ECE-249

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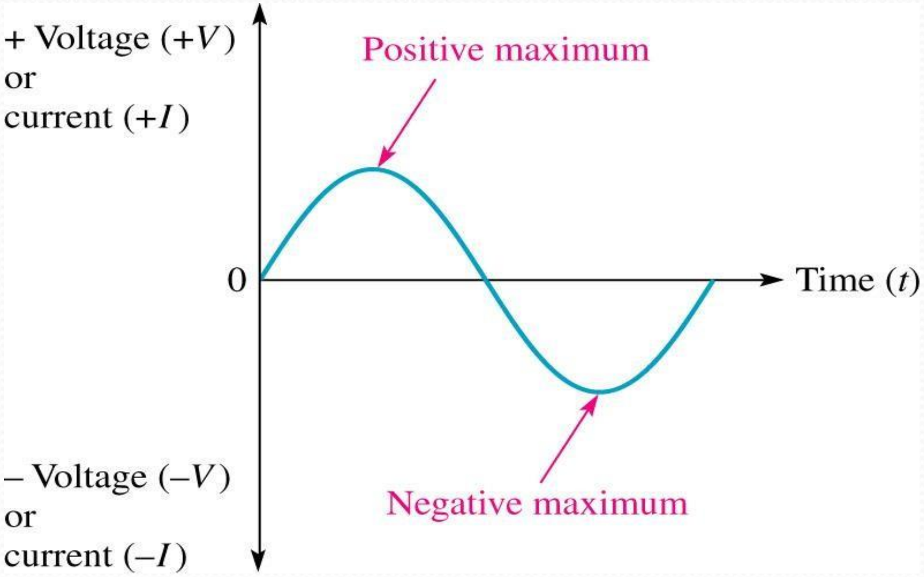
Unit-2

Fundamentals of A.C. circuits



Contents:

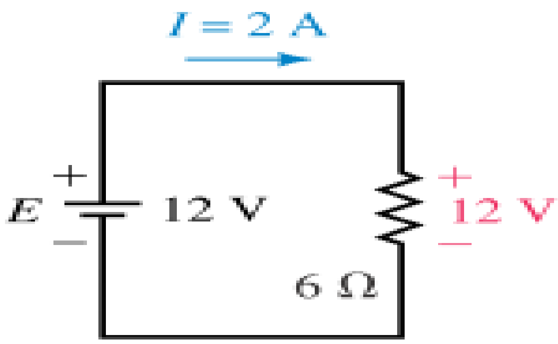
- Alternating Current
- Generating Ac Voltages
- Determination of Frequency (f) In The Ac Generator Fundamental
- Periodic Voltage Or Current Waveform
  - Average Value
  - Root Mean Square (RMS) Value
  - Average And RMS Values Of Sinusoidal Voltage Waveform
- Single-phase AC Supply
  - Purely Resistive Circuit (R Only)
  - Purely Inductive Circuit (L Only)
  - Purely Capacitive Circuit (C Only)

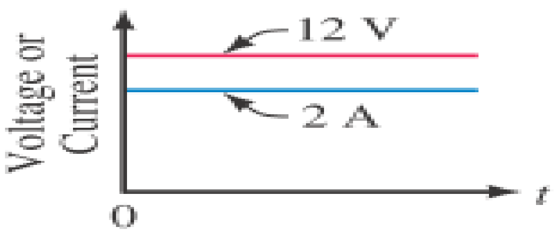
Cont:

- series A.C. circuit
  - R–L series A.C. circuit
  - R–C series A.C. circuit
  - R–L–C series A.C. circuit
- Three-phase AC Circuits
  - Generation of Three-phase Balanced Voltages
  - Delta(Δ)-Star(Y) conversion and Star-Delta conversion
  - Delta(Δ)-Star(Y) conversion and Star-Delta conversion impedance conduction

AC Fundamentals

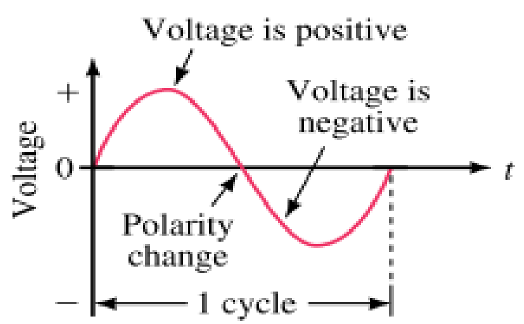
- Previously you learned that DC sources have fixed polarities and constant magnitudes and thus produce currents with constant value and unchanging direction





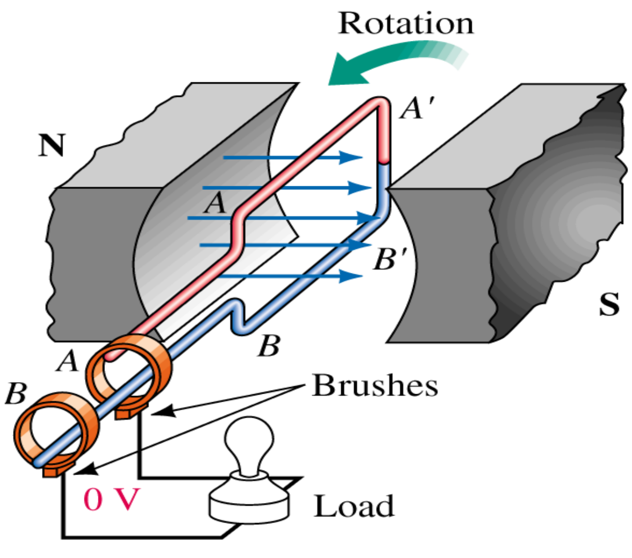
(b) Voltage and current versus time for dc

- In contrast, the voltages of ac sources alternate in polarity and vary in magnitude and thus produce currents that vary in magnitude and alternate in direction.

