Relation

Let A and B be two sets, then the relation R is the subset of $A\times B$.

i.e., $\boxed{R \subseteq A\times B}$

eg: $A = \{1, 2\}$, $B = \{a, b\}$

$A\times B = \{1, 2\} \times \{a, b\}$

$A\times B = \{(1, a), (1, b), (2, a), (2, b)\}$

$R_1 = \{(1, a), (2, a)\}$

$R_2 = \{(2, a), (2, b)\}$

$R_3 = \{(2, a)\}$

Counting of relations

$n(A) = 2$, $n(B) = 2$

$n(A\times B) = 2\times 2 = 4$

$A\times B \to$ 2 2 – –

$R = \{$

Total no of Relation $= 2\times 2\times 2\times 2$

$= 2^4 = 2^{2\times \boxed{2}} = 2^{n(A)\times n(B)}$

—X—

Q Let A and B be two sets having n and m elements then find the total no of Relation from the set A to set B.

Soln: no. of elements in set A = n

no. of elements in set B = m

Total no. of Relation $= 2^{n(A)\times n(B)}$

$= \boxed{2^{n\cdot m}} = 2^{\boxed{mn}}$

Total no. of Relation from A to A $= 2^{n(A)\times n(A)} = 2^{n\times n} = 2^{n^2}$

Total no of Relation from Set $\underline{A}$ to Set $\underline{A}$ $= 2^{n(A)\times n(A)} = 2^{n\times n} = 2^{n^2}$

—×—

① <u>Reflexive relation</u>: Let A be a non-empty set, then the relation $\underline{R}$ defined on $\underline{A\times A}$ is called reflexive.

if $\underline{a}R\underline{a}$ $\forall$ $\underline{a}\in \underline{A}$

② <u>Symmetric relation</u>: Let A be a non-empty set, then the relation R defined on $A\times A$ is called Symmetric

if $\underline{(a,b)}\in R$ then $\underline{(b,a)}\in \underline{R}$

③ <u>Transitive Relation</u>:→ Let A be a non-empty set, then the relation R defined on $A\times A$ is called transitive

if $(a,b)\in R$ and $\underline{(b,c)\in R}$ $\Rightarrow$ $(a,c)\in R$

—×—

Q! Let $A=\{1,2,3\}$,

$R=\{(1,1),\ (2,2),\ (1,2),\ (2,3),\ (1,3)\}$

Check is this relation <u>(i)</u> Reflexive <u>(ii)</u> Symmetric (iii) Transitive.

<u>Sol^n</u>: $A=\{\underline{1},\underline{2},\underline{3}\}$

$R=\{\underline{(1,1)},\ \underline{(2,2)},\ (1,2),\ (2,3),\ (1,3)\}$

① <u>Reflexive</u>

As $(3,3)\notin R$ $\Rightarrow$ Relation is not reflexive.

② <u>Symmetric</u>

$(1,2)\in R$ but $(\underline{2,1})\notin R$

$\Rightarrow$ Relation is not Symmetric

③ <u>Transitive</u>

As $(1,2)\in R$, $(2,3)\in R$ $\Rightarrow$ $(1,3)\in R$

(3) Transitive

As $(1,2) \in R$, $(2,3) \in R \Rightarrow (1,3) \in R$

$\Rightarrow$ Relation is transitive.

Q2 Let $\mathbb{N}$ be the set of natural nos. define a relation R on $\mathbb{N} \times \mathbb{N}$ by $R = \{(a,b) : a \le b\}$. Prove that this relation is reflexive, transitive but not symmetric.

Solⁿ: $R = \{(a,b) : a \le b\}$

(i) Reflexive.

As every element is less than or equal to itself

i.e, $a \le a \quad \forall\, a \in \mathbb{N}$

$\Rightarrow (a,a) \in R \quad \forall\, a \in \mathbb{N}$

This relation is reflexive.

(ii) Transitive

Let $(a,b) \in R$ and $(b,c) \in R$

$a \le b$ and $b \le c$

$\Rightarrow a \le b \le c \Rightarrow a \le c$

$\Rightarrow (a,c) \in R$

This relation is transitive.

