<u>Question No. 2</u>

<u>Solution to Question No. 1 part B:</u>

B.1.1 Obtain a mathematical model to describe population of each countries at any time:
The Equations are,

$$A' = \frac{1}{100}[A(r-h) + aH]$$
$$H' = \frac{1}{100}[H(r-a) + hA]$$

B.1.2 Convert the above model into a second order ODE and solve manually to get expressions for A(t) and H(t):

$$A'' = \frac{1}{100}[A'(r-h) + aH'] - (3)$$
$$H'' = \frac{1}{100}[H'(r-a) + hA'] - (4)$$
$$100A'' = A'(r-h) + a\frac{1}{100}[H(r-a) + hA] - (5)$$

Which becomes,

$$10000A'' - 100A'[2r - (a+h)] + A[r^2 - r(a+h)] = 0$$

Which is a Second Order Constant Coefficient Ordinary Differential Equation

$$10000m^2 - 100[2r - (a+h)]m + [r^2 - r(a+h)] = 0$$

The roots for this equation are

$$m_{1,2} = \frac{100[2r-(a+h)] \pm \sqrt{\{100[2r-(a+h)]\}^2 - 4 \times 10000 \times [r^2 - r(a+h)]}}{2 \times 10000}$$
$$\lambda_1 = \frac{r}{100}; \lambda_2 = \frac{r-a-h}{100}$$

Hence the solution for $A(t)$ is,0

$$A(t) = k_1 e^{t \times \frac{r}{100}} + k_2 e^{t \times \frac{r-a-h}{100}}$$
$$H(t) = k_1 \frac{h}{a} e^{t \times \frac{r}{100}} - k_2 e^{t \times \frac{r-a-h}{100}}$$

B.1.3 Compute the population of both the countries after 10 years for the given data:
Taking the given values

$$r = 0.45\%, A_0 = 6000000, H_0 = 6850000, h = 0.05\%, a = 0.08\%$$

Substituting these into (16)

$$A(10) = \left(\frac{0.08(6000000 + 6850000)}{0.08 + 0.05}\right) e^{10\times\frac{0.45}{100}} + \left(\frac{0.05 \times 6000000 - 0.08 \times 6850000}{0.08 + 0.05}\right) e^{10\times\frac{0.45-(0.08+0.05)}{100}}$$

$$H(10) = \left(\frac{0.05(6000000 + 6850000)}{0.08 + 0.05}\right) e^{10\times\frac{0.45}{100}} - \left(\frac{0.05 \times 6000000 - 0.08 \times 6850000}{0.08 + 0.05}\right) e^{10\times\frac{0.45-(0.08+0.05)}{100}}$$

B.1.4 Comment on the results obtained and conclude:

Write your comments here.