OBJECTIVE TYPE QUESTIONS

Choose the correct alternative:

1. The value of $\lim\limits_{(x,y)\to(0,0)} (x+y)\sin\dfrac{1}{(x+y)}, x \neq 0, y \neq 0$ is

   (*i*) limit does not exist (*ii*) 0 (*iii*) 1 (*iv*) –1 **Ans.** (*i*)

2. The value of the $\lim\limits_{(x,y)\to(0,0)} \dfrac{x+\sqrt{y}}{\sqrt{(x^2+y)}}, x \neq 0, y \neq 0$ is

   (*i*) limit does not exist (*ii*) 0 (*iii*) 1 (*iv*) –1

   (*AMIETE, Dec.* 2007) **Ans.** (*iii*)

3. The value of $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 y}{x^4+y^2}$ is

   (*i*) 0 (*ii*) $\dfrac{1}{2}$ (*iii*) 1 (*iv*) Does not exist **Ans.** (*iv*)

4. The value of $\lim\limits_{(x,y)\to(0,0)} \dfrac{x.\sin(x^2+y^2)}{x^2+y^2}$ is

   (*i*) 0 (*ii*) 1 (*iii*) –1 (*iv*) Does not exist **Ans.** (*i*)

5. The value of limit $\lim\limits_{\substack{x\to 1\\ y\to 1}} \dfrac{8x^2 y}{x^2+y^2+5}$ is

   (*i*) $\dfrac{3}{7}$ (*ii*) $\dfrac{8}{5}$ (*iii*) $\dfrac{8}{7}$ (*iv*) None of these **Ans.** (*iii*)

6. The value of $\lim\limits_{\substack{x\to 1\\ y\to 2}} \dfrac{4xy}{6x^2+y^2}$ is

   (*i*) $\dfrac{4}{5}$ (*ii*) $\dfrac{2}{3}$ (*iii*) $\dfrac{3}{10}$ (*iv*) None of these **Ans.** (*i*)

7. The value of $\lim\limits_{\substack{y\to 0\\ x\to 1}} \dfrac{2x^2+y}{4x-y}$ is

   (*i*) $\dfrac{3}{2}$ (*ii*) $\dfrac{1}{2}$ (*iii*) 1 (*iv*) None of these **Ans.** (*ii*)

8. The value of $\lim\limits_{\substack{x\to 0\\ y\to 0}} \dfrac{2x^2+y}{4x^2-y}$ is

   (*i*) –1 (*ii*) $\dfrac{1}{2}$ (*iii*) 1 (*iv*) Limit does not exist **Ans.** (*iv*)

9. The value of $\lim\limits_{\substack{y\to 0\\ x\to 0}} \dfrac{2x^2+y}{4x^2-y}$ is

   (*i*) $\dfrac{3}{4}$ (*ii*) $\dfrac{2}{1}$ (*iii*) $\dfrac{1}{2}$ (*iv*) None of these **Ans.** (*iv*)

10. If $u = x^2 + y^2$ then the value of $\dfrac{\partial^2 u}{\partial x \partial y}$ is equal to

    (*i*) 0 (*ii*) 2 (*iii*) $2x + 2y$ (*iv*) $y\,x^{y-1}$

    (*A.M.I.E.T.E. Dec. 2008*) **Ans.** (*i*)

11. If $u = y^x$, then $\dfrac{\partial u}{\partial x}$ is

    (*i*) xy^{x-1} (*ii*) 0 (*iii*) $y^x \log y$ (*iv*) none of these **Ans.** (*iii*)

12. If $u = \log\left(\frac{x^2}{y}\right)$, then the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is

(*i*) $2u$ (*ii*) u (*iii*) 0 (*iv*) 1

(*A.M.I.E.T.E. Dec. 2008*) **Ans.** (*iv*)

13. If $x = r\cos\theta$, $y = r\sin\theta$, then

(*i*) $\frac{\partial x}{\partial r} = \frac{\partial r}{\partial x}$ (*ii*) $\frac{\partial x}{\partial \theta} = 0$ (*iii*) $\frac{\partial x}{\partial r} = 0$ (*iv*) $\frac{\partial x}{\partial r} = \frac{1}{\partial r/\partial x}$ **Ans.** (*i*)

14. If $u = y^x$ then $\frac{\partial u}{\partial y}$ is

(*i*) xy^{x-1} (*ii*) $y^x \log y$ (*iii*) 0 (*iv*) none of these **Ans.** (*i*)

15. If $u = x^y$ then the value of $\frac{\partial u}{\partial y}$ is equal to

(*i*) 0 (*ii*) $x^y \log(x)$ (*iii*) xy^{x-1} (*iv*) yx^{y-1}

(*A.M.I.E.T.E. Dec. 2007*) **Ans.** (*ii*)

16. If $x = r\cos\theta$, $y = r\sin\theta$, then $\frac{\partial r}{\partial x}$ is equal to

(*i*) $\sec\theta$ (*ii*) $\sin\theta$ (*iii*) $\cos\theta$ (*iv*) $\operatorname{cosec}\theta$ **Ans.** (*iii*)

