MA423 Lab-03

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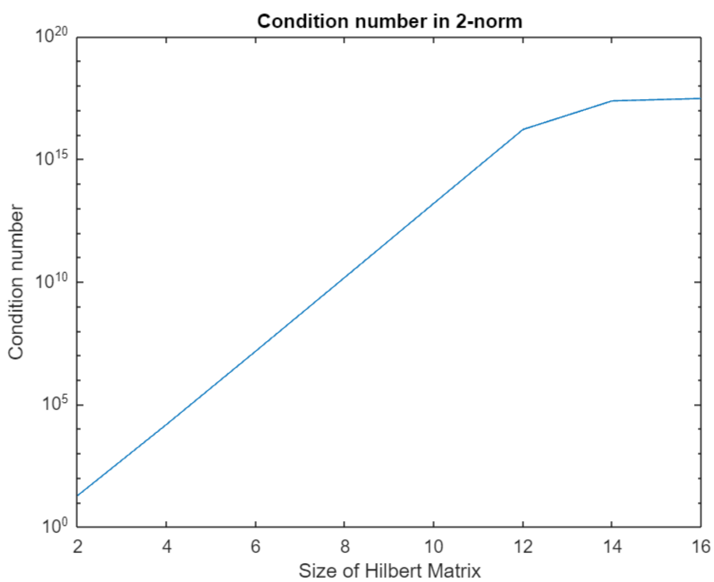
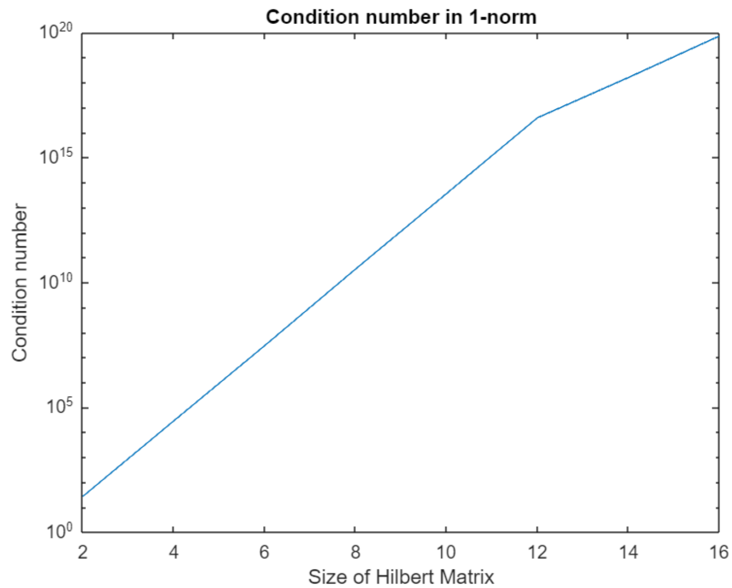
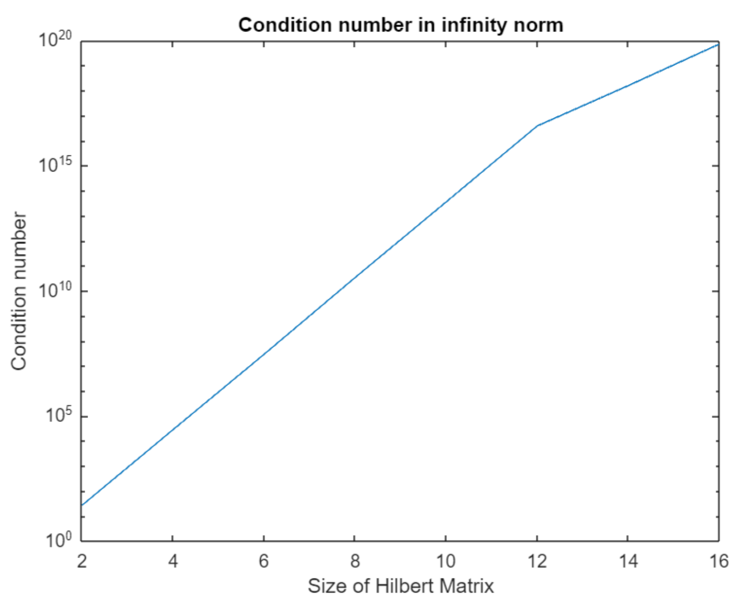
Question 1

The workspace for this question is saved in q1.mat and code file is saved as q1.m
In all the 3 plots, we notice a linear relation between log(cond(H)) and n.
The condition number increases exponentially with matrix size.
For very large values of n, the matrix becomes highly ill conditioned and due to errors curve deviates from linearity.