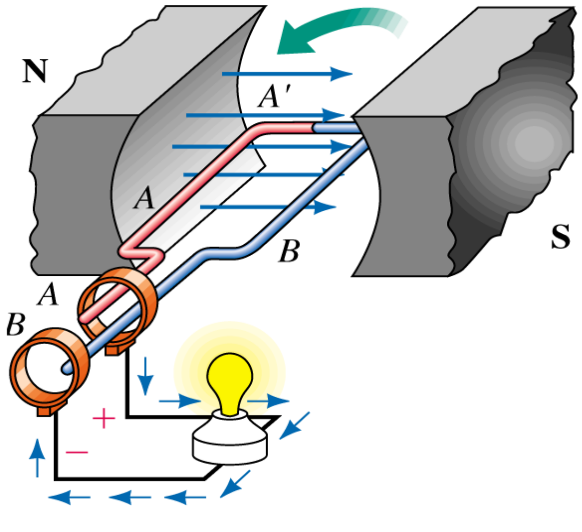


Generating AC Voltages

- Generators convert rotational energy to electrical energy.
- Generate alternative emf by rotating a coil within a stationary magnetic field.
- Another way to rotating magnetic field a within a stationary coil.



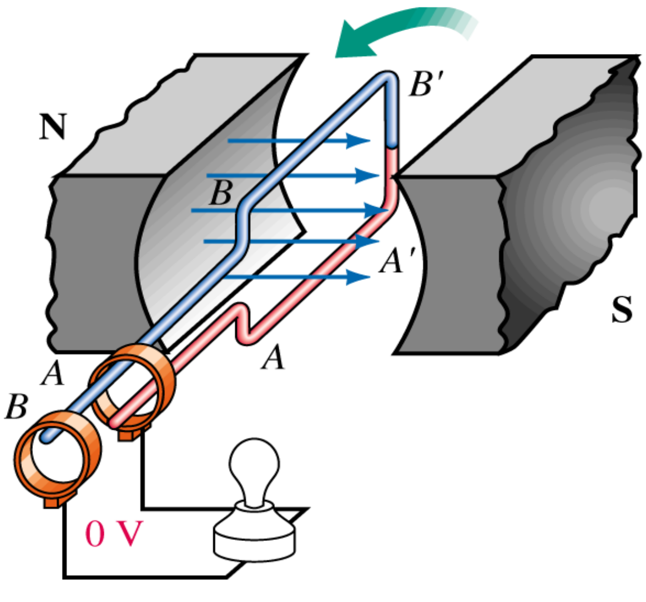
(a) 0° Position: Coil sides move parallel to flux lines. Since no flux is being cut, induced voltage is zero.



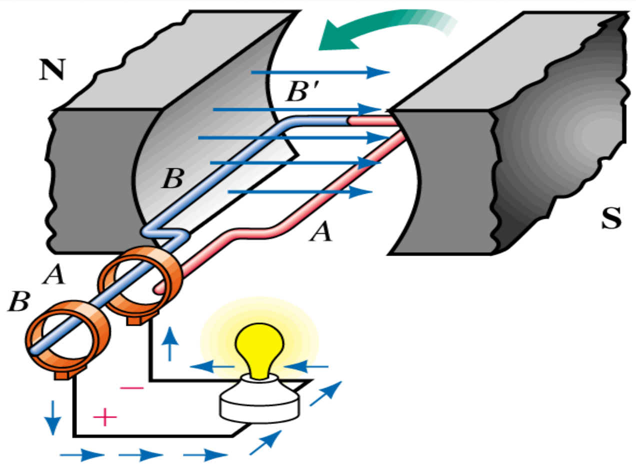
(b) 90° Position: Coil end *A* is positive with respect to *B*. Current direction is out of slip ring *A*.

Cont….

- The armature has an induced voltage, which is connected through slip rings and brushes to a load.
- The armature loops are wound on a magnetic core



(c) 180° Position: Coil again cutting no flux. Induced voltage is zero.

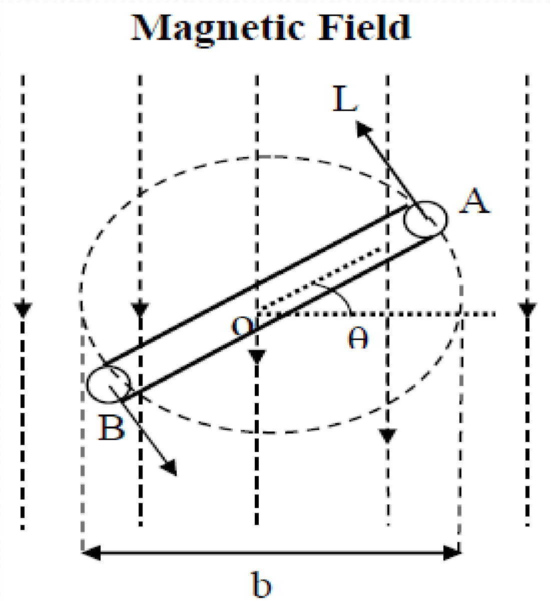


(d) 270° Position: Voltage polarity has reversed, therefore, current direction reverses.

- A multi-turn coil is placed inside a magnet with an air gap as shown in above Fig.
- The flux lines are from North Pole to South Pole. The coil is rotated at an angular speed,

$$\omega = 2\pi n \text{ (rad/S)}$$

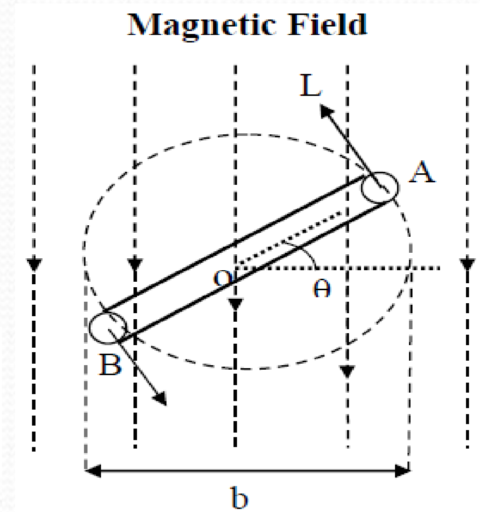
- N is rotating a rectangular coil in a counter clockwise direction with a angular velocity of ω radians per sec in a uniform magnetic field.
- The instant of coincidence of the plane of the coil with X-axis. At this instant max flux, Φ_{max} link with the coil



- The coil assume the position, as shown in fig , after moving the counter clockwise for t sec.
- The angle θ through which the coil has rotated in sec = ω t.

