Fourier Series :-

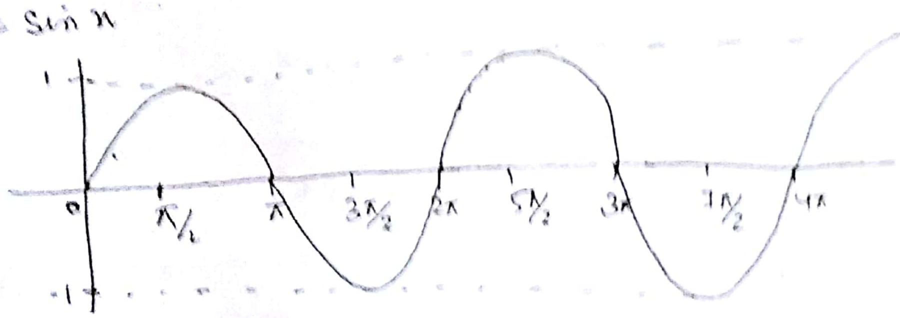
Real fourier series
Complex fourier series
even & odd function of fourier series
Fourier transform
Vector plots
Built in commands to find laplace transform

Periodic function :-

A function is said to be periodic, if $f(x+T) = f(x), \forall x$, smallest such positive 'T' is called periodic function of the ~~f(x)~~ periodic function $f(x)$.

$y = \sin(x) = \sin(x+2\pi) = \sin x$

$y = \sin x$



$\sin(x+2\pi) = \sin x = f(x)$

$\sin(x+4\pi) = \sin(x+2\pi+2\pi) = \sin x$

$f(x+6\pi) = \sin(x+6\pi) = \sin(x+4\pi+2\pi) = \sin x$

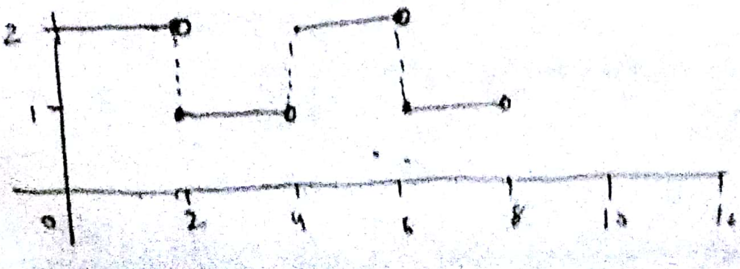
$\vdots$

$f(x+2\pi) = \sin(x+2n\pi) = \sin x = f(x)$

Ex:- plot the periodic function,

$$f(x) = \begin{cases} 2, & 0 \le x < 2 \\ 1 & 2 \le x < 4 \end{cases}$$

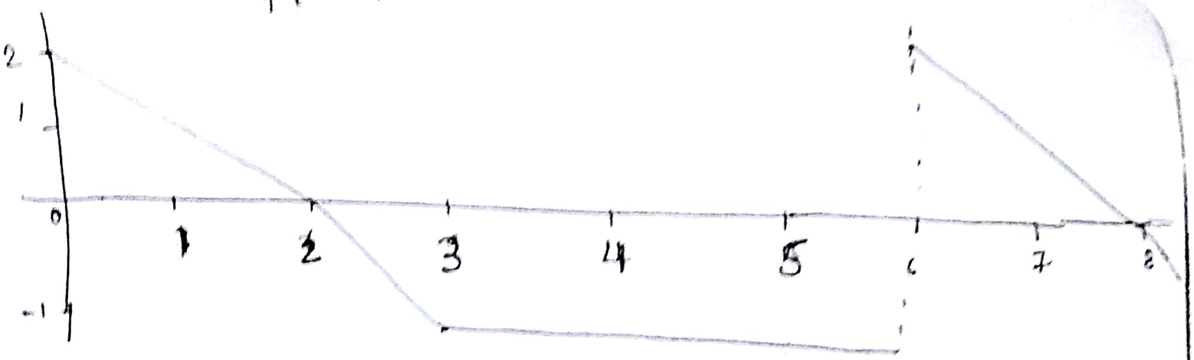
$f(x+4) = f(x)$



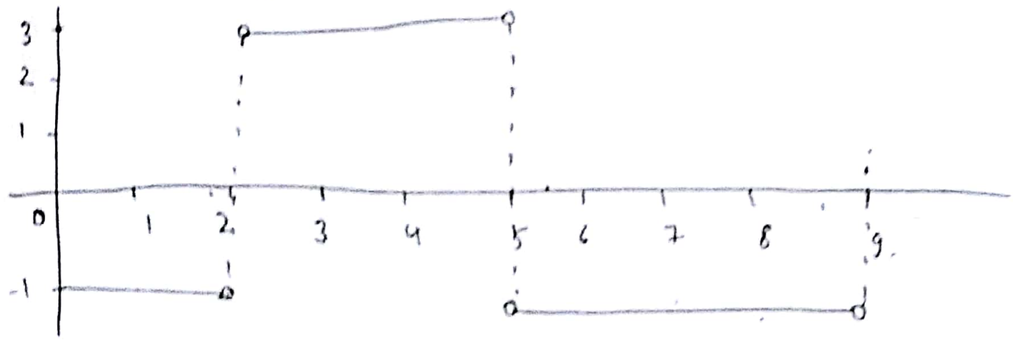
2) plot the function :-

$$f(x) = \begin{cases} 2-x & 0 \le x < 3 \\ -1 & 3 \le x < 6 \end{cases}$$

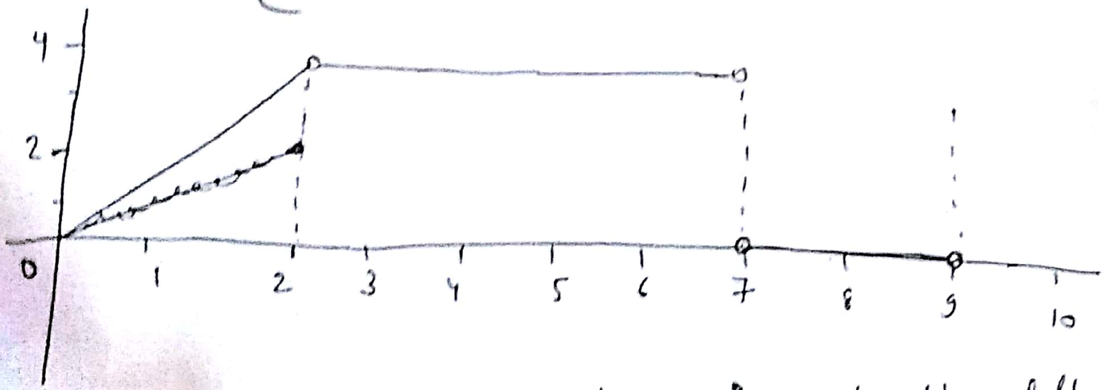
$$f(x+6) = f(x)$$



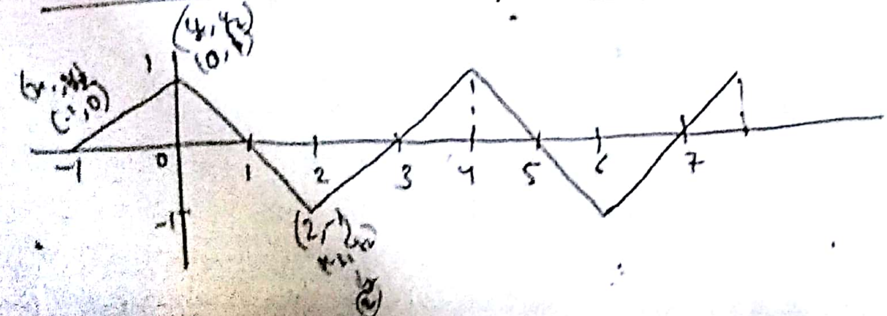
3) $f(x) = \begin{cases} -1 & 0 < x < 2 \\ 3 & 2 < x < 5 \\ -1 & 5 < x < 9 \end{cases}$ $f(x+9)$



4) $f(x) = \begin{cases} x & 0 < x < 2 \\ 4 & 2 < x < 7 \\ 0 & 7 < x < 9 \end{cases}$.