17. If $u = \tan^{-1}(x+y)$, then $(u_x - u_y)$ equals

(*i*) 0 (*ii*) 1 (*iii*) -1 (*iv*) $\sin x \cos y$ **Ans.** (*i*)

18. If $P = r\tan\theta$, then $\frac{\partial P}{\partial r}$ is equal to

(*i*) $\tan\theta$ (*ii*) $\sec^2\theta$ (*iii*) $\tan\theta + r\sec^2\theta$ (*iv*) $\frac{1}{2}\tan\theta$ **Ans.** (*i*)

19. If $Q = r\cot\theta$, then $\frac{\partial Q}{\partial r}$ is equal to

(*i*) $\cot\theta$ (*ii*) $-\operatorname{cosec}^2\theta$ (*iii*) $\cot\theta - r\operatorname{cosec}^2\theta$ (*iv*) $\frac{1}{2}\cot\theta$ **Ans.** (*i*)

20. If $f(x, y, z) = 0$, then the value of $\frac{\partial x}{\partial y}\cdot\frac{\partial y}{\partial z}\cdot\frac{\partial z}{\partial x}$ is:

(*i*) 1 (*ii*) -1 (*iii*) 0 (*iv*) None of these **Ans.** (*ii*)

21. If $f(x, y) = 0$, then $\frac{dy}{dx}$ is equal to

(*i*) $\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}$ (*ii*) $\frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial x}}$ (*iii*) $-\frac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial x}}$ (*iv*) $-\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}$ **Ans.** (*iv*)

22. If $f(x, y, z) = \frac{x^2}{y^2} + \frac{y^2}{z^2} + \frac{z^2}{x^2}$, then $x f_x + y f_y + z f_z$ is

(*i*) 0 (*ii*) -1 (*iii*) 1 (*iv*) 2 **Ans.** (*i*)

23. If $x = r\cos\theta$, $y = r\sin\theta$ then $\frac{\partial r}{\partial x}$ is equal to

(*i*) $\sec\theta$ (*ii*) $\sin\theta$ (*iii*) $\cos\theta$ (*iv*) $\operatorname{cosec}\theta$ **Ans.** (*iii*)

24. If $u = ax^2 + 2hxy + by^2$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to

(*i*) $2u$ (*ii*) u (*iii*) 0 (*iv*) None of these **Ans.** (*i*)

25. If $P = s\tan\theta$, $q = s\cot\theta$, then

(*a*) $\frac{\partial P}{\partial s}$ is equal to

(*i*) $\tan\theta$ (*ii*) $\sec^2\theta$ (*iii*) $\tan\theta + s\sec^2\theta$ (*iv*) $\frac{1}{2}\tan\theta$ **Ans.** (*i*)

(*b*) $\frac{\partial q}{\partial s}$ is equal to

(*i*) $\cot\theta$ (*ii*) $-\operatorname{cosec}^2\theta$ (*iii*) $\cot\theta - s\operatorname{cosec}^2\theta$ (*iv*) $\frac{1}{2}\cot\theta$ **Ans.** (*i*)

(*c*) $\frac{\partial s}{\partial p}$ is equal to

(*i*) $\cot\theta$ (*ii*) $\cos^2\theta$ (*iii*) $\frac{1}{\tan\theta + s\sec^2\theta}$ (*iv*) $\frac{1}{2}\cot\theta$ **Ans.** (*iv*)

(*d*) $\frac{\partial s}{\partial q}$ is equal to

(iii) 0 (iv) None of these **Ans.** (i)

25. If $P = s\tan\theta$, $q = s\cot\theta$, then

(a) $\dfrac{\partial P}{\partial S}$ is equal to

(i) $\tan\theta$ (ii) $\sec^2\theta$ (iii) $\tan\theta + s\sec^2\theta$ (iv) $\dfrac{1}{2}\tan\theta$ **Ans.** (i)

(b) $\dfrac{\partial q}{\partial s}$ is equal to

(i) $\cot\theta$ (ii) $-\operatorname{cosec}^2\theta$ (iii) $\cot\theta - s\operatorname{cosec}^2\theta$ (iv) $\dfrac{1}{2}\cot\theta$ **Ans.** (i)

(c) $\dfrac{\partial s}{\partial p}$ is equal to

(i) $\cot\theta$ (ii) $\cos^2\theta$ (iii) $\dfrac{1}{\tan\theta + s\sec^2\theta}$ (iv) $\dfrac{1}{2}\cot\theta$ **Ans.** (iv)

(d) $\dfrac{\partial s}{\partial q}$ is equal to

(i) $\tan\theta$ (ii) $-\sin^2\theta$ (iii) $\dfrac{1}{\cot\theta + s\sec^2\theta}$ (iv) $\dfrac{1}{2}\tan\theta$ **Ans.** (iv)

26. If $u = f\left(\dfrac{x}{y}\right)$ then

(i) $x\dfrac{\partial u}{\partial x} - y\dfrac{\partial u}{\partial y} = 0$ (ii) $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 0$

(iii) $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = u$ (iv) $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 1$ **Ans.** (ii)

(U.P. I Sem. Jan 2011)