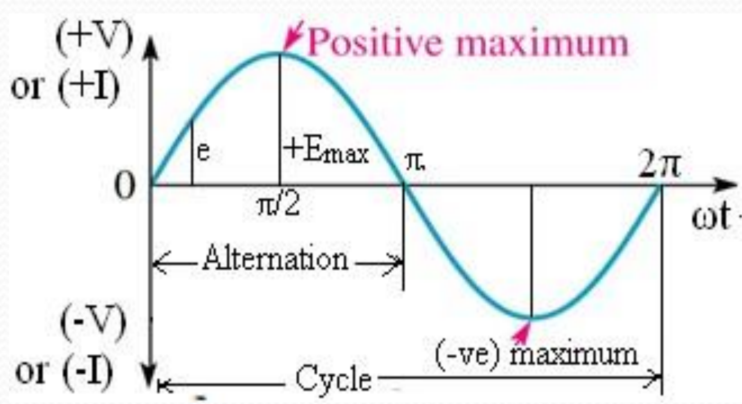
- The component of flux along perpendicular to the coil = Φ_{max} Cos ωt.
- Flux linkage of the coil at the instant = no. of turns on coil x linkage flux

  $N \ \Phi_{max} \cos \omega t$
- emf induced in a coil is equal to the rate of change of the flux linkage with minus sign .
- emf induced is max at any instant.



$$e = -\frac{d}{dt}[N\Phi_{max}\cos \omega t] \quad = \quad N\Phi_{max}\frac{d}{dt}[-\cos \omega t]$$

$$e = N\Phi_{max}\,\omega \sin \omega t$$



- Instantaneous emf , e = $E_{max} \sin \omega t$

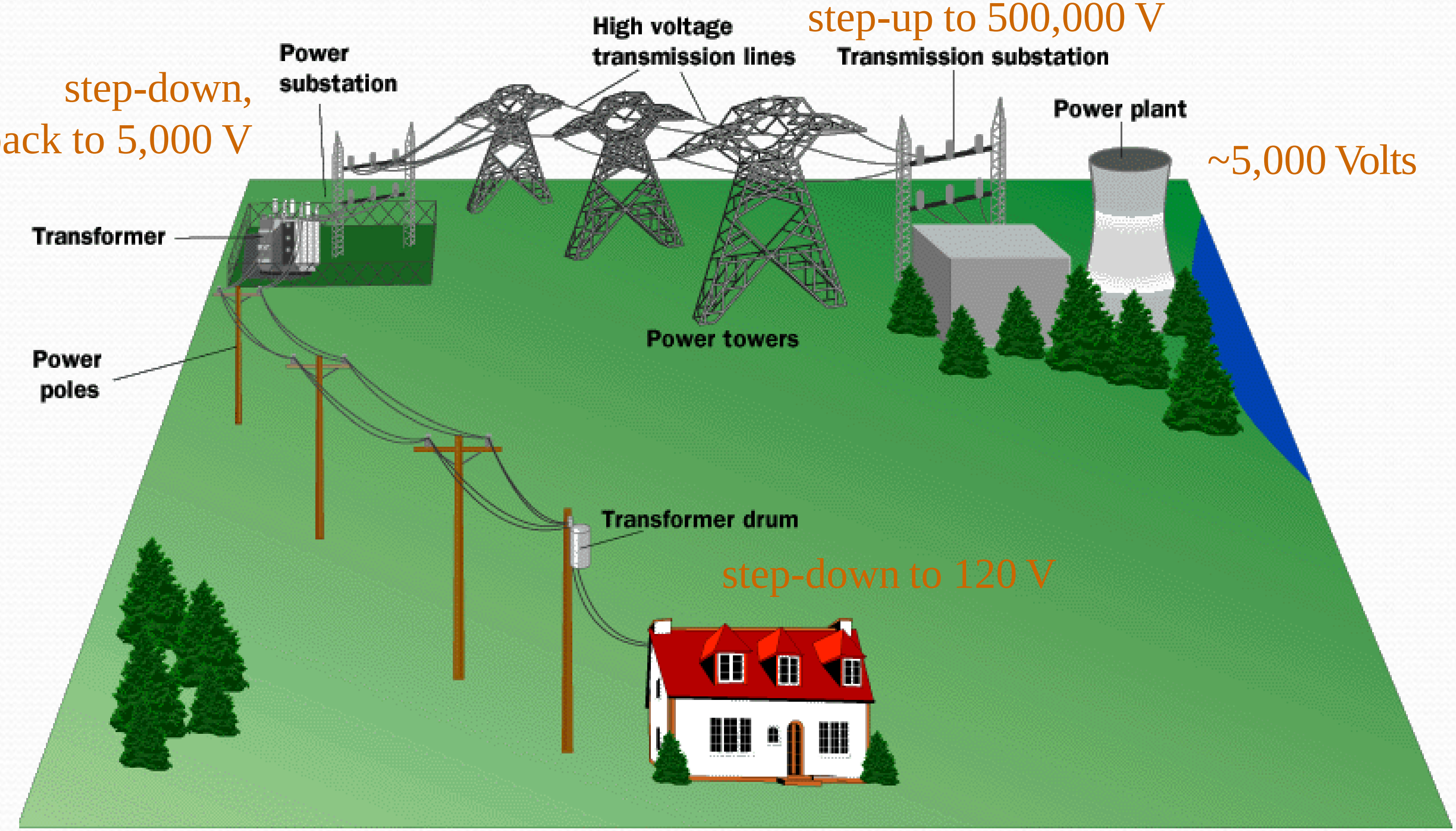
A way to provide high efficiency, safe low voltage:

step-down, back to 5,000 V

step-up to 500,000 V

~5,000 Volts

step-down to 120 V



High Voltage Transmission Lines
Low Voltage to Consumers

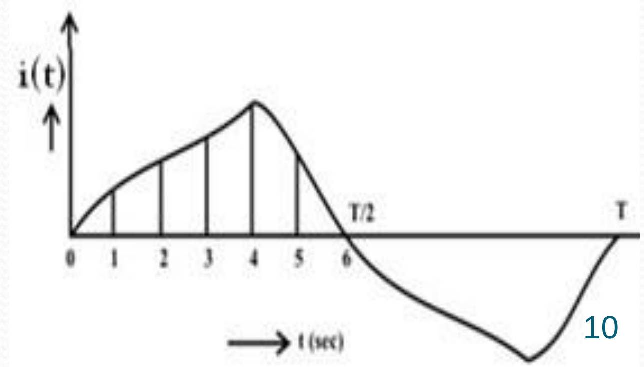
Periodic Voltage or Current Waveform

- Average value
  - The current waveform shown in Fig, is periodic in nature, with time period, T. It is positive for first half cycle, while it is negative for second half cycle.
  - The average value of the waveform, i(t) is defined as

$$I_{av} = \frac{Area\ over\ half\ cycle}{Time\ period\ of\ half\ cycle} = \frac{1}{T/2}\int_0^{T/2} i(t)\,dt = \frac{2}{T}\int_0^{T/2} i(t)\,dt$$

