

Scalable and Interpretable Graph Modeling with Graph Grammars

Satyaki Sikdar

Ph.D. Defense

Committee: Dr. Tim Weninger, Chair

Dr. David Chiang · Dr. Peter Kogge · Dr. Danai Koutra

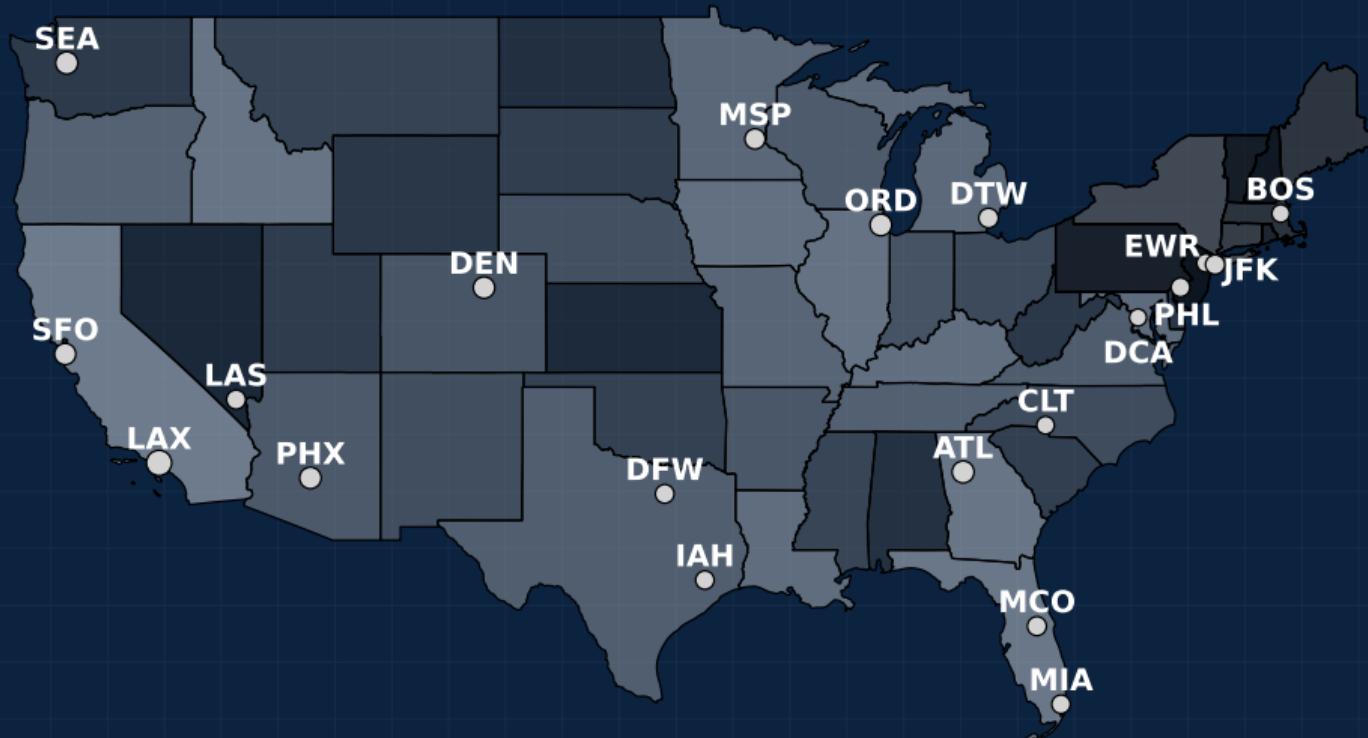
A dark, atmospheric photograph of Prague's Old Town Bridge Tower and the Charles Bridge at dusk or night. The image is overlaid with a light gray grid. In the center, the words "Introduction" are written in a white, sans-serif font.

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US Flights Network in 2011

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Infinity Mirror Test

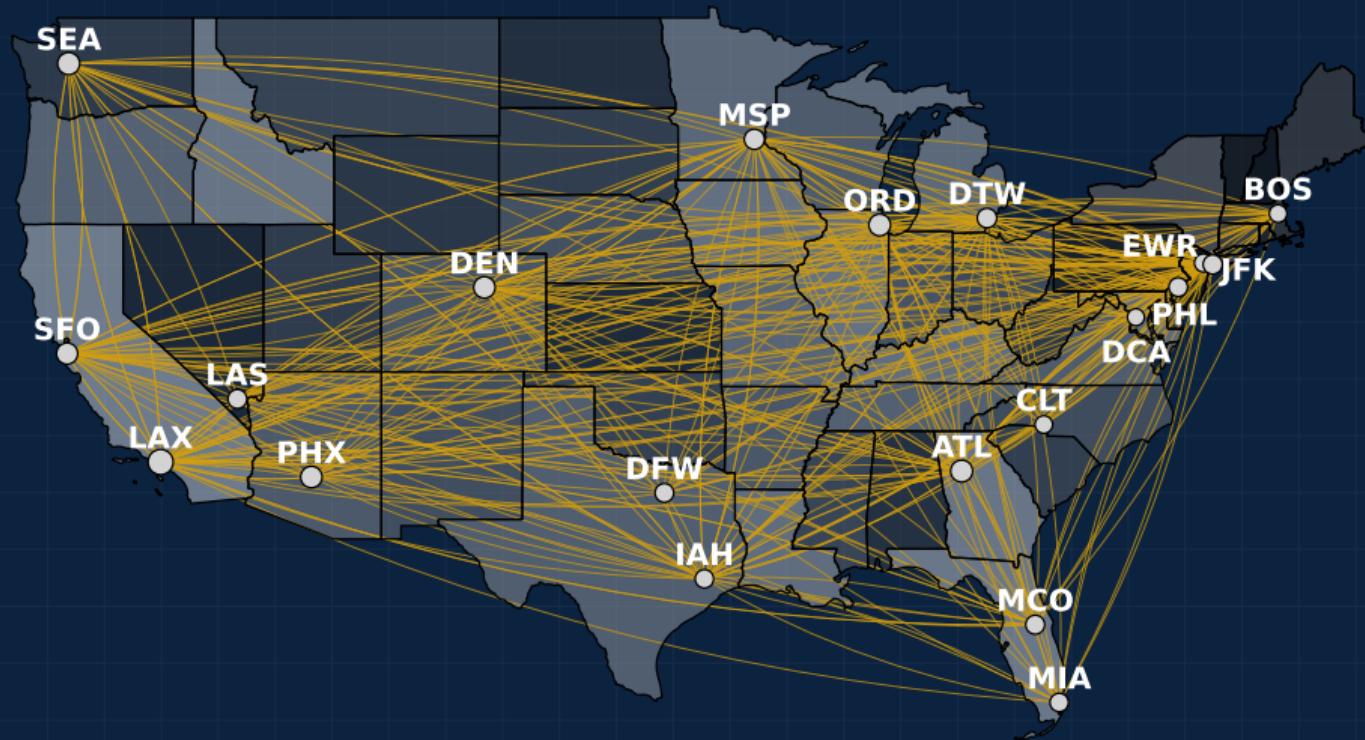
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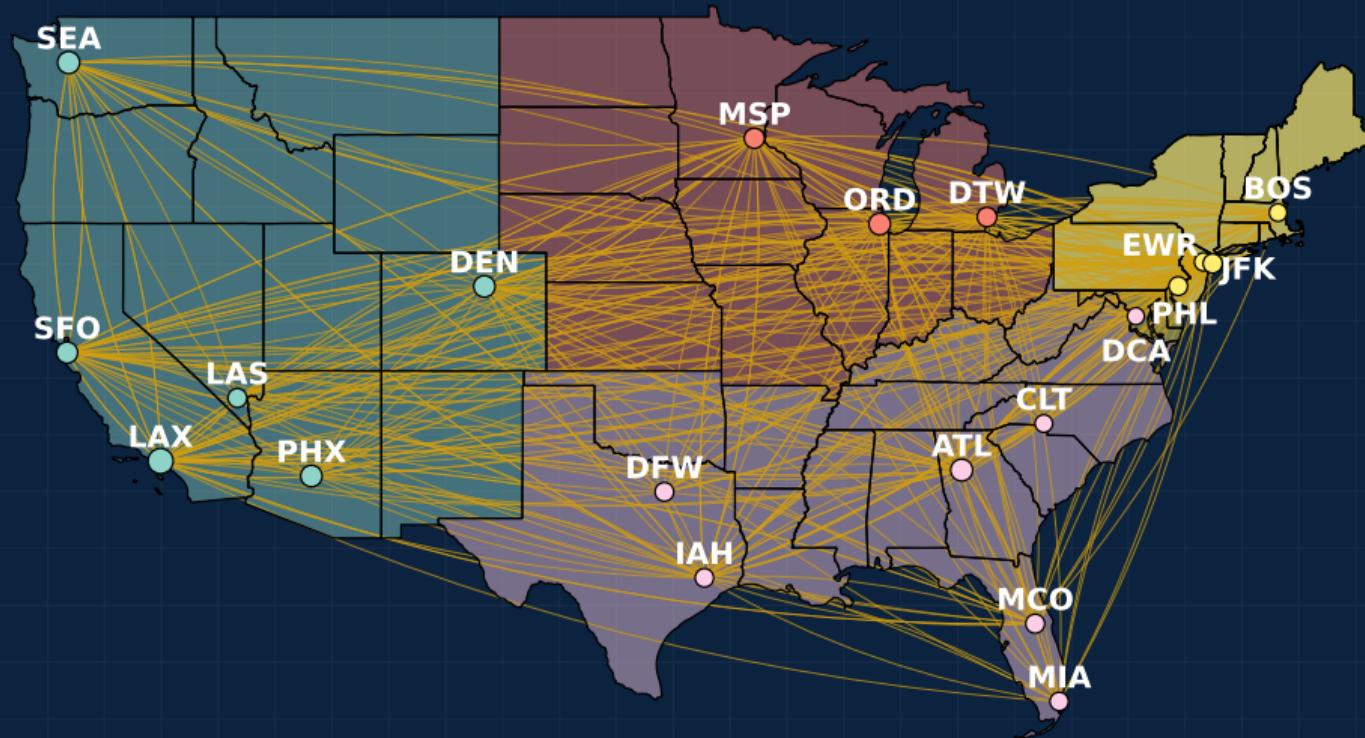
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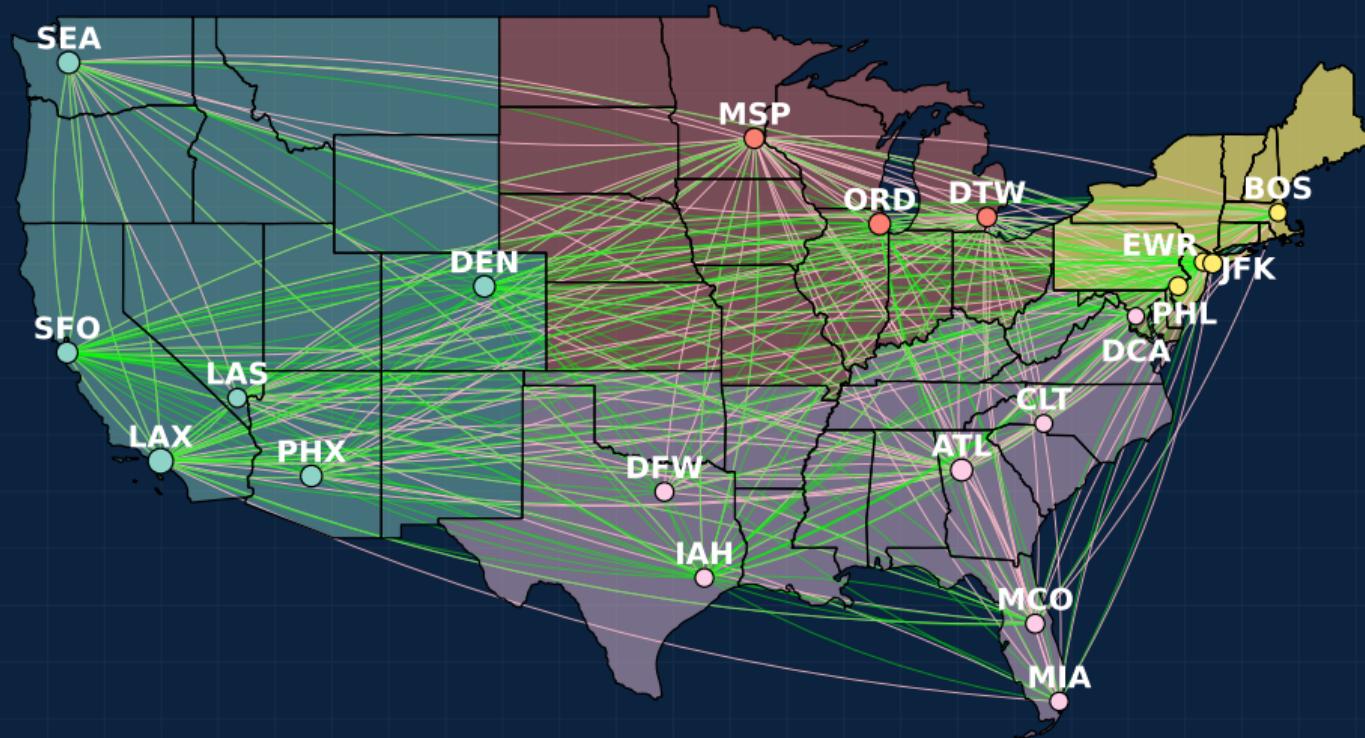
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Subgraph Mining

- ◆ Extract *statistically significant* patterns



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Subgraph Mining

- ◆ Extract *statistically significant* patterns

Graph Representation Learning

- ◆ Learn low-dimensional *embedding* of nodes



Graph Generative Models

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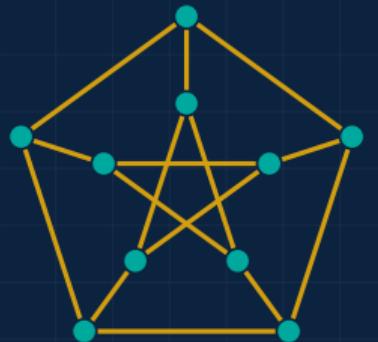
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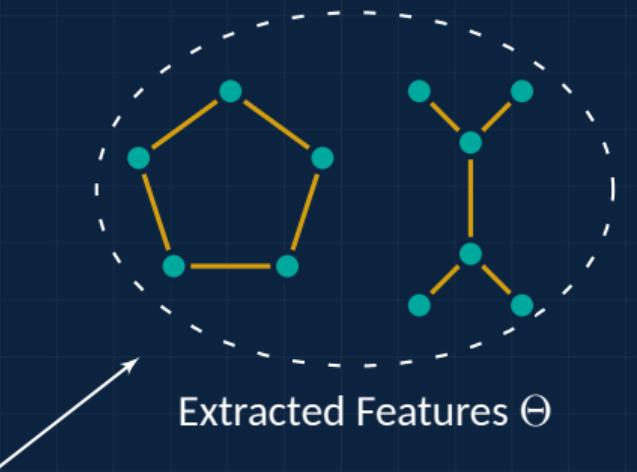
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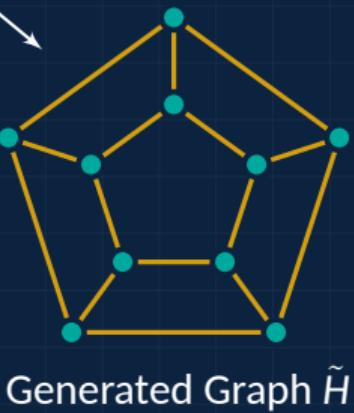
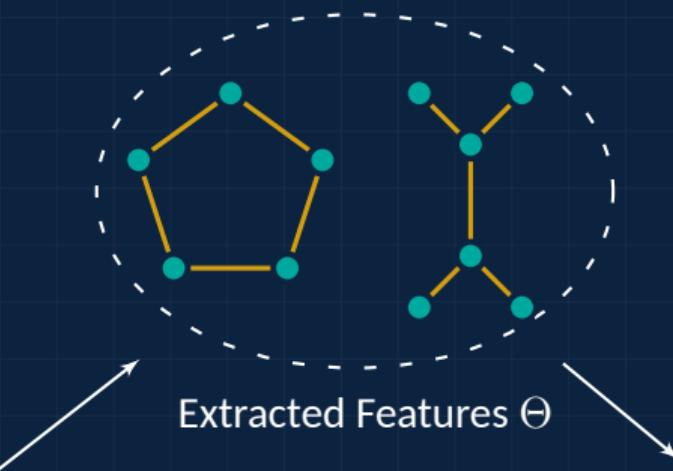
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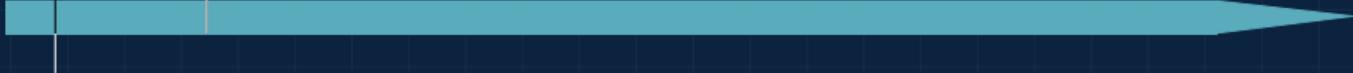
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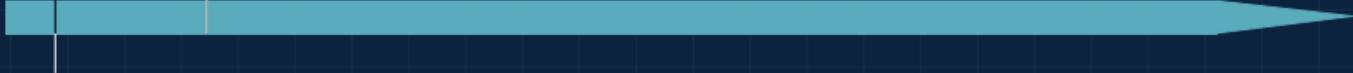
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2001

