ASSIGNMENT

Course Code	BSC207A
Course Name	Engineering Mathematics - 3
Programme	B.Tech
Department	CSE
Faculty	FET

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Reg. No	17ETCS002159
Semester/Year	03/2018
Course Leader/s	Dr. Mahadev Channakote

<table>
<tr><th colspan="4">Declaration Sheet</th></tr>
<tr><td>Student Name</td><td colspan="3">Satyajit Ghana</td></tr>
<tr><td>Reg. No</td><td colspan="3">17ETCS002159</td></tr>
<tr><td>Programme</td><td>B.Tech</td><td>Semester/Year</td><td>03/2018</td></tr>
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<tr><td>Course Title</td><td colspan="3">Engineering Mathematics - 3</td></tr>
<tr><td>Course Date</td><td></td><td>to</td><td></td></tr>
<tr><td>Course Leader</td><td colspan="3">Dr. Mahadev Channakote</td></tr>
</table>

Declaration

The assignment submitted herewith is a result of my own investigations and that I have conformed to the guidelines against plagiarism as laid out in the Student Handbook. All sections of the text and results, which have been obtained from other sources, are fully referenced. I understand that cheating and plagiarism constitute a breach of University regulations and will be dealt with accordingly.

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<tr><td>Signature of the Student</td><td></td><td>Date</td><td></td></tr>
<tr><td>Submission date stamp
(by Examination & Assessment Section)</td><td colspan="3"></td></tr>
</table>

Signature of the Course Leader and date	Signature of the Reviewer and date

Question No. 1

Solution to Question No. 1:

1.1 Obtain the mathematical model:

The General form of an Ordinary Differential Equation for a Spring-Mass system is:

$$m\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + kx = f(t)$$

Where,

$m: mass\ of\ the\ attached\ load$

$k: spring\ constant$

$\beta: damping\ constant$

$f(t): external\ force$

From the given data, $m = 8\ kg, k = 5\frac{N}{m}, f(t) = 5\cos 2t\ , \beta\frac{dx}{dt} = 2y'(t)$

Here the displacement variable is taken as y

Now, the equation becomes

$$8\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 5y(t) = 5\cos 2t \qquad -(1)$$

1.2 Solution of the model using Laplace transform:

We can write the equation (1) in a simplified form as

$$8y'' + 2y' + 5y = 5\cos 2t \qquad -(2)$$

Applying Laplace transforms to equation (2), we obtain,

$$8 \times \mathcal{L}\{y''\} + 2 \times \mathcal{L}\{y'\} + 5 \times \mathcal{L}\{y\} = 5 \times \mathcal{L}\{\cos 2t\} \qquad -(3)$$

Using the property,

$$\mathcal{L}\left\{\frac{d^n y(t)}{dt^n}\right\} = s^n F(s) - s^{n-1}y(0) - s^{n-2}y'(0) - s^{n-3}y''(0)\ .\ .\ .\ . - sy^{n-2}(0) - y^{n-1}(0)$$

Applying this property in (3)

$$8 \times [s^2Y(s) - sy(0) - y'(0)] + 2 \times [sY(s) - y(0)] + 5 \times Y(s) = 5 \times \frac{s}{s^2+4} \qquad -(4)$$

i. Given Initial Conditions

$y(0) = 0, y'(0) = 0$

Substituting these values in (4), we obtain

$$8 \times [s^2Y(s)] + 2 \times [sY(s)] + 5 \times Y(s) = 5 \times \frac{s}{s^2+4}$$

$$Y(s) = \frac{5s}{(s^2+4)(8s^2+2s+5)} \quad -(5)$$

Taking Inverse Laplace transform of (5),

$$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{5s}{(s^2+4)(8s^2+2s+5)}\right\}$$

Splitting the terms

$$y(t) = \mathcal{L}^{-1}\left\{\frac{As+B}{(s^2+4)} + \frac{Cs+D}{(8s^2+2s+5)}\right\} \quad -(6)$$

Where A, B, C, D are arbitrary constants

$$8A + C = 0$$

$$2A + 8B + D = 0$$

$$5A + 2B + 4C = 5$$

$$5B + 4D = 0$$

This forms a system of Linear Equations

$$\begin{pmatrix} 8 & 0 & 1 & 0 \\ 2 & 8 & 0 & 1 \\ 5 & 2 & 4 & 0 \\ 0 & 5 & 0 & 4 \end{pmatrix} \times \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 5 \\ 0 \end{pmatrix}$$

Solving this obtains:

```
>> inv([8 0 1 0; 2 8 0 1; 5 2 4 0; 0 5 0 4])*[0;0;5;0]

ans =

     -27/149
       8/149
     216/149
     -10/149
```

$$A = -\frac{27}{149}; B = \frac{8}{149}; C = \frac{216}{149}; D = -\frac{10}{149}$$