Q) Write the periodic function of the following:-



$$f(x) = \begin{cases} x+1 & -1 \le x < 0 \\ 1-x & 0 \le x < 2 \\ x-3 & 2 \le x < 3 \end{cases}$$

$(x_1, y_1) = (0, 1)$
$(x_2, y_2) = (2, -1)$

$f(x) =$

$$y - y_1 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

$$y - 0 = \left(\frac{1-0}{0-(-1)}\right)(x - (-1))$$

$$y = x + 1$$

$(x_1, y_1) = (2, -1)$
$(x_2, y_2) = (3, 0)$

$$y + 1 = \frac{0+1}{3-2}(x-2)$$

$$y = x - 2 - 1$$

$$y = x - 3$$

Q/ Write the periodic function of the graph

$$f(x) = \begin{cases} 0 & 0 \le x < 4 \\ x-4 & 4 \le x < 6 \\ 8-x & 6 \le x < 8 \end{cases}$$

$(x_1, y_1) = (4, 0)$
$(x_2, y_2) = (6, 2)$

$$y - 0 = \left(\frac{2-0}{6-4}\right)(x-4)$$

$$y = x - 4$$

$(x_1, y_1) = (6, 2)$
$(x_2, y_2) = (4, 0)$

$$y - 2 = \left(\frac{0-2}{4-6}\right)(x-6)$$

$$y - 2 = x - 6$$

$$y = x - 6 + 2$$

(5) plot $f(x) = \sin(x)$ in $(0, 2\pi)$

→ f = @(x) sin(x).
x = linspace(0, 2*pi, 500)
y = f(x);
plot(x,y).

(6) $f(x) = \begin{cases} x & 0 \le x < 3 \\ x^2 & 3 \le x < 5 \end{cases}$

→ f = @(x) x.*(0<=x & x<3) + x.^2 .*(3<=x & x<5).

x = linspace(0,5,100)
y = f(x);
plot(x,y)

>> A=[0 1 2];
>> repmat (A,1,2)

ans:-

0 1 2 0 1 2

>> repmat (A,2,2)

ans

0 1 2 0 1 2
0 1 2 0 1 2

(Q) plot f(x) in (0,15).

>> f=@(x) x.*(0<=x & x<3) + x.^2.*(3<=x & x<5);
>> x = linspace (0,5,100);
>> y = f(x),
>> ry = repmat (y,1,3);
>> rx = linspace (0,15, length (ry))
>> plot (rx, ry)

$\frac{15-0}{5} = \frac{15}{5} = 3$

(Q) $f(x) = \begin{cases} x & 0 \leq x < 3 \\ x^2 & 3 \leq x < 5 \end{cases}$

$\frac{-15-15}{5} = \frac{30}{5} = 6$

(Q) Plot the periodic function:-

$f(x) = \begin{cases} \pi - x & -\pi \leq x < 0 \\ \pi & 0 \leq x < \pi \end{cases}$

Plot in $(-\pi, 3\pi)$.

~~f=@(pi-x).~~

>> f=@(x) (pi-x).*(-pi<=x & x<0) + pi.*(0<=x & x<pi).

> x=linspace (-pi, pi,500);

>> y=f(x);

$\frac{3\pi-(-\pi)}{2\pi}$

>> ry = repmat (y,1,2);

>> rx = linspace (-pi, 3*pi, length (ry));

$\frac{4\pi}{2\pi} = 2$

plot (rx,ry).

Q. $f(x) = \begin{cases} -x-\pi & -\pi \le x < 0 \\ x+\pi & 0 \le x \le \pi \end{cases}$

(i) plot $(-\pi, 3\pi)$ ↔ plot $f(x)$
(ii) plot $(-6\pi, 6\pi)$ ← plot $f(x)$.
(iii) plot $f(x)$ in plot $(-12\pi, 12\pi)$.

Q. $f(x) = \begin{cases} x+\pi/2 & -\pi \le x < 0 \\ \pi/2 - x & 0 < x < \pi \end{cases}$

(i) plot $(-\pi, 3\pi)$.
(ii) plot $(-6\pi, 6\pi)$.

Fourier Series :-

Defn :- A function $f(x)$ is defined on the interval $(d, d+T)$, where $f(x+nT) = f(x)$, then

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{(2n\pi x)}{T} + \sum_{n=1}^{\infty} b_n \sin\left(\frac{2n\pi x}{T}\right)$$

This series is known as the fourier series of the $f(x)$; where a_0, a_n & b_n are called fourier co-efficients are given by eulers formulae.

where, $a_0 = \frac{2}{T}\int_d^{d+T} f(x)\,dx$.

$$a_n = \frac{2}{T}\int_d^{d+T} f(x)\cos\left(\frac{2n\pi x}{T}\right)dx.$$

$$b_n = \frac{2}{T}\int_d^{d+T} f(x)\sin\left(\frac{2n\pi x}{T}\right)dx.$$

Ex: Find the fourier serie for the function $f(x) = x$ in the interval $(0, 2\pi)$ & $f(x+2\pi) = f(x)$

Soln :- The fourier series for the function $f(x) = x$ is given by;

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \quad -①.$$

where;

$$a_0 = \frac{2}{T}\int_d^{d+T} f(x)\,dx = \frac{1}{\pi}\int_0^{2\pi} (x)\,dx = \frac{1}{\pi}\left[\frac{x^2}{2}\right]_0^{2\pi}$$

$$\frac{1}{\pi}\left[\frac{4\pi^2}{2} - 0\right] = \frac{1}{\pi} \times 2\pi^2 = \underline{2\pi}.$$

$\boxed{a_0 = 2\pi}$

$\boxed{\int uv\,dx = u\int v\,dx - u'\iint v\,dx\,dx + u''\iiint v\,dx \ldots + -}$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos nx\,dx = \frac{1}{\pi}\int_0^{2\pi} x\cdot\cos nx\,dx$$

$$= \frac{1}{\pi}\left[\left(x\cdot\cancel{\frac{\sin nx}{n}}^{0}\right) - (1)\left(-\frac{\cos x}{n^2}\right)\right]_0^{2\pi}$$

$\sin n\pi = 0$
$\sin 2n\pi = 0$
$\sin(0) = 0$

$$= \frac{1}{\pi n^2}\left[\cos nx\right]_0^{2\pi} = \frac{1}{n^2\pi}\cdot[\cos 2n\pi - \cos(0)]$$

$\therefore \boxed{a_n = 0}$.

$\because \cos 2n\pi = \cos 0 = 1.$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} x\sin nx\,dx.$$

$$= \frac{1}{\pi}\left[x\left(\frac{-\cos nx}{-n}\right) - (1)\left(\cancel{\frac{\sin nx}{n^2}}^{0}\right)\right]_0^{2\pi}$$

$$= -\frac{1}{n\pi}\left[x\cos nx\right]_0^{2\pi} = -\frac{1}{n\pi}\left[2\pi\cos 2n\pi - 0\right].$$

$$= -\frac{1}{n\pi}\left[2\pi(1)\right]$$

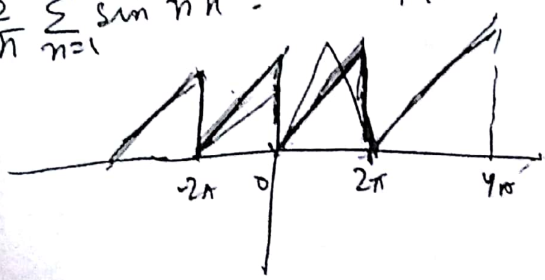
$\boxed{\cos 2n\pi = 1}$.

