Thursday, August 27, 2020 10:54 AM

Q1) Solve the recurrence relation:

$$a_{n+2} + 3a_{n+1} + 2a_n = 2$$

Solⁿ The given recurrence relation is

$$a_{n+2} + 3a_{n+1} + 2a_n = 2$$

$$E^2 a_n + 3Ea_n + 2a_n = 2$$

$$(E^2 + 3E + 2)a_n = 2$$

$\therefore$ Characteristic eqn is

$$r^2 + 3r + 2 = 0$$

$$r^2 + 2r + r + 2 = 0$$

$$r(r+2) + 1(r+2) = 0$$

$$(r+1)(r+2) = 0$$

$$r = -1, -2$$

$$a_n^{(h)} = c_1(-1)^n + c_2(-2)^n$$

$$a_n^{(p)} = \frac{1}{E^2+3E+2}(2)$$

$$= \frac{1}{(1)^2+3(1)+2}(2)$$

$$= \frac{2}{1+3+2} = \frac{\cancel{2}}{\cancel{6}} = \frac{1}{3}$$

$$a_n = a_n^{(h)} + a_n^{(p)}$$

$$= c_1(-1)^n + c_2(-2)^n + \frac{1}{3}$$

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Q2 Solve the recurrence relation

$$a_{n+2} + 4a_{n+1} + 4a_n = 20$$

Solⁿ: The given recurrence relation is

$$a_{n+2} + 4a_{n+1} + 4a_n = 20$$

$$E^2 a_n + 4Ea_n + 4a_n = 20$$

$$(E^2 + 4E + 4)a_n = 20$$

$\therefore$ Its Characteristic eqn is

$$r^2 + 4r + 4 = 0$$

$$r^2 + 2r + 2r + 4 = 0$$

$$r(r+2) + 2(r+2) = 0$$

$$(r+2)(r+2) = 0$$

$$r = -2, -2$$

$\therefore$ $a_n^{(h)} = (c_1 + c_2 n)(-2)^n$

$$a_n^{(p)} = \frac{1}{E^2+4E+4}(20)$$

$$= \frac{1}{(1)^2+4(1)+4} 20$$

$$= \frac{20}{9}$$

$$a_n = a_n^{(h)} + a_n^{(p)}$$

$$= (c_1 + c_2 n)(-2)^n + 20/9$$

Q3 Solve the recurrence relation.

$\therefore$ The characteristic eqn is.

$$r^2 - 5r + 6 = 0$$

Q3 Solve the recurrence relation.

$$a_n - 5a_{n-1} + 6a_{n-2} = 7^n$$

Sol^n The given recurrence relation is.

$$a_n - 5a_{n-1} + 6a_{n-2} = 7^n$$

$$E^2(a_n) - 5E^2(a_{n-1}) + 6E^2(a_{n-2}) = E^2(7^n)$$

$$a_{n+2} - 5a_{n+1} + 6a_n = 7^{n+2}$$

$$E^2 a_n - 5Ea_n + 6a_n = 7^{n+2}$$

$$(E^2 - 5E + 6)a_n = 7^{n+2}$$

$\therefore$ The characteristic eqn is:

$$r^2 - 5r + 6 = 0$$

$$r^2 - 3r - 2r + 6 = 0$$

$$r(r-3) - 2(r-3) = 0$$

$$(r-2)(r-3) = 0$$

$$r = 2, 3$$

$$a_n^{(H)} = c_1(2)^n + c_2(3)^n$$

$$a_n^{(p)} = \frac{1}{E^2 - 5E + 6}(7^{n+2})$$

$$= \frac{1}{E^2 - 5E + 6} 49(7^n)$$

$$= \frac{1}{(7)^2 - 5\times 7 + 6} 49(7)^n$$

$$= \frac{1}{49 - 35 + 6} 49(7)^n$$

$$= \frac{49}{20}(7)^n$$

$$a_n = a_n^{(H)} + a_n^{(p)}$$

$$= c_1(2)^n + c_2(3)^n + \frac{49}{20}(7)^n$$

Q4 Solve the recurrence relation

$$a_{n+2} - 6a_{n+1} + 5a_n = 2^n$$

$$a_n = c_1(1)^n + c_2(2)^n - \frac{2^n}{3}$$

Results of Binomial expansion

① $(1+x)^{-1} = 1 - x + x^2 - x^3 + \cdots\cdots \infty$

② $(1-x)^{-1} = 1 + x + x^2 + x^3 + \cdots\cdots \infty$

③ $(1+x)^{-2} = 1 - 2x + 3x^2 \cdots\cdots \infty$

④ $(1-x)^{-2} = 1 + 2x + 3x^2 + \cdots\cdots \infty$

① $n^{[1]} = n^1$

① $n^{[1]} = n^1$

② $n^{[2]} + n^{[1]} = n^2$

Q Solve the recurrence relation

$$a_{n+2} - 5a_{n+1} + 6a_n = 2n^2 - 6n - 1$$

Sol^n The given recurrence relation is.

$$a_{n+2} - 5a_{n+1} + 6a_n = 2n^2 - 6n - 1$$

$$E^2 a_n - 5Ea_n + 6a_n = 2n^2 - 6n - 1$$

$$(E^2 - 5E + 6)a_n = 2n^2 - 6n - 1$$

Its characteristic eqn is.

$$r^2 - 5r + 6 = 0$$

$$r^2 - 3r - 2r + 6 = 0$$

$$r(r-3) - 2(r-3) = 0$$

$$(r-2)(r-3) = 0$$

$$r = 2, 3$$

$$a_n^{(h)} = c_1(2)^n + c_2(3)^n$$

$$a_n^{(p)} = \frac{1}{E^2 - 5E + 6}(2n^2 - 6n - 1) \qquad (E = 1 + \Delta)$$

$$= \frac{1}{(1+\Delta)^2 - 5(1+\Delta) + 6}\left[2(n^{[2]} + n^{[1]}) - 6n^{[1]} - 1\right]$$

$$= \frac{1}{1 + \Delta^2 + 2\Delta - 5 - 5\Delta + 6}\left[2n^{[2]} + 2n^{[1]} - 6n^{[1]} - 1\right]$$

$$= \frac{1}{\Delta^2 - 3\Delta + 2}\left[2n^{[2]} - 4n^{[1]} - 1\right]$$

$$= \frac{1}{2 - 3\Delta + \Delta^2}\left[2n^{[2]} - 4n^{[1]} - 1\right]$$

$$= \frac{1}{2-3\Delta+\Delta^2}\left[2n^{[2]} - 4n^{[1]} \right]$$

$$= \frac{1}{2}\,\frac{1}{\left(1-\frac{3\Delta}{2}+\frac{\Delta^2}{2}\right)}\left[2n^{[2]}-4n^{[1]}-1\right]$$

$$= \frac{1}{2}\,\frac{1}{\left[1-\left(\frac{3\Delta}{2}-\frac{\Delta^2}{2}\right)\right]}\left[2n^{[2]}-4n^{[1]}-1\right]$$

$$\left[(1-x)^{-1} = 1+x+x^2+x^3+ \dots \infty\right]$$

$$= \frac{1}{2}\left[1-\left(\frac{3\Delta}{2}-\frac{\Delta^2}{2}\right)\right]^{-1}\left[2n^{[2]}-4n^{[1]}-1\right]$$

$$= \frac{1}{2}\left[1+\left(\frac{3\Delta}{2}-\frac{\Delta^2}{2}\right)+\left(\frac{3\Delta}{2}-\frac{\Delta^2}{2}\right)^2 - \dots\right]\left(2n^{[2]}-4n^{[1]}-1\right)$$

$$= \frac{1}{2}\left[1+\frac{3\Delta}{2}-\frac{\Delta^2}{2}+\frac{9}{4}\Delta^2\right]\left(2n^{[2]}-4n^{[1]}-1\right)$$

$$= \frac{1}{2}\left[1+\frac{3\Delta}{2}+\frac{7}{4}\Delta^2\right]\left(2n^{[2]}-4n^{[1]}-1\right)$$

$$= \frac{1}{2}\left[2n^{[2]}-4n^{[1]}-1+\frac{3}{2}\Delta\left[2n^{[2]}-4n^{[1]}-1\right]+\frac{7}{4}\Delta^2\left[2n^{[2]}-4n^{[1]}-1\right]\right]$$

