nature of roots are as follows:

In brief Nature of roots and corresponding C.F.

Sl	*Nature of Roots of A.E.*	*Roots*	*C.F.*
1.	Real (ational) and Distinct roots	m_1, m_2, m_3	$C_1e^{m_1x} + C_2e^{m_2x} + C_3e^{m_3x}$
2.	Repeated roots	$m_1 = m_2$, $m_1 = m_2 = m_3$	$(C_1 + C_2x)\ e^{m_1x}$ $(C_1 + C_2x + C_3x^2)\ e^{m_1x}$
3.	Complex roots	$m_1 = \alpha + i\beta$ $m_2 = \alpha - i\beta$	$e^{\alpha x}\ [C_1 \cos \beta x + C_2 \sin \beta x]$
4.	Repeated Complex roots	$m_1 = m_2 = \alpha + i\beta$ $m_3 = m_4 = \alpha - i\beta$	$e^{\alpha x}\ [(C_1 + C_2x) \cos \beta x + (C_3 + C_4x)\sin \beta x]$
5.	Irrational roots	$m_1 = a + \sqrt{b}$ $m_2 = a - \sqrt{b}$	$e^{ax}\left[C_1 \cosh \sqrt{b}\, x + C_2 \sinh \sqrt{b}x\right]$
6.	Repeated irrational roots	$m_1 = m_2 = a + \sqrt{b}$ $m_3 = m_4 = a - \sqrt{b}$	$e^{ax}\left[(C_1 + C_2x)\cosh \sqrt{b}x + (C_3 + C_4x)\sinh \sqrt{b}x\right]$

auxiliary equation are (rational) and distinct.

OBJECTIVE TYPE QUESTIONS

Choose the correct alternative :

1. The particular intergral of $\frac{d^2y}{dx^2} + \frac{6dy}{dx} + 9y = 5e^{3x}$ is

(*a*) $\frac{5e^{3x}}{18}$ (*b*) $\frac{e^{3x}}{36}$ (*c*) $5e^{3x}$ (*d*) $\frac{5e^{3x}}{36}$ **Ans.** (*d*)

2. The particular integral of $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = e^x$ is

(*a*) e^x (*b*) $\frac{x}{e^3}$ (*c*) e^{x^2} (*d*) $\frac{x}{e^2}$ **Ans.** (*b*)

3. The particular integral of $2\frac{d^2y}{dx^2}+\frac{dy}{dx}+3y=e^{2x}$ is

(a) $\frac{e^x}{13}$ (b) $\frac{e^{2x}}{13}$ (c) $\frac{2x}{e^{27}}$ (d) None of these **Ans.** (b)

4. The particular integral of $4\frac{d^2y}{dx^2}+\frac{3dy}{dx}+2y=e^{3x}$ is

(a) $\frac{e^{3x}}{7}$ (b) $\frac{e^{2x}}{47}$ (c) $\frac{e^{3x}}{27}$ (d) $\frac{e^{3x}}{47}$ **Ans.** (d)

5. The particular integral of $2\frac{d^2y}{dx^2}+3\frac{dy}{dx}+y=e^{-x}$ is

(a) $-x\,e^{-x}$ (b) $x\,e^{-x}$ (c) $-x\,e^{x}$ (d) xe^{x^2} **Ans.** (a)

6. The particular integral of $\frac{d^2y}{dx^2}-4y=e^{2x}$ is

(a) $x\,e^{2x}$ (b) $2\,x\,e^{2x}$ (c) $\frac{xe^{2x}}{2}$ (d) $\frac{xe^{2x}}{4}$ **Ans.** (d)

7. Particular integral of $\frac{d^2y}{dx^2}+y=x^2$ is

(a) x^2 (b) x^2+2 (c) x^2-2 (d) None of these **Ans.** (c)

8. Particular integral of $\frac{d^2y}{dx^2}+\frac{dy}{dx}+y=x^2$ is

(a) x^2-2x (b) x^2+2x (c) x^2+2 (d) x^2-2 **Ans.** (a)

