17. Evaluate $\iint \frac{dydx}{\sqrt{1-2x^2-y^2}}$ over the first quadrant in the ellipse $2x^2 + y^2 = 1$.

(*M.U. II Semester 2003*) **Ans.** $\frac{\pi}{2\sqrt{2}}$.

OBJECTIVE TYPE QUESTIONS

Choose the correct alternative:

1. The value of the integral $\iint xy\,(x+y)\,dx\,dy$ over the area between $y = x^2$ and $y = x$ is

(*i*) $\frac{3}{56}$ (*ii*) $\frac{47}{56}$ (*iii*) $\frac{33}{56}$ (*iv*) $\frac{23}{56}$ **Ans.** (*i*)

2. The integral $\iint\limits_{x^2+y^2 \le 1} \frac{1}{\pi}(x^2+y^2)\,dx\,dy$ equals

(*i*) 0 (*ii*) 1 (*iii*) 1/3 (*iv*) 1/2 **Ans.** (*iv*)

3. $\int \frac{\sqrt{\tan x}}{\sin x \cos x}\,dx$ is equal to

(*i*) $2\sqrt{\tan x} + C$ (*ii*) $2\sqrt{\cot x} + C$ (*iii*) $\frac{\sqrt{\tan x}}{2} + C$ (*iv*) None of these

Ans. (*i*)

4. Value of the integral $\int_{-a}^{a} \int_0^{\sqrt{a^2-x^2}} dx\,dy$ is equal to

(*i*) $4a$ (*ii*) $2a$ (*iii*) 0 (*iv*) None of these

Ans. (*iv*)

5. The value of $\int_1^0 \int_0^1 (x+y)\,dx\,dy$ is equal to

(*i*) 1 (*ii*) −1 (*iii*) 2 (*iv*) 0 **Ans.** (*i*)

6. The value of $\int_0^1 \int_0^x e^x \, dx \, dy$ is

(*i*) -1 (*ii*) 0 (*iii*) 1 (*iv*) 2 Ans. (*iii*)

7. The value of $\int_{-a}^{a} \left[\int_0^x dy \right] dx$ is

(*i*) 1 (*ii*) 2 (*iii*) 3 (*iv*) 0 Ans. (*iv*)

8. The value of $\int_0^1 dx \int_0^x e^{\frac{y}{x}} \, dy$ is

(*i*) $\frac{1}{2}(e-1)$ (*ii*) $(e+1)$ (*iii*) $(e-1$ (*iv*) $\frac{1}{2}(e+1)$

Ans. (*i*)

9. The value of $\int_0^{\pi} \int_0^{a(1-\cos\theta)} r^3 \sin\theta \, dr \, d\theta$ is

(*i*) $\frac{15}{16}a^4$ (*ii*) $\frac{8a^4}{5}$ (*iii*) a^4 (*iv*) $\frac{16}{15}$ Ans. (*ii*)

10. The value of $\int_0^{\pi} \left[\int_{2\sin\theta}^{4\sin\theta} r^3 \, dr \right] d\theta$ is

(*i*) 22.5 π (*ii*) 22 π (*iii*) 10.5 π (*iv*) π Ans. (*i*)

11. The value of integral $\int_0^2 \int_0^x (x+y) \, dx \, dy$ is equal to

(*i*) -4 (*ii*) 3 (*iii*) 4 (*iv*) -3 Ans. (*iii*)

12. $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} dx \, dy$ is equal to

(*i*) $\int_0^{\pi} \int_0^{2a\cos\theta} r \, dr \, d\theta$ (*ii*) $\int_0^{\frac{\pi}{2}} \int_0^{2a\cos\theta} r \, dr \, d\theta$

(*iii*) $\int_0^{\frac{\pi}{2}} \int_0^{2a\sin\theta} r \, dr \, d\theta$ (*iv*) None of these Ans. (*iii*)

13. The value of $\int_0^{\pi} \int_0^{a(1+\cos\theta)} r^2 \sin\theta \, . \, d\theta \, dr$ is

(*i*) a^3 (*ii*) $\frac{4}{3}\pi^3$ (*iii*) $\frac{4}{3}a^3$ (*iv*) $\frac{1}{3}a^3$ Ans. (*iii*)

14. The value of integral $\int_0^1 \int_{x^2}^{2-x} xy \, dx \, dy$ is equal to

(*i*) $\frac{3}{4}$ (*ii*) $\frac{3}{8}$ (*iii*) $\frac{3}{5}$ (*iv*) $\frac{3}{7}$ Ans. (*ii*)

15. The value of the integral $\int_0^{a/2} \int_0^{\sqrt{a^2-x^2}} dy \, dx$ is equal to

(*i*) πa^2 (*ii*) $\frac{\pi a^2}{8}$ (*iii*) $\frac{\pi a^2}{4}$ (*iv*) None of these

Ans. (*iv*)

Fill in the blanks:

16. $\int_1^0 \int_0^1 (x + y)\, dx\, dy$ = ...

