RAMAIAH UNIVERSITY OF APPLIED SCIENCES

Assignment

Course Code	CSC301A
Course Name	Discrete Mathematics-II
Programme	B.Tech
Department	CSE
Faculty	FET

Name of the Student	Satyajit Ghana
Reg. No.	17ETCS002159
Semester/Year	5/2017
Course Leader(s)	Ms. Pallavi R. Kumar

Declaration Sheet

Student Name	Satyajit Ghana		
Reg. No	17ETCS002159		
Programme	B.Tech	Semester/Year	5/2017
Course Code	CSC301A		
Course Title	Discrete Mathematics-II		
Course Date		To	
Course Leader	Ms. Pallavi R. Kumar		

Declaration

The assignment submitted herewith is a result of my own investigations and that I have conformed to the guidelines against plagiarism as laid out in the Student Handbook. All sections of the text and results, which have been obtained from other sources, are fully referenced. I understand that cheating and plagiarism constitute a breach of University regulations and will be dealt with accordingly.

Signature of the Student		Date	
Submission date stamp (by Examination & Assessment Section)			

Signature of the Course Leader and date	Signature of the Reviewer and date

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1 Question 1

Solution to Question No. 1 Part A

1.1 Specification of constrains for the scheduling problem

Given,

There are 200 students that need to take up Lab Exams for CN, CS and OS. There are 2 Labs available each day for the exam, 103C and 103D, and each lab can take up to 3 slots per day. We need to create a time table for the examination of each student in minimum number of days.

Hard Constraints:

- The constraints being, each student has to take up all the lab exams.
- At a given time slot a student cannot give more than 1 exam simultaneously.

Soft Constraints:

- At a given time slot, only one subject examination can take place, for example at Slot #1 for both the Labs, only CN will be conducted. This is to reduce the number of question papers that the teachers have to prepare. In the answer the number of days of exams will be the number of question papers that the teacher of each subject has to prepare.
- The students shouldn't have to change classrooms at all. Their PC's remain the same.

1.2 Assumptions made, if any, along with justification

Some assumptions that we are going to make are, we are not constrained to conduct only one subject in a lab in a given day, i.e for example Lab 103C, we can conduct CN for slot #1, CS for slot #2, and OS for slot #3. This has been done to simplify the graph and reduce the number of constrained, it would be possible to solve the graph manually now.

Another simplification done is, the batch is assigned to a lab room, all the subject exams for that batch has to be done in that room, this is to reduce the room changes that the student has to do for the exam, the seating arrangement hence can be constant, making it easier for the examination department, for example B1, B3, B5 and B7 will give exam in Lab 103C, so the computers assigned to each student is same throughout the subjects lab exams.

1.3 The graph model used and the colouring algorithm for it

We are to allocate 200 students in batches of 30 each, since the capacity of each lab is 30, hence we will have 7 batches of capacities `[30 30 30 30 30 30 20]` named `[B1, B2, B3, B4, B5, B6, B7]`. B1-B6 have 30 students each, and B7 has 20 students.

Now we have 2 Labs, based on our soft constrains, we'll have to conduct CN, OS and CS exams in each of the labs, we'll call the exams in Lab1 as `[CN1, OS1, CS1]` and in Lab2 as `[CN2, OS2, CS2]`.

The 7 Batches are round-robin divided into the two labs, since our soft constraints says that we have to conduct all the exams for one batch in the same lab. We'll assign `[B1, B3, B5, B7]` to Lab1 and `[B2, B4, B6]` to Lab2.

The following dependency table is generated for the subjects and the batches.

