

4. If $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then

(i) $AB = 0$

(iii) AB and BA are not defined

(ii) $BA = 0$

(iv) None of these

Ans. (iv)

5. If $A = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix}$, then AB is

(i) $\begin{bmatrix} 4 & 2 \\ 0 & 2 \end{bmatrix}$

(ii) $\begin{bmatrix} -4 & 2 \\ 1 & 2 \end{bmatrix}$

(iii) $\begin{bmatrix} -4 & 2 \\ 0 & 2 \end{bmatrix}$

(iv) $\begin{bmatrix} 4 & 2 \\ 1 & 2 \end{bmatrix}$

Ans. (iii)

6. Let A and B , be two matrices, then

(i) $AB = BA$

(ii) $AB \neq BA$

(iii) $AB < BA$

(iv) $AB > BA$

Ans. (ii)

7. In matrices:

(i) $(A + B)^2 = A^2 + 2AB + B^2$

(ii) $(A + B)^2 = A^2 + B^2$

(iii) $(A + B)^2 \neq A^2 + 2AB + B^2$

(iv) $(A + B)^2 = A^2 + 2BA + B^2$

Ans. (iii)

8. If $f(x) = x^2 + 3x + 4$, then $f(A) =$

(i) $A^2 + 3A + 4I$

(ii) $A^2 + 3A + 4$

(iii) $A^2 + 3A$

(iv) $(A + 4)(A + I)$

Ans. (i)

9. In matrix multiplication of two matrix A and matrix B ,

(i) $AB = BA$

(ii) $AB \neq BA$

(iii) $AB = 2B$

(iv) None of these

Ans. (ii)

10. The value of the determinant $\begin{bmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{bmatrix}$ where w is the cube root of unity equals

(i) 0

(ii) 1

(iii) w

(iv) w^2

Ans. (i)

11. If T_p , T_q and T_r are the p th, q th, r th terms of an AP then $\begin{bmatrix} T_p & T_q & T_r \\ p & q & r \\ 1 & 1 & 1 \end{bmatrix}$ equals

(i) 1

(ii) -1

(iii) 0

(iv) None of these

Ans. (iii)

Elementary Row and Column Transformation

is called :

(i) Singular

(ii) Unit matrix

(iii) Inverse

(iv) None of the above

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16. If A is non-singular matrix of order $n \times n$ then $|\text{adj}(A)|$ is equal to
(i) 1
(ii) 0

(iii) $|A|^{n-1}$

Ans. (iii)

17. Which of the following matrices is non-singular ?

(i) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix}$

(iv) $\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$

Ans. (iii)

18. If A is a skew-symmetric matrix of odd order, then the determinant of A is
(i) -1.
(ii) 0.

(iii) 1.

(iv) a real number.

Ans. (ii)

19. If A and B are square matrices of equal order and λ, μ are numbers, then $\lambda A + \mu B$ is
(i) symmetric if A is symmetric and B is skew-symmetric

Ans. (ii)

(ii) symmetric if A and B are both symmetric

(iii) symmetric if both A and B are skew-symmetric

(iv) symmetric if B is symmetric and A is skew-symmetric

Ans. (ii)

20. If $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$, then $A^n =$

(i) $\begin{bmatrix} 1 & 1 \\ 0 & n \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 0 \\ n & 1 \end{bmatrix}$

(iii) $\begin{bmatrix} n & 0 \\ 1 & 1 \end{bmatrix}$

(iv) $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$

Ans. (iv)

21. If $r \begin{bmatrix} 5 \\ 2 \end{bmatrix} + s \begin{bmatrix} 6 \\ 3 \end{bmatrix} = \begin{bmatrix} 27 \\ 12 \end{bmatrix}$, then

(i) $r = 3, s = 2$

(ii) $r = 2, s = 3$

(iii) $r = 3, s = -2$

(iv) $r = -3, s = 2$

Ans. (i)

