UNIT-4:

LOGIC GATE
And
Boolean Algebra

Logic Gate

- Digital systems are said to be constructed by using logic gates.
- These gates are the AND, OR, NOT, NAND, NOR, EXOR and EXNOR gates.
- The basic operations are described with the aid of truth tables.

- Boolean functions practically implemented by using electronic gates

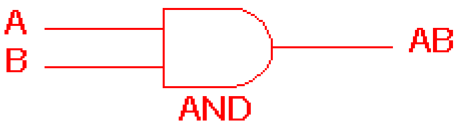
- Generally logic gate have 2 input 1 output
- Gate **INPUTS** are driven by voltages having two nominal values,

  0V logic 0 and 5V logic 1
- The **OUTPUT** of a gate provides two nominal values of voltage only

  0V logic 0 and 5V logic 1

Logic Gate

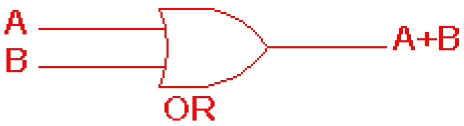
AND gate



2 Input AND gate		
A	B	A.B
0	0	0
0	1	0
1	0	0
1	1	1

high output (1) only if **all** its inputs are high

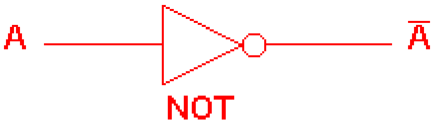
OR gate



2 Input OR gate		
A	B	A+B
0	0	0
0	1	1
1	0	1
1	1	1

high output (1) if **one or more** of its inputs are high.

NOT gate



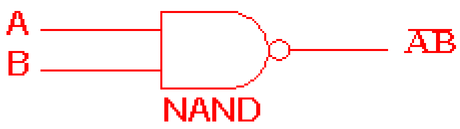
NOT gate	
A	$\overline{A}$
0	1
1	0

produces an inverted version of the input at its output.

Logic Gate

NAND GATE

AND gate followed by a NOT gate.

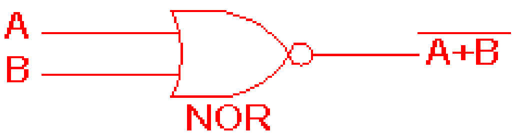


2 Input NAND gate		
A	B	$\overline{A.B}$
0	0	1
0	1	1
1	0	1
1	1	0

high output (1) only if **all** its inputs are low

NOR GATE

OR gate followed by a NOT gate.



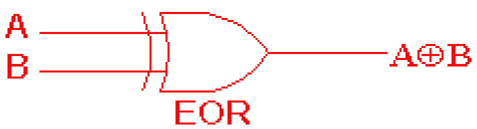
2 Input NOR gate		
A	B	$\overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0

Low output (0) if **one or more** of its inputs are high.