Table 1 Lab1 Dependency Table

Lab1-Subject	Batches
CN1	B1, B3, B5, B7
OS1	B1, B3, B5, B7
CS1	B1, B3, B5, B7

Table 2 Lab2 Dependency Table

Lab2-Subject	Batches
CN2	B2, B4, B6
OS2	B2, B4, B6
CS2	B2, B4, B6

For Table1 Dependency Table we generate the Undirected Graph,

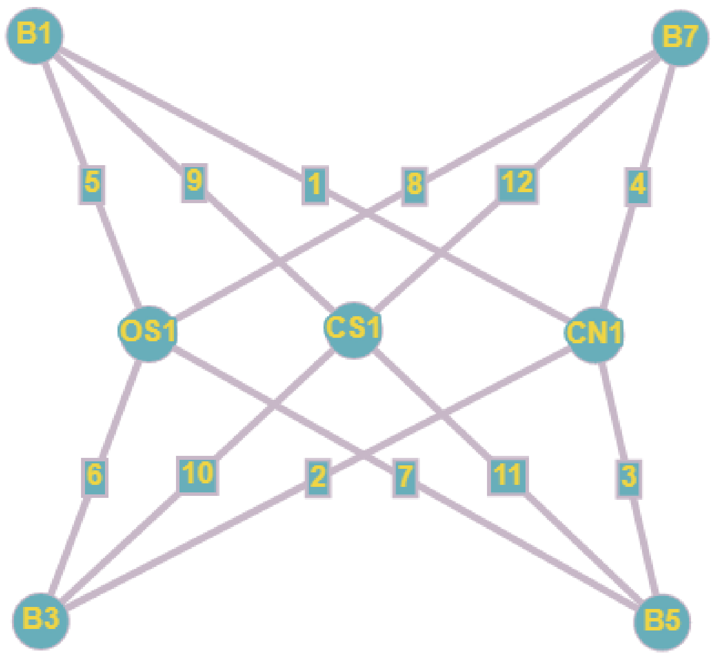


Figure 1-1 Lab1 dependency graph

For the above dependency graph, the allocation can be done by edge colouring, but we'll convert the edge colouring problem into vertex coloring problem, by assigning number to edges and creating an adjacency edges as adjacency vertex graph as below.

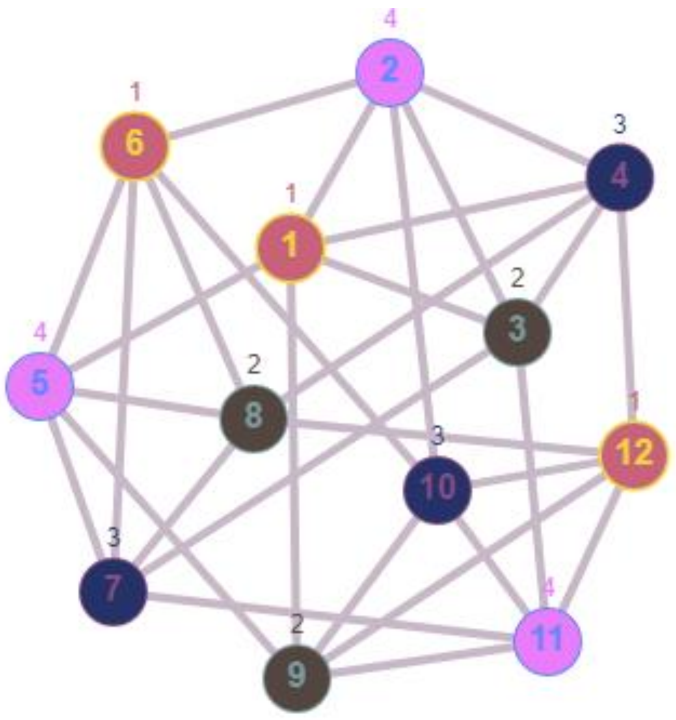


Figure 1-2 Lab1 coloured graph

Similarly, we generate the dependency graph from the table and convert the edge colouring problem into vertex colouring problem.

