

Number Systems

NUMBER SYSTEM

Work with the
world of numbers



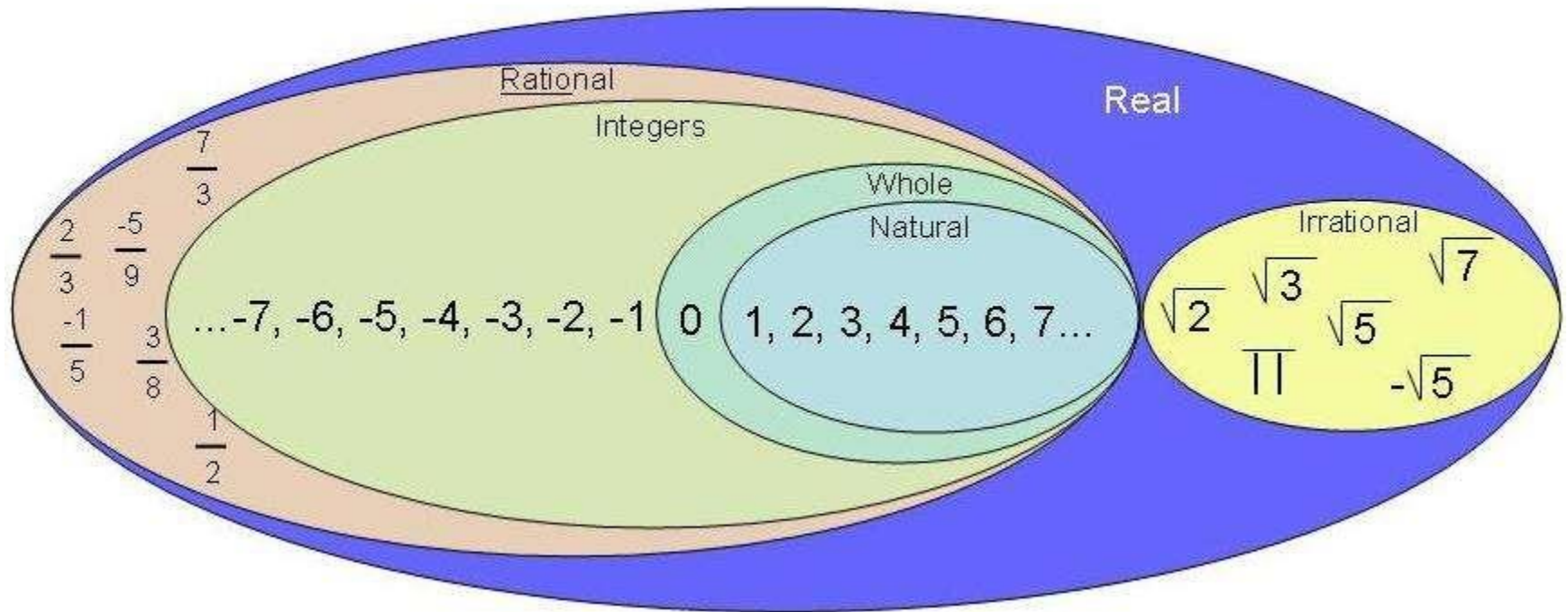
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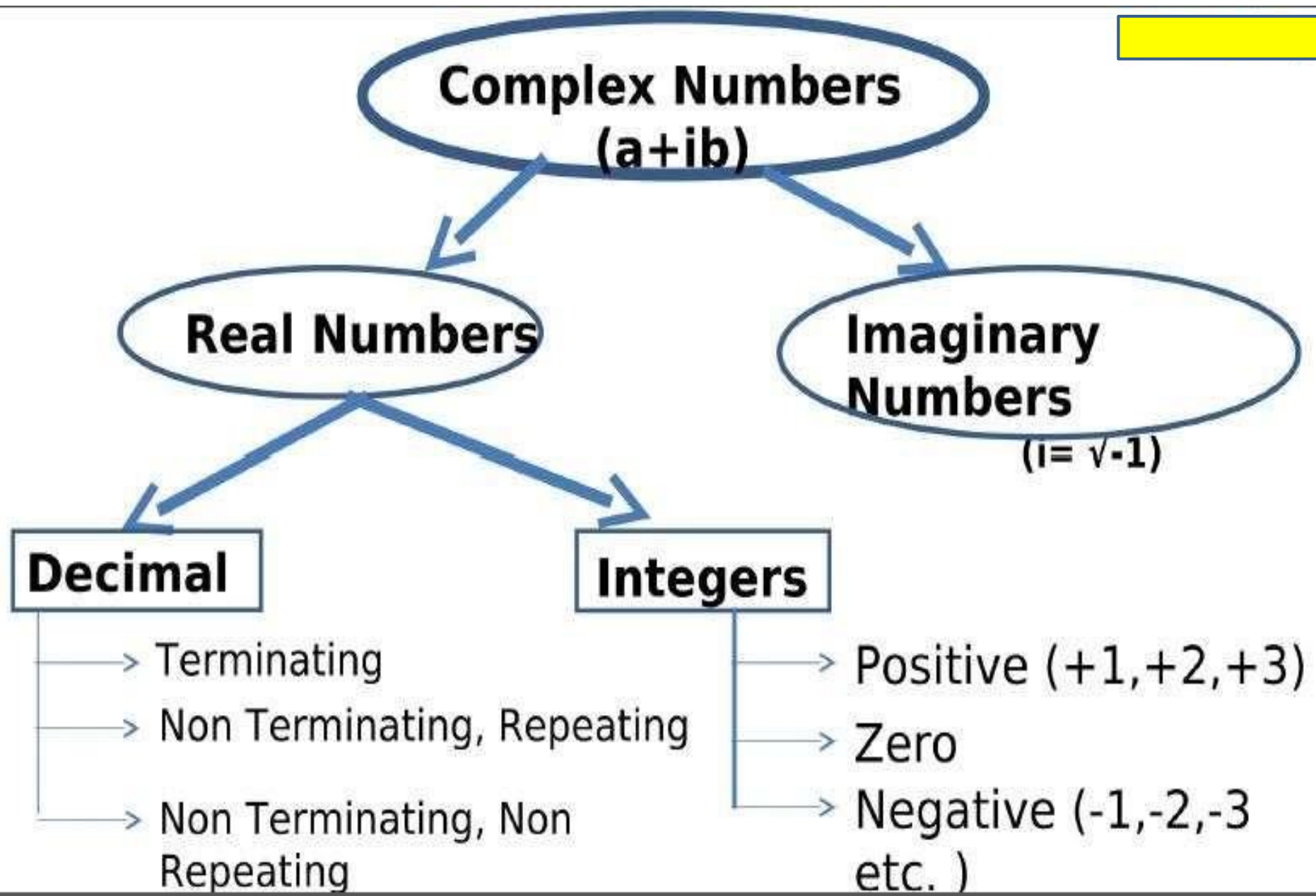
- TYPES OF NUMBERS
- Conversion of a decimal number to fraction
- DIVISIBILITY RULE
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- FACTORS AND MULTIPLES
 - i) Number of factors
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Types of numbers

