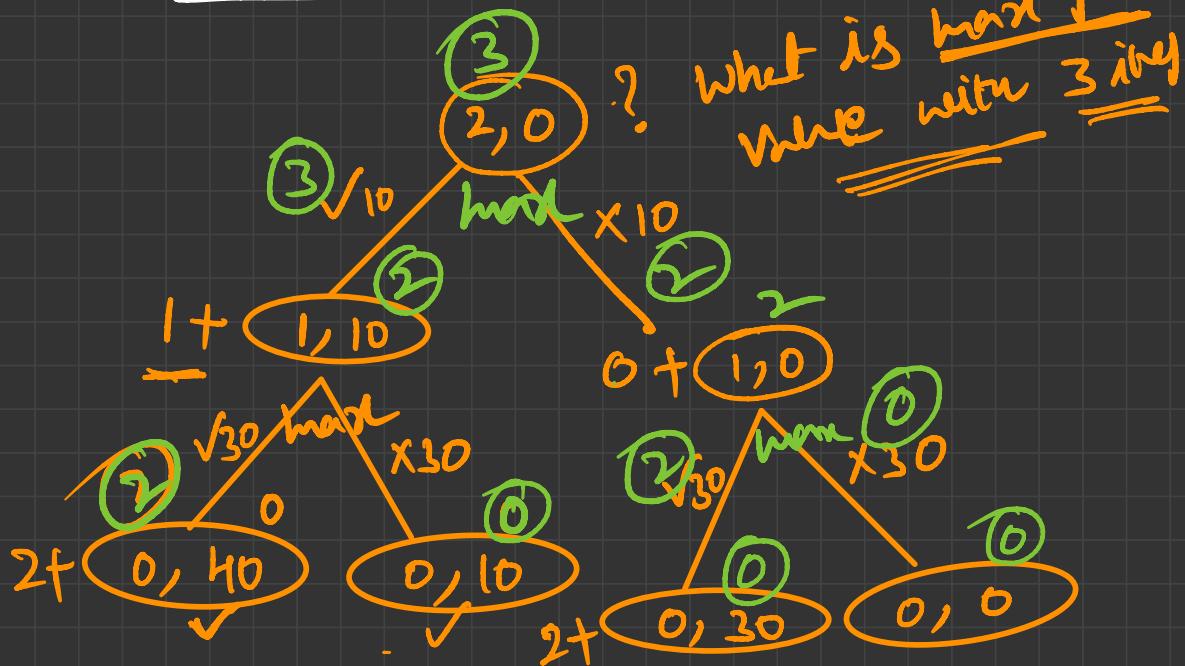
$$b = 1900$$

0	1	2
1950	30	10
5	2	1



2

what is mark fav
value with 3 ike



$$40 + 1950 = 1990$$

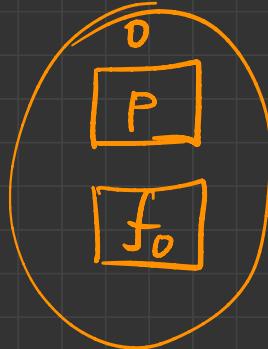
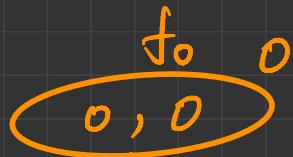
$$b = 1900$$

$$S + P_0 > b$$

$$S + P_0 < 2001$$

$$S \leq b$$

b



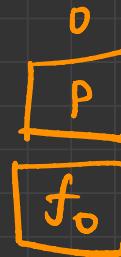
✓ P ✓✓

✓ P ✗

✓ — ?

$\lceil S \rfloor$

An oval containing the symbols 0 and S .



S

$\underline{S > b}$

b ~~pick~~ $S + P \leq b$ → ✓

P ~~pick~~ $S + P > b$ $\&$ $S + P < \underline{b + 200}$
 $\&$ $S + P \geq 2001$ ✗✓

