Assignment

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Course Name	Signals and Systems
Programme	B.Tech
Department	CSE
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Reg. No.	17ETCS002159
Semester/Year	04/2017
Course Leader(s)	Ms. Prafulla Kumari

Declaration Sheet

<table>
<tr><td>Student Name</td><td colspan="3">Satyajit Ghana</td></tr>
<tr><td>Reg. No</td><td colspan="3">17ETCS002159</td></tr>
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Declaration

The assignment submitted herewith is a result of my own investigations and that I have conformed to the guidelines against plagiarism as laid out in the Student Handbook. All sections of the text and results, which have been obtained from other sources, are fully referenced. I understand that cheating and plagiarism constitute a breach of University regulations and will be dealt with accordingly.

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Signature of the Course Leader and date	Signature of the Reviewer and date

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Solution to Question No. 1 Part A

"Discrete Cosine Transform for Audio Signal Processing"

1.1 Introduction

The Discrete Fourier Transform (DFT) and Discrete Cosine Transform (DCT) perform similar functions: they both decompose a finite-length discrete-time vector into a sum of scaled-and-shifted basis functions. The difference between the two is the type of basis function used by each transform; the DFT uses a set of harmonically-related complex exponential functions, while the DCT uses only (real-valued) cosine functions.

The DFT is widely used for general spectral analysis applications that find their way into a range of fields. It is also used as a building block for techniques that take advantage of properties of signals' frequency-domain representation, such as the overlap-save and overlap-add fast convolution algorithms.

The DCT is frequently used in lossy data compression applications, such as the JPEG image format. The property of the DCT that makes it quite suitable for compression is its high degree of "spectral compaction;" at a qualitative level, a signal's DCT representation tends to have more of its energy concentrated in a small number of coefficients when compared to other transforms like the DFT. This is desirable for a compression algorithm; if you can approximately represent the original (time- or spatial-domain) signal using a relatively small set of DCT coefficients, then you can reduce your data storage requirement by only storing the DCT outputs that contain significant amounts of energy.

Multiresolution analysis and wavelets provide a convenient framework for compression of audio files. Common audio formats like MP3, AAC and Ogg Vorbis were developed before this time and use a mathematical operation called the Discrete Cosine Transform (DCT) to transform the audio data to a form that lends itself well to compression.

1.2 MATLAB Example

This example shows how to compress a speech signal using the discrete cosine transform (DCT)

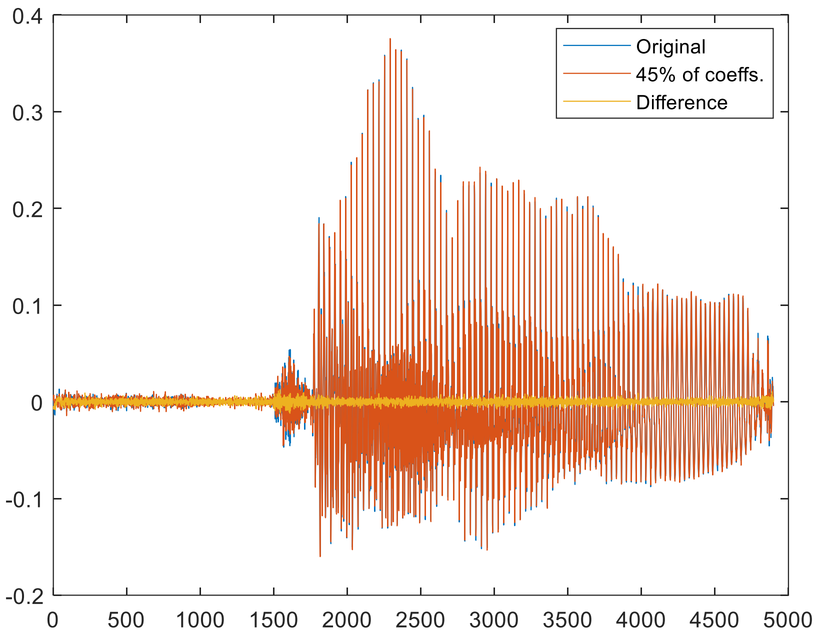
```
load(fullfile(matlabroot,'examples','signal','strong.mat'))

x = her';
X = dct(x);
[XX,ind] = sort(abs(X),'descend');
```