Real Number System





Real Numbers : All numbers which can be represented on number line are called Real numbers. Or we can say that all numbers from $-\infty$ to $+\infty$ are called Real numbers.

Decimal Numbers: Decimal Numbers are classified into 3 categories.

1. Terminating Decimals (e.g. 0.5, 0.2, 1.5 etc.)
2. Non terminating but repeating decimals (e.g. 0.3333.....)
3. Non Terminating and non repeating (π , $\sqrt{2}$, $\sqrt{3}$)

First two decimal number's type are Rational Number and third type is Irrational.

Rational Numbers : All numbers which can be written in the form of p/q , where $q \neq 0$ are called Rational Numbers. All other numbers are Irrational.

$0.5 = 5/10$, So terminating decimals are Rational Numbers.

$0.333333.....=1/3$, So Non terminating but repeating decimals are also Rational.

But Non terminating and non repeating decimals are Irrational numbers.

Note: Here one should know that value of π is not $22/7$ which we generally use for our convenience.

Rational no. b/w a and b = $(ak+b)/(k+1)$

Irrational no. b/w a and b = \sqrt{ab} ,

Positive Integers: Positive integers can be categorized in many ways.

1. Prime numbers: Numbers having exactly two factors are called prime numbers. They have factors as 1 and the number itself. e.g. 2, 3, 5 etc.

2. Composite Numbers: Numbers having more than two factors are called composite numbers. e.g. 4, 6, 8

3. Neither Prime nor composite: 1 is neither prime nor composite as it has only one factor.

- 2 is the smallest Prime number and the only prime number which is even.

Even number: Numbers divisible by 2 are even numbers. e.g. 2,4,6,8 etc.

Odd numbers: Numbers not divisible by 2 are odd numbers.

Co-Prime numbers: Set of two numbers having $HCF=1$ e.g. (2,3) , (5,7) etc.

Perfect number: If the sum of all the factors of a number (excluding that number) is equal to that number. Then that number is called perfect number.
E.g. $6 = 1, 2, 3, 6$ adding factor $sum = 1 + 2 + 3 = 6$

Important rules related to Even and Odd numbers:

