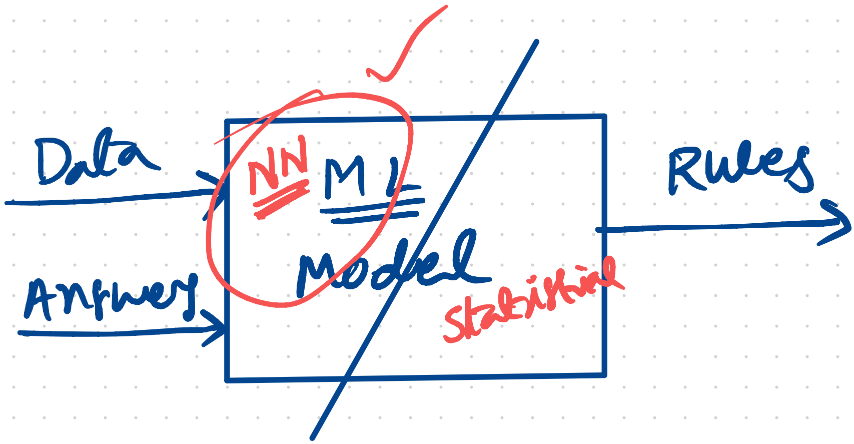


What is Neural Network?

Def:

$$A : \mathbb{R}^n \longrightarrow \mathbb{R}$$

$$A(x) = wx^T + b$$

where $w, x \in \mathbb{R}^n$, $b \in \mathbb{R}$

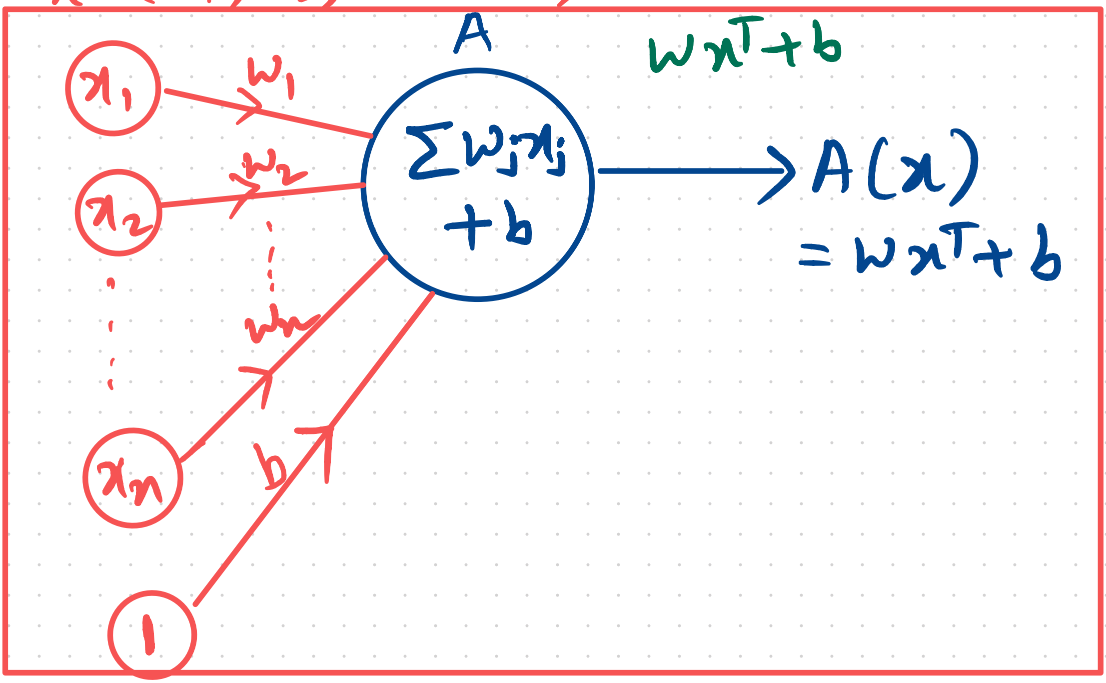
$$\boxed{\underline{A^n} = \left\{ A : \mathbb{R}^n \longrightarrow \mathbb{R} \;:\; A(x) = wx^T + b,\; w, x \in \mathbb{R}^n,\; b \in \mathbb{R} \right\}}$$

$$A : \mathbb{R}^n \longrightarrow \mathbb{R}$$

$$A(x) = wx^T + b$$

where $w, x \in \mathbb{R}^n$, $b \in \mathbb{R}$

$x = (x_1, x_2, \ldots x_n)$



$$w \cdot x = wx^T = \sum_{j=1}^{n} w_j x_j = w_1 x_1 + w_2 x_2 + \cdots w_n x_n$$

$$A(x) = \sum_{j=1}^{n} w_j x_j + 1 \cdot b$$

Def:

