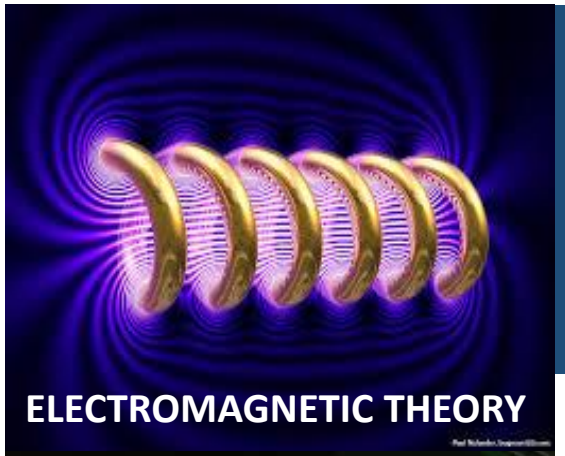

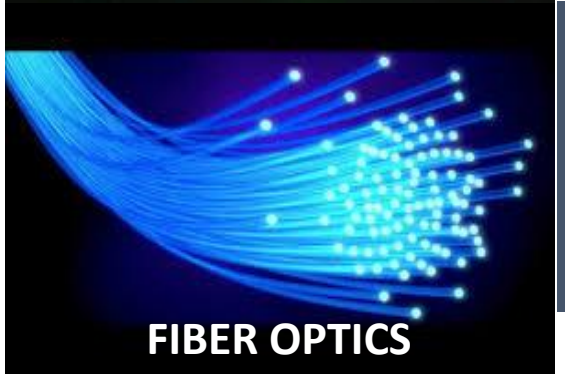

Engineering Physics
PHY-109
Solid State Physics-2

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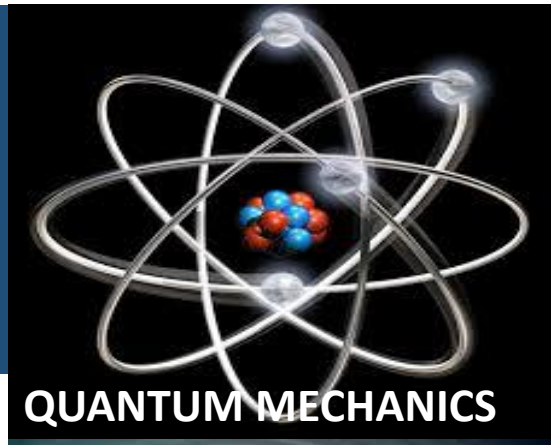
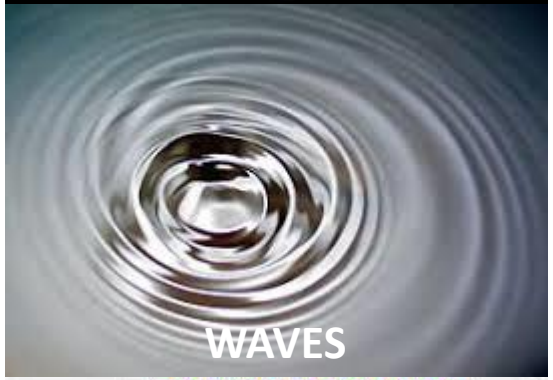
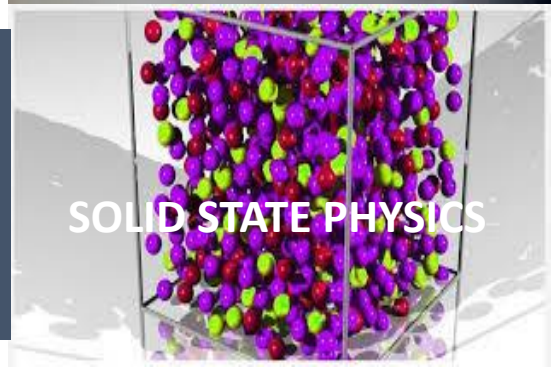
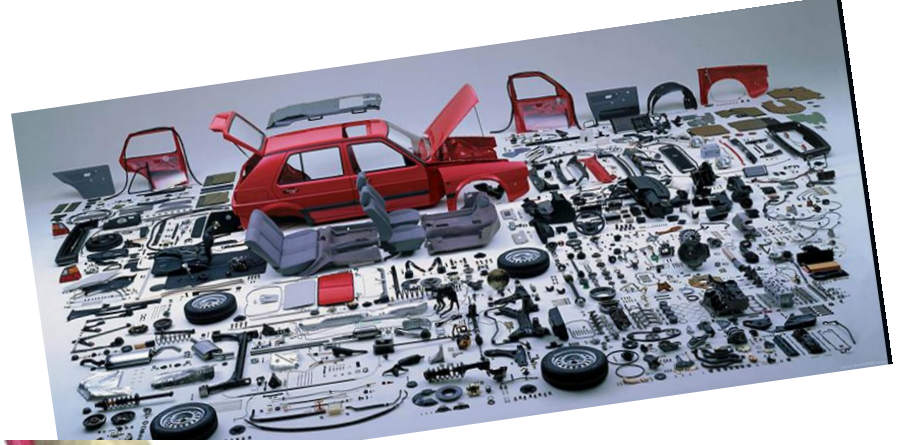