27. If $u = x^3 e^{-\frac{x}{y}}$ then $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x\partial y} + y^2\dfrac{\partial^2 u}{\partial y^2}$ is equal to

(i) $3u$ (ii) $6u$ (iii) $9u$ (iv) $-u$ **Ans.** (ii)

28. If $f(x, y) = \begin{cases} \dfrac{x^2 + xy}{x + y} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$ then $f_x(0, 0)$ equals

(i) -1 (ii) 0 (iii) 1 (iv) 1/2 **Ans.** (iii)

29. If $z = F(x^l y^k)$ satisfies the equation $x\dfrac{\partial z}{\partial x} - 2y\dfrac{\partial z}{\partial y} = 0$, then $\dfrac{l}{k}$ equals

(i) 1 (ii) 2 (iii) 3 (iv) 4 **Ans.** (ii)

30. If $Z = g(x^a y^b)$ satisfies the equation $2x\dfrac{\partial z}{\partial x} - 3y\dfrac{\partial z}{\partial y} = 0$ then $\dfrac{b}{a}$ satisfies

(i) $3b^2 = 4a^2$ (ii) $3a^2 = 4b^2$ (iii) $4b^2 = 9a^2$ (iv) $9b^2 = 4a^2$ **Ans.** (iv)

31. If $z = f(x + ct) + g(x - ct)$, then

(i) $z_{tt} = z_{xx}$ (ii) $z_t = z_x$ (iii) $z_{tt} = c^2 z_{xx}$ (iv) $z_{xx} = c^2 z_{tt}$ **Ans.** (iii)

32. If $u = x^2 - y^2$, $v = xy$ then $\dfrac{\partial x}{\partial u}$ equals

(i) $\dfrac{x}{2(x^2 + y^2)}$ (ii) $\dfrac{y}{2(x^2 + y^2)}$ (iii) $\dfrac{y}{x^2 + y^2}$ (iv) $\dfrac{x}{x^2 + y^2}$ **Ans.** (i)

33. If $z = f(x^j y^k)$ satisfies the equation $x\dfrac{\partial z}{\partial x} - 2y\dfrac{\partial z}{\partial y} = 0$, then $\dfrac{j}{k}$ equals

(i) 1 (ii) 2 (iii) 3 (iv) 4 **Ans.** (ii)

34. If $u = \sin^{-1}\left(\dfrac{x}{y}\right)$ then $\dfrac{\partial x}{\partial u}$ equals to

(i) $\dfrac{1}{\sqrt{y^2 - x^2}}$ (ii) $\dfrac{1}{\sqrt{x^2 - y^2}}$ (iii) $\sqrt{1 - x^2}$ (iv) None of these **Ans.** (i)

35. If $u = \tan^{-1}\left(\dfrac{y}{x}\right)$ then $\left(\dfrac{\partial u}{\partial y}\right)$ equals to

(i) $\dfrac{x^2}{x^2 - y^2}$ (ii) $\dfrac{x}{x^2 + y^2}$ (iii) $\dfrac{y}{x^2 + y^2}$ (iv) None of these **Ans.** (ii)

36. If $u = \dfrac{1}{2}\log(x^2 + y^2)$ then $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$ is equal to

36. If $u = \frac{1}{2}\log(x^2+y^2)$ then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to

(i) $2u$ (ii) u (iii) x^2+y^2 (iv) 1 Ans. (iv)

37. If $x = r\cos\theta$, $y = r\sin\theta$, then

(i) $\left(\frac{\partial r}{\partial x}\right)_y = -\left(\frac{\partial x}{\partial r}\right)_\theta$ (ii) $r\left(\frac{\partial x}{\partial r}\right)_\theta = -\left(\frac{\partial y}{\partial \theta}\right)_r$

(iii) $\left(\frac{\partial x}{\partial \theta}\right)_r = r^2\left(\frac{\partial \theta}{\partial x}\right)_y$ (iv) None of these Ans. (i

38. If $v = (x^2+y^2+z^2)^{-\frac{1}{2}}$, then $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} + z\frac{\partial v}{\partial z} =$

(i) $-v$ (ii) v (iii) $2v$ (iv) None of these Ans. (i

(*R.G.P.V., Bhopal, Feb. 2005*)

39. $u = \frac{x}{x^2+y^2}$ then $\frac{\partial u}{\partial y}$ is equal to

(i) $\frac{xy}{(x^2+y^2)^2}$ (ii) $\frac{2xy}{(x^2+y^2)^2}$ (iii) $\frac{-2xy}{(x^2+y^2)^2}$ (iv) $\frac{xy}{(x^2-y^2)^2}$ Ans. (iii)

40. If $u = x^2\tan^{-1}\left(\frac{y}{x}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ at $x = y = 1$ is

(i) $\frac{\pi}{4}$ (ii) $\frac{\pi}{2}$ (iii) π (iv) $-\frac{\pi}{4}$ Ans. (ii)

41. If $u = \frac{x^2+y^2+xy}{x+y}$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ equals

(i) 0 (ii) 1 (iii) u (iv) $2u$ Ans. (iii)

42. If $z = \log[(x^3+y^3)/(x+y)]$, then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$ is equal to

(i) 2 (ii) $2z$ (iii) $2e^z$ (iv) 0 Ans. (i)

