RAMAIAH UNIVERSITY OF APPLIED SCIENCES

ASSIGNMENT

Course Code	BSC207A
Course Name	Engineering Mathematics - 3
Programme	B.Tech
Department	CSE
Faculty	FET

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Semester/Year	03/2018
Course Leader/s	Dr Mahadev Channakote

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<tr><td>Student Name</td><td colspan="5">Satyajit Ghana</td></tr>
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Declaration

The assignment submitted herewith is a result of my own investigations and that I have conformed to the guidelines against plagiarism as laid out in the Student Handbook. All sections of the text and results, which have been obtained from other sources, are fully referenced. I understand that cheating and plagiarism constitute a breach of University regulations and will be dealt with accordingly.

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Question No. 1

Solution to Question No. 1:

1.1 Determining the values of a, b, c, for which the given ideal fluid is irrotational:

Given the vector field as,

$$F(x,y,z) = \left(y^a \sin x + z^b\right)i - (3y^2 \cos x - ce^{cy}z)j + (5xz^4 + e^{7y})k \quad -(1)$$

For the given ideal fluid to be irrotational the curl of the vector has to be zero,

$$\nabla \times \vec{F} = \vec{0}$$

$$\begin{vmatrix} i & j & k \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ y^a \sin x + z^b & -3y^2 \cos x + ce^{cy}z & 5xz^4 + e^{7y} \end{vmatrix} = \vec{0}$$

Therefore,

$$\left[\frac{\partial}{\partial y}(5xz^4 + e^{7y}) - \frac{\partial}{\partial z}(-3y^2 \cos x + ce^{cy}z)\right] i - \left[\frac{\partial}{\partial x}(5xz^4 + e^{7y}) - \frac{\partial}{\partial z}\left(y^a \sin x + z^b\right)\right] j$$
$$+ \left[\frac{\partial}{\partial x}(-3y^2 \cos x + ce^{cy}z) - \frac{\partial}{\partial y}\left(y^a \sin x + z^b\right)\right] k = \vec{0}$$

$$(7e^{7y} - ce^{cy})\hat{\imath} - \left(5z^4 - bz^{b-1}\right)\hat{\jmath} + (3y^2 \sin x - ay^{a-1} \sin x)\hat{k} = \vec{0} \quad -(2)$$

By comparing the LHS and RHS of Equation (2),

$$7e^{7y} - ce^{cy} = 0$$
$$7e^{7y} = ce^{cy}$$
$$\boldsymbol{c = 7}$$

$$5z^4 - bz^{b-1} = 0$$
$$5z^4 = bz^{b-1}$$
$$\boldsymbol{b = 5}$$

$$3y^2 \sin x - ay^{a-1} \sin x = 0$$
$$3y^2 = ay^{a-1}$$
$$\boldsymbol{a = 3}$$

Hence, for $(a,b,c) = (3,5,7)$ the given vector field F is irrotational

$$F(x,y,z) = (y^3 \sin x + z^5)i - (3y^2 \cos x - 7e^{7y}z)j + (5xz^4 + e^{7y})k$$

1.2 Verify whether the irrotational vector field is incompressible or not:

The condition for the conservation of mass or the continuity equation of compressible fluid flow is given by

$$\frac{\partial \rho}{\partial t} + \mathrm{div}(\rho \mathrm{v}) = 0 - (3)$$

If the flow is steady then $\frac{\partial \rho}{\partial t} = 0$, and if the fluid is incompressible then ρ is constant, equation (3) becomes,

$$\mathrm{div}\,\mathrm{v} = 0$$

Where div is defined as,

$$\mathrm{div}\,\mathrm{v} = \nabla \cdot \mathrm{v} = \frac{\partial v_1}{\partial x} + \frac{\partial v_2}{\partial y} + \frac{\partial v_3}{\partial z}$$

