CHAPTER 21

Elementary Row and Column Transformation

21.1 DEFINITION

Let us consider a set of simultaneous equations,

$$x + 2y + 3z + 5t = 0$$
$$4x + 2y + 5z + 7t = 0$$
$$3x + 4y + 2z + 6t = 0.$$

Now we write down the coefficients of x, y, z, t of the above equations and enclose them within brackets and then we get

$$A = \begin{bmatrix} 1 & 2 & 3 & 5 \\ 4 & 2 & 5 & 7 \\ 3 & 4 & 2 & 6 \end{bmatrix}$$

The above system of numbers, arranged in a rectangular array in rows and columns and bounded by the brackets, is called a matrix.

It has got 3 rows and 4 columns and in all $3 \times 4 = 12$ elements. It is termed as 3×4 matrix, to be read as [3 by 4 matrix]. In the double subscripts of an element, the first subscript determines the row and the second subscript determines the column in which the element lies, a_{ij} lies in the ith row and jth column.

21.2 USE OF MATRICES

Matrices are generally used in solving simultaneous equations and linear transformation. The theory of matrices is also applied in differential equations, astronomy, mechanics, theory of electric circuits etc.

21.3 VARIOUS TYPES OF MATRICES

(*i*) **Row Matrix.** If a matrix has only one row and any number of columns, it is called a *Row matrix, e.g.,*

$$[2\ 7\ 3\ 9]$$

(*b*) **Column Matrix.** A matrix, having one column and any number of rows, is called a *Column matrix, e.g.,* $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

(*c*) **Null Matrix or Zero Matrix.** Any matrix, in which all the elements are zeros, is called a *Zero matrix* or *Null matrix e.g.,*

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(*d*) **Square Matrix.** A matrix, in which the number of rows is equal to the number of columns, is called a square matrix *e.g.*,

$$\begin{bmatrix} 2 & 5 \\ 1 & 4 \end{bmatrix}$$

(*e*) **Diagonal Matrix.** A square matrix is called a diagonal matrix, if all its non-diagonal elements are zero *e.g.*,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

(*f*) **Scalar matrix.** A diagonal matrix in which all the diagonal elements are equal to a scalar, say (k) is called a scalar matrix.
For example;

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \begin{bmatrix} -6 & 0 & 0 & 0 \\ 0 & -6 & 0 & 0 \\ 0 & 0 & -6 & 0 \\ 0 & 0 & 0 & -6 \end{bmatrix}$$

i.e., $A = [a_{ij}]_{n \times n}$ is a scalar matrix if $a_{ij} = \begin{cases} 0, & \text{when } i \neq j \\ k, & \text{when } i = j \end{cases}$

(*g*) **Unit or Identity Matrix.** A square matrix is called a unit matrix if all the diagonal elements are unity and non-diagonal elements are zero *e.g.*,

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

(*h*) **Symmetric Matrix.** A square matrix will be called symmetric, if for all values of i and j, $a_{ij} = a_{ji}$ *i.e.*, $A' = A$

e.g., $$\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$$

(*i*) **Skew Symmetric Matrix.** A square matrix is called skew symmetric matrix, if
(1) $a_{ij} = -a_{ji}$ for all values of i and j, or $A' = -A$
(2) All diagonal elements are zero, *e.g.*,

$$\begin{bmatrix} 0 & -h & -g \\ h & 0 & -f \\ g & f & 0 \end{bmatrix}$$

(*j*) **Triangular Matrix.** (Echelon form) A square matrix, all of whose elements below the leading diagonal are zero, is called an *upper triangular matrix.* A square matrix, all of whose elements above the leading diagonal are zero, is called *a lower triangular matrix*
e.g.,

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & 4 & 1 \\ 0 & 0 & 6 \end{bmatrix}$$

Upper triangular matrix

$$\begin{bmatrix} 2 & 0 & 0 \\ 4 & 1 & 0 \\ 5 & 6 & 7 \end{bmatrix}$$

Lower triangular matrix

(*k*) **Transpose of a Matrix.** If in a given matrix A, we interchange the rows and the corresponding columns, the new matrix obtained is called the transpose of the matrix A and is denoted by A' or A^T *e.g.*,

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 0 & 5 \\ 6 & 7 & 8 \end{bmatrix}, \quad A' = \begin{bmatrix} 2 & 1 & 6 \\ 3 & 0 & 7 \\ 4 & 5 & 8 \end{bmatrix}$$

(*l*) **Orthogonal Matrix.** A square matrix A is called an orthogonal matrix if the product of the matrix A and the transpose matrix A' is an identity matrix *e.g.*,

$$A.\, A' = I$$

if $|A| = 1$, matrix A is proper.

(*m*) **Conjugate of a Matrix**

Let
$$A = \begin{bmatrix} 1+i & 2-3i & 4 \\ 7+2i & -i & 3-2i \end{bmatrix}$$

Conjugate of matrix A is $\bar{A}$

$$\bar{A} = \begin{bmatrix} 1-i & 2+3i & 4 \\ 7-2i & i & 3+2i \end{bmatrix}$$

(*n*) **Matrix A^{θ}.** Transpose of the conjugate of a matrix A is denoted by A^{θ}.

