



# Random Walks for Segmentation

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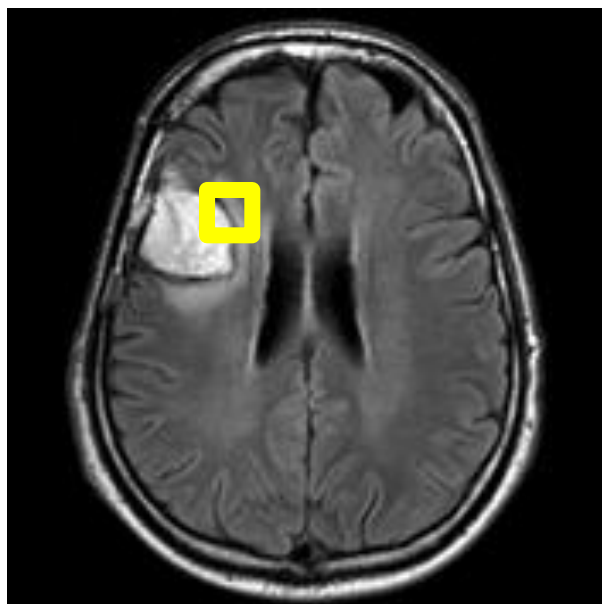


# Contents

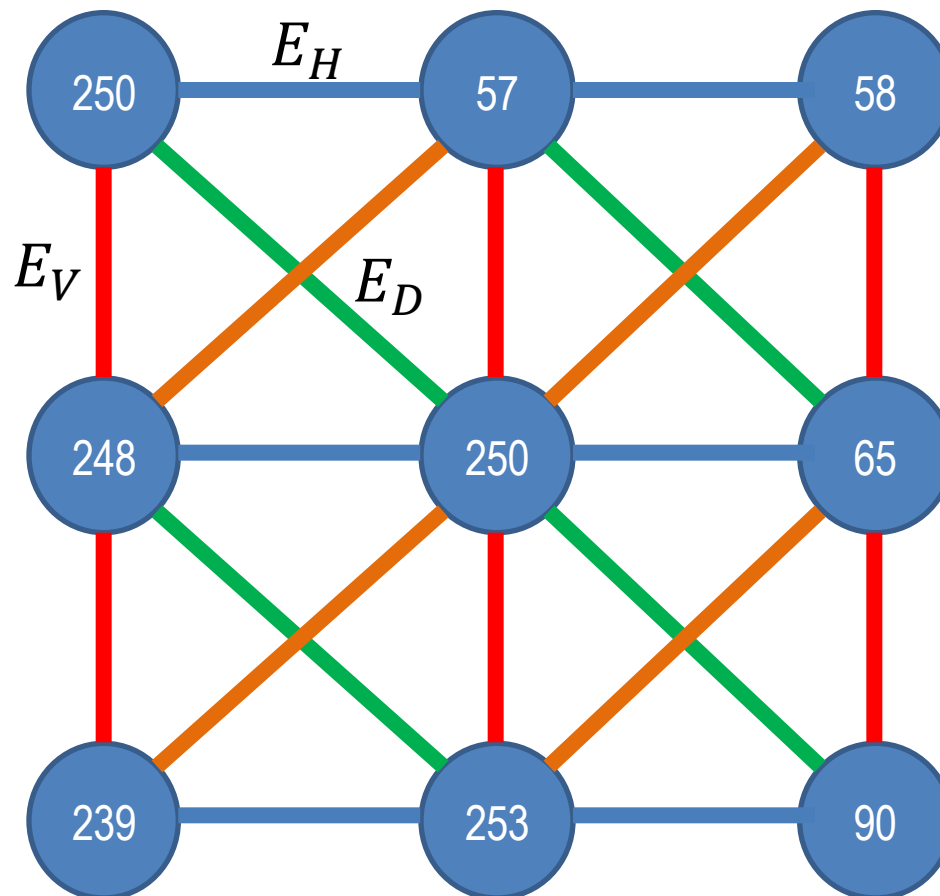
- Graph equivalent of an image
- Random walks as a statistical phenomenon
- Electrical circuit equivalent of image
- Random walks via electrical network
- Random walks solver
- Example segmentation