22. Given $A = \begin{bmatrix} 1 & 0 \\ 2 & -1 \end{bmatrix}$, which of the following results is true ?

(i) $A^2 = I$

(ii) $A^2 = 2I$

(iii) $2A^2 = I$

(iv) $A^2 = A$

Ans. (i)

23. If $A = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 0 \\ 0 & 4 \end{bmatrix}$, then $|AB|$ is equal to

(i) -24

(ii) 24

(iii) Not defined

(iv) None of these

Ans. (i)

24. If the matrix $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $C = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}$ then

(i) $C = A \cos \theta - B \sin \theta$

(ii) $C = A \sin \theta + B \cos \theta$

(iii) $C = A \sin \theta - B \cos \theta$

(iv) $C = A \cos \theta + B \sin \theta$

Ans. (iv)

25. Let I be the unit matrix of order n and $\text{adj.}(2I) = 2^k I$. Then k equals
(i) 1
(ii) 2

(iii) $n - 1$

(iv) n

Ans. (iii)

26. Which of the following matrices is not invertible ?

(i) $\begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(iii) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(iv) None of these Ans. (i)

27. With 1, ω , ω^2 as cube roots of unity, inverse of which of the following matrices exists ?

(i) $\begin{bmatrix} 1 & \omega \\ \omega & \omega^2 \end{bmatrix}$

(ii) $\begin{bmatrix} \omega^2 & 1 \\ 1 & \omega \end{bmatrix}$

(iii) $\begin{bmatrix} \omega & \omega^2 \\ \omega^2 & 1 \end{bmatrix}$

(iv) None of these Ans. (iv)

28. If A and B are two square matrices of same order then $\text{adj}(AB)$ is

(i) $(\text{Adj } A)(\text{Adj } B)$

(ii) $(\text{Adj } B)(\text{Adj } A)$

(iii) $A \text{ adj } B + (\text{adj } A) B$

(iv) None of these

Ans. (ii)

29. If $A \begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$, where $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then A is

(i) $\begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix}$

(ii) $\begin{bmatrix} 0 & 1 \\ 2 & -1 \end{bmatrix}$

(iii) $\begin{bmatrix} 2 & 1 \\ -1 & 0 \end{bmatrix}$

(iv) $\begin{bmatrix} 2 & 1 \\ -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}$

Ans. (iv)

30. If $A \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$, where $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then A is

(i) $\begin{bmatrix} 2 & 1 \\ 5 & -1 \end{bmatrix}$

(ii) $\begin{bmatrix} 2 & 5 \\ 1 & -1 \end{bmatrix}$

(iii) $\begin{bmatrix} 2 & -5 \\ -1 & 1 \end{bmatrix}$

(iv) $\begin{bmatrix} 0 & 5 \\ -1 & 1 \end{bmatrix}$

Ans. (ii)

31. If a matrix A satisfies a relation $A^2 + A - I = 0$, then

(i) A^{-1} exists

(ii) A^{-1} does not exist

(iii) A^{-1} exists and is equal to $I + A$

(iv) A^{-1} exists and is equal to I , where I is an identity matrix

Ans. (iii)

32. Matrix A has x rows and $x + 5$ columns. Matrix B has y rows and $11 - y$ columns. Both AB and BA exist. Which of the following values for x and y are possible ?

(i) $x = 2, y = 6$

(ii) $x = 3, y = 8$

(iii) $x = 4, y = 4$

(iv) $x = 8, y = 3$

Ans. (ii)

33. If $I + A + A^2 + \dots + A^K = 0$, then A^{-1} equal to

(i) A^K

(ii) A^{K-1}

(iii) A^{K+1}

(iv) $I + A$

Ans. (i)

34. Let $A = \begin{bmatrix} 1 & 0 & 0 \\ \alpha & 1 & 0 \\ \beta & \gamma & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 4 & 1 \end{bmatrix}$ then

(i) A is row equivalent to B only if $\alpha = 2, \beta = 3, \gamma = 4$

(ii) A is row equivalent to B only if $\alpha \neq 0, \beta \neq 0, \gamma = 0$

(iii) A is not equivalent to B

(iv) A is row equivalent to B for all values of α, β, γ

Ans. (i)