Applying divergence on equation (1)

$$\frac{\partial}{\partial x}(y^3 \sin x + z^5) - \frac{\partial}{\partial y}(3y^2 \cos x - 7e^{7y}z) + \frac{\partial}{\partial z}(5xz^4 + e^{7y})$$

$$y^3 \cos x - 6y \cos x + 49e^{7y}z + 20xz^3 \neq 0$$

Hence the given vector field is not incompressible

1.3 Obtain the scalar potential $\boldsymbol{\phi}$ such that $\boldsymbol{F(x, y, z) = \nabla\phi}$:

$$\overrightarrow{F} = \nabla \overrightarrow{\phi}$$

$$(y^3 \sin x + z^5)i - (3y^2 \cos x - 7e^{7y}z)j + (5xz^4 + e^{7y})k = \frac{\partial \phi}{\partial x}i + \frac{\partial \phi}{\partial y}j + \frac{\partial \phi}{\partial z}k \quad - (4)$$

Equating the individual terms

$$\frac{\partial \phi}{\partial x} = y^3 \sin x + z^5 \quad - (5)$$

$$\frac{\partial \phi}{\partial y} = -3y^2 \cos x + 7e^{7y}z \quad - (6)$$

$$\frac{\partial \phi}{\partial z} = 5xz^4 + e^{7y} \quad - (7)$$

Solving (5)

$$\phi = \int (y^3 \sin x + z^5) \partial x$$

$$\phi = -y^3 \cos x + z^5 x + P(y, z) \quad - (8)$$

Solving (6)

$$\phi = \int (-3y^2 \cos x + 7e^{7y}z) \partial y$$

$$\phi = -y^3 \cos x + e^{7y}z + Q(x, z) \quad - (9)$$

Solving (7)

$$\phi = \int (5xz^4 + e^{7y})\, \partial z$$

$$\phi = xz^5 + e^{7y}z + R(x, y) \quad - (10)$$

Comparing equation (8) and (9)

$$P(y,z) = e^{7y}z$$
$$Q(x,z) = xz^5$$

Comparing equation (9) and (10)

$$R(x,y) = -y^3 \cos x$$

Therefore,

$$\phi = -y^3 \cos x + e^{7y}z + z^5 x$$

1.4 Plot the given vector field in the intervals $-4 \leq x \leq 4, -4 \leq y \leq 4, -4 \leq z \leq 4,$:

```
X = linspace(-4, 4, 20); Y = linspace(-4, 4, 20); Z = linspace(-4, 4, 20);
[x, y, z] = meshgrid(X, Y, Z);

u = y.^3.*sin(x)+z.^5;
v = -3.*y.^2.*cos(x)+7.*exp(7.*y).*z;
w = 5.*x.*z.^4+exp(7.*y);

%% Quiver Plot

quiver3(x, y, z, u, v, w, 'LineWidth', 1);
hold on;
grid on;
box on;
title('Quiver Plot', 'Interpreter', 'latex')

%% Stream Slice

streamslice(x, y, z, u, v, w, 4, 4, -4);
grid on;
hold on;
box on;
axis([-4 4 -4 4 -4 4]);
view(-50, 30);
title('Stream Slice', 'Interpreter', 'latex')
```