17. $\int_0^1 \int_0^x e^x\, dx\, dy$ = ...

18. $\int_{-a}^{a} \left[\int_0^x dy \right] dx$ = ...

19. $\int_0^1 \int_{e^x}^{e} \frac{dy\, dx}{\log y}$ = ...

20. $\int_0^a \int_y^a \frac{x\, dx\, dy}{x^2 + y^2}$ = ...

21. $\int_0^1 \int_{2y}^{2} e^{2x}\, dx\, dy$ = ...

22. $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2\, dy\, dx$ = ...

Match the following:

23. (*a*) $\int_0^1 \int_0^y xye^{-x^2}\, dx\, dy$ (*p*) $\frac{1}{2}$

(*b*) $\int_0^1 \int_0^{x^2} e^{\frac{y}{x}}\, dy\, dx$ (*q*) $\left[\frac{\pi a}{4} - a \tan^{-1} \frac{1}{a}\right]$

(*c*) $\int_1^{\log 8} \int_0^{\log y} e^{x+y}\, dx\, dy$ (*r*) $\frac{1}{4e}$

(*d*) $\int_0^a \int_{\frac{x}{a}}^{x} \frac{x\, dy\, dx}{x^2 + y^2}$ (*s*) $8 \log 8 - 16 + e$

Ans. (*a*) → (*r*), (*b*) → (*p*), (*c*) → (*s*), (*d*) → (*q*)

24. (*a*) $\int_0^{\frac{\pi}{2}} \left[\int_0^{a \cos\theta} r\sqrt{a^2 - r^2}\, dr \right] d\theta$ (*p*) $\frac{4}{3}a^3$

(*b*) $\int_0^{\pi} \int_0^{a(1-\cos\theta)} r^2 \sin\theta\, dr\, d\theta$ (*q*) $\frac{21}{4} + e^4 - e^3$

(*c*) $\int_1^x \int_3^2 (xy + e^y)\, dy\, dx$ (*r*) $\frac{\pi a^3}{6}$

(*d*) $\int_0^a \int_0^{\sqrt{a^2-y^2}} \sqrt{a^2 - x^2 - y^2}\, dx\, dy$ (*s*) $\frac{a^3}{18}(3\pi - 4)$

Ans. (*a*) → (*s*), (*b*) → (*p*), (*c*) → (*q*), (*d*) → (*r*)

Ans.

Ans. 1

Ans. 0

Ans. $e - 1$

Ans. $\frac{\pi a}{4}$

Ans. $\frac{e^4 - 1}{4}$

Ans. $\frac{\pi}{16}$

2. $\int_0^{\frac{\pi}{2}} \int_0^{\sqrt{\sin\theta}} r\, d\theta\, dr$ is equal to

(*i*) $\frac{1}{2}$ (*ii*) $-\frac{1}{2}$ (*iii*) 1 (*iv*) -1 **Ans.** (*i*)

3. $\int_{-a}^{a} \left[\int_0^x dy \right] dx$ is equal to

(*i*) $-a$ (*ii*) a (*iii*) 0 (*iv*) $\frac{a}{2}$ **Ans.** (*iii*)

4. $\int_0^{2\pi} d\theta \int_0^1 e^{2r}\, dr$ is equal to

(*i*) $(e^2 - 1)$ (*ii*) $\frac{\pi}{2}(e^2 - 1)$ (*iii*) $\pi(e^2 - 1)$ (*iv*) $2\pi(e^2 - 1)$ **Ans.** (*iii*)

5. The transformations $x + y = u$, $y = uv$ transform the area element $dy\, dx$ into $|J|\, du\, dv$, where $|J|$ is equal to

(*i*) 1 (*ii*) u (*iii*) -1 (*iv*) none of these **Ans.** (*ii*)

6. $\int_1^{\log 8} \int_0^{\log y} e^{x+y}\, dx\, dy =$

(*i*) $8 \log 8 + 16 + e$ (*ii*) $8 \log 8 - 16 - e$
(*iii*) $8 \log 8 - 16 + e$ (*iv*) $\log 8 - 16 + e$ **Ans.** (*iii*)

7. $\iint_D (x^2 + y^2)\, dx\, dy = ?$, where D is bounded by $y = x$ and $y^2 = 4x$.

