U1 (important for mcqs)
Properties of expectations , variance and co variance

1.
If a and b are constants, then

$$E(aX+b) = aE(X) + b.$$

$E(b) = b.$
$E(aX) = aE(X)$

2.

$E[g(X) \pm h(X)] = E[g(X)] \pm E[h(X)].$

3.

$E[g(X,Y) \pm h(X,Y)] = E[g(X,Y)] \pm E[h(X,Y)].$

4.

$E[X \pm Y] = E[X] \pm E[Y].$

5.

Let X and Y be two independent random variables. Then

$$E(XY) = E(X)E(Y).$$

6. $\mathrm{Cov}(aX, bY) = ab\,\mathrm{Cov}(X,Y).$

7
[covariance of independent random variable is zero]

Let X and Y be two independent random variables. Then $\sigma_{XY} = 0.$

8.(properties on variance)

If X and Y are random variables with joint probability distribution $f(x, y)$ and a, b, and c are constants, then

$$\sigma^2_{aX+bY+c} = a^2\sigma^2_X + b^2\sigma^2_Y + 2ab\sigma_{XY}.$$

i.e. Var(aX + bY + c)= a²Var(X) + b²Var(Y) +2ab Cov(X,Y)

Corollary

: Setting $b = 0$, we see that

$$\sigma^2_{aX+c} = a^2\sigma^2_X = a^2\sigma^2.$$

: Setting $a = 1$ and $b = 0$, we see that

$$\sigma^2_{X+c} = \sigma^2_X = \sigma^2.$$

: Setting $b = 0$ and $c = 0$, we see that

$$\sigma^2_{aX} = a^2\sigma^2_X = a^2\sigma^2.$$

9.

If X and Y are independent random variables, then

$$\sigma^2_{aX-bY} = a^2\sigma^2_X + b^2\sigma^2_Y.$$

Example 4.22: If X and Y are random variables with variances $\sigma^2_X = 2$ and $\sigma^2_Y = 4$ and covariance $\sigma_{XY} = -2$, find the variance of the random variable $Z = 3X - 4Y + 8$.

Solution:

$$\begin{aligned}\sigma_Z^2 &= \sigma_{3X-4Y+8}^2 = \sigma_{3X-4Y}^2\\ &= 9\sigma_X^2 + 16\sigma_Y^2 - 24\sigma_{XY}\\ &= (9)(2) + (16)(4) - (24)(-2) = 130.\end{aligned}$$

Example 4.23: Let X and Y denote the amounts of two different types of impurities in a batch of a certain chemical product. Suppose that X and Y are independent random variables with variances $\sigma_X^2 = 2$ and $\sigma_Y^2 = 3$. Find the variance of the random variable $Z = 3X - 2Y + 5$.

Solution:

$$\begin{aligned}\sigma_Z^2 &= \sigma_{3X-2Y+5}^2 = \sigma_{3X-2Y}^2\\ &= 9\sigma_x^2 + 4\sigma_y^2\\ &= (9)(2) + (4)(3) = 30.\end{aligned}$$

mcq

Two continuous random variables X and Y are related as

Y = 2X + 3

Let σ_X^2 and σ_Y^2 denote the variances of X and Y, respectively. The variances are related as

1. $\sigma_Y^2 = 4\sigma_X^2$
2. $\sigma_Y^2 = 2\sigma_X^2$
3. $\sigma_Y^2 = 25\sigma_X^2$
4. $\sigma_Y^2 = 5\sigma_X^2$

Ans 1

Remember

Properties of Variance:

1) $V[K] = 0$, Where K is some constant.

2) $V[cX] = c^2 V[X]$

3) $V[aX + b] = a^2 V[X]$

4) $V[aX + bY] = a^2 V[X] + b^2 V[Y] + 2ab \, Cov(X,Y)$

$Cov.(X,Y) = E[XY] - E[X].E[Y]$