$\text{odd} \pm \text{odd} = \text{even};$

$\text{even} \pm \text{even} = \text{even};$

$\text{even} \pm \text{odd} = \text{odd}$

$\text{odd} \times \text{odd} = \text{odd};$

$\text{even} \times \text{even} = \text{even};$

$\text{even} \times \text{odd} = \text{even}.$

$\text{odd}(\text{any number}) = \text{odd}$

$\text{even}(\text{any number}) = \text{even}$

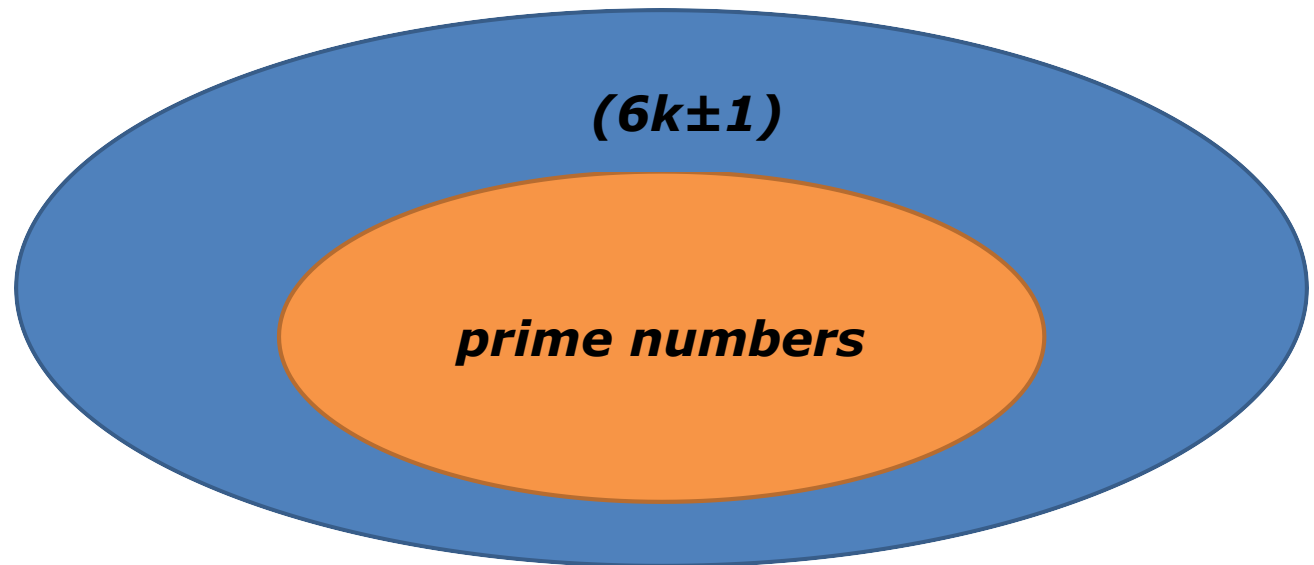
How to find if a number is prime or not?

N is a prime number if it is not divisible by numbers lesser than \sqrt{N} .

Example: 191 is a prime number since it is not divisible by 2, 3, 5, 7, 11 and 13 [numbers less than $\sqrt{191}$ (≈ 14)].

Note: *Prime numbers* will always be in the form $(6k \pm 1)$ where $k = 1, 2, 3, \dots$

But not all $(6k \pm 1)$ will be a prime number.



Conversion of a decimal number to fraction:

Example:

$$3.\overline{713} =$$

Solution:

$$3.\overline{713} = 3 + \frac{713}{999} = \frac{2997 + 713}{999} = \frac{3710}{999}$$

Example:

$$12.\overline{345} =$$

Solution:

Here only 45 are recurring.

$$\text{Therefore, } 12.\overline{345} = 12 + \frac{345 - 3}{990} = 12 + \frac{342}{990} = 12 + \frac{38}{110} = 12 + \frac{19}{55} = \frac{679}{55}$$

Q. Find the rational form of the recurring rational
0.2333333333

[A] $11/99$

[B] $1/3$

[C] $7/30$

[D] $13/30$

Q. Convert $37.565656565656\dots$ into P/Q

Divisibility Rules

Divisibility rule of 2 : Numbers which ends with even number or zero is always divisible by 2.

Example: 122, 246, 230, 458 etc.

Divisibility rule of 3 : A number is divisible by 3 if sum of it's digit is divisible by 3.

Example: 1296, 342, etc.

1296, Sum of digits= $1+2+9+6=18$, which is divisible by 3.

Divisibility rule of 4: If the last two digit of a number are divisible by 4 or numbers ending with two or more zeros then that number is divisible by 4.

Example: 2332, 1240, 2500, 816000, etc. are divisible by 4.

Divisibility rule of 6: Now 6 is a composite number and whenever we discuss the divisibility rule of a composite number then we break that composite number into its two Co-Prime factors. For example 6 has (2,3) as its Co-Prime factors.

If a number is divisible by both 2 and 3 it means that number is also divisible by 6.

Example: 612, 2532, 5250 etc.

Divisibility rule of 8: A number is divisible by 8, if its last three digits

Divisibility rule of 9 : If sum of all digits of a number is divisible by 9, the number is also divisible by 9.

Example: 1296, 369, 1440, 25254 etc.

Divisibility rule of 10: if a number is ending with 0, it is divisible by 10.

Example : 1220, 320, 2500, 450 etc.