$f : \mathbb{R}^n \longrightarrow \mathbb{R}$, $G : \mathbb{R} \longrightarrow \mathbb{R}$ continuous function

$$f(x) = \sum_{j=1}^{m} \beta_j \, G(A_j(x)) \quad , \quad A_j \in A^n$$

$$= \sum_{j=1}^{m} \beta_j \, (G \circ A_j)(x)$$

$$\boxed{\begin{aligned} \Sigma_G = \Big\{ f : \mathbb{R}^n \longrightarrow \mathbb{R} : f(x) = \sum_{j=1}^{m} \beta_j \, G(A_j(x)) \\ x \in \mathbb{R}^n, \ \beta_j \in \mathbb{R}, \ A_j \in A^n, \\ m = 1, 2, 3, \ldots \Big\} \end{aligned}}$$

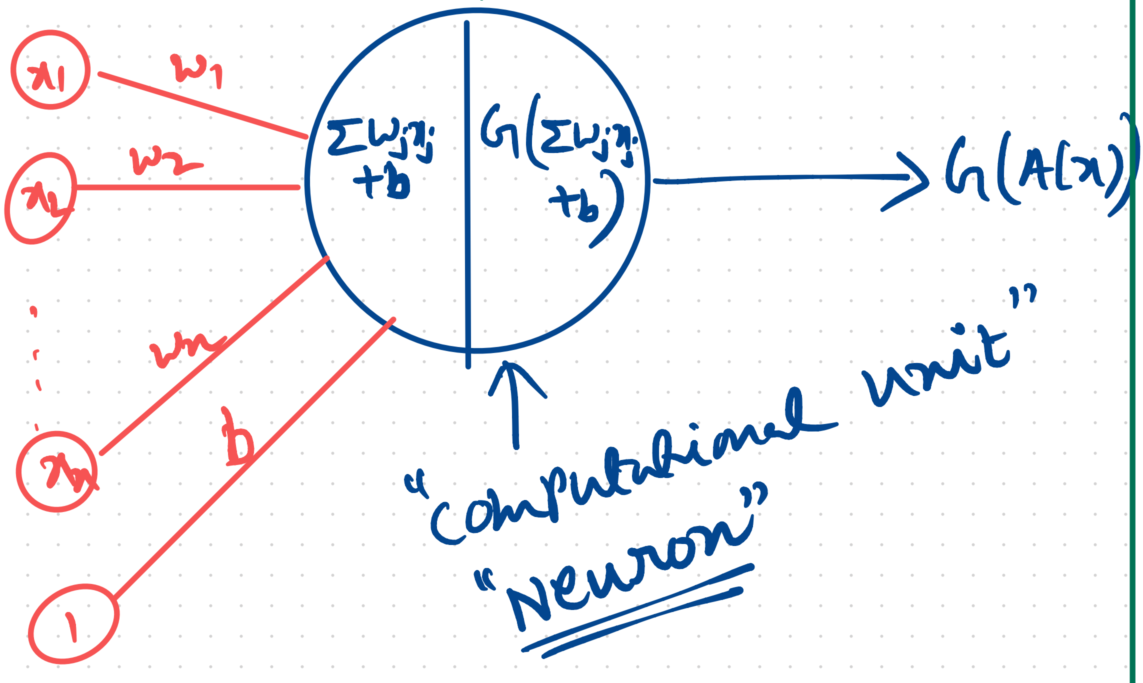
Σ <u>nets</u>

$$f(x) = \sum_{j=1}^{m} \beta_j \, G(A_j(x)) \quad , \quad A_j \in A^n$$

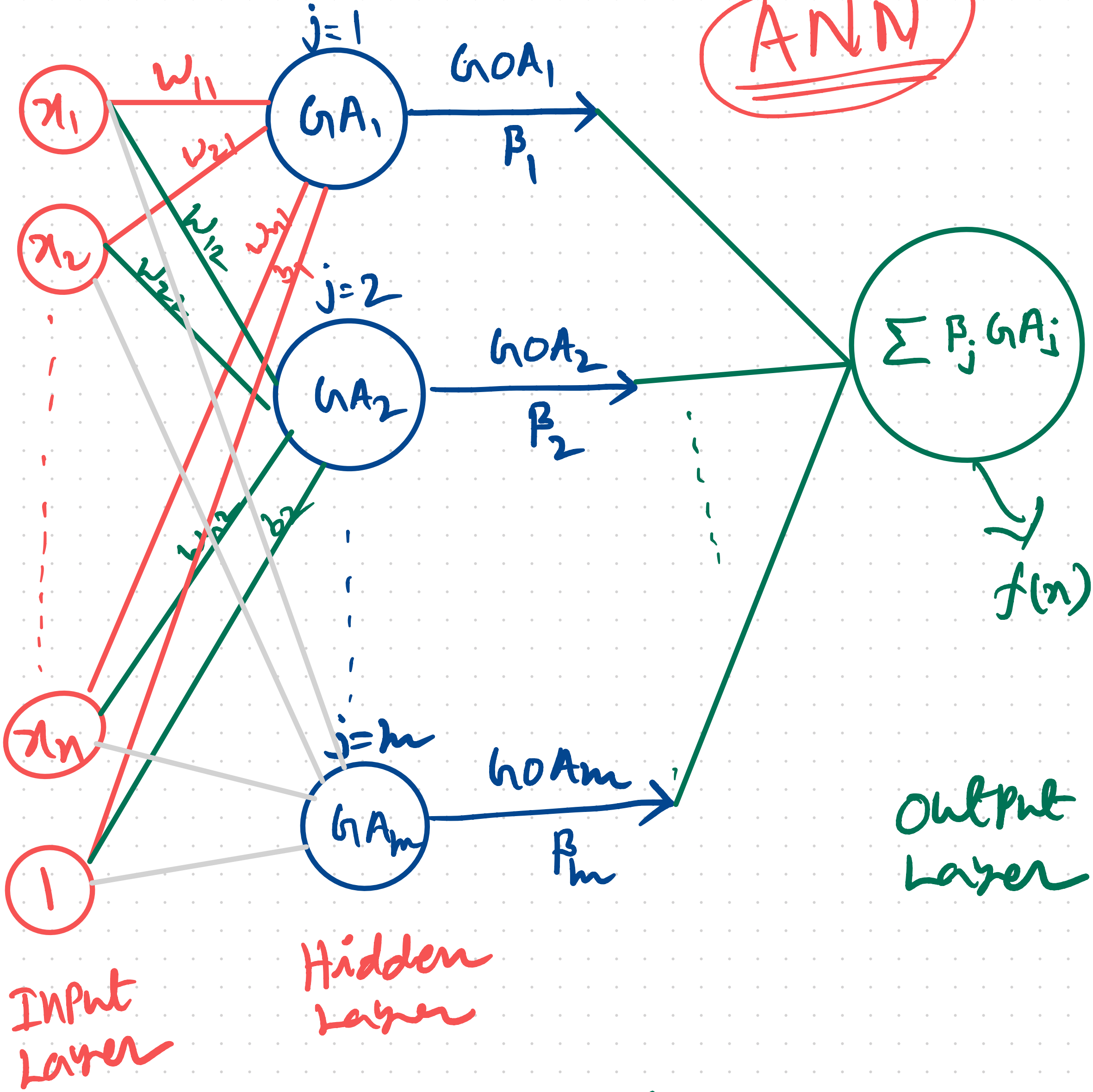
$$f(x) = \sum_{j=1}^{m} \beta_j \, \underline{\underline{(G \circ A_j)(x)}} \qquad \beta_j \in \mathbb{R}$$

$$(G \circ A)(x) = G(A(x))$$

$$\equiv$$



ANN



$$GA_j = GoA_j$$

$$(GA_j)(x) = (GoA_j)(x)$$

"Single Hidden Layer NN."

Def:

$f : \mathbb{R}^n \longrightarrow \mathbb{R}$

$$f(x) = \sum_{j=1}^{m} \beta_j \prod_{k=1}^{l_j} G(A_{jk}(x))$$

$\Sigma\Pi^G$

$$= \left\{ f : \mathbb{R}^n \longrightarrow \mathbb{R} : f(x) = \sum_{j=1}^{m} \beta_j \prod_{k=1}^{l_j} G(A_{jk}(x)) \right.$$

$$\left. x \in \mathbb{R}^n,\ \beta_j \in \mathbb{R},\ A_{jk} \in A^n \right.$$

$$\left. l_j \in \mathbb{N},\ m = 1, 2, 3 \ldots\ldots, \right\}$$

$\Sigma\Pi^G$

$\Sigma\Pi$ Nets