

# Problems on Probability

1. What is the Chance that a leap year selected at random will contain 53 Sundays?

**Solution.** In a leap year (which consists of 366 days) there are 52 complete weeks and 2 days over. The following are the possible combinations for these two 'over' days:

(i) Sunday and Monday, (ii) Monday and Tuesday, (iii) Tuesday and Wednesday, (iv) Wednesday and Thursday, (v) Thursday and Friday, (vi) Friday and Saturday, and (vii) Saturday and Sunday.

In order that a leap year selected at random should contain 53 Sundays, one of the two 'over' days must be Sunday. Since out of the above 7 possibilities, 2 viz., (i) and (vii), are favourable to this event,

$$\therefore \text{Required probability} = \frac{2}{7}$$

2. A bag contains 3 red, 6 white and 7 blue balls. What is the probability that two balls drawn are white and blue?

**Solution.** Total number of balls =  $3 + 6 + 7 = 16$ .

Now, out of 16 balls, 2 can be drawn in  ${}^{16}C_2$  ways.

$$\therefore \text{Exhaustive number of cases} = {}^{16}C_2 = \frac{16 \times 15}{2} = 120.$$

Out of 6 white balls 1 ball can be drawn in  ${}^6C_1$  ways and out of 7 blue balls 1 ball can be drawn in  ${}^7C_1$  ways. Since each of the former cases can be associated with each of the latter cases, total number of favourable cases is :  ${}^6C_1 \times {}^7C_1 = 6 \times 7 = 42$ .

$$\therefore \text{Required probability} = \frac{42}{120} = \frac{7}{20}.$$

3. What is the probability of getting 9 cards of the same suit in one hand at a game of bridge?

**Solution.** One hand in a game of bridge consists of 13 cards.

$\therefore$  Exhaustive number of cases =  $^{52}C_{13}$

Number of ways in which, in one hand, a particular player gets 9 cards of one suit are  $^{13}C_9$  and the number of ways in which the remaining 4 cards are of some other suit are  $^{39}C_4$ . Since there are 4 suits in a pack of cards, total number of favourable cases =  $4 \times ^{13}C_9 \times ^{39}C_4$ .

$$\therefore \quad \text{Required probability} = \frac{4 \times ^{13}C_9 \times ^{39}C_4}{^{52}C_{13}}$$

*A committee of 4 people is to be appointed from 3 officers of the production department, 4 officers of the purchase department, two officers of the sales department and 1 chartered accountant. Find the probability of forming the committee in the following manner:*

- (i) There must be one from each category.*
- (ii) It should have at least one from the purchase department.*
- (iii) The chartered accountant must be in the committee.*

ANS(i)(A)  $8/70$     (B)  $9/70$     (C)  $8/15$     (D) NONE



**Solution.** There are  $3+4+2+1=10$  persons in all and a committee of 4 people can be formed out of them in  $^{10}C_4$  ways. Hence exhaustive number of cases is

$$^{10}C_4 = \frac{10 \times 9 \times 8 \times 7}{4!} = 210$$

(i) Favourable number of cases for the committee to consist of 4 members, one from each category is :

$$^4C_1 \times ^3C_1 \times ^2C_1 \times 1 = 4 \times 3 \times 2 = 24$$

$$\therefore \text{Required probability} = \frac{24}{210} = \frac{8}{70}$$

(ii)  $P$  [Committee has at least one purchase officer]  
 $= 1 - P$  [Committee has no purchase officer]

In order that the committee has no purchase officer, all the 4 members are to be selected from amongst officers of production department, sales department and chartered accountant, i.e., out of  $3+2+1=6$  members and this can be done in

$$^6C_4 = \frac{6 \times 5}{1 \times 2} = 15 \text{ ways. Hence}$$

$$P \text{ [ Committee has no purchase officer ]} = \frac{15}{210} = \frac{1}{14}$$

$$\therefore P \text{ [ Committee has at least one purchase officer ]} = 1 - \frac{1}{14} = \frac{13}{14}$$

(iii) Favourable number of cases that the committee consists of a chartered accountant as a member and three others are :

$$1 \times {}^9C_3 = \frac{9 \times 8 \times 7}{1 \times 2 \times 3} = 84 \text{ ways,}$$

since a chartered accountant can be selected out of one chartered accountant in only 1 way and the remaining 3 members can be selected out of the remaining  $10 - 1 = 9$  persons in  ${}^9C_3$  ways. Hence the required probability =  $\frac{84}{210} = \frac{2}{5}$ .

15. If the letters of the word 'REGULATIONS' be arranged at random, what is the chance that there will be exactly 4 letters between R and E?

**Solution.** (a) The word '*REGULATIONS*' consists of 11 letters. The two letters *R* and *E* can occupy  ${}^{11}P_2$ , i.e.,  $11 \times 10 = 110$  positions.

The number of ways in which there will be exactly 4 letters between *R* and *E* are enumerated below:

(i) *R* is in the 1st place and *E* is in the 6th place.

(ii) *R* is in the 2nd place and *E* is in the 7th place.

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(vi) *R* is in the 6th place and *E* is in the 11th place.

Since *R* and *E* can interchange their positions, the required number of favourable cases is  $2 \times 6 = 12$

$\therefore$  The required probability  $= \frac{12}{110} = \frac{6}{55}$ .

If the letters of the word ATTEMPT are written down at random, the chance that all TS are consecutive is...

(a)  $1/42$

(b)  $6/7$

(c)  $1/7$

(d)  $1/8$

*The sum of two non-negative quantities is equal to  $2n$ . Find the chance that their product is not less than  $\frac{3}{4}$  times their greatest product.*

**Solution.** Let  $x > 0$  and  $y > 0$  be the given quantities so that  $x + y = 2n$ .

We know that the product of two positive quantities whose sum is constant is greatest when the quantities are equal. Thus the product of  $x$  and  $y$  is maximum when  $x = y = n$ .

$$\therefore \text{Maximum product} = n \cdot n = n^2$$

$$\begin{aligned} \text{Now } P \left[ xy \leq \frac{3}{4} n^2 \right] &= P \left[ xy \geq \frac{3}{4} n^2 \right] = P \left[ x (2n - x) \geq \frac{3}{4} n^2 \right] \\ &= P [(4x^2 - 8nx + 3n^2) \leq 0] \\ &= P [(2x - 3n)(2x - n) \leq 0] \\ &= P \left[ x \text{ lies between } \frac{n}{2} \text{ and } \frac{3n}{2} \right] \end{aligned}$$

$$\therefore \text{Favourable range} = \frac{3n}{2} - \frac{n}{2} = n$$

$$\text{Total range} = 2n$$

$$\therefore \text{Required probability} = \frac{n}{2n} = \frac{1}{2}$$