Let
$$A = \begin{bmatrix} 1+i & 2-3i & 4 \\ 7+2i & -i & 3-2i \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} 1-i & 2+3i & 4 \\ 7-2i & +i & 3+2i \end{bmatrix}$$

$$(\bar{A})' = \begin{bmatrix} 1-i & 7-2i \\ 2+3i & i \\ 4 & 3+2i \end{bmatrix}$$

$$A^{\theta} = \begin{bmatrix} 1-i & 7-2i \\ 2+3i & i \\ 4 & 3+2i \end{bmatrix}$$

(*o*) **Unitary Matrix.** A square matrix A is said to be unitary if

$$A^{\theta} A = I$$

e.g.
$$A = \begin{bmatrix} \frac{1+i}{2} & \frac{-1+i}{2} \\ \frac{1+i}{2} & \frac{1-i}{2} \end{bmatrix}, \quad A^{\theta} = \begin{bmatrix} \frac{1-i}{2} & \frac{1-i}{2} \\ \frac{-1-i}{2} & \frac{1+i}{2} \end{bmatrix}, \quad A \cdot A^{\theta} = I$$

(*p*) **Hermitian Matrix.** A square matrix $A = (a_{ij})$ is called Hermitian matrix, if every *i-jth* element of A is equal to conjugate complex *j-ith* element of A.

In other words, $a_{ij} = \bar{a}_{ji}$

e.g.
$$\begin{bmatrix} 1 & 2+3i & 3+i \\ 2-3i & 2 & 1-2i \\ 3-i & 1+2i & 5 \end{bmatrix}$$

Necessary and sufficient condition for a matrix A to be Hermitian is that $A = A^{\theta}$ *i.e.* conjugate transpose of A

$$\Rightarrow \quad A = (\bar{A})'.$$

(*q*) **Skew Hermitian Matrix.** A square matrix $A = (a_{ij})$ will be called a Skew Hermitian matrix if every *i*-*j*th element of *A* is equal to negative conjugate complex of *j*-*i*th element of *A*.

In other words,
$$a_{ij} = -\bar{a}_{ji}$$

All the elements in the principal diagonal will be of the form
$$a_{ii} = -\bar{a}_{ii} \quad \text{or} \quad a_{ii} + \bar{a}_{ii} = 0$$

If $\quad a_{ii} = a + ib \quad$ then $\quad \bar{a}_{ii} = a - ib$

$(a + ib) + (a - ib) = 0 \quad \Rightarrow \quad 2a = 0 \Rightarrow a = 0 \quad \left[aii + \bar{a}\,ii + 0\right]$

So, a_{ii} is pure imaginary $\Rightarrow a_{ii} = 0 + ib = ib$

Hence, all the diagonal elements of a Skew Hermitian Matrix are either zeros or pure imaginary.

e.g.
$$\begin{bmatrix} i & 2-3i & 4+5i \\ -(2+3i) & 0 & 2i \\ -(4-5i) & 2i & -3i \end{bmatrix}$$

The necessary and sufficient condition for a matrix *A* to be Skew Hermitian is that
$$A^\theta = -A$$
$$(\bar{A})' = -A$$

(*r*) **Idempotent Matrix.** A matrix, such that $A^2 = A$ is called Idempotent Matrix.

$$e.g.\ A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}, A^2 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}\begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} = A$$

(*s*) **Periodic Matrix.** A matrix *A* will be called a Periodic Matrix, if
$$A^{k+1} = A$$

where *k* is a +ve integer. If *k* is the least + ve integer, for which $A^{k+1} = A$, then *k* is said to be the period of *A*. If we choose $k = 1$, we get $A^2 = A$ and we call it to be idempotent matrix.

(*t*) **Nilpotent Matrix.** A matrix will be called a Nilpotent matrix, if $A^k = 0$ (null matrix) where *k* is a +ve integer ; if however *k* is the least +ve integer for which $A^k = 0$, then *k* is the *index* of the nilpotent matrix.

$$e.g.,\ A = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}, A^2 = \begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix}\begin{bmatrix} ab & b^2 \\ -a^2 & -ab \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

A is nilpotent matrix whose index is 2.

(*u*) **Involuntary Matrix.** A matrix *A* will be called an Involuntary matrix, if $A^2 = I$ (unit matrix). Since $I^2 = I$ always $\quad \therefore$ Unit matrix is involuntary.

(*v*) **Equal Matrices.** Two matrices are said to be equal if

(*i*) They are of the same order.

(*ii*) The elements in the corresponding positions are equal.

Thus if
$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}, B = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

Here
$$A = B$$

(*w*) **Singular Matrix.** If the determinant of the matrix is zero, then the matrix is known as singular matrix e.g. $A = \begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$ is singular matrix, because $|A| = 6 - 6 = 0$.

OBJECTIVE TYPE QUESTIONS

Choose the correct answer:

1. Let A and B be any two matrices such that $AB = 0$ and A is non-singular.
   Then (*i*) $B = 0$; (*ii*) B is also non-singular; (*iii*) $B = A$; (*iv*) B is singular. **Ans.** (*iv*)

2. If $AB = 0$ and $BA \neq 0$ then necessarily
   (*i*) $A = 0$ (*ii*) $B = 0$ (*iii*) $A = 0, B = 0$ (*iv*) $A \neq 0,\ B \neq 0$
   Ans. (*iv*)

3. Let A and B be two matrices, such that $A = 0$, $AB = 0$, the equation always implies that
   (*i*) $B = 0$ (*ii*) $B \neq 0$ (*iii*) $B = -A$ (*iv*) $B = A'$ **Ans.** (*ii*)

4. If $A=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$, $B=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$, then

(*i*) $AB=0$ (*ii*) $BA=0$

(*iii*) AB and BA are not defined (*iv*) None of these **Ans.** (*iv*)