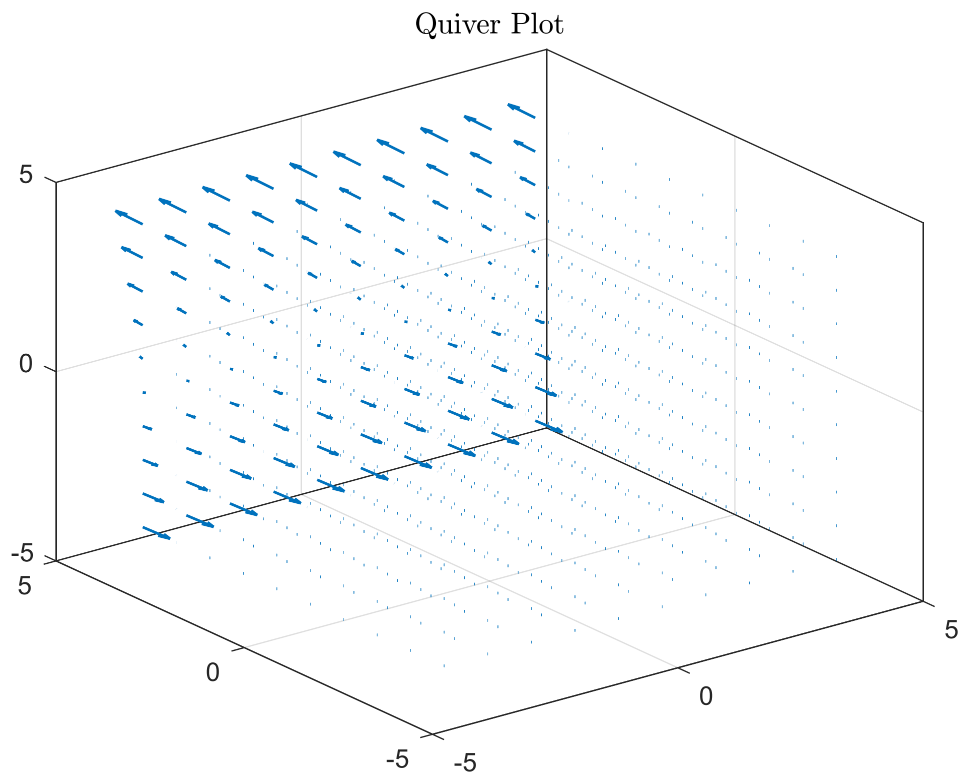

Figure A1.1: Quiver Plot

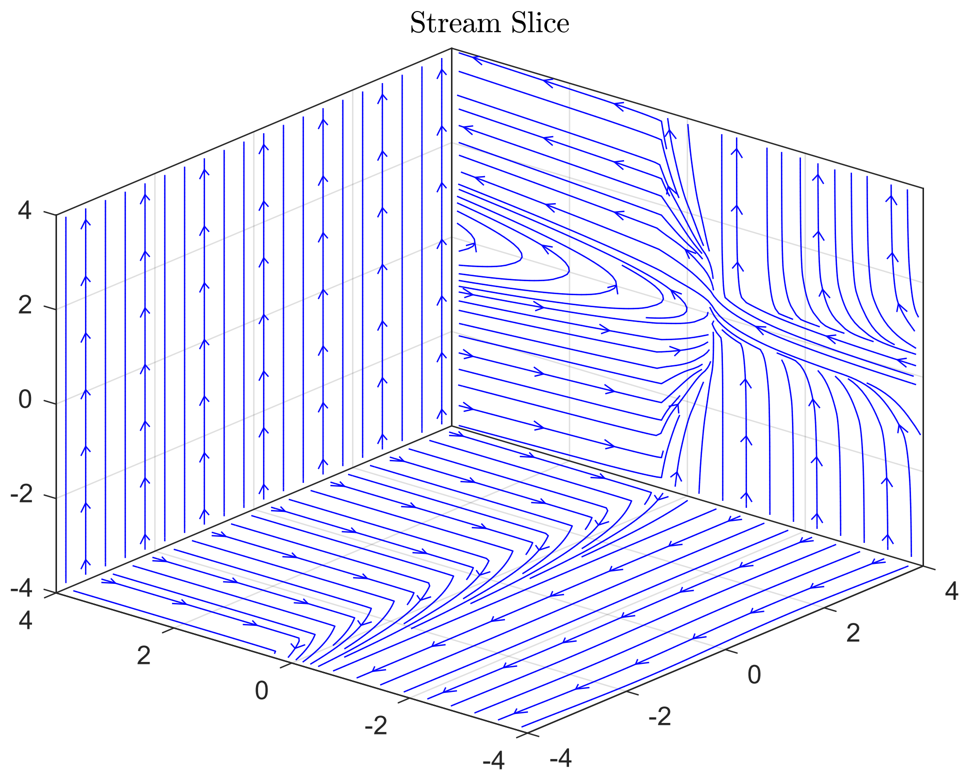

Figure A1.2: StreamSlice Plot

1.5 Conclusion:

The given vector is an ideal fluid, which is irrotational for certain values of $a, b, c = 3,5,7$, i.e. the curl of the vector is found to be zero at those values of constants. For these values, the given fluid is not incompressible and the divergence of the vector is non-zero. In physical terms, the divergence of a three-dimensional vector field is the extent to which the vector field flow behaves like a source at a given point. It is a local measure of its "outgoingness" – the extent to which there is more of some quantity exiting an infinitesimal region of space than entering it.