$\therefore \boxed{b_n = -2/n}$

$$f(x) = \pi + \sum_{n=1}^{\infty}(0)\cos nx + \sum_{n=1}^{\infty} -\frac{2}{n}\sin nx.$$

$$f(x) = \pi - \frac{2}{n}\sum_{n=1}^{\infty}\sin nx.$$

$f(x) = x$ $\quad 0, 2\pi.$



```
function [ ] = Fourier_CS (K, T)
syms x
W = (2* pi)/T;
n = 1: K
f(x) = x^2;
a0 = (2/T) * int(f, x, 0, 2*pi);
an = (2/T) * int (f* cos (n*W*x), x, 0, 2*pi);
bn = (2/T) * int( f * sin (n*w*x), x, 0, 2*pi);
V1 = [a0, an, bn];
V2 = [1/2, cos (n* w*x), sin (n* W*x)];
FS = sum (V1.* V2);
FS = vpa (FS, 5)
x = linspace (0, 4*pi, 500);
y = eval (FS);
plot (x, y, 'r');
g = @(x) x.^2;
x1 = linspace (0, 2*pi, 500)
y1 = g(x1);
ry1 = repmat (y1, 1, 2);
rx1 = linspace (0, 4*pi, length (ry1));
hold on;
plot (rx1, ry1, 'b');
end.
```

Given; $f(x) = \begin{cases} 1, & 0 \le x < \pi \\ x & \pi \le x < 2\pi \end{cases}$

plot $f(x)$ in $(0, 6\pi)$

$$a_0 = \frac{2}{T}\int_0^{2\pi} f(x)\,dx = \frac{2}{T}\left[\int_0^{\pi} f(x)\,dx + \int_{\pi}^{2\pi} f(x)\,dx\right]$$

```
function [ ] = Fourier_CS2 (k,T)
syms x
w = (2 * pi) /T;
n = 1:k
f1 = 1;
f2 = x;
a0 = (2/T)* (int (f1, x, 0, pi) + int (f2, x,
                                      pi, 2*pi));

an = (2/T) * ( int (f1 * cos (n* w* x), x ,0, pi)
          + int ( f2* cos (n* w * x), x , pi, 2*pi));

bn = (2/T) * (int (f1 * sin (n* w* x) , x, 0, pi)+
          int(f2* sin (n* w * x), x, pi, 2*pi));

V1 = [a0, an, bn];
V2 = [1/2, cos(n* w* x), sin (n* w* x)];
FS = sum (V1 .* V2);
FS = vpa (FS, 5)
x = linspace (0, 6* pi, 500);
y = eval (FS);
~~...~~ plot (x,y,'r');
~~...~~ g =@(x) 1.*(0<=x & x<pi) + x.*(pi <=x
                                   & x <= 2* pi);

x1 = linspace (0, 2*pi, 500);
y1 = g(x1);
ry1 = repmat (y1, 1 3);
rx1 = linspace (0, 6* pi, length (ry1));
plot (rx1, ry1, 'b');
end.
```

Ex:- ① $f(x) = \begin{cases} x/2, & 0 \le x \le \pi \\ \pi - x/2 & \pi \le x \le 2\pi \end{cases}$ $f(x+2\pi) = f(x)$

plot in the interval $(-4\pi, 4\pi)$.

② $f(x) = x + x^2, \quad -\pi \le x \le \pi$

$f(x+2\pi) = f(x)$ &

plot in the interval $(-3\pi, 3\pi)$.

$f(-x) = f(x)$ i.e. even.

$f(-x) = -f(x)$ odd.

$f(x) = \sin x$.

$f(-x) = \sin(-x)$

$= -\sin x$.

$f(-x) = -f(x)$.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega x + \sum_{n=1}^{\infty} b_n \sin n\omega x.$$

$$a_0 = \frac{2}{T}\int_{d}^{d+T} f(x)\,dx.$$

$$a_n = \frac{2}{T}\int_{d}^{d+T} f(x)\cos n\omega x\,dx.$$

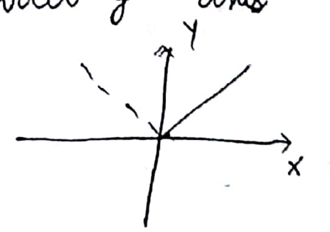
$$b_n = \frac{2}{T}\int_{d}^{d+T} f(x)\sin n\omega x\,dx.$$

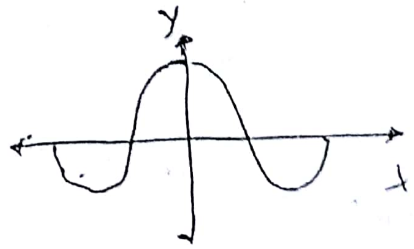
Even and Odd function :-

A function $f(x)$ is said to be even if $f(-x) = f(x)$.

Ex:- x^2, $\cos x$, $\sec x$, etc are all even function

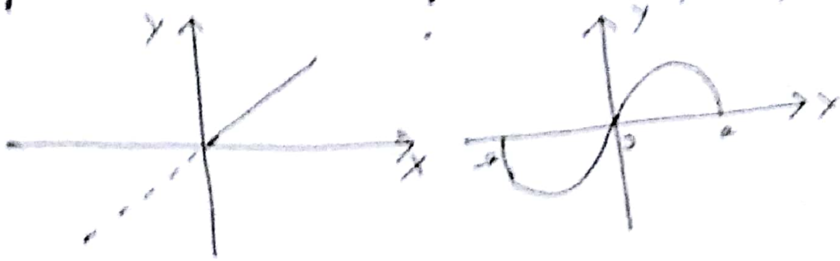
Graphically and even function is symmetrical about y - axis.





A function $f(x)$ is said to be odd if $f(-x) = -f(x)$.

Ex: x, x^3, $\sin x$, $\operatorname{cosec} x$, etc are all odd functions. Graphically an odd function is symmetrical about origin.



Property '1':-

The product of two odd function or the products of two even function is even, while the product of even and odd function is odd.

Property 2:-

By definite Integral

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$$

Fourier series for even and odd functions so defined in $(-l, l)$ or $(-\pi, \pi)$.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{l}\right)$$

$$a_0 = \frac{2}{T}\int_{d}^{d+T} f(x)\,dx = \frac{1}{l}\int_{-l}^{l} f(x)\,dx$$

$$a_n = \frac{2}{T}\int_{d}^{d+T} f(x)\cos\left(\frac{n\pi x}{l}\right) dx$$

$$= \frac{1}{l}\int_{-l}^{l} f(x)\cos\left(\frac{n\pi x}{l}\right) dx.$$

$$b_n = \frac{2}{T}\int_{d}^{d+T} f(x)\cos\left(\frac{n\pi x}{l}\right) dx.$$

$$= \frac{1}{l}\int_{-l}^{l} f(x)\sin\left(\frac{n\pi x}{l}\right) dx.$$

Case I:-

Suppose $f(x)$ is even

$$a_0 = \frac{1}{l}\int_{-l}^{l} f(x)\,dx$$

$$\boxed{\int_{-a}^{a} f(x)\,dx = \int_{0}^{a} f(x)\,dx}$$

if $f(x)$ is even

$= 0$ if $f(x)$ is odd

$$a_0 = \frac{2}{l}\int_{0}^{l} f(x)\,dx.$$

$$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cdot \cos n\left(\frac{n\pi x}{l}\right)dx.$$

$$a_n = \frac{2}{l}\int_{0}^{l} f(x)\cdot \cos\left(\frac{n\pi x}{l}\right)dx.$$

(product of two even function is even)

$$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx.$$

$$\boxed{b_n = 0}$$

Suppose $f(x)$ is odd.

$$a_0 = \frac{1}{l}\int_{-l}^{l} f(x)\,dx.$$

$$a_0 = \frac{2}{l}\int_{0}^{l} f(x)\,dx = 0.$$

(By property 2)

$$a_n = \frac{1}{l}\int_{-l}^{l} f(x)\cdot \cos n\left(\frac{n\pi x}{l}\right)dx.$$

$$a_n = \frac{2}{l}\int_{0}^{l} f(x)\cos\left(\frac{n\pi x}{l}\right)dx = 0$$

{By property 1 & 2}

$$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx.$$

$$b_n = \frac{2}{l}\int_{0}^{l} f(x)\sin\left(\frac{n\pi x}{l}\right)dx.$$

NOTE:- To check the given function odd or even

replace x by $a+b-x$.

i.e.; $f(a+b-x) = f(x)$ is even

if $f(a+b-x) = -f(x)$ then $f(x)$ is odd.

