ASSIGNMENT

Course Code	BSC104A
Course Name	Engineering Mathematics - 2
Programme	B.Tech
Department	CSE
Faculty	FET

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Semester/Year	02/1st
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Declaration Sheet			
Student Name	Satyajit Ghana		
Reg. No	17ETCS002159		
Programme	B.Tech	Semester/Year	02/1st
Course Code	BSC104A		
Course Title	Engineering Mathematics - II		
Course Date		to	
Course Leader			

Declaration

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Contents

<u>Question No. 1</u>

<u>Solution to Question No. 1 part A:</u>

A.1 Description of a Model:

<u>Predator-Prey Model</u>

In the study of dynamics of a single specie population in ecosystem, we would generally take into consideration factors affecting this single population such as natural growth or the growth rate and the carrying capacity of the environment. But in a bit more generalist the population of a specie would depend on several other factors, one of which is the predator or the prey population. Species compete, evolve and disperse, seek resources in order to survive. As Darwin says "Survival of the Fittest".

Depending upon the environmental conditions and the type of species living in that environment, this model can have application in forms of resource - consumer, parasite - host, tumor cells – immune system, plant – herbivore. When seemingly competitive interactions are carefully examined, they are often in fact some forms of predator – prey interaction in disguise.

<u>Let's take a General Predator – Prey Model</u>

Consider two population species each of whose population at a reference time t is given by $x(t)$ and $y(t)$ respectively. The functions taken here x and y are taken to be continuous functions.
Let the function $f(x,y)$ give the per capita growth rate of the first specie and $g(x,y)$ give the per capita growth rate of the second specie.

Now the rate of change of population of a specie is directly dependent on this growth rate and increases according to this growth rate, the rate of change is increasing, hence a positive sign is included.

$$Rate\ of\ Change\ of\ Population = Current\ Population \times Growth\ Rate$$

$$\frac{dx(t)}{dt} = x \times f(x,y) - (1)$$

And similarly

$$\frac{dy(t)}{dt} = y \times g(x,y) - (2)$$

Equations (1) and (2) are Coupled System of Linear Differential Equations and can be solved by any usual method unless it gets more complex. This is a very general form of the model, where some factors such as the death rate of two species are not considered. If we consider even these factors it becomes a $Lotka - Volterra\ Model$

Lotka - Volterra Model

Consider some parameters for this model, such as,

The parameter a be the growth rate of the species x, which we consider as the prey, in the absence of interaction with the predators or the y species. Prey numbers are diminished by these interactions. The per capita growth rate decreases with increasing y, possibly it may become negative.

The parameter b measures the impact of predation on x.

The parameter c is the death rate of species y in the absence of interaction with species x.

The term px denoted the net growth of the predator population in response to the size of the prey population.

If we modify $f(x, y)$ and $g(x, y)$ as following

$$f(x, y) = a - by$$

And

$$g(x, y) = px - c$$

Substituting these into (1) and (2)

Our Predator-Prey Model becomes,

$$\frac{dx(t)}{dt} = \big(a - by(t)\big) \times x(t)$$
$$\frac{dy(t)}{dt} = (px(t) - c) \times y(t)$$

Or simply,

$$\boxed{\begin{aligned} x' &= (a - by)x - (3) \\ y' &= (px - c)y - (4) \end{aligned}}$$

These Equations are known as the Lotka-Volterra Equations, which are a pair of first order nonlinear differential equations.

The assumptions of this model are

- The predator species is totally dependent on the prey species as its only food supply.

- The prey species has an unlimited food supply and no thread to its growth other than the specific predator.
- The rate at which predators encounter prey is jointly proportional to the sizes of the two populations.
- A fixed proportion of encounters leads to the death of the prey.

A.2 The Solution of the Model:

Since the discussed model in A1. forms a system of linear differential equations, but due to the simplicity of this model, t can be eliminated to give an implicit solution. The equations have a periodic solutions and do not have a simple expression in terms of the usual trigonometric functions, although they are quite tractable.

So if we divide equation (4) by (3).

$$\frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{y(px-c)}{x(a-by)}$$

$$\frac{dy}{dx} = \frac{y(px-c)}{x(a-by)}$$

This is a Variable Separable Ordinary Differential Equation,

$$\int \frac{a-by}{y}dy = \int \frac{px-c}{x}dx$$

$$\int \frac{a}{y}dy - \int bdy = \int pdx - \int \frac{c}{x}dx$$

$$a\ln y - by = px - c\ln x + k$$

Where k is an arbitrary constant that can be determined from the initial conditions of the population of the two species.

$$k = a\ln y + c\ln x - (by+px)$$

Or,

$$V(x,y) = a\ln y + c\ln x - (by+px) - (5)$$

Where $V(x,y)$ is constant of motion.