(*i*) $\frac{768}{25}$ (*ii*) $\frac{768}{35}$ (*iii*) $\frac{708}{35}$ (*iv*) $\frac{68}{35}$ **Ans.** (*ii*)

8. $\iint_D x^3 y\, dxdy$, where D is the region enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in the first quadrant.

(*i*) $\frac{b^2a^4}{24}$ (*ii*) $\frac{b^3a^4}{24}$ (*iii*) $\frac{ba^4}{24}$ (*iv*) $\frac{b^2a^2}{24}$ **Ans.** (*i*)

9. $\int_0^3 \int_x^{4x-x^2} y\, dx\, dy =$

(*i*) $\frac{54}{7}$ (*ii*) $\frac{54}{17}$ (*iii*) $\frac{34}{5}$ (*iv*) 54 **Ans.** (*iv*)

10. $\int_0^1 \int_y^{10y} \sqrt{xy - y^2}\, dxdy = \ldots\ldots$

(*i*) 6 (*ii*) 4 (*iii*) 5 (*iv*) 16 **Ans.** (*i*)

11. $\int_0^1 \int_{e^x}^{e} \frac{dxdy}{\log y} =$

(*i*) $e + 1$ (*ii*) $e - 1$ (*iii*) e (*iv*) e^{-1} Ans. (*ii*)

12. $\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y}\, dy\, dx$

(*i*) 1 (*ii*) 2 (*iii*) 3 (*iv*) 4 Ans. (*i*)

13. $\iint f(x, y)\, dx\, dy = J \iint f(r, \theta)\, dr\, d\theta$, where $J =$

(*i*) r^2 (*ii*) $\frac{\partial (x, y)}{\partial (r, \theta)}$ (*iii*) $\frac{\partial (r, \theta)}{\partial (x, y)}$ (*iv*) r, θ Ans. (*ii*)

On Changing the order of integration:

14. $\int_0^1 \int_0^x f(x, y) dy\, dx = ...$ Ans. $\int_0^1 \int_y^1 f(x, y)\, dx\, dy$

15. $\int_0^a \int_0^{\sqrt{a^2 - x^2}} f(x, y)\, dx\, dy = ...$ Ans. $\int_0^a \int_0^{\sqrt{a^2 - y^2}} f(x, y)\, dx\, dy$

16. $\int_0^\infty \int_0^x \frac{e^{-y}}{y} dx\, dy = ...$ Ans. $\int_0^\infty \frac{e^{-y}}{y} dy \int_y^\infty dx$

17. $\int_0^2 \int_1^{e^x} dy\, dx = ...$ Ans. $\int_1^{e^2} \int_{x = \log y}^2 dx\, dy$,

18. The value of $\int_0^e d\theta \int_0^\theta e^{\frac{r}{\theta}} dr$ is ..... Ans. $\frac{e^2}{2}(e - 1)$

Indicate Ture/False for the following statements:

19. For $\int_0^\infty \int_x^\infty f(x\, y)\, dx\, dy$, the change of order of integration is

(*i*) $\int_0^\infty \int_0^\infty f(x\, y)\, dx\, dy$ True/False (*ii*) $\int_0^\infty \int_0^\infty f(x\, y)\, dx\, dy$ True/False

(*iii*) $\int_0^\infty \int_0^\infty f(x\, y)\, dx\, dy$, True/False (*iv*) $\int_0^\infty \int_0^x f(x\, y)\, dx\, dy$ True/False

(U.P.I. Sem., Dec. 2009) **Ans.** (*i*) False (*ii*) False (*iii*) True (*iv*) False

20. Match the following:

(*i*) $\iint dx\, dy$ (*p*) $3\left(\frac{\pi^2}{8} - 1\right)$

(*ii*) $\int_0^{\frac{\pi}{2}} \int_0^x yx \sin x\, dx\, dy$ (*q*) $\frac{\lceil n}{2}$

(*iii*) $\int_0^{\frac{\pi}{4}} \int_0^1 r \cos^2 \theta\, dr\, d\theta$ (*r*) $\iint r\, d\theta dr$

(*iv*) $\int_0^1 \int_0^\infty x^{n-1} e^{-x} y\, dx\, dy$ (*s*) $\frac{1}{16}[2 + \pi]$