Factorizing $8s^2 + 2s + 5$ becomes $8\left[\left(s+\frac{1}{8}\right)^2 + \left(\frac{\sqrt{39}}{8}\right)^2\right]$ and substituting these values, Equation (6) becomes,

$$y(t) = -\frac{27}{149} \times \mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\} + \frac{8}{149} \times \mathcal{L}^{-1}\left\{\frac{1}{s^2+4}\right\} + \frac{216}{149}\mathcal{L}^{-1}\left\{\frac{s}{8\left[\left(s+\frac{1}{8}\right)^2 + \left(\frac{\sqrt{39}}{8}\right)^2\right]}\right\} - \frac{10}{149}\mathcal{L}^{-1}\left\{\frac{1}{8\left[\left(s+\frac{1}{8}\right)^2 + \left(\frac{\sqrt{39}}{8}\right)^2\right]}\right\}$$

Simplifying it,

$$y(t) = \frac{1}{149}\left[-27 \times \mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\} + 8 \times \mathcal{L}^{-1}\left\{\frac{1}{s^2+4}\right\} + 27 \times \mathcal{L}^{-1}\left\{\frac{s}{\left(s+\frac{1}{8}\right)^2 + \left(\frac{\sqrt{39}}{8}\right)^2}\right\} - \frac{5}{4} \times \mathcal{L}^{-1}\left\{\frac{1}{\left(s+\frac{1}{8}\right)^2 + \left(\frac{\sqrt{39}}{8}\right)^2}\right\}\right]$$

$$y(t) = \frac{1}{149}\left[-27\cos 2t + 4\sin 2t + 27e^{-\frac{t}{8}}\cos\left[\left(\frac{\sqrt{39}}{8}\right)t\right] - \frac{37}{\sqrt{39}}e^{-\frac{t}{8}}\sin\left[\left(\frac{\sqrt{39}}{8}\right)t\right]\right] \quad -(7)$$

ii. Given Initial Conditions

$y(0) = 0, y'(0) = 5$

Substituting these values in (4), we obtain

$$8 \times [s^2Y(s) - 5] + 2 \times [sY(s)] + 5 \times Y(s) = 5 \times \frac{s}{s^2+4}$$

$$Y(s) \times (8s^2 + 2s + 5) = \frac{5s}{s^2+4} + 40$$

$$Y(s) \times (8s^2 + 2s + 5) = \frac{5s + 40s^2 + 160}{s^2+4}$$

$$Y(s) = \frac{5(8s^2 + s + 32)}{(s^2+4)(8s^2+2s+5)} \quad -(8)$$

Taking Inverse Laplace transform of (8),

$$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{5(8s^2 + s + 32)}{(s^2+4)(8s^2+2s+5)}\right\}$$

Splitting the terms

$$y(t) = \mathcal{L}^{-1}\left\{\frac{As+B}{(s^2+4)} + \frac{Cs+D}{(8s^2+2s+5)}\right\} \quad -(9)$$

Where A, B, C, D are arbitrary constants

$$8A + C = 0$$

$$2A + 8B + D = 40$$

$$5A + 2B + 4C = 5$$

$$5B + 4D = 160$$

This forms a system of Linear Equations

$$\begin{pmatrix} 8 & 0 & 1 & 0 \\ 2 & 8 & 0 & 1 \\ 5 & 2 & 4 & 0 \\ 0 & 5 & 0 & 4 \end{pmatrix} \times \begin{pmatrix} A \\ B \\ C \\ D \end{pmatrix} = \begin{pmatrix} 0 \\ 40 \\ 5 \\ 160 \end{pmatrix}$$

Solving this obtains:

```
>> inv([8 0 1 0; 2 8 0 1; 5 2 4 0; 0 5 0 4])*[0;40;5;160]

ans =

    -27/149
      8/149
    216/149
   5950/149
```

$$A = -\frac{27}{149}; B = \frac{8}{149}; C = \frac{216}{149}; D = \frac{5950}{149}$$

Factorizing $8s^2+2s+5$ becomes $8\left[\left(s+\frac{1}{8}\right)^2+\left(\frac{\sqrt{39}}{8}\right)^2\right]$ and substituting these values, Equation (9) becomes,

$$y(t)=-\frac{27}{149}\times\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\}+\frac{8}{149}\times\mathcal{L}^{-1}\left\{\frac{1}{s^2+4}\right\}+\frac{216}{149}\mathcal{L}^{-1}\left\{\frac{s}{8\left[\left(s+\frac{1}{8}\right)^2+\left(\frac{\sqrt{39}}{8}\right)^2\right]}\right\}+\frac{5950}{149}\mathcal{L}^{-1}\left\{\frac{1}{8\left[\left(s+\frac{1}{8}\right)^2+\left(\frac{\sqrt{39}}{8}\right)^2\right]}\right\}$$