Divisibility rule of 12: Again it is a composite number whose two Co-Prime factors are (3,4)

We can say that if a number is divisible by 3 and 4 both then that number is also divisible by 12.

Example : 468, 1152, 1020 etc.

Any other numbers can be written in terms of the numbers whose divisibility is already known.

Example: $15 = 3 \times 5$

$$18 = 2 \times 9$$

$$33 = 3 \times 11$$

Note: The numbers expressed should be co-prime (i.e., the HCF of the two numbers should be 1)

Example: $40 = 4 \times 10$ is wrong because $\text{HCF}(4,10)$ is 2.
 $\therefore 40 = 5 \times 8$ because $\text{HCF}(5,8)$ is 1.

Question: If number $1792N$ is divisible by 2. How many values N can take?

- A. 4
- B. 5
- C. 3
- D. 6

Question: What should come in place of K if 563K5 is divisible by 9?

A. 7

B. 8

C. 9

D. 2

Question: For what values of P number 345472P34 is exactly divisible by 9.

- A. 3
- B. 4
- C. 6
- D. 7

Question: For what values of N number 9724N is exactly divisible by 6.

[A] 2 & 8

[B] 4 & 6

[C] 2 & 6

[D] 6 & 8

Divisibility rule of 11 : A number is divisible by 11 if the difference between the sum of digits at odd places and sum of digits at even places is either 0 or divisible by 11.

Example: 10593, 9372 etc.

For 10593

$$\begin{aligned} &(\text{Sum of digits at odd places}) - (\text{Sum of digits at even places}) = (3+5+1) - \\ &(9+0) = 9 - 9 = 0 \end{aligned}$$

For 9372

$$(2+3) - (9+7) = 5 - 16 = -11 \text{ which is divisible by 11.}$$

Question: For what values of N number 857N32 is exactly divisible by 11.

A. 1

B. 0

C. 3

D. 4

Question: What should come in place x if $4857x$ is divisible by 88?

A. 6

B. 8

C. 2

D. 4

Remainder theorem

$$\begin{array}{r} 4 \text{ — Quotient} \\ \text{Divisor } 6 \overline{) 25 \text{ — Dividend}} \\ \underline{24} \\ 1 \text{ — Remainder} \end{array}$$

$$\text{Dividend} = (\text{Divisor} \times \text{Quotient} + \text{remainder})$$

Question: In a division process divisor is 5 times of quotient and remainder is 3 times of quotient. If remainder is 15. Find dividend.

[A] 140

[B] 150

[C] 170

[D] 190

Question: When a number is divided by 102 leaves remainder 87. If the same no. is divided by 17. Find remainder.

- A. 1
- B. 5
- C. 7
- D. 2

Important Rules

1. If x/D Remainder = R , then $2x/D$ Remainder = $2R/D$
2. If x/D Remainder = R , then $\frac{x^2}{D}$ Remainder = R^2/D
3. If x/D Remainder = R , then $\frac{x^3}{D}$ Remainder = R^3/D
4. If $\frac{A \times B}{D}$ then Remainder = $\frac{Ar \times Br}{D}$ (product of individual remainders)
5. If $\frac{A+B}{D}$ then Remainder = $\frac{Ar+Br}{D}$ (Sum of individual remainders)

Question: Find remainder for $\frac{63 \times 78}{9}$

Question: Find remainder for $\frac{63 + 78}{9}$

Question: When a number divided by a divisor m then remainder is 20. If the twice of that number is divided by the same divisor m then the remainder is 9. Find the divisor m .

Important Formulas:

1. $\frac{(x+a)^n}{x}$ then remainder = a^n

2. $\frac{(tx+a)^n}{x}$ then remainder = a^n

3. $\frac{(x+1)^n}{x}$ then remainder = $1^n = 1$

4. $\frac{(x-a)^n}{x}$ then remainder = $(-1)^n$ if power even then $R=1$
if Power Odd then $R=-1$ or $x-1$

5. $\frac{(tx+1)^n}{x}$ then remainder = $1^n = 1$

6. $\frac{(tx-1)^n}{x}$ then remainder = $(-1)^n$ if power even then $R=1$
if Power Odd then $R=-1$ or $x-1$

Remainder Cyclicity:

Example: What is the remainder when 2^{202} is divided by 7?

$$2^1/7 = R(2)$$

$$2^2/7 = R(4)$$

$$2^3/7 = R(1)$$

The next three remainder values will be the same. i.e., The remainder pattern is 2,4,1, 2,4,1, 2,4,1.....The size of the pattern is 3.

Now divide the power by number of repeating values (3) to choose the remainder.

Choose the nth value in the cycle if the remainder is n except for the last value whose remainder should be 0.

$$202/3 = R(1).$$

The 1st value in the cycle is 2.

