Here is the link to the problem : Robot Capture-the-Flag

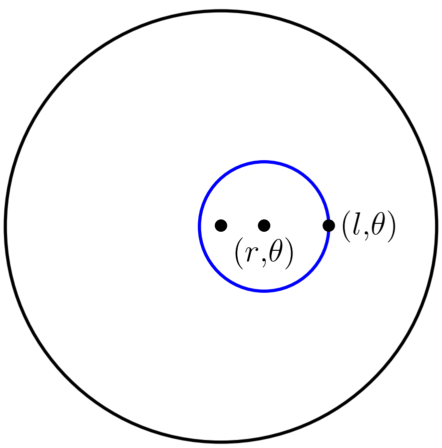
Let us use polar co-ordinates i.e., radius and angle instead of $x$ and $y$ coordinates.

Think of the flag position to be $(r,\theta)$.
Erin and Aaron have the knowledge of $\theta$ and $r$ respectively. Consider their chosen positions to be $(r_1,\theta_1)$ and $(r_2,\theta_2)$.

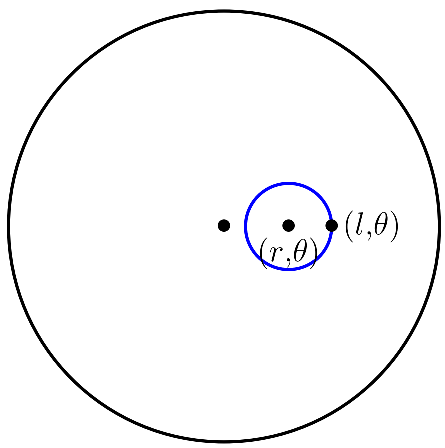
Consider the distance between polar co-ordinates to be represented by function $\mathbf{D}$.
A useful result we can use is $\mathbf{D}((r,\theta),(r_1,\theta)) \leq \mathbf{D}((r,\theta),(r_1,\alpha))$ for any $\alpha$, $r$, $r_1$.
The action of choosing $\theta$ for Erin will be a weakly dominant strategy over choosing any other angular position. Here, as $\theta$ is known to Erin, she will choose $\theta_1 = \theta$. And given that Erin chooses a fixed distance from the centre, let us call it to be $l$.

So, Erin's position = $(l,\theta)$. Take Aaron's position to be $(x,\theta_r)$.
The problem comes down to comparison between $\mathbf{D}((r,\theta),(l,\theta))$ and $\mathbf{D}((r,\theta),(x,\theta_r))$

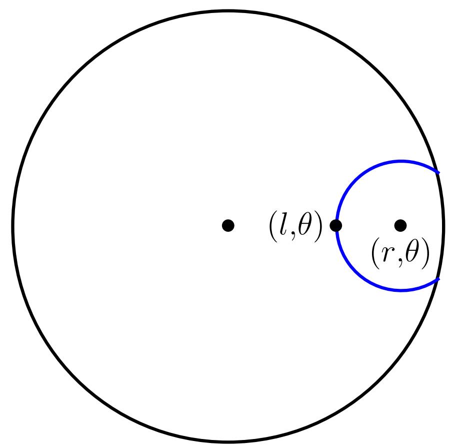
If Aaron chooses a point within the smaller circle below i.e, the circle that contains all the points that are closer to the flag than Erin's position, he will win. Let us call the circle **C**. For various cases of $r$ and $l$, **C** is shown in the 3 diagrams.




