Neural Networks

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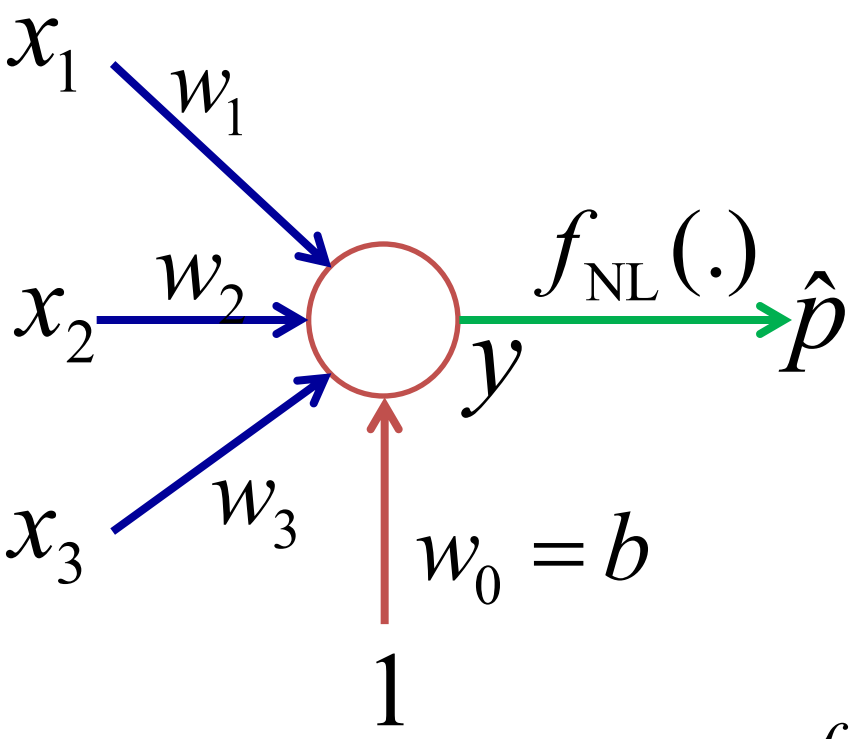
Simple Neuron Model



$$y = w_0 + w_1 x_1 + w_2 x_2 + w_3 x_3$$

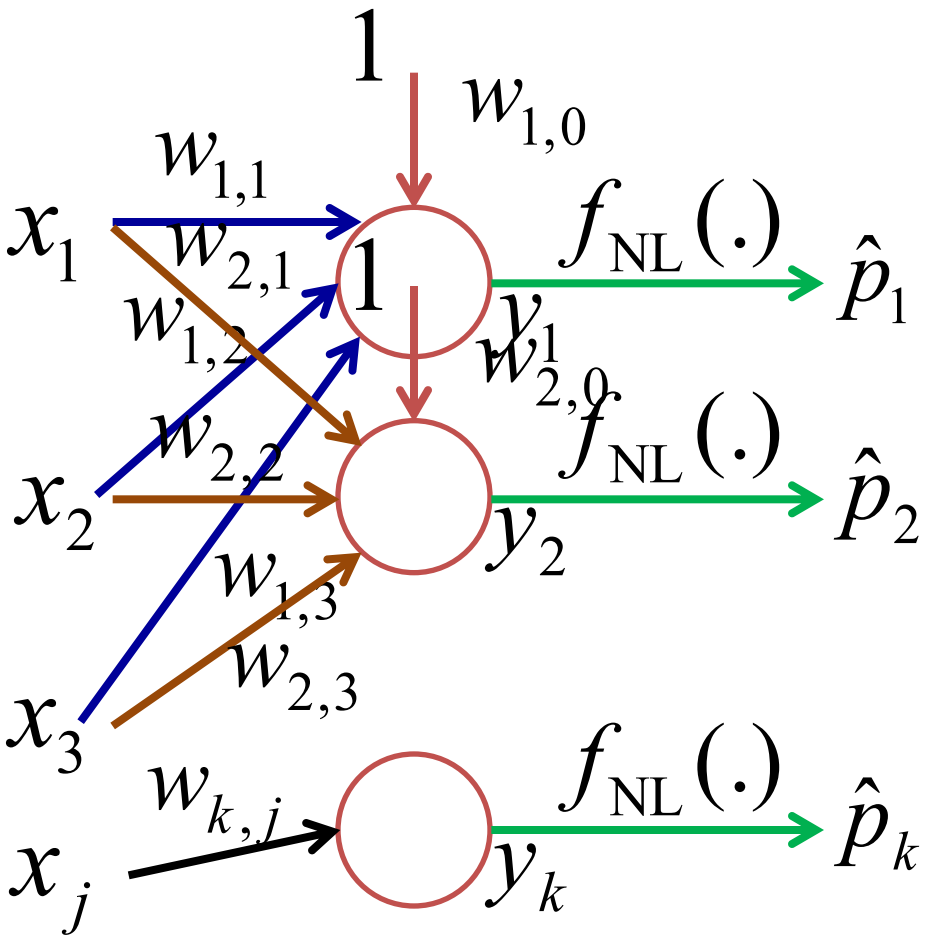
$$y = \begin{bmatrix} \mathbf{w} & \mathbf{b} \end{bmatrix} \begin{bmatrix} \mathbf{x} & 1 \end{bmatrix}^T$$