Ans. (*i*) → (*r*), (*ii*) → (*p*), (*iii*) → (*s*), (*iv*) → (*q*)

3. If a circle $x^2 + y^2 = a^2$ is rotated about x-axis, the volume generated is

(*i*) πa^3 (*ii*) $2\pi a^2$ (*iii*) $\frac{4}{3}\pi a^3$ (*iv*) $\frac{2}{3}\pi a^3$ Ans. (*iii*)

4. If a circle in positive quadrant is rotated about y–axis is

(*i*) $\frac{4}{3}\pi a^3$ (*ii*) $\frac{2}{3}\pi a^3$ (*iii*) $4\pi a^3$ (*iv*) πa^3 Ans. (*ii*)

5. If the area enclosed by $y = x$, $y = 0$ and $x = a$ is revolved about x–axis, the volume generated is

(*i*) πa^3 (*ii*) $2\pi a^3$ (*iii*) $\frac{2\pi a^3}{3}$ (*iv*) $\frac{\pi}{3}a^3$ Ans. (*iv*)

6. The volume of the solid generated by revolving the segment of $x + y = 2a$ between the axes about x-axis is

(*i*) $\frac{8}{3}\pi a^3$ (*ii*) $8\pi a^3$ (*iii*) $\frac{1}{3}\pi a^3$ (*vi*) $\frac{4}{3}\pi a^3$ Ans. (*i*)

7. An area, being surrounded by a closed curve, revolves about a line. The volume of the solid of revolution shall depend on

(*i*) the length of the curve

(*ii*) path described by centre of gravity of the area

(*iii*) length of the line

(*iv*) area enclosed

Ans. (*ii*)

8. The area bounded by the circle $r = 4$ is

(*i*) 16π (*ii*) 17π (*iii*) 18π (*vi*) 19π Ans. (*i*)

9. The area bounded by the cardioid $r = 2(1 + \cos\theta)$ is

(*i*) 16π (*ii*) 6π (*iii*) 5π (*vi*) π Ans. (*ii*)

10. The formula of area in polar co-ordinates is

(*i*) $\iint d\theta\, dr$ (*ii*) $\iint r^2 d\theta\, dr$ (*iii*) $\iint r d\theta\, dr$ (*vi*) $\iint \frac{1}{r} d\theta\, dr$

Ans. (*iii*)

11. If A is the area under the curve $y = \sin x$ above x-axis in the interval $[0, \pi/4]$, then the area included between $y = \cos x$, and x-axis in the interval $[0, \pi/4]$ is given by

(*i*) A (*ii*) $\pi/2 - A$ (*iii*) $1 - A$ (*iv*) None of these.

Ans. (*iii*)

12. If A is the area under the curve $y = \sin x$, above x-axis st. $0 \le x \le \pi/2$, then the area under the curve $y = \sin 2x$, $0 \le x \le \pi/2$, is

(*i*) A (*ii*) $2A$ (*iii*) $A/2$ (*iv*) $1 + A$ Ans. (*i*)

13. If A is the area under the curve $y = \cos x$, above x-axis, $0 \le x \le \pi/3$, then the area under the curve $y = \cos 2x$ in the same interval is

(*i*) A (*ii*) $2A$ (*iii*) $A/2$ (*iv*) $\left(\frac{\sqrt{3}}{2}\right)A$

Ans. (*iii*)

14. If A_1 and A_2 are the areas between the x-axis and the curves $y = \sin^n x$ and $y = \cos^n x$ in the interval $[0, \pi/2]$ respectively, then

(*i*) $A_2 = 1 - A_1$ (*ii*) $A_2 = A_1$ (*iii*) $A_2 = 2A_1$ (*iv*) None of these.

Ans. (*ii*)

15. The area bounded by the rectangular hyperbola $xy = c^2$, the axis of x, and the ordinates $x = c$ and $x = 2c$ is

(*i*) $c^2 \log 2$ (*ii*) $c \log 2$ (*iii*) $2c \log 2$ (*iv*) None of these.

Ans. (*i*)

16. The area bounded by the curve $x = 3 + \cos\theta$, $y = 4 \sin\theta$, is

(*i*) 7π (*ii*) 2π (*iii*) 4π (*iv*) None of these.