NAND and NOR gates are called *universal*

Logic Gate

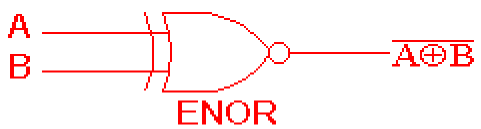
EXOR GATE/XOR



2 Input EXOR gate		
A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

high output (1) for different input

EXNOR GATE/XNOR

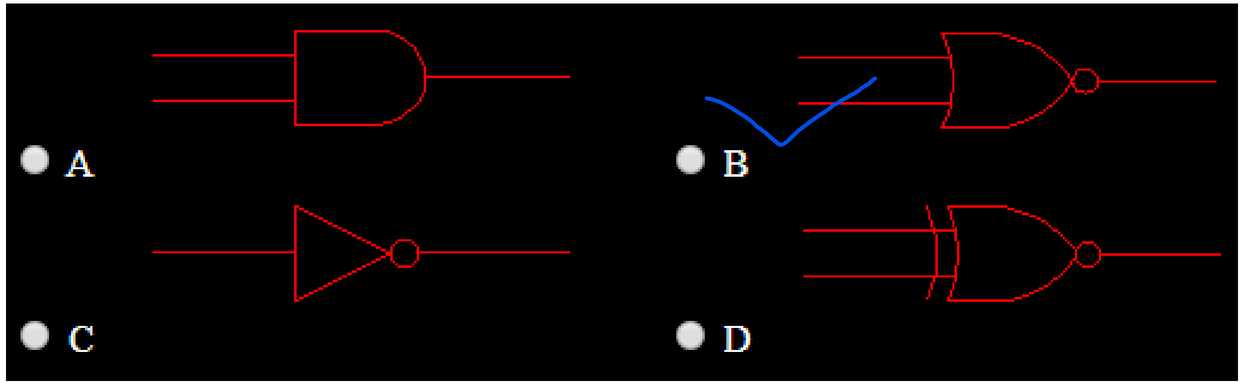


2 Input EXNOR gate		
A	B	$\overline{A \oplus B}$
0	0	1
0	1	0
1	0	0
1	1	1

High output (1) for same input

Practice Questions

Which of the following symbols represents a NOR gate?



Which one of the following truth tables represents the behavior a NAND gate?

A

2 Input NAND gate		
A	B	$\overline{A.B}$
0	0	1
0	1	0
1	0	0
1	1	0

B

2 Input NAND gate		
A	B	$\overline{A.B}$
0	0	0
0	1	1
1	0	1
1	1	0

C

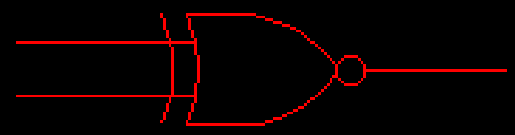
2 Input NAND gate		
A	B	$\overline{A.B}$
0	0	0
0	1	0
1	0	0
1	1	1

D

2 Input NAND gate		
A	B	$\overline{A.B}$
0	0	1
0	1	1
1	0	1
1	1	0

Practice Questions

What type of logic gate does this symbol represent?



- Exclusive OR
- Exclusive NOR
- OR
- NOR

What type of logic gate's behavior does this truth table represent?

?			
A	B	C	?
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

- 2 input OR
- 3 input OR
- 3 input EXOR
- 4 input EXOR

Practice Questions

The output of an AND gate with three inputs, A, B, and C, is HIGH when _________.

A. A = 1, B = 1, C = 0
B. A = 0, B = 0, C = 0
C. A = 1, B = 1, C = 1
D. A = 1, B = 0, C = 1

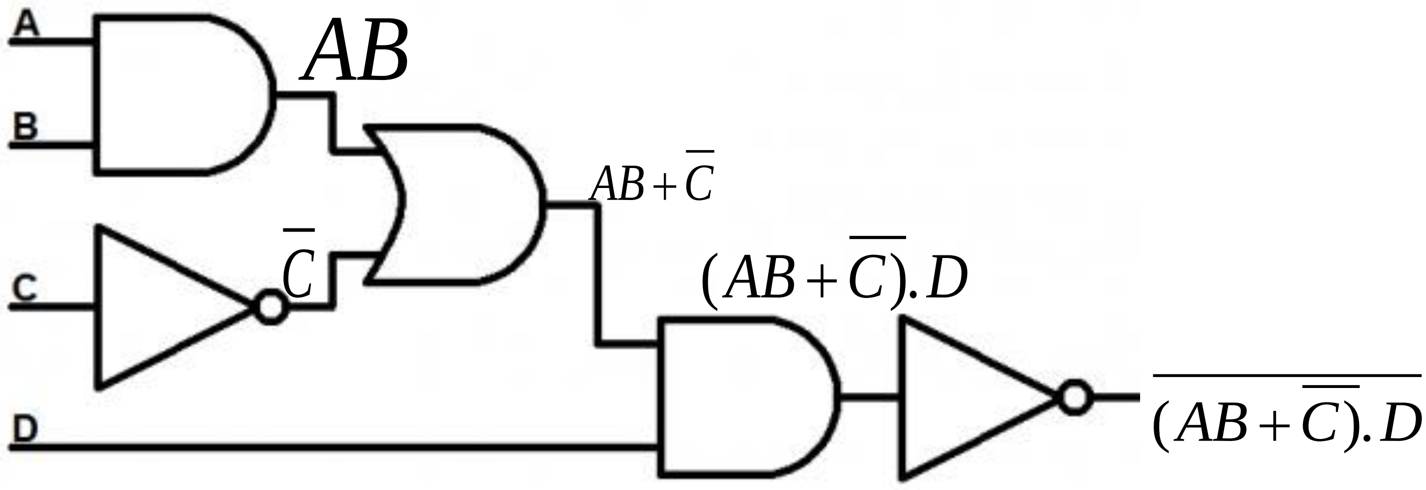
If the input to a NOT gate is A and the output is X, then _________.

