RAMAIAH UNIVERSITY OF APPLIED SCIENCES

ASSIGNMENT

Course Code	ECC201A
Course Name	Signals and Systems
Programme	B.Tech
Department	CSE
Faculty	FET

Name of the Student	Satyajit Ghana
Reg. No	17ETCS002159
Semester/Year	04/2019
Course Leader/s	Ms Prafulla Kumari

Declaration Sheet			
Student Name	Satyajit Ghana		
Reg. No	17ETCS002159		
Programme	B.Tech	Semester/Year	04/2019
Course Code	ECC201A		
Course Title	Signals and Systems		
Course Date		to	
Course Leader	Ms. Prafulla Kumari		

Declaration

The assignment submitted herewith is a result of my own investigations and that I have conformed to the guidelines against plagiarism as laid out in the Student Handbook. All sections of the text and results, which have been obtained from other sources, are fully referenced. I understand that cheating and plagiarism constitute a breach of University regulations and will be dealt with accordingly.

Signature of the Student		Date	
Submission date stamp (by Examination & Assessment Section)			

Signature of the Course Leader and date	Signature of the Reviewer and date

Question No. 1

Solution to Question No. 1 Part A:

A 1.1 Introduction:

Time series analysis comprises methods for analyzing time series data in order to extract meaningful statistics and other characteristics of the data. Time series forecasting is the use of a model to predict future values based on previously observed values.

The fundamental idea for time series analysis is to decompose the original time series (sales, stock market trends, etc.) into several independent components. Typically, business time series are divided into the following four components:

- Trend– overall direction of the series i.e. upwards, downwards etc.
- Seasonality– monthly or quarterly patterns
- Cycle– long term business cycles
- Irregular remainder– random noise left after extraction of all the components

Linear difference equations play an important role in the time series models. The properties of these models often depend on the characteristics of the roots of these difference equations (Wei, 2006). Higher order difference equations arise quite naturally in economic analysis (Enders, 2010).

The objective of evaluation of time series models is based on analysis of difference equations containing stochastic (random) components, in order to forecast the observed phenomena in the future (Arneric and Kordic, 2010).

Difference equation is parametric stochastic process model that express the value of a variable as a function of its own lagged value and other variable. Arneric and Kordic (2010), estimated difference equation it will be examined whether Croatia is implementing a stable policy of exchange rates.

A 1.2 Role of difference equation in the analysis of time series:

Put simply, a difference equation is an expression that relates the value of a function at one point in time to its values at other points in time. It is a deterministic relationship between the current value X_t and its past values X_{t-i} with $i > 0$. In some cases, it may also contain the current and past values of a "forcing" or "driving" variable. A first order difference equation involved only one lagged variable:

$$X_t = \phi X_{t-1} + ba_t + c_t$$

Where a_t is a forcing variable, which follows a well defined probability distribution.

Let's take the example of Random Walk Model,

A random walk is defined as a process where the current value of a variable is composed of the past value
plus an error term defined as a white noise (a normal variable with zero mean and variance one). Algebraically a random walk is represented as follows:

$$y_t = y_{t-1} + \epsilon_t$$

In others words, it predicts that all future values will equal the last observed value. Finance is a very important application area, because there is a very good theoretical reason for believing that prices of assets for which
speculative markets exist ought to behave like random walks. Namely, if it were easy to predict whether the market would go up or down tomorrow, it should have already gone that way today.

The relevance of the random walk model is that many economic time series follow a pattern that resembles a trend model. Furthermore, if two time series are independent random walk processes then the relationship between the two does not have an economic meaning.

A 1.3 Conclusion:

The difference equation is a formula for computing an output sample at time n based on past and present input samples and past output samples in the time domain.

When used for discrete-time physical modeling, the difference equation may be referred to as an explicit finite difference scheme

Most time series are time-dependent. That is, the present value depends on its history

- For example, last year's GDP definitely will affect this year's GDP; yesterday's stock price will affect today's price.
- Difference equation relates the present value y_t to its past values $y_{t-1}, y_{t-2}, \ldots$
- Difference equation is a major component of a dynamic economic model

Solution to Question No. 1 Part B:

Given:

`ASCII String:` **`VN`**

`Transmission rate: 1Mbps`

`Amplitude: 3.5V`

`Frequency for bit 1: 10Mhz`

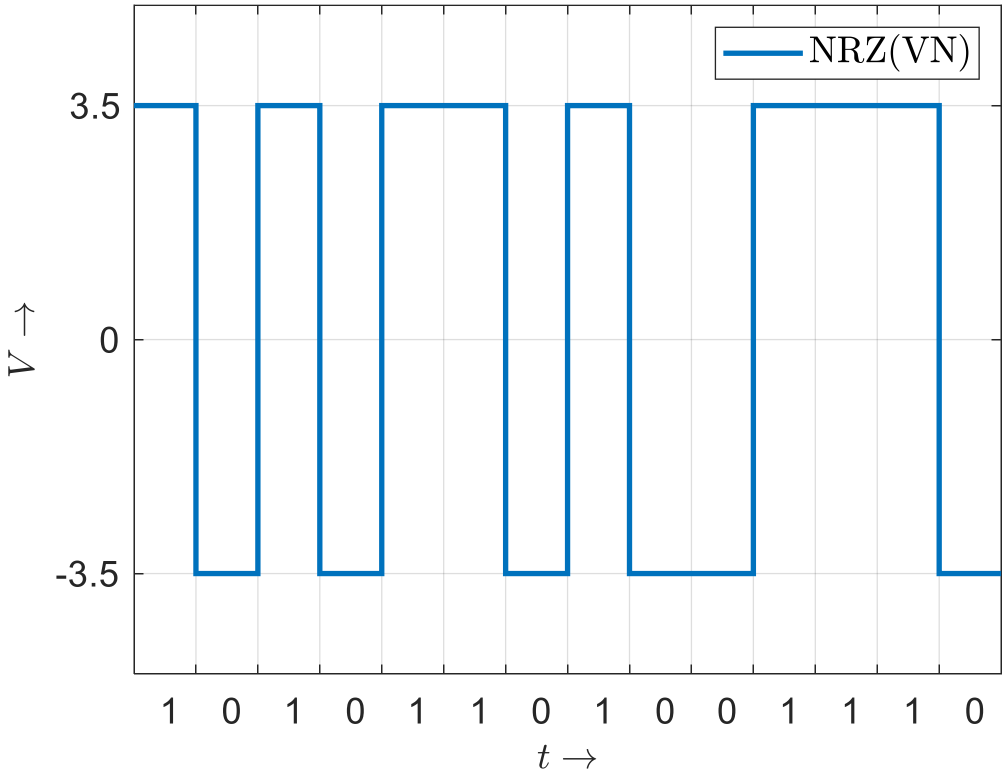
`Frequency for bit 0: 5Mhz`

B 1.1 Determine the NRZ encoding of the text string

The given 2-character ASCII string is first converted to its binary representation,