9. Particular integral of $2\frac{d^2y}{dx^2}-y=x^3$ is

(a) x^3+12x (b) $-(x^3+12x)$ (c) x^3+12 (d) $-x^3-12$ **Ans.** (b)

10. Particular integral of $\frac{d^2y}{dx^2}+y=1$ is

(a) 0 (b) x (c) 1 (d) x^2 **Ans.** (c)

11. The particular integral of $(D^2+4)\,y=\cos 2x$ is

(a) $\frac{x}{4}\cos 2x$ (b) $\frac{x}{4}\sin 2x$ (c) $x\cos 2x$ (d) $\frac{1}{4}\sin 2x$ **Ans.** (b)

12. The particular integral of $\frac{d^2y}{dx^2}+y=\sin x$ is

(a) $-\frac{x}{2}\cos x$ (b) $\frac{x}{2}\cos x$ (c) $-\frac{x}{2}\sin x$ (d) None of these **Ans.** (a)

13. The particular integral of $\frac{d^2y}{dx^2}-y=\cos x$ is

(a) $-\frac{1}{2}\cos x$ (b) $\frac{1}{2}\cos x$ (c) $-\frac{1}{2}\sin x$ (d) None of these **Ans.** (a)

14. The particular integral of $\frac{d^2y}{dx^2}+\frac{dy}{dx}+y=\sin 2x$ is

(a) $3\sin 2x+2\cos 2x$ (b) $\frac{1}{13}(3\sin 2x+2\cos 2x)$

(c) $13\,(3\sin 2x+2\cos 2x)$ (d) $-\frac{1}{13}(3\sin 2x+2\cos 2x)$ **Ans.** (d)

15. The particular integral of $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = e^x \sin x$ is

(a) $-\frac{e^x}{13}(3\cos x - 2\sin x)$

(b) $\frac{e^x}{13}(3\cos x - 2\sin x)$

(c) $e^x(3\cos x - 2\sin x)$

(d) None of these

Ans. (a)

16. The particular integral of $\frac{d^2y}{dx^2} + y = e^{2x}\cos x$ is

(a) $\frac{e^x}{4}(\sin x + \cos x)$

(b) $e^{2x}(\sin x + \cos x)$

(c) $\frac{e^{2x}}{8}(\sin x + \cos x)$

(d) None of these

Ans. (c)

17. The particular integral of $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = \sin^2 x$ is

(a) $\frac{1}{2} - \frac{2}{13}(2\sin 2x - 2\cos 2x)$

(b) $\frac{1}{2} + \frac{2}{13}(2\sin 2x - 3\cos 2x)$

(c) $\frac{2}{13}(2\sin 2x - 3\cos 2x)$

(d) $\frac{1}{2} + \frac{1}{26}(-2\sin 2x + 3\cos 2x)$

Ans. (d)

18. Particular integral of $(D^2 + a^2)\, y = \cos ax$ is :

(a) $\frac{-x}{2a}\cos a\,x$

(b) $\frac{1}{2a}\cos a\,x$

(c) $\frac{1}{2a}\sin a\,x$

(d) $\frac{x}{2a}\sin a\,x$

Ans. (d)

19. e^{iax} is equal to

(a) $\sin a\,x + i\cos ax$

(b) $\cos ax - i\ \sin ax$

(c) $\sin ax - i\cos ax$

(d) $\cos ax + i\ \sin ax$

Ans. (d)

(R.G.P.V. Bhopal, 1Semester June,2007)

20. $\frac{1}{f(D)}x^n$ sin ax is equal to

(a) $e^{iax}\frac{1}{f(D+ia)}x^n$

(b) Imaginary part of $e^{iax}\frac{1}{f(D+ia)}x^n$

(c) Real part of $e^{iax}\frac{1}{f(D+ia)}x^n$

(d) None of these

Ans. (b)

21. $\frac{1}{f(D)}x^n$ cos ax is equal to

(a) Real part of $e^{iax}\frac{1}{f(D+ia)}x^n$

(b) Imaginary part of $e^{iax}\frac{1}{f(D+ia)}x^n$

(c) $e^{iax}\frac{1}{f(D+ia)}x^n$

(d) None of thee

Ans. (a)