Simplifying it,

$$y(t)=\frac{1}{149}\left[-27\times\mathcal{L}^{-1}\left\{\frac{s}{s^2+4}\right\}+8\times\mathcal{L}^{-1}\left\{\frac{1}{s^2+4}\right\}+27\times\mathcal{L}^{-1}\left\{\frac{s}{\left(s+\frac{1}{8}\right)^2+\left(\frac{\sqrt{39}}{8}\right)^2}\right\}+\frac{2975}{4}\times\mathcal{L}^{-1}\left\{\frac{1}{\left(s+\frac{1}{8}\right)^2+\left(\frac{\sqrt{39}}{8}\right)^2}\right\}\right]$$

$$y(t)=\frac{1}{149}\left[-27\cos 2t+4\sin 2t+27e^{-\frac{t}{8}}\cos\left[\left(\frac{\sqrt{39}}{8}\right)t\right]+\frac{5923}{\sqrt{39}}e^{-\frac{t}{8}}\sin\left[\left(\frac{\sqrt{39}}{8}\right)t\right]\right] \quad -(10)$$

1.3 Plotting solutions

```
syms t
w = sqrt(39)/8;
a1 = -27*cos(2*t)+4*sin(2*t);
a2 = cos(w*t);
a3 = sin(w*t);
a4 = exp(-t/8);
y1 = (1/149)*(a1+27*a4*a2-(37/sqrt(39))*a4*a3);
y2 = (1/149)*(a1+27*a4*a2+(5923/sqrt(39))*a4*a3);
fplot(y1, [0 5], 'LineWidth', 2);
hold on;
fplot(y2, [0 5], 'LineWidth', 2);
grid on;
legend('Sol 1', 'Sol 2');
```

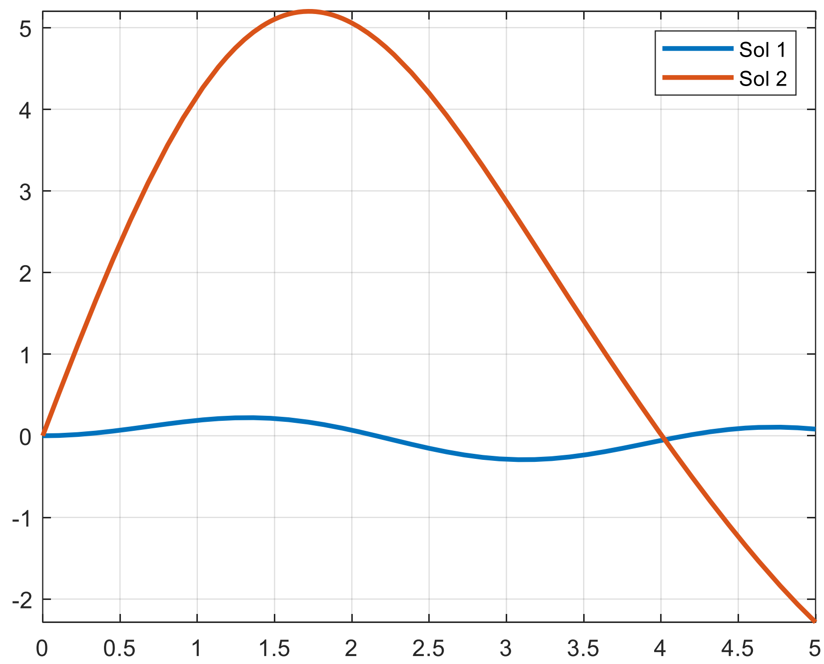


Figure 1.1 Comparison between the solutions

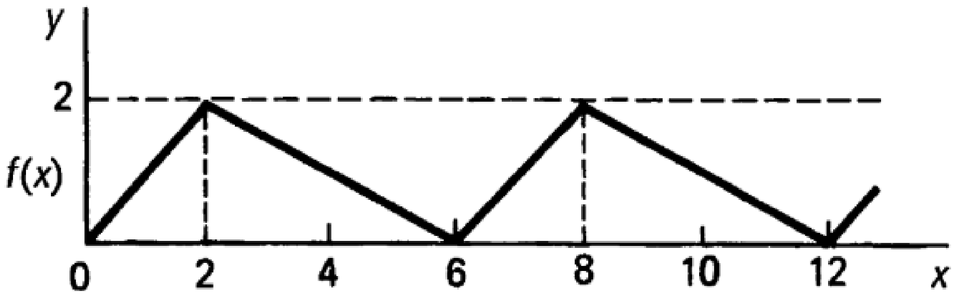
From the graph comparing Solution 1 and Solution 2, it can be observed that when an initial velocity is given to the mass, the displacement is more, the oscillations are more, but when the initial velocity is zero, the dampening force is more and hence there is not much of amplitude compared to the case when there is an initial velocity.