A. X = A
B. A=~A
C. X = 0
D. none of the above

How many inputs of a four-input AND gate must be HIGH in order for the output of the logic gate to go HIGH?

A. any one of the inputs
B. any two of the inputs
C. any three of the inputs
D. all four inputs

Circuit with Logic Gate



Draw circuit for $Y = AB + \overline{A}C$

Boolean Algebra

Analyze and simplify the digital (logic) circuits

Commutative law

(i) $A.B = B.A$ (ii) $A + B = B + A$

Associative law

(i) $(A.B).C = A.(B.C)$ (ii) $(A + B) + C = A + (B + C)$

Distributive law

$$A.(B + C) = A.B + A.C$$

AND law

(i) $A.0 = 0$ (ii) $A.1 = A$

(iii) $A.A = A$ (iv) $A.\overline{A} = 0$

OR law

(i) $A + 0 = A$ (ii) $A + 1 = 1$

(iii) $A + A = A$ (iv) $A + \overline{A} = 1$

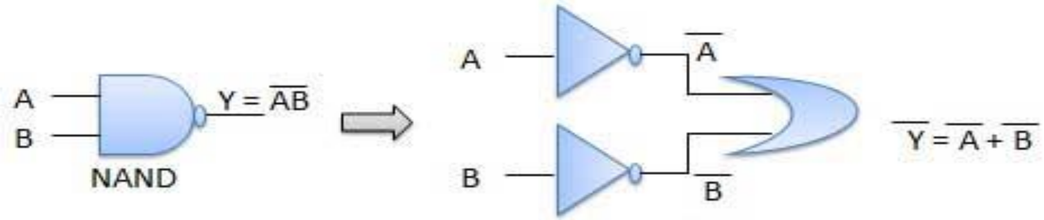
INVERSION law

$$\overline{\overline{A}} = A$$

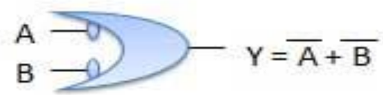
De Morgan Law

$$\overline{A.B} = \overline{A} + \overline{B}$$

NAND = Bubbled OR



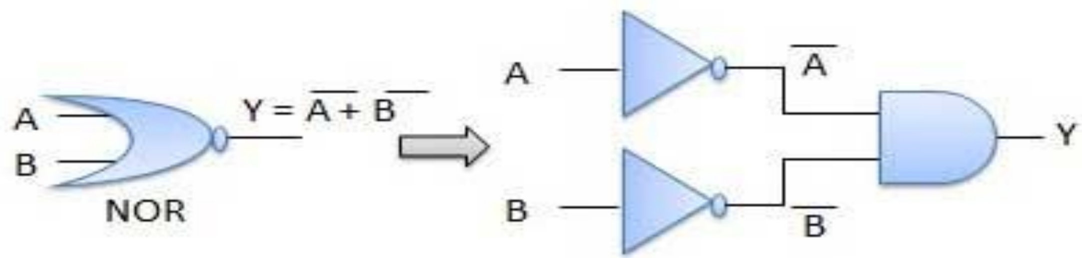
