Question No. B.1

Solution to Question No. B.1:

Given equation: $y\,(n) = 4y\,(n-1) - 3y(n-2) + x(n)$ $\quad\ldots\ldots.(i)$

$x(n) = 8{\cdot}\cos(\pi n) \qquad y(0) = 5; \qquad y(1) = 6$

B.1.1 Formulate and solve the difference equation using the given data:

Provided difference equation is: $y\,(n) - 4y\,(n-1) + 3y(n-2) = x(n)$

For Natural solution: we'll assume whole input term to be $'0'$

$$\therefore\, y\,(n) - 4y\,(n-1) + 3y(n-2) = 0$$

Corresponding Auxiliary equation: $m^0 - 4m^{-1} + 3m^{-2} = 0$

$$\therefore\, m^2 - 4m + 3 = 0$$

Solving the above auxiliary equation we get:

The roots are: $m_1 = 3 \qquad m_2 = 1$

Therefore the solution of this differential equation is:

Natural Solution: $y^n(n) = C_1 3^n + C_2 2^n$

For Forced Solution that is due to input,

$$\because\, x(n) = 8\cos(n\pi) \qquad \therefore\, y^p(n) = k_1\cos(n\pi) + k_2\sin(n\pi)$$

$$y^p(n) = k_1\cos(n\pi) \qquad \because\ \sin(n\pi) = \begin{cases} 0 & \forall n \in \{0,1..\infty\} \\ sin(n\pi) & \text{otherwise} \end{cases}$$

Now, putting the value of $y^p(n)$ in eq. (i) we get,

$$k_1\cos(n\pi) \;=\; 4k_1\cos((n-1)\pi) - 3k_1\cos((n-2)\pi) + 8\cos(n\pi)$$

$$k_1\cos(n\pi) \;=\; 4k_1\cos(n\pi-\pi) - 3k_1\cos(n\pi-2\pi) + 8\cos(n\pi)$$

$$k_1\cos(n\pi) - 4k_1\cos(n\pi-\pi) + 3k_1\cos(n\pi-2\pi) \;=\; 8\cos(n\pi) \qquad \ldots\ldots(ii)$$

For $\cos(n\pi-\pi) \;=\; \cos(n\pi)\cos(\pi) \;+\; \sin(n\pi)sin(\pi)$

$$= \cos(n\pi)\cos(\pi) \qquad \because\ \sin(n\pi) = \begin{cases} 0 & \forall n \in \{0,1..\infty\} \\ sin(n\pi) & \text{otherwise} \end{cases}$$

$$= -\cos(n\pi) \qquad \because \cos(n\pi) = \begin{cases} 1 & \forall n \in \text{even numbers} \\ -1 & \forall n \in \text{odd numbers} \end{cases}$$

Similarly, for $\cos(n\pi - 2\pi)$;

$$= \cos(n\pi)\cos(2\pi) = \cos(n\pi)$$

Putting the value of above two equations in eq. (ii), we get;

$$k_1 \cos(n\pi) + 4k_1 \cos(n\pi) + 3k_1 \cos(n\pi) \;=\; 8\cos(n\pi)$$

$$\cos(n\pi)\cdot \left(k_1 + 4k_1 + 3k_1\right) \;=\; 8\cos(n\pi)$$

$$8k_1 = 8 \qquad\qquad \therefore k_1 = 1$$

$$\boxed{y^p(n) = \cos(n\pi)}$$

Therefore the total solution is: $y^t(n) = y^n(n) + y^p(n)$

$$\therefore y^t(n) = C_1 3^n + C_2 2^n + \cos(n\pi)$$

Satisfying the initial condition we get,

$$y(0) \;= 5 \;= C_1 + C_2 + 1 \qquad\qquad \therefore C_1 + C_2 = 4 \qquad\qquad .....(iii)$$

$$y(1) = 6 = 3C_1 + 2C_2 + \cos(\pi) \qquad\qquad \therefore 3C_1 + 2C_2 = 7 \qquad\qquad .....(iv)$$

