

Course MEEN 489/689
Homework 5
Due 11-26-18

An initial velocity distribution representing a compression wave is given by the following:

$$u(x, 0) = \begin{cases} 1 & , x < 0.25 \\ 1.25 - x & , 0.25 \leq x \leq 1.25 \\ 0 & , x > 1.25 \end{cases}$$

The inviscid Burgers is used to solve for the wave propagation within a domain of $0 < x < 4$. The solution is sought up to $t=6$. Note that the initial wave is compressed (steepens) with time and subsequently forms a shock wave. Within the specified time and space intervals, no shock reflection occurs, and therefore, the boundary conditions are simply specified as

$$\begin{aligned} u(0, t) &= 1 \\ u(4, t) &= 0 \end{aligned}$$

Solve the problem by a low order (e.g., Godunov, upwind, etc) and a high order (Lax-Wendroff, McCormack method, Ruge-Kutta with artificial dissipation or TVD, etc) method.

Use a spatial step of $\Delta x=0.05$ and obtain solution for different CFL numbers. Discuss the effect of CFL number (time step size) on stability. Plot the solution for all methods at time levels 0, 2, 4, and 6. No need to write a report. Just mention what methods you have used, a brief explanation of figures, and your code as appendix.

Hint: Use the non-linear version of the methods.

Remark: You will be using the same methods to solve the dam-break problem in your final. This will help you get ready by first applying these methods to a simpler equation.