If we would like to sketch the level curves of this, let's consider $a = b = c = p = 1$

$$V(x, y) = \ln y + \ln x - (x + y)$$

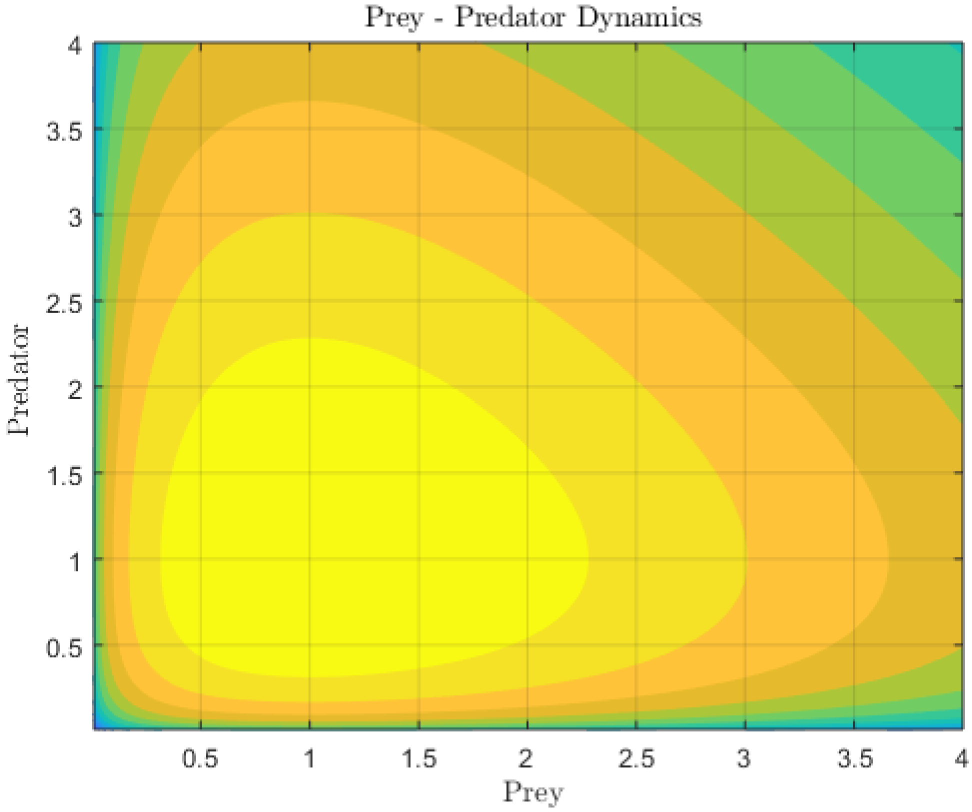

Figure A1.1 Prey-Predator dynamics as described by the level curves of a conserved quantity.

There are equilibria at $x = 1, y = 1$ and at $x = 0, y = 0$, in this simulation the data are $a = b = c = p = 1$

A.3 Comment on the results:

From the results obtained in A2, the right hand side of the identity is referred to as a conservation law at $t = 0$, since it is constant along any solution. Having a conserved quantity facilitates visualizing solutions. In the Figure A1.1 the level curves of the surface is drawn. It can be observed that in the first quadrant of the xy plane. The contours describe the solutions of the system, and since all of these curves are closed curves, the solutions are said to be in periodic oscillations, but as said earlier these oscillations cannot be expressed in form of normal trigonometric equations.

If $a > 0$, there are two equilibria, $x = 0, y = 0$, i.e. extinction, and $x = \frac{c}{p}, y = \frac{a}{b}$, i.e. coexistence, and the surface had a single peal at the latter equilibrium, The contour lines describe a classic prey-predator cycles observed in ecological systems.

Question No. 2

Solution to Question No. 1 part B:

B.1.1 Obtain a mathematical model to describe population of each countries at any time:

Considering that Austria and Hungary have the same natural growth rate of r percent per year, Emigration rate from Austria to Hungary h percent per year and from Hungary to Austria a percent per year.

Let the population at any time t year of Austria be $A(t)$ and that of Hungary be $H(t)$.

$$Rate\ of\ change\ of\ Population = Growth - Emigration + Immigration$$

Since due to Emigration the population will decrease, the rate of change will be negative, while that of Growth and Immigration increase the population of the country hence the rate of change is taken as positive.

$Austria \rightarrow Hungary = \left(\frac{h}{100}\right)A(t)$

$Hungary \rightarrow Austria = \left(\frac{a}{100}\right)H(t)$

Now,

$$\frac{dA(t)}{dt} = \left(\frac{r}{100}\right)A(t) - \left(\frac{h}{100}\right)A(t) + \left(\frac{a}{100}\right)H(t)$$
$$\frac{dH(t)}{dt} = \left(\frac{r}{100}\right)H(t) - \left(\frac{a}{100}\right)H(t) + \left(\frac{h}{100}\right)A(t)$$

Simplifying it a bit further to make it a little more pretty

$$A' = \frac{1}{100}[A(r-h) + aH] - (1)$$
$$H' = \frac{1}{100}[H(r-a) + hA] - (2)$$

