

Day-5: 27 June 2023

1980-90

"Let  $G$  be any continuous non-constant function from  $\mathbb{R}$  to  $\mathbb{R}$ .

Then  $\Sigma^n(G)$  is uniformly dense on compacta in  $C(\mathbb{R}^n)$ ".

" $\Sigma^n(G)$  networks are capable of arbitrarily accurate approximation to any real-valued continuous function over a compact set"

"Universal Approximation Theorem"

# Break down the goal into tasks

- Understand terminology in ML ✓
- Mathematical foundation of ML
  - Mathematical Definitions
  - Universal Approximation Theorem
- Mathematical Model of Artificial Neural Network(ANN)
  - Feed Forward
  - Backward Propagation
  - Activation and loss functions
- Implement algorithm for ANN in python
- **Problem-1:** Binary Classifier
- Introduction to python packages
- **Problem-2:** Digit Recognition
- **Problem-3:** Solving a Differential Equation



# Computers vs Human Brain

$$24321578392311 \times 6578413655462 + \frac{52345\sqrt{95665742124}}{\sqrt{1+\frac{57551872}{301785}}}$$

Computers

vs

Human Brain

Easy

Difficult

# Computers vs Human Brain



3



Computers

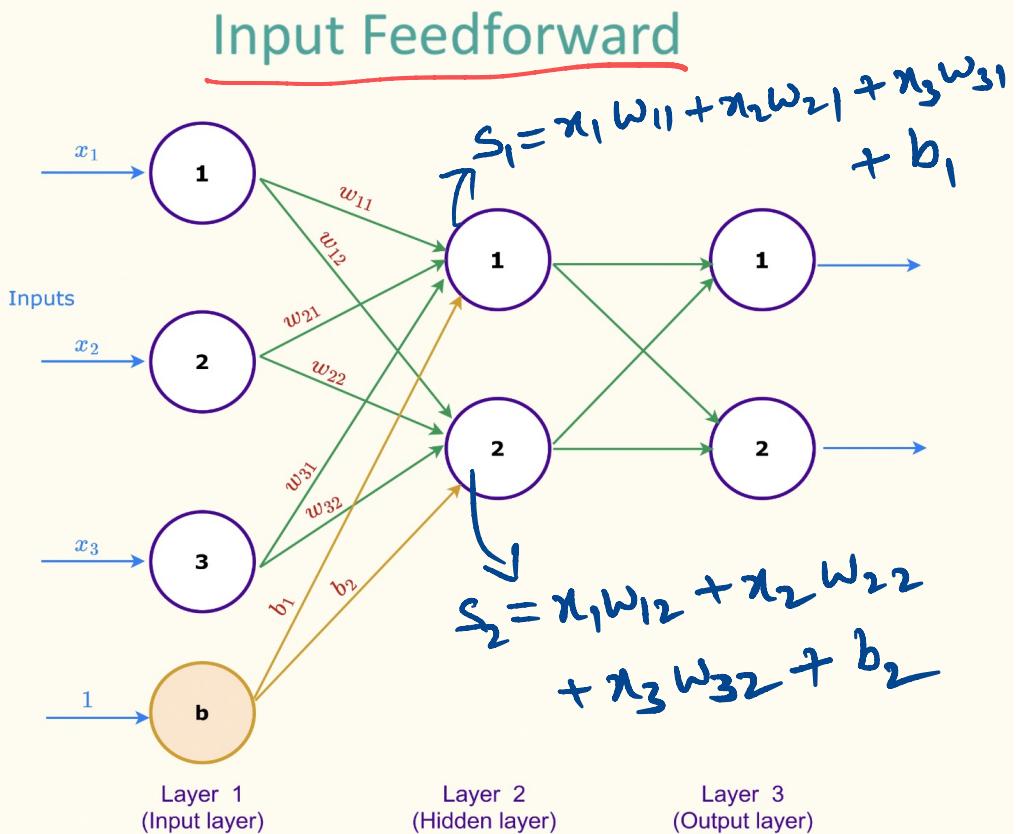
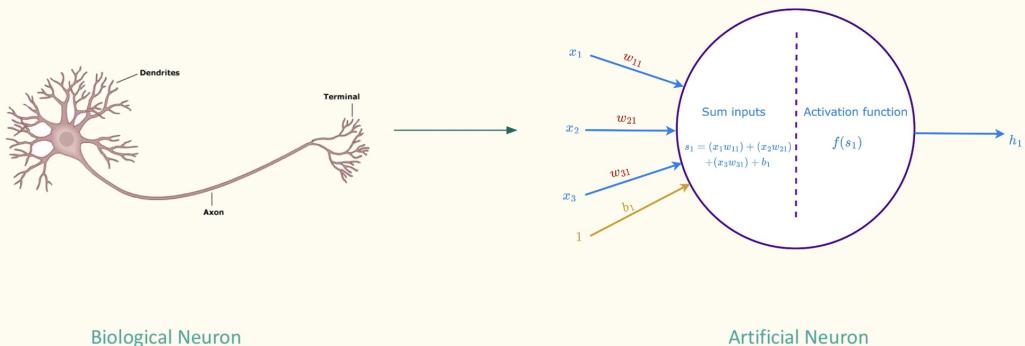
vs