5. If $A=\begin{bmatrix}2 & 1\\ 2 & 3\end{bmatrix}$, $B=\begin{bmatrix}-3 & 1\\ 2 & 0\end{bmatrix}$, then AB is

(*i*) $\begin{bmatrix}4 & 2\\ 0 & 2\end{bmatrix}$ (*ii*) $\begin{bmatrix}-4 & 2\\ 1 & 2\end{bmatrix}$ (*iii*) $\begin{bmatrix}-4 & 2\\ 0 & 2\end{bmatrix}$ (*iv*) $\begin{bmatrix}4 & 2\\ 1 & 2\end{bmatrix}$ **Ans.** (*iii*)

6. Let A and B, be two matrices, then

(*i*) $AB=BA$ (*ii*) $AB \neq BA$ (*iii*) $AB < BA$ (*iv*) $AB > BA$ **Ans.** (*ii*)

7. In matrices:

(*i*) $(A+B)^2 = A^2 + 2AB + B^2$ (*ii*) $(A+B)^2 = A^2 + B^2$

(*iii*) $(A+B)^2 \neq A^2 + 2AB + B^2$ (*iv*) $(A+B)^2 = A^2 + 2BA + B^2$ **Ans.** (*iii*)

8. If $f(x) = x^2 + 3x + 4$, then $f(A) =$

(*i*) $A^2 + 3A + 4I$ (*ii*) $A^2 + 3A + 4$ (*iii*) $A^2 + 3A$ (*iv*) $(A+4)(A+I)$

Ans. (*i*)

9. In matrix multiplication of two matrix A and matrix B,

(*i*) $AB = BA$ (*ii*) $AB \neq BA$ (*iii*) $AB = 2B$ (*iv*) None of these

Ans. (*ii*)

10. The value of the determinant $\begin{bmatrix}1 & w & w^2\\ w & w^2 & 1\\ w^2 & 1 & w\end{bmatrix}$ where w is the cube root of unity equals

(*i*) 0 (*ii*) 1 (*iii*) w (*iv*) w^2 **Ans.**(*i*)

11. If Tp, Tq and Tr are the pth, qth, rth terms of an AP then $\begin{bmatrix}Tp & Tq & Tr\\ p & q & r\\ 1 & 1 & 1\end{bmatrix}$ equals

(*i*) 1 (*ii*) –1 (*iii*) 0 (*iv*) None of these

Ans. (*iii*)

12. The value of $\begin{bmatrix}x+a & x & x\\ x & x+a & x\\ x & x & x+a\end{bmatrix}$ is equal to

(*i*) $3a^2x$ (*ii*) $a^2(3x-a)$ (*iii*) $a^2(3x+a)$ (*iv*) $3ax^2$ **Ans.** (*iii*)

13. If $C = AB$ where $A=\begin{bmatrix}1 & 2\\ 4 & 3\end{bmatrix}$, $B=\begin{bmatrix}\cos n\theta & \sin n\theta\\ -\sin n\theta & \cos n\theta\end{bmatrix}$, then det C is equal to

(*i*) 5 (*ii*) –5 (*iii*) $\cos n\theta$ (*iv*) $\sin n\theta$

Ans. (*ii*)

14. If A, B are square matrices of the same size then

(*i*) $(AB)' = A'B'$, (*ii*) $(AB)' = B'A'$, (*iii*) $(AB)' = AB$, (*iv*) $(AB)' = BA$

Ans. (*ii*)

15. A be a square matrix. If there exists a matrix B such that $AB = I = BA$ where I is a unit matrix then B is called :

is called :

(*i*) Singular (*ii*) Unit matrix (*iii*) Inverse (*iv*) None of the above

Ans. (*iii*)

16. If A is non-singular matrix of order $n \times n$ then $|\text{adj}(A)|$ is equal to

(*i*) 1 (*ii*) 0 (*iii*) $|A|^{n-1}$ (*iv*) None of these

Ans. (*iii*)

17. Which of the following matrices is non-singular ?

(*i*) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ (*ii*) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (*iii*) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$ (*iv*) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$ Ans. (*ii*)

18. If A is a skew-symmetric matrix of odd order, then the determinant of A is

(*i*) – 1. (*ii*) 0. (*iii*) 1. (*iv*) a real number. Ans. (*ii*)

19. If A and B are square matrices of equal order and λ, μ are numbers, then $\lambda A + \mu B$ is

(*i*) symmetric if A is symmetric and B is skew-symmetric

(*ii*) symmetric if A and B are both symmetric

(*iii*) symmetric if both A and B are skew-symmetric

(*iv*) symmetric if B is symmetric and A is skew-symmetric

Ans. (*ii*)

20. If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, then $A^n =$

(*i*) $\begin{bmatrix} 1 & 1 \\ 0 & n \end{bmatrix}$ (*ii*) $\begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix}$ (*iii*) $\begin{bmatrix} n & 0 \\ 1 & 1 \end{bmatrix}$ (*iv*) $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ Ans. (*iv*)

21. If $r\begin{bmatrix} 5 \\ 2 \end{bmatrix} + s\begin{bmatrix} 6 \\ 3 \end{bmatrix} = \begin{bmatrix} 27 \\ 12 \end{bmatrix}$, then

(*i*) $r = 3, s = 2$ (*ii*) $r = 2, s = 3$ (*iii*) $r = 3, s = -2$ (*iv*) $r = -3, s = 2$ Ans. (*i*)

22. Given $A = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$, which of the following results is true ?

(*i*) $A^2 = I$ (*ii*) $A^2 = 2I$ (*iii*) $2A^2 = I$ (*iv*) $A^2 = A$ Ans. (*i*)

