

# Digital Image Fundamentals



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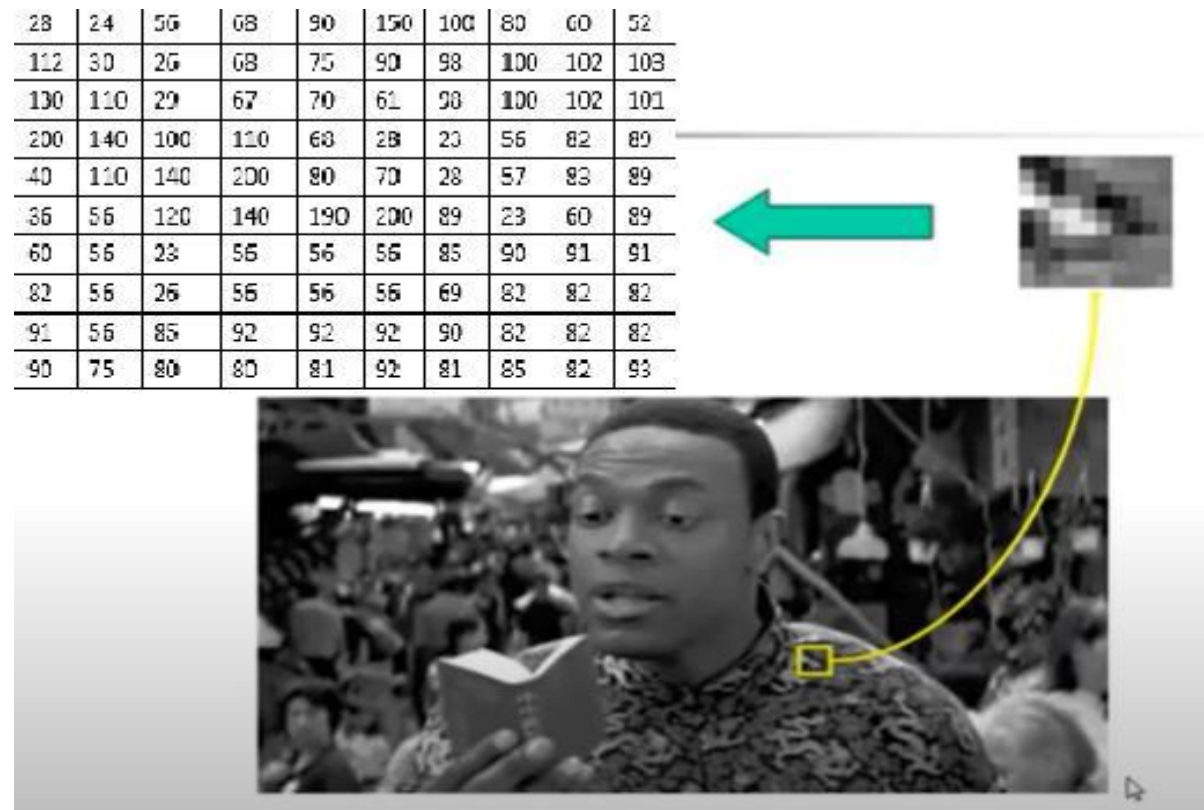
# Digital Image Fundamentals

- At the end of these lectures, students will be able to:
  - Understand Image sensing and Acquisition.
  - Distinguish between various Image Acquisition techniques.
  - Describe Image Formation Model.
  - Distinguish between Spatial and Gray-Level Resolution.
  - Describe a basic relationships between the pixels.



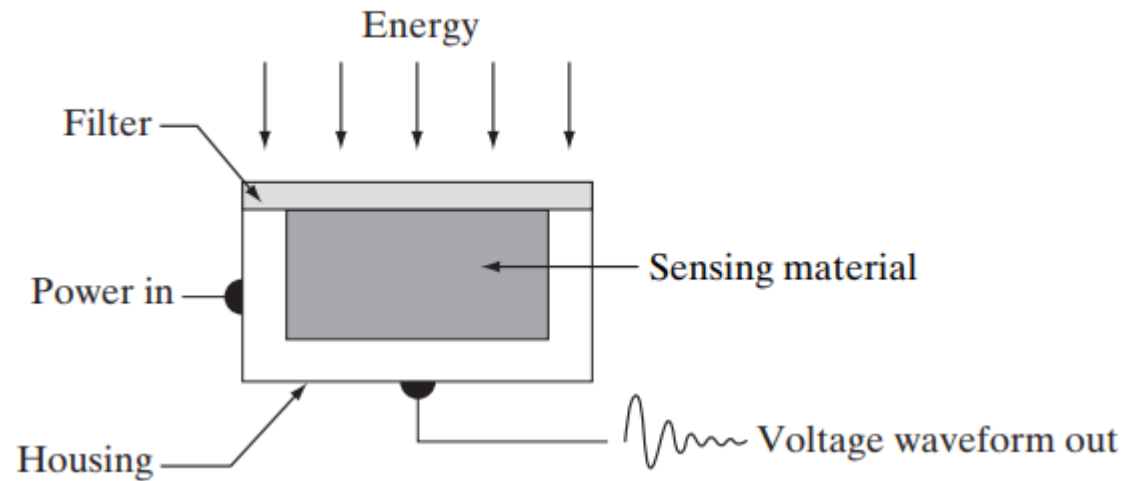
# Digital Image

- A digital image is an image composed of picture elements, also known as *pixels*.
- Each pixel has a finite, discrete value which represents the intensity or gray level at the position defined by spatial co-ordinates (x and y).

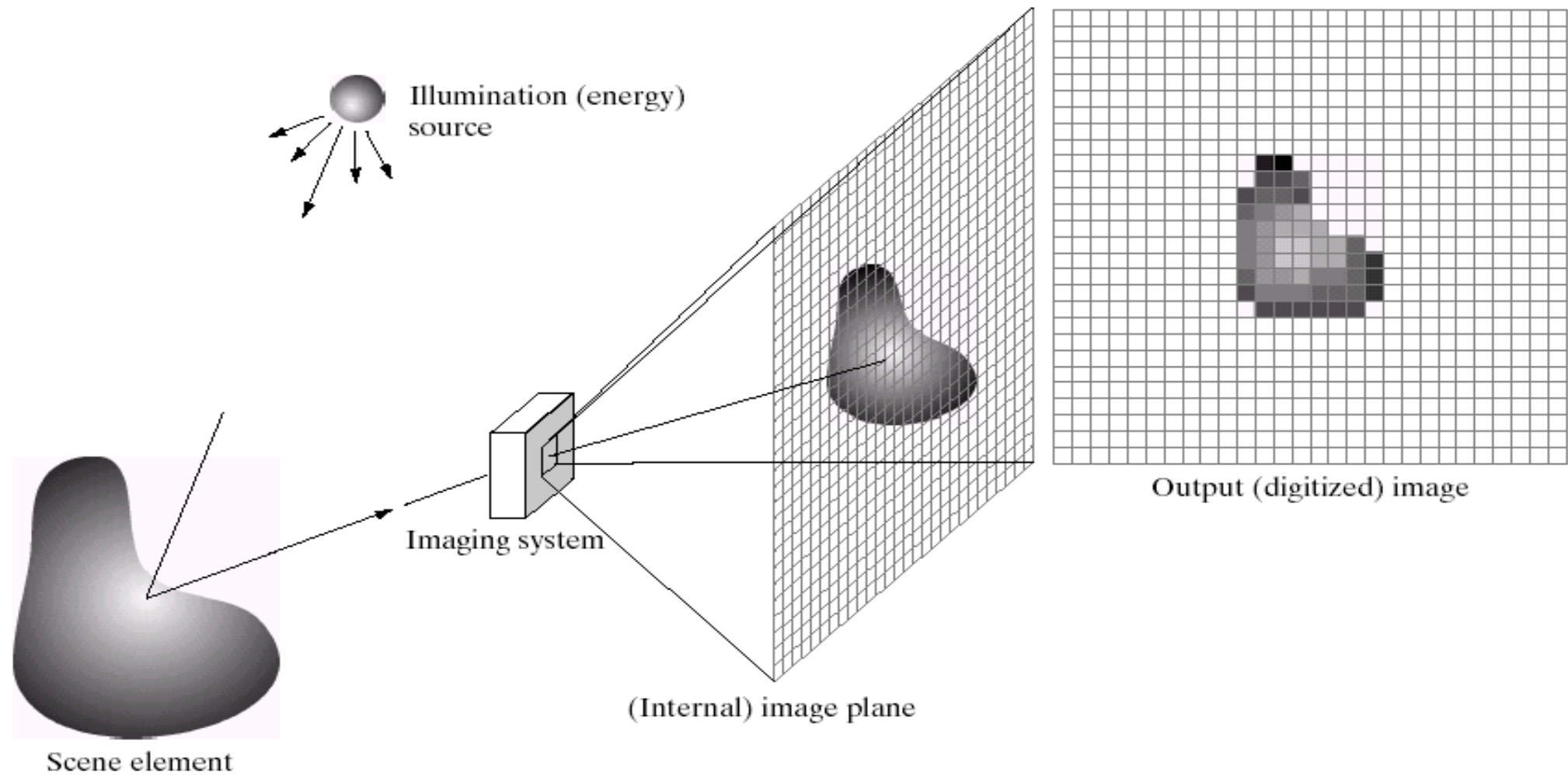


# Image Sensing and Digital Image Acquisition

- Single imaging sensor



# Image Sensing & Acquisition



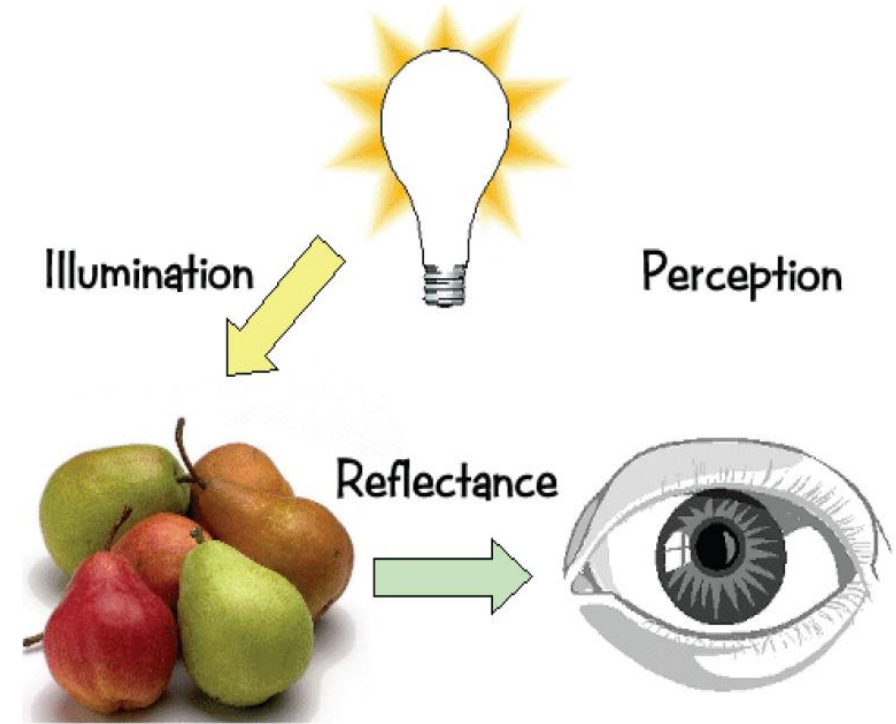
a b c d e

**FIGURE 2.15** An example of the digital image acquisition process. (a) Energy (“illumination”) source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.



# Simple Image Formation Model

- Simple image formation
  - $f(x,y) = i(x,y)r(x,y)$
  - $i(x,y)$ : illumination (determined by ill. Source)
    - $0 < i(x,y) < \infty$
    - $i(x,y) = 90,000 \text{ lm/m}^2$  (clear day),  $0.1 \text{ lm/m}^2$  (evening)
    - $i(x,y) = 10,000 \text{ lm/m}^2$  (cloudy day),
  - $r(x,y)$  reflectance (determined by imaged object)
    - $0 < r(x,y) < 1$
    - $0.01$  for black velvet
    - $0.65$  for stainless steel
- In real situation
  - $L_{min} \leq L = f(x,y) \leq L_{max}$ 
    - $L_{min} = i_{min} * r_{min}$
    - $L_{max} = i_{max} * r_{max}$
    - $L$ : gray level

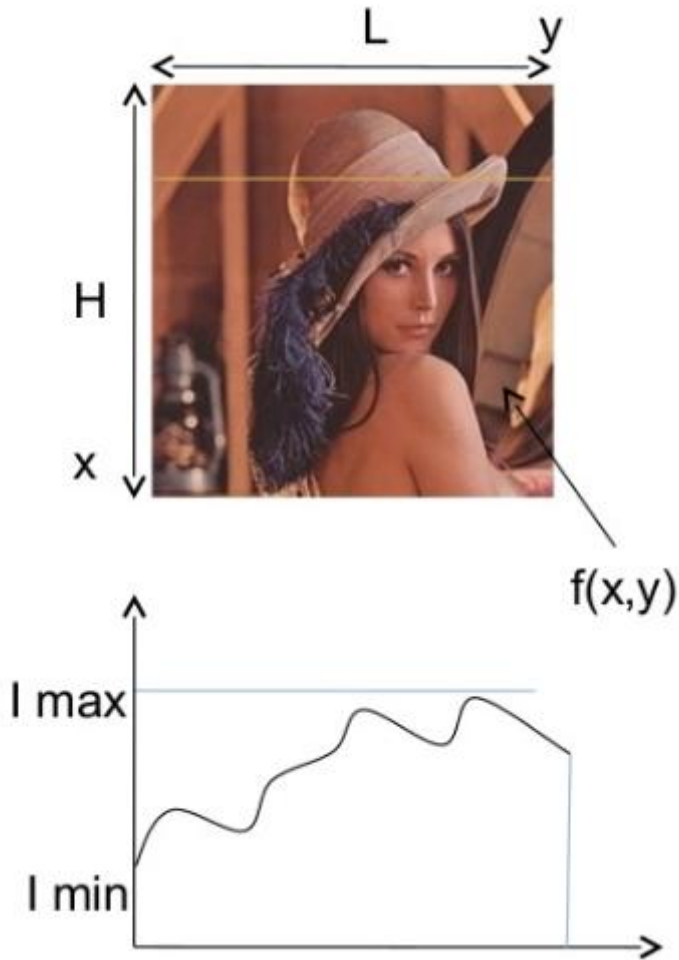


# Image Digitization

- Why do we need Digitization?
- What is Image Digitization ?
- How to Digitize an Image