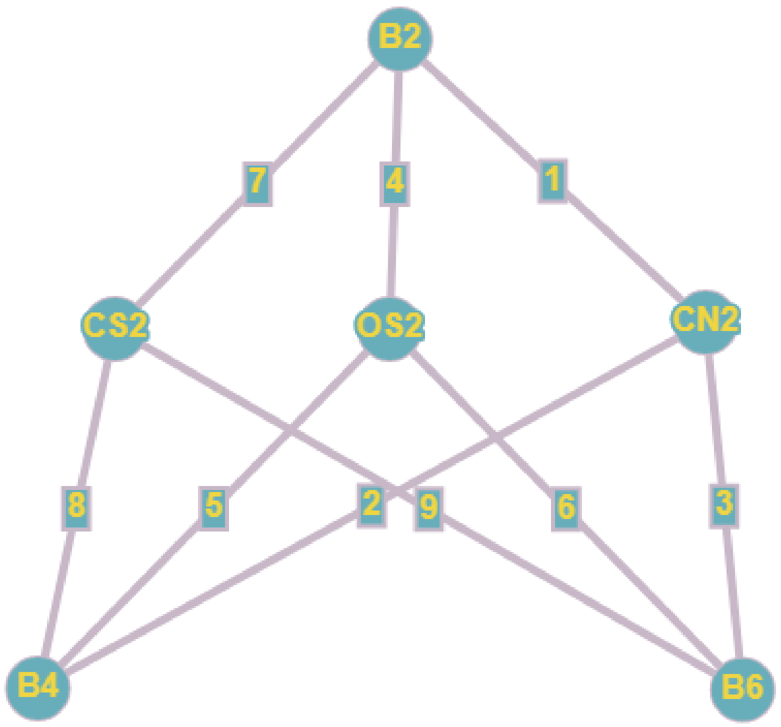


Figure 1-3 Lab2 dependency graph

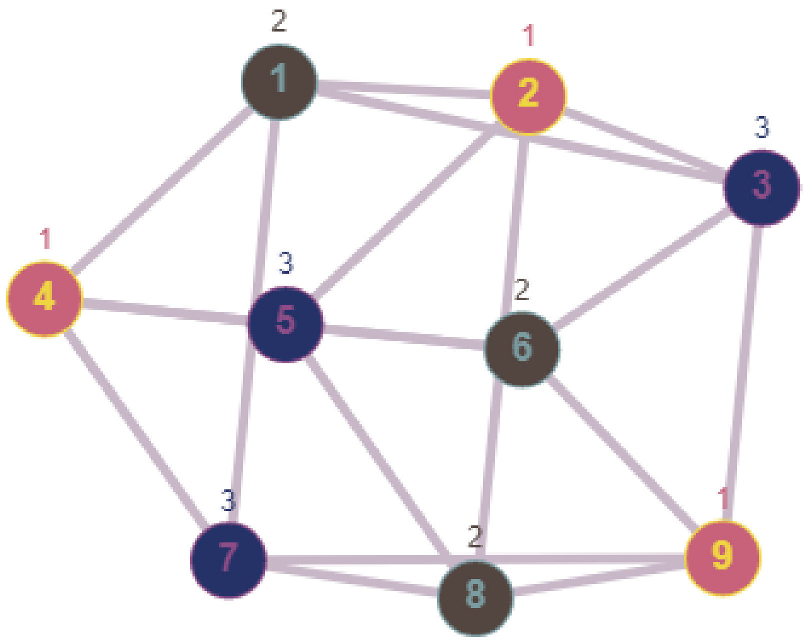


Figure 1-4 Lab2 Coloured Graph

The vertex colouring problem was solved using Backtracking algorithm as described below.

```
If all colors are assigned,
    print vertex assigned colors
Else
    a. Trying all possible colors, assign a color to the vertex
    b. If color assignment is possible, recursivelty assign colors to next
vertices
    c. If color assignment is not possible, de-assign color, return False
```

The source code for the `C++` program and the output for the graph colourings is attached in Appendix A.

1.4 The generation of a timetable based on the colouring carried out

Based on the colours assigned to the edges, we go to that edge number in the dependency graph and see the vertex associated with that edge and hence generate the time table.

Table 3 Allocation for Lab1 - 103C

	Day 1	Day 2	Day 3	Day 4
Slot #1 - OS	B3	B1	B7	B5
Slot #2 - CN	B5	B7	B1	B3
Slot #3 - CS	B7	B5	B3	B1

Table 4 Allocation for Lab2 - 103D

	Day 1	Day 2	Day 3
Slot #1 - OS	B2	B6	B4
Slot #2 - CN	B4	B2	B6
Slot #3 - CS	B6	B4	B2

2 Question 2

Solution to Question 1 Part B

2.1 Specification of constraints for the external examiner assignment problem

The external examiner assignment problem is to minimize the Total Enumeration paid to the examiners. The question is a continuation of the previous scheduling problem so the constraints needs to be formulated accordingly. There are 4 Days, so we need to assign Examiners for all those days and for the two labs.

