

Laboratory 12: Sorting Algorithms part-2

CSC205A Data structures and Algorithms Laboratory B. Tech. 2015

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Introduction and Purpose of Experiment

- Sorting provide us with means of organising information to facilitate the retrieval of specific data.
- Searching methods are designed to take advantage of the organisation of information.
- By solving these problems students will be able to use sorting algorithms to sort a randomly ordered set of numbers, and search for key element.



Aim and objectives

Aim:

- To design and develop C programs to sort the given data using quick sort and merge sort, different sorting techniques

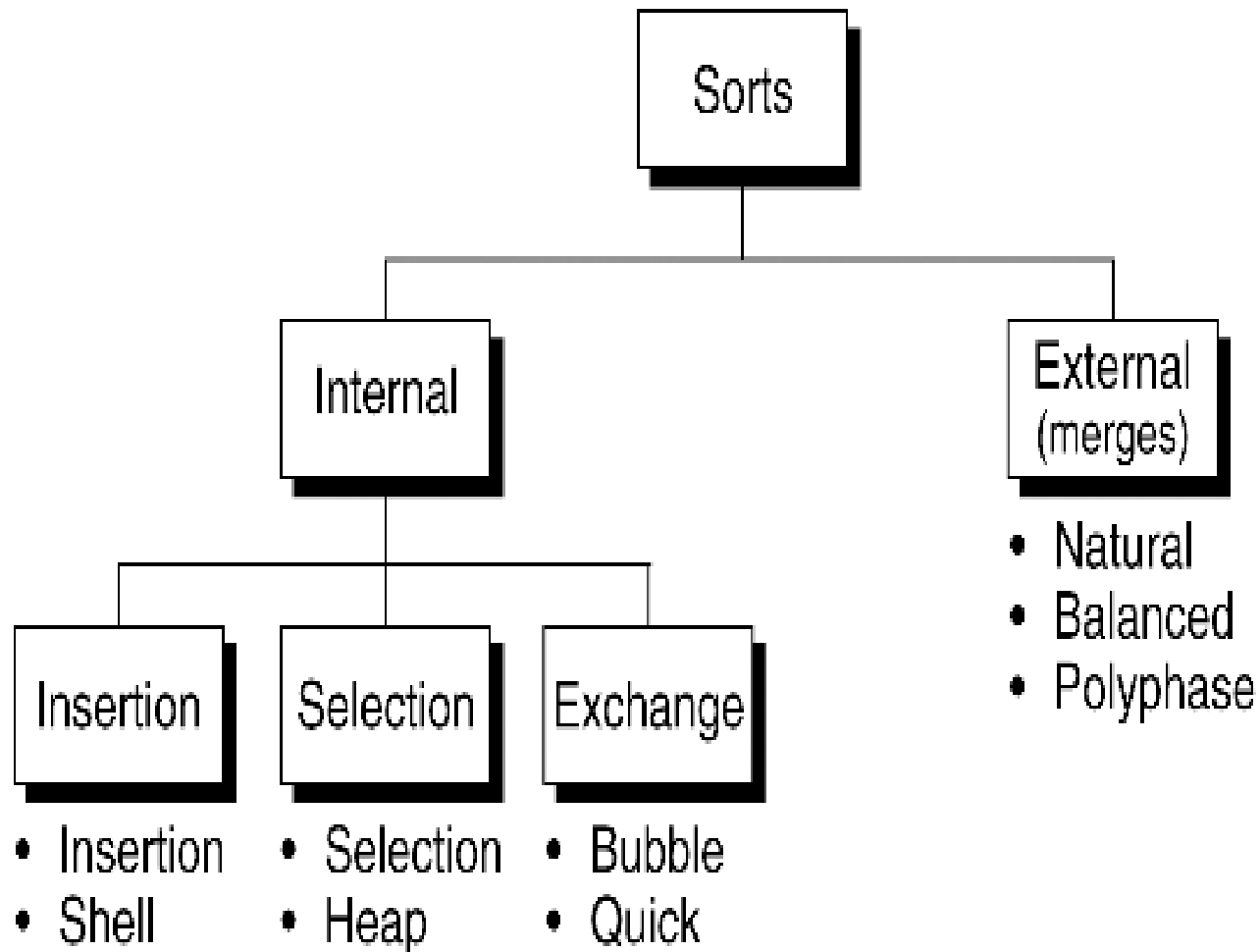
Objectives:

At the end of this lab, the student will be able to

- Create C programs using sorting algorithms such as **quick sort**
- Create C programs using sorting algorithms such as **merge sort**
- Analyse the efficiency of implemented sort algorithms



Sort Classifications



Bubble Sort



"Bubbling Up" the Largest Element

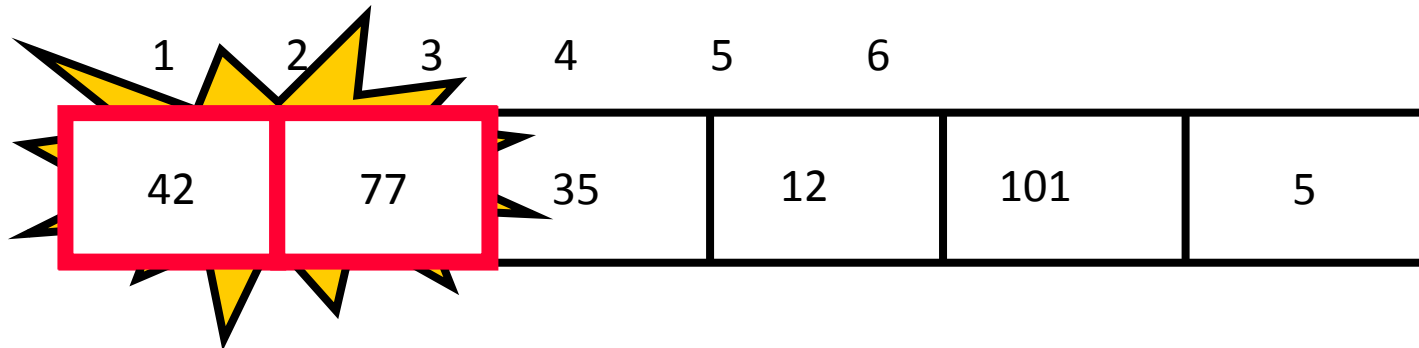
- **Traverse a collection of elements**
 - Move from the front to the end
 - “Bubble” the **largest value** to the end using **pair-wise comparisons and swapping**

| | | | | | |
|----|----|----|----|-----|---|
| 1 | 2 | 3 | 4 | 5 | 6 |
| 77 | 42 | 35 | 12 | 101 | 5 |



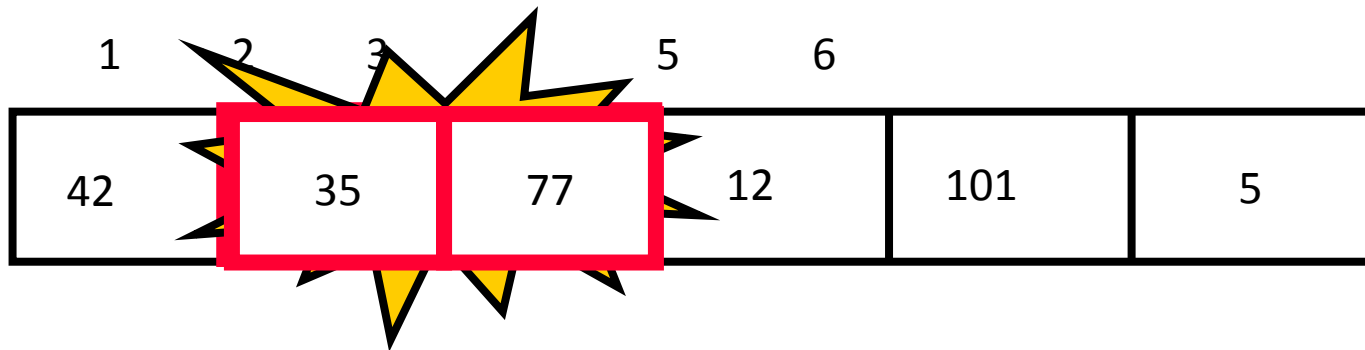
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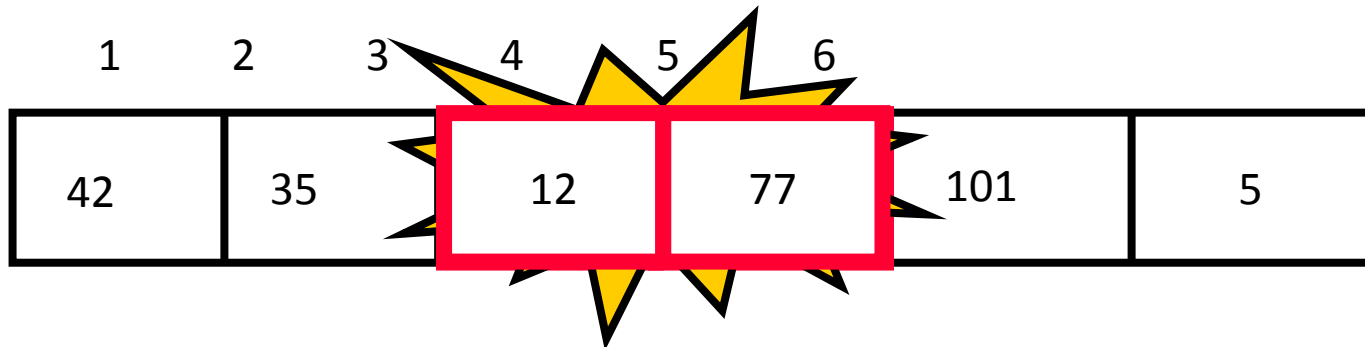
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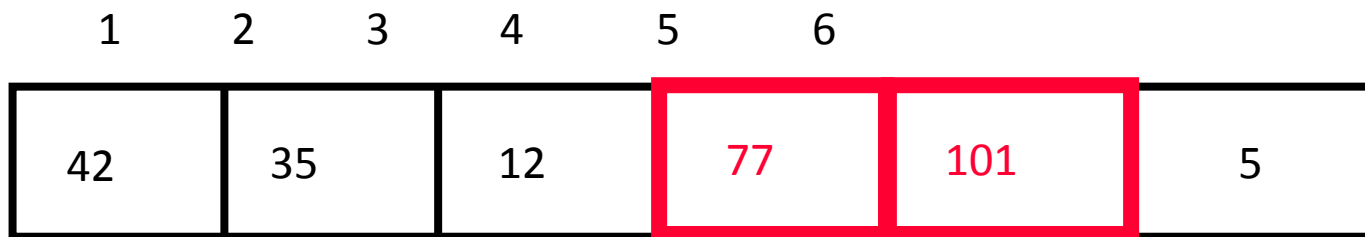
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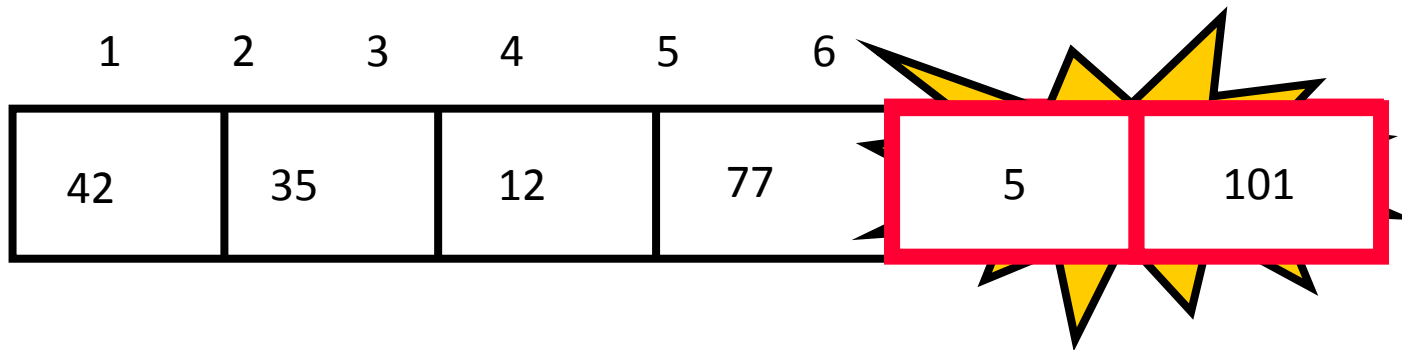


No need to swap



"Bubbling Up" the Largest Element

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| | | | | | |
|----|----|----|----|---|-----|
| 1 | 2 | 3 | 4 | 5 | 6 |
| 42 | 35 | 12 | 77 | 5 | 101 |

Largest value correctly placed



The “Bubble Up” Algorithm

```
index <- 1
```

```
last_compare_at <- n - 1
```

```
loop
```

```
  exitif(index > last_compare_at)
```

```
  if(A[index] > A[index + 1]) then
```

```
    Swap(A[index], A[index + 1])
```

```
  endif
```

```
  index <- index + 1
```

```
endloop
```



No, Swap isn't built in.

```
Procedure Swap(a, b isoftype in/out  
  Num)
```

```
  t isoftype Num
```

```
  t <- a
```

```
  a <- b
```

```
  b <- t
```

```
endprocedure // Swap
```



Items of Interest

- Notice that only the largest value is correctly placed
- All other values are still out of order
- So we need to **repeat this process**

| | | | | | |
|----|----|----|----|---|-----|
| 1 | 2 | 3 | 4 | 5 | 6 |
| 42 | 35 | 12 | 77 | 5 | 101 |

Largest value correctly placed

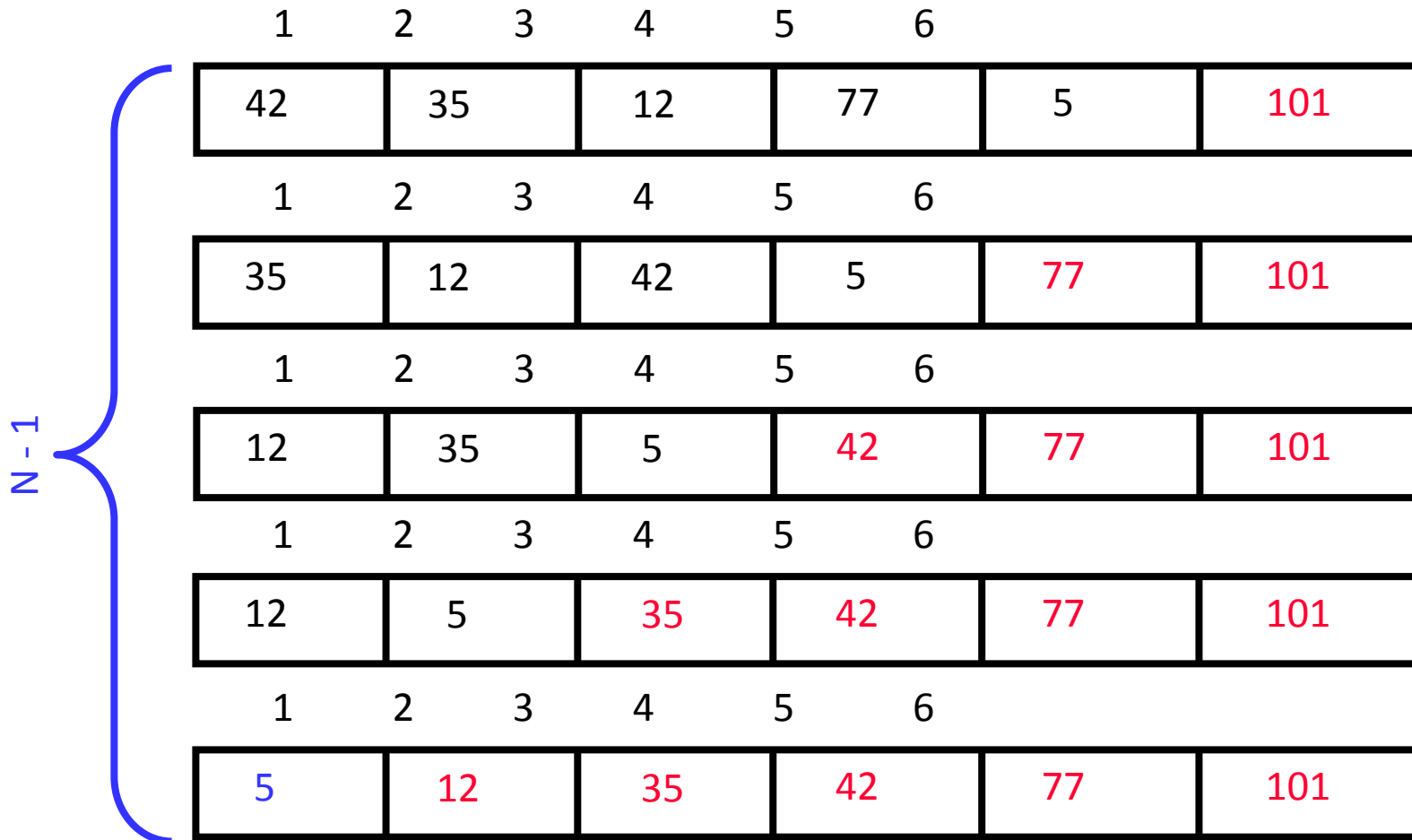


Repeat “Bubble Up” How Many Times?

- If we have N elements...
- And if each time we bubble an element, we place it in its correct location...
- Then we **repeat the “bubble up” process $N - 1$ times.**
- This **guarantees we’ll correctly place all N elements.**



“Bubbling” All the Elements



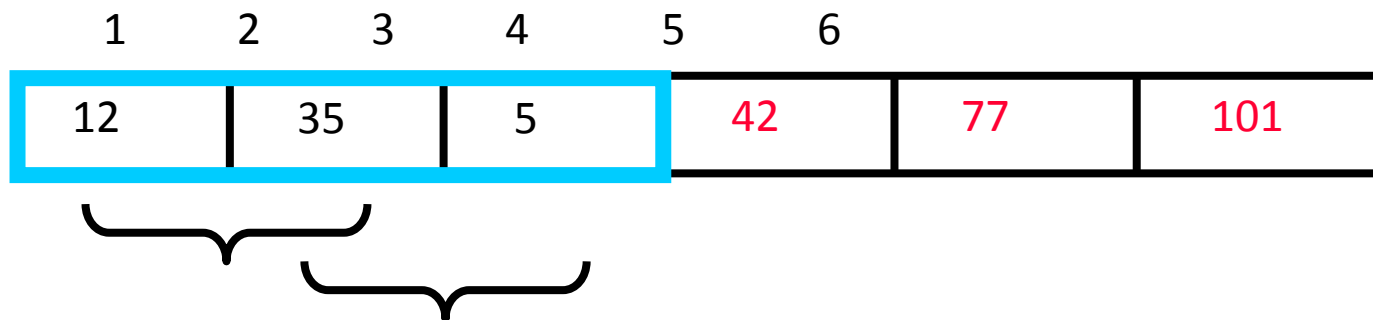
Reducing the Number of Comparisons

| | | | | | |
|----|----|----|----|-----|-----|
| 1 | 2 | 3 | 4 | 5 | 6 |
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Reducing the Number of Comparisons

- On the N^{th} “bubble up”, we only need to do **MAX-N comparisons**.
- For example:
 - This is the 4th “bubble up”
 - MAX is 6
 - Thus we have **2 comparisons** to do



Putting It All Together



```
N is ... // Size of Array
```

```
Arr_Type defines a Array[1..N] of Num
```

```
Procedure Swap(n1, n2 is of type int Num)
```

```
    temp is of type Num
```

```
    temp <- n1
```





```
    n1    <- n2
```

```
    n2    <- temp
```

```
end procedure // Swap
```



```
procedure Bubblesort(A is of type int Arr_Type)
  to_do, index is of type Num
  to_do <- N - 1
```

```
  loop ← 
    exitif(to_do = 0)
    index <- 1
    loop ← 
      exitif(index > to_do)
      if(A[index] > A[index + 1]) then
        Swap(A[index], A[index + 1])
      endif
      index <- index + 1
    endloop ← 
    to_do <- to_do - 1
  endloop ← 
```

Inner loop

Outer loop

```
end procedure // Bubblesort
```



Already Sorted Collections?

- What if the collection was already sorted?
- What if only a few elements were out of place and after a couple of “bubble ups,” the collection was sorted?
- We want to be able to **detect this** and “stop early”!

| | | | | | |
|---|----|----|----|----|-----|
| 1 | 2 | 3 | 4 | 5 | 6 |
| 5 | 12 | 35 | 42 | 77 | 101 |



Using a Boolean “Flag”

- We can use a boolean variable to determine if any swapping occurred during the “bubble up.”
- If no swapping occurred, then we know that the collection is already sorted!
- This boolean “flag” needs to be reset after each “bubble up.”

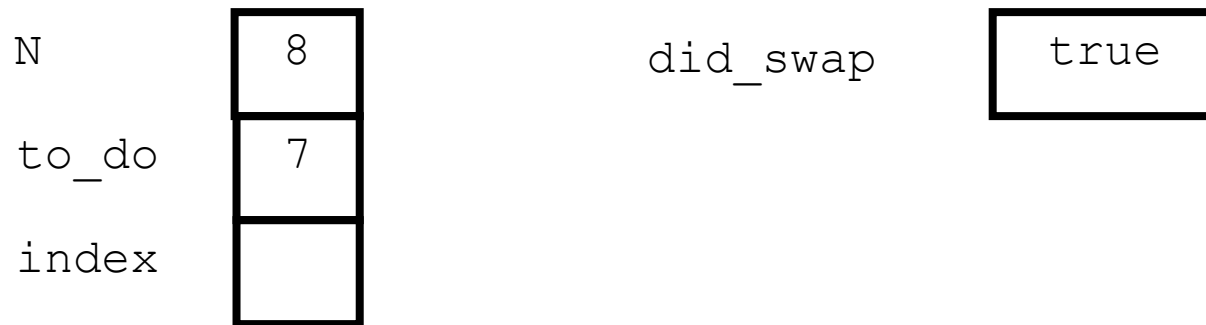



```
did_swap isotype Boolean
did_swap <- true
```

```
loop
  exitif ((to_do = 0) OR NOT(did_swap))
  index <- 1
  did_swap <- false
  loop
    exitif(index > to_do)
    if(A[index] > A[index + 1]) then
      Swap(A[index], A[index + 1])
      did_swap <- true
    endif
    index <- index + 1
  endloop
  to_do <- to_do - 1
endloop
```



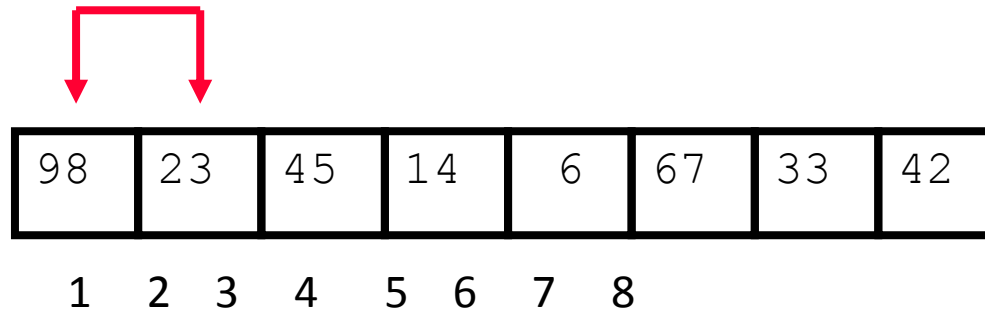
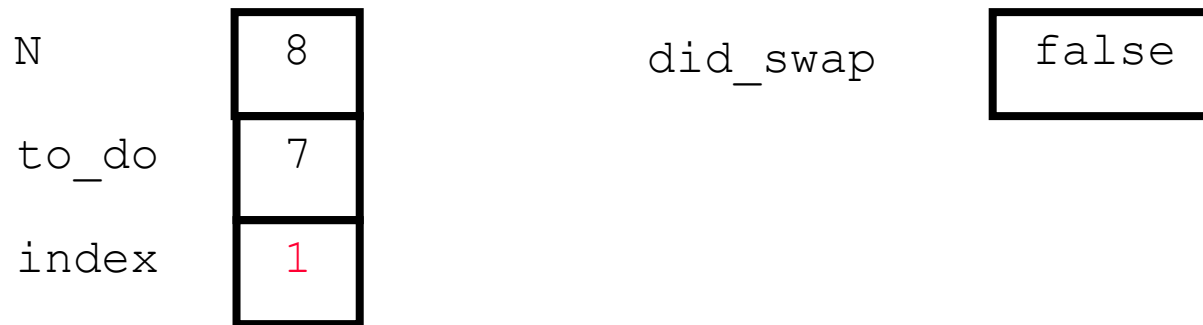
An Animated Example



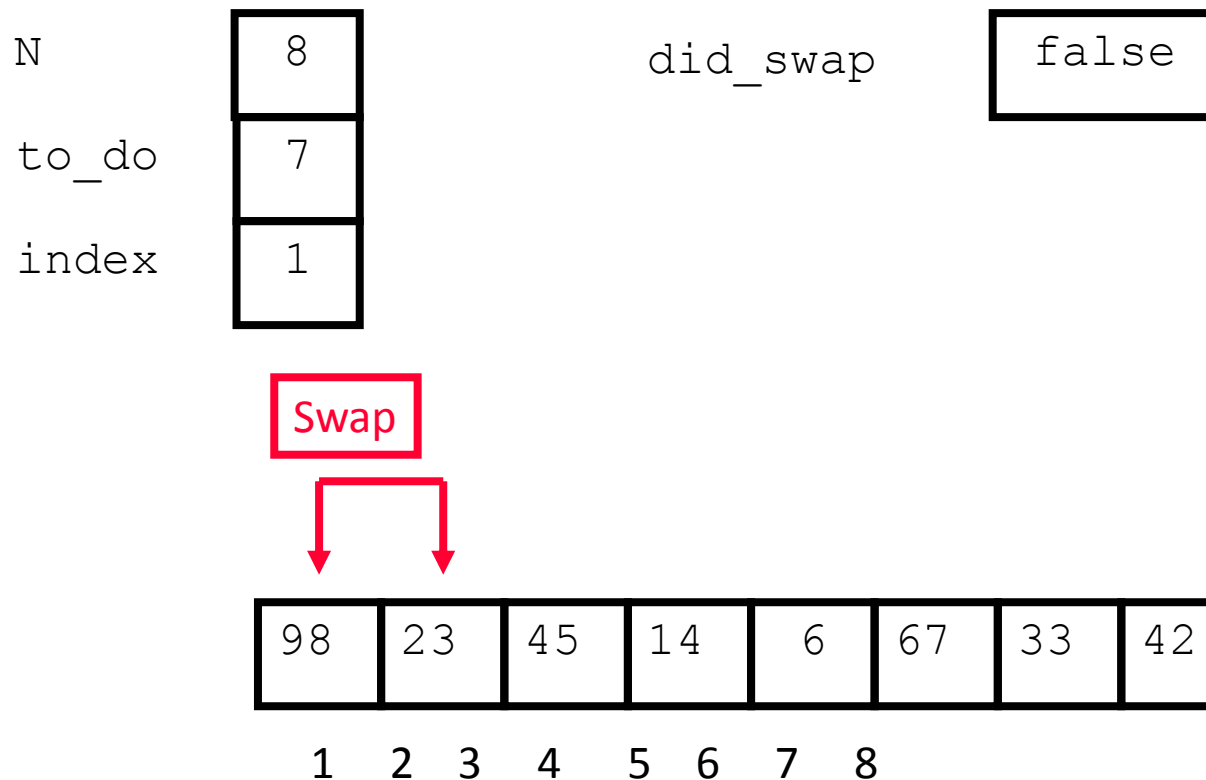
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|----|----|----|----|---|----|----|----|
| 98 | 23 | 45 | 14 | 6 | 67 | 33 | 42 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |



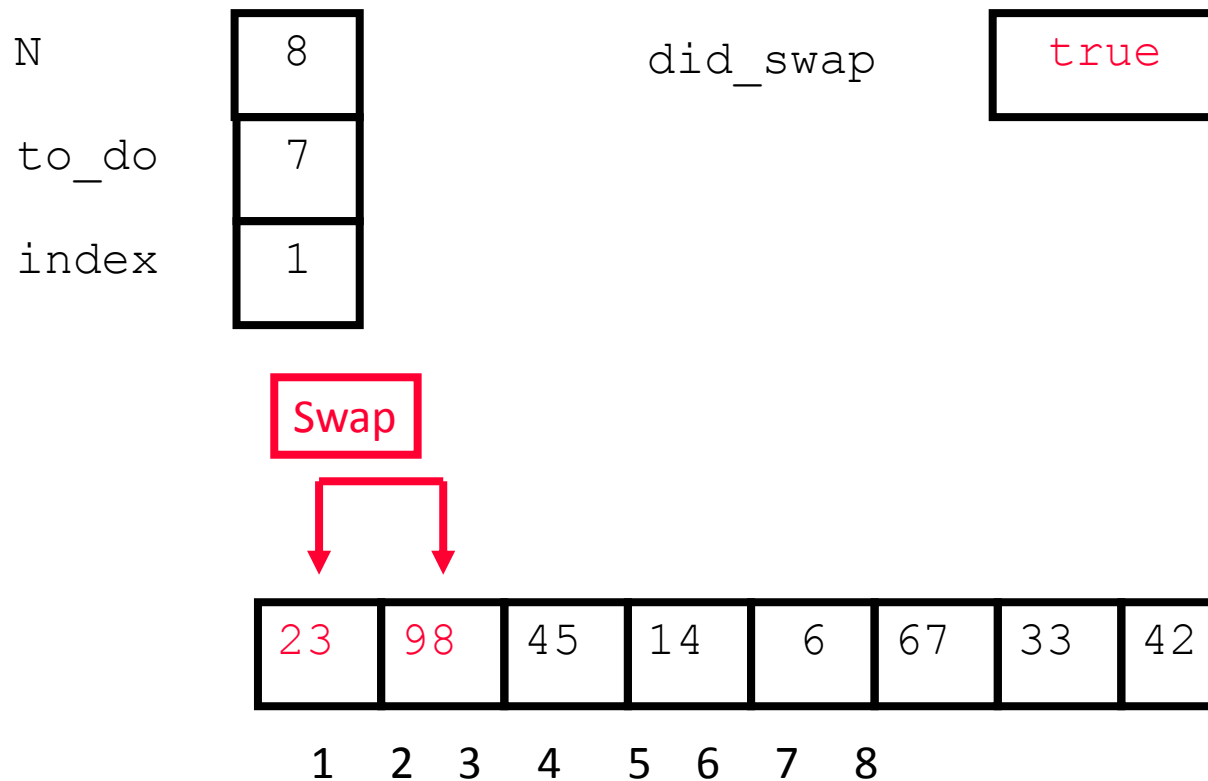
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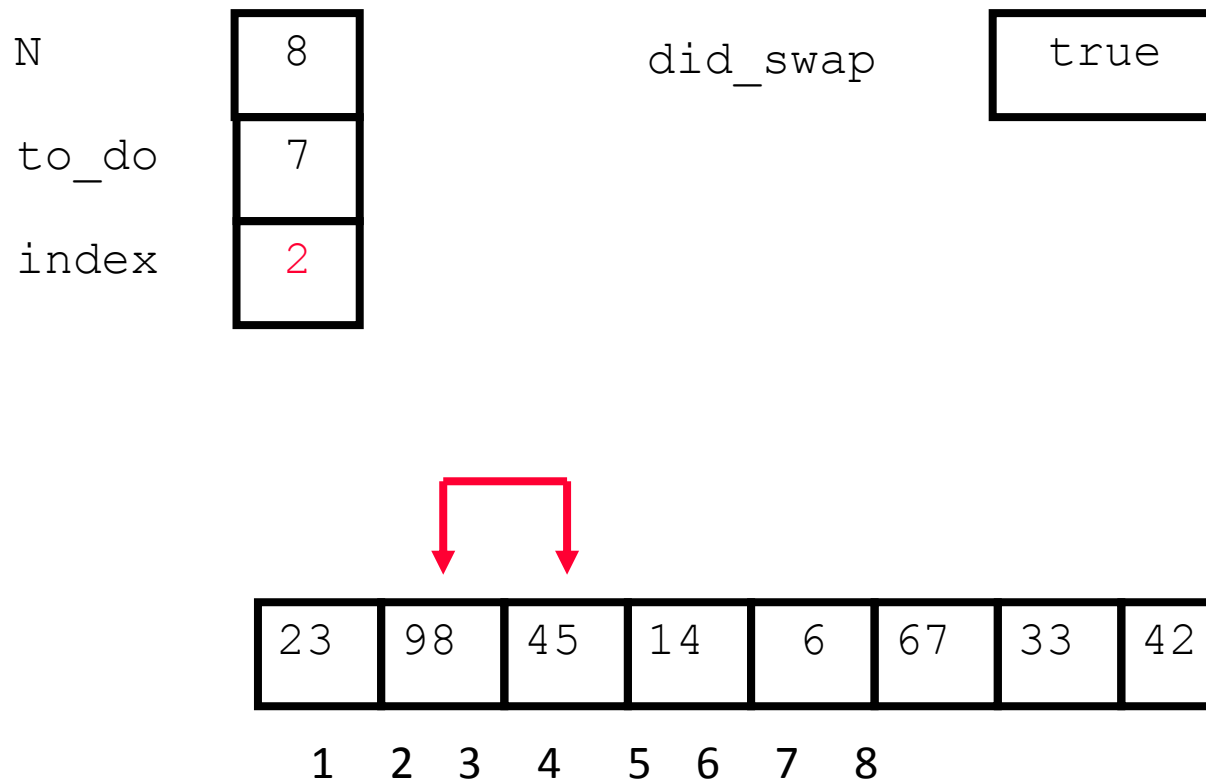
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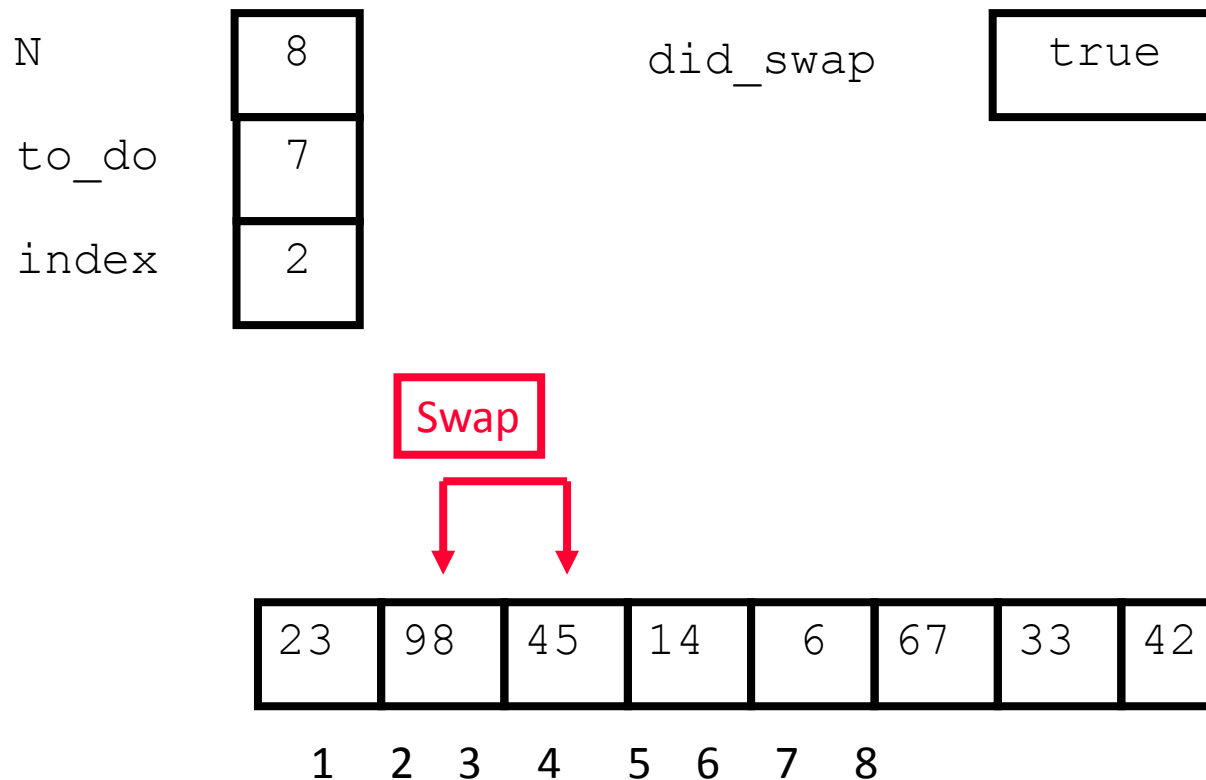
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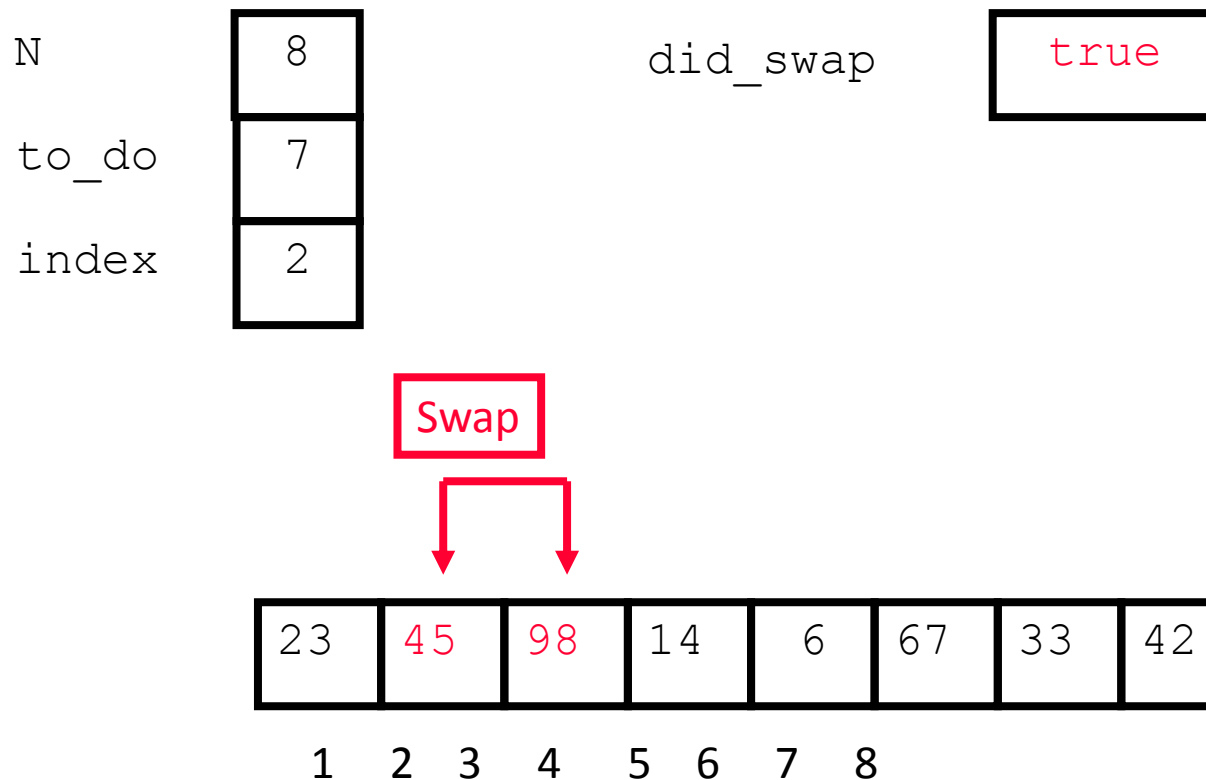
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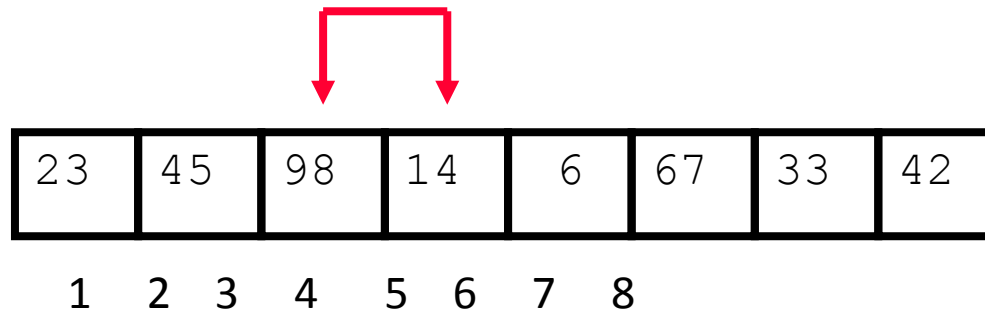
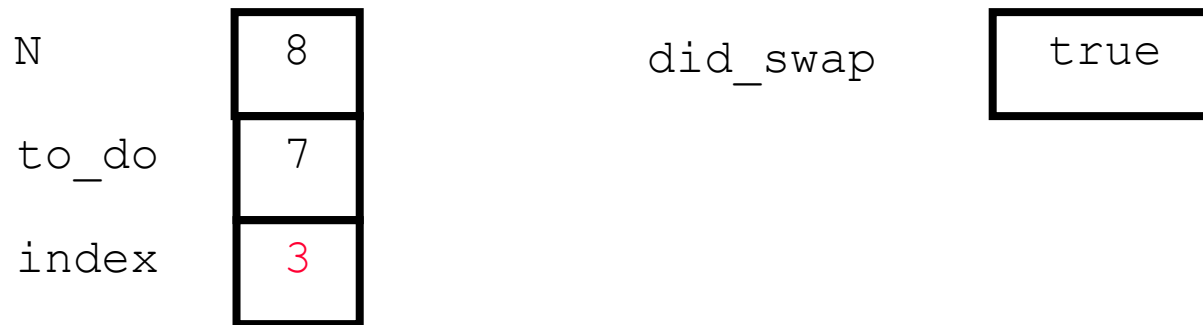
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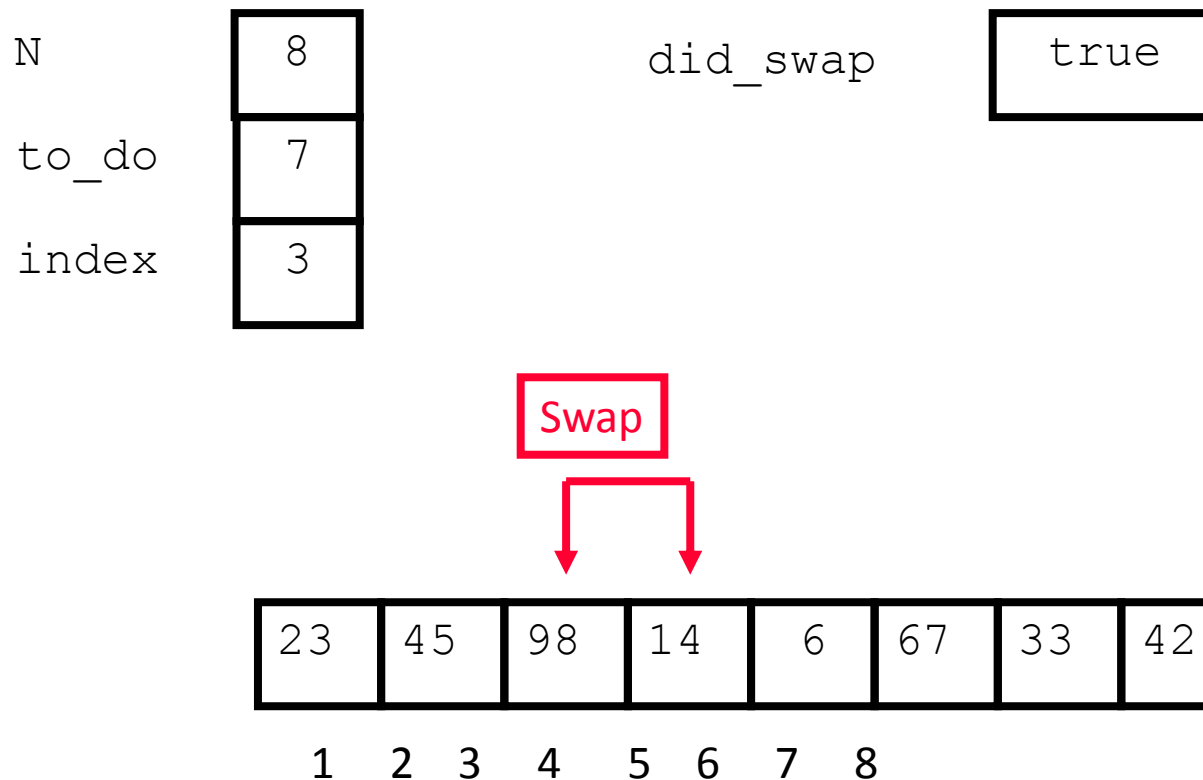
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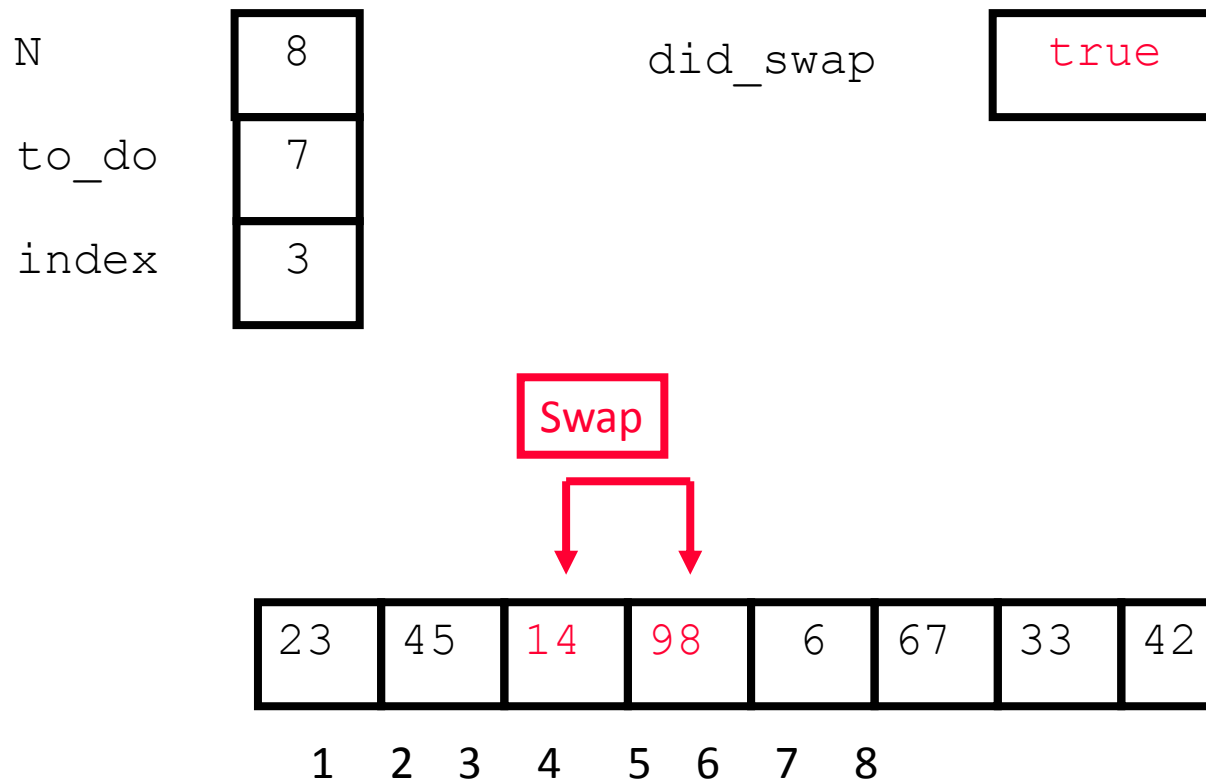
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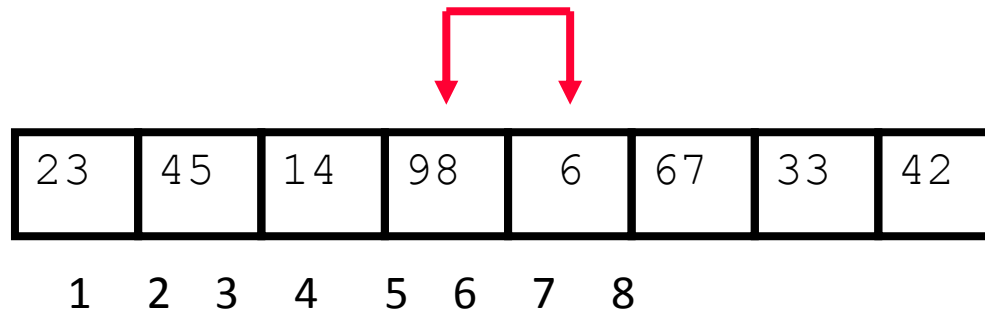
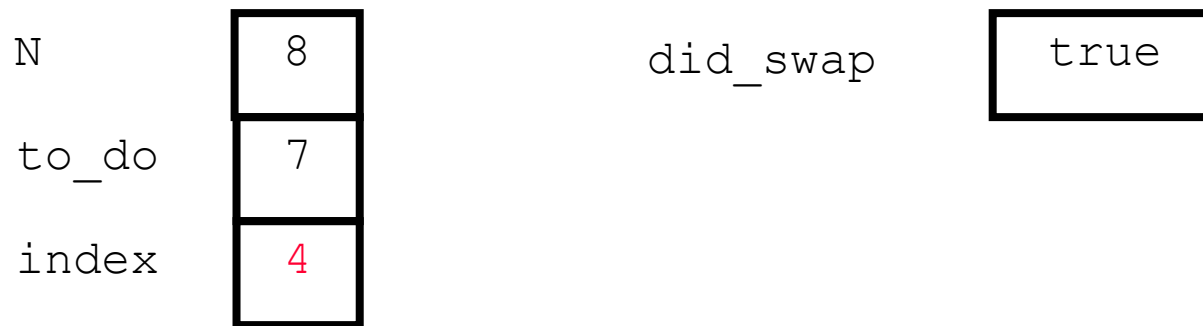
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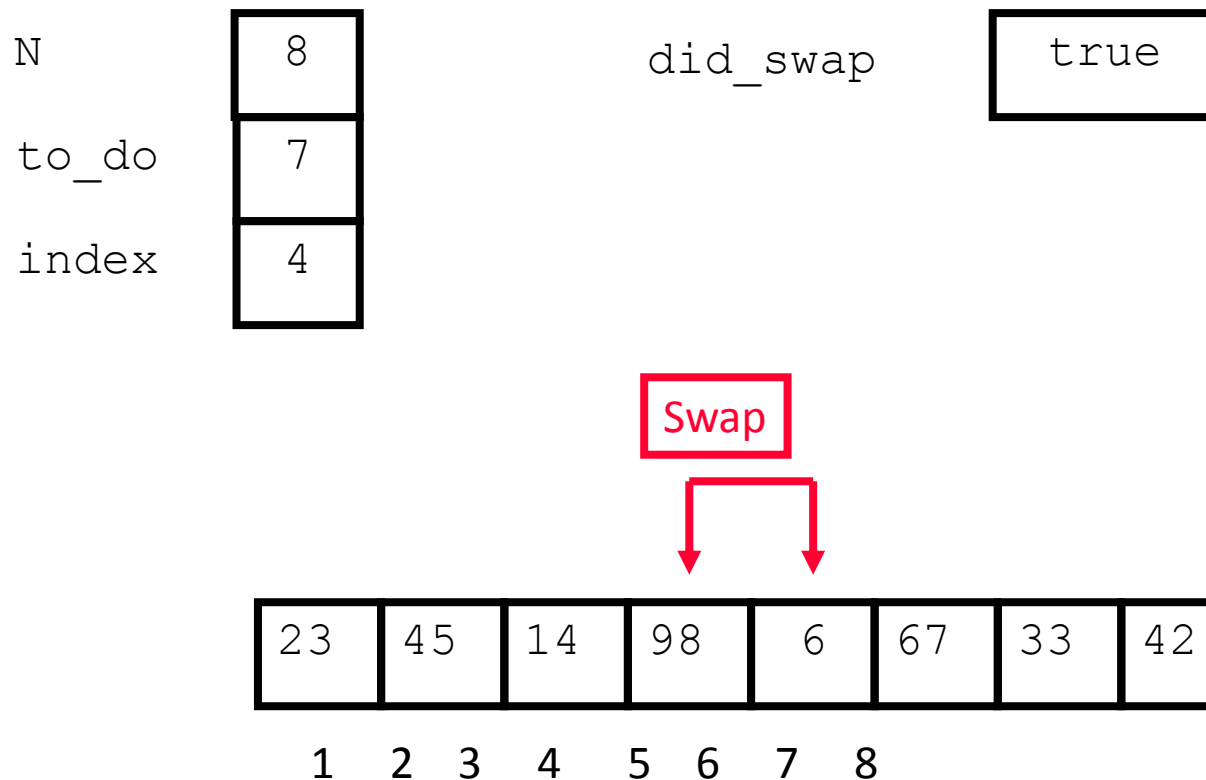
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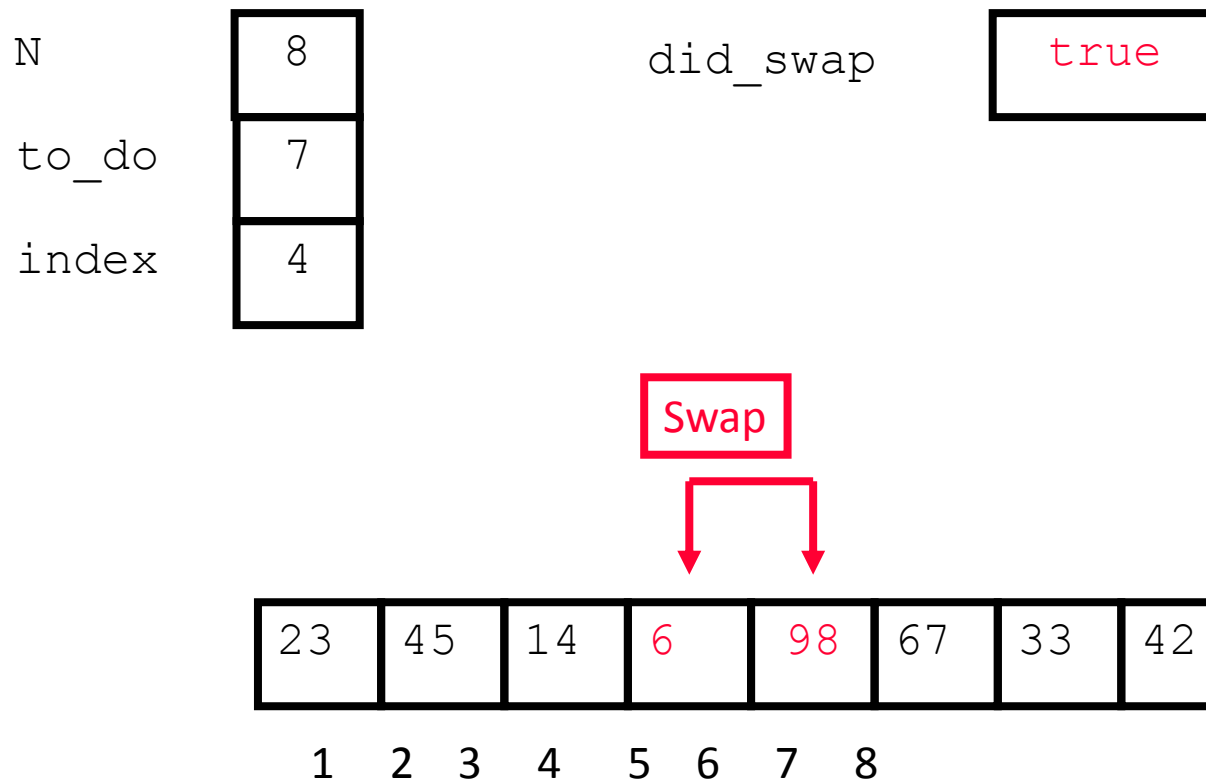
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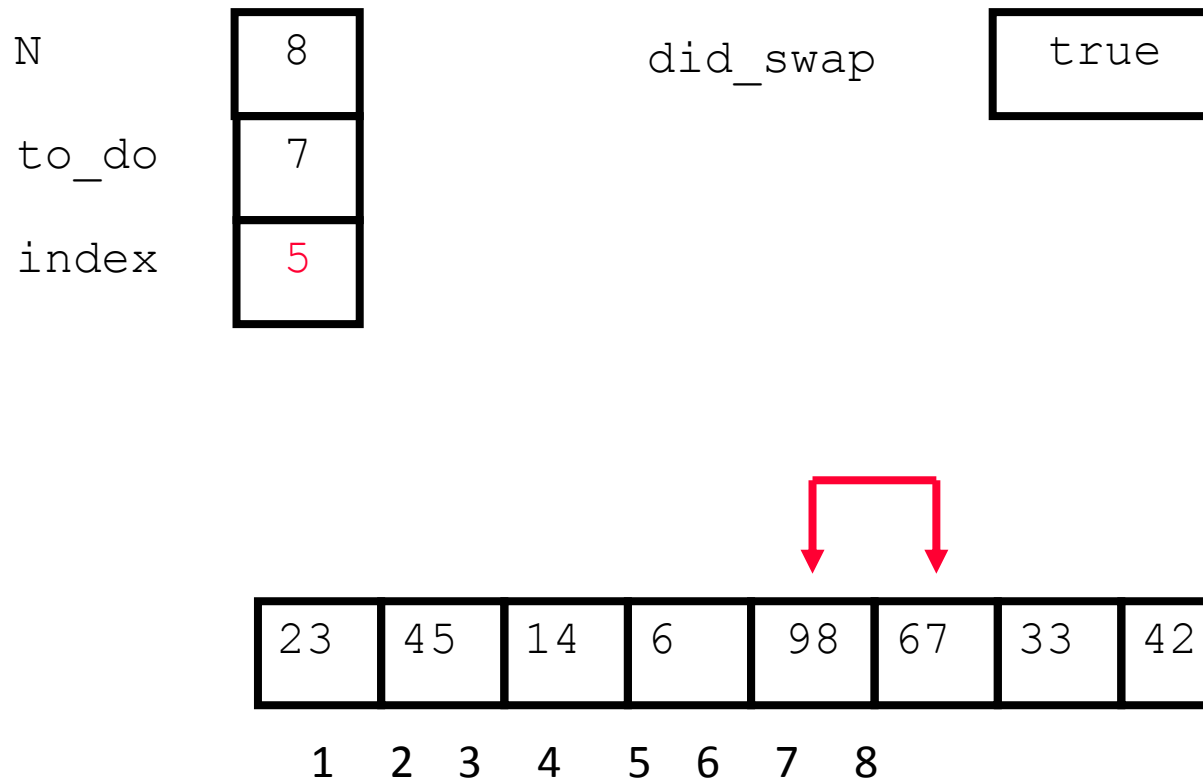
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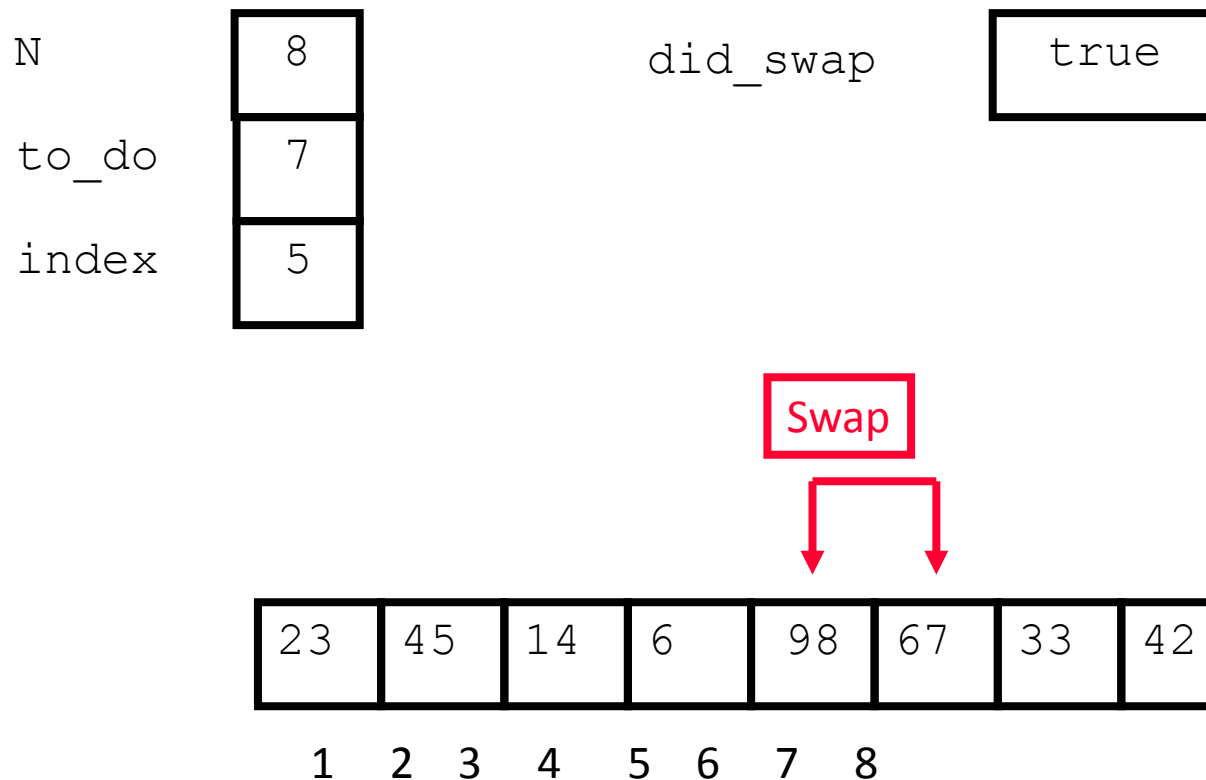
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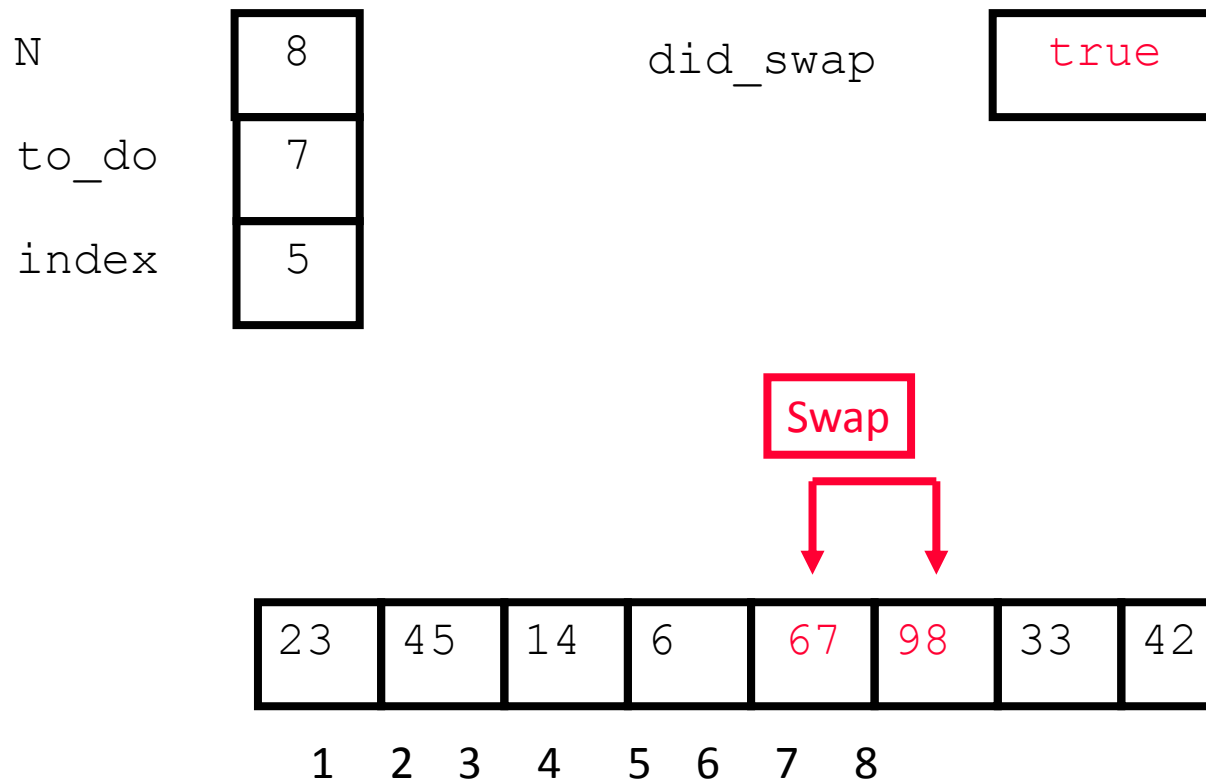
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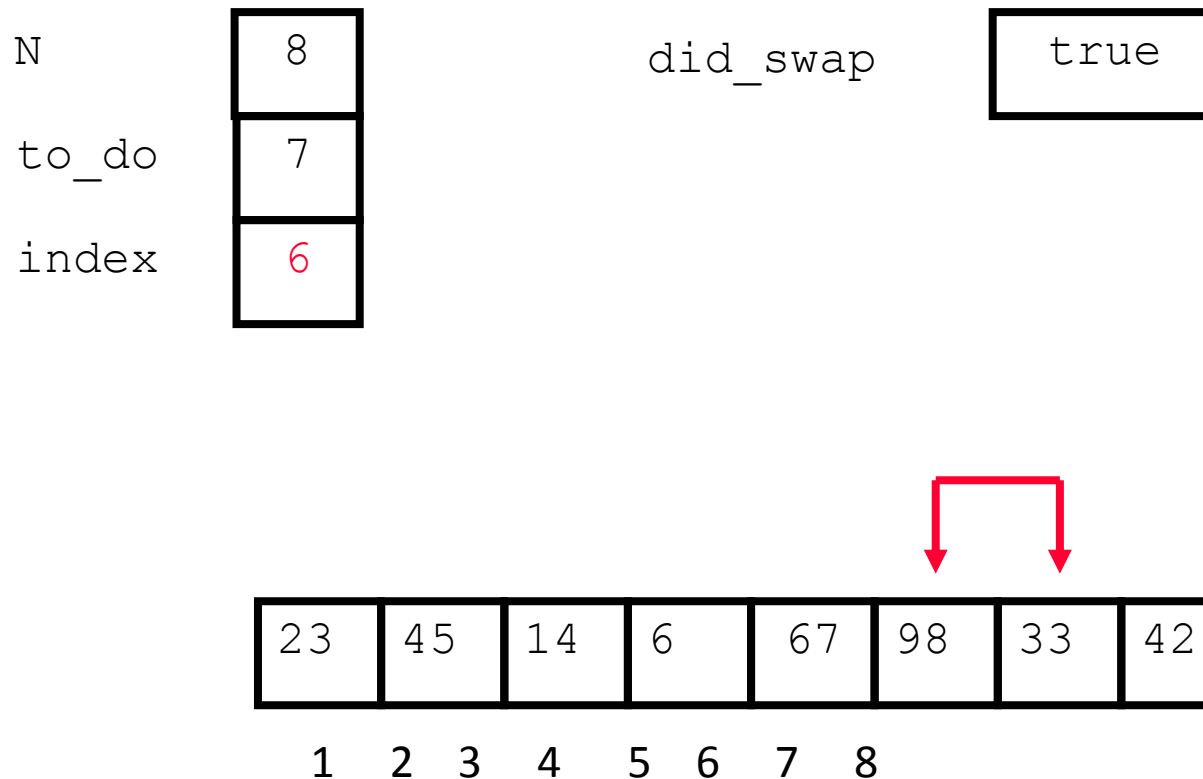
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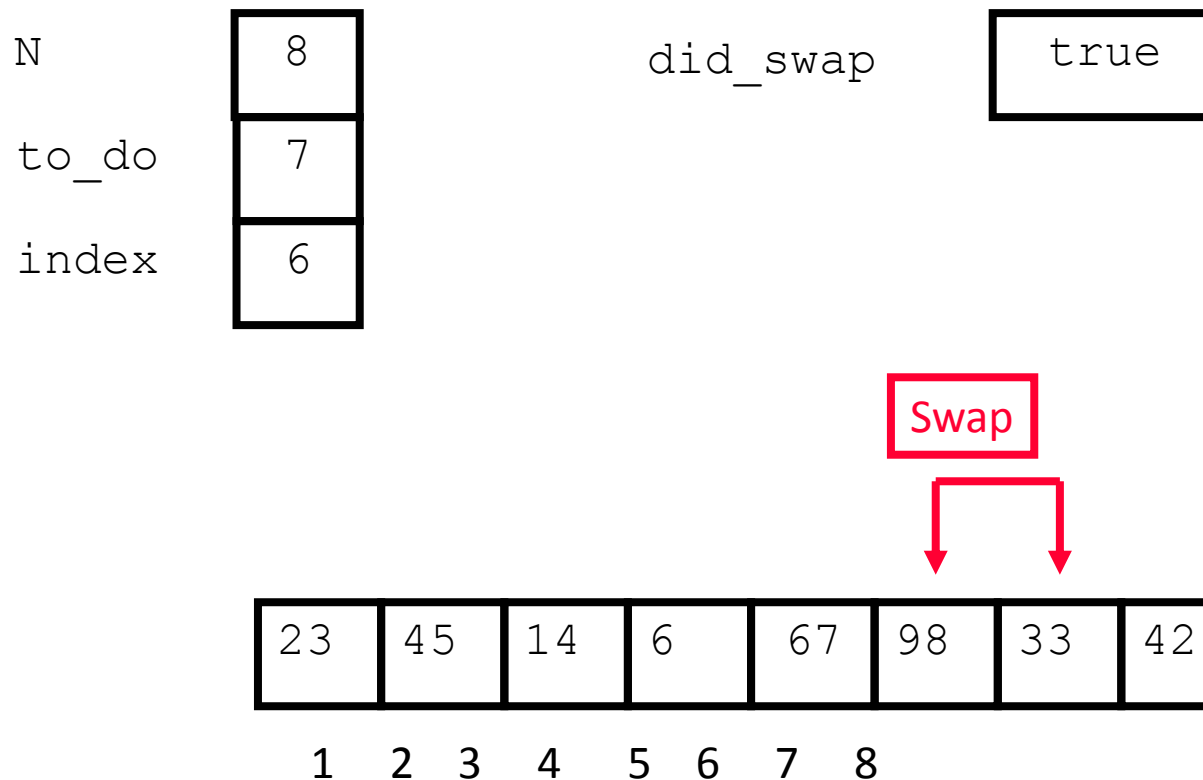
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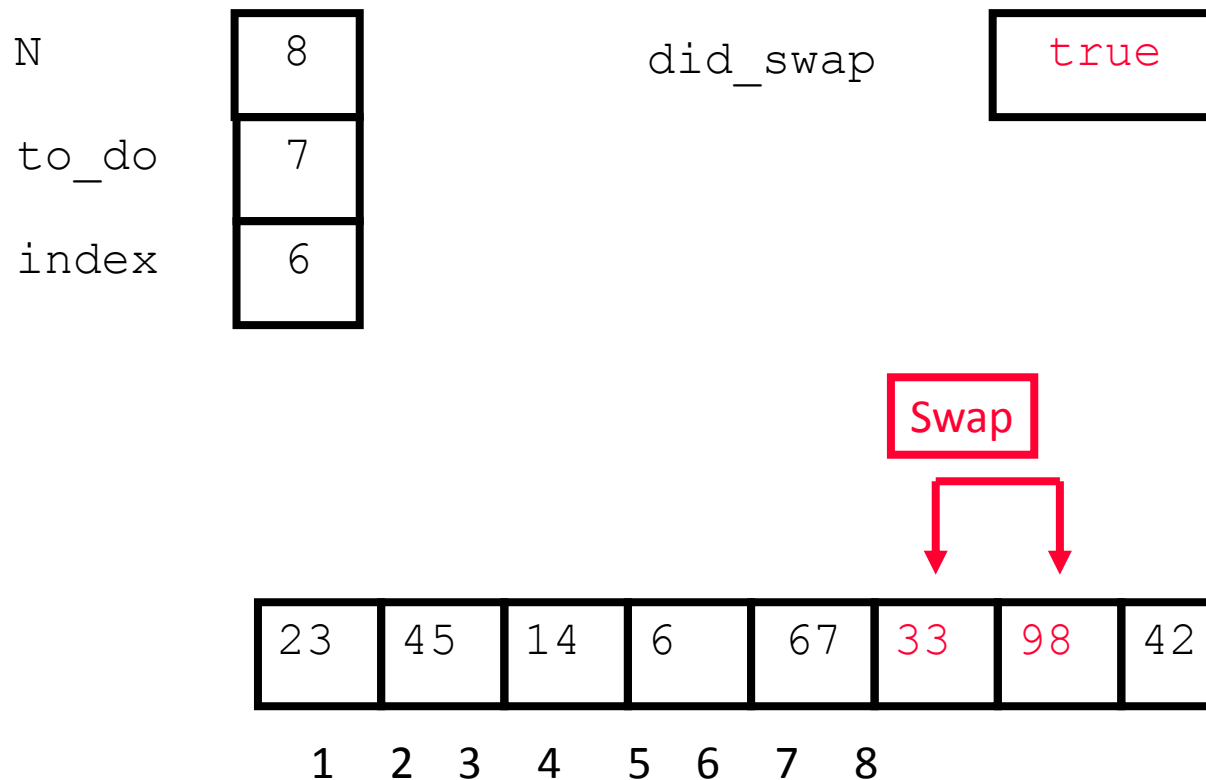
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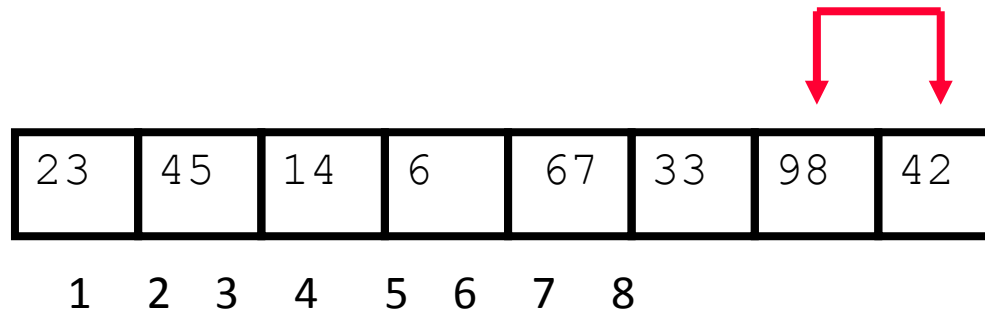
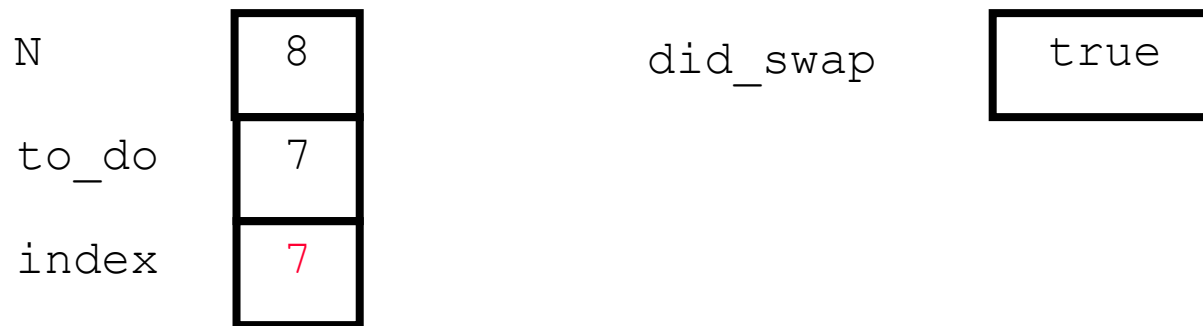
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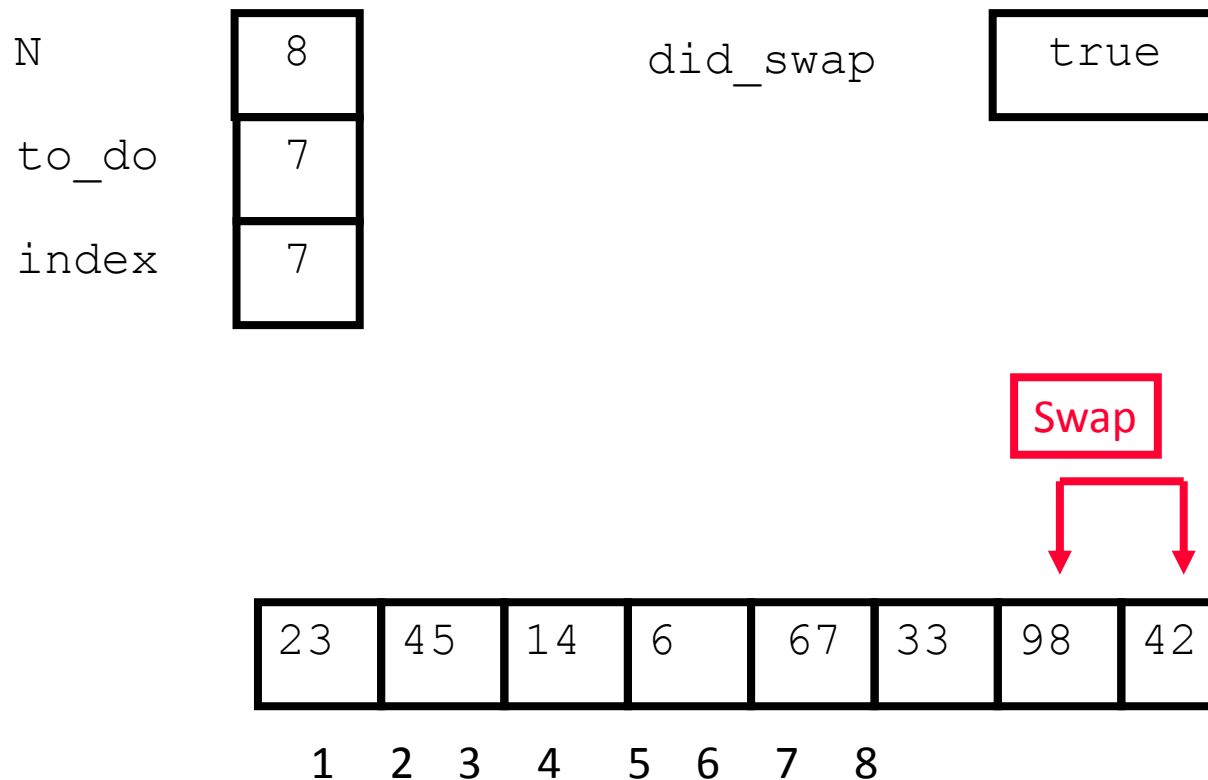
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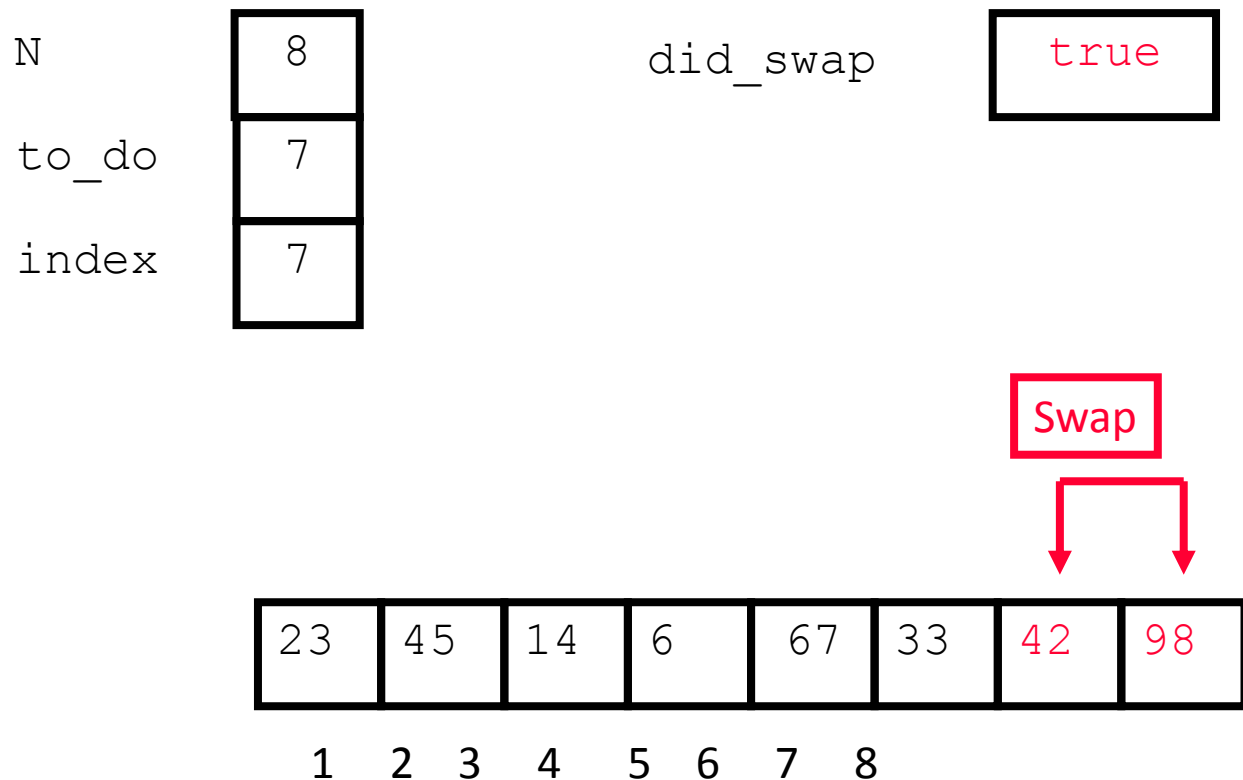
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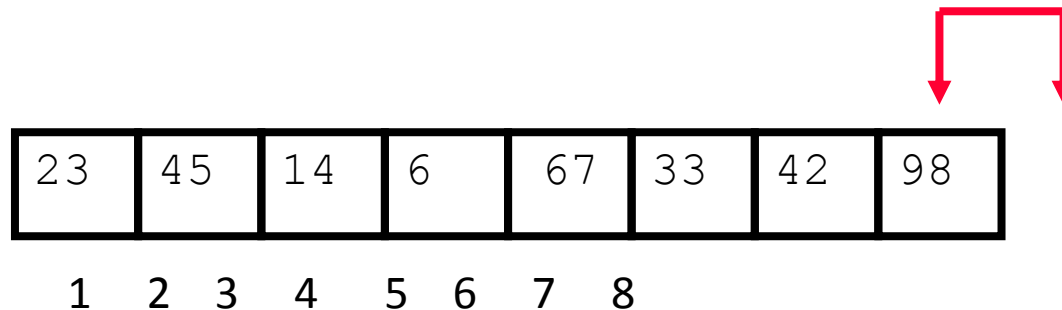
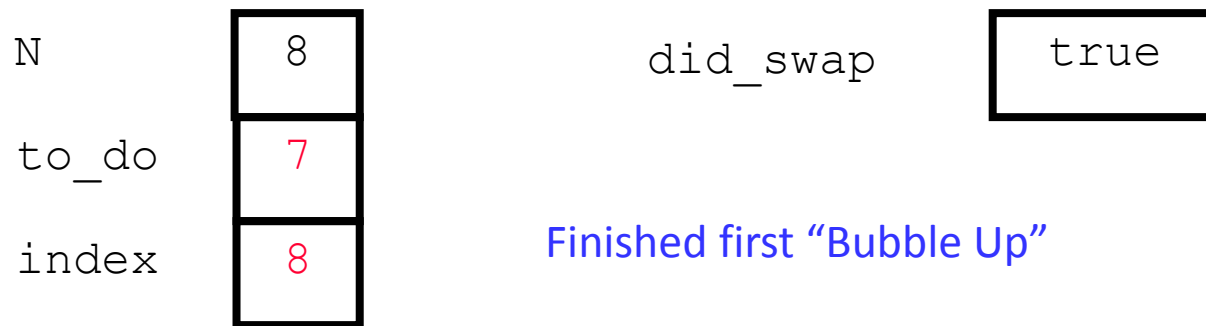
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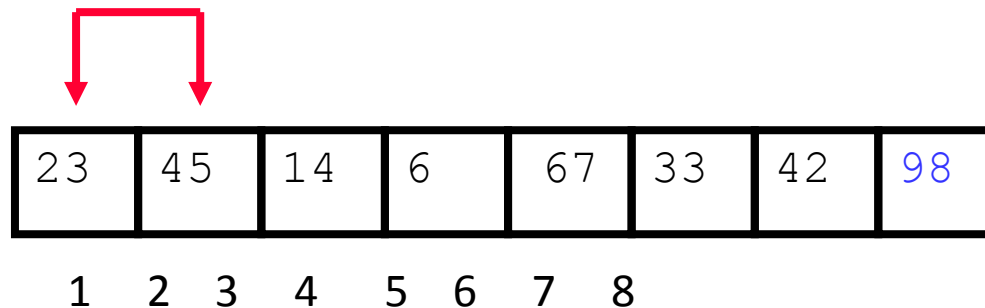
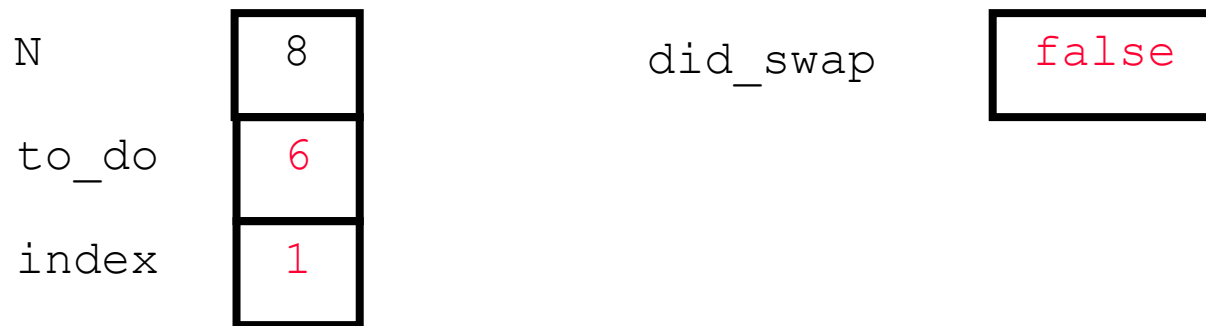
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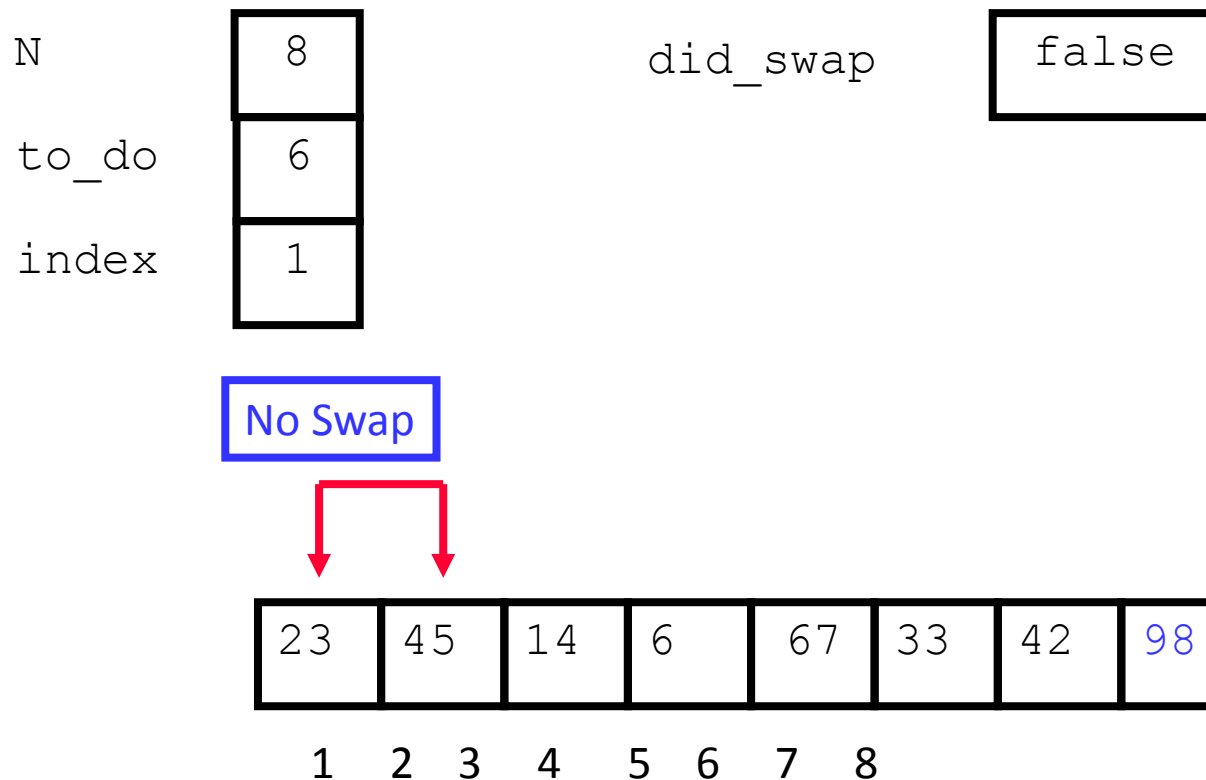
After First Pass of Outer Loop