Barabási-Albert



1998

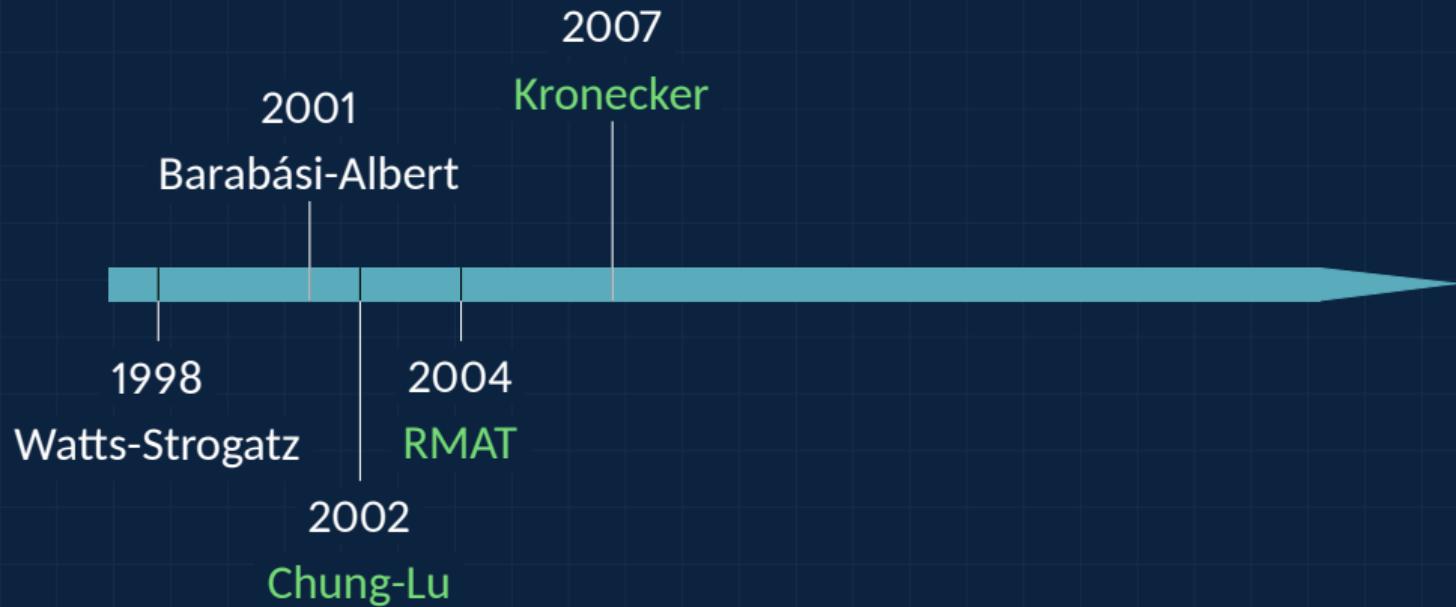
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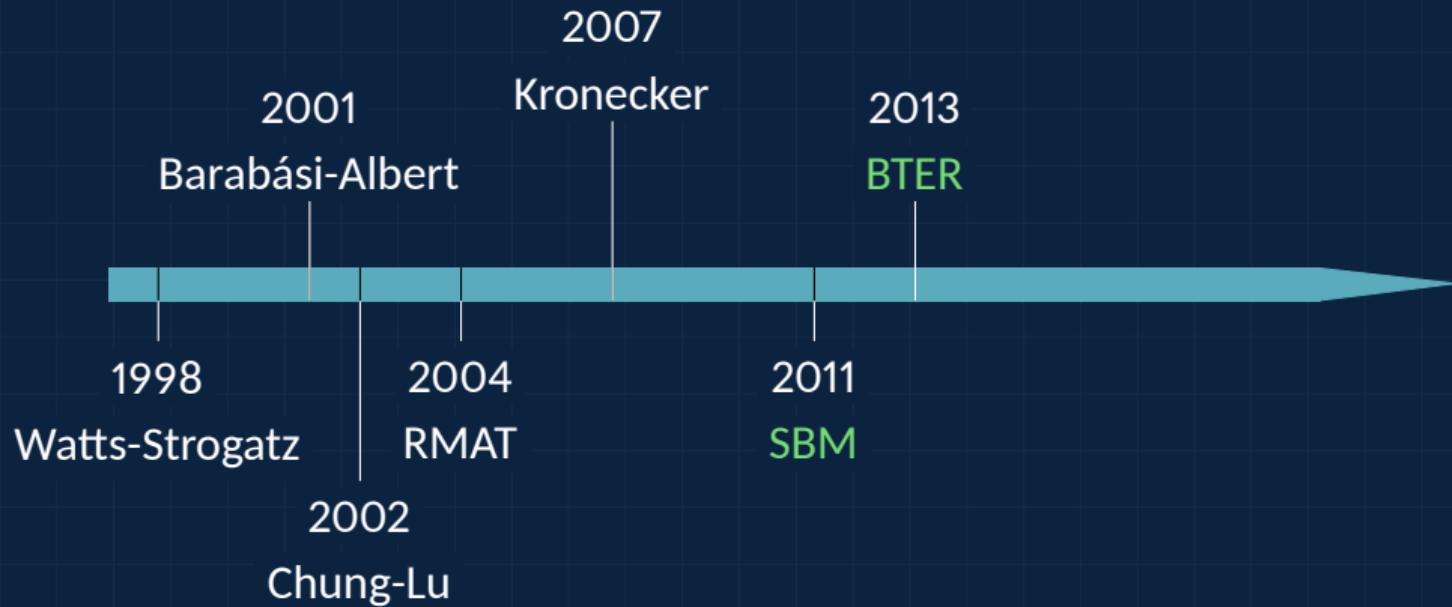
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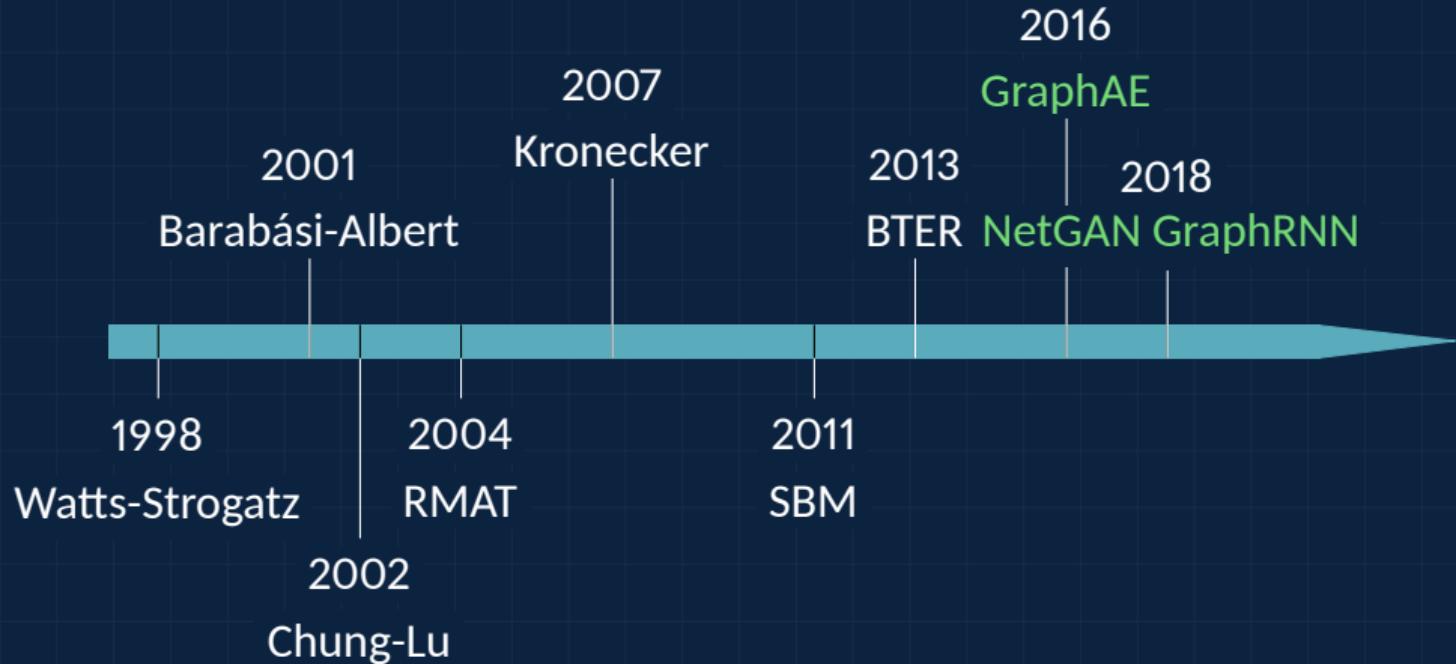
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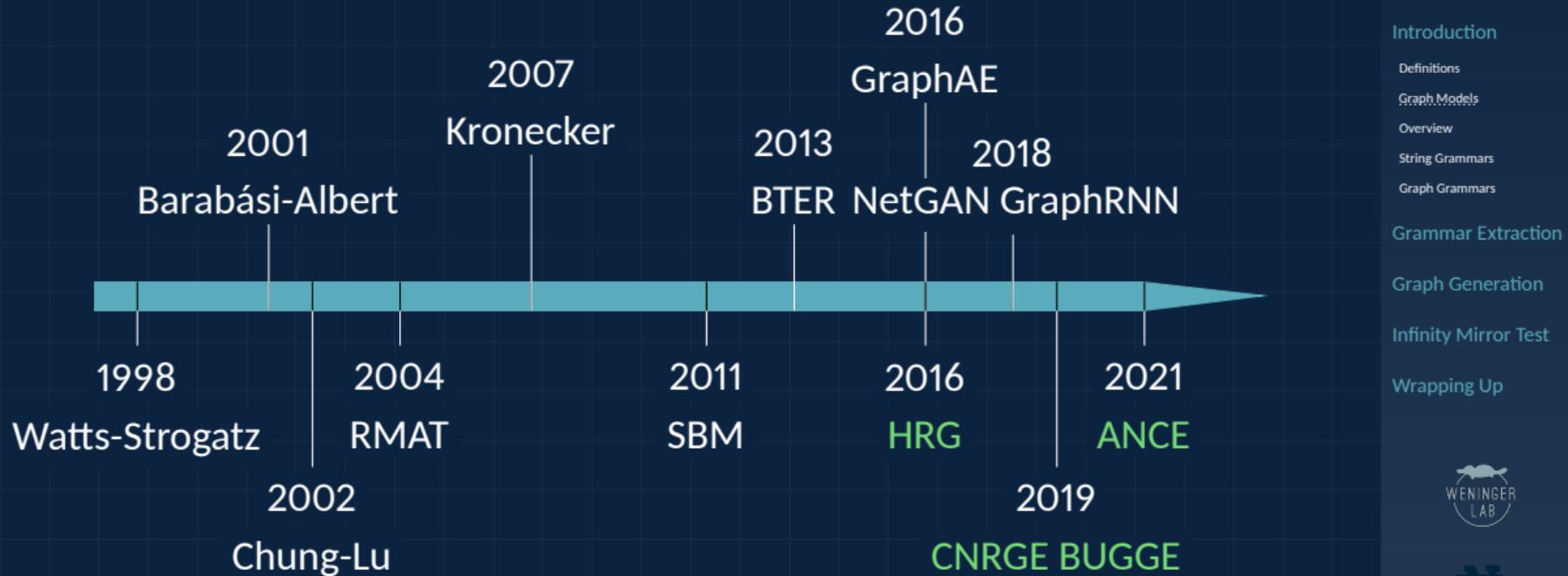
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A Fork in the Road

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Thesis Statement

Scalably and methodically extracting graph grammar rules provide a unique insight into understanding the inner workings of real-world graphs. Furthermore, these rules can generate accurate copies of the input, allowing for highly interpretable models.

Analyzing the performance of graph models undergoing a stress test reveals and amplifies the latent biases and sheds new light on their inner workings.

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List of Publications

- ICGT 18 *Synchronous Hyperedge Replacement Graph Grammars*
- ICDM 19 *Modeling Graphs with Vertex Replacement Grammars*
- BigData 19 *Towards Interpretable Graph Modeling with Vertex Replacement Grammars*
- WSDM 21 *Joint Subgraph-to-Subgraph Transitions: Generalizing Triadic Closure for Powerful and Interpretable Graph Modeling*
- TKDE 21 *The Infinity Mirror Test for Graph Models*
- WSDM 22* *Attributed Vertex Replacement Graph Grammars*

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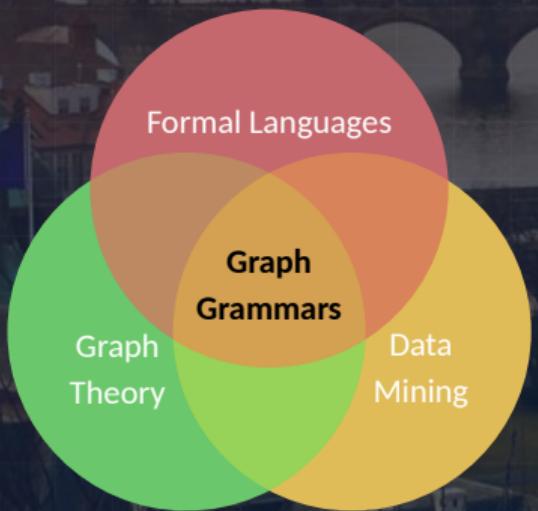
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ICDM 19 *Modeling Graphs with Vertex Replacement Grammars*

WSDM 22* *Attributed Vertex Replacement Graph Grammars*

The Big Picture

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Studying Complex Systems

- ◆ Discovering the *building blocks*
- ◆ Figuring how the pieces *fit* together

Formalism

- ◆ String grammars for natural *language* processing
- ◆ Graph grammars for *graph* mining



String Grammars

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CFG Production Rules

- ◆ $S \rightarrow NP\ VP$
- ◆ $NP \rightarrow \text{the}\ N$
- ◆ $VP \rightarrow V\ NP$
- ◆ $N \rightarrow \text{cat} \mid \text{song}$
- ◆ $V \rightarrow \text{sings} \mid \text{eats}$

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$S \Rightarrow NP\ VP$



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$S \Rightarrow NP\ VP \Rightarrow \text{the}\ N\ VP$



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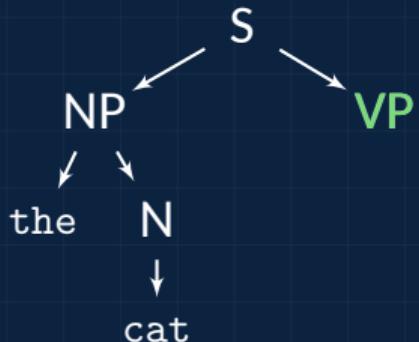
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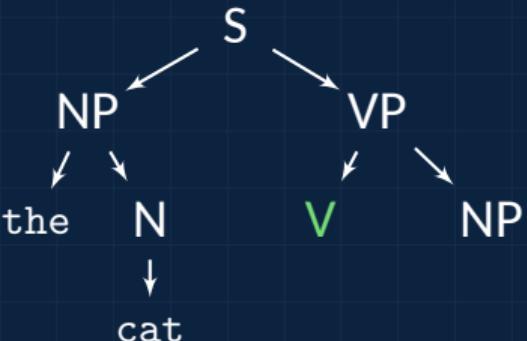
$S \Rightarrow NP\ VP \Rightarrow \text{the}\ N\ VP \Rightarrow \text{the}\ \text{cat}\ VP$



String Grammars

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$S \Rightarrow NP\ VP \Rightarrow \text{the}\ N\ VP \Rightarrow \text{the}\ \text{cat}\ VP \Rightarrow \text{the}\ \text{cat}\ V\ NP$

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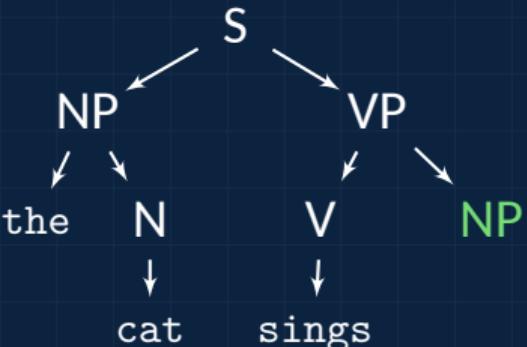
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$S \Rightarrow NP\ VP \Rightarrow \text{the}\ N\ VP \Rightarrow \text{the}\ \text{cat}\ VP \Rightarrow \text{the}\ \text{cat}\ V\ NP$
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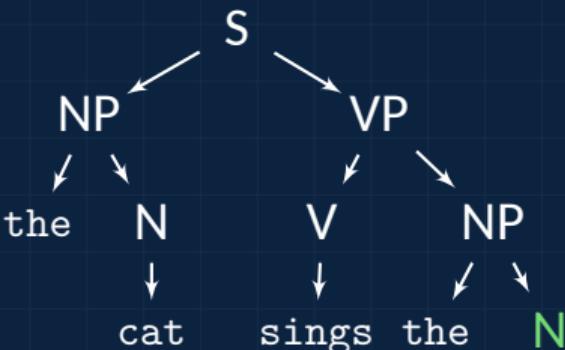
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$S \Rightarrow NP\ VP \Rightarrow \text{the}\ N\ VP \Rightarrow \text{the}\ \text{cat}\ VP \Rightarrow \text{the}\ \text{cat}\ V\ NP$
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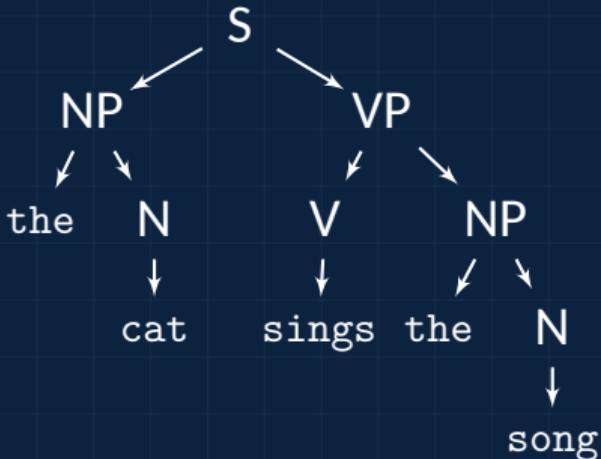
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$S \Rightarrow NP\ VP \Rightarrow \text{the}\ N\ VP \Rightarrow \text{the}\ \text{cat}\ VP \Rightarrow \text{the}\ \text{cat}\ V\ NP$
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 $\Rightarrow \text{the}\ \text{cat}\ \text{sings}\ \text{the}\ \text{song}$

Context Free Graph Grammars

NCE Production Rules

- ◆ $[0] \rightarrow [2] \text{---} [2]$
- ◆ $[2] \rightarrow [2] \text{---} \bullet \text{---} [2]$
- ◆ $[2] \rightarrow \bullet$



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- ◆ $[0] \rightarrow [2] \text{---} [2]$
- ◆ $[2] \rightarrow [2] -\bullet- [2]$
- ◆ $[2] \rightarrow \bullet$



$\begin{matrix} S \\ 0 \end{matrix} \Rightarrow$

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- ◆ $[2] \rightarrow [2] - \bullet - [2]$
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$$[0] \Rightarrow [2] \text{---} [2]$$

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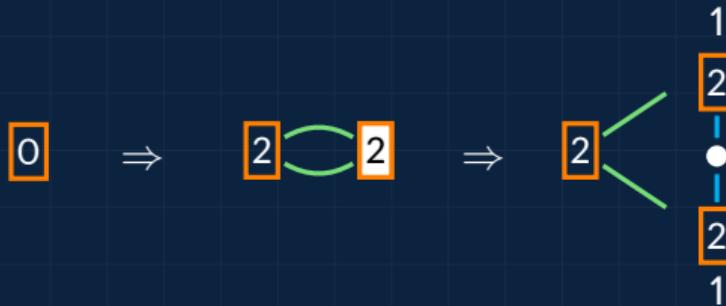
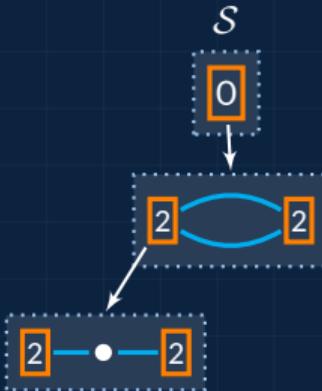
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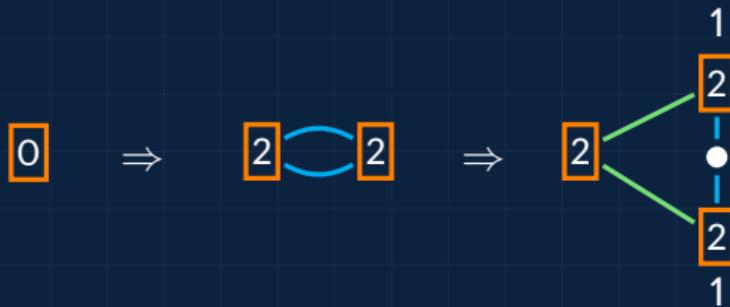
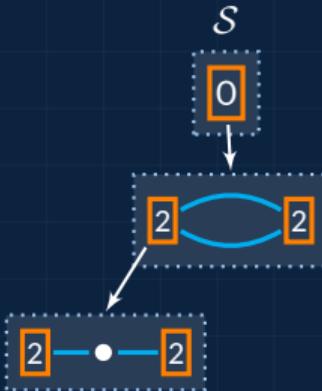
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- ◆ $[0] \rightarrow [2] \xrightarrow{\text{double loop}} [2]$
- ◆ $[2] \rightarrow [2] - \bullet - [2]$
- ◆ $[2] \rightarrow \bullet$



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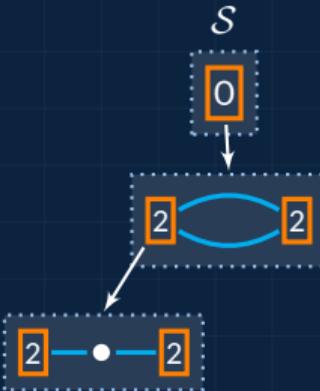
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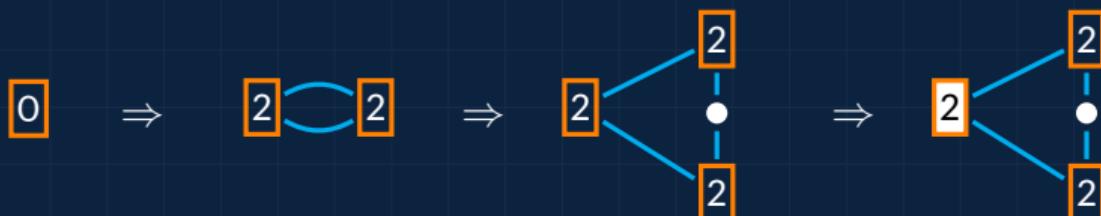
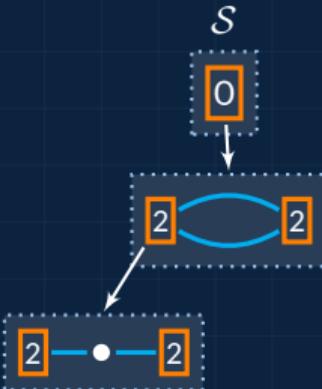
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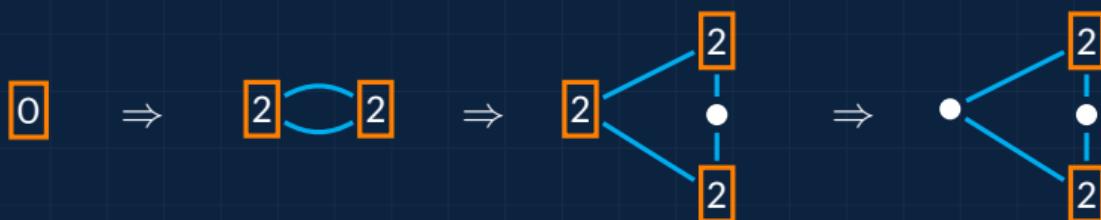
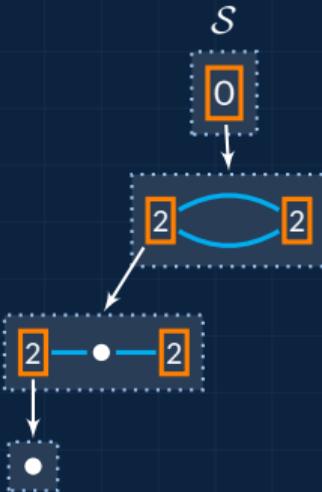
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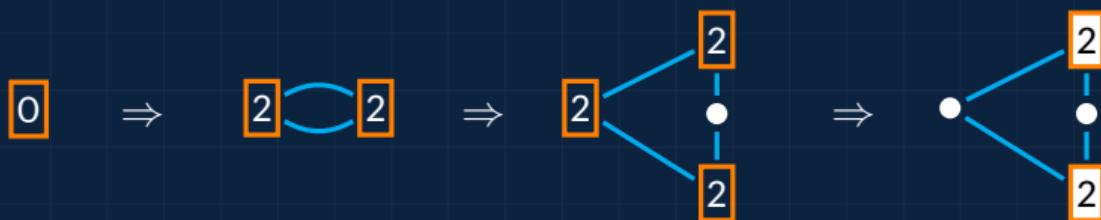
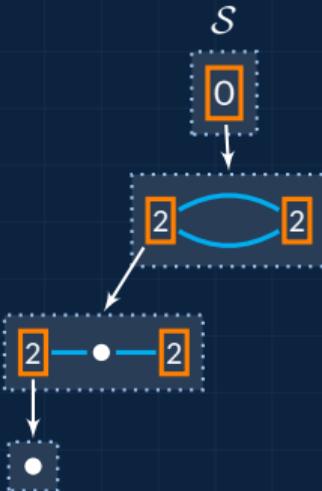
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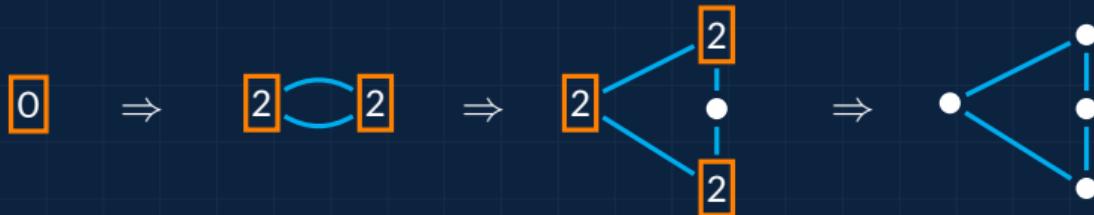
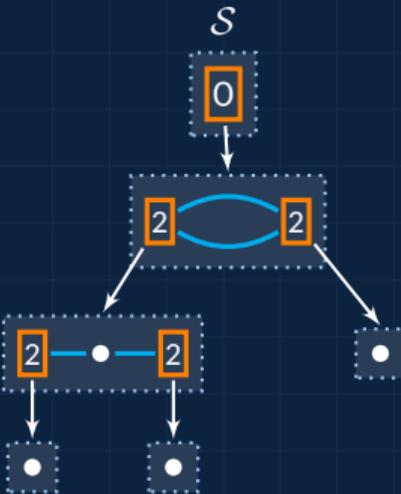
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Context Free Graph Grammars