The remuneration table is as follow:

Teacher	Availability	Renumeration per Day
A	3 Days	Rs. 2000
B	1 Day	Rs. 3400
C	4 Days	Rs. 3000
D	2 Days	Rs. 2500

- Based on the previous solution in Question 1, the Examiners have to be assigned.
- No examiner can be invited for more than three days.
- The examiners are paid per day.
- An examiner cannot examine for two classes simultaneously.
- The total Renumeration has to be minimized.

2.2 Assumptions made, if any, along with justification

Hard Constraints:

From Section 1, we have 2 labs, 4 days of examination in Lab1 and 3 days of examination in Lab2. At a time, an examiner can only examine for one Lab, so we need another examiner for the other lab, so it can be thought of $3 \times 2 + 1$ Slots, or 7 “Examiner Days”, that’s what we will call it. 4 Days of exams in our time table needs 7 Examiner Days, since there are parallel exams going on.

Soft Constraints:

The same examiner can examine for OS, CN and CS, this is to make the work simpler, also this constrain should have be presumed to be true as well.

2.3 Formulation of the Linear Programming Problem (LPP)

Let the number of days Examiner A comes = x_1

Let the number of days Examiner B comes = x_2

Let the number of days Examiner C comes = x_3

Let the number of days Examiner D comes = x_4

From the cost table in 2.1 we need to minimize the total cost, i.e.

$$Z = 2x_1 + 3.4x_2 + 3x_2 + 2.5x_4$$

No Examiner can come more than 3 days, so

$$\begin{gathered} x_1 \leq 3 \\ x_2 \leq 1 \\ x_3 \leq 3 \\ x_4 \leq 2 \end{gathered}$$

From the assumption made in 2.2 we need the examiners for at least 7 days,

$$x_1 + x_2 + x_3 + x_4 \geq 4$$

The LPP Problem can thus be presented as,

MIN $Z = 2x_1 + 3.4x_2 + 3x_2 + 2.5x_4$

x_1	$0x_2$	$0x_3$	$0x_4$	3
$0x_1$	x_2	$0x_3$	$0x_4$	3
$0x_1$	$0x_2$	x_3	$0x_4$	3
$0x_1$	$0x_2$	$0x_3$	x_4	3
$-x_1$	$-x_2$	$-x_3$	$-x_4$	4

2.4 Method used to solve the LPP and the reason to choose the method

Simplex Minimization is used to solve the problem, because it is simple and can be programmed easily. Dual wasn't taken here because the number of slack variables is not reduced by converting, so there wasn't any significant benefit to convert and solve.

$$\text{Min Z} = 2\text{x}_1 + 3.4x_2 + 3x_3 + 2.5x_4$$

The problem is converted to canonical form by adding slack variables, surplus and artificial variables as appropriate.

1. As the constraint-1 is of type $\leq$ we should add slack variable S_1

2. As the constraint-2 is of type $\leq$ we should add slack variable S_2

3. As the constraint-3 is of type $\leq$ we should add slack variable S_3

4. As the constraint-4 is of type $\leq$ we should add slack variable S_4

5. As the constraint-5 is of type $\geq$ we should subtract surplus variable S_5 and add artificial variable A_1

After introducing these, the equations become,

$$\text{Min Z} = 2\text{x}_1 + 3.4x_2 + 3x_3 + 2.5x_4 + 0S_1 + 0S_2 + 0S_3 + 0S_5 + MA_1$$

subject to

$$\begin{aligned}
&x_1 + S_1 = 3\\
&x_2 + S_2 = 1\\
&x_3 + S_3 = 3\\
&x_4 + S_4 = 2\\
&x_1 + x_2 + x_3 + x_4 - S_5 + A_1 = 7\\
&x_1, x_2, x_3, x_4, S_1, S_2, S_3, S_4, S_5, A_1 \geq 0
\end{aligned}$$