- In this case, only half cycle, or half of the time period, is to be used for computing the average value, as the average value of the waveform over full cycle is zero (0).
- If the half time period (T/2) is divided into 6 equal time intervals (ΔT)

$$I_{av} = \frac{(i_1 + i_2 + i_3 + \cdots i_6)\Delta T}{6 \cdot \Delta T} = \frac{(i_1 + i_2 + i_3 + \cdots i_6)}{6}$$



Root Mean Square (RMS) value

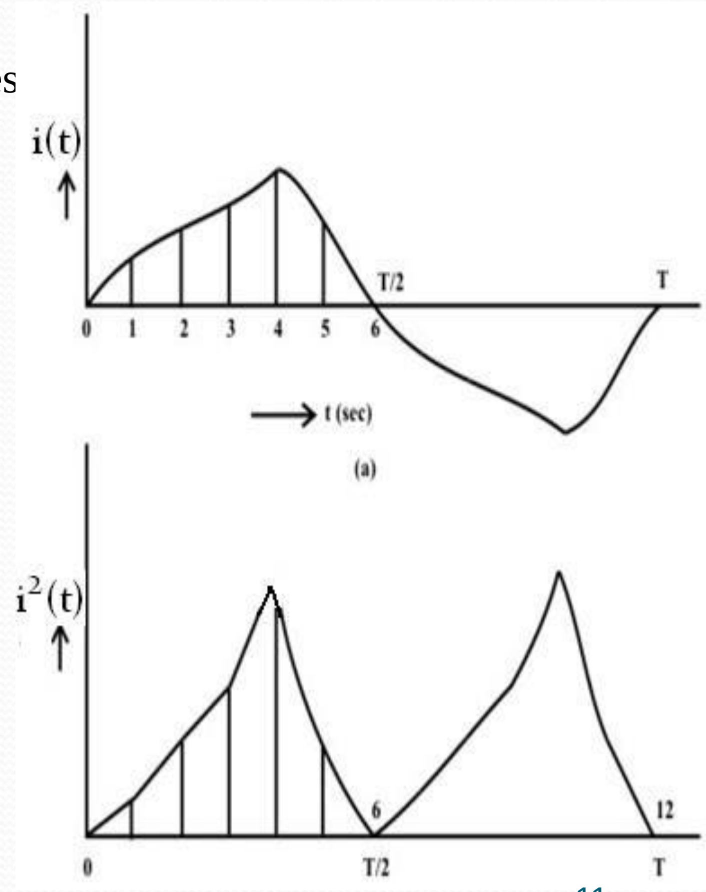
- For this current in half time period subdivided into 6 time intervals as given above, in the resistance R, the average value of energy dissipated is given by
  - Heat produce during interval = $I^2 R \frac{T}{n}$ joules

$$\propto \left[\frac{(i_1^2 + i_2^2 + i_3^2 + \cdots i_6^2)}{6} \right] R$$

$$I^2 R\, \Delta T = \left[\frac{(i_1^2 + i_2^2 + i_3^2 + \cdots i_n^2)\, \Delta T}{n \cdot} \right] R$$

$$I = \sqrt{\frac{(i_1^2 + i_2^2 + i_3^2 + \cdots i_n^2)\, \Delta T}{n \cdot \Delta T}} = \sqrt{\frac{\text{Area of } i^2 \text{ curve over half cycle}}{\text{Time period of half cycle}}}$$

$$= \sqrt{\frac{1}{T/2} \int_0^{T/2} i^2\, dt} = \sqrt{\frac{2}{T} \int_0^{T/2} i^2\, dt}$$



Average Values of Sinusoidal Voltage Waveform

- The average value of sine wave over the complete cycle is zero.

- The half wave value of sinusoidal current is

$$I = I_{max}\ \sin\omega t$$

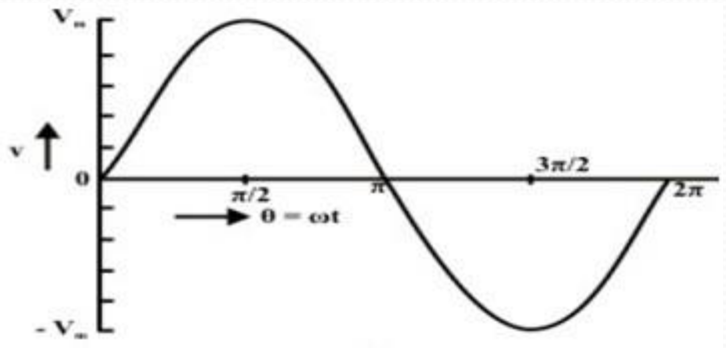
- For half cycle when ωt varies from 0 to π

$$I_{AV} = \frac{area\ of\ half\ cycle}{\pi}$$

$$I_{AV} = \frac{1}{\pi}\int_0^{\pi} i\,d(\omega t)$$

$$I_{AV} = \frac{1}{\pi}\int_0^{\pi} I_{max}\ \sin\omega t\ d(\omega t)$$

$$I_{AV} = \frac{I_{max}}{\pi}[-\cos\omega t] = \frac{2\ I_{max}}{\pi} = 0.637\ I_{max}$$



- Similarly

$$V_{AV} = \frac{2\ V_{max}}{\pi} = 0.637\ V_{max}$$

RMS Values of Sinusoidal Voltage Waveform

- The waveform of the voltage v(t), and the square of waveform, $v^2(t)$, are shown in figures 12.4a and 12.4b respectively.

