Synchronous Hyperedge Replacement Graph Grammars

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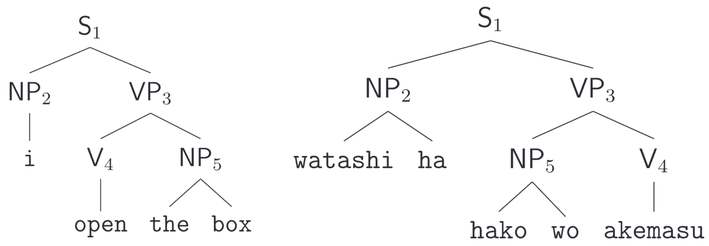
Introduction

Discovering the underlying structures present in real world graphs is a fundamental scientific problem. Learning the LEGO-like building blocks of real world networks helps us gain insights into the mechanisms that underlie network growth and evolution. We find that PSHRG rules effectively capture growth patterns found in temporal graphs and thus can be used to predict the future evolution of a temporal graph.

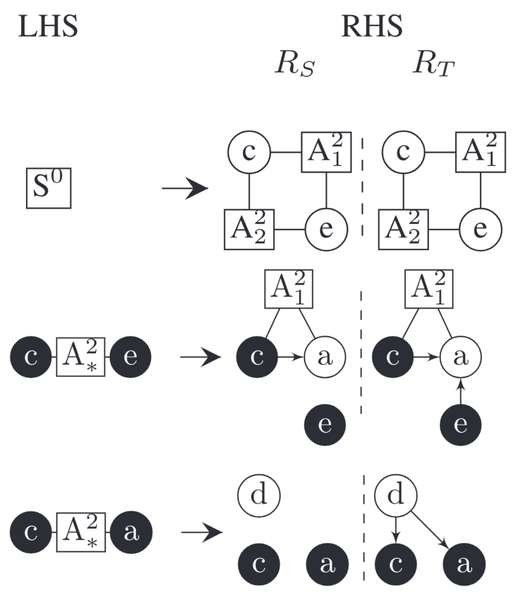
Synchronous Grammars

String grammars

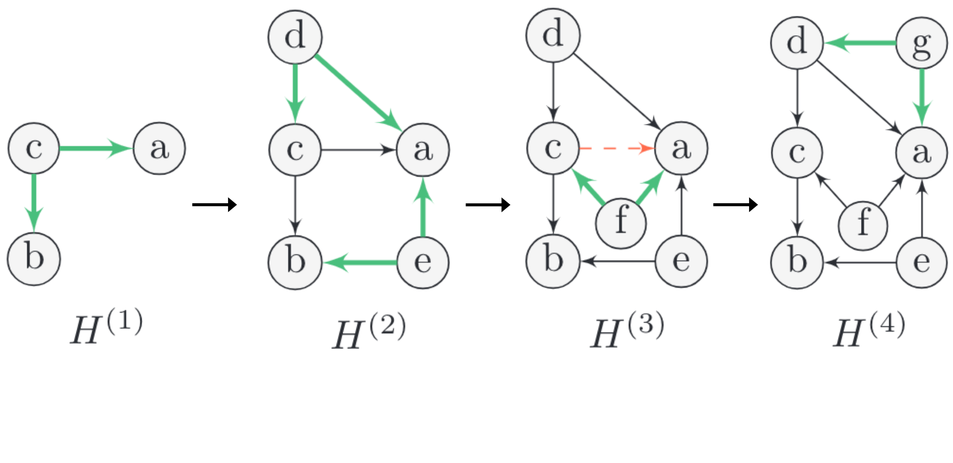
S	→	NP_1 VP_2	:	NP_1 VP_2
VP	→	V_1 NP_2	:	NP_2 V_1
NP	→	i	:	watashi ha
NP	→	the box	:	hako wo
V	→	open	:	akemasu