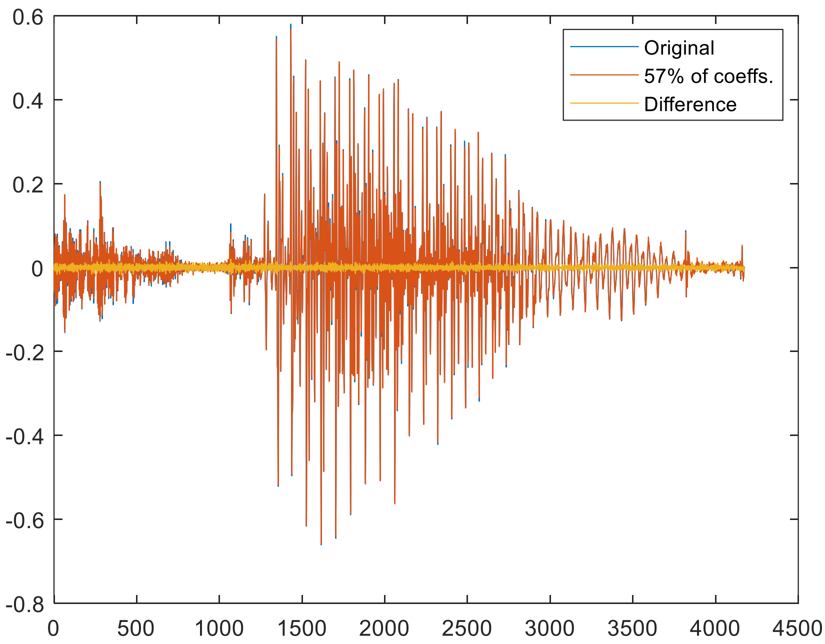
Use the discrete cosine transform to compress the female voice signal. Decompose the signal into DCT basis vectors. There are as many terms in the decomposition as there are samples in the signal. The expansion coefficients in vector X measure how much energy is stored in each of the components. Sort the coefficients from largest to smallest.

```
need = 1;
while norm(X(ind(1:need)))/norm(X)<0.999
   need = need+1;
end

xpc = need/length(X)*100;


X(ind(need+1:end)) = 0;
xx = idct(X);
```

Set to zero the coefficients that contain the remaining 0.1% of the energy. Reconstruct the signal from the compressed representation. Plot the original signal, its reconstruction, and the difference between the two.

```
figure(1)
plot([x;xx;x-xx]')
legend('Original',[int2str(xpc) '% of coeffs.'],'Difference', ...
       'Location','best')

y = him';
Y = dct(y);

[YY,ind] = sort(abs(Y),'descend');

need = 1;
while norm(Y(ind(1:need)))/norm(Y)<0.999
   need = need+1;
end

ypc = need/length(Y)*100;

Y(ind(need+1:end)) = 0;
yy = idct(Y);

figure(2)
plot([y;yy;y-yy]')
legend('Original',[int2str(ypc) '% of coeffs.'],'Difference', ...
       'Location','best')
```





In both cases, about half of the DCT coefficients suffice to reconstruct the speech signal reasonably. If the required energy fraction is 99%, the number of necessary coefficients reduces to about 20% of the total. The resulting reconstruction is inferior but still intelligible.

Analysis of these and other samples suggests that more coefficients are needed to characterize the man's voice than the woman's.

2 Question 2

Solution to Question 1 Part B

2.1 Formulate and solve the Difference Equation using the given data

Given that the traffic during the current hour depends on the traffic during past two hours and on external factors. The external factors are found to be of the form $A\cos\Omega n$.

