<u>**Question No. 3**</u>

<u>Solution to Question No. 2 part B:</u>

B.2.1 ODE and solution:

The loan amount is Rs $50{,}00{,}000$ taken at an interest rate of 8.5% per annum. And the repayments are made at a monthly rate of $40{,}000(1+\frac{t}{120})$, where t is the number of months since the load was made.

Let $S(t)$ denote the amount of debt at any time t, assuming the compounding takes place continuously,

$$\begin{aligned} &change\ of\ the\ principle\ amount \\ &\qquad = [rate\ of\ new\ debt\ to\ interest] - [rate\ at\ which\ the\ debt\ is\ repaid] \end{aligned}$$

$$\frac{dS(t)}{dt} = \frac{0.085}{12} \times S(t) - 4 \times 10^4 \times \left(1 + \frac{t}{120}\right)$$

Rearranging this, the DE becomes a First Order Linear Ordinary Differential Equation,

$$\frac{dS(t)}{dt} - \frac{0.085}{12} S(t) = -4 \times 10^4 \left(1 + \frac{t}{120}\right)$$

$$I.F = e^{\int -\frac{0.085}{12} dt} = e^{-\frac{0.085}{12}t}$$

$$S(t)e^{-\frac{0.085}{12}t} = -4 \times 10^4 \int \left(1 + \frac{t}{120}\right) e^{-\frac{0.085}{12}t} dt$$

$$S(t)e^{-\frac{0.085}{12}t} = -4 \times 10^4 \left\{\int e^{-\frac{0.085}{12}t} dt + \frac{1}{120}\int te^{-\frac{0.085}{12}t}\, dt\right\}$$

Solving by-parts, and let $a = \frac{0.085}{12}$

$$S(t)e^{-at} = -4 \times 10^4 \left\{\frac{e^{-at}}{-a} + \frac{1}{120}\left[-\frac{e^{-at}}{a^2}\ (at+1)\right]\right\}$$

$$S(t)e^{-at} = -4 \times 10^4 \left(-\frac{e^{-at}}{a} - \frac{at \times e^{-at}}{120 \times a^2} - \frac{e^{-at}}{120}\right) + C$$

Dividing by e^{-at} both sides,

$$S(t) = 4 \times 10^4 \left(\frac{1}{a} + \frac{t}{120a} + \frac{1}{120a^2}\right) + C_1 e^{at}$$

Initially, i.e. at $t = 0\ months$, the amount paid is zero and the remaining load is the loan taken.

$$S(0) = 50{,}00{,}000$$

$$5 \times 10^6 = 4 \times 10^4 \left(\frac{1}{a} + \frac{1}{120a^2}\right) + C_1$$

Substituting the value of a

$$C_1 = 5 \times 10^6 - 4 \times 10^4 \left(\frac{12}{0.085} + \frac{12^2}{120 \times 0.085^2}\right)$$

$$C_1 = -7290657.439$$

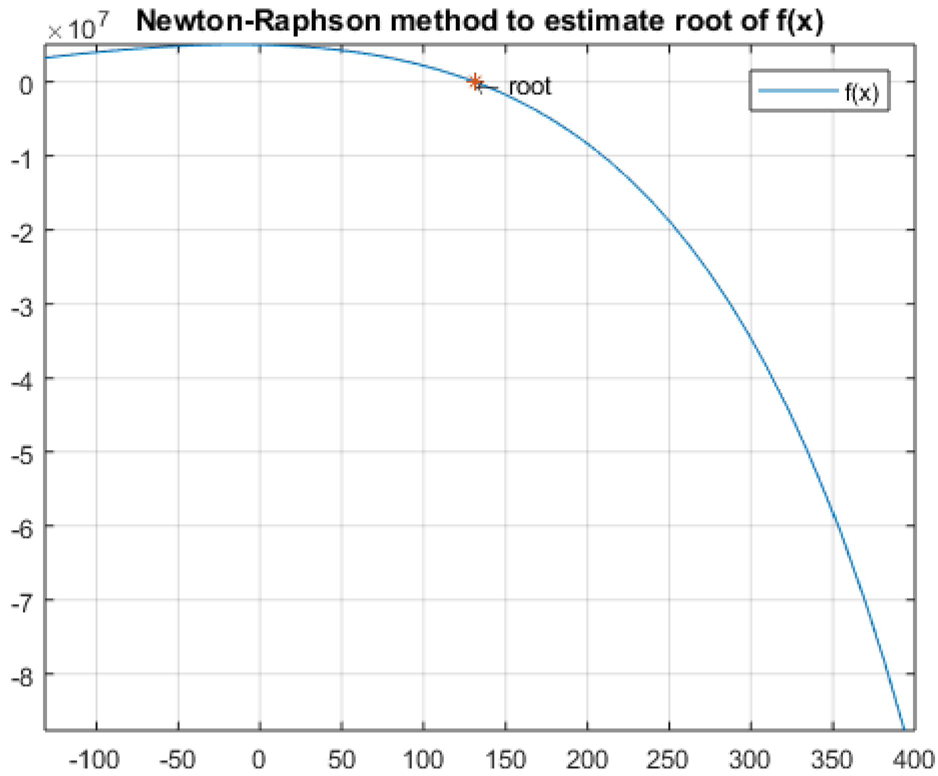
Now the Differential Equation becomes,

$$S(t) = 47058.82353 \times t - 7290657.439 \times e^{0.007083333 \times t} + 12290657.44$$

B.2.2 Time required to repay the loan completely:

The Loan will be paid completely when $S(t)$ or the Loan at month t becomes zero, using Newton Rapson method to solve the above Transcendental Equation,

```
>> func

func =

  function_handle with value:

    @(x)47058.82353.*x-7290657.439.*exp(0.007083333.*x)+12290657.44

>> diff

diff =

  function_handle with value:

    @(x)47058.82353-7290657.439.*exp(0.007083333.*x).*0.007083333

>> newton_raphson_improved(func, diff, 10, 100)
 Iteration      Root                        Error
        1      137.92173084409410000000    27.49510945%
        2      131.42702286477987000000     4.94168386%
        3      131.18661527503198000000     0.18325619%
        4      131.18629517303140000000     0.00024401%
        5      131.18629517246450000000     0.00000000%
        6      131.18629517246447000000     0.00000000%

Root has converged at 6th iteration with at least 10 decimal precision for
the function :
    @(x)47058.82353.*x-7290657.439.*exp(0.007083333.*x)+12290657.44

x = 131.186295172464470000000000000000000000000000000
>>
```



$$t = 131.1863\ months$$

Hence on the 132^{nd} $Month$ the Loan will be repaid completely.

B.2.3 Amount of load paid after 10 years and 15 years:

```
>> func

func =

  function_handle with value:

    @(x)47058.82353.*x-7290657.439.*exp(0.007083333.*x)+12290657.44

>> func(10*12)

ans =

     8.801532202753760e+05

>> func(15*12)

ans =

    -5.329838816651752e+06
```

>>

B.2.4 Comments and conclusion:
Students are expected to draw conclusions based on the discussions and suggestions (not to exceed 100 words)