# Graph Equivalent of Image

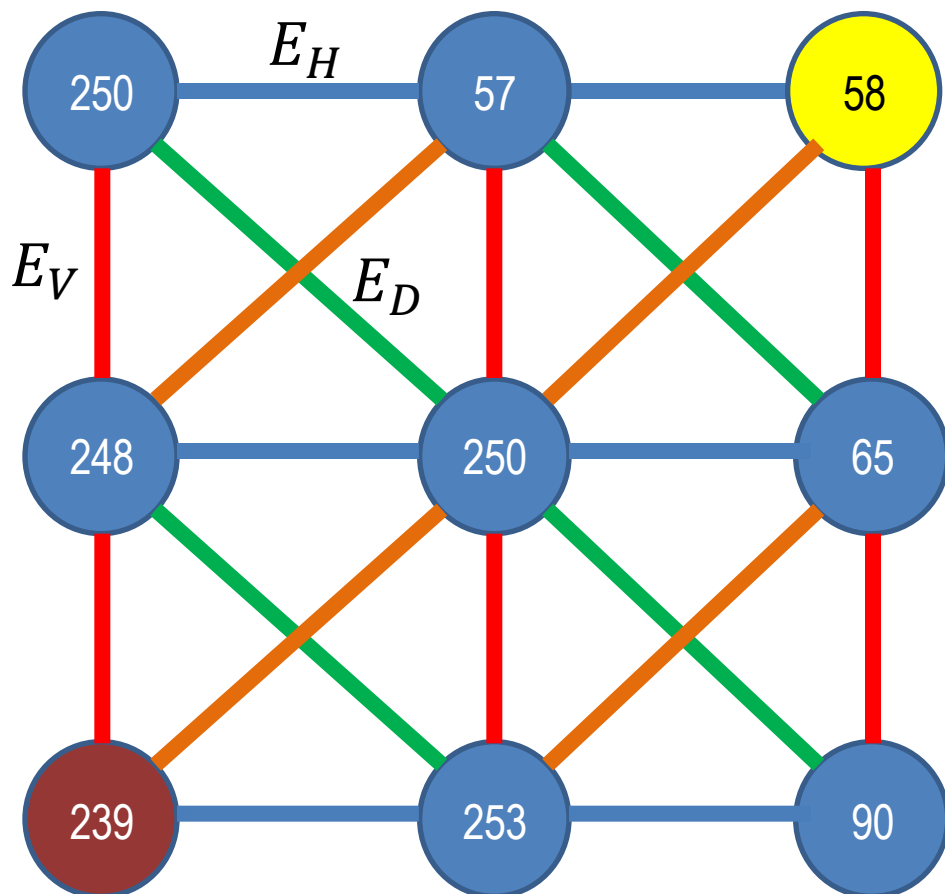


250	57	58
248	250	65
239	253	90





# Random Walk (Drunkard's walk)



$$G = (V, E) \quad E = V \times V$$

$$p(\omega_1 | \text{drunkard})$$

$$p(\omega_2 | \text{drunkard})$$

$$y_n = \operatorname{argmax}(p_n(\omega_k | \mathbf{x}))$$



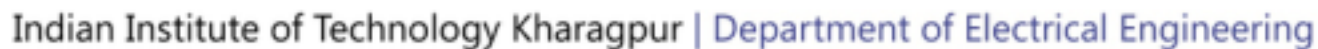
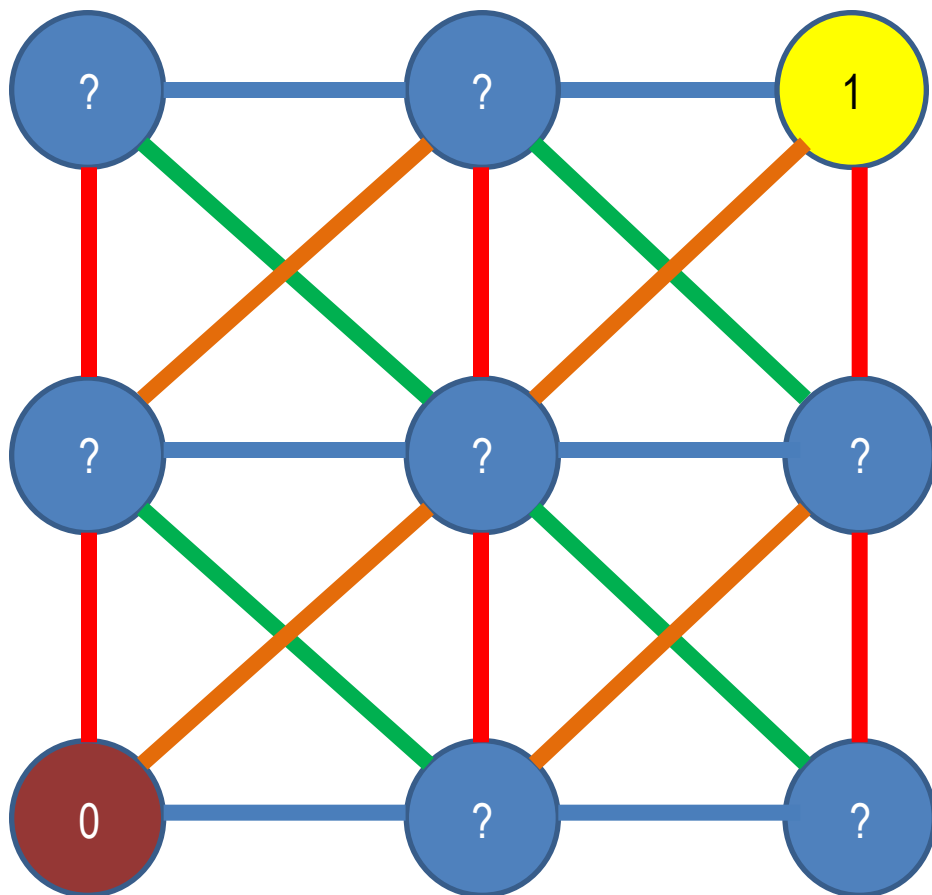