NAND $\equiv$ Bubbled OR



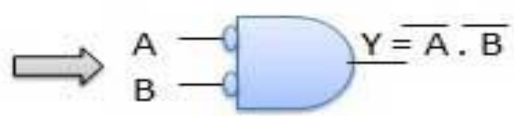
Bubbled OR

$$\overline{A + B} = \overline{A}.\overline{B}$$

NOR = Bubbled AND



NOR $\equiv$ Bubbled AND



Bubbled AND

Simplification

$$Simplify \quad C + \overline{BC}$$

$$C + (\overline{B} + \overline{C})$$

$$(C + \overline{C}) + \overline{[B]}$$

$$1 + \overline{B}$$

$$1$$

$$Simplify\ F = ABC + A + A\overline{B}C$$

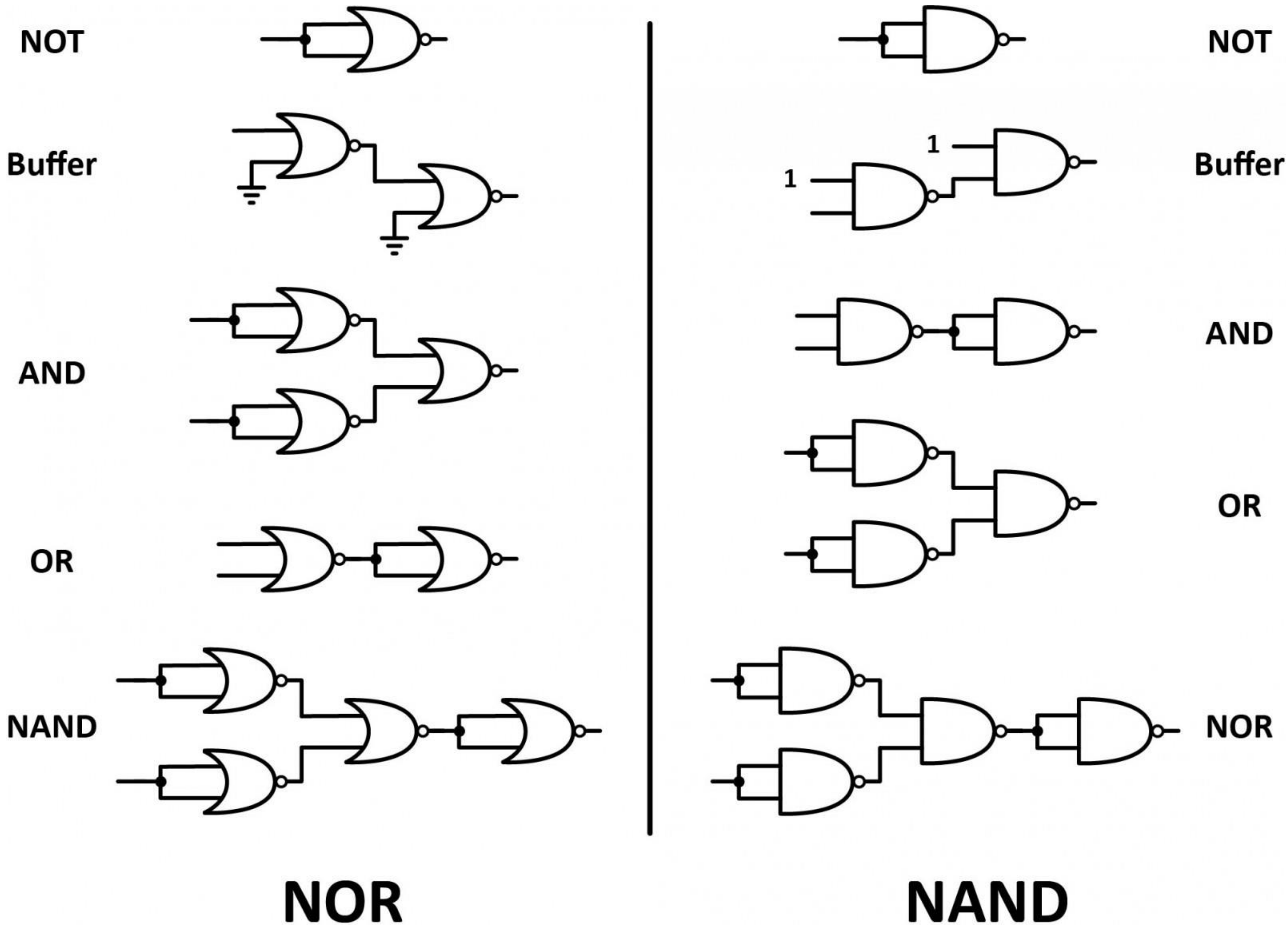
$$AC(B + \overline{B}) + A$$

$$AC + A$$

$$A(C + 1)$$

$$A$$

Logic Gate Implement with NAND-NOR



SOP-POS

Boolean function is an algebraic form of Boolean expression

Sum-of-Products (SOP) - variables are operated by AND (product) are OR(sum) together
Product-of-sums (POS) - variables are operated by OR (sum) are AND (product) together

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

SOP Expression

- Write AND term for each input combination produces HIGH
- Write the input variables for 1 and compliment for 0.
- OR the AND terms to obtain the output function.