NCE Production Rules

- $0 \rightarrow [2] \xrightarrow{\text{loop}} [2]$
- $[2] \rightarrow [2] - \bullet - [2]$
- $[2] \rightarrow \bullet$



Graph Grammar Pipeline

Scalable and
Interpretable Graph
Modeling with
Graph Grammars

bit.ly/satyaki-slides

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String Grammars

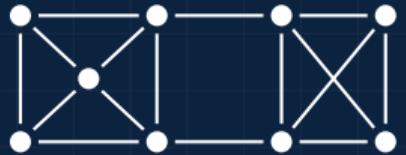
Graph Grammars

Grammar Extraction

Graph Generation

Infinity Mirror Test

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Input Graph H



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Grammar Extractor



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Input Graph H



Bottom-up, Clustering-based

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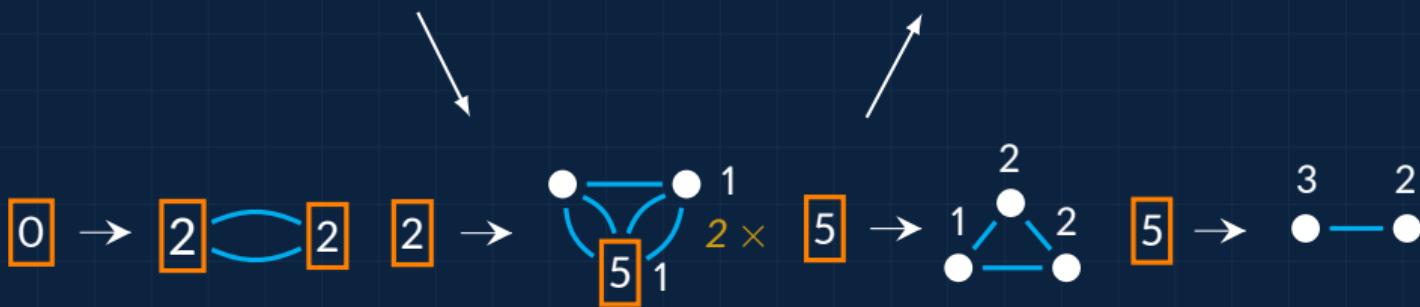
Wrapping Up



Input Graph H

Grammar Extractor

Graph Generator



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Random, Attribute-aware



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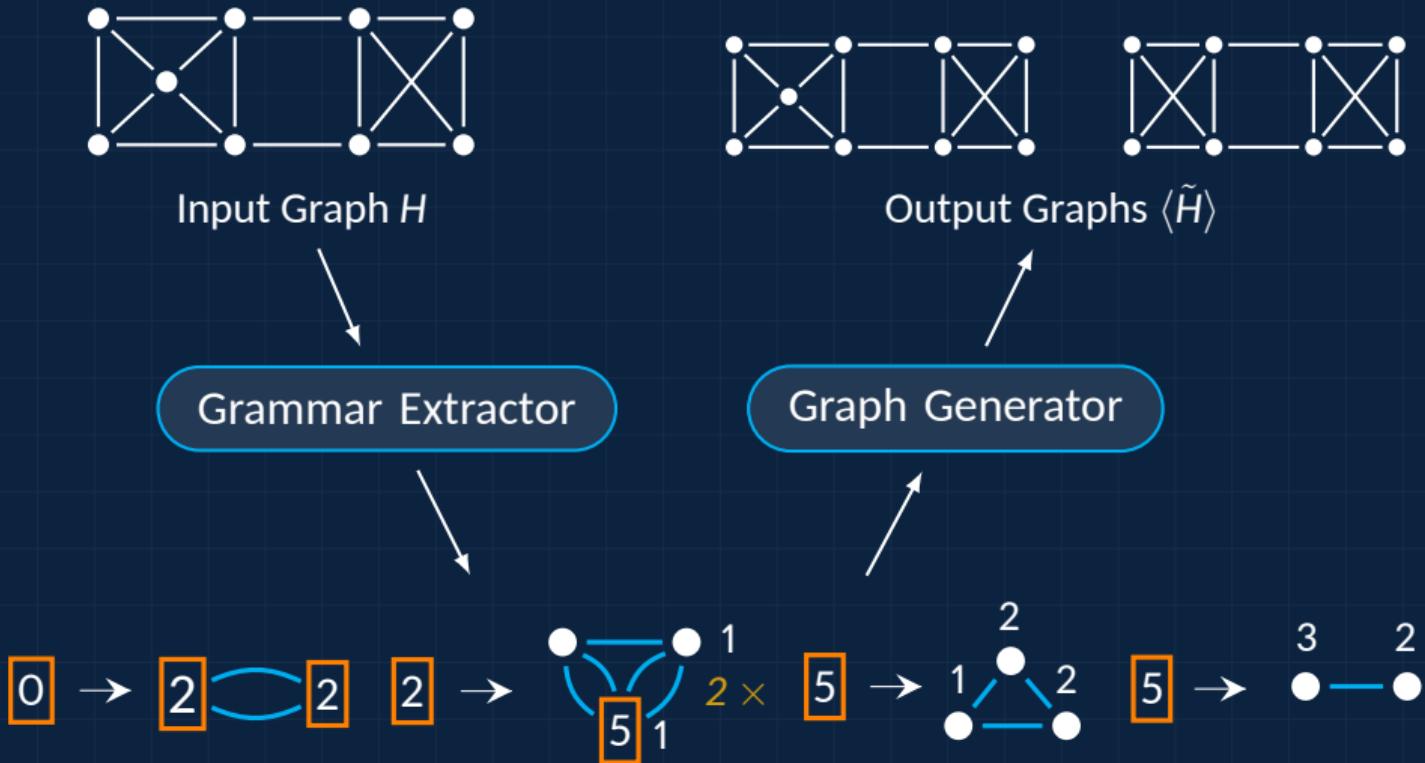
Graph Grammars

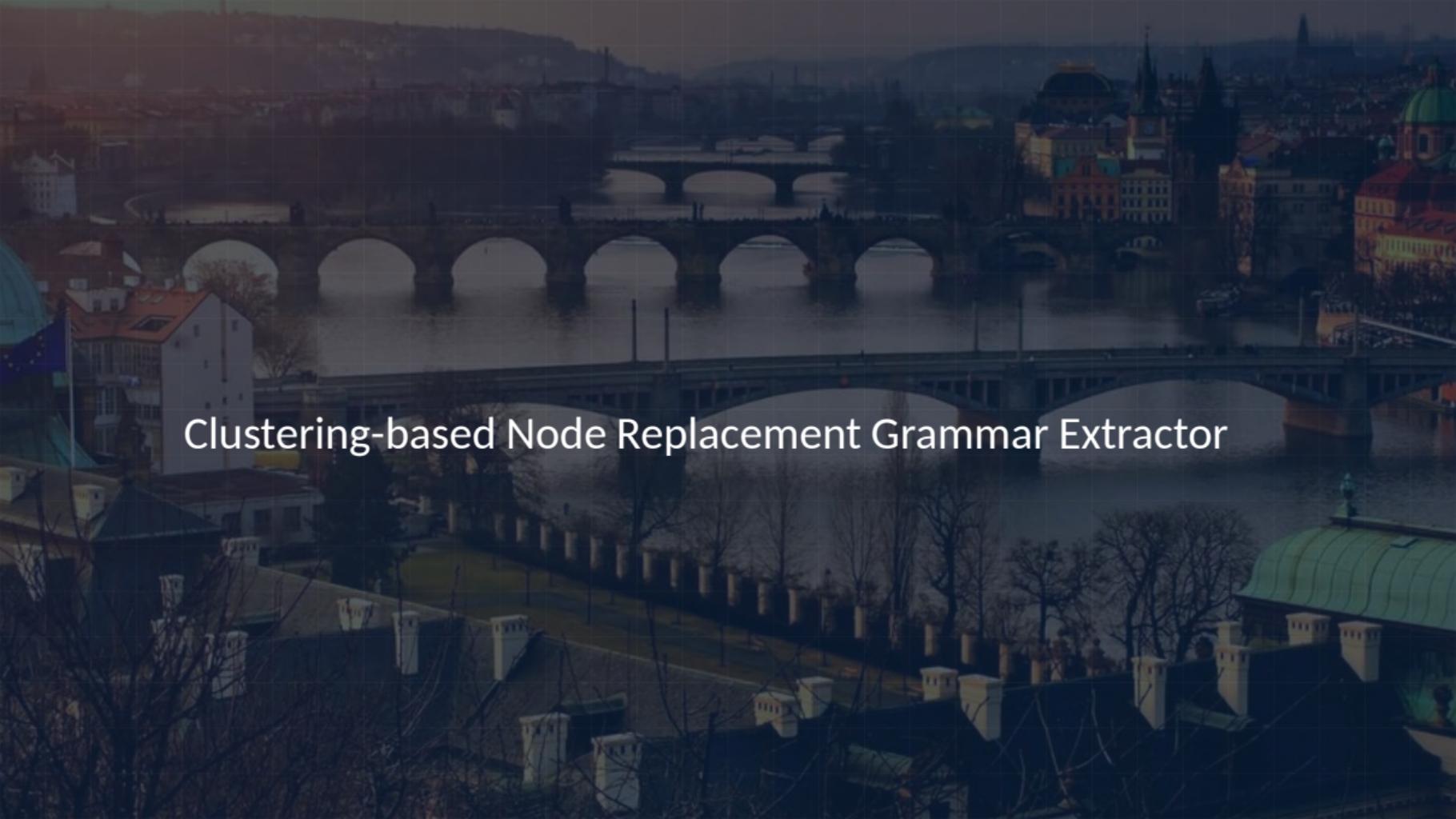
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The background image is a dark, atmospheric photograph of a city skyline at dusk or night. It features a bridge with multiple arches spanning a body of water, with lights reflecting on the water's surface. In the foreground, there are silhouettes of bare trees and the rooftops of buildings. The overall mood is moody and architectural.

Clustering-based Node Replacement Grammar Extractor

Hierarchical Graph Clustering

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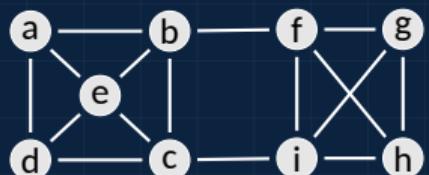
Rule Extraction

Attributed Graphs

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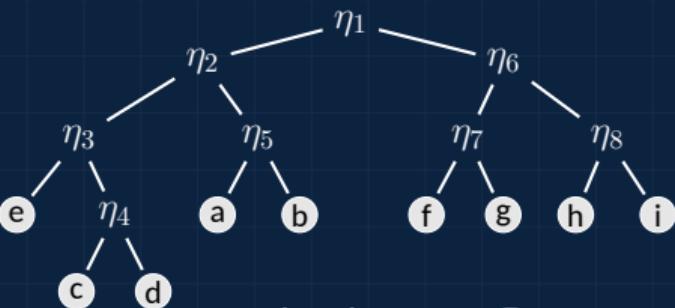
Infinity Mirror Test

Wrapping Up



Graph H

Leiden
Louvain
Spectral
MinCut



A dendrogram \mathcal{D}

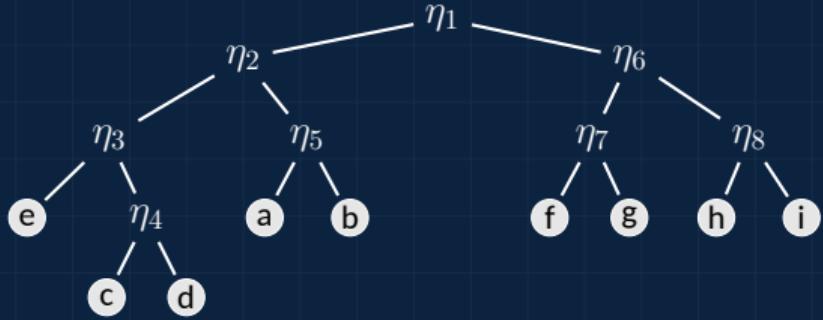


Extracting NCE Rules I

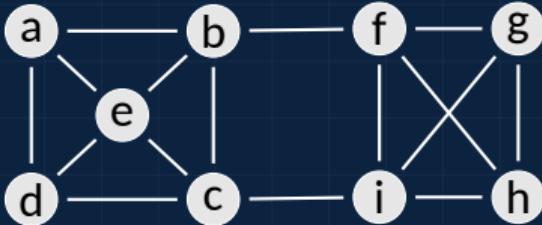
$(\mu = 3)$

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Current Dendrogram \mathcal{D}



Current Graph H

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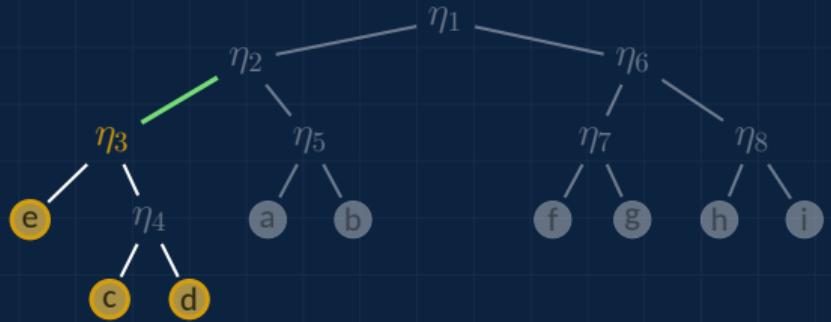
Infinity Mirror Test

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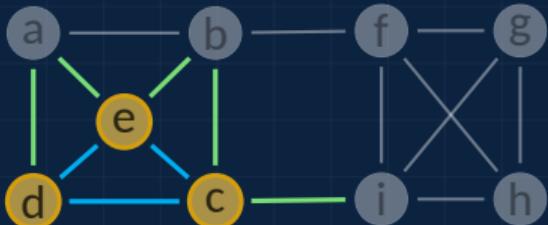


Extracting NCE Rules I

$(\mu = 3)$



Current Dendrogram \mathcal{D}



Current Graph H



Extracted NCE Rule

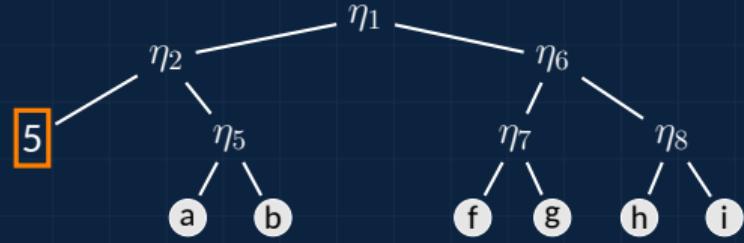


Extracting NCE Rules I

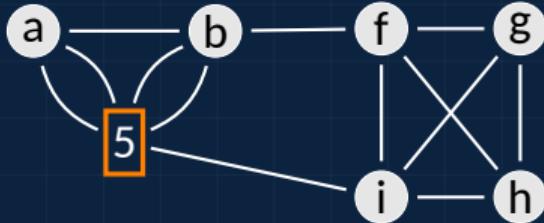
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Updated Dendrogram \mathcal{D}'



Updated Graph H'

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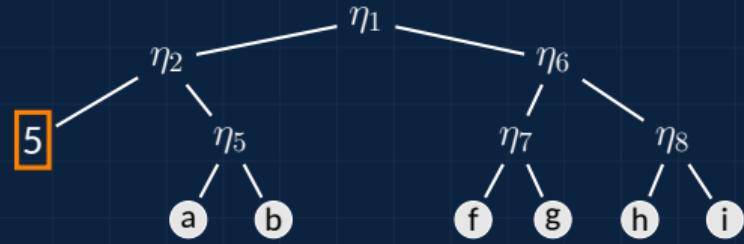
Infinity Mirror Test

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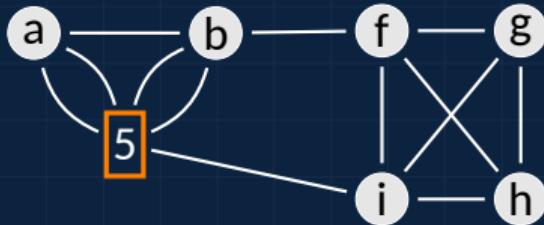


Extracting NCE Rules II

$(\mu = 3)$



Current Dendrogram \mathcal{D}



Current Graph H

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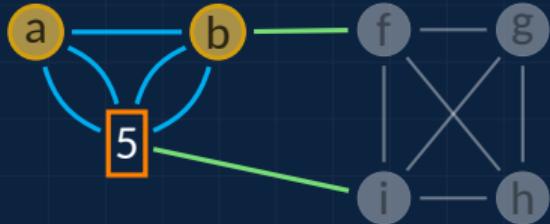


Extracting NCE Rules II

($\mu = 3$)



Current Dendrogram \mathcal{D}



Current Graph H



Extracted NCE Rule

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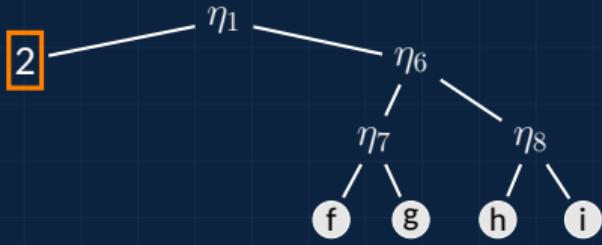


Extracting NCE Rules II

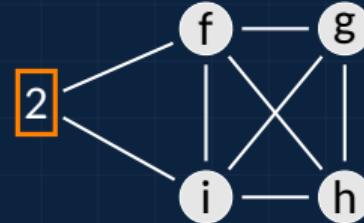
$$(\mu = 3)$$

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Updated Dendrogram \mathcal{D}'



Updated Graph H'

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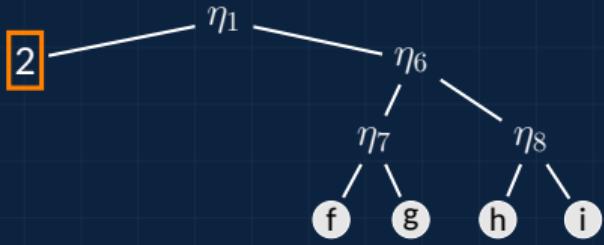
Graph Generation

Infinity Mirror Test

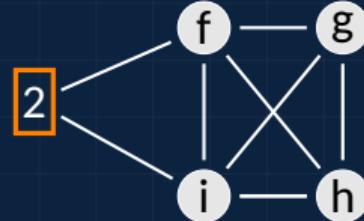
Wrapping Up

Extracting NCE Rules III

$(\mu = 3)$



Current Dendrogram \mathcal{D}



Current Graph H



Extracting NCE Rules III

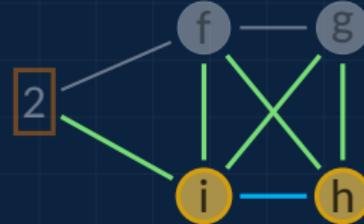
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Current Dendrogram \mathcal{D}



Current Graph H



Extracted NCE Rule

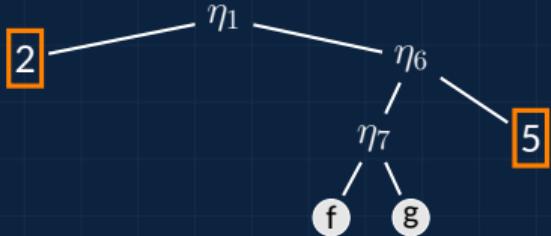


Extracting NCE Rules III

$(\mu = 3)$

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Updated Dendrogram \mathcal{D}'



Updated Graph H'

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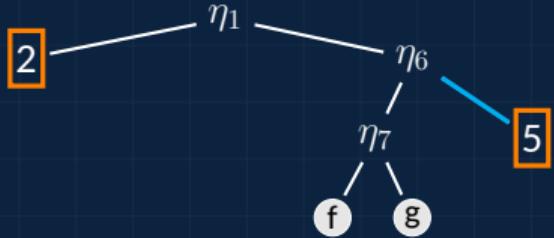


Extracting NCE Rules IV

($\mu = 3$)

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Current Dendrogram \mathcal{D}



Current Graph H

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Extracting NCE Rules IV

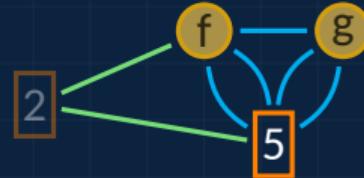
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Current Dendrogram \mathcal{D}