Fill up the blanks:

OBJECTIVE TYPE QUESTIONS

The rank of $\begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{bmatrix}$ is equal to

Ans. $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

The rank of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$ is equal to

Ans. 3

Choose the correct alternative:

The rank of the diagonal matrix $\begin{bmatrix} -1 & & & \\ & 0 & & \\ & & 1 & \\ & & & 0 \\ & & & & 0 \\ & & & & & 4 \end{bmatrix}$ is

(i) 1,

(ii) 2,

(iii) 3,

(iv) 4

Ans. (iii)

$\begin{bmatrix} 2 & -4 & 6 \end{bmatrix}$

5. If A is a non-zero column vector, ($n \times 1$), then the rank of matrix AA^T is

(i) 0, (ii) 1, (iii) $n - 1$, (iv) n .

Ans. (ii)

6. The rank of matrix $\begin{bmatrix} \mu & -1 & 0 \\ 0 & \mu & -1 \\ -1 & 0 & \mu \end{bmatrix}$ is 2, for μ equal

(i) any row number (ii) 3 (iii) 1 (iv) 2

Ans. (iii)

7. If P and Q are non-singular matrices, then for Matrix ' M ', which of the following statements are correct?

(i) Rank $(PMQ) > \text{Rank } M$
 (ii) Rank $(PMQ) = \text{Rank } M$
 (iii) Rank $(PMQ) > \text{Rank } M$
 (iv) Rank $(PMQ) = \text{Rank } M + \text{Rank } (PQ)$

Ans. (ii)

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8. Rank of singular matrix of order 4, can be at the most.

(i) 1 (ii) 2 (iii) 3 (iv) 4

Ans. (iii)

9. The value of μ for which the rank of the matrix $A = \begin{bmatrix} \mu & -1 & 0 & 0 \\ 0 & \mu & -1 & 0 \\ 0 & 0 & \mu & -1 \\ -6 & 11 & -6 & 1 \end{bmatrix}$ is equal to 3 is

(i) 0 (ii) 1 (iii) 4 (iv) -1

Ans. (ii)

10. If the rank of an $n \times n$ matrix A is $(n - 1)$, then the system of equations $Ax = b$ has

(i) $(n - 1)$ parameter family of solutions
 (ii) one parameter family of solutions
 (iii) no solution
 (iv) a unique solution

Ans. (i)

11. The rank of matrix $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 5 & 6 & 0 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ is

(i) 4 (ii) 3 (iii) 1 (iv) 0

Ans. (i)

Indicate True or False for the following:

12. The rank of the matrix $\begin{bmatrix} 1 & 2 & 5 & 6 \\ 2 & 4 & 10 & 12 \\ -1 & -2 & -5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ is 1.

Ans. True

13. The rank of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ is 2.

Ans. True

14. The rank of a matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is 2.

Ans. False

15. The rank of $\begin{bmatrix} 100 & 90 & 20 \\ 10 & 2 & 2 \\ 0 & 0 & 1 \end{bmatrix}$ is 1.

Ans. False

16. The rank of matrix A and A' are equal.

17. The rank of matrix A and $100A$ are not equal.

Ans. True

Ans. False

18. The rank of $\begin{bmatrix} 1 & 2 & 0 & -2 & -4 \\ 2 & 3 & 4 & 5 & 6 \\ 3 & 5 & 4 & 3 & 2 \end{bmatrix}$ and $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ are not equal.