Note: While finding the remainder pattern if the remainder becomes 1, then the process can be stopped as it will always repeat after 1.

$$\therefore 2^{202}/7 = R(2)$$

Note: Do not cancel any numerator value with the denominator value as the remainder will differ.

$$R(6/4) \neq R(3/2)$$

$$6/4 = R(2)$$

$$\text{But } 3/2 = R(1)$$

If you want to cancel out numerator and denominator by a **certain value** then at last we also need to multiply the remainder by the **same value** in order to get correct remainder.

Q) What is the remainder when 3^7 is divided by 8?

- A. 3
- B. 4
- C. 5
- D. 7

Q) Remainder when 17^{23} is divided by 16?

- A. 1
- B. 2
- C. 3
- D. 4

Q) Remainder when 35^{113} is divided by 9?

- A. 1
- B. 8
- C. 3
- D. 4

Q) Remainder when 2^{33} is divided by 9?

- A. 1
- B. 4
- C. 8
- D. 5

Q) Remainder when 2^{99} is divided by 10?

- A. 1
- B. 4
- C. 2
- D. 8

Q) Remainder when 5^{500} is divided by 500?

[A] 125

B. 1

C. 5

[D] 250

Unit Digit Concept

Right most digit of a number is called Unit digit.

For e.g. 278×623 what will be the unit digit?

Unit digit questions can be asked in two ways:

1. Simple Product type Questions

e.g. What will be the unit digit of $123 \times 456 \times 789$.

2. Power Type Questions

e.g. Find the unit digit of $(127)^{23}$

It can also be the mixture of both.

We can categories in three category:

- 1. Numbers ending with (0, 1, 5, 6)**
- 2. Numbers ending with (4,9)**
- 3. Numbers ending with (2,3,7,8)**

Each category follow a certain rule.

1. **Numbers ending with (0, 1, 5, 6) :** Any number ending with 0, 1, 5, 6 raised to power any number (Except 0) will always have same number at unit place respectively.

For e.g. $(2350)^{234}$, $(531)^{34}$, $(245)^{321}$, $(776)^{321}$

2. Numbers ending with (4 and 9) : Cyclicity of 4 and 9 is 2. It means their unit digit repeats after every 2 powers. So we can say

$$(4)^{odd} = 4 \text{ at unit place}$$

$$(4)^{Even} = 6 \text{ at unit place}$$

$$(9)^{Odd} = 9 \text{ at unit place}$$

$$(9)^{Even} = 1 \text{ at unit place}$$

3. Numbers ending with (2,3,7,8) : All numbers have cyclicity of 4, it means after every 4th power the unit digit pattern will be same.

In these type of questions we will divide the power by cyclicity (i.e. 4) so that we will know how many cycles have been completed and we will try to find remainder. And unit digit will be $(2,3,7,8)^{\text{Rem}}$

Note: If remainder is zero then we take highest power of that cycle which is 4. or we can say $(2,3,7,8)^4$

	Power			
Base	1	2	3	4
2	2	4	8	6
3	3	9	7	1
7	7	9	3	1
8	8	4	2	6
4	4	6		
9	9	1		

Number	<u>Cyclicity</u>
1	1
2	4
3	4
4	2
5	1
6	1
7	4
8	4
9	2
10	1

Choose the n th value in the cycle if the remainder is n except for the last value whose remainder should be 0.

Example : What is the unit digit of $(123)^{42}$?

The unit digit pattern of 3 repeats four times. So find the remainder when the power value is divided by 4.

$$42/4 = R(2)$$

2nd value in 3 cycle is 9.

∴ Unit digit of $(123)^{42}$ is 9

Question: Find the unit digit of $(2354)^{1048}$

[A] 4

[B] 6

[C] 8

[D] None of these

Question: Find the unit digit of $(248)^{1587}$

[A] 2

[B] 4

[C] 8

[D] 6

Question: Find the unit digit of $(127)^{223}$

[A] 7

[B] 9

[C] 3

[D] 1

Question: Find the unit digit of $(456)^{87} \times (307)^{42}$

[A] 1

[B] 4

[C] 5

[D] 7

Factors

Factors of a number are the values that divides the number completely.

Example: Factors of 10 are 1, 2, 5 and 10.

Multiple of a number is the product of that number and any other whole number.

Example: multiples of 10 are 10, 20, 30,.....

1. Total number of factors
2. Sum of Factors
3. Product of Factors
4. Prime factors
5. Composite factors
6. Even factors
7. Odd factors
8. In how many ways a number can be written as a product of its two factors
9. In how many ways a number can be written as a product of its two co-prime factors

1. Total Number of factors:

- Take any number “N” and it is to be converted into **product of prime numbers** (*Prime factorization*) i.e.
- $N = A^p \times B^q \times C^r$ here A, B, C are prime numbers and p, q, and r were respective powers of that prime numbers.
- **Total numbers of factors for** $N = (p + 1)(q + 1)(r + 1)$.