Not Pick

$$\underline{S \leq b} \quad \checkmark$$

$$S > b$$

✓ ✓ ✓

$$(S) > b$$

$$S + P > b \quad \text{and} \quad S + P \leq 2000$$

Without

$$S \leq b$$
$$S > b$$
$$S \leq b + 200$$
$$S \geq 2001$$

Lark

$$\underline{S + P} \quad \checkmark$$

$$S + P \leq b \quad \checkmark$$

\checkmark

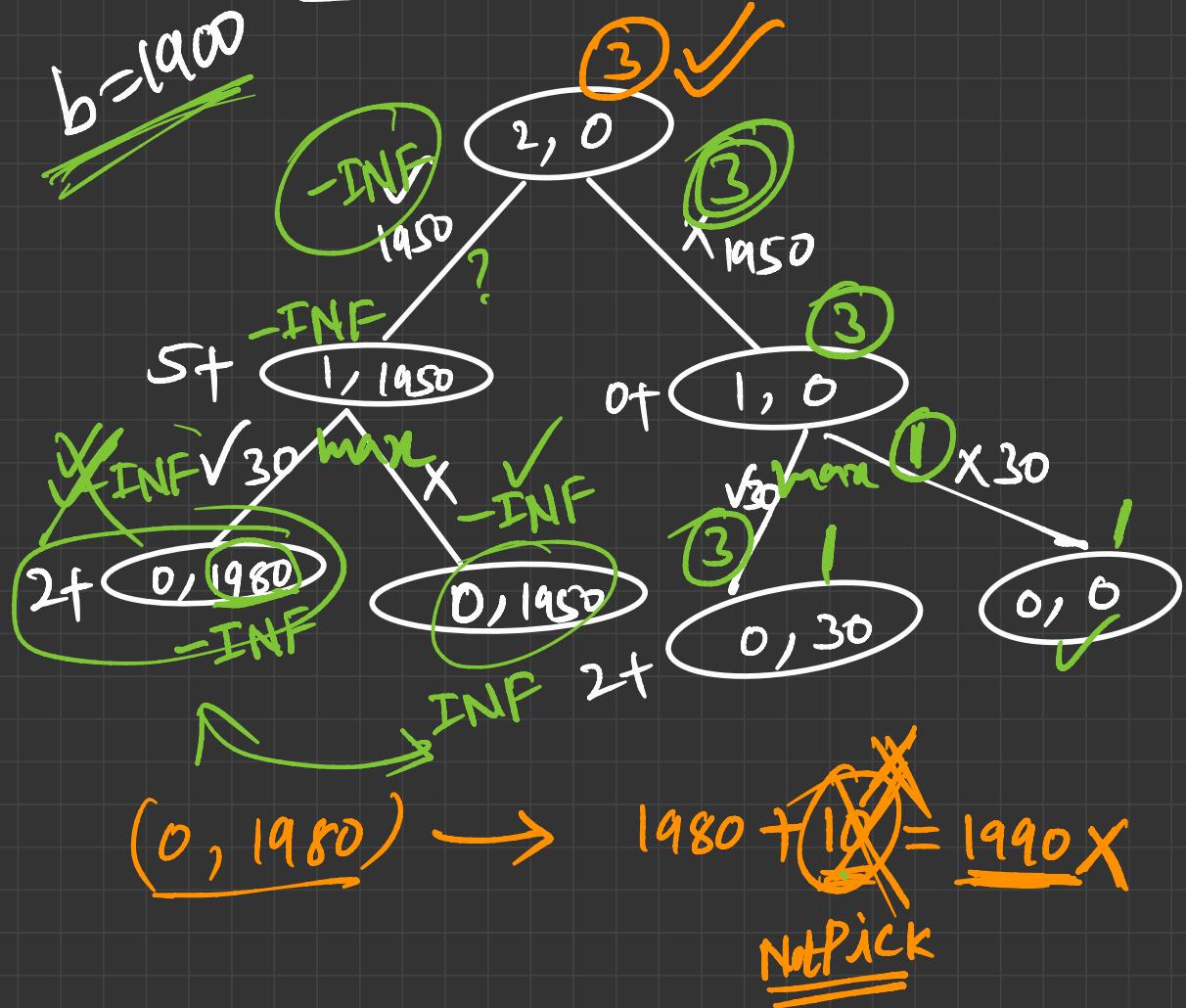
$$S + P > b \quad \text{and} \quad \begin{array}{l} S + P \leq b + 200 \\ S + P \geq 2001 \end{array}$$

$$b > 1800 \quad \checkmark$$

$$b = 1900$$

0	1	2
10	30	1950 ✓
1	2	5 ✓

~~b = 1900~~



$f(i, s) :$

$f(n-1, o)$

if ($i == 0$) {

if ($s + p[0] \leq b \quad || \quad b > 1800 \quad \&\& \quad s + p[0] \geq 2001$
 $\quad \quad \quad \&\& \quad s + p[0] \leq b + 200$)

return $F[0]$

if ($b > 1800 \quad \&\& \quad s \leq b + 200 \quad \&\& \quad s \geq 2001$)

return 0

if ($s > b$) return -INF ✓

return 0

}

Take = -INF

if ($p[i] \leq b \quad || \quad \frac{s + p[i] \leq b + 200 \quad \&\&}{b > 1800}$)

Take = $F[i] + f(i-1, s + p[i])$

notTake = $f(i-1, s)$

return max (Take, notTake)