Time period, $T = 1/f = (2\pi)/\omega$; in angle $(\omega T = 2\pi)$

Half time period, $T/2 = 1/(2f) = \pi/\omega$; in angle $(\omega T/2 = \pi)$

$v(\theta) = V_m \sin\theta \quad for \quad \pi \le \theta \le 0;$

$$V_{\text{rms}} = \left[\frac{1}{\pi}\int_0^{\pi} v^2 \, d\theta\right]^{\frac{1}{2}} = \left[\frac{1}{\pi}\int_0^{\pi} V_m^2 \sin^2\theta \, d\theta\right]^{\frac{1}{2}} = \left[\frac{V_m^2}{\pi}\int_0^{\pi} \frac{1}{2}(1-\cos 2\theta)\, d\theta\right]^{\frac{1}{2}}$$

$$V_{\text{rms}} = \left[\frac{V_m^2}{2\pi}\left(\theta - \frac{1}{2}\sin 2\theta\right)\Big|_0^{\pi}\right]^{\frac{1}{2}} = \left[\frac{V_m^2}{2\pi}\pi\right]^{\frac{1}{2}} = \frac{V_m}{\sqrt{2}} = 0.707\, V_m$$

or, $V_m = \sqrt{2}\, V_{\text{rms}}$

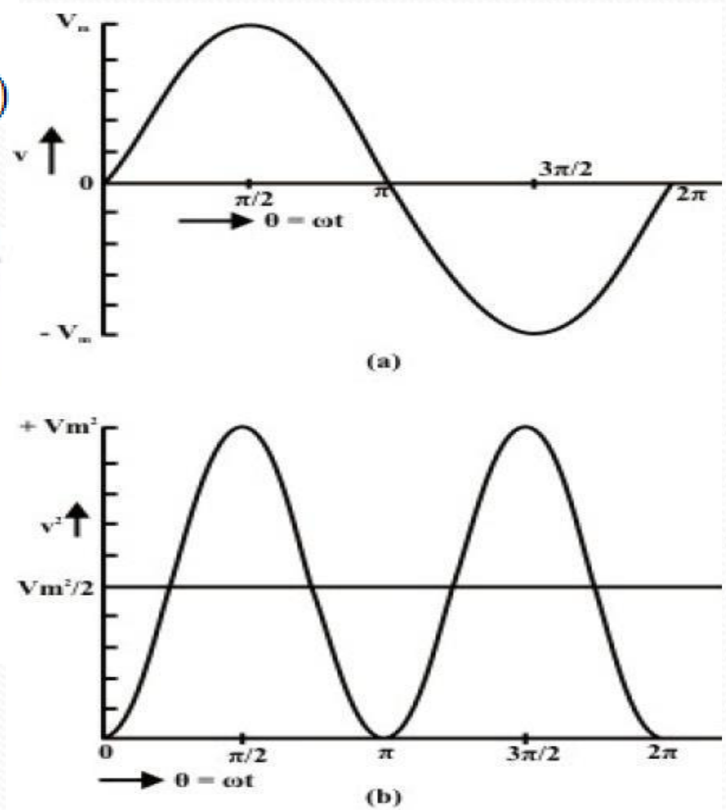


Fig. 12.4 Sinusoidal voltage waveform
(a) Voltage (v), (b) Square of voltage (v²)

- Similarly for current RMA value

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = 0.707\, I_{max}$$

Form factors

- The form factor of an alternating quantity is define as the ratio of RMS value to the average value.

$$\text{Form factor} = \frac{RMS\ value}{Average\ value} = \frac{0.707\ V_m}{0.637\ V_m} = 1.11$$

- **<u>Peak value:-</u>**
- The peck factor of an alternating quantity is define as the ratio of maximum value to the average value

$$\text{Peak factor (P.E.)} = \frac{Maximum\ value}{RMS\ value}$$

$$= \frac{Peak\ Value}{Peak\ Value/\sqrt{2}} = \sqrt{2}$$

$$\text{P.F.} = 1.414$$

- **<u>NOTE:-</u>**
  - The rms value is always greater than the average value.
  - Except for a rectangular waveform, in which case the heating effect remains constant, so that the average and the rms values are same

Question

Q1) determine the average value and RMS value of sinusoidal current of peak value 40A.

Solution:- Imax = 40A

$$I_{rms} = \frac{I_{max}}{\sqrt{2}}$$

$$I_{av} = 0.637\, I_{max}$$

Q2) write the instantaneous value for a 50Hz sinusoidal voltage supply for domestic purposes at 230V.

Solution:- given value Vrms = 230 V. , f = 50 Hz

$$V(t) = V_{max} \sin \omega t$$

$$V_{max} = \sqrt{2} \times V_{rms}$$

$$\omega = 2\pi f$$

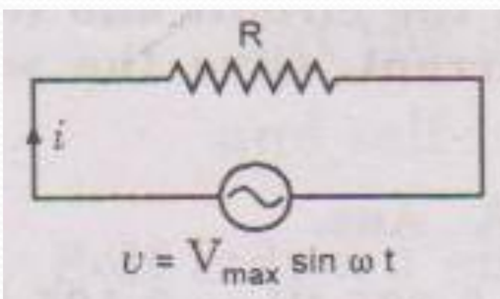
Single-phase AC Supply

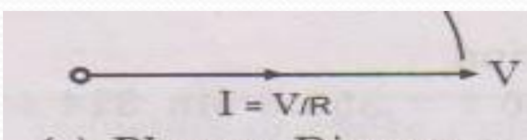
- Purely resistive circuit (R only)
  - The instantaneous value of the current though the circuit is given by