Ans. (*ii*)

17. The line which divides the area of curvilinear triangle bounded by $y = 2x - x^2$, $y = 0$, $x = 1$, into two equal areas, is

(*i*) $y = x$ (*ii*) $y = x/3$ (*iii*) $y = 2x/3$ (*iv*) $y = 2x/5$

Ans. (*iii*)

18. The area bounded by the two curves $y = x^2$, $y^2 = x$ is

(*i*) $\frac{1}{3}$ (*ii*) 2/3 (*iii*) 4/3 (*iv*) None of these.

Ans. (*i*)

19. The area common to the two ellipses $a^2x^2 + b^2y^2 = 1$, $b^2x^2 + a^2y^2 = 1$, where $0 < a < b$ is

(*i*) $\frac{4}{ab}\tan^{-1}\frac{a}{b}$ (*ii*) $\frac{1}{ab}\tan^{-1}\frac{a}{b}$ (*iii*) $\frac{4}{ab}\tan^{-1}\frac{b}{a}$ (*iv*) None of these.

Ans. (*i*)

20. The area enclosed by the curve $|x| + |y| = 2$ is

(*i*) 2 (*ii*) 4 (*iii*) 8 (*iv*) None of these.

Ans. (*iii*)

21. The area bounded by the line $y = x$, x-axis and the ordinates $x = -1$ and $x = 2$.

(*i*) 2 (*ii*) 5 (*iii*) 5/2 (*iv*) None of these.

Ans. (*iii*)

22. The area of a circle centred at (1, 2) and passing through (4, 6) is

(*i*) 5π (*ii*) 10π (*iii*) 25π (*iv*) None of these.

Ans. (*iii*)

23. The area between the parabola $y^2 = ax$ and its latus rectum is

(*i*) $\frac{a^2}{3}$ (*ii*) $\frac{a^2}{4}$ (*iii*) $\frac{4a^2}{3}$ (*iv*) $\frac{8a^2}{3}$ Ans. (*iii*)

24. The area bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ is

(*i*) 3π (*ii*) 4π (*iii*) 5π (*iv*) 6π Ans. (*iv*)

25. The area bounded by the circle $x^2 + y^2 = 16$ is

(*i*) 15π (*ii*) 16π (*iii*) 17π (*iv*) 18π Ans. (*ii*

EXERCISE 17.1

Evaluate the following :

1. $\int_{-1}^{1}\int_{-2}^{2}\int_{-3}^{3} dx\,dy\,dz$ *(M.U., II Semester 2002)* **Ans. 48**

2. $\int_{0}^{4}\int_{0}^{x}\int_{0}^{x+y} z\,dz\,dy\,dx$ *(R.G.P.V. Bhopal I Sem. 2003)* **Ans. 70**

3. $\int_{1}^{2}\int_{0}^{1}\int_{-1}^{1}(x^2+y^2+z^2)\,dx\,dy\,dz$ **Ans. 6**

Ans. $\frac{\pi}{2}a^3$.

OBJECTIVE TYPE QUESTIONS

1. The volume of the integral $\iiint_E xyz \; dx \; dy \; dz$, over the domain E bounded by planes $x = 0$, $y = 0$, $x + y + z = 1$ is

(*i*) $\frac{1}{20}$ (*ii*) $\frac{1}{40}$ (*iii*) $\frac{1}{720}$ (*iv*) $\frac{1}{800}$ Ans. (*iii*)

2. The triple integral $\iiint_T dx \; dy \; dz$ gives

(*i*) Volume of region T
(*ii*) Area of region T
(*ii*) Surface area of region T
(*iv*) Density of region T (*A.M.I.E.T.E. 2002*)

Ans. (*i*)

3. The volume of the solid under the surface $az = x^2 + y^2$ and whose base R is the circle $x^2 + y^2 = a^2$ is given as

(*i*) $\frac{\pi}{2a}$ (*ii*) $\frac{\pi a^3}{2}$ Ans. (*ii*)

(*iii*) $\frac{4}{3}\pi a^3$ (*iv*) None of the above. [*U.P., I. Sem. Dec. 2008*]

4. The Value of integral $\int_{-1}^{1} \int_{0}^{z} \int_{x-z}^{x+z} (x + y + z) \; dy \; dx \; dz$ is

(*i*) 2π (*ii*) 2 (*iii*) -2 (*iv*) 0 Ans. (*iv*)

5. The value of $\int_0^1 \int_0^1 \int_0^1 (x^2 + y^2 + z^2) \; dz \; dy \; dx$ is

(*i*) 1 (*ii*) 1/3 (*iii*) 2/3 (*iv*) 3 Ans. (*i*)