These are a coupled System of Differential Equations,

Where

$$\frac{dA(t)}{dt} = A';\ A(t) = A$$
$$\frac{dH(t)}{dt} = H'; H(t) = H$$

B.1.2 Convert the above model into a second order ODE and solve manually to get expressions for A(t) and H(t):

Differentiating (1) and (2) w.r.t t,

$$A'' = \frac{1}{100}[A'(r-h) + aH'] - (3)$$

$$H'' = \frac{1}{100}[H'(r-a) + hA'] - (4)$$

From (3) and (2)

$$100A'' = A'(r-h) + a\frac{1}{100}[H(r-a) + hA] - (5)$$

Rearranging (1)

$$H = \frac{100A' - A(r-h)}{a} - (6)$$

From (5) and (6)

$$100A'' = A'(r-h) + a\frac{1}{100}\left[\left(\frac{100A' - A(r-h)}{a}\right)(r-a) + hA\right]$$

$$10000A'' = 100A'(r-h) + 100A'(r-a) - A(r-h)(r-a) + ahA$$

$$10000A'' = 100A'[2r-(a+h)] - A[r^2 - r(a+h)]$$

$$10000A'' - 100A'[2r-(a+h)] + A[r^2 - r(a+h)] = 0$$

Which is a Second Order Constant Coefficient Ordinary Differential Equation

Taking $= \frac{d}{dt}$, the differential operator

$$\{10000D^2 - 100[2r-(a+h)]D + [r^2 - r(a+h)]\}A = 0$$

The characteristic equation for this is,

$$10000\lambda^2 - 100[2r-(a+h)]\lambda + [r^2 - r(a+h)] = 0$$

The roots for this equation are

$$\lambda_{1,2} = \frac{100[2r-(a+h)] \pm \sqrt{\{100[2r-(a+h)]\}^2 - 4 \times 10000 \times [r^2 - r(a+h)]}}{2 \times 10000}$$

$$\lambda_{1,2} = \frac{2r-(a+h) \pm \sqrt{(a+h)^2}}{200}$$

$$\lambda_{1,2} = \frac{2r-(a+h) \pm (a+h)}{200}$$

$$\lambda_1 = \frac{r}{100}; \lambda_2 = \frac{r-(a+h)}{100}$$

Hence the solution for $A(t)$ is,

$$A(t) = k_1 e^{t \times \frac{2r-(a+h)+(a+h)}{200}} + k_2 e^{t \times \frac{2r-(a+h)-(a+h)}{200}}$$

$$A(t) = k_1 e^{t\times\frac{r}{100}} + k_2 e^{t\times\frac{r-(a+h)}{100}} - (7)$$

Taking the roots as λ_1 and λ_2

$$A = k_1 e^{t\lambda_1} + k_2 e^{t\lambda_2} - (8)$$

Differentiating this,

$$A' = k_1\lambda_1 e^{t\lambda_1} + k_2\lambda_2 e^{t\lambda_2} - (9)$$

From (1)

$$H = \frac{100A' - A(r-h)}{a} - (10)$$

Substituting (8) and (9) in (10)

$$H = \frac{1}{a}\left[100k_1\lambda_1 e^{t\lambda_1} + 100k_2\lambda_2 e^{t\lambda_2} - (r-h)k_1 e^{t\lambda_1} - (r-h)k_2 e^{t\lambda_2}\right]$$

$$H = \frac{1}{a}\left\{k_1 e^{t\lambda_1}\left[100\lambda_1 - (r-h)\right] + k_2 e^{t\lambda_2}\left[100\lambda_2 - (r-h)\right]\right\}$$

$$H = \frac{1}{a}\left\{k_1 e^{t\times\frac{r}{100}}\left[100\times\frac{r}{100} - (r-h)\right] + k_2 e^{t\times\frac{r-(a+h)}{100}}\left[100\times\frac{r-(a+h)}{100} - (r-h)\right]\right\}$$

$$H(t) = k_1\frac{h}{a}e^{t\times\frac{r}{100}} - k_2 e^{t\times\frac{r-(a+h)}{100}} - (11)$$

Given the initial conditions $A(0) = A_0$ and $H(0) = H_0$

Substituting these into (7) and (11)

$$A_0 = k_1 + k_2 - (12)$$

$$H_0 = k_1\frac{h}{a} - k_2 - (13)$$

Adding (12) and (13)

$$A_0 + H_0 = k_1\left(1 + \frac{h}{a}\right)$$

$$k_1 = \frac{aA_0 + aH_0}{a+h} - (14)$$

Substituting this into (12)

$$k_2 = A_0 - \frac{aA_0 + aH_0}{a+h}$$

$$k_2 = \frac{hA_0 - aH_0}{a+h} - (15)$$

From (14), (15), (7) & (11)

$$A(t) = \left(\frac{a(A_0 + H_0)}{a+h}\right)e^{t\times\frac{r}{100}} + \left(\frac{hA_0 - aH_0}{a+h}\right)e^{t\times\frac{r-(a+h)}{100}}$$

$$H(t) = \left(\frac{h(A_0 + H_0)}{a+h}\right)e^{t\times\frac{r}{100}} - \left(\frac{hA_0 - aH_0}{a+h}\right)e^{t\times\frac{r-(a+h)}{100}}$$