1.4 Conclusion

Laplace Transform converts a function with time domain into frequency domain, this property can be used to solve Initial Value Problems or Ordinary Differential Equations, since it makes it easier to solve. Some Equations that cannot be solved using usual methods can be solved using Laplace Transforms.

Question No. 2

Solution to Question No. 2:



2.1 Writing periodic function:

The given function is periodic function with a period of 6, i.e. after every 6 consecutive values of x, it starts repeating,

The function can be broken down form $[0, 6]$ as a function of two lines, one from $[0, 2]$ and other from $[2,6]$

The first line passes through $(0,0)$ and $(2,2)$, the equation of line is

$$y - 0 = \frac{2}{2}(x - 0)$$

$$y = x$$

The second line passes through $(2, 2)$ and $(6,0)$, the equation of line is

$$y - 2 = \frac{2 - 0}{2 - 6}(x - 2)$$

$$y = 3 - \frac{x}{2}$$

Hence the function is:

$$f(x) = \begin{cases} x, & 0 \le x < 2 \\ 3 - \frac{x}{2}, & 2 \le x < 6 \end{cases} \qquad - (1)$$

$$f(x + 6) = f(x)$$

The period of the function is 6.

2.2 MATLAB function for plotting periodic function, plot from [-24, 24]:

```
function [] = PlotPiecewise(function1, function2, pieceLimit1, pieceLimit2,
plotInterval)
%PLOTPIECEWISE Plots the funtion1, and function2 piecewise
%   Author: Satyajit Ghana
%   USAGE: PlotPiecewise(@(x) x, @(x) 3-x./2, 2, 6, [0 12])
%   plotInterval is optional

    %% default arguments
if nargin < 5
    plotInterval = [0 pieceLimit2];
end
```

```
    %% initialize
    l1 = pieceLimit1;
    l2 = pieceLimit2;
    timePeriod = l2;
    T = timePeriod;

    %% function
    FOriginal = @(x) function1(mod(x, T)).*(0 <= mod(x, T) & mod(x, T) < l1) +
function2(mod(x, T)).*(l1 <= mod(x, T) & mod(x, T) < l2);

    %% plot
    fplot(FOriginal, plotInterval, 'LineWidth', 2);
    title('$ $ PieceWise Function Plot', 'Interpreter', 'latex');
    legend({'$ $ Function'}, 'Interpreter', 'latex', 'Location', 'best');

end


f1 = @(x) x;
l1 = 2;
f2 = @(x) 3 - x./2;
l2 = 6;
PlotPiecewise(f1, f2, l1, l2, [-24 24]);
```

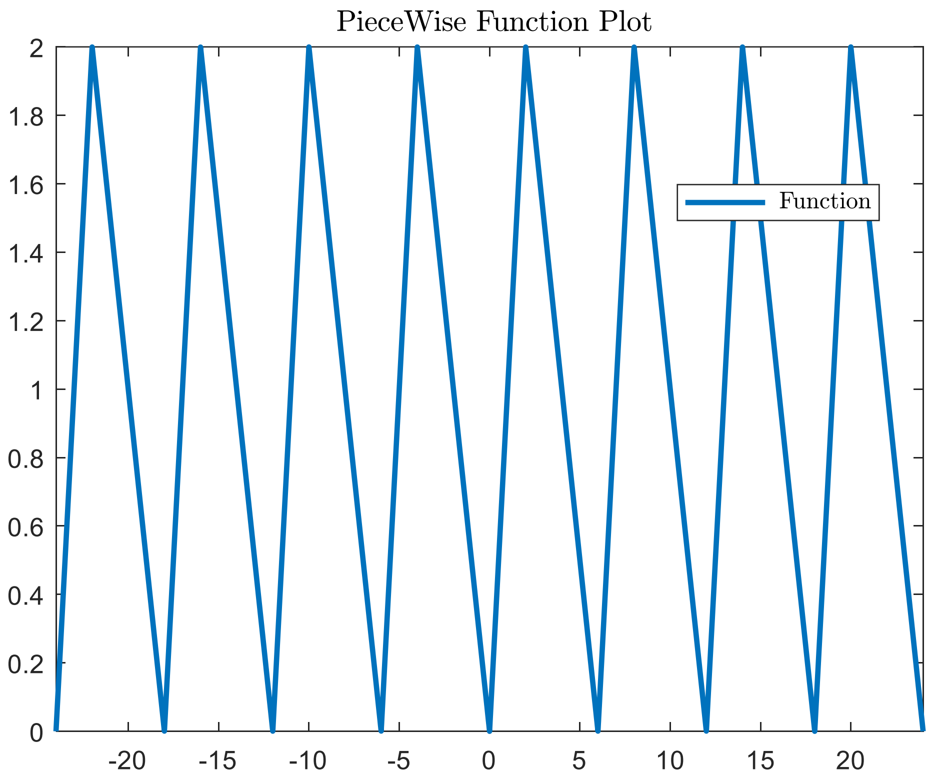


Figure 1.2 PieceWise Plot

2.3 Checking whether the periodic function is even or odd:

The function f is said to be even if the graph of f is symmetric with respect to y axis, and f is an odd function if the function is symmetric with respect to the origin.