Current Graph H



Extracted NCE Rule



Extracting NCE Rules IV

($\mu = 3$)

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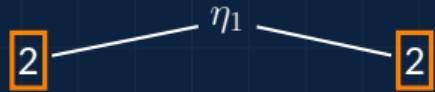
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Updated Dendrogram \mathcal{D}'



Updated Graph H'



Extracted NCE Rule





Current Dendrogram \mathcal{D}

$(\mu = 3)$



Current Graph H

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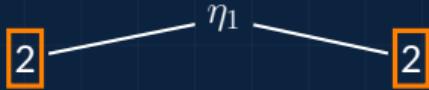


Extracting NCE Rules V

($\mu = 3$)

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Current Dendrogram \mathcal{D}



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($\mu = 3$)

0

0

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0



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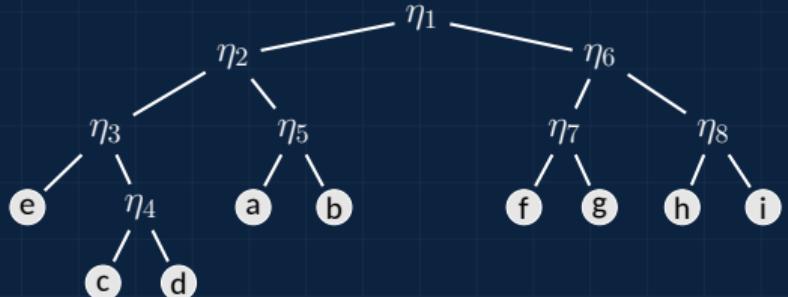
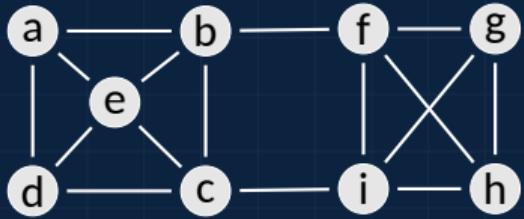
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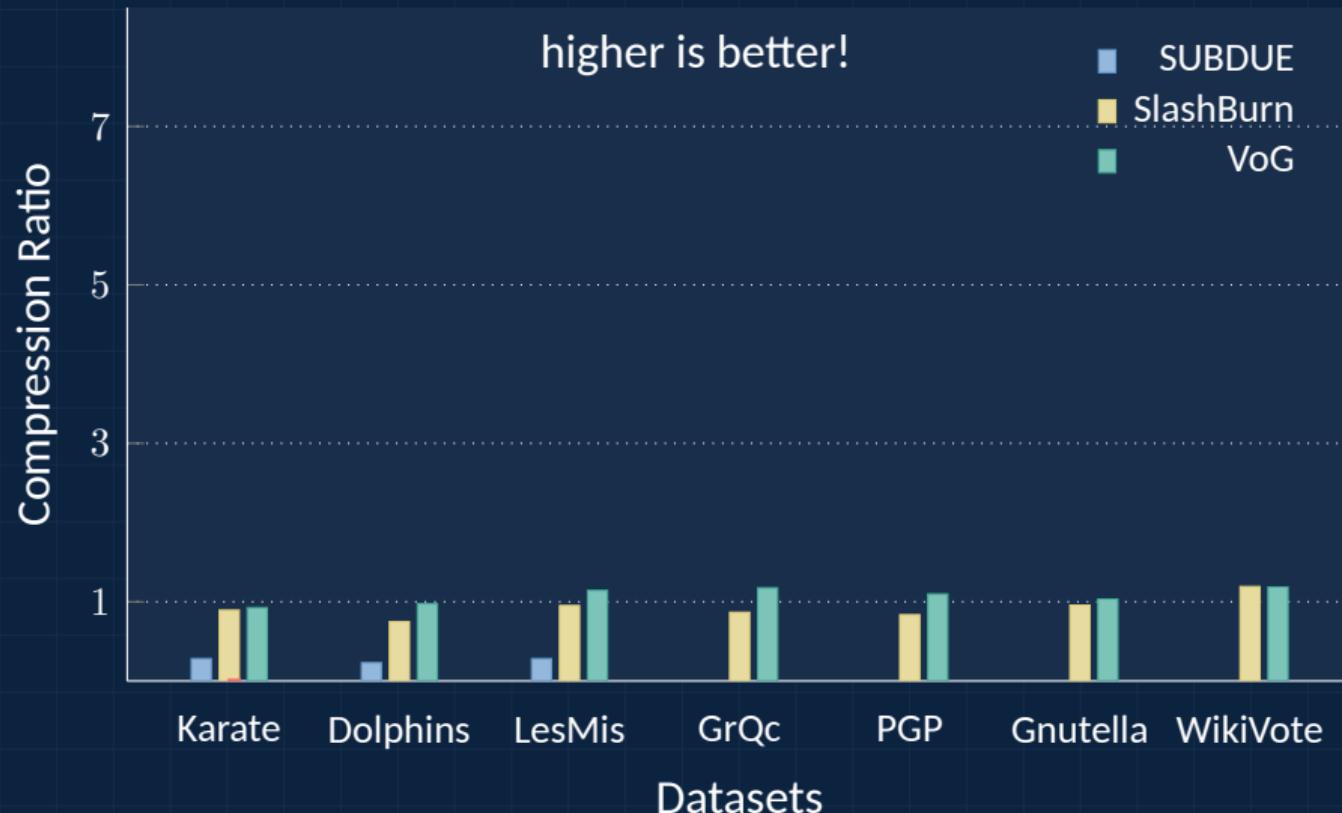
Extracted NCE Grammar



Extracted NCE rules

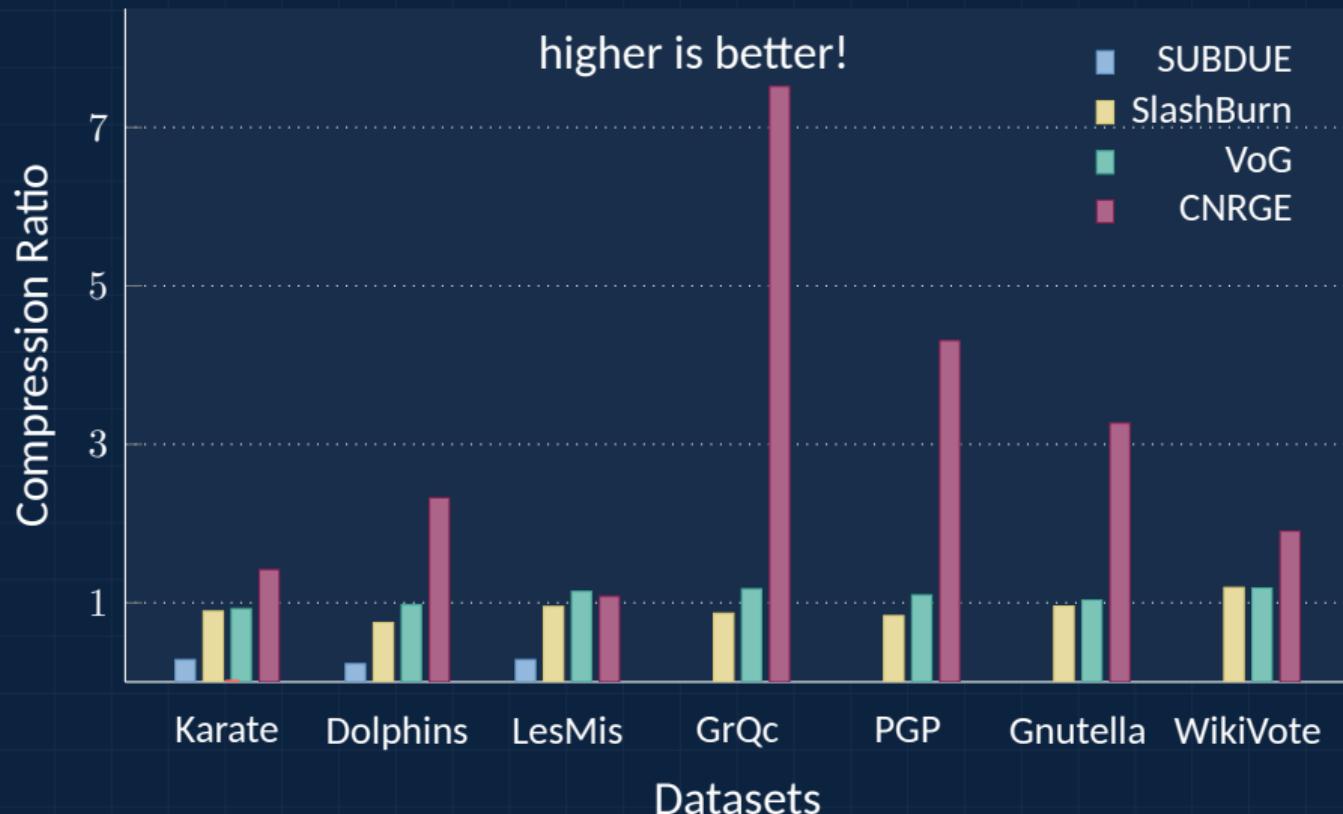


Model Size Comparison





Model Size Comparison



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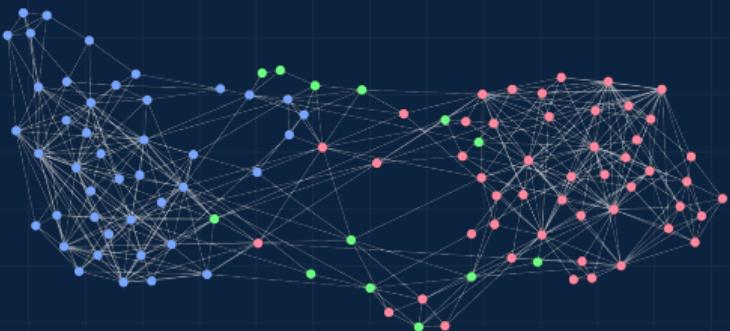
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Homophily in PolBooks graph



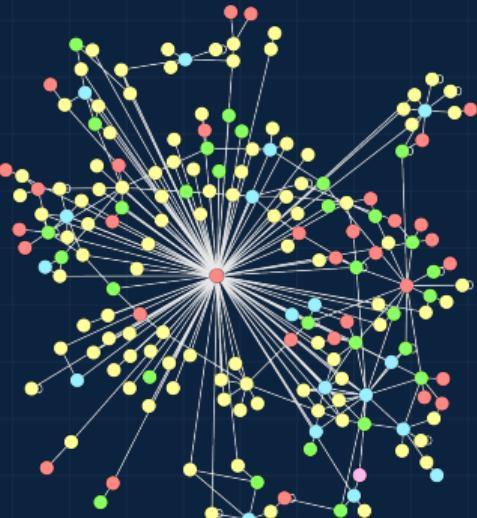
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Homophily in PolBooks graph



Heterophily in Texas CS website



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Real-world Grammars: Cora



Node Distribution



Edge Mixing Matrix

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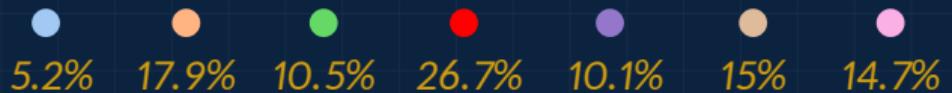
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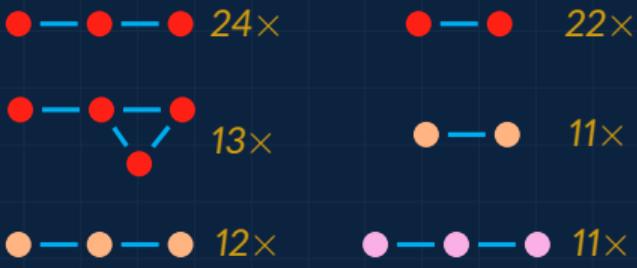
Real-world Grammars: Cora



Node Distribution



Edge Mixing Matrix



6 Most-frequent Rule RHSs

Real-world Grammars: Chameleon

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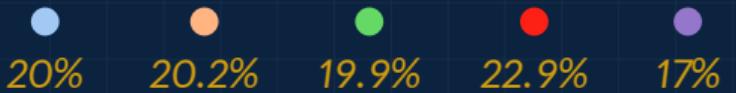
Graph Generation

Infinity Mirror Test

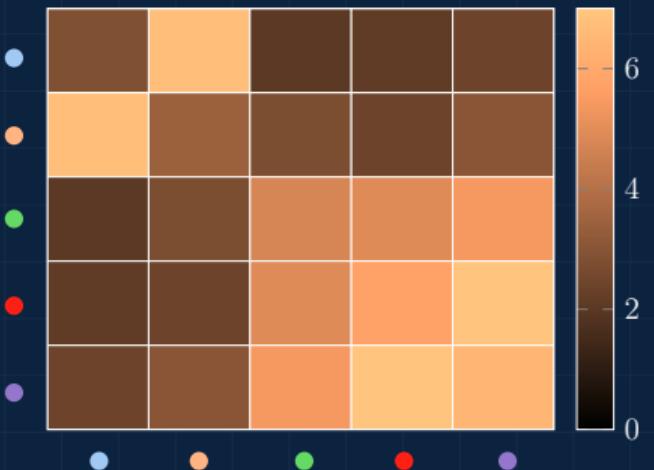
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Node Distribution



Edge Mixing Matrix

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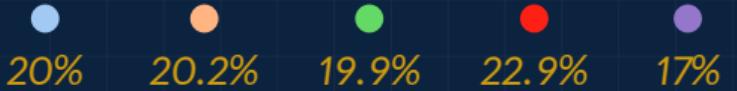
Rule Extraction

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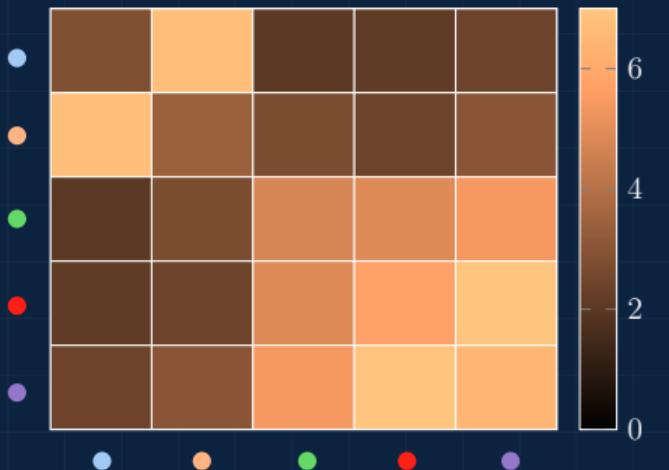
Graph Generation

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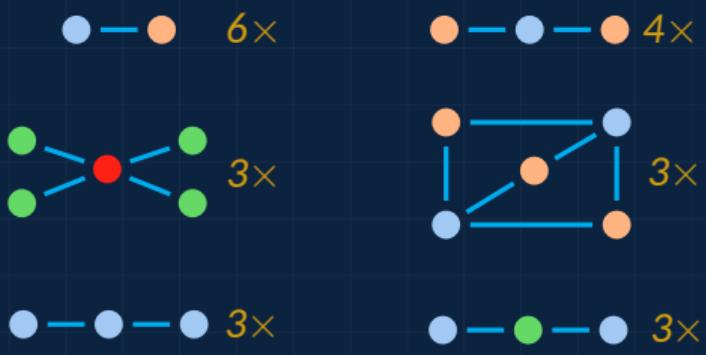
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Node Distribution

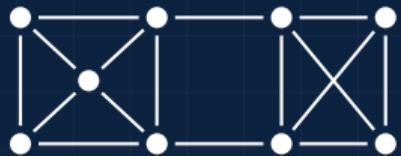


Edge Mixing Matrix



6 Most-frequent Rule RHSs

Graph Grammar Pipeline Revisited



Input Graph H



Output Graphs $\langle \tilde{H} \rangle$

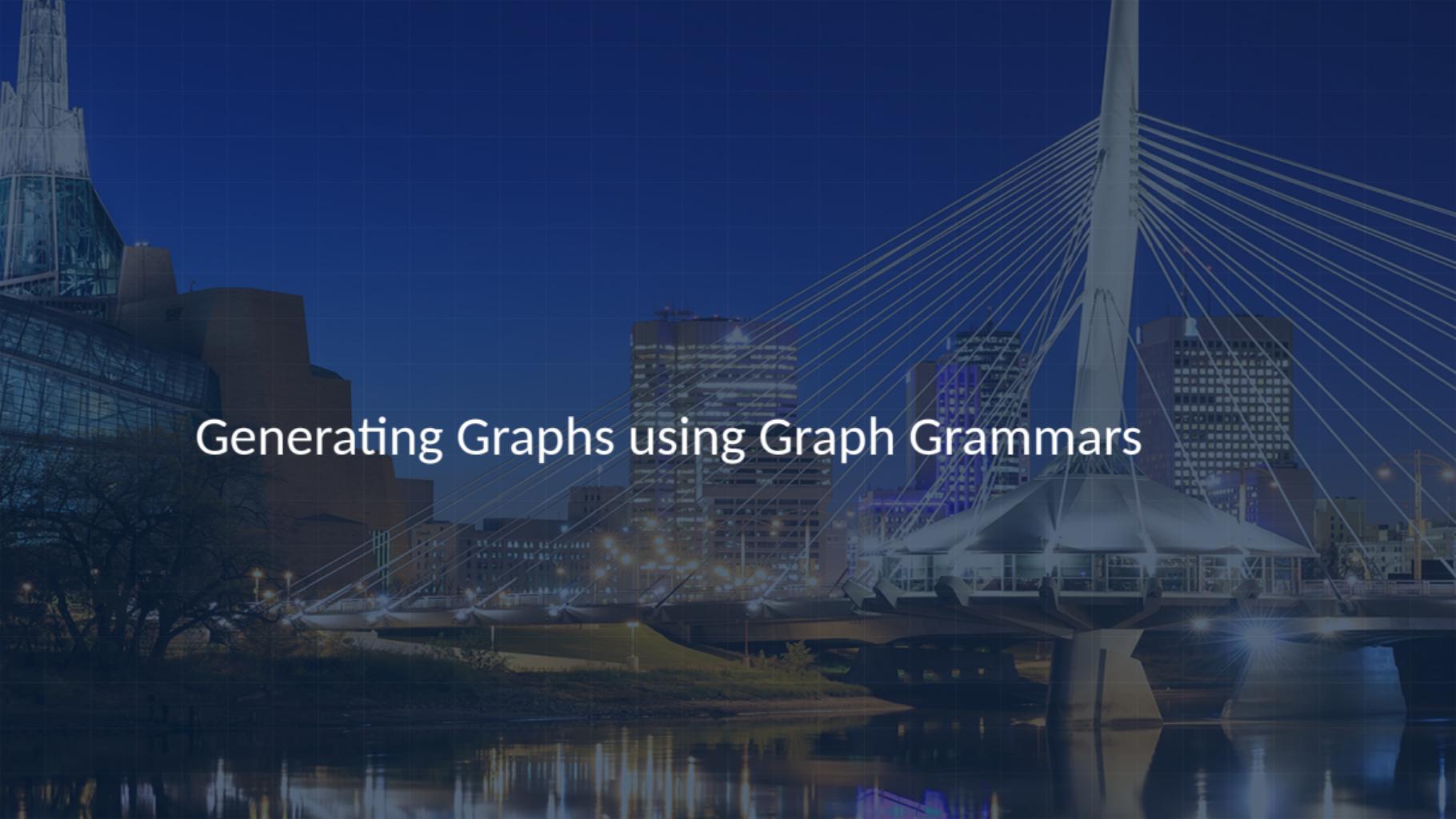


Grammar Extractor

Graph Generator



Generating Graphs using Graph Grammars



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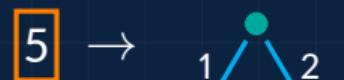
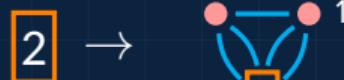
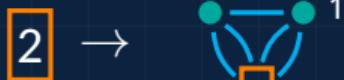
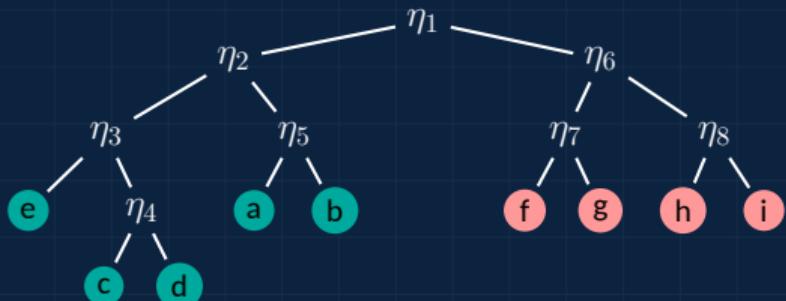
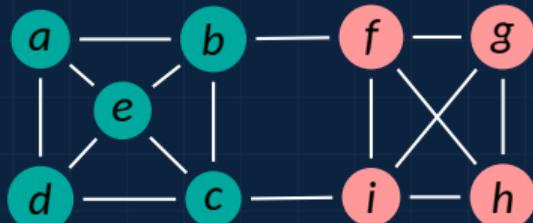
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Attributed NCE Grammars



Extracted ANCE rules

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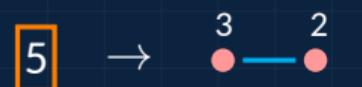
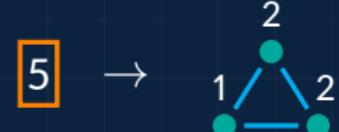
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Generating Graphs from an ANCE I



Extracted ANCE Rules

\mathcal{S}
0

Current Graph H'