The Difference Equation can be formulated as,

$$\begin{aligned} y\ n\ &= a_1 y\ n-1\ + a_2 y\ n-2\ + x\ n \\ x\ n\ &= A\cos\Omega n \end{aligned}$$

The values given are:

$$\begin{aligned} a_1 &= 4 \\ a_2 &= -3 \\ A &= 5 \\ y\ -1\ &= 3 \\ y\ -2\ &= 7 \\ \Omega &= \pi \end{aligned}$$

Hence the Equation becomes,

$$y\ n\ - 4y\ n-1\ + 3y\ n-2\ = 5\cos\pi n \qquad -(1)$$

The Auxiliary Equation for (1) is,

$$\begin{aligned} m^2 - 4m + 3 &= 0 \\ m-1\ \ m-3\ &= 0 \\ m_1 &= 1, m_2 = 3 \end{aligned}$$

The Natural Response is,

$$y_N\ n\ = c_1\ 1\ ^n + c_2\ 3\ ^n$$

Given,

$$x\;n\; = 5\cos\pi n = 5\;{-1}^{\;n}$$

The Forced Response is,

$$y_F\;n\; = c\;{-1}^{\;n}$$
$$y_F\;n-1\; = c\;{-1}^{\;n-1}$$
$$y_F\;n-2\; = c\;{-1}^{\;n-2}$$

Substituting these in (1)

$$\begin{aligned}
&c\;{-1}^{\;n} - 4c\;{-1}^{\;n-1} + 3c\;{-1}^{\;n-2} = 5\;{-1}^{\;n}\\
&c\;\;{-1}^{\;n} - 4\;{-1}^{\;n-1} + 3\;{-1}^{\;n-2}\; = 5\;{-1}^{\;n}\\
&c\;{-1}^{\;n}\left(1 + \frac{-4}{-1^{\;1}} + \frac{3}{-1^{\;2}}\right) = 5\;{-1}^{\;n}\\
&c\;8\; = 5\\
&c = \frac{5}{8}
\end{aligned}$$

$$y_F\;n\; = \frac{5}{8}\;{-1}^{\;n}$$

The Total Response is,

$$\begin{aligned}
y_T\;n\; &= y_N\;n\; + y_F\;n\\
y_T\;n\; &= c_1\;1^{\;n} + c_2\;3^{\;n} + \frac{5}{8}\;{-1}^{\;n} \qquad -(2)
\end{aligned}$$

Given the boundary conditions, $y\;{-1}\; = 3, y\;{-2}\; = 7$

Substituting these values into (2)

$$\begin{aligned}
y\;{-1}\; &= c_1\;1^{\;-1} + c_2\;3^{\;-1} + \frac{5}{8}\;{-1}^{\;-1}\\
3 &= c_1 + \frac{c_2}{3} - \frac{5}{8}\\
c_1 + \frac{c_2}{3} &= \frac{29}{8} \qquad -(3)
\end{aligned}$$