$-(16)$

B.1.3 Compute the population of both the countries after 10 years for the given data:
Taking the given values

$$r = 0.45\%, A_0 = 6000000, H_0 = 6850000, h = 0.05\%, a = 0.08\%$$

Substituting these into (16)

$$A(10) = \left(\frac{0.08(6000000 + 6850000)}{0.08 + 0.05}\right) e^{10\times\frac{0.45}{100}} + \left(\frac{0.05 \times 6000000 - 0.08 \times 6850000}{0.08 + 0.05}\right) e^{10\times\frac{0.45-(0.08+0.05)}{100}}$$

$$H(10) = \left(\frac{0.05(6000000 + 6850000)}{0.08 + 0.05}\right) e^{10\times\frac{0.45}{100}} - \left(\frac{0.05 \times 6000000 - 0.08 \times 6850000}{0.08 + 0.05}\right) e^{10\times\frac{0.45-(0.08+0.05)}{100}}$$

Using MATLAB for computing these,

```
syms A(t) H(t)
syms C1 C2 r a h Ao Ho

C1=(a*Ao+a*Ho)/(a+h)
C2=(h*Ao-a*Ho)/(a+h)

r = 0.45;
a = 0.08;
h = 0.05;
Ao = 6000000;
Ho = 6850000;

A(t) = C1*exp((t*r)/100) + C2*exp(t*(r-(a+h))/100);
H(t) = C1*h*exp((t*r)/100)/a-C2*exp(t*(r-(a+h))/100);

t = 10;

vpa(subs(A),10)
vpa(subs(H),10)

OUTPUT:
ans(t) =

6301940.759


ans(t) =

7139517.241
```

$$A(10) = 6301940.759$$
$$H(10) = 7139517.241$$

Population of Austria is 6301941 and that of Hungary is 7139517.

B.1.4 Comment on the results obtained and conclude:

The Solution to this population types question is that of a Logistics Population Equation, the two countries depends on each other's population and hence are said to be coupled. These Coupled equations form a system of Linear Differential Equations and can be solved by the Eigen value method, or by eliminating one of the variables in an equation, hence forming a differential equation of second order in one variable, this is then solved to find the solution to population function of that country, using this solution the population function of the other country is also calculated.

Solution to Question No. 2 part B:

B.2.1 All possible angular speeds and corresponding deflections $y(x)$:

$$\frac{d}{dx}\left[T(x)\frac{dy}{dx}\right] + \rho\omega^2 y = 0$$

Given $T(x) = x^2$

$$\frac{d}{dx}\left[x^2\frac{dy}{dx}\right] + \rho\omega^2 y = 0$$

$$x^2\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + \rho\omega^2 y = 0$$

$$[x^2D^2 + 2xD + \rho\omega^2]y = 0$$

This is a Cauchy-Euler Differential Equation,

Using the transformation, $\ln x = z; x = e^z$

$$\overline{D} = \frac{d}{dz}$$

$$[\,\overline{D}\left(\overline{D} - 1\right) + 2\overline{D} + \rho\omega^2\,]y = 0$$

$$[\overline{D}^2 + \overline{D} + \rho\omega^2]y = 0$$

The Auxiliary Equation in λ is,

$$\lambda^2 + \lambda + \rho\omega^2 = 0$$

The Roots for this are,

$$\lambda_{1,2} = \frac{-1 \pm \sqrt{1 - 4\rho\omega^2}}{2}$$

It's given that $\rho\omega^2 > 0.25$, so $1 - 4\rho\omega^2 < 0$, due to this $\lambda_{1,2}$ are imaginary,

$$\lambda_{1,2} = -\frac{1}{2} \pm \frac{i\sqrt{4\rho\omega^2 - 1}}{2}$$

This is of the form $a \pm ib$

Which has the solution

$$y = e^{az}\{k_1\sin(bz) + k_2\cos(bz)\}$$

Where $a = -\frac{1}{2}$ and $b = \frac{\sqrt{4\rho\omega^2 - 1}}{2}$

$$y = e^{-\frac{1}{2}z}\left\{k_1\sin\left(z \times \frac{\sqrt{4\rho\omega^2 - 1}}{2}\right) + k_2\cos\left(z \times \frac{\sqrt{4\rho\omega^2 - 1}}{2}\right)\right\}$$

Since $x = e^z$ and $\ln x = z$

$$y = x^{-\frac{1}{2}}\left\{k_1 \sin\left(\ln x \times \frac{\sqrt{4\rho\omega^2 - 1}}{2}\right) + k_2 \cos\left(\ln x \times \frac{\sqrt{4\rho\omega^2 - 1}}{2}\right)\right\} - (1)$$