$$v = V_m \sin \omega t$$

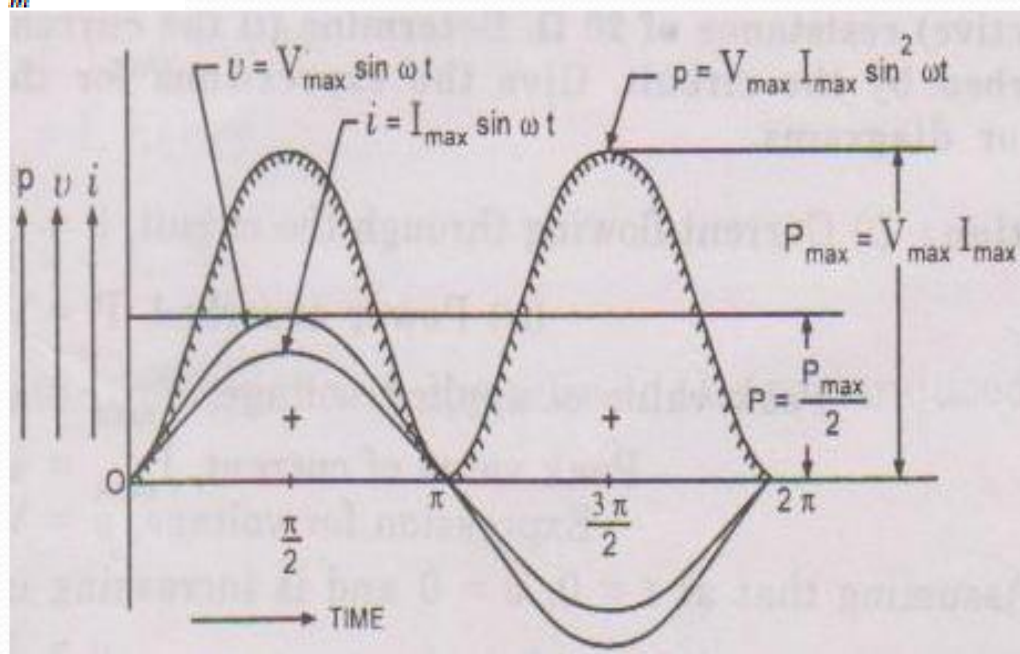
  - Im and Vm are the maximum values of current and voltage respectively

$$i = \frac{v}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$$





(c) Phasor Diagram



Purely inductive circuit (L only)

- For the circuit , the current i, is obtained by the procedure described here

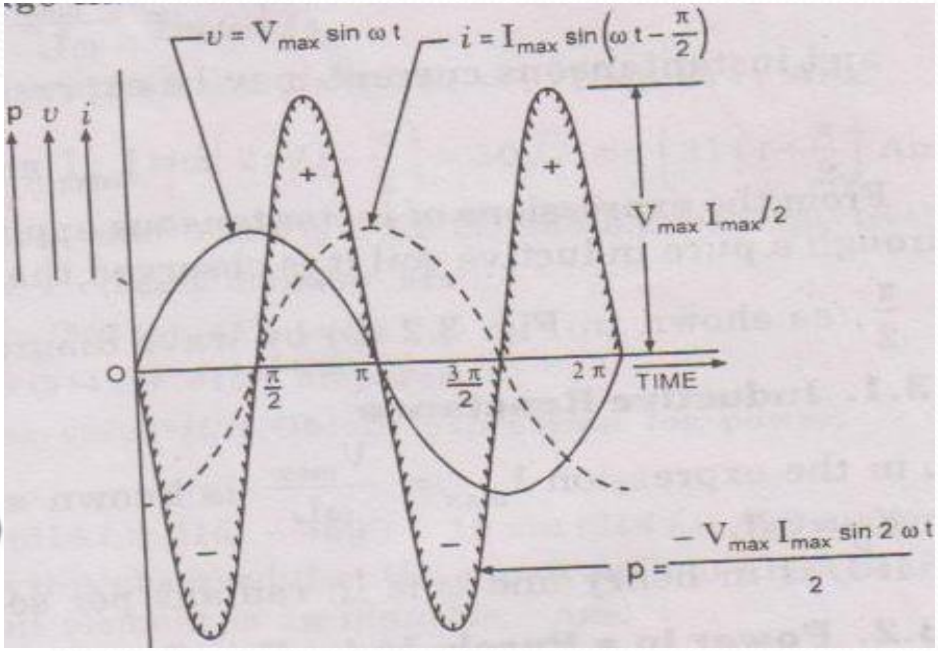
$$\text{As } v = L\frac{di}{dt} = V_m \sin \omega t$$

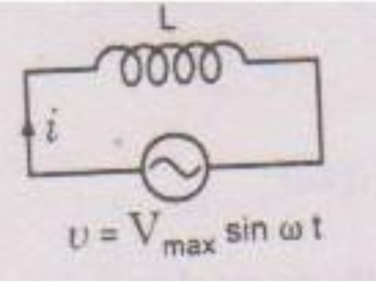
$$di = \frac{V}{L}\sin(\omega t)\, dt$$

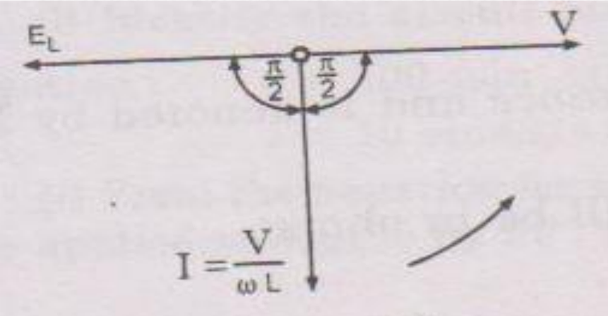
Integrating,

$$i = -\frac{V_m}{\omega L}\cos \omega t = \frac{V_m}{\omega L}\sin(\omega t - 90^\circ)$$

$$= I_m \sin(\omega t - 90^\circ)$$







(c) Phasor Diagram

Purely capacitive circuit (C only)

