MCQ BASED ON RECURRENECE RELATIONS

1. Consider the recurrence relation $a_1=4$, $a_n=5n+a_{n-1}$. The value of a_{64} is _________

a) 10399
b) 23760
c) 75100
d) 53700

Answer: a
Explanation: $a_n=5n+a_{n-1}$
$= 5n + 5(n-1) + \ldots + a_{n-2}$
$= 5n + 5(n-1) + 5(n-2) + \ldots + a_1$
$= 5n + 5(n-1) + 5(n-2) + \ldots + 4$ [since, a1=4]
$= 5n + 5(n-1) + 5(n-2) + \ldots + 5.1 - 1$
$= 5(n + (n-1) + \ldots + 2 + 1) - 1$
$= 5 * n(n+1)/ 2 - 1$
$a_n = 5 * n(n+1)/ 2 - 1$
Now, n=64 so the answer is $a_{64} = 10399$.

2. Determine the solution of the recurrence relation $F_n=20F_{n-1} - 25F_{n-2}$ where $F_0=4$ and $F_1=14$.

a) $a_n = 14*5^{n-1}$

b) $a_n = 7/2*2^n-1/2*6^n$

c) $a_n = 7/2*2^n-3/4*6^{n+1}$

d) $a_n = 3*2^n-1/2*3^n$

Answer: b
Explanation: The characteristic equation of the recurrence relation is → $x^2-20x+36=0$
So, $(x-2)(x-18)=0$. Hence, there are two real roots $x_1=2$ and $x_2=18$. Therefore the solution to the recurrence relation will have the form: $a_n=a2^n+b18^n$. To find a and b, set n=0 and n=1 to get a system of two equations with two unknowns: $4=a2^0+b18^0=a+b$ and $3=a2^1+b6^1=2a+6b$. Solving this system gives b=-1/2 and a=7/2. So the solution to the recurrence relation is,
$a_n = 7/2*2^n-1/2*6^n$.

3. What is the recurrence relation for 1, 7, 31, 127, 499?

a) $b_{n+1}=5b_{n-1}+3$
b) $b_n=4b_n+7!$
c) $b_n=4b_{n-1}+3$
d) $b_n=b_{n-1}+1$

Answer: c
Explanation: Look at the differences between terms: 1, 7, 31, 124,…. and these are growing by a factor of 4. So, $1\cdot 4=4$, $7\cdot 4=28$, $31\cdot 4=124$, and so on. Note that we always end up with 3 less than the next term. So, $b_n=4b_{n-1}+3$ is the recurrence relation and the initial condition is $b_0=1$.

4. If $S_n=4S_{n-1}+12n$, where $S_0=6$ and $S_1=7$, find the solution for the recurrence relation.

a) $a_n=7(2^n)-29/6n6^n$

b) $a_n=6(6^n)+6/7n6^n$

c) $a_n=6(3^{n+1})-5n$

d) $a_n=nn−2/6n6^n$

Answer: b

Explanation: The characteristic equation of the recurrence relation is → $x^2−4x-12=0$

So, (x-6)(x+2)=0. Only the characteristic root is 6. Therefore the solution to the recurrence relation will have the form: $a_n=a.6^n+b.n.6^n$. To find a and b, set n=0 and n=1 to get a system of two equations with two unknowns: $6=a6^0+b.0.6^0=a$ and $7=a6^1+b.1.6^1=2a+6b$. Solving this system gives a=6 and b=6/7. So the solution to the recurrence relation is, $a_n=6(6^n)−6/7n6^n$.

5. Find the value of a_4 for the recurrence relation $a_n=2a_{n-1}+3$, with $a_0=6$.

a) 320
b) 221
c) 141
d) 65

Answer: c

Explanation: When n=1, $a_1=2a_0+3$, Now $a_2=2a_1+3$. By substitution, we get $a_2=2(2a_0+3)+3$.
Regrouping the terms, we get $a_4=141$, where $a_0=6$.

6. The solution to the recurrence relation $a_n=a_{n-1}+2n$, with initial term $a_0=2$ are _________

a) 4n+7
b) 2(1+n)
c) $3n^2$
d) 5*(n+1)/2

Answer: b

Explanation: When n=1, $a_1=a_0+2$. By substitution we get, $a_2=a_1+2 \Rightarrow a_2=(a_0+2)+2$ and so on. So the solution to the recurrence relation, subject to the initial condition should be $a_n=2+2n=2(1+n)$.

7. Determine the solution for the recurrence relation $b_n=8b_{n-1}−12b_{n-2}$ with $b_0=3$ and $b_1=4$.

a) $7/2*2^n−1/2*6^n$
b) $2/3*7^n-5*4^n$
c) $4!*6^n$
d) $2/8^n$

Answer: a

Explanation: Rewrite the recurrence relation $b_n-8b_{n-1}+12b_{n-2}=0$. Now from the characteristic equation: $x^2−8x+12=0$ we have x: (x−2)(x−6)=0, so x=2 and x=6 are the characteristic roots. Therefore the solution to the recurrence relation will have the form: $b_n=b2^n+c6^n$. To find b and c, set n=0 and n=1 to get a system of two equations with two unknowns: $3=b2^0+c6^0=b+c$, and $4=b2^1+c6^1=2b+6c$. Solving this system gives c=-1/2 and b=7/2. So the solution to the recurrence relation is, $b_n=7/2*2^n−1/2*6^n$.

8. What is the solution to the recurrence relation $a_n=5a_{n-1}+6a_{n-2}$?

a) $2n^2$
b) 6n
c) (3/2)n
d) n!*3

Answer: b

Explanation: Check for the left side of the equation with all the options into the recurrence relation. Then, we get that 6n is the required solution to the recurrence relation $a_n=5a_{n-1} + 6a_{n-2}$.

9. Determine the value of a2 for the recurrence relation $a_n = 17a_{n-1} + 30n$ with $a_0=3$.

a) 4387
b) 5484

c) 238
d) 1437

Answer: d
Explanation: When n=1, $a_1=17a_0+30$, Now $a_2=17a_1+30*2$. By substitution, we get $a_2=17(17a_0+30)+60$. Then regrouping the terms, we get $a_2=1437$, where $a_0=3$.

10. Determine the solution for the recurrence relation $a_n = 6a_{n-1}−8a_{n-2}$ provided initial conditions $a_0=3$ and $a_1=5$.

a) $a_n = 4 * 2^n – 3^n$

b) $a_n = 3 * 7^n – 5*3^n$

c) $a_n = 5 * 7^n$

d) $a_n = 3! * 5^n$

Answer: b
Explanation: The characteristic polynomial is $x^2−6x+8$. By solving the characteristic equation, $x^2−6x+8=0$ we get x=2 and x=4, these are the characteristic roots. Therefore we know that the solution to the recurrence relation has the form $a_n=a*2^n+b*4^n$, for some constants a and b. Now, by using the initial conditions a_0 and a_1 we have: a=7/2 and b=-1/2. Therefore the solution to the recurrence relation is: $a_n = 4 * 2^n – 1*3^n = 7/2 * 2^n – 1/2*3^n$.

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