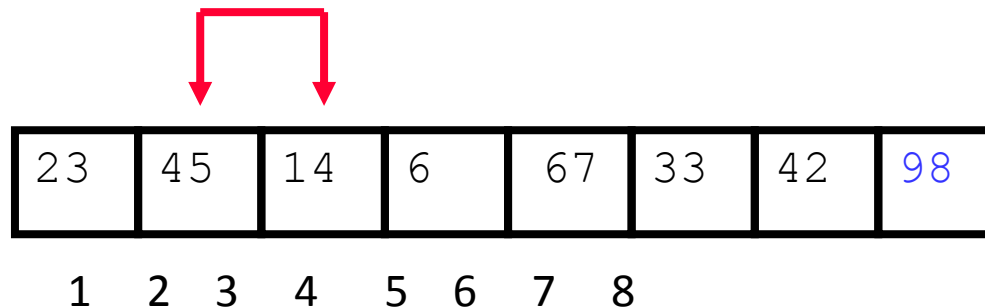
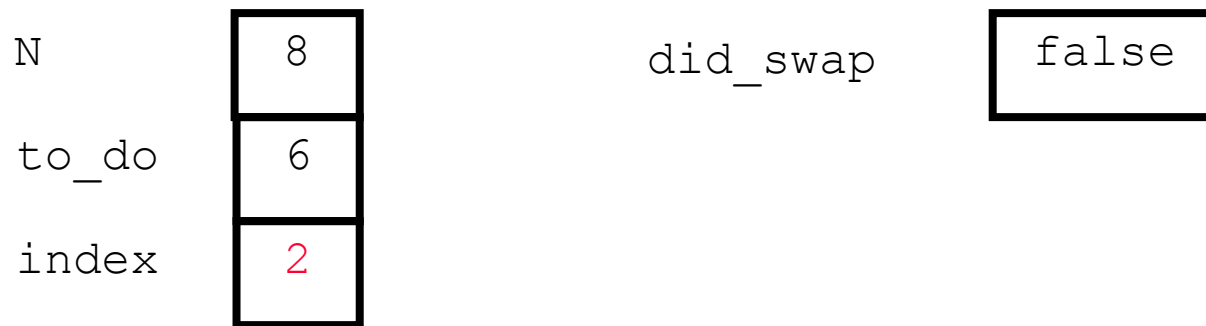
The Second “Bubble Up”