The current i, in the circuit (Fig. 14.3a), is,

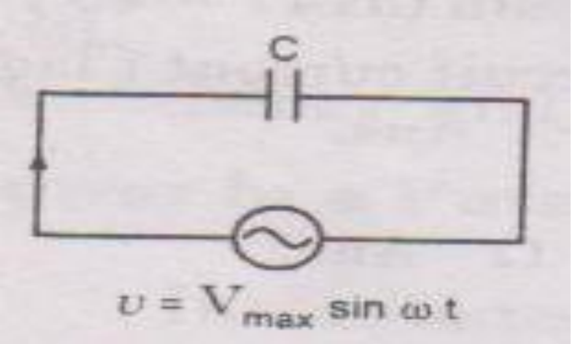
$$i = C\frac{dv}{dt}$$

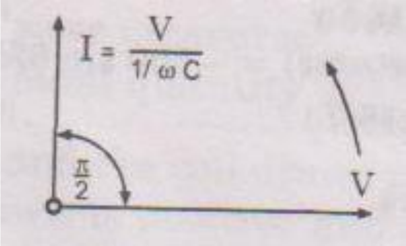
Substituting $v = V_m \sin\omega t$, i is

$$i = C\frac{d}{dt}\left(V_m \sin\omega t\right) = \omega C V_m \cos\omega t$$

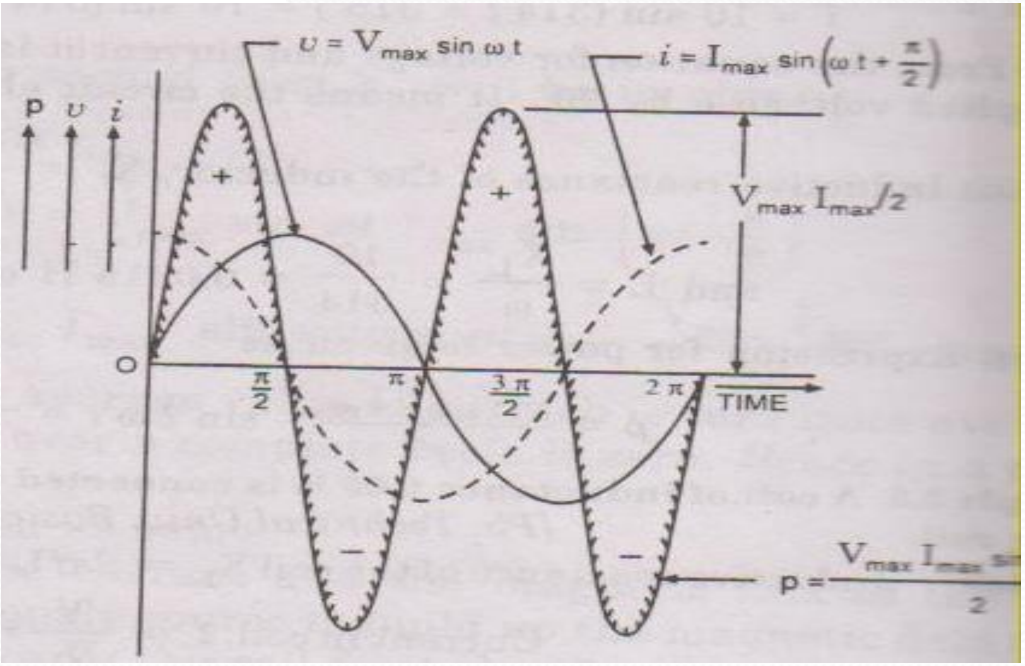
$$= \omega C V_m \sin(\omega t + 90^\circ)$$