Next Graph \hat{H}

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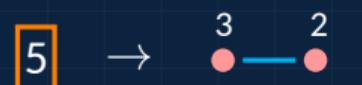
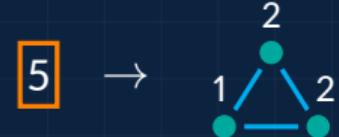
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Extracted ANCE Rules

\mathcal{S}
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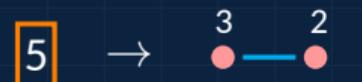
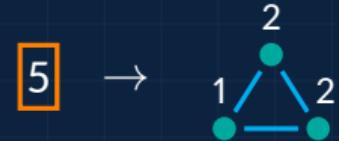
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Extracted ANCE Rules

\mathcal{S}
[0]

Current Graph H'



Next Graph \hat{H}

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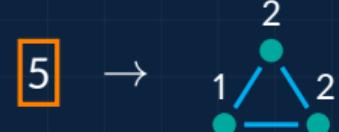
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Generating Graphs from an ANCE II



Extracted ANCE Rules



Current Graph H'

Next Graph \hat{H}



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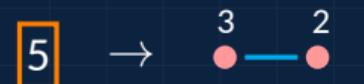
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Generating Graphs from an ANCE II



Extracted ANCE Rules



Current Graph H'



Next Graph \hat{H}

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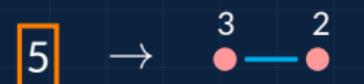
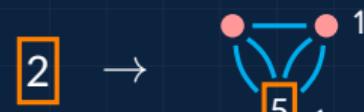
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Current Graph H'



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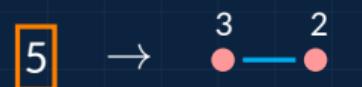
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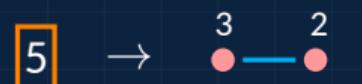
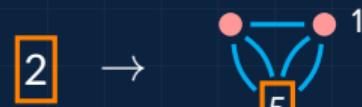
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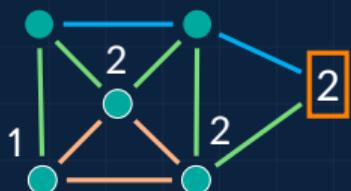
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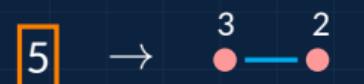
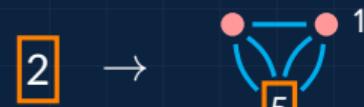
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$$\boxed{0} \rightarrow \boxed{2} \text{ } \boxed{2}$$

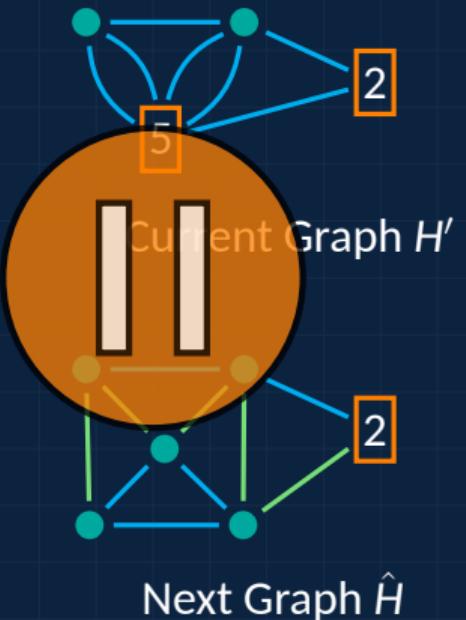
$$\boxed{2} \rightarrow \begin{array}{c} \text{---} \\ | \\ \text{---} \\ | \\ \text{---} \end{array} \boxed{5} \boxed{1}$$

$$\boxed{2} \rightarrow \begin{array}{c} \text{---} \\ | \\ \text{---} \\ | \\ \text{---} \end{array} \boxed{5} \boxed{1}$$

$$\boxed{5} \rightarrow \begin{array}{c} \text{---} \\ | \\ \text{---} \\ | \\ \text{---} \end{array} \boxed{1} \boxed{2}$$

$$\boxed{5} \rightarrow \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \boxed{3} \boxed{2}$$

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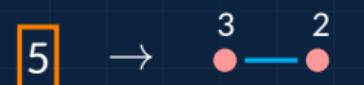
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Generating Graphs from an ANCE IV

$$\boxed{0} \rightarrow \boxed{2} \text{---} \boxed{2}$$

$$\boxed{2} \rightarrow \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \boxed{5} \quad \boxed{1}$$

$$\boxed{2} \rightarrow \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \boxed{5} \quad \boxed{1}$$

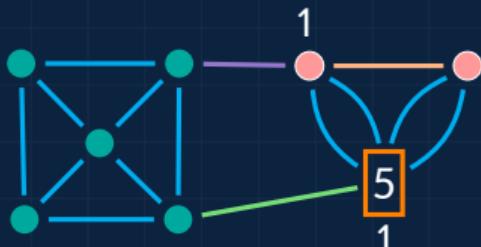
$$\boxed{5} \rightarrow \begin{array}{c} 2 \\ | \\ \text{---} \end{array} \quad \boxed{1} \quad \boxed{2}$$

$$\boxed{5} \rightarrow \begin{array}{c} 3 \\ | \\ \text{---} \end{array} \quad \boxed{2}$$

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Generating Graphs from an ANCE IV

$$\boxed{0} \rightarrow \boxed{2} \text{---} \boxed{2}$$

$$\boxed{2} \rightarrow \boxed{5} \text{---} \boxed{1}$$

$$\boxed{2} \rightarrow \boxed{5} \text{---} \boxed{1}$$

$$\boxed{5} \rightarrow \boxed{1} \text{---} \boxed{2}$$

$$\boxed{5} \rightarrow \boxed{3} \text{---} \boxed{2}$$

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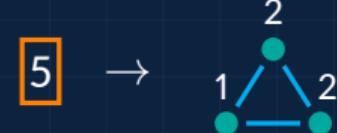
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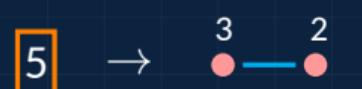
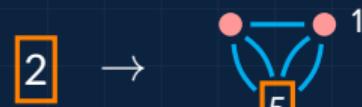
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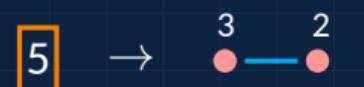
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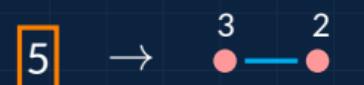
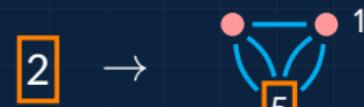
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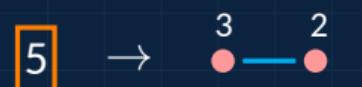
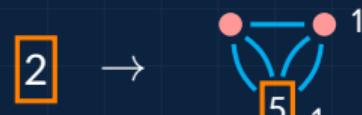
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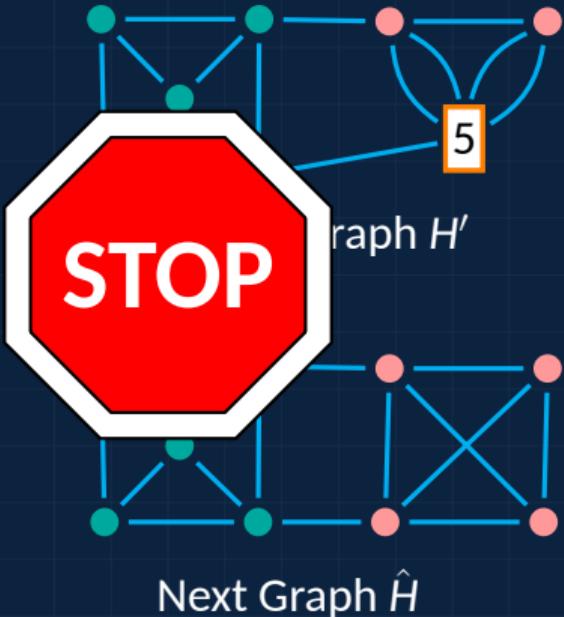
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Language of Graphs Described by a Grammar

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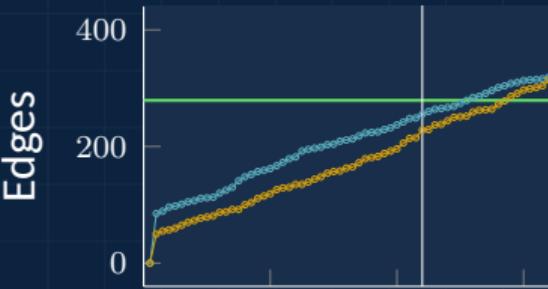
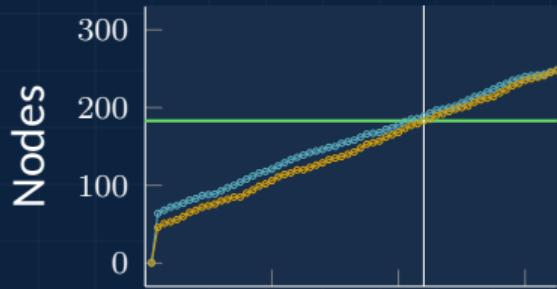
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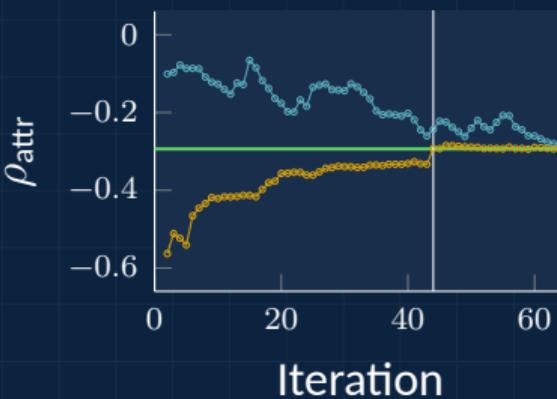
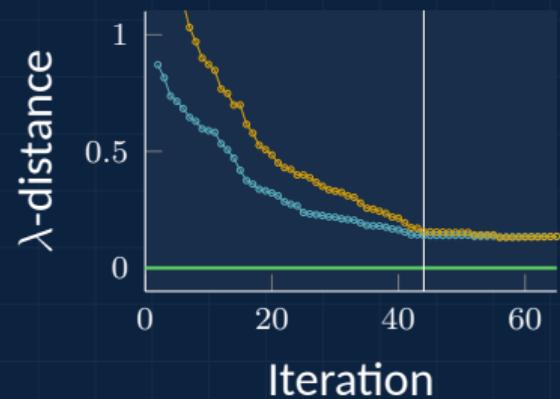
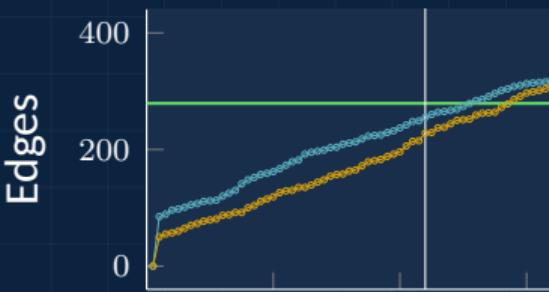
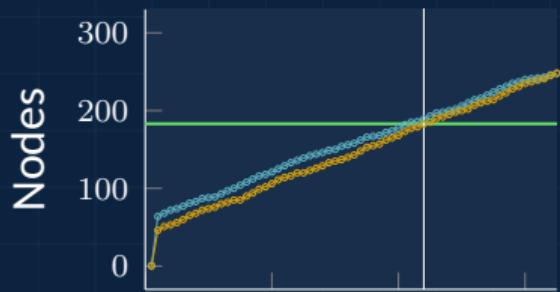
Examining the Graph Growth Process



—●— Whole Graph —●— Terminal Graph —— Input Graph Metric —— Input Graph Size

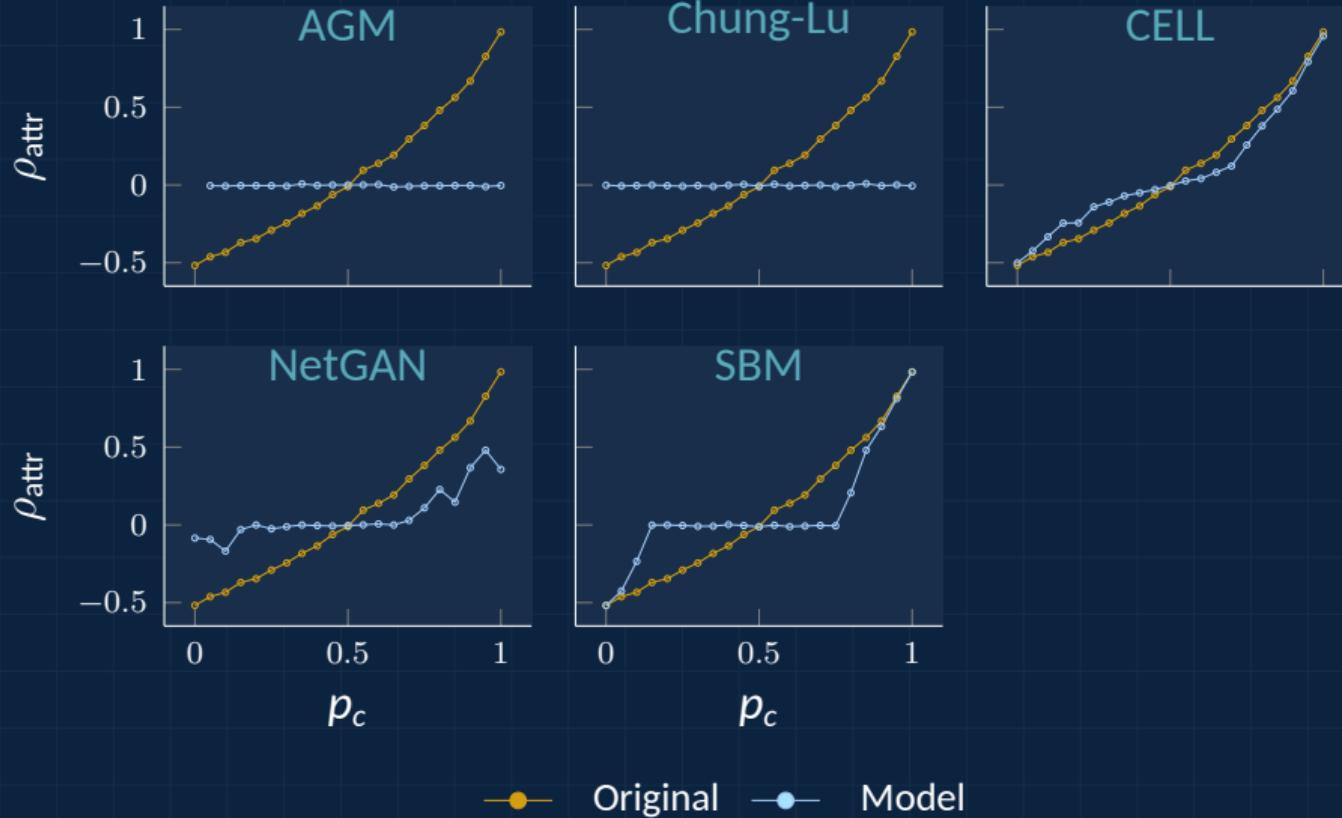


Examining the Graph Growth Process



—●— Whole Graph —●— Terminal Graph —— Input Graph Metric —— Input Graph Size

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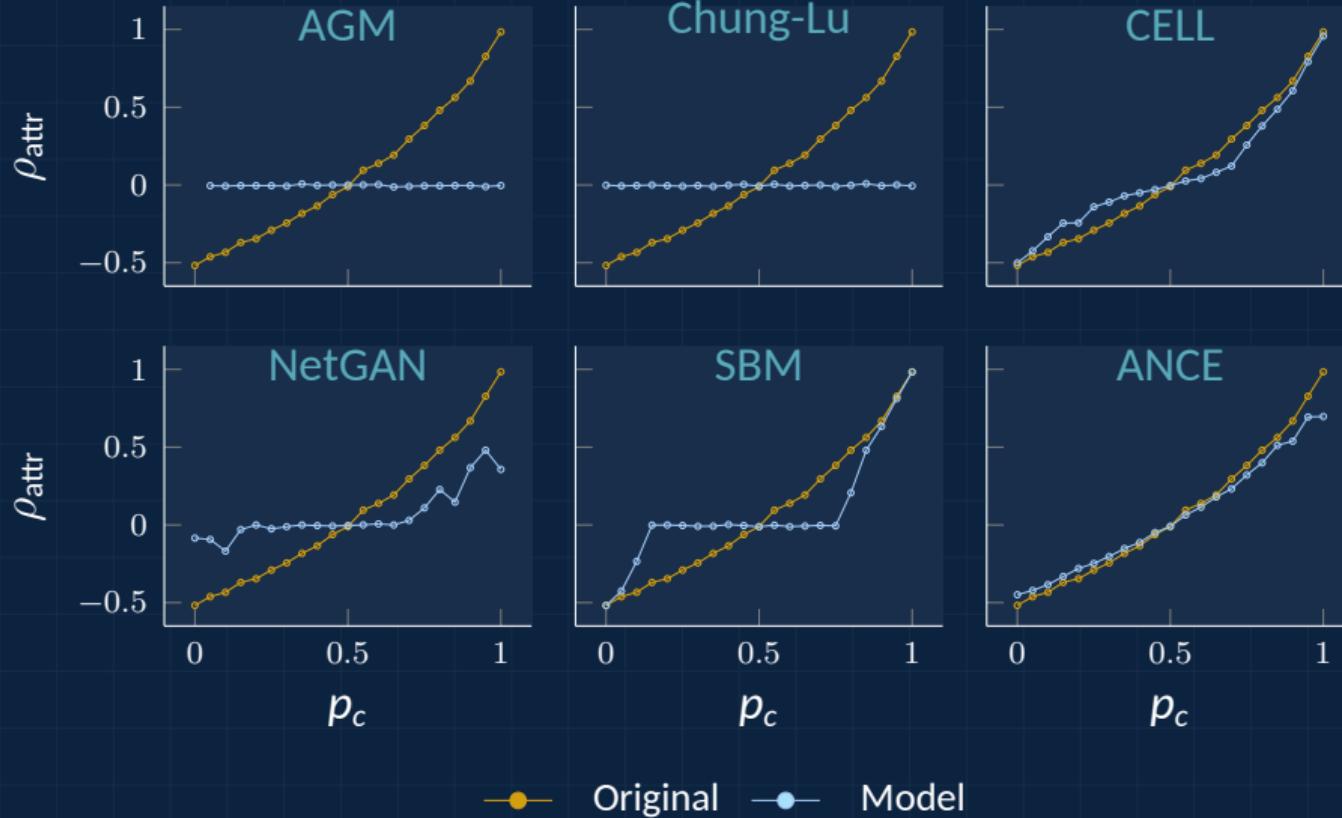
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Main Takeaways

- ◆ *Simple* and *interpretable* formalism from formal languages

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Main Takeaways

- ◆ *Simple* and *interpretable* formalism from formal languages
- ◆ *Scalable* to medium-large graphs

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Main Takeaways

- ◆ *Simple* and *interpretable* formalism from formal languages
- ◆ *Scalable* to medium-large graphs
- ◆ *Faithful* graph generation: topology and attribute

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Main Takeaways

- ◆ *Simple* and *interpretable* formalism from formal languages
- ◆ *Scalable* to medium-large graphs
- ◆ *Faithful* graph generation: topology and attribute

Shortcomings

- ◆ Dependence on the *dendrogram*



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Main Takeaways

- ◆ *Simple* and *interpretable* formalism from formal languages
- ◆ *Scalable* to medium-large graphs
- ◆ *Faithful* graph generation: topology and attribute

Shortcomings

- ◆ Dependence on the *dendrogram*
- ◆ Rule extraction process is *non-deterministic*

The background image shows a coastal city at sunset. On the left, a long stretch of beach meets the ocean. In the center, a dense cluster of modern skyscrapers rises from a mix of lower residential and commercial buildings. To the right, a river or waterway flows through a more residential area with smaller houses and trees. The sky is filled with warm, orange and yellow clouds, suggesting a sunrise or sunset. A faint grid pattern is visible across the entire image.