`V = 86 = 1010110`

`N = 78 = 1001110`




```
a = 3.5.*[1 -1 1 -1 1 1 -1 1 -1 -1 1 1 1 -1 -1];
x = [0:length(a)-1];
stairs(x, a, 'LineWidth', 2);
ax1 = gca;
set(ax1, 'FontSize', 14);
xticks(0:length(a)-1);
xticklabels({'        1', '        0', '        1', '        0', '        1', '
1', '        0',...
    '        1', '        0', '        0', '        1', '        1', '        1', '
0'});
```

```
yticks([-3.5 0 3.5])
axis([0 length(a)-1 -5 5])
grid on;
xlabel('$t \rightarrow$', 'Interpreter', 'latex', 'FontSize', 14)
ylabel('$V \rightarrow$', 'Interpreter', 'latex', 'FontSize', 14)
legend({'NRZ(VN)'}, 'Interpreter', 'latex', 'FontSize', 14);
```

B 1.2 Determine and plot the Continuous Time (CT) signal for the entire string:

Transmission Rate = 1 $Mbps = 10^6 bps$

Bit Duration = $\frac{1}{Transmission\ Rate} = 10^{-6}s$

Number of cycles in a bit 0 with 10 Mhz frequency,

$$= bit\ duration \times frequency$$

$$= 10^{-6} \times 10 \times 10^6$$

$$= 10\ Cycles$$

Number of cycles in a bit 1 with 5 Mhz frequency,

$$= bit\ duration \times frequency$$

$$= 10^{-6} \times 5 \times 10^6$$

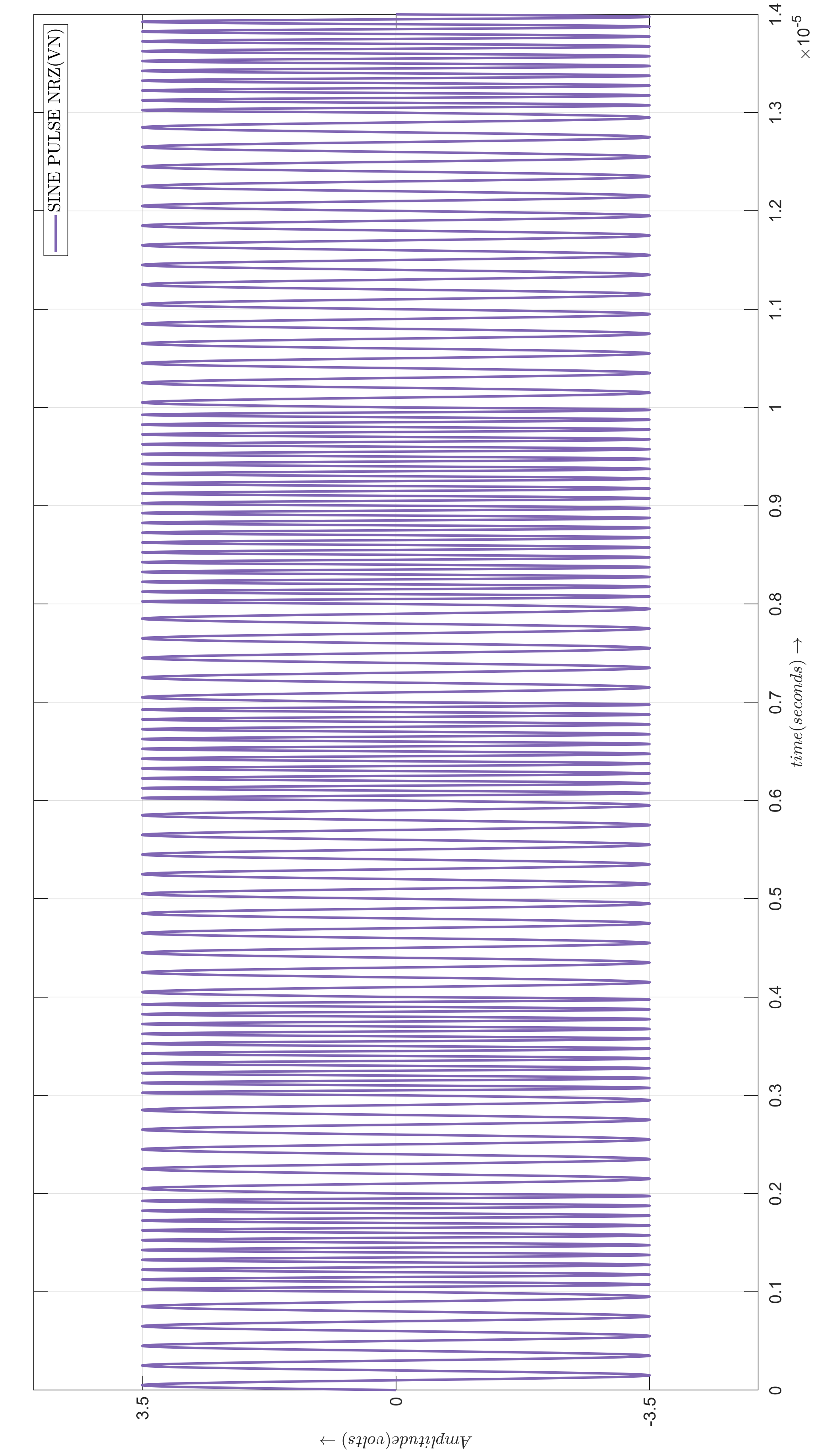
$$= 5\ Cycles$$

```
freq_zero = 10*10^6;
freq_one = 5*10^6;
bit_duration = 10^-6;
bits = [1 0 1 0 1 1 0 1 0 0 1 1 1 0];
A = 3.5;
time = 0:bit_duration:length(bits)*bit_duration;
syms t;
for i = 1:length(bits)

    if bits(i) == 0
        f = freq_zero;
    else
        f = freq_one;
    end

    fplot(A*sin(2*pi*f*t), [time(i) time(i+1)], 'color', [.5 .4 .7], 
'LineWidth', 2);
    hold on;
end

axis([0 max(time) -5 5])
xticks(time)
yticks([-3.5 0 3.5])
ax1 = gca;
set(ax1, 'FontSize', 14);
grid on;
xlabel('$t \rightarrow$', 'Interpreter', 'latex', 'FontSize', 14)
ylabel('$V \rightarrow$', 'Interpreter', 'latex', 'FontSize', 14)
legend({'SINE PULSE NRZ(VN)'}, 'Interpreter', 'latex', 'FontSize', 14);
```


SINE PULSE NRZ(VN)
3.5
0
-3.5
Amplitude(volts) →
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1.1
1.2
1.3
1.4
×10^-5
time(seconds) →

B 1.3 Compute the energy for the characters in the text and energy per character of the CT Signal