6. The volume of the sphere $r = 2$ is

(*i*) π (*ii*) 32π (*iii*) $\frac{\pi}{3}$ (*iv*) $\frac{32\pi}{3}$ Ans. (*iv*)

7. The volume of the cylinder $x^2 + y^2 = \frac{25}{4}$, $z = 4$ & $z = 0$ is

(*i*) 23 π (*ii*) 24 π (*iii*) 25 π (*iv*) 26 π Ans. (*iii*)

8. The volume of the cylinder $r = 16$, $z = 0$ and $z = 3$ is

(*i*) $\frac{768}{3}\pi$ (*ii*) 768 π (*iii*) 256 π (*iv*) 48 π Ans. (*ii*)

9. A triangle ABC is rotated about x-axis, where A (4, 3), B (0, 0) and C (8, 0). The volume of the solid generated is

(*i*) 6π (*ii*) 12π (*iii*) 24π (*iv*) none of these **Ans.** (*iii*)

10. The volume of the solid generated by revolving the area bounded by $x = 1$, $x = 2$, $y = 1$, $y = 2$ about x-axis is

(*i*) 3π (*ii*) 6π (*iii*) π (*iv*) none of these **Ans.** (*i*)

11. In spherical coordinates, $dx\,dy\,dz$ is equal to

(*i*) $r\,d\theta\,d\phi\,dr$ (*ii*) $r\sin\theta\,d\theta\,d\phi\,dr$ (*iii*) $r^2\sin\theta\,d\theta\,d\phi\,dr$ (*iv*) $r^2\,d\theta\,d\phi\,dr$ **Ans.** (*iii*)

12. The formula for calculating surface area is

(*i*) $S = \iint_A \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 \times \left(\frac{\partial z}{\partial y}\right)^2 + 1}\,dx\,dy$ (*ii*) $S = \iint_A \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 \times \left(\frac{\partial z}{\partial y}\right)^2}\,dx\,dy$

(*iii*) $S = \iint_A \left[\left(\frac{\partial z}{\partial x}\right)^2 \times \left(\frac{\partial z}{\partial y}\right)^2 + 1\right] dx\,dy$ (*iv*) $S = \iint_A \left[\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + \left(\frac{\partial z}{\partial z}\right)^2\right] dx\,dy$ **Ans.** (*i*)

13. The value of $\int_0^1 \int_0^x \int_0^{x+y} dx\,dy\,dz$ is

(*i*) 1 (*ii*) $\frac{1}{3}$ (*iii*) $\frac{1}{4}$ (*iv*) $\frac{1}{2}$ **Ans.** (*iv*)

14. The value of $\int_0^a dx \int_0^{\sqrt{a^2-x^2}} dy \int_0^{\sqrt{a^2-x^2-y^2}} dz$ is

(*i*) $4\pi a^2$ (*ii*) $\frac{\pi a^3}{6}$ (*iii*) $4\pi a^3$ (*iv*) $\frac{\pi}{3}a^2$ **Ans.** (*ii*)

15. The surface of the solid generated by revolving the area enclosed by curve $x^2 + y^2 = 16$ about $x = 4$ is

(*i*) 64π (*ii*) $32\pi^2$ (*iii*) 32π (*iv*) $64\pi^2$ **Ans.** (*iv*)

16. The value of intergral $\int_0^2 \int_1^3 \int_1^2 xy^2 z\,dx\,dy\,dz$ is equal to

(*i*) 22 (*ii*) 26 (*iii*) 5 (*iv*) 25 **Ans.** (*ii*)

Fill up the blanks:

17. $\int_0^{\pi} \int_0^{2\pi} \int_0^1 r^2 (r^2 \sin\theta\,d\theta\,d\phi\,dr) = \ldots\ldots$ **Ans.** $\frac{4\pi}{5}$

18. In spherical coordinates $dx\,dy\,dz = \ldots\ldots$ **Ans.** $r^2\sin\theta\,dr\,d\theta\,d\phi$

19. $\int_{-1}^1 \int_{-2}^2 \int_{-3}^3 dx\,dy\,dz = \ldots$ **Ans.** 48

20. The formula for the volume in spherical coordinates is ..... **Ans.** $\iiint_V r^2 \sin\theta\,dr\,d\theta\,d\phi$

21. $\int_0^a \int_0^{\frac{bx}{a}} \int_0^{c+xy} dz\,dy\,dx = \ldots\ldots$ **Ans.** $\frac{ab}{8}(4c + ab)$