$$y\ \ -2\ \ = c_1\ \ 1\ ^{-2} + c_2\ \ 3\ ^{-2} + \frac{5}{8}\ \ -1\ ^{-2}$$

$$7 = c_1 + \frac{c_2}{9} + \frac{5}{8}$$

$$c_1 + \frac{c_2}{9} = \frac{51}{8} \qquad\qquad -(4)$$

From (3) and (4),

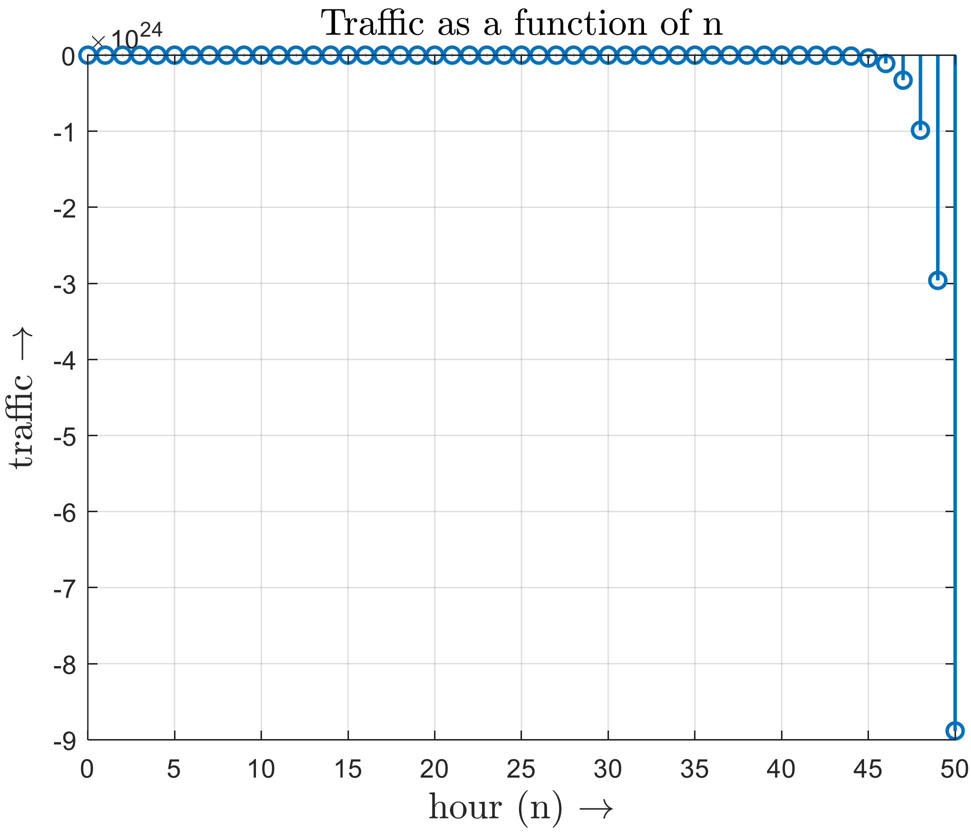
$$c_1 = \frac{62}{8}$$

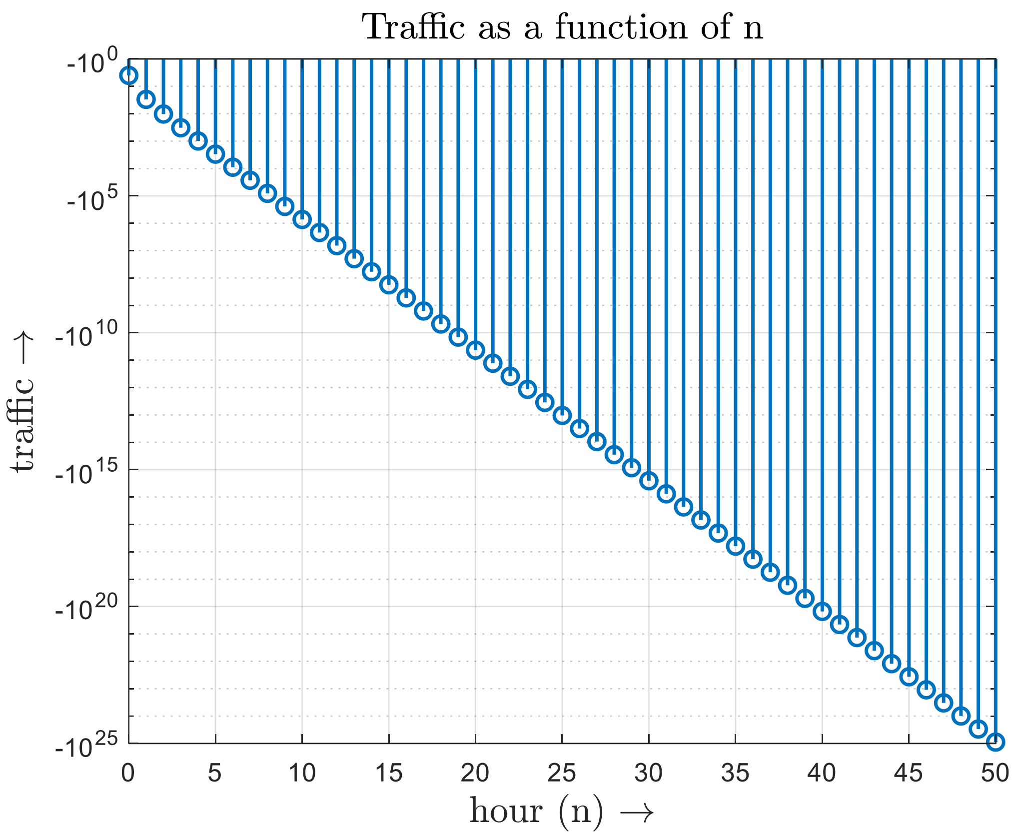
$$c_2 = -\frac{99}{8}$$

Hence the Final Equation becomes,

$$y_T\ \ n\ \ = \frac{62}{8}\ \ 1\ ^{n} - \frac{99}{8}\ \ 3\ ^{n} + \frac{5}{8}\ \ -1\ ^{n}$$