22. $\frac{1}{D-a}\phi(x)$ is equal to

(a) $e^{ax}\int \phi(x)\,e^x\,dx$

(b) $\int e^{-ax}\phi(x)\,dx$

(c) $e^{ax}\int e^{-ax}\phi(x)\,dx$

(d) $e^{ax}\int \phi(x)\,dx$

Ans. (c)

23. $\frac{1}{D-3i}\sec 3x$ is equal to

(a) $e^{3ix}\int e^{-3ix}\sec 3x\,dx$ (b) $\int e^{-3ix}\sec 3x\,dx$

(c) $e^{3ix}\int e^{-3ix}dx$ (d) $e^{-3ix}\int e^{3ix}\sec 3x\,dx$ **Ans.** (a)

24. The complementary function for the solution of the differential equation $2x^2y'' + 3xy' - 3y = x^3$ is obtaned as

(a) $Ax + Bx^{-3/2}$ (b) $Ax + Bx^{3/2}$

(c) $Ax^2 + Bx$ (d) $Ax^{3/2} + Bx^{3/2}$ **Ans.** (a)

25. Solution of $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$ is

(a) $y = C_1e^x + C_2e^{2x}$ (b) $y = C_1 + (C_2 + C_3x)e^x$

(c) $y = (C_1 + C_2x + C_3x^2)e^{-x}$ (d) $y = C_1 + C_2e^x$ **Ans.** (b)

(A.M.I.E.T.E. June 2009)

26. The basis of solutions for the differential equation $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = 0$ are

(a) $x, x\,I_n x,$ (b) $I_n x, e^x$

(c) $\frac{1}{x}, \frac{1}{x^2}$ (d) $\frac{1}{x^2}e^x, xI_n x$ *(R.G.P.V., Bhopal, June, 2008)*

Ans. (a)

27. The complementary function for the solution of the differential equation $2x^2y'' + 3xy' - 3y = x^3$ is obtained as

(a) $Ax + Bx^{-3/2}$ (b) $Ax + Bx^{3/2}$

(c) $Ax^2 + Bx$ (d) $Ax^{-3/2} + Bx^{3/2}$ **Ans.** (a)

28. The substitutions in solving the homogeneous linear equations are

(a) $x = e^z,\ x\frac{dy}{dx} = Dy, x^2\frac{d^2y}{dx^2} = D(D-1)y$, where $D = \frac{d}{dz}$

(b) $z = e^x,\ x\frac{dy}{dx} = Dy, x^2\frac{d^2y}{dx^2} = D(D-1)y$, where $D = \frac{d}{dz}$

(c) $x = e^{-z},\ \frac{dy}{dx} = Dy, x^2\frac{d^2y}{dx^2} = D(D-1)y$, where $D = \frac{d}{dz}$

(d) None of these **Ans.**(a)

29. The complementary function for the solution of the differential equation.

$x^2y'' + xy' + y = \log x \sin(\log x)$

(a) $C_1 + C_2 \sin(\log x)$ (b) $C_1\cos(\log x) + C_2\sin(\log x)$

(c) $C_1\cos(\log x) + C_2$ (d) $(C_1 + C_2x)\sin(\log x)$ **Ans.**(b)

30. The integrating factor of $\cos^2 x\frac{dy}{dx} + y = \tan x$ is

(a) $e^{\sec x}$ (b) $e^{-\tan x}$ (c) $e^{\sin x}$ (d) $e^{\tan x}$ **Ans.**(d)

31. The integrating factor of $x(x-1)\frac{dy}{dx} - (x-2)y = x^2(2x-1)$ is

(a) $\frac{x^3-1}{x}$ (b) $\frac{x^3+1}{x}$ (c) $\frac{x-1}{x^2}$ (d) $\frac{x+1}{x^3}$ **Ans.**(c)

32. Solution $(x - y - 1)\frac{dy}{dx} = 1$ is

(a) $x - y - z = c\,e^{y}$ (b) $x + y = c\,e^{y}$ (c) $x - y - 2 = e^{y}$ (d) $x - y - 2 = c\,e^{y}$ **Ans.**(a)