$$f \text{ is even if } f(-x) = f(x) \text{ or } f(-x) - f(x) = 0$$
$$f \text{ is odd if } f(-x) = -f(x) \text{ or } f(-x) + f(x) = 0$$

We know

$$f(x) = \begin{cases} x, & 0 \le x < 2 \\ 3 - \dfrac{x}{2}, & 2 \le x < 6 \end{cases}$$

To find $f(-x)$ replace x with $-x$ in (1)

$$f(-x) = \begin{cases} -x, & 0 \ge x > -2 \\ 3 + \dfrac{x}{2}, & -2 \ge x > -6 \end{cases}$$

$f(x) + f(-x)$

$$f(x) + f(-x) = \begin{cases} 3 + \dfrac{x}{2}, & -6 \le x < -2 \\ -x, & -2 \le x < 0 \\ x, & 0 \le x < 2 \\ 3 - \dfrac{x}{2}, & 2 \le x < 6 \end{cases}$$

Since $f(x) + f(-x) \neq 0$, the given function is an not an odd function$-(2)$

$f(-x) - f(x)$

$$f(-x) - f(x) = \begin{cases} 3 + \dfrac{x}{2}, & -6 \le x < -2 \\ -x, & -2 \le x < 2 \\ \dfrac{x}{2} - 3, & 2 \le x < 6 \end{cases}$$

Since $f(-x) - f(-x) \neq 0$, the given function is an not an even function$-(3)$

From (2) and (3) it's concluded that $f(x)$ is neither an even function nor an odd function.

2.4 Finding the Fourier series expansion for the given wave:

If the function is defined on the interval $[a, b]$ then the Fourier series for $F(x)$ is given by

$$F(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{L} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{L} \quad - (1)$$

Where

$$L = \frac{b-a}{2}$$

$$a_0 = \frac{1}{L}\int_a^b F(x)dx$$

$$a_n = \frac{1}{L}\int_a^b F(x)\cos\frac{n\pi x}{L}dx$$

$$b_n = \frac{1}{L}\int_a^b F(x)\sin\frac{n\pi x}{L}dx$$

$$f(x) = \begin{cases} x, & 0 \le x < 2 \\ 3 - \frac{x}{2}, & 2 \le x < 6 \end{cases}$$

$a = 0, b = 6$

$$L = \frac{6-0}{2} = 3$$

$$a_0 = \frac{2}{6}\int_0^6 f(x)dx$$

$$a_0 = \frac{1}{3}\left\{\int_0^2 xdx + \int_2^6 \left(3 - \frac{x}{2}\right)dx\right\}$$

$$a_0 = \frac{1}{3}\{2+4\}$$

$$a_0 = 2 \qquad -(2)$$

$$a_n = \frac{1}{3}\int_0^6 f(x)\cos\frac{n\pi x}{3}dx$$

$$a_n = \frac{1}{3}\left\{\int_0^2 x\cos\frac{n\pi x}{3}dx + \int_2^6 \left(3-\frac{x}{2}\right)\cos\frac{n\pi x}{3}dx\right\}$$

$$a_n = \frac{1}{3}\left\{\frac{1}{\pi^2 n^2}\left[6\pi n\sin\frac{2\pi n}{3} + 9\cos\frac{2\pi n}{3} - 9\right] + \frac{1}{\pi^2 n^2}\left[3\left(\sin\frac{2\pi n}{3}\right)\times\left(3\sin\frac{4\pi n}{3} - 2\pi n\right)\right]\right\}$$

$$a_n = \frac{1}{3}\left(\frac{6\pi n\sin\left(\frac{2\pi n}{3}\right) + 9\cos\left(\frac{2\pi n}{3}\right) - 9}{\pi^2 n^2} + \frac{3\sin\left(\frac{2\pi n}{3}\right)\left(3\sin\left(\frac{4\pi n}{3}\right) - 2\pi n\right)}{\pi^2 n^2}\right) \qquad -(3)$$

$$b_n = \frac{1}{3}\int_0^6 f(x)\sin\frac{n\pi x}{3}dx$$

$$b_n = \frac{1}{3}\left\{\int_0^2 x\sin\frac{n\pi x}{3}dx + \int_2^6 \left(3-\frac{x}{2}\right)\sin\frac{n\pi x}{3}dx\right\}$$