NOTE :

$$f(x) = \begin{cases} \phi(x) & -\pi < x < 0 \\ \psi(x) & 0 < x < \pi \end{cases}$$

if $\phi(a+b-x) = \psi(x)$, then $f(x)$ is even.

if $\phi(a+b-x) = -\psi(x)$, then $f(x)$ is odd.

Ex :- Express $f(x) = x$ as a fourier series in $(-\pi, \pi)$.

Soln :- The fourier series for the function $f(x)$ is :-

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx .$$

Let us check for the odd & even.

$$f(x) = x$$

$$f(-\pi + \pi - x) = -\pi + \pi - x$$

$$f(-x) = -x$$

hence the given function is odd ;

Consequently $a_0 = 0$, $a_n = 0$.

$$b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx \, dx .$$

$$b_n = \frac{2}{\pi} \int_0^{\pi} x \sin nx \, dx$$

By the Bernoulli's rule of integration, we have .

$$b_n = \frac{2}{\pi} \left[x \cdot - \frac{\cos nx}{n} - (1) \left(- \frac{\sin nx}{n^2} \right) \right]_0^{\pi}$$

$$b_n = \frac{-2}{\pi n} \left[x \cos nx \right]_0^{\pi} \quad (\because \sin n\pi = \sin 0 = 0) .$$

$$= \frac{-2}{\pi n} \left[\pi \cos n\pi - 0 \right] .$$

$$= -\frac{2}{\pi n} \left[\pi (-1)^n \right]$$

$= -\frac{2}{n}(-1)^n = \frac{2}{n}(-1)^{n+1}$

Substituting the value of a_0, a_n, b_n in the above series:

$$f(x) = 0 + 0 + \sum_{n=1}^{\infty} \frac{2}{n}(-1)^{n+1} \sin nx$$

Express :-

$$f(x) = \begin{cases} 1 + \frac{2x}{\pi}, & \text{in } -\pi < x < 0 \\ 1 - \frac{2x}{\pi}, & \text{in } 0 < x < \pi \end{cases}$$ in fourier series

Soln:- $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$,

∴ Let us check for odd or even

Here, $\phi(x) = 1 + \frac{2x}{\pi}$.

$\psi(x) = 1 - \frac{2x}{\pi}$.

$\phi(-\pi + \pi - x) = 1 + \frac{2}{\pi}(-\pi + \pi - x)$.

$\phi(-x) = 1 - \frac{2x}{\pi}$

$\therefore \phi(-x) = \psi(x)$.

Hence, it is even function, consequently $b_n = 0$

$\therefore a_0 = \frac{2}{l}\int_0^l f(x)\,dx$

$a_0 = \frac{2}{\pi}\int_0^\pi f(x)\,dx$.

$a_0 = \frac{2}{\pi}\int_0^\pi \left(1 - \frac{2x}{\pi}\right)dx$.

$a_0 = \frac{2}{\pi}\left[x - \frac{x^2}{\pi}\right]_0^\pi$

$= \frac{2}{\pi}[\pi - \pi] = 0$

$a_n = \frac{2}{\pi}\int_0^\pi f(x) \cos nx\,dx$

$a_n = \frac{2}{\pi}\int_0^\pi \left(1 - \frac{2x}{\pi}\right) \cos nx\,dx$.

$$= \frac{2}{\pi}\left[\left(1-\frac{2x}{\pi}\right)\left(\frac{\sin nx}{n}\right) - \left(-\frac{2}{\pi}\right)\left(-\frac{\cos nx}{n^2}\right)\right]_0^\pi$$

$$= \cancel{\frac{2}{\pi}} - \frac{4}{\pi^2 n^2}\left[\cos nx\right]_0^\pi$$

$$= \frac{-4}{\pi^2 n^2}\left[\cos n\pi - \cos n(0)\right]$$

$$= \frac{-4}{\pi^2 n^2}\left[(-1)^n - 1\right].$$

$$= \frac{4}{\pi^2 n^2}\left[1-(-1)^n\right]$$

But $(1-(-1)^n) = 2$ or $n = 1,3,5, \ldots$

0 or $n = 2,4,6 \ldots$

$\therefore a_n = \frac{8}{n^2\pi^2}$ for $n = 1,3,5 \ldots$

Therefore, substituting the value of a_0, a_n & b_n in the above series

$$f(x) = 0 + \sum_{n=1,3,5\ldots}^{\infty} \frac{8}{\pi^2 n^2} \cos nx.$$

(6) Obtain the fourier series for the function

$$f(x) = \begin{cases} 2-x & \text{in } 0 \le x \le 4 \\ x-6 & \text{in } 4 \le x \le 8 \end{cases}$$

Soln: $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l}$

$$\boxed{(0, 2l) = (0, 8) \quad 2l = 8 \quad l = 4.}$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{4}$$

$$+ \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{4}.$$

Let us check for the odd & even

Here; $\phi(x) = 2-x.$

$\psi(x) = x-6.$

~~$\phi(a+b-x)$~~

$$\phi(0+8-x) = 2-(0+8-x).$$

$$= x-6 = \psi(x).$$

$\therefore \phi(-x) = \psi(x)$.

Hence it is even function.

Consequently $b_n = 0$.

$(0, 2l) = (0, 8)$

$2l = 8$

$l = 4$.

$a_0 = \frac{1}{8}\int_0^8$

$$a_0 = \frac{2}{l}\int_0^l f(x)\,dx$$

$$a_0 = \frac{2}{4}\int_0^4 f(x)\,dx = \frac{1}{2}\int_0^4 (2-x)\,dx$$

$$a_0 = \frac{1}{2}\left[2x - \frac{x^2}{2}\right]_0^4$$

$$a_0 = \frac{1}{2}[8-8] = 0$$

$$a_n = \frac{1}{2}\int_0^4 (2-x)\cos\left(\frac{n\pi x}{4}\right)dx$$

$$a_n = \frac{1}{2}\left[(2-x)\cancel{\frac{\sin\left(\frac{n\pi x}{4}\right)}{\left(\frac{n\pi x}{4}\right)}} - (-1)\left(\frac{-\cos\frac{n\pi x}{4}}{\left(\frac{n\pi}{4}\right)^2}\right)\right]_0^4$$

$$a_n = -\frac{1}{2}\cdot\frac{16}{n^2\pi^2}\left(\cos\frac{n\pi x}{4}\right)_0^4$$

$$a_n = \frac{-8}{n^2\pi^2}\left(\cos n\pi - \cos 0\right)$$

$$\therefore = \frac{-8}{n^2\pi^2}\left[(-1)^n - 1\right]$$

$$= \frac{-8}{n^2\pi^2}\left[1 - (-1)^n\right]$$

$$= \frac{16}{n^2\pi^2} \text{ for } n = 1, 3, 5, \ldots$$

Complex Fourier Series :-

Let $f(x)$ is defined in $(d, d+T)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega x) + \sum_{n=1}^{\infty} b_n \sin(n\omega x) \quad \text{—— (1)}$$