The Given Initial Conditions are $y(1) = 0$ and $y(e) = 0$, the length of the string extends from 1 to e

Using $y(1) = 0$ in (1)

$$0 = 1^{-\frac{1}{2}}\left\{k_1 \sin\left(\ln 1 \times \frac{\sqrt{4\rho\omega^2 - 1}}{2}\right) + k_2 \cos\left(\ln 1 \times \frac{\sqrt{4\rho\omega^2 - 1}}{2}\right)\right\}$$

$$\ln 1 = 0; \sin 0 = 0; \cos 0 = 1;$$

$$0 = 1^{-\frac{1}{2}}\{k_2\}$$

$$0 = k_2$$

Using $y(e) = 0$

$$0 = e^{-\frac{1}{2}}\left\{k_1 \sin\left(\ln e \times \frac{\sqrt{4\rho\omega^2 - 1}}{2}\right) + k_2 \cos\left(\ln e \times \frac{\sqrt{4\rho\omega^2 - 1}}{2}\right)\right\}$$

$$\ln e = 1$$

$$0 = e^{-\frac{1}{2}}\left\{k_1 \sin\left(\frac{\sqrt{4\rho\omega^2 - 1}}{2}\right) + k_2 \cos\left(\frac{\sqrt{4\rho\omega^2 - 1}}{2}\right)\right\}$$

Also from previous we know $k_2 = 0$

$$\sin\left(\frac{\sqrt{4\rho\omega^2 - 1}}{2}\right) = 0$$

Or $k_1 = 0$

Since we are interested in obtaining non-trivial solutions we will ignore the case of $k_1 = 0$

Let

$$v = \frac{\sqrt{4\rho\omega^2 - 1}}{2} - (2)$$

Then

$$\sin v = 0$$

This has solutions,

$$v = \pi \pm k\pi$$

Where $k \in \mathbb{N}$, $k = 0,1,2,3,4$

$$v = 0, \pm\pi, \pm 2\pi, \pm 3\pi \ldots -(3)$$

Since we know that $\rho\omega^2 > 0.25$

$$\rho\omega^2 > \frac{1}{4}$$

From (2)

$$4v^2 + 1 = 4\rho\omega^2$$

$$\rho\omega^2 = \frac{1}{4} + v^2 - (4)$$

From (2) and (4)

$$\frac{1}{4} + v^2 > \frac{1}{4}$$

This implies,

$$v > 0$$

From this and (3)

$$\pi, 2\pi, 3\pi, 4\pi\ .\ .\ . = n\pi - (5)$$

Where $n \in \mathbb{N}$

From (2) and (5)

$$\frac{\sqrt{4\rho\omega^2 - 1}}{2} = n\pi$$

$$4\rho\omega^2 - 1 = 4n^2\pi^2$$

$$4\rho\omega^2 = 4n^2\pi^2 + 1 \qquad - (6)$$

$$\omega^2 = \frac{n^2\pi^2}{\rho} + \frac{1}{4\rho}$$

In General,

$$\boxed{\omega = \pm\sqrt{\frac{n^2\pi^2}{\rho} + \frac{1}{4\rho}}}$$

Taking $n = 1$

$$\omega = \pm\sqrt{\frac{\pi^2}{\rho} + \frac{1}{4\rho}}$$

Taking $n = 2$

$$\omega = \pm\sqrt{\frac{4\pi^2}{\rho} + \frac{1}{4\rho}}$$

From (6) and (1)

$$\boxed{y(x) = x^{-\frac{1}{2}} k_1 \sin(n\pi \times \ln x)}$$

$$n \in \mathbb{N}$$

B.2.2 Plot deflection curves in the given interval:

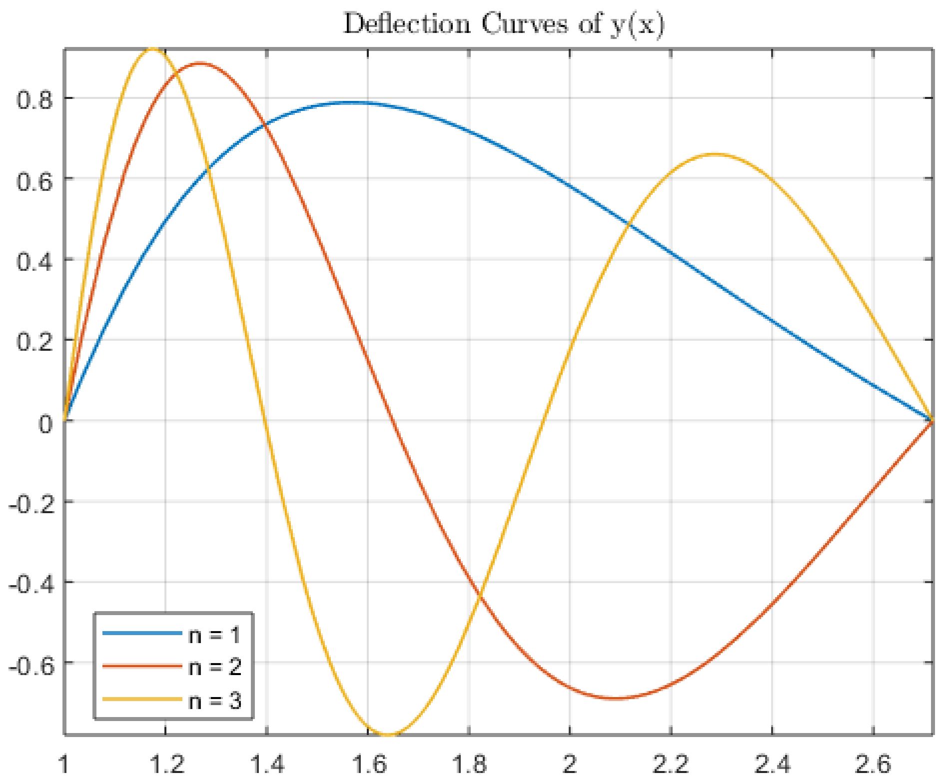

Figure B2.2 Deflection Curves for n=1,2,3

```
for i = 1:1:3
    fplot(@(x) x.^(-0.5).*sin(i.*pi.*log(x)), [1 exp(1)], 'LineWidth', 1.2);
    hold on;
end
legend('n = 1', 'n = 2', 'n = 3', 'Location','best');
title('$ $ Deflection Curves of y(x)','Interpreter','latex');
```

B.2.3 Comment:

Observing from the deflection curves of $y(x)$ we can see that as we increase n the number of points at which $y'(x) = 0$ increases, when $n = 1$, there is one point at which $y'(x) = 0$, when $n = 2$, there are two points at which $y'(x) = 0$ so on and so forth. All of which are solutions for the given Differential Equation.

Solution to Question No. 3 part B:

B.3.1 Determine the area of the sector brake-pad:

Area of the Break Pad can be calculating by integrating a small element arc of width dr and at a distance r from the centre.

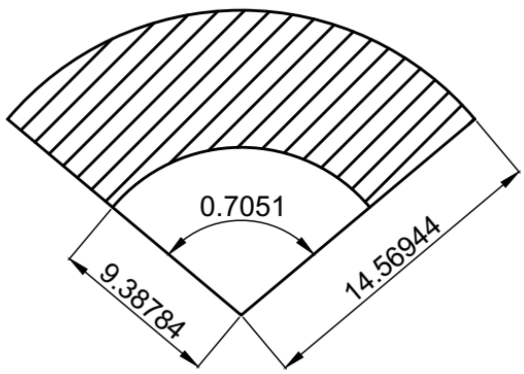

Figure B3.1 Break Pad Illustration

Area of this small elemental strip is $r\theta_p dr$

Hence the area of the shaded region will be integrating this from r_e to r_o

$$A = \int_{r_e}^{r_o} r\theta_p dr$$

$$A = \theta_p \left(\frac{r_o^2 - r_e^2}{2}\right)$$

Substituting the given values,

$$A = \frac{0.7051}{2} \times (14.56944^2 - 9.38784^2)$$

$$A = 43.76451417\ cm^2 - (1)$$

B.3.2 Write a MATLAB function using Simpson's one-third rule to determine the approximate T:

```
function [] = simpsons_one_third(lower_bound, upper_bound, func, partitions)
%SIMPSON'S ONE-THIRD rule for Numerical Integration
%   Author : Satyajit Ghana
% USAGE : simpsons_one_third(lower_bound, upper_bound, function,
number_of_partitions)

% If the number of partitions is not given then its assumed to be
% (upper_bound-lower_bound)*100

% the func should be values of function at points, or a function_handle

%%
```

```
if nargin < 4
    partitions = (upper_bound-lower_bound)*100;
end

%%
if isa(func, 'sym') | isa(func, 'function_handle')
if mod(partitions, 2)
    fprintf('The number of Partitions must be even\n');
    partitions = partitions + 1;
    fprintf('Taking %d Partitions\n', partitions);
end
end
%%
n = partitions;
a = lower_bound;
b = upper_bound;
f = func;

h = (b-a)/n;

%%
if isa(func, 'sym')
    try
        func_hand = matlabFunction(func);
        f = func_hand;
    catch ME
        fprintf('The Function should be a function_handle\n');
        disp(ME);
        return;
    end
end

if isa(func, 'function_handle')
    x = a:h:b;
    I = (h/3)*(f(x(1))+f(x(n+1)) + 4*sum(f(x(2:2:n))) + 2*sum(f(x(3:2:n-1))));
elseif ismatrix(func)
    if isa(func, 'char')
        fprintf('Characters ? Seriously ?\n');
        return;
    end
    if length(func) ~= n
        %fprintf('The number of paritions and value points of functions do not
match\n');
        %return;
    end
    n = numel(func) - 1;
    h = (b-a)/n;
    I = (h/3)*(f(1)+f(end) + 4*sum(f(2:2:end)) + 2*sum(f(3:2:end-2)));
    %I = (h/3)*(f(1)+f(n+1) + 4*sum(f(2:2:n)) + 2*sum(f(3:2:n-1)));
else
    fprintf('The given Function is neither a function_handle nor a Matrix of
Values of Function\n');
    return;
end

%%
fprintf('The value of Integral Computed using Simpson''s one-third rule');
if isa(func, 'function_handle')
    fprintf(' for %s ', func2str(func));
end
```

```
fprintf(' is ');
disp(vpa(I, 10));
end
```