The Energy of the Non-Periodic signal obtained in B1.2 is given by

$$E = \int_{-\infty}^{\infty} (A \sin \omega t)^2 dt$$

Where,

$A: amplitude\ of\ the\ signal$

$\omega: 2\pi f\ angular\ frequency$

$t: time$

$$E = A^2 \int_{-\infty}^{\infty} \sin^2 2\pi f t\, dt$$

$$= A^2 \int_{-\infty}^{\infty} \frac{1}{2} - \frac{\cos 4\pi f t}{2} dt$$

$$= \frac{A^2}{2} \int_{-\infty}^{\infty} dt - \left. \frac{\frac{A^2}{2} \sin 4\pi f t}{4\pi f} \right|_{-\infty}^{\infty} \quad [\sin 2n\pi = 0]$$

$$= \frac{A^2}{2} \int_{-\infty}^{\infty} dt$$

Now we can compute it for the entire signal,

$$E = \frac{A^2}{2} \left(\int_{0}^{1\times10^{-6}} dt + \int_{1\times10^{-6}}^{2\times10^{-6}} dt + \int_{2\times10^{-6}}^{3\times10^{-6}} dt + \int_{3\times10^{-6}}^{4\times10^{-6}} dt + \int_{4\times10^{-6}}^{5\times10^{-6}} dt + \int_{5\times10^{-6}}^{6\times10^{-6}} dt + \int_{6\times10^{-6}}^{7\times10^{-6}} dt \right.$$
$$+ \int_{7\times10^{-6}}^{8\times10^{-6}} dt + \int_{8\times10^{-6}}^{9\times10^{-6}} dt + \int_{9\times10^{-6}}^{10\times10^{-6}} dt + \int_{10\times10^{-6}}^{11\times10^{-6}} dt + \int_{11\times10^{-6}}^{12\times10^{-6}} dt + \int_{12\times10^{-6}}^{13\times10^{-6}} dt$$
$$\left. + \int_{13\times10^{-6}}^{14\times10^{-6}} dt \right)$$

$$E = \frac{3.5^2}{2} \times (14 \times 10^{-6})$$

$$E = 85.75 \times 10^{-6} J$$

Since there are 2 characters, Energy per character is,

$$E_{character} = \frac{85.75}{2} \times 10^{-6}$$

$$E_{character} = 42.875 \times 10^{-6} J$$

Question No. 3

Solution to Question No. 2 Part B:

Given data:

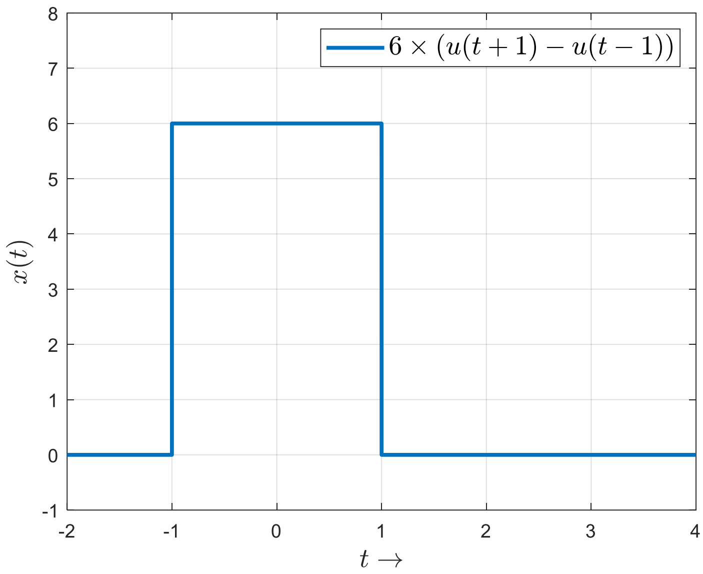
$x(t)$: Amplitude 6, from t = -1 to t = 1

$h_1(t)$: Amplitude 2, from t = 0 to t = 2

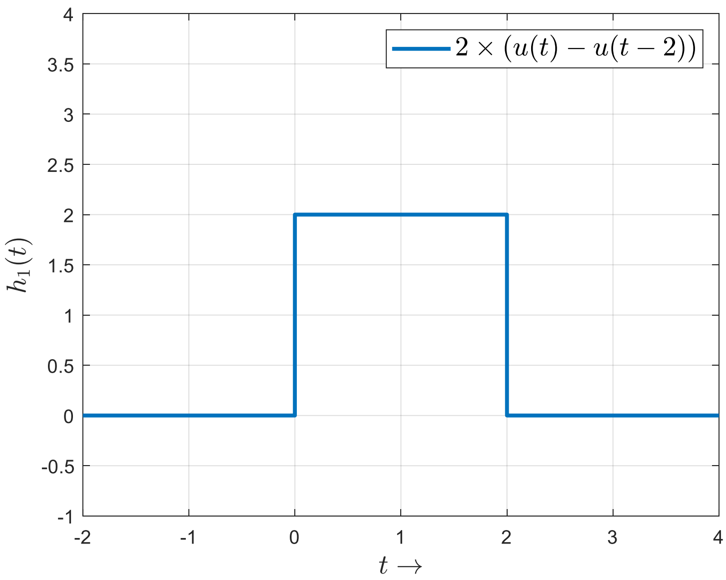
$h_2(t)$: Amplitude 4, from t = 2 to t = 3

Plots:

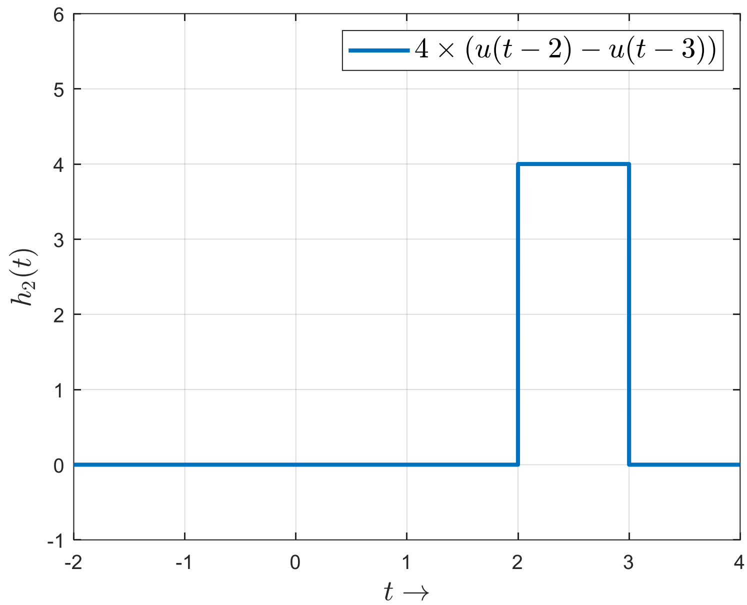
$x(t)$



$h_1(t)$

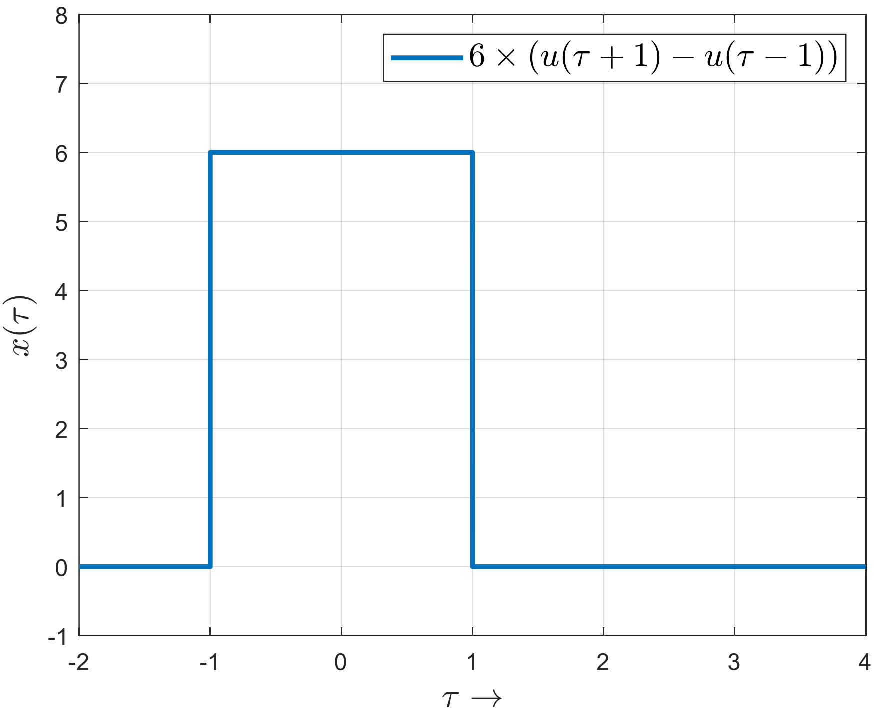


$h_2(t)$



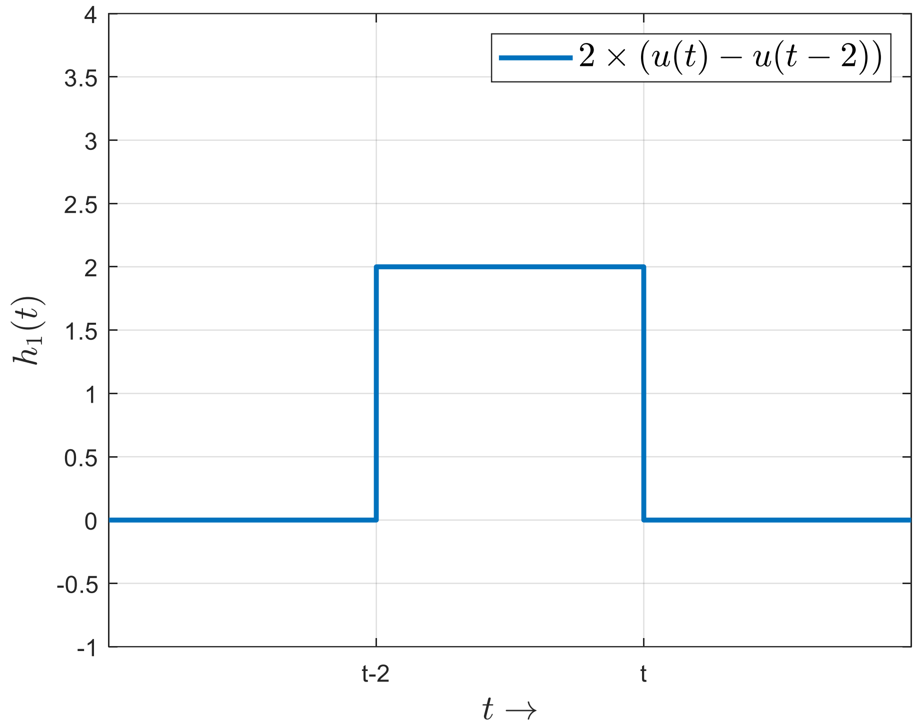
B 2.1 Compute and plot the signal received by the second link:

The given signal $x(t)$ is in terms of the variable t, which is first converted to the variable τ, to give, $x(\tau)$:



Now to get $h_1(t-\tau)$, first the signal $h_1(t)$ is converted to $h_1(\tau)$ and then reflection of it is done to get $h_1(-\tau)$, to this formed signal we add t to get our final signal,

$h_1(t-\tau)$:



Comparing the values of overlaps from the x-axis we obtain,

<table>
<tr><td>$t \leq -1$</td><td>$t \leq -1$</td><td rowspan="8">$t \leq -1$
$-1 \leq t \leq 1$
$1 \leq t \leq 3$
$t \geq 3$</td></tr>
<tr><td>$t \geq -1$</td><td>$t \geq -1$</td></tr>
<tr><td>$t \leq 1$</td><td>$t \leq 1$</td></tr>
<tr><td>$t \geq 1$</td><td>$t \geq 1$</td></tr>
<tr><td>$t-2 \leq -1$</td><td>$t \leq 1$</td></tr>
<tr><td>$t-2 \geq -1$</td><td>$t \geq 1$</td></tr>
<tr><td>$t-2 \leq 1$</td><td>$t \leq 3$</td></tr>
<tr><td>$t-2 \geq 1$</td><td>$t \geq 3$</td></tr>
</table>

CASE 1

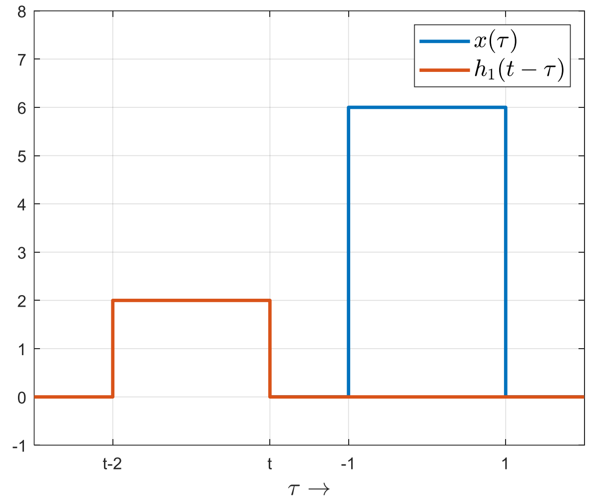
Range :

$$t \leq -1$$

Terms involved :

$$t \leq -1$$

$$y_1(t) = 0$$



CASE 2

Range :