$\omega = \frac{2\pi}{T}$

$$e^{ix} = \cos x + i\sin x$$

$$e^{-ix} = \cos x - i\sin x$$

$$e^{ix} + e^{-ix} = 2\cos x$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2}; \quad \cos(n\omega x) = \frac{e^{in\omega x} + e^{-in\omega x}}{2}$$

$(a) - (b) \Rightarrow$

$$e^{ix} - e^{-ix} = 2i\sin x$$

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i};$$

$$\sin(n\omega x) = \frac{e^{in\omega x} - e^{-in\omega x}}{2i}$$

Substituting $\cos(n\omega x)$ and $\sin(n\omega x)$ in eqn (1)

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \left[\frac{e^{in\omega x} + e^{-in\omega x}}{2}\right] + \sum_{n=1}^{\infty} b_n \left[\frac{e^{in\omega x} - e^{-in\omega x}}{2i}\right]$$

$$= \frac{a_0}{2} + \sum_{n=1}^{\infty} \left\{\left(\frac{a_n}{2} + \frac{b_n}{2i}\right) e^{in\omega x} + \left(\frac{a_n}{2} - \frac{b_n}{2i}\right) e^{-in\omega x}\right\}$$

$\frac{1}{i} = \frac{1}{i} \times \frac{i}{i}$

$\frac{i}{i^2} = -i$

$i^2 = -1$

$z = x + iy$

$\bar{z} = x - iy$

$$= \frac{a_0}{2} + \sum_{n=1}^{\infty} \left\{ \left[\frac{a_n - ib_n}{2}\right] e^{in\omega x} + \left[\frac{a_n + ib_n}{2}\right] e^{-in\omega x} \right\}$$

Put $\frac{a_0}{2} = c_0$, $c_n = \left[\frac{a_n - ib_n}{2}\right]$

$$c_{-n} = \left[\frac{a_n + ib_n}{2}\right]$$

$$= c_0 + \sum_{n=1}^{\infty} c_n e^{inwx} + \sum_{n=1}^{\infty} c_{-n} e^{-inwx}$$

$$= c_0 + \sum_{n=1}^{\infty} c_n e^{inwx} + \sum_{n=-1}^{-\infty} c_n e^{inwx}$$

$$\boxed{f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inwx}} \rightarrow$$ This is known as Complex fourier series.

Derive the formula for c_n :-

Consider; $c_n = \frac{1}{2}\left[a_n - ib_n\right]$.

$$a_n = \frac{2}{T}\int_d^{d+T} f(x)\cos(nwx)\,dx$$

$$b_n = \frac{2}{T}\int_d^{d+T} f(x)\sin(nwx)\,dx$$

$$\Rightarrow c_n = \frac{1}{2}\left[\frac{2}{T}\int_d^{d+T} f(x)\cos(nwx)\,dx - i\frac{2}{T}\int_d^{d+T} f(x)\sin(nwx)\,dx\right]$$

$$c_n = \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{T}\int_d^{d+T} f(x)\left[\cos(nwx) - i\sin(nwx)\right]dx$$

$$c_n = \frac{1}{T}\int_d^{d+T} f(x)\left[e^{-inwx}\right]dx$$

$\because e^{-inwx} = \cos(nwx) - i\sin(nwx)$

Put $n = 0$.

$$\boxed{c_0 = \frac{1}{T}\int_d^{d+T} f(x)\,dx}$$

NOTE:- Complex fourier series in the interval $(d, d+T)$ is given by :-

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inwx}$$

where;

$$c_n = \frac{1}{T}\int_d^{d+T} f(x)\,e^{-inwx}\,dx.$$

Conversion formulas :-

$$a_n - i b_n = 2c_n \quad \text{—} (I)$$

$$a_n + i b_n = 2c_{-n} \quad \text{—} (II)$$

add (I) & (II)

$$\cancel{2}a_n = \cancel{2}(c_n + c_{-n})$$

$$\boxed{a_n = c_n + c_{-n}}$$

sub (I) & (II)

$$a_n - i b_n = 2c_n \quad - (I)$$

$$a_n + i b_n = 2c_{-n} \quad - (II)$$

$$-\cancel{2} i b_n = \cancel{2}(c_n - c_{-n})$$

$$b_n = \left(-\frac{1}{i}\right)(c_n - c_{-n})$$

$$\boxed{b_n = i(c_n - c_{-n})}$$

$$\frac{1}{i} = \frac{1}{i} \times \frac{i}{i}$$

$$\frac{i}{i^2} = -i$$

$$i^2 = -1$$

$$\therefore \frac{1}{i} = -i$$

$$\boxed{\begin{aligned} \frac{1}{i} &= -i \\ \left(-\frac{1}{i}\right) &= -(-i) \\ &= i \end{aligned}}$$

Q. obtain a complex fourier series for the function $f(x) = x$ in $(-\pi, \pi)$

Solⁿ:- Here,

complex fourier series is given by

$$f(x) = \sum_{-\infty}^{\infty} c_n e^{inwx} \quad \text{—} (I)$$

where; $c_n = \frac{1}{T} \int_{d}^{d+T} f(x) e^{-inwx} \, dx$.

here; $T = 2\pi$, $w = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$.

$$c_n = \frac{1}{2\pi} \int_{-\pi}^{\pi} x e^{-inx} \, dx$$

$$C_n = \frac{1}{2\pi}\left\{ x\left(\frac{e^{-inx}}{-in}\right) - (1)\left(\frac{e^{-inx}}{(-in)^2}\right)\right\}_{x=-\pi}^{\pi}$$

$$C_n = \frac{1}{2\pi}\left\{\frac{1}{-in}\left[\pi e^{-in\pi} - (-\pi)\, e^{in\pi}\right] - \frac{1}{(in)^2}\left[\cancel{e^{-in\pi} - e^{in\pi}}\right]^{0}\right.$$

$$\boxed{\begin{array}{l} e^{-in\pi} = \cos n\pi - i\cancel{\sin n\pi}^{0} \\ e^{in\pi} = \cos n\pi + i\cancel{\sin n\pi}^{0} \\ \\ e^{-in\pi} - e^{in\pi} = \cos n\pi - \cos n\pi = 0 \\ e^{-in\pi} + e^{in\pi} = \cos n\pi + \cos n\pi = 2\cos n\pi \end{array}}$$

$$C_n = \frac{1}{2\pi}\left\{\frac{1}{-in}\left[\pi e^{-in\pi} - (-\pi)e^{in\pi}\right]\right.$$

$$C_n = \frac{1}{2\cancel{\pi}}\left(\frac{\cancel{\pi}}{-in}\right)\left[e^{-in\pi} + e^{in\pi}\right].$$

$$C_n = \frac{1}{2(-in)}\left[2\cos n\pi\right]$$

$$C_n = \frac{\cos n\pi}{-in} = \frac{i\cos n\pi}{n} = \frac{i}{n}(-1)^n.$$

$$\boxed{C_n = \frac{i}{n}(-1)^n} \quad n \neq 0.$$

C_n is not defined at $n=0$ whereas it is defined for all the values. So, we have to find C_0.

$$C_n = \frac{1}{T}\int_d^{d+T} f(x)\, e^{-in\omega x}\, dx$$

$$C_0 = \frac{1}{T}\int_d^{d+T} f(x)\, dx.$$

$$C_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} x\, dx = \frac{1}{2\pi}\left[\frac{x^2}{2}\right]_{-\pi}^{\pi}$$

$$C_0 = \frac{1}{4\pi}\left[-\pi^2 - (-\pi^2)\right].$$

$$C_0 = \frac{1}{4\pi}\left(-\pi^2 + \pi^2\right)$$

$$C_0 = \frac{0}{4\pi} = 0.$$

$$\boxed{C_0 = 0}$$

Required complex fourier series is

$$f(x) = \sum_{-\infty}^{\infty} c_n e^{inwx}.$$

$$f(x) = c_0 + \sum_{\substack{-\infty \\ n\neq 0}}^{\infty} c_n e^{inwx}$$

$$f(x) = 0 + \sum_{\substack{-\infty \\ n\neq 0}}^{\infty} \frac{i(-1)^n}{n} e^{inwx}$$

$$\boxed{f(x) = \sum_{\substack{-\infty \\ n\neq 0}}^{\infty} \frac{i(-1)^n}{n} e^{inwx}}$$

(Q) obtain a complex fourier series for the function ; $f(x) = \begin{cases} -K & -\pi < x < 0 \\ K & 0 \leq x < \pi \end{cases}$.

