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| **Course Name** | Engineering Mathematics - 2 |
| **Programme** | B.Tech |
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| --- | --- |
| **Name of the Student** | Satyajit Ghana |
| **Reg. No** | 17ETCS002159 |
| **Semester/Year** | 02/1st |
| **Course Leader/s** | Dr. Sumanth Bharadwaj |

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| **Declaration Sheet** | | | | | | | | |
| Student Name | Satyajit Ghana | | | | | | | |
| Reg. No | 17ETCS002159 | | | | | | | |
| Programme | B.Tech | | | | | Semester/Year | 02/1st | |
| Course Code | BSC104A | | | | | | | |
| Course Title | Engineering Mathematics - II | | | | | | | |
| Course Date |  | | to | |  | | | |
| Course Leader |  | | | | | | | |
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| Signature of the Student | |  | | | | | Date |  |
| Submission date stamp  (by Examination & Assessment Section) | |  | | | | | | |
| Signature of the Course Leader and date | | | | Signature of the Reviewer and date | | | | |
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# **Question No. 1**

**Solution to Question No. 1 part A:**

## A.1 Description of a Model:

**Predator-Prey Model**

In the study of dynamics of a single specie population in ecosystem, we would generally take into consideration factors affecting this single population such as natural growth or the growth rate and the carrying capacity of the environment. But in a bit more generalist the population of a specie would depend on several other factors, one of which is the predator or the prey population. Species compete, evolve and disperse, seek resources in order to survive. As Darwin says “Survival of the Fittest”.

Depending upon the environmental conditions and the type of species living in that environment, this model can have application in forms of resource - consumer, parasite - host, tumor cells – immune system, plant – herbivore. When seemingly competitive interactions are carefully examined, they are often in fact some forms of predator – prey interaction in disguise.

**Let’s take a General Predator – Prey Model**

Consider two population species each of whose population at a reference time is given by and respectively. The functions taken here and are taken to be continuous functions.

Let the function give the per capita growth rate of the first specie and give the per capita growth rate of the second specie.

Now the rate of change of population of a specie is directly dependent on this growth rate and increases according to this growth rate, the rate of change is increasing, hence a positive sign is included.

And similarly

Equations and are Coupled System of Linear Differential Equations and can be solved by any usual method unless it gets more complex. This is a very general form of the model, where some factors such as the death rate of two species are not considered. If we consider even these factors it becomes a

**Lotka - Volterra Model**

Consider some parameters for this model, such as,

The parameter be the growth rate of the species, which we consider as the prey, in the absence of interaction with the predators or the species. Prey numbers are diminished by these interactions. The per capita growth rate decreases with increasing, possibly it may become negative.

The parameter measures the impact of predation on .

The parameter is the death rate of species in the absence of interaction with species .

The term denoted the net growth of the predator population in response to the size of the prey population.

If we modify and as following

And

Substituting these into and

Our Predator-Prey Model becomes,

Or simply,

These Equations are known as the Lotka-Volterra Equations, which are a pair of first order nonlinear differential equations.

The assumptions of this model are

* The predator species is totally dependent on the prey species as its only food supply.
* The prey species has an unlimited food supply and no thread to its growth other than the specific predator.
* The rate at which predators encounter prey is jointly proportional to the sizes of the two populations.
* A fixed proportion of encounters leads to the death of the prey.

## A.2 The Solution of the Model:

Since the discussed model in A1. forms a system of linear differential equations, but due to the simplicity of this model, t can be eliminated to give an implicit solution. The equations have a periodic solutions and do not have a simple expression in terms of the usual trigonometric functions, although they are quite tractable.

So if we divide equation by .

This is a Variable Separable Ordinary Differential Equation,

Where is an arbitrary constant that can be determined from the initial conditions of the population of the two species.

Or,

Where is constant of motion.