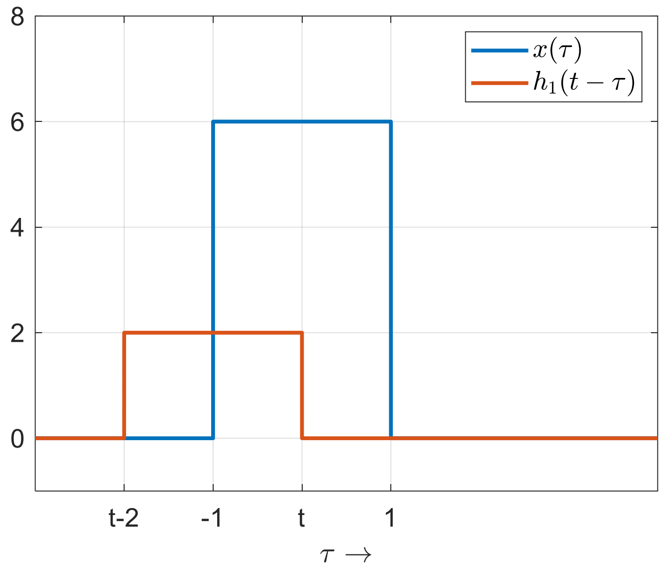
$$-1 \leq t \leq 1$$

Terms involved :

$$t \geq -1, t \leq 1, t-2 \leq -1$$

$$y_1(t) = \int_{-1}^{t} (2)(6)d\tau$$

$$y_1(t) = 12t + 12$$



CASE 3

Range :

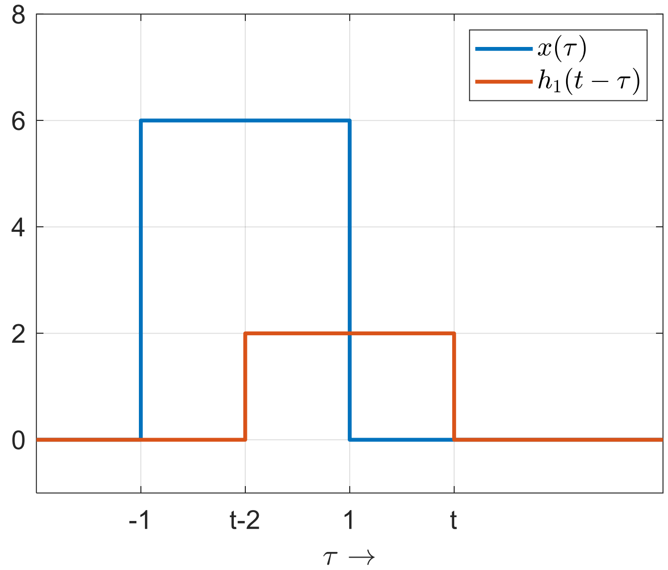
$$1 \leq t \leq 3$$

Terms involved :

$$t \geq 1, t-2 \geq -1, t-2 \leq 1$$

$$y_1(t) = \int_{t-2}^{t} (2)(6)d\tau$$

$$y_1(t) = 36 - 12t$$



CASE 4

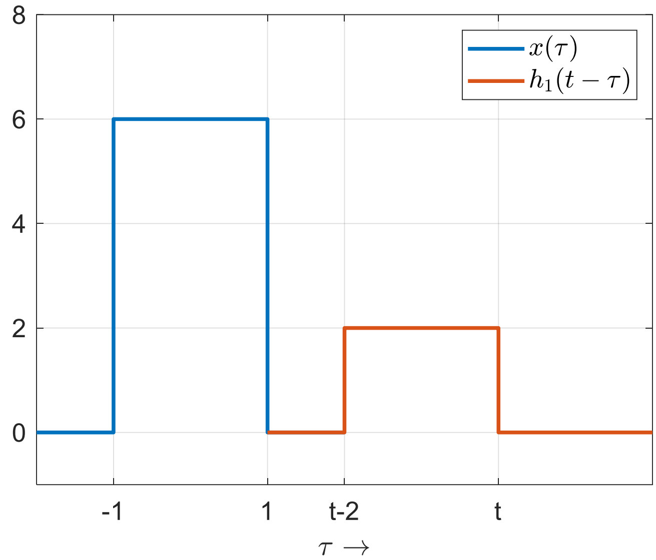
Range :

$$t \geq 3$$

Terms involved :

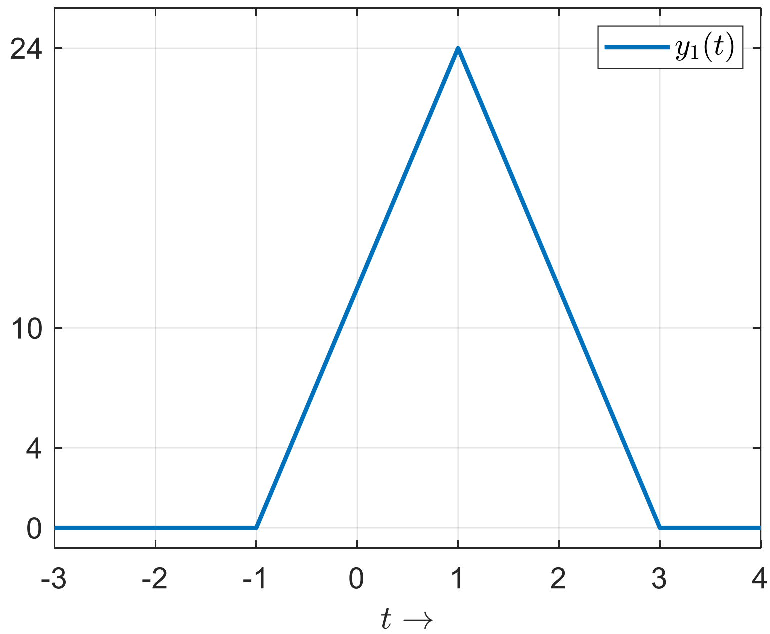
$$t-2 \geq 1$$

$$y_1(t) = 0$$



Hence the output from the first convolution is:

$$y_1(t) = (12t+12) \times \big(u(t+1) - u(t-1)\big) + (36-12t) \times (u(t-1) - u(t-3))$$

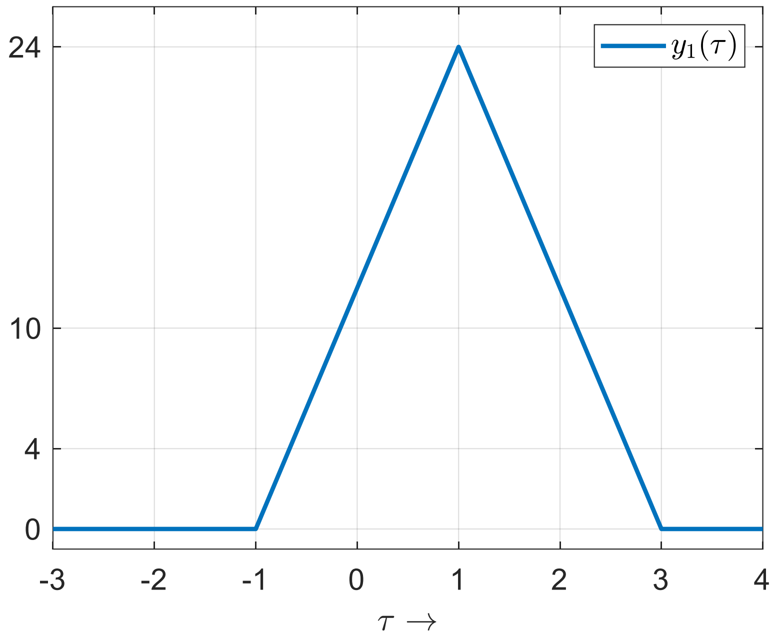



```
%% plot for y1(t)
syms t
hold off;
y1 = (12*t+12).*(heaviside(t+1)-heaviside(t-1)) + (36-12*t).*(heaviside(t-1)-
heaviside(t-3));
fplot(y1, [-3 4], 'LineWidth', 2)
xlabel('$\tau \rightarrow$', 'Interpreter', 'latex', 'FontSize', 14)
legend({'$y_1(\tau)$'}, 'Interpreter', 'latex', 'FontSize', 14);
grid on;
ax1 = gca;
set(ax1, 'FontSize', 14);
axis([-3 4 -1 26])
yticks([0 4 10 24])
```