$$b_n = \frac{1}{3}\left\{\frac{1}{\pi^2 n^2}\left[9\sin\frac{2\pi n}{3} - 6\pi n\cos\frac{2\pi n}{3}\right] + \frac{3}{2\pi^2 n^2}\left[3\sin\frac{2\pi n}{3} - 3\sin 2\pi n + 4\pi n\cos\frac{2\pi n}{3}\right]\right\}$$

$$b_n = \frac{1}{3}\left(\frac{9\sin\left(\frac{2\pi n}{3}\right) - 6\pi n\cos\left(\frac{2\pi n}{3}\right)}{\pi^2 n^2} + \frac{3\left(3\sin\left(\frac{2\pi n}{3}\right) - 3\sin(2\pi n) + 4\pi n\cos\left(\frac{2\pi n}{3}\right)\right)}{2\pi^2 n^2}\right) \qquad -(4)$$

From (1), (2), (3) and (4)

$$f(x) = 1 + \sum_{n=1}^{\infty} \frac{1}{3}\left(\frac{6\pi n \sin\left(\frac{2\pi n}{3}\right) + 9\cos\left(\frac{2\pi n}{3}\right) - 9}{\pi^2 n^2} + \frac{3\sin\left(\frac{2\pi n}{3}\right)\left(3\sin\left(\frac{4\pi n}{3}\right) - 2\pi n\right)}{\pi^2 n^2}\right) \times \cos\frac{n\pi x}{3}$$

$$+ \sum_{n=1}^{\infty} \frac{1}{3}\left(\frac{9\sin\left(\frac{2\pi n}{3}\right) - 6\pi n \cos\left(\frac{2\pi n}{3}\right)}{\pi^2 n^2} + \frac{3\left(3\sin\left(\frac{2\pi n}{3}\right) - 3\sin(2\pi n) + 4\pi n \cos\left(\frac{2\pi n}{3}\right)\right)}{2\pi^2 n^2}\right) \times \sin\frac{n\pi x}{3}$$

2.5 MATLAB function for Fourier series expansion for N=5:

```
function [FTransform] = FourierSeriesPW(function1, function2, pieceLimit1,
pieceLimit2, precision, plotInterval)
% Author : Satyajit Ghana
% Arguments:
%   function1 : The first function
%   function2 : The second function
%   pieceLimit1 : end limit of function1
%   pieceLimit2 : end limit of function2
%   precision : value of n in fourier transform
%   plotInterval !optional default: [0 pieceLimit2]: Interval to plot
%
% USAGE Exaxmple: FourierSeriesPW(@(x) x, @(x) (3-x/2, 2, 6, 50, 6, [0 20]))

    %% default arguments
    if nargin < 6
        plotInterval = [0 pieceLimit2];
    end

    %% initialize
    f1 = function1;
    f2 = function2;
    l1 = pieceLimit1;
    l2 = pieceLimit2;
    K = precision;
    timePeriod = l2;
    T = timePeriod;
    syms x;
    w = (2*pi)/T;
    n = 1:K;

    FOriginal = @(x) function1(mod(x, T)).*(0 <= mod(x, T) & mod(x, T) < l1) +
function2(mod(x, T)).*(l1 <= mod(x, T) & mod(x, T) < l2);

    %% calculate the constants

    % a better version
    %a0 = (2/T)*integral(@(x) FOriginal(x), 0, T);
    % fill with zeros to improve performance
    %an = zeros(1, K);
    %bn = zeros(1, K);
    %for i = 1:K
    %     an(i) = (2/T)*integral(@(x) FOriginal(x).*cos(i.*w.*x), 0, T);
    %     bn(i) = (2/T)*integral(@(x) FOriginal(x).*sin(i.*w.*x), 0, T);
    %end

    % plain-old way
```

```
    a0 = (2/T)*(int(f1, x, 0, l1) + int(f2, x, l1, l2));
    an = (2/T)*(int(f1*cos(n*w*x), x, 0, l1) + int(f2*cos(n*w*x), l1, l2));
    bn = (2/T)*(int(f1*sin(n*w*x), x, 0, l1) + int(f2*sin(n*w*x), l1, l2));
    v1 = [a0 an bn];
    v2 = [1/2 cos(n*w*x) sin(n*w*x)];

    FTransform = v1.*v2;

    %% fourier series
    FTransform = vpa(simplify(sum(FTransform)));

    %% plot the actual function
    fplot(FOriginal, plotInterval, 'LineWidth', 2);
    hold on;

    %% plot the series
    fplot(sym(FTransform), plotInterval, 'LineWidth', 2);
    grid on;

    legend({'$ $ Original Function', 'Fourier Series Function'}, 'Interpreter',
'latex', 'Location', 'best');
    title(strcat('$ $ Fourier Series for PieceWise Function for N = $ $',
num2str(precision)), 'Interpreter', 'latex');

end
```