(iii) Symmetric

Let $(a,b) \in R$

$a \le b$

$\not\Rightarrow b \le a$

$(b,a) \notin R$

$$\left(\begin{array}{c} 4 \le 8 \\ 8 \not\le 4 \end{array}\right)$$

$\Rightarrow$ Relation is not symmetric

— x —

Q Let L be the set of all lines in a plane define a relation

① Let L be the set of all lines in a plane define a relation R on $L \times L$ by $R = \{(l_1, l_2) : l_1 \perp l_2\}$. Prove that this relation is symmetric but it is neither reflexive nor transitive.

Sol^n $R = \{(l_1, l_2) : l_1 \perp l_2\}$

① Reflexive

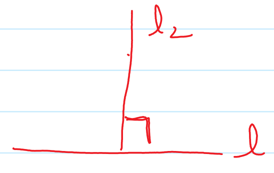
No line can be $\perp$ to itself

i.e, $l_1 \not\perp l_1$

$\Rightarrow (l_1, l_1) \notin R \quad \forall\ l_1 \in L$

$\Rightarrow$ This relation is not reflexive.

② Symmetric



Let $(l_1, l_2) \in R$

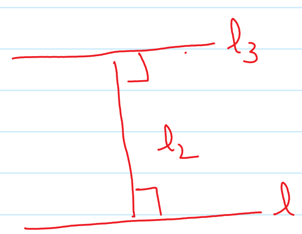
$\Rightarrow l_1 \perp l_2$

$\Rightarrow l_2 \perp l_1$

$\Rightarrow (l_2, l_1) \in R$

$\Rightarrow$ This relation is symmetric.

③ Transitive



Let $(l_1, l_2) \in R$ and $(l_2, l_3) \in R$

$l_1 \perp l_2$ and $l_2 \perp l_3$

$\not\Rightarrow l_1 \perp l_3$

$\Rightarrow (l_1, l_3) \notin R$

$\Rightarrow$ This relation is not transitive.

—— X ——

• Equivalence relation: Let A be a non-empty set. Define a relation R on $A \times A$ then this relation is called an equivalence relation if it is called an equivalence relation if it is

1. Reflexive
2. Symmetric
3. Transitive

$a|b = \frac{b}{a}$ ✓

Q. Let $\mathbb{N}$ be the set of natural no.s. define a relation R on $N \times N$ as $R = \{(a,b) : \underline{a|b}\}$. Check that relation is not an equivalence relation.

Sol $R = \{(a,b) : a|b\}$

1. Reflexive

Every element divides itself

i.e, $a|a \quad \forall a \in \mathbb{N}$

$\Rightarrow (a,a) \in R$

$\Rightarrow$ Relation is reflexive.

2. Symmetric

Let $(a,b) \in R$

$\Rightarrow a|b$

$\not\Rightarrow b|a$

$4|8$

$8 \nmid 4$

$\Rightarrow (b,a) \notin R$

$\Rightarrow$ Relation is not Symmetric.

$\therefore$ We can say that this relation is not an equivalence relation.

—×—

Q. Let $\mathbb{Z}$ be the set of Integers define a Relation R on $\mathbb{Z}$ by $R = \{(a,b) : 2 \text{ divides } a-b\}$. Prove that this relation is an equivalence relation.

Sol $R = \{(a,b) : \underline{2} \text{ divides } a-b\}$

$\frac{0}{2}$

1. Reflexive.

We know that $2|0$ ✓

$2|a-a \quad \forall a \in \mathbb{Z}$

$2 \mid a-a \quad \forall a \in \mathbb{Z}$

2 divides $a-a \quad \forall a \in \mathbb{Z}$

$(a,a) \in R$

$\Rightarrow$ Relation is reflexive.

② Symmetric

Let $(a,b) \in R$

$\Rightarrow 2 \mid (a-b)$, Then $\exists$ integer k s.t

$\frac{a-b}{2} = k \quad (k \in \mathbb{Z})$

$a-b = 2k$

Multiply both sides by -1

$-(a-b) = 2(-k)$

$(b-a) = 2(-k)$

$2 \mid (b-a)$

$(b,a) \in R$

$\Rightarrow$ Relation is Symmetric

$2|4,\ 2|6 \Rightarrow 2|(4+6)$

③ Transitive.

Let $(a,b) \in R$ and $(b,c) \in R$

$2 \mid (a-b)$ and $2 \mid (b-c)$

$2 \mid (a-\cancel{b}) + (\cancel{b}-c)$

$2 \mid (a-c)$

$\Rightarrow (a,c) \in R$

$\Rightarrow$ This relation is Transitive.