23. If $A = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}, B = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$, then $|AB|$ is equal to

(*i*) –24 (*ii*) 24 (*iii*) Not defined (*iv*) None of these Ans. (*i*)

24. If the matrix $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, C = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$ then

(*i*) $C = A\cos\theta - B\sin\theta$ (*ii*) $C = A\sin\theta + B\cos\theta$

(*iii*) $C = A\sin\theta - B\cos\theta$ (*iv*) $C = A\cos\theta + B\sin\theta$ Ans. (*iv*)

25. Let I be the unit matrix of order n and adj. $(2I) = 2^k I$. Then k equals

(*i*) 1 (*ii*) 2 (*iii*) $n - 1$ (*iv*) n. Ans. (*iii*)

26. Which of the following matrices is not invertible ?

(*i*) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$ (*ii*) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (*iii*) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (*iv*) None of these **Ans.** (*i*)

27. With 1, ω, ω^2 as cube roots of unity, inverse of which of the following matrices exists ?

(*i*) $\begin{bmatrix} 1 & \omega \\ \omega & \omega^2 \end{bmatrix}$ (*ii*) $\begin{bmatrix} \omega^2 & 1 \\ 1 & \omega \end{bmatrix}$ (*iii*) $\begin{bmatrix} \omega & \omega^2 \\ \omega^2 & 1 \end{bmatrix}$ (*iv*) None of these **Ans.** (*iv*)

28. If A and B are two square matrices of same order then *adj* (AB) is

(*i*) $(Adj\ A)\ (Adj\ B)$ (*ii*) $(Adj\ B)\ (Adj\ A)$

(*iii*) $A\ adj\ B + (adj\ A)\ B$ (*iv*) None of these **Ans.** (*ii*)

29. If $A\begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$, where $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then A is

(*i*) $\begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}$ (*ii*) $\begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}$ (*iii*) $\begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$ (*iv*) $\begin{bmatrix} 2 & 1 \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$ **Ans.** (*iv*)

30. If $A\begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$, where $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then A is

(*i*) $\begin{bmatrix} 2 & 1 \\ 5 & -1 \end{bmatrix}$ (*ii*) $\begin{bmatrix} 2 & 5 \\ 1 & -1 \end{bmatrix}$ (*iii*) $\begin{bmatrix} 2 & -5 \\ -1 & 1 \end{bmatrix}$ (*iv*) $\begin{bmatrix} 0 & 5 \\ -1 & 1 \end{bmatrix}$ **Ans.** (*ii*)

31. If a matrix A satisfies a relation $A^2 + A - I = 0$, then

(*i*) A^{-1}exists (*ii*) A^{-1} does not exist

(*iii*) A^{-1}exists and is equal to $I + A$

(*iv*) A^{-1} exists and is equal to I, where I is an identity matrix **Ans.** (*iii*)

32. Matrix A has x rows and $x + 5$ columns. Matrix B has y rows and $11 - y$ columns. Both AB and BA exist. Which of the following values for x and y are possible ?

(*i*) $x = 2, y = 6$ (*ii*) $x = 3, y = 8$ (*iii*) $x = 4, y = 4$ (*iv*) $x = 8, y = 3$ **Ans.** (*ii*)

33. If $I + A + A^2 + ... + A^K = 0$, then A^{-1} equal to

(*i*) A^K (*ii*) A^{K-1} (*iii*) A^{K+1} (*iv*) $I + A$ **Ans.** (*i*)

34. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ \alpha & 1 & 0 \\ \beta & \gamma & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix}$ then

(*i*) A is row equivalent to B only if $\alpha = 2, \beta = 3, \gamma = 4$

(*ii*) A is row equivalent to B only if $\alpha \neq 0, \beta \neq 0, \gamma = 0$

(*iii*) A is not equivalent to B

(*iv*) A is row equivalent to B for all values of α, β, γ **Ans.** (*i*)

35. A is a skew symmetric matrix. Then for all vectors X. $X^T A X$ has a value.

(i) 0

(ii) greater than zero

(iii) purely imaginary

(iv) equal to the largest eigenvalue of A. **Ans.** (i)

Fill up the blanks :

36. If A, B, C are non-singular $n \times n$ matrices, then $(ABC)^{-1} =$ .... **Ans.** $C^{-1} B^{-1} A^{-1}$

37. If $A^2 + A - I = 0$, then $A^{-1} =$ ...... **Ans.** $I + A$

Indicate true or false for the following:

38. $\begin{bmatrix} 0 & -1 & -2 \\ -1 & 0 & -3 \\ -2 & -3 & 0 \end{bmatrix}$ is a skew matrix. **Ans.** False

39. $\begin{bmatrix} 0 & 4 & 5 \\ -4 & 0 & -6 \\ -5 & 6 & 0 \end{bmatrix}$ is symmetric matrix. **Ans.** False

40. If the transpose of matrix A is A', then $A + A'$ is symmetric matrix. **Ans.** True

41. If the transpose of the matrix A is A', then $A - A'$ is skew symmetric matrix. **Ans.** True

Match the following:

42. (i) $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 0 \end{bmatrix}$ (p) Symmetric matrix

(ii) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (q) Orthogonal matrix

(iii) $[AA' = I]$ (r) Unit matrix

(iv) $\begin{bmatrix} 0 & -5 & 6 \\ 5 & 0 & -1 \\ -6 & 1 & 0 \end{bmatrix}$ (s) Skew symmetric matrix