# Why Digitization



No of pixel along x axis  $0 \leq x \leq H$

No of pixel along y axis  $0 \leq y \leq L$

Intensity at  $f(x,y)$   $I_{\min} \leq f(x,y) \leq I_{\max}$

$I_{\min} \rightarrow$  minimum intensity value

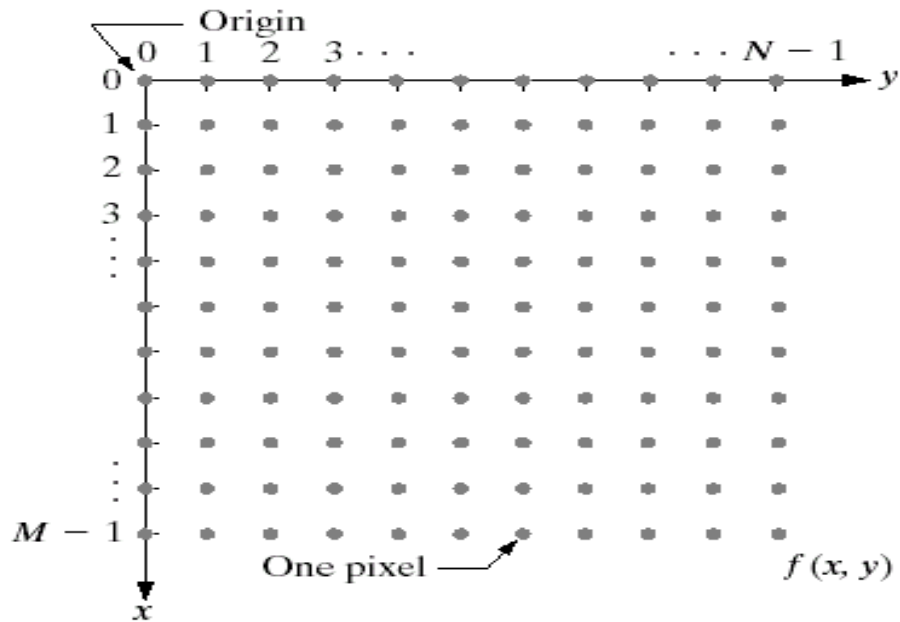
$I_{\max} \rightarrow$  maximum intensity value





# Digitization

- Digitization means Sampling & Quantization
- An image can be represented by a 2D matrix which has finite no. of values in rows and columns.



$$f(x, y) \cong \begin{bmatrix} f(0,0) & f(0,1) & \dots & f(0,N-1) \\ f(1,0) & f(1,1) & \dots & f(1,N-1) \\ \vdots & \vdots & \ddots & \vdots \\ f(M-1,0) & f(M-1,1) & \dots & f(M-1,N-1) \end{bmatrix}$$

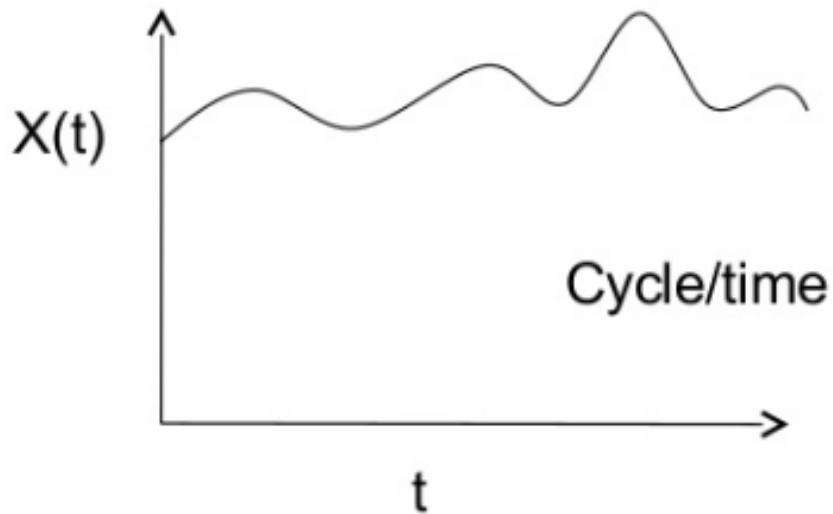


# Digitization

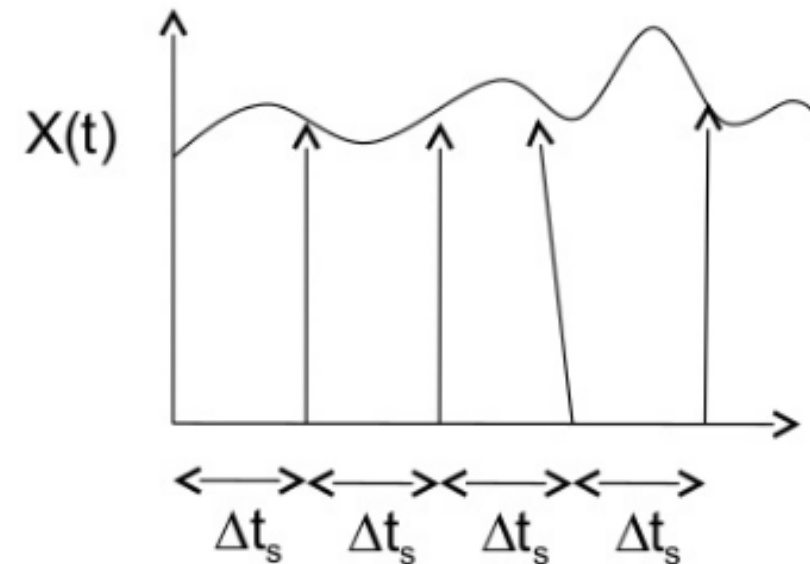


# Sampling

- Image representation by a 2D matrix
- A 1D analog signal can be represented as



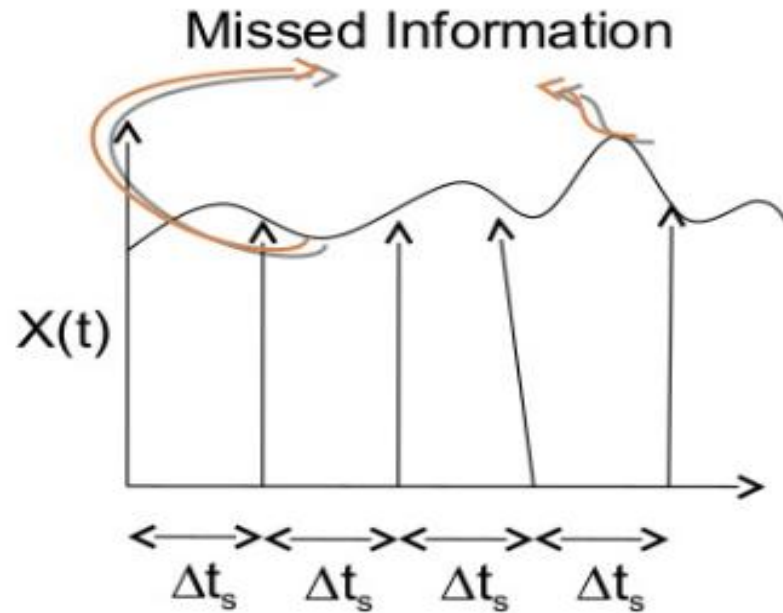
Rather by taking value at every point  
we are taking value at some interval  
 $\Delta$



Sampling frequency  $f_s = 1 / \Delta t_s$

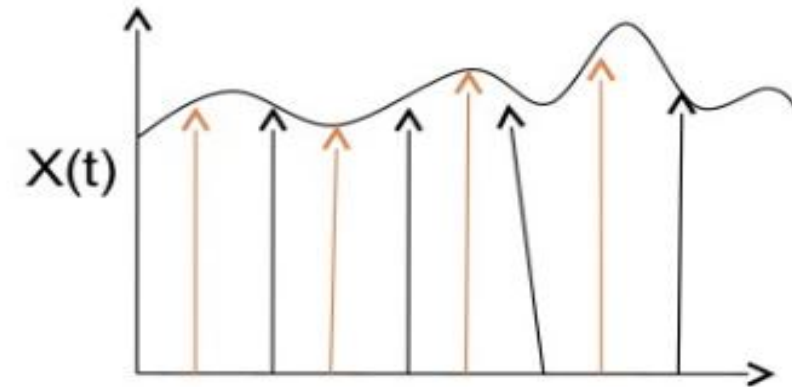


# Sampling



**Problem** is some information can be missed when we taking value at interval  $\Delta t$

**Solution** is increasing the sampling frequency or decreasing the sampling interval  $\Delta t$

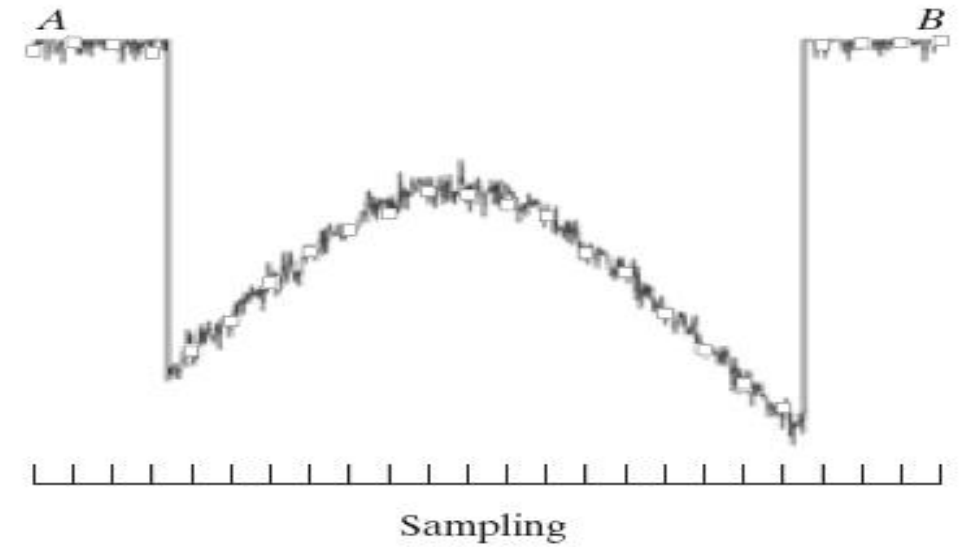
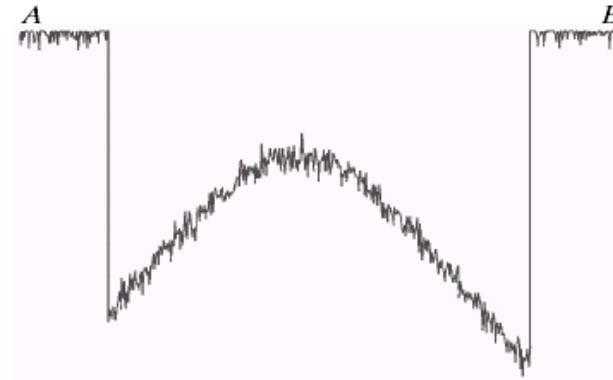
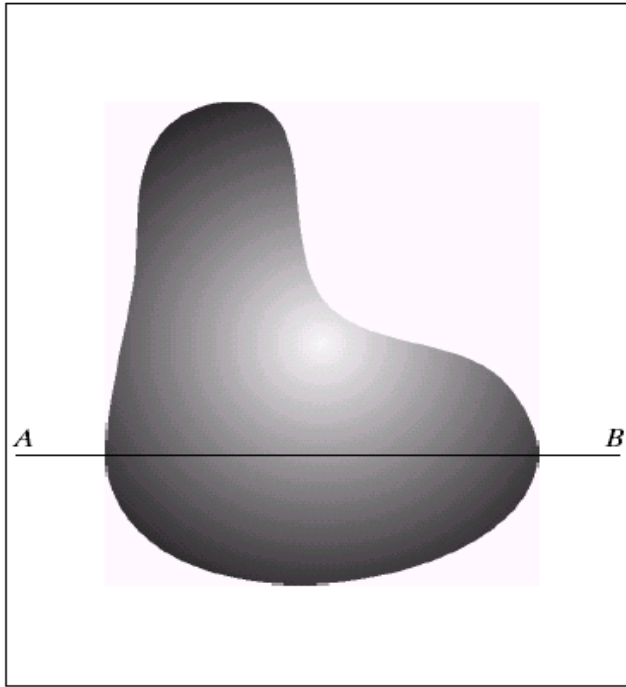


Here we take interval  $\Delta t'_s = \Delta t / 2$

Sampling frequency  
 $f'_s = 1 / \Delta t'_s = 2 / \Delta t = 2f_s$



# Sampling



# Quantization

- Digitizing the Amplitude values called image quantization.
- Sampling limits established by no. of sensors, but quantization limits by color levels.