$\therefore$ This relation is an equivalence relation.

—x—

Q① Let $\mathbb{R}$ be the set of real nos. Define a Relation $R = \{(a,b) : |a| = |b|\}$

Prove that this relation is an equivalence relation.

Sol^n $R = \{(a,b) : |a| = |b|\}$

① Reflexive Relation.

② Symmetric

Let $(a,b) \in R$

③ Transitive.

Let $(a,b) \in R$ and $(b,c) \in R$

① Reflexive Relation.

As we know that

$|a| = |a| \quad \forall a \in \mathbb{R}$

$\Rightarrow (a,a) \in R \quad \forall a \in \mathbb{R}$

This relation is reflexive

② Symmetric

Let $(a,b) \in R$

$|a| = |b|$

or $|b| = |a|$

$(b,a) \in R$

This relation is symmetric

③ Transitive.

Let $(a,b) \in R$ and $(b,c) \in R$

$|a| = |b|$ and $|b| = |c|$

$|a| = |b| = |c|$

$\Rightarrow |a| = |c|$

$\Rightarrow (a,c) \in R$

$\therefore$ This Relation is transitive

Hence, we can say that this relation is an equivalence relation.

—x—

$R = \{(a,b) : a \le b\}$

This relation is not symmetric

Let $(a,b) \in R$

$\Rightarrow a \le b$

$\not\Rightarrow b \le a$

$(b,a) \notin R$

$4 \le 8$

$8 \le 4$

$\therefore$ This relation is not an equivalence relation.

① Let R be the relation over the set $N \times N$ and is defined by $(a,b) R (c,d) \Leftrightarrow$ $\cancel{a+d = b+c}$ $ad = bc$. Prove that this relation is an equivalence relation.

Sol^n ① Reflexive

As we know that

$a+b = b+a$

$(a,b) R (a,b) \quad \forall (a,b) \in N \times N$

$\Rightarrow$ Relation is reflexive

Reflexive.

As we know that

$a+b = b+a$

$(a,b) R (a,b) \quad \forall (a,b) \in N \times N$

② Symmetric

Let $(a,b) R (c,d)$

$a+d = b+c$

$d+a = c+b$

$c+b = d+a$

$(c,d) R (a,b)$

Transitive

Let $(a,b) R (c,d)$ and $(c,d) R (e,f)$

$a+d = b+c \rightarrow ①$

$c+f = d+e \rightarrow ②$

Adding ① & ②, we get

$a+\cancel{d}+\cancel{c}+f = b+\cancel{c}+\cancel{d}+e$

$$c+b = d+a$$

$$(c,d)\,R\,(a,b)$$

$\Rightarrow$ This relation is symmetric

$$a+\cancel{d}+\cancel{c}+f = b+\cancel{c}+\cancel{d}+e$$

$$a+f = b+e$$

$$(a,b)\,R\,(e,f)$$

$\Rightarrow$ Relation is transitive

$\therefore$ Relation is an equivalence relation

—x—

Anti-Symmetric relation: Let A be a non-empty set, then the relation R is called anti-symmetric if

$(a,b) \in R$ and $(b,a) \in R$ then $\boxed{a=b}$

eg: $A = \{1, 2, 3\}$

$R = \{(1,1), (2,2), \}$

① Reflexive (This relation is not reflexive)

② Anti-symmetric (yes)

How to express a relation in matrix form.

$A = \{1, 2, 3\}$

$\rightarrow R = \{(1,1), (1,2), (2,3), (2,2)\}$

$(1,2) \in R$, $(2,3) \in R$

$(1,3) \notin R$

$$M_R = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}_{3\times 3}$$

a_{12}, a_{13}, a_{21}, a_{23}, a_{31}, a_{32}

	1	2	3
1	(1,1)	(1,2)	(1,3)
2	(2,1)	(2,2)	(2,3)
3	(3,1)	(3,2)	(3,3)

3×3

$$M_R = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}_{4 \times 4}$$

find the Relation

$A = \{1, 2, 3, 4\}$

$R = \{(1,1), (1,4), (2,2), (2,4), (3,1), (3,2), (4,1), (4,4)\}$

Def^n of Partial order relation (P.O.R)

Let A be a non-empty set, then the Relation R on A is called Partial order relation if it is

1. Reflexive
2. ~~Symmetric~~
2. Anti-symmetric
3. Transitive.