RAMAIAH UNIVERSITY OF APPLIED SCIENCES

ASSIGNMENT

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Programme	B.Tech
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Semester/Year	04/2019
Course Leader/s	Dr. Somashekhara G.

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<tr><td>Student Name</td><td colspan="4">Satyajit Ghana</td></tr>
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Declaration

The assignment submitted herewith is a result of my own investigations and that I have conformed to the guidelines against plagiarism as laid out in the Student Handbook. All sections of the text and results, which have been obtained from other sources, are fully referenced. I understand that cheating and plagiarism constitute a breach of University regulations and will be dealt with accordingly.

Signature of the Student		Date	
Submission date stamp (by Examination & Assessment Section)			

Signature of the Course Leader and date	Signature of the Reviewer and date

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<u>Question No. 1</u>

<u>Solution to Question No. 1:</u>

Q 1.1 Determine the Integral Surface that passes through the straight lines $x + y = 1$ and $z = 1$:

The given Partial Differential Equation is:

$$x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z \quad - (1)$$

Lagrange's Auxiliary Equation for (1) is:

$$\frac{dx}{x(y^2 + z)} = \frac{dy}{-y(x^2 + z)} = \frac{dz}{(x^2 - y^2)z}$$

Choosing the Multipliers as $\left(\frac{1}{x}, \frac{1}{y}, \frac{1}{z}\right)$

$$e.r = \frac{\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz}{\frac{1}{x}x(y^2 + z) - \frac{1}{y}y(x^2 + z) + \frac{1}{z}(x^2 - y^2)z}$$

$$e.r = \frac{\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz}{y^2 + z - x^2 - z + x^2 - y^2}$$

$$e.r. = \frac{\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz}{0} = 0$$

Hence one of the solutions is

$$\int \frac{1}{x}dx + \int \frac{1}{y}dy + \int \frac{1}{z}dz = 0$$

$$\ln x + \ln y + \ln z = \ln c_1$$

$$\ln xyz = \ln c_1$$

$$xyz = c_1 \quad - (2)$$

Choosing the Multipliers as $(x, y, -1)$

$$e.r = \frac{xdx + ydy - dz}{x \times x(y^2 + z) - y \times y(x^2 + z) - (x^2 - y^2)z}$$

$$e.r = \frac{xdx + ydy - dz}{x^2y^2 + x^2z - y^2x^2 - y^2z + x^2z - y^2z}$$

$$e.r. = \frac{xdx + ydy - dz}{0} = 0$$

Hence the other solution is

$$\int xdx + \int ydy - \int dz = 0$$

$$\frac{x^2}{2} + \frac{y^2}{2} - z = c_2$$

$$x^2 + y^2 - 2z = 2c_2$$

$$x^2 + y^2 - 2z = c_2^* - (3)$$

From (2) and (3) the solutions are:

$$u = xyz = c_1$$
$$v = x^2 + y^2 - 2z = c_2 - (4)$$

The General Solution being

$$\phi(xyz, x^2 + y^2 - 2z)$$

To find the integral surface we take t as the parameter, the given equation of straight lines are $x + y = 1$ and $z = 1$

$$x = t$$
$$y = 1 - t$$
$$z = 1 \quad - (5)$$

Using the parametric form (5) in (4)

$$t - t^2 = c_1$$
$$t^2 + 1 + t^2 - 2t - 2 = 2t^2 - 2t - 1 = c_2$$

Rearranging the equations

$$t^2 - t = -c_1$$
$$2(t^2 - t) - 1 = c_2$$

Which becomes

$$-2c_1 - 1 = c_2$$

Or

$$2c_1 + c_2 + 1 = 0$$

Substituting the values for c_1 and c_2 the Integral Surface is,

$$2xyz + x^2 + y^2 - 2z + 1 = 0$$

Q 1.2 Plot the Integral Surface:

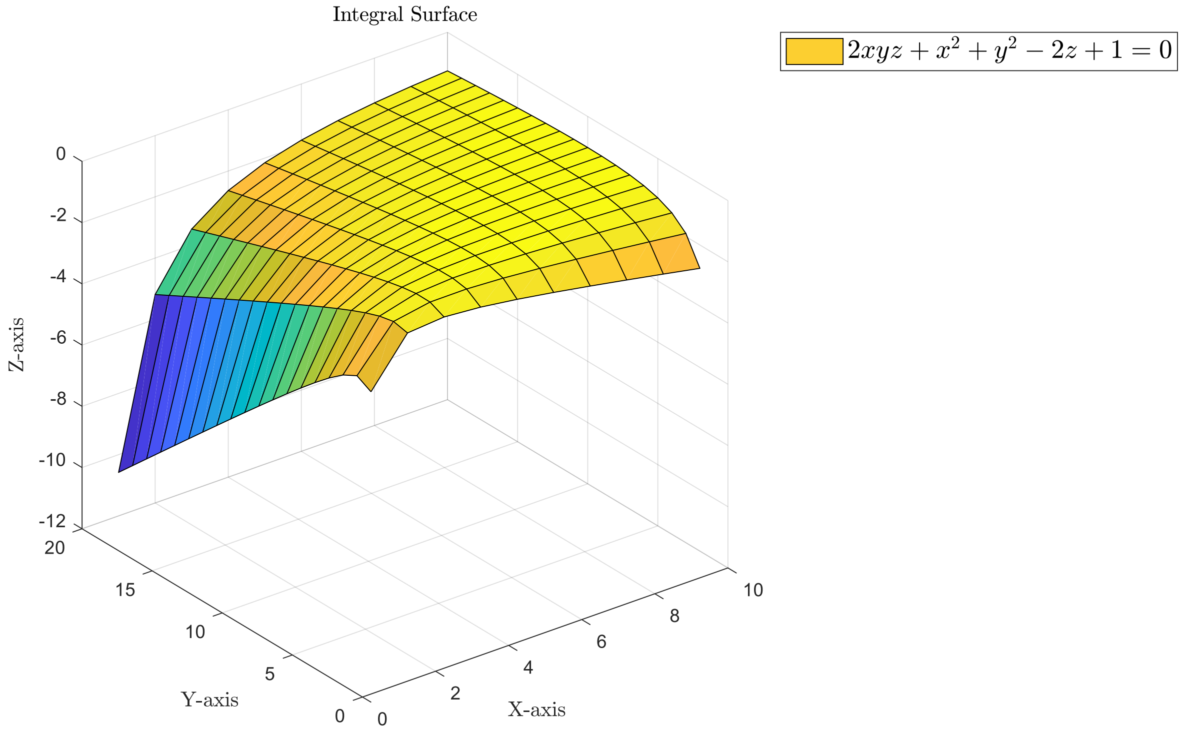


Figure 1 Integral Surface plot

```
[X,Y]= meshgrid(1:10,2:20);
Z=(X.^2+Y.^2+1)./(2-(2*X.*Y));
surf(X,Y,Z)
title('Integral Surface', 'Interpreter', 'latex');
legend({'$2xyz+x^2+y^2-2z+1=0$'}, 'Interpreter', 'latex', 'FontSize',
14);
xlabel('X-axis', 'Interpreter', 'latex');ylabel('Y-axis', 'Interpreter',
'latex');zlabel('Z-axis', 'Interpreter', 'latex')
```

