Quiz 4

Let P: If Sahil bowls, Saurabh hits a century and Q: If Raju bowls, Sahil gets out on first ball. Now if P is true and Q is false, then which of the following can be true?

(A) Raju bowled and Sahil got out on first ball.

(B) Raju did not bowled.

(C) Sahil bowled and Saurabh hits a century.

(D) Sahil bowled and Saurabh got out.

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Answer: (C)

Quiz 5

The truth value '9 is prime then 3 is even' is

TRUE

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FALSE

Answer: TRUE.

Quiz 6

Let P: We should be honest., Q: We should be dedicated., R: We should be overconfident. Then 'We should be honest or dedicated but not overconfident.' is best represented by?

(A)$\sim P \vee \sim Q \vee R$.

(B)$P \wedge \sim Q \wedge R$.

(C)$P \vee Q \wedge R$.

(D)$P \vee Q \wedge \sim R$.

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(B)$P \wedge \sim Q \wedge R$.

(C)$P \vee Q \wedge R$.

(D)$P \vee Q \wedge \sim R$.

Answer: (D)

Topics of the day......PROPOSITIONAL EQUIVALENCES???

- Tautology, Contradiction and Contingency
- Logical Equivalence
- De Morgan Law
- Quiz

Propositional Equivalences

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Two logical statements are said to be equivalent if they have the same truth values in all cases.

This fact of logical equivalence helps us in proving a mathematical result by replacing one expression with another equivalent expression, without changing the truth value of the original compound statement.

Tautology, Contradiction and Contingency

Tautology : A compound proposition that is always true, no matter what the truth values of the propositional variable that occur in it, is called a *tautology*.

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Contradiction : A compound proposition that is always false is called a *contradiction*.

Contingency : A compound proposition that is neither a tautology nor a contradiction is called a *tautology*.

Example of Tautology and Contradiction

p	$\sim p$	$p \vee \sim p$	$p \wedge \sim p$

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p	$\sim p$	$p \vee \sim p$	$p \wedge \sim p$
T	F	T	F
F	T	T	F

Example of Contingency

p	q	p∨q	p∧q

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p	q	$p \vee q$	$p \wedge q$
T	T	T	T

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Logical Equivalence

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Definition 1 : Compound propositions that have the same truth values in all possible cases are called logically equivalent.

Definition 2 : The compound propositions p and q are called logically equivalent if $p \leftrightarrow q$ is a **tautology**.

Notation : If the compound propositions p and q are logically equivalent, then, in notation form, we write $p \equiv q$.

Important Remark about Logical Equivalence

The symbol $\equiv$ is not a logical connective, and $p \equiv q$ is not a compound proposition. It implies that $p \leftrightarrow q$ is a tautology.

Example of logical equivalence

$\sim(p \vee q)$ and $\sim p \wedge \sim q$ are logically equivalent compound propositions. It can be proved with the help of a truth table as follows :

p	q	$p \vee q$	$\sim(p \vee q)$	$\sim p$	$\sim q$	$\sim p \wedge \sim q$

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T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Some important logical equivalence

Identity laws: $p \wedge T \equiv p$ $p \vee F \equiv p$

Domination laws: $p \vee T \equiv T$ $p \wedge F \equiv F$

Idempotent laws: $p \vee p \equiv p$ $p \wedge p \equiv p$

Double negation law: $\sim(\sim)p \equiv p$

Commutative laws: $p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$

Associative laws: $(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

Some more . . .

Distributive laws: $(p \vee q) \wedge r \equiv (p \vee q) \wedge (p \vee r)$ $\quad$ $(p \wedge q) \vee r \equiv (p \wedge q) \vee (p \wedge r)$

Absorption laws: $p \vee (p \wedge q) \equiv p$ $\quad$ $p \wedge (p \vee q) \equiv p$

Negation laws: $p \vee \sim p \equiv T$ $\quad$ $p \wedge \sim p \equiv F$

De Morgan Laws-Logical equivalences

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First statement : $\sim(p \wedge q) \equiv \sim p \vee \sim q$

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The statements of De Morgan laws are written as follows :

First statement : $\sim(p \wedge q) \equiv \sim p \vee \sim q$

Second statement : $\sim(p \vee q) \equiv \sim p \wedge \sim q$

Quiz 1

The compound propositions p and q are logically equivalent if

(A) $p \leftrightarrow q$ is a tautology.

(B) $p \rightarrow q$ is a tautology.

(C) $\sim(p \vee q)$ is a tautology

(D) $\sim p \vee \sim q$ is a tautology.

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(D) $\sim p \vee \sim q$ is a tautology.

Answer: (A). From the definition of logical equivalence.

Quiz 2

If p is any statement, then which of the following is a tautology?

(A) $p \wedge F$.

(B) $p \vee F$.

(C) $p \vee \sim p$.

(D) $p \wedge T$.

Quiz 2

If p is any statement, then which of the following is a tautology?

(A) $p \wedge F$.

(B) $p \vee F$.

(C) $p \vee \sim p$.

(D) $p \wedge T$.

Answer: (C). Since $p \vee \sim p$ is always true.

Quiz 3

If p is any statement, then which of the following is not a contradiction?

(A) $p \wedge \sim p$.

(B) $p \vee F$.

(C) $p \wedge F$.

(D) None of the above.

Quiz 3

If p is any statement, then which of the following is not a contradiction?

(A) $p \wedge \sim p$.

(B) $p \vee F$.

(C) $p \wedge F$.

(D) None of the above.

Answer: (B). Since $p \vee F$ is NOT always false.

Quiz 4

The compound proposition $p \to q$ is logically equivalent to

(A) $\sim p \vee \sim q$.

(B) $p \vee \sim q$.

(C) $\sim p \vee q$.

(D) $\sim p \wedge q$.

The compound proposition $p \rightarrow q$ is logically equivalent to

(A) $\sim p \vee \sim q$.

(B) $p \vee \sim q$.

(C) $\sim p \vee q$.

(D) $\sim p \wedge q$.

Answer: (C). Since $(p \rightarrow q) \leftrightarrow (\sim p \vee q)$ is a tautology.