$$\hat{p} = f_{NL}(y)$$

$$f_{\mathrm{NL}}(y) = \frac{1}{1 + \exp(-y)}$$

$$f_{\mathrm{NL}}(y) = \tanh(y) = \frac{\exp(y) - \exp(-y)}{\exp(y) + \exp(-y)}$$

Neural Network Formulation



$$y_1 = \begin{bmatrix} \mathbf{w} & \mathbf{b} \end{bmatrix} \begin{bmatrix} \mathbf{x} & 1 \end{bmatrix}^T$$

$$\hat{p}_1 = f_{NL}(y_1)$$

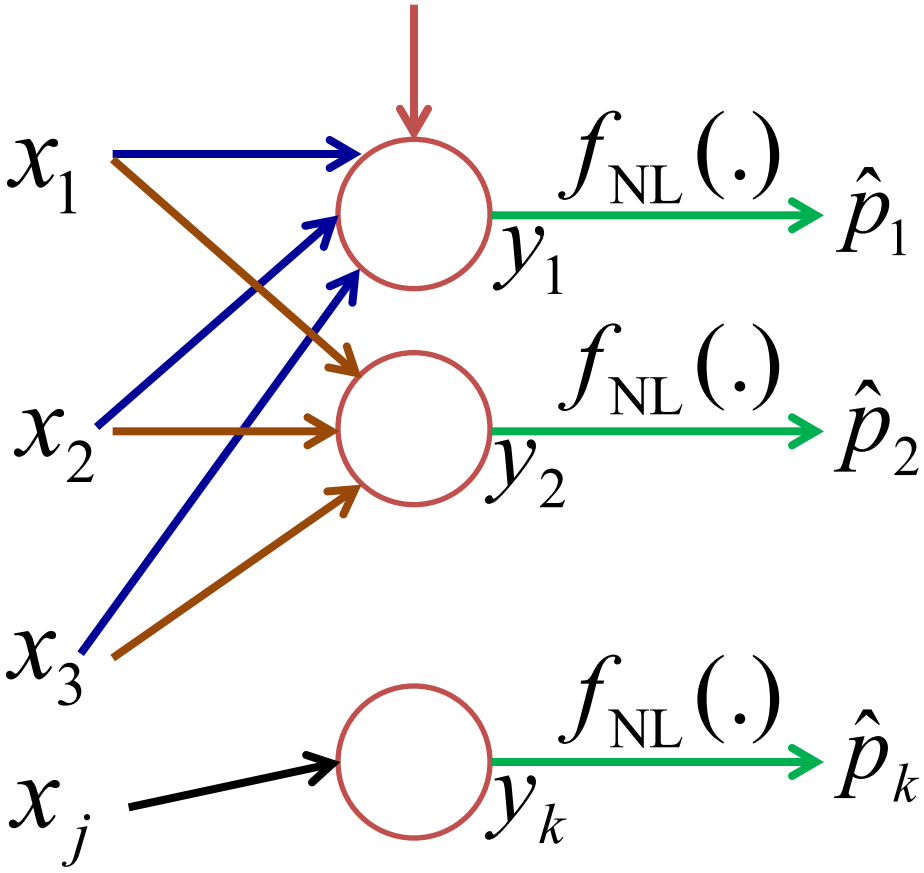
$$y_2 = \begin{bmatrix} \mathbf{w} & \mathbf{b} \end{bmatrix} \begin{bmatrix} \mathbf{x} & 1 \end{bmatrix}^T$$

