REG. NO.: 11920ZMTH94890

COURSE CODE : MTH166

COURSE TITLE : DIFFERENTIAL EQUATIONS AND VECTOR CALCULUS

Time Allowed: 01 hr *Max. Marks: 40*

Read the following instructions carefully before attempting the question paper.

1. Match the Paper Code shaded on the OMR Sheet with the Paper code mentioned on the question paper and ensure that both are the same.

2. This paper contains 40 questions of 1 mark each. 0.25 marks will be deducted for each wrong answer.

3. Do not write or mark anything on the question paper except your registration no. on the designated space.

4. Submit the question paper and the rough sheet(s) along with the OMR sheet to the invigilator before leaving the examination hall.

Q1. If $M(x,y)dx + N(x,y)dy = 0$ is an exact differential equation, then its solution can be written as:

(a) $\int_{y=constant} Mdx + \int(Terms\ of\ N\ not\ containing\ x)dy = c$

(b) $\int_{x=constant} Mdy + \int(Terms\ of\ N\ not\ containing\ x)dy = c$

(c) $\int_{y=constant} Mdx + \int(Terms\ of\ N\ containing\ x)dy = c$

(d) $\int_{y=constant} Mdx + \int Ndy = c$

Q2. The equation $(x^2 - ay)dx + (y^2 - ax)dy = 0$ is:

(a) An exact differential equation
(b) Homogeneous differential equation
(c) Non-exact differential equation
(d) Bernoulli equation

Q3. The equation $ye^{xy}dx + (xe^{xy} + 2y)dy = 0$ is:

(a) An exact differential equation
(b) Homogeneous differential equation
(c) Non-exact differential equation
(d) Bernoulli equation

Q4. The equation: $y\sin 2x\, dx - (1 + y^2 + cos^2x)dy = 0$ is:

(a) An exact differential equation
(b) Homogeneous differential equation
(c) Non-exact differential equation
(d) Bernoulli equation

Q5. The expression $\frac{xdy-ydx}{y^2}$ is equal to:

(a) $d\left(\frac{y}{x}\right)$ (b) $-d\left(\frac{x}{y}\right)$ (c) $d[log\left(\frac{y}{x}\right)]$ (d) $d\left(tan^{-}\frac{y}{x}\right)$

Q6. The expression $\frac{xdy-ydx}{xy}$ is equal to:

(a) $d\left(\frac{y}{x}\right)$ (b) $d[log\left(\frac{y}{x}\right)]$ (c) $-d\left(\frac{x}{y}\right)$ (d) $d\left(tan^{-}\frac{y}{x}\right)$

Q7. The solution of differential equation $xdx + ydy = a^2\left(\frac{xdy-ydx}{x^2+y^2}\right)dx$ is:

(a) $tan^{-1}\left(\frac{y}{x}\right) + ax = c$
(b) $x^2 + y^2 = 2a^2tan^{-1}\left(\frac{y}{x}\right) + c$
(c) $x^2 + y^2 + ax = c$
(d) $x^2 + y^2 + tan^{-1}\left(\frac{y}{x}\right) + ax = c$

Q8. The general solution of the differential equation $xp^2 - yp + a = 0$ is:

(a) $y = cx - e^c$ (b) $y = cx + \frac{a}{c}$ (c) $y = cx + \sqrt{a^2c^2 + b^2}$ (d) $y = cx - sin^{-1}c$

Q9.The general solution of the differential equation $p = log(px - y)$ is:

(a) $y = cx + \sqrt{a^2c^2 + b^2}$
(b) $y = cx + \frac{a}{c}$
(c) $y = cx - e^c$
(d) $y = cx - sin^{-1}c$

Q10.The solution of differential equation $ydx - xdy + 3x^2y^2e^{x^3}dx = 0$ is:

(a) $x^2 + y^2 + ax = c$
(b) $\frac{e^x}{y} + x^2 = c$
(c) $\frac{x}{y} + e^{x^3} = c$
(d) $x^2 + y^2 + tan^{-1}\left(\frac{y}{x}\right) + ax = c$

Q11.Integrating Factor (I.F.) for the equation $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)dy = 0$ is:

(a) $\frac{1}{x^4}$ (b) $\frac{1}{2x^2y^2}$ (c) $\frac{1}{x^2y^2}$ (d) $\frac{1}{y}$

Q12.Integrating Factor (I.F.) for the equation $(1 + xy)ydx + (1 - xy)xdy = 0$ is:

(a) $\frac{1}{x^4}$ (b) $\frac{1}{x^2y^2}$ (c) $\frac{1}{2x^2y^2}$ (d) $\frac{1}{y}$

Q13.Integrating Factor (I.F.) for the equation $(y^2x + y)dx + 2(x^2y^2 + x + y^4)dy = 0$ is:

(a) $\frac{1}{x}$ (b) $\frac{1}{2x^2y^2}$ (c) $\frac{1}{x^4}$ (d) $\frac{1}{y}$

Q14.Let $\{y_1, y_2\}$ be the basis of $(1 + x^2)y'' - 2xy' + 12y = 0$, then $W(y_1, y_2) =$

(a) e^x (b) x^2 (c) x (d) $1 + x^2$

Q15.If $y = e^t$ is solution of $\frac{d^2y}{dt^2} + \frac{dy}{dt} + ky = 0$, then value of k is

(a) 2 (b) -2 (c) 4 (d) -1

Q16.The general solution of $y^{iv} - 8y'' + 16y = 0$ is

(a) $(c_1 + c_2x)e^{2x} + (c_3 + c_4x)e^{-2x}$
(b) $(c_1 + c_2x)\cos 2x + (c_3 + c_4x)\sin 2x$
(c) $e^x\big((c_1 + c_2x)\cos 2x + (c_3 + c_4x)\sin 2x\big)$
(d) $c_1e^{2x} + c_2e^{-2x} + c_3\cos 2x + c_4\sin 2x$

Q17.Let $f_1 = x^2, f_2 = 1 + 3x^2, f_3 = x^4, f_4 = 1$, then Wronskian $W(f_1, f_2, f_3, f_4) =$

(a) $6x$ (b) 2 (c) $3 - 4x^2$ (d) 0

Q18.In which of the following interval, differential equation $(3x - x^2)y'' + xy' = 0$ is normal?

(a) $[0,3]$ (b) $[0,2]$ (c) $[1,2]$ (d) $[3,5]$

Q19.For the initial value problem $y'' - 4y' + 3y = 0, y(0) = 1, y'(0) = -1$, difference of arbitrary constant is

(a) 1 (b) 2 (c) 3 (d) 0

Q20.Which of the following interval guarantee the existence and uniqueness of solution to initial value problem$(1 - x^2)y'' - 2xy' + 20y = 0,\ y(0) = 1, y'(0) = 2$?

(a) $[-3,0]$ (b) $[0,2]$ (c) $[0,0.5]$ (d) $[-1,4]$

Q21.Which of the following functions are linearly independent?

(a) $\{x^2, x, 1 - 2x, 1\}$
(b) $\{1, \sin^2 x, \cos^2 x\}$
(c) $\{\cos x, \cos 2x, \cos 3x\}$
(d) $\{2x,\ x^2, 4x + x^2\}$

Q22.The general solution of $y''' + 16y'' = 0$ is

(a) $(c_1 + c_2x) + c_2e^{4x}$
(b) $(c_1 + c_2x) + c_2e^{-16x}$
(c) $c_1 + c_2e^{4x} + c_3e^{-4x}$
(d) $c_1 + c_2\cos 4x + c_3\sin 4x$

Q23.What is lowest order of homogeneous linear homogeneous differential equation with constant coefficient whose particular solution is $(1 - x)e^{2x} + 2x^2e^{-2x}$

(a) 2 (b) 3 (c) 4 (d) 5

Q24.Roots of auxiliary equation of a homogeneous linear DE with real constant coefficients having $xe^{-x}+(3+x)e^{2x}$ as its particular solution are

(a) 1, 1, 3 (b) −1, 2, 2, 3 (c) −1, −1, 2, 2 (d) −1, 2, 2

Q25.The differential equation whose two linear independent solutions are e^{-2x}, e^{2x} is

(a) $y'-2y=0$ (b) $y''-4y=0$ (c) $y''-4y'+4y=0$ (d) $y''+4y=0$

Q26.The linear homogeneous differential equation with basis $\{e^x, \cos x, \sin x\}$ is given by

(a) $y'''-3y''+4y'-2y=0$ (b) $y'''-y''+y'-y=0$

(c) $y'''+y'=0$ (d) $y'''-y=0$

Q27. The P.I. of the differential equation $(D^3+9)y=(x-1)/5$ is

a) $\frac{x-1}{5}$ b) $\frac{x-1}{9}$ c) $\frac{x-1}{45}$ d) $-\frac{x-1}{45}$

Q28. The P.I. of the differential equation $(D^2+4D+4)y=9e^x$ is

a) e^x b) e^{2x} c) $9e^x$ d) None of these

Q29. The P.I. of the differential equation $(D^2-1)y=x^3+2x-1$ is

a) $-(x^3-4x-1)$ b) $-(x^3+8x-1)$ c) (x^3+8x-1) d) (x^3-4x-1)

