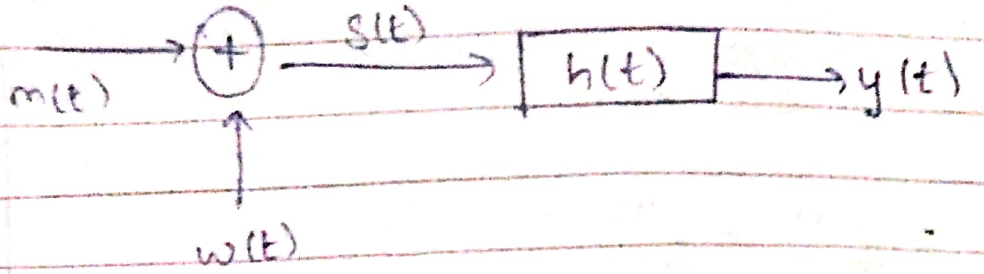


$m(t) \rightarrow$ message signal

$w(t) \rightarrow$ water mark signal

$s(t) = m(t)+w(t)$ which is secret signal

$h(t) \rightarrow$ impulse response for the LTI system

$y(t) \rightarrow$ output for the system

2.1] Data is from assignment 1 B.2.1 [B.2 question]

$x(t)$ is $m(t)$

$h_1(t)$ is $w(t)$

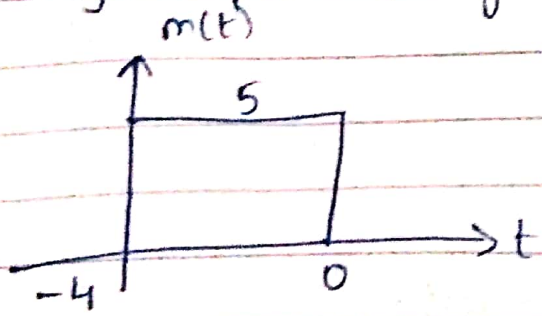
$h_2(t)$ is $h(t)$

$$y(t) = s(t) * h(t)$$
$$= h(t) * [m(t)+w(t)]$$

Taking laplace transform on both side

$$Y(s) = H(s)\,[M(s)+W(s)] \longrightarrow (1)$$

Finding laplace transform for the following signals



$$m(t) = 5\,[u(t+4) - u(t)]$$

Laplace of $u(t+4) = \int_{-\infty}^{\infty} e^{-st}\, u(t+4)\, dt$

$$= \int_{-4}^{\infty} e^{-st}$$

$$= \left.\frac{e^{-st}}{-s}\right]_{-4}^{\infty}$$

$$= \frac{e^{-\infty}}{-s} - \frac{e^{4s}}{-s}$$

$$= \frac{e^{4s}}{s}$$

L.T of $u(t) = \int_{-\infty}^{\infty} e^{-st} u(t)\, dt$

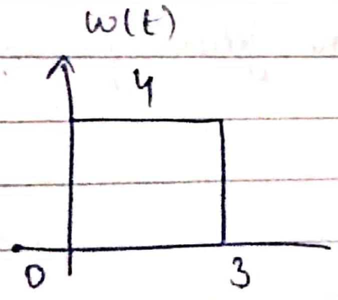
$$= \int_{0}^{\infty} e^{-st}\, dt$$

$$= \left.\frac{e^{-st}}{-s}\right]_{0}^{\infty}$$

$$= \frac{e^{-\infty}}{-s} - \frac{e^{-0}}{-s}$$

$$= \frac{1}{s}$$

$$M(s) = 5\left[\frac{e^{4s}}{s} - \frac{1}{s}\right]$$



$w(t) = 4[u(t) - u(t-3)]$

L.T of $u(t) = \int_{-\infty}^{\infty} e^{-st} u(t)\, dt$

$$= \int_{0}^{\infty} e^{-st}\, dt$$

$$= \left[\frac{e^{-st}}{-s}\right]_{0}^{\infty}$$

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$$= \frac{e^{-\infty} - e^{-0}}{-s}$$