$$\hat{p}_2 = f_{NL}(y_2)$$

$$\mathbf{y} = \begin{bmatrix} \mathbf{w} & \mathbf{b} \end{bmatrix} \begin{bmatrix} \mathbf{x} & 1 \end{bmatrix}^T$$

$$\hat{\mathbf{p}} = f_{NL}(\mathbf{y})$$

Error in Prediction



$$e_1 = |p_1 - \hat{p}_1|$$

$$e_2 = |p_2 - \hat{p}_2|$$

$$E = \|\mathbf{p} - \hat{\mathbf{p}}\|$$

Error Backpropagation

$$\begin{array}{lll} \mathbf{x}_1 & \mathbf{p}_1 & \hat{\mathbf{p}}_1 \\ \mathbf{x}_2 & \mathbf{p}_2 & \hat{\mathbf{p}}_2 \\ \mathbf{x}_3 & \mathbf{p}_3 & \hat{\mathbf{p}}_3 \\ \vdots & \vdots & \vdots \\ \mathbf{x}_n & \mathbf{p}_n & \hat{\mathbf{p}}_n \end{array}$$

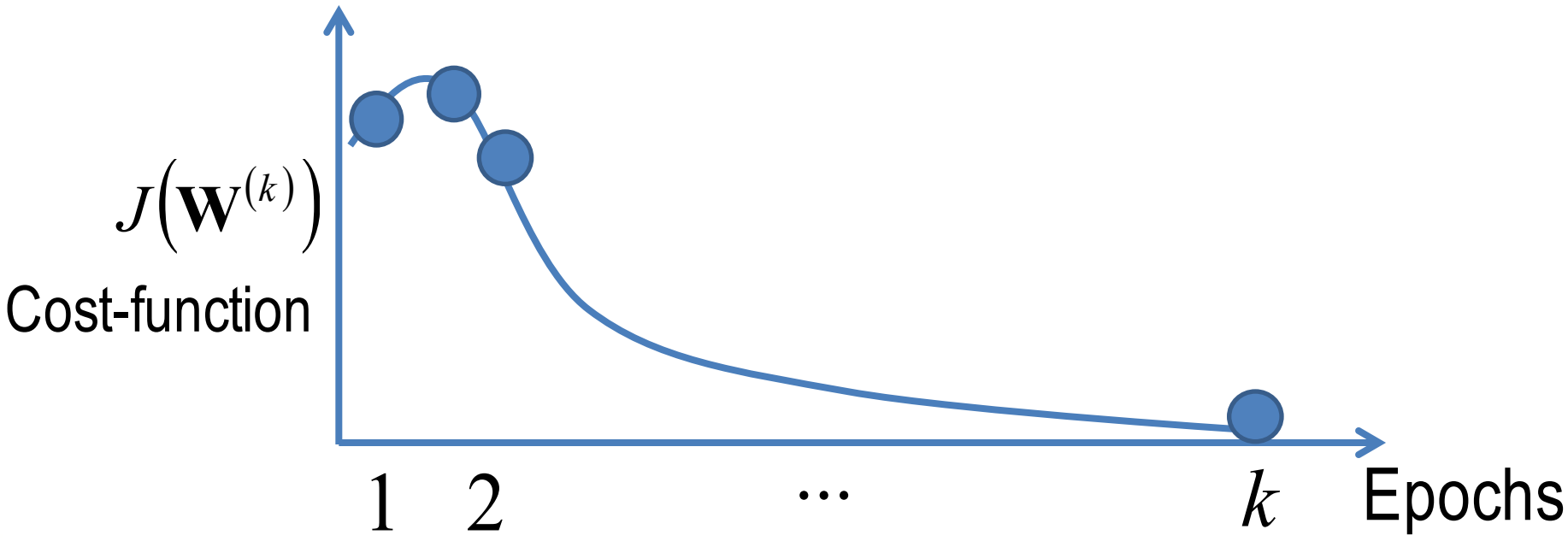
$$J(\mathbf{W}) = \sum_n \left\| \mathbf{p}_n - \hat{\mathbf{p}}_n \right\|$$