Ans. (i) → (p)
(ii) → (r)
(iii) → (q)
(iv) → (s)

Ans. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

OBJECTIVE TYPE QUESTIONS

Fill up the blanks:

1. The rank of $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is equal to ....

Ans. 3

2. The rank of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$ is equal to ....

Ans. 1

Choose the correct alternative:

3. The rank of the diagonal matrix $\begin{bmatrix} -1 & & & & & \\ & 0 & & & & \\ & & 1 & & & \\ & & & 0 & & \\ & & & & 0 & \\ & & & & & 4 \end{bmatrix}$ is

(*i*) 1, (*ii*) 2, (*iii*) 3, (*iv*) 4

Ans. (*iii*)

4. The rank of the matrix $\begin{bmatrix} 2 & -4 & 6 \\ -1 & 2 & -3 \\ 3 & -6 & 9 \end{bmatrix}$ is

(*i*) 3, (*ii*) 2, (*iii*) 0, (*iv*) 1

Ans. (*iv*)

5. If A is a non-zero column vector, ($n \times 1$), then the rank of matrix AA^T is

(*i*) 0, (*ii*) 1, (*iii*) $n - 1$, (*iv*) n. **Ans.** (*ii*)

6. The rank of matrix $\begin{bmatrix} \mu & -1 & 0 \\ 0 & \mu & -1 \\ -1 & 0 & \mu \end{bmatrix}$ is 2, for μ equal

(*i*) any row number (*iv*) 3 (*iii*) 1 (*iv*) 2 **Ans.** (*iii*)

7. If P and Q are non-singular matrices, then for Matrix 'M', which of the following statements are correct ?

(*i*) Rank (PMQ) > Rank M

(*ii*) Rank (PMQ) = Rank M

(*iii*) Rank (PMQ) > Rank M

(*iv*) Rank (PMQ) = Rank M + Rank (PQ) **Ans.** (*ii*)

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8. Rank of singular matrix of order 4, can be at the most.

(*i*) 1 (*ii*) 2 (*iii*) 3 (*iv*) 4 **Ans.** (*iii*)

9. The value of μ for which the rank of the matrix $A = \begin{bmatrix} \mu & -1 & 0 & 0 \\ 0 & \mu & -1 & 0 \\ 0 & 0 & \mu & -1 \\ -6 & 11 & -6 & 1 \end{bmatrix}$ is equal to 3 is

(*i*) 0 (*ii*) 1 (*iii*) 4 (*iv*) –1

Ans. (*ii*)

10. If the rank of an $n \times n$ matrix A is ($n - 1$), then the system of equations $A\,x = b$ has

(*i*) ($n - 1$) parameter family of solutions .

(*ii*) one parameter family of solutions

(*iii*) no solution

(*iv*) a unique solution **Ans.** (*i*)

11. The rank of matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 0 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ is

(*i*) 4 (*ii*) 3 (*iii*) 1 (*iv*) 0 **Ans.** (*i*)

Indiacte True or False for the following:

12. The rank of the matrix $\begin{bmatrix} 1 & 2 & 5 & 6 \\ 2 & 4 & 10 & 12 \\ -1 & -2 & -5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ is 1. **Ans.** True

13. The rank of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ is 2. **Ans.** True

14. The rank of a matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is 2. **Ans.** False

15. The rank $\begin{bmatrix} 100 & 90 & 20 \\ 10 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ is 1.

Ans. False

16. The rank of matrix A and A' are equal.

Ans. True

17. The rank of matirx A and $100\,A$ are not equal.

Ans. False

18. The rank of $\begin{bmatrix} 1 & 2 & 0 & -2 & -4 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 5 & 4 & 3 & 2 \end{bmatrix}$ and $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ are not equal.

Ans. False

Match the following:

19. Rank of

(*i*) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ (*a*) 2

(*ii*) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (*b*) 3

(*iii*) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (*c*) 0

(*iv*) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ (*d*) 1

(*v*) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ (*e*) 4

Ans. (*i*) → (*d*),
(*ii*) → (*a*),
(*iii*) → (*b*),
(*iv*) → (*c*),
(*v*) → (*e*)

Ans. $u = -1, v = 2, w = 1$

OBJECTIVE TYPE QUATIONS

Choose the correct answers:

1. $(A\ B)^{-1}$ is equal to
(i) $A^{-1}\ B^{-1}$ (ii) $B^{-1}\ A^{-1}$ (iii) $(A')^{-1}\ (B')^{-1}$ (iv) $A^{-1}\ (B')^{-1}$ **Ans. (ii)**

2. The characteristic of an orthogonal matrix A is
(i) $A^{-1} . A = I$ (ii) $A.\ A^{-1} = I$ (iii) $A'.\ A^{-1} = I$ (iv) $A.A' = I$ **Ans. (iv)**

3. Inverse of $\begin{bmatrix} 4 & 3 \\ -7 & 1 \end{bmatrix}$ is
(i) $\begin{bmatrix} \frac{1}{4} & \frac{1}{3} \\ \frac{-1}{7} & 1 \end{bmatrix}$ (ii) $\frac{1}{25}\begin{bmatrix} 4 & 3 \\ -7 & 1 \end{bmatrix}$ (iii) $\frac{1}{25}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (iv) $\frac{1}{25}\begin{bmatrix} 1 & -3 \\ 7 & 4 \end{bmatrix}$ **Ans. (iv)**