Syllabus

- ✓ **Free electron theory (Introduction).**
- ✓ **Diffusion and drift current (qualitative).**
- ✓ Fermi energy, Fermi-Dirac distribution function.
- ✓ Band theory of solids - formation of allowed and forbidden energy bands.
- ✓ Semiconductors and insulators, Fermi level for intrinsic and extrinsic semiconductors.
- ✓ Direct and indirect band gap semiconductors.
- ✓ Concept of effective mass - electrons and holes.
- ✓ Hall effect (with derivation).



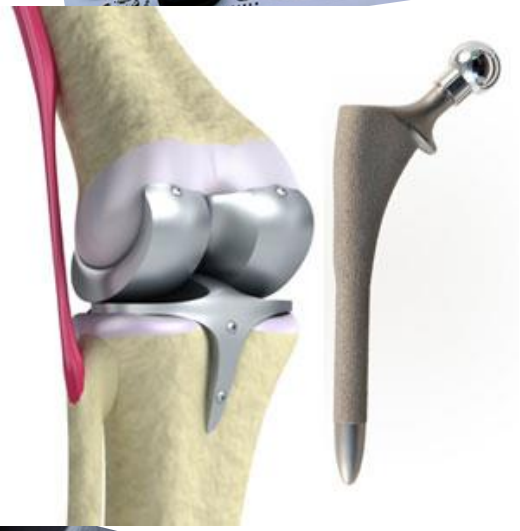
Solid

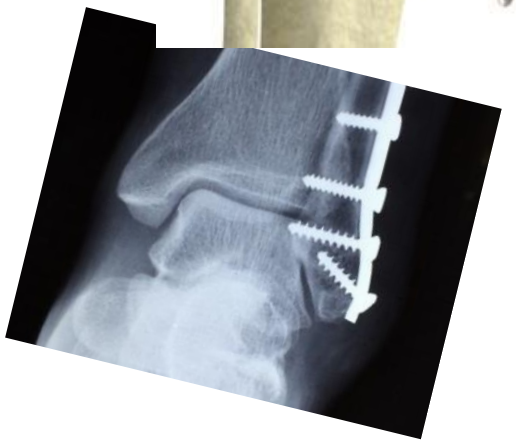
Metals:

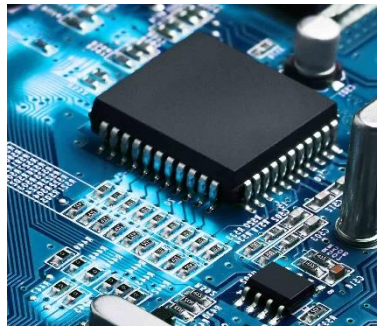
Na, Fe, Cu, Al, Au, Ag, Ti, Ni, Pt etc.

Semiconductor/Insulators:

Si, Ge, GaAs, BN, Al_2O_3, ZnO, TiO_2, SiO_2, CdTe, PbS, ZnS etc.







Metals

- ✓ The simplest metals are the alkali metals with one valence electrons such as Na, K, Li etc.
- ✓ The electronic configuration of sodium is Na- $\mathbf{1s^2\ 2s^2\ 2p^6\ 3s^1}$.
- ✓ The last electron behaves as a conduction electrons in metal which is also called the free electron.
- ✓ The free electron behaves like the molecules in a perfect gas and are called free electron gas.
- ✓ The free electron theory considers the force between the conduction electrons and the ion cores are neglected so that the total energy of the electron is all kinetic and the potential energy is taken to be zero. Therefore the motion of the electrons within the metal is free because there are no collisions, similar to the molecules of an ideal gas.

- ✓ It was believed that many physical properties of metals including electrical and thermal conductivities can be understood by considering free electron model. The works of Lorentz and Drude are important to cite here.

Properties of Metals

- ✓ Metals obey ohm's law which says that in steady state the current density is proportional to the applied electric field strength E, i.e.

$$J \propto E$$
$$\boldsymbol{J = \sigma E}$$

Here σ is defined as the electrical conductivity of the medium.

- ✓ Metals possesses high electrical and thermal conductivities.
- ✓ At low temperature, the resistivity ρ is proportional to the fifth power of absolute temperature i.e. $\rho \propto T^5$ where, $\rho = 1/\sigma$
- ✓ The resistance of the metals increases with the rise in temperature. i.e. they have positive temperature coefficient. The resistance of certain metals vanishes at absolute zero and they exhibit the phenomenon of superconductivity.
- ✓ The resistivity is inversely proportional to the pressure for most of the metals i.e. $\rho = 1/P$.