From the quiver plot of the field from $-4 \leq x \leq 4, -4 \leq y \leq 4, -4 \leq z \leq 4$ it can be observed that the vector field flips its direction from the plane $z = 0$. The magnitude is higher at the ends compared to the plane $z = 0$.

Question No. 2

Solution to Question No. 2:

Given Data

t (sec)	0	T/6	T/3	T/2	2T/3	5T/6	T
A (ampere)	1.98	1.30	1.05	1.30	−0.88	−0.25	1.98

2.1 Write the Fourier series expansion for the given data points up to third harmonics using Harmonic analysis method:

Length of the interval is 6, we have $2l = 6$, or $l = \frac{6}{2} = 3$

$$A(t) = \frac{a_0}{2} + \sum a_n \cos\left(\frac{2n\pi t}{6}\right) + \sum b_n \sin\left(\frac{2n\pi t}{6}\right)$$

Writing $\frac{2\pi t}{6} = \theta$

The Fourier series of the given function up to third harmonics is given by,

$$A(t) = \frac{a_0}{2} + a_1 \cos\theta + b_1 \sin\theta + a_2 \cos 2\theta + b_2 \sin 2\theta + a_3 \cos 3\theta + b_3 \sin 3\theta$$

Where,

$$a_0 = \frac{1}{N}\sum A(t)$$

$$a_n = \frac{2}{N}\sum A(t)\cos n\theta$$

$$b_n = \frac{2}{N}\sum A(t)\sin n\theta$$

Let $A(t) = y$

Taking $T = 2\pi$

t	θ	y	y cos θ	y sin θ	y cos 2θ	y sin 2θ	y cos 3θ	y sin 3θ
0	0	1.980	1.980	0.000	1.980	0	1.980	0
$\frac{T}{6}$	$\frac{2\pi}{6}$	1.300	0.650	1.125	−0.650	0.125	−1.300	-0.041×10^{-14}
$\frac{T}{3}$	$\frac{4\pi}{6}$	1.050	−0.525	0.909	−0.525	−0.909	1.050	-0.067×10^{-14}
$\frac{T}{2}$	$\frac{6\pi}{6}$	1.300	−1.300	0.000	1.300	0	−1.300	-0.047×10^{-14}
$\frac{2T}{3}$	$\frac{8\pi}{6}$	−0.880	0.440	0.762	0.440	−0.762	−0.880	-0.113×10^{-14}
$\frac{5T}{6}$	$\frac{10\pi}{6}$	−0.250	−0.125	0.216	0.125	0.216	0.250	-0.015×10^{-14}

$$\sum y = 4.5$$
$$\sum y \cos\theta = 1.120$$
$$\sum y \sin\theta = 3.013$$
$$\sum y \cos 2\theta = 2.670$$
$$\sum y \sin 2\theta = -0.329$$
$$\sum y \cos 3\theta = -0.2$$
$$\sum y \sin 3\theta = -5.502 \times 10^{-16}$$

$$a_0 = \frac{1}{N}\sum y = \frac{1}{6} \times 4.5 = 0.75$$
$$a_1 = \frac{2}{N}\sum y \cos\theta = \frac{2}{6} \times 1.120 = 0.3733$$
$$b_1 = \frac{2}{N}\sum y \sin\theta = \frac{2}{6} \times 3.013 = 1.0045$$
$$a_2 = \frac{2}{N}\sum y \cos 2\theta = \frac{2}{6} \times 2.670 = 0.8900$$
$$b_2 = \frac{2}{N}\sum y \sin 2\theta = \frac{2}{6} \times -0.329 = -0.1096$$
$$a_3 = \frac{2}{N}\sum y \cos 3\theta = \frac{2}{6} \times -0.2 = -0.6667$$
$$b_3 = \frac{2}{N}\sum y \sin 3\theta = \frac{2}{6} \times -5.502 \times 10^{-16} = -1.8342 \times 10^{-16}$$

