DISCRETE-TIME AND CONTINUOUS-TIME APPROACHES TO DYNAMIC MICROSIMULATION RECONSIDERED

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Technical Paper No. 13

October 1997

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National Centre for Social and Economic Modelling
• Faculty of Management • University of Canberra •

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ISSN 1328-9136
ISBN 0 85889 657 5

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Core funding for NATSEM is provided by the federal Department of Health and Family Services.

Abstract

In this paper the continuous-time competing-risk approach to dynamic microsimulation modelling and approaches based on a discrete-time framework are compared in a systematic way. Besides the basic modelling approaches the possibilities to extend the models to include quantitative and qualitative dependent variables, to use macroeconomic explanatory variables, and to account for dependencies between micro-units are discussed. However, most attention is paid to the problems of causality, of simultaneity and of stochastic dependencies between the partial processes in multivariate models. The main conclusion is that a discrete-time framework with comparatively short time periods appears to be best suited for causal modelling in dynamic microsimulation models.

Author note

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Acknowledgments

The author would like to especially thank Hans Bækgaard, Anthony King, Marion Kuschel and Joachim Wilde for their discussion on various aspects of the problems that are presented in this paper. Nevertheless any remaining faults are the author’s responsibility.

General caveat

NATSEM research findings are generally based on estimated characteristics of the population. Such estimates are usually derived from the application of microsimulation modelling techniques to microdata based on sample surveys.

These estimates may be different from the actual characteristics of the population because of sampling and nonsampling errors in the microdata and because of the assumptions underlying the modelling techniques.

The microdata do not contain any information that enables identification of the individuals or families to which they refer.

Contents

1 Some questions to start with

Traditionally, most dynamic microsimulation models have been based on the discrete-time transition-probability approach. In such models, the state of the micro-units is considered for discrete time periods and transitions in the individual states between succeeding time periods are modelled conditional on the state in the period, on the values of exogenous variables and on the parameters of the model. This is basically a direct extension to the microdata case of the concept of transition models that has been used for a long time in the social sciences to model the dynamics of grouped data.

On the other hand, there have been attempts to use a continuous-time approach to microsimulation. One example is the SOCSIM model that has been developed by Hammel et al. at Berkeley (Hammel 1990). Another new model is the demographic PopSim part of the DYNAMOD model being developed at NATSEM (Antcliff 1993). These models can be regarded as duration models that model the timing of events and thereby changes in the state variables. Models of this type have been used for a long time in demography, in biometric research and in analysing the reliability of technical systems. Only during the past two decades have these models gained more interest in a wider social science context.

The main conceptual advantage of dynamic microsimulation over more aggregated models such as transition models at the group level is that it allows more detailed modelling of processes that operate on the level of the micro-units, the interactions between such processes, and their dependency on exogenous factors. A typical application, for instance, is the simulation of the labour supply of different members of a household and the responses to changes in wage rates, tax rates and transfer programs. While the group level allows only a few explanatory variables to be considered jointly, the micro-unit approach allows the use of very detailed behavioural relations for each individual. On the other hand, the microsimulation technique in general implies a higher computational burden because of the larger number of units that are considered and the more complex model structure. It is thus of crucial importance for the practical value of a dynamic microsimulation model that it allows the modelling of complex structural relationships at a reasonable cost in terms of the ease of model specification and the computational burden of the simulation.

In a recent paper, Bækgaard and King (1995) discussed the relative merits of the two approaches with particular emphasis on the specific implementation of the continuous-time approach that has been used in DYNAMOD. They compared the continuous-time and transition-probability approaches with regard to the solutions that the two approaches provide to a set of problems. Of major importance to them are how simultaneous events can be handled, the implications for the ordering events, aggregate influences on micro-unit behaviour, the data requirements of the models, the relative efficiency of the simulation methods, the flexibility and transparency of the model structure, and the methods used for handling microdata. While Antcliff (1993) had been rather optimistic about the relative merits of the continuous-time approach, Bækgaard and King concluded that there is no such clear advantage and that it may be preferable for practical purposes to use a discrete-time model with a comparatively short time period.

Some of the same issues will be discussed, though along somewhat different lines. After describing the general structure of the two model types, some theoretical and statistical properties of the approaches are discussed in more detail. Of particular interest are the questions of how causality, simultaneity, and heterogeneity of the micro-units can be handled in the two approaches.

2 The basic model structure

2.1 Continuous-time models

The basic approach in a continuous-time event-driven model is to simulate the event history for each micro-unit — that is, the timing of different types of events (see Lawless 1982 and Lancaster 1990). In general, a random process is assumed to generate the events being considered, with the probability density for experiencing an event at a given point in time depending on a set of explanatory variables. These may include the state of the unit and the time since the last event of a given type. Additional explanatory variables can be used to model the effects of the environment on the micro-unit. These dependencies can be modelled conveniently using hazard rates that give the instantaneous conditional probability density for experiencing the event conditional on the values of the explanatory variables given that the event has not yet been observed. Alternatively, an accelerated failure-time specification can be used to model the 'speed' of the corresponding process conditional on the explanatory variables. In principle, both specifications can be transformed into each other. However, one of the two specifications may become rather complicated, depending on the specific model structure. In the following, only the hazard rate specification will be discussed because of its greater flexibility.

In general, the hazard rate may depend on the current state of the unit being considered as well as its history. Additional explanatory variables can be used to model the effects of the environment on the unit. In principle the explanatory variables may vary continuously over time. However, a general specification with continuous-time explanatory variables results in a rather complex model structure that cannot be handled easily. Thus, in practice the covariates are usually assumed to be constant over finite time intervals and specific functional forms such as the Weibull or the Gompertz specifications are used to model duration dependence. As an approximation to more complex patterns, hazard rates can also be specified as a step function that is piecewise constant for given time intervals and a given state of the micro-unit. Then, the events are constrained to follow the exponential distribution within each time interval. This specification has been adopted in DYNAMOD and SOCSIM. It simplifies the model structure substantially and in many cases this simplification is essential to obtain a manageable model structure. However, from a theoretical point of view the assumption may be rather restrictive, especially if long periods are used in the model.

If a piecewise constant hazard rate $r(t) = r(t_i, x(t_i) | t_i \le t < t_{i+1}) = r_i$ is considered — that is, the hazard rate is assumed to be constant in each time interval (t_i, t_{i+1}) — the survival function $S(t \mid x)$ of the process that gives the probability of observing an event after time t has a simple exponential form:

$$
\begin{aligned}
S(t|x) &= \Pr\left\{T \geq t|x(z), z = t_0, \ldots\right\} \\
&= \exp\left\{-\int_0^t r(u)du\right\} \\
&= \exp\left\{-\sum_{i=1}^m r_i(t_i - t_{i-1})\right\} \\
&= \prod_{i=1}^m S_i(t_i - t_{i-1}|x(t_{i-1}))
\end{aligned}
$$

The survival function defines the conditional probability distribution of the time of the next event in the process. In a Monte Carlo simulation a random draw from this distribution can be obtained without difficulty by first drawing a random number u that is uniformly distributed over [0,1] and then deriving the value of t for which $u = S(t \mid x)$. If an event has been generated, the state of the micro-unit is updated accordingly and the procedure can proceed to the simulation of the next event conditional on the new state. Thus, the event history for an individual unit can be simulated in a straightforward sequential way.