$$F(SOP) = A'BC + AB'C + ABC' + ABC$$

POS Expression

- Write OR term for each input combination produces LOW
- Write the input variables for 0 and complement for 1
- AND the OR terms to obtain the output function

$$F(POS) = F = (A + B + C)(A + B + C')(A + B' + C)(A' + B + C)$$

POS is compliment of SOP

Min Term –Max Term

Variables			Min terms	Max terms
A	B	C	m_i	M_i
0	0	0	A' B' C' = m 0	A + B + C = M 0
0	0	1	A' B' C = m 1	A + B + C' = M 1
0	1	0	A' B C' = m 2	A + B' + C = M 2
0	1	1	A' B C = m 3	A + B' + C' = M 3
1	0	0	A B' C' = m 4	A' + B + C = M 4
1	0	1	A B' C = m 5	A' + B + C' = M 5
1	1	0	A B C' = m 6	A' + B' + C = M 6
1	1	1	A B C = m 7	A' + B' + C' = M 7

$$Write\ SOP \exp ression\ for \min term\ F(A,B,C) = \sum m(1,2,3)$$

$$In\ binary \quad 01 \quad 10 \quad 11$$

$$= \overline{A}B + A\overline{B} + AB$$

$$Write\ POS \exp ression\ for \min term\ F(A,B,C) = \pi M(1,2,3)$$

$$In\ binary = 01 \quad 10 \quad 11$$

$$(A+\overline{B}).(\overline{A}+B).(\overline{A}+\overline{B})$$

$$Write\,SOP\,\exp ression\ \ for\,\min term\ \ F(A,B)=\sum m(1,2,3)$$

$$In\,binary\quad 01\qquad 10\qquad 11$$

$$=\overline{A}B+A\overline{B}+AB$$

$$Write\,SOP\,\exp ression\ \ for\ \ F(A,B,C)=\sum m(2,4,6,7)$$

$$In\,binary\quad 010\qquad 100\qquad 110\qquad 111$$

$$\overline{A}B\overline{C}+A\overline{B}\,\overline{C}+AB\overline{C}+ABC$$

SOP-POS Conversion

- Convert the SOP expression to an equivalent POS expression:

$$\overline{A}\,\overline{B}\,\overline{C} + \overline{A}B\overline{C} + \overline{A}BC + A\overline{B}C + ABC$$

  - The evaluation is as follows:

$$000 + 010 + 011 + 101 + 111$$

  - There are 8 possible combinations. The SOP expression contains five of these, so the POS must contain the other 3 which are: 001, 100, and 110.

$$(A + B + \overline{C})(\overline{A} + B + C)(\overline{A} + \overline{B} + C)$$

MCQ

A small circle on the output of a logic gate is used to represent

A) NOT
B) BUF

Output will be a LOW for any case when one or more inputs are zero

A) AND
B) OR

How many two-input AND and OR gates are required to realize Y = CD+EF+G
A) 2,3
B) 2,2

Which is XNOR gate equation

A) AB +(~A)(~B)

B) ~AB + A(~B)

If one input of XOR gate is connected to high terminal, equivalent to

A) NOT

B) BUF

Which is not correct

A) A.1=1

B) A+A=A

C) A+1=1

D) A.A=A

POS is compliment of SOP
A) True
B) False

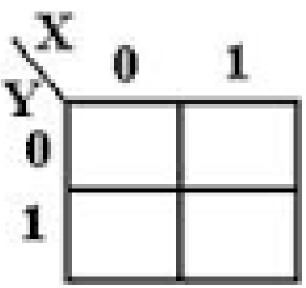
min term when x=0, y=0 and z=1
A) x'y'z
B) x+y+z’