Now,

$$A(t) = 0.75 + 0.3733\cos t + 1.0045\sin t + 0.8900\cos 2t - 0.1096\sin 2t - 0.6667\cos 3t - 1.8342 \times 10^{-16}\sin 3t$$

2.2 Write a MATLAB function to estimate the Fourier coefficients of first and second harmonics to be fitted above data:

harmonic_analysis.m

```
function [HS, a0, a, b] = harmonic_analysis(x, y, nh)
%HARMONIC_ANALYSIS
% Author : Satyajit Ghana
% Computes the Harmonic function for a given set of values (x_i, y_i)
% Conditions for x : must be of period 2*pi, angles should be given in
% radians
%
% USAGE : x = linspace(0, 10*pi/6, 6)
% y = [1.98 1.30 1.05 1.30 -0.88 -0.25]
% HS = harmonic_analysis(x, y, 3)
%
% Extra Params : number of harmonics, default : 2

% handle the default parameter
if nargin < 2
    nh = 2;
```

```
end

T = 2 * pi;
w = (2*pi)/T;

%x = (pi/180)*x;
syms t;

a0 = 2 * mean(y);
HS = a0/2;

h = nh; % number of harmonics
for i=1:h
    a(i) = 2 * mean(y.*cos(i*w*x));
    b(i) = 2 * mean(y.*sin(i*w*x));

    HS = HS + a(i) * cos(i*w*t) + b(i) * sin(i*w*t);
end

fplot(HS, [0 x(end)], 'LineWidth', 2)
hold on;
plot(x, y ,'*', 'LineWidth', 2)
grid on;

title('$ $ Harmonic Analysis', 'Interpreter', 'latex')
legend({'Harmonic Function', 'Data Points'}, 'Interpreter', 'latex')
xlabel('x $\rightarrow$', 'Interpreter', 'latex')
ylabel('f(x) $\rightarrow$', 'Interpreter', 'latex')

end
```

matlab_script.m

```
X = linspace(0, 10*pi/6, 6);
Y = [1.98 1.30 1.05 1.30 -0.88 -0.25];
for i=1:1:2
    [HS, a0, a, b] = harmonic_analysis(X, Y, i);
    disp("Harmonic Function for " + num2str(i) + " harmonics");
    disp(vpa(HS, 4));
    disp('Coefficients : ')
    disp(a0);
    disp(a);
    disp(b);
end
```

OUTPUT

```
Harmonic Function for 1 harmonics
0.3733*cos(t) + 1.005*sin(t) + 0.75

Coefficients :
    1.5000

    0.3733

    1.0046
```

```
Harmonic Function for 2 harmonics
0.89*cos(2.0*t) - 0.1097*sin(2.0*t) + 0.3733*cos(t) + 1.005*sin(t) +
0.75

Coefficients :
    1.5000

    0.3733    0.8900

    1.0046   -0.1097
```

2.3 Calculate the least square error between the observation and approximation by Fourier series:

$$E = \sum_{i=1}^{n} |O_i - F_i|^2$$

LSE.m

```
function [error] = LSE(y_actual, y_estimated)
%LSE Computes the least square error between the actual and the
estimated
%data points
% Author : Satyajit Ghana
%   uses sum(( y_actual - y_estimated)^2) the sum of square error
ya = y_actual;
ye = y_estimated;

error = sum((ya-ye).^2);

fprintf("Sum of Errors : %.12f\n", error);

end
```

matlab_script.m

```
X = linspace(0, 10*pi/6, 6);
Y = [1.98 1.30 1.05 1.30 -0.88 -0.25];
[HS, a0, a, b] = harmonic_analysis(X, Y, 3);
HS = matlabFunction(HS);
LSE(Y, HS(X));
```