Ans. False

Match the following:

9. Rank of

(i) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

(a) 2

(ii) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(b) 3

(iii) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(c) 0

(iv) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

(d) 1

(v) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(e) 4

Ans. (i) \rightarrow (d),

(ii) \rightarrow (a),

(iii) \rightarrow (b),

(iv) \rightarrow (c),

(v) \rightarrow (e)

10. The following system of equations given

$$x + 2y + z = 0$$

$$2x + ay + az = 2$$

$$x + y + 2z = 1$$

The value of a , for which nontrivial solution exists is

(i) $a = 3$

(ii) $a = 6$

(iii) $a \neq 3$

(iv) $a \neq 6$

Ans. (iii)

11. The solution of the equations

$$5x + 3y + 3z = 48$$

$$2x + 6y - 3z = 18$$

$$8x - 3y + 2z = 21$$

(i) $x = 3, y = 5, z = 6$

(ii) $x = 0, y = 5, z = 3$

(iii) $x = 3, y = 0, z = 6$

(iv) $x = 3, y = 5, z = 0$

Ans. (i)

12. The system of equations

$$x - y + z = -\lambda$$

$$x + y + z = \lambda$$

$$-x + y - z = \lambda$$

has

(i) unique solution

(ii) infinitely many solutions

(iii) no solution

(iv) None of these

Ans. (ii)

13. The solution of the simultaneous equations

$$x + y + z = 3, \quad 2x + y - z = 2 \quad \text{and} \quad 3x + 2y + 2z = 7$$

(i) $x = 0, y = 1, z = 2$

(ii) $x = 1, y = 1, z = 1$

(iii) $x = y = z = 0$

(iv) $x = 1, y = 2, z = 3$

Ans. (ii)

14. The solution of the simultaneous equations

$$2x + y + z = 7, \quad 3x + y + z = 8 \quad \text{and} \quad 5x + 6x - z = 14$$

(i) $x = 1, y = 2, z = 3$

(ii) $x = 0, y = 1, z = 2$

(iii) $x = 2, y = 3, z = 2$

(iv) $x = 2, y = 3, z = 4$

Ans. (i)

15. Let $v_1 = (1, 1, 0, 1)$, $v_2 = (1, 1, 1, 1)$, $v_3 = (4, 4, 1, 1)$ and $v_4 = (1, 0, 0, 1)$ be elements or R_4 . The set of vectors $\{v_1, v_2, v_3, v_4\}$ is

(i) linearly independent

(ii) linearly dependent

(iii) null

(iv) none of these

Ans. (ii)

16. The system of equations $2x - y = 3$, $x - 3y = 4$ and $x + 2y = 1$ has

(i) a unique solution

(ii) infinitely many solutions

(iii) no solution

(iv) none of these

Ans. (iii)

Match the following

In the system of equation $AX = B$ and $A, B = C$

- | | |
|---|--|
| (a) If the rank of $A \neq$ rank of C | (p) consistent with unique solution |
| (b) If the rank of $A =$ rank of $C =$ number of unknowns | (q) Infinite solutions consistent with |
| (c) If the rank $A =$ rank of $C <$ No. of unknowns | (r) have a solution |
| (d) The solution of $AX = 0$ is always | (s) Inconsistent |

Ans. (a) \rightarrow (s)

(b) \rightarrow (p)

(c) \rightarrow (q)

(d) \rightarrow (r)

4. The simultaneous equations

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

(i) $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(p) No solution

(ii) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} = \frac{c_1}{c_2}$

(q) Unique solution

(iii) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

(r) Infinity many solutions

(iv) $\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

(s) None of these

Ans. (i) \rightarrow (r)

(ii) \rightarrow (s)

(iii) \rightarrow (p)

(iv) \rightarrow (q)

Fill up the blanks.

25. The solution of the following system of equations $x + 2y + 3z = 6$

$$2x + y - z = 2$$

$$x - 3y + 5z = 3$$

$$x = 1, y = \text{---} \text{ and } z = \text{---}$$

Ans. $y = 1, z = 1$

OBJECTIVE TYPE QUESTIONS

Tick (✓) the correct answer:

1. If λ_1, λ_2 and λ_3 are the eigen values of the matrix

$$\begin{bmatrix} -2 & -9 & 5 \\ -5 & -10 & 7 \\ -9 & -21 & 14 \end{bmatrix} \text{ then } \lambda_1 + \lambda_2 + \lambda_3 \text{ is equal to}$$

- (i) -16 (ii) 2 (iii) -6 (iv) -14

Ans. (ii)

2. The matrix $A = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$ is given and the eigen values of $4A^{-1} + 3A + 2I$ are