Wiedemann and Franz law:

The law states that, for metals, the ratio of thermal and electrical conductivities is directly proportional to the absolute temperature.

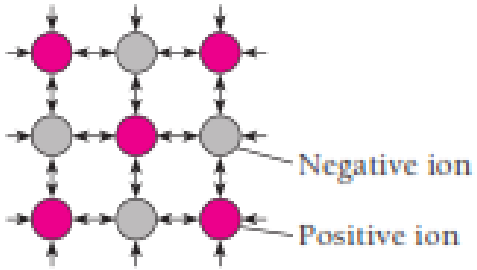
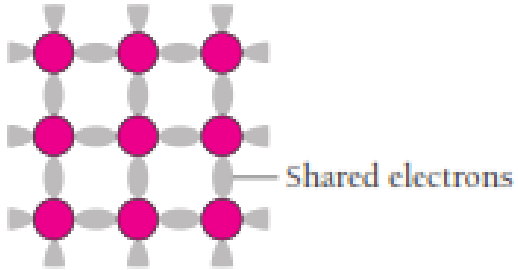
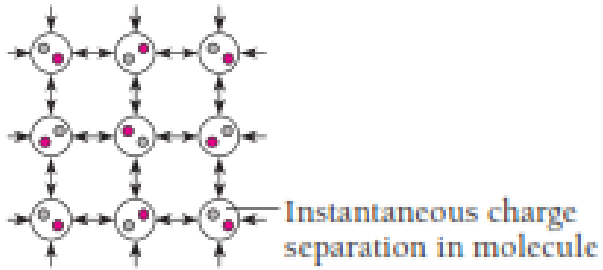
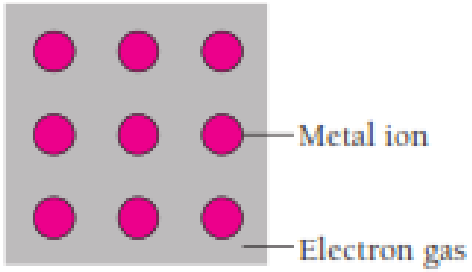
$$\frac{K}{\sigma} \propto T$$

Or,

$$\frac{K}{\sigma T} = \text{constant}$$

Comparison of bonds

Table 10.1 Types of Crystalline Solids. The cohesive energy is the work needed to remove an atom (or molecule) from the crystal and so indicates the strength of the bonds holding it in place.

Type	Ionic	Covalent	Molecular	Metallic
Lattice				
Bond	Electric attraction	Shared electrons	Van der Waals forces	Electron gas
Properties	Hard; high melting points; may be soluble in polar liquids such as water; electrical insulators (but conductors in solution)	Very hard; high melting points; insoluble in nearly all liquids; semi-conductors (except diamond, which is an insulator)	Soft; low melting and boiling points; soluble in covalent liquids; electrical insulators	Ductile; metallic luster; high electrical and thermal conductivity
Example	Sodium chloride, NaCl $E_{cohesive}$ = 3.28 eV/atom	Diamond, C $E_{cohesive}$ = 7.4 eV/atom	Methane, CH_4 $E_{cohesive}$ = 0.1 eV/molecule	Sodium, Na $E_{cohesive}$ = 1.1 eV/atom

Classical Picture and Lorentz-Drude Model

- 1900, Drude explained electrical and thermal conductivity of metals using the concept of mobile free electrons.
- Later, Drude and Lorentz proposed the free electron theory (Lorentz-Drude theory).

The assumptions of free electron theory are:

- Metal consists of ion cores (nucleus and core electrons).
- The free (valence) electrons in a metal are treated as an ideal gas of free particles.
- The valence electrons surround the ion core and are free to move within the metaland consequently responsible for the conductivity.

- The electrons obey Maxwell-Boltzmann statistics: the free electrons are in thermal
- equilibrium with a Maxwell-Boltzmann velocity distribution =>$v_{rms} = \sqrt{\frac{3k_B T}{m}}$

Ohm's Law

Metallic bonding occurs when the reduction in electron potential energy outbalances the increase in electron KE that accompanies it. The more valence electrons per atom, the higher the average KE of the free electrons, but without a commensurate drop in their potential energy. For this reason nearly all the metallic elements are found in the first three groups of the periodic table.

Ohm's Law

When the potential difference across the ends of a metal conductor is V, the resulting current I is, within wide limits, directly proportional to V. This empirical observation, called **Ohm's law,** is usually expressed as

Ohm's law

$$I = \frac{V}{R} \tag{10.9}$$

Here R, the **resistance** of the conductor, depends on its dimensions, composition, and temperature, but is independent of V. Ohm's law follows from the free-electron model of a metal.