4. In the matrix equation $\begin{bmatrix} 3 & -1 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$ the values of x and y are
(i) $x = 3, y = -1$ (ii) $x = 2, y = 5$ (iii) $x = 1, y = -1$ (iv) $x = -1, y = 1$ **Ans. (iii)**

5. An $n \times n$ homogeneous system of equations $AX = 0$ is given. The rank of A is $r < n$. Then the system has
(i) $n - r$ independent solutions (ii) r independent solutions
(iii) no solution (iv) $n - 2r$ independent solutions. **Ans. (i)**

6. The value of K for which the system of equations
$4x + 2y - 5y = 0$
$x + Ky + 2z = 0$
$2x + y - z = 0$
has a non-zero solution is
(i) $\frac{1}{2}$ (ii) $-\frac{1}{2}$ (iii) 1 (iv) -1 **Ans. (i)**

7. The condition for consistency of simultaneous equation $AX = B$ is
(i) Rank A = Rank C (ii) Rank $A \neq$ Rank C
(iii) Rank A = Rank B (iv) None of these **Ans. (i)**

8. The name of the method for solving simultaneous equations by determinant is
(i) Cramers rule (ii) Laplace Rule
(iii) Newton's method (iv) None of these **Ans. (i)**

9. A system of equations (algebraic) is said to be inconsistent if
(i) it has more than one solution (ii) it has no solution
(iii) it has one solution (iv) None of these **Ans. (ii)**

10. The following system of equations given

$x + 2y + z = 0$

$2x + ay + az = 2$

$x + y + 2z = 1$

The value of a, for which nontrivial solution exists is

(*i*) $a = 3$ (*ii*) $a = 6$ (*iii*) $a \neq 3$ (*iv*) $a \neq 6$ **Ans.** (*iii*)

11. The solution of the equations

$5x + 3y + 3z = 48$

$2x + 6y - 3z = 18$

$8x - 3y + 2z = 21$ is

(*i*) $x = 3, y = 5, z = 6$ (*ii*) $x = 0, y = 5, z = 3$

(*iii*) $x = 3, y = 0, z = 6$ (*iv*) $x = 3, y = 5, z = 0$ **Ans.** (*i*)

12. The system of equations

$x - y + z = -\lambda$

$x + y + z = \lambda$

$-x + y - z = \lambda$ has

(*i*) unique solution (*ii*) infinitely many solutions

(*iii*) no solution (*iv*) None of these **Ans.** (*ii*)

13. The solution of the simultaneous equations

$x + y + z = 3,\quad 2x + y - z = 2$ and $3x + 2y + 2z = 7$ is

(*i*) $x = 0, y = 1, z = 2$ (*ii*) $x = 1,\ y = 1,\ z = 1$

(*iii*) $x = y = z = 0$ (*iv*) $x = 1,\ y = 2,\ z = 3$ **Ans.** (*ii*)

14. The solution of the simultaneous equations

$2x + y + z = 7,\quad 3x + y + z = 8$ and $5x + 6x - z = 14$ is

(*i*) $x = 1,\ y = 2,\ z = 3$ (*ii*) $x = 0,\ y = 1,\ z = 2$

(*iii*) $x = 2,\ y = 3,\ z = 2$ (*iv*) $x = 2,\ y = 3,\ z = 4$ **Ans.** (*i*)

15. Let $v_1 = (1, 1, 0, 1)$, $v_2 = (1, 1, 1, 1)$, $v_3 = (4, 4, 1, 1)$ and $v_4 = (1, 0, 0, 1)$ be elements or R_4. The set of vectors $\{v_1, v_2, v_3, v_4\}$ is

(*i*) linearly independent (*ii*) linearly dependent

(*iii*) null (*iv*) none of these **Ans.** (*ii*)

16. The system of equations $2x - y = 3$, $x - 3y = 4$ and $x + 2y = 1$ has

(*i*) a unique solution (*ii*) infinitely many solutions

(*iii*) no solution (*iv*) none of these. **Ans.** (*iii*)

17. If $3x + 2y + z = 0$, $x + 4y + z = 0$, $2x + y + 4z = 0$, be a system of equations then

(*i*) System is inconsistent

(*ii*) it has only trivial solution

(*iii*) it can be reduced to a single equation thus solution does not exist

(*iv*) Dterminant of the coefficient matrix is zero. **Ans.** (*ii*)

Indicate Ture or False for the following

18. Inverse of a singular matrix is possible. **Ans.** False

19. In equation $AX = B$ has no solution if A is singular. **Ans.** True

20. Homogeneous equations are always consistant **Ans.** True

21. If the rank of A and C are not equal then the equation are the consistent. **Ans.** False

22. If the rank of A = Rank of C = No. of unknowns, then the solution will be unique **Ans.** True

Match the following

23. In the system of equation $AX = B$ and $A, B = C$

(a) If the rank of $A \neq$ rank of C	(p) consistent with unique solution
(b) If the rank of A = rank of C = number of unknowns	(q) Infinite solutions consistent with
(c) If the rank A = rank of $C <$ No. of unknowns	(r) have a solution
(d) The solution of $AX = 0$ is always	(s) Inconsistant

Ans. (a) → (s)
(b) → (p)
(c) → (q)
(d) → (r)

24. The symultaneous equations

$$a_1x + b_2y + c_1 = 0$$
$$a_2x + b_2y + c_2 = 0$$

(i) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	(p) No solution
(ii) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} = \frac{c_1}{c_2}$	(q) Unique solution
(iii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	(r) Infinity many solutions
(iv) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	(s) None of these

Ans. (i) → (r)
(ii) → (s)
(iii) → (p)
(iv) → (q)

Fill up the blanks.