Example: 3600

Step 1: Prime factorize the given number

$$3600 = 36 \times 100$$

$$= 6^2 \times 10^2$$

$$= 2^2 \times 3^2 \times 2^2 \times 5^2$$

$$= 2^4 \times 3^2 \times 5^2$$

Step 2: Add 1 to the powers and multiply.

$$(4+1) \times (2+1) \times (2+1)$$

$$= 5 \times 3 \times 3$$

$$= 45$$

\therefore Number of factors of 3600 is 45.

Question: Find the total number of factors of 330.

- A. 2
- B. 8
- C. 16
- D. 32

Question: Find the total number of factors of 1560.

- A. 6
- B. 16
- C. 12
- D. 32

2. Sum of factors:

Example: 45

Step 1: Prime factorize the given number

$$45 = 3^2 \times 5^1$$

Step 2: Split each prime factor as sum of every distinct factors.

$$(3^0 + 3^1 + 3^2) \times (5^0 + 5^1)$$

The following result will be the sum of the factors
= 78

Question: Find the sum of factors of 24.

- A. 60
- B. 46
- C. 56
- D. 59

Question: Find the sum of factors of 98.

[A] 161

[B] 171

[C] 160

[D] None

3. Product of factors:

Product of factors of a number N is given by $N^{(\text{Total factor}/2)}$

Question: Find the product of factors of 24.

4. Prime factors:

Number of Prime factors used in prime factorization are prime factors.

5. Composite Factors:

(Total factors – Prime factors)-1

Question: Find the number of prime factors of 300.

- A. 2
- B. 3
- C. 4
- D. 5

Question: Find the number of composite factors of 42.

- A. 1
- B. 4
- C. 5
- D. 3

Even and Odd factors: We can find any one of two values and other we can find from subtracting from the total number of factors.

6.Odd factors: To find odd factors we will not consider power of 2 as it is a even Prime number.

Note: Do not consider even prime factor

7.Even factors: We can also find even factor directly. After prime factorization we will increase all power by 1 except power of 2. It will remain constant.

Odd Factors:

Example: 4500

$$4500 = 45 \times 100 = 9 \times 5 \times 10 \times 10 = 3 \times 3 \times 5 \times 5 \times 2 \times 5 \times 2$$

$$4500 = 2^2 \times 3^2 \times 5^3 \text{ Here consider } A = 2, B = 3, C = 5, p = 2, q = 2 \text{ and } r = 3$$

Here identifying that odd number are 3 and 5

$$\text{Numbers of odd factors of number } 4500 = (q + 1)(r + 1) = 3 \times 4 = 12$$

Note: Do not consider even prime factor

Question: How many factors of 340 are even.

- A. 12
- B. 16
- C. 8
- D. 6

Question: How many factors of 408 are even.

- A. 12
- B. 16
- C. 8
- D. 10

8. In how many ways a number can be written as a product of its two factors:

It is given by $(\text{Total Factors}/2)$

Note: If the total number of factors are odd then Product of two number= $(\text{total factors}+1)/2$

9. In how many ways a number can be written as a product of its two co-prime factors:

It is given by $2^{(N - 1)}$

Where N = no. of prime factors of that number

Question: In How many ways 320 can be written as a product of its two co-prime factors .

A. 2

B. 6

C. 1

D. 5

Question: In How many ways 12 can be written as a product of its two co-prime factors .

A. 2

B. 6

C. 8

D. 6

Factors will occur in pairs for the numbers except perfect squares.

Example 1: A non perfect square number- 10

$$1 \times 10 = 10$$

$$2 \times 5 = 10$$

‘ \therefore Factors of 10 are 1, 2, 5 and 10.

Non perfect squares will have even number of factors

Example 2: A perfect square number- 16

$$1 \times 16 = 16, 2 \times 8 = 16, 4^2 = 16$$

\therefore Factors of 16 are 1, 2, 4, 8 and 16.

Every **perfect square** will have **odd number of factors** because its square root number will pair with itself.

This has odd number of factors because 4 will pair with itself.

Every **perfect square** will have **odd number of factors** because its square root number will pair with itself.

Example 3: A prime square number- 49

The factors of 49 are 1, 7 and 49.

Prime square number will have exactly **3 factors** (1, that number itself and square root of that number).

If **N** is a **prime square number** then the factors are 1, N and \sqrt{N} .

Trailing zeros (Number of zeros at the end)

Concept: In multiplication zero can be produced when we have a pair of 2 and 5. So, in order to find no. of zeros at the end, the pair of 2 and 5 need to be counted.

Example: $2^3 \times 5 \times 9 \times 8 \times 24 \times 15$ Find number of zeros at the end.

How to find Trailing zeros in any $n!$:

Ex: Find number of zeros in $10!$

Answer: $10! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10$

Note: For any factorial value number of 5s are always less than number of 2s. So our concern will be to find number of 5.