33. *P.I.* of $(D^2 + 4)\,y = \sin 3x$ is

(a) $-\frac{1}{10}\sin 3x$ (b) $\frac{1}{2}\sin 3x$ (c) $\frac{1}{3}\sin 2x$ (d) $-\frac{1}{5}\sin 3x$ **Ans.**(d)

34. *P.I.* of $(D^2 - 2D + 1)\,y = e^{x}$ is

(a) $\frac{x^2}{2}e^{x}$ (b) $\frac{x}{2}e^{x}$ (c) $\frac{x}{4}e^{x^2}$ (d) $\frac{x^2}{5}e^{2x}$ **Ans.**(a)

35. On putting $x = e^{z}$, the transformed differential equation of $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = x$ is

(a) $\frac{d^2y}{dz^2} - y = e^{x}$ (b) $\frac{d^2y}{dz^2} + y = e^{z}$ (c) $\frac{dy}{dz} + y = e^{z}$ (d) $\frac{dy}{dz} - y = e^{z^2}$ **Ans.** (b)

36. The linear differential equation in standard form is

(a) $\frac{dy}{dx} + py = Q$ where P and Q are the functions of x only

(b) $\frac{dy}{dx} + py = Q$, where P and Q are the functions of y only

(c) $\frac{dy}{dx} + py = Q$, where P is function of x and Q is function of y

(d) $\frac{dy}{dx} + py = Q$, where P is funtion y and Q is a function of x. **Ans.** (a)

37. Let y_1, y_2 be two linearly independent solutions of the differential equations $yy'' - (y)^2 = 0$. Then $c_1y_1 + c_2y_2$, where c_1, c_2 are contants is a solution of this differential equation for

(a) $c_1 = c_2 = 0$ only (b) $c_1 = 0$ or $c_2 = 0$

(b) no value of c_1, c_2 (d) all real c_1, c_2 **Ans.** (d)

38. The solution of the differential equation $y'' + 2y' + y = 0$, $y(0) = 1$, $y'(0) = -1$ is

(a) xe^{-x} (b) $-xe^{-x}$ (c) $-e^{-x}$ (d) e^{-x} **Ans.** (d)

39. The solution of the differential equation $y'' + a^2y = 0$; $y(0) = 0$, $y'(0) = a$ is

(a) $\cos x$ (b) $\sin x$ (c) $\cos ax$ (d) $\sin ax$ **Ans.** (d)

40. Which of the following is not a solution of the equation $\frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$

(a) e^{2x} (b) e^{-3x} (c) $3e^{2x} + 5e^{-3x}$ (d) $c_1e^{2x} + c_2e^{-3x} + 1$ **Ans.** (d)

41. The complementary function of $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = 0$ is

(a) $y = (c_1 + c_2x)e^{3x}$ (b) $y = (c_1 + c_2)e^{3x}$ (c) $y = (c_1 + c_2x)e^{-3x}$ (d) $y = (c_1 + c_2)e^{-3x}$ **Ans.** (a)

42. The equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = 0$ has the solution

(a) $c_1e^{x} + c_2e^{2x}$ (b) $c_1e^{-x} + c_2e^{2x}$ (c) $c_1e^{x}\cos(2x + c_2)$ (d) $c_1e^{-x}\sin(2x + c_2)$ **Ans.** (c)

43. Complementary function of $(D^4 + 2D^3 - 3D^2)y = x^2$ is:

(a) $c_1x + c_2x^2 + c_3e^x + c_4e^{-3x}$ (b) $(c_1 + c_2x) + c_3e^x + c_4e^{-x}$

(c) $(c_1 + c_2x) + c_3e^x + c_4e^{-3x}$ (d) $(c_1 + c_2x)e^x + c_3e^{2x} + c_4e^{-3x}$ **Ans. (c)**

(R.G.P.V., Bhopal, I Semester June 2007)