Iter 1:		C_j	2	3.4	3	2.5	0	0	0	0	0	M	
B	C_B	X_B	x_1	x_2	x_3	x_4	S_1	S_2	S_3	S_4	S_5	A_1	$MinRatio$ $\frac{X_B}{x_1}$
S_1	0	3	(1)	0	0	0	1	0	0	0	0	0	$\frac{3}{1} = 3$
S_2	0	1	0	1	0	0	0	1	0	0	0	0	--
S_3	0	3	0	0	1	0	0	0	1	0	0	0	--
S_4	0	2	0	0	0	1	0	0	0	1	0	0	--

A_1	M	7	1	1	1	1	0	0	0	0	-1	1	$\frac{7}{1} = 7$
$Z = 7M$		Z_j	M	M	M	M	0	0	0	0	-M	M	
		$Z_j - C_j$	M-2	M-3.4	M-3	M-2.5	0	0	0	0	-M	0	

Positive Maximum $Z_j - C_j$ is $M - 2$ and its column index is 1. So, the entering variable is x_1.

Minimum ration is 3 and its row index is 1. So the leaving basis variable is S_1

Pivot element is 1

Now, $R_5 \rightarrow R_5 - R_1$

Iter 2:		C_j	2	3.4	3	2.5	0	0	0	0	0	M	
B	C_B	X_B	x_1	x_2	x_3	x_4	S_1	S_2	S_3	S_4	S_5	A_1	$MinRatio$ $\frac{X_B}{x_4}$
x_1	2	3	1	0	0	0	1	0	0	0	0	0	--
S_2	0	1	0	1	0	0	0	1	0	0	0	0	--
S_3	0	3	0	0	1	0	0	0	1	0	0	0	--
S_4	0	2	0	0	0	(1)	0	0	0	1	0	0	$\frac{2}{1} = 2$
A_1	M	4	0	1	1	1	-1	0	0	0	-1	1	$\frac{4}{1} = 4$
$Z = 4M + 6$		Z_j	2	M	M	M	-M+2	0	0	0	-M	M	
		$Z_j - C_j$	0	M-3.4	M-3	M-2.5	-M+2	0	0	0	-M	0	

Positive Maximum $Z_j - C_j$ is $M - 2.5$ and its column index is 4. So, the entering variable is x_4.

Minimum ration is 2 and its row index is 4. So, the leaving basis variable is S_4

The pivot element is 1

Now, $R_5 \rightarrow R_5 - R_4$

Iter 3:		C_j	2	3.4	3	2.5	0	0	0	0	0	M	
B	C_B	X_B	x_1	x_2	x_3	x_4	S_1	S_2	S_3	S_4	S_5	A_1	$MinRatio$ $\frac{X_B}{x_3}$
x_1	2	3	1	0	0	0	1	0	0	0	0	0	--
S_2	0	1	0	1	0	0	0	1	0	0	0	0	--
S_3	0	3	0	0	1	0	0	0	1	0	0	0	$\frac{3}{1} = 3$
x_4	2.5	2	0	0	0	1	0	0	0	1	0	0	--
A_1	M	2	0	1	(1)	0	-1	0	0	-1	-1	1	$\frac{2}{1} = 2$
$Z = 2M + 11$		Z_j	2	M	M	2.5	-M+2	0	0	-M+2.5	-M	M	
		$Z_j - C_j$	0	M-3.4	M-3	0	-M+2	0	0	-M+2.5	-M	0	

Positive Maximum $Z_j - C_j$ is $M - 3$ and its column index is 3. So, the entering variable is x_3.