If different processes are considered, the hazard rate for one process will in general depend on state variables that are related to other processes. For instance, the process of fertility may depend on marital status, or female labour force participation may depend on the number and the age of any children in the family. Thus, events in one process will affect the probability that an event will occur in other processes. This can be taken into account by using a competing risk framework for the model: if an event is observed for one process, it censors the time to the next event for the other processes. That is, all other processes are considered to be interrupted at that point in time and the probability distribution of the time to the next event is re-evaluated, conditional on the new state of the micro-unit that resulted from the event that has taken place. Thus, given the current state of a unit, only the first event out of the set of possible events is actually simulated in one step of the simulation and all other processes are censored.

An important implication of a strictly continuous time scale is that the probability that two events occur at the same time is zero. As a consequence, different events are mutually exclusive for a given point in time and the probability of observing any one out of a set of events is given by the sum of the individual probabilities. Correspondingly, the hazard rate for observing any event in a model with multiple processes is defined as the sum of the hazard rates for the individual processes. Thus, the joint survival function for k processes is given by:

$$
\begin{aligned}
S(t_1,\ldots,t_k|x,t_1=\ldots=t_k=t) &= Pr\left\{T_j \geq t, \forall j=1,\ldots,k|x(z),0\leq z\leq t\right\}\\
&= exp\left\{-\int_0^t\left[\sum_{j=1}^k r_j(u)\right]du\right\}\\
&= exp\left\{-\sum_{j=1}^k\left[\int_0^t r_j(u)du\right]\right\}\\
&= \prod_{j=1}^k S_j(t|x_j)
\end{aligned}
$$

According to the last line in the above expression, the survival function can be factored into the product of partial survival functions for the individual processes. This implies stochastic independence of the different partial processes — a property that can be used to decompose the simulation of competing risks into the independent simulation of the partial processes. The different processes are then linked via the event that occurs first and censors all other processes. Since the survival function of each partial process can be factored into conditional partial survival functions corresponding to subperiods, the simulation can be carried out stepwise from one censoring event to the next.

This property follows from the mathematical condition that any two random events will occur at the same point in time with a probability of unity. However, from a substantive point of view this may be regarded as questionable. For instance, buying property and raising credit for this purpose might be considered to be simultaneous events since the sequence of signing the contracts does not matter for the purpose of a model. In this case the two events result from the same decision and they are linked in a deterministic way. In the context of competing risks, this implies that the hazard rate of observing one event, given that the other event has been observed, will approach infinity. Since such hazard rates cannot be modelled easily, it is preferable to regard events of this type as a single composite event that is modelled by a single hazard rate function.

The conditional stochastic independence of the partial processes gives rise to a very simple structure for the simulation model even if a large number of processes is being considered. Starting from the initial state of the micro-units, the conditional probability distribution of the time to the next event is derived for each process being considered in the model and the time of the next event is simulated by first drawing a uniformly distributed [0,1] random number and then computing the corresponding duration from the partial survival function. As far as competing risks are considered, the first event to occur is selected in the next step and all other processes are marked as being censored. The state of the unit is changed and the new conditional survival functions are derived for all competing processes. Then the simulation of the next event is repeated. In this way a random draw from the space of all possible event histories for a unit is obtained.

Given the conditional independence of the partial processes, the specification of the simulation model can be decomposed into relations or modules corresponding to the different processes that are conditionally independent given the state of the unit. This simplifies the process of model specification considerably, since the different parts of the model can be developed independently and are linked together only in the simulation process. Also the structure of the full model is rather flexible and can be changed easily since changes in the specification of one partial process do not affect the specification of the other processes being considered. Additional partial processes can be added to the model just by specifying the corresponding conditional relations and adding them to the model structure. Thus, given the property of conditional independence, the continuous-time competing-risk approach provides a comparatively simple framework for dynamic microsimulation.

2.2 Discrete-time models

In discrete-time models, only the outcomes for discrete time periods are considered and no reference is made to the timing of events within a period (see, for example, Allison 1982). As an additional simplification, all events are often assumed to occur at only discrete points in time and the time intervals in between are not considered explicitly in the model. Thus, an aggregation over time is applied, resulting in a loss of information about the event history within the time period. However, the number of events of a given type that a micro-unit experiences within a time period may still be recorded. If, for instance, an annual period is being considered, it may be important to model the number of unemployment spells within the year, even if the timing of the spells is not considered.

Basically, a discrete-time model considers the changes in the state variables between the start and the end of a time period without reference to the process in between. This can be done either by directly modelling the state of a unit at the end of the time period or by modelling the transitions between subsequent points in time. Since states and transitions are linked by definition, the two approaches are equivalent if only single (net) transitions are considered. If the new state is explained by the model, the corresponding transition is defined by the change in the state variables and vice versa. Thus both specifications are in principle exchangeable. However, since the number of possible transitions between states exceeds the number of states, a model specification for the states will in general be simpler than a specification for transitions between the states. On the other hand, modelling transitions may be more appropriate from a substantive point of view if, for instance, transitions occur with a comparatively high frequency.

A common assumption is to model transitions between different states by a random process, with transition probabilities conditional on the state in previous periods and on other exogenous variables. In addition, the time index of the period may be used as an explanatory variable in order to model duration dependence. Thus the basic model specification commonly consists of a function that gives the conditional probability to observe a given transition in a given period depending on the values of the explanatory variables. Traditionally, table functions have been used for this purpose, though structural models developed in econometrics can also be used. In this latter case the transition probability is usually modelled using a logit or a probit specification. In the case of a simple dichotomous outcome, a binary model can be specified for the outcome x_t in period t, with z_t being the vector of explanatory variables, y_t being an unobservable, latent variable, and ε_t being a random error term:

$$y_t = \beta' z_t + \varepsilon_t$$

$$x_t = \delta(y_t) = \begin{cases} 1 & \text{if } y_t \le 0 \\ 0 & \text{else} \end{cases} \quad \Leftrightarrow \quad \Pr(x_t = 1 | z_t, \beta) = \Pr(\varepsilon_t \le -\beta' z_t) = F(-\beta' z_t)$$

The specific form of the model depends on the distributional assumptions for the error term. The most common specifications are the logit and the probit models that are obtained if an extreme value distribution or the normal distribution is assumed. Both models can be generalised easily to processes that allow more than two outcomes. In this case a multinomial logit or probit model results.

Given the parameter estimates and the values of the explanatory variables, a Monte Carlo simulation of the outcome x_t can be performed for each unit in the sample. The standard approach for a univariate model is to generate a random draw from a uniform (0,1) distribution. If the value exceeds $F(-\beta' z_t)$, the event is assumed to occur; otherwise $x_t = 0$ is assumed. As an alternative simulation procedure, for each unit a value of ε_t can be generated randomly from the corresponding distribution and then the condition for $x_t = 1$ is evaluated. This second approach requires some more calculations for generating the random numbers but it can be generalised more easily to a multivariate setting.

Model specifications based on simple table functions, which provide the probabilities conditional on the values of explanatory variables, can be integrated into this structural framework by specifying dummy variables as explanatory variables that correspond to the different table entries. If the logit specification is used, this amounts to a log-linear representation of the corresponding table of conditional probabilities.