For $N = 5$

```
N = 5;
disp(FourierSeriesPW(@(x) x, @(x) 3-x./2, 2, 6, N));
```

OUTPUT:

```
1.0 - 0.049357529423872216065957222760213*cos(4.1887902047863909846168578443727*x +
0.52359877559829887307710723054658) -
0.19743011769548886426382889104085*sin(2.0943951023931954923084289221863*x +
1.0471975511965977461542144610932) -
0.031588818831278218282212622566536*sin(5.2359877559829887307710723054658*x +
1.0471975511965977461542144610932) -
0.78972047078195545705531556416 34*cos(1.0471975511965977461542144610932*x +
0.52359877559829887307710723054658)
```

2.6 Plotting the Fourier series expansion and periodic function for N = 10, N = 20:

i. N = 10

```
f1 = @(x) x;
l1 = 2;
f2 = @(x) 3 - x./2;
l2 = 6;
disp(FourierSeriesPW(f1, f2, 2, 6, 10, [-24 24]))
```

```
1.0 - 0.049357529423872216065957222760213*cos(4.1887902047863909846168578443727*x +
0.52359877559829887307710723054658) -
0.19743011769548886426382889104085*sin(2.0943951023931954923084289221863*x +
```

```
1.0471975511965977461542144610932)                                    -
0.012339382355968054016489305690053*sin(8.3775804095727819692337156887453*x      +
1.0471975511965977461542144610932)                                    -
0.0078972047078195545705531556416 34*cos(10.471975511965977461542144610932*x      +
0.52359877559829887307710723054658)                                   -
0.031588818831278218282212622566536*sin(5.2359877559829887307710723054658*x      +
1.0471975511965977461542144610932)                                    -
0.016116744301672560348067664574763*cos(7.3303828583761842230795012276522*x      +
0.52359877559829887307710723054658)                                   -
0.78972047078195545705531556416 34*cos(1.0471975511965977461542144610932*x       +
0.52359877559829887307710723054658)
```