Condition number in 1-norm
Condition number
Size of Hilbert Matrix
10^20
10^15
10^10
10^5
10^0
2
4
6
8
10
12
14
16


Condition number in infinity norm
Condition number
Size of Hilbert Matrix
10^20
10^15
10^10
10^5
10^0
2
4
6
8
10
12
14
16

Question 2

The workspace for this question is saved in q2.mat and code file is saved as q2.m

n	Cond(H)	$\|x - x1\|/\|x\|$	$\|x - x2\|/\|x\|$	$\|x - x3\|/\|x\|$
8	1.52576*10^{10}	3.77061*10^{-7}	3.482749*10^{-7}	4.159412*10^{-7}
10	1.60249*10^{13}	8.30276*10^{-5}	5.97886*10^{-5}	8.015817*10^{-5}
12	1.628373*10^{16}	4.4353197*10^{-2}	5.892829*10^{-2}	2.988729*10^{-2}

(a)

For n = 8, accuracy is upto 6 decimal digits.
For n = 10, accuracy is upto 4 decimal digits.
For n = 12, accuracy is low (only 1 decimal digit).

(b)

All 3 methods generate nearly the same error, there isn't a considerable difference between the accuracy of the three.

(c)

The accuracy does agree with the value predicted by the rule-of-thumb analysis.

Question 3

The workspace for this question is saved in q3.mat and code file is saved as q3.m

Here, $\|r\|/\|b\|$ = 1.177026*10^{-16} but $\|x - xt\|/\|x\|$ = 6.73276*10^{-5}
Hence, a small $\|r\|/\|b\|$ does not imply that that the $\|x - xt\|/\|x\|$ is also small in magnitude

Question 4

The workspace for this question is saved in q4.mat and code file is saved as q4.m

W is wilkinson matrix here.
For n = 32,

Method	Cond(W)	Forward error	Residual error
GEPP	32	1.770514*10^{-9}	1.590365*10^{-9}
QR	32	7.140198*10^{-16}	4.561348*10^{-16}

For n = 64,

Method	Cond(W)	Forward error	Residual error
GEPP	64	0.709741	0.213358
QR	64	3.265515*10^{-15}	8.975923*10^{-16}

norm(x - x1,inf)/norm(x,inf)

(a)

For both values of n, QR factorization method has a lower forward error. ($\|x - \hat{x}\|/\|b\|$)

(b)

For both values of n, QR factorization method has a lower residual error ($\|r\|/\|b\|$).

(c)

The conditional numbers are 32 and 64, both of which are less than 10^2. So, t = 2 and the method is backward stable in the QR case.
Using the rule of thumb analysis, the solution is correct to atleast s - t decimal places.
QR method seems to be predicting the solution reasonably well.

(d)

As the errors ($\|x - \hat{x}\|/\|b\|$) and ($\|r\|/\|b\|$) are of the order 10^{-16}, the QR factorization method is backward stable.

Question 5

The workspace for this question is saved in q5.mat and code file is saved as q5.m

The 1,2 and infinity norms are tabulated for matrices of different sizes.
The larger values of norm are produced by GENP.

n	Method	1-norm	2-norm	Infinity norm
20	GENP	0.0088	0.0046	0.0103
	GEPP	1.5482*10^{-16}	8.6331*10^{-17}	1.7647*10^{-16}
40	GENP	0.0051	0.0019	0.0051
	GEPP	3.3657*10^{-16}	1.2429*10^{-16}	2.8947*10^{-16}
60	GENP	0.0055	0.0016	0.0055
	GEPP	4.7081*10^{-16}	1.1907*10^{-16}	3.5357*10^{-16}
80	GENP	0.0149	0.0038	0.0140
	GEPP	4.5919*10^{-16}	1.3002*10^{-16}	4.8005*10^{-16}
100	GENP	0.0533	0.0133	0.0538
	GEPP	5.7516*10^{-16}	1.3178*10^{-16}	4.8224*10^{-16}