$$= \frac{1}{s}$$

L.T of $u(t-3) = \int_{-\infty}^{\infty} e^{-st} u(t-3)\, dt$

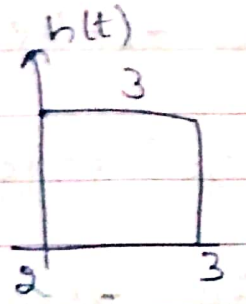
$$= \int_{3}^{\infty} e^{-st}\, dt$$

$$= \left. \frac{e^{-st}}{-s} \right]_{3}^{\infty}$$

$$= \frac{e^{-\infty}}{-s} - \frac{e^{-3s}}{-s}$$

$$= \frac{e^{-3s}}{s}$$

$$\therefore W(s) = 4\left[\frac{1}{s} - \frac{e^{-3s}}{s}\right]$$



$h(t) = 3\left[u(t-2) - u(t-3)\right]$

L.T of $u(t-2) = \int_{-\infty}^{\infty} e^{-st} u(t-2)\, dt$

$$= \int_{2}^{\infty} e^{-st}\, dt$$

$$= \left. \frac{e^{-st}}{-s} \right]_{2}^{\infty}$$

$$= \frac{e^{-\infty}}{-s} - \frac{e^{-2s}}{-s}$$

$$= \frac{e^{-2s}}{s}$$

$L.T$ of $u(t-3) = \int_{-\infty}^{\infty} e^{-st} u(t-3)\,dt$

$$= \int_{3}^{\infty} e^{-st}\,dt$$

$$= \left. \frac{e^{-st}}{-s} \right]_{3}^{\infty}$$

$$= \frac{e^{-\infty}}{-s} - \frac{e^{-3s}}{-s}$$

$$= \frac{e^{-3s}}{s}$$

$$H(s) = 3\left[\frac{e^{-2s}}{s} - \frac{e^{-3s}}{s}\right]$$

$$Y(s) = H(s)\,[M(s) + W(s)]$$

$$= 3\left[\frac{e^{-2s}}{s} - \frac{e^{-3s}}{s}\right]\left[\frac{5e^{4s}}{s} - \frac{5}{s} + \frac{4}{s} - \frac{4e^{-3s}}{s}\right]$$

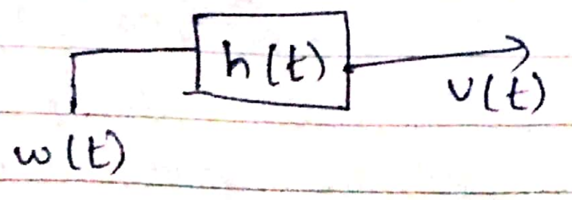
$$= \left[\frac{3e^{-2s}}{s} - \frac{3e^{-3s}}{s}\right]\frac{1}{s}\left[5e^{4s} - 1 - 4e^{-3s}\right]$$

$$= \frac{1}{s}\left[3e^{-2s} - 3e^{-3s}\right]\frac{1}{s}\left[5e^{4s} - 1 - 4e^{-3s}\right]$$

$$= \frac{1}{s^2}\left[15e^{2s} - 3e^{-2s} - 12e^{-5s} - 15e^{s} + 3e^{-3s} + 12e^{-6s}\right]$$

$$= \frac{1}{s^2}\left[15e^{2s} - 3e^{-2s} - 12e^{-5s} - 15e^{s} + 3e^{-3s} + 12e^{-6s}\right]$$

B.2.2



$v(t) = w(t) * h(t)$

Taking Laplace transform

$V(s) = W(s)\,H(s)$

$$V(s) = \left[\frac{4}{s} - \frac{4e^{-3s}}{s}\right]\left[\frac{3e^{-2s}}{s} - \frac{3e^{-3s}}{s}\right]$$

$$= \frac{12e^{-2s}}{s^2} - \frac{12e^{-3s}}{s^2} - \frac{12e^{-5s}}{s^2} + \frac{12e^{-6s}}{s^2}$$

$$= \frac{1}{s^2}\left[12e^{-2s} - 12e^{-3s} - 12e^{-5s} + 12e^{-6s}\right]$$

B.2.3. Consider $Y(s) = H(s)[M(s) + W(s)]$ —①

$V(s) = W(s)\,H(s)$ —②

① − ②

$$Y(s) - V(s) = H(s)\,[M(s) + W(s)] - W(s)\,H(s)$$

$$Y(s) - V(s) = H(s)\,M(s)$$

$$M(s) = \frac{Y(s) - V(s)}{H(s)}$$

$$= \frac{\frac{1}{s^2}\left[15e^{2s} - 3e^{-2s} - 12e^{-5s} - 15e^{s} + 3e^{-3s} + 12e^{-6s}\right] - \frac{1}{s^2}\left[12e^{-2s} - 12e^{-3s} - 12e^{-5s} + 12e^{-6s}\right]}{\frac{3e^{-2s}}{s} - \frac{3e^{-3s}}{s}}$$