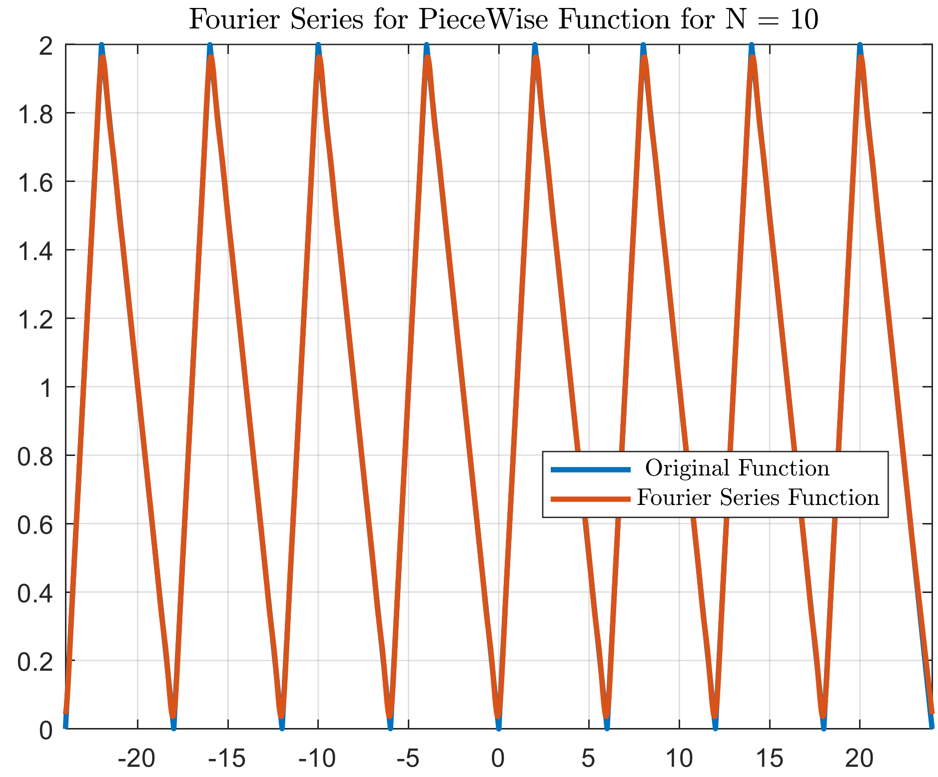


Figure 1.3 Fourier Series Plot for N = 10

i. <u>N = 20</u>

```
f1 = @(x) x;
l1 = 2;
f2 = @(x) 3 - x./2;
l2 = 6;
disp(FourierSeriesPW(f1, f2, 2, 6, 20, [-24 24]))
```

```
1.0 -
0.049357529423872216065957222760213*cos(4.1887902047863909846168578443727*x +
0.52359877559829887307710723054658) -
0.0030848455889920135041223264225133*cos(16.755160819145563938467431377491*x +
0.52359877559829887307710723054658) -
0.0021875913318059707951670791251064*cos(19.896753472735357176930074760 77*x +
0.52359877559829887307710723054658) -
```

```
0.19743011769548886426382889104085*sin(2.0943951023931954923084289221863*x +
1.0471975511965977461542144610932) -
0.012339382355968054016489305690053*sin(8.3775804095727819692337156887453*x +
1.0471975511965977461542144610932) -
0.0078972047078195545705531556416 34*cos(10.471975511965977461542144610932*x +
0.52359877559829887307710723054658) -
0.0027325967847126486403298116407038*sin(17.802358370342161684621645838584*x +
1.0471975511965977461542144610932) -
0.031588818831278218282212622566536*sin(5.2359877559829887307710723054658*x +
1.0471975511965977461542144610932) -
0.0019743011769548886426382889104085*sin(20.943951023931954923084289221863*x +
1.0471975511965977461542144610932) -
0.0065266154610078963393001286294496*sin(11.519173063162575207696359072025*x +
1.0471975511965977461542144610932) -
0.0046729021939760677932267193145763*cos(13.613568165555770700004787994211*x +
0.52359877559829887307710723054658) -
0.016116744301672560348067664574763*cos(7.3303828583761842230795012276522*x +
0.52359877559829887307710723054658) -
0.0040291860754181400870169161436908*sin(14.660765716752368446159002455304*x +
1.0471975511965977461542144610932) -
0.78972047078195545705531556416 34*cos(1.0471975511965977461542144610932*x +
0.52359877559829887307710723054658)
```

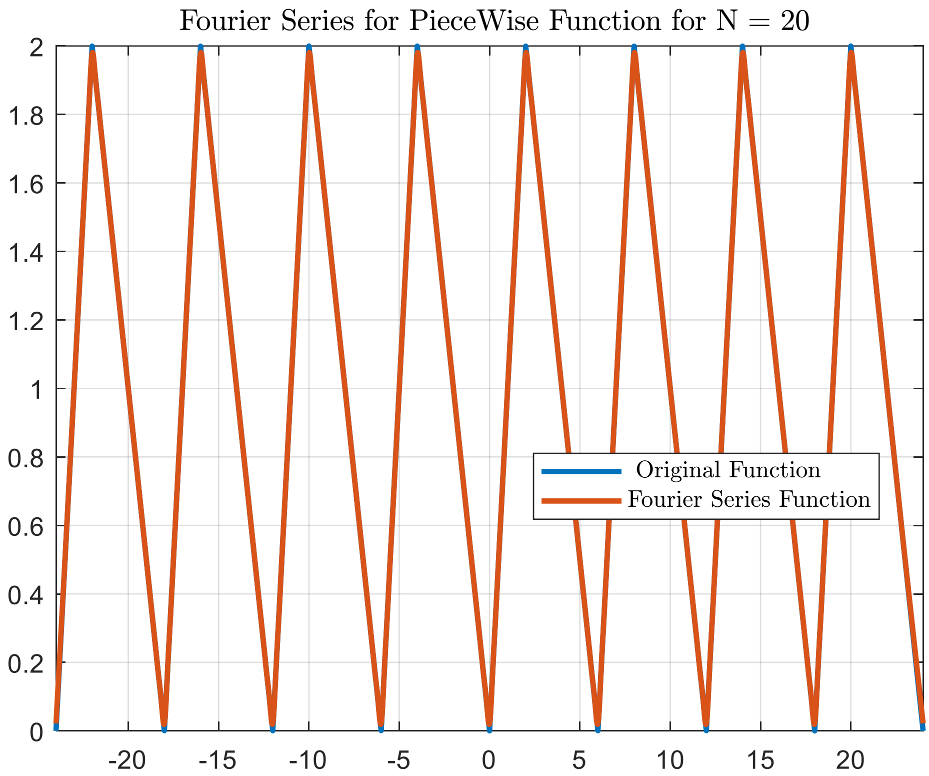


Figure 1.4 Fourier Series Plot for N = 20

2.7 Conclusion:

Fourier Series is an expansion for a periodic function, it’s very useful since it can breakup arbitrary periods into a set of simple terms that can easily be plugged in, solved individually and then recombine them to obtain the solution. The Series is a combination of sinusoidal and cosinusoidal terms which when added together with respective scaling forms the periodic function. Since they are continuous and continuously differentiable it makes the work easier.

Bibliography