B 2.2 Compute and plot the signal received at the destination:

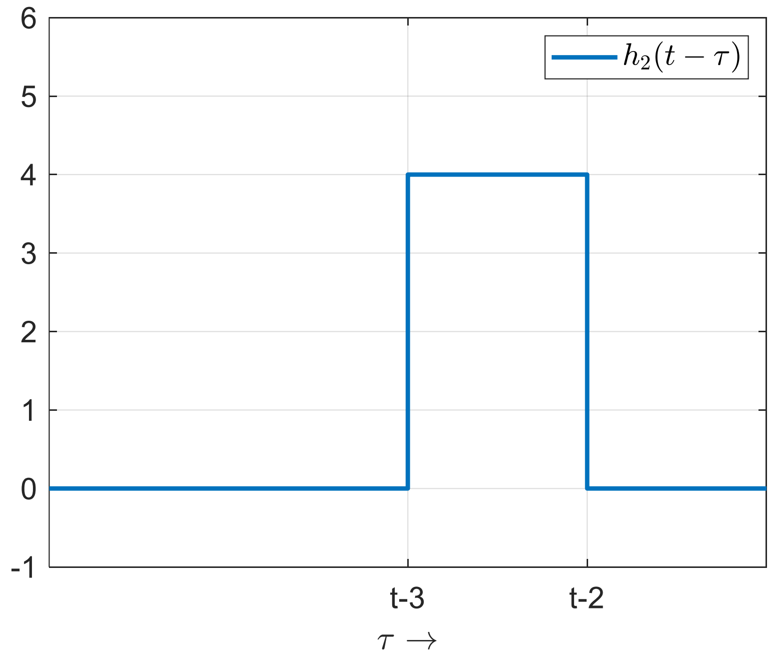
The output from the previous link is the input for the second link which will give the signal received at the destination,

$y_1(\tau)$:



Now to get $h_2(t-\tau)$, first the signal $h_2(t)$ is converted to $h_2(\tau)$ and then reflection of it is done to get $h_2(-\tau)$, to this formed signal we add t to get our final signal,

$h_2(t-\tau)$:



Comparing the values of overlaps from the x-axis we obtain,

<table>
<tr><td>$t-2 \leq -1$</td><td>$t \leq 1$</td><td rowspan="12">$t \leq 1$
$1 \leq t \leq 2$
$2 \leq t \leq 3$
$3 \leq t \leq 4$
$4 \leq t \leq 5$
$5 \leq t \leq 6$
$t \geq 6$</td></tr>
<tr><td>$t-2 \geq -1$</td><td>$t \geq 1$</td></tr>
<tr><td>$t-2 \leq 1$</td><td>$t \leq 3$</td></tr>
<tr><td>$t-2 \geq 1$</td><td>$t \geq 3$</td></tr>
<tr><td>$t-2 \leq 3$</td><td>$t \leq 5$</td></tr>
<tr><td>$t-2 \geq 3$</td><td>$t \geq 5$</td></tr>
<tr><td>$t-3 \leq -1$</td><td>$t \leq 2$</td></tr>
<tr><td>$t-3 \geq -1$</td><td>$t \geq 2$</td></tr>
<tr><td>$t-3 \leq 1$</td><td>$t \leq 4$</td></tr>
<tr><td>$t-3 \geq 1$</td><td>$t \geq 4$</td></tr>
<tr><td>$t-3 \leq 3$</td><td>$t \leq 6$</td></tr>
<tr><td>$t-3 \geq 3$</td><td>$t \geq 6$</td></tr>
</table>

CASE 1

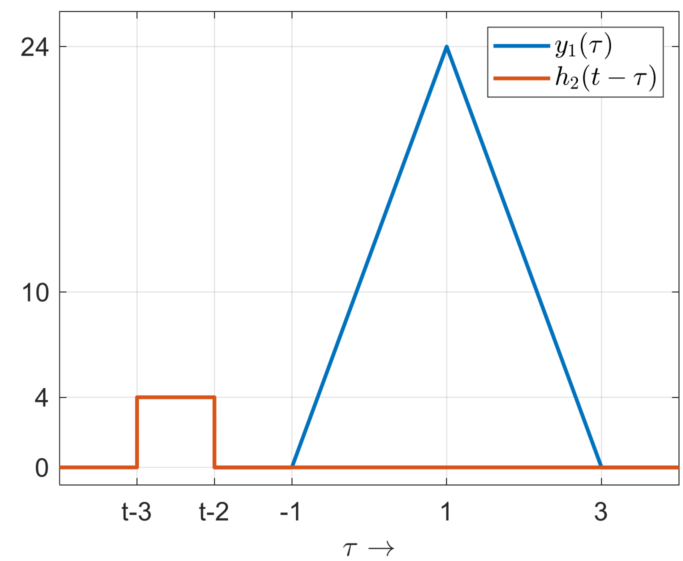
Range :

$$t \leq 1$$

Terms involved :

$$t-2 \leq -1$$

$$y(t) = 0$$



CASE 2

Range :

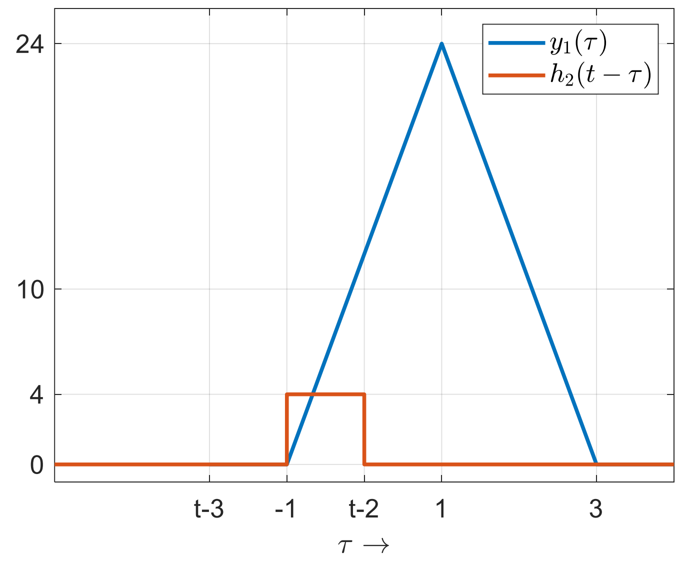
$$1 \leq t \leq 2$$

Terms involved :

$$t-2 \geq -1, t-3 \leq -1$$

$$y(t) = \int_{-1}^{t-2} (12\tau + 12)(4) d\tau$$

$$y(t) = 24(t-1)^2$$



CASE 3

Range :

