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Date
Page

## (1) t-test for single mean:

Suppose we want to test:

(i) If a random sample $x_i$ $(i=1,2,\ldots n)$ of size $n$ has been drawn from a normal population with specified mean, say $\mu_0$, or

(ii) If the sample mean differs significantly from the hypothetical value $\mu_0$ of the population mean.

Under the null hypothesis, $H_0$:

(i) The sample has been drawn from the population with mean $\mu_0$ or

(ii) There is no significant difference between the sample mean $\bar{x}$ and the population mean $\mu_0$.

the statistic

$$t = \frac{\bar{x}-\mu_0}{S/\sqrt{n}}$$

where $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ and $S^2 = \frac{1}{(n-1)}\sum_{i=1}^{n}(x_i-\bar{x})^2$

follow student's t-distribution with $(n-1)$ degree of freedom.

Assumptions for Student's t-test

The following assumptions are made in the student's t-test

(i) The parent population from which the sample is drawn is normal.

(ii) The sample observations are independent, i.e. the sample is random

(iii) The population standard deviation $\sigma$ is unknown.

(1) A machinist is making engine parts with axle diameters of 0.700 inch. A random sample of 10 parts shows a mean diameter of 0.742 inch with a standard deviation of 0.040 inch. Compute the statistic you would use to test whether the work is meeting the specification. Also state how would you proceed.

Sol: [ Observation - Here we note that a sample of size 10 is taken whose mean and standard deviation is given, based on this we want to test whether it is coming from a population with the hypothetical mean (hypothetical mean in this case is the axle diameter 0.700 inch)

**Solution.** Here we are given :

$$\mu = 0{\cdot}700 \text{ inche}, \quad \bar{x} = 0{\cdot}742 \text{ inche}, \quad s = 0{\cdot}040 \text{ inche} \quad \text{and} \quad n = 10$$

*Null Hypothesis,* $H_0 : \mu = 0{\cdot}700$, *i.e., the product is conforming to specifications.*

*Alternative Hypothesis,* $H_1 : \mu \neq 0{\cdot}700$

*Test Statistic.* Under $H_0$, the test statistic is : $t = \dfrac{\bar{x} - \mu}{\sqrt{S^2/n}} = \dfrac{\bar{x} - \mu}{\sqrt{s^2/(n-1)}} \sim t_{(n-1)}$

$$\therefore \qquad t = \frac{\sqrt{9}\,(0{\cdot}742 - 0{\cdot}700)}{0{\cdot}040} = 3{\cdot}15$$

**How to proceed further.** Here the test statistic '$t$' follows Student's $t$-distribution with $10 - 1 = 9$ *d.f.* We will now compare this calculated value with the tabulated value of $t$ for 9 *d.f.* and at certain level of significance, say 5%. Let this tabulated value be denoted by $t_0$.

(*i*) If calculated '$t$', *viz.,* $3{\cdot}15 > t_0$, we say that the value of $t$ is significant. This implies that $\bar{x}$ differs significantly from $\mu$ and $H_0$ is rejected at this level of significance and we conclude that the product is not meeting the specifications.

(*ii*) If calculated $t < t_0$, we say that the value of $t$ is not significant, *i.e.,* there is no significant difference between $\bar{x}$ and $\mu$. In other words, the deviation $(\bar{x} - \mu)$ is just due to fluctuations of sampling and null hypothesis $H_0$ may be retained at 5% level of significance, *i.e.,* we may take the product conforming to specifications.

(2) Observation In the previous problem, we were testing whether the sample mean is equal to hypothetical mean, that is why the alternative hypothesis was $\mu \neq 0.700$ (Two tailed)

→ But in the next problem, we will test whether the advertising campaign is successful. So the null hypothesis being the hypothesis of no difference is

$H_0: \mu = 146.3$

But the alternative hypothesis is

$H_1: \mu > 146.3$ (Right tail)

Since we are expecting that the advertising campaign is successful.

**Example 16·6.** *The mean weekly sales of soap bars in departmental stores was 146·3 bars per store. After an advertising campaign the mean weekly sales in 22 stores for a typical week increased to 153·7 and showed a standard deviation of 17·2. Was the advertising campaign successful ?*

**Solution.** We are given : $n = 22, \bar{x} = 153{\cdot}7, s = 17{\cdot}2$.

*Null Hypothesis.* The advertising campaign is not successful, *i.e.*, $H_0 : \mu = 146{\cdot}3$

*Alternative Hypothesis,* $H_1 : \mu > 146{\cdot}3$ (Right-tail).

*Test Statistic.* Under $H_0$, the test statistic is : $t = \dfrac{\bar{x} - \mu}{\sqrt{s^2/(n-1)}} \sim t_{22-1} = t_{21}$

$$\therefore \qquad t = \frac{153{\cdot}7 - 146{\cdot}3}{\sqrt{(17{\cdot}2)^2/21}} = \frac{7{\cdot}4 \times \sqrt{21}}{17{\cdot}2} = 9{\cdot}03$$

**Conclusion.** Tabulated value of $t$ for 21 *d.f.* at 5% level of significance for *single-tailed test* is 1·72. Since calculated value is much greater than the tabulated value, it is highly significant. Hence we reject the null hypothesis and conclude that the advertising campaign was definitely successful in promoting sales.