Classical Picture and Lorentz-Drude Model

- We begin by assuming that
- the free electrons in a metal, like the molecules in a gas, move in random directions and undergo frequent collisions.
- These collisions are not billiard-ball collisions with other electrons but represent the scattering of Electron waves by irregularities in the crystal structure, both defects such as impurity atoms and also atoms temporarily out of place as they vibrate.
- As we will see later, the Atoms of a perfect crystal lattice do not scatter free Electron waves except under certain specific circumstances.

Electrical Properties of Metals

- If λ is the mean free path between the collisions of a free electron, the average time between collisions is

Collision time $$\tau = \frac{\lambda}{v_F}$$

The quantity v_F is the electron velocity that corresponds to the Fermi energy ϵ_F, since only electrons at or near the top of their energy distribution can be accelerated (see Sec. 9.10). This average time is virtually independent of an applied electric field **E** because v_F is extremely high compared with the velocity change such a field produces. In copper, for instance, $\epsilon_F = 7.04$ eV and so

$$v_F = \frac{2\epsilon_F}{m} = \sqrt{\frac{(2)(7.04\text{ eV})(1.60 \times 10^{-19}\text{ J/eV})}{9.11 \times 10^{-31}\text{ kg}}} = 1.57 \times 10^6\text{ m/s}$$

The superimposed **drift velocity** v_d due to an applied electric field, however, is usually less than 1 mm/s.

Find the drift velocity v_d of the free electrons in a copper wire whose cross-sectional area is $A = 1.0\ \text{mm}^2$ when the wire carries a current of 1.0 A. Assume that each copper atom contributes one electron to the electron gas.

Solution

The wire contains n free electrons per unit volume. Each electron has the charge e and in the time t it travels the distance $v_d t$ along the wire, as in Fig. 10.16. The number of free electrons in the volume $Av_d t$ is $nAv_d t$, and all of them pass through any cross section of the wire in the time t. Thus the charge that passes through this cross section in t is $Q = nAev_d t$, and the corresponding current is

$$I = \frac{Q}{t} = nAev_d$$

The drift velocity of the electrons is therefore

$$v_d = \frac{I}{nAe}$$

From Example 9.8 we know that, in copper, $n = N/V = 8.5 \times 10^{28}$ electrons/m^3, and here $I = 1.0$ A and $A = 1.0\ \text{mm}^2 = 1.0 \times 10^{-6}\ \text{m}^2$. Hence

$$v_d = \frac{1.0\ \text{A}}{(8.5 \times 10^{28}\ \text{m}^{-3})(1.0 \times 10^{-6}\ \text{m}^2)(1.6 \times 10^{-19}\ \text{C})} = 7.4 \times 10^{-4}\ \text{m/s}$$

But if the free electrons have so small a drift velocity, why does an electric appliance go on as soon as its switch is closed and not minutes or hours later? The answer is that applying a potential difference across a circuit very rapidly creates an electric field in the circuit, and as a result all the free electrons begin their drift almost simultaneously.

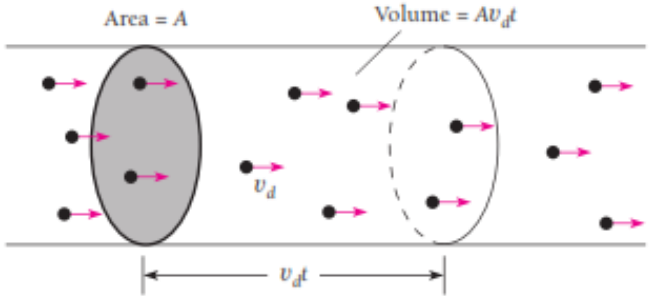


Figure 10.16 The number of free electrons in a wire that drift past a cross-section of the wire in the time t is $nV = nAv_d t$, where n is the number of free electrons/m^3 in the wire.

Let N be the number of atoms and consider that each atom contribute one free electron, then the number of free electrons will also be N.

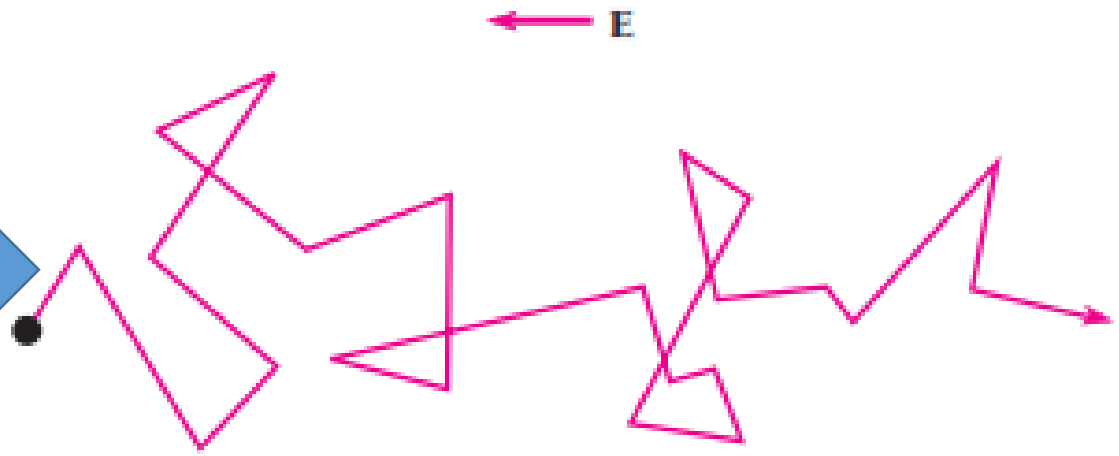
Consider V is the volume of the system, then the number of free electrons per unit volume is