For counting 5s in any $n!$ value we divide n by powers of 5.

$$= \left\lfloor \frac{n}{5} \right\rfloor + \left\lfloor \frac{n}{5^2} \right\rfloor + \left\lfloor \frac{n}{5^3} \right\rfloor + \left\lfloor \frac{n}{5^4} \right\rfloor + \dots$$

Question: Find number of trailing zeros in $100!$

- A. 24
- B. 25
- C. 26
- D. 27

Question: Find number of trailing zeros in $154!$

- A. 35
- B. 31
- C. 34
- D. 37

Question: Find number of trailing zeros in $56!$

- A. 13
- B. 11
- C. 12
- D. 6

Question: Find number of trailing zero in $100! + 200!$

- A. 24
- B. 49
- C. 73
- D. None of these

Question: Find number of trailing zero in $100! \times 200!$

- A. 24
- B. 49
- C. 73
- D. None of these

LCM AND HCF

LCM- Least Common Multiple

LCM of given numbers X,Y,Z is the Least number which is exactly divisible by all the given numbers X,Y,Z.

OR we can say it is the least multiple of all the given numbers.

For e.g. 12= 12, 24, 36, 48, 60, 72, 84, 96.....

16= 16, 32, 48, 64, 80, 96.....

So there are lots of multiple of 12 and 16 which will be common but the least multiple which is common is 48. That is the LCM of 12 and 16.

FINDING L.C.M. OF BIG NUMBERS

Step 1 Find all the prime factors of both numbers.

Step 2 Multiply all the prime factors of the larger number by those prime factors of the smaller number that are not already included

HCF- Highest Common Factor

HCF of given numbers X, Y, Z is the greatest number which exactly divides all the given numbers X, Y, Z . Sometimes called **Greatest Common divisor(G.C.D.)**

For e.g. $12 = 1, 2, 3, 4, 6, 12$

$16 = 1, 2, 4, 8, 16$

So there three common factor= $1, 2, 4$ but the Greatest factor is 4. That is the HCF of 12 and 16.

FINDING THE H.C.F. OF BIG NUMBERS

For larger numbers you can use the following method:

Step 1 Find all prime factors of both numbers.

Step 2 Write both numbers as a multiplication of prime numbers.

Step 3 Find which factors are repeating in both numbers and multiply them to get H.C.F

Important Formulae

Product of Two numbers = HCF \times LCM

$$\text{HCF of Fraction} = \frac{\text{HCF of numerators}}{\text{LCM of Denominator}}$$

$$\text{LCM of Fraction} = \frac{\text{LCM of numerators}}{\text{HCF of Denominator}}$$

LCM Word Problems

Question	Answer
The Least number which is exactly divisible by x, y and z	$\text{LCM}(x, y, z)$
The least number which when divided by x, y, z leaves the same remainder R in each case	$\text{LCM}(x, y, z) + R$
The least Number which when divided by x, y and z, leaves remainder a, b and c respectively	$\text{LCM}(x, y, z) - K$ Where $K = (x-a) = (y-b) = (z-c)$

HCF Word Problems

Question	Answer
The greatest number that will exactly divide x, y and z	$\text{HCF}(x, y, z)$
The greatest number that will divide x, y, z and leaves remainder R in each case	$\text{HCF}(x-y, y-z, z-x)$ OR $\text{HCF}(x-R, y-R, z-R)$
The greatest Number that will divide x, y and z, leaving remainder a, b and c respectively	$\text{HCF}(x-a, y-b, z-c)$

Q) Find the lowest common multiple of 24, 36 and 40.

[A] 120

[B] 240

[C] 360

[D] 480

Q) The least number which is exactly divisible by 8, 16, 40 and 80 is:

[A] 16

[B] 120

C. 80

D. None

Q) The greatest number that will exactly divide 36 and 84 is:

- A. 4
- B. 6
- C. 12
- D. 18

Q) The greatest possible length which can be used to measure exactly the lengths 7 m, 3 m 85 cm, 12 m 95 cm is:

- A. 15
- B. 25
- C. 35
- D. 42

Q) Four bells ring at an interval 3min, 4min, 5min and 6 minutes respectively. If all the four bells ring at 9am first, when will it ring again?

Q) The H.C.F. of two numbers is 11 and their L.C.M. is 7700.

If one of the numbers is 275, then the other is:

[A] 308

[B] 310

[C] 312

[D] None

Q) The H.C.F of $9/10$, $12/25$, $18/35$, and $21/40$ is?

[A] $3/1400$

[B] $5/1400$

[C] $7/1400$

[D] None

Q) Which of the following fraction is the largest? $7/8$, $13/16$, $31/40$, $63/80$

[A] $7/8$

[B] $13/16$

[C] $31/40$

[D] $63/80$

Q) Three number are in the ratio of 3 : 4 : 5 and their L.C.M. is 2400. Their H.C.F. is:

A. 40

B. 80

[C] 120

[D] 200

Q) The ratio of two numbers is $3 : 4$ and their H.C.F. is 4. Their L.C.M. is:

A. 12

B. 16

C. 24

D. 48

Q) The least number, which when divided by 12, 15, 20 and 54 leaves in each case a remainder of 8 is:

[A] 504

[B] 536

[C] 544

[D] 548

Q) Find the smallest number, which when divided by 3, 4 and 5 leaves remainder 1, 2 and 3 respectively?