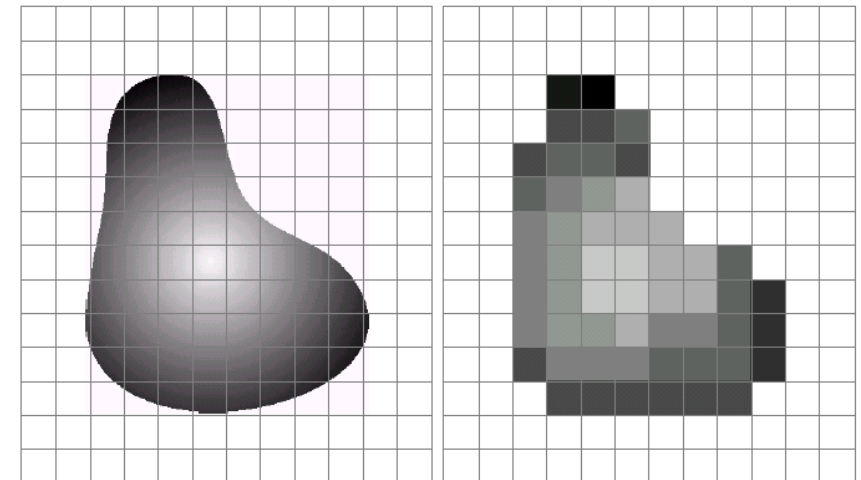
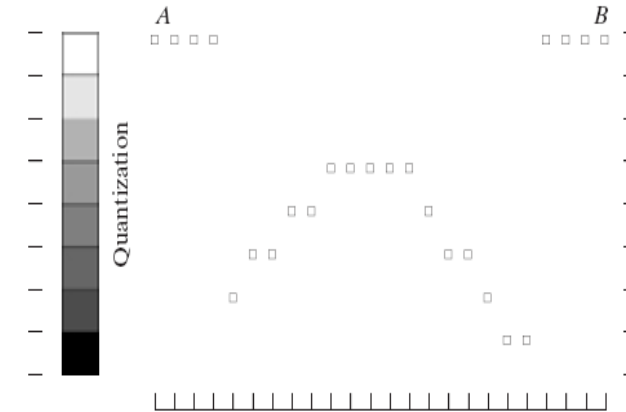
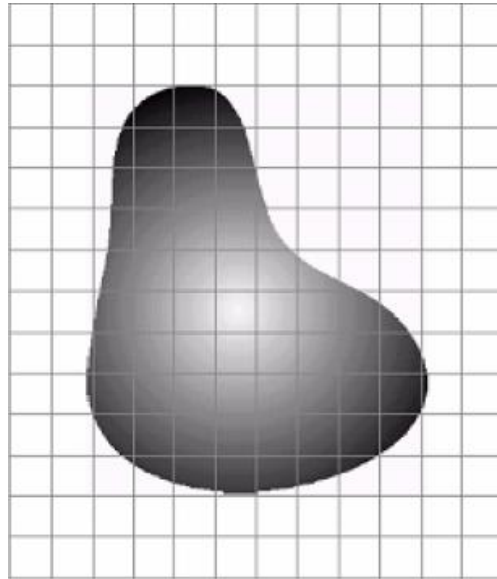


Image Quantization



# Example



99	71	61	51	49	40	35	53	86	99
93	74	53	56	48	46	48	72	85	102
101	69	57	53	54	52	64	82	88	101
107	82	64	63	59	60	81	90	93	100
114	93	76	69	72	85	94	99	95	99
117	108	94	92	97	101	100	108	105	99
116	114	109	106	105	108	108	102	107	110
115	113	109	114	111	111	113	108	111	115
110	113	111	109	106	108	110	115	120	122
103	107	106	108	109	114	120	124	124	132

**CAMERA**



**DIGITIZER**



A set of number  
in 2D grid

Samples the analog data and digitizes it.

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# Image Sampling & quantization

**Consider an image which has :**

$M * N$  : size of the image

$L$  : Number of discrete gray levels in this image

$$L = 2^k \quad \text{Where } k \text{ is any positive integer}$$

The total number of bits needed to store this image  $b$  :

$$b = M * N * K,$$

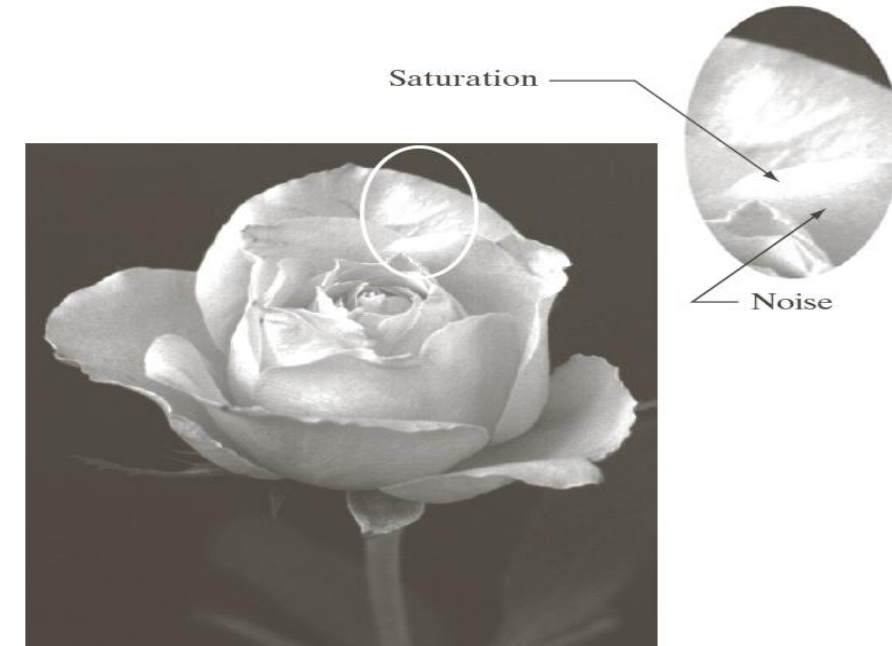
$$\text{If } M = N, \text{ then } b = N^2 * K$$





# Image Sampling & quantization

- The dynamic range is the ratio of the maximum (determined by saturation) measurable intensity to the minimum (limited by noise) detectable intensity.
- Contrast is defined as the difference in intensity between the highest and the lowest intensity levels in an image.

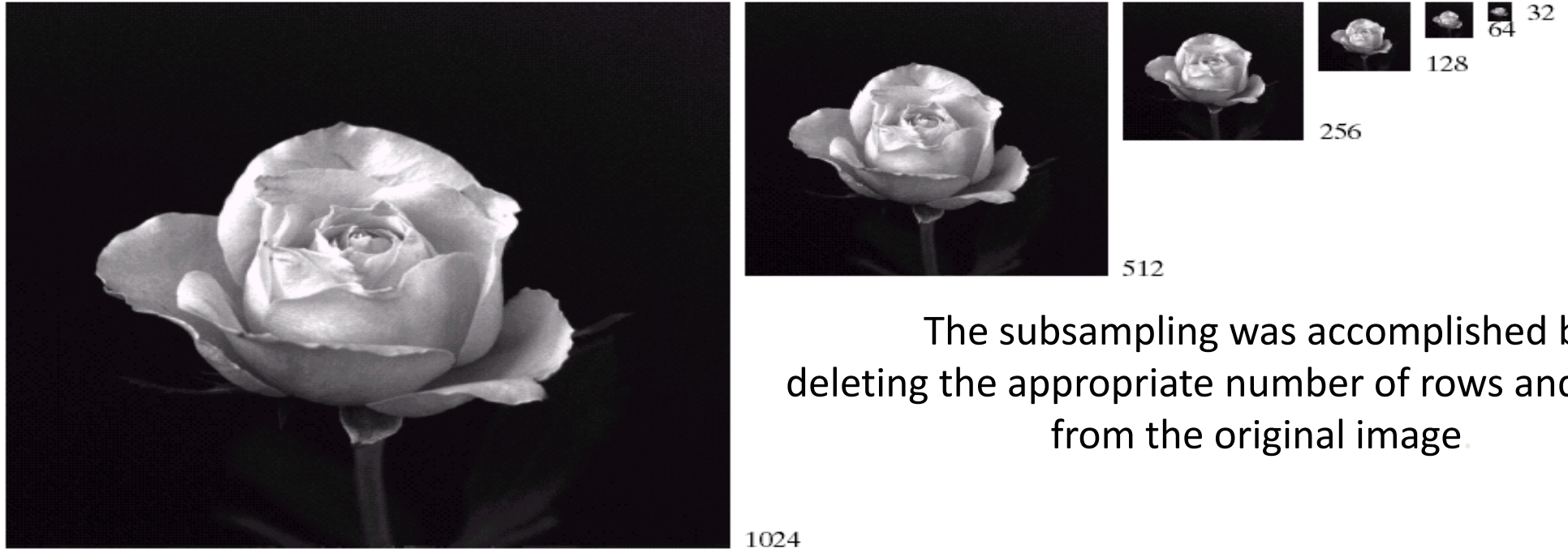


# Image Sampling & quantization

- **Spatial resolution:**
  - # of samples per unit length or area.
  - Lines and distance: Line pairs per unit distance
  - Dpi- dots per inch
- **Gray level resolution:**
  - Number of bits per pixel.
  - Usually 8 bits.
  - Color image has 3 image planes to yield  $8 \times 3 = 24$  bits/pixel.



# Image Sampling & quantization



The subsampling was accomplished by deleting the appropriate number of rows and columns from the original image.

Spatial Image Resolutions

No. of gray levels ( $K$ ) is constant(8-bits images).

No. of samples ( $N$ ) is reduced (No. of sensors)



# Image Sampling & quantization



Comparison between all image sizes



# Image Sampling & quantization

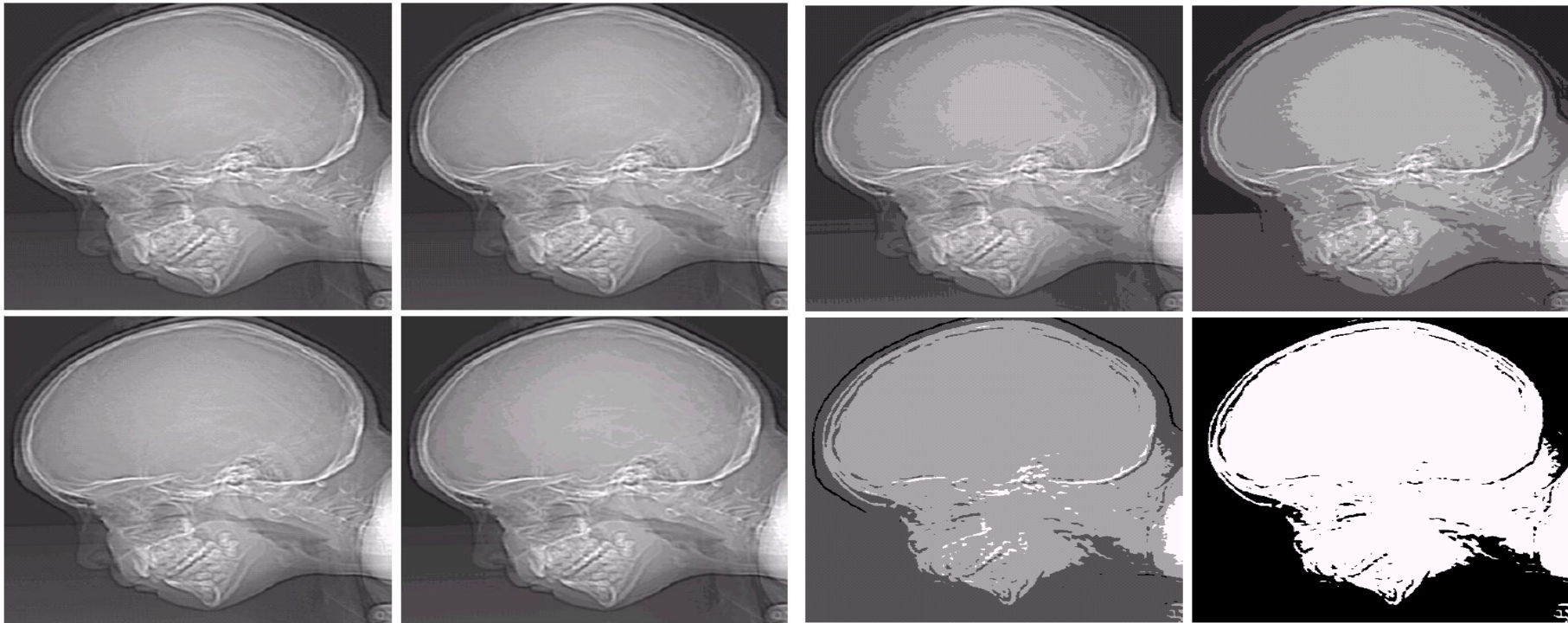


a b  
c d

**FIGURE 2.20** Typical effects of reducing spatial resolution. Images shown at: (a) 1250 dpi, (b) 300 dpi, (c) 150 dpi, and (d) 72 dpi. The thin black borders were added for clarity. They are not part of the data.



# Image Sampling & quantization



Gray Level Image Resolutions

No. of samples (N) is constant, but gray levels (K) decreases.



# Sampling and Quantization

- How many samples to take?
  - Number of pixels (samples) in the image
  - Nyquist rate
- How many gray-levels to store?
  - At a pixel position (sample), number of levels of color/intensity to be represented

# Sampling and Quantization

- How many samples to take?
  - The Nyquist Rate
  - Samples must be taken at a rate that is twice the frequency of the highest frequency component to be reconstructed.
  - Under-sampling: sampling at a rate that is too coarse, i.e., is below the Nyquist rate.
  - Aliasing: artefacts that result from under-sampling.