If we would like to sketch the level curves of this, let’s consider

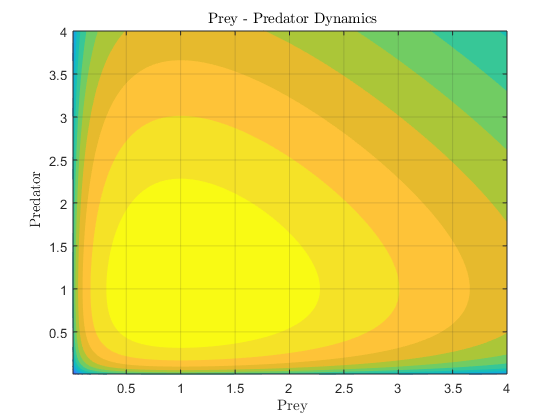
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Figure A1.1 Prey-Predator dynamics as described by the level curves of a conserved quantity.

There are equilibria at and at , in this simulation the data are

## A.3 Comment on the results:

From the results obtained in A2, the right hand side of the identity is referred to as a conservation law at , since it is constant along any solution. Having a conserved quantity facilitates visualizing solutions. In the Figure A1.1 the level curves of the surface is drawn. It can be observed that in the first quadrant of the plane. The contours describe the solutions of the system, and since all of these curves are closed curves, the solutions are said to be in periodic oscillations, but as said earlier these oscillations cannot be expressed in form of normal trigonometric equations.

If , there are two equilibria, , i.e. extinction, and , i.e. coexistence, and the surface had a single peal at the latter equilibrium, The contour lines describe a classic prey-predator cycles observed in ecological systems.

# **Question No. 2**

**Solution to Question No. 1 part B:**

## B.1.1 Obtain a mathematical model to describe population of each countries at any time:

Considering that Austria and Hungary have the same natural growth rate of percent per year, Emigration rate from Austria to Hungary percent per year and from Hungary to Austria percent per year.

Let the population at any time year of Austria be and that of Hungary be .

Since due to Emigration the population will decrease, the rate of change will be negative, while that of Growth and Immigration increase the population of the country hence the rate of change is taken as positive.

Now,

Simplifying it a bit further to make it a little more pretty

These are a coupled System of Differential Equations,

Where

## B.1.2 Convert the above model into a second order ODE and solve manually to get expressions for A(t) and H(t):

Differentiating and w.r.t ,

From and

Rearranging

From and

Which is a Second Order Constant Coefficient Ordinary Differential Equation

Taking , the differential operator

The characteristic equation for this is,

The roots for this equation are

Hence the solution for is,

Taking the roots as and

Differentiating this,

From

Substituting and in

Given the initial conditions and

Substituting these into and

Adding and

Substituting this into

From

## B.1.3 Compute the population of both the countries after 10 years for the given data:

Taking the given values

Substituting these into

Using MATLAB for computing these,

syms A(t) H(t)

syms C1 C2 r a h Ao Ho

C1=(a\*Ao+a\*Ho)/(a+h)

C2=(h\*Ao-a\*Ho)/(a+h)

r = 0.45;

a = 0.08;

h = 0.05;

Ao = 6000000;

Ho = 6850000;

A(t) = C1\*exp((t\*r)/100) + C2\*exp(t\*(r-(a+h))/100);

H(t) = C1\*h\*exp((t\*r)/100)/a-C2\*exp(t\*(r-(a+h))/100);

t = 10;

vpa(subs(A),10)

vpa(subs(H),10)

OUTPUT:

ans(t) =

6301940.759

ans(t) =

7139517.241

Population of Austria is and that of Hungary is .

## B.1.4 Comment on the results obtained and conclude:

The Solution to this population types question is that of a Logistics Population Equation, the two countries depends on each other’s population and hence are said to be coupled. These Coupled equations form a system of Linear Differential Equations and can be solved by the Eigen value method, or by eliminating one of the variables in an equation, hence forming a differential equation of second order in one variable, this is then solved to find the solution to population function of that country, using this solution the population function of the other country is also calculated.