Given,

$$T = \frac{\int_{r_e}^{r_0} T(r) r \theta_p dr}{A}$$

Let $T(r)r = F(r)$, then

$$T = \frac{\theta_p \int_{r_e}^{r_0} F(r) dr}{A}$$

```
x = [9.3878 9.9060 10.4242 10.9423 11.4605 11.9786 12.4968 13.0150 13.5331
14.0513 14.5694];
y = [338 423 474 506 557 573 601 622 651 661 671];

simpsons_one_third(x(1), x(end), y.*x)
```

```
OUTPUT
The value of Integral Computed using Simpson's one-third rule is 35258.30218
```

From this

$$\theta_p \int_{r_e}^{r_0} F(r) dr = 0.7051 \times 35258.30218 = 24860.62887 - (2)$$

From (1) and (2)

$$T = \frac{24860.62887}{43.76451417}$$

$$T = 568.0545°C$$

B.3.3 Comment on the result:

When the integral to be computed has been given in the form of a vector, i.e. precomputed functional values are different points of independent variable, the number of partitions are limited to one less than the length of the vector, and due to this the precision of the numerical integral therefore obtained cannot be further harnessed. Although instead of one-third rule, three-eighth rule could also be used and so one and so forth to increase the precision a little further, but this wouldn’t have much of an impact on the numerical integral than increasing the number of partitions.

Question No. 5

Solution to Question No. 4 part B:

B.4.1 MATLAB function for Exponential Curve and Parabolic Fit by method of least squares:

```
function func = exponential_fit( x, y )
%EXPONENTIAL_FIT
%   Author : Satyajit Ghana
%USAGE: exponential_fit(xi, yi)
%   where xi and yi are the data points given

n = length(x);
Y = log(y);
S = [x; ones(1,n)];

A = S*S';
B = S*Y';
D = A\B;

func = @(t) exp(D(1)*t+D(2));

error = 0;
for k=1:length(x)
    error = error + (func(x(k))-y(k))^2;
end

func = subs(func);
disp('Exponential fit for the gien data points is : ');
disp(vpa(func,6));
disp('With a least square error sum as')
disp(error)

%%
plot(x, y, 'r*');
hold on;
fplot(func, [(min(x)-(x(2)-x(1))) (max(x)+(x(2)-x(1)))],'LineWidth',1.2);
axis tight
title('$ $ Exponential Curve fit for the given data
points','Interpreter','latex');
legend('Data Points',char(vpa(func, 4)),'Location','best');

func = matlabFunction(vpa(func, 8));

end


function func = quadratic_fit( x, y )
%QUADRATIC_FIT
%   Author : Satyajit Ghana
%USAGE: quadratic_fit(xi, yi)
%   where xi and yi are the data points given

n = length(x);
S = [x.*x; x; ones(1,n)];
A = S*S';
```

```
B = S*y';
C = A\B;

func = @(t) C(1)*t^2 + C(2)*t + C(3);

error = 0;
for k=1:length(x)
    error = error + (func(x(k))-y(k))^2;
end

func = subs(func);
disp('Qudratic fit for the gien data points is : ');
disp(vpa(func, 6))
disp('With a least square error sum as')
disp(error)

%%
plot(x,y,'r*');
hold on;
fplot(func, [(min(x)-(x(2)-x(1))) (max(x)+(x(2)-x(1)))], 'LineWidth', 1.2);
axis tight
title('$ $ Quadratic Fit for the given data points','Interpreter','latex')
legend('Data Points',char(vpa(subs(func),4)),'Location','best');

func = matlabFunction(vpa(func, 8));

end
```

A2_B4.m

```
x = [50 450 780 1200 1700 2100 2800];
y = [28 30 32 36 41 50 59];

exponential_fit(x, y);
quadratic_fit(x, y);
```

OUTPUT:

```
Exponential fit for the gien data points is :
exp(0.000282683*t + 3.27415)

With a least square error sum as
   11.6502

Qudratic fit for the gien data points is :
0.00000261173*t^2 + 0.00419059*t + 27.5032

With a least square error sum as
    6.9378
```