44. Complementary function of $(D^2 - 4)y = 0$ is:

(a) $c_1e^{2x} + c_2e^{2x}$ (b) $(c_1 + c_2x)e^{2x}$

(c) $c_1e^{2x} + c_2e^{-2x}$ (d) $c_1 \sin 2x + c_2 \cos 2x$ **Ans. (c)**

(R.G.P.V., Bhopal, I Semester June 2006)

45. The complementary function for the solution of the differential equation $2x^2y'' + 3xy' - 3y - x^3$ is obtained as

(a) $Ax + Bx^{-3/2}$ (b) $Ax + Bx^{3/2}$

(c) $Ax^2 + Bx$ (d) $Ax^{-3/2} + bx^{3/2}$ **Ans. (a)**

46. The solution of the differential equation $\frac{d^2y}{dx^2} + y = 0$ satisfying the initial conditions $y(0) = 1, y(\pi/2) = 2$ is

(a) $y = 2 \cos x + \sin x$ (b) $y = \cos x + 2\sin x$

(c) $y = \cos x + \sin x$ (d) $y = 2 \cos x + 2\sin x$ **Ans. (b)**

47. The value of α so that $e^{\alpha y^2}$ is an integrating factor of the differential equation $\left(e^{\frac{y^2}{2}} - xy\right)dy - dx = 0$ is

(a) -1 (b) 1 (c) $\frac{1}{2}$ (d) $-\frac{1}{2}$ **Ans. (d)**

48. The solution of $(D^2 + 2D + 2)y = 0, y(0) = 0, y'(0) = 1$ is

(a) $e^x \sin x$ (b) $e^{-x} \cos x$ (c) $e^{-x} \sin x$ (d) $e^x \cos x$ **Ans. (c)**

49. The solution of the differential equation $(D^4 - 6D^3 + 12D^2 - 8D)y = 0$ is given by

(a) $y = a + (b + cx + dx^2)e^{2x}$ (b) $y = (b + cx + dx^2)e^{2x}$

(c) $y = a + bx + cx^2 + dx^3$ (d) $y = a + bx + cx^2 + de^{2x}$ **Ans. (a)**

50. The particular integral of the differential equation $\frac{d^2y}{dx^2} + a^2y = \sin ax$ is

(a) $-\frac{x}{2a}\cos ax$ (b) $\frac{x}{2a}\cos ax$ (c) $-\frac{ax}{2}\cos ax$ (d) $\frac{ax}{2}\cos ax$ **Ans. (a)**

51. The solution of the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{3x}$ is

(a) $y = ae^x + be^{2x} + \frac{1}{2}e^{3x}$ (b) $y = ae^{-x} + be^{-2x} + \frac{1}{2}e^{3x}$

(c) $y = ae^x + be^{-2x} + \frac{1}{2}e^{3x}$ (d) $y = ae^{-x} + be^{2x} + \frac{1}{2}e^{3x}$ **Ans. (a)**

52. The roots of the auxiliary equation of the differential equation $\frac{d^2y}{dt^2} - 6\frac{dy}{dt} + 9y = 4e^{3t}$ are

(a) 3, – 3 (b) 3, 3 (c) –3, –3 (d) None of these **Ans. (b)**

(GBTU 2011)

Fill in the blanks in each of the following

53. Particular integral of $(D^2 - 4D + 4)\, y = \sin 2x$ is .... *(GBTU 2011)* **Ans.** $\frac{1}{8}\cos 2x$

54. The solution of $\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 4y = 0$ is ..... **Ans.** $y = c_1 e^{x} + (c_2 + c_3 x)e^{2x}$

55. The particular integral of $(D^2 + 9)\, y = \sin 3x$ is ..... **Ans.** $P.I. = -\frac{x}{6}\cos 3x$

56. The particular integral of $(D^2 + 6D + 5)\, y = 4e^x$ is ..... **Ans.** $P.I. = \frac{1}{3}e^x$

57. The solution of $x^2\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$ is **Ans.** $c_1 + c_2x^2$

58. The particular intagral of $\frac{d^2y}{dx^2} + y = 2\cos h\, 3x$ is ..... **Ans.** $-\frac{1}{4}\cosh 3x$

59. The differential equation whose auxiliary equation has the roots 0, – 1, – 1 is .....
Ans. $(D^3 + 2D^2 + D)\, y = 0$