2.2 Plot the traffic as a function of n = 0, 1, 2 . . . 50






```
syms y(n)
y(n) = (62/8)-(99/8).*((3).^n)+(5/8).*((-1).^n);
n = 0:1:50;
stem(n, subs(y, n), 'linewidth', 1.3);
set(gca, 'YScale', 'log')

grid on;
title('Traffic as a function of n', 'Interpreter', 'latex', 'FontSize', 14)
xlabel('hour (n) $\rightarrow$', 'Interpreter', 'latex', 'FontSize', 14)
ylabel('traffic $\rightarrow$', 'Interpreter', 'latex', 'FontSize', 14)
```

2.3 Comment on the variability of the traffic using the solution

Analyzing from the graph obtained in 2.2, the Traffic from hour n = 0 to hour n = 50 increases in the negative direction in an exponential scale, when the graph is plotted in logarithmic scale it becomes linear.

If we ignore the sign and focus on the magnitude of the solution obtained, we can conclude that the Traffic increases with n exponentially. The values at different time intervals obtained are,

```
[ -4, -730723, -43148956954, -2547902759671273, -150451110055829493904, -888398759
7686675786031823]
```

These values are at hour n = 0, 10, 20, 30, 40, 50.

As we can observe that at n = 0 the traffic is 4, while in 10 hours it increases rapidly to 730723 and keeps on increasing rapidly to 43148956954 in the following hours.

If we continue to plot the graph for n > 50, the traffic keeps on increasing, there is no point where the direction of the derivative of the graph changes, hence it is a monotonously increasing function for n > 0.

3 Question 3

Solution to Question 2 Part B

Given data:

$m\ \ t\ $: Amplitude 6, from t = -1 to t = 1

$w(t)$: Amplitude 2, from t = 0 to t = 2

$h(t)$: Amplitude 4, from t = 2 to t = 3

3.1 Compute the Laplace Transforms $\boldsymbol{W\ \ s\ }, \boldsymbol{H\ \ s\ }$ and $\boldsymbol{Y(s)}$

The given data signals can be represented in the form of unit step or Heaviside functions,

$$\begin{aligned} m\ \ t\ \ &= 6[u\ \ t+1\ \ - u\ \ t-1\ \ ] \\ w\ \ t\ \ &= 2[u\ \ t\ \ - u\ \ t-2\ \ ] \\ h\ \ t\ \ &= 4[u\ \ t-2\ \ - u(t-3)] \end{aligned}$$

Now the Laplace transform of the individual signals can be calculated,

$$\mathcal{L}\{m(t)\} = 6[\mathcal{L}\{u(t+1)\} - \mathcal{L}\{u(t-1)\}]$$

$\begin{aligned} \mathcal{L}\{u\ \ t+1\ \ \} &= \int_{-\infty}^{\infty} e^{-st} u\ \ t+1\ \ dt \\ &= \int_{-1}^{\infty} e^{-st} dt \\ &= \frac{e^{-st}}{-s}\Big\|_{-1}^{\infty} \\ &= \frac{e^{-\infty} - e^{s}}{-s} \\ &= \frac{e^{s}}{s} \end{aligned}$	$\begin{aligned} \mathcal{L}\{u\ \ t-1\ \ \} &= \int_{-\infty}^{\infty} e^{-st} u\ \ t-1\ \ dt \\ &= \int_{1}^{\infty} e^{-st} dt \\ &= \frac{e^{-st}}{-s}\Big\|_{1}^{\infty} \\ &= \frac{e^{-\infty} - e^{-s}}{-s} \\ &= \frac{e^{-s}}{s} \end{aligned}$