- The pixel values of the following 5x5 image are represented by 8-bit integers:

$$f = \begin{bmatrix} 123 & 162 & 200 & 147 & 93 \\ 137 & 157 & 165 & 232 & 189 \\ 151 & 155 & 152 & 141 & 130 \\ 205 & 101 & 100 & 193 & 115 \\ 250 & 50 & 75 & 88 & 100 \end{bmatrix}$$

- Determine (f) with gray level resolution of  $2^k$ , when
  - $K = 5$
  - $K = 3$



- Dividing the image by 2 will reduce its gray level resolution by one bit.
- Hence to reduce the gray level resolution from 8-bit to 5-bit, we have to reduce 3-bits.
- 8bits – 5bits = 3 bits will be reduced
- Thus, we divide the 8-bit image by 8 ( $2^3$ ) to get the following 5-bit image:

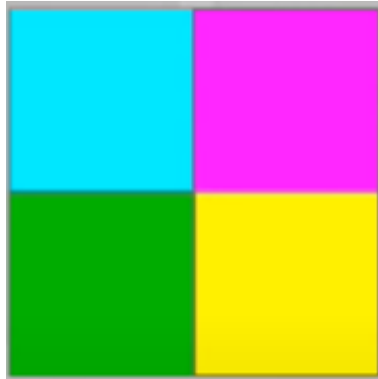
$$f = \begin{bmatrix} 15 & 20 & 25 & 18 & 11 \\ 17 & 19 & 20 & 29 & 23 \\ 18 & 19 & 19 & 17 & 16 \\ 25 & 12 & 12 & 24 & 14 \\ 31 & 6 & 9 & 10 & 12 \end{bmatrix}$$



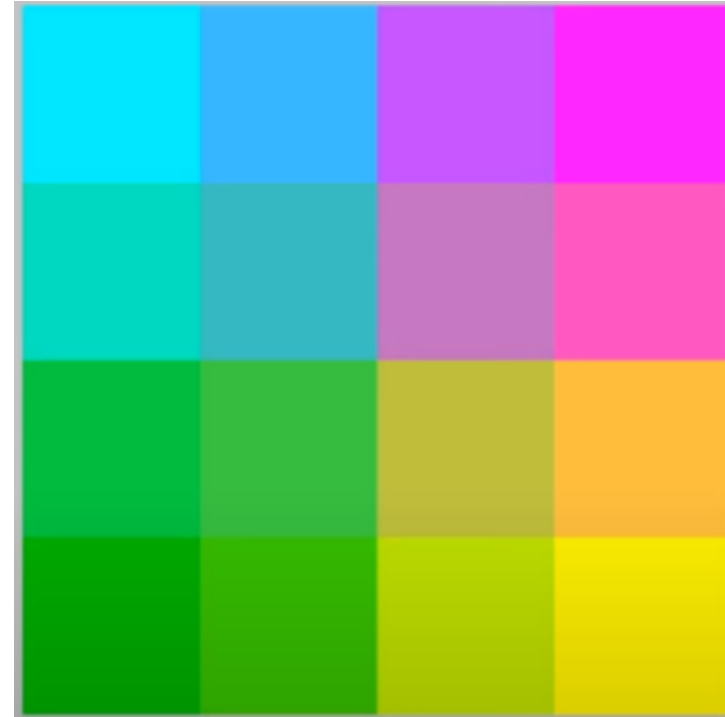
- Similarly, to obtain 3-bit image, we divide the 8-bit image by  $(32) 2^5$

$$f = \begin{bmatrix} 3 & 5 & 6 & 4 & 2 \\ 4 & 4 & 5 & 7 & 5 \\ 4 & 4 & 4 & 4 & 4 \\ 6 & 3 & 3 & 6 & 3 \\ 7 & 1 & 2 & 2 & 3 \end{bmatrix}$$





**Original Image**



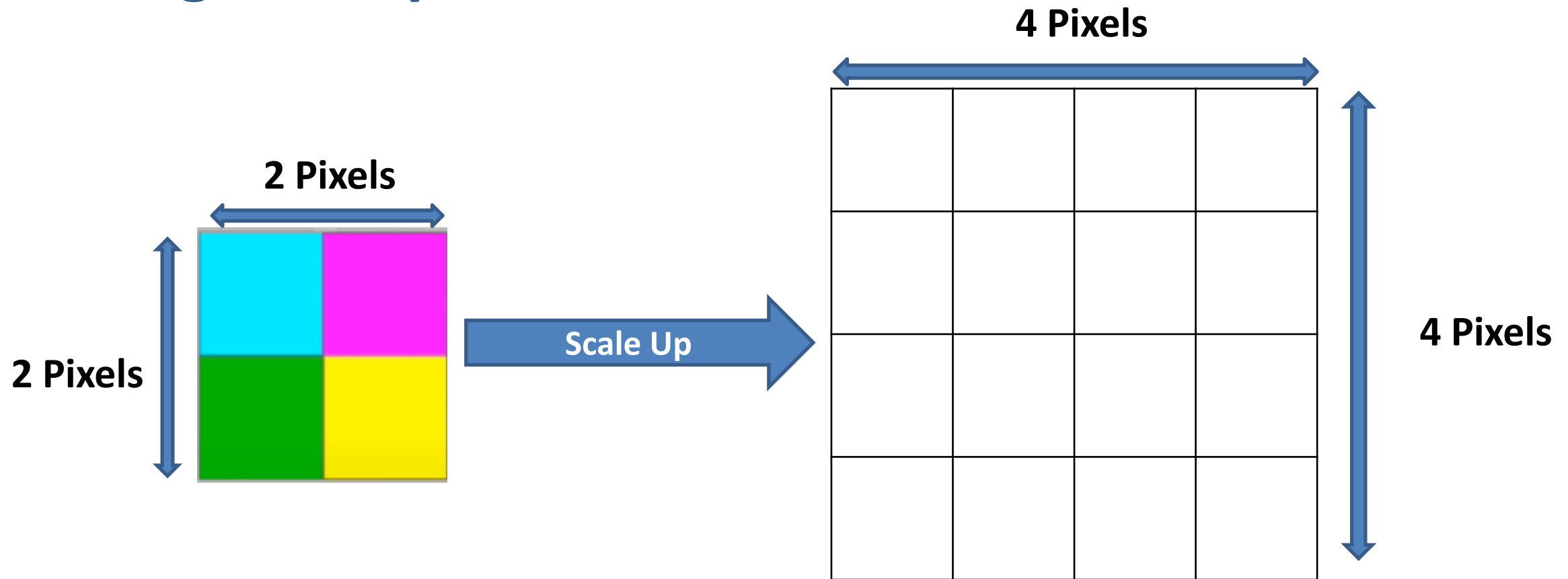
**New Image**



# Image Interpolation



# Image Interpolation

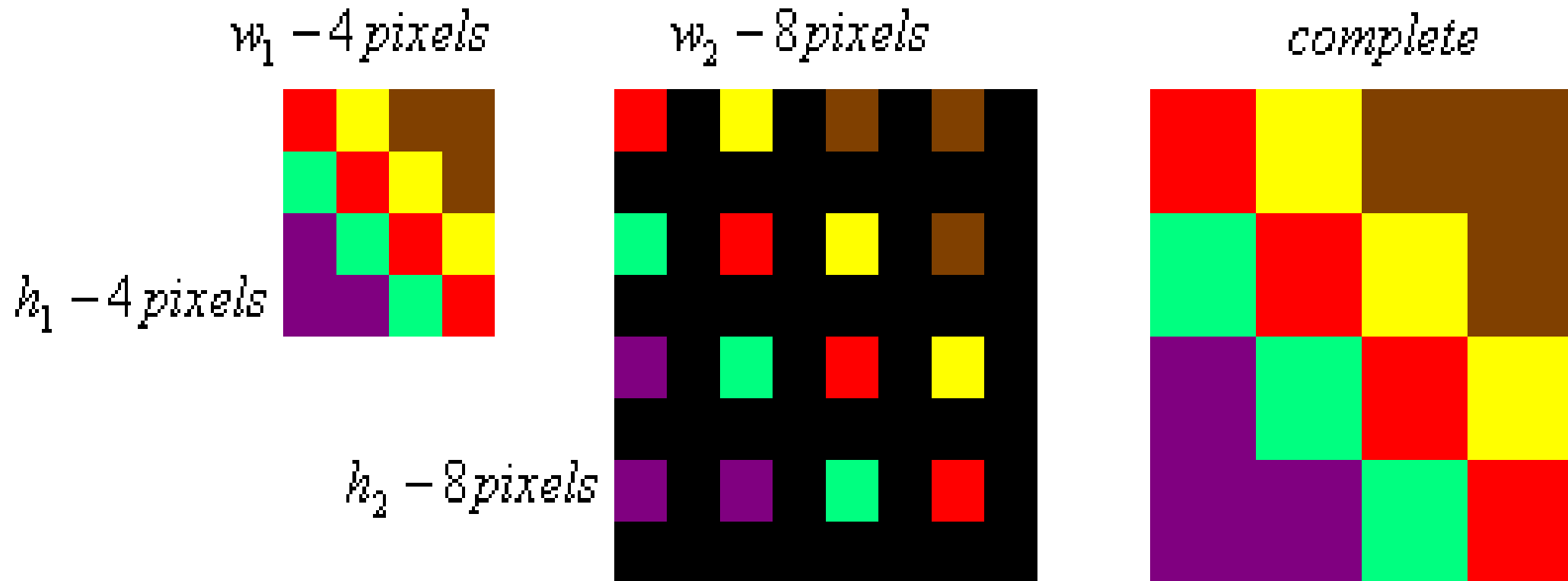


# Image Interpolation

- Process of using known data to estimate unknown values
  - *e.g.*, zooming, shrinking, rotating, and geometric correction
- **Interpolation** (sometimes called *resampling*) — an imaging method to increase (or decrease) the number of pixels in a digital image.



# Image Interpolation





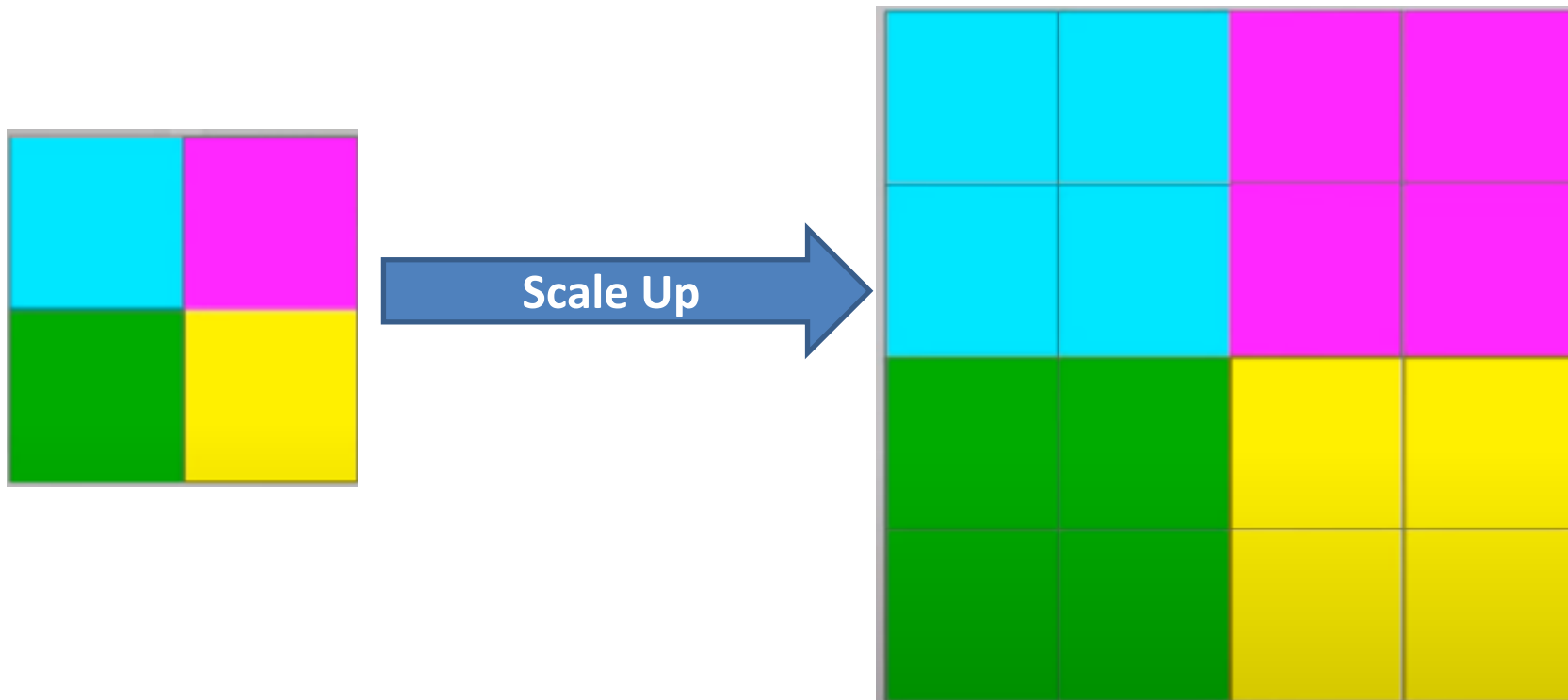
# Image Interpolation

- Three types
  - Nearest Neighbor Interpolation
  - Bi-linear Interpolation
  - Bi-cubic Interpolation



# Nearest Neighbor Interpolation

- Is performed by repeating pixel values



# Nearest Neighbor Interpolation

- The example below shows 8-bit image zooming by 2x (2 times) using nearest neighbor interpolation.

$$\begin{bmatrix} 69 & 50 & 80 \\ 45 & 60 & 66 \\ 30 & 55 & 80 \end{bmatrix} = \begin{bmatrix} 69 & 69 & 50 & 50 & 80 & 80 \\ 45 & 45 & 60 & 60 & 66 & 66 \\ 30 & 30 & 55 & 55 & 80 & 80 \end{bmatrix} = \begin{bmatrix} 69 & 69 & 50 & 50 & 80 & 80 \\ 69 & 69 & 50 & 50 & 80 & 80 \\ 45 & 45 & 60 & 60 & 66 & 66 \\ 45 & 45 & 60 & 60 & 66 & 66 \\ 30 & 30 & 55 & 55 & 80 & 80 \\ 30 & 30 & 55 & 55 & 80 & 80 \end{bmatrix}$$

