Active Contours for Segmentation

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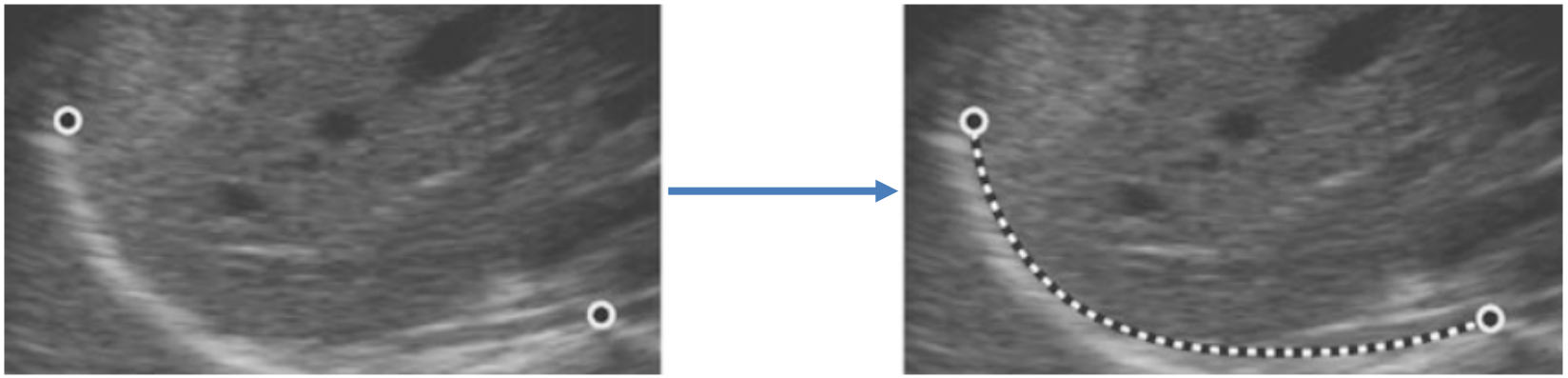
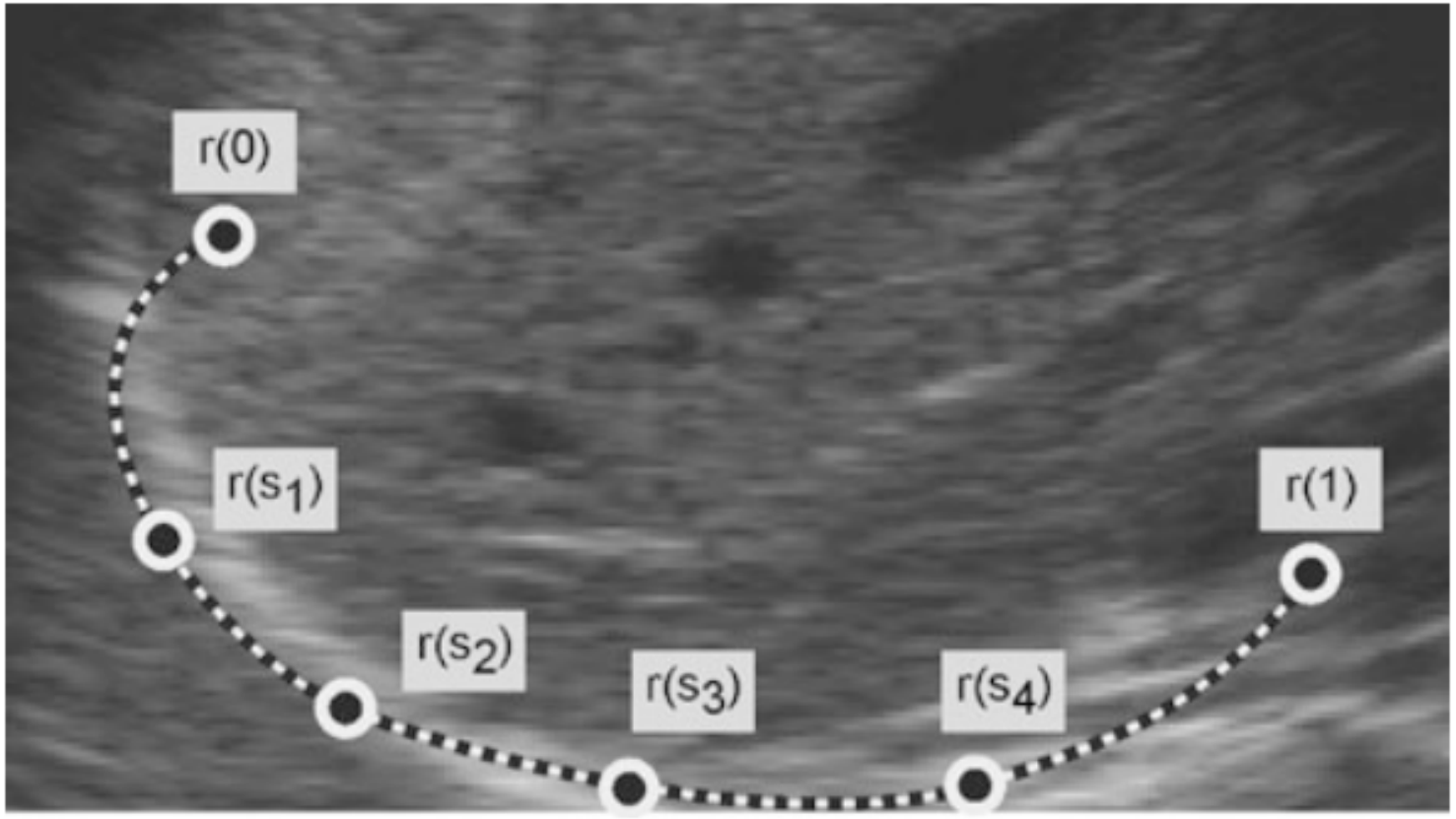
Objective