The basic model structure for a single transition can be generalised to a multivariate approach that models transitions for more than one state variable in the same period. Essentially, such a model is constructed by combining the equations describing the partial processes for different state variables into a system of equations. However, in a multivariate model additional dependencies may occur since the probability of observing a transition in one state variable may depend on the transitions in other state variables. This can be taken into account by introducing the vectors of all latent variables y_t and of all qualitative outcomes x_t as additional explanatory variables for the j-th transition.

$$\left.\begin{aligned} y_{jt} &= \alpha_j' y_t + \beta_j' z_t + \gamma_j' x_t + \varepsilon_{jt} \\ x_{jt} &= \delta(y_{jt}) = \begin{cases} 1 & \text{if } y_{jt} \le 0 \\ 0 & \text{else} \end{cases} \end{aligned}\right\} \quad j = 1,\dots,k; \qquad \text{with} \quad \alpha_{jj} = \gamma_{jj} = 0.$$

In matrix notation, the structural form of this simultaneous equation model can be written as:

$$\begin{aligned} Ay_t &= Bz_t + Cx_t + \varepsilon_t \\ x_t &= D(y_t) \end{aligned}$$

with the reduced form:

$$y_t = \Pi_1 z_t + \Pi_2 x_t + \nu_t \qquad \text{with } \Pi_2 = 0 \text{ or lower triangular}$$

$$x_t = D(y_t)$$

This specification allows for structural interdependencies between different partial processes since a process j may depend on the outcomes for other processes in the same time period. However, the model structure must conform to a consistency constraint that follows from the definition of probability. It can be shown (see Schmidt 1981) that in general a recursive dependency structure between the qualitative outcomes must be maintained in order to achieve logical consistency. If the qualitative outcome x_{jt} of one process enters as an explanatory variable into other processes, the outcomes of those processes must not affect the probability to observe the outcome of the first process. Otherwise the probability to observe the first outcome would depend on that outcome and the model would give inconsistent results. Thus, multivariate models of this type are restricted to a recursive structure with regard to the observable outcomes in the same period. However, if the (contemporary) qualitative outcomes are not used as explanatory variables, a simultaneous dependency structure may be specified for the latent variables y_t.

In addition to the structural dependencies, the joint probability of observing a specific pattern of transitions in a period depends on the correlation structure of the random terms. If a multivariate probit specification is used with a multivariate normal distribution of the error terms, such stochastic dependencies can be modelled by a nondiagonal covariance matrix of the error terms ε_t. The same is not true, however, for a multivariate logit model since no simple multivariate generalis-ation of the extreme value distribution is available. Still, parameter estimation poses some problems even for the multivariate normal distribution because of the difficulties in integrating that distribution. Thus, in practice, conditionally independent error terms are often assumed. However, this implies that no correlation of unobserved heterogeneity between different processes can be taken into account in the model.

Given the parameter estimates and the correlation structure of the error terms, the simulation of a multivariate transition model can be performed in a similar way to the univariate case. First, a vector of values of the error terms is generated from their joint probability distribution. Due to the recursive structure in x_t, the equations of the reduced form can be solved first for the latent variables y_{jt} and then the qualitative outcomes x_{jt} can be evaluated sequentially.

As long as only net changes between different states are of interest, transition models have a simple structure. However, conceptual problems arise if more than one transition may occur for a state variable within the time interval being considered since the sequence of the events is not modelled explicitly. One might consider adding an equation that explains the number of events, depending on the type of transition and other explanatory variables. But it is not clear how such an ad hoc solution can be linked to an underlying theoretical model in a meaningful way. The problem is that, in general, the conditional probability of observing another transition in the same period will change when a transition has been observed. But this cannot be modelled in a discrete-time framework since even the order of transitions within a period is not identified due to the aggregation over time. The only solution is to reduce the length of the time interval being considered in the model. At the limit this implies the adoption of a continuous-time approach. Thus, if processes with a high frequency of events must be modelled, the discrete-time approach is of limited use.

3 Extensions to the basic model structure

For practical purposes, the basic structure of both the continuous-time hazard rate approach and the discrete-time transition models is too restrictive. In general, continuous quantitative variables must be included in dynamic microsimulation models in addition to discrete variables that correspond to qualitative events. In household models, for instance, flow variables such as income, taxes and consumption, or stocks such as wealth, must be considered. A second problem is that, if economic variables such as labour force participation, income or consumption are included in a microsimulation model, the corresponding processes will in general also depend on aggregated variables such as labour demand, unemployment rates, wage rates, prices and other macro variables. As a consequence, the basic model structure must be extended to allow for both continuous dependent variables and aggregated explanatory variables.

Also there may be dependencies between different micro-units that affect, for example, labour force participation, earnings and consumption. In this case, the behaviour of different members of the same household is linked via the budget constraint of the household. This implies, for instance, that changes in the earnings of the husband may influence the labour supply of the wife and that the retirement decision of a couple is affected by the interrelated decisions of the spouses. In order to model such processes in an adequate way, a model structure that allows for such dependencies between different micro-units is required.

3.1 Continuous dependent variables

An event-driven model can be generalised to include flow variables such as income if the intensity of the flow rather than its level is being considered. While the level of a flow variable is defined for only finite time periods, the flow intensity refers to a point in time like the hazard rate. Thus flow intensities and hazard rates can be integrated into a consistent model framework.

Like hazard rates, flow intensities can be modelled for a given point in time, conditional on the state variables of the unit and on other exogenous variables such as time. The flow over a time period is then obtained by integrating the flow intensity over that period. However, in general a model of this type will be difficult to solve if the flow intensity is a continuous function of time. Only if the flow intensities are assumed to be step functions of time with jumps at discrete points in time can a comparatively simple model structure be obtained. In this case a flow intensity can be considered as a state variable and its change is modelled as an event in the competing risk framework.

More difficulties arise in a continuous-time framework with regard to stock variables that change as a result of the flows over time. Even if constant flow intensities are assumed, the corresponding stocks will change continuously over time. One solution to this problem might be to introduce ‘discrete-time bookkeeping’ in the sense that the state variables representing stocks are updated at given time intervals.

Otherwise, a differential equation framework must be used to specify the model structure. This results in rather complicated model structures that are difficult to solve, since differential equations for stocks and flows must be combined with events that result in discrete jumps in qualitative variables at some points in time.

Due to this problem and to the fact that macroeconomic data are generally available for only discrete periods, macroeconomic models are usually specified for discrete time intervals. Only a few attempts have been made to specify and estimate continuous-time models in econometrics (Gandolfo 1993). Due to the formal complexity of continuous-time models, it is common to use a discrete-time approximation to solve the model even if a continuous-time approach has been used for the theoretical model structure. This is, for instance, true for models of the systems dynamics type. Basically, a continuous-time approach is used to specify the model, but the estimation and simulation of the model are then based on a discrete-time approximation.

In a discrete-time model structure, flows can be handled easily by specifying additional equations for the flows during the period. In the same way, variables representing stocks can be included without problems since only discrete points in time are being considered. Since stocks and flows are related by definition, the dynamics of the stocks can be derived easily from the equations for the flows. As an alternative, the stock at the end of a period may also be explained directly by an equation. The corresponding net flow is then derived from the change in the stock variable. In both cases the behavioural relation explaining the stock or the flow may depend in principle on all variables in the model as long as the model structure can be identified. Which of the two approaches is more appropriate depends on the character of the problem and on the empirical data that are available for modelling.