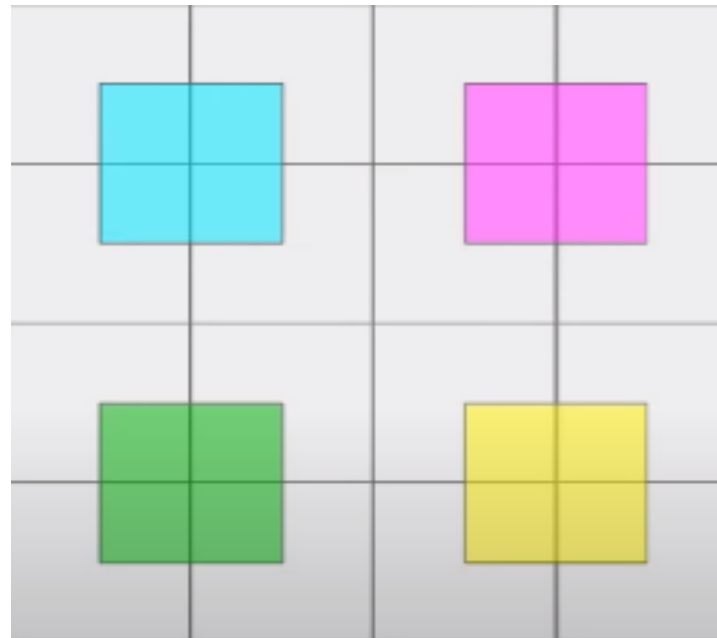
Original image

image with rows expanded

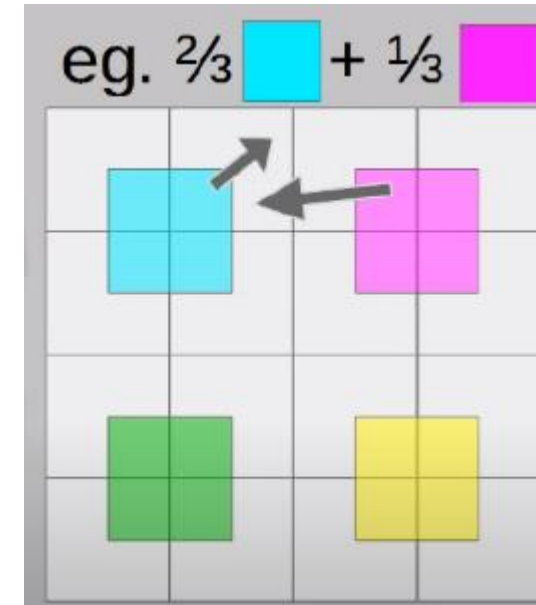
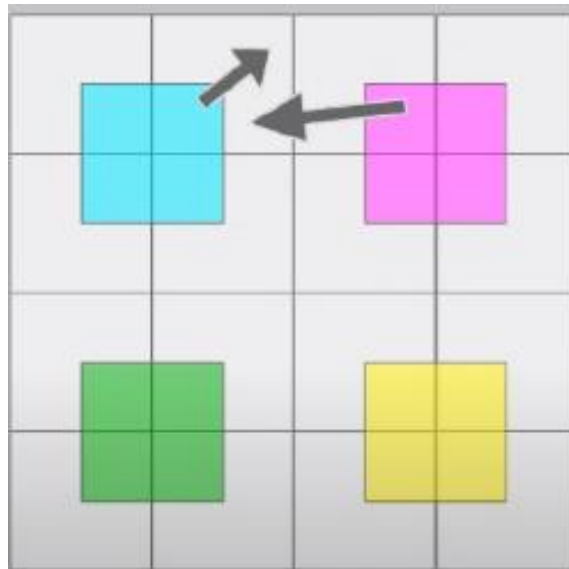
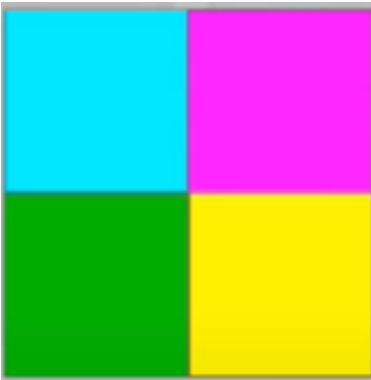
image with rows and  
columns expanded



- How to make color appear smoother

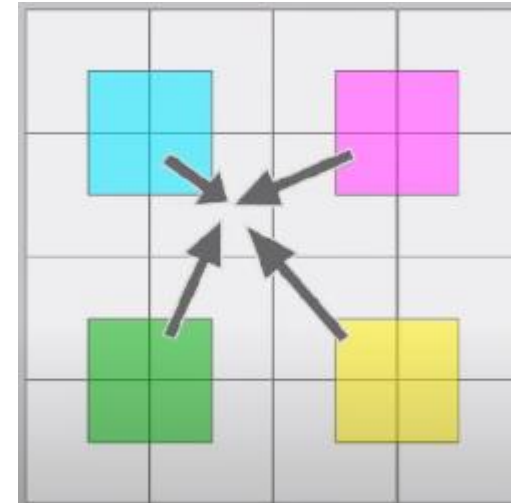


- How to make color appear smoother



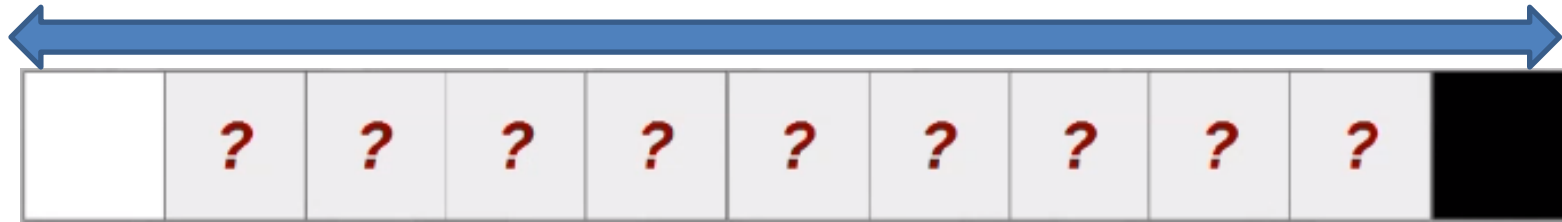
# Bi-linear Interpolation

- Each output takes the weighted average of 4 input pixels



# Linear Interpolation

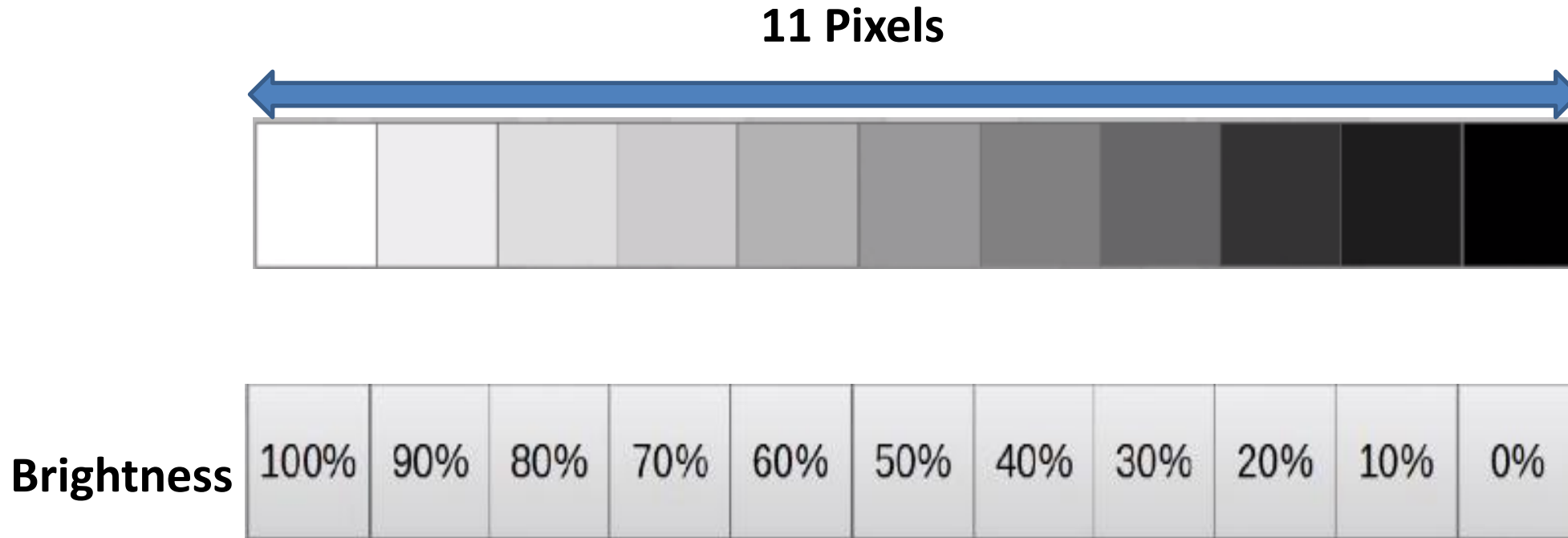
11 Pixels



Brightness



# Linear Interpolation



**Linear: Change between each successive square is same**



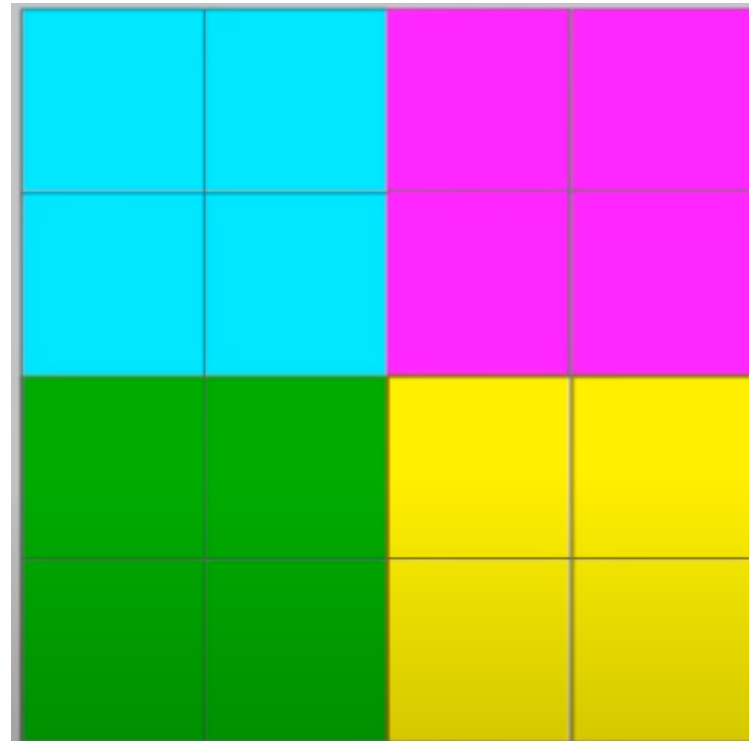
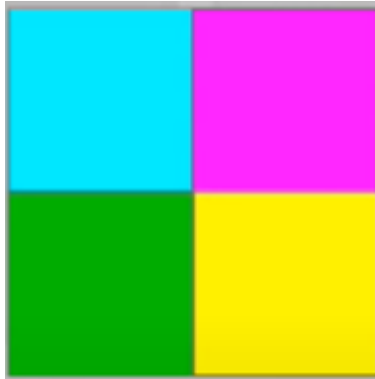


# Bi-linear Interpolation

- Linear Interpolation: An interpolation in which rate of change is constant
- Bi-linear Interpolation: Perform linear interpolation twice
  - Once in each direction



# Bi-linear Interpolation



# Bi-linear Interpolation

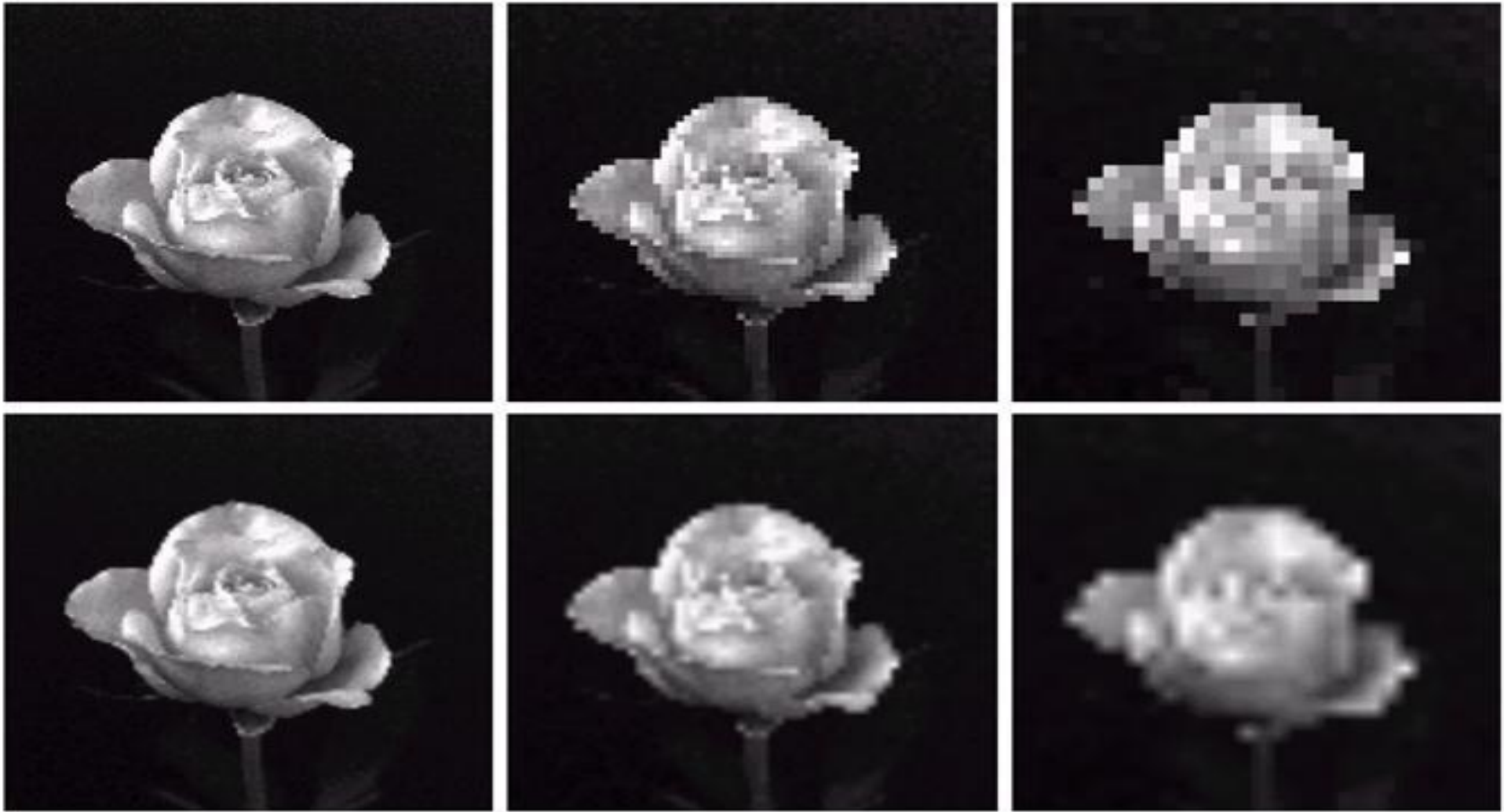
- 8-bit image zooming by 2 (2 times) using bi-linear interpolation