The Infinity Mirror Test for Graph Models

Infinity Mirror Test for Duck Vader

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Key Idea

Forcing a model to fit and re-fit the generated graphs **amplifies** biases



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Key Idea

Forcing a model to fit and re-fit the generated graphs **amplifies** biases

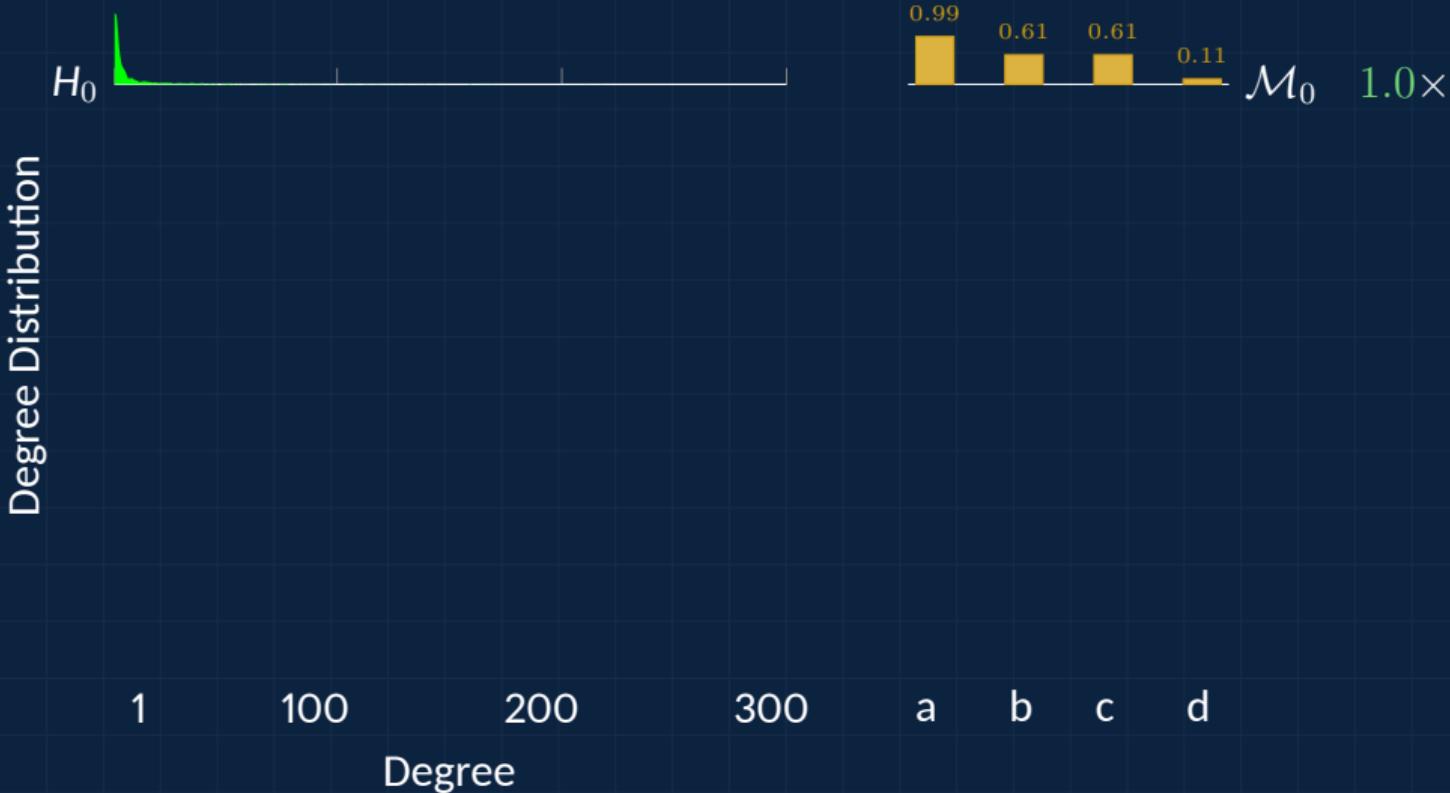
Evaluation Plan

Compare H_i and H_0 to observe
incremental degradation



Kronecker Degree Distribution Ridge Plot

$$\mathcal{M} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$



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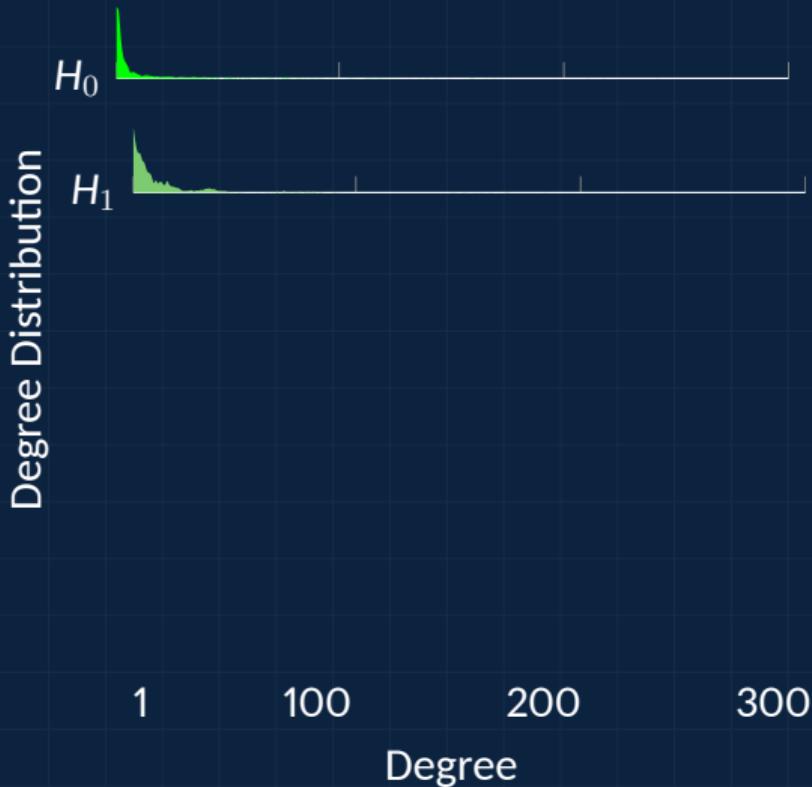
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Kronecker Degree Distribution Ridge Plot

$$\mathcal{M} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$



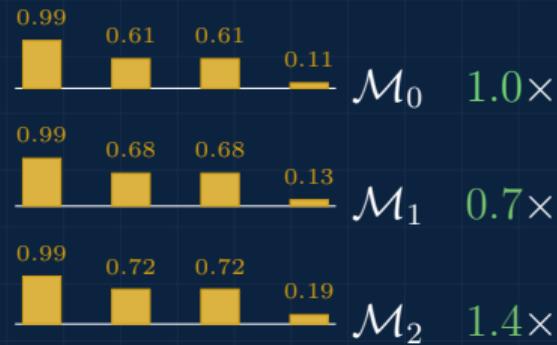
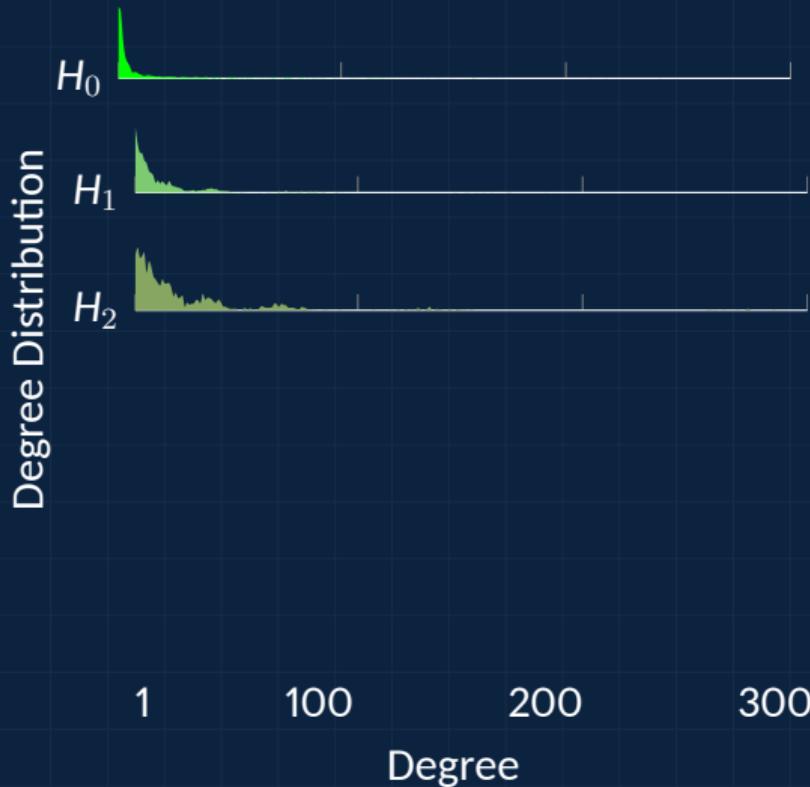
a b c d





Kronecker Degree Distribution Ridge Plot

$$\mathcal{M} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

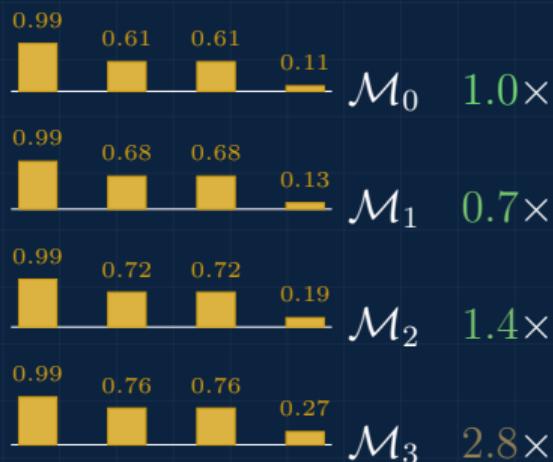
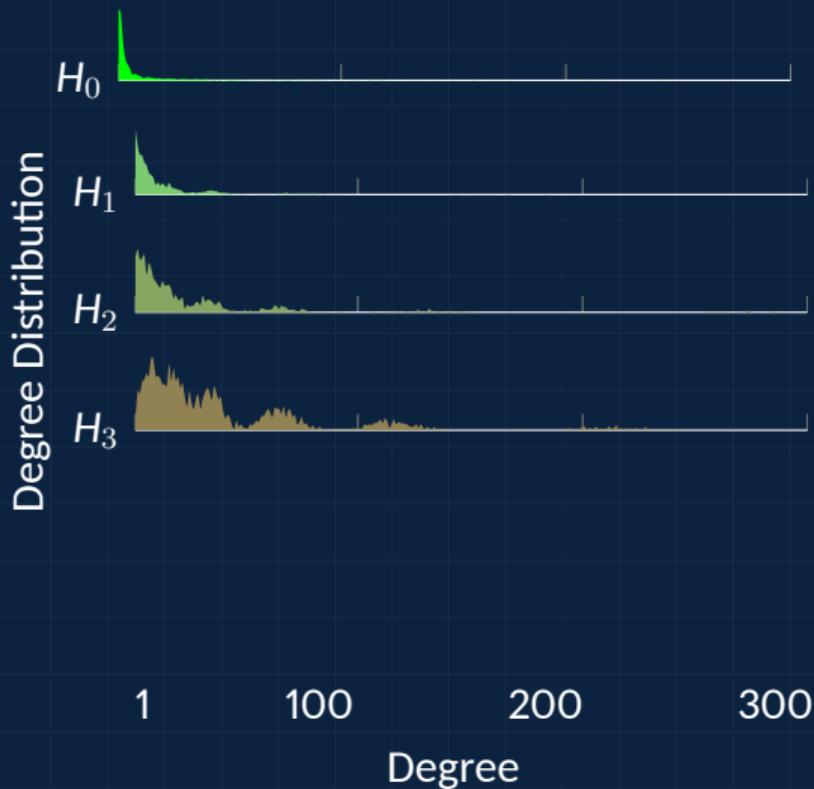


a b c d



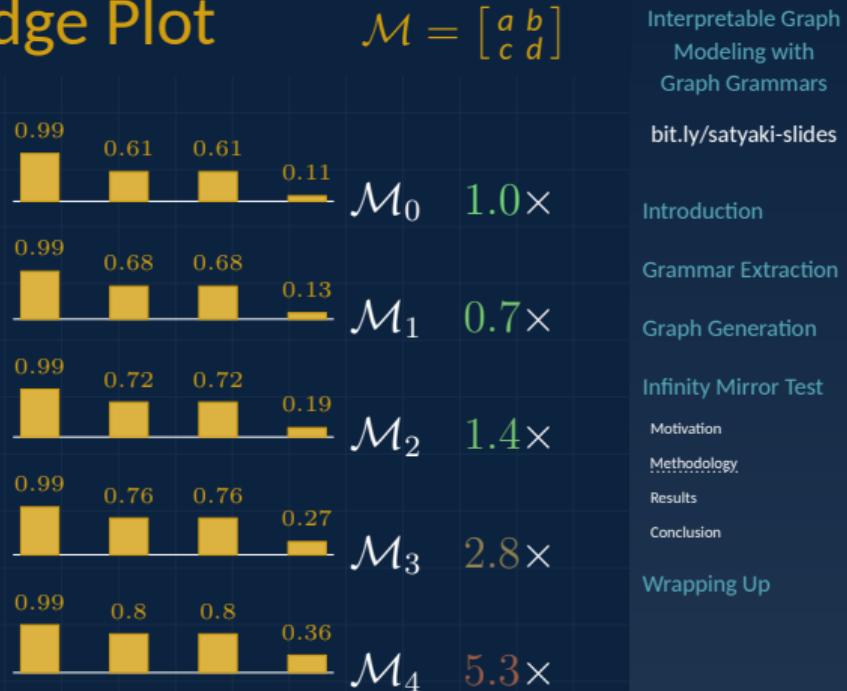
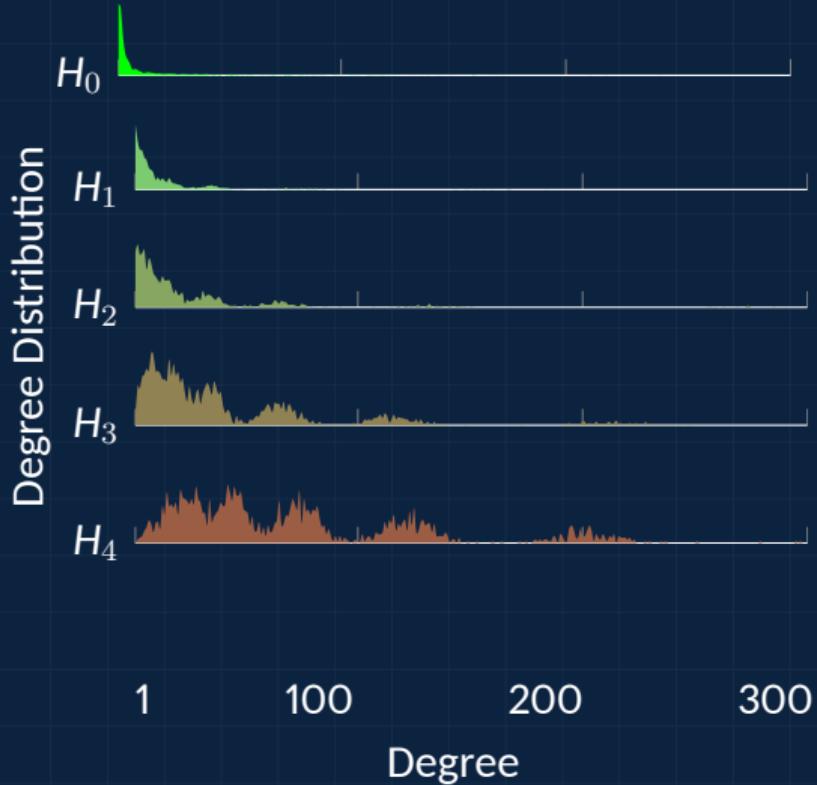
Kronecker Degree Distribution Ridge Plot

$$\mathcal{M} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$



a b c d

Kronecker Degree Distribution Ridge Plot



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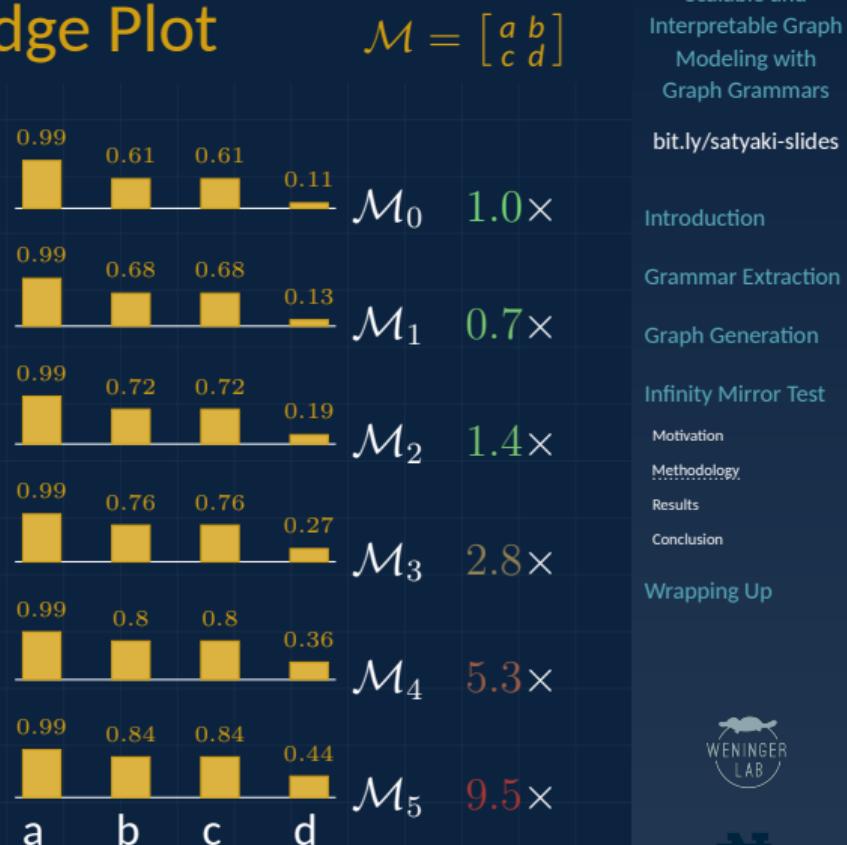
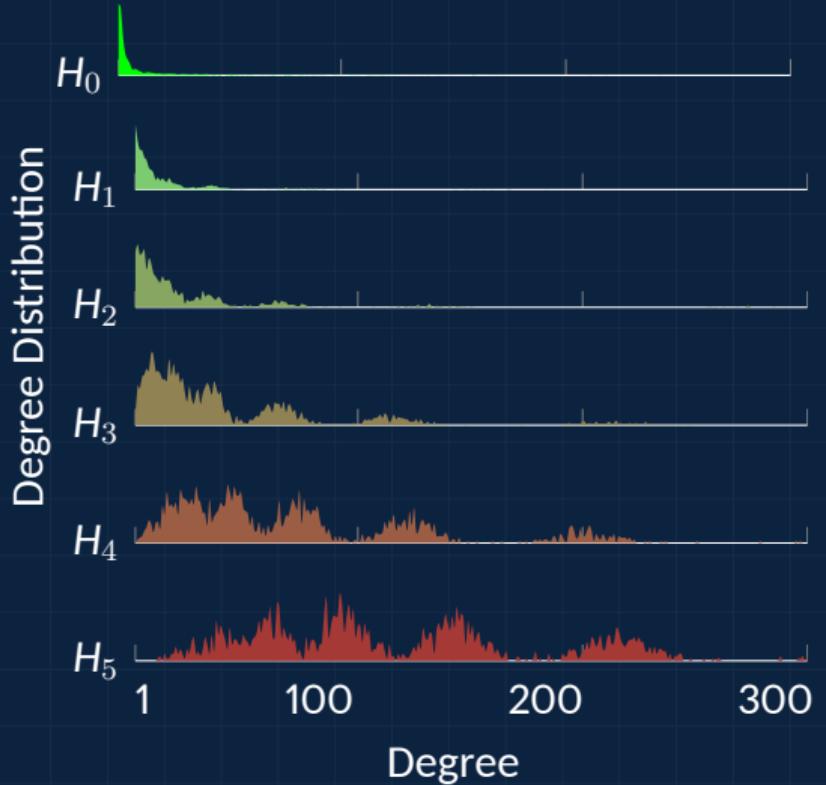
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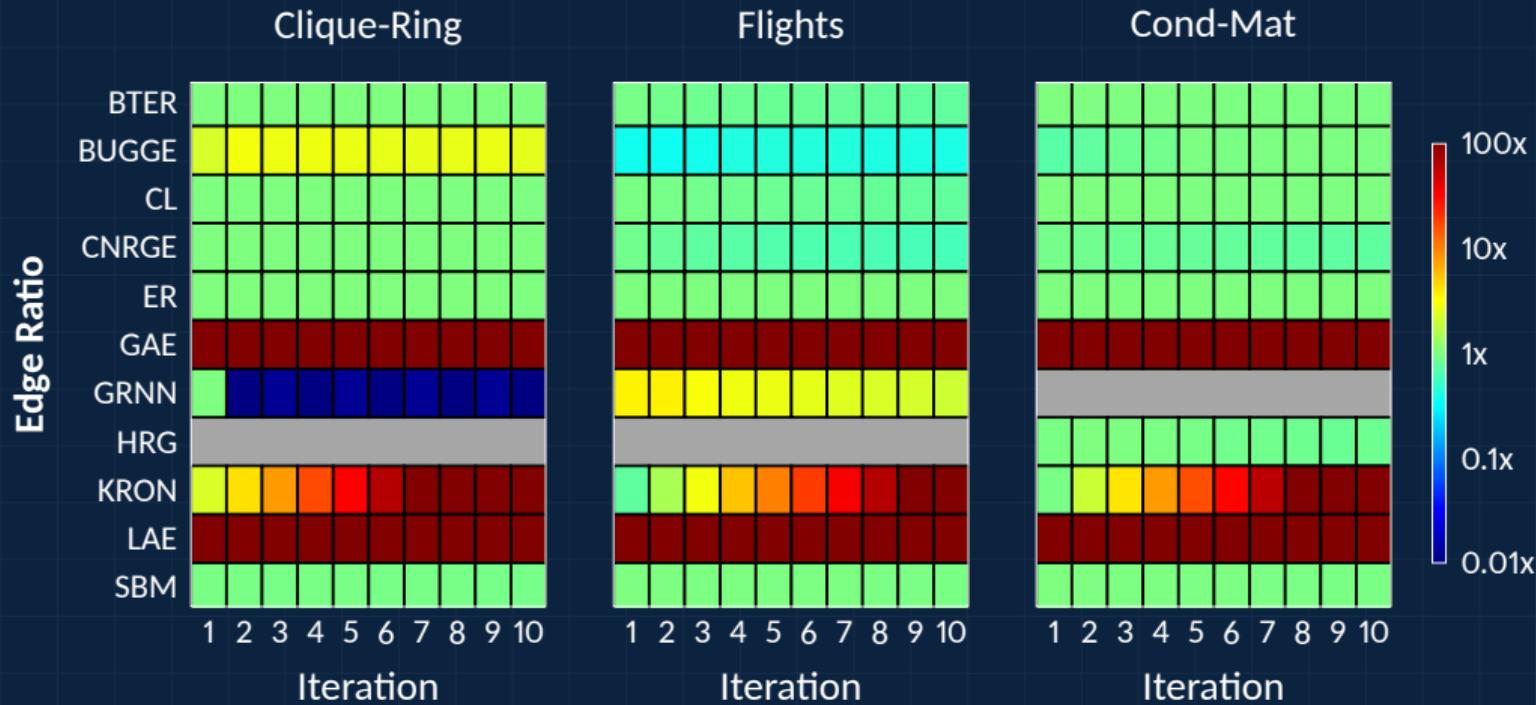
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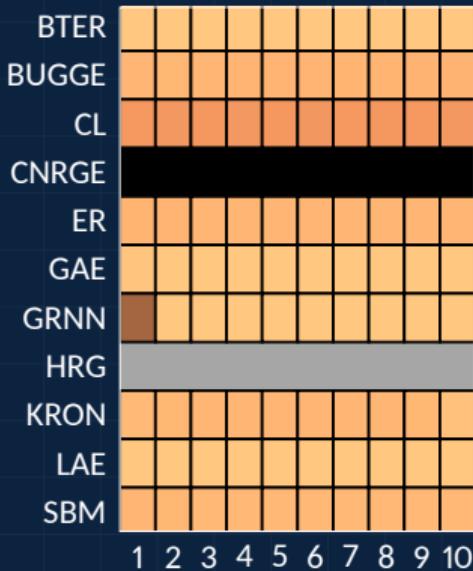
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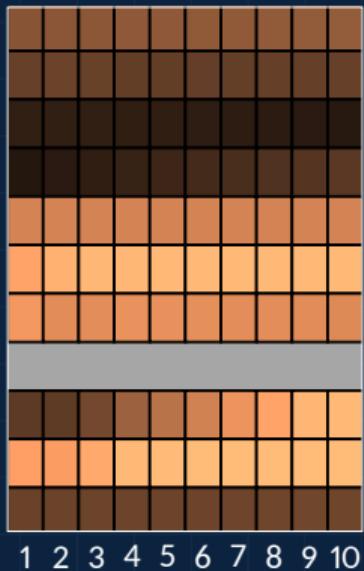
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PageRank Divergence