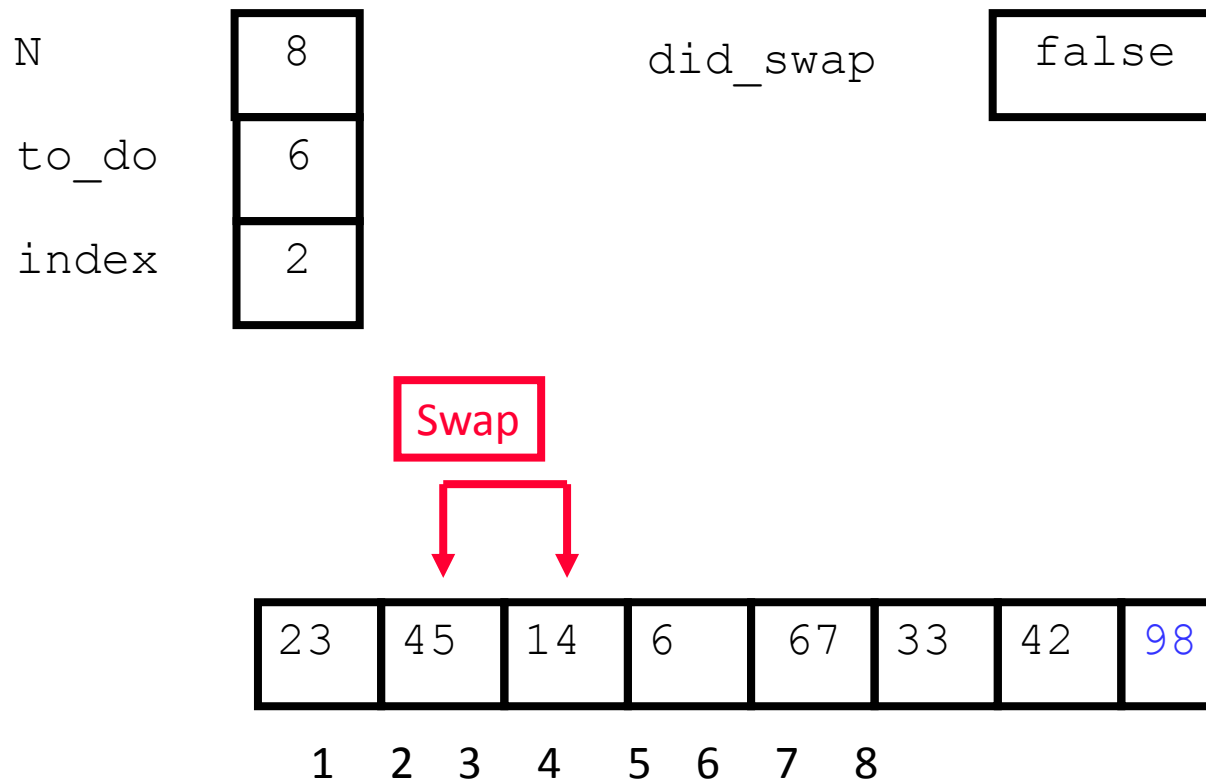
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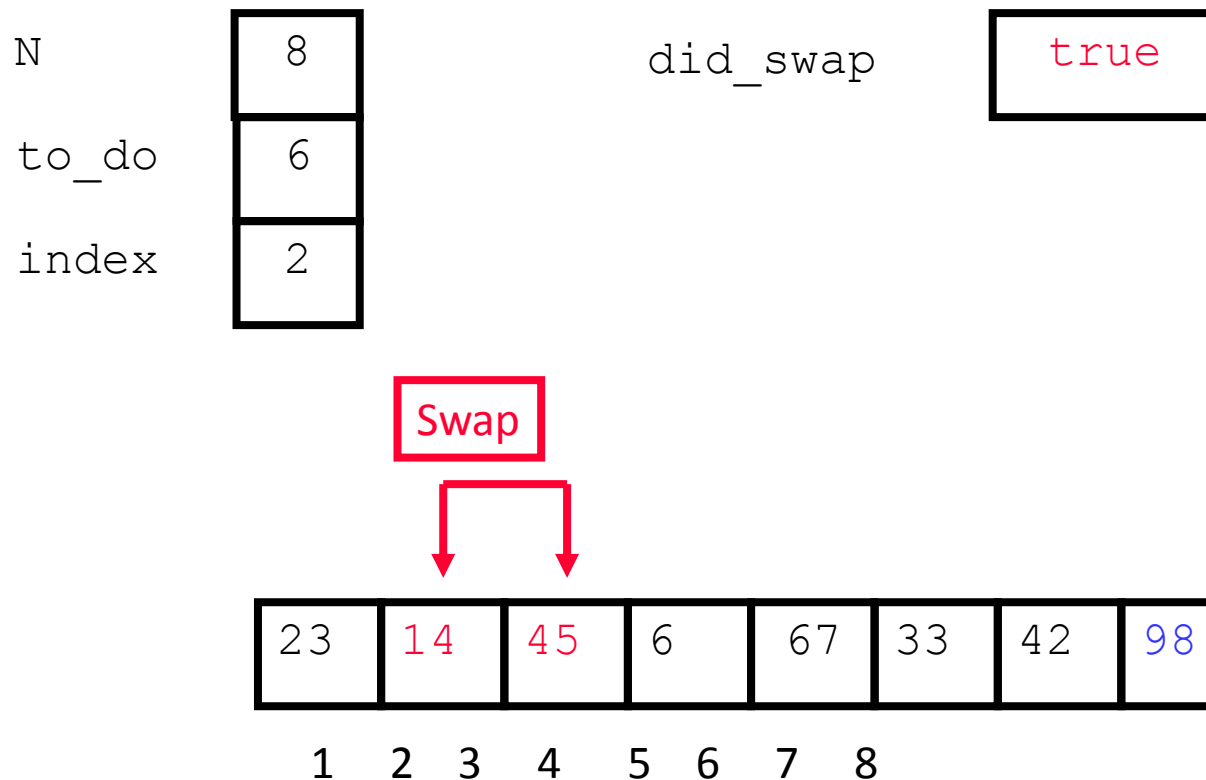
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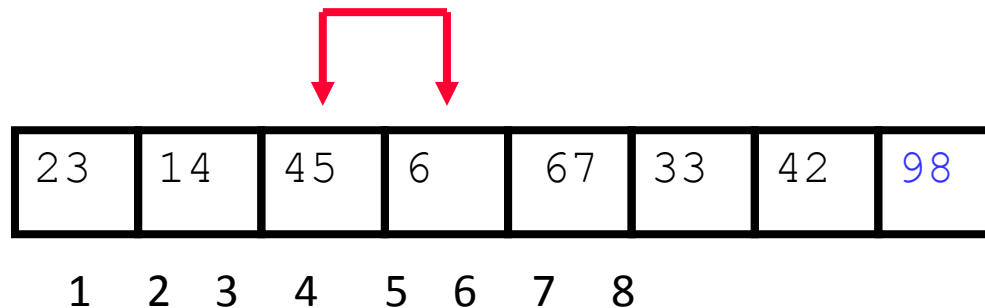
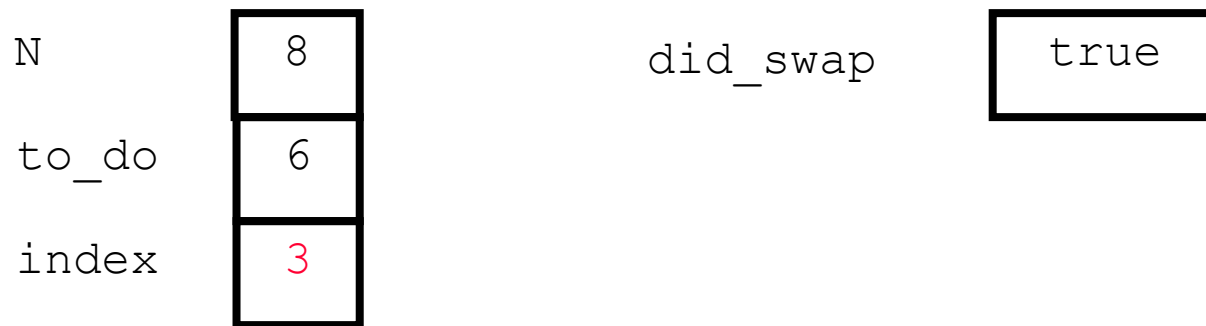
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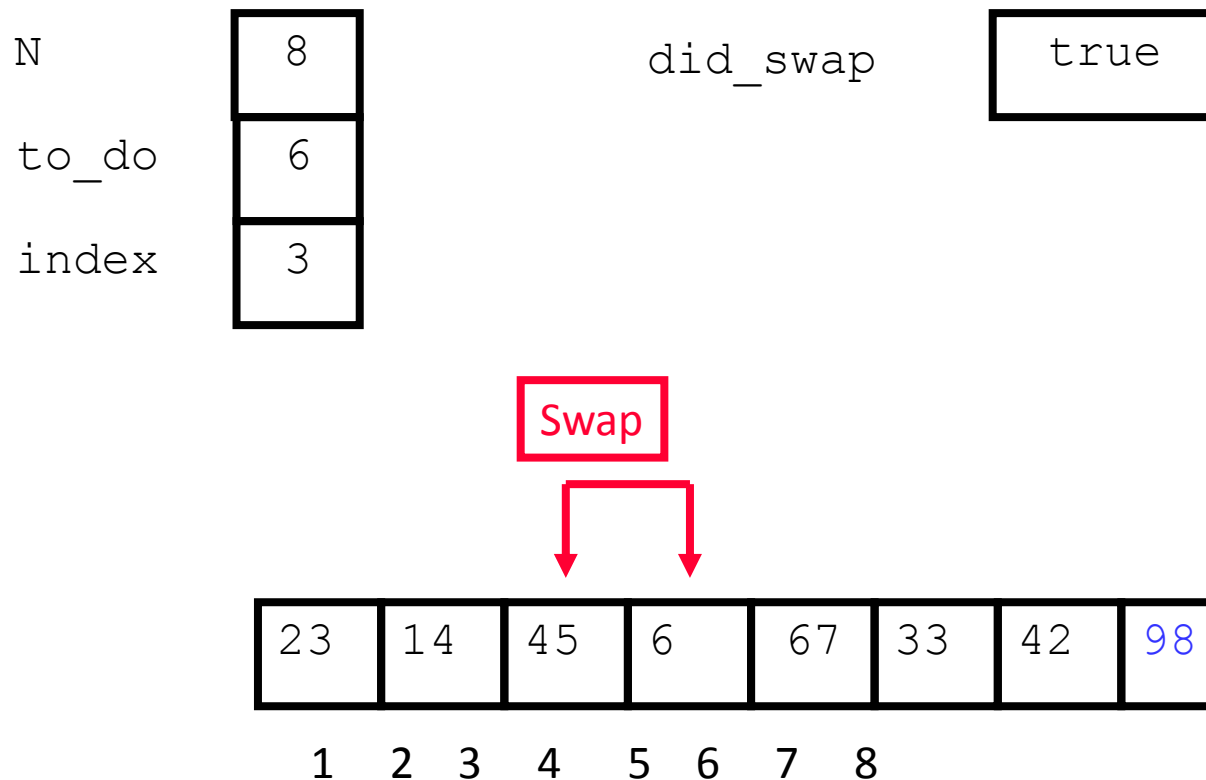
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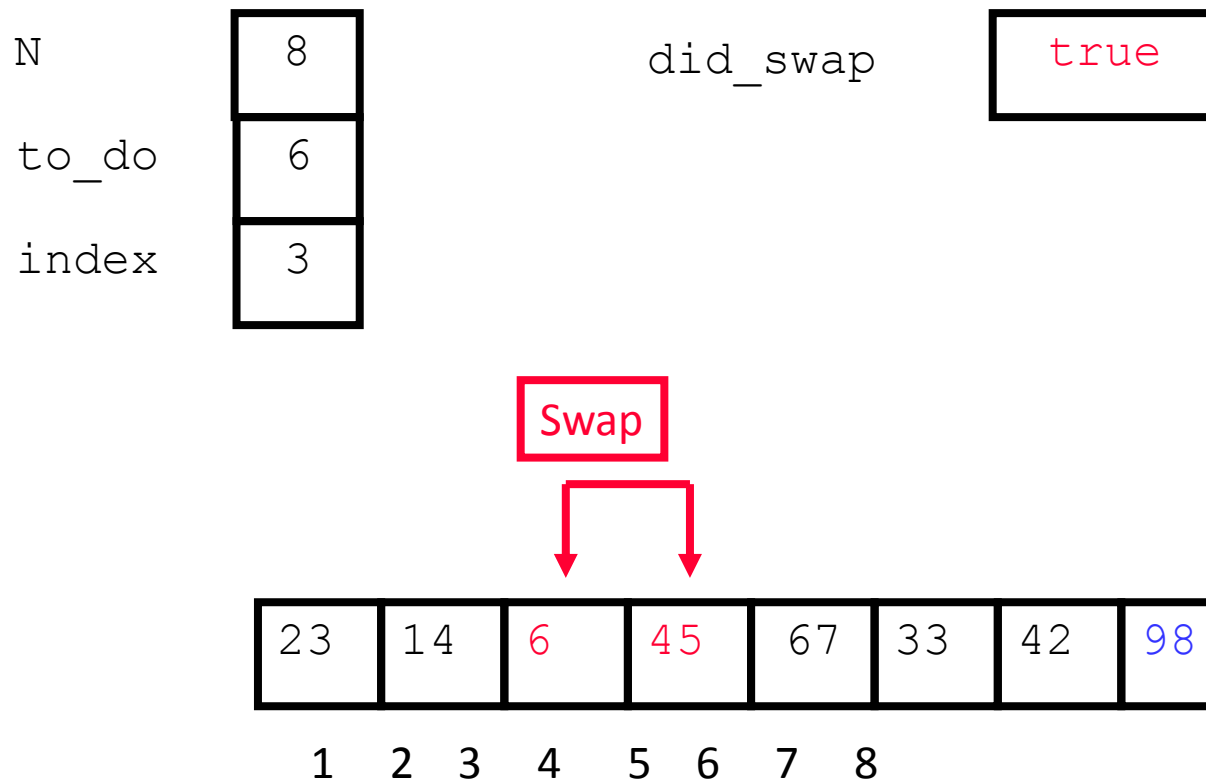
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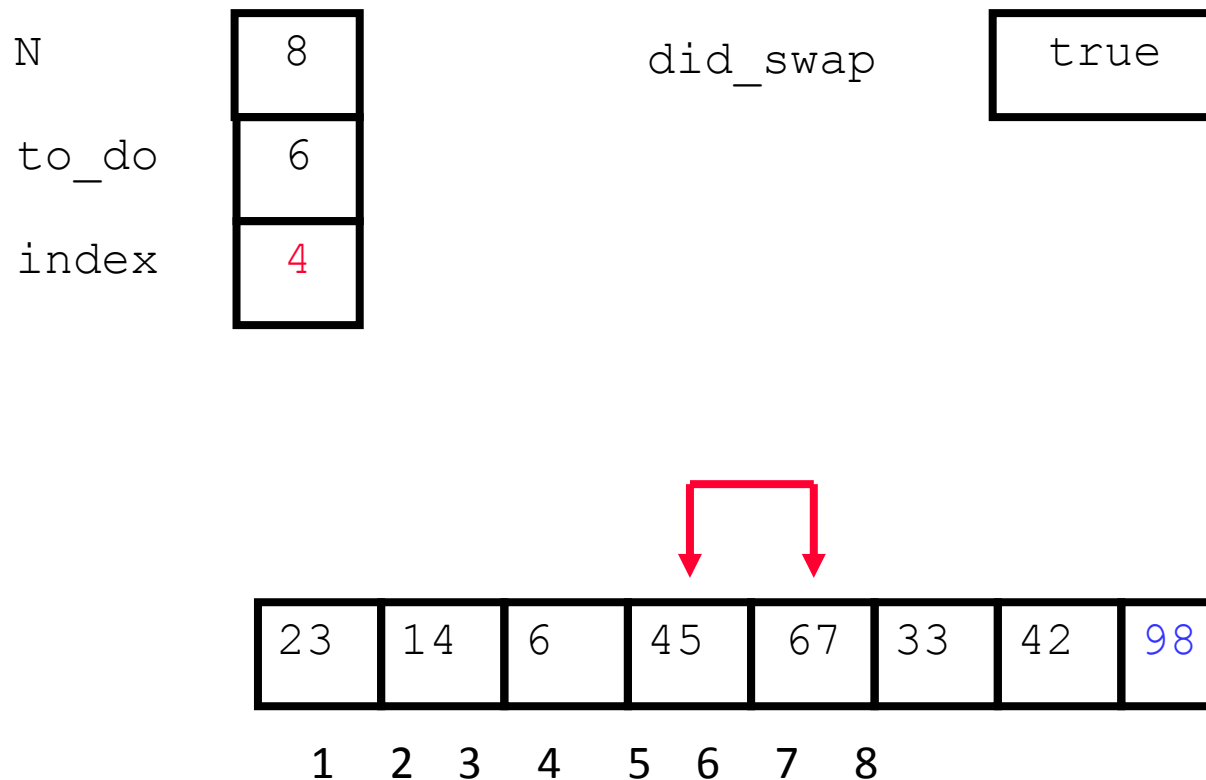
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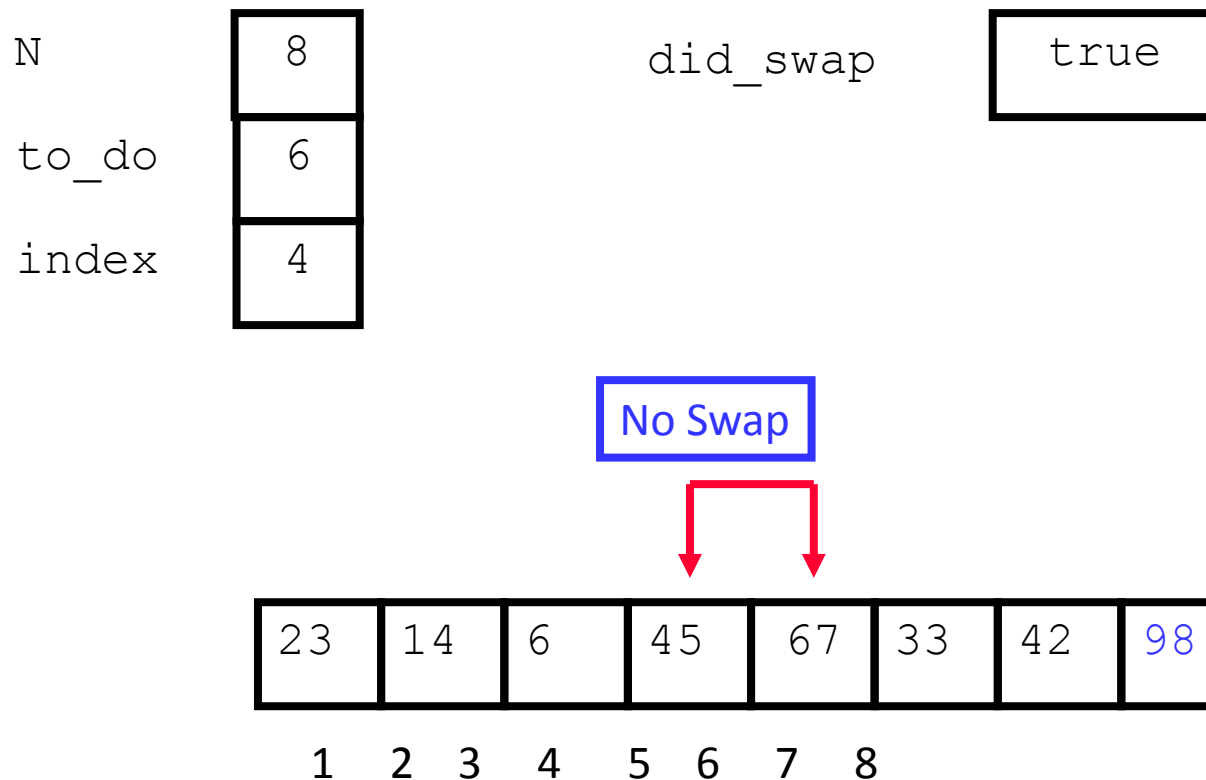
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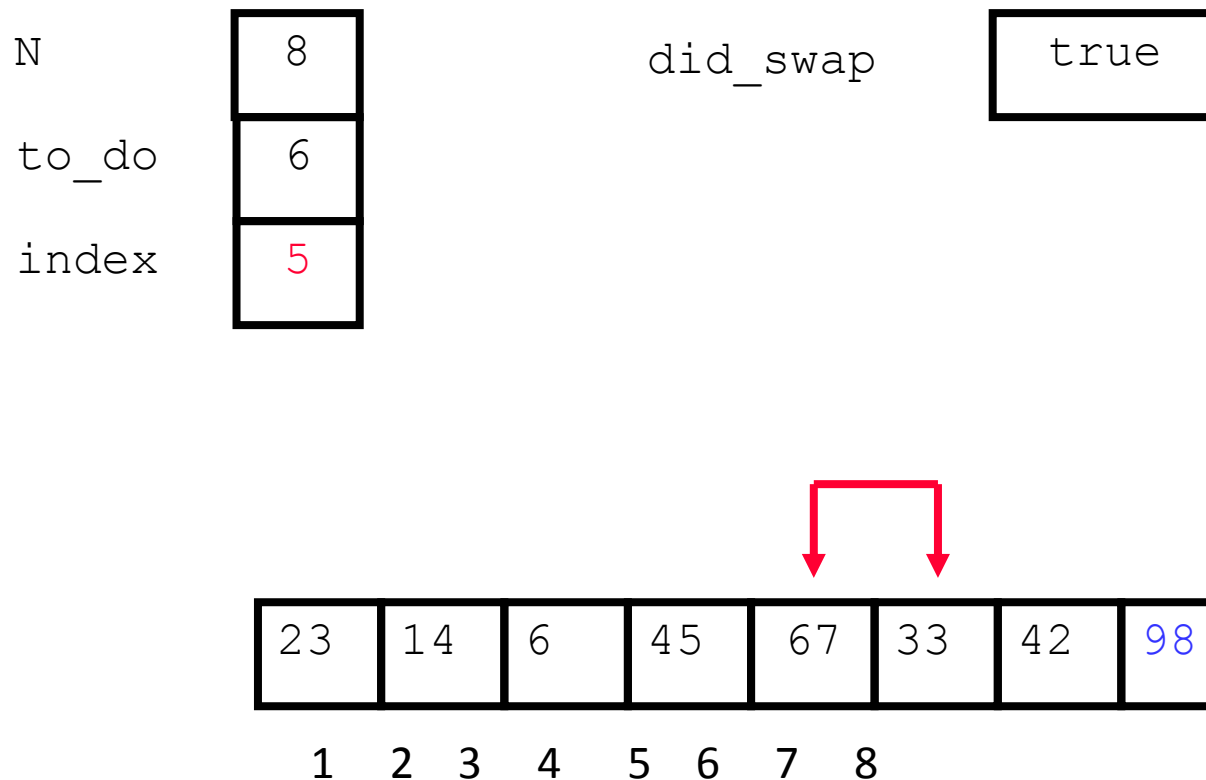
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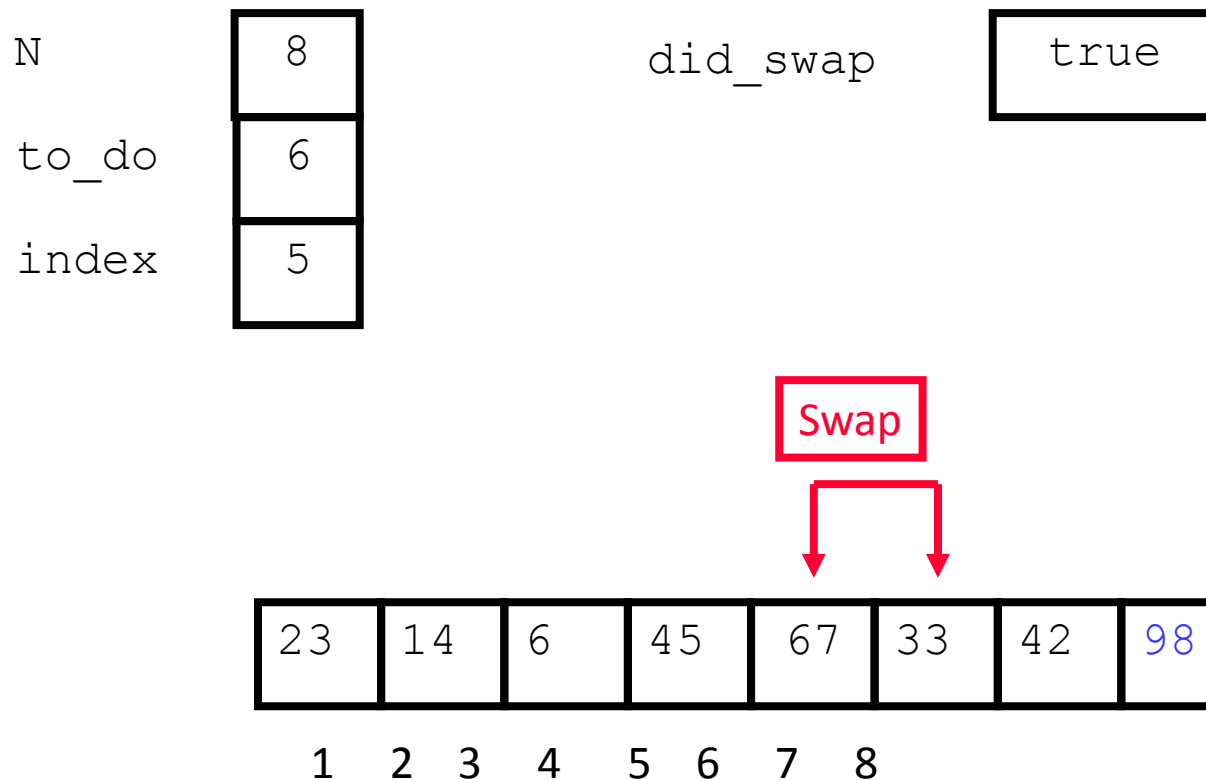
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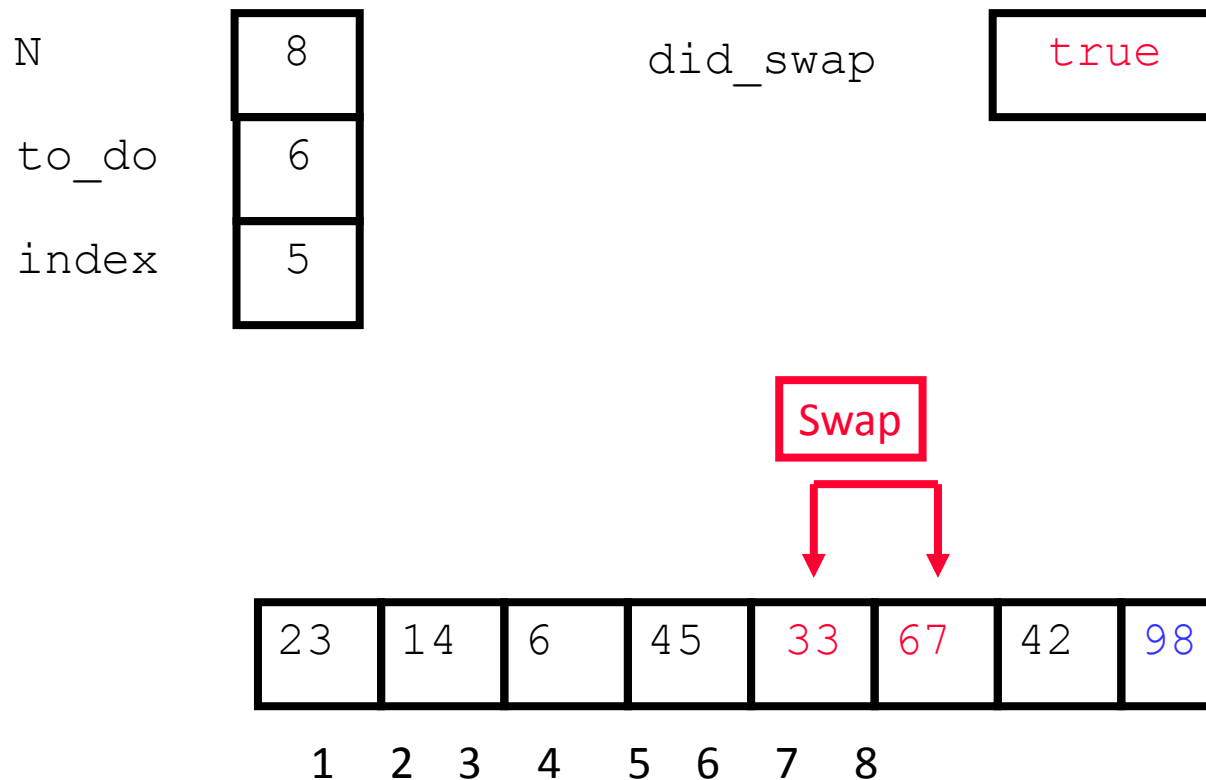
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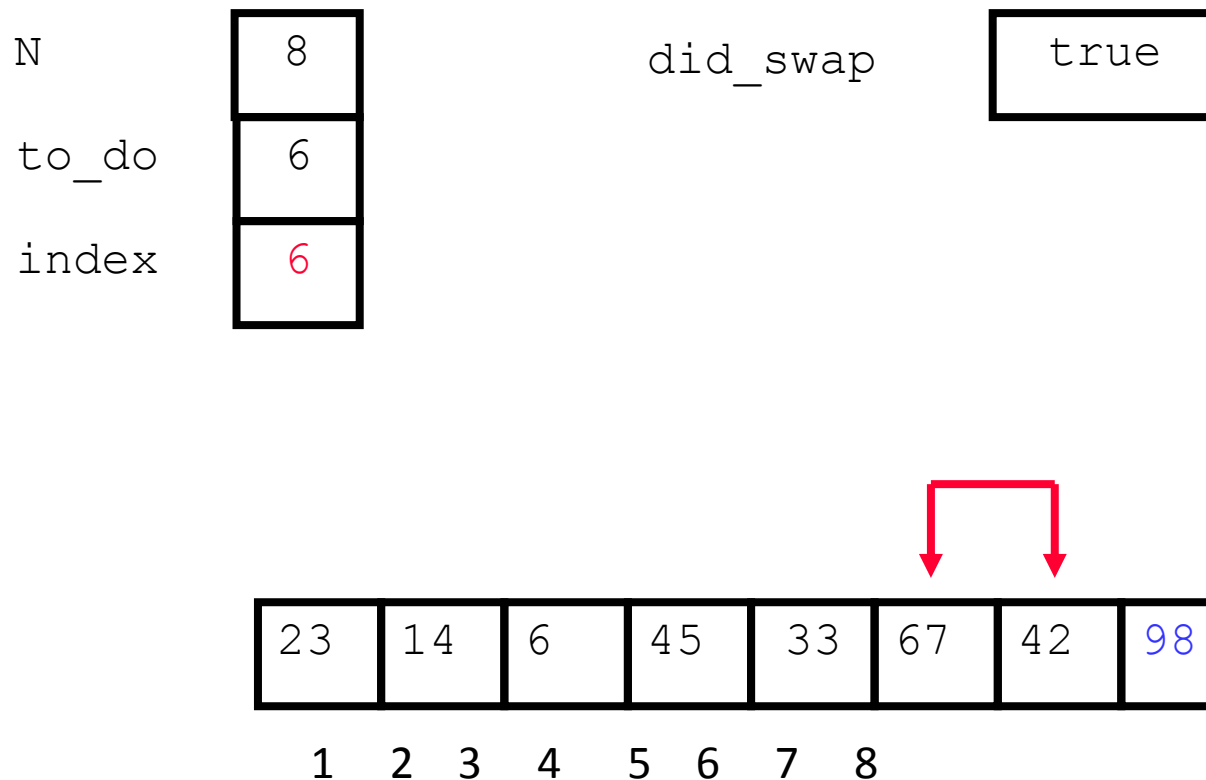
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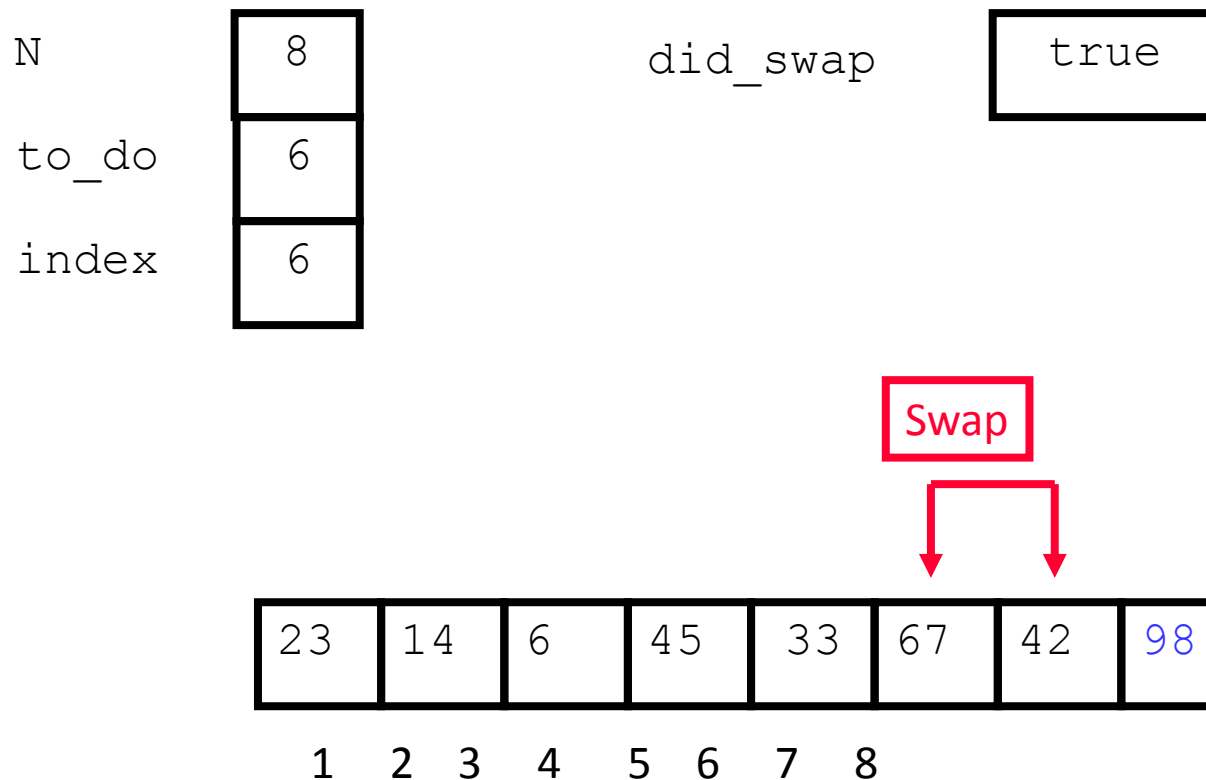
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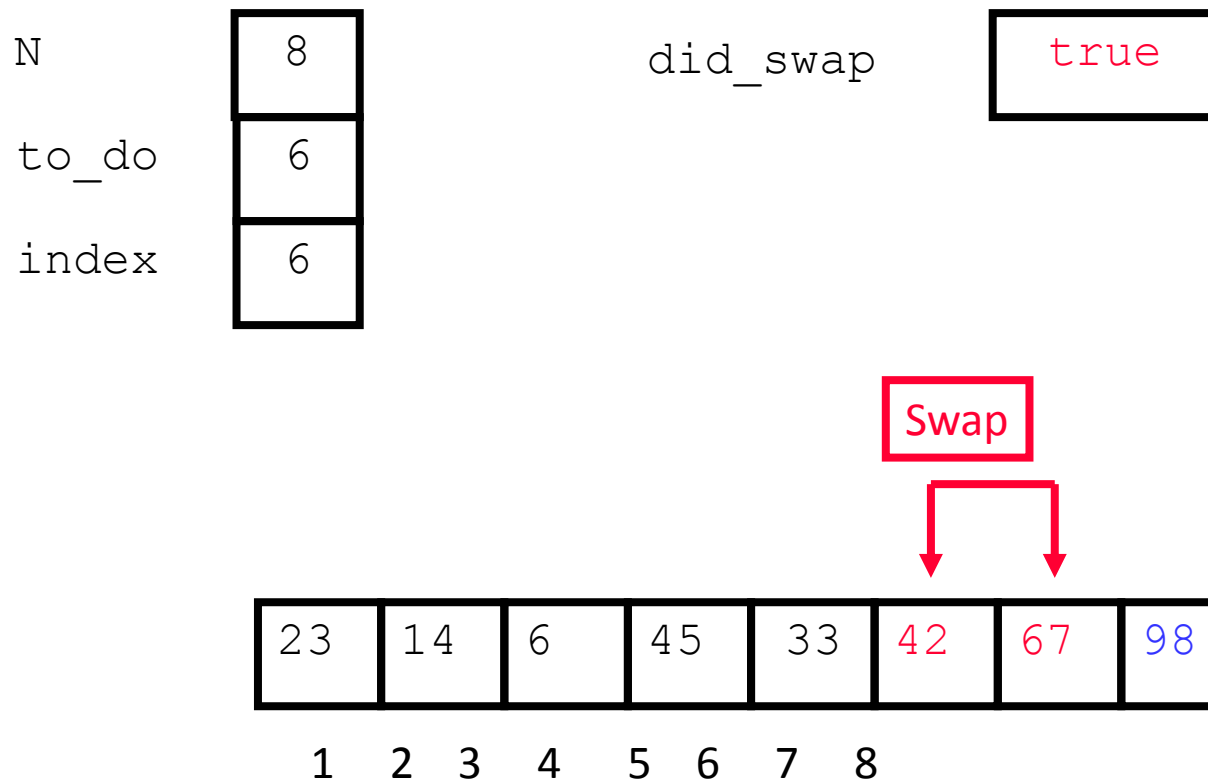
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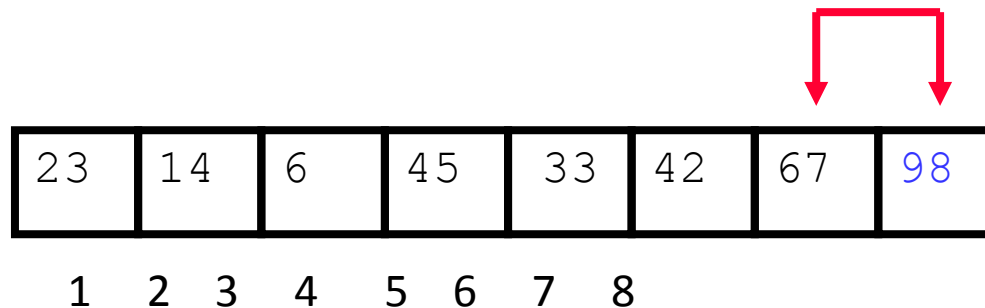
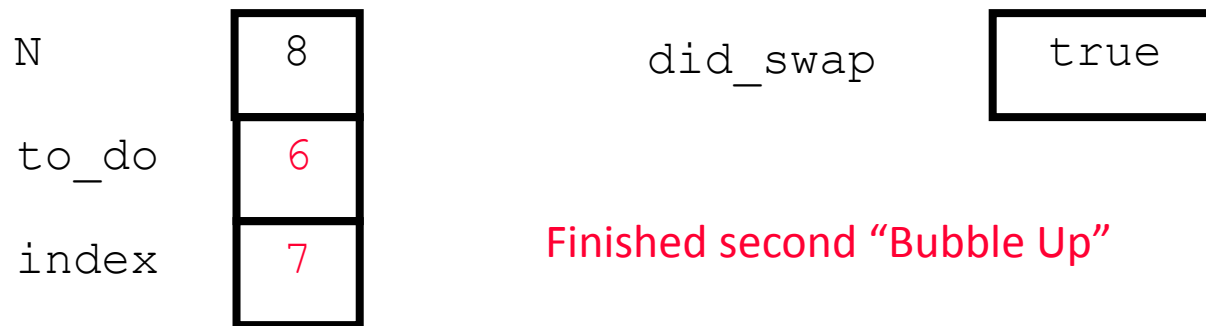
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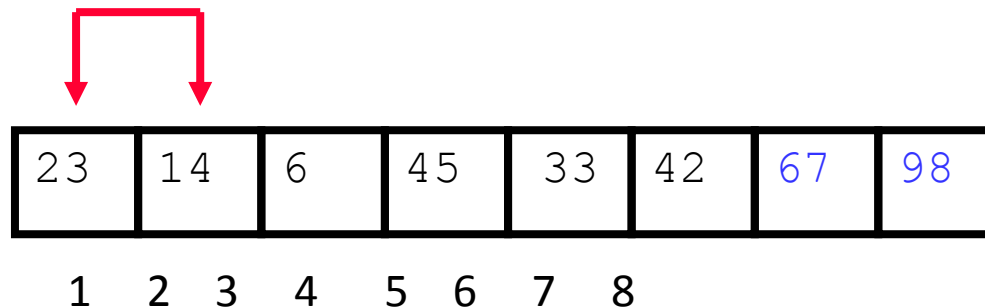
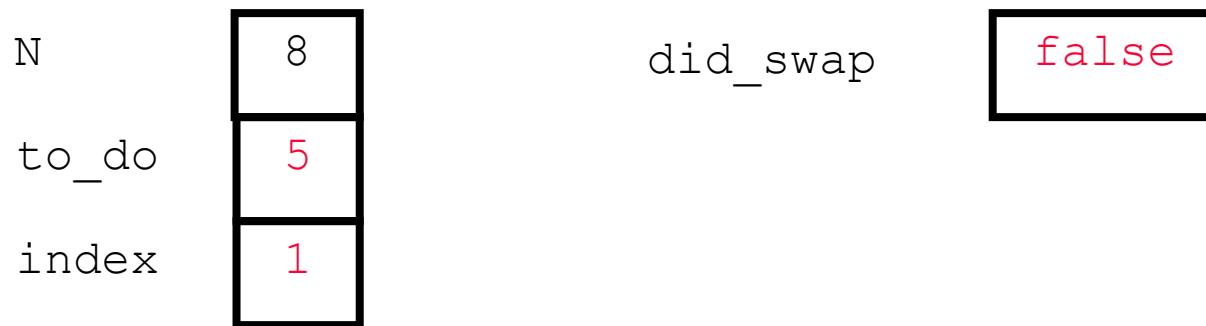
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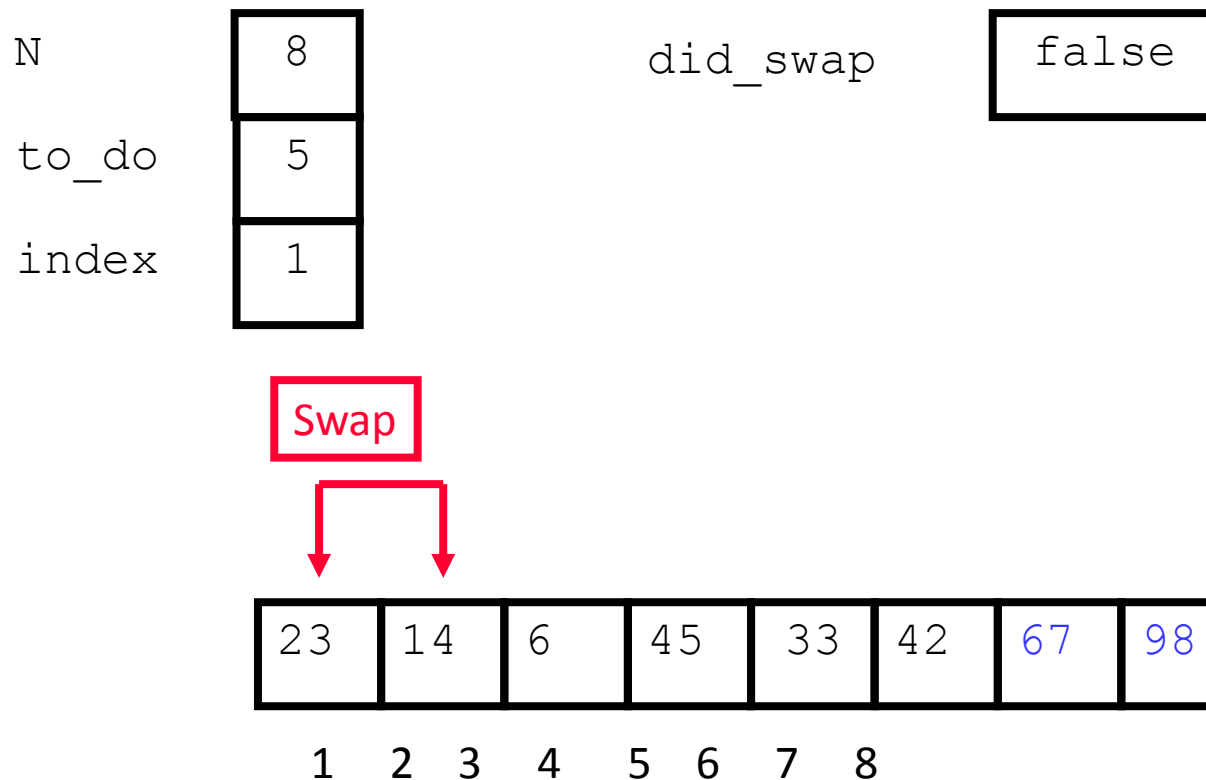
After Second Pass of Outer Loop