$$\mathbf{W} = \arg\min_{\mathbf{w}} \left\{ J(\mathbf{W}) \right\}$$

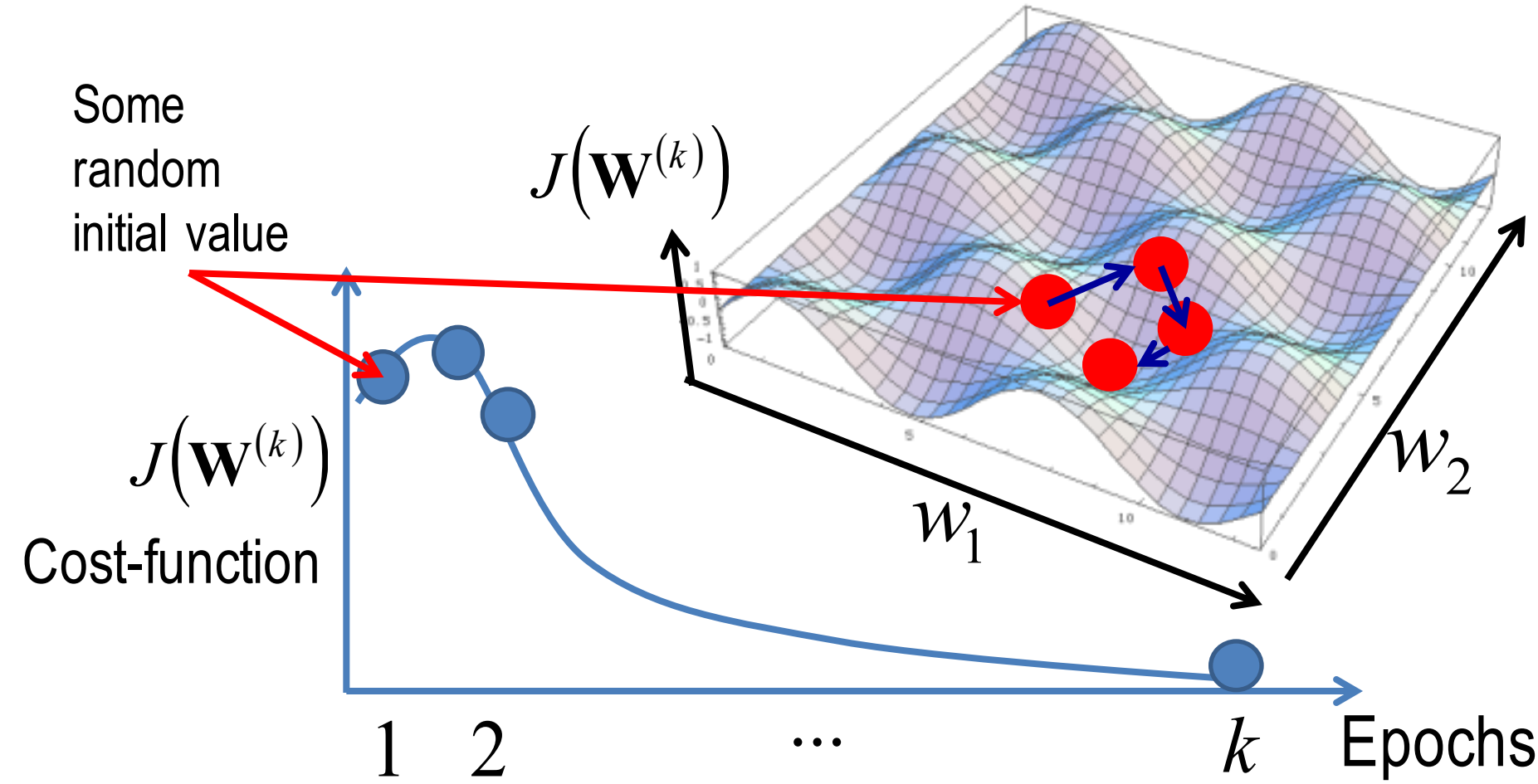
$$\mathbf{W}^{(k+1)} = \mathbf{W}^{(k)} - \frac{\partial}{\partial \mathbf{W}^{(k)}} J(\mathbf{W})$$

Gradient Descent Learning

$$\mathbf{w}^{(k+1)} = \mathbf{w}^{(k)} - \frac{\partial}{\partial \mathbf{w}^{(k)}} J(\mathbf{w})$$



Understanding Gradient Descent



Take Home Messages

- Haykin, Simon, *Neural Networks and Learning Machines*, 2001.
- Toolboxes
  - Matlab – Neural Network Toolbox (nprtool)
  - Python – Theano
  - Lua – Torch, nn, cuDNN, nngraph