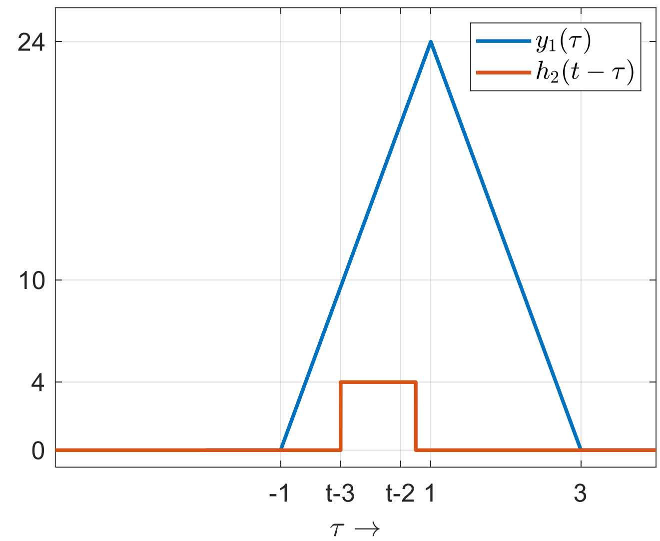
$$2 \leq t \leq 3$$

Terms involved :

$$t-3 \geq -1, t-2 \leq 1$$

$$y(t) = \int_{t-3}^{t-2} (12\tau + 12)(4) d\tau$$

$$y(t) = 48t - 72$$



CASE 4

Range :

$$3 \le t \le 4$$

Terms involved :

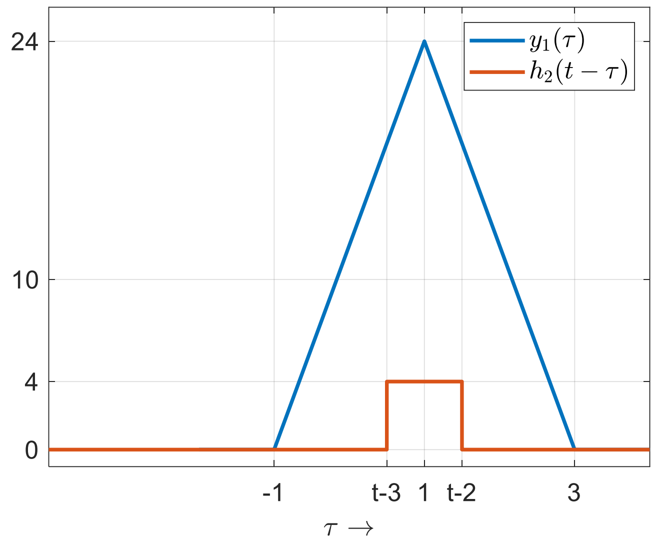
$$t-2 \ge 1, t-3 \le 1$$

$$y(t) = \int_{t-3}^{1} (12\tau + 12)(4)d\tau$$

$$+ \int_{1}^{t-2} (36 - 12\tau)(4)d\tau$$

$$y(t) = -24(t-3)(t-7) - 24t(t-4)$$

$$y(t) = -48t^2 + 336t - 504$$



<u>CASE 5</u>

Range :

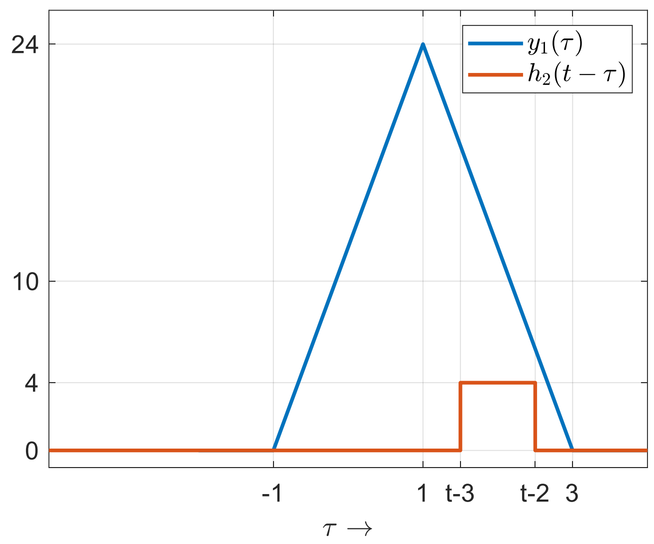
$$4 \le t \le 5$$

Terms involved :

$$t-3 \le 1, t-2 \le 3$$

$$y(t) = \int_{t-3}^{t-2} (36 - 12\tau)(4)d\tau$$

$$y(t) = 264 - 48t$$



<u>CASE 6</u>

Range :

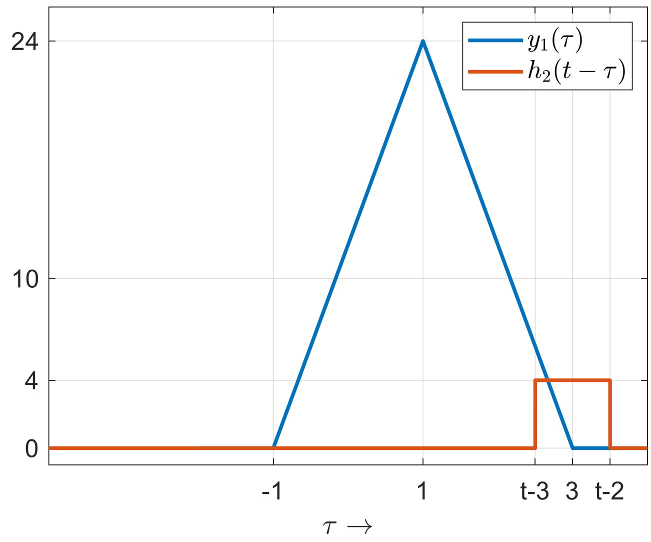
$$5 \le t \le 6$$

Terms involved :