Minimum ration is 2 and its row index is 5. So, the leaving basis variable is A_1

The pivot element is 1

Now, $R_3 \rightarrow R_3 - R_5$

Iter 4:		C_j	2	3.4	3	2.5	0	0	0	0	0	
B	C_B	X_B	x_1	x_2	x_3	x_4	S_1	S_2	S_3	S_4	S_5	$MinRatio$
x_1	2	3	1	0	0	0	1	0	0	0	0	
S_2	0	1	0	1	0	0	0	1	0	0	0	
S_3	0	1	0	-1	0	0	1	0	1	1	1	
x_4	2.5	2	0	0	0	1	0	0	0	1	0	
x_3	3	2	0	1	1	0	-1	0	0	-1	-1	
$Z = 17$		Z_j	2	3	3	2.5	-1	0	0	-0.5	-3	
		$Z_j - C_j$	0	-0.4	0	0	-1	0	0	-0.5	-3	

Since all $Z_j - C_j \leq 0$

Hence the optimal solution is arrived with the value of variables as:

$$\mathrm{x}_1 = 3 \text{ Days}, \mathrm{x}_2 = 0 \text{ Days}, \mathrm{x}_3 = 2 \text{ Days}, \mathrm{x}_4 = 2 \text{ Days}$$

$$\text{MIN Z} = \text{Rs. } 17\text{K}$$

The Teacher allocation will be as follows

	Day 1	Day 2	Day 3	Day 4
Slot #1 - OS	A	A	A	D
Slot #2 - CN	A	A	A	D
Slot #3 - CS	A	A	A	D

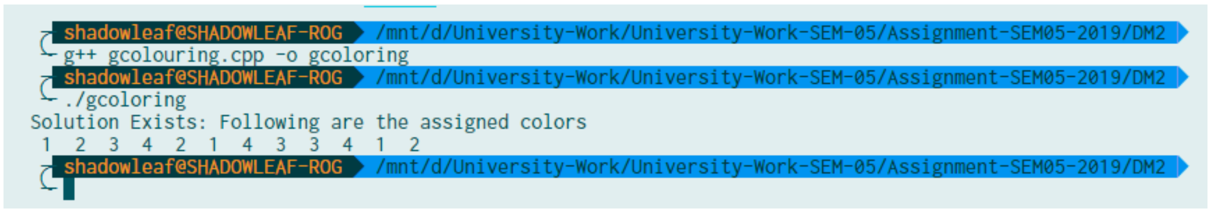
	Day 1	Day 2	Day 3
Slot #1 - OS	C	C	D
Slot #2 - CN	C	C	D
Slot #3 - CS	C	C	D

The total cost being Rs. 17,000

Appendix A

```c
#include <stdbool.h>
#include <stdio.h>

// Number of vertices in the graph
#define V 12

void print_colors(int color[]);

bool is_safe(int v, bool graph[V][V], int color[], int c) {
    for (int i = 0; i < V; i++)
        if (graph[v][i] && c == color[i])
            return false;
    return true;
}

bool graph_coloring_recurse(bool graph[V][V], int m, int color[], int v) {
    if (v == V)
        return true;

    /* Consider this vertex v and try different colors */
    for (int c = 1; c <= m; c++) {
        /* Check if assignment of color c to v is fine*/
        if (is_safe(v, graph, color, c)) {
            color[v] = c;

            /* recur to assign colors to rest of the vertices */
            if (graph_coloring_recurse(graph, m, color, v + 1) == true)
                return true;

            /* If assigning color c doesn't lead to a solution
            then remove it */
            color[v] = 0;
        }
    }

    /* If no color can be assigned to this vertex then return false */
    return false;
}

bool graph_coloring(bool graph[V][V], int m) {
    // Initialize all color values as 0. This initialization is needed
    // correct functioning of is_safe()
    int color[V];
    for (int i = 0; i < V; i++)
        color[i] = 0;
    // Call graph_coloring_recurse() for vertex 0
    if (graph_coloring_recurse(graph, m, color, 0) == false) {
        printf("Solution does not exist");
        return false;
    }

    // Print the solution
    print_colors(color);
    return true;
}

void print_colors(int color[]) {
    printf(
        "Solution Exists:"
        " Following are the assigned colors \n");
    for (int i = 0; i < V; i++)
        printf(" %d ", color[i]);
    printf("\n");
}

// driver program to test above function
int main() {
    bool graph[V][V] = {0, 1, 1, 1, 1, 0, 0, 0, 1, 0, 0, 0,
                        1, 0, 1, 1, 0, 1, 0, 0, 0, 1, 0, 0,
                        1, 1, 0, 1, 0, 0, 1, 0, 0, 0, 1, 0,
                        1, 1, 1, 0, 0, 0, 0, 1, 0, 0, 0, 1,
                        1, 0, 0, 0, 0, 1, 1, 1, 1, 0, 0, 0,
                        0, 1, 0, 0, 1, 0, 1, 1, 0, 1, 0, 0,
                        0, 0, 1, 0, 1, 1, 0, 1, 0, 0, 1, 0,
                        0, 0, 0, 1, 1, 1, 1, 0, 0, 0, 0, 1,
                        1, 0, 0, 0, 1, 0, 0, 0, 0, 1, 1, 1,
                        0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 1, 1,
                        0, 0, 1, 0, 0, 0, 1, 0, 1, 1, 0, 1,
                        0, 0, 0, 1, 0, 0, 0, 1, 1, 1, 1, 0};
    int m = 4;  // Number of colors
    graph_coloring(graph, m);
    return 0;
}
```