$$M\ \ s\ \ = 6\left[\frac{e^{s}}{s} - \frac{e^{-s}}{s}\right]$$

$$= \frac{6}{s}[e^{s} - e^{-s}]$$

$$\mathcal{L}\{w(t)\} = 2[\mathcal{L}\{u(t)\} - \mathcal{L}\{u(t-2)\}]$$

$\begin{aligned} \mathcal{L}\{u\ t\ \} &= \int_{-\infty}^{\infty} e^{-st} u\ t\ dt \\ &= \int_{0}^{\infty} e^{-st} dt \\ &= \frac{e^{-st}}{-s}\bigg\rvert_{0}^{\infty} \\ &= \frac{e^{-\infty} - e^{-0s}}{-s} \\ &= \frac{1}{s} \end{aligned}$	$\begin{aligned} \mathcal{L}\{u\ t-2\ \} &= \int_{-\infty}^{\infty} e^{-st} u\ t-2\ dt \\ &= \int_{2}^{\infty} e^{-st} dt \\ &= \frac{e^{-st}}{-s}\bigg\rvert_{2}^{\infty} \\ &= \frac{e^{-\infty} - e^{-2s}}{-s} \\ &= \frac{e^{-2s}}{s} \end{aligned}$

$$\begin{aligned} W\ s\ &= 2\left[\frac{1}{s} - \frac{e^{-2s}}{s}\right] \\ &= \frac{2}{s}[1 - e^{-2s}] \end{aligned}$$

$$\mathcal{L}\{h(t)\} = 4[\mathcal{L}\{u(t-2)\} - \mathcal{L}\{u(t-3)\}]$$

$\begin{aligned} \mathcal{L}\{u\ t-2\ \} &= \int\limits_{-\infty}^{\infty} e^{-st} u\ t-2\ dt \\ &= \int\limits_{2}^{\infty} e^{-st} dt \\ &= \frac{e^{-st}}{-s}\Big\vert_{2}^{\infty} \\ &= \frac{e^{-\infty}-e^{-2s}}{-s} \\ &= \frac{e^{-2s}}{s} \end{aligned}$	$\begin{aligned} \mathcal{L}\{u\ t-3\ \} &= \int\limits_{-\infty}^{\infty} e^{-st} u\ t-3\ dt \\ &= \int\limits_{3}^{\infty} e^{-st} dt \\ &= \frac{e^{-st}}{-s}\Big\vert_{3}^{\infty} \\ &= \frac{e^{-\infty}-e^{-3s}}{-s} \\ &= \frac{e^{-3s}}{s} \end{aligned}$

$$\begin{aligned} H\ s\ &= 4\left[\frac{e^{-2s}}{s} - \frac{e^{-3s}}{s}\right] \\ &= \frac{4}{s}[e^{-2s} - e^{-3s}] \end{aligned}$$

$$\begin{aligned} Y\ s\ &= H\ s\ \cdot [M\ s\ + W\ s\ ] \\ &= \frac{4}{s}[e^{-2s} - e^{-3s}] \times \left(\frac{6}{s}[e^{s} - e^{-s}] + \frac{2}{s}[1 - e^{-2s}]\right) \\ &= \frac{1}{s^2}\{\ 4e^{-2s} - 4e^{-3s}\ \}\ 6e^{s} - 6e^{-s} + 2 - 2e^{-2s} \\ &= \frac{8}{s^2}\{e^{-5s} + 2e^{-4s} - 4e^{-3s} - 2e^{-2s} + 3e^{-s}\} \end{aligned}$$

3.2 Compute the response $V\ s$ of the system when only $w(t)$ is transmitted over $h(t)$

Given,

$$v\ t\ = w\ t\ * h(t)$$

Taking Laplace Transform on both sides

$$V\ s\ = W\ s\ \cdot H\ s$$

$$
\begin{aligned}
V\ \ s\ \ &= 2\left[\frac{1}{s} - \frac{e^{-2s}}{s}\right] \times 4\left[\frac{e^{-2s}}{s} - \frac{e^{-3s}}{s}\right] \\
&= \frac{8}{s^2}\left[e^{-5s} - e^{-4s} - e^{-3s} + e^{-2s}\right]
\end{aligned}
$$

