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| **ASSIGNMENT** | |
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| **Course Name** | Engineering Mathematics - 3 |
| **Programme** | B.Tech |
| **Department** | CSE |
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| **Semester/Year** | 03/2018 |
| **Course Leader/s** | Dr. Mahadev Channakote |

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| **Declaration Sheet** | | | | | | | | |
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| **Declaration**  The assignment submitted herewith is a result of my own investigations and that I have conformed to the guidelines against plagiarism as laid out in the Student Handbook. All sections of the text and results, which have been obtained from other sources, are fully referenced. I understand that cheating and plagiarism constitute a breach of University regulations and will be dealt with accordingly. | | | | | | | | |
| Signature of the Student | |  | | | | | Date |  |
| Submission date stamp  (by Examination & Assessment Section) | |  | | | | | | |
| Signature of the Course Leader and date | | | | Signature of the Reviewer and date | | | | |
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# 

# **Question No. 1**

**Solution to Question No. 1:**

## 1.1 Obtain the mathematical model:

The General form of an Ordinary Differential Equation for a Spring-Mass system is:

Where,

From the given data,

Here the displacement variable is taken as

Now, the equation becomes

## 1.2 Solution of the model using Laplace transform:

We can write the equation in a simplified form as

Applying Laplace transforms to equation , we obtain,

Using the property,

Applying this property in

1. Given Initial Conditions

Substituting these values in , we obtain

Taking Inverse Laplace transform of ,

Splitting the terms

Where are arbitrary constants

This forms a system of Linear Equations

Solving this obtains:

>> inv([8 0 1 0; 2 8 0 1; 5 2 4 0; 0 5 0 4])\*[0;0;5;0]

ans =

-27/149

8/149

216/149

-10/149

Factorizing becomes and substituting these values, Equation becomes,

Simplifying it,

1. Given Initial Conditions

Substituting these values in , we obtain

Taking Inverse Laplace transform of ,

Splitting the terms

Where are arbitrary constants

This forms a system of Linear Equations

Solving this obtains:

>> inv([8 0 1 0; 2 8 0 1; 5 2 4 0; 0 5 0 4])\*[0;40;5;160]

ans =

-27/149

8/149

216/149

5950/149

Factorizing becomes and substituting these values, Equation becomes,

Simplifying it,

## 1.3 Plotting solutions

syms t

w = sqrt(39)/8;

a1 = -27\*cos(2\*t)+4\*sin(2\*t);

a2 = cos(w\*t);

a3 = sin(w\*t);

a4 = exp(-t/8);

y1 = (1/149)\*(a1+27\*a4\*a2-(37/sqrt(39))\*a4\*a3);

y2 = (1/149)\*(a1+27\*a4\*a2+(5923/sqrt(39))\*a4\*a3);

fplot(y1, [0 5], 'LineWidth', 2);

hold on;

fplot(y2, [0 5], 'LineWidth', 2);

grid on;

legend('Sol 1', 'Sol 2');



Figure 1.1 Comparison between the solutions

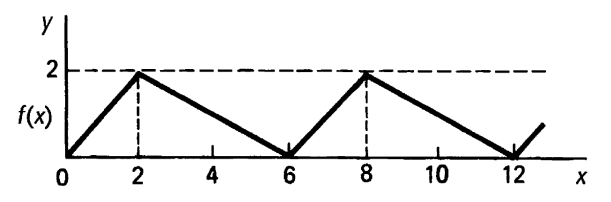
From the graph comparing Solution 1 and Solution 2, it can be observed that when an initial velocity is given to the mass, the displacement is more, the oscillations are more, but when the initial velocity is zero, the dampening force is more and hence there is not much of amplitude compared to the case when there is an initial velocity.

## 1.4 Conclusion

Laplace Transform converts a function with time domain into frequency domain, this property can be used to solve Initial Value Problems or Ordinary Differential Equations, since it makes it easier to solve. Some Equations that cannot be solved using usual methods can be solved using Laplace Transforms.

# **Question No. 2**

**Solution to Question No. 2:**



## 2.1 Writing periodic function:

The given function is periodic function with a period of 6, i.e. after every 6 consecutive values of x, it starts repeating,

The function can be broken down form as a function of two lines, one from and other from

The first line passes through and , the equation of line is

The second line passes through and , the equation of line is

Hence the function is:

The period of the function is 6.