Y=AB+BC+AC is
A) SOP
B) POS

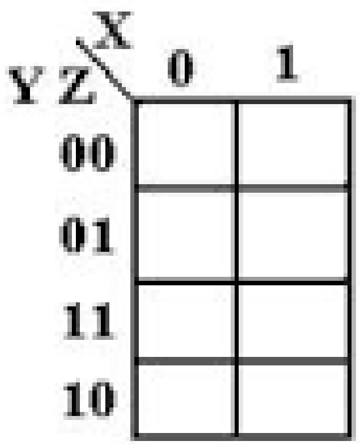
Y=(A+B)(B+C)(C+A) shows
A) POS
B) SOP

K-MAP

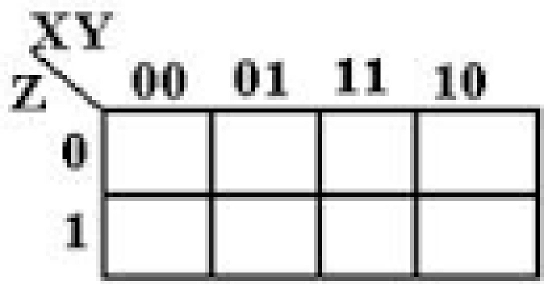
Karnaugh map is a tool for simplification of Boolean algebra
K-Map diagram is made up of squares
K-map is a graphical representation of SOP (Minterm)
K-Map extensively reduce the calculation and provides best minimized solution
K-map solve the expression with grouping of neighbor cells



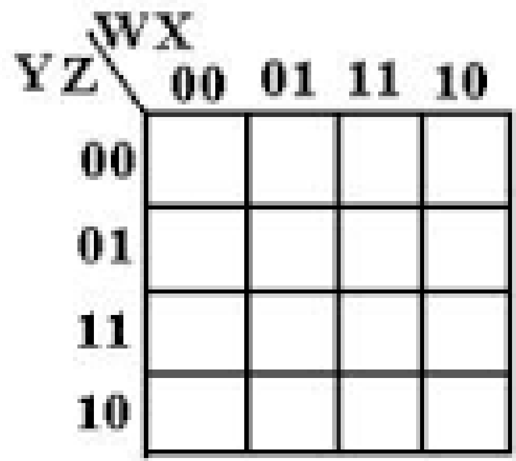
2 variable



3 variable



3 variable



4 variable

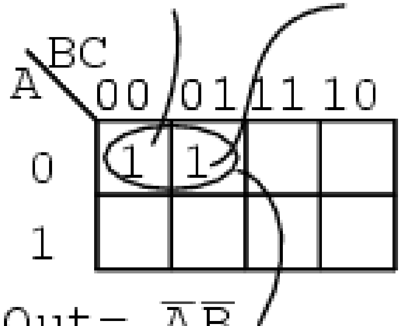
Kmap Simplification Rule

1) Construct k-map and place 1's in the squares according to the truth table.
2) Groupings can contain only 1s
3) Groups can be formed only at right angles; diagonal groups are not allowed.
4) The number of 1's in a group must be a power of 2
5) The groups must **be made as large** as possible.
6) Groups can overlap and wrap around the sides of the Kmap.
7) Every group puts a term in the solution

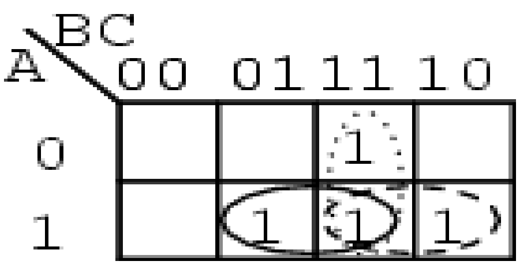
Optimized Solution
Minimum number of group
Each group covers maximum possible squares

Example

Out = $\overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,C$

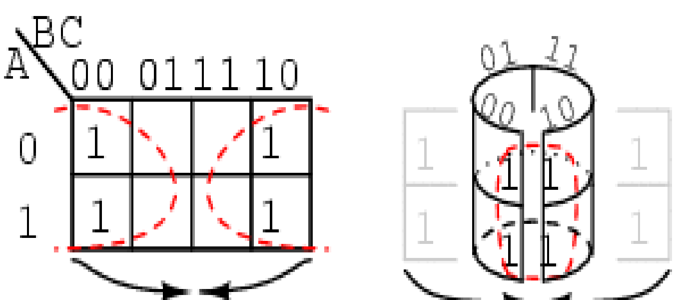


Out = $\overline{A}\,\overline{B}$



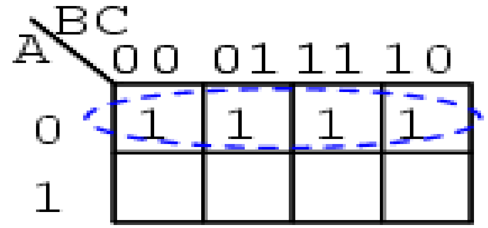
Output = AB + BC + AC

Out = $\overline{A}\,\overline{B}\,\overline{C} + A\,\overline{B}\,\overline{C} + \overline{A}\,B\,\overline{C} + A\,B\,\overline{C}$