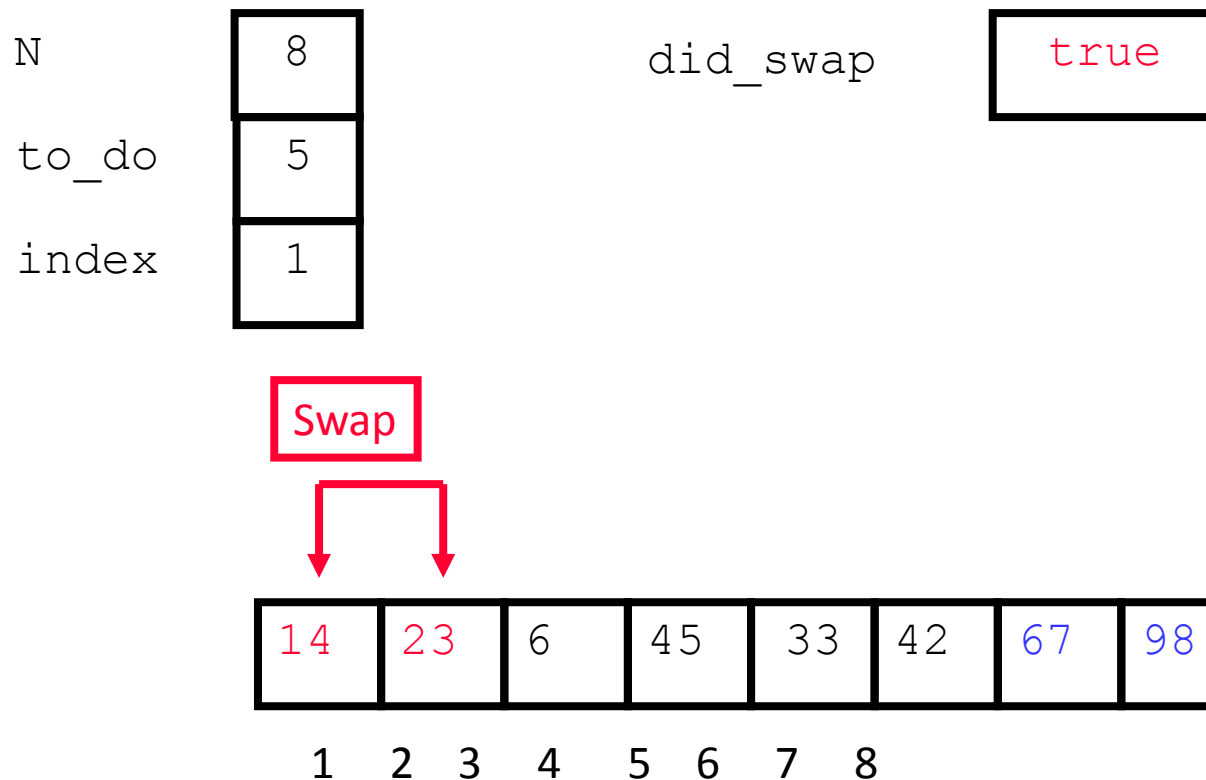
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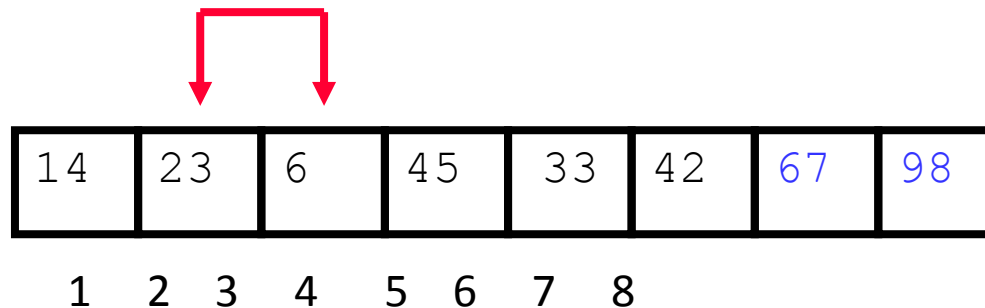
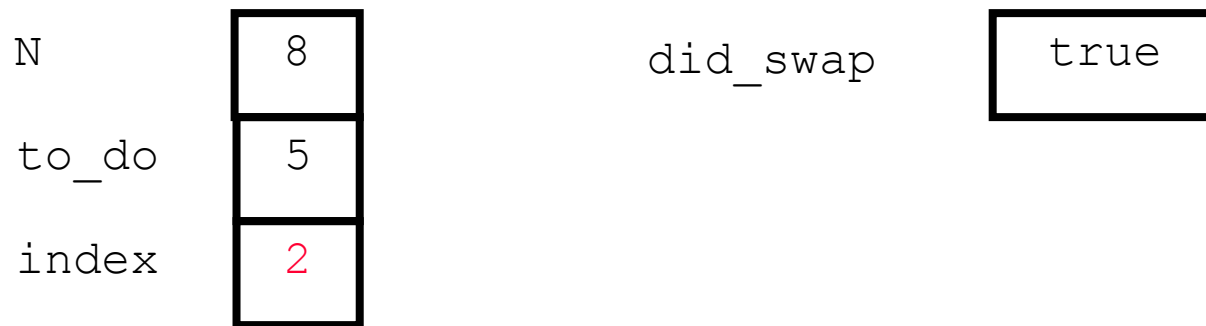
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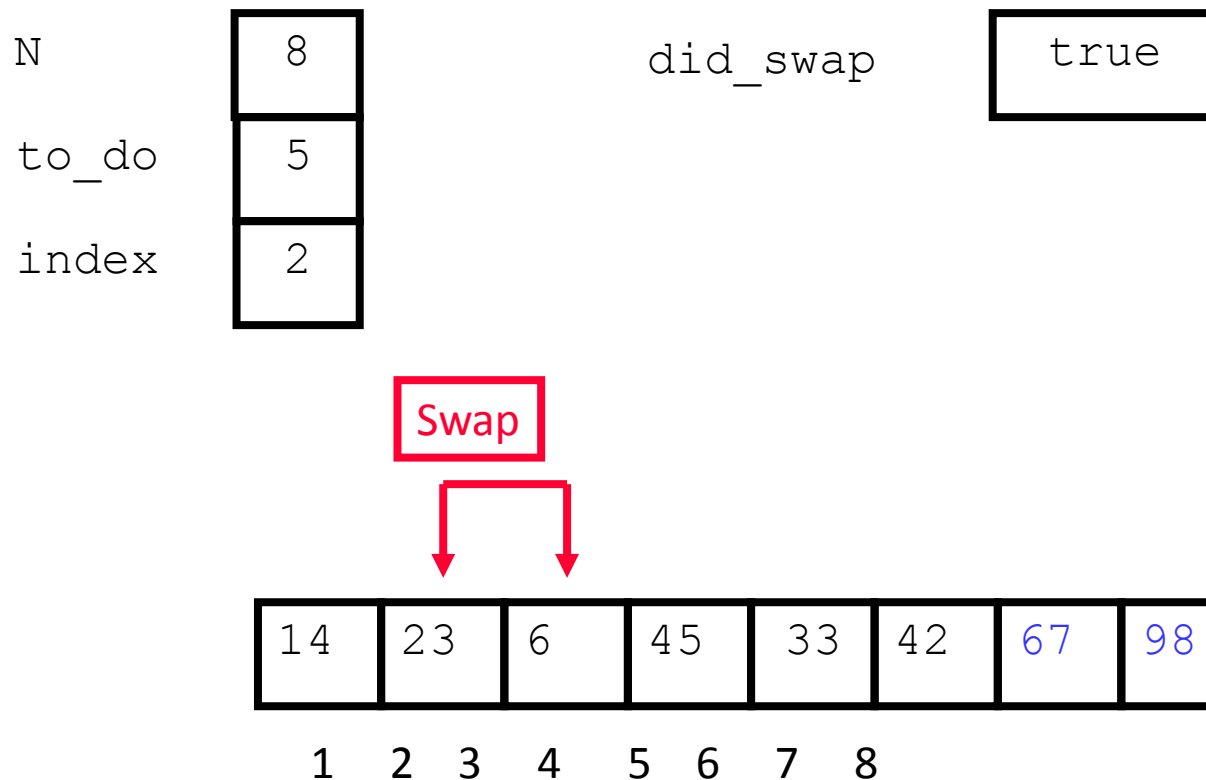
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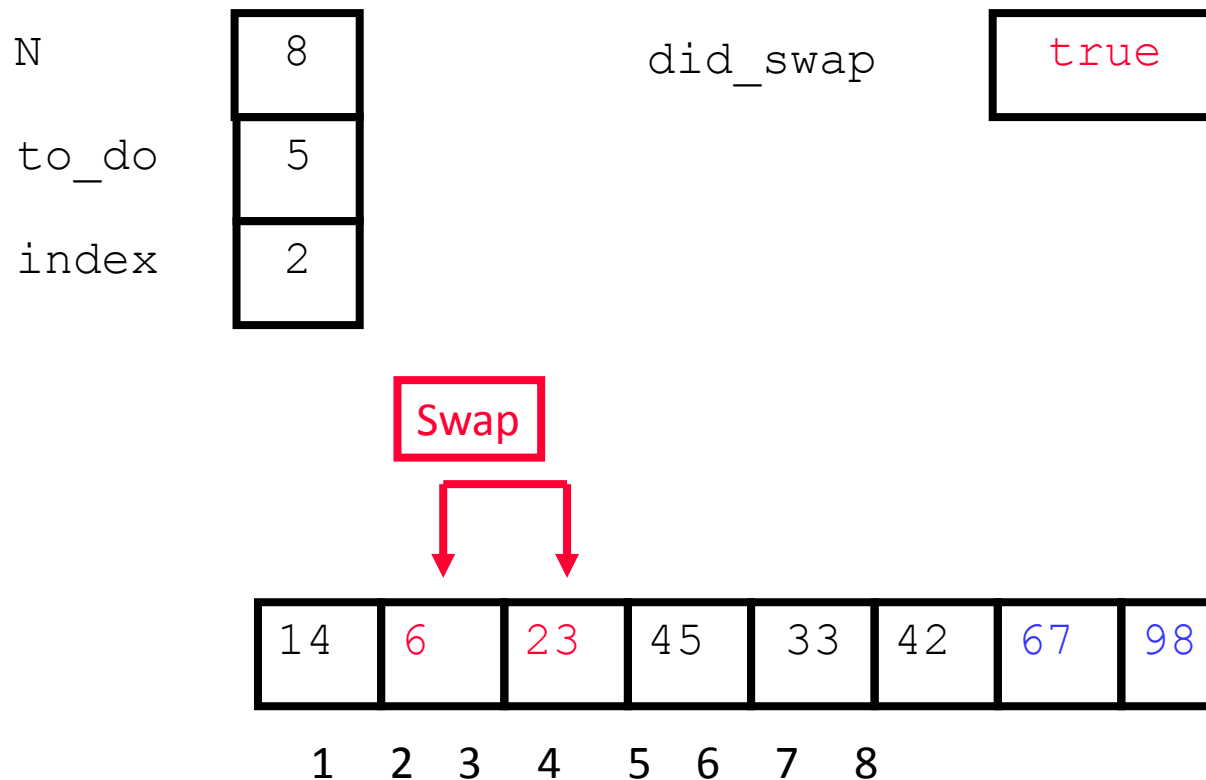
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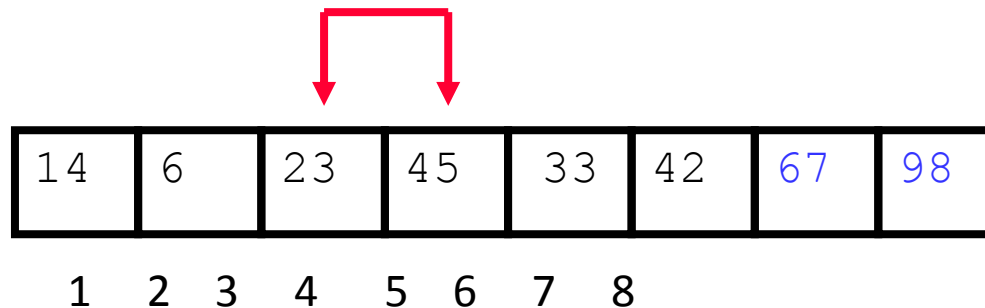
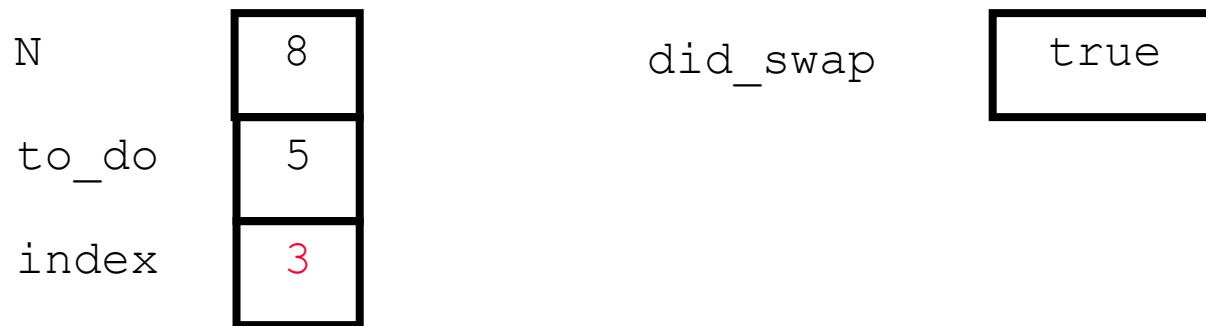
The Third “Bubble Up”