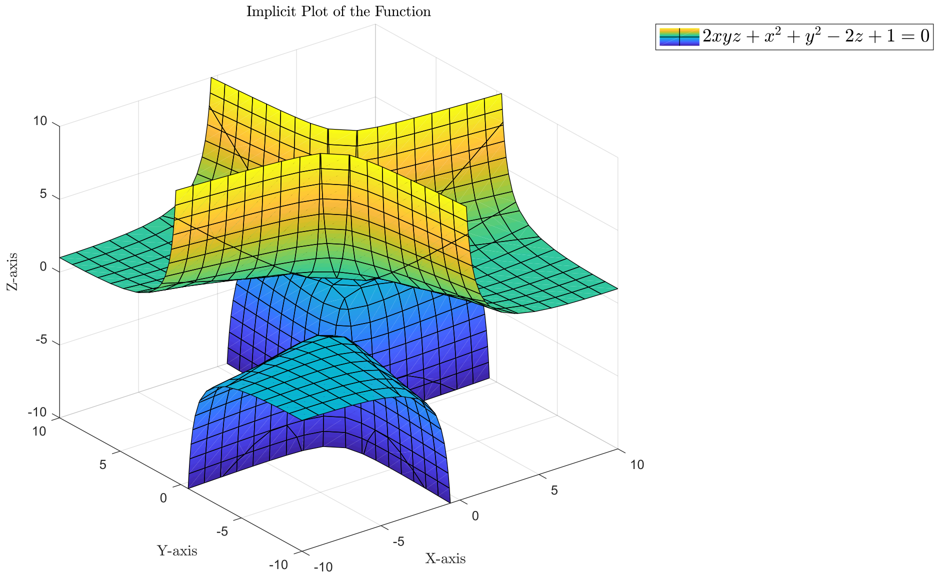
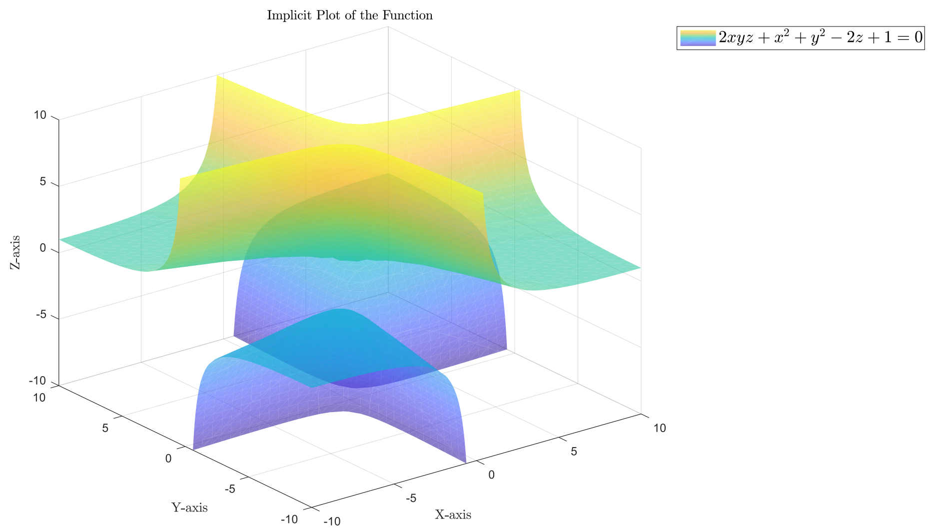


Figure 2 Integral Surface plot using fimplicit3

```
f = @(x,y,z) 2.*x.*y.*z + x.^2 + y.^2 - 2.*z + 1;
interval = [-10 10 -10 10 -10 10];
fimplicit3(f, interval, 'MeshDensity', 20);
title('Implicit Plot of the Function', 'Interpreter', 'latex')
legend({'$2xyz+x^2+y^2-2z+1=0$'}, 'Interpreter', 'latex', 'FontSize', 14)
xlabel('X-axis', 'Interpreter', 'latex');ylabel('Y-axis', 'Interpreter',
'latex');zlabel('Z-axis', 'Interpreter', 'latex')
grid on;
```




```
f = @(x,y,z) 2.*x.*y.*z + x.^2 + y.^2 - 2.*z + 1;
interval = [-10 10 -10 10 -10 10];
fimplicit3(f, interval, 'EdgeColor', 'none', 'FaceAlpha', 0.6);
title('Implicit Plot of the Function', 'Interpreter', 'latex')
legend({'$2xyz+x^2+y^2-2z+1=0$'}, 'Interpreter', 'latex', 'FontSize', 14)
xlabel('X-axis', 'Interpreter', 'latex');ylabel('Y-axis', 'Interpreter',
'latex');zlabel('Z-axis', 'Interpreter', 'latex')
grid on;
```