Figure 1 shows a graph visualization of a 3x3 grid of nodes. The nodes are labeled with IDs: 250, 57, 58 (top row); 248, 250, 65 (middle row); 239, 253, 90 (bottom row). The nodes are connected by edges of different colors: red, green, and orange. The weights of the edges are labeled:  $1.51 \times 10^{-84}$  (red),  $0.37$  (orange),  $1.00$  (green),  $1.12 \times 10^{-83}$  (orange),  $9.11 \times 10^{-4}$  (red),  $1.38 \times 10^{-11}$  (red),  $1.62 \times 10^{-71}$  (orange), and  $0.13$  (red). To the right of the grid, two nodes labeled  $z_p$  and  $z_q$  are connected by a red edge with weight  $w_{pq} = e^{-\beta|z_p - z_q|}$ . Below this, it is stated that  $\beta = 1$ .



# Random Walks via Electrical Circuit



$$d_p = \sum_q w_{pq}$$

$$L_{pq} = \begin{cases} d_p, & \text{if } p = q \\ -w_{pq}, & \text{if } v_p, v_q \text{ adjacent} \\ 0, & \text{otherwise} \end{cases}$$

$$L = \begin{bmatrix} L_M & B \\ B^T & L_U \end{bmatrix}$$

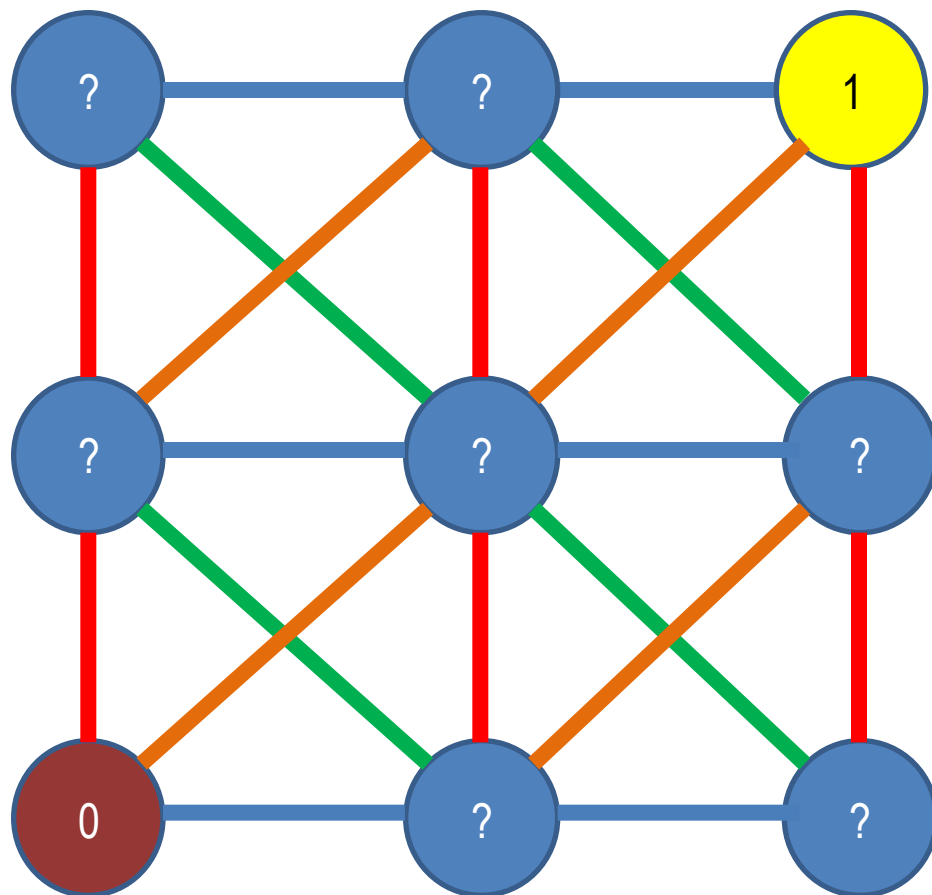


# Random Walks Solver

$$m_q^\omega = \begin{cases} 1, & \text{if } Q(v_q) = \omega \\ 0, & \text{if } Q(v_q) \neq \omega \end{cases}$$