- (i) 6, 15 (ii) 9, 12 (iii) 9, 15 (iv) 7, 15

Ans. (iii)

3. If a square matrix A has an eigen value λ , then an eigen value of the matrix $(kA)^T$ where $k \neq 0$ is a scalar, is

- (i) λ/k (ii) k/λ (iii) $k\lambda$ (iv) None of these

Ans. (iii)

4. For the matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$ the sum of the eigen values is

- (i) -1 (ii) 0 (iii) 3 (iv) 5

Ans. (i)

5. A 3×3 real matrix has an eigen value i then its other two eigen values can be

- (i) 0, 1 (ii) -1, i (iii) $2i, -2i$ (iv) 0, $-i$

Ans. (iv)

6. Let A be a square matrix. Then $\lambda = 0$ is an eigen value of A if and only if

- (i) A is non-singular (ii) A is of even order

- (iii) A is of odd order (iv) A is singular

Ans. (iv)

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Eigen Values, Eigen Vectors, Cayley Hamilton Theorem, Diagonalisation

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7. The matrix A is defined as $A = \begin{bmatrix} -1 & 0 & 0 \\ 2 & -3 & 0 \\ 1 & 4 & 2 \end{bmatrix}$. The eigen values of A^2 are

- (i) -1, -9, -4 (ii) 1, 9, 4
(iii) -1, -3, 2 (iv) 1, 3, -2

Ans. (ii)

8. If the matrix $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix}$ then the eigen values of $A^2 + 5A + 8I$, are

- (i) -1, 27, -8 (ii) 1, 3, -2 (iii) 2, 50, -10 (iv) 2, 50, 10

Ans. (iii)

9. Let P be a real square matrix of order n and n is odd. Then

- (i) at least one eigen value of A is real (ii) one eigen value of A is zero

- (iii) one eigen value of A is 1 (iv) A has no real eigen values

Ans. (i)

10. Two of the eigen values of a 3×3 matrix, whose determinant equals 4, are -1 and +2 the third eigen value of the matrix is equal to

- (i) -2 (ii) -1 (iii) 1 (iv) 2

Ans. (i)

11. The matrix A has eigen values $\lambda_i \neq 0$. Then $A^{-1} - 2I + A$ has eigen values

- (i) $1 + 2\lambda_i + \lambda_i^2$ (ii) $\frac{1}{\lambda_i} - 2 + \lambda_i$ (iii) $1 - 2\lambda_i + \lambda_i^2$ (iv) $1 - \frac{2}{\lambda_i} + \frac{1}{\lambda_i^2}$

Ans. (ii)

12. The eigen values of a matrix A are 1, -2, 3. The eigen values of $3I - 2A + A^2$ are

- (i) 2, 11, 6 (ii) 3, 11, 18 (iii) 2, 3, 6 (iv) 6, 3, 11

Ans. (i)

13. If A is a singular hermitian matrix, then the least eigen value of A^2 is

- (i) 0 (ii) 1 (iii) 2 (iv) None of these

20. The product of the eigen values of the matrix

$$A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{pmatrix} \text{ is}$$

(i) 3

(ii) 8

(iii) 1

(iv) -1 **Ans. (ii)**

21. If $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & 0 & -2 \end{bmatrix}$, then the eigen value of A^2 are

(i) 1, 2, 3

(ii) -1, 2, 3

(iii) 1, 4, 9

(iv) -1, 4, 9 **Ans. (iii)**

22. The eigenvalues of the matrix $\begin{bmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{bmatrix}$ are

(i) -1, 2 and 1

(ii) 0, 1 and 2

(iii) -1, -2 and 4

(iv) 1, 1 and -1

Ans. (i)

Considering the following choose the correct alternative:

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \text{ if } U_1, U_2 \text{ and } U_3 \text{ are columns matrices satisfying.}$$

$$AU_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, AU_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}, AU_3 = \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix} \text{ and } U \text{ is } 3 \times 3 \text{ matrix whose columns are } U_1, U_2, U_3 \text{ then}$$

answer the following equations.

