

Heat Equation using Finite Difference Method

$$c^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$$

$$u_{i,j+1} = a \cdot u_{i+1,j} + (1 - 2a) \cdot u_{i,j} + a \cdot u_{i-1,j}$$

$$a = \frac{k c^2}{h^2}$$

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function u = heat_equation(x0, xn, t0, tn, h, k, c, f)
x = x0:h:xn;
t = t0:k:tn;

u = zeros(length(x), length(t));
u(:, 1) = f(x);

a = k * c^2 / h^2;

if a > 0.5
    fprintf('Method Fails');
    return
end

for j = 1:length(t)-1
    for i = 2:length(x)-1
        u(i, j+1) = a * u(i+1, j) + (1-2*a)*u(i, j) + a * u(i-1, j);
    end
end

figure
surf(t, x, u);
xlabel('x');
ylabel('t');
zlabel('u');

title('Heat Equation');

figure
for j = 1:length(t)
    plot(x, u(:, j));
    hold on;
end
hold off;
```

end