lecture3

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Topics of the lecture

lecture-3

- Exclusive OR connective
- Conditional-disjunction equivalence
- Proof using logical equivalences
- Predicates
- Quantification and Quantifiers
- Quiz

p	q	$p \to q$	$p \leftrightarrow q$
T	T	T	T
T	F	F	F
F	T	T	F
F	F	T	T

About Conditional Statement

$p \to q \to$ conc.

hyp

In a conditional statement p→q, p is called "hypothesis" or "antecedent" and q is called "conclusion" or "consequent".

If antecedent then consequent.

Exclusive OR

A connective in logic that yields the truth value "T" if exactly one of the propositions involved are true, otherwise false. It is also known as "XOR".

$p \oplus q \equiv \sim(p \leftrightarrow q)$

$\vee, \wedge, \sim, \rightarrow, \leftrightarrow$

$\oplus$

$p \oplus q$

p	q	$p \oplus q$	$p \leftrightarrow q$	$\sim(p \leftrightarrow q)$
T	T	F	T	F
T	F	T	F	T
F	T	T	F	T
F	F	F	T	F

Exclusive OR

A connective in logic that yields the truth value "T" if exactly one of the propositions involved are true, otherwise false. It is also known as "XOR". The truth table for "XOR" connective is as follows:

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Conditional-disjunction equivalence

① $\boxed{p \rightarrow q \equiv \sim p \vee q}$ or ② $\boxed{p \rightarrow q \leftrightarrow \sim p \vee q \text{ is a tautology}}$

$p \rightarrow q \equiv \sim p \vee q$

p	q	$\sim p$	$p \rightarrow q$	$\sim p \vee q$
T	T	F	T	T
T	F	F	F	F

$p \rightarrow q \leftrightarrow \sim p \vee q$

T

T

tautology

T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

T
T
T

tautology

Conditional-disjunction equivalence

$p \to q \equiv \sim p \vee q$ or $p \to q \leftrightarrow \sim p \vee q$ is a tautology

p	q	$p\to q$	$\sim p$	$\sim p \vee q$	$p\to q \leftrightarrow \sim p\vee q$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

Use of logical equivalences

$\sim(p\to q) \equiv p \wedge \sim q$ Truth table //

$p \to q \equiv \sim p \vee q$

$\sim(p\vee q) \equiv \sim p \wedge \sim q$

Prove that $\sim(p\to q)$ and $p\wedge \sim q$ are logically equivalent.

Pf: $\sim(p \to q) \equiv \sim(\sim p \vee q)$ [Conditional- disjunction equivalence]

$\equiv \sim(\sim p) \wedge \sim q$ [De-Morgans law]

$\equiv p \wedge \sim q$ [Double negation law]

Proved //

Use of logical equivalences

Prove that ∼(p→q) and p∧ ∼q are logically equivalent.

Proof:

$$\sim(p \to q) \equiv \sim(\sim p \vee q) \qquad \text{[Using conditional-disjunction equivalence]}$$
$$\equiv \sim(\sim p) \wedge \sim q \qquad \text{[Using De Morgan's law]}$$
$$\equiv p \wedge \sim q \qquad \text{[Using double negation law].}$$

Predicates

Let us consider the following statement : "x is greater than 3". It has two parts. The first part is the variable x, which is called the subject of the statement. The second part "is greater than 3" is called the predicate.

He is 31 years old.
↓ subject ↓ predicate

x is greater than 3.
↓ subject ↓ predicate

Predicates

Let us consider the following statement : "x is greater than 3". It has two parts. The first part is the variable x, which is called the subject of the statement. The second part "is greater than 3" is called the predicate.

statement

Predicate is the property that the subject of a ~~subject~~ can have.

Notation

$P(x)$: x is greater than 3 .

K

We denote the statement "x is greater than 3" by $P(x)$, where P denotes the **predicate** "is greater than 3" and x is the variable. The statement $P(x)$ is also said to be the value of the propositional function P at x. Once a value has been assigned to the variable x, the statement $P(x)$ becomes a **proposition** and has a **truth value**.