Soln Here ;

Complex fourier series is given by

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inwx} \quad \text{--- (1)}$$

where; $c_n = \frac{1}{T}\int_{d}^{d+T} f(x) e^{-inwx} dx$.

here ; $T = 2\pi$, $w = \frac{2\pi}{T} = \frac{2\pi}{2\pi} = 1$.

$$c_n = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x) e^{-inx} dx.$$

$$c_n = \frac{1}{2\pi}\int_{-\pi}^{0} f(x) e^{-inx} dx + \int_{0}^{\pi} f(x) e^{-inx} dx.$$

$$c_n = \frac{1}{2\pi}\int_{-\pi}^{0} -K e^{-inx} dx + \int_{0}^{\pi} K e^{-inx} dx.$$

$$\cancel{C_n = \frac{1}{2\pi}\left[(-1)\left(\frac{e^{-inx}}{-in}\right)\right]}$$

$$C_n = \frac{1}{2\pi} k \left[(-1)\left(\frac{e^{-inx}}{-in}\right)_{-\pi}^{0} + \left(\frac{e^{-inx}}{-in}\right)_{0}^{\pi}\right].$$

$$C_n = \frac{k}{2\pi}\left\{\frac{1}{in}\left[1 - e^{in\pi}\right] - \frac{1}{in}\left[e^{-in\pi} - 1\right]\right\}.$$

$$\boxed{\begin{aligned} &\because e^{in\pi} = \cos n\pi + i \sin n\pi = \cos n\pi = (-1)^n. \\ &\because e^{-in\pi} = \cos n\pi - i \sin n\pi = \cos n\pi = (-1)^n. \end{aligned}}$$

$$C_n = \frac{K}{2\pi}\left\{\frac{1}{in}\left[1 - (-1)^n\right] - \frac{1}{in}\left[(-1)^n - 1\right]\right\}$$

$$C_n = \frac{K}{2\pi} \times \frac{1}{in}\left\{1 - (-1)^n - (-1)^n + 1\right]$$

$$C_n = \left(\frac{K}{2\pi}\right)\left(\frac{1}{in}\right)\left[2 - 2(-1)^n\right].$$

$$C_n = \frac{\cancel{2}K}{(\cancel{2}\pi)(in)}\left[1 - (-1)^n\right]$$

$$\boxed{C_n = \frac{-Ki}{\pi}\left[\frac{1 - (-1)^n}{n}\right]} \quad n \neq 0$$

C_n is not defined at $n = 0$ whereas it is defined for the values. So we have to find C_0

$$C_0 = \frac{1}{T}\int_{d}^{d+T} f(x)\,dx.$$

$$C_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx.$$

$$C_0 = \frac{1}{2\pi}\left\{\int_{-\pi}^{0} f(x)\,dx + \int_{0}^{\pi} f(x)\,dx\right\}.$$

$$C_0 = \frac{1}{2\pi}\left[\int_{-\pi}^{0} (-k)\,dx + \int_{0}^{\pi} k\,dx\right].$$

$$C_0 = \frac{k}{2\pi}\left\{\left[-x\right]_{-\pi}^{0} + \left[x\right]_{0}^{\pi}\right\}.$$

$$\boxed{C_0 = 0}$$

Required Complex fourier Series is.

$$f(x) = \sum_{-\infty}^{\infty} C_n e^{inwx}$$

$$f(x) = C_0 + \sum_{\substack{-\infty \\ n\neq 0}}^{\infty} C_n e^{inwx}$$

$$f(x) = 0 + \sum_{\substack{-\infty \\ n\neq 0}}^{\infty} \frac{-Ki}{\pi}\left[\frac{1-(-1)^n}{n}\right]$$

$$\boxed{f(x) = \sum_{\substack{-\infty \\ n\neq 0}}^{\infty} \frac{-Ki}{\pi}\left[\frac{1-(-1)^n}{n}\right]}$$

Harmonic Analysis:-

It the process of contructing fourier series for the ~~proc~~ given data.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$= \frac{a_0}{2} + a_1 \cos x + b_1 \sin x + a_2 \cos 2x + b_2 \sin 2x + a_3 \cos 3x + b_3 \sin 3x + \cdots$$

$$= \frac{a_0}{2} + \underbrace{a_1 \cos x + b_1 \sin x}_{\text{first harmonic}} + \underbrace{a_2 \cos 2x + b_2 \sin 2x}_{\text{Second harmonic}}$$

$$a_0 = \frac{1}{\pi}\int_d^{d+2\pi} f(x)\,dx$$

$$a_0 = \frac{2}{N}\Sigma y.$$

The mean value of function $y=f(x)$ is defined in the range (a,b).

$$\int_a^b f(x)\,dx.$$

$$a_0 = 2\left[\frac{1}{d+2\pi-\pi}\int_d^{d+2\pi} f(x)\,dx\right]$$

$= 2 \times$ [mean value of $y = f(x)$] $= \frac{2\Sigma y}{N}$

$$a_n = 2\left[\int_{d}^{d+2\pi} f(x) \cos nx \, dx\right]$$

$= 2 \times$ [mean value of $y \cos nx$] $= \frac{2\Sigma y \cos nx}{N}$

$b_n = 2$ [mean value of $y \sin nx$].

$$b_n = \frac{2}{N}\Sigma y \sin nx$$

Suppose, we have set of N values of $y = f(x)$ having a period 2π with equidistant value of x defined in the interval ~~of~~ $d \le x < 2\pi$ or $d < x \le 2\pi$ If the value of x is given $x = d$ and $x = d + 2\pi$, we omit one of them value

∴ The value of a_0 and a_n and b_n can be obtained by using the principle

we know that the mean value of $y = f(x)$ defined in the range (a, b),

then $\frac{1}{b-a}\int_a^b f(x)\,dx$.

By using above principle, we have

$$a_0 = \frac{1}{\pi}\int_{d}^{d+2\pi} f(x)\,dx$$

$$a_0 = 2 \times \left[\frac{1}{d+2\pi-d}\int_{d}^{d+2\pi} f(x)\,dx\right]$$

$= 2 \times$ [mean value of $f(x)$]

$$a_0 = 2 \times \frac{\Sigma y}{N}$$

$$a_n = \frac{2}{N}\Sigma y \cos nx$$

$$b_n = \frac{2}{N}\Sigma y \sin nx$$

Q.) Compute the first two harmonics of the fourier series of f(x) given the following table :-

	60°	120°	180°	240°	300°	360°
x	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
f(x)	1.0	1.4	1.9	1.7	1.5	1.2	1.0

Soln :- The fourier series for the function f(x) is $f(x) = \frac{a_0}{2} \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$.

$$f(x) = \frac{a_0}{2} + a_1 \cos x + b_1 \cos x + a_2 \cos 2x + b_2 \sin 2x$$

$$a_0 = \frac{2}{N} \Sigma y \quad a_1 = \frac{2}{N} \Sigma y \cos x \,, \quad a_2 = \frac{2}{N} \Sigma y \cos 2x$$

MATLAB
cosd(60)
y.*cosd(x)
y.*sind(x)
y.*sind(2*x)
sum(...)

$$b_1 = \frac{2}{N} \Sigma y \sin x \quad b_2 = \frac{2}{N} \Sigma y \sin 2x$$

Here, ~~first values~~ $f(0) = f(2\pi) = 1.0$, hence we omit the last value.