# **Question No. 3**

**Solution to Question No. 2 part B:**

## B.2.1 All possible angular speeds and corresponding deflections:

Given

This is a Cauchy-Euler Differential Equation,

Using the transformation,

The Auxiliary Equation in is,

The Roots for this are,

It’s given that , so , due to this are imaginary,

This is of the form

Which has the solution

Where and

Since and

The Given Initial Conditions are and , the length of the string extends from to

Using in

Using

Also from previous we know

Or

Since we are interested in obtaining non-trivial solutions we will ignore the case of

Let

Then

This has solutions,

Where ,

Since we know that

From

From and

This implies,

From this and

Where

From and

In General,

Taking

Taking

From and

## B.2.2 Plot deflection curves in the given interval:

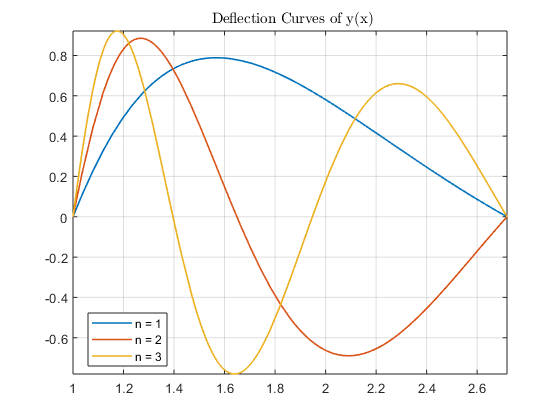


Figure B2.2 Deflection Curves for n=1,2,3

for i = 1:1:3

fplot(@(x) x.^(-0.5).\*sin(i.\*pi.\*log(x)), [1 exp(1)], 'LineWidth', 1.2);

hold on;

end

legend('n = 1', 'n = 2', 'n = 3', 'Location','best');

title('$ $ Deflection Curves of y(x)','Interpreter','latex');

## B.2.3 Comment:

Observing from the deflection curves of we can see that as we increase the number of points at which increases, when , there is one point at which , when , there are two points at which so on and so forth. All of which are solutions for the given Differential Equation.

# **Question No. 4**

**Solution to Question No. 3 part B:**

## B.3.1 Determine the area of the sector brake-pad:

Area of the Break Pad can be calculating by integrating a small element arc of width and at a distance from the centre.

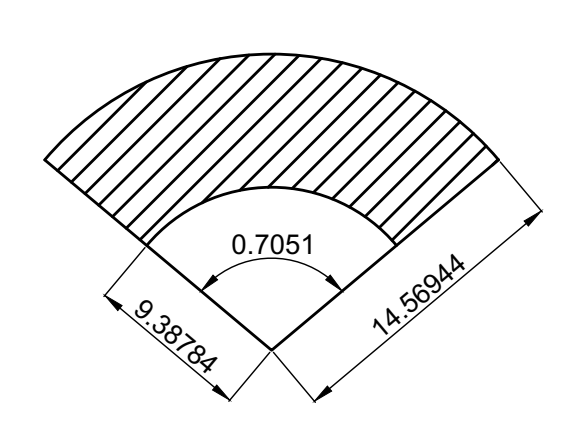


Figure B3.1 Break Pad Illustration

Area of this small elemental strip is

Hence the area of the shaded region will be integrating this from to

Substituting the given values,

## B.3.2 Write a MATLAB function using Simpson’s one-third rule to determine the approximate T:

function [] = simpsons\_one\_third(lower\_bound, upper\_bound, func, partitions)

%SIMPSON's ONE-THIRD rule for Numerical Integration

% Author : Satyajit Ghana

% USAGE : simpsons\_one\_third(lower\_bound, upper\_bound, function, number\_of\_partitions)

% If the number of partitions is not given then its assumed to be

% (upper\_bound-lower\_bound)\*100

% the func should be values of function at points, or a function\_handle

%%

if nargin < 4

partitions = (upper\_bound-lower\_bound)\*100;

end

%%

if isa(func, 'sym') | isa(func, 'function\_handle')

if mod(partitions, 2)

fprintf('The number of Partitions must be even\n');

partitions = partitions + 1;

fprintf('Taking %d Partitions\n', partitions);

end

end

%%

n = partitions;

a = lower\_bound;

b = upper\_bound;

f = func;

h = (b-a)/n;