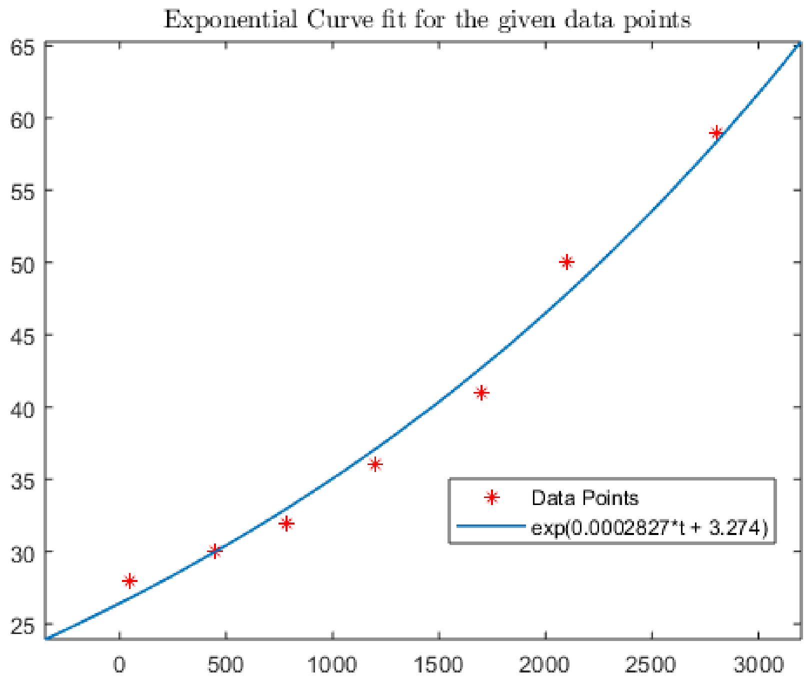

Figure B4.1 Exponential Curve Fit

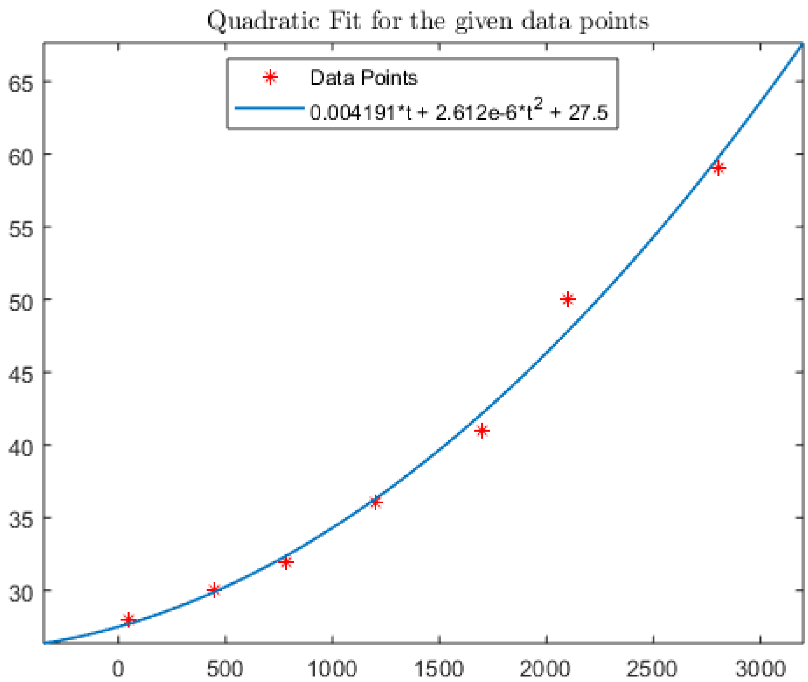

Figure B4.2 Quadratic Curve Fit

B.4.2 Estimate the day, when the raw materials received is 1500kg, using exponential fit obtained:

```
x = [50 450 780 1200 1700 2100 2800];
y = [28 30 32 36 41 50 59];

func = exponential_fit(x, y);
fprintf('The Day on which 1500kg of raw material was recieved : \n');
disp(func(1500));
```

OUTPUT:

```
Exponential fit for the gien data points is :
exp(0.000282683*t + 3.27415)

The Day on which 1500kg of raw material was recieved :
   40.3734
```

Hence approximately on the 41st day the raw materials received is 1500kg using the curve obtained from exponential fit.

B.4.3 Compare the exponential and parabolic fit obtained and determine the best fit among these two for the given data:

From B4.1

```
Exponential fit for the gien data points is :
exp(0.000282683*t + 3.27415)

With a least square error sum as
   11.6502

Qudratic fit for the gien data points is :
0.00000261173*t^2 + 0.00419059*t + 27.5032

With a least square error sum as
    6.9378
```

As it can be observed that Quadratic fit has the least amount of error in fitting the curve through the points, hence Parabolic Fit is the best fit for the given data points. From the Plot obtained in B4.1 it can be clearly seen that the curve passes more closely through the data points in quadratic fit than in exponential fit.

1. Frank Hoppensteadt (2006), Scholarpedia, 1(10):1563, Courant Institute of Mathematical Sciences, New York.