If equations for stocks or flows are included in a discrete-time transition model, a mixed quantitative–qualitative model results with some equations explaining observable quantitative variables while other equations correspond to the qualitative transition outcomes. Models of this type have been discussed in the econometric literature and efficient estimation techniques have been derived (Blundell and Smith 1993; Maddala 1983). Some problems still remain in estimating such models, because of computational difficulties related to the multivariate normal distribution, but substantial progress has already been made in overcoming these technical problems and they can be expected to be solved.

Thus, comparing the two modelling concepts, the discrete-time approach offers some advantages over continuous-time models with regard to the integration of quantitative variables. Basically, stocks and flows are easier to handle conceptually in a discrete-time framework. Rather elaborate methods have been developed in econometrics for discrete-time models while only few results are available for the continuous-time framework. A less complex model structure is obtained in the continuous-time framework if constant flow intensities for finite time intervals are assumed. However, this is a rather restrictive assumption that amounts to the transformation of the problem into a discrete-time framework. Also, no simultaneous dependencies between hazard rates and flows can be accommodated in such a model if the flow intensities are assumed to be constant over a finite time interval but the hazard rates may change within the interval.

3.2 Dependencies on macro variables

Dependencies of the micro processes on macro variables can be accommodated easily in a continuous-time competing-risk framework if the changes of the macro variables occur only at discrete points in time. In this case, the macro framework can be modelled by additional pseudo processes with events that correspond to changes in the macro variables. These pseudo processes compete with the micro processes in the sense that the micro processes are censored if a change in a macro variable takes place that will affect the hazard rates of the micro processes. Thus, from a technical point of view, the integration of macro variables poses no problem if changes occur only at discrete points in time.

The assumption of piecewise constant macro variables appears to be far less restrictive than a similar assumption for the dependent variables at the micro-unit level since aggregated macro information is in general only available for discrete periods. However, one might question whether it is consistent in a strict sense with the underlying assumptions of a continuous-time micro model.

Pseudo macro processes that are used to take account of changes in the macro variables or other more technical problems are in general of a deterministic type. In a simulation model it is generally well known in advance when the next macro ‘event’ will occur. Since the micro processes will be censored at that point in time, there is no need to evaluate the survival function of a micro process beyond this point in time. It is sufficient to simulate events that occur before the macro event. Basically, the introduction of a ‘time step’ for the macro variables imposes a discrete-time meta-structure on the continuous-time simulation problem. The continuous-time simulation is carried out sequentially within the periods for which the macro variables are assumed to be constant.

In a discrete-time model, the inclusion of aggregated macro variables in the set of explanatory variables does not involve any problems either if they are defined for the same period as the micro model. If a shorter period is used at the macro level, an aggregation over time will be required for flow variables, which can be done easily. However, if the period of the macro model exceeds that of the micro model, additional assumptions, such as interpolation and smoothing, are required in order to disaggregate the macro information into values that are compatible with the shorter period being used at the micro-unit level.

Thus, there are slight advantages in the continuous-time specification at the micro-unit level if constant flow intensities are assumed for fixed time intervals at the macro level. Given this assumption, a continuous-time micro model can accommodate any periodicity of the macro data. For discrete-time models, on the other hand, some aggregation or disaggregation of the macro variables is required if the periodicity of the macro data does not coincide with that of the micro model.

3.3 Structural dependencies between micro-units

In principle, dependencies between different micro-units can be accommodated in any microsimulation model by the use of a larger simulation unit that contains all individual units that are mutually dependent. In this way the dependency between micro-units is transformed into a dependency that occurs within the simulation unit. For instance, in microsimulation models of private households, all individuals belonging to the same household are simulated jointly — that is, the household is used as the simulation unit. Then all variables of all household members are available for the simulation procedures at the same time and dependencies between household members can easily be taken into account.

In this sense, allowing for dependencies between different micro-units in a competing risk framework boils down to combining all processes of all units that may be interrelated into the same set of competing risks. This allows making a process for one individual unit depend on the state of another unit in the same way as dependencies between processes are modelled for a single unit. At the limit, all processes for all units in the sample might be considered as potential competing risks, giving much freedom to simulate dependencies all over the sample. At least in principle, one could even conceive of the simulation of interactions in markets based on this concept, with a micro-unit making an offer at some point in time and allowing other units to respond.

For the implementation of a microsimulation model this would imply that groups of units must be defined corresponding to the level at which competing risks are to be applied jointly. If large groups are chosen, the administrative overhead of the simulation will increase since it is necessary to identify the next event to occur for all processes and for all units being considered simultaneously. Since the probability of observing an event within a time interval increases with the number of competing risks that are considered, the resources that are required to administer the competing processes may become substantial. However, the experience with DYNAMOD indicates that there may be feasible solutions to this problem.

In a discrete-time transition model, structural dependencies between micro-units can be taken into account in a similar way. Here, the equations for the different micro-units are linked to form a larger set of equations with some dependencies between them. In general, the joint solution for all micro-units that are interrelated is obtained by solving this system of linked equations. However, this may become infeasible if a large number of units or processes is considered at the same time. An additional, and more technical, problem is that the number of units that are related may vary over the sample. In this case equation systems of varying size must be handled.

A simple solution to this problem is to assume a recursive relationship between the individuals. This approach allows the equation system to be solved recursively for the different individuals, and has been used in some models. But recursiveness in individual behaviour is a rather restrictive assumption, especially if longer periods are being considered. Thus it would be preferable to use a simulation model that allows simultaneity between individual units. But a general simultaneous framework for all units in the sample will not be feasible if a sample of sufficient size is being used. This implies that some lower level simulation units such as households must be defined in discrete-time models so that the equation system can be handled in a feasible way.

In this respect, the discrete-time approach appears to be somewhat more restrictive than an event-driven model. In addition, some assumptions must be made with regard to the functional form of the equations in order to provide methods for solving the equation system. If rather general numerical methods are used that give flexibility with regard to the form of the equations, substantial computational resources may be needed to solve the model. Thus, in this respect the continuous-time event-driven approach seems in general to have some advantages over discrete-time models.

The disadvantage of discrete-time models will be reduced, however, if models with short periods are being considered. In this case, simultaneous dependencies between micro-units can be transformed into time-recursive relationships with one unit reacting to changes in the state of another unit in only the following periods. If such a specification is used, no contemporaneous simultaneities exist and the model can be solved in a simple recursive way. Thus, the choice of a short time scale will allow substantial simplifications of discrete-time models. However, the computational costs of the simulation will increase proportionally with the frequency of model evaluations.

4 Theoretical and statistical properties

The question of how the continuous-time and the discrete-time approaches can be extended to allow for model specifications that are more in line with the requirements of microsimulation applications is only one aspect of the suitability of the two approaches. The general theoretical and statistical properties of the two approaches and their implications for the model structures are also of interest. One issue in this context is how to handle causal relations between different outcomes in a multivariate framework in the different model specifications. A second issue is related to the fact that empirical models are only imperfect approximations of the unknown true structure. Thus the question of how the resulting errors can be handled is of major importance for applied work. Finally, the two approaches differ with regard to the available estimation techniques and the data requirements.