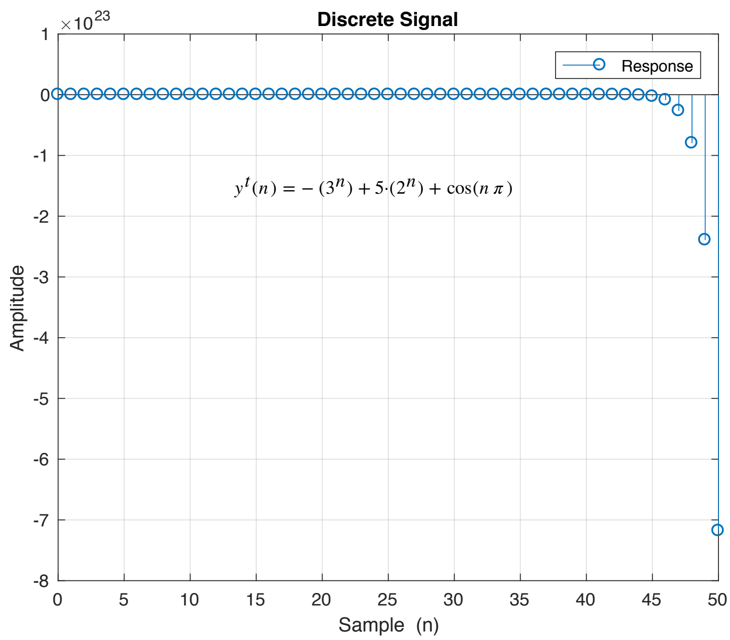
Solving above two equations we get,

$$C_1 = (-1) \qquad\qquad C_2 = 5$$

$$\boxed{\therefore y^t(n) = -(3^n) + 5(2^n) + \cos(n\pi)}$$

B.1.2 Plot the traffic as a function of n:

Given $n = \left(0,1..50\right)$

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>> n = 0:1:50;
>> y =@(n) -(3.^n)+5.*(2.^n)+cos(n.*pi);
>> y1 = y(n);
>> figure(1),clf,stem(n,y(n),'b'),grid,xlabel('Sample (n)'),ylabel('Amplitude')
>> title('Discrete Signal')
>> legend('Response')
>>
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B.1.3 Comment on the variability of the traffic using the solution

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>> disp(y(n))
   1.0e+23 *

  Columns 1 through 13

    0.0000     0.0000     0.0000     0.0000          0    -0.0000    -0.0000    -0.0000
-0.0000    -0.0000    -0.0000    -0.0000    -0.0000

  Columns 14 through 26

   -0.0000    -0.0000    -0.0000    -0.0000    -0.0000    -0.0000    -0.0000    -0.0000
-0.0000    -0.0000    -0.0000    -0.0000    -0.0000

  Columns 27 through 39

   -0.0000    -0.0000    -0.0000    -0.0000    -0.0000    -0.0000    -0.0000    -0.0000
-0.0000    -0.0000    -0.0000    -0.0000    -0.0000

  Columns 40 through 51

   -0.0000    -0.0001    -0.0004    -0.0011    -0.0033    -0.0098    -0.0295    -0.0886
-0.2659    -0.7977    -2.3930    -7.1790

>> vpa(y1,10)

ans =

[ 5.0, 6.0, 12.0, 12.0, 0, -84.0, -408.0, -1548.0, -5280.0, -17124.0, -53928.0,
-166908.0, -510960.0, -1553364.0, -4701048.0, -14185068.0, -42719040.0,
-128484804.0, -386109768.0, -1159640028.0, -3481541520.0, -10449867444.0,
-31360088099.0, -94101235799.0, -2.823456504e11, -8.471208373e11, -2.541530284e12,
-7.624926396e12, -2.287545028e13, -6.862769301e13, -2.058857634e14,
-6.176626589e14, -1.852998714e15, -5.559017617e15, -1.66770958e16, -5.00313733e16,
-1.500942917e17, -4.502832187e17, -1.350850343e18, -4.052552404e18,
-1.215765996e19, -3.647298538e19, -1.094189671e20, -3.282569234e20,
-9.847708142e20, -2.954312531e21, -8.862937768e21, -2.658881366e22,
-7.976644167e22, -2.392993264e23, -7.178979821e23]

>>
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At staring at $(t = 0)$ the traffic slowly increased in $(+)$ve direction till 12 then suddenly decreased to 0 in a single step from then it exponentially increased in the $(-)$ve direction till -7.1789×10^{23}. As n incremented from 45 the traffic shows huge variability[from graph].

Seeing overall to the graph we get to know that the traffic increased in $(-)$ve direction exponentially.

B.1.4 Conclusions

From this we concluded that the difference equation can be useful in analysing the input—output relationship in LTI—system. As this provides the information regrading the output for each of its input discretely.