3.3 Recover the message signal $m(t)$ from $Y(s)$

Consider,

$$
\begin{aligned}
Y\ \ s\ \ &= H\ \ s\ \ \cdot [M\ \ s\ \ + W\ \ s\ \ ] \\
&= H\ \ s\ \ \cdot M\ \ s\ \ + H\ \ s\ \ \cdot W\ \ s\ \ \quad -(1)
\end{aligned}
$$

Also,

$$
V\ \ s\ \ = W\ \ s\ \ \cdot H\ \ s\ \ \quad -(2)
$$

$1\ \ -(2)$,

$Y\ \ s\ \ - V\ \ s\ \ = H\ \ s\ \ \cdot M\ \ s$

$$
\begin{aligned}
M\ \ s\ \ &= \left[\frac{Y\ \ s\ \ - V\ \ s}{H\ \ s}\right] \\
&= \frac{\frac{8}{s^2}\{e^{-5s} + 2e^{-4s} - 4e^{-3s} - 2e^{-2s} + 3e^{-s}\} - \frac{8}{s^2}[e^{-5s} - e^{-4s} - e^{-3s} + e^{-2s}]}{\frac{4}{s}[e^{-2s} - e^{-3s}]} \\
&= \frac{\frac{24}{s^2}\{e^{-4s} - e^{-3s} - e^{-2s} + e^{-s}\}}{\frac{4}{s}[e^{-2s} - e^{-3s}]} \\
&= \frac{6}{s}[e^{s} - e^{-s}] \\
&= \frac{12}{s}\sinh s
\end{aligned}
$$

Inverse Laplace Transform on both sides to obtain $m(t)$.

$$
\begin{aligned}
m\ \ t\ \ &= \mathcal{L}^{-1}\left\{\frac{6}{s}[e^{s} - e^{-s}]\right\} \\
&= 6\left\{\mathcal{L}^{-1}\left\{\frac{e^{s}}{s}\right\} - \mathcal{L}^{-1}\left\{\frac{e^{-s}}{s}\right\}\right\} \\
&= 6[u\ \ t+1\ \ - u(t-1)]
\end{aligned}
$$

3.4 Analyze the effect of change in operation between the message and the watermark on the response $w(t)$ and $m(t)$ are convolved instead of being added.