## 2.2 MATLAB function for plotting periodic function, plot from [-24, 24]:

function [] = PlotPiecewise(function1, function2, pieceLimit1, pieceLimit2, plotInterval)

%PLOTPIECEWISE Plots the funtion1, and function2 piecewise

% Author: Satyajit Ghana

% USAGE: PlotPiecewise(@(x) x, @(x) 3-x./2, 2, 6, [0 12])

% plotInterval is optional

%% default arguments

if nargin < 5

plotInterval = [0 pieceLimit2];

end

%% initialize

l1 = pieceLimit1;

l2 = pieceLimit2;

timePeriod = l2;

T = timePeriod;

%% function

FOriginal = @(x) function1(mod(x, T)).\*(0 <= mod(x, T) & mod(x, T) < l1) + function2(mod(x, T)).\*(l1 <= mod(x, T) & mod(x, T) < l2);

%% plot

fplot(FOriginal, plotInterval, 'LineWidth', 2);

title('$ $ PieceWise Function Plot', 'Interpreter', 'latex');

legend({'$ $ Function'}, 'Interpreter', 'latex', 'Location', 'best');

end

f1 = @(x) x;

l1 = 2;

f2 = @(x) 3 - x./2;

l2 = 6;

PlotPiecewise(f1, f2, l1, l2, [-24 24]);

****

Figure 1.2 PieceWise Plot

## 2.3 Checking whether the periodic function is even or odd:

The function is said to be even if the graph of is symmetric with respect to y axis, and is an odd function if the function is symmetric with respect to the origin.

We know

To find replace with in

Since , the given function is an not an odd function

Since , the given function is an not an even function

From and it’s concluded that is neither an even function nor an odd function.

## 2.4 Finding the Fourier series expansion for the given wave:

If the function is defined on the interval then the Fourier series for is given by

Where

From and

## 2.5 MATLAB function for Fourier series expansion for N=5:

function [FTransform] = FourierSeriesPW(function1, function2, pieceLimit1, pieceLimit2, precision, plotInterval)

% Author : Satyajit Ghana

% Arguments:

% function1 : The first function

% function2 : The second function

% pieceLimit1 : end limit of function1

% pieceLimit2 : end limit of function2

% precision : value of n in fourier transform

% plotInterval !optional default: [0 pieceLimit2]: Interval to plot

%

% USAGE Exaxmple: FourierSeriesPW(@(x) x, @(x) (3-x/2, 2, 6, 50, 6, [0 20]))

%% default arguments

if nargin < 6

plotInterval = [0 pieceLimit2];

end

%% initialize

f1 = function1;

f2 = function2;

l1 = pieceLimit1;

l2 = pieceLimit2;

K = precision;

timePeriod = l2;

T = timePeriod;

syms x;

w = (2\*pi)/T;

n = 1:K;

FOriginal = @(x) function1(mod(x, T)).\*(0 <= mod(x, T) & mod(x, T) < l1) + function2(mod(x, T)).\*(l1 <= mod(x, T) & mod(x, T) < l2);

%% calculate the constants

% a better version

%a0 = (2/T)\*integral(@(x) FOriginal(x), 0, T);

% fill with zeros to improve performance

%an = zeros(1, K);

%bn = zeros(1, K);

%for i = 1:K

% an(i) = (2/T)\*integral(@(x) FOriginal(x).\*cos(i.\*w.\*x), 0, T);

% bn(i) = (2/T)\*integral(@(x) FOriginal(x).\*sin(i.\*w.\*x), 0, T);

%end

% plain-old way

a0 = (2/T)\*(int(f1, x, 0, l1) + int(f2, x, l1, l2));

an = (2/T)\*(int(f1\*cos(n\*w\*x), x, 0, l1) + int(f2\*cos(n\*w\*x), l1, l2));

bn = (2/T)\*(int(f1\*sin(n\*w\*x), x, 0, l1) + int(f2\*sin(n\*w\*x), l1, l2));

v1 = [a0 an bn];

v2 = [1/2 cos(n\*w\*x) sin(n\*w\*x)];

FTransform = v1.\*v2;

%% fourier series

FTransform = vpa(simplify(sum(FTransform)));

%% plot the actual function

fplot(FOriginal, plotInterval, 'LineWidth', 2);

hold on;

%% plot the series

fplot(sym(FTransform), plotInterval, 'LineWidth', 2);

grid on;

legend({'$ $ Original Function', 'Fourier Series Function'}, 'Interpreter', 'latex', 'Location', 'best');

title(strcat('$ $ Fourier Series for PieceWise Function for N = $ $', num2str(precision)), 'Interpreter', 'latex');

end

For

N = 5;

disp(FourierSeriesPW(@(x) x, @(x) 3-x./2, 2, 6, N));