Human Brain

Difficult

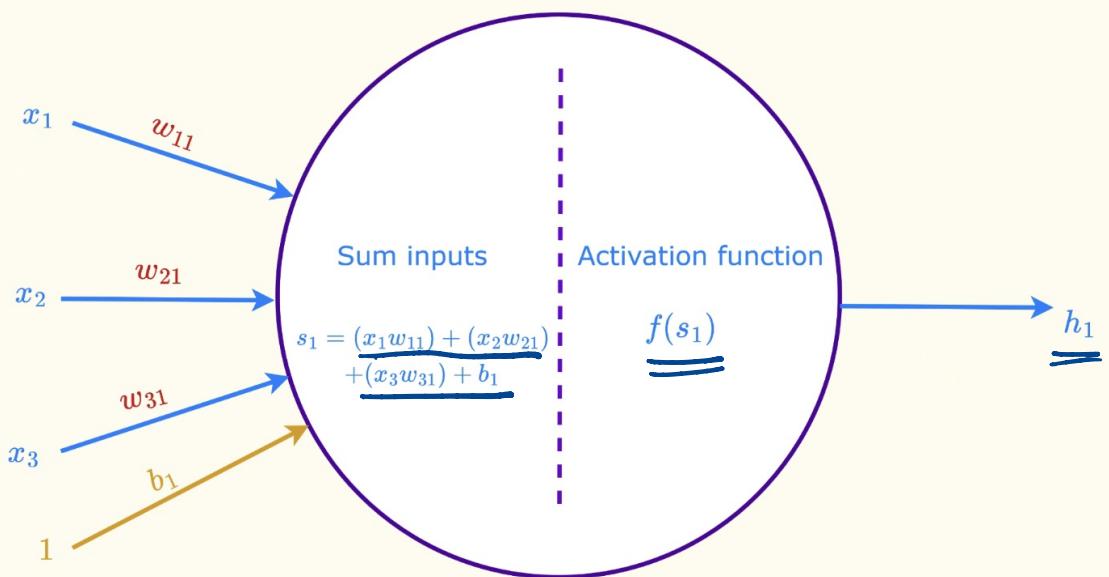
Easy

## Biological Neuron vs Artificial Neuron



Activation function (Sigmoid)

