Axioms of Probability

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(1) Let S be the sample space and A and B be two mutually exclusive events. Then

(I) $0 \le P(A) \le 1$, $0 \le P(B) \le 1$

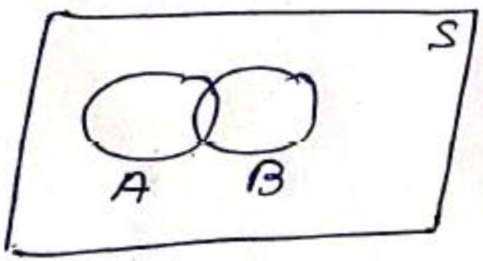
(II) $P(S) = 1$

(III) $P(A \cup B) = P(A) + P(B)$

(2) Law of addition of Probabilities

If A and B are any two events associated with a random experiment, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



For, three events A, B and C

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

(3) If A is any event associated with an experiment, then

$$P(\text{not } A) = P(\overline{A}) = 1 - P(A)$$

(4) If $A \subset B$, that is, the event A implies the event B, then

$$P(A) \leq P(B)$$

Ex: Two dice are tossed once. Find the probability of getting an even number on the first dice or a total of 8.

Sol: Define the events

A: Getting an even number on the first dice

B: Getting a total of 8

$A = \{(2,1), \dots (2,6), (4,1), \dots (4,6), (6,1), \dots (6,6)\}$

$B = \{(2,6), (3,5), (4,4), (5,3), (6,2)\}$

$A \cap B = \{(2,6), (4,4), (6,2)\}$

$P(A) = \frac{18}{36} = \frac{1}{2}$, $P(B) = \frac{5}{36}$, $P(A \cap B) = \frac{3}{36} = \frac{1}{12}$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= \frac{1}{2} + \frac{5}{36} - \frac{1}{12} = \frac{5}{9}$$

Ex: From a pack of well shuffled cards, one card is drawn. Find the probability that this card is either a king or an ace.

(a) $\frac{1}{13}$ (b) $\frac{2}{13}$ (c) $\frac{3}{13}$ (d) $\frac{4}{13}$

Sol: A: Card drawn is king

B: Card drawn is an ace

The events A and B are mutually exclusive

$$P(A) = \frac{4}{52} = \frac{1}{13}, \quad P(B) = \frac{4}{52} = \frac{1}{13}$$

$$P(A \cup B) = P(A) + P(B) = \frac{1}{13} + \frac{1}{13} = \frac{2}{13}$$

(3) A bag contains 4 red and 3 black balls. A second bag contains 2 red and 4 black balls. One bag is selected at random. From the selected bag one ball is drawn. Find the probability that the ball drawn is red.

Sol: Required probability $= \frac{1}{2} \cdot \frac{4}{7} + \frac{1}{2} \cdot \frac{1}{3} = \frac{19}{42}$

Ex: If A, B, C are mutually exclusive and exhaustive events associated with a random experiment and $P(B) = (0.6)(P(A))$ and $P(C) = (0.2)(P(A))$ then find $P(A)$

(a) $\frac{1}{9}$ (b) $\frac{4}{9}$ (c) $\frac{5}{9}$ (d) none of these

Sol: $P(A) + P(B) + P(C) = 1$

$\Rightarrow P(A) + (0.6)\, P(A) + (0.2)\, P(A) = 1$

$\Rightarrow P(A)(1 + 0.6 + 0.2) = 1$

$\Rightarrow P(A) = \dfrac{1}{1.8} = \dfrac{10}{18} = \dfrac{5}{9}$

Ex: The probability that atleast one of the events A and B occurs is 0.8 and the probability that both the events occur simultaneously is 0.25. Find the probability $P(\overline{A}) + P(\overline{B})$.

(a) 0.65 (b) 0.85 (c) 0.95 (d) 0.5

Sol: $P(A \cup B) = 0.8$, $P(A \cap B) = 0.25$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(A) + P(B) = P(A \cup B) + P(A \cap B)$$

$$= 0.8 + 0.25 = 1.05$$

$$\Rightarrow 1 - P(\overline{A}) + 1 - P(\overline{B}) = 1.05$$

$$\Rightarrow 2 - (P(\overline{A}) + P(\overline{B})) = 1.05$$

$$\Rightarrow P(\overline{A}) + P(\overline{B}) = 2 - 1.05 = 0.95$$

Ex: The word ASSASIN is given. It is rearranged so that the three S's come consecutively. Find the probability of this event.

Sol:

$$n(s) = \frac{\lfloor 7}{\lfloor 3 \ \lfloor 2} = \frac{7 \times 6 \times 5 \times 4}{2 \times 1} = 420$$

$$n(E) = \frac{\lfloor 5}{\lfloor 2} = 5 \times 4 \times 3 = 60$$

$$p(E) = \frac{60}{420} = \frac{1}{7}$$

Ex: If a number of two digits is formed with the digits 2, 3, 5, 7, 9 without repetition of digits, what is the probability that the number formed is 35

(a) $\frac{1}{16}$ (b) $\frac{1}{20}$ (c) $\frac{1}{18}$ (d) $\frac{1}{120}$

Sol: $p(E) = \frac{n(E)}{n(S)} = 1/20$ $\quad n(E) = 1, \; n(S) = {}^5P_2 = \frac{\lfloor 5}{\lfloor 3} = 20$

Ex: There are 4 envelops corresponding to 4 letters. If the letters are placed in the envelopes at random, what is the probability that all the letters are not placed in the right envelopes?

(a) $\frac{1}{4}$ (b) $\frac{1}{24}$ (c) $\frac{23}{24}$ (d) none of these

Sol: $n(S) = \underline{\lfloor 4} = 24$

E: The event that all the letters are placed in the right envelope.

$$P(\overline{E}) = 1 - P(E) = 1 - \frac{n(E)}{n(S)} = 1 - \frac{1}{24} = \frac{23}{24}$$

Ex: The odds in Favour of standing first of three students appearing at an examination are 1:2, 2:5, and 1:7 respectively. What is the probability that either of them will stand first.

Sol: Let the three students be P, Q, R.

Let A, B, C denote the events of standing first of the three students P, Q, R respectively

$P(A) = \frac{1}{1+2} = \frac{1}{3}$, $P(B) = \frac{2}{2+5} = \frac{2}{7}$

$P(C) = \frac{1}{1+7} = \frac{1}{8}$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

$$= \frac{1}{3} + \frac{2}{7} + \frac{1}{8} = \frac{125}{168}$$