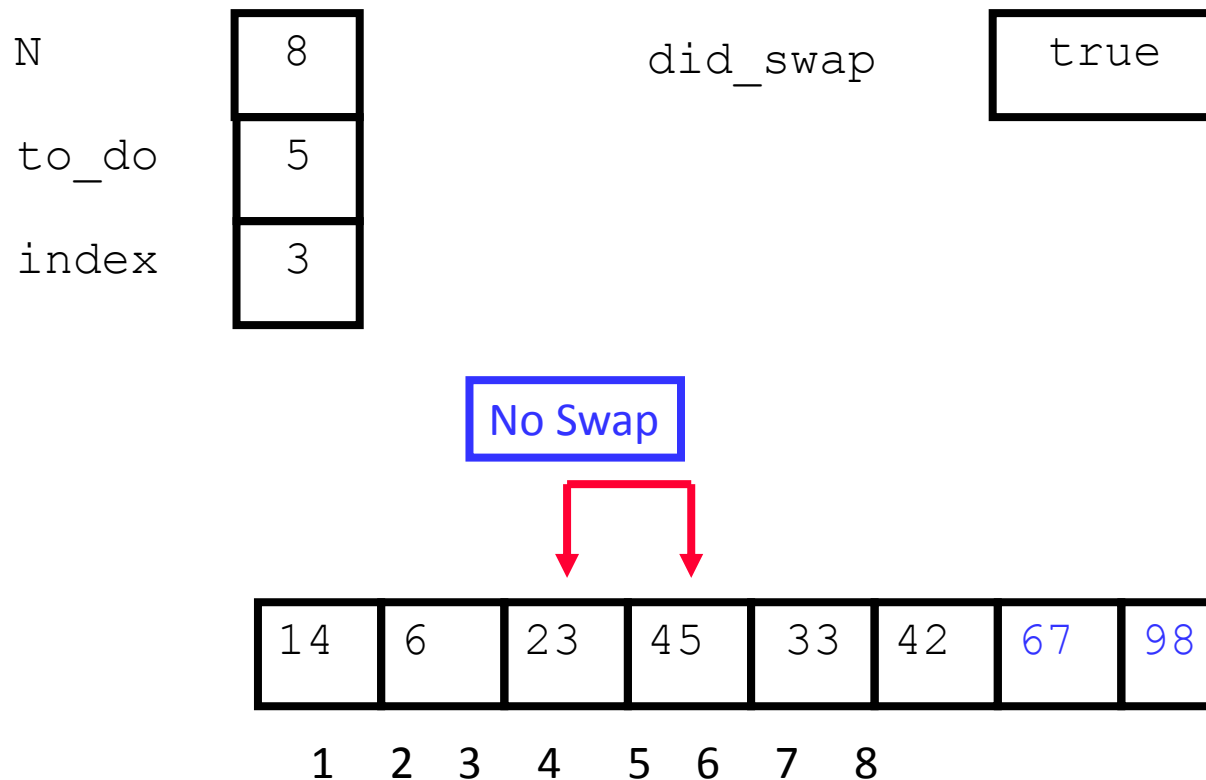
The Third “Bubble Up”



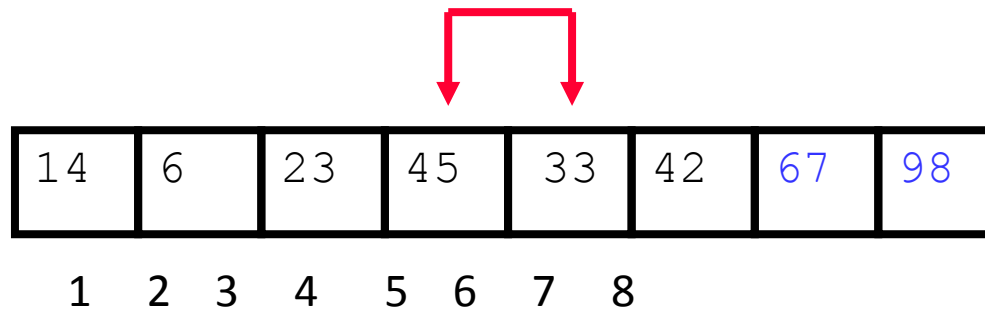
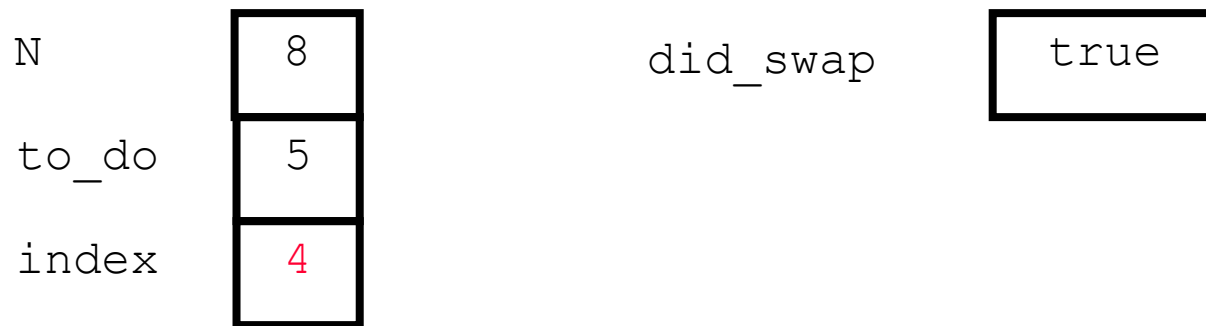
The Third “Bubble Up”



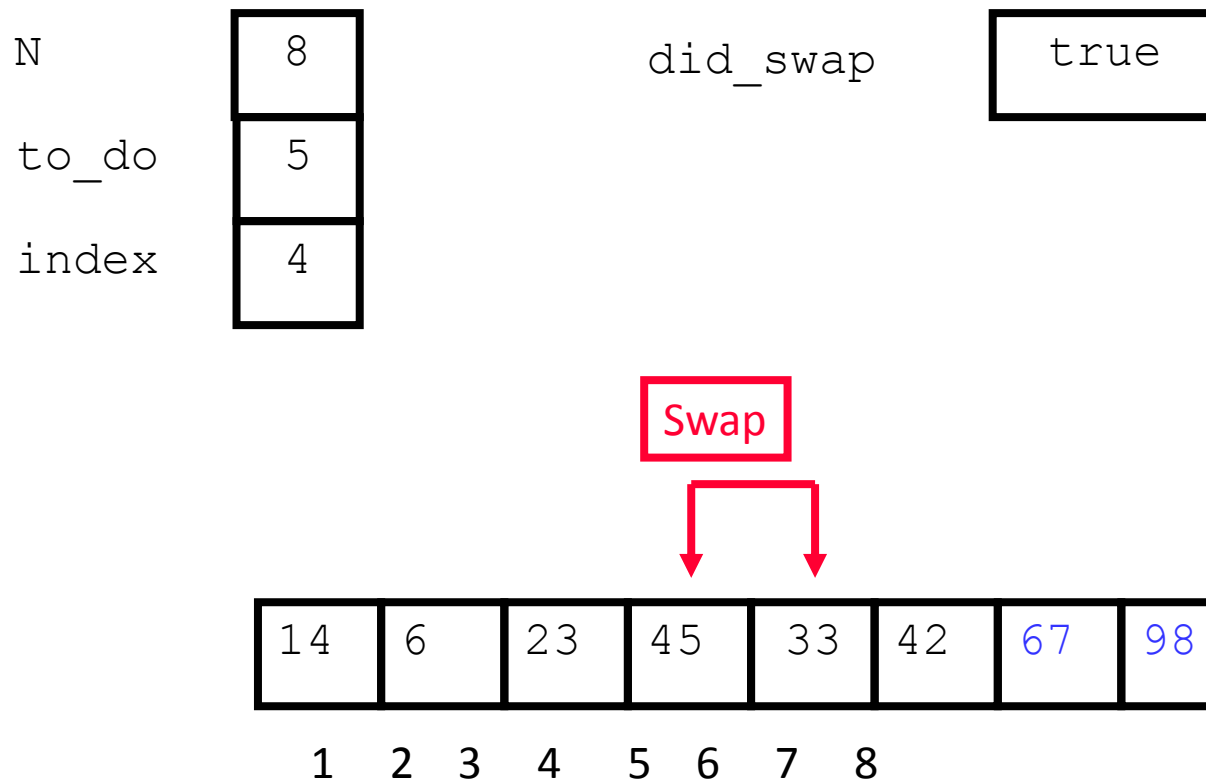
The Third “Bubble Up”



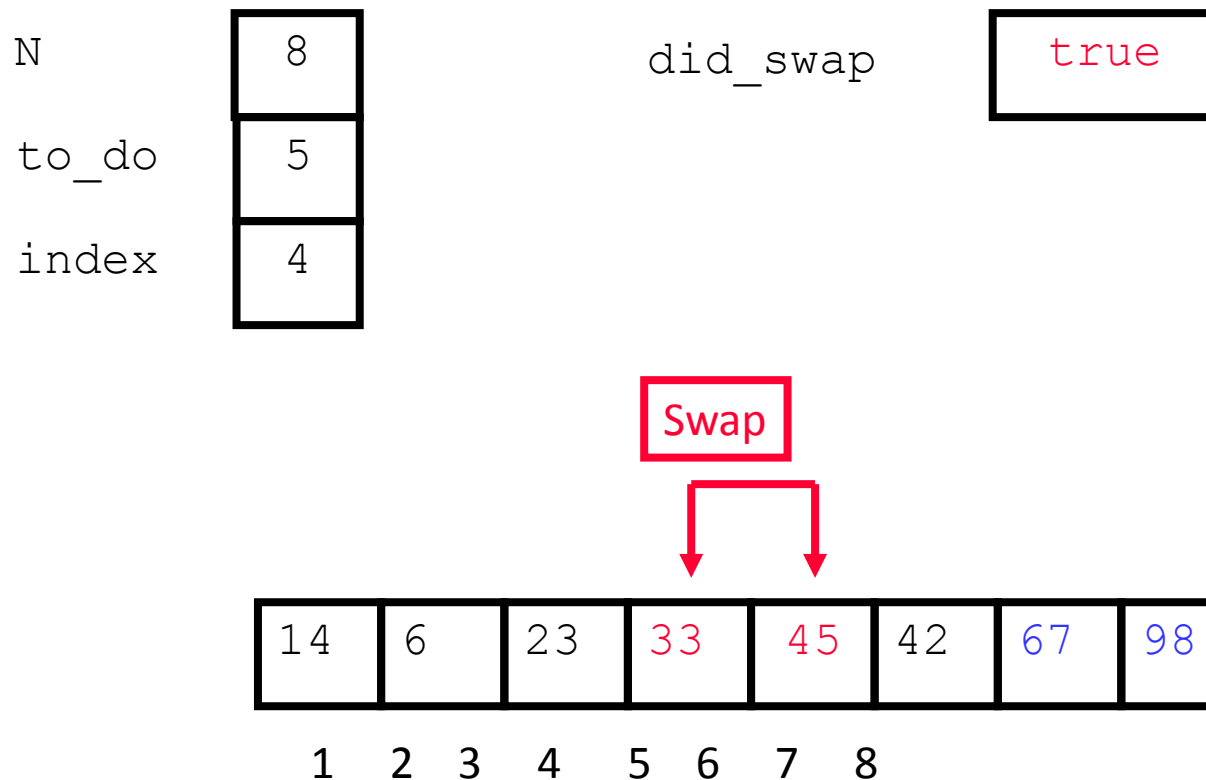
The Third “Bubble Up”



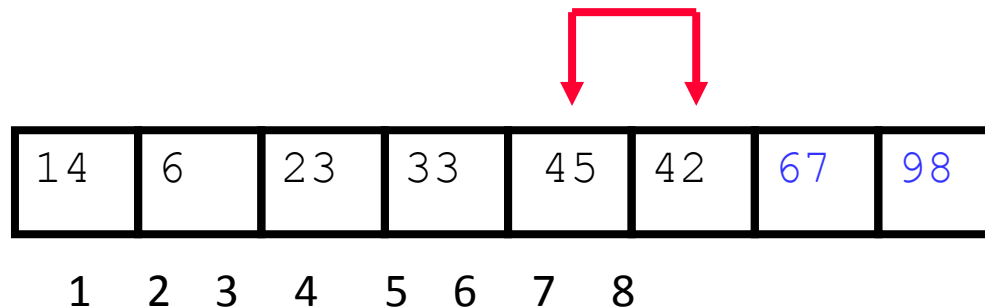
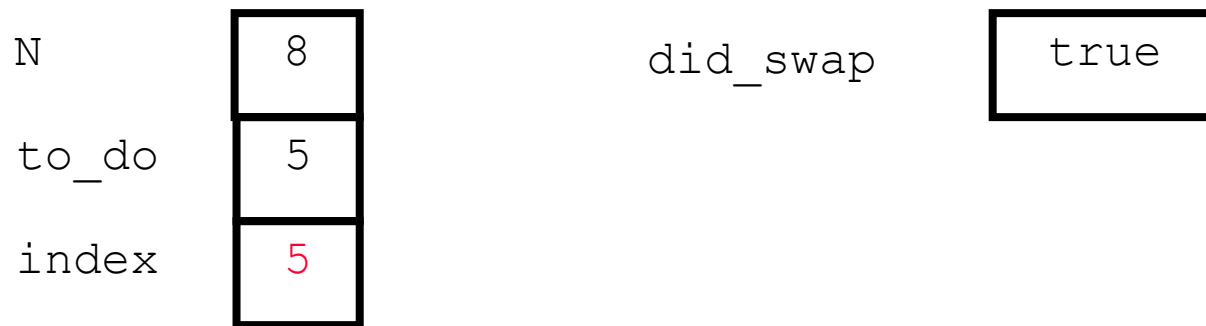
The Third “Bubble Up”



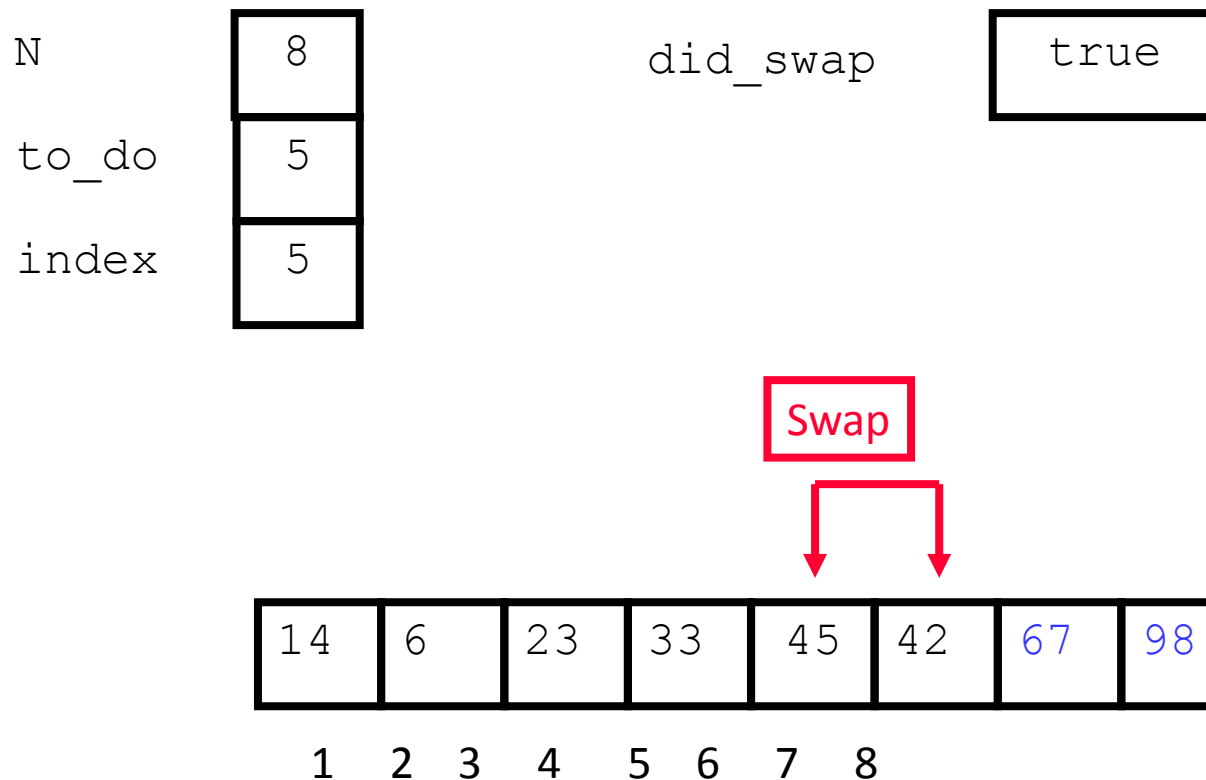
The Third “Bubble Up”



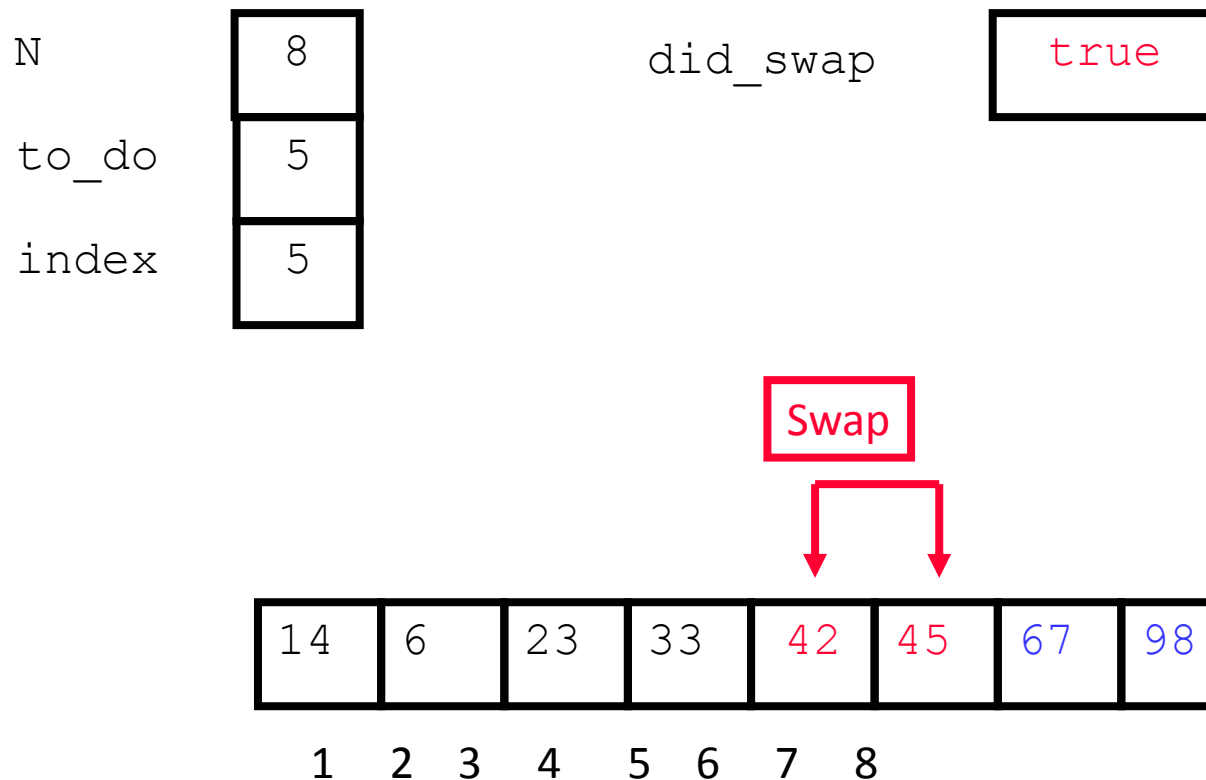
The Third “Bubble Up”



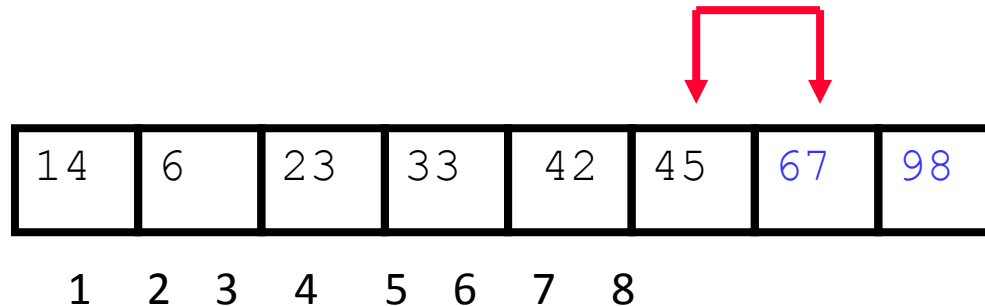
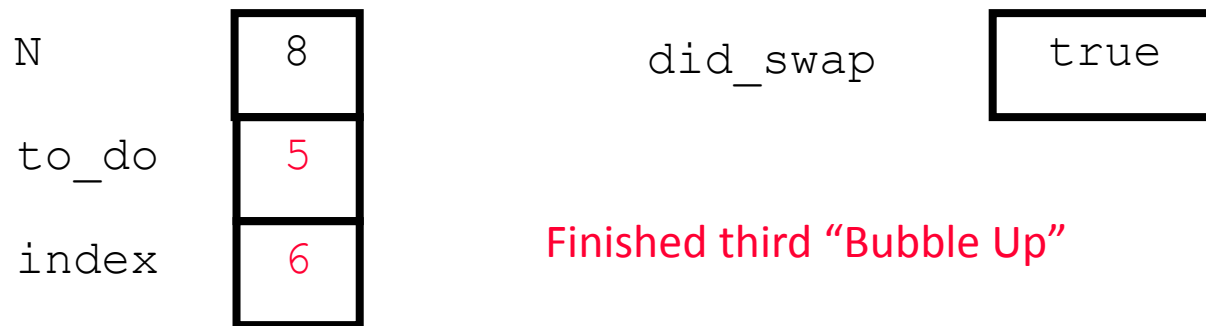
The Third “Bubble Up”



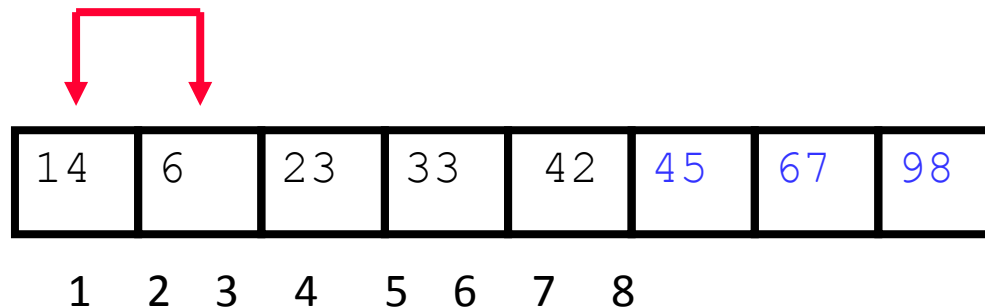
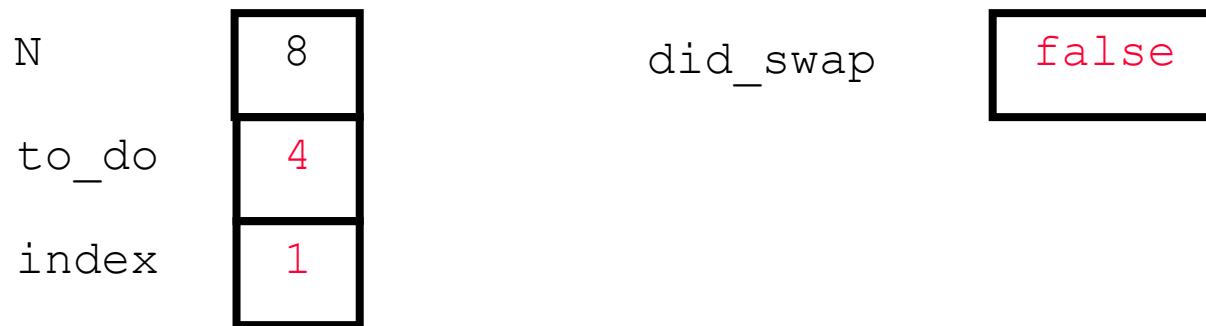
The Third “Bubble Up”



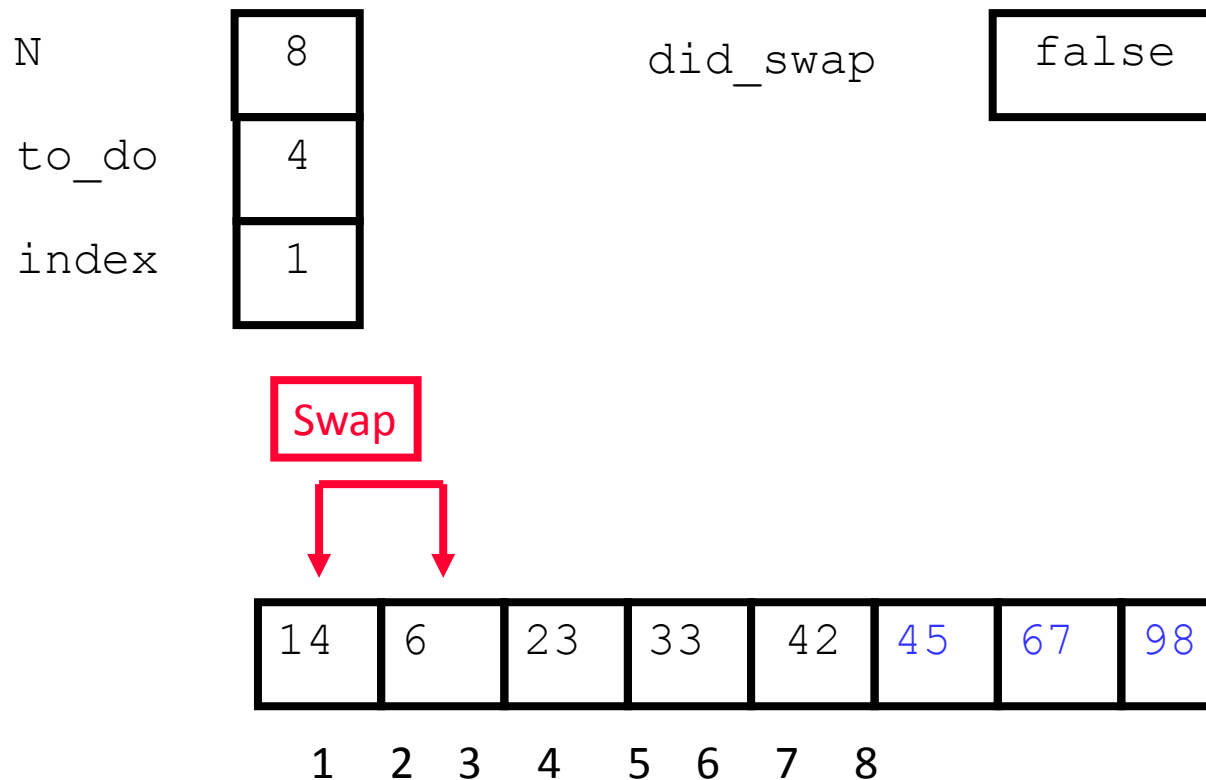
After Third Pass of Outer Loop



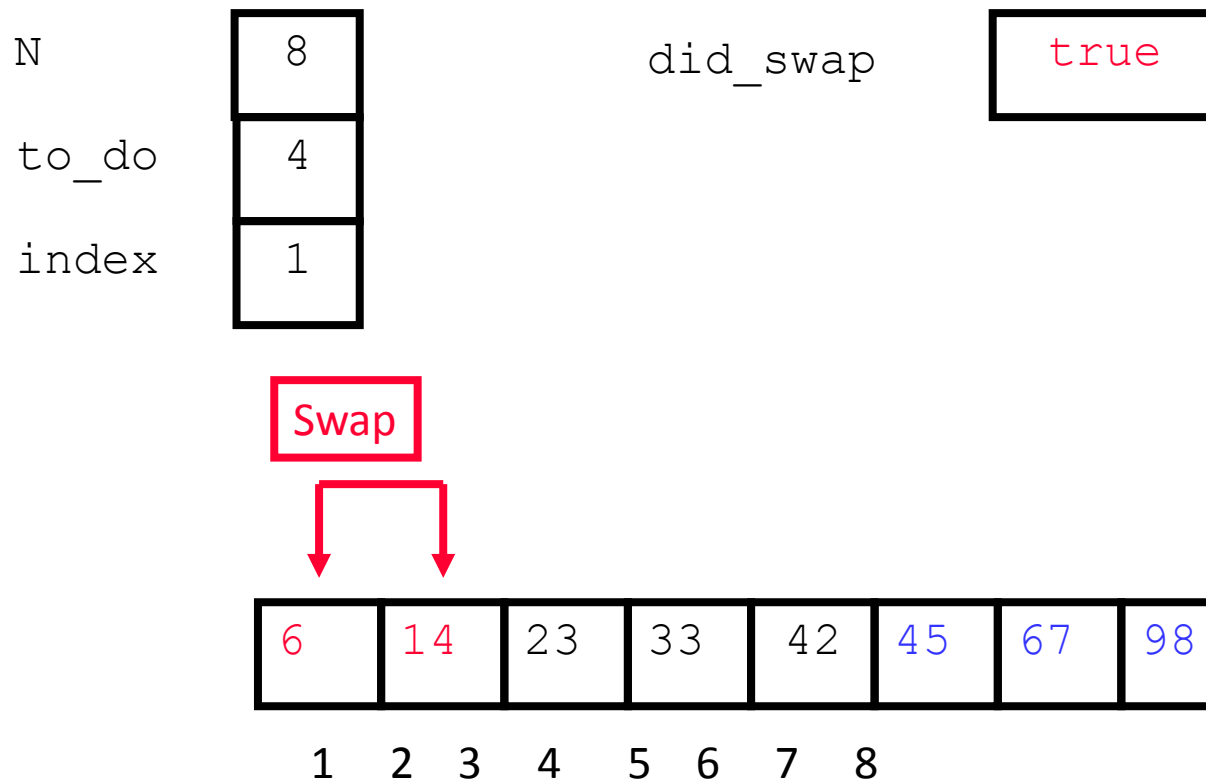
The Fourth “Bubble Up”