$$y\ t\ = m\ t\ * w\ t\ * h(t)$$

Taking Laplace Transform on both sides,

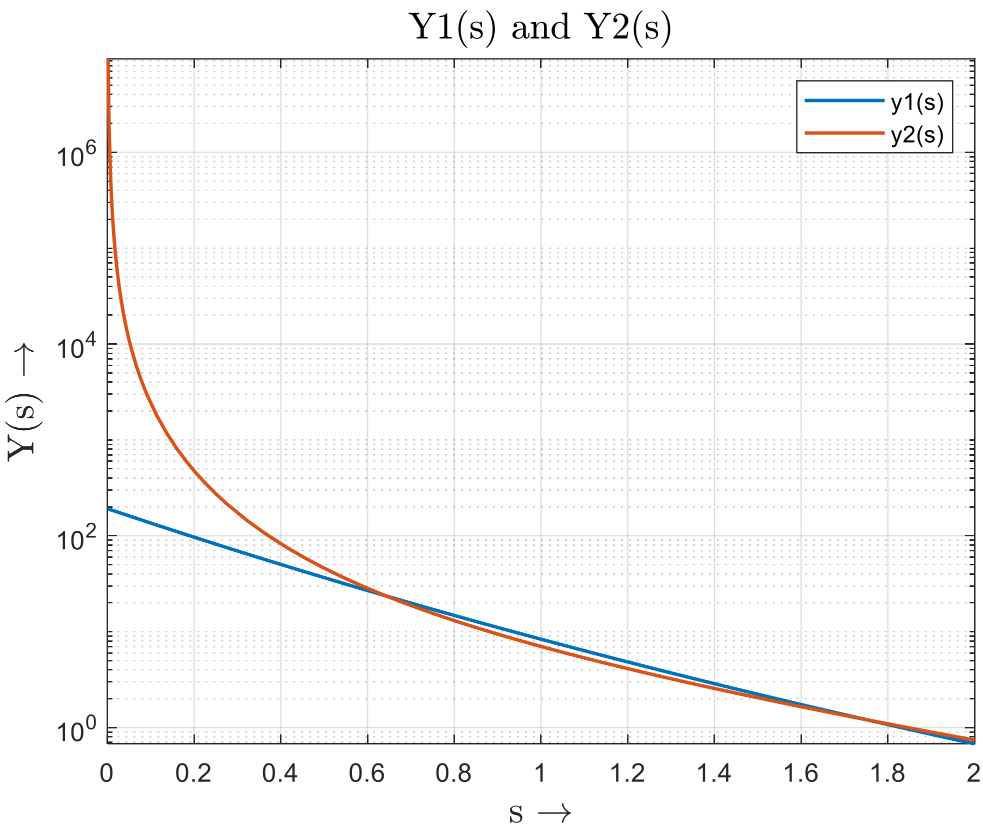
$$\begin{aligned} Y\ s\ &= M\ s\ \cdot W\ s\ \cdot H\ s \\ &= \frac{6}{s}[e^{s} - e^{-s}] \times \frac{2}{s}[1 - e^{-2s}] \times \frac{4}{s}[e^{-2s} - e^{-3s}] \\ &= \frac{48}{s^3}[-e^{-6s} + e^{-5s} + 2e^{-4s} - 2e^{-3s} - e^{-2s} + e^{-s}]\ - (1) \end{aligned}$$

This is the $Y\ s$ obtained when the operation is convolved.

$$Y\ s\ = \frac{8}{s^2}\{e^{-5s} + 2e^{-4s} - 4e^{-3s} - 2e^{-2s} + 3e^{-s}\} - (2)$$

This is the Y(s) obtained when the operation is added.

We can compare the two by plotting the graph of each of them,




```
syms y(s)
```

```
y1 = ((48)/(s^3))*(-exp(-6*s)+exp(-5*s)+2*exp(-4*s)-2*exp(-3*s)-exp(-2*s)+exp(-s));
fplot(y1, [0 2], 'LineWidth', 1.3)
hold on;
y2 = ((8)/(s^2))*(exp(-5*s)+2*exp(-4*s)-2*exp(-2*s)+3*exp(-s));
fplot(y2, [0 2], 'LineWidth', 1.3)
hold off;

grid on;
legend('y1(s)', 'y2(s)')
set(gca, 'YScale', 'log')

title('Y1(s) and Y2(s)', 'Interpreter', 'latex', 'FontSize', 14)
xlabel('s $\rightarrow$', 'Interpreter', 'latex', 'FontSize', 14)
ylabel('Y(s) $\rightarrow$', 'Interpreter', 'latex', 'FontSize', 14)
```

From the Figure we can see that Y_2 has a high magnitude for small values of x, while Y_1 has lower magnitude compared to Y_2, obviously Y1 and Y2 signals aren't same.

Even in a semi-log graph Y2 is non-linear while Y1 is linear.

Some problems that might be cause when transmitting Y2 signal (signals are added) is that to transmit such high values of voltages we would require more precision; hence a greater number of samples would have to be taken when transmitting over a digital network and higher bandwidth would be required if transmitting over a analog network.

While in Y1 signal (signals are convoluted) the amplitude is relatively lower and would be preferred for transmission.

Although to get back the message from Y1 is more complicated than Y2, since the Inverse Laplace Transform of Y2 is simpler, in Y1, we have 1/s^2 terms subtracted with 1/s^3 term, hence s will have to be multiplied with the left terms, making the Inverse Laplace more tedious to compute.

Bibliography

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2. Audio Compression in Practice, UiOUiO Universitetet i Oslo.

   https://www.uio.no/studier/emner/matnat/math/MAT-INF1100/h07/undervisningsmateriale/kap6.pdf