$$\begin{bmatrix} 69 & 50 & 80 \\ 45 & 60 & 66 \\ 30 & 55 & 80 \end{bmatrix} = \begin{bmatrix} 69 & 59 & 50 & 65 & 80 \\ 45 & 52 & 60 & 63 & 66 \\ 30 & 42 & 55 & 67 & 80 \end{bmatrix} = \begin{bmatrix} 69 & 59 & 50 & 65 & 80 \\ 57 & 55 & 55 & 64 & 73 \\ 45 & 52 & 60 & 63 & 66 \\ 37 & 47 & 57 & 65 & 73 \\ 30 & 42 & 55 & 67 & 80 \end{bmatrix}$$

Original image    image with rows expanded

image with rows and  
columns expanded





a	b	c
d	e	f

 Top row: images zoomed from  $128 \times 128$ ,  $64 \times 64$ , and  $32 \times 32$  pixels to  $1024 \times 1024$  pixels, using nearest neighbor gray-level interpolation. Bottom row: same sequence, but using bilinear interpolation.

# Bi-cubic interpolation

- Bi-cubic interpolation (16 coefficients)

$$v(x, y) = \sum_{i=0}^3 \sum_{j=0}^3 a_{ij} x^i y^j$$

- Better job of preserving finer details
- Standard in commercial image editing programs



# Shrinking

- Shrinking may be viewed as under sampling
- Performed by row deletion

$$\begin{bmatrix} 69 & 69 & 50 & 50 & 80 & 80 \\ 69 & 69 & 50 & 50 & 80 & 80 \\ 45 & 45 & 60 & 60 & 66 & 66 \\ 45 & 45 & 60 & 60 & 66 & 66 \\ 30 & 30 & 55 & 55 & 80 & 80 \\ 30 & 30 & 55 & 55 & 80 & 80 \end{bmatrix} = \begin{bmatrix} 69 & 69 & 50 & 50 & 80 & 80 \\ 45 & 45 & 60 & 60 & 66 & 66 \\ 30 & 30 & 55 & 55 & 80 & 80 \end{bmatrix} = \begin{bmatrix} 69 & 50 & 80 \\ 45 & 60 & 66 \\ 30 & 55 & 80 \end{bmatrix}$$

Original image

image with rows deleted

image with rows  
and columns deleted

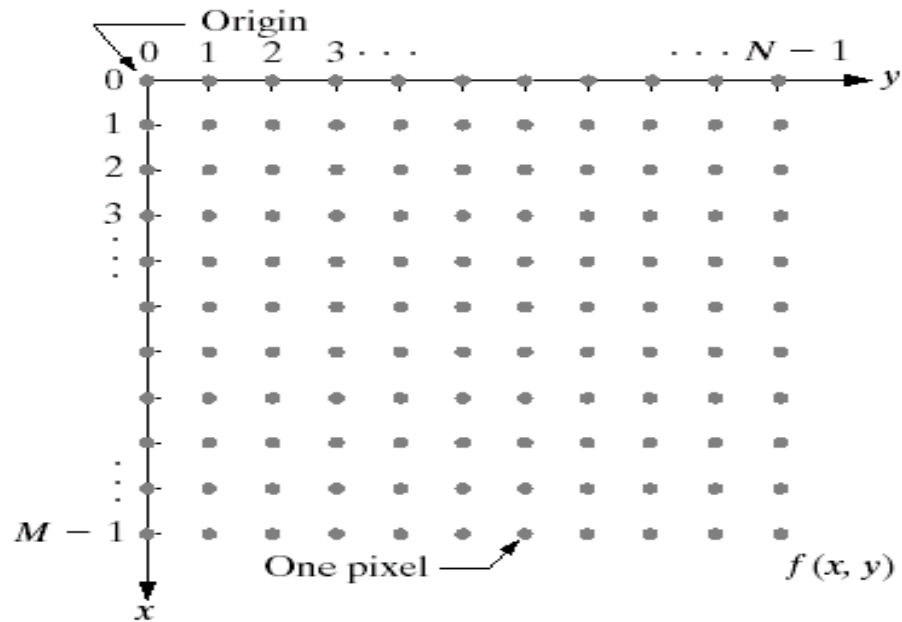


# Pixel Relationships



# Image Representation

- An image can be represented by a 2D matrix which has finite no. of values in rows and columns.



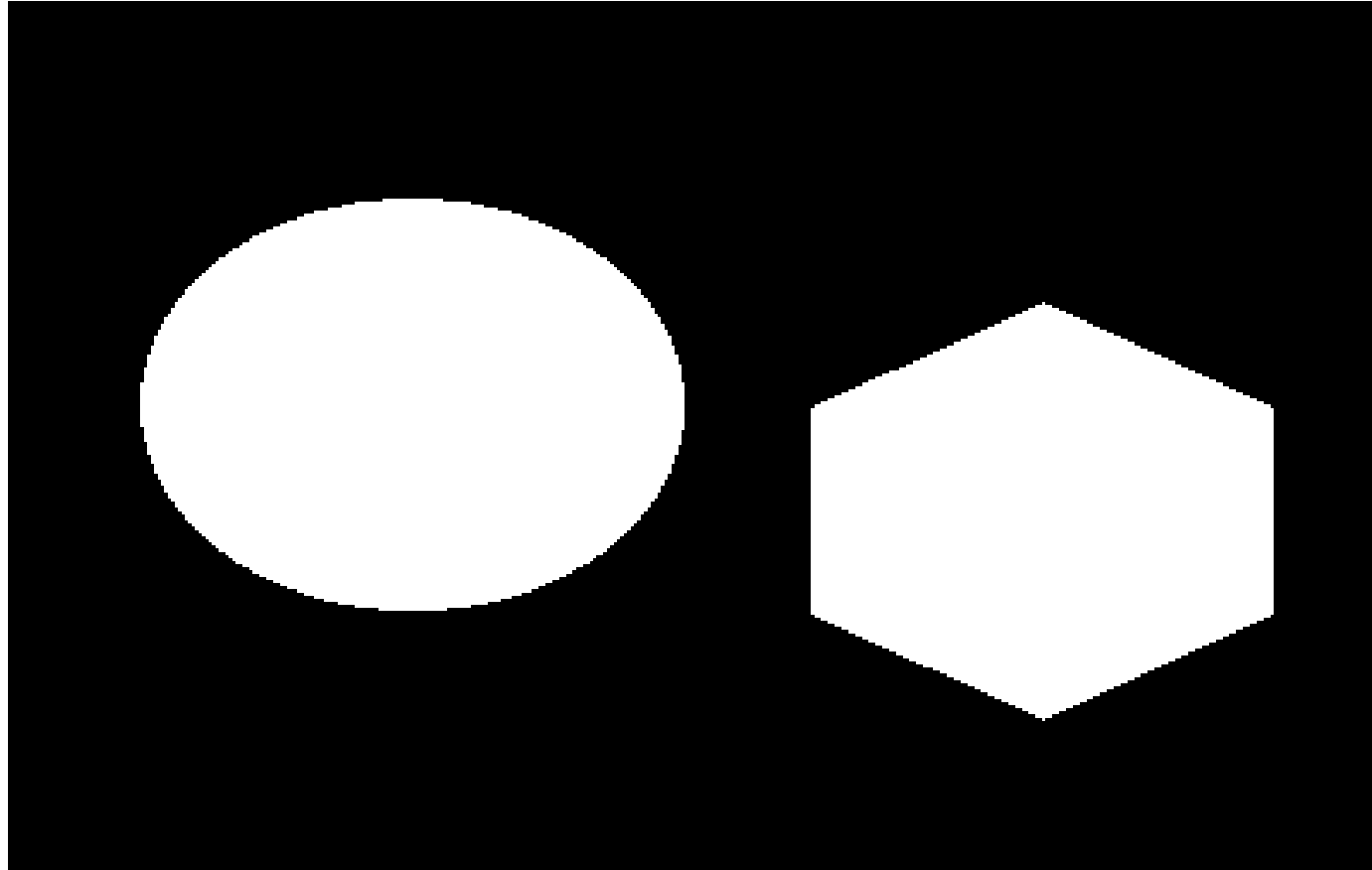
$$f(x, y) \cong \begin{bmatrix} f(0,0) & f(0,1) & \dots & f(0,N-1) \\ f(1,0) & f(1,1) & \dots & f(1,N-1) \\ \vdots & \vdots & \ddots & \vdots \\ f(M-1,0) & f(M-1,1) & \dots & f(M-1,N-1) \end{bmatrix}$$





112	110	109	0	0	0	0	0	0
116	114	108	0	0	0	0	0	0
107	115	110	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	200	215	214
0	0	0	0	0	0	201	212	204
0	0	0	0	0	0	208	210	207





# Relationship Between Pixels

- Neighborhood
- Adjacency
- Paths
- Connectivity
- Regions and boundaries

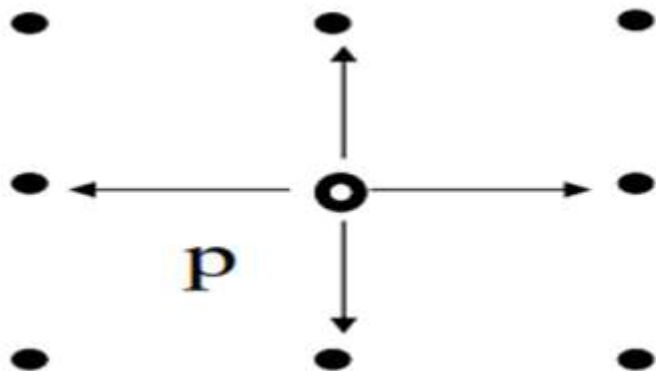


$$f(x,y) = \begin{bmatrix} f(0,0) & f(0,1) & f(0,2) & f(0,3) & f(0,4) & \dots \\ f(1,0) & f(1,1) & f(1,2) & f(1,3) & f(1,4) & \dots \\ f(2,0) & f(2,1) & f(2,2) & f(2,3) & f(2,4) & \dots \\ f(3,0) & f(3,1) & f(3,2) & f(3,3) & f(3,4) & \dots \\ | & | & | & | & | & \dots \\ | & | & | & | & | & \dots \end{bmatrix}$$

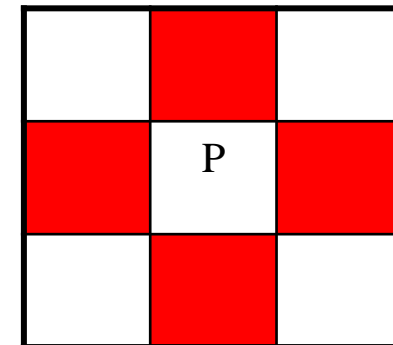


# Neighbors

- Any pixel  $p(x, y)$  has two vertical and two horizontal neighbors, given by  $(x-1, y)$ ,  $(x+1, y)$ ,  $(x, y-1)$ , and  $(x, y+1)$
- This set of pixels are called the 4-neighbors of  $P$ , and is denoted by  $N_4(p)$

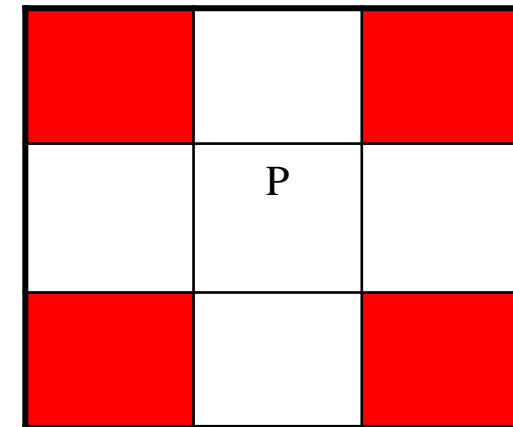
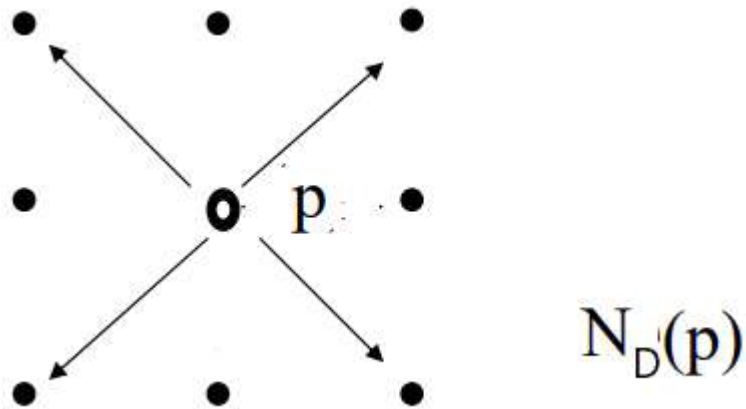


