MTH166

Lecture-12

Solution of Higher Order Homogeneous LDE with Constant Coefficients-II

<u>Topic:</u>

Solution of Higher order Homogeneous LDE with Constant coefficients

<u>Learning Outcomes:</u>

1. Formulation of 3^{rd} order and 4^{th} order homogeneous LDE when roots are given.

Formulation of LDE: $ay''' + by'' + cy' + dy = 0$ when Roots are given:

Let the three given roots be: m_1, m_2 and m_3.

Then required 3rd order homogeneous LDE is:

$y''' -$ (sum of roots taken one at a time) $y'' +$ (sum of roots taken one at a time) y' $-$ (Product of roots)$y = 0$

i.e. $y''' - (m_1 + m_2 + m_3)y'' + (m_1m_2 + m_2m_3 + m_3m_1)y' - (m_1m_2m_3)y = 0$

or

Then required 3rd order homogeneous LDE is:

$(D - m_1)(D - m_2)(D - m_3)y = 0$ where $D \equiv \frac{d}{dx}$

Formulation of LDE: $\boldsymbol{ay^{iv} + by''' + cy'' + dy' + ey = 0}$ when Roots are given:

Let the four given roots be: $m_1, m_2, m_3$and m_4.

Then required 4th order homogeneous LDE is:

$(D - m_1)(D - m_2)(D - m_3)(D - m_4)y = 0$ where $D \equiv \frac{d}{dx}$

Find a homogeneous LDE with constant coefficients of lowest order which has the following particular solution:

Q 1. $5 + e^{x} + 2e^{3x}$

Sol. Here: $5 + e^{x} + 2e^{3x} = 5e^{0x} + e^{1x} + 2e^{3x}$

So, $m_1 = 0, m_2 = 1, m_3 = 3$

Then required 3rd order homogeneous LDE is:

$y''' - (m_1 + m_2 + m_3)y'' + (m_1m_2 + m_2m_3 + m_3m_1)y' - (m_1m_2m_3)y = 0$

$\Rightarrow y''' - (0 + 1 + 3)y'' + (0 + 3 + 0)y' - (0)y = 0$

$\Rightarrow y''' - 4y'' + 3y' = 0$ **Answer.**

Q 2. $xe^{-x} + e^{2x}$

Sol. Here: $xe^{-x} + e^{2x} = (0 + 1x)e^{-x} + e^{2x}$ $[(c_1 + c_2 x)e^{m_1 x} + c_3 e^{m_2 x}]$

So, $m_1 = -1, m_2 = -1, m_3 = 2$

Then required 3rd order homogeneous LDE is:

$y''' - (m_1 + m_2 + m_3)y'' + (m_1 m_2 + m_2 m_3 + m_3 m_1)y' - (m_1 m_2 m_3)y = 0$

$\Rightarrow y''' - (-1 - 1 + 2)y'' + (1 - 2 - 2)y' - (2)y = 0$

$\Rightarrow y''' - 3y' - 2y = 0$ **Answer.**

Q 3. $e^{-x} + \cos 5x + 3 \sin 5x$

Sol. Here: $e^{-x} + \cos 5x + 3 \sin 5x = c_1 e^{m_1 x} + e^{\alpha x}[c_2 \cos \beta x + c_3 \sin \beta x]$

So, $m_1 = -1, m_2 = 0 + 5i, m_3 = 0 - 5i$

Then required 3rd order homogeneous LDE is:

$y''' - (m_1 + m_2 + m_3)y'' + (m_1 m_2 + m_2 m_3 + m_3 m_1)y' - (m_1 m_2 m_3)y = 0$

$\Rightarrow y''' - (-1 + 5i - 5i)y'' + (-5i + 25 + 5i)y' - (25i^2)y = 0$

$\Rightarrow y''' + y'' + 25y' + 25y = 0$ **Answer.**

Q 4. $1 + x + e^x - 3e^{3x}$

Sol. Here: $1 + x + e^x - 3e^{3x} = (1 + x)e^{0x} + e^x - 3e^{3x}$

So, $m_1 = 0, m_2 = 0, m_3 = 1, m_4 = 3$

Then required 4th order homogeneous LDE is:

$(D - m_1)(D - m_2)(D - m_3)(D - m_4)y = 0$ where $D \equiv \frac{d}{dx}$

$\Rightarrow (D - 0)(D - 0)(D - 1)(D - 3)y = 0$

$\Rightarrow D^2(D^2 - 4D + 3)y = 0$

$\Rightarrow (D^4 - 4D^3 + 3D^2)y = 0$

$\Rightarrow y^{iv} - 4y''' + 3y'' = 0$ **Answer.**

Polling Questions:

Q1. If e^{x}, e^{4x} are solutions of differential equation $y'' + a(x)y' + b(x)y = 0$**,** then the values of $a(x)$ and $\mathrm{b}(x)$ are:

(A) $a(x) = -5, b(x) = 4$

(B) $a(x) = 5, b(x) = 4$

(C) $a(x) = -5, b(x) = -4$

(D) $a(x) = 5, b(x) = -4$

Q2. The intervals on which the differential equation $y' = 3\frac{y}{x}$ is normal are:

(A) $(-\infty, 0), (0, \infty)$ **(B)** $(-\infty, \infty)$

(C) $(-\infty, 1), (1, \infty)$ **(D)** None of these.

Q3. If $y = e^{at}$ is solution of $y'' - 5y' + 4y = 0$, then possible value of a is:

(A) $a = 2$ **(B)** $a = 3$

(C) $a = 4$ **(D)** $a = 5$

Q4. The general solution of $y'' - 9y = 0$ is:

(A) $Ae^{-3x} + Be^{3x}$

(B) $Ae^{3x} + Be^{3x}$

(C) $Ae^{-3x} + Be^{-3x}$

(D) None of these

Q5. The general solution of $y'' + 4y = 0$ is:

(A) $(Acos2x + Bsin\ 2x)$

(B) $Ae^{2x} + Be^{-2x}$

(C) $(A + Bx)e^{-2x}$

(D) None of these

Q6. The Wronskian of functions: $(1, \sin x, \cos x)$is:

(A) 0

(B) 1

(C) -1

(D) None of these

Q7. The general solution of $y'' - 10y' + 25y = 0$ is:

(A) $Ae^{4x} + Be^{5x}$

(B) $Ae^{5x} + Be^{5x}$

(C) $Ae^{4x} + Be^{7x}$

(D) $Ae^{5x} + Bxe^{5x}$