25. The solution of the following system of equations $x + 2y + 3z = 6$

$2x + y - z = 2$

$x - 3y + 5z = 3$ is

$x = 1$, y = ——— and z = ———

Ans. $y = 1, z = 1$

OBJECTIVE TYPE QUESTIONS

Tick (√) the correct answer:

1. If λ_1, λ_2 and λ_3 are the eigen values of the matrix

$$\begin{bmatrix} -2 & -9 & 5 \\ -5 & -10 & 7 \\ -9 & -21 & 14 \end{bmatrix}$$ then $\lambda_1 + \lambda_2 + \lambda_3$ is equal to

(*i*) -16 (*ii*) 2 (*iii*) -6 (*iv*) -14 **Ans.** (*ii*)

2. The matrix $A = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$ is given and the eigen values of $4A^{-1} + 3A + 2I$ are

(*i*) 6, 15 (*ii*) 9, 12 (*iii*) 9, 15 (*iv*) 7, 15 **Ans.** (*iii*)

3. If a square matrix A has an eigen value λ, then an eigen value of the matrix $(kA)^T$ where $k \neq 0$ is a scalar, is

(*i*) λ / k (*ii*) k / λ (*iii*) $k\lambda$ (*iv*) None of these **Ans.** (*iii*)

4. For the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ the sum of the eigen values is

(*i*) -1 (*ii*) 0 (*iii*) 3 (*iv*) 5 **Ans.** (*i*)

5. A 3×3 real matrix has an eigen value i then its other two eigen values can be

(*i*) 0, 1 (*ii*) $-1, i$ (*iii*) $2i, -2i$ (*iv*) $0, -i$ **Ans.** (*iv*)

6. Let A be a square matrix. Then $\lambda = 0$ is an eigen value of A if and only if

(*i*) A is non-singular (*ii*) A is of even order

(*iii*) A is of odd order (*iv*) A is singular **Ans.** (*iv*)

7. The matrix A is defined as $A=\begin{bmatrix}-1 & 0 & 0\\ 2 & -3 & 0\\ 1 & 4 & 2\end{bmatrix}$. The eigen values of A^2 are

(*i*) –1, –9, –4 (*ii*) 1, 9, 4
(*iii*) –1, –3, 2 (*iv*) 1, 3, –2 **Ans.** (*ii*)

8. If the matrix $A=\begin{bmatrix}-1 & 2 & 3\\ 0 & 3 & 5\\ 0 & 0 & -2\end{bmatrix}$ then the eigen values of $A^3 + 5A + 8I$, are

(*i*) –1, 27, –8 (*ii*) 1, 3, –2 (*iii*) 2, 50, –10 (*iv*) 2, 50, 10 **Ans.** (*iii*)

9. Let P be a real square matrix of order n and n is odd. Then

(*i*) at least one eigen value of A is real (*ii*) one eigen value of A is zero
(*iii*) one eigen value of A is 1 (*iv*) A has no real eigen values **Ans.** (*i*)

10. Two of the eigen values of a 3 × 3 matrix, whose determinant equals 4, are –1 and +2 the third eigen value of the matrix is equal to

(*i*) –2 (*ii*) –1 (*iii*) 1 (*iv*) 2 **Ans.** (*i*)

11. The matrix A has eigen values $\lambda_i \neq 0$. Then $A^{-1} - 2I + A$ has eigen values

(*i*) $1+2\lambda_i+\lambda_i^2$ (*ii*) $\frac{1}{\lambda_i}-2+\lambda_i$ (*iii*) $1-2\lambda_i+\lambda_i^2$ (*iv*) $1-\frac{2}{\lambda_i}+\frac{1}{\lambda_i^2}$
Ans. (*ii*)

12. The eigen values of a matrix A are 1, –2, 3. The eigen values of $3I - 2A + A^2$ are

(*i*) 2, 11, 6 (*ii*) 3, 11, 18 (*iii*) 2, 3, 6 (*iv*) 6, 3, 11 **Ans.** (*i*)

13. If A is a singular hermitian matrix, then the least eigen value of A^2 is

(*i*) 0 (*ii*) 1 (*iii*) 2 (*iv*) None of these
Ans. (*i*)

14. If λ is an eigen value of the matrix 'M' then for the matrix $(M - \lambda I)$, which of the following statement (*s*) is/are coorrect ?

(*i*) Skew symmetric (*ii*) Non singular (*iii*) Singular (*iv*) None of these **Ans.** (*iii*)
(*U.P., I Sem. Dec. 2009*)

15. If A is a skew symmetric matrix, then for all vectors X, $X^T A X$ has a value

(*i*) 0 (*ii*) greater than zero
(*iii*) purely imaginary (*iv*) equal to the largest eigen value of A. **Ans.** (*i*)

16. A square matrix A is idempotent if :

(*i*) $A' = A$ (*ii*) $A' = -A$ (*iii*) $A^2 = A$ (*iv*) $A^2 = I$ **Ans.** (*iii*)
(*R.G.P.V. Bhopal, I Semester June,* 2007)

17. If a square matrix U such that $\overline{U}' = U^{-1}$ then U is

(*i*) Orthogonal (*ii*) Unitary (*iii*) Symmetric (*iv*) Hermitian **Ans.** (*ii*)
(*R.G.P.V. Bhopal, I Semester June,* 2007)