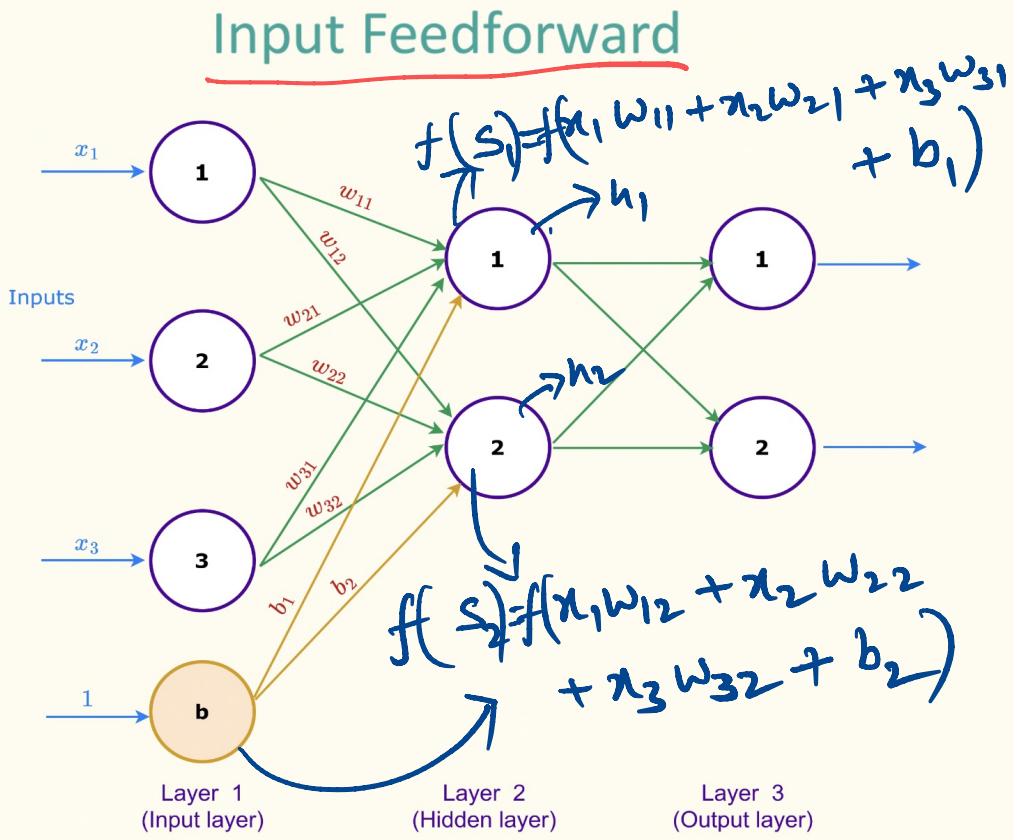
$$f(x) = \frac{1}{1+e^{-x}}$$



$$S_1 = x_1 w_{11} + x_2 w_{21} + x_3 w_{31} + b_1$$

$$S_2 = x_1 w_{12} + x_2 w_{22} + x_3 w_{32} + b_2$$

$$\begin{bmatrix} S_1 \\ S_2 \end{bmatrix} = \begin{bmatrix} w_{11} & w_{21} & w_{31} \\ w_{12} & w_{22} & w_{32} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$



$$h_1 = f(s_1) = f(x_1 w_{11} + x_2 w_{21} + x_3 w_{31} + b_1)$$

$$h_2 = f(s_2) = f(x_1 w_{12} + x_2 w_{22} + x_3 w_{32} + b_2)$$

$$\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} w_{11} & w_{21} & w_{31} \\ w_{12} & w_{22} & w_{32} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$