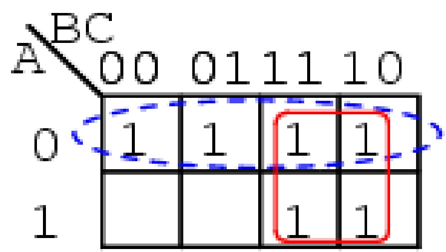
Out = $\overline{C}$

Out = $\overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,C + \overline{A}\,B\,C + \overline{A}\,B\,\overline{C}$



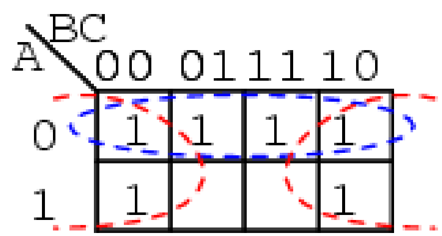
Out = $\overline{A}$

Out = $\overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,C + \overline{A}\,B\,C + \overline{A}\,B\,\overline{C} + ABC + A\,B\,\overline{C}$



Out = $\overline{A} + B$

Out = $\overline{A}\,\overline{B}\,\overline{C} + \overline{A}\,\overline{B}\,C + \overline{A}\,B\,C + \overline{A}\,B\,\overline{C} + A\,\overline{B}\,\overline{C} + A\,B\,\overline{C}$

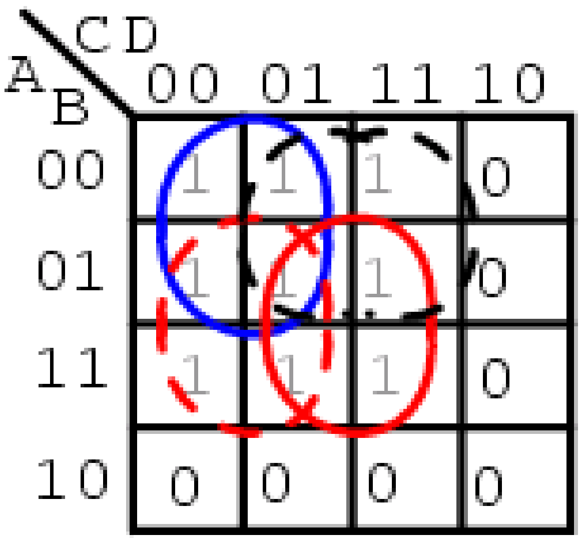


Out = $\overline{A} + \overline{C}$

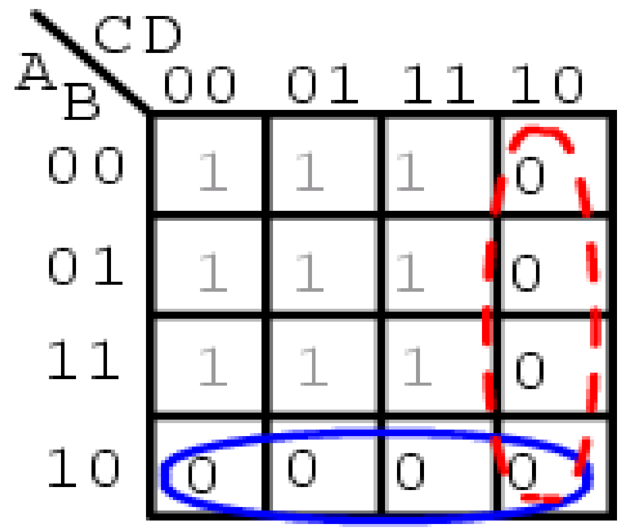
$$Out = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + \bar{A}\bar{B}CD$$
$$+ \bar{A}B\bar{C}\bar{D} + \bar{A}B\bar{C}D + \bar{A}BCD$$
$$+ AB\bar{C}\bar{D} + AB\bar{C}D + ABCD$$

$$f(A,B,C,D) = \sum m(0,1,3,4,5,7,12,13,15)$$

$$f(A,B,C,D) = \prod M(2,6,8,9,10,11,14)$$



$$\bar{A}\bar{C} + \bar{A}D + B\bar{C} + BD$$



$$f(A,B,C,D) = (\bar{A}+B)(\bar{C}+D)$$