Heat Equation using Finite Difference Method

$$u_{i,j+1} = a \cdot u_{i+1,\,j} + (1 - 2a)u_{i,j} + a \cdot u_{i-1,j}$$

```matlab
clear all;

% params
f = @(x) x .* (4-x);

c_sq = 1/2;

h = 1;
k = 1;

tn = 3;
xn = 4;

% function

x = 0:h:xn;
t = 0:k:tn;

% a = k * c_sq / h^2;
a = 0.5;

u = zeros(length(x), length(t));
u(:, 1) = f(x);

for j = 1:length(t)-1
    for i = 2:length(x)-1
        u(i, j+1) = a * u(i+1, j) + (1-2*a) * u(i, j) + a * u(i-1, j);
    end
end

for j = 1:length(t)
    for i = 1:length(x)
        fprintf('value at u(%d, %d) is %f\n', i-1, j-1, u(i, j));
    end
    fprintf('\n');
end
```

```
value at u(0, 0) is 0.000000
value at u(1, 0) is 3.000000
value at u(2, 0) is 4.000000
value at u(3, 0) is 3.000000
value at u(4, 0) is 0.000000

value at u(0, 1) is 0.000000
value at u(1, 1) is 2.000000
value at u(2, 1) is 3.000000
value at u(3, 1) is 2.000000
value at u(4, 1) is 0.000000

value at u(0, 2) is 0.000000
value at u(1, 2) is 1.500000
value at u(2, 2) is 2.000000
value at u(3, 2) is 1.500000
```

```
value at u(4, 2) is 0.000000

value at u(0, 3) is 0.000000
value at u(1, 3) is 1.000000
value at u(2, 3) is 1.500000
value at u(3, 3) is 1.000000
value at u(4, 3) is 0.000000
```
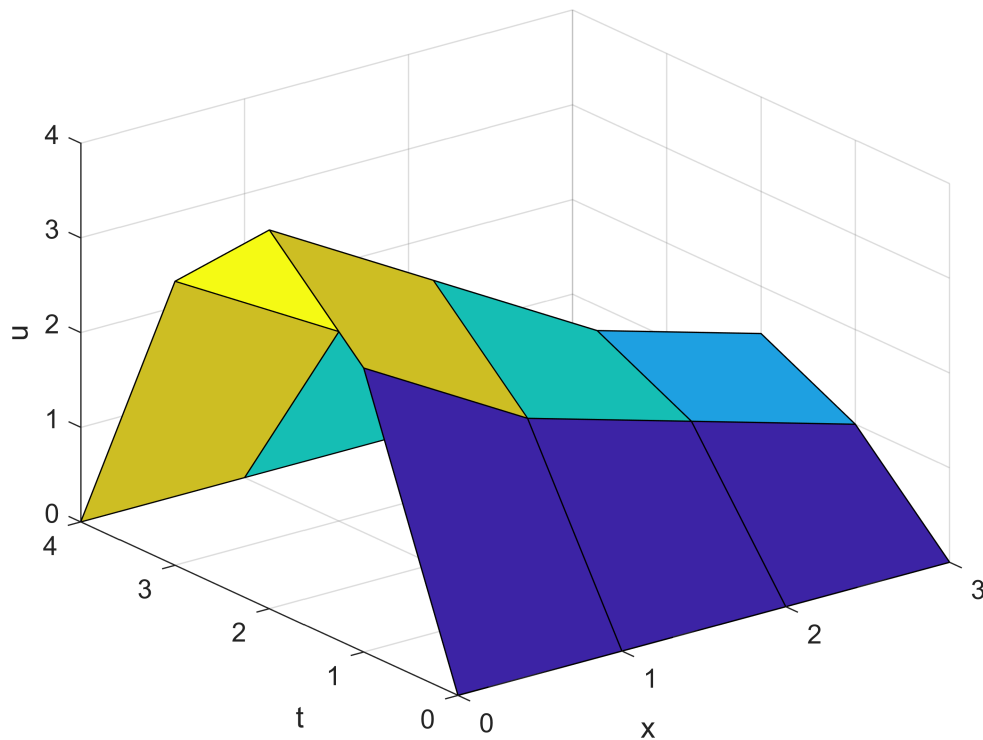
```
disp(u)
```

```
                0                   0                   0                   0
3.000000000000000   2.000000000000000   1.500000000000000   1.000000000000000
4.000000000000000   3.000000000000000   2.000000000000000   1.500000000000000
3.000000000000000   2.000000000000000   1.500000000000000   1.000000000000000
                0                   0                   0                   0
```

```
surf(t, x, u);
xlabel('x');
ylabel('t');
zlabel('u');
```



```
for j = 1:length(t)
    plot(x, u(:, j));
    hold on;
end
hold off;
```