%%

if isa(func, 'sym')

try

func\_hand = matlabFunction(func);

f = func\_hand;

catch ME

fprintf('The Function should be a function\_handle\n');

disp(ME);

return;

end

end

if isa(func, 'function\_handle')

x = a:h:b;

I = (h/3)\*(f(x(1))+f(x(n+1)) + 4\*sum(f(x(2:2:n))) + 2\*sum(f(x(3:2:n-1))));

elseif ismatrix(func)

if isa(func, 'char')

fprintf('Characters ? Seriously ?\n');

return;

end

if length(func) ~= n

%fprintf('The number of paritions and value points of functions do not match\n');

%return;

end

n = numel(func) - 1;

h = (b-a)/n;

I = (h/3)\*(f(1)+f(end) + 4\*sum(f(2:2:end)) + 2\*sum(f(3:2:end-2)));

%I = (h/3)\*(f(1)+f(n+1) + 4\*sum(f(2:2:n)) + 2\*sum(f(3:2:n-1)));

else

fprintf('The given Function is neither a function\_handle nor a Matrix of Values of Function\n');

return;

end

%%

fprintf('The value of Integral Computed using Simpson''s one-third rule');

if isa(func, 'function\_handle')

fprintf(' for %s ', func2str(func));

end

fprintf(' is ');

disp(vpa(I, 10));

end

Given,

Let , then

x = [9.3878 9.9060 10.4242 10.9423 11.4605 11.9786 12.4968 13.0150 13.5331 14.0513 14.5694];

y = [338 423 474 506 557 573 601 622 651 661 671];

simpsons\_one\_third(x(1), x(end), y.\*x)

OUTPUT

The value of Integral Computed using Simpson's one-third rule is 35258.30218

From this

From and

## B.3.3 Comment on the result:

When the integral to be computed has been given in the form of a vector, i.e. precomputed functional values are different points of independent variable, the number of partitions are limited to one less than the length of the vector, and due to this the precision of the numerical integral therefore obtained cannot be further harnessed. Although instead of one-third rule, three-eighth rule could also be used and so one and so forth to increase the precision a little further, but this wouldn’t have much of an impact on the numerical integral than increasing the number of partitions.

# **Question No. 5**

**Solution to Question No. 4 part B:**

## B.4.1 MATLAB function for Exponential Curve and Parabolic Fit by method of least squares:

function func = exponential\_fit( x, y )

%EXPONENTIAL\_FIT

% Author : Satyajit Ghana

%USAGE: exponential\_fit(xi, yi)

% where xi and yi are the data points given

n = length(x);

Y = log(y);

S = [x; ones(1,n)];

A = S\*S';

B = S\*Y';

D = A\B;

func = @(t) exp(D(1)\*t+D(2));

error = 0;

for k=1:length(x)

error = error + (func(x(k))-y(k))^2;

end

func = subs(func);

disp('Exponential fit for the gien data points is : ');

disp(vpa(func,6));

disp('With a least square error sum as')

disp(error)

%%

plot(x, y, 'r\*');

hold on;

fplot(func, [(min(x)-(x(2)-x(1))) (max(x)+(x(2)-x(1)))],'LineWidth',1.2);

axis tight

title('$ $ Exponential Curve fit for the given data points','Interpreter','latex');

legend('Data Points',char(vpa(func, 4)),'Location','best');

func = matlabFunction(vpa(func, 8));

end

function func = quadratic\_fit( x, y )

%QUADRATIC\_FIT

% Author : Satyajit Ghana

%USAGE: quadratic\_fit(xi, yi)

% where xi and yi are the data points given

n = length(x);

S = [x.\*x; x; ones(1,n)];

A = S\*S';

B = S\*y';