4.1 Causality and model structure

An important property of the continuous-time competing-risk approach is that, given its assumptions, it implies a simple sequential relationship between different events. Since at any point in time only a single event can occur, and if in addition the standard assumption is invoked that an event at time t can depend on only past history and the current state of a unit but not on the future, a well-defined sequential causal order of events over time results. Thus, in a competing-risk framework, the probability of observing an event at time t will depend on only events that have already taken place; it will not depend on future events. This simplifies the task of specifying the model structure substantially. All processes are modelled depending on the current state of the unit and on its history. Accordingly there is no need to specify a direct causal relationship between different events.

The simplicity of the competing-risk model depends crucially on the conditional independence of the partial processes given the state of the micro-unit. If this holds, the partial hazard rate for one partial process depends on only the state of the micro-unit but not on the hazard rates for other partial processes. This allows the specification of independent partial models for the different processes being considered, with the information on the other processes being provided by the state variables of the unit. This property is often regarded as a major advantage of a continuous-time competing-risk framework over discrete-time approaches. However, the assumption of conditional independence implies some rather strong restrictions for the properties of the model.

Most importantly, conditional independence of the partial processes implies a property of the independence-of-irrelevant-alternatives type (IIA). For two partial processes i and j in a competing risk model the ratio of the probability densities f_i and f_j for observing events i or j at time t depends on only those explanatory variables that enter into the hazard rates for these two processes, and is independent from other explanatory variables that enter the hazard rate of a third process:

$$\frac{f_i(t|x_i)}{f_j(t|x_j)} = \left.\frac{-\partial S(t|x)/\partial t_i}{-\partial S(t|x)/\partial t_j}\right|_{t_i=t_j=t} = \frac{r_i(t,x_i)S(t|x)}{r_j(t,x_j)S(t|x)} = \frac{r_i(t,x_i)}{r_j(t,x_j)}$$

This implies that the probability densities of the two events will change proportionally if the hazard rate of a third process is changed by a factor that does not enter the hazard rate of the two processes i and j. For instance, in the case of job mobility, the transition rates to two job categories will change proportionally if the rate of becoming unemployed changes due to a change in a variable that does not enter the mobility rates. This property may be regarded as questionable from a substantive point of view since, in practice, not all explanatory variables can be included in the model specification. However, in order to obtain a model structure that does not show the IIA property, a multivariate survival function that allows for stochastic dependencies between the partial processes must be used.

The IIA property results from the assumption that all relevant explanatory variables have been included in the specification of the partial hazard rates. In this case the partial processes are conditionally independent and changes in one hazard rate will change the probability density of other events proportionally. However, if this assumption does not hold, additional latent factors exist that affect the partial hazards. If, in addition, the latent factors are correlated for different processes, the events corresponding to these processes will no longer be conditionally independent given the observed explanatory variables. In this case, a specification allowing for stochastic dependencies that does not imply the proportionality property would be more appropriate. This problem will be discussed in some more detail in the next section.

In discrete-time models, on the other hand, changes in different state variables may occur during the same interval. Also, at least in principle, multiple events of the same kind may occur during the same period. But, since the timing of events within a period is not considered in a discrete-time model, no causal order is defined for these transitions if no further assumptions are made. Basically, the different transitions should be considered to be jointly dependent on the explanatory factors. As a consequence, one would in general expect stochastic dependencies between transitions corresponding to different processes. For instance, in an annual model, changes in labour force participation and fertility will in general not be independent. But without further assumptions it is not possible to tell for a given year whether fertility has caused a change in labour force participation, a change in labour force participation has affected fertility or both result from a joint decision. In general, if more than one transition is observed in the same period, a causal order of the transitions cannot be identified from time-discrete data without further assumptions.

A general solution would consist of simulating the different processes on the basis of the joint probability distribution of the different outcomes. This requires a model structure that allows the derivation of the joint probability distribution of the outcomes, conditional on the values of the explanatory variables of the model. Any dependencies between the partial processes that are not captured by the systematic part of the model will then result in stochastic dependencies between the partial outcomes. Since in a discrete-time framework, dependencies that occur within the same period cannot be captured by a model that is conditional on only the state of the unit at the start of the period, such stochastic dependencies will generally occur in discrete-time models. Only if these stochastic dependencies are appropriately taken into account in the model will the model reflect the full dependency structure of the partial processes.

This implies that, in addition to the systematic part of the model structure, the correlation structure of the outcomes must also be considered in the simulation process. Otherwise the part of the dependency structure not captured by the systematic part of the model is ignored in the simulation. The consequence will typically be that the correlation between the outcomes in the simulation is biased downward. This may result in biased estimates of the distribution of some characteristics in the population, such as household income. To avoid this the conditional correlation structure of the outcomes must also be estimated in addition to the parameters of the systematic part of the model. As a consequence a discrete-time microsimulation model cannot in general be given a simple modular structure with independent simulation modules for different partial processes.

A simple model structure is obtained only if additional restrictive assumptions are made about the causal order of the partial processes within a time interval. In early microsimulation models a fixed recursive structure of simulation modules was used as an ad hoc solution, with the outcomes for the partial processes being simulated conditional on the outcomes that have been derived earlier in the sequence but with no feedback to earlier steps. This allows the specification of unidirectional dependencies between the different outcomes but no general dependency pattern. Thus a recursive structure implies rather restrictive assumptions that may be questionable from a substantive point of view, especially if long periods such as a year are being considered.

A recursive specification has been justified by some authors using the argument that, from a statistical point of view, any joint probability distribution can be factored into a recursive sequence of conditional probability distributions. This is true by definition. But a given recursive decomposition of the joint probability distribution implies that on each step the probability of the outcome is defined conditional on the outcomes of all processes that have been already modelled on the steps. This poses problems for applied modelling, since it is in general difficult to obtain estimates of the conditional probabilities from empirical data that are consistent with such a recursive structure. The basic problem is that transition probabilities are required that are conditional on the transitions modelled earlier in the sequence and are marginal with regard to those transitions modelled later on. In addition, the recursive transition probabilities for one process will in general depend on the explanatory variables used to model the other processes.

As an alternative a randomised recursive order of the processes has been used in the DARMSTADT model and the models derived from it (Hellwig 1988). But this is only a partial solution to the problem, since interdependencies are still not being allowed for and the random selection of the simulation sequence is in general inconsistent with the assumptions used to estimate the parameters of the model. Typically, in the estimation of the transition probabilities, a given set of conditioning variables must be assumed that refer the start of the period. But in a simulation based on a randomised recursive order, the conditioning variables are defined by a random process. Thus, the randomisation approach implies some inconsistencies and does not amount to much more than an ad hoc solution.

An advantage of a recursive model structure is that it allows the use of table functions with nonparametric estimates of the transition probabilities in the simulation. However, due to the ‘curse of dimensionality’, such a nonparametric simulation approach becomes generally infeasible if more complex dependency structures are being considered for a large number of partial processes. The number of possible combinations of different transitions increases exponentially with the number of different processes that are simulated in the model. As a consequence, both the estimation of transition probabilities and the simulation of the transitions will become infeasible in most cases if a large number of transitions is to be considered jointly since not enough empirical observations will be available to estimate the transition probabilities. Thus, in general, parametric assumptions on the form of the joint probability distribution of the outcomes will be required if more complex dependency patterns of the outcomes are to be taken into account.