OUTPUT:

1.0 - 0.049357529423872216065957222760213\*cos(4.1887902047863909846168578443727\*x + 0.52359877559829887307710723054658) - 0.19743011769548886426382889104085\*sin(2.0943951023931954923084289221863\*x + 1.0471975511965977461542144610932) - 0.031588818831278218282212622566536\*sin(5.2359877559829887307710723054658\*x + 1.0471975511965977461542144610932) - 0.7897204707819554570553155641634\*cos(1.0471975511965977461542144610932\*x + 0.52359877559829887307710723054658)

## 2.6 Plotting the Fourier series expansion and periodic function for N = 10, N = 20:

1. N = 10

f1 = @(x) x;

l1 = 2;

f2 = @(x) 3 - x./2;

l2 = 6;

disp(FourierSeriesPW(f1, f2, 2, 6, 10, [-24 24]))

1.0 - 0.049357529423872216065957222760213\*cos(4.1887902047863909846168578443727\*x + 0.52359877559829887307710723054658) - 0.19743011769548886426382889104085\*sin(2.0943951023931954923084289221863\*x + 1.0471975511965977461542144610932) - 0.012339382355968054016489305690053\*sin(8.3775804095727819692337156887453\*x + 1.0471975511965977461542144610932) - 0.007897204707819554570553155641634\*cos(10.471975511965977461542144610932\*x + 0.52359877559829887307710723054658) - 0.031588818831278218282212622566536\*sin(5.2359877559829887307710723054658\*x + 1.0471975511965977461542144610932) - 0.016116744301672560348067664574763\*cos(7.3303828583761842230795012276522\*x + 0.52359877559829887307710723054658) - 0.7897204707819554570553155641634\*cos(1.0471975511965977461542144610932\*x + 0.52359877559829887307710723054658)



Figure 1.3 Fourier Series Plot for N = 10

1. N = 20

f1 = @(x) x;

l1 = 2;

f2 = @(x) 3 - x./2;

l2 = 6;

disp(FourierSeriesPW(f1, f2, 2, 6, 20, [-24 24]))

1.0 - 0.049357529423872216065957222760213\*cos(4.1887902047863909846168578443727\*x + 0.52359877559829887307710723054658) - 0.0030848455889920135041223264225133\*cos(16.755160819145563938467431377491\*x + 0.52359877559829887307710723054658) - 0.0021875913318059707951670791251064\*cos(19.89675347273535717693007476077\*x + 0.52359877559829887307710723054658) - 0.19743011769548886426382889104085\*sin(2.0943951023931954923084289221863\*x + 1.0471975511965977461542144610932) - 0.012339382355968054016489305690053\*sin(8.3775804095727819692337156887453\*x + 1.0471975511965977461542144610932) - 0.007897204707819554570553155641634\*cos(10.471975511965977461542144610932\*x + 0.52359877559829887307710723054658) - 0.0027325967847126486403298116407038\*sin(17.802358370342161684621645838584\*x + 1.0471975511965977461542144610932) - 0.031588818831278218282212622566536\*sin(5.2359877559829887307710723054658\*x + 1.0471975511965977461542144610932) - 0.0019743011769548886426382889104085\*sin(20.943951023931954923084289221863\*x + 1.0471975511965977461542144610932) - 0.0065266154610078963393001286294496\*sin(11.519173063162575207696359072025\*x + 1.0471975511965977461542144610932) - 0.0046729021939760677932267193145763\*cos(13.613568165555770700004787994211\*x + 0.52359877559829887307710723054658) - 0.016116744301672560348067664574763\*cos(7.3303828583761842230795012276522\*x + 0.52359877559829887307710723054658) - 0.0040291860754181400870169161436908\*sin(14.660765716752368446159002455304\*x + 1.0471975511965977461542144610932) - 0.7897204707819554570553155641634\*cos(1.0471975511965977461542144610932\*x + 0.52359877559829887307710723054658)



Figure 1.4 Fourier Series Plot for N = 20

## 2.7 Conclusion:

Fourier Series is an expansion for a periodic function, it’s very useful since it can breakup arbitrary periods into a set of simple terms that can easily be plugged in, solved individually and then recombine them to obtain the solution. The Series is a combination of sinusoidal and cosinusoidal terms which when added together with respective scaling forms the periodic function. Since they are continuous and continuously differentiable it makes the work easier.

**Bibliography**

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