Question No. 2

Solution to Question No. 1 Part B:

Q 2.1 Write the mathematical form of the logistic model:

Logistic growth deals with growth rates that are directly proportional to both of the quantities P and $K - P$,

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

Which is the differential equation for logistic growth,

Where,

P : $Population\ at\ any\ time\ t$

K : $Carrying\ capacity$

The given values are:

$$P(0) = P_0 = 50\ million$$
$$growth\ rate = 0.5\ million\ per\ year$$
$$K = 5000\ million$$

NOTE: We'll be using one million as the base unit for population, and year as the base unit for time.

case 1: taking r as the relative increase in the population

Since the initial population is small compared to the carrying capacity, we take the initial relative growth rate $\frac{growth\ rate}{initial\ population} = \left(\frac{0.5}{50}\right)$ to be an estimate of r.

Hence the Logistic Model becomes,

$$\frac{dP}{dt} = \frac{0.5}{50}P\left(1 - \frac{P}{5000}\right)$$

$$\frac{dP}{dt} = 0.01P\left(1 - \frac{P}{5000}\right) - (1)$$

case 2: taking r as the absolute increase in the population

We take r as it is without any modification to form,

$$\frac{dP}{dt} = 0.5P\left(1 - \frac{P}{5000}\right) - (2)$$

Q 2.2 Obtain the exact solution of the mathematical problem defined in Q2.1:

<u>case 1: taking r as the absolute increase in the population</u>

In order to solve (2) we separate the variables first and integrate both sides:

$$\int \frac{5000dP}{P(5000-P)} = \int 0.5dt$$

Separating the integrand by partial fractions we have,

$$\frac{5000}{P(5000-P)} = \frac{1}{P} + \frac{1}{5000-P}$$

Therefore,

$$\int \frac{dP}{P} + \int \frac{dP}{5000-P} = \int 0.5dt$$

$$\ln|P| - \ln|5000-P| = 0.5t + C$$

$$\ln\left|\frac{5000-P}{P}\right| = -0.5t - C$$

$$\frac{5000-P}{P} = C_1 e^{-0.5t}\ ; here\ C_1 = e^{-C}$$

When $t = 0$, $P = P_0 = 50$

$$C_1 = \frac{5000-50}{50}$$

$$C_1 = 99$$

Therefore,

$$\frac{5000-P}{P} = 99 \times e^{-0.5t}$$

$$5000 - P = 99P \times e^{-0.5t}$$

$$P = \frac{5000}{1+99e^{-0.5t}} - (3)$$

<u>case 2: taking r as the relative increase in the population</u>

$$P = \frac{5000}{1+99e^{-0.01t}} - (4)$$

Q 2.3 Write a MATLAB function to obtain the population (in millions) after 12 years in steps of:

Assuming r to be the absolute increase in the population, we use the Runge-Kutta 4th order approximation on (3).

rk4order.m

```
function [x, y] = rk4order(x0, y0, xn, h, f)
% Author : Satyajit Ghana 17ETCS002159
% RK4ORDER Runge-Kutta 4th-Order Approximation
%   The Runge-Kutta method finds approximate value of y for a given x.
% Only first order ordinary differential equations can be solved by
using
% the Runge Kutta 4th order method.
% Params:
% x0 = x(0)
% y0 = y(0)
% xn = x at which value of y is to be approximated
% h = step length
% f = slope as a function of x and y

    x = x0:h:xn;
    n = length(x);
    y(1) = y0;

    for i = 1:n-1
        k1 = h * f(x(i), y(i));
        k2 = h * f(x(i)+h/2, y(i)+k1/2);
        k3 = h * f(x(i)+h/2, y(i)+k2/2);
        k4 = h * f(x(i)+h, y(i)+k3);

        y(i+1) = y(i)+(1/6)*(k1+2*k2+2*k3+k4);
    end
end
```

(i) One year

```
P = @(t, P) 0.5.*P.*(1-P./5000);
t0 = 0;
P0 = 50;

%% taking tn = 12th year, step = 1 year
tn = 12;
h = 1;

[x, y] = rk4order(t0, P0, tn, h, P);

fprintf("taking tn = 12 years, step = 1 year\n");
fprintf("The population after 12 years is %10.5f million\n", y(end))
```

