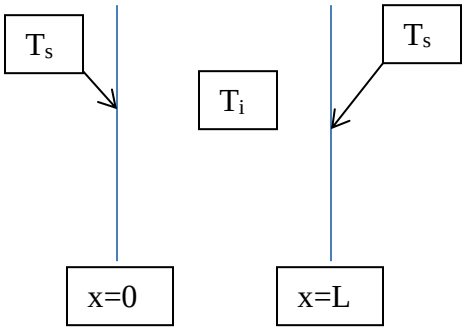
Course MEEN 489/689
Homework 4
Due 10-17-18

A wall of thickness 2 ft and infinite in the other directions has an initial uniform temperature (T_i) of 100°F. The surface temperatures (T_s) at the two sides are suddenly increased and maintained at 400°F. The wall is composed of nickel steel (40% Ni) with a diffusivity of α=0.4 ft^2/hr. We are interested in computing the temperature distribution within the wall as a function of time.

The governing equation is unsteady heat equation:

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$



The analytical solution for this problem, subject to the imposed initial and boundary conditions, is

$$T = T_s + 2(T_i - T_s) \sum_{m=1}^{\infty} e^{-(\frac{m\pi}{L})^2 \alpha t} \left[\frac{1-(-1)^m}{m\pi} \sin(\frac{m\pi x}{L}) \right]$$

A) What is the type of the PDE?

B) Nondimensionalize the equation and the analytical solution. Hint: $T^* = (T - T_s)/(T_i - T_s)$

C) Solve the non-dimensional equation using (a) FTSC and (b) Crank-Nicolson and plot the temperature profile for all x location at each 0.1 hr interval from 0.0 to 1 hr. (Note: do not confuse non-dimensional and dimensional times)

D) Numerically investigate the stability of the FTSC and Crank-Nicolson methods. How does it relate to $\alpha \frac{\Delta t}{\Delta x^2}$?

E) Show the temporal and spatial order of accuracy of each method. Hint: plot error at t=1 hr w.r.t Δt and Δx in log-log scale. Compute the error at t=1 hr as follows:

$$error = \frac{1}{IM} \sqrt{\sum_{i=1}^{IM} (T_{(i)}^{computed} - T_{(i)}^{analytical})^2}$$

Computer Project Report Format:
I encourage you to discuss this project with your classmates, but all computer coding, analysis and reports should be done individually. The best way to really learn how to put together a CFD computer program is to do it yourself. Please type up your reports and include the format summarized on the next page.

Report Format

Please write-up findings of your study in a report using the following guidelines. **The report will be graded out of 20 points in total. Five of the 20 points is on the overall format and presentation of the report. Any report that does not follow these guidelines will therefore be graded for less than full credit.**

Title Page – A page with the title of your study, your name, course info. and date.

Introduction – Background information and a short summary of the problem you studied (~1-2 pages). Include the cases you conducted in a well-organized table or set of tables. Make sure to include a caption(s) with your table(s).

Method of Solution – A summary of the difference equations used, sketch of stencils, implementation of boundary conditions. Please DO NOT list your code in this section. Include a listing of the computer program as an appendix. (~ 2-4 pages)

Discussion of Results – Include all plots and discussion in this section. When writing your discussion, please use the following rules of thumb. (~5-10 pages)

- If you include a figure, make sure to refer to it in the discussion, *e.g.*, "Figure 1 shows the overall heat release as a function of time for cases 1 and 2." There should be no figures in your report that are not discussed. If the word "figure" is the first word of a sentence, such as the previous example, then spell it out. If the word "figure" is used in the middle of a sentence then use the abbreviation "Fig. ". The same rules apply for the word "equation", e.g., "Substituting Eq. (1) into Eq. (4) results in the following." vs. "Equation (5) summarizes the governing equations." The word "Table" always is spelled out.
- Be quantitative with your discussion, *e.g*., "Comparing Figs. (3) and (4), it is clear that the error is reduced by 35% using the Crank-Nicholson method." **DO NOT** use statements such as: "Using the higher order centered difference stencil reduced the error a lot." or "Much more time is required using an implicit method than the explicit method.". These statements are too qualitative. Be quantitative by specifying how much something has changed.
- Please keep your figures to a minimum by plotting several results on one plot. For instance, if you are comparing different numerical methods then group your results by plotting results at the time, spatial location or both. **Credit will be deducted for reports that have excessive figures.**
- Please make sure that all plots and tables have numbered captions. The caption should contain short distinct descriptions and should be complete sentences. <u>Please do not place discussion in the captions.</u>
- On plotting figures …..
    - All figures should have axis labels (with units), numbered major tick marks, minor tick marks and a legend. Each line should have a distinct line type. Color is optional.
    - Please make sure minor tick marks are chosen for easy reading of the plots. Choose minor tick marks as 0.25, 0.2, 1, etc. **DO NOT** choose values such as 0.23, 0.56, *etc.*
    - A major tick mark and a label should end of each axis.
    - Make sure all labels on the axes and the legend are clear and large enough to be easily read.

Summary and Conclusions - Briefly summarize your findings and any conclusions. Please do not simply cut-n-paste from the discussion. Try to put your finding into some larger relevance with regard to the theory. Did the results agree well with the theory always? What appears to be the largest limitation? (< 1 page).

Appendices – Include any lengthy hand calculations and/or code listing as an appendix.