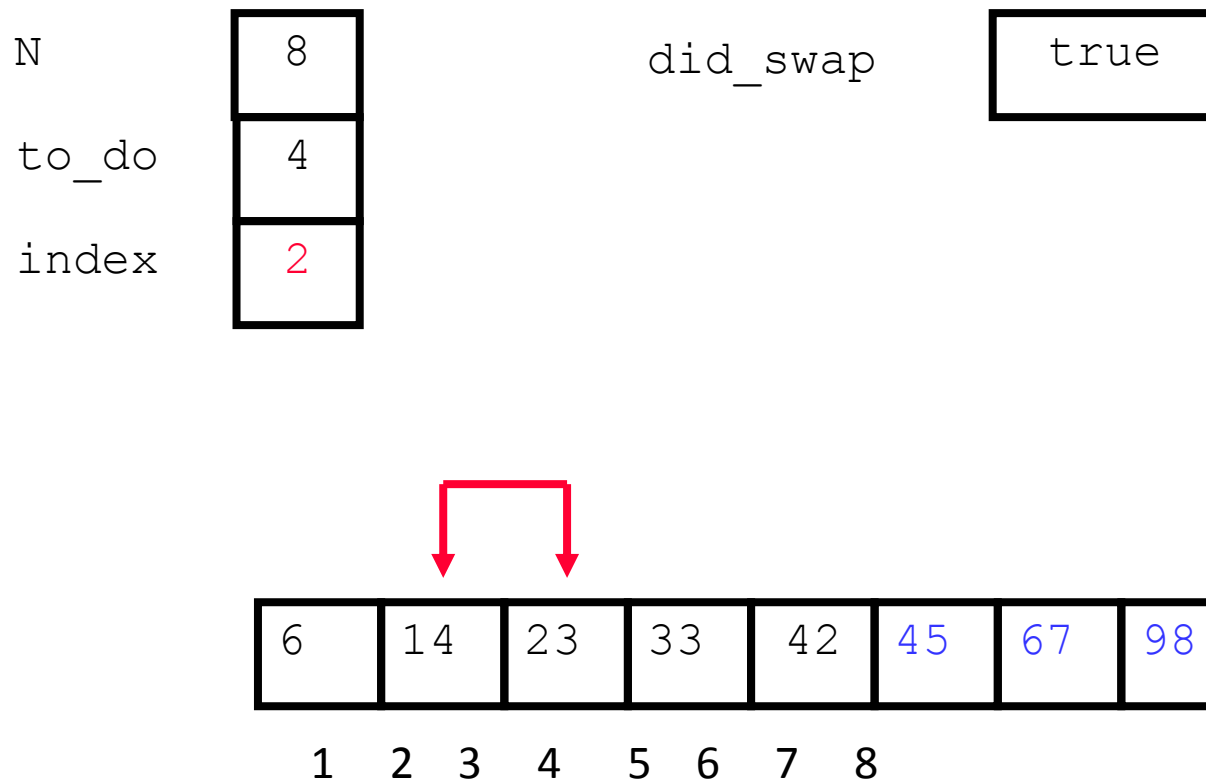
The Fourth “Bubble Up”



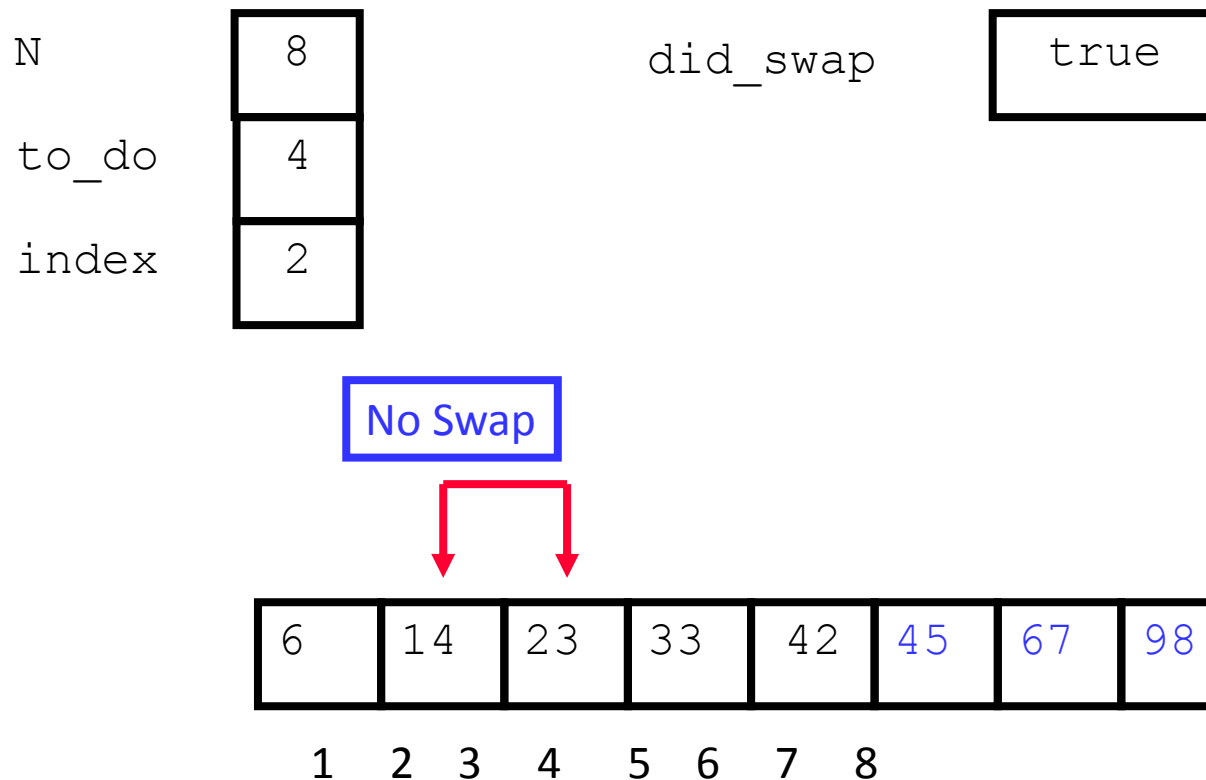
The Fourth “Bubble Up”



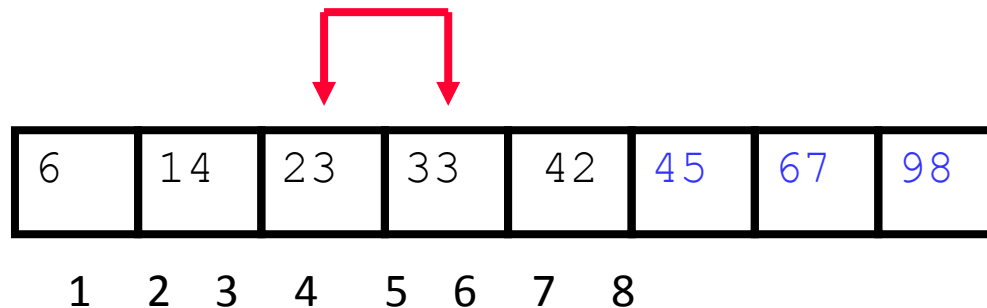
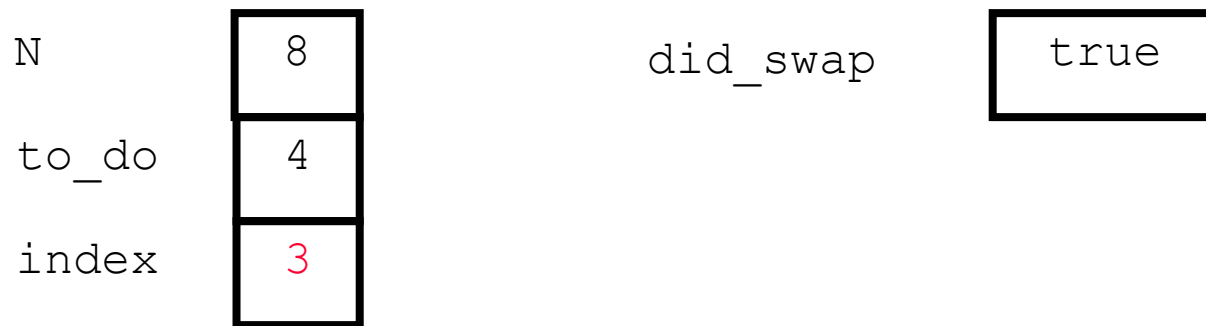
The Fourth “Bubble Up”



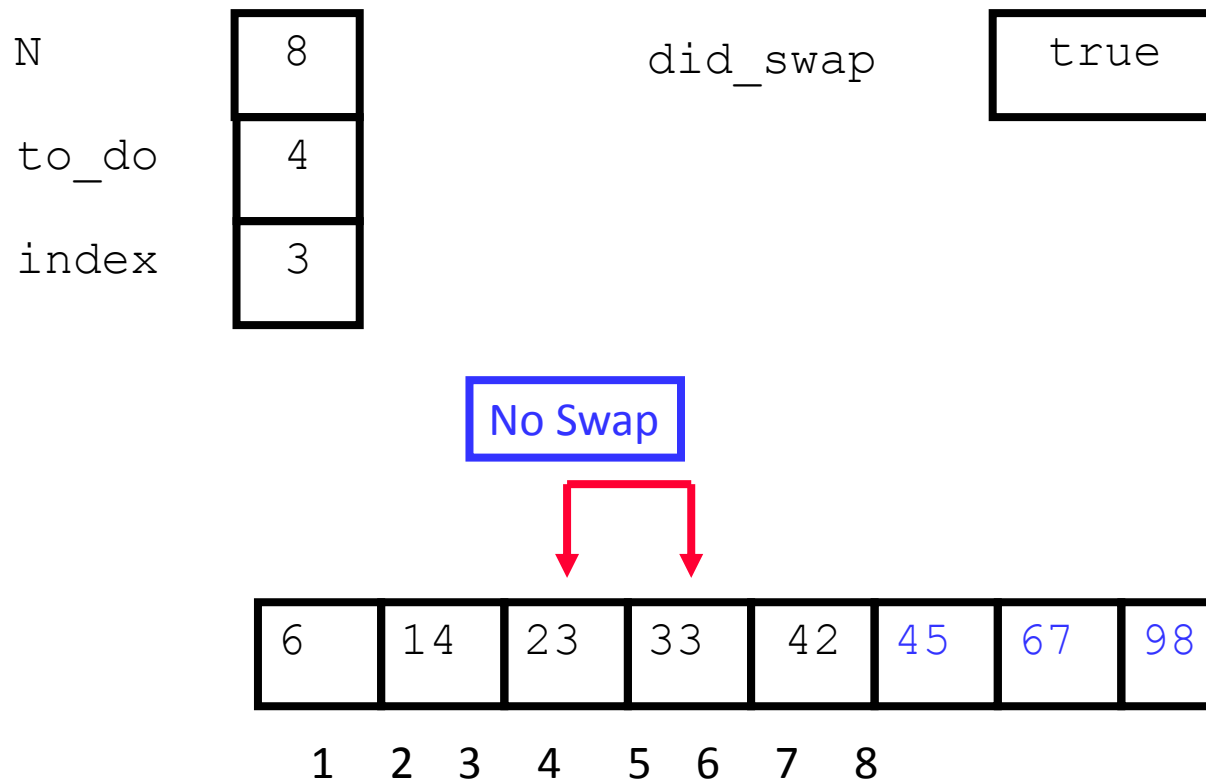
The Fourth “Bubble Up”



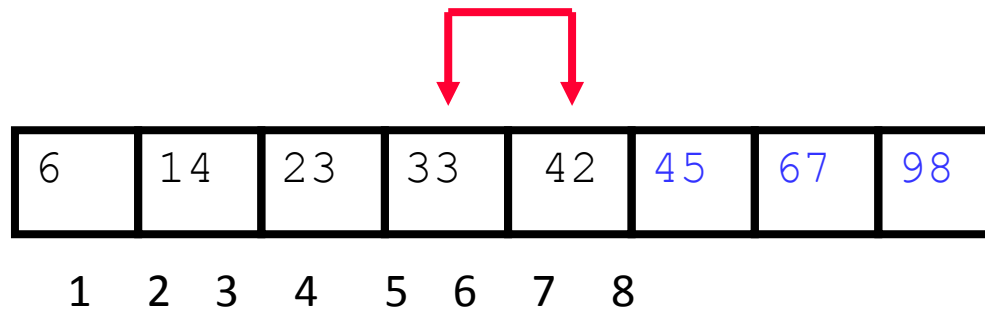
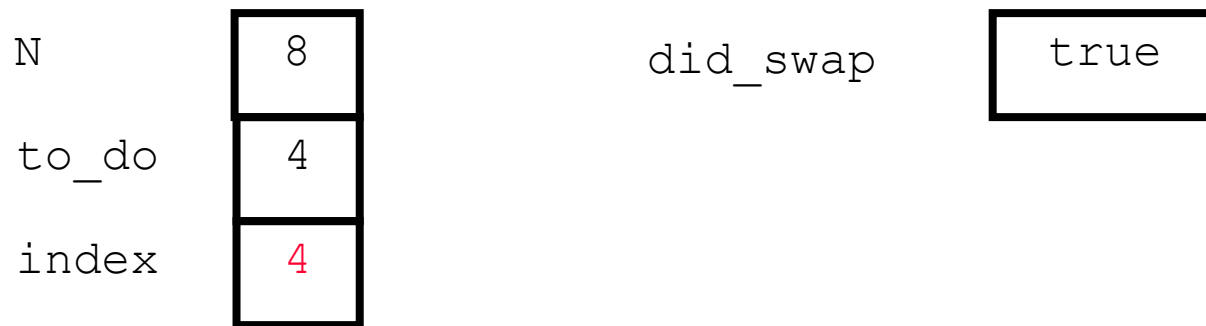
The Fourth “Bubble Up”



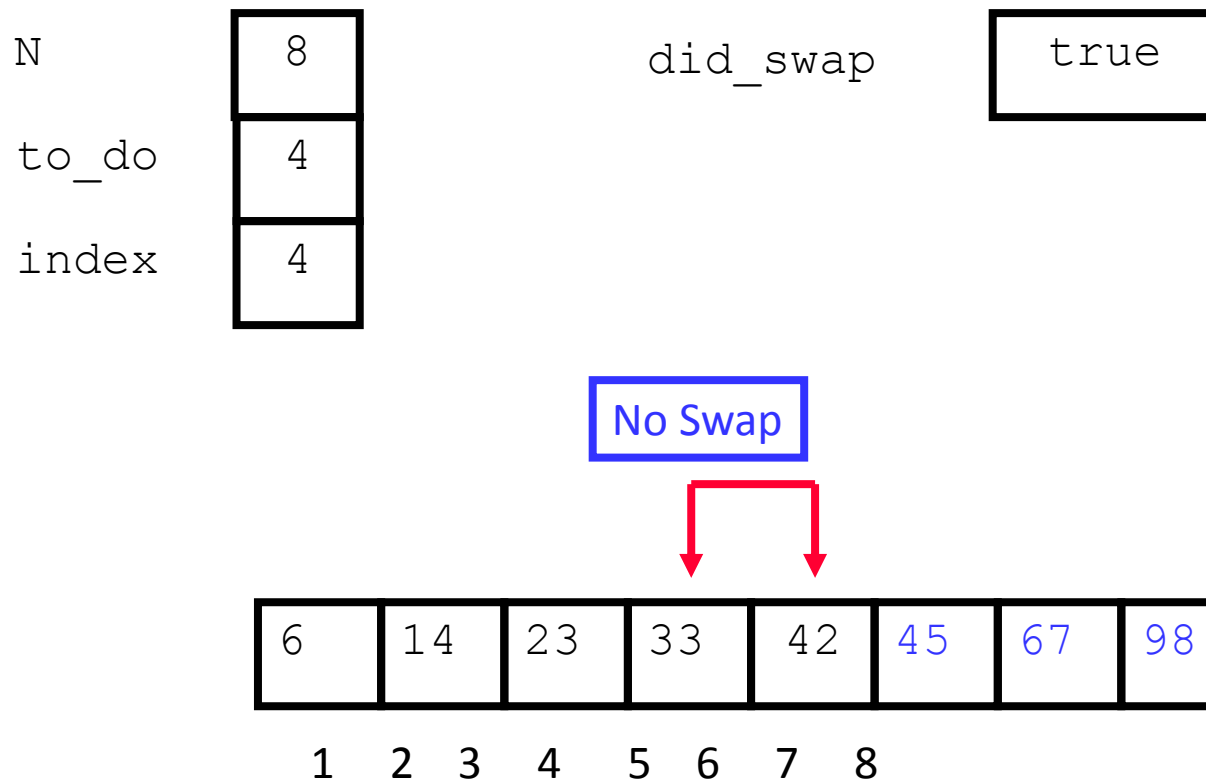
The Fourth “Bubble Up”



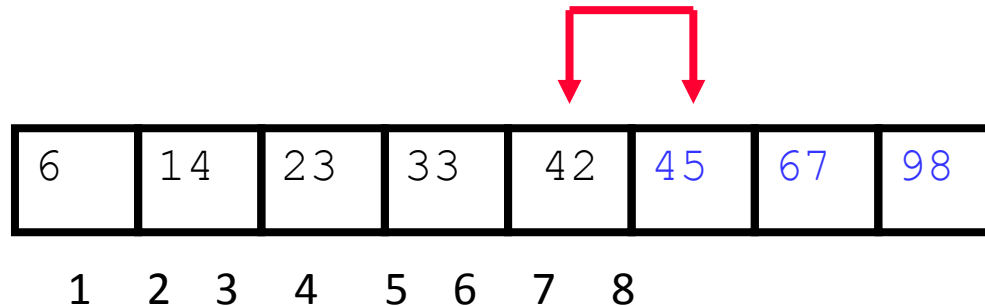
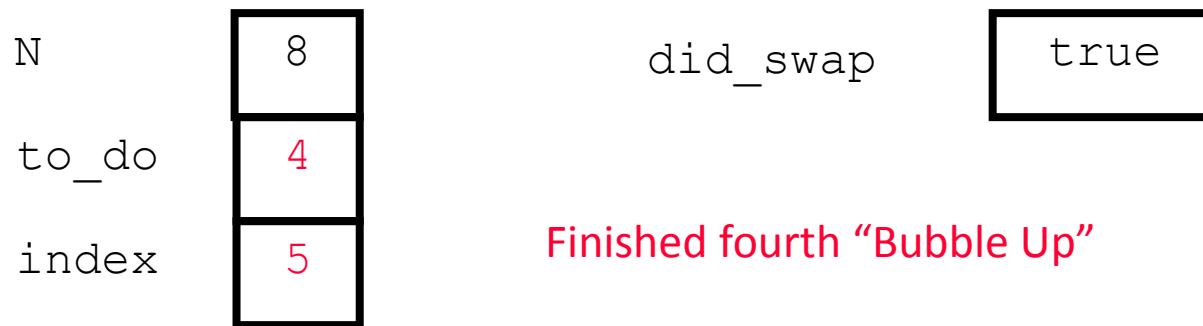
The Fourth “Bubble Up”



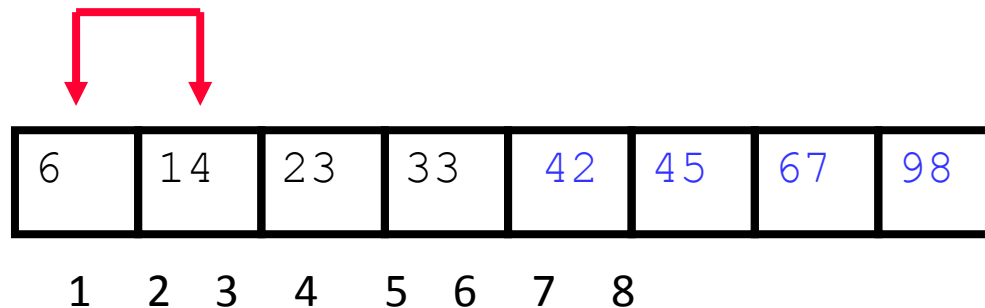
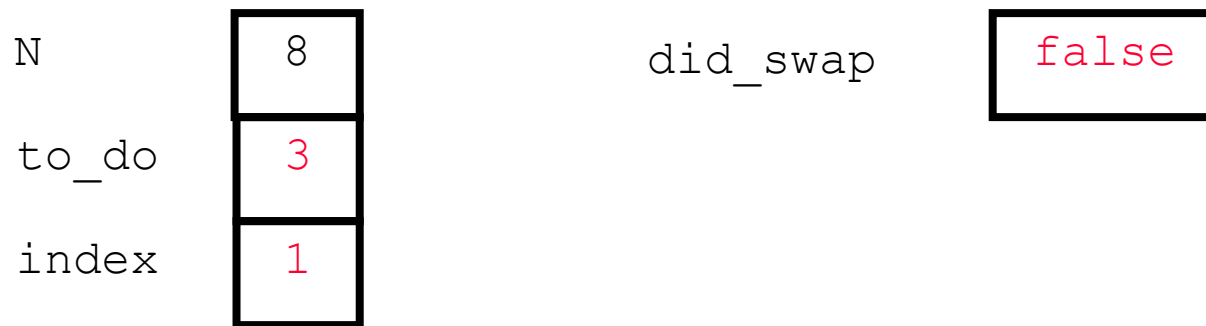
The Fourth “Bubble Up”



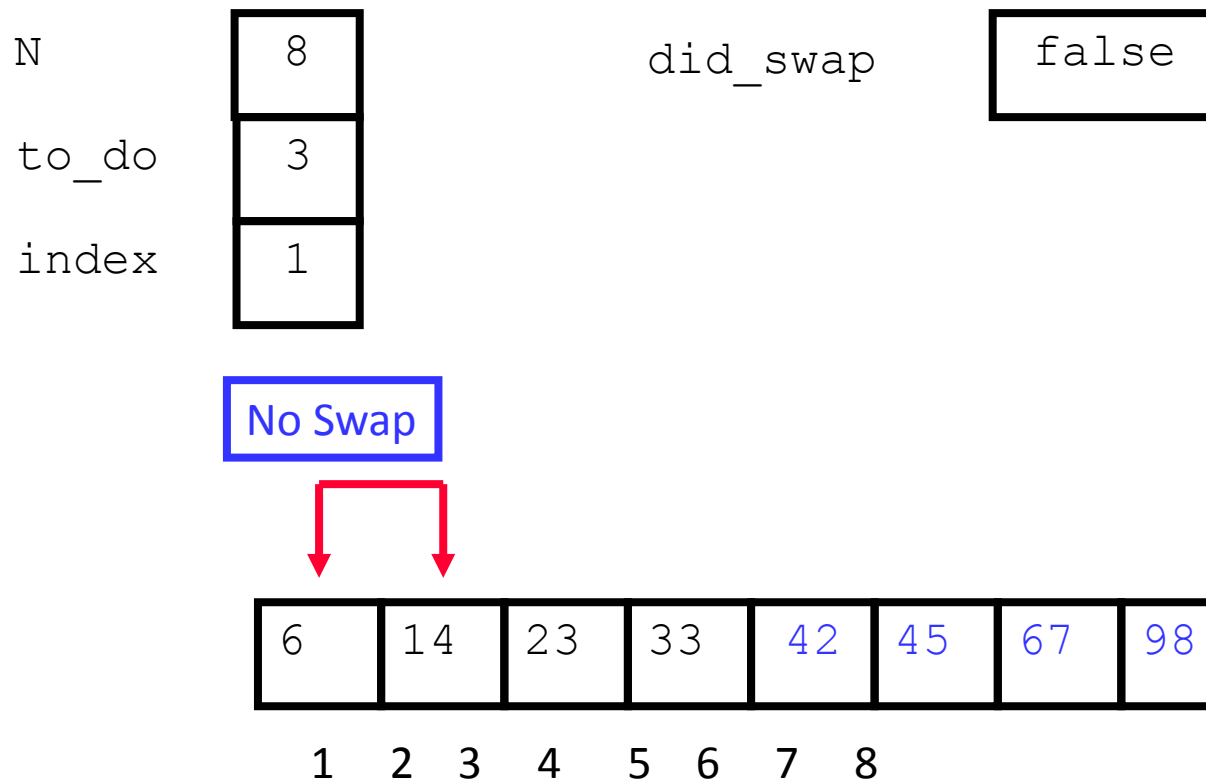
After Fourth Pass of Outer Loop



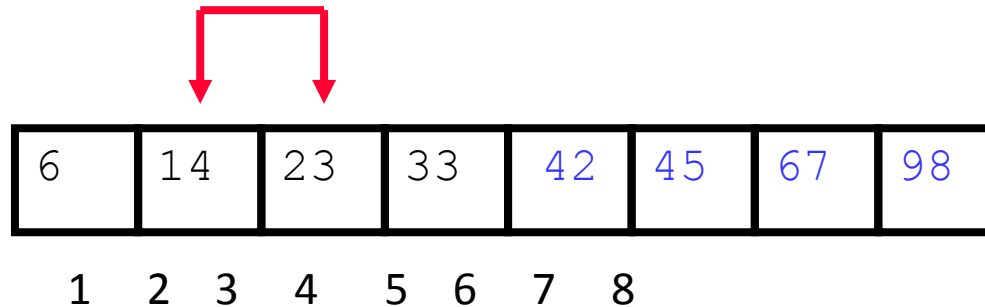
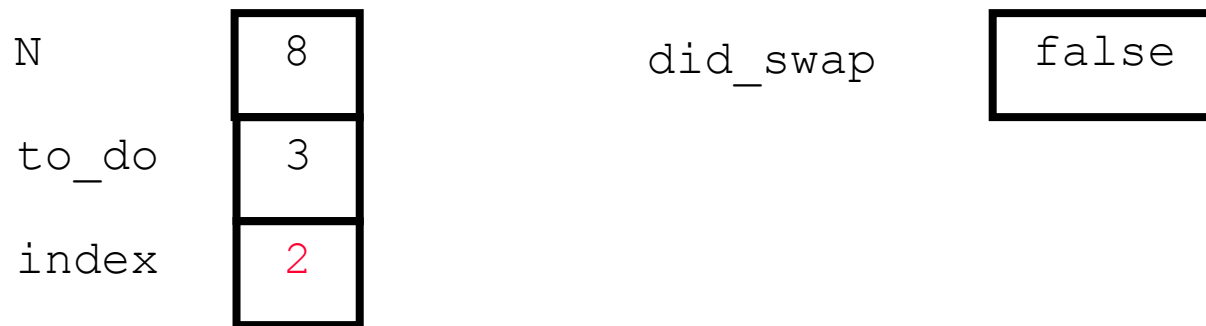
The Fifth “Bubble Up”



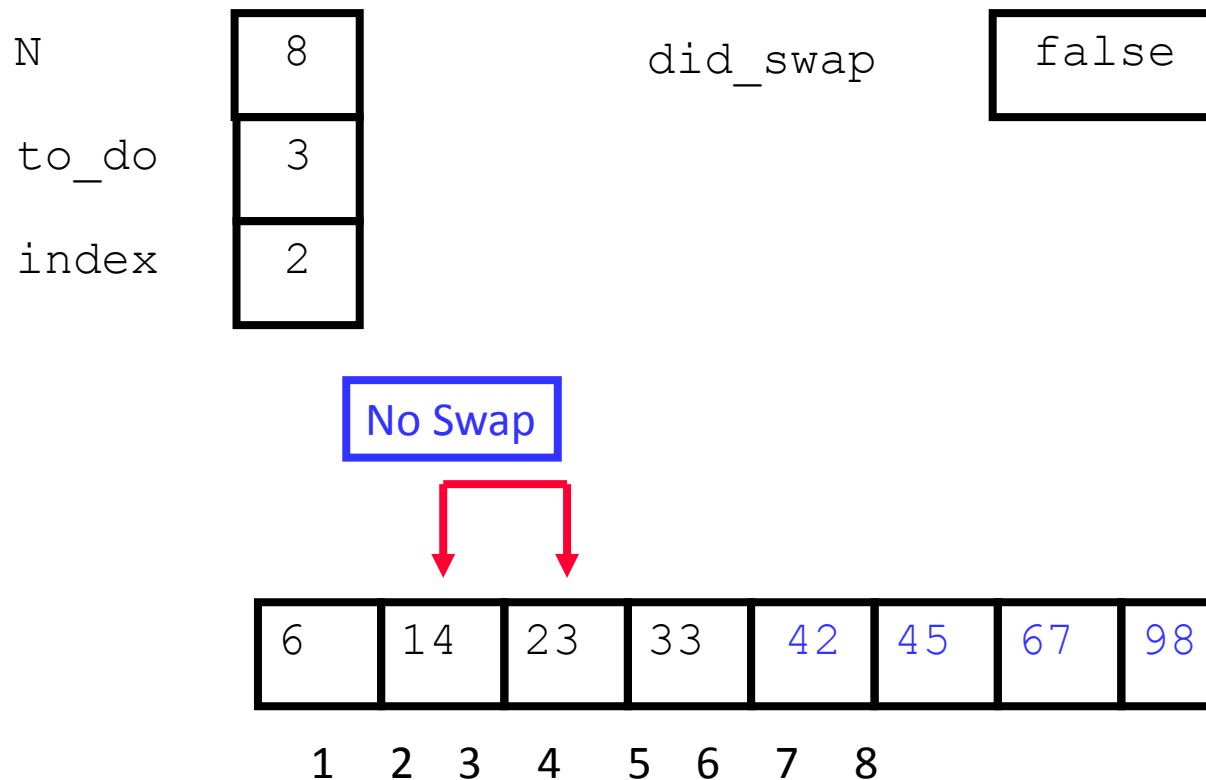
The Fifth “Bubble Up”