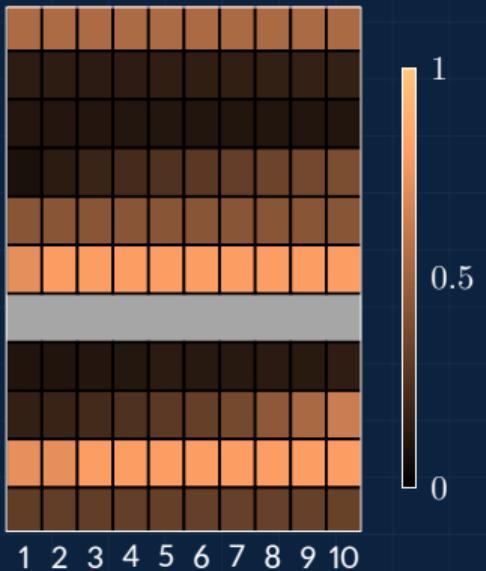
Clique-Ring



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Cond-Mat



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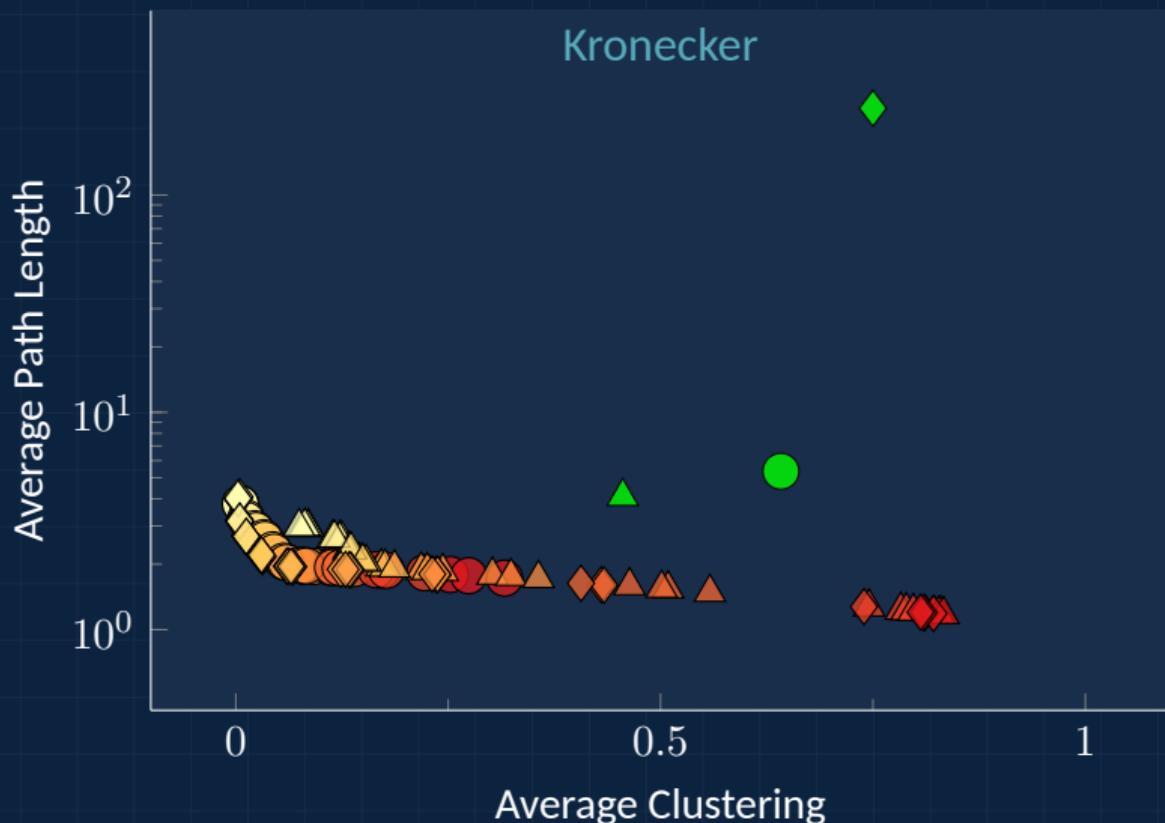
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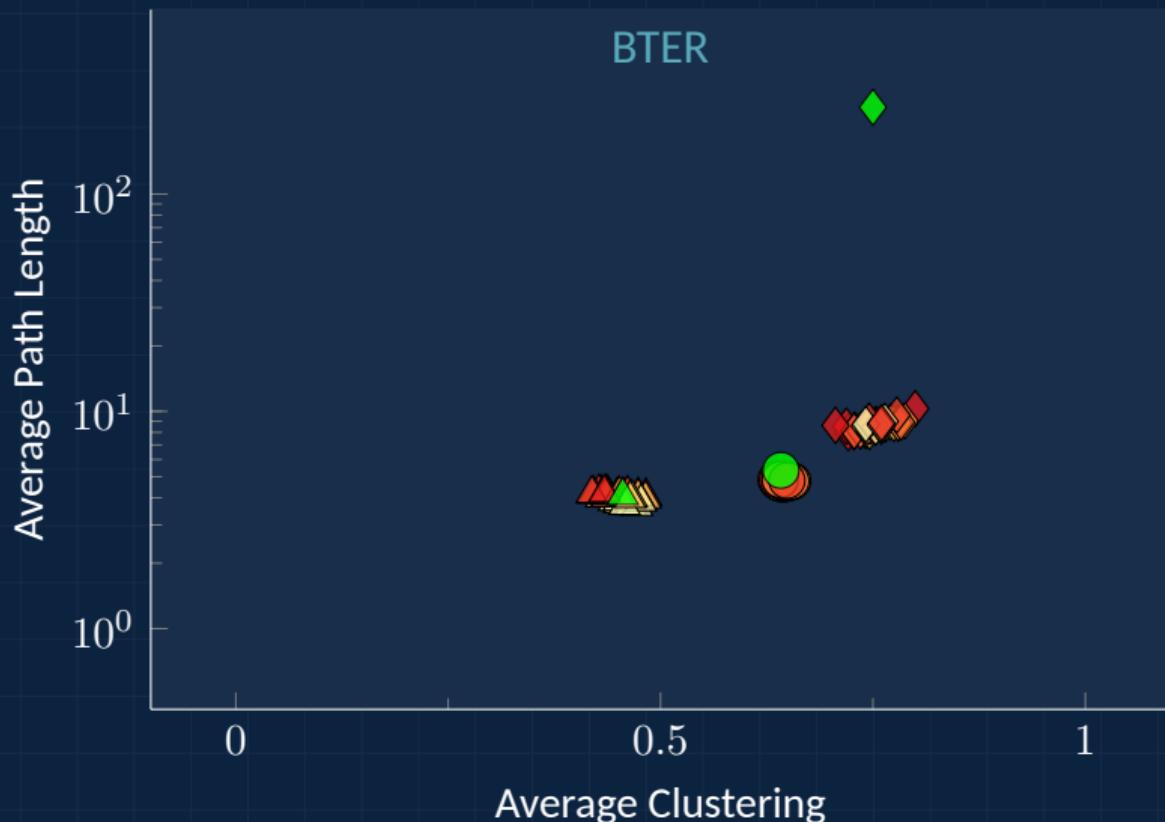
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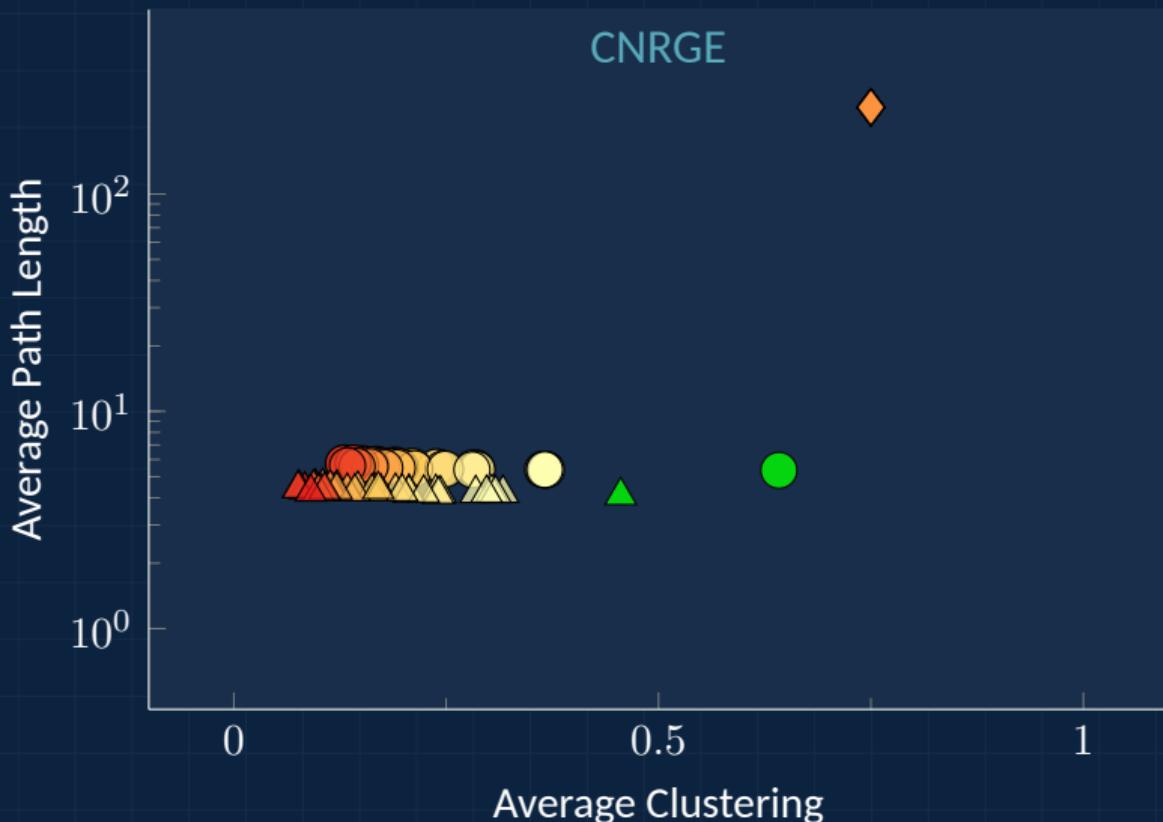
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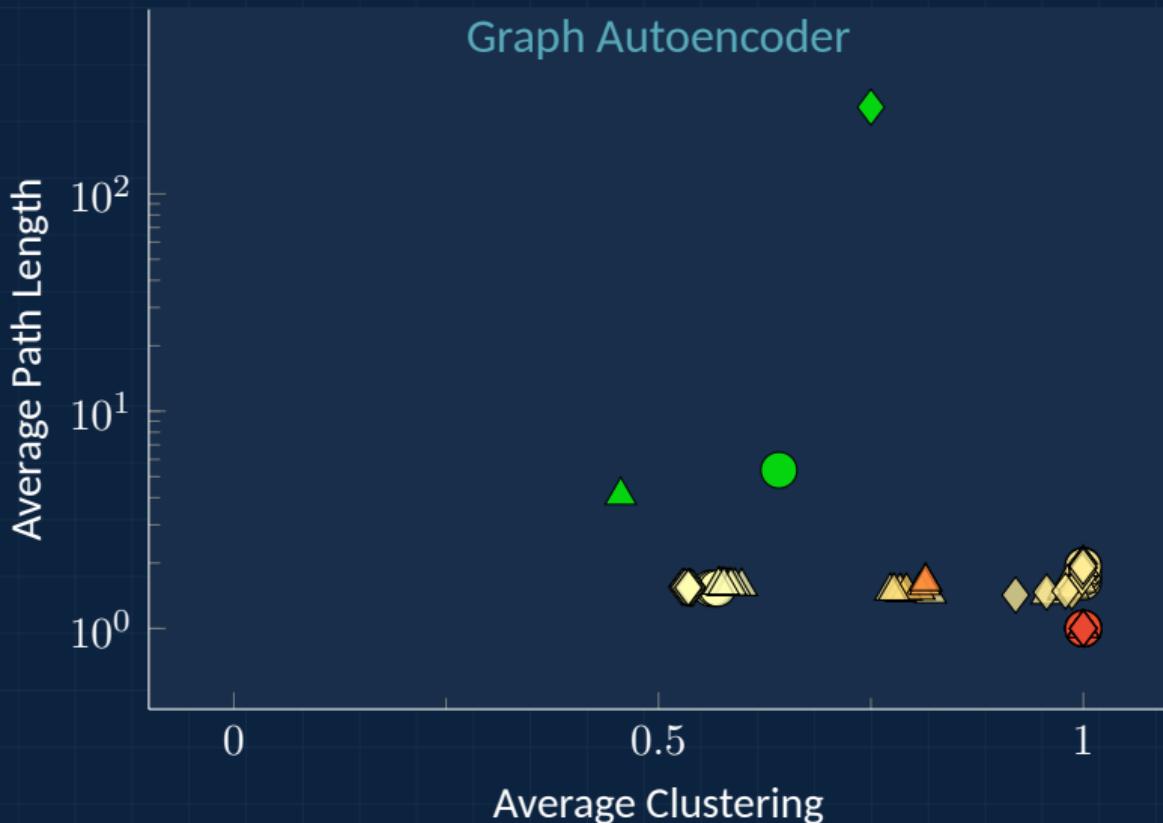
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Main Takeaways

- ◆ *Confirms* previously known biases in Kronecker graphs



Takeaways

Scalable and
Interpretable Graph
Modeling with
Graph Grammars

bit.ly/satyaki-slides

Introduction

Grammar Extraction

Graph Generation

Infinity Mirror Test

Motivation

Methodology

Results

Conclusion

Wrapping Up



Main Takeaways

- ◆ *Confirms* previously known biases in Kronecker graphs
- ◆ *Uncovers unique* distortion patterns in popular graph models

Takeaways

Scalable and
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Modeling with
Graph Grammars

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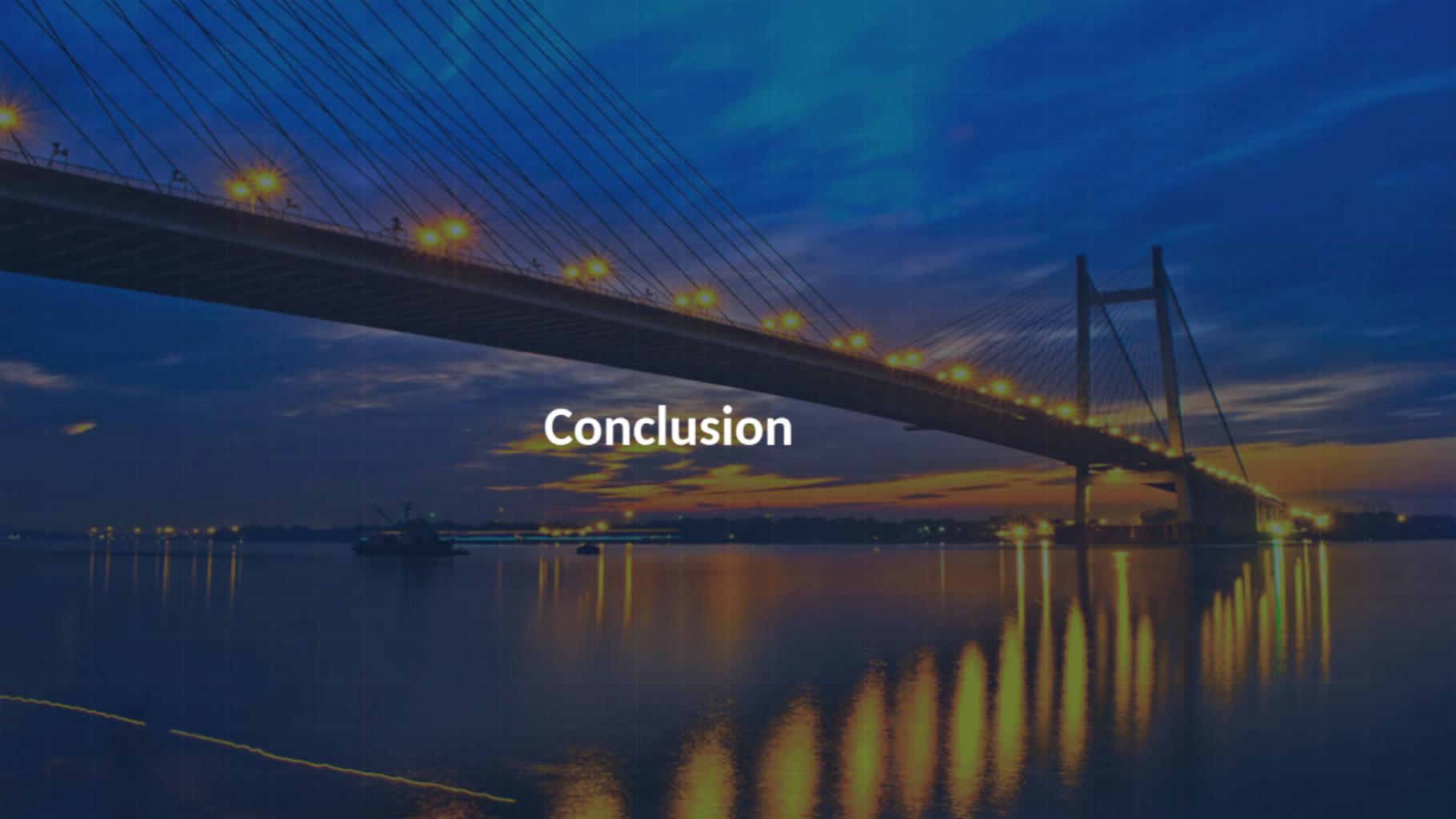


Main Takeaways

- ◆ *Confirms* previously known biases in Kronecker graphs
- ◆ Uncovers *unique* distortion patterns in popular graph models
- ◆ A tool to design *better*, more parsimonious models



Conclusion



Vertex Replacement Grammars

- ◆ A new class of *scalable, interpretable* graph models
- ◆ Preserves both *topology* and *attribute* similarities

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What's Next



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Summary

Vertex Replacement Grammars

- ◆ A new class of *scalable, interpretable* graph models
- ◆ Preserves both *topology* and *attribute* similarities

Infinity Mirror Test

- ◆ A *novel* stress-test for graph models
- ◆ Reveals new *biases* prompting further investigation

What's Next

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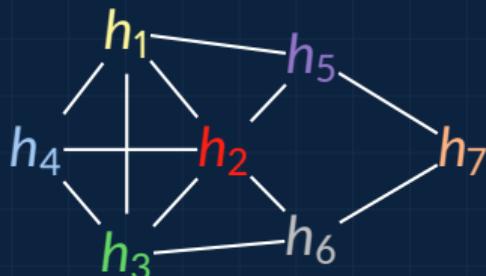
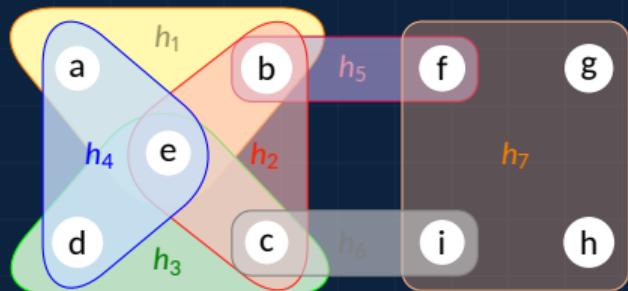
Summary