$\therefore N = 6$.

Let us construct the table :-

x	f(x)	cos x	y cos x	y sin x	y cos 2x
0°	1.0	1	1.0	0	1.
60°	1.4	0.5	0.7	1.2124	-0.7
120°	1.9	-0.5	-0.95	1.6454	-0.95
180°	1.7	-1	-1.7	0	1.7
240°	1.5	-0.5	-0.75	-1.2990	-0.75
300°	1.2	0.5	0.6	-1.0392	-0.6
	$\Sigma y = 8.7$		$\Sigma y \cos x = -1.1$	$\Sigma y \sin x = 0.5196$	$\Sigma y \cos 2x = -0.3$

$y \sin 2x$
0
1.2124
-1.6454
0
1.2990
-1.0392
$\Sigma y \sin 2x = -0.1732$

first two → a_1, b_1
a_2, b_2
first three → a_1, b_1
a_2, b_2
a_3, b_3

$a_0 = \frac{2}{N} \Sigma y = \frac{2}{6}(8.7) = 2.9$

$a_1 = \frac{2}{N} \Sigma y \cos x = \frac{2}{6}(-1.1) = -0.3667$

$a_2 = \frac{2}{N} \Sigma y \cos 2x = \frac{2}{6}(-0.3) = -0.1$

$b_1 = \frac{2}{N} \Sigma y \sin x = \frac{2}{6}(0.5196) = 0.1732$

$b_2 = \frac{2}{N} \Sigma y \sin 2x = \frac{2}{6}(-0.1732) = -0.0577$

$f(x) = 1.45 + (-0.3667) \cos x + (-0.1) \cos 2x + (-0.0577) \sin 2x + (0.1732) \sin x$

$a_0 = 2 \text{ mean } (f(x))$

$a_0 = 2 \text{ mean } y$

$\boxed{a_0 = 2 * \left(\frac{\Sigma y}{N}\right)}$

$a_n = 2 * \text{ mean } (f(x) \cos(n \omega x))$

$\boxed{a_n = 2 * \left(\frac{\Sigma y \cos(n \omega x)}{N}\right)}$

$b_n = 2 * \text{ mean } (f(x) \sin(n \omega x))$

$\boxed{b_n = 2 * \frac{\Sigma y \sin(n \omega x)}{N}}$

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n \omega x) + \sum_{n=1}^{\infty} b_n \sin(n \omega x)$

where, $\omega = \frac{2\pi}{T}$

```
x = [0   60   120   180   240   300];
y=[1   1.4,   1.9   1.7   1.5   1.2];
x = (pi/180)*x; → (when x is in degree we have
T = 2*pi;           to use this else
w = (2*pi/T);       when x is not in degree
                    remove it.
syms t
a0 = 2*mean(y);
HS = (a0/2);
h=2;
for i=1:h
    a(i) = 2*mean(y.*cos(i*w*x));
    b(i) = 2*mean(y.*sin(i*w*x));
    HS=HS+a(i)*cos(i*w*t)+b(i)*sin(i*w*t);
end
HS = vpa(HS, 4);
disp(HS)
plot(x,y,'*')
hold on
t = linspace(x(1), x(end), 1000);
y1= eval(HS);
plot(t, y1, 'r').
```

Q given;

x	0	1	2	3	4	5
y	9	18	24	28	26	20

Find fourier series upto 3rd harmonics.

Sol^n: rewrite.

x	0	1	2	3	4	5	6
y	9	18	24	28	26	20	9

assume that at x=6,
y=9.

$T = 6 - 0 = 6$.

$w = \frac{2\pi}{T} = \frac{2\pi}{6} = \pi/3$.

$h = 3$.

```
x = [0 1 2 3 4 5];
y = [0 18.24 28 26 20];
% x = (pi/180)*x .
T = 6 ;
w = (2* pi/T);
syms t
a0 = 2* mean (y);
HS = (a0/2);
h = 3;   % number of harmonics
for i = 1:h
    a(i) = 2* mean (y.* cos(i* w* x));
    b(i) = 2* mean (y.* sin (i* w* x));
    HS = HS + a(i)* cos (i* w* t) + b(i)* sin (i*w
        * t)

end
HS = vpa (HS, 4);
disp (HS)
plot (x, y, '*').
hold on
t= linspace (x(1), x(end), 1000);
y1 = eval (HS);
plot (t, y1, 'r')
```

Plot $F = 2y\hat{i} + x\hat{j} + z\hat{k}$ in $-1 \leq x \leq 1$.
$-1 \leq y \leq 1$.
$-1 \leq z \leq 1$.

```
x = linspace (-1,1,10);
y = linspace (-1,1,10);
z = linspace (-1,1,10);
[X,Y,Z] = meshgrid (x,y,z);
f1 = 2*Y;
f2 = X;
f3 = Z;
quiver3(X,Y,Z,f1,f2,f3)
```

$\vec{F} = xy\hat{i} + yz\hat{j} + z\cdot zx\hat{k}$.

f1 = X.*Y , f2 = Y.*Z , f3 = Y.Z.*X

$\vec{F} = (x\cos e^{x})\hat{i} + (y\sin x)\hat{j} + x^2 z\hat{k}$.

```
f1 = X.* cos(X).* exp(X);
f2 = Y.* sin(X);
f3 = X.^2.*Z;
```

Complex Fourier Series :-

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inwx}$$

$$c_n = \frac{1}{T}\int_{d}^{d+T} f(x) e^{-inwx} \, d(x)$$

Question :-

Q1 given; $f(x) = x$ in $(-\pi, \pi)$

(a) Find complex F.S in $(-\pi, \pi)$

(b) Plot given function & CFS in $(-\pi, \pi)$.

Q2 given; $f(x) = x$ in $(-\pi, \pi)$.

(a) Find complex CFS in $(-3\pi, \pi)$.

(b) Plot given function and CFS in $(-3\pi, \pi)$.

Q3 given; $f(x) = \begin{cases} -1 & -\pi \le x < 0 \\ 1 & 0 \le x \le \pi \end{cases}$

(a) Find CFS in $(-\pi, 5\pi)$

(b) Plot CFS & given function in $(-\pi, 5\pi)$.

Plot :- $\vec{F} = \dfrac{-x\hat{i} - y\hat{j} - z\hat{k}}{\sqrt{x^2+y^2+z^2}}$ in $-3 \le x \le 3$

$-3 \le y \le 3$

$-3 \le z \le 3$

```
x = linspace (-3, 3, 10);
y = linspace (-3, 3, 10);
z = linspace (-3, 3, 10);
[X,Y,Z] = meshgrid (x, y, z, f1 f2 f3)
f1 = -x./sqrt (x.^2 + y.^2 + z.^2);
f2 = -y./sqrt (x.^2 + y.^2 + z.^2)
f3 = -z./sqrt (x.^2+y.^2+z.^2);
quiver3 (x, y, z, f1, f2, f3).
```

$T = 2\pi$

Fourier Transform:-

We know that;

$$\mathcal{L}\{f(t)\} = \int_{-\infty}^{\infty} e^{-st} f(t)\, dt.$$

where; $s = \sigma + i w$

$\sigma \rightarrow$ system analysis. $\quad i w \rightarrow$ frequency analysis.