$$n = \frac{N}{V}$$

Electrical Properties of Metals

- A potential difference V across the ends of a conductor of length L produces an electric field of magnitude E=V/L in the conductor. This field exerts a force of ‘eE’ on a free electron in the conductor, whose acceleration is
- $a = \frac{F}{m} = \frac{eE}{m}$

An electric field produces a general drift superimposed on the random motion of a free electron. The electron’s path between collisions is actually slightly curved because of the acceleration due to the field.



- When the electron undergoes a collision, it bounce back in an arbitrary direction and on the average, no longer has a component of velocity parallel to E. Imposing the field E on the free electron gas in a metal superimposes a general drift on the faster but random motions of the electron. We can therefore ignore the electron’s motion at the Fermi velocity in calculating the drift velocity v_d.

Electrical Properties of Metals

After each collision, the electron is accelerated for some time interval Δt before the next collision, and at the end of the interval has traveled $\frac{1}{2}a\,\Delta t^2$. When the electron has made many collisions, its average displacement will be $\overline{X} = \frac{1}{2}a\,\overline{\Delta t^2}$, where $\overline{\Delta t^2}$ is the average of the squared time intervals. Because of the way Δt varies, $\overline{\Delta t^2} = 2\tau^2$. Hence $\overline{X} = a\tau^2$ and the drift velocity is $\overline{X}/\tau = a\tau$, so that

Collision time $\tau = \dfrac{\lambda}{v_F}$

Drift velocity
$$v_d = a\tau = \left(\frac{eE}{m}\right)\left(\frac{\lambda}{v_F}\right) = \frac{eE\lambda}{mv_F} \qquad (10.12)$$

$a = \dfrac{F}{m} = \dfrac{eE}{m}$

In Example 10.2 we found that the current I in a conductor of cross-sectional area A in which the free electron density is n is given by

$$I = nAev_d$$

$I = \dfrac{Q}{t} = nAcv_d$

Using the value of v_d from Eq. (10.12) gives

$v_d = \dfrac{I}{nAc}$

$$I = \frac{nAe^2E\lambda}{mv_F}$$

Since the electric field in the conductor is $E = V/L$,

$$I = \left(\frac{ne^2\lambda}{mv_F}\right)\left(\frac{A}{L}\right)V \qquad (10.14)$$

Electrical Properties of Metals

Drift velocity $v_d = a\tau = \left(\frac{eE}{m}\right)\left(\frac{\lambda}{v_F}\right) = \frac{eE\lambda}{mv_F}$

