EXAMPLE 2

Find the temperature $u(x, t)$ in a metal rod of length 25 cm that is insulated on the ends as well as on the sides and whose initial temperature distribution is $u(x, 0) = x$ for $0 < x < 25$.

The temperature in the rod satisfies the heat conduction problem (1), (3), (24) with $L = 25$. Thus, from Eq. (35), the solution is

$$u(x, t) = \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n e^{-n^2\pi^2\alpha^2 t/625} \cos\frac{n\pi x}{25}, \tag{39}$$

where the coefficients are determined from Eq. (37). We have

$$c_0 = \frac{2}{25}\int_0^{25} x\, dx = 25 \tag{40}$$

and, for $n \geq 1$,

$$\begin{aligned} c_n &= \frac{2}{25}\int_0^{25} x \cos\frac{n\pi x}{25}\, dx \\ &= 50(\cos n\pi - 1)/(n\pi)^2 = \begin{cases} -100/(n\pi)^2, & n \text{ odd;} \\ 0, & n \text{ even.} \end{cases} \end{aligned} \tag{41}$$

Thus

$$u(x, t) = \frac{25}{2} - \frac{100}{\pi^2} \sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n^2} e^{-n^2\pi^2\alpha^2 t/625} \cos(n\pi x/25) \tag{42}$$

is the solution of the given problem.

For $\alpha^2 = 1$ Figure 10.6.2 shows plots of the temperature distribution in the bar at several times. Again the convergence of the series is rapid so that only a relatively few terms are needed to generate the graphs.

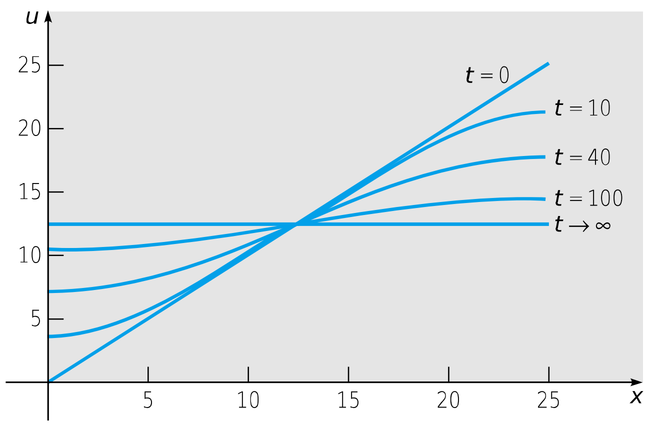

FIGURE 10.6.2 Temperature distributions at several times for the heat conduction problem of Example 2.