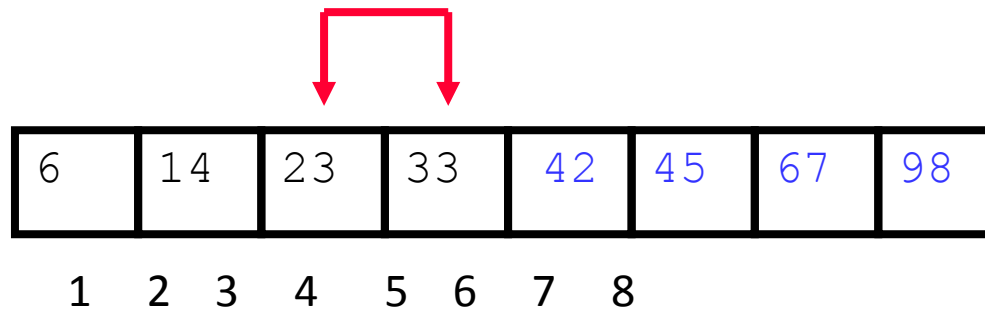
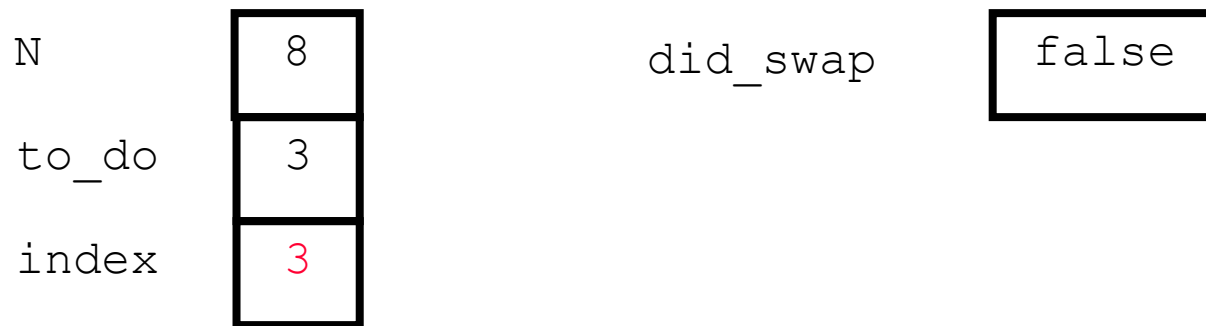
The Fifth “Bubble Up”



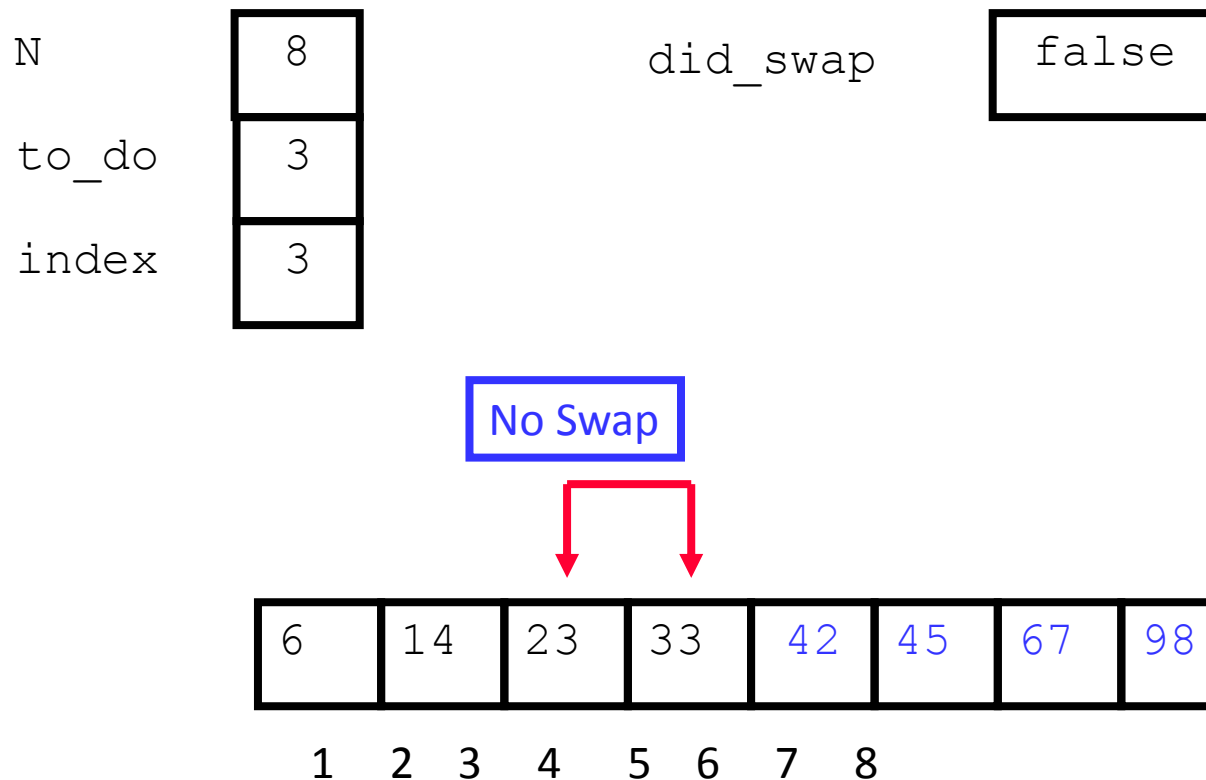
The Fifth “Bubble Up”



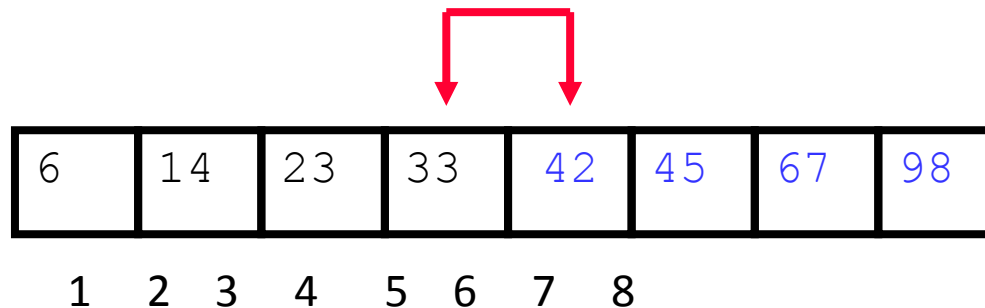
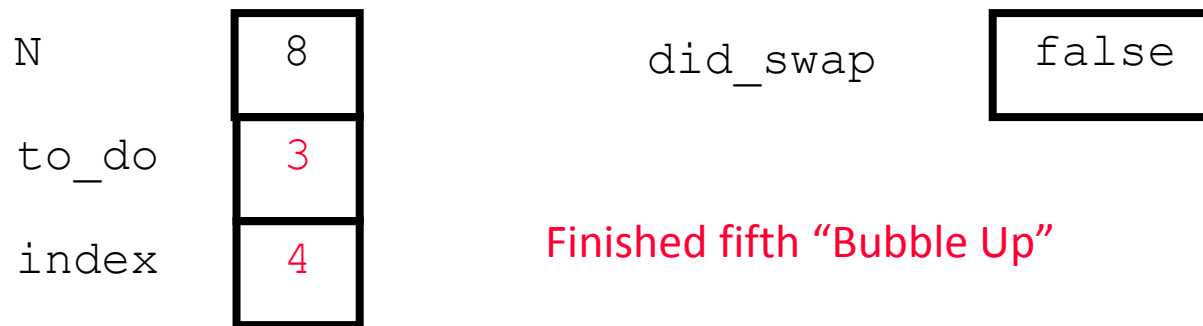
The Fifth “Bubble Up”



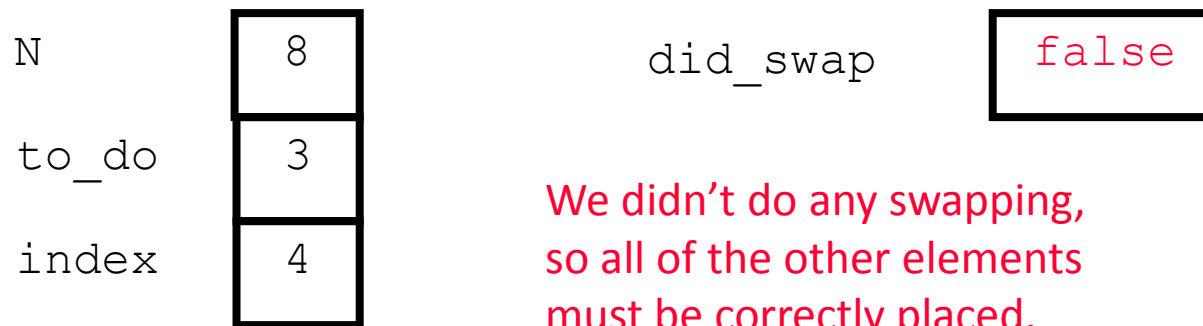
The Fifth “Bubble Up”



After Fifth Pass of Outer Loop



Finished “Early”



We didn't do any swapping,
so all of the other elements
must be correctly placed.

We can “skip” the last two
passes of the outer loop.

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|---|----|----|----|----|----|----|----|
| 6 | 14 | 23 | 33 | 42 | 45 | 67 | 98 |
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |



Summary

- “Bubble Up” algorithm will **move largest value to its correct location** (to the right)
- Repeat “Bubble Up” until all elements are correctly placed:
 - **Maximum of N-1 times**
 - Can finish early if **no swapping** occurs
- We reduce the number of elements we compare each time one is correctly placed



Mergesort



Sorting

- **Sorting takes an unordered collection and makes it an ordered one.**

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|----|----|----|----|-----|---|
| 1 | 2 | 3 | 4 | 5 | 6 |
| 77 | 42 | 35 | 12 | 101 | 5 |

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|---|----|----|----|----|-----|
| 1 | 2 | 3 | 4 | 5 | 6 |
| 5 | 12 | 35 | 42 | 77 | 101 |



Divide and Conquer

- **Divide and Conquer cuts the problem in half each time, but uses the result of both halves:**
 - cut the problem in half until the problem is trivial
 - solve for both halves
 - combine the solutions



Mergesort

- A divide-and-conquer algorithm:
- Divide the unsorted array into 2 halves until the sub-arrays only contain one element
- Merge the sub-problem solutions together:
 - Compare the sub-array's first elements
 - Remove the smallest element and put it into the result array
 - Continue the process until all elements have been put into the result array

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| 37 | 23 | 6 | 89 | 15 | 12 | 2 | 19 |
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Algorithm

Mergesort(Passed an array)

if array size > 1

Divide array in half

Call Mergesort on first half.

Call Mergesort on second half.

Merge two halves.

Merge(Passed two arrays)

Compare leading element in each array

Select lower and place in new array.

(If one input array is empty then place
remainder of other array in output array)



Algorithm

Mergesort(Passed an array)

if array size > 1

Divide array in half

Call Mergesort on first half.

Call Mergesort on second half.

Merge two halves.

Merge(Passed two arrays)

Compare leading element in each array

Select lower and place in new array.

(If one input array is empty then place
remainder of other array in output array)



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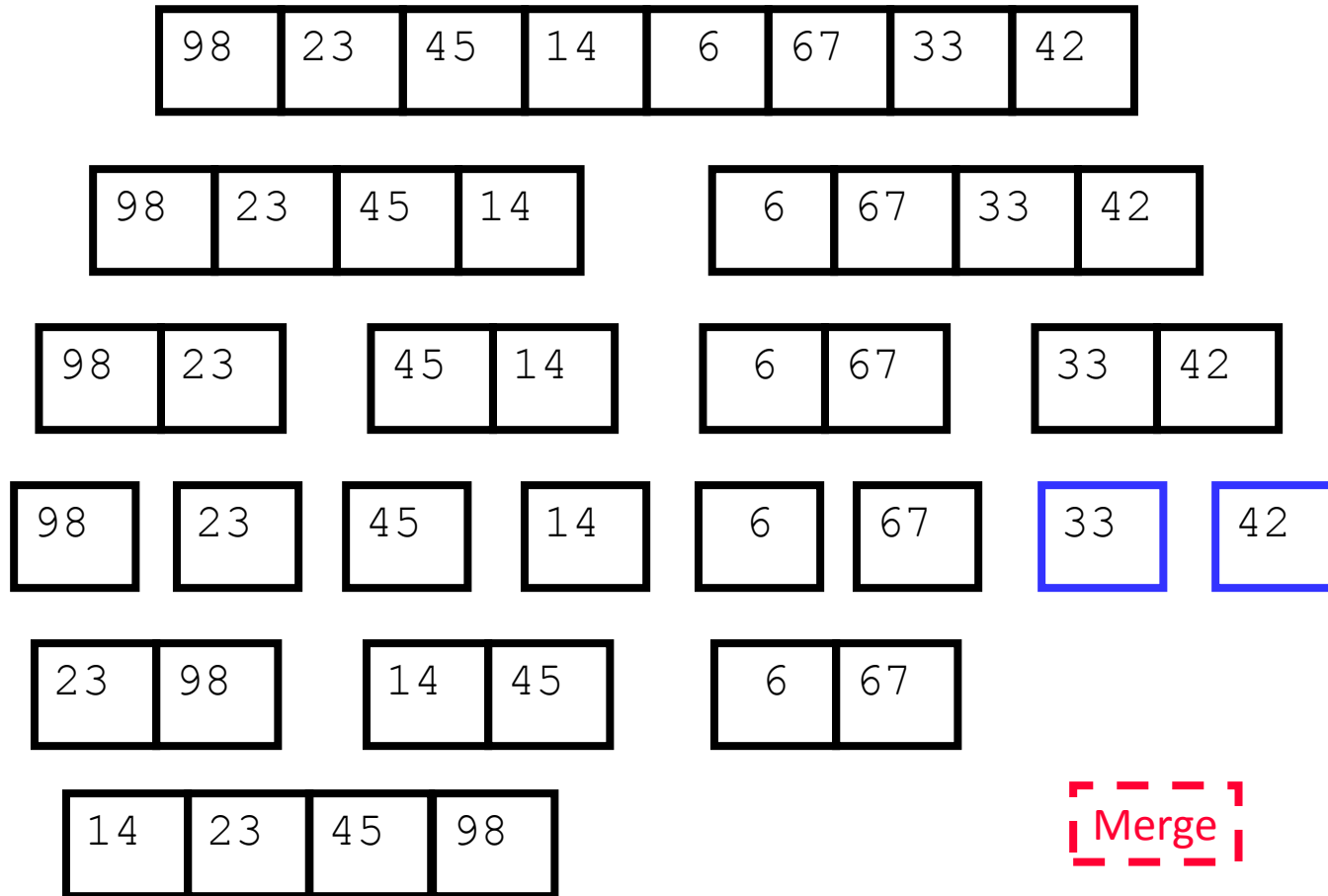
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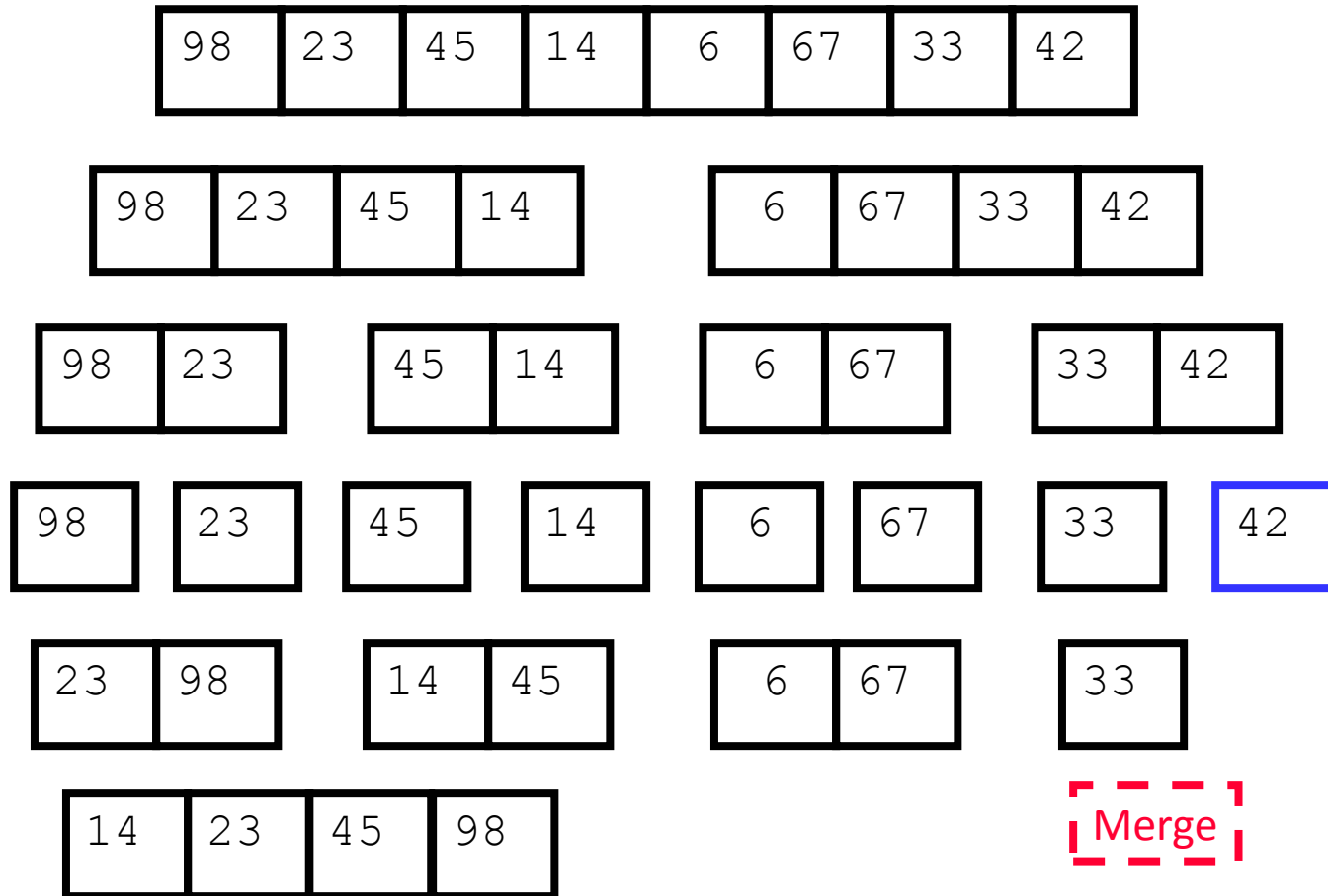
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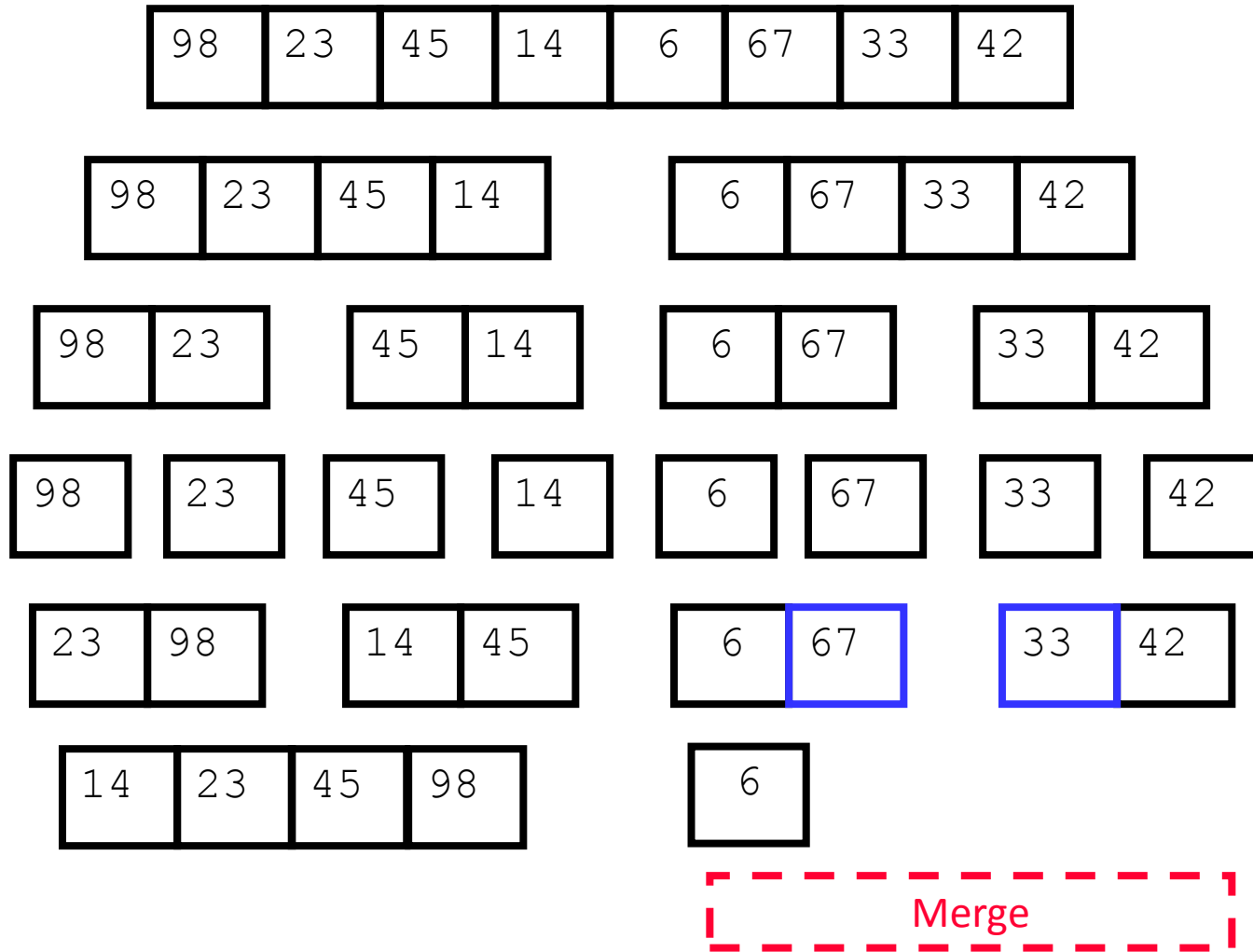
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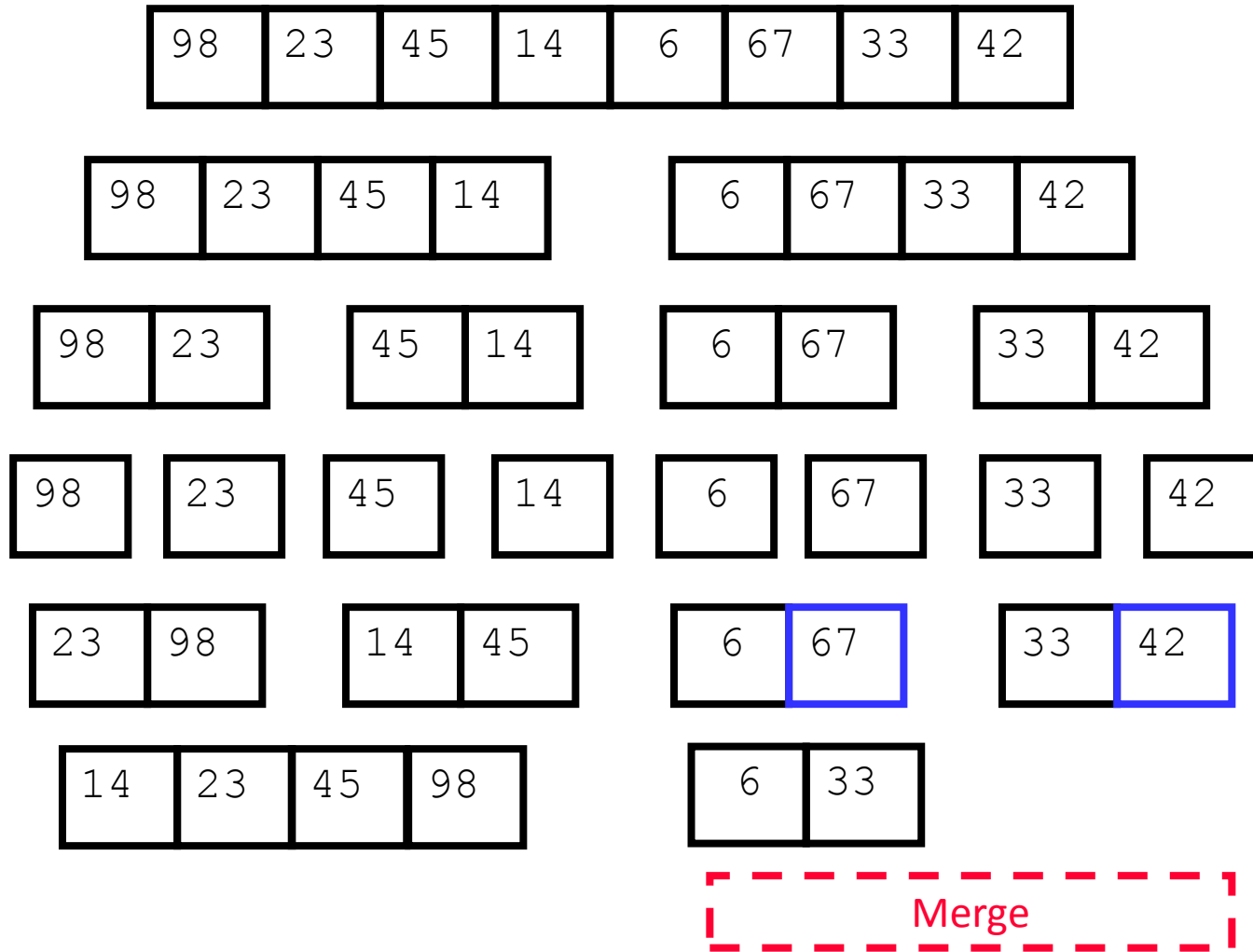
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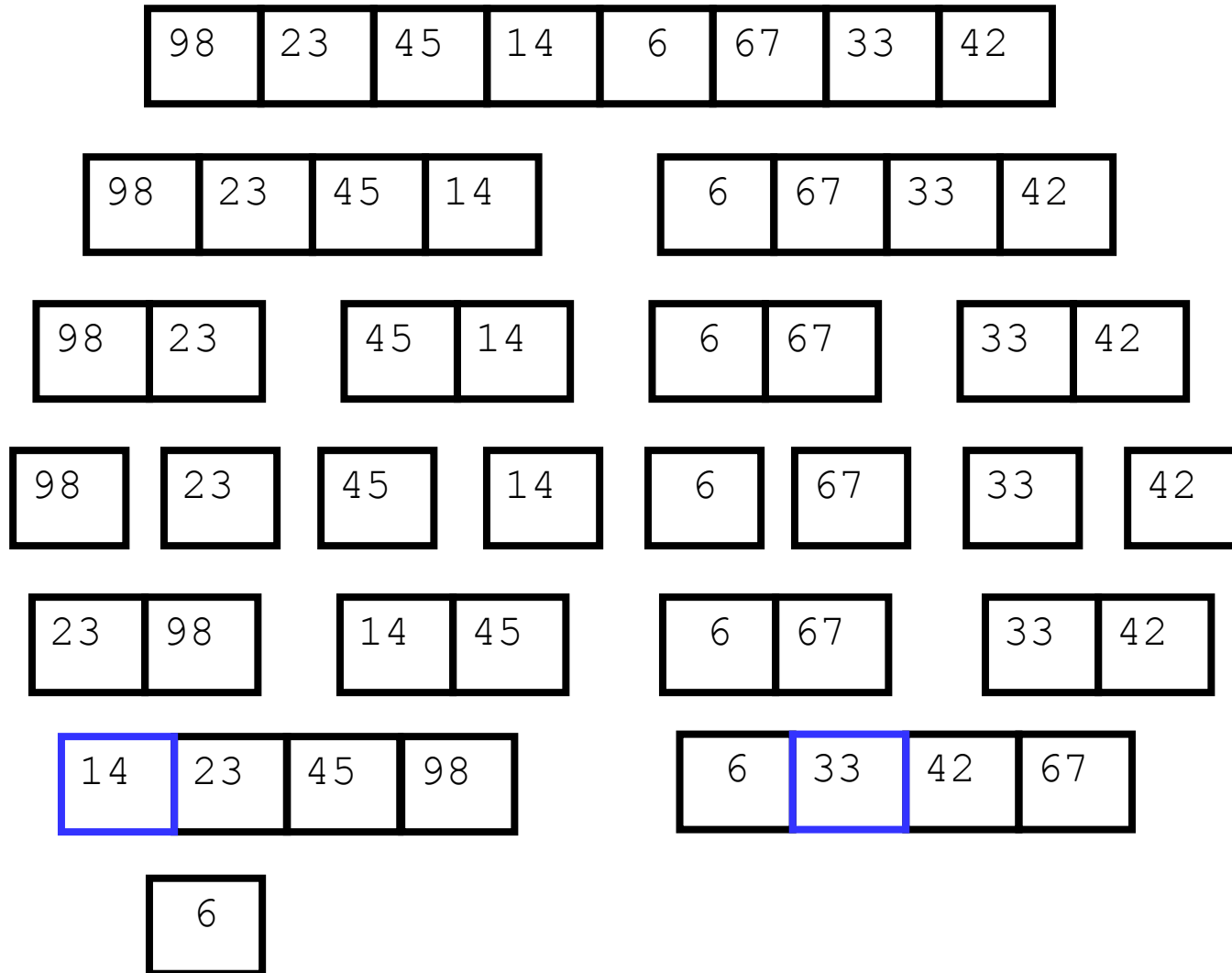
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Summary

- **Divide** the unsorted collection **into two**
- **Until the sub-arrays only contain one element**
- **Then merge the sub-problem solutions together**



Exercise

- Given a randomly ordered set of n numbers, design and develop an algorithm to sort them into non-descending order using bubble sort and merge sort compare their efficiency. Tabulate the output for various inputs and verify against expected values. Analyse the efficiency of both the algorithms. Describe your learning along with the limitations of both, if any. Suggest how these can be overcome.



Key factors for discussion and analysis

- Implement bubble sort and sort random integers
- Implement merge sort and sort random integers
- Analyse the performance of both
- List out the advantages and disadvantages of both



Results and Presentations

- Calculations/Computations/Algorithms

The calculations/computations/algorithms involved in each program has to be presented

- Presentation of Results

The results for all the valid and invalid cases have to be presented

- Analysis and Discussions

how the data is manipulated or transformed, what are the key operations involved. Errors encounters and how they are resolved.

- Conclusions

Summary



Comments

- Limitations of Experiments

Outline the loopholes in the program, data structures or solution approach.

- Limitations of Results

Present the test cases; justify if the program is tested correctly considering all the outcomes. Mention what is not tested, if any.

- Learning happened

What is the overall learning happened

- Conclusions

Summary



References

- Gilberg, R. F., and Forouzan, B. A. (2007): A Pseudocode Approach With C, 2nd edn. Cengage Learning

