Solving Recurrence Relation and Verifying

```
format long
```

Given the recurrence relation : $s_n = a_1 s_{n-1} + a_2 s_{n-2} + a_3 s_{n-3}$

Given values of s

```
s = [44 -3.87138 16.4883 -12.9416 9.9215 -12.3688]
```

```
s = 1×6
  44.000000000000000  -3.871380000000000  16.488299999999999 -12.941599999999999 ···
```

To find the values of a_1, a_2, a_3, we substitute the values of $n = 3, 4, 5$ to obtain 3 equations with 3 variables

```
A = [s(3) s(2) s(1);
     s(4) s(3) s(2);
     s(5) s(4) s(3)]
```

```
A = 3×3
  16.488299999999999  -3.871380000000000  44.000000000000000
 -12.941599999999999  16.488299999999999  -3.871380000000000
   9.921500000000000 -12.941599999999999  16.488299999999999
```

```
B = [s(4);
     s(5);
     s(6)]
```

```
B = 3×1
 -12.941599999999999
   9.921500000000000
 -12.368800000000000
```

Solving for the values of a_1, a_2, a_3

```
C = A\B
```

```
C = 3×1
   0.286223877779394
   0.747586388319374
  -0.335608049363139
```

Given the recurrence relation

$s_n = a_1 s_{n-1} + a_2 s_{n-2} + a_3 s_{n-3}$

```
syms r
```

```
eqn = r^3-C(1)*r^2-C(2)*r-C(3) == 0;
```

The Characteristic Equation

```
vpa(eqn, 10)
```

ans = $r^3 - 0.2862238778\, r^2 - 0.7475863883\, r + 0.3356080494 = 0.0$

```
soln = solve(eqn);
rs = vpa(soln, 15)
```

rs =

$$\begin{pmatrix} -0.921050902503191 \\ 0.601916591678999 \\ 0.605358188603586 \end{pmatrix}$$

Verifying if the roots found are correct

```
subs(eqn, rs)
```

ans =

$$\begin{pmatrix} 3.2754503563770089151419460804602\ 10^{-33} = 0 \\ 3.2754397034594545881614051137854\ 10^{-33} = 0 \\ 3.2754397034594545881614051137854\ 10^{-33} = 0 \end{pmatrix}$$

```
rts = roots([1 -C(1) -C(2) -C(3)])'
```

```
rts = 1×3
  -0.921050902503191   0.605358188603577   0.601916591679009
```

Solving

$S_n = b_1 r_1^n + b_2 r_2^n + b_3 r_3^n$

n = 0, 1, 2

$S_0 = b_1 r_1^0 + b_2 r_2^0 + b_3 r_3^0$
$S_1 = b_1 r_1^1 + b_2 r_2^1 + b_3 r_3^1$
$S_2 = b_1 r_1^2 + b_2 r_2^2 + b_3 r_3^2$

```
R = [rts.^0; rts.^1; rts.^2]
```

```
R = 3×3
   1.000000000000000   1.000000000000000   1.000000000000000
  -0.921050902503191   0.605358188603577   0.601916591679009
   0.848334765001942   0.366458536509404   0.362303583338475
```

```
Sc = [s(1); s(2); s(3)]
```

```
Sc = 3×1
  44.000000000000000
  -3.871380000000000
  16.488299999999999
```

```
B = R\Sc
```

```
B = 3×1
10^3 ×
   0.015999949514285
  -1.739979185917188
   1.767979236402903
```

```
syms n;
B = B'
```

```
B = 1×3
10^3 ×
   0.015999949514285  -1.739979185917188   1.767979236402903
```

The Explicit Relation of the Recurrence Relation

```
Sn = vpa(B(1)*(rts(1)^n)+B(2)*(rts(2)^n)+B(3)*(rts(3)^n),10)
```

Sn = $1767.979236\,0.6019165917^n - 1739.979186\,0.6053581886^n + 15.99994951\,\left(-0.9210509025\right)^n$

Substituting the values of n = 0, 1, 2, 3, 4, 5 into this explicit equation

```
vpa(subs(Sn, [0 1 2 3 4 5]), 10)
```

ans = $\left(44.0 \quad -3.87138 \quad 16.4883 \quad -12.9416 \quad 9.9215 \quad -12.3688\right)$

Verifying with the actual values given

```
vpa(s, 10)
```

ans = $\left(44.0 \quad -3.87138 \quad 16.4883 \quad -12.9416 \quad 9.9215 \quad -12.3688\right)$

Verification of the answer using Symbolic Math

Substituting n = n-1, n-2, n-3 into the equation obtained.

```
snm1 = subs(Sn, n-1);
snm2 = subs(Sn, n-2);
snm3 = subs(Sn, n-3);
```

substituting these obtained values into $s_n = a_1 s_{n-1} + a_2 s_{n-2} + a_3 s_{n-3}$

```
SnOri = C(1)*snm1 + C(2)*snm2 + C(3)*snm3;
pretty(vpa(SnOri, 10))
```

```
                          n - 1.0                              n - 3.0                            n - 2.0
506.0378729 0.6019165917        - 5.369711846 (-0.9210509025)        + 1321.717212 0.6019165917

                              n - 1.0                           n - 2.0                            n - 3.0
   - 498.0235898 0.6053581886        - 1300.784755 0.6053581886        - 593.3480628 0.6019165917

                              n - 3.0                              n - 1.0                              n - 2.0
   + 583.9510205 0.6053581886        + 4.579567594 (-0.9210509025)        + 11.96134447 (-0.9210509025)
```

```
SnOri = vpa(simplify(SnOri),10);
SnObt = vpa(simplify(Sn), 10);
```

where SnOri is the substituted equation and SnObt is the obtained equation, and comparing them using `isequal`

```
isequal(round(SnObt), round(SnOri))
```

```
ans = logical
   1
```

Thus we conclude that the obtained Explicit Relation is equivalent to the Recurrence Relation