Find the boundary given the two end points?

Active Contour

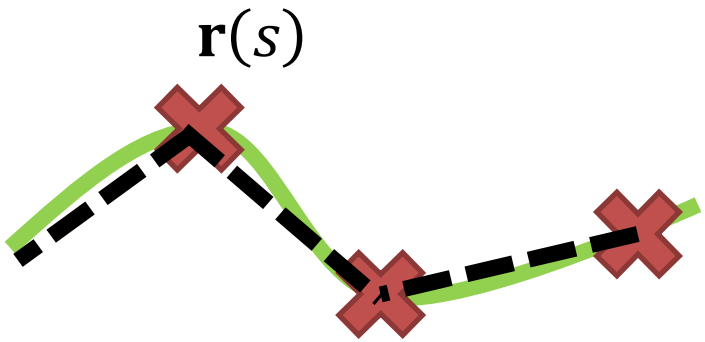


Definitions

$$E_{internal}(\mathbf{r}(s)) = w_1 \frac{\partial \mathbf{r}(s)}{\partial s} + w_2 \frac{\partial^2 \mathbf{r}(s)}{\partial s^2}$$

Elasticity

Stiffness



$$E_{external}(\mathbf{r}(s)) = -\|\nabla f(\mathbf{r}(s))\|$$

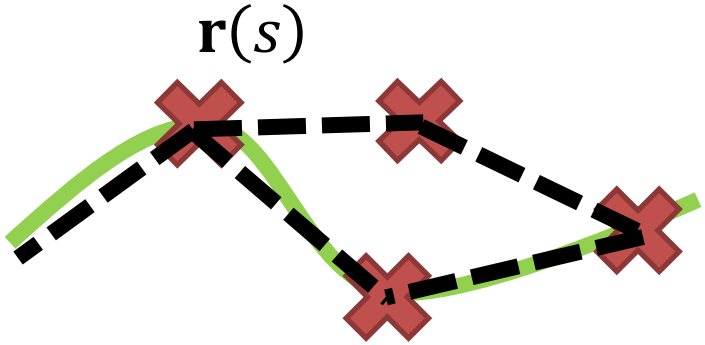
Image intensity

$$J(\mathcal{C}) = \int_{\mathcal{C}} \left(E_{internal}(\mathbf{r}(s)) + E_{external}(\mathbf{r}(s)) \right) ds$$