60. The complementary function of the differential equation $x^2y'' + xy' + y = \log x^2$ is .....
Ans. $(c_1 \cos \log x + c_2 \sin \log x)$

61. To transform $x\frac{d^2y}{dx^2} + \frac{dy}{dx} = \frac{2y}{x}$ into the linear differential equation with constant coefficients, we put $x =$. **Ans.** e^z

62. The no. of arbitrary constants in the general solution of a differential equation is equal to ..... of the differential equation. **Ans.** Order

Match the following

63.

(*i*) $\frac{1}{f(D)}e^{ax}$	(*a*) $\frac{1}{f(-a^2)}\cos ax$
(*ii*) $\frac{1}{f(D^2)}\cos ax$	(*b*) $[f(D)]^{-1}x^m$
(*iii*) $\frac{1}{f(D)}x^m$	(*c*) $e^{ax}\frac{1}{f(D+a)}\phi(x)$
(*iv*) $\frac{1}{f(D)}e^{ax}\phi(x)$	(*d*) $\frac{1}{f(a)}e^{ax}$

Ans. (*i*) → (*d*)
(*ii*) → (*a*)
(*iii*) → (*b*)
(*iv*) → (*c*)

Indicate True or False for the following :

64. The *P.I.* of $\frac{d^2y}{dx^2} + y = \sin 3x$ is $-\frac{1}{8}\sin 3x$. **Ans.** True

65. The *P.I.* of $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = e^{3x}$ is $\frac{1}{21}e^{3x}$ **Ans.** False

66. The *P.I.* of $\frac{1}{f(D)}e^{2x}\phi(x) = e^{2x}\frac{1}{f(D+2)}\phi(x)$. **Ans.** True

67. The *P.I.* of $\frac{d^2y}{dx^2} + y = x^2$ is x^2. **Ans.** False

OBJECTIVE TYPE QUESTIONS

Choose the correct answer:

1. If $\frac{dx}{dt} + 4y = 0$ and $\frac{dy}{dt} - 4x = 0$ then $x =$

   (*i*) $-A \sin 4t + B \cos 4t$ (*ii*) $A \sin 4t - B \cos 4t$

   (*iii*) $A \sin 4t + B \cos 4t$ (*iv*) None of these **Ans. (*i*)**

2. If $\frac{dx}{dt} + 5y = 0$ and $\frac{dy}{dt} - 5x = 0$ then $y =$

   (*i*) $A \cos 5t - B \sin 5t$ (*ii*) $A \cos 5t + B \sin 5t$

   (*iii*) $-A \cos 5t + B \sin 5t$ (*iv*) None of these **Ans. (*ii*)**

3. If $\frac{dx}{dt} - y = t$ and $\frac{dy}{dt} = t^2 - x$ then $x =$

   (*i*) $c_1 \cot t + c_2 \sin t + t^2 - 1$ (*ii*) $c_1 \cos t + c_2 \sin t - t^2 + 1$

   (*iii*) $c_1 \cos t + c_2 \sin t + t^2 - 1$ (*iv*) $c_1 \cos t - c_2 \sin t - t^2 + 1$ **Ans. (*iii*)**

4. If $\frac{dx}{dt} = 3x + 2y$, $\frac{dy}{dt} = 5x + 3y$, then $x =$

   (*i*) $e^{3t} (c_1 \cos h \sqrt{10}\, t + c_2 \sin h \sqrt{10}\, t)$ (*ii*) $(c_1 \cos h \sqrt{10}\, t + c_2 \sin h \sqrt{10}\, t)$

   (*iii*) $e^{-3t} (c_1 \cos h \sqrt{10}\, t + c_2 \sin h \sqrt{10}\, t)$ (*iv*) $e^{3t} (c_1 \cos h \sqrt{10}\, t - c_2 \sin h \sqrt{10}\, t)$ **Ans. (*i*)**