$$t - 2 \ge 3, t - 3 \le 3$$

$$y(t) = \int_{t-3}^{3} (36 - 12\tau)(4) d\tau$$

$$y(t) = 24(t - 6)^2$$



CASE 7

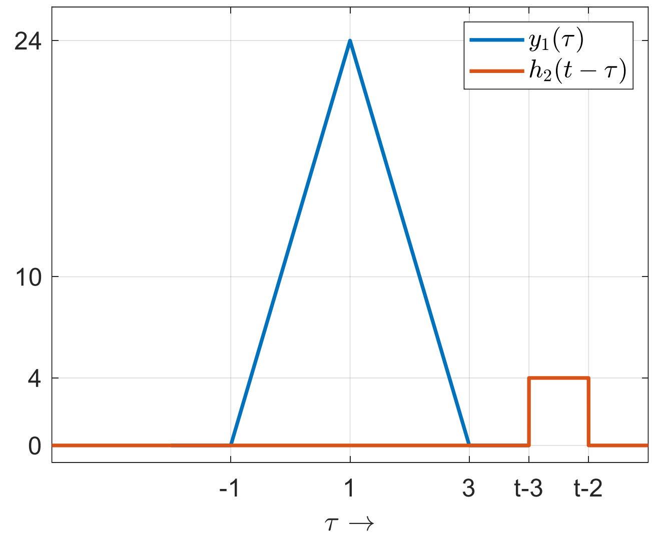
Range :

$$t \ge 6$$

Terms involved :

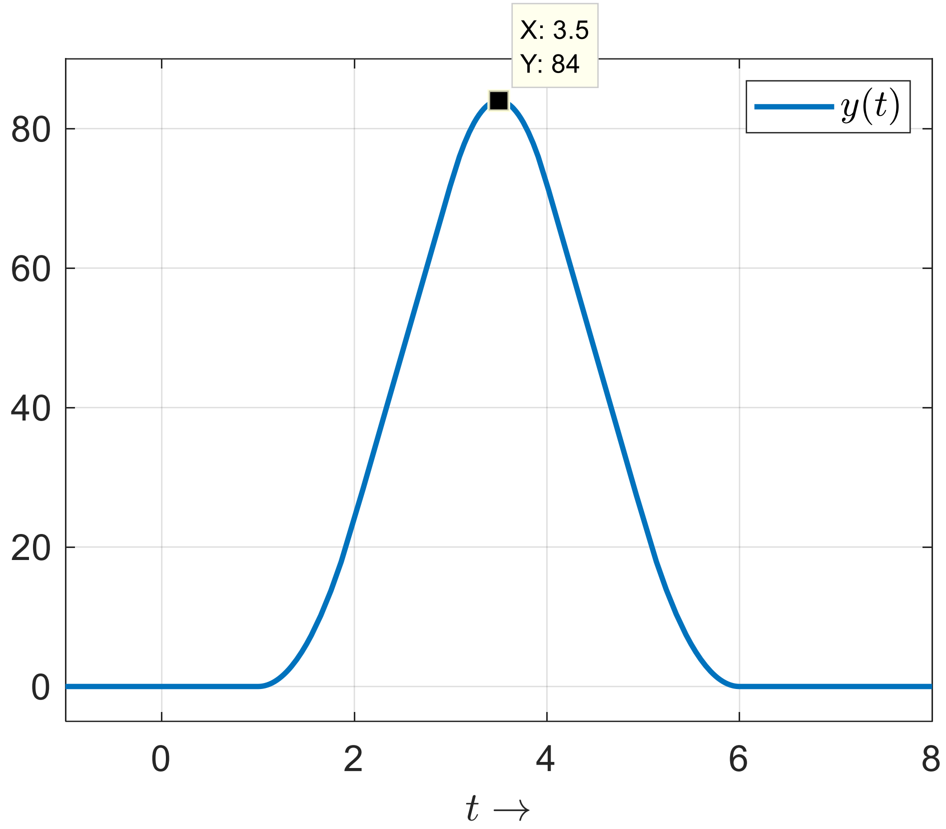
$$t - 3 \ge 3$$

$$y(t) = 0$$



Hence the output from the second convolution is:

$$y(t) = \begin{cases} 24(t-1)^2; 1 \le t \le 2 \\ 48t - 72; 2 \le t \le 3 \\ -48t^2 + 336t - 504; 3 \le t \le 4 \\ 264 - 48t; 4 \le t \le 5 \\ 24(t-6)^2; 5 \le t \le 6 \\ 0;\ otherwise \end{cases}$$




```
%% plot for y(t)
syms t;
y = (24*(t-1).^2).*(heaviside(t-1)-heaviside(t-2)) + ...
    (48*t-72).*(heaviside(t-2)-heaviside(t-3)) + ...
    (-48*t.^2+336*t-504).*(heaviside(t-3)-heaviside(t-4)) + ...
    (264-48*t).*(heaviside(t-4)-heaviside(t-5)) + ...
    (24*(t-6).^2).*(heaviside(t-5)-heaviside(t-6));
fplot(y , [-1 8], 'LineWidth', 2);
axis([-1 8 -5 90]);
grid on;
xlabel('$t \rightarrow$', 'Interpreter', 'latex', 'FontSize', 14)
ax1 = gca;
set(ax1, 'FontSize', 14);
legend({'$y(t)$'}, 'Interpreter', 'latex', 'FontSize', 14);
```

B 2.3 Obtain the formula for the overall impulse response of the network:

The impulse response at first link:

$$y_1(t) = \int_{-\infty}^{\infty} x(\tau)h_1(t-\tau)d\tau$$

$$y(t) = \int_{-\infty}^{\infty} y_1(\sigma)h_2(t-\sigma)d\sigma - (1)$$

Which can be written as,

$$y(t) = \int_{\sigma=-\infty}^{\infty} \left(\int_{\tau=-\infty}^{\infty} x(\tau) h_1(\sigma - \tau) d\tau \right) h_2(t - \sigma) d\sigma - (2)$$

Put

$$\sigma - \tau = \lambda \rightarrow \sigma = \lambda + \tau$$
$$d\sigma = d\lambda$$

When

$$\sigma = -\infty, \lambda = -\infty - \tau = -\infty$$
$$\sigma = \infty, \lambda = \infty - \tau = \infty$$

Substituting in equation (2)

$$y(t) = \int_{\lambda=-\infty}^{\infty} \left(\int_{\tau=-\infty}^{\infty} x(\tau) h_1(\lambda) d\tau \right) h_2(t - \lambda - \tau) d\lambda$$

Rearranging,

$$y(t) = \int_{\tau=-\infty}^{\infty} x(\tau) \int_{\lambda=-\infty}^{\infty} h_1(\lambda)\ h_2(t - \lambda - \tau) d\lambda d\tau$$

$$y(t) = \int_{\tau=-\infty}^{\infty} x(\tau)(h_1 * h_2)(t - \tau) d\tau$$

$$y(t) = \int_{\tau=-\infty}^{\infty} x(\tau) h(t - \tau) d\tau$$

The Overall impulse response is

$$h(t) = \int_{-\infty}^{\infty} h_1(\tau) h_2(t - \tau) d\tau = h_1(t) * h_2(t)$$

Bibliography

1. https://ccrma.stanford.edu/~jos/fp/Difference_Equation_I.html
2.