Unit 1: Linear Algebra

(**Book:** Advanced Engineering Mathematics by Jain and Iyengar, **Chapter-3**)

Topic:

Inverse of Matrices

Learning Outcomes:

Finding the inverse of a matrix using Gauss-Jordan Method..

Inverse of a Matrix:

You already know:

Let A be a non-singular matrix ($|A| \neq 0$), then Inverse of matrix A is given by:

$$A^{-1} = \frac{adj.A}{|A|}$$

New: Gauss-Jordan Method

$$[\boldsymbol{A} \quad |\boldsymbol{I}] \quad \xrightarrow{Apply\ row(column) operations} \quad \left[\boldsymbol{I} \quad \middle| \boldsymbol{A}^{-1}\right]$$

Where I stands for Identity matrix.

By Gauss-Jordan method:

(a) If A is any non singular matrix then its inverse can be calculated

$$[A \quad | I] \xrightarrow{\hspace{1em}} \overline{Apply\ elementry\ row\ operations} \quad \left[I \quad \middle| A^{-1}\right]$$

(b) Solution of system of equations AX=B can also be obtained

$$[A \quad | B] \xrightarrow{\hspace{1em}} \overline{Apply\ elementry\ row\ operations} \quad [I \quad | X]$$

Problem 1. Using Gauss-Jordan method, find the inverse of the following:

$$A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$$

Solution. By Gauss-Jordan method:

$$[\boldsymbol{A} \quad |\boldsymbol{I}] = \left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 3 & 2 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2-3R_1} \left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 0 & -10 & -3 & 1 \end{array}\right] \xrightarrow{R_2/-10}$$

$$\left[\begin{array}{cc|cc} 1 & 4 & 1 & 0 \\ 0 & 1 & 3/10 & -1/10 \end{array}\right]$$

$$\xrightarrow{R_1 - 4R_2} \left[\begin{array}{cc|cc} 1 & 0 & -2/10 & 4/10 \\ 0 & 1 & 3/10 & -1/10 \end{array}\right]$$

$$= \left[\boldsymbol{I} \;\middle|\; \boldsymbol{A}^{-1}\right]$$

Hence, $\boldsymbol{A}^{-1} = \begin{bmatrix} -2/10 & 4/10 \\ 3/10 & -1/10 \end{bmatrix}$ **Answer.**

Problem 2. Using Gauss-Jordan method, find the inverse of the following:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

Solution. By Gauss-Jordan method:

$$[\boldsymbol{A} \quad |\boldsymbol{I}] = \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 1 & 3 & 4 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 2 & 3 & 0 & 1 & 0 \\ 1 & 3 & 4 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2-R_1, R_3-R_1} \left[\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 2 & 3 & -1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_3-2R_2, R_1-R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 2 & -1 & 0 \\ 0 & 1 & 2 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 & -2 & 1 \end{array}\right]$$

$$\xrightarrow{R_2+2R_3, R_1-R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & 1 & -3 & 2 \\ 0 & 0 & -1 & 1 & -2 & 1 \end{array}\right]$$

$$\xrightarrow{R_3/-1}\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & -1 \\ 0 & 1 & 0 & 1 & -3 & 2 \\ 0 & 0 & 1 & -1 & 2 & -1 \end{array}\right] = \left[\boldsymbol{I} \;\middle|\; \boldsymbol{A^{-1}}\right]$$

Hence, $\boldsymbol{A^{-1}} = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -3 & 2 \\ -1 & 2 & -1 \end{bmatrix}$ **Answer.**

Problem 3. Using Gauss-Jordan method, find the inverse of the following:

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 3 & 3 \\ 0 & 1 & 2 \end{bmatrix}$$

Solution. By Gauss-Jordan method:

$$[\boldsymbol{A} \quad |\boldsymbol{I}] = \left[\begin{array}{ccc|ccc} 2 & 3 & 1 & 1 & 0 & 0 \\ 1 & 3 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_2} \left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 0 & 1 & 0 \\ 2 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 0 & 1 & 0 \\ 2 & 3 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2-2R_1} \left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 0 & 1 & 0 \\ 0 & -3 & -5 & 1 & -2 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_2\leftrightarrow R_3} \left[\begin{array}{ccc|ccc} 1 & 3 & 3 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \\ 0 & -3 & -5 & 1 & -2 & 0 \end{array}\right]$$

$$\xrightarrow{R_3+3R_2, R_1-3R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & -3 & 0 & 1 & -3 \\ 0 & 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & -2 & 3 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -3 & 0 & 1 & -3 \\ 0 & 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & -2 & 3 \end{array}\right]$$

$$\xrightarrow{R_2-2R_3, R_1+3R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -5 & 6 \\ 0 & 1 & 0 & -2 & 4 & -5 \\ 0 & 0 & 1 & 1 & -2 & 3 \end{array}\right] = \left[\boldsymbol{I} \quad \middle| \boldsymbol{A^{-1}}\right]$$

Hence, $\boldsymbol{A^{-1}} = \begin{bmatrix} 3 & -5 & 6 \\ -2 & 4 & -5 \\ 1 & -2 & 3 \end{bmatrix}$ **Answer.**