

# MTH-401: DISCRETE MATHEMATICS

Department of Mathematics  
Lovely Professional University

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# List of topics for today's class

- Proposition
- Propositional logic
- Logical connectives
- Truth tables

# What is Logic?

Logic is the basis of all mathematical reasoning, and of all automated reasoning. The rules of logic specify the meaning of mathematical statements. These rules help us understand and reason with statements

# Importance of Mathematical Logic

The rules of logic give precise meaning to mathematical statements. These rules are used to distinguish between valid and invalid mathematical arguments. Apart from its importance in understanding mathematical reasoning, logic has numerous applications in Computer Science, varying from design of digital circuits, to the construction of computer programs and verification of correctness of programs.

# Proposition

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(c) Dog is an animal.

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# Proposition

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It cannot have both the both the truth values "TRUE" and "FALSE".

The truth value "TRUE" and "FALSE" are denoted BY "T" and "F" respectively.

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For example, in terms of propositional logic, the claims, “if the moon is made of cheese then basketballs are round,” and “if spiders have eight legs then Sam walks with a limp” are exactly the same. They are both implications: statements of the form,  $P \rightarrow Q$ .



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Generally there are five connectives :

- Conjunction( $\wedge$ )
- Disjunction( $\vee$ )
- Negation( $\sim$ )
- Conditional( $\rightarrow$ , or  $\Rightarrow$ )
- Biconditional( $\leftrightarrow$ , or  $\Leftrightarrow$ )

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$P$  : James bought a shirt and  $Q$  : James bought a book.

Then, with the help of logical connectives defined in the previous page, we can write the following compound propositions :

$P \wedge Q$  : James bought a shirt and a book.

$P \vee Q$  : James bought a shirt or a book.

$\sim P$  : James did not buy a shirt,  $\sim Q$  : James did not buy a book.

$P \rightarrow Q$  : If James bought a shirt, then he bought a book.

$P \leftrightarrow Q$  : James bought a shirt if and only if he bought a book.

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A truth table shows how the truth and falsity of a compound statement depends on the truth or falsity of the simple statements from which it is constructed.

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The TRUTH TABLES for the 5 connectives are as follows.



# TRUTH TABLE FOR LOGICAL CONNECTIVES

P	Q	$P \wedge Q$	$P \vee Q$	$\sim P$	$P \rightarrow Q$	$P \leftrightarrow Q$
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T	T	T	T	F	T	T

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T	F	F	T	F	F	F

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T	T	T	T	F	T	T
T	F	F	T	F	F	F
F	T	F	T	T	T	F

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T	T	T	T	F	T	T
T	F	F	T	F	F	F
F	T	F	T	T	T	F
F	F	F	F	T	T	T

# Quiz 1

Which of the following statement is a proposition?

- (A) Get me a glass of milkshake.
- (B) God bless you!.
- (C) What is the time now?
- (D) The only odd prime number is 2.

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Which of the following statement is a proposition?

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Answer: (D)

## Quiz 2

2. The truth value of ' $4+3=7$  or 5 is not prime' is

TRUE

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Answer: TRUE.

## Quiz 3

Which of the following option is true?

- (A) If the Sun is a planet, elephants will fly.
- (B)  $3 + 2 = 8$  if  $5 - 2 = 7$ .
- (C)  $1 > 3$  and 3 is a positive integer.
- (D)  $-2 > 3$  or 3 is a negative integer.

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Answer: (A)

## Quiz 4

Let P: If Sahil bowls, Saurabh hits a century and Q: If Raju bowls, Sahil gets out on first ball. Now if P is true and Q is false, then which of the following can be true?

- (A) Raju bowled and Sahil got out on first ball.
- (B) Raju did not bowled.
- (C) Sahil bowled and Saurabh hits a century.
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Answer: (C)

## Quiz 5

The truth value '9 is prime then 3 is even' is

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FALSE

Answer: TRUE.

## Quiz 6

Let P: We should be honest., Q: We should be dedicated., R: We should be overconfident. Then 'We should be honest or dedicated but not overconfident.' is best represented by?

- (A)  $\sim P \vee \sim Q \vee R$ .
- (B)  $P \wedge \sim Q \wedge R$ .
- (C)  $P \vee Q \wedge R$ .
- (D)  $P \vee Q \wedge \sim R$ .



## Quiz 6

Let P: We should be honest., Q: We should be dedicated., R: We should be overconfident. Then 'We should be honest or dedicated but not overconfident.' is best represented by?

(A)  $\sim P \vee \sim Q \vee R.$

(B)  $P \wedge \sim Q \wedge R.$

(C)  $P \vee Q \wedge R.$

(D)  $P \vee Q \wedge \sim R.$

Answer: (D)