43. If $z = \frac{x^3+y^3}{xy}$, then the value of $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$ equals

(i) 1 (ii) $2z$ (iii) $\frac{x^3+y^3}{xy}$ (iv) None of these Ans. (iii)

44. Let $u(x, y) = x^2\tan^{-1}\left(\frac{y}{x}\right) - y^2\tan^{-1}\left(\frac{x}{y}\right), x \neq 0, y \neq 0$ then $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$ equals

(i) 0 (ii) $2u$ (iii) u (iv) $3u$ Ans. (ii)

45. If $u = \frac{x^2y^2}{x^2+y^2}\log\frac{y}{x}$ and $v = \cos^{-1}\left(\frac{xy}{x^2-y^2}\right)$ and $z = u + v$ then $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y}$ equals

(i) $4v$ (ii) $4u$ (iii) $2u$ (iv) $4u + v$ Ans. (iii)

46. If $u = \frac{x^3+y^3}{x+y}$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is equal to

(i) 0 (ii) u (iii) $2u$ (iv) $3u$ Ans. (iii)

47. Euler's Theorem on Homogeneous function if z is a Homogeneous of x, y of order n, then:

(i) $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = nz$ (ii) $x^2\frac{\partial z}{\partial y} + y^2\frac{\partial z}{\partial x} = nz$

(iii) $x\frac{\partial z}{\partial y} + y\frac{\partial z}{\partial x} = nz$ (iv) $y^2\frac{\partial z}{\partial x} + x^2\frac{\partial z}{\partial y} = nz$ Ans. (i)

(*R.G.P.V., Bhopal, Feb. 2006*)

$x^{1/4} + y^{1/4}$

48. If $u = \dfrac{x^{1/4} + y^{1/4}}{x^{1/5} + y^{1/5}}$, then the value of $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$ is

(*i*) $4u$ (*ii*) $5u$ (*iii*) $20u$ (*iv*) $\dfrac{1}{20}u$ **Ans.** (*iv*)

(*R.G.P.V., Bhopal, Ist Semester, June 2007*)

Fill in the blanks:

49. If $u = xf\left(\dfrac{y}{x}\right) + g\left(\dfrac{y}{x}\right)$ then $x^2\dfrac{\partial^2 u}{\partial x^2} + 2xy\dfrac{\partial^2 u}{\partial x \partial y} + y^2\dfrac{\partial^2 u}{\partial y^2} =$ .... **Ans.** 0

50. If $x = e^r \cos\theta$, $y = e^r \sin\theta$, then $\dfrac{\partial\theta}{\partial x}$ is equal to .... and $\dfrac{\partial\theta}{\partial y}$ is equal to ....

Ans. $\dfrac{\partial\theta}{\partial x} = -\dfrac{\sin^2\theta}{y}$, $\dfrac{\partial\theta}{\partial y} = \dfrac{\cos^2\theta}{x}$

51. If $z = x^3 \cos(y/x)$ then $x\dfrac{\partial z}{\partial x} + y\dfrac{\partial z}{\partial y}$ is equal to .... **Ans.** $3z$

52. If $x = u + v$, $y = u - v$, then $\dfrac{\partial u}{\partial x}$ is equal to .... and $\dfrac{\partial v}{\partial y}$ is equal to ....

Ans. $\dfrac{\partial u}{\partial x} = \dfrac{1}{2}$, $\dfrac{\partial v}{\partial y} = -\dfrac{1}{2}$

53. If $u = \log\left(\dfrac{x}{y}\right) + \tan\left(\dfrac{x}{y}\right)$ then $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} =$ .... **Ans.** 0

54. If $u = x^3 \cos\left(\dfrac{y}{x}\right)$, then $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$ is equal to .... **Ans.** $3u$

55. State, whether the statement is

(*i*) If $u = \dfrac{x^2 + y^2}{x^2 - y^2} + 4$ then $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = 4$

(*i*) True (*ii*) False (*iii*) Could be either (*iv*) Do not know. **Ans.** (*ii*)

(*ii*) If $u = \dfrac{x^3 - x^2y + xy^2 + y^3}{x^2 - xy - y^2}$ then $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = u$

(*i*) True (*ii*) False (*iii*) Could be either (*iv*) Do not know. **Ans.** (*i*)

(*c*) If $u = \log_e \dfrac{x^4 - y^4}{x^3 + y^3}$, then $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \dfrac{4}{3}$

(*i*) True (*ii*) False (*iii*) Could be either (*iv*) Do not know. **Ans.** (*ii*)

(*d*) If $f(x, y) = \dfrac{1}{x^3} + \dfrac{1}{x^2y} + \dfrac{1}{x^3 + 5y^3}$, then $x\dfrac{\partial f}{\partial x} + y\dfrac{\partial f}{\partial y} + 3f = 0$

(*i*) True (*ii*) False (*iii*) Could be either (*iv*) Do not know. **Ans.** (*i*)

5. Show that the plane $ax + by + cz + d = 0$ touches the surface $px^2 + qy^2 + 2z = 0$, if $\dfrac{a^2}{p} + \dfrac{b^2}{q} + 2cd = 0$.