If $\sigma = 0$

$$F(f(t)) = \int_{-\infty}^{\infty} e^{-iwt} f(t)\, dt.$$

$$F(f(t)) = \int_{-\infty}^{\infty} e^{iwt} f(t) dt = F(w)$$

$F^{-1} F(w) = f(t).$

$F^{-1} \rightarrow$ Inverse Fourier transform

Fourier Transform :-

Introduction:-

The fourier transform is a tool that breaks ~~its down~~ a wave form in terms of (a function or signal) in to alternate representation characterised by sine and cosines.

(i) The fourier transform shows that any waveform can be rewritten as sum of sinosoidal function

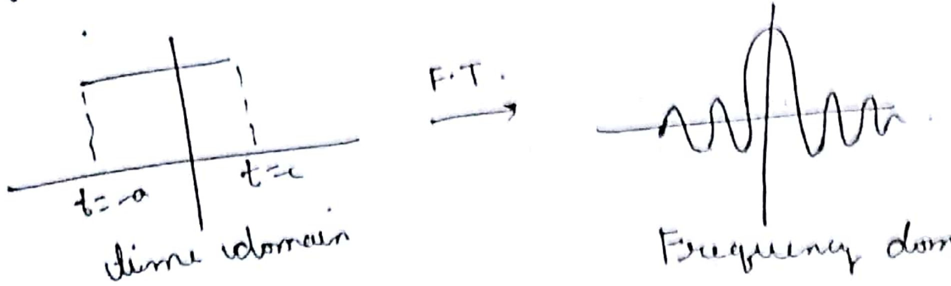
(ii) The fourier transform has many wide application that include image compression, image filtering and image analysis.

(iii) The fourier tranform can also be used to solve the ordinary differential equations.

Def^n :- The fourier transform of a function $f(t)$, $-\infty \leq t \leq \infty$ is denoted by $F(w)$ and is defined as $F(f(t)) = F(w) = \int_{-\infty}^{\infty} e^{iwt} f(t)\, dt$ is called fourier tranform of $f(t)$.

Then, the inverse fourier transform $F(w)$ is defined as $F^{-1}[F(w)] = f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-iwt} f(w)\, dt$.

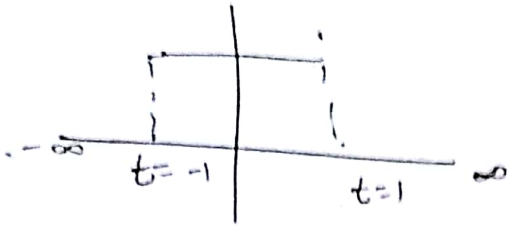
~~Properties of fourier transform :-~~



~~Thus~~ Fourier Transform is a function which transform from time domain to frequency domain.

(Q.) Find fourier transform of $f(t) = \begin{cases} 1 & f(t) \leq 1 \\ 0 & f(t) > 1 \end{cases}$.

$$f(t) = \begin{cases} 1 & -1 \leq t \leq 1 \\ 0 & \text{otherwise} \end{cases}$$



syms w t

f = 1* (heaviside (t - (-1)) - heaviside (t - 1))

FS = fourier (f, t, w).

%

$$f(t) = \begin{cases} 1 - |t| & t \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$f(t) = \begin{cases} 1 - (-t) & -1 \leq t < 0 \\ 1 - t & 0 \leq t < 1 \\ 0 & \text{otherwise} \end{cases}$$

$-1 \leq t \leq 1.$

$\Downarrow$

$(-1 \leq t < 0, 0 \leq t \leq 1)$

syms w t

f = (1 + t)* (heaviside (t+1) - heaviside (t - 0)) + (1-t)* (heaviside (t - 0) - heaviside (t - 1));

FT = fourier (f, t, w).

or

f = (1 - abs(t))* heaviside (t+1) - heaviside (t - 1));

Find FT;

$$f(t) = \begin{cases} t \cdot e^{t} & t > 0 \\ 0 & t \leq 0. \end{cases}$$

or.

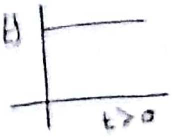
$$f(t) = \begin{cases} t \cdot e^{t} & 0 < t < \infty \\ 0 & \text{otherwise}) \end{cases}$$

(9) $f(t) = \begin{cases} 2-t & 0 < t < 4 \\ t-6 & 4 \le t < 8 \end{cases}$

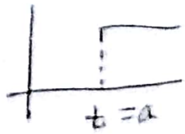
(10) $f(t) = \begin{cases} 1 & 0 \le t < \pi \\ \sin t & t \geq \pi \end{cases}$

Laplace Transform :-

$$H(t) = \begin{cases} 1 & t \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

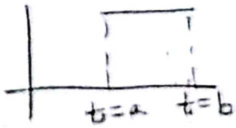


$$H(t-a) = \begin{cases} 1 & t \geq a \\ 0 & \text{otherwise} \end{cases}$$

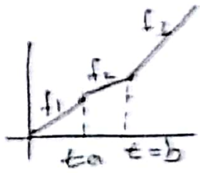


Top hat function :-

$$H(t-a) - H(t-b) = \begin{cases} 1 & a \leq t \leq b \\ 0 & \text{otherwise} \end{cases}$$



$$f(t) = \begin{cases} f_1(t) & 0 \leq t < a \\ f_2(t) & a \leq t < b \\ f_3(t) & t \geq b \end{cases}$$



$$f(t) = f_1(t)\,(H(t-0) - H(t-a)) + f_2(t)\,(H(t-a) - H(t-b)) + f_3(t)\,(H(t-b)).$$

Important.

① Find the laplace transformation of (sin t) by using in-built command.

→
```
syms s t
f = sin(t);
laplace(f, t, s)
```

③ L (sin 2t * cos 5t).

④ $L\left(\frac{\sin t}{t}\right)$ ds.

⑤ $L(t^2 \sin t)$

② $L(e^{2t})$.

→
```
syms s t
f = exp(2*t);
laplace(f, t, s)
```

⑥ $f(t) = \begin{cases} t^2 & 0 \leq t < \pi \\ t & t \geq \pi \end{cases}$

→
```
syms s t
f = t^2 * (heaviside(t-0) - heaviside(t-pi))
      + t* (heaviside(t-pi));
laplace(f, t, s).
```

$$f(t) = \begin{cases} \cos t & 0 \le t < \pi \\ \sin t & t \ge \pi \end{cases}$$

→ syms s t

```
f = cos(t) * (heaviside (t-0) - heaviside (t-pi)) +
        sin (t) * (heaviside (t - pi));
laplace (f, t, s)
```

(Q) $f(t) = \begin{cases} 1 & 0 \le t < 1 \\ t & 1 \le t < 2 \\ t^2 & t \ge 2 \end{cases}$ find laplace transform.

→ syms s t

```
f = 1 * (heaviside (t - 0) - heaviside (t-1)) +
        t * (heaviside (t-1) - heaviside (t-2)) +
        t^2 * (heaviside (t-2));
laplace (f, t, s)
```

(Q) $f(t) = \begin{cases} \cos(t) & 0 \le t < \pi \\ 1 & \pi \le t \le 2\pi \\ \sin(t) & t \ge 2\pi \end{cases}$

→ syms s t

```
f = cos(t) * (heaviside (t - 0) - heaviside (t-pi)) +
        1 * (heaviside (t-pi) - heaviside (t-2*pi)) +
        sin(t) * (heaviside (t - 2*pi));
laplace (f, t, s)
```

(Q) $f(t) = \begin{cases} \sin t & 0 \le t < \pi \\ \sin 2t & \pi \le t < 2\pi \\ \sin 3t & t \ge 2\pi \end{cases}$

(Q) $f(t) = \begin{cases} e^{2t} & 0 < t < 1 \\ 2 & t > 1 \end{cases}$