Lecture 1 MTH302 (unit 1)

Topics: concept of Random variable, Discrete probability distribution, Continuous probability distribution.

Definition 1: A random variable is a function that associates a real number with each element in the sample space.

(leave 5-6 lines)

Example 1

Example 1:- Let X be random variable giving the number of heads in the three tosses of a coin. List the elements of the sample space S and to each sample point assign a value x of X.

Example 1 will be discussed in the class (Leave 10-12 lines)

Example 2 : (exercise question)

3.3 Let W be a random variable giving the number of heads minus the number of tails in three tosses of a coin. List the elements of the sample space S for the three tosses of the coin and to each sample point assign a value w of W.

Example 2 will be discussed in the class (leave 10-12 lines)

Definition 2 : If a sample space contains a finite number of possibilities or an unending sequence with as many elements as there are whole numbers, it is called a discrete sample space .

Definition 3: If a sample space contains an infinite number of possibilities equal to the number of points on a line segment, it is called a continuous sample space .

Example 3 (exercise question)

3.4 A coin is flipped until 3 heads in succession occur. List only those elements of the sample space that require 6 or less tosses. Is this a discrete sample space? Explain.

Example 3 will be discussed in the class (leave 12-14 lines)

Discrete Probability Distributions

Definition 4:

The set of ordered pairs (x, f (x)) is a probability function , probability mass function (pmf), or probability distribution (pdf) of the discrete random variable X if, for each possible outcome x ,

1. $f(x) \geq 0,$
2. $\sum_{x} f(x) = 1,$
3. $P(X = x) = f(x).$

Continuous Probability Distributions

Definition 5:

The function **f** (**x**) is a probability density function (pdf) for the continuous random variable **X** , defined over the set of real numbers, if

1. $f(x) \geq 0$, for all $x \in R$.
2. $\int_{-\infty}^{\infty} f(x)\, dx = 1.$
3. $P(a < X < b) = \int_{a}^{b} f(x)\, dx.$

Remark:-

Example 4: (exercise question)

3.8 Find the probability distribution of the random variable W in Exercise 3.3, assuming that the coin is biased so that a head is twice as likely to occur as a tail.

Example 4 will be discussed in the class (leave half page)

Example 5 (exercise question)

3.5 Determine the value c so that each of the following functions can serve as a probability distribution of the discrete random variable X:

(a) $f(x) = c(x^2 + 4)$, for $x = 0, 1, 2, 3;$

(b) $f(x) = c\binom{2}{x}\binom{3}{3-x}$, for $x = 0, 1, 2.$

Example 5(a) part will be discussed in the class (b) will be home work (leave 1 page)

Example 6 (exercise question)

3.6 The shelf life, in days, for bottles of a certain prescribed medicine is a random variable having the density function

$$f(x) = \begin{cases} \frac{20{,}000}{(x+100)^3}, & x > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the probability that a bottle of this medicine will have a shell life of

(a) at least 200 days;

(b) anywhere from 80 to 120 days.

Example 6(a) part will be discussed in the class (b) will be home work (leave 1 page)

Example 7 (exercise question)

3.7 The total number of hours, measured in units of 100 hours, that a family runs a vacuum cleaner over a period of one year is a continuous random variable X that has the density function

$$f(x) = \begin{cases} x, & 0 < x < 1, \\ 2 - x, & 1 \leq x < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the probability that over a period of one year, a family runs their vacuum cleaner

(a) less than 120 hours;

(b) between 50 and 100 hours.

Eg 7 HW question hint will be given in the class (leave 1page)

Example 8 (exercise question)

3.11 A shipment of 7 television sets contains 2 defective sets. A hotel makes a random purchase of 3 of the sets. If x is the number of defective sets purchased by the hotel, find the probability distribution of X. Express the results graphically as a probability histogram.

Eg 8 HW question hint will be given in the class (leave 1page)