Since the electric field in the conductor is $E = V/L$,

$$I = \left(\frac{ne^2\lambda}{mv_F}\right)\left(\frac{A}{L}\right)V$$

$$I = nAev_d$$

$$I = \frac{nAe^2E\lambda}{mv_F}$$

This formula becomes Ohm's law if we set

$V = IR$ or $I = \frac{V}{R}$

Resistance of metal conductor

$$R = \left(\frac{mv_F}{ne^2\lambda}\right)\frac{L}{A}$$

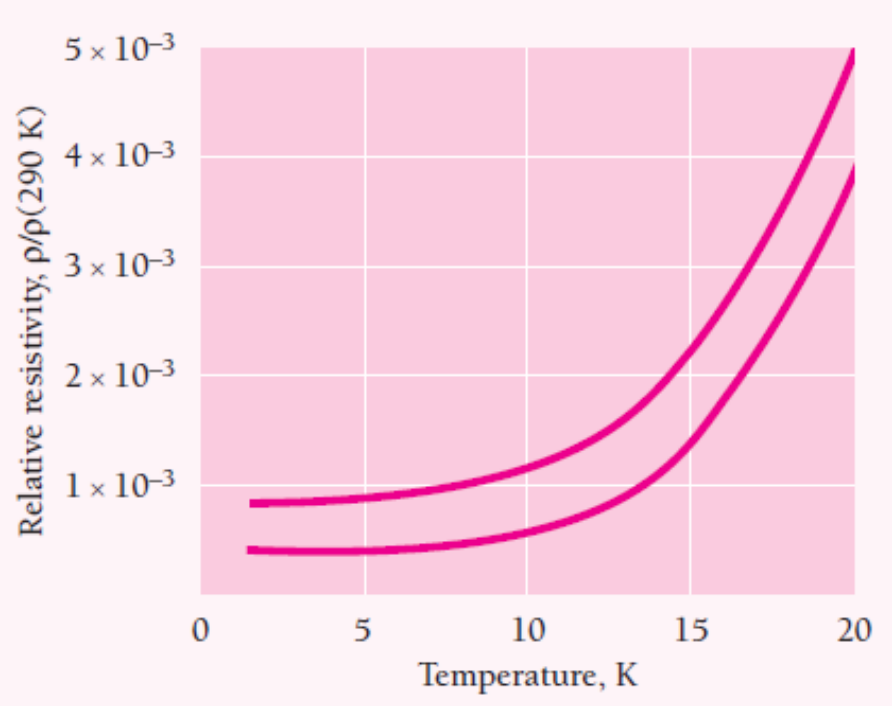
The quantity in parentheses is known as the **resistivity** ρ of the metal and is a constant for a given sample at a given temperature:

Resistivity $\rho = \frac{mv_F}{ne^2\lambda}$

Electrical Properties of Metals: Resistivity

- The scattering of free electron waves in a metal that leads to its electric resistance is caused both by structural defects and by ions out of place as they vibrate.
- Imperfections of the former kind do not depend on temperature but on the purity of the metal and on its history. The resistivities of cold-worked metals (such as "hard drawn" wires) are lowered by annealing because the number of defects is thereby decreased.
- On the other hand, lattice vibrations increase in amplitude with increasing temperature, and their contribution to resistivity accordingly goes up with temperature. Thus the resistivity of a metal is

$$\rho = \rho_i + \rho_T$$



ρ_i is the residual resistivity, due to the scattering by impurities. Independent of temperature.
At zero temperature, the residual resistivity ρ_i dominates.

ρ_T is the resistivity due to the scattering by phonons (lattice vibrations). Depends on temperature.
As temperature increases, resistivity of metals increases due to the increase of lattice vibrations
=>the scattering of electron with phonons increases.

Example

The resistivity of copper at 20°C is $\rho = 1.72 \times 10^{-8}\ \Omega \cdot \text{m}$. Estimate the mean free path λ between collisions of the free electrons in copper at 20°C.

Solution

$n=N/V$

In Example 9.8 we found that the free electron density in copper is $n = 8.48 \times 10^{28}\ \text{m}^{-3}$, and earlier in this section we saw that the Fermi velocity there is $v_F = 1.57 \times 10^{6}$ m/s. Solving Eq. (10.16) for λ gives

$$\lambda = \frac{m v_F}{n e^2 \rho} = \frac{(9.11 \times 10^{-31}\ \text{kg})(1.57 \times 10^{6}\ \text{m/s})}{(8.48 \times 10^{28}\ \text{m}^{-3})(1.60 \times 10^{-19}\ \text{C})^2(1.72 \times 10^{-8}\ \Omega \cdot \text{m})}$$

$$= 3.83 \times 10^{-8}\ \text{m} = 38.3\ \text{nm}$$

The ions in solid copper are 0.26 nm apart, so a free electron travels past nearly 150 of them, on the average, before being scattered.

Electrical Properties of Metals

Mean free path (λ): The mean distance between two successive collision.

Mean collision time (τ): The mean time between two successive collision.