OUTPUT:

```
>>
taking tn = 12 years, step = 1 year
The population after 12 years is 4013.93960 million
```

(ii) 6 months

```
P = @(t, P) 0.5.*P.*(1-P./5000);
t0 = 0;
P0 = 50;

%% taking tn = 12th year, step = 0.5 year or 6 months
```

```
tn = 12;
h = 0.5;

[x, y] = rk4order(t0, P0, tn, h, P);

fprintf("taking tn = 12 years, step = 0.5 year or 6 months\n");
fprintf("The population after 12 years is %10.5f million\n", y(end))
```

OUTPUT:

```
>>
taking tn = 12 years, step = 0.5 year or 6 months
The population after 12 years is 4014.72435 million
```

Q 2.4 Plot the exact solution and numerical solution obtained in Q2.2 and Q2.3 in the same graph.

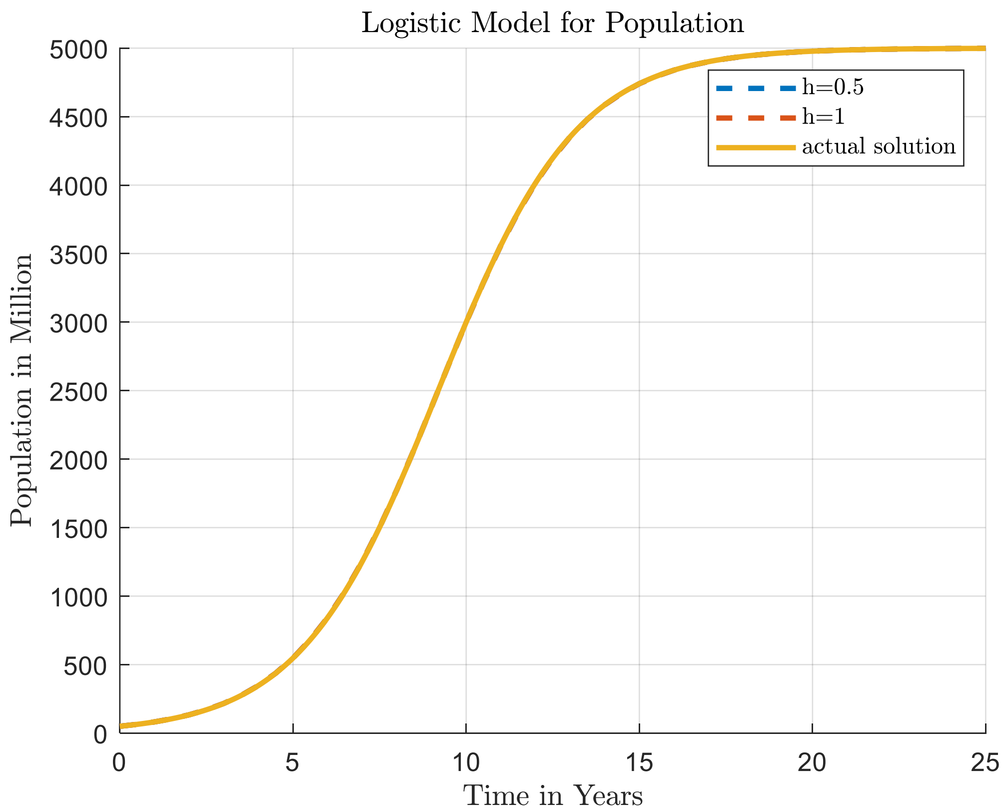


Figure 3 Logistic Model for Population

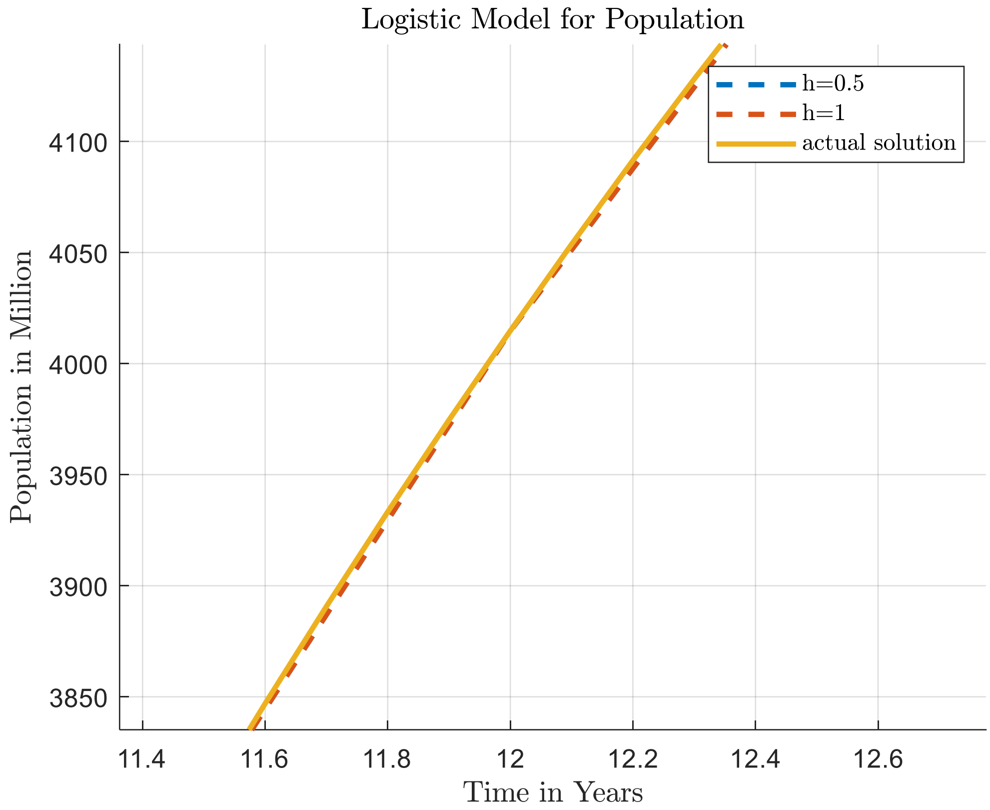


Figure 4 Logistic Model with finer data points

```
%% plotting the approximated solution and actual solution
tn = 25;

% taking step size as 6 months, and 1 year
h1 = 0.5;
h2 = 1;

[x1, y1] = rk4order(t0, P0, tn, h1, P);
[x2, y2] = rk4order(t0, P0, tn, h2, P);

% actual solution
aP = @(t) 5000./(1+99.*exp(-0.5.*t));

