Probability

The probability theory provides a mathematical model to study the uncertain situations.

Random Experiment:

The word "experiment" is used to describe an act which can be repeated under some given conditions.

Random experiments are those experiments whose result depend on chance.

Outcome: The result of a random experiment is called an outcome.

Examples: (I) Tossing of a coin
(II) Throwing a six-faced die
(III) A single throw of two dice

Event: In theory of probability, the term event is used to denote any pheomenon which occurs in a random experiment. In effect, one or more outcomes are said to constitute an "event".

Events may be 'elementary' or composite. An event is said to be elementary, if it cannot be decomposed into simpler events. A composite event is an aggregate of several elementary events.

Examples: (1) Toss of a coin, H and T are elementary events

(11) When 2 coins are tossed, the event "both heads" is an elementary event (HH). But "one head and one tail" is a composite event consisting of elemetary events (HT, TH)

Sample space: Sample space of a random experiment is the set of all the possible outcomes, that is, the set of all elementary events of that experiment.

Example 1 If three coins are tossed simultaneously. then describe the sample space.

$$S = \{HHH, HHT, HTH, HTT, TTT, TTH, THT, THH\}$$

Example 2: Two balls are to be drawn simultaneously from a set of 3 red and 2 white balls. Find the sample space (Find the $n(S)$)

(a) 6 (b) 10 (c) 5 (d) none of these.

Sure event (Universal event) : Since every outcome belongs to S, the event represented by the set S always occurs. Therefore, the event represented by S is called a sure event.

Impossible Event : An empty set ϕ is always a subset of a set S. Hence, the empty set ϕ can always be considered as representing an event of an experiment. The event represented by ϕ is called an Impossible event.

Equally likely events : Let S be the sample space of a random experiment. If all the elementary events of S have the same chance of occuring then events are said to be equally likely events

Mutually exclusive events : Two events E_1 and E_2 are said to be mutually exclusive if $E_1 \cap E_2 = \phi$

Mutually Exhaustive Events

Let S be the sample space of a random experiment and $A_1, A_2, \ldots A_m$ be the events defined on the sample space. If $A_1 \cup A_2 \cup \cdots \cup A_m = S$, then the events are said to be exhaustive.

If Furthur $A_i \cap A_j = \phi$, $i \neq j$, then the events are said to be mutually exclusive and exhaustive.

Probability of an Event :

$$P(E) = \frac{\text{Number of elements Favourable to } E}{\text{Total number of equally likely elementary events}}$$

$$P(E) = \frac{n(E)}{n(S)} \quad \text{———} (*)$$

$P(\phi) = 0$ and $P(S) = 1$

$$\boxed{0 \leq P(E) \leq 1}$$

Example: Three coins are tossed simultaneously. What is the probability that atleast two tails occur?

Sol: The sample space

$S = \{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT\}$

E: atleast two tails occur

$E = \{HTT, THT, TTH, TTT\}$

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{8} = \frac{1}{2}.$$

Example: Two dice with faces marked 1, 2, 3, 4, 5, 6 are thrown simultaneously and the points on the dice are multiplied together. Find the probability that the product is 12.

(a) $\frac{1}{4}$ (b) $\frac{1}{6}$ (c) $\frac{1}{9}$ (d) $\frac{1}{8}$

Sol:

$$n(S) = 36$$

$$E = \{(2,6), (3,4), (4,3), (6,2)\}$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{4}{36} = \frac{1}{9}$$

Example: A batch contains 10 articles of which 4 are defective. If 3 articles are chosen at random. What is the probability that none of them is defective?

<u>Sol:</u>

$$n(S) = {}^{10}C_3 = \frac{\lfloor 10}{\lfloor 3 \lfloor 7} = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 120$$

$$n(E) = {}^{6}C_3 = \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = 20$$

$$P = \frac{20}{120} = \frac{1}{6}$$

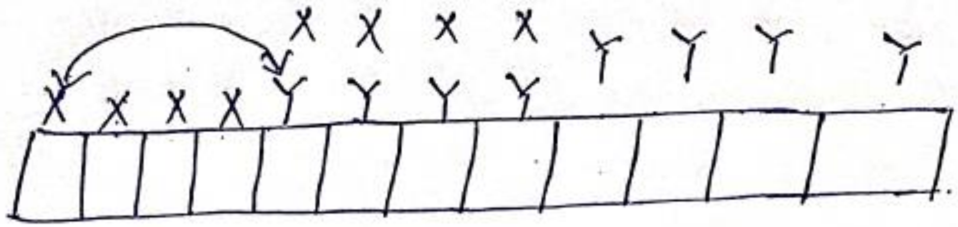
Example: what is the probability that all 3 children in a family have different birthdays?
(Assume, 1 year = 365 days)

<u>Sol</u>:

$$P = \frac{365 \times 364 \times 363}{365 \times 365 \times 365} = 0.992$$

Example: X and Y stand in a line at random with 10 other people. What is the probability that there are 3 people between X and Y?

Sol:



$$P = \frac{2 \times 8 \times \lfloor 10}{\lfloor 12} = \frac{2 \times 8}{12 \times 11} = \frac{4}{33}$$

Example: If 10 persons are arranged at random (I) in a line (II) in a ring. Find the probability that 2 particular persons will be next to each other.

Sol: (i) $P = \dfrac{\lfloor 9 \times \lfloor 2}{\lfloor 10} = \dfrac{1}{5}$

(ii) $P = \dfrac{\lfloor 8 \times \lfloor 2}{\lfloor 9} = \dfrac{2}{9}$