$N_4(p)$



# Neighbors

- The four diagonal neighbors of  $p(x, y)$  are given by  $(x-1, y-1)$ ,  $(x+1, y+1)$ ,  $(x+1, y-1)$ , and  $(x-1, y+1)$ .
- This set is denoted by  $N_D(p)$



# Neighbors

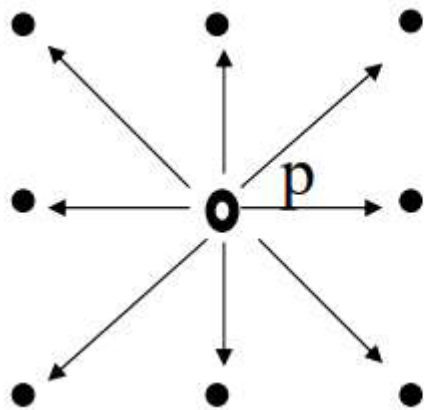
$$f(x,y) = \begin{bmatrix} f(0,0) & f(0,1) & f(0,2) & f(0,3) & f(0,4) & \dots \\ f(1,0) & f(1,1) & f(1,2) & f(1,3) & f(1,4) & \dots \\ f(2,0) & f(2,1) & f(2,2) & f(2,3) & f(2,4) & \dots \\ f(3,0) & f(3,1) & f(3,2) & f(3,3) & f(3,4) & \dots \\ | & | & | & | & | & \dots \\ | & | & | & | & | & \dots \end{bmatrix}$$



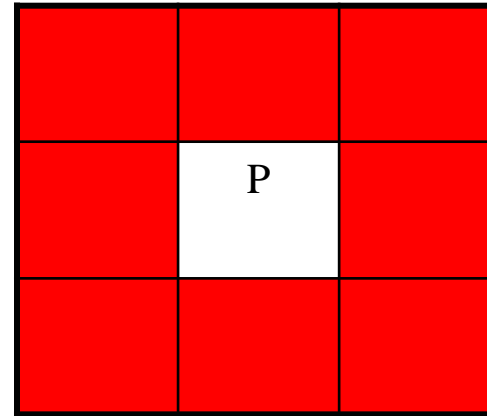
# Neighbors

- The points  $N_4(p)$  and  $N_D(p)$  are together known as 8-neighbors of the point P, denoted by  $N_8(p)$

$$N_8(p) = N_4(p) \cup N_D(p)$$



$N_8(p)$





# Adjacency

- Let  $V$  be the set of intensity used to define adjacency; e.g.  $V=\{1\}$  if we are referring to adjacency of pixels with value 1 in a binary image with 0 and 1.
- In a gray-scale image, for the adjacency of pixels with a range of intensity values of say, 100 to 120, it follows that  $V=\{100,101,102,...,120\}$ .
- We consider three types of adjacency :

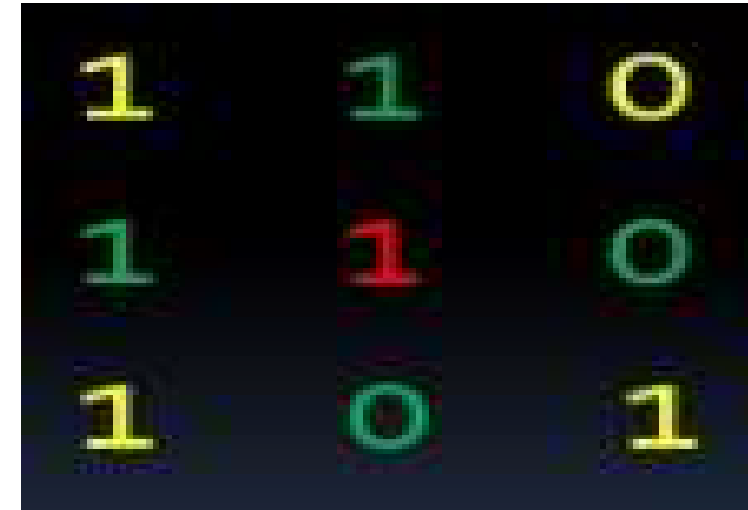


## 4- adjacency

- Let  $V$  be set of binary values used to define adjacency.
- Two pixels  $p$  and  $q$  with values from  $V$  are 4- adjacency if  $q$  is in the set  $N_4(p)$ .

$p$  in RED color

$q$  can be any value in GREEN color

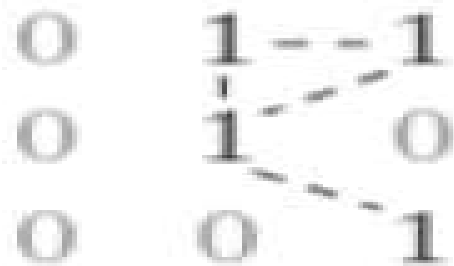
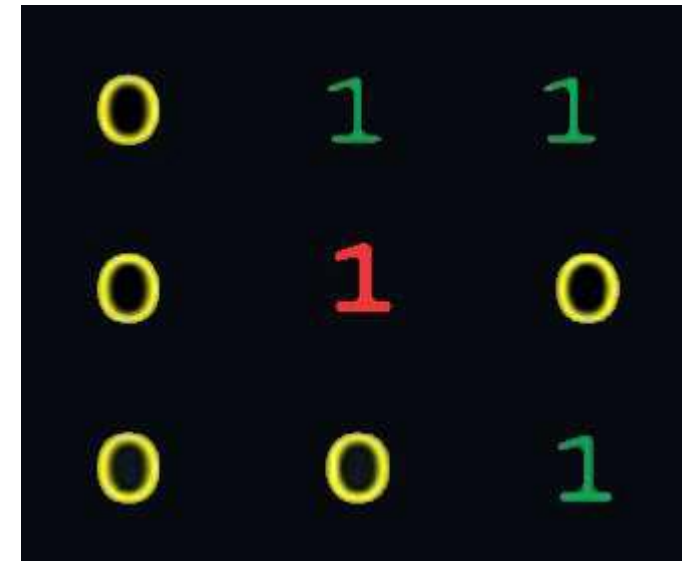


## 8- adjacency

- Two pixels  $p$  and  $q$  with values from  $V=\{1\}$  are 8- adjacency if  $q$  is in the set  $N_8(p)$ .

$p$  in RED color

$q$  can be any value in GREEN color



# m- adjacency (mixed adjacency)

- Two pixels  $p$  and  $q$  with values from  $V$  are m- adjacency if

(i)  $q$  is in  $N_4(p)$

OR

(ii)  $q$  is in  $N_D(p)$  and  $N_4(p) \cap N_4(q)$  is empty

e.g.  $V = \{1\}$



# m- adjacency (mixed adjacency)

Two pixels  $p$  and  $q$  with the values from set 'V' are m-adjacent if

(i)  $q$  is in  $N_4(p)$  e.g.  $V = \{1\}$

i. b & c

0 a	1 b	1 c
0 d	1 e	0 f
0 g	0 h	1 i

ii. b & e

0 a	1 b	1 c
0 d	1 e	0 f
0 g	0 h	1 i



# m- adjacency (mixed adjacency)

(ii)  $q$  is in  $N_D(p)$  and the set  $[N_4(p) \cap N_4(q)]$  is empty

e.g.  $V = \{1\}$

iii.  $e$  &  $i$

0 a	1 b	1 c
0 d	1 e	0 f
0 g	0 h	1 i

m-adjacent

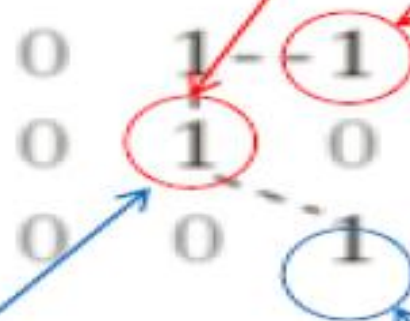
iv.  $e$  &  $c$

0 a	1 b	1 c
0 d	1 e	0 f
0 g	0 h	1 i

Not m-adjacent



Not  $m$ -connected. They have a common 4-connected neighbor.

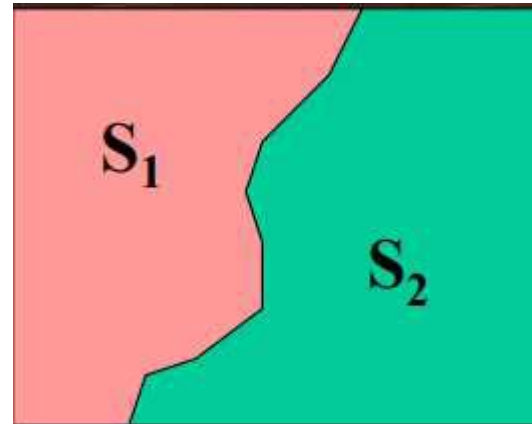


$m$ -adjacency

$m$ -connected. They do not have any common 4-connected neighbor.



- Two image subsets  $S_1$  and  $S_2$  are adjacent if some pixel in  $S_1$  is adjacent to some pixel in  $S_2$





# Paths and Paths length

- A path from pixel  $p$  with coordinates  $(x, y)$  to pixel  $q$  with coordinates  $(s, t)$  is a sequence of distinct pixels with coordinates:

Where

- $(x_0, y_0), (x_1, y_1), (x_2, y_2) \dots (x_n, y_n),$
- $(x_0, y_0) = (x, y)$  and  $(x_n, y_n) = (s, t)$  and
- $(x_i, y_i)$  is adjacent to  $(x_{i-1}, y_{i-1})$   $1 \leq i \leq n$
- $(x_0, y_0)$



# Paths and Paths length

- If  $(x_0, y_0) = (x_n, y_n)$ , the path is ***closed path***.
- We can define 4-, 8-, and m-paths based on type of adjacency used.



# Paths and Paths length

0	1	1
0	2	0
0	0	1

0	1	1
0	2	0
0	0	1

8 - adjacent

0	1	1
0	2	0
0	0	1

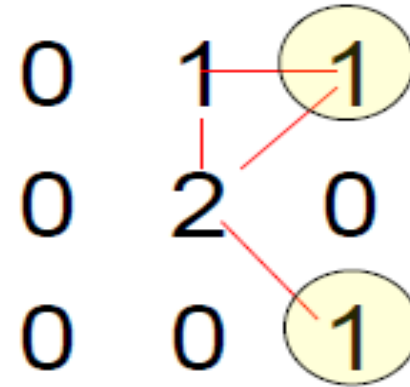
m - adjacent

$V = \{1, 2\}$

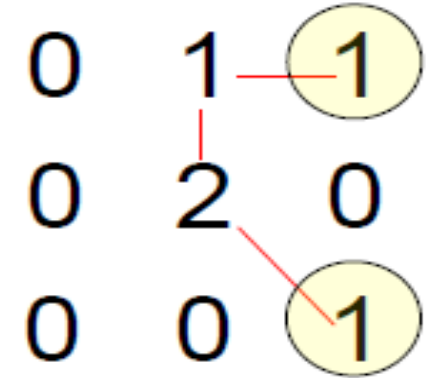


# Paths and Paths length

0	1	1
0	2	0
0	0	1



**8 – adjacent**



**m - adjacent**

The 8-path from (1,3) to (3,3)

(i) (1,3), (1,2), (2,2), (3,3)

(ii) (1,3), (2,2), (3,3)

The m-path from (1,3) to (3,3)

(1,3), (1,2), (2,2), (3,3)



- Example # 1: Consider the image segment shown in figure. Compute length of the shortest-4, shortest-8 & shortest-m paths between pixels p & q where,  $V = \{1, 2\}$ .



# Shortest 4-path

- $V = \{1, 2\}$



# Shortest 8-path

- $V = \{1, 2\}$

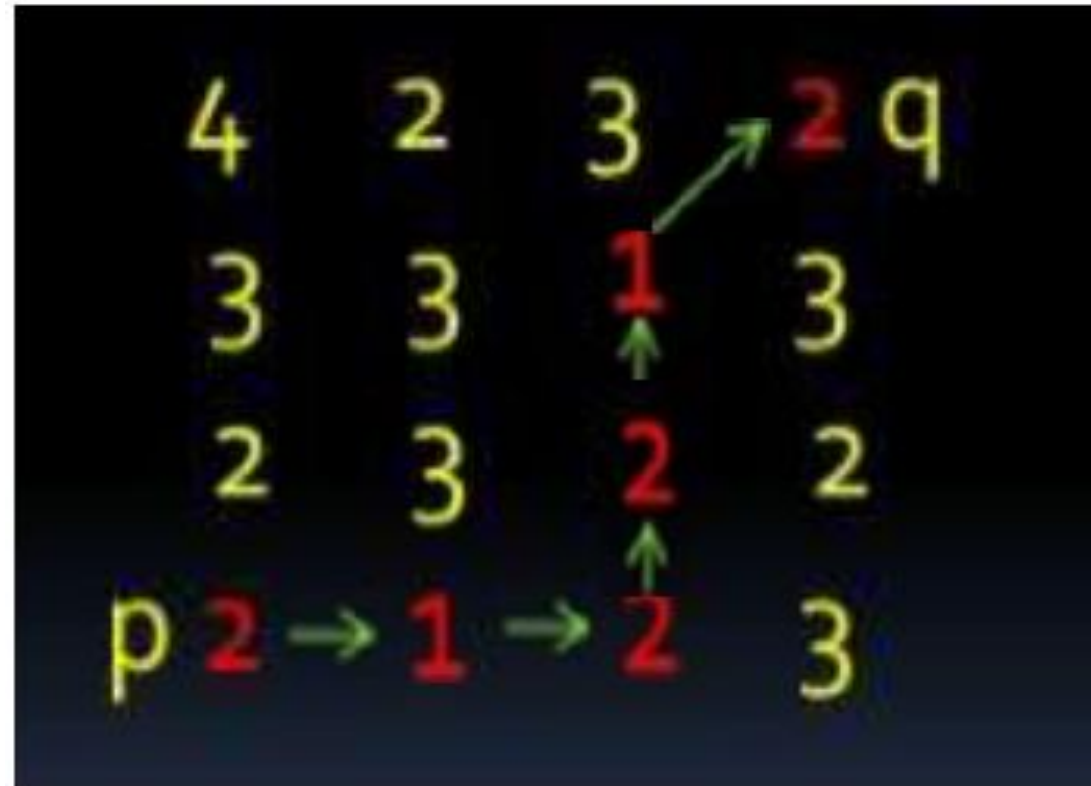


So, shortest 8-path = 4



# Shortest m-path

- $V = \{1, 2\}$



So, shortest m-path = 5





# Connected Components

- Let  $S$  be a subset of pixels in an image.
  - Two pixels  $p$  and  $q$  are said to be connected in  $S$  if there exists a path between them consisting entirely of pixels in  $S$
  - For any pixel  $p$  in  $S$ , the set of pixels that are connected to it in  $S$  is called a connected component of  $S$ .
  - If it only has one connected component, then set  $S$  is called a connected set.