- A. 60
- B. 53
- C. 58
- D. None

Q) The greatest number which on dividing 1657 and 2037 leaves remainders 6 and 5 respectively, is:

[A] 123

[B] 127

[C] 235

[D] 305

Q) Find the greatest number that will divide 43, 91 and 183 so as to leave the same remainder in each case.

- A. 4
- B. 7
- C. 9
- D. 13

Q) The product of two numbers is 4107. If the H.C.F. of these numbers is 37, then the greater number is:

[A] 101

[B] 107

[C] 111

[D] 185

ARITHMETIC PROGRESSION

An Arithmetic Progression (A.P.) is a sequence in which the difference between any two consecutive terms is constant.

Let a = first term, d = common difference

- Then n th term

$$a_n = a + (n - 1)d$$

A diagram illustrating the relationship between the last term and the n th term from the end in an arithmetic progression. It features a horizontal line with a red question mark at the left end and the text "Last term (l)" at the right end. A yellow arrow points from the right towards the question mark, with the text " n^{th} term from the end" written below it. Below the diagram, a blue box contains the formula:

$$n^{\text{th}} \text{ term from end} = l - (n - 1)d$$

Sum of an A.P

The sum of n terms of an A.P. whose first term is a and common difference is d , is given by

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

The sum of n terms of an A.P. whose first term is a and last term is l is given by the formula:

$$S_n = \frac{n}{2} [a + l]$$

AM (Arithmetic mean)

If a, b, c are in AP then the arithmetic mean is given by

$$b = (a+c)/2$$

Inserting AM

To insert k means between a and b the formula for common difference is given by

$$d = (b-a) / (k+1)$$

For example: Insert 4 AM's between 4 and 34

$$d = (34 - 4) / (4+1)$$

$$= 30/5$$

$$= 6$$

∴ The means are $4+6=10$

$$10+6=16$$

$$16+6=22$$

$$22+6=28$$

Question: 1,3,5, 7, Which term of this AP is 55?

A. 25th

B. 26th

C. 27th

D. 28th

Question: Find the 15th term of the series 20,15,10,.....

[A] -45

[B] -50

[C] -55

[D] 0

Question: How many terms are there in the AP 20, 25, 30,
..... 130?

- A. 21
- B. 22
- C. 23
- D. 24

Question: Find the sum of the series 5,8,11,..... 221

[A] 8249

[B] 8239

[C] 7886

[D] 9000

Question: Find the sum of all 2-digit numbers, which are exactly divisible by 9?

[A] 525

[B] 565

[C] 575

[D] 585

Question: Find the first term of an AP whose 8th and 12th terms are 39 and 59 respectively?

- A. 3
- B. 4
- C. 5
- D. 6

GEOMETRIC PROGRESSION

A geometric sequence are powers r^k of a fixed number r , such as 2^k and 3^k . The general form of a geometric sequence is

The n -th term of a geometric sequence with initial value a and common ratio r is given by

$$a_n = a r^{n-1}.$$

Such a geometric sequence also follows the recursive relation

$$a_n = r a_{n-1} \text{ for every integer } n \geq 1.$$

General term of a GP is $T_n = ar^{n-1}$

Sum of first n terms of G.P:

a. $S_n = \frac{a(r^n - 1)}{r - 1}$ where $r > 1$

b. $S_n = \frac{a(1 - r^n)}{1 - r}$ where $r < 1$

c. $S_n = na$ where $r = 1$

Sum of infinite G.P:

If a G.P. has **infinite terms** and $-1 < r < 1$ or $|x| < 1$,

Sum of infinite G.P is $S_\infty = \frac{a}{1 - r}$

GM (Geometric mean)

If a, b, c are in GP Then the GM is given by $\mathbf{b = \sqrt{ac}}$

Inserting GM

To insert k means between a and b the formula for common ratio is given by

$$r = (b/a)^{1/(k+1)}$$

For example: Insert 4 GM's between 2 and 486

$$r = (486/2)^{1/(4+1)} = (243)^{1/5} = 3$$

\therefore the means are $2 \times 3 = 6$

$$6 \times 3 = 18$$

$$18 \times 3 = 54$$

$$54 \times 3 = 162$$

Question: How many terms are there in the sequence

5, 20, 80, 320, 20480?

A. 5

B. 6

C. 7

D. 8

Question: If the first and fifth term of a GP are 16 and 81 respectively then find the fourth term?

- A. 18
- B. 24
- C. 36
- D. 54

Question: Find the sum of the series 2, 4, 8, 16.... 256.

[A] 510

[B] 1020

[C] 520

[D] None

Next Class Average.