OBJECTIVE TYPE QUESTIONS

Choose the correct answer :

1. If $f = x^2 + y^2$, $x = r + 3s$, $y = 2r - s$, then $\dfrac{\partial f}{\partial r}$ is

(*i*) $4x + 2y$ (*ii*) $2x + y$ (*iii*) $2x + 4y$ (*iv*) $x + 4y$ **Ans.** (*iii*)

2. If $f = x + 4y$, $x = 2s + t$, $y = s + 2t$, then $\dfrac{\partial f}{\partial t}$ is

(*i*) 9 (*ii*) 8 (*iii*) 7 (*iv*) -7 **Ans.** (*i*)

3. If $z = xy$, $x = e^r \cos\theta$, $y = e^\theta \sin r$, then $\dfrac{\partial z}{\partial r}$ is

(*i*) $xy - x e^\theta \cos r$ (*ii*) $xy + x e^\theta \cos r$

(*iii*) $xy + x e^\theta \sin r$ (*iv*) $xy + y e^\theta \cos r$ **Ans.** (*ii*)

4. If $z = x^2 + y^2$ and $x = r + t$, $y = r^2 + t^2$, then $\dfrac{\partial z}{\partial t}$ is

(iii) $xy + x\,e^{\theta}\sin r$ (iv) $xy + y\,e^{\theta}\cos r$ **Ans.** (ii)

4. If $z = x^2 + y^2$ and $x = r + t,\ y = r^2 + t^2$, then $\dfrac{\partial z}{\partial t}$ is

(i) $x + 6yt$ (ii) $2x + 2yt$ (iii) $x + 4yt$ (iv) $2x + 4yt$ **Ans.** (iv)

5. If $z = x + y,\ x = e^{r\cos\theta},\ y = e^{r\sin\theta}$, then $\dfrac{\partial z}{\partial \theta}$ is

(i) $r(\cos\theta\, e^{r\cos\theta} - \sin\theta\, e^{r\sin\theta})$ (ii) $r(\cos\theta\, e^{r\sin\theta} - \sin\theta\, e^{r\cos\theta})$

(iii) $r\,e^{r}(\cos\theta - \sin\theta)$ (iv) $r(\cos\theta\, e^{r\sin\theta} + \sin\theta\, e^{r\cos\theta})$ **Ans.** (ii)

6. If $z = x^2y^2$, and $x = s\log r;\ y = r\log s$ then $\dfrac{\partial z}{\partial r}$ is

(i) $2xy\left(\dfrac{xs}{r} + y\log s\right)$ (ii) $2xy\,(ys + x\log s)$

(iii) $2xy\left(\dfrac{ys}{r} + x\log s\right)$ (iv) $2xy\left(\dfrac{ys}{r} - x\log s\right)$ **Ans.** (iii)

7. If $z = f(x, y),\ x = r\cos\theta,\ y = r\sin\theta$, then $\dfrac{\partial z}{\partial r}$ is

(i) $\dfrac{\partial f}{\partial x}\cos\theta + \dfrac{\partial f}{\partial y}\sin\theta$ (ii) $\dfrac{\partial f}{\partial x}\sin\theta + \dfrac{\partial f}{\partial y}\cos\theta$

(iii) $\dfrac{\partial f}{\partial x}\cos\theta - \dfrac{\partial f}{\partial y}\sin\theta$ (iv) $\dfrac{\partial f}{\partial x}\sin\theta - \dfrac{\partial f}{\partial y}\cos\theta$ **Ans.** (i)

8. If $z = f(x, y),\ x = r\cos\theta,\ y = r\sin\theta$, then $\dfrac{1}{r}\dfrac{\partial z}{\partial \theta} =$

(i) $\dfrac{\partial f}{\partial x}\sin\theta + \dfrac{\partial f}{\partial y}\cos\theta$ (ii) $-\dfrac{\partial f}{\partial x}\sin\theta + \dfrac{\partial f}{\partial y}\cos\theta$

(iii) $\dfrac{\partial f}{\partial x}\cos\theta + \dfrac{\partial f}{\partial y}\sin\theta$ (iv) $\dfrac{\partial f}{\partial x}\cos\theta - \dfrac{\partial f}{\partial y}\sin\theta$ **Ans.** (ii)

9. If $z = f(x, y),\ x = s + t,\ y = s - t$ then $\dfrac{\partial z}{\partial s}$ is equal to

(i) $\dfrac{\partial f}{\partial x} + \dfrac{\partial f}{\partial y}$ (ii) $\dfrac{\partial f}{\partial x} - \dfrac{\partial f}{\partial y}$

(iii) $\dfrac{\partial f}{\partial y} - \dfrac{\partial f}{\partial x}$ (iv) None of these **Ans.** (i)

10. If $z = x + y,\ x = 2t,\ y = t^2$ then $\dfrac{\partial z}{\partial t}$ is equal to

(i) $2 + 2t$ (ii) $2 - 2t$ (iii) $2t - 2$ (iv) $2t + 5$ **Ans.** (i)