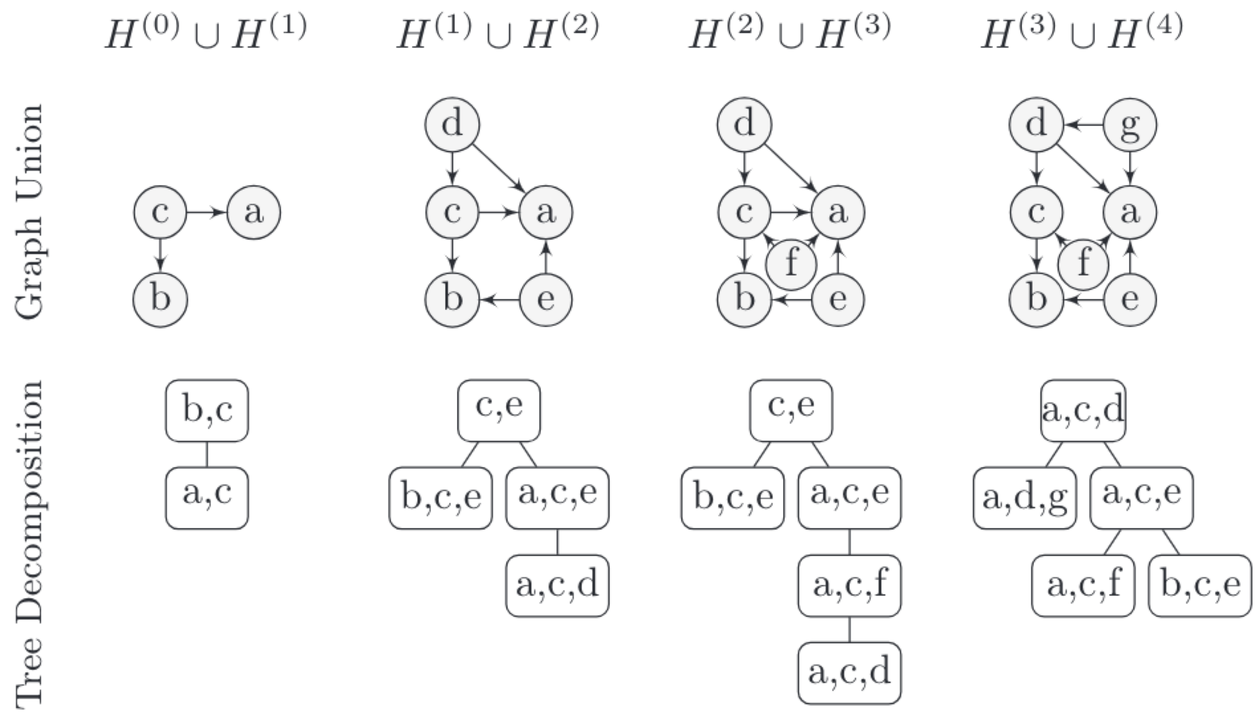


Graph grammars



Temporal Dynamics of Networks



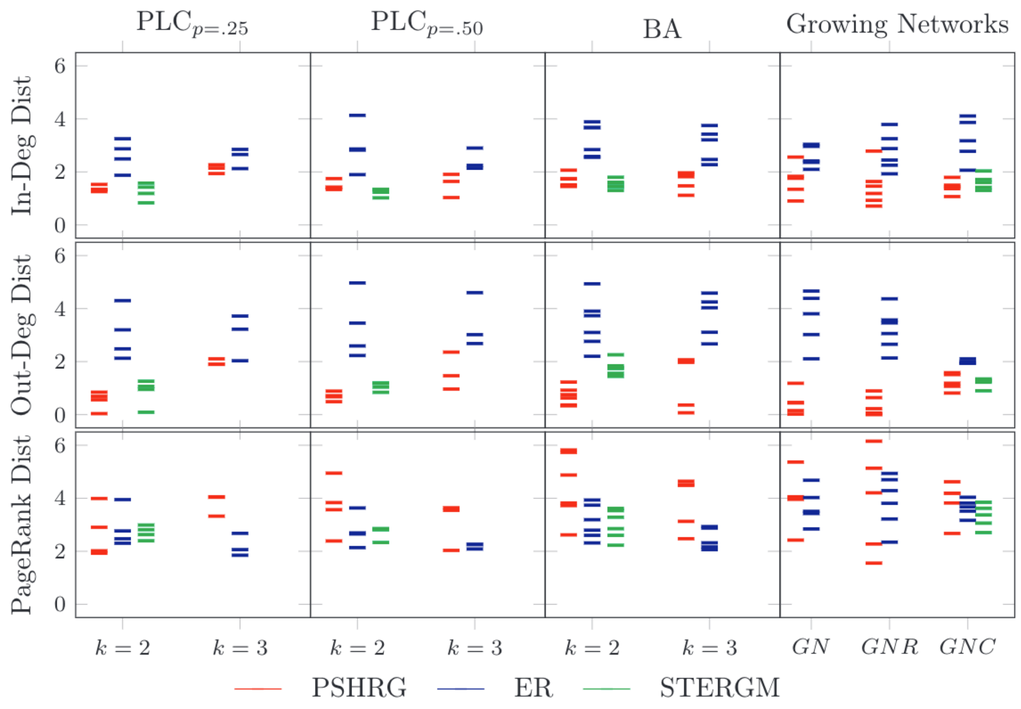


Exploring the Grammar

Graphs generated by the BA process have well known properties. Most notably, the generative process is easy to visually identify.
The addition of outgoing wedges in line 8 and the attachment of an edge where one already exist in line 9 indicates that the extracted PSHRG grammar is able to capture the essence of the dynamic processes used to generate the BA graphs.

		R_S		R_T	BA	ER_1	ER_2	ER_3
Static Rules	1		:		0.051	0.157	0.171	0.153
	2		:		0.133	0.545	0.542	0.412
	3		:		0.000	0.006	0.017	0.007
	4		:		0.260	0.047	0.034	0.029
	5		:		0.032	0.028	0.030	0.017
	6		:		0.025	0.009	0.000	0.002
BA Rules	7		:		0.243	0.164	0.166	0.325
	8		:		0.097	0.000	0.000	0.001
	9		:		0.155	0.012	0.004	0.014
Other Rules	10		:		0.001	0.000	0.000	0.002
	11		:		0.000	0.021	0.031	0.028
	12		:		0.000	0.006	0.006	0.008
	13		:		0.000	0.006	0.005	0.007

Results



Conclusion

The present work presents a method to extract synchronous grammar rules from a temporal graph. We find that the synchronous probabilistic hyperedge replacement grammar, with RHSs containing *synchronized* source- and target-PHRGs, is able to clearly and succinctly represent the graph dynamics found in the graph process. This allows us to accurately predict the future growth of the graph.

Acknowledgement

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