$P(x)$

$x = K$ $P(K)$

$P(x) \rightarrow$ proposition

truth value.

$f(x)$

$P(x)$

$x = K$

$P(K)$ T or F.

Example

$P(x): x > 3$
$P(4): 4 > 3$

Let $P(x)$ denote the statement "$x > 3$". What are the truth values of $P(4)$ and $P(2)$?

$P(4): 4 > 3$ True.

$P(2): 2 > 3$ False.

Example

Let $P(x)$ denote the statement "$x > 3$". What are the truth values of $P(4)$ and $P(2)$?

Solution: $P(4)$: $4 > 3$. TRUE.

Example

Let $P(x)$ denote the statement "$x > 3$". What are the truth values of $P(4)$ and $P(2)$?

Solution: $P(4) : 4 > 3$. TRUE. $P(2) : 2 > 3$. FALSE

Quantification and quantifiers

- When the variables in a propositional function are assigned values, the resulting statement becomes a proposition with a certain truth value.

$P(x)$

$x = k$

$P(k)$ - True or False.

Quantification and quantifiers

1) Universal quantifiers
2) Existential quantifiers

When the variables in a propositional function are assigned values, the resulting statement becomes a proposition with a certain truth value.

The way of creating a proposition from a propositional function is called a **quantification**.

Many
few
some
none

quantifiers

Few of the students are present.

For all values of x

There exists $x \in D$ s.t

Quantification and quantifiers

When the variables in a propositional function are assigned values, the resulting statement becomes a proposition with a certain truth value.

The way of creating a proposition from a propositional function is called a **quantification**

Quantification expresses the extent to which a predicate is true over a range of elements. The words, *some*, *many*, *none*, *and few* are used in quantification. These words are termed as **quantifiers**.

Predicate calculus

The area of logic that deals with predicates and quantifiers is called **predicate calculus**

Types of quantifiers

Universal quantification:Here, predicate is true for every element under consideration.

$\forall x \in \mathbb{N}$, $2x$ is even.

Types of quantifiers

Universal quantification:Here, predicate is true for every element under consideration.

Existential quantification:Here, there is one or more element under consideration for which the predicate is true.

$\exists x \in \mathbb{Z}$ s.t $x^2 - 4 = 0$.

$\rightarrow$ $x = \pm 2$

Quiz 1

What is a procedure that returns a value that signals true or false?

A. List
B. Predicate
C. Quantifier
D. Data

Quiz 1

What is a procedure that returns a value that signals true or false?

A. List
B. Predicate
C. Quantifier
D. Data

Answer: B.

Quiz 2

$p \vee q$

In a disjunction, even if one of the statements is false, the whole disjunction is still

A. False
B. Negated
C. True
D. Both True and False

Quiz 2

In a disjunction, even if one of the statements is false, the whole disjunction is still

A. False

B. Negated

C. True

D. Both True and False

Answer: C.

Quiz 3

The symbolization for a conjunction is

A. p→q

B. p& q and ∧

C. p∨q or disjunction

D. ∼p

Quiz 3

The symbolization for a conjunction is

A. $p \rightarrow q$

B. p& q

C. $p \vee q$

D. $\sim p$

Answer: B.

Quiz 4

$p \rightarrow q$ conse

ante

A conditional is false only when the antecedent is

A. true and the consequent is false

B. false and the consequent is false

C. true and the consequent is true

D. false and the consequent is true

$p \rightarrow q$

T F

T T

F T

F F

T

Quiz 4

A conditional is false only when the antecedent is

A. true and the consequent is false

B. false and the consequent is false

C. true and the consequent is true

D. false and the consequent is true

Answer: A.

Quiz 5

Let $P(x)$ denote "x is a prime number". Then, the truth values of $P(4)$ and $P(91)$ are

A. T and F

B. T and T

C. F and T

D. F and F

P(4) : 4 is a prime no. False

P(91) : 91 is a prime no. False

$91 = 13 \cdot 7$.

Quiz 5

Let $P(x)$ denote "x is a prime number". Then, the truth values of $P(4)$ and $P(91)$ are

A. T and F

B. T and T

C. F and T

D. F and F

Answer: D