11. If $z = x^2 + y^2,\ x = t^2$ and $y = t^3$ then $\dfrac{\partial z}{\partial t}$ is equal to

(i) $4t^3 - 6t^5$ (ii) $4t^3 + 6t^5$ (iii) $6t^3 - 4t^5$ (iv) $6t - 5$ **Ans.** (ii)

12. If $z = 2x + 3y,\ x = \sin\theta$ and $y = \cos\theta$ then $\dfrac{\partial z}{\partial \theta}$ is equal to

(i) $2\cos\theta + 3\sin\theta$ (ii) $3\sin\theta - 2\cos\theta$

(iii) $2\cos\theta - 3\sin\theta$ (iv) $3\cos\theta + 2\sin\theta$ **Ans.** (iii)

13. If $z = x^2y^2,\ x = t$ and $y = 2t$ then $\dfrac{\partial z}{\partial t}$ is equal to

(i) $2xy(2x - y)$ (ii) $xy(2x + y)$

(iii) $2xy(x + 2y)$ (iv) $2xy(2x + y)$ **Ans.** (iv)

14. If $z = x^3y^3$ then $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2}$ is equal to

(i) $6xy(x^2 + y^2)$ (ii) $6xy(x + y)$ (iii) $6xy(x - y)$ (iv) $xy(x^2 + y^2)$ **Ans.** (i)

15. If $z = \sqrt{xy}$ then $\dfrac{\partial^2 z}{\partial x \partial y}$ is equal to

(i) $4z$ (ii) $\dfrac{1}{4z}$ (iii) $\dfrac{z}{4}$ (iv) $\dfrac{4}{z}$ **Ans.** (ii)

16. If $u = x^2 + y^2 + z^2,\ x = r\sin\theta\cos\phi,\ y = r\sin\theta\sin\phi$ and $z = r\cos\theta$ then $\dfrac{\partial u}{\partial r}$ is equal to

(i) ... (ii) $2r$ (iii) r^2 (iv) $2r^2$ **Ans.** (ii)

(i) $6xy(x^2 + y^2)$ (ii) $6xy(x + y)$ (iii) $6xy(x - y)$ (iv) $xy(x$ ...

15. If $z = \sqrt{xy}$ then $\frac{\partial^2 z}{\partial x \partial y}$ is equal to

(i) $4z$ (ii) $\frac{1}{4z}$ (iii) $\frac{z}{4}$ (iv) $\frac{4}{z}$ Ans. (ii)

16. If $u = x^2 + y^2 + z^2$, $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$ and $z = r\cos\theta$ then $\frac{\partial u}{\partial r}$ is equal to

(i) r (ii) $2r$ (iii) r^2 (iv) $2r^2$ Ans. (ii)

17. If $y = e^x + \sin x$, then $\frac{d^2y}{dx^2}$ is equal to .

(i) $e^x + \sin x$ (ii) $e^x - \sin x$ (iii) $e^x - \cos x$ (iv) None of these

Ans. (ii)

18. If $y = \tan x + \sec x$ then $\frac{d^2y}{dx^2}$ is equal to

(i) $\sec x(\tan^2 x + \sec^2 x)$ (ii) $\sec x(\sec x \tan x + \tan^2 x \sec^2 x)$

(iii) $\sec x(2\sec x \tan x + \tan^2 x + \sec^2 x)$ (iv) $2\sec x \tan x + \tan^2 x + \sec^2 x$

Ans. (iii)

19. If $f(x, y, z) = 0$, then $\frac{\partial x}{\partial y} \cdot \frac{\partial y}{\partial z} \cdot \frac{\partial z}{\partial x}$ is equal to

(i) 1 (ii) 2 (iii) -1 (iv) 0 Ans. (iii)

20. If $z = f(x, y)$ where $x = \phi(t)$, $y = \psi(t)$, then $\frac{dz}{dt}$ is equal to

(i) $\frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$ (ii) $\frac{\partial z}{\partial x}\frac{\partial x}{\partial t} - \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial t}$

(iii) $\frac{\partial z}{\partial x} + \frac{\partial y}{\partial t}\frac{\partial z}{\partial y}$ (iv) $\frac{dx}{dt} + \frac{\partial z}{\partial t}\frac{dx}{dt}$ Ans. (i)

21. If $f(x, y) = 0$, then $\frac{dy}{dx}$ is equal to

(i) $\dfrac{\frac{\partial y}{\partial f}}{\frac{\partial x}{\partial f}}$ (ii) $-\dfrac{\frac{\partial f}{\partial y}}{\frac{\partial f}{\partial x}}$ (iii) $-\dfrac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}$ (iv) $\frac{\partial y}{\partial x} \cdot \frac{\partial f}{\partial y}$ Ans. (iii)

22. If $f(x, y) = 0$ and $\phi(y, z) = 0$, then $\frac{\partial f}{\partial y} \cdot \frac{\partial \phi}{\partial z} \cdot \frac{\partial z}{\partial x}$ is equal to

(i) $\frac{\partial x}{\partial y} \cdot \frac{\partial \phi}{\partial y}$ (ii) $\frac{\partial f}{\partial x} \cdot \frac{\partial y}{\partial \phi}$ (iii) $\frac{\partial f}{\partial y} \cdot \frac{\partial \phi}{\partial x}$ (iv) $\frac{\partial f}{\partial x} \cdot \frac{\partial \phi}{\partial y}$ Ans. (iv)