**Example 16·7.** *A random sample of 10 boys had the following I.Q.'s : 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Do these data support the assumption of a population mean I.Q. of 100 ? Find a reasonable range in which most of the mean I.Q. values of samples of 10 boys lie.*

**Solution.** *Null hypothesis,* $H_0$ : *The data are consistent with the assumption of a mean I.Q. of 100 in the population, i.e.,* $\mu = 100$.

*Alternative hypothesis,* $H_1 : \mu \neq 100$.

*Test Statistic. Under* $H_0$, the test statistic is : $t = \dfrac{(\bar{x} - \mu)}{\sqrt{S^2/n}} \sim t_{(n-1)}$,

where $\bar{x}$ and $S^2$ are to be computed from the sample values of I.Q.'s.

TABLE 16·1 : CALCULATIONS FOR SAMPLE MEAN AND S.D.

| $x$ | $(x - \bar{x})$ | $(x - \bar{x})^2$ |
|---|---|---|
| 70 | − 27·2 | 739·84 |
| 120 | 22·8 | 519·84 |
| 110 | 12·8 | 163·84 |
| 101 | 3·8 | 14·44 |
| 88 | − 9·2 | 84·64 |
| 83 | − 14·2 | 201·64 |
| 95 | − 2·2 | 4·84 |
| 98 | 0·8 | 0·64 |
| 107 | 9·8 | 96·04 |
| 100 | 2·8 | 7·84 |
| Total 972 | | 1833·60 |

Here $n = 10$, $\bar{x} = \dfrac{972}{10} = 97{\cdot}2$ and $S^2 = \dfrac{1833{\cdot}60}{9} = 203{\cdot}73$

$$\therefore \qquad |t| = \frac{|97{\cdot}2 - 100|}{\sqrt{203{\cdot}73/10}} = \frac{2{\cdot}8}{\sqrt{20{\cdot}37}} = \frac{2{\cdot}8}{4{\cdot}514} = 0{\cdot}62$$

Tabulated $t_{0{\cdot}05}$ for $(10 - 1)$, *i.e.*, 9 *d.f.* for two-tailed test is 2·262.

*Conclusion.* Since calculated $t$ is less than tabulated $t_{0{\cdot}05}$ for 9 *d.f.*, $H_0$ may be accepted at 5% level of significance and we may conclude that the data are consistent with the assumption of mean I.Q. of 100 in the population.

The 95% confidence limits within which the mean I.Q. values of samples of 10 boys will lie are given by :

$$\bar{x} \pm t_{0{\cdot}05}\, S/\sqrt{n} = 97{\cdot}2 \pm 2{\cdot}262 \times 4{\cdot}514 = 97{\cdot}2 \pm 10{\cdot}21 = 107{\cdot}41 \text{ and } 86{\cdot}99$$

Hence the required 95% confidence interval is [86·99, 107·41].

**Remark. *Aliter for computing*** $\bar{x}$ ***and*** $S^2$. Here we see that $\bar{x}$ comes in fractions and as such the computation of $(x - \bar{x})^2$ is quite laborious and time consuming. In this case we use the method of step deviations to compute $\bar{x}$ and $S^2$, as given below.

**Example 16·8.** *The heights of 10 males of a given locality are found to be 70, 67, 62, 68, 61, 68, 70, 64, 64, 66 inches. Is it reasonable to believe that the average height is greater than 64 inches ? Test at 5% significance level assuming that for 9 degrees of freedom P ($t > 1{\cdot}83$) = 0·05.*

**Solution.** *Null Hypothesis,* $H_0 : \mu = 64$ inches

*Alternative Hypothesis,* $H_1 : \mu > 64$ inches

TABLE 16·2 : CALCULATIONS FOR SAMPLE MEAN AND S.D.

| $x$ | 70 | 67 | 62 | 68 | 61 | 68 | 70 | 64 | 64 | 66 | Total 660 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $x - \bar{x}$ | 4 | 1 | −4 | 2 | −5 | 2 | 4 | −2 | −2 | 0 | 0 |
| $(x - \bar{x})^2$ | 16 | 1 | 16 | 4 | 25 | 4 | 16 | 4 | 4 | 0 | 90 |

$$\bar{x} = \frac{\Sigma x}{n} = \frac{660}{10} = 66; \quad S^2 = \frac{1}{n-1} \Sigma (x - \bar{x})^2 = \frac{90}{9} = 10$$

*Test Statistic.* Under $H_0$, the test statistic is :

$$t = \frac{\bar{x} - \mu}{\sqrt{S^2/n}} = \frac{66 - 64}{\sqrt{10/10}} = 2,$$

which follows Student's $t$-distribution with $10 - 1 = 9$ *d.f.*

Tabulated value of $t$ for 9 *d.f.* at 5% level of significance for single (right) tail-test is 1·833. (This is the value $t_{0 \cdot 10}$ for 9 *d.f.* in the two-tailed tables given at the end of the chapter.)

*Conclusion.* Since calculated value of $t$ is greater than the tabulated value, it is significant. Hence $H_0$ is rejected at 5% level of significance and we conclude that the average height is greater than 60 inches.