Q30. The P.I. of the differential equation $\frac{d^2y}{dx^2}+\frac{dy}{dx}+y=e^x\sin x$ is

a) $-\frac{e^x}{13}(3\cos x-2\sin x)$ b) $\frac{e^x}{13}(3\cos x-2\sin x)$

c) $-e^x(3\cos x-2\sin x)$ d) $-\frac{e^x}{13}(3\cos x+2\sin x)$

Q31. $P.I.=\frac{1}{f(D)}x^2\cos 2x$ is equal to

a) real part of $e^{ix}\frac{1}{f(D+i)}x^2$ b) Imaginary part of $\frac{1}{f(D+i)}x^2e^{ix}$

c) real part of $\frac{1}{f(D)}x^2e^{2ix}$ d) Imaginary part of $\frac{1}{f(D)}x^2e^{2ix}$

Q32.. $P.I.=\frac{1}{f(D)}e^{ax}\phi(x)$ is equal to

) $e^{ax}\frac{1}{f(D+a)}\phi(x)$ b) $e^{ax}\frac{1}{f(D-a)}\phi(x)$

) $e^{ax}\frac{1}{f(D+ia)}\phi(x)$ d) $e^{ax}\frac{1}{f(D-ia)}\phi(x)$

Q33. $\frac{1}{(D^2+2D+1)}\left(\frac{e^{-x}}{(x+2)}\right)=$

a) $e^{x}[(x+2)\log(x+2)-x]$

b) $e^{x}[(x+2)\log(x+2)+x]$

c) $e^{-x}[(x+2)\log(x+2)-x]$

d) $e^{-x}[(x+2)\log(x+2)+x]$

Q34. $\frac{1}{(D+a)}\phi(x)=$

a) $e^{ax}\int e^{-ax}\phi(x)dx$

b) $e^{-ax}\int e^{ax}\phi(x)dx$

c) $e^{ax}\int e^{ax}\phi(x)dx$

d) None of these

Q35. The Cauchy Euler differential $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=e^x$ can be converted into a linear differential equation with constant coefficient by the substitution $x=e^z$. Which of the following is the converted differential equation?

a) $(D^2+2D+3)y=e^{e^z}$

b) $(D^2+3D+2)y=e^{e^z}$

c) $(D^2-2D+3)y=e^{e^z}$

d) $(D^2+2D-3)y=e^{e^z}$

Q36. The P.I. of the differential equation $(2D^2-1)y=x^3$; is

a) x^3+12x

b) $-x^3+12x$

c) x^3-12x

d) $-(x^3+12x)$

Q37. If the P.I. of a differential equation $f(D)y=\phi(x)$ by variation of parameter is uy_1+vy_2 then

a) $u=\int\frac{y_2\phi(x)}{y_1y_2'-y_1'y_2}dx$

b) $u=-\int\frac{y_2\phi(x)}{y_1y_2'-y_1'y_2}dx$

c) $u=\int\frac{y_2\phi(x)}{y_1y_2'+y_1'y_2}dx$

d) $u=-\int\frac{y_2\phi(x)}{y_1y_2'+y_1'y_2}dx$

Q38. The trial solution for finding the P.I. of the differential equation $(D^3+3D^2+3D+1)y=e^{3x}\sin 4x$

a) $Ae^{3x}\sin 4x$

b) $Ae^{3x}\cos 4x$

c) $Ae^{3x}(\sin 4x+\cos 4x)$

d) Both (a) and (b)

Q39. The trial solution for finding the P.I. of the differential equation $(D^2+3D+1)y=x^3+2x^2$

a) $C_0+C_1x+C_2x^2$

b) $C_0+C_1x+C_3x^2+C_4x^3$

c) $C_0+C_1x+C_3x^3$

d) $C_0x+C_1x^2+Cx^3$

Q40. The trial solution for finding the P.I. of the differential equation $(D^3+4D^2+3D)y=2xe^{-5x}$

a) $(C_0+C_1x)e^{-5x}$

b) $(C_0+C_1x+C_3x)e^{-5x}$

c) $(C_0+C_1x)e^{5x}$

d. None of these

-- End of Question Paper --