23. If $f(x, y) = 0$, then $\frac{d^2y}{dx^2}$ is equal to

(i) $-\frac{q^2 r - 2pqs + p^2 t}{q^3}$ (ii) $\frac{q^2 r - 2pqs + p^2 t}{q}$

(iii) $\frac{q^2 r - 2pqs - p^2 t}{q^3}$ (iv) $\frac{q^2 r - 2pqs + p^2 t}{q^3}$ Ans. (i)

24. The equation of the tangent plane to the surface $x^2 + y^2 + z^2 = 14$ at (1, 2, 3) is

(i) $2x + 4y + 6z = 14$ (ii) $x + 2y + 3z = 0$

(iii) $x + 2y + 3z = 1$ (iv) $x + 2y + 3z = 14$ Ans. (iv)

25. The equation of the normal to the tangent plane $x^2 + y^2 + z^2 = 6$ at $(-1, -2, -1)$ is

(i) $\frac{x+1}{2} = \frac{y+2}{-4} = \frac{z+1}{-2}$ (ii) $\frac{x+1}{1} = \frac{y+2}{2} = \frac{z+1}{1}$

(iii) $\frac{x+1}{1} = \frac{y+2}{2} = \frac{z+1}{-1}$ (iv) $\frac{x+1}{1} = \frac{y+2}{-2} = \frac{z+1}{1}$ Ans. (ii)

3. If $\frac{\partial (u, v)}{\partial (x, y)} \times \frac{\partial (x, y)}{\partial (u, v)}$ is equal to

(i) 1 (ii) − 1 (iii) zero (iv) none of these **Ans. (i)**

4. If $x = r\cos\theta$, $y = r\sin\theta$ then $J\left(\frac{x, y}{r, \theta}\right) J\left(\frac{r, \theta}{x, y}\right)$ is equal to

(i) 1 (ii) −1 (iii) 0 (iv) none of these **Ans. (i)**

5. If $x = r\cos\theta$, $y = r\sin\theta$, then $\frac{\partial (r, \theta)}{\partial (x, y)}$ is equal to

(i) 1 (ii) r (iii) $\frac{1}{r}$ (iv) 0 **Ans. (iii)**

6. If $x = r\cos\theta$, $y = r\sin\theta$, $z = z$ then $\frac{\partial (x, y, z)}{\partial (r, \theta, z)}$ is equal to

(i) r (ii) $\frac{1}{r}$ (iii) $r^2 \sin\theta$ (iv) none of these **Ans. (i)**

7. Jacobian $\frac{(u, v)}{(r, s)}$ Jacobian $\frac{(r, s)}{(x, y)} =$

(i) $J\frac{(x, y)}{(u, v)}$ (ii) $J\frac{(ur, vs)}{(rx, xy)}$ (iii) $J\frac{(u, v)}{(x, y)}$ (iv) $J\frac{(u, y)}{(x, v)}$ **Ans. (iii)**

8. Cylindrical coordinates are

(i) $x = r\sin\theta$, $y = r\cos\theta$, $z = z$

(ii) $x = r\cos\theta$, $y = r\sin\theta$, $z = z$

(iii) $x = r\cos\theta$, $y = r\sin\theta$, $z = r\sin\theta\cos\theta$

(iv) $x = r\cos\theta$, $y = r\sin\theta$, $z = \cos\theta$ **Ans. (ii)**

9. Spherical coordinates are

(i) $x = r\sin\theta$, $\cos\phi$, $y = r\cos\theta\sin\phi$, $z = r\sin\theta$

(ii) $x = r\cos\theta$, $y = r\sin\theta$, $z = r\cos\phi$

(iii) $x = r\cos\theta$, $\sin\phi$, $y = r\cos\theta$, $\cos\phi$ $z = r\sin\theta$

(iv) $x = r\sin\theta$, $\cos\phi$, $y = r\sin\theta$, $\sin\theta$ $z = r\cos\theta$ **Ans. (ii)**

10. If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, then the value of the Jacobian $\frac{\partial (x, y, z)}{\partial (r, \theta, \phi)} =$

(i) r^2 (ii) $r^2\sin\theta$ (iii) $r^2\cos\theta$ (iv) $r^2\cos\phi$ **Ans. (ii)**

Fill in the blanks:

11. If $x = r\cos\theta$, $y = r\sin\theta$, then the value of the Jacobian $\frac{\partial (x, y)}{\partial (r, \theta)}$ is .... **Ans. r**

12. If $u = x(1 - y)$, $v = xy$, then the value of the Jacobian $\frac{\partial (u, v)}{\partial (x, y)}$ is .... **Ans. x**

13. $\frac{\partial (u, v)}{\partial (r, s)} \times \frac{\partial (r, s)}{\partial (x, y)} =$ .... **Ans.** $\frac{\partial (u, v)}{\partial (x, y)}$

14. $\frac{\partial (x, y)}{\partial (r, \theta)} \times \frac{\partial (r, \theta)}{\partial (x, y)} =$ .... **Ans.** 1

Indicate True or False for the following:

15. If $u = 2axy$, $v = a(x^2 - y^2)$, where $x = r\cos\theta$, $y = r\sin\theta$ then Jacobian $\frac{\partial (x, y)}{\partial (r, \theta)}$ is $-4a^2r^3$.