Convergence Criteria

$$\min \rightarrow \int_{\mathcal{C}} E_{external}\big(\mathbf{r}(s)\big)ds$$

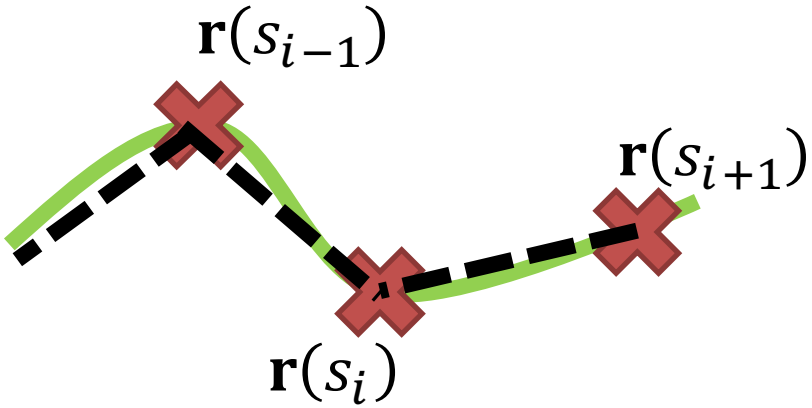
$$\min \rightarrow \int_{\mathcal{C}} E_{internal}\big(\mathbf{r}(s)\big)ds$$



$$\min \rightarrow J(\mathcal{C}) = \int_{\mathcal{C}} \Big(E_{internal}\big(\mathbf{r}(s)\big) + E_{external}\big(\mathbf{r}(s)\big)\Big)ds$$

$$-w_1 \frac{1}{\partial s}\left(\frac{\partial \mathbf{r}(s)}{\partial s}\right) + w_2 \frac{1}{\partial s^2}\left(\frac{\partial^2 \mathbf{r}(s)}{\partial s^2}\right) + \nabla E_{external}\big(\mathbf{r}(s)\big) = 0$$

Solver



$$h = \|\mathbf{r}(s_i) - \mathbf{r}(s_{i-1})\|$$

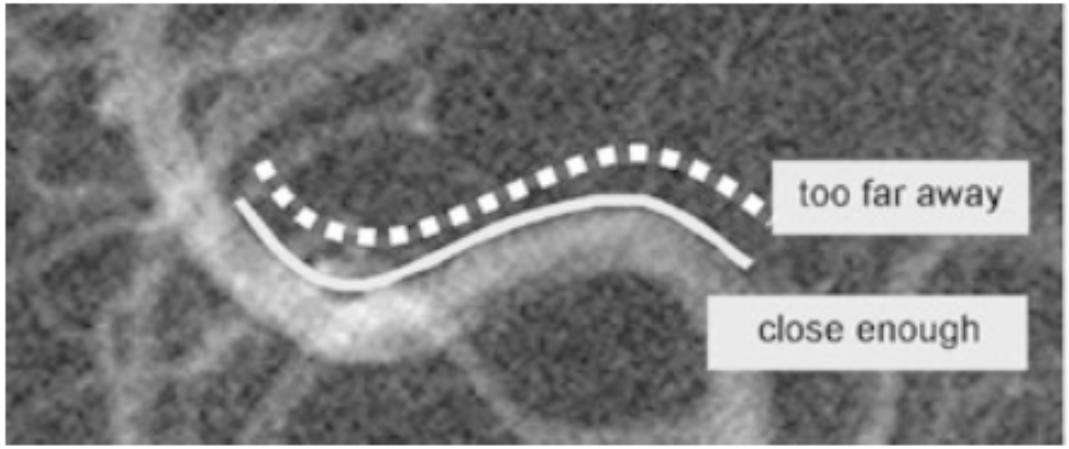
$$\frac{\partial \mathbf{r}(s_i)}{\partial s} \approx \frac{\mathbf{r}(s_i) - \mathbf{r}(s_{i-1})}{h}$$