23. The value of $|U|$ is

(i) 3

(ii) -3

(iii) $\frac{3}{2}$

(iv) 2

Ans. (i)

[Hint : Let U_1 be $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ so that

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$\text{Similarly, } U_2 = \begin{bmatrix} 2 \\ -1 \\ -4 \end{bmatrix}, U_3 = \begin{bmatrix} 2 \\ -1 \\ -3 \end{bmatrix}$$

$$\text{Hence, } U = \begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -3 \end{bmatrix} \text{ and } |U| = 3.$$

[Hint : Moreover $\text{adj. } U = \begin{bmatrix} -1 & -2 & 0 \\ -7 & -5 & -3 \\ 9 & 6 & 3 \end{bmatrix}$.

Hence, $U^{-1} = \frac{\text{adj } U}{3}$ and sum of the elements of $U^{-1} = 0$

25. The value of $[3 \ 2 \ 0] U \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$ is

(i) 5

(ii) $\frac{5}{2}$

(iii) 4

(iv) $\frac{3}{2}$

Ans. (i)

[Hint : The value of $[3 \ 2 \ 0] U \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$

$$= [3 \ 2 \ 0] \begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -1 \\ 1 & -4 & -3 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} = [-1 \ 4 \ 4] \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix} = -3 + 8 = 5$$

Fill up the blanks.

26. If the eigen values of the matrix A are 1, 2, 3 then, the eigen values of $(A \cdot P)$ are

Ans. 1, 2, 3

27. The eigen values of A are 2, 3, 4 then the eigen values of A^2 are

Ans. 4, 9, 16

28. The eigen values of A are 2, 3, 1 then the eigen values of $A^2 + A$ are

Ans. 6, 12, 2

29. If the eigen values of A are 1, 1, 1 then the eigen values of $A^2 + 2A + 3I$ are

Ans. 6, 6, 6

30. If the eigen values of A are 4, 6, 9 then the eigen values of A^{-1} are

Ans. $\frac{1}{4}, \frac{1}{6}, \frac{1}{9}$

Indicate True or False for the following:

31. The elements of modal matrix are the eigen vectors of the corresponding eigen values. Ans. True

32. $P^{-1}AP = \text{The diagonal matrix.}$

Ans. True

33. $A^6 = PD^6P^{-1}$

Ans. True

34. If $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$ then $A^{100} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Ans. False

35. If $A = \begin{bmatrix} 1 & -2 & -3 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then $A^{-1} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Ans. False

36. Conjugate of 2 is 2.

Ans. True

37. Conjugate of i is $-i$.

Ans. True

38. If the eigen value of A is 2, then the eigen value of $A^3 + 2A^2 + A + I$ is 10.

Ans. False

Fill up the blanks:

39. The characteristic roots of a skew hermitian matrix is either..... or

Ans. 0, Pure imaginary

40. The modulus of each characteristic roots of a unitary matrix is **Ans. unity**
41. If λ is an eigen value of an orthogonal matrix, then the other eigen value of the same orthogonal matrix is **Ans. $\frac{1}{\lambda}$**
42. The characteristic roots of a Hermitian matrix or all..... **Ans. real**
43. The characteristic root of a triangular matrix is **Ans. Diagonal element**
44. If a characteristic roots of a matrix is zero, then the matrix is **Ans. Singular**
45. If A and P be square matrices of the same type and if P is invertible, then the matrices A and $P^{-1}AP$ have characteristic roots. **Ans. Same**

Match the following:

46. (i) The eigen vectors X of a matrix A , is not

(a) $X_1' X_2 = 0$

(ii) Two eigen vectors X_1 and X_2 are called orthogonal if

(b) $\sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}$

(iii) Normalised form of vectors $\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix}$ is obtained on

dividing each element by

(c) unique

(iv) Every square matrix satisfies its own

(d) Characteristic equation

Ans. (i) \rightarrow (c)

(ii) \rightarrow (a)

(iii) \rightarrow (b)

(iv) \rightarrow (d)