Ans. True

16. If $x = \sqrt{vw}$, $y = \sqrt{wu}$, $z = \sqrt{uv}$ and $u = r\sin\theta\cos\phi$, $v = r\sin\theta\sin\phi$, $w = r\cos\theta$ then the value of the Jacobian $\frac{\partial (x, y, z)}{\partial (r, \theta, \phi)}$ is $-\frac{1}{4}$. **Ans. False**

...then Jacobian $\frac{\partial (u, v)}{}$ is $(x + y)^{-1}$ **Ans. True**

(i) $x = r \sin\theta, \cos\phi, y = r\cos\theta \sin\phi, z = r\sin\theta$

(ii) $x = r\cos\theta, y = r\sin\theta, z = r\cos\phi$

(iii) $x = r\cos\theta, \sin\phi, y = r\cos\theta, \cos\phi\; z = r\sin\theta$

(iv) $x = r\sin\theta,\ \cos\phi,\ y = r\sin\theta, \sin\theta\; z = r\cos\theta$ **Ans.** (*ii*)

10. If $x = r\sin\theta\cos\phi, y = r\sin\theta\sin\phi, z = r\cos\theta$, then the value of the Jacobian $\dfrac{\partial(x,y,z)}{\partial(r,\theta,\phi)} =$

(i) r^2 (ii) $r^2\sin\theta$ (iii) $r^2\cos\theta$ (iv) $r^2\cos\phi$ **Ans.** (*ii*)

Fill in the blanks:

11. If $x = r\cos\theta, y = r\sin\theta$, then the value of the Jacobian $\dfrac{\partial(x,y)}{\partial(r,\theta)}$ is .... **Ans.** r

12. If $u = x(1-y), v = xy$, then the value of the Jacobian $\dfrac{\partial(u,v)}{\partial(x,y)}$ is .... **Ans.** x

13. $\dfrac{\partial(u,v)}{\partial(r,s)} \times \dfrac{\partial(r,s)}{\partial(x,y)} = \ldots\ldots\ldots$ **Ans.** $\dfrac{\partial(u,v)}{\partial(x,y)}$

14. $\dfrac{\partial(x,y)}{\partial(r,\theta)} \times \dfrac{\partial(r,\theta)}{\partial(x,y)} = \ldots\ldots\ldots$ **Ans.** 1

Indicate True or False for the following:

15. If $u = 2axy$, $v = a(x^2 - y^2)$, where $x = r\cos\theta$, $y = r\sin\theta$ then Jacobian $\dfrac{\partial(x,y)}{\partial(r,\theta)}$ is $-4a^2r^3$. **Ans.** True

16. If $x = \sqrt{vw}, y = \sqrt{wu}, z = \sqrt{uv}$ and $u = r\sin\theta\cos\phi$, $v = r\sin\theta\sin\phi$, $w = r\cos\theta$ then the value of the Jacobian $\dfrac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$ is $-\dfrac{1}{4}$. **Ans.** False

17. If $u = x + y$, $y = uv$ then Jacobian $\dfrac{\partial(u,v)}{\partial(x,y)}$ is $(x+y)^{-1}$ **Ans.** True

18. If $u = \dfrac{x+y}{1-xy}$, $v = \tan^{-1}x + \tan^{-1}y$ then Jacobian $\dfrac{\partial(u,v)}{\partial(x,y)}$ is 0. **Ans.** True

19. If $x = u(1-v)$, $y = uv$ then the value of Jacobian $\dfrac{\partial(x,y)}{\partial(u,v)}$ is $\dfrac{1}{u}$ **Ans.** False

Match the following:

20.

(i) $\dfrac{\partial(r,\theta,z)}{\partial(x,y,z)}$	(a) $\dfrac{\partial(u,v)}{\partial(r,s)} \times \dfrac{\partial(r,s)}{\partial(x,y)}$
(ii) $\dfrac{\partial(r,\theta,\phi)}{\partial(x,y,z)}$	(b) 1
(iii) $\dfrac{\partial(u,v)}{\partial(x,y)}$	(c) $\dfrac{1}{r^2\sin\theta}$
(iv) $\dfrac{\partial(u,v)}{\partial(x,y)} \times \dfrac{\partial(x,y)}{\partial(u,v)}$	(d) $\dfrac{1}{r}$

Ans. (*i*) → (*d*)
(*ii*) → (*c*)
(*iii*) → (*a*)
(*iv*) → (*b*)

21.

(i) $u = x^2,\quad v = y^2$	(a) $J(u,v) = x - y$
(ii) $u = x + y,\quad v = xy$	(b) $J(u,v) = -29$
(iii) $u = x + y,\quad v = \dfrac{y}{x+y}$	(c) $J(u,v) \dfrac{1}{x+y}$
(iv) $u = 3x + 5y,\quad v = 4x - 3y$	(d) $J(u,v) = 4xy$

Ans. (*i*) → (*d*)