$$= \frac{1}{2}\left[2n^{[2]}-4n^{[1]}-1+\frac{3}{2}\left(4n^{[1]}-4\right)+\frac{7}{\cancel{4}}\,\cancel{4}\right]$$

$$= \frac{1}{2}\left[2n^{[2]}-4n^{[1]}+\frac{3}{\cancel{2}}\times\cancel{4}^{\,2}\left(n^{[1]}-1\right)+7-1\right]$$

$$= \frac{1}{2}\left[2n^{[2]}-4n^{[1]}+6n^{[1]}-6+7-1\right] = \frac{1}{2}\left[2n^{[2]}+2n^{[1]}+0\right]$$

$$= \frac{1}{2}\left[2n(n-1)+2n+0\right]$$

$$= \frac{1}{2}\left[2n^2-\cancel{2n}+\cancel{2n}+0\right]$$

$$= \frac{1}{2}\left[2n^2+0\right] = n^2$$

$$a_n = a_n^{(h)} + a_n^{(p)}$$

$$= c_1(2)^n + c_2(3)^n + n^2 + 2.$$

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Q Find the P.S of $\dfrac{1}{E^2+E}(n^2+3)$

Soln

$$P.S = \frac{1}{E^2+E}(n^2+3)$$

$$= \frac{1}{(1+\Delta)^2+(1+\Delta)}\left[n^{[2]} + n^{[1]} + 3\right]$$

$$= \frac{1}{1+\Delta^2+2\Delta+1+\Delta}\left[n^{[2]} + n^{[1]} + 3\right]$$

$$= \frac{1}{\Delta^2+3\Delta+2}\left[n^{(2)} + n^{[1]} + 3\right]$$

$$= \frac{1}{2+3\Delta+\Delta^2}\left[n^{[2]} + n^{[1]} + 3\right]$$

$$= \frac{1}{2}\,\frac{1}{\left[1+\frac{3\Delta}{2}+\frac{\Delta^2}{2}\right]}\left[n^{[2]} + n^{[1]} + 3\right]$$

$$= \frac{1}{2}\left[1+\left(\frac{3\Delta}{2}+\frac{\Delta^2}{2}\right)\right]^{-1}\left[n^{[2]} + n^{[1]} + 3\right]$$

$$= \frac{1}{2}\left[1-\left(\frac{3\Delta}{2}+\frac{\Delta^2}{2}\right)+\left(\frac{3\Delta}{2}+\frac{\Delta^2}{2}\right)^2 - \cdots\right]\left[n^{[2]} + n^{[1]} + 3\right]$$

$$= \frac{1}{2}\left[1-\frac{3\Delta}{2} \underbrace{-\frac{\Delta^2}{2}+\frac{9}{4}\Delta^2}\right]\left[n^{[2]} + n^{[1]} + 3\right]$$

$$= \frac{1}{2}\left[1-\frac{3\Delta}{2}+\frac{7}{4}\Delta^2\right]\left[n^{[2]} + n^{[1]} + 3\right]$$

$$= \frac{1}{2}\left[n^{[2]} + n^{[1]} + 3 - \frac{3}{2}\Delta\left(n^{[2]} + n^{[1]} + 3\right) + \frac{7}{4}\Delta^2\left[n^{[2]} + n^{[1]} + 3\right]\right]$$

$$= \frac{1}{2}\left[n^{[2]} + n^{[1]} + 3 - \frac{3}{2}\left[2n^{[1]}+1\right] + \frac{7}{\cancel{4}\,2}(2)\right]$$

$$= \frac{1}{2}\left[n^{[2]} + n^{[1]} + 3 - 3n^{[1]} - \frac{3}{2} + \frac{7}{2}\right] = \frac{1}{2}\left[n^{[2]} - 2n^{[1]} + 5\right]$$

$$= \frac{1}{2}\left[n^{[2]} + \underline{n^{[1]}} + \frac{3}{2} - \underline{3n^{[1]}} - \frac{3}{2} + \frac{7}{2} \right] = \frac{1}{2}\left[n^{[2]} - 2n^{[1]} + 5 \right]$$

$$= \frac{1}{2}\left[n(n-1) - 2n + 5 \right] = \frac{1}{2}\left[n^2 - n - 2n + 5 \right]$$

$$= \frac{1}{2}\left[n^2 - 3n + 5 \right]$$