If the continuous-time and the discrete-time approaches are compared with regard to the implications for modelling causal relationships, the basic continuous-time approach appears to be much more simple but also much less flexible than discrete-time models. The strictly sequential order of events in a continuous-time framework is an appealing property but it rests on the assumption of a perfect model specification with conditionally independent partial processes. If this assumption is violated, the standard competing-risk approach will no longer be adequate.

Discrete-time models, on the other hand, do not require the assumption of conditional independence. On the contrary, due to the aggregation over time and the resulting loss of information on the ordering of events within a period, one would even expect stochastic dependencies to exist between partial outcomes in a discrete-time model. Since in practice such dependencies cannot be modelled appropriately using a simple nonparametric approach, more complex model structures will be required, at least if a large number of partial processes is being considered. Thus the structure of a discrete-time model will be more complex than that of a basic competing-risk model. It does, however, accommodate stochastic dependencies that are not allowed for in the other approach.

4.2 Specification errors and unobserved heterogeneity

The problem of stochastic dependencies between the partial processes becomes even more pronounced if specification errors and unobserved heterogeneity of the micro-units are considered. In practice, the specification of models will in general be imperfect since not all relevant factors can be included in the model and the functional form of the relations represents only an approximation to the unknown true structure. In econometric models, these errors are represented by the error variables of the relations that are assumed to be random variables. In micro-simulation models, these errors can be decomposed into error components. In the simplest case an error component that is specific to the micro-unit and a residual error component are assumed. The individual specific error component represents unobserved heterogeneity of the micro-units — differences in the dependent variable that are not explained by the observed explanatory variables but are caused by unobserved latent characteristics of the unit.

In the context of multivariate models, unobserved errors may result in stochastic dependencies between partial processes in the model, even if the processes are conditionally independent when all factors are controlled for. An example is the age at first marriage and the age at the birth of the first child. One would expect that young couples are heterogeneous with regard to their propensity to marry and to have children. If this heterogeneity affects the age at marriage and the birth of the first child in a similar way, a positive correlation between the two events will be observed even if, for instance, education is being controlled for. Similar correlations may also exist, for instance, for job mobility and the risk of unemployment.

Let $F_j(y_j \mid z,u_j)$ be the conditional distribution function for outcome j with conditional independence of the partial processes given the values of the observed variables in z and the values of the unobserved latent variables u. The joint distribution of the error terms u_j is given by M(u). If the errors in u are not independent for the partial processes, stochastic dependencies will result for the joint distribution function $F(y_1,\ldots,y_k \mid z)$ that is still conditional on z but is marginal with regard to u:

$$\begin{aligned} F(y_1,\ldots,y_k \mid z) &= \int \prod_{j=1}^{k} F_j(y_j \mid z,u_j)\,dM(u) \\ &\neq \prod_{j=1}^{k} \int F_j(y_j \mid z,u_j)\,dM_j(u_j) = \prod_{j=1}^{k} F_j(y_j \mid z) \end{aligned}$$

Thus, given the imperfections of empirical models, the assumption of conditional independence of the partial processes given only the values of the observable conditioning variables is at least questionable. In general, one would expect that some of the omitted variables will affect more than one of the partial processes and as a consequence one would expect stochastic dependencies between the different outcomes. If this is true, a model that assumes conditional independence and neglects the correlation structure of the latent errors in the model will produce biased results since the simulation is based on incorrect distributional assumptions.

Unobserved heterogeneity of the micro-units not only generates stochastic dependencies between different partial processes but also between multiple outcomes of the same process. If, for instance, transitions into unemployment are considered, the duration of subsequent employment spells will be shorter for individuals with a higher propensity to become unemployed due to some unobserved factors. Here one would expect a positive correlation between the duration of employment spells. In order to generate an unbiased estimate of the effects of unemployment in the population, such dependencies must be included in the simulation of the micro-units.

In principle, there are two options for obtaining unbiased simulation results. One way is to apply the joint probability distribution of the outcomes that is marginal with regard to the unobserved factors and allows for stochastic dependencies between the outcomes that result from these factors. In principle, this joint distribution is obtained by 'integrating out' the effects of the unobserved factors in the model. A simulation based on this marginal distribution will provide unbiased results since the dependencies that result from the unobserved factors are taken into account by the marginal distribution function.

The other option is to estimate the distribution of the latent variables and to perform the Monte Carlo simulation conditional on values for these latent variables that are generated at random from their joint distribution. If conditional independence of the partial processes holds given the values of the unobserved factors, this simulation procedure would be simpler to perform than a simulation based on the marginal distribution. However, it requires an estimate of the joint distribution of the latent variables in the model.

In a continuous-time framework the impact of unobserved heterogeneity factors can be taken into account by adding these variables to the specification of the hazard rate of the partial processes or directly to the corresponding partial survival functions. Then the joint survival function that is marginal with regard to the latent variables is obtained by integrating out the heterogeneity factors. If $S_i(t \mid x_i,\mu_i)$ is the conditional survival function for process i, conditional on the observable variables in x_i and on the heterogeneity component μ_i, and $M(\mu)$ is the joint distribution of the heterogeneity components, the marginal joint survival function for two processes i and j is obtained according to:

$$
\begin{aligned}
S^*_{jk}(t|x) &= \int \left\{ S_j(t|x,\mu_j) S_k(t|x,\mu_k) \right\} dM(\mu_j,\mu_k) \\
&\neq \left\{ \int \left\{ S_j(t|x,\mu_j) \right\} dM(\mu_j) \right\} \left\{ \int \left\{ S_k(t|x,\mu_k) \right\} dM(\mu_k) \right\} \\
\text{if} \quad & dM(\mu_j,\mu_k) \neq dM(\mu_j) dM(\mu_k)
\end{aligned}
$$

If the mixing distribution $M(\mu)$ cannot be factored into independent distributions for the heterogeneity factors, the joint survival function cannot be factored into the product of the partial marginal survival functions. In this case the conditional independence of the partial processes given only the values of the observed variables is lost.

This implies that either the simulation must be based on the joint marginal survival function, allowing for stochastic dependencies between the partial events, or a simulation approach must be adopted that explicitly conditions on the values of the heterogeneity factors. The problem with the first simulation approach based on the marginal survival function is that, even for standard specifications of the hazard rate and of the mixing distribution of the latent factors, the joint marginal probability distribution of the events has a rather complex form. Up to now the marginal distribution has been derived for only special cases that are based on rather restrictive assumptions.

From a technical point of view the second approach has a simpler structure but requires an explicit estimate of the mixing distribution of the heterogeneity components. In principle, one could randomly generate values for the heterogeneity factors from their joint distribution and then perform the simulation conditional on these values in addition to the values of the observed variables. Due to the conditional independence of the partial processes, the simple structure of the basic competing-risk model is preserved and the simulation could be conducted in a comparatively simple way. However, as will be discussed in the next section, neither the dependency structure of the marginal survival function nor the mixing distribution of a competing-risk model can be identified from empirical data without further assumptions.