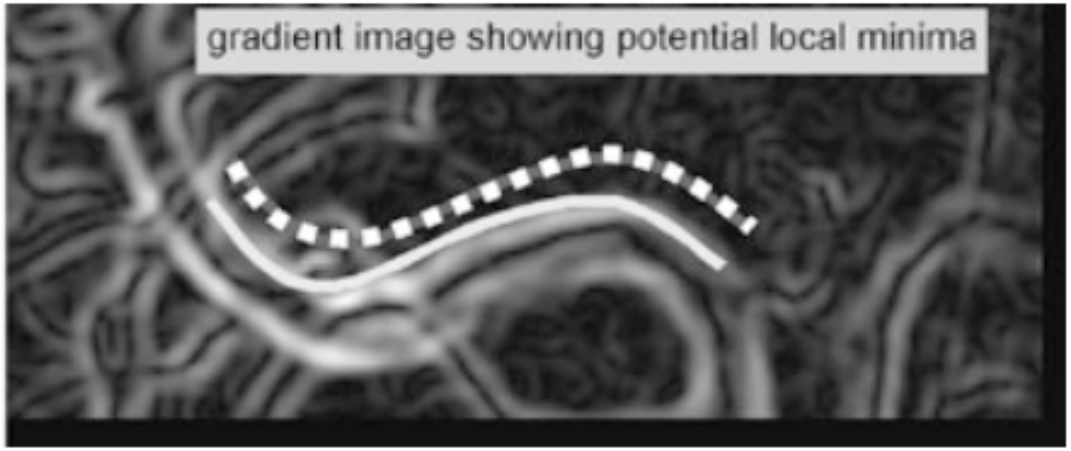
$$\frac{\partial^2 \mathbf{r}(s_i)}{\partial s^2} \approx \frac{\mathbf{r}(s_{i+1}) - 2\mathbf{r}(s_i) + \mathbf{r}(s_{i-1})}{h^2}$$

$$x^{(t+1)} = (\mathbf{A} - \gamma \mathbf{I})^{-1}\left(x^{(t)} - \frac{\partial E\left(\mathbf{r}(s^t)\right)}{\partial x}\right)$$

$$y^{(t+1)} = (\mathbf{A} - \gamma \mathbf{I})^{-1}\left(y^{(t)} - \frac{\partial E\left(\mathbf{r}(s^t)\right)}{\partial y}\right)$$

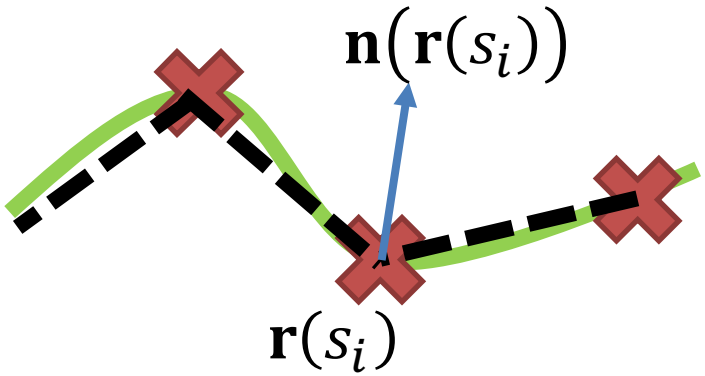
Effect of the Model





Balloon Model

$$\mathbf{f}_{external}\big(\mathbf{r}(s)\big) = k_1\,\mathbf{n}\big(\mathbf{r}(s)\big) - k_2\,\frac{\nabla E_{external}\big(\mathbf{r}(s)\big)}{\left\|\nabla E_{external}\big(\mathbf{r}(s)\big)\right\|}$$



Effect of Balloon Model

Take home message

- K.D. Toennies, *Guide to Medical Image Analysis* [Chap. 9], Advances in Computer Vision and Pattern Recognition, Springer-Verlag, 2012.