$$= \frac{\frac{15e^{2s}}{s^2} - \frac{3e^{-2s}}{s^2} - \cancel{\frac{12e^{-5s}}{s^2}} - \frac{15e^{s}}{s^2} + \frac{3e^{-3s}}{s^2} + \cancel{\frac{12e^{-6s}}{s^2}} - \frac{12e^{-2s}}{s^2} + \frac{12e^{-3s}}{s^2} + \cancel{\frac{12e^{-5s}}{s^2}} - \cancel{\frac{12e^{-6s}}{s^2}}}{\frac{3e^{-2s}}{s} - \frac{3e^{-3s}}{s}}$$

$$= \frac{\frac{15e^{2s}}{s^2} - \frac{3e^{-2s}}{s^2} - \frac{15e^{s}}{s^2} + \frac{3e^{-3s}}{s^2} - \frac{12e^{-2s}}{s^2} + \frac{12e^{-3s}}{s^2}}{\frac{3e^{-2s}}{s} - \frac{3e^{-3s}}{s}}$$

$$= \frac{\frac{15e^{2s}}{s^2} - \frac{15e^{-2s}}{s^2} - \frac{15e^{s}}{s^2} + \frac{15e^{-3s}}{s^2}}{\frac{3e^{-2s}}{s} - \frac{3e^{-3s}}{s}}.$$

Here numerator can be written as product of denominator & $M(s)$.

After expanding this product we will get back the value same as numerator.

$\therefore$ numerator can be written as

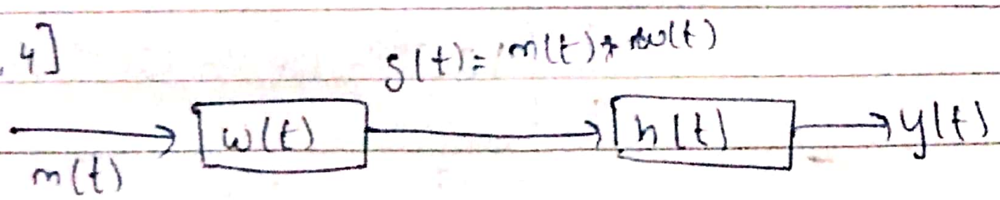
$$= \frac{\left(\frac{3e^{-2s}}{s} - \frac{3e^{-3s}}{s}\right)\left(\frac{5e^{4s}}{s} - \frac{5}{s}\right)}{\cancel{\left(\frac{3e^{-2s}}{s} - \frac{3e^{-3s}}{s}\right)}}.$$

$$M(s) = \frac{5e^{4s}}{s} - \frac{5}{s}.$$

Taking inverse laplace transform.

$m(t) = 5\,[u(t+4) - u(t)]$

[3.2.4]



$y(t) = s(t) * h(t)$

$y(t) = m(t) * w(t) * h(t)$

Taking Laplace transform on both side

$Y(s) = M(s)\,W(s)\,H(s)$

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$$= \left[\frac{5e^{4s}}{s} - \frac{5}{s}\right]\left[\frac{4}{s} - \frac{4e^{-3s}}{s}\right]\left[\frac{3e^{-2s}}{s} - \frac{3e^{-3s}}{s}\right]$$

$$= \frac{20e^{4s}}{s^2} - \frac{20e^{s}}{s^2} - \frac{20}{s^2} + \frac{20e^{-3s}}{s^2}\left[\frac{3e^{-2s}}{s} - \frac{3e^{-3s}}{s}\right]$$

$$= \frac{60e^{2s}}{s^3} - \frac{60e^{s}}{s^3} - \frac{60e^{-s}}{s^3} + \cancel{\frac{60e^{-2s}}{s^3}} - \cancel{\frac{60e^{-2s}}{s^3}}$$

$$+ \frac{60e^{-3s}}{s^3} + \frac{60e^{-5s}}{s^3} - \frac{60e^{-6s}}{s^3}$$

$$= \frac{1}{s^3}\left[60e^{2s} - 60e^{s} - 60e^{-s} + 60e^{-3s} + 60e^{-5s} - 60e^{-6s}\right]$$

Write analysis

Analyse the effect of change in operation b/w the message & the watermark on the response.

Atleast comment 1 or 2 para on the above.