C = A\B;

func = @(t) C(1)\*t^2 + C(2)\*t + C(3);

error = 0;

for k=1:length(x)

error = error + (func(x(k))-y(k))^2;

end

func = subs(func);

disp('Qudratic fit for the gien data points is : ');

disp(vpa(func, 6))

disp('With a least square error sum as')

disp(error)

%%

plot(x,y,'r\*');

hold on;

fplot(func, [(min(x)-(x(2)-x(1))) (max(x)+(x(2)-x(1)))], 'LineWidth', 1.2);

axis tight

title('$ $ Quadratic Fit for the given data points','Interpreter','latex')

legend('Data Points',char(vpa(subs(func),4)),'Location','best');

func = matlabFunction(vpa(func, 8));

end

**A2\_B4.m**

x = [50 450 780 1200 1700 2100 2800];

y = [28 30 32 36 41 50 59];

exponential\_fit(x, y);

quadratic\_fit(x, y);

**OUTPUT:**

Exponential fit for the gien data points is :

exp(0.000282683\*t + 3.27415)

With a least square error sum as

11.6502

Qudratic fit for the gien data points is :

0.00000261173\*t^2 + 0.00419059\*t + 27.5032

With a least square error sum as

6.9378

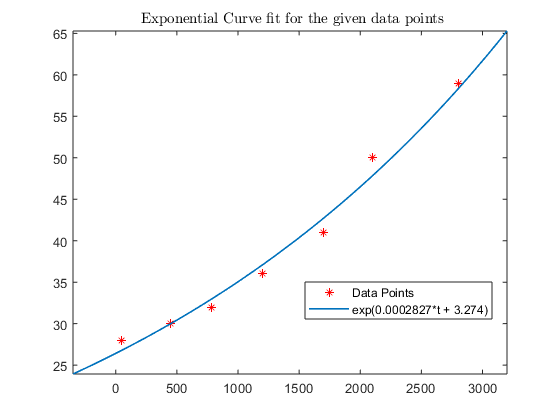


Figure B4.1 Exponential Curve Fit

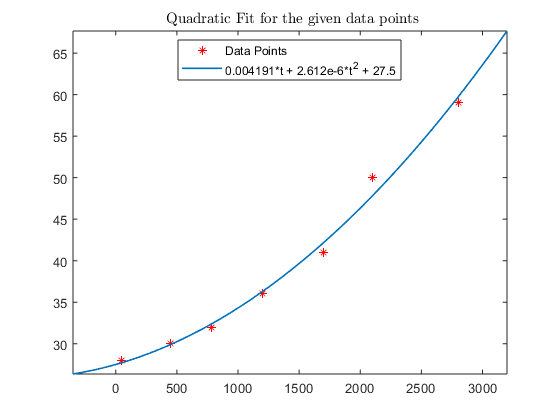


Figure B4.2 Quadratic Curve Fit

## B.4.2 Estimate the day, when the raw materials received is 1500kg, using exponential fit obtained:

x = [50 450 780 1200 1700 2100 2800];

y = [28 30 32 36 41 50 59];

func = exponential\_fit(x, y);

fprintf('The Day on which 1500kg of raw material was recieved : \n');

disp(func(1500));

**OUTPUT:**

Exponential fit for the gien data points is :

exp(0.000282683\*t + 3.27415)

The Day on which 1500kg of raw material was recieved :

40.3734

Hence approximately on the 41st day the raw materials received is 1500kg using the curve obtained from exponential fit.

## B.4.3 Compare the exponential and parabolic fit obtained and determine the best fit among these two for the given data:

From B4.1

Exponential fit for the gien data points is :

exp(0.000282683\*t + 3.27415)

With a least square error sum as

11.6502

Qudratic fit for the gien data points is :

0.00000261173\*t^2 + 0.00419059\*t + 27.5032

With a least square error sum as

6.9378

As it can be observed that Quadratic fit has the least amount of error in fitting the curve through the points, hence Parabolic Fit is the best fit for the given data points. From the Plot obtained in B4.1 it can be clearly seen that the curve passes more closely through the data points in quadratic fit than in exponential fit.

**Bibliography**

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