18. The sum of the eigen values of $\begin{pmatrix}1 & 0 & 0\\ 0 & 3 & -1\\ 0 & -1 & 3\end{pmatrix}$ is equal to

(*i*) 6 (*ii*) – 8 (*iii*) 7 (*iv*) – 6 **Ans.** (*iii*)

19. If λ is an eigen value of a non-singular matrix A then the eigen value of A^{-1} is

(*i*) $1/\lambda$ (*ii*) λ (*iii*) $-\lambda$ (*iv*) $-1/\lambda$ **Ans.** (*i*)

20. The product of the eigen values of the matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{pmatrix} \text{ is}$$

(*i*) 3 (*ii*) 8 (*iii*) 1 (*iv*) – 1 **Ans.** (*ii*)

21. If $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix}$, then the eigen value of A^2 are

(*i*) 1, 2, 3 (*ii*) –1, 2, 3 (*iii*) 1, 4, 9 (*iv*) – 1, 4, 9 **Ans.** (*iii*)

22. The eigenvalues of the matrix $\begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ are

(*i*) –1, 2 and 1 (*ii*) 0, 1 and 2 (*iii*) –1, –2 and 4 (*iv*) 1, 1 and–1

Ans. (*i*)

Considering the following choose the correct alternative:

$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}$ if U_1, U_2 and U_3 are columns matrices satisfying.

$AU_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, AU_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}, AU_3 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$ and U is 3 × 3 matrix whose columns are U_1, U_2, U_3 then answer the following equations.

23. The value of $|U|$ is

(*i*) 3 (*ii*) –3 (*iii*) $\frac{3}{2}$ (*iv*) 2 **Ans.** (*i*)

[Hint : Let U_1 be $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ so that

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

Similarly, $U_2 = \begin{bmatrix} 2 \\ -1 \\ -4 \end{bmatrix}$, $U_3 = \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}$

Hence, $U = \begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -3 \end{bmatrix}$ and $|U| = 3$.

24. The sum of the elements of U^{-1} is

(*i*) –1 (*ii*) 0 (*iii*) 1 (*iv*) 3 **Ans.** (*ii*)

[**Hint :** Moreover adj. $U = \begin{bmatrix} -1 & -2 & 0 \\ -7 & -5 & -3 \\ 9 & 6 & 3 \end{bmatrix}$.

Hence, $U^{-1} = \frac{adj\ U}{3}$ and sum of the elements of $U^{-1} = 0$]

25. The value of $[3\ 2\ 0]\ U \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$ is

(*i*) 5 (*ii*) $\frac{5}{2}$ (*iii*) 4 (*iv*) $\frac{3}{2}$ **Ans.** (*i*)

[**Hint :** The value of $[3\ 2\ 0]\ U \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$

$$= [3\,2\,0] \begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -3 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} = [-1\ 4\ 4] \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} = -3 + 8 = 5]$$

Fill up the blanks.

26. If the eigen values of the matrix A are 1, 2, 3 then, the eigen values of $(A. I^2)$ are .... **Ans.** 1, 2, 3

27. The eigen values of A are 2,. 3, 4 then the eigen values of A^2 are ..... **Ans.** 4, 9, 16

28. The eigen values of A are 2, 3, 1 then the eigne values of $A^2 + A$ are ..... **Ans.** 6, 12, 2

29. If the eigen values of A are 1, 1, 1 then the eigen values of $A^2 + 2A + 3I$ are..... **Ans.** 6, 6, 6

30. If the eigen values of A are 4, 6, 9 then the eigen values of A^{-1} are ...... **Ans.** $\frac{1}{4}, \frac{1}{6}, \frac{1}{9}$

Indicate True of False for the following:

31. The elements of modal matrix are the eigen vectors of the corresponding eigen values. **Ans.** True

32. $P^{-1} AP$ = The diagonal matrix. **Ans.** True

33. $A^6 = PD^6 P^{-1}$ **Ans.** True

34. If $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ then $A^{100} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **Ans.** False

35. If $A = \begin{bmatrix} 1 & -2 & -3 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then $A^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **Ans.** False

36. Conjugate of 2 is 2. **Ans.** True

37. Conjugate of i is $-i$. **Ans.** True

38. If the eigen value of A is 2, then the eigen value of $A^3 + 2A^2 + A + I$ is 10. **Ans.** False

Fill up the blanks:

39. The characteristic roots of a skew hermitian matrix is eighter...... or .... **Ans.** 0, Pure imaginary

40. The modulus of each characteristic roots of a unitary matrix is .... **Ans.** unity

41. If λ is an eigen value of an orthogonal matrix, then the other eigen value of the same orthogonal matrix is .... **Ans.** $\frac{1}{\lambda}$

42. The characteristic roots of a Hermitian matrix or all........ **Ans. real**

43. The characteristic root of a triangular matrix is .... **Ans.** Diagonal element

44. If a characteristic roots of a matrix is zero, then the matrix is .... **Ans.** Singular

45. If A and P be square matrices of the same type and if P is investible, then the matrices A and $P^{-1}AP$ have .... characteristic roots. **Ans.** Same

Match the following:

46. (*i*) The eigen vectors X of a matrix A, is not — (*a*) $X_1' X_2 = 0$

(*ii*) Two eigen vectors X_1 and X_2 are called orthogonal if — (*b*) $\sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}$

(*iii*) Normalised form of vectors $\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix}$ is obtained on dividing each element by — (*c*) unique

(*iv*) Every square matrix satisfies its own — (*d*) Characteristic equation

Ans. (*i*) → (*c*)
(*ii*) → (*a*)
(*iii*) → (*b*)
(*iv*) → (*d*)