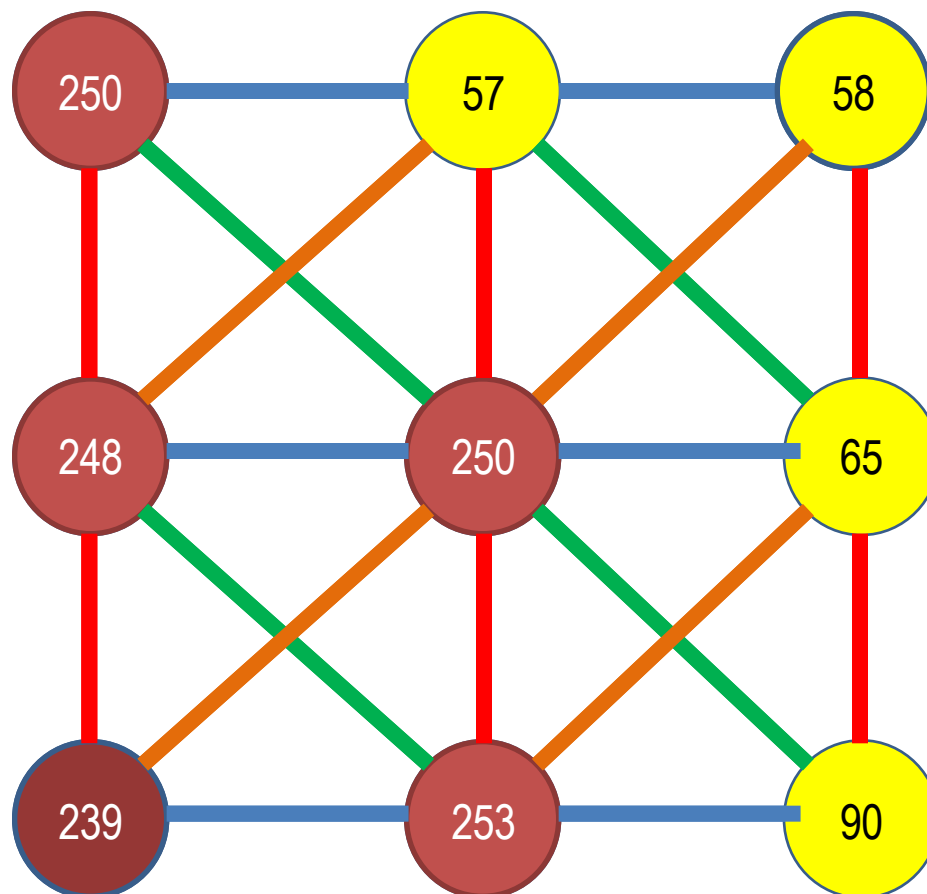
$$L_U X = -B^T M$$

$$p(\omega | \mathbf{x}) = x_q^\omega$$





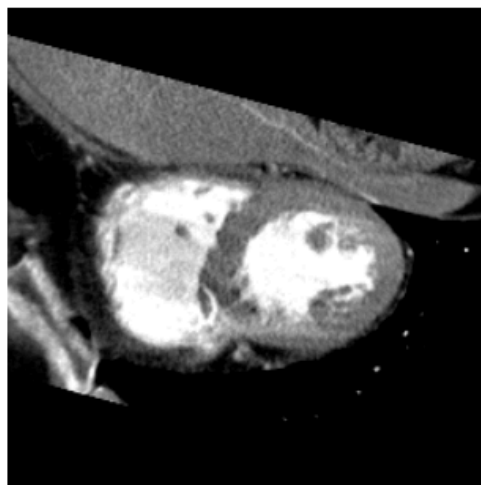
# Random Walks Solver



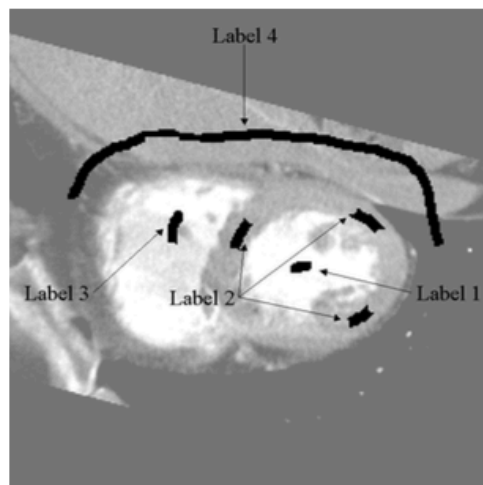




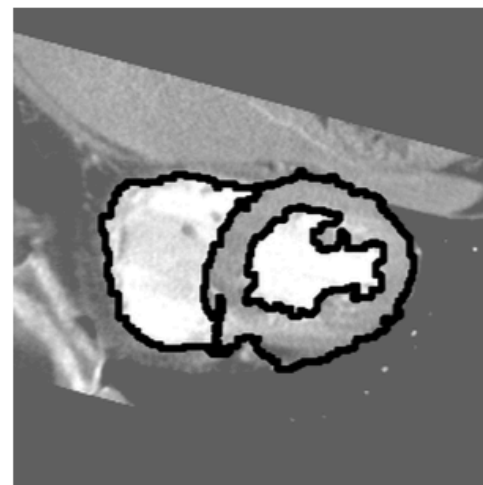
# Some Examples



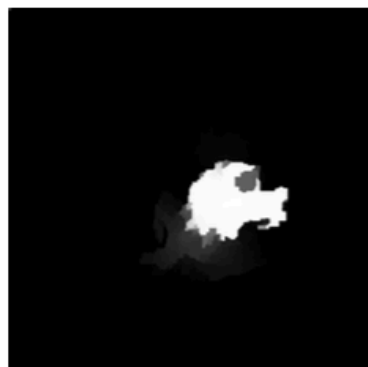
(a) Original



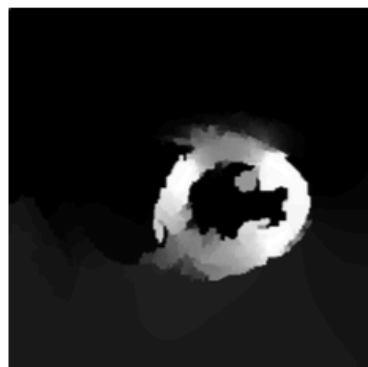
(b) Seeds indicating four objects



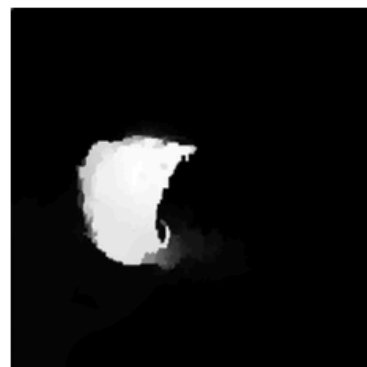
(c) Resulting segmentation



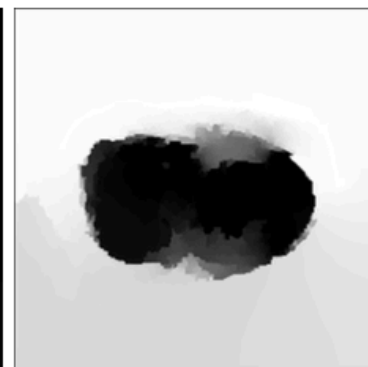
(d) Label 1 probabilities



(e) Label 2 probabilities



(f) Label 3 probabilities



(g) Label 4 probabilities



# Take home message

- L. Grady, "Random Walks for Image Segmentation", *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol. 28, no. 11, pp. 1768-1783, 2006.
- Toolboxes
  - Matlab:  
<http://cns.bu.edu/~lgrady/software.html>
  - Python: skimage.segmentation.random\_walker