OUTPUT

```
Sum of Errors : 0.006666666667
```

2.4 Plot the observed data points using symbol '*' and approximation by Fourier series as solid line in same graph:

```
X = linspace(0, 10*pi/6, 6);
Y = [1.98 1.30 1.05 1.30 -0.88 -0.25];
[HS, a0, a, b] = harmonic_analysis(X, Y, 3);
HS = matlabFunction(HS);
fplot(HS, [0 x(end)], 'LineWidth', 2)
hold on;
plot(X, Y ,'*', 'LineWidth', 2)
grid on;

title('$ $ Harmonic Analysis', 'Interpreter', 'latex')
```

```
legend({'Harmonic Function', 'Data Points'}, 'Interpreter', 'latex')
xlabel('x $\rightarrow$', 'Interpreter', 'latex')
ylabel('f(x) $\rightarrow$', 'Interpreter', 'latex')
```

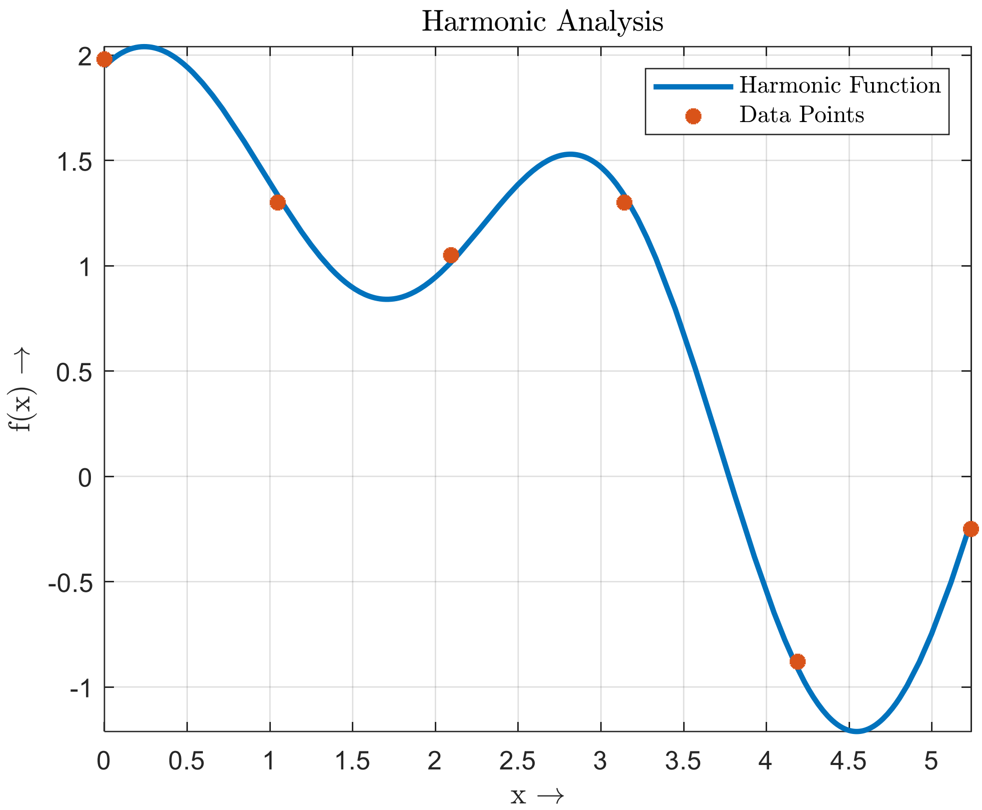

Figure B1.1 Harmonic Function Plot

2.5 Conclusion:

The given values of x and y are discrete data points, Fourier Series for these discrete data points can be performed using Harmonic Analysis. The given data is a data points of time and the current in the circuit. The first and the last data points are same, and hence omitted. The time period is taken as 2π. And the harmonic analysis was done for the first three harmonics. The function obtained has some deviation from the actual data points, this is quantified using a least square error, which was calculated and found to be very less, i.e. the curve fits the data points very well. The data points and the function are then plotted on a graph for graphical interpretation. Harmonic Analysis is a great way for curve fitting with very less computation and less error.