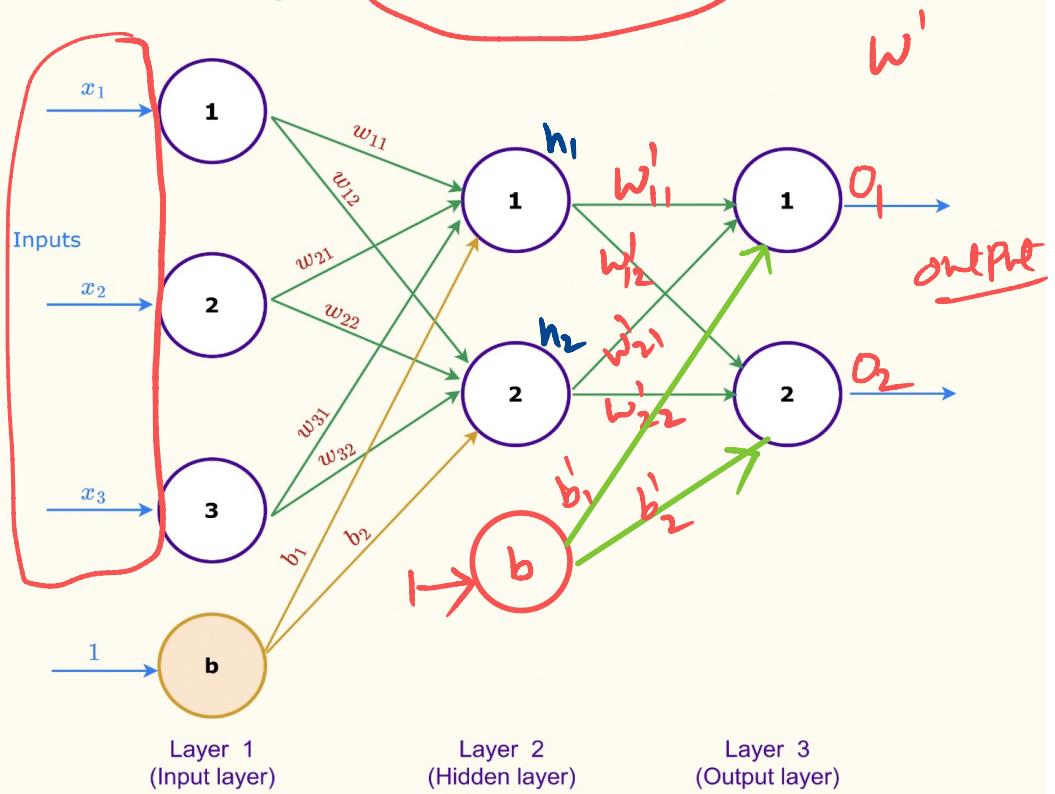
$$s_H = w_{IH} \cdot x_I + b_I$$

$$h_1 = f(s_1)$$

$$h_2 = f(s_2)$$

$$H_{\text{hidden}} = f(s_H)$$

# Input Feedforward



$$H_{\text{hidden}} = \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

$$O_1 = f(h_1 w'_{11} + h_2 w'_{21} + b'_1)$$

$$O_2 = f(h_1 w'_{12} + h_2 w'_{22} + b'_2)$$

$$\begin{pmatrix} o_1 \\ o_2 \end{pmatrix} = f \left( \begin{bmatrix} w_{11} & w_{21} \\ w_{12} & w_{22} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \right)$$