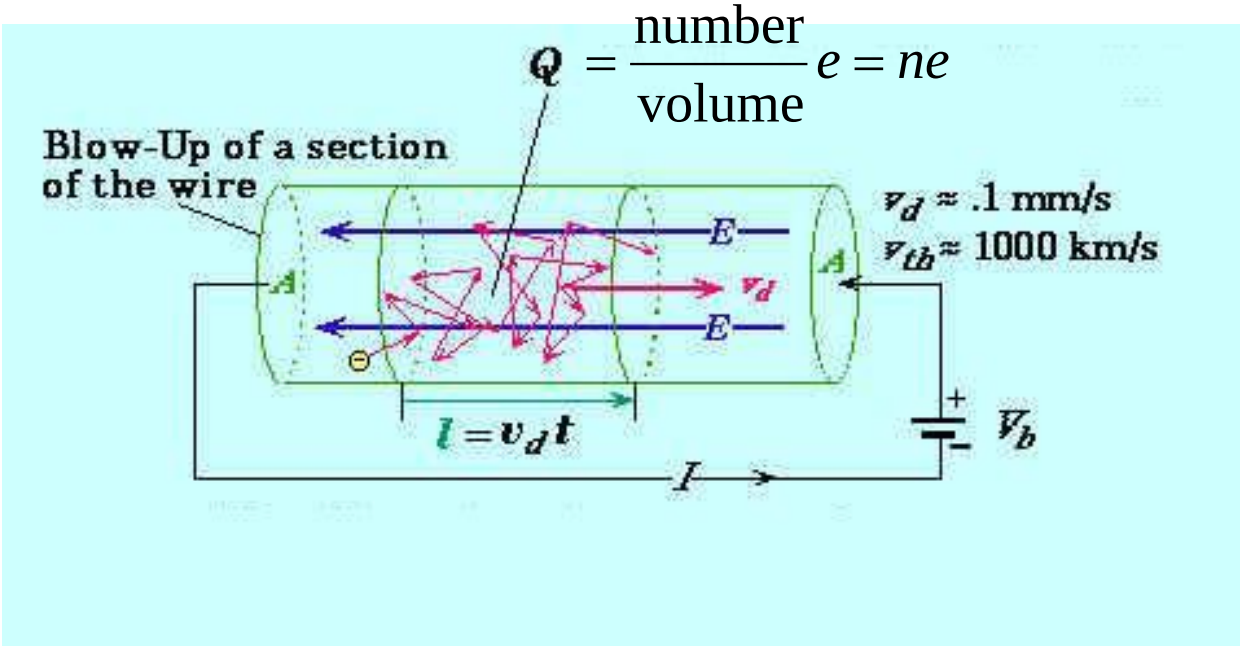
Drift velocity: motion of charge carriers (electron or hole) in electric field

$$v_d = \frac{\lambda}{\tau}$$

Mobility: $\mu = \frac{v_d}{E}$; unit: $cm^2\ V^{-1}\ sec^{-1}$

Electrical conductivity: $\boldsymbol{\sigma} = \frac{\boldsymbol{J}}{\boldsymbol{E}}$; J and E are current density and electric field. Unit: $(ohm\ m)^{-1}$

The relation between $\boldsymbol{\sigma}$ and $\boldsymbol{\mu}$ is: $\boldsymbol{\sigma = ne\mu}$



Electron mobility is the measure of how fast an **electron** can move through a metal or semiconductor under the influence of external electric field.

Mobility μ is defined as the magnitude of drift velocity per unit electric field. μ=E|vd|.

Electrical Properties of Metals

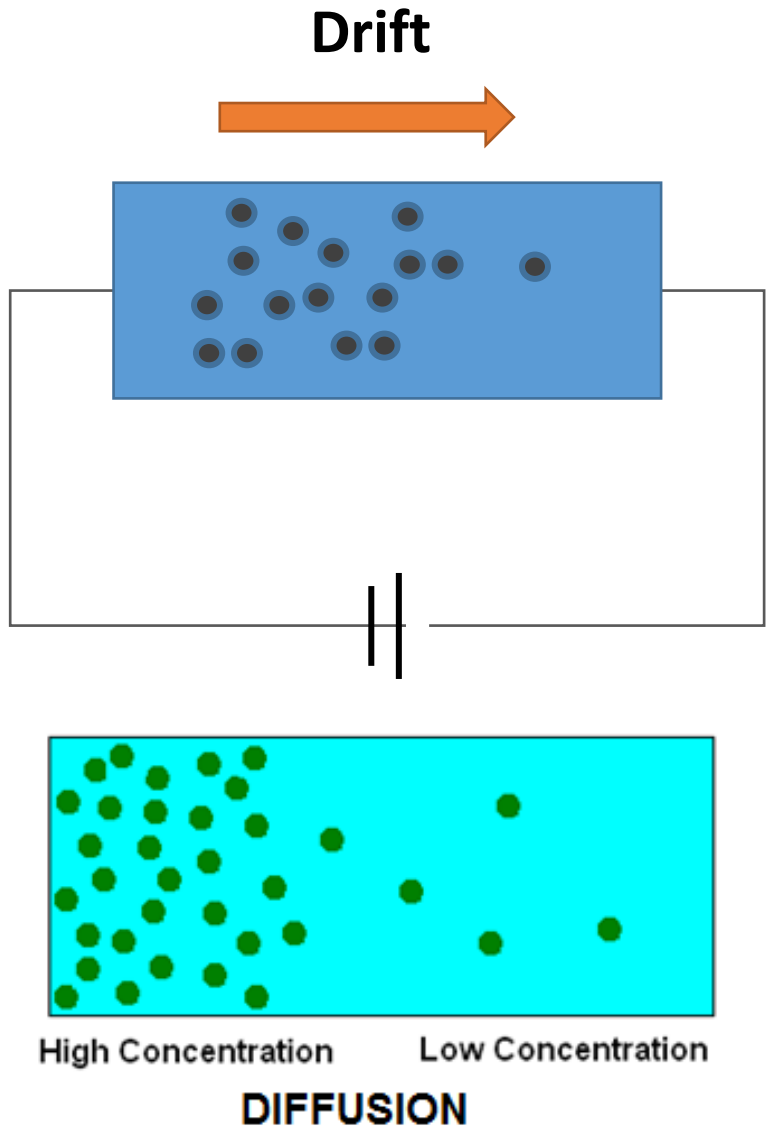
Drift current: Current due to drift of charge carriers in electric field.

$$J = nev_d$$

Diffusion current: Current due to diffusion of charge carriers from heavily concentrated region to the low concentrated region.

$$J = eD_e \frac{dn}{dx}$$

D_e and $\frac{dn}{dx}$ are diffusion coefficient and concentration gradient.