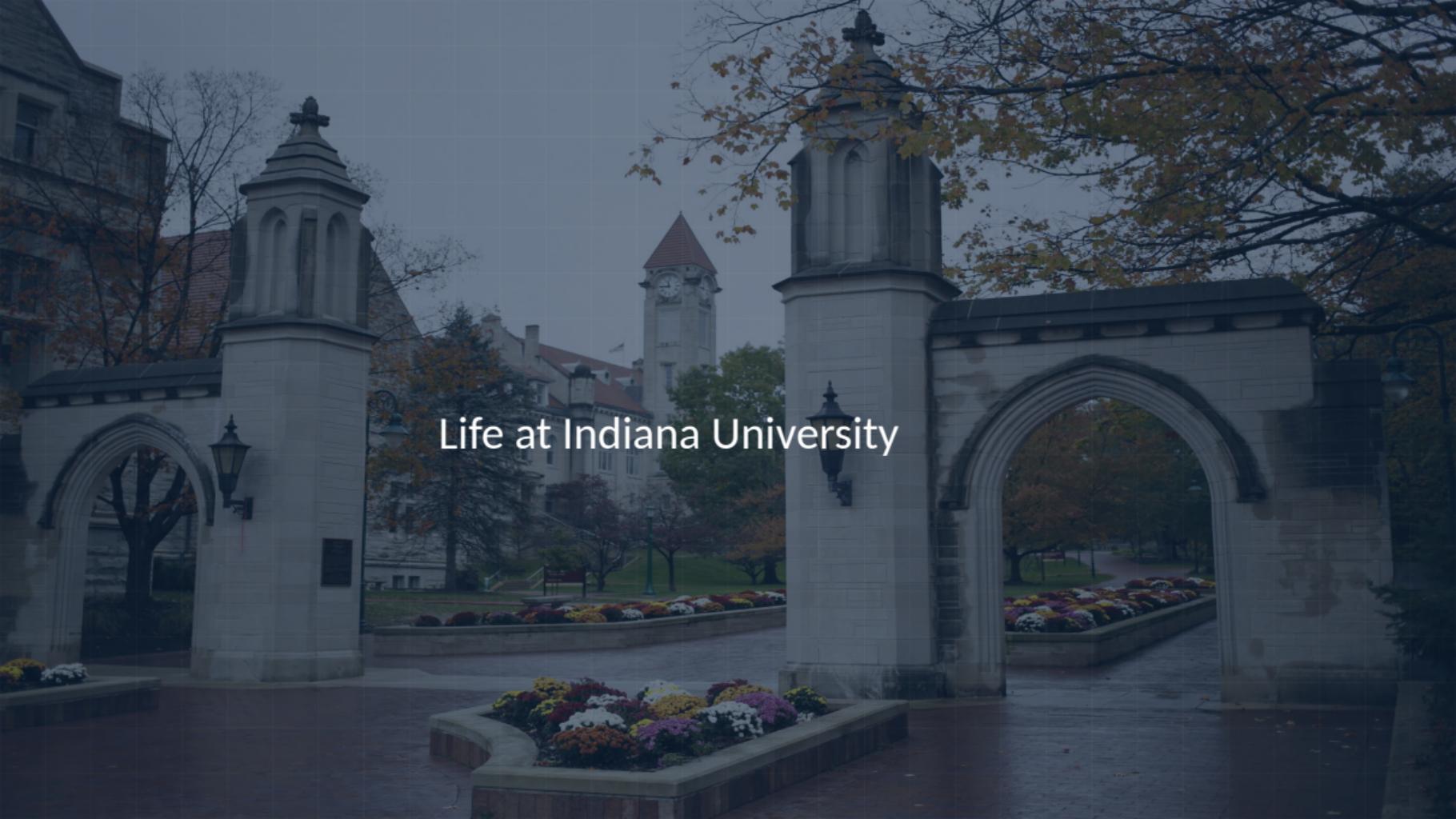
What's Next



Hypergraph Grammars

- ◆ New extraction method for HRGs *bypassing* tree decompositions
- ◆ Should improve both *stability* and scalability



A photograph of Indiana University's campus. In the foreground, there are two large, white Gothic-style stone arches. Between them is a planter filled with colorful autumn flowers. In the background, a tall clock tower with a red roof and a white clock face stands prominently. Other buildings with red roofs are visible behind it. Bare trees with yellow and orange leaves are scattered throughout the scene, suggesting it's fall. The overall atmosphere is peaceful and academic.

Life at Indiana University

What's Next

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What's Next

Postdoctoral Fellow @ IU

- ◆ Work with *Santo Fortunato* on the MINERVA project
- ◆ Understanding the *process* of scientific discovery
- ◆ Studying the *dynamics* of citation and collaboration networks



An aerial photograph of the University of Notre Dame campus in South Bend, Indiana. The image shows the iconic golden dome of the Main Building, the Basilica of the Sacred Heart with its blue-roofed spires, and various other Gothic-style buildings. The campus is surrounded by lush green trees and is located near a body of water. A large, semi-transparent dark overlay covers the entire image, with the words "Thank You!" centered in white.

Thank You!

Minimum Description Length Principle

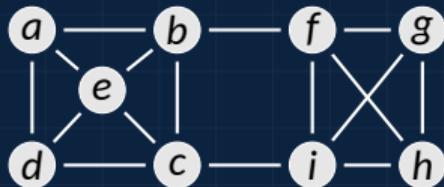
The Core Idea

- ◆ The best model is the one that leads to the best compression

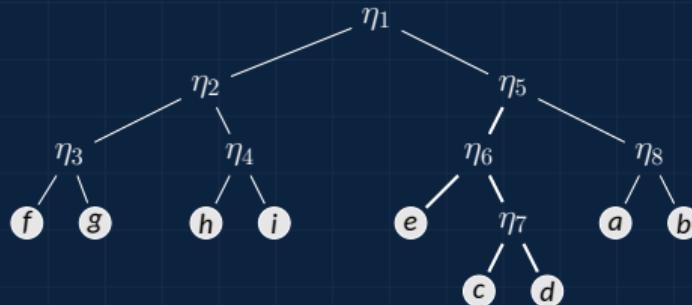
$$\mathcal{M}^* = \operatorname{argmin} \{ \mathcal{L}(\mathcal{M}) + \mathcal{L}(\mathcal{D} | \mathcal{M}) \}$$

- ◆ $\mathcal{L}(\mathcal{M})$ is the length of the model
- ◆ $\mathcal{L}(\mathcal{D} | \mathcal{M})$ is the length of data compressed by the model





(a) Example graph H
with 9 nodes and 16
edges



(b) An example dendrogram \mathcal{D}



(c) All possible rules that can be obtained from \mathcal{D}

Comparing Graphs

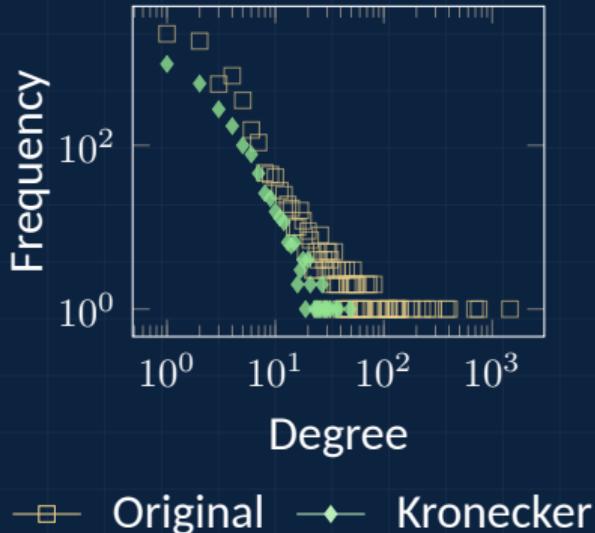
Distribution Based

- ◆ Degree distribution
- ◆ PageRank centrality
- ◆ ...

Topology Based

- ◆ Graph Edit Distance (GED)
- ◆ Graphlet Correlation Distance (GCD)
- ◆ DeltaCon - Personalized PR
- ◆ ...

Routers Degree Distribution



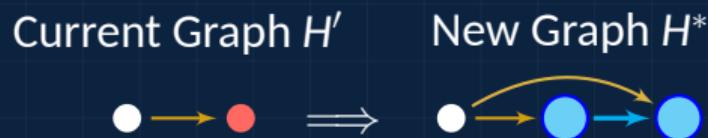
Kemp-Tenenbaum (KT) Graph Grammars

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(A) A KT grammar rule



(B) Example of rule application

- replaced $\bullet \rightarrow$ new \rightarrow boundary



Bottom-up Graph Grammar Extraction (BUGGE)

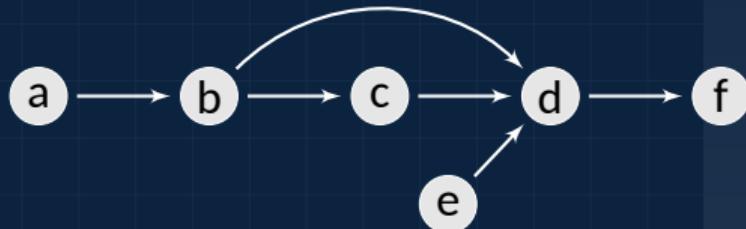
($\mu = 2$)

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(1) Enumerate Subgraphs

- ◆ Specify *max* rule size μ
- ◆ Find *connected* sets of up to size μ



Enumerating all connected size μ subgraphs in H

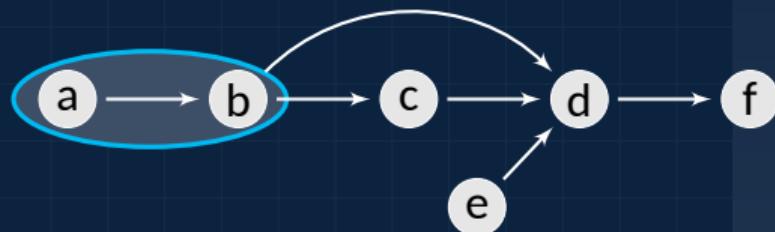


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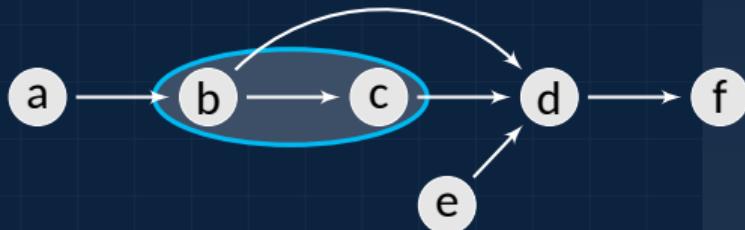


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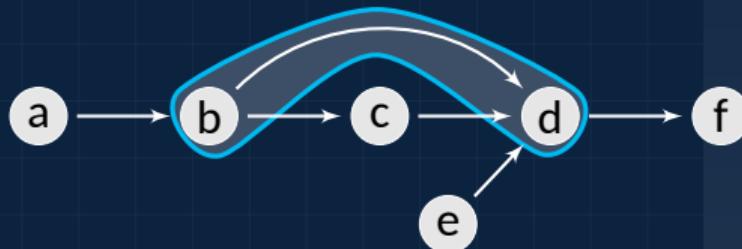


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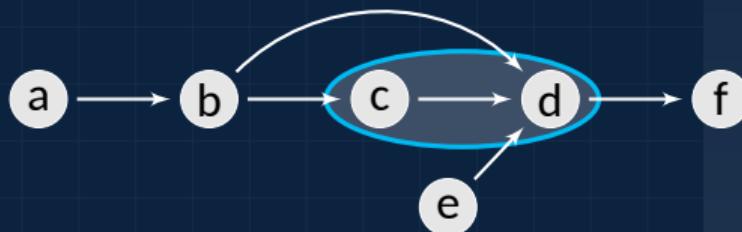


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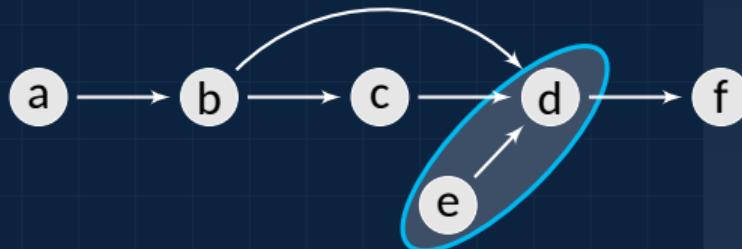


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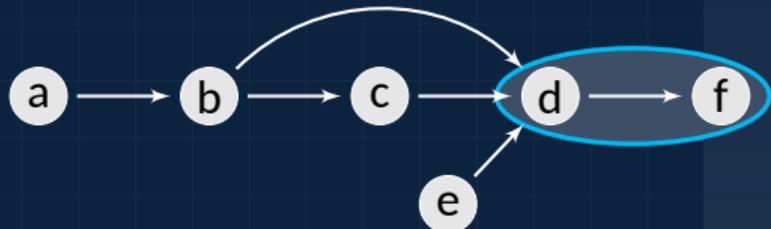
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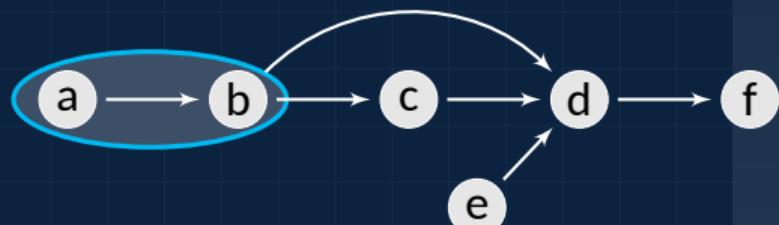
(2) Best Rule Selection

Use MDL principle to find the rule
which *compresses* the graph the
most

LHS RHS



(A) Best rule R



(B) R appears 4 times in Graph H



Bottom-up Graph Grammar Extraction (BUGGE)

$(\mu = 2)$

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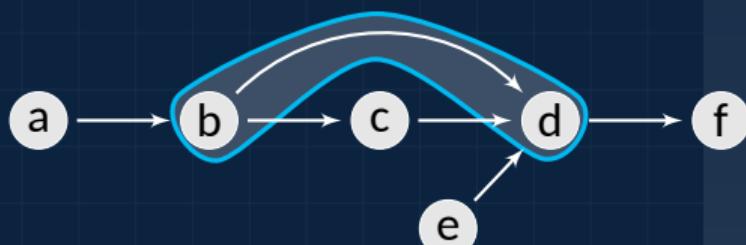
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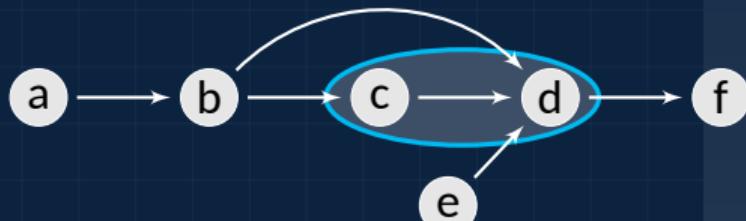
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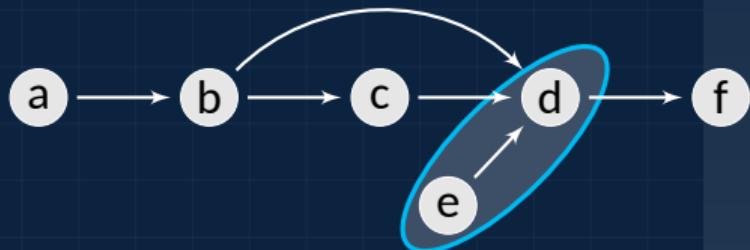
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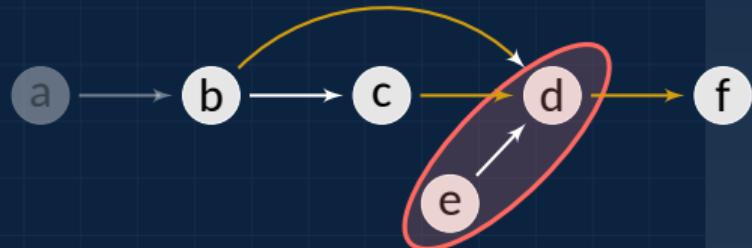
(3) Extract Best Rule

- ◆ Match rule **RHS** in the graph
- ◆ *Collapse* matched nodes
- ◆ *Repeat* until the graph is empty

LHS RHS



(A) Best rule R



(B) Current graph H



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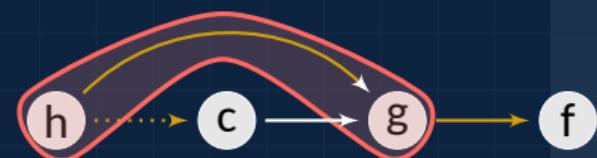
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k



Results from Synthetic Graphs

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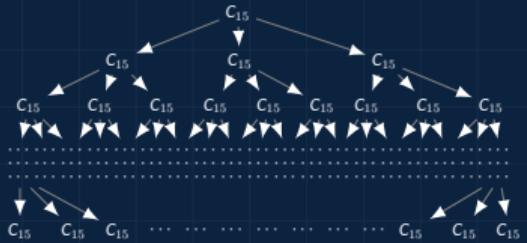
Synthetic
Graphs



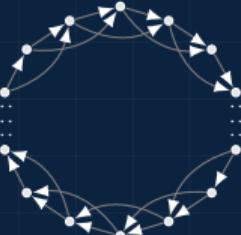
BUGGE
Rules



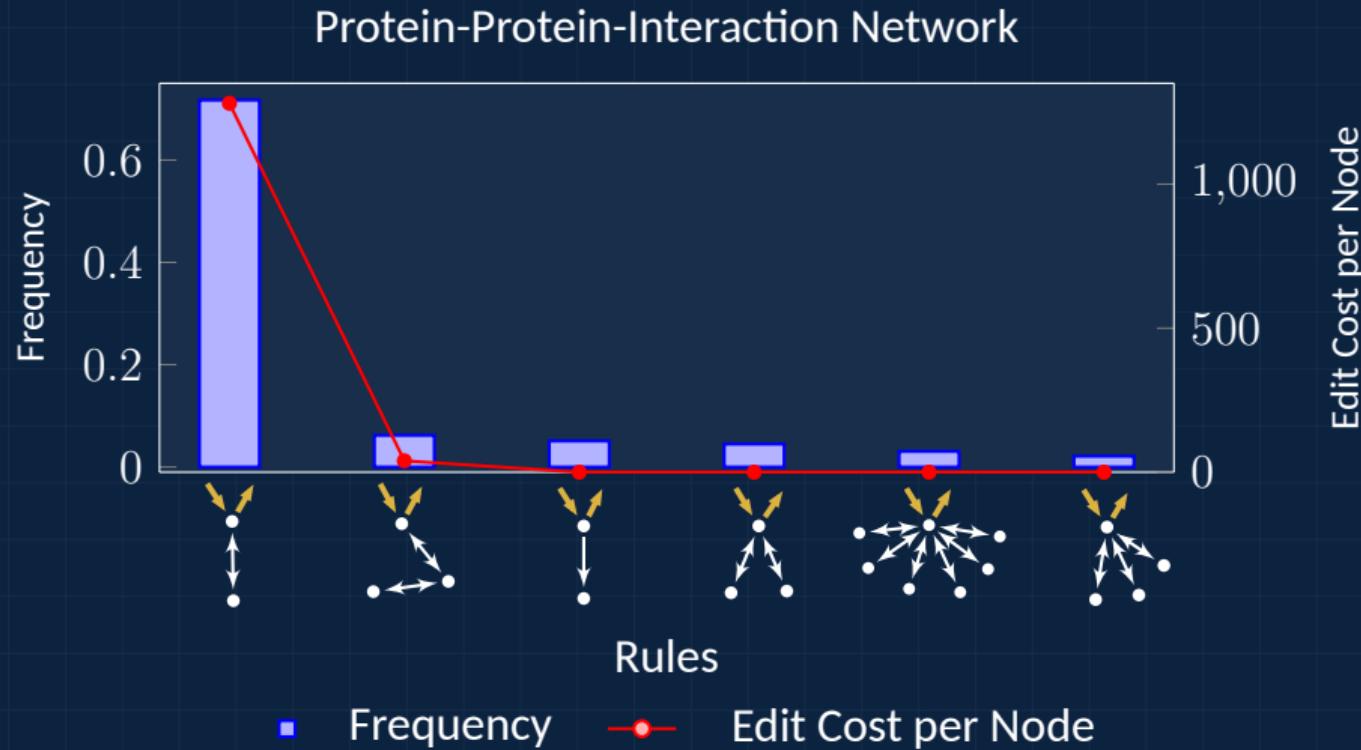
II: Tree of Directed C_{15} Rings



III: Regular Ring Lattice



Most Frequent Rules Extracted from a PPI Network



CNRG Results

	EuCore		PolBlogs		OpenFlights	
	GCD	λ -dist	GCD	λ -dist	GCD	λ -dist
ChungLu	0.409	0.803	0.466	1.234	1.1116	0.614
HRG	0.229	8.091	1.196	4.407	1.2442	2.761
DC-SBM	0.180	2.057	0.262	4.186	0.8414	3.534
BTER	-	-	0.352	7.505	0.832	4.936
Kronecker	0.3164	11.802	1.302	14.31	1.83	10.459
CNRGE	0.233	4.969	0.212	4.276	0.2832	3.581



CNRG Results

	GrQc		PGP		Gnutella	
	GCD	λ -dist	GCD	λ -dist	GCD	λ -dist
ChungLu	2.657	0.389	2	0.64	1.02	0.42
HRG	1.99	4.41	-	-	2	5
DC-SBM	2.065	2.202	1.39	2.29	-	-
BTER	2.231	0.439	1.61	0.832	1.10	0.474
Kronecker	3.87	5.468	2.882	3.54	3.31	5.96
CNRGE	1.067	0.723	0.448	1.329	0.41	0.20



Dendrogram Selection

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