**Case 1**: $0 < r < l/2$



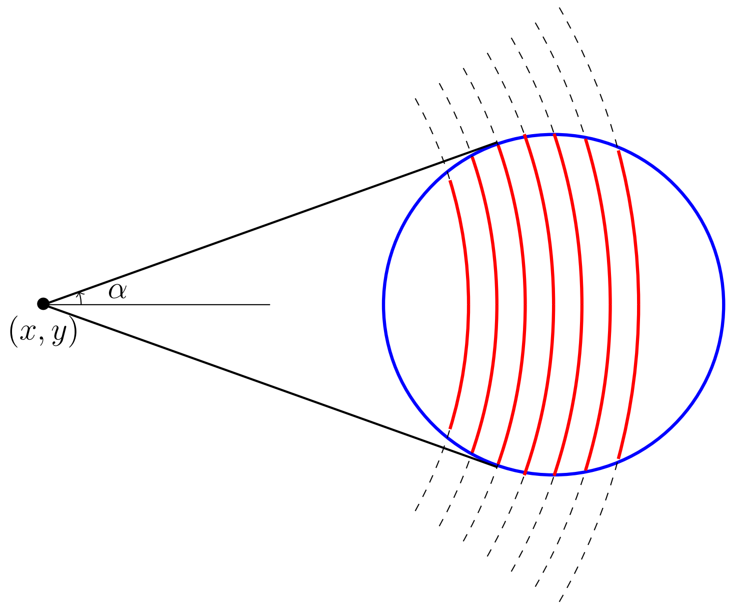

**Case 2**: $l/2 < r < l$




**Case 3**: $l < r < 1$

**Case 1**:
If $2r \leq l$, the origin lies in the circle **C**, Aaron can just choose the origin that will fetch him victory.
**Cases 2,3**:
But if $2r \geq l$, Aaron must choose a $x$ that has the highest probability of being in the circle **C**.
**C** is defined by $r$ and $l$ but not $\theta$ as both flag and Erin are on same angular positions.
For a given distance from the centre, any $\theta_r$ chosen will give him the same probability of winning as the $\theta$ is a uniform random variable on $[0,2\pi]$. So, he can choose angular position at random.
So, the prob. of Aaron's position being in **C** will be dependent on $x$ solely. Let us call the circle made by Aaron's positions i.e., circle of radius x as **X**. For a given $x$, only some of the angles will belong inside **C** as seen below. For a given $x$, the probability is equal to the angle made by the arc of **X** on **C**.

To maximise the angle, $x$ will be chosen such that **X** intersects with **C** perpendicularly. The radius of **X** will be a tangent to **C**.
So, the ideal choice for Aaron is to take x = $\sqrt(l(2r-l))$




Calculating the probability,
**Case 1**, the probability of Aaron's victory is 1.
**Cases 2 and 3**, the probability of Aaron's victory is equal to the angle made by the arc over $\pi$ i.e., $\frac{arcsin\frac{|r-l|}{r}}{\pi}$.

Integrating the probability over values of $r$ from 0 to 1 gives $P(l)$

$$P(l) = \int_0^{l/2} 1(2r)dr + \int_{l/2}^1 \frac{arcsin\frac{|r-l|}{r}}{\pi}\frac{2\pi r}{\pi}dr$$

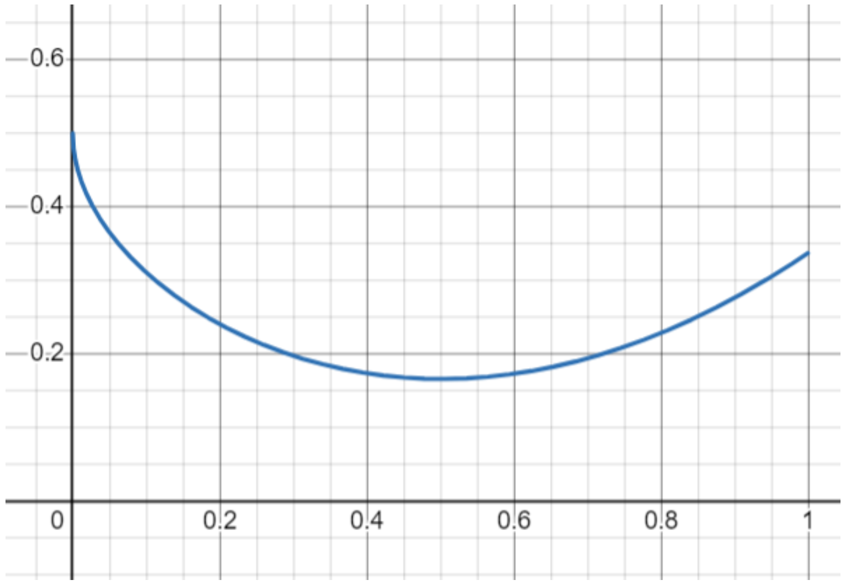
$$= \int_0^{l/2} 1(2r)dr + \int_{l/2}^{l} \frac{arcsin\frac{l-r}{r}}{\pi}(2r)dr + \int_{l}^{1} \frac{arcsin\frac{r-l}{r}}{\pi}(2r)dr$$

$$= \frac{l^2}{4} + \frac{2l^2}{\pi}\int_0^1 \frac{arcsin(u)}{(1+u)^3}du + \frac{2l^2}{\pi}\int_0^1 \frac{arcsin(u)}{(1-u)^3}du$$

$$= (\frac{4}{3\pi} + \frac{1}{8})l^2 + \frac{3arcsin(1-l) - (l+1)\sqrt(1-(1-l)^2)}{3\pi}$$

The probability P($l$) of Aaron's victory in terms of $l$ is plotted in the next page.

So, Aaron's chance of winnining depends on Erin's choice.
Both players are aware of each other's strategies. As Erin is aware of Aaron's chance of winning, she will try to minimise $P(l)$ by choosing a suitable value for $l$.




Take derivative of $P(l)$ and put it to 0.

$$P^{'}(l) = 0$$

which gives

$$l^3.(1 + (4 + \frac{3\pi}{8})^2) = 3l + 2$$

We get $l$ to be 0.501307
And the corresponding probability of Aaron winning is $P(0.501307) = \mathbf{0.16619}$