$$O_i = f(w_{HO}^T h_{\text{hidden}} + B_H)$$

ANN\_function\_approx

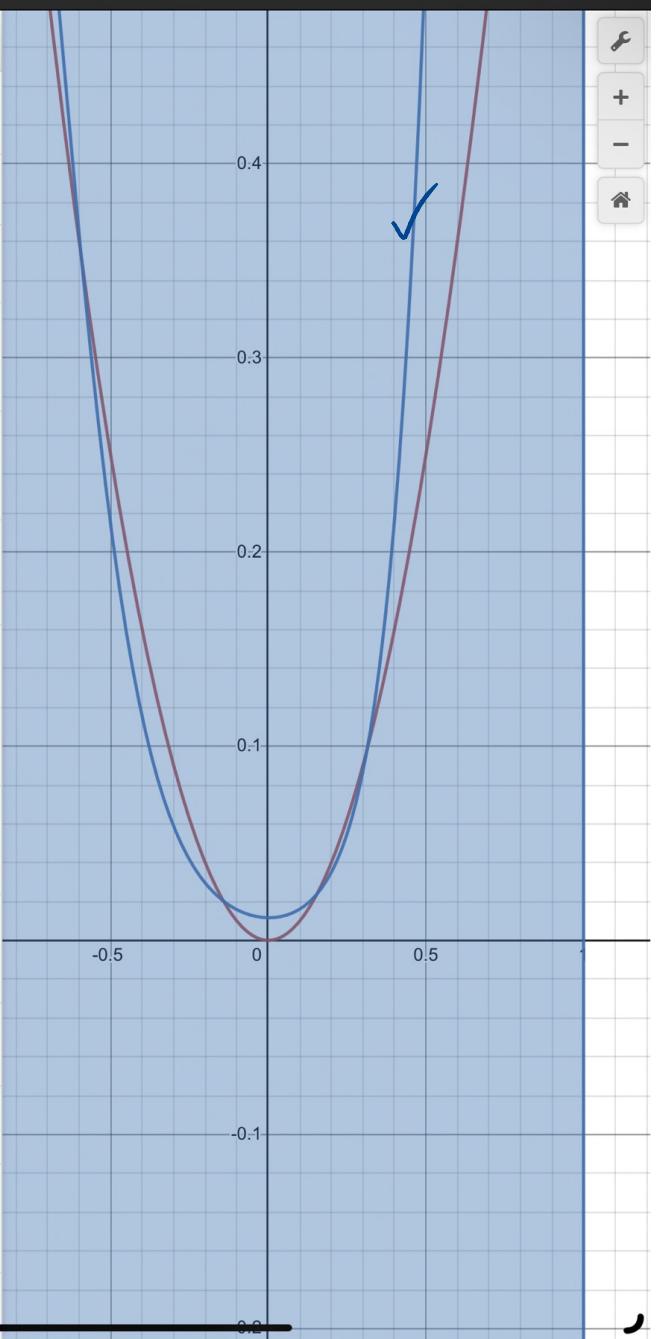
Save

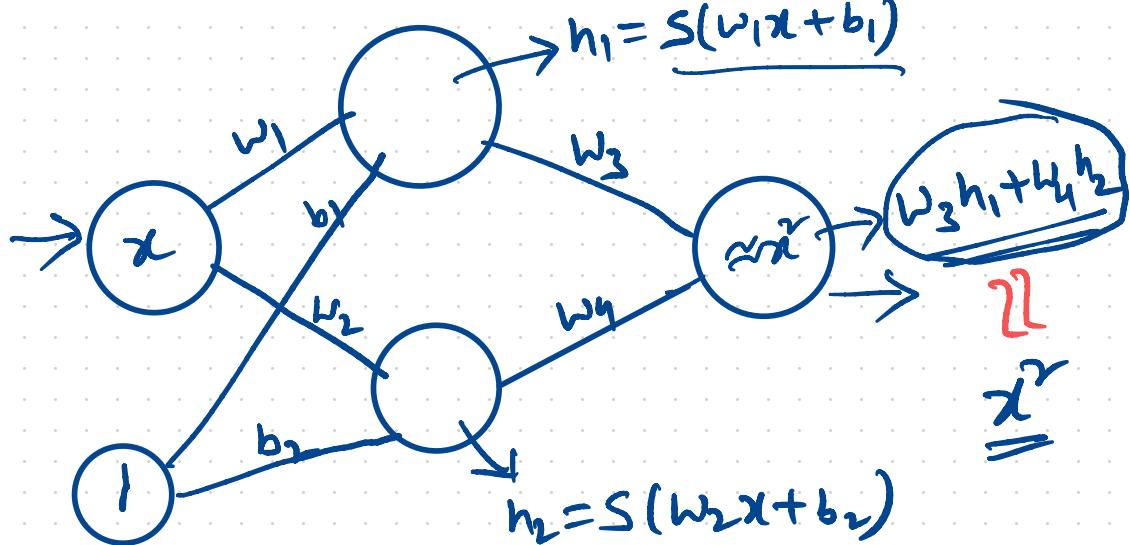
desmos

Satyaprasad



	$f(x) = x^2$	
	$-1 \leq x \leq 1$	
	$f_1(x) = w_1x + b_1$	
	$w_1 = 9.6$	
	$b_1 = -6.5$	
	$s(x) = \frac{1}{1 + e^{-x}}$	
	$h_1(x) = s(f_1(x))$	
	$f_2(x) = w_2x + b_2$	
	$w_2 = -7.4$	
	$b_2 = -5$	
	$h_2(x) = s(f_2(x))$	
	$N(x) = w_3h_1(x) + w_4h_2(x)$	
	$w_3 = 3.3$	
	$w_4 = 1$	





$$w_1 = 9 \cdot 6$$

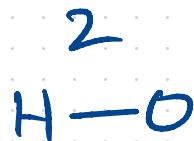
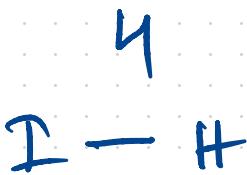
$$w_2 = -7 \cdot 4$$

$$b_1 = -6 \cdot 5$$

$$b_2 = -5$$

$$w_3 = 3 \cdot 3$$

$$w_4 = 1$$



" $\min(E(x))$ "