The problem of specification errors and unobserved heterogeneity also occurs with discrete-time models. In its simplest form, unobserved heterogeneity can also be included in a discrete-time model by assuming an error component structure for the error terms in the different equations of the model. Let μ_{ij} and η_{ijt} indicate an individual specific and a residual error component in equation j of individual i in period t. Then the structural form of a transition model is given by the following equation system:

$$y_{ijt} = \beta_j' z_{ijt} + \mu_i + \eta_{ijt} \quad , \ j = 1, \ldots, k$$

and for qualitative outcomes:

$$x_{ijt} = \begin{cases} 1 & \text{if } y_{ijt} \leq 0 \\ 0 & \text{else} \end{cases} \quad \Leftrightarrow \quad \Pr(x_{ijt} = 1 | z_{ijt}, \beta_j) = \Pr(\mu_i + \eta_{ijt} \leq -\beta_j' z_{ijt})$$

This model specification is conditional on the unobserved heterogeneity factors μ_{ij}. The marginal model is obtained if a new random variable is defined as the sum of the individual specific and the residual error components in the different equations, $\varepsilon_{ijt} = \mu_{ij} + \eta_{ijt}$. Even if the residual error components η_{ijt} are independent random variables but the individual specific components are correlated over the equations, the error terms ε_{ijt} in the marginal model will be correlated. This implies that, conditional on the explanatory variables in z, the outcomes of different equations are in general not independent if unobserved heterogeneity exists.

Again, the simulation can be based either on the marginal model with the error terms ε_{ijt} or on the model specification that explicitly takes the error components μ_{ij} and η_{ijt} into account. If the error components μ_{ij} and η_{ijt} are assumed to be distributed according to a normal distribution, the errors of the marginal model are also multivariate normal. Thus, given this additional assumption, the error distribution of the marginal model can be obtained at least in principle much more easily than in the case of a continuous-time hazard rate model with unobserved heterogeneity.

In general, the basic implications of unobserved heterogeneity are very similar for continuous-time and discrete-time models. In both cases it will result in stochastic dependencies between the different outcomes for a given micro-unit. Since unobserved heterogeneity can be assumed present in almost any micro model, the assumption of conditional independence of partial processes, given the values of the observed explanatory variables in the model, can in general not be maintained. As a consequence, model specifications that allow for stochastic dependencies between the outcomes should be applied in practice.

4.3 Modelling stochastic dependencies

Discrete-time models with a generalised correlation structure of the partial outcomes can be derived by the use of structural models with a parametric joint distribution of the error terms in the different equations. Due to its simplicity, a multivariate normal distribution is usually specified for the error terms, resulting in a multivariate probit model for the transitions. In such a model, structural dependencies between different processes can also be modelled by using the latent dependent variable or the observable qualitative variable of one process as an explanatory variable for another process. In principle, structural simultaneity of the processes can be modelled in this way. However, due to the consistency constraint for probabilities, no qualitative outcome for a partial process may affect its own probability. This implies a recursive causal order for the qualitative outcomes.

This recursivity constraint does not really restrict the structure of the model since remaining stochastic dependencies are modelled via the correlation structure of the error terms. On the other hand, the recursivity of the systematic part of the model simplifies the simulation procedure. It implies that the simulation of the partial processes may be conducted in a recursive way based on their joint conditional probability distribution. Thus, the framework of a multivariate probit model with general error structure allows the modelling of stochastic dependencies between transitions in a discrete-time model.

Some deficiencies do remain, however. Firstly, the assumption of a multivariate normal distribution may not always be appropriate. Also, problems occur with regard to the estimation procedure for models with a general correlation structure due to the difficulties in integrating multi-dimensional normal distributions. But most of these problems are of a more technical nature and some progress in overcoming them has already been made. For instance, an iterative estimation procedure can be used, with the parameters of the systematic part being estimated in a first step from the marginal distribution for each process. In a second step, estimates for the correlation structure can be derived from a conditional likelihood approach (Muthén 1983). As an alternative, maximum likelihood estimators based on simulation techniques can be used for a joint estimation of the structural parameters of the model and the correlation structure of the latent variables. Provided that a sufficient number of observations with information on the partial processes being considered is available the correlation structure of the error terms can be estimated by both methods.

For competing-risk models, on the other hand, estimating the dependency structure between the partial processes poses problems. Due to a non-identifiability property, dependencies between partial processes cannot be identified from empirical data without further assumptions. It can easily be shown that any multivariate survival function can be factored into the product of partial survival functions for conditionally independent partial processes — that is, an observationally equivalent survival function with independent partial processes (see, for example, Tsiatis 1975).

Let $S(t_1,...,t_k \mid x)$ be the joint survival function of k dependent processes that is conditional on the observable variables x but marginal with regard to unobserved factors. The hazard rate of process i at time t is defined by:

$$r_i(t,x) = \left.\frac{\partial \log S(t_1,\ldots,t_k|x)}{\partial t_i}\right|_{t_1=\ldots=t_k=t} = \frac{1}{S(t_1,\ldots,t_k|x)}\frac{\partial}{\partial t_i}S(t_1,\ldots,t_k|x)\bigg|_{t_1=\ldots=t_k=t}$$

Based on the hazard rates for all partial processes, a survival function S* can be defined that is observationally equivalent to $S(t_1,...,t_k \mid x)$ and that implies conditional independence of the partial processes — that can be factored into the product of partial survival functions S_j^*:

$$S^*(t_1,\ldots,t_k \mid x, t_1=\ldots=t_k) = \exp\left\{-\int_0^t \sum_{j=1}^k r_j(u,x)du\right\}$$
$$= \prod_{j=1}^k \exp\left\{-\int_0^t r_j(u,x)du\right\}$$
$$= \prod_{j=1}^k S_j^*(t \mid x)$$

For $t=t_1=...=t_k$, the survival function S^* is identical to the original survival function S. In general there will be differences between S and S* if different durations are considered for the partial processes. But since only the first event is actually observed in a competing-risk setting and the durations for all other processes are censored, no observations with different partial durations are available. Thus, no information is available to discriminate empirically between S and S^*. As a consequence, stochastic dependencies between the partial processes cannot be identified from the empirical data alone. For every model with dependent processes there is always an observationally equivalent model specification with independent partial processes.

This result has been used as an argument in favour of models implying conditional independence. However, this solution is not as simple as it may look at a first glance. Firstly, the non-identifiability argument implies that the hazard rate of each partial process in the conditionally independent representation generally will depend on all explanatory variables for all processes in the model. As a consequence, the partial processes may not be specified independently. If, for instance, the specification is changed for one partial process by adding an additional variable, the hazard rates for all other processes should be re-estimated including with the additional variable included in the specification. If, on the other hand, the coefficients are restricted to zero for some variables — that is, these variables are assumed not to affect the hazard rate of a partial process — some independence assumption is implied.

In addition, the functional form of the hazard rates in the independent model may become rather complex if it is used to represent a marginal survival function with a general dependency structure. If, on the other hand, the functional form of the hazard rates is restricted, for instance, to a simple log-linear specification, the observational equivalence may no longer hold and dependencies between the partial processes may prevail. Thus a model specification with conditionally independent partial risks will in general not solve the problem.

However, the use of competing-risk models with dependent risks also poses severe problems. Because of the non-identifiability property, additional a priori assumptions are required to make a model with dependent processes estimable at all (see, for example, Lancaster 1990, p. 154f.). To date, only a few specifications that allow for dependent risks and that can be handled in a feasible way are available. But even if such a specification is adopted, some ambiguity prevails. Estimates for the dependency structure of the partial processes depend on the identifying assumptions that have been introduced. For different sets of assumptions, different estimates of the dependency structure will be obtained. Thus, it is difficult to discriminate between alternative model specifications. On the other hand, alternative specifications may lead to different conclusions about the impact of exogenous variables on the processes being simulated.