x = 0:0.1:tn;
y = aP(x);

hold on;
plot(x1, y1, '--', 'LineWidth', 2);
plot(x1, y1, '--', 'LineWidth', 2)
plot(x, y, 'LineWidth', 2);

legend({'h=0.5', 'h=1', 'actual solution'}, 'Interpreter', 'latex');
title('Logistic Model for Population', 'Interpreter', 'latex');
xlabel('Time in Years', 'Interpreter', 'latex')
ylabel('Population in Million', 'Interpreter', 'latex')

grid on;
```

Q 2.5 Comment on the solution and graph

The solution that was obtained by using Runge-Kutta 4th order Approximation is almost same as that of the Actual Solution with very little error. From Visual inspection the data points are very close and almost undistinguishable.

The Root Mean Square Error can be computed to compare the results numerically. Here the step size is taken as 6 months for both the Runge-Kutta 4th order and the actual solution.

```
%% RMS error
tn = 100;
h = 0.5;
x = 0:h:tn;
y = aP(x);
[x1, y1] = rk4order(t0, P0, tn, h, P);
rmse = sqrt(mean((y-y1).^2));
fprintf('Root Mean Square Error is : %10.5f\n', rmse);
```

OUTPUT:

```
Root Mean Square Error is :    0.02050
```

Bibliography

1.