# Regions

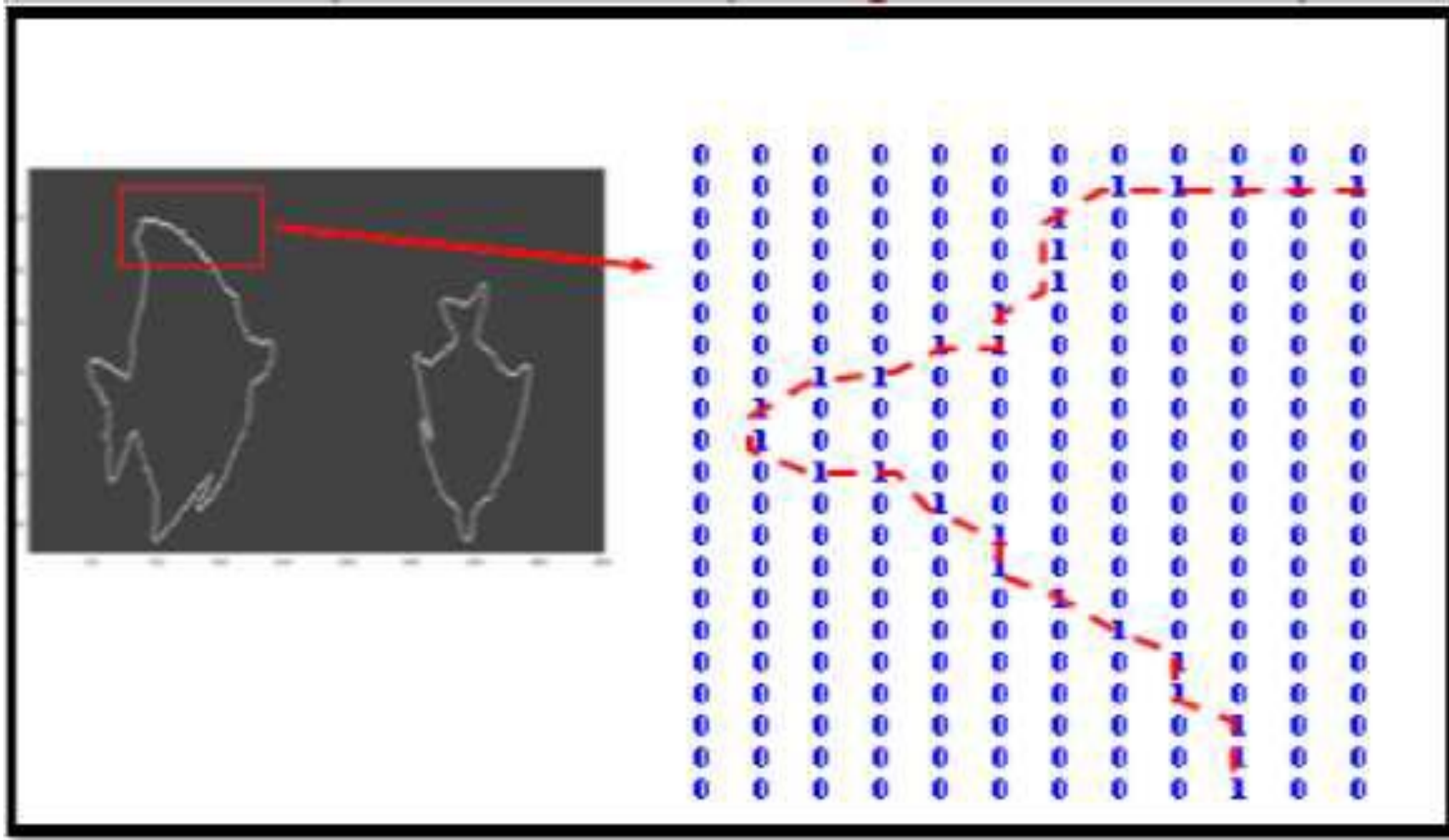
- Let  $R$  be a subset of pixels in an image. We call  $R$  a region of the image if  $R$  is a connected set
- Two regions are said to be adjacent if their union forms a connected set.



# Boundary (or border)

- The ***boundary of the region  $R$  is the set of pixels in the region *that**** have one or more neighbors that are not in  $R$ .
- If  $R$  happens to be an entire image, then its boundary is defined as the set of pixels in the first and last rows and columns of the image.

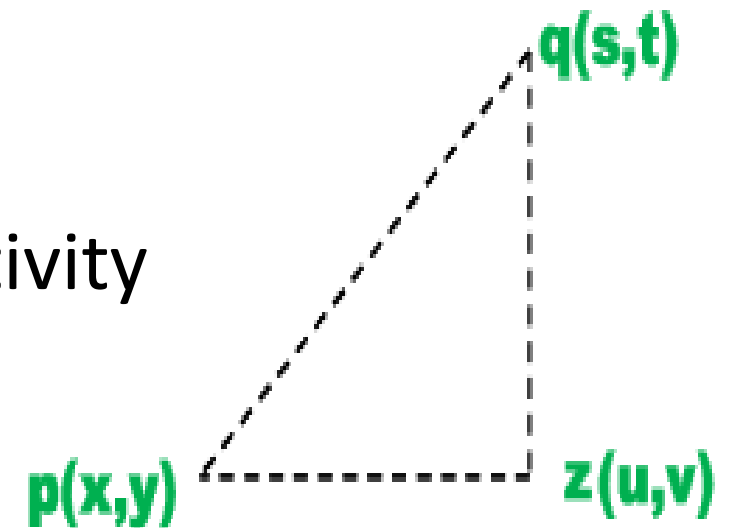




# Distance Measures

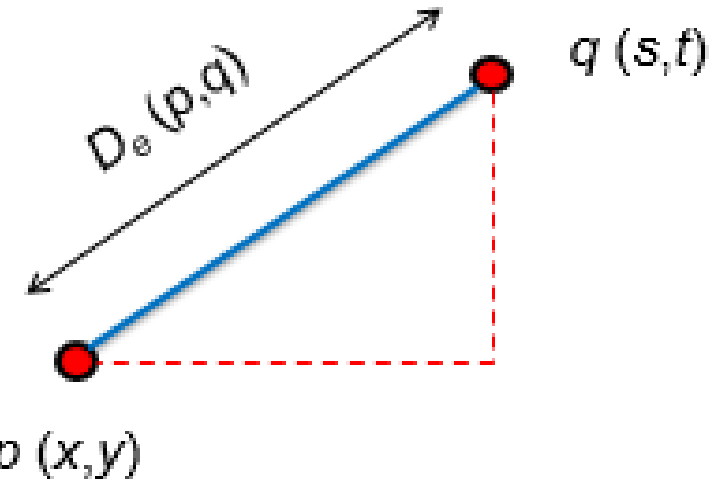


- Given pixels  $p$ ,  $q$  and  $z$  with coordinates  $(x, y)$ ,  $(s, t)$ ,  $(u, v)$  respectively, the distance function  $D$  has following properties:
  - $D(p, q) \geq 0$  [ $D(p, q) = 0$ , iff  $p = q$ ]  $\longrightarrow$  reflexivity
  - $D(p, q) = D(q, p)$   $\longrightarrow$  symmetry
  - $D(p, z) \leq D(p, q) + D(q, z)$   $\longrightarrow$  transitivity

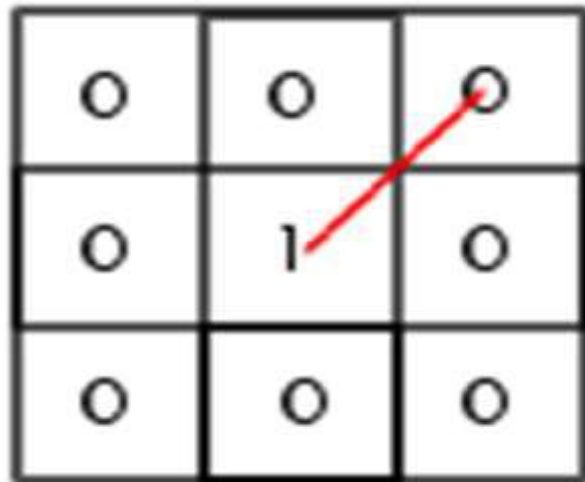


# Euclidean Distance

$$D_e(p, q) = \left[ (x - s)^2 + (y - t)^2 \right]^{1/2}$$



The Euclidean distance is the straight-line distance between two pixels.



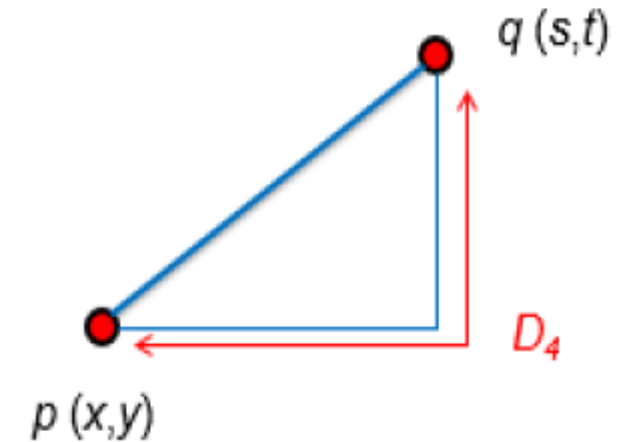
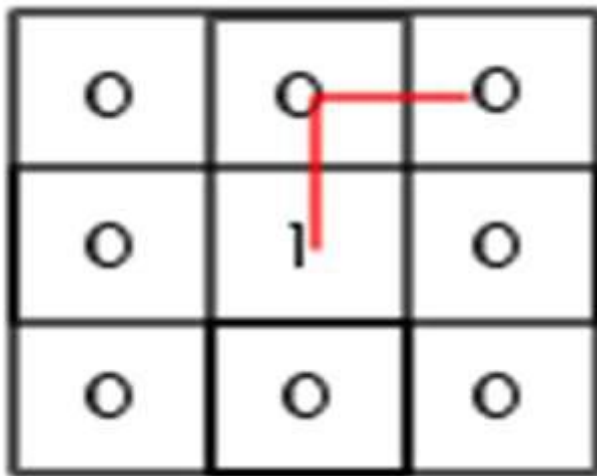
1.41	1.0	1.41
1.0	0.0	1.0
1.41	1.0	1.41



# City Block Distance

The City Block distance metric measures the path between the pixels based on a 4-connected neighborhood. Pixels whose edges touch are 1 unit apart; pixels diagonally touching are 2 units apart

$$D_4(p, q) = |x - s| + |y - t|$$

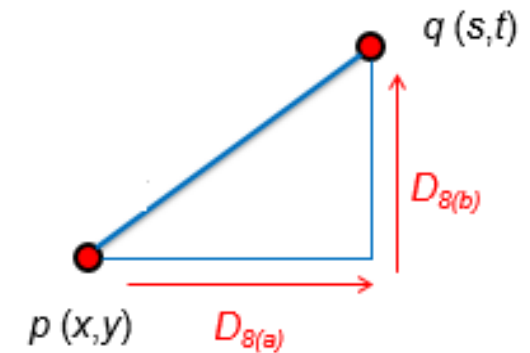
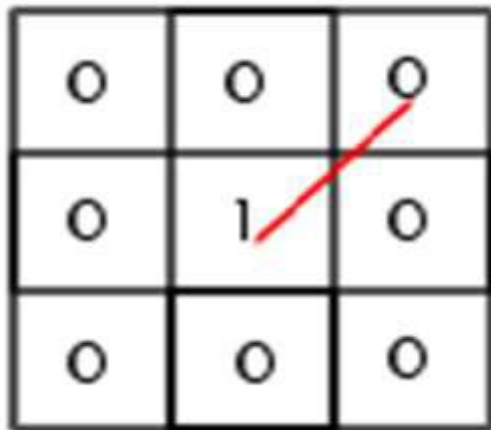




# Chess Board Distance

- The Chessboard distance metric measures the path between the pixels based on an 8- connected neighborhood. Pixels whose edges or corners touch are 1 unit apart.

$$D_8(p, q) = \max(|x - s|, |y - t|).$$



$$D_8 = \max(D_{8(a)}, D_{8(b)})$$



# Summary

- Image sensing and Acquisition.
- Image Formation Model.
- Image Digitization
- Basic relationships between the pixels.
- Distance Measures



*Thank  
you!*