If the properties of competing-risk models are compared with the properties of discrete-time models, a clear advantage of the latter becomes obvious as far as the modelling of stochastic dependencies between partial processes is concerned. It is much simpler to account for such dependencies in a discrete-time approach. However, if the assumption of conditional independence of the partial processes holds, the continuous-time framework offers some advantages. But since one would expect some dependencies to exist in practical applications simply because of imperfections of the model, this property is less important for practical modelling.

4.4 Estimation procedures and data requirements

Basically, both the data requirements and the complexity of the estimation procedures are higher for continuous-time models. The main reason is that continuous-time models use more information than discrete-time models since they are based on the exact timing of events while in discrete-time models states and transitions are considered for only finite periods.

In principle, parameter estimation in the continuous-time context requires information on the exact timing of the events. But in general, empirical data on socioeconomic processes are measured on only a discrete-time basis. This can be taken into account in maximum likelihood estimators by using a likelihood function that is based on the probability implied by the continuous-time model to observe the events during the observed time intervals. However, this adds to the complexity of the estimators, especially if more than one event is observed within the same period.

As an ad hoc solution, additional assumptions about the timing of events within the measurement intervals can be introduced. Standard assumptions are to assign the events to a given point in time in the interval, such as the centre or the end of the interval, or to make some distributional assumptions, such as a uniform distribution of events in the interval. The problem is that the parameter estimates may be quite sensitive to such assumptions if the measurement intervals are long compared with the durations being modelled.

Thus continuous-time models can be estimated from discrete-time data but the estimation procedure may become rather complex or additional simplifying assumptions that may affect the estimates may be required. In comparison, discrete-time models are less demanding in their data requirements and, at least for basic model specifications, standard estimation procedures that can easily be applied are available (see, for example, Hill, Axinn and Thornton 1993). This is a major advantage of the discrete-time approach.

If a multivariate setting is considered with more than a single process being modelled, the differences in the complexity of the estimation procedures become even more pronounced. If multivariate discrete-time models are formulated in the framework of the multivariate normal theory, many standard results and procedures are available that make even rather complex model structures manageable. Efficient estimators for mixed multivariate discrete-time models have been developed in econometrics during the past decade. There are still problems to be solved, but much progress has already been achieved.

For multivariate continuous-time models, the situation is even less favourable if the assumption of conditional independence of the partial processes is not invoked. Firstly, to date no general model specification has been developed for dependent processes. In general, only models with a single process are considered in the theory and only a few rather specific model specifications for two dependent processes have been suggested (see, for example, Lawless 1982). Thus, given the current state of the methods, it will generally be necessary to assume conditional independence of the partial processes for multivariate continuous-time models. In addition, if unobserved heterogeneity is taken into account, rather complex models and estimation procedures result, even if only a single process is being considered.

Basically, longitudinal data are required for dynamic models in both continuous and discrete time. Only if at least two observations are available for the individual units can transitions be modelled. Similarly, for continuous-time models, information on the events during at least some time interval is required. To some extent, aggregated data for groups can also be used to estimate transition rates or hazard rates conditional on the grouping of variables. But if unobserved heterogeneity of the individual units is to be taken into account, longitudinal data on individual micro-units are generally required to identify the mixing distribution of the heterogeneity factor. Thus, in this case, panel data are required for both continuous-time and discrete-time models.

A conceptual problem arises with regard to discrete-time models if the information on transitions is recorded in the data conditional on the state of the unit immediately preceding the transition and not conditional on the state at the beginning of the observation period. This is sometimes true for process-generated data such as data on marriages or fertility. But labour market data may also be of that type. The problem is that, since in a discrete-time model the sequence of events is not defined within a given period, transitions cannot be modelled conditional on the state immediately preceding the transition and, instead, the state at the beginning of the period must be used for conditioning.

Thus, conceptual inconsistencies may exist between the concept of the discrete-time model and the measurement approach used for the empirical observations. This will result in biased estimates, especially if longer periods are considered with more than one transition and, accordingly, with more than one change of the state within a period. If comparatively short periods are considered on the other hand, such a bias will be reduced and it will vanish at the limit if a shift is made to a continuous-time approach. Thus, a continuous-time model is advantageous if empirical observations are available that are conditional on the state at the time of an event. However, the disadvantage of a discrete-time model will generally be minor if comparatively short periods are applied.

5 Conclusions

Since only some theoretical and statistical properties have been discussed in this paper, no general conclusions should be drawn about the relative merits of the continuous-time and the discrete-time approaches to dynamic microsimulation modelling. However, some of the arguments make a strong case in favour of a discrete-time modelling approach.

A first general result is that the conceptual simplicity of the continuous-time competing-risk approach is lost to a large degree if unobserved heterogeneity and other factors that result in dependencies between partial processes are taken into account. This is a common problem in applied microsimulation modelling due to the approximate nature of the models being used in practice. The structure of continuous-time models becomes rather involved in this case, with no general model specification available and feasible specifications requiring rather specific assumptions. Also, parameter estimation becomes even more complicated in this case. Thus, for a continuous-time model to be operational, rather restrictive assumptions are required, such as the conditional independence of partial processes and a specific functional form of duration dependence, like the Weibull specification. However, these assumptions may be questionable from a substantive point of view.

On the other hand, these problems can be dealt with in a multivariate normal discrete-time framework without great additional problems. During the past decade, comparatively general model specifications and corresponding estimation procedures have been developed in econometrics that can be used to specify structural models in a discrete-time framework. The assumption of a multivariate normal error structure may still be questionable from a substantive point of view, but it fades in comparison with the restrictive assumptions that are required for a feasible model in the continuous-time framework.

However, the discrete-time approach also suffers from some deficiencies. Most of them are related to the fact that the timing of events within a given period is not defined for a discrete-time model. Causal ordering is not well defined for a multivariate model and the causal relationships between different partial processes within a period are not obvious. In general, stochastic dependencies between different transitions will occur as a result of factors that have not been included in the model. Thus, the specification of the dependency structure between different processes becomes an important issue in discrete-time models. Another aspect that should not be neglected is that conceptual problems arise if empirical data being used for estimation are based on a different measurement concept using the current state of the unit as conditioning variables.

These problems are less important if comparatively short periods are considered. In this case the probability that more than one event will occur within the period becomes small. This implies that stochastic dependencies between different processes become less important, since a larger part of the dependency can be taken into account by the conditioning variables in a time-recursive way. Also, eventual conceptual discrepancies between the model and the database are less problematic, since the state at the time of an event will coincide with the state at the beginning of the period for most cases if short periods are considered.

Thus from a theoretical and statistical point of view, a discrete-time approach based on comparatively short periods appears to be better suited to dynamic microsimulation modelling than a continuous-time framework, given the problems that must be tackled in practical modelling and the methods that are currently available. Basically, this should not be a surprise, since such dynamic models to some extent represent a compromise between discrete-time and continuous-time modelling. Since, at least in principle, a continuous-time model can be understood as the limiting case of a discrete-time model, discrete-time models with a sufficiently short period should have properties that are not far from those of continuous-time models while maintaining the practical advantages of the discrete-time approach.

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