OUTPUT for Graph 1



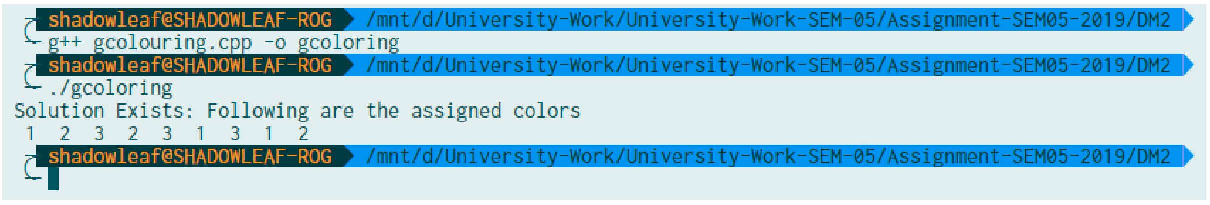
```
shadowleaf@SHADOWLEAF-ROG  /mnt/d/University-Work/University-Work-SEM-05/Assignment-SEM05-2019/DM2
g++ gcolouring.cpp -o gcoloring
shadowleaf@SHADOWLEAF-ROG  /mnt/d/University-Work/University-Work-SEM-05/Assignment-SEM05-2019/DM2
./gcoloring
Solution Exists: Following are the assigned colors
 1  2  3  4  2  1  4  3  3  4  1  2
shadowleaf@SHADOWLEAF-ROG  /mnt/d/University-Work/University-Work-SEM-05/Assignment-SEM05-2019/DM2
```

As we can see 4 colours were used

Source Code changes for Graph 2

```
bool graph[V][V] = {0, 1, 1, 1, 0, 0, 1, 0, 0,
                    1, 0, 1, 0, 1, 0, 0, 1, 0,
                    1, 1, 0, 0, 0, 1, 0, 0, 1,
                    1, 0, 0, 0, 1, 1, 1, 0, 0,
                    0, 1, 0, 1, 0, 1, 0, 1, 0,
                    0, 0, 1, 1, 1, 0, 0, 0, 1,
                    1, 0, 0, 1, 0, 0, 0, 1, 1,
                    0, 1, 0, 0, 1, 0, 1, 0, 1,
                    0, 0, 1, 0, 0, 1, 1, 1, 0,};
```

Output for Graph 2



```
shadowleaf@SHADOWLEAF-ROG  /mnt/d/University-Work/University-Work-SEM-05/Assignment-SEM05-2019/DM2
g++ gcolouring.cpp -o gcoloring
shadowleaf@SHADOWLEAF-ROG  /mnt/d/University-Work/University-Work-SEM-05/Assignment-SEM05-2019/DM2
./gcoloring
Solution Exists: Following are the assigned colors
 1  2  3  2  3  1  3  1  2
shadowleaf@SHADOWLEAF-ROG  /mnt/d/University-Work/University-Work-SEM-05/Assignment-SEM05-2019/DM2
```

As we can see only 3 colours were used, which are the same as it was shown in the graph figure.

Bibliography

1. https://www.geeksforgeeks.org/m-coloring-problem-backtracking-5/
2. http://stephanie-w.github.io/brainscribble/m-coloring-problem.html