Wiedemann-Franz Law (1853)

- Wiedemann-Franz law: ratio of thermal conductivity (k) to the electrical conductivity (σ) is proportional to the absolute temperature

$$\frac{k}{\sigma T} = L$$

The thermal conductivity (k) is defined as:

$$q = k\frac{dT}{dx}$$

q is heat flow per unit time per unit area (W/m^2),

k is thermal conductivity and unit is W/(K.m).

L is Lorentz number = 2.45×10^{-8} WΩ/K^2

(Ludwig) Lorenz Number (derived via quantum mechanical treatment)

$$L = \frac{\kappa}{\sigma}\frac{1}{T} = \frac{\pi^2 k_B^{\,2}}{3e^2} = 2.45\times10^{-8}\ \frac{\mathrm{W}\cdot\Omega}{\mathrm{K}^2}$$

Lorenz number in 10^-8 Watt ohm/K^2		
Metal	273K	373K
Ag	2.31	2.37
Au	2.35	2.40
Cd	2.42	2.43
Cu	2.23	2.33
Ir	2.49	2.49
Mo	2.61	2.79
Pb	2.47	2.56
Pt	2.51	2.60
Sn	2.52	2.49
W	3.04	3.20
Zn	2.31	2.33

	N/V $\times10^{22}$ /cm^3	k_F $\times10^8$ /cm	v_F $\times10^8$ cm/s	ε_F eV	T_F $\times10^4$ K
Li	**4.70**	**1.11**	**1.29**	**4.72**	**5.48**
Na	**2.65**	**0.92**	**1.07**	**3.23**	**3.75**
Cu	**8.45**	**1.36**	**1.57**	**7.00**	**8.12**
Au	**5.90**	**1.20**	**1.39**	**5.51**	**6.39**
Be	**24.20**	**1.93**	**2.23**	**14.14**	**16.41**
Al	**18.06**	**1.75**	**2.02**	**11.63**	**13.49**
Pb	**13.20**	**1.57**	**1.82**	**9.37**	**10.87**

Wiedemann-Franz law

The theory was nevertheless considered to be on the right track, both because it gave the correct form of Ohm's law and also because it accounted for the **Weidemann-Franz law**. This empirical law states that the ratio K/σ (where $\sigma = 1/\rho$) between thermal and electric conductivities is the same for all metals and is a function only of temperature. If there is a temperature difference ΔT between the sides of a slab of material Δx thick whose cross-sectional area is A, the rate $\Delta Q/\Delta t$ at which heat passes through the slab is given by

$$\frac{\Delta Q}{\Delta t} = -KA\frac{\Delta T}{\Delta x}$$

where K is the thermal conductivity. According to the kinetic theory of a classical gas applied to the electron gas in the Drude-Lorentz model,

$$K = \frac{kn v_{\text{rms}} \lambda}{2}$$

Wiedemann-Franz law

From Eq. (10.16) with v_F replaced by v_{rms},

$$\sigma = \frac{1}{\rho} = \frac{ne^2\lambda}{mv_{rms}}$$

Hence the ratio between the thermal and electric resistivities of a metal is

$$\frac{K}{\sigma} = \left(\frac{knv_{rms}\lambda}{2}\right)\left(\frac{mv_{rms}}{ne^2\lambda}\right) = \frac{kmv_{rms}^2}{2e^2}$$

According to Eq. (9.15), $v_{rms}^2 = 3kT/m$, which gives

$$\frac{K}{\sigma T} = \frac{3k^2}{2e^2} = 1.11 \times 10^{-8}\ \mathrm{W \cdot \Omega/K^2}$$

This ratio does not contain the electron density n or the mean free path λ, so $K/\sigma T$ ought to have the same constant value for all metals, which is the Weidemann-Franz law. To be sure, the above value of $K/\sigma T$ is incorrect because it is based on a Maxwell-Boltzmann distribution of electron velocities. When Fermi-Dirac statistics are used, the result is

$$\frac{K}{\sigma T} = \frac{\pi^2 k^2}{3e^2} = 2.45 \times 10^{-8}\ \mathrm{W \cdot \Omega/K^2}$$

which agrees quite well with experimental findings.