$$= I_m \sin(\omega t + 90^\circ)$$





(c) Phasor Diagram



- From our earlier discussions we know that

$$v = V_m \sin(\omega t + \phi)$$

where V_m is the **peak voltage**

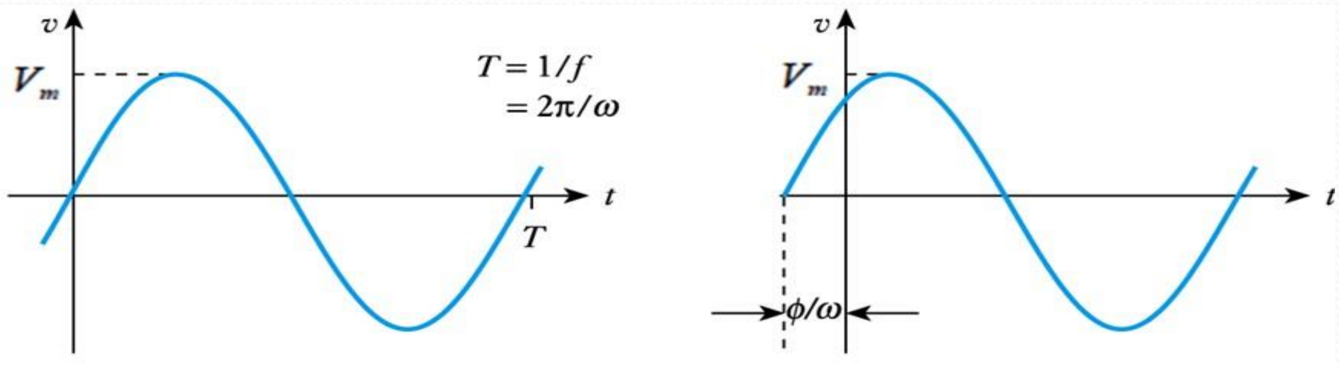
ω is the **angular frequency**

ϕ is the **phase angle**

- Since $\omega = 2\pi f$ it follows that the period T is given by

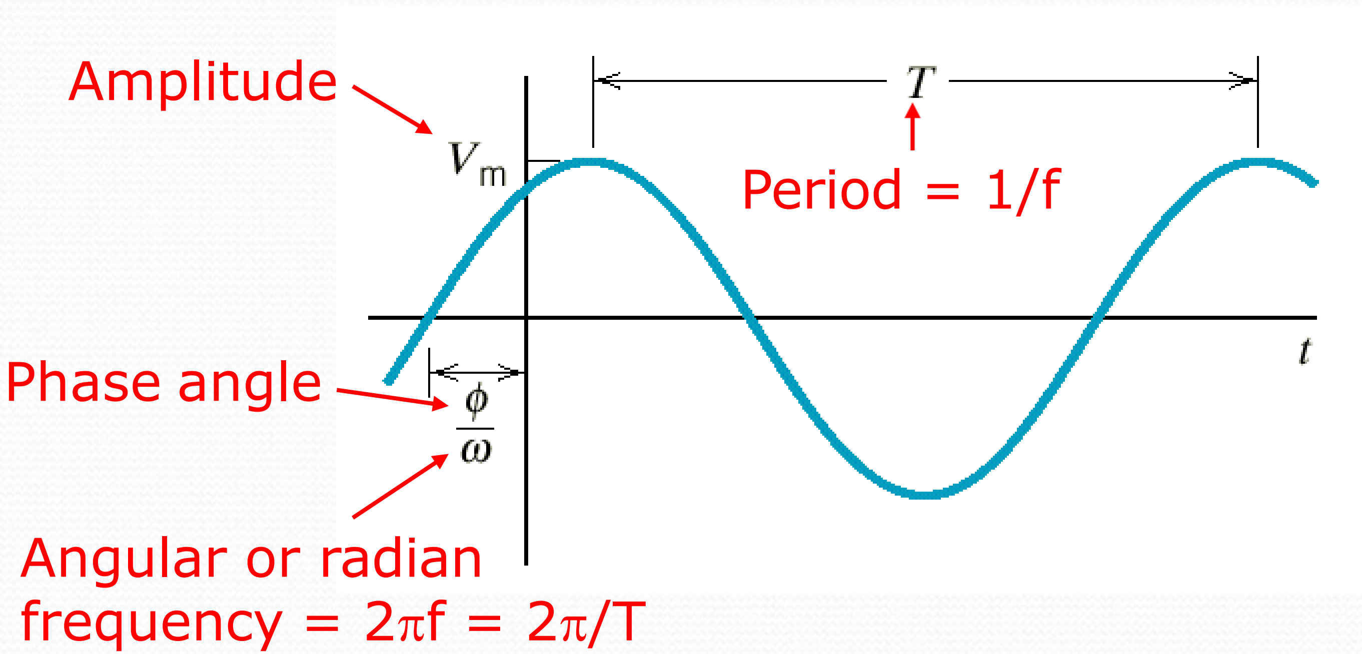
$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

- If ϕ is in radians, then a time delay t is given by ϕ/ω as shown below



(a) $v = V_m \sin(\omega t)$ (b) $v = V_m \sin(\omega t + \phi)$

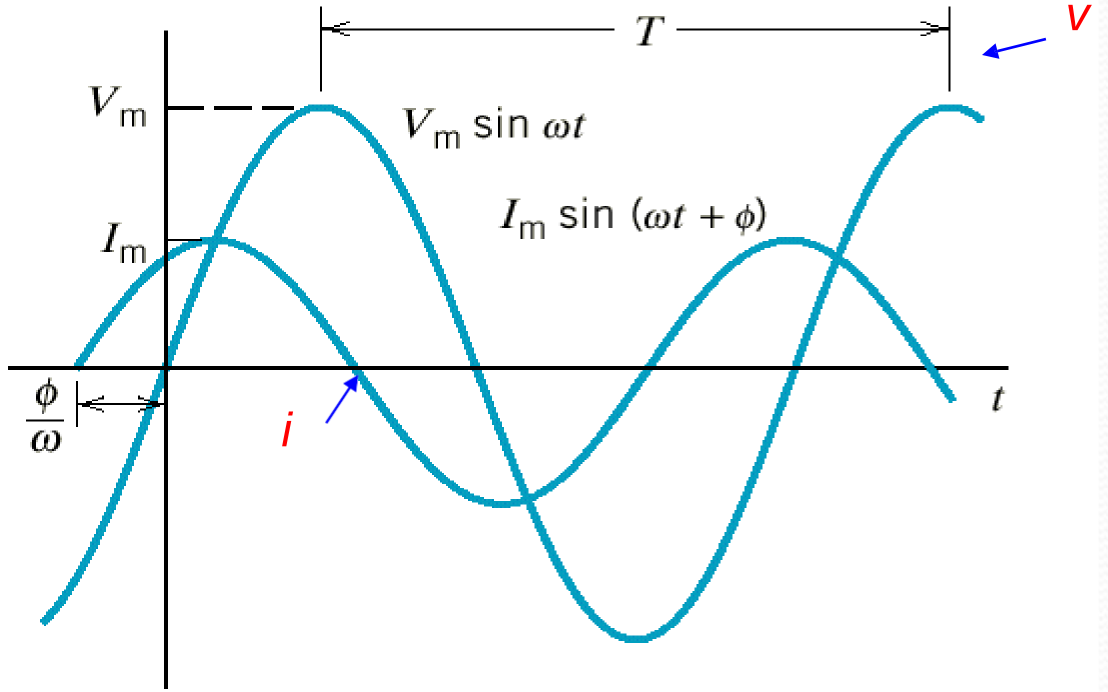
- Sinusoidal signals are characterised by their **magnitude**, their **frequency** and their **phase**



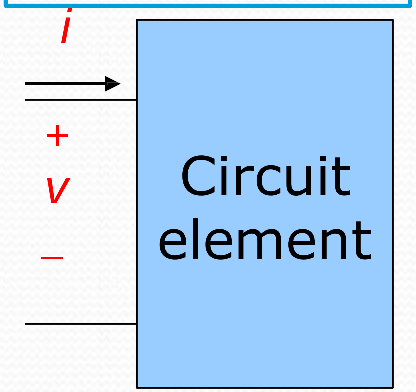
Example

$$\cos\theta = \sin\left(\theta + \frac{\pi}{2}\right)$$

$$\sin\theta = \cos\left(\theta - \frac{\pi}{2}\right)$$



Voltage and current of a circuit element.



The current *leads* the voltage by ϕ radians.

OR

The voltage *lags* the current by ϕ radians.

Mathematical representation of phasor

- Phasor can be representation in two way.
  - Rectangular form
  - Polar form

Polar form:-

- The instantaneous voltage $v_s = V_m \sin(\omega t + \phi)$ can be represented in polar form.

$$v_s = V_m \angle \phi$$

- For example v(t) = 20 sin(2πft + 60).
  - Then it represented in the polar form

$$v(t) = 20 \angle 60^\circ \text{ volt}$$

For Rectangular form:-

- The instantaneous voltage $v_s = V_m \sin(\omega t + \phi)$ can be represented in Rectangular form.

$$v(t) = x + jy$$

Where 'x' is x component of the phasor $= V_m \cos\varphi$

'y' is y component of the phasor $= V_m \sin\varphi$

- $V(t) = V_m \cos\varphi + j V_m \sin\varphi$

Example: $v(t) = 20 \sin(2\pi ft + 60)$.

$$v(t) = 20 \angle 60^\circ \text{ volt}$$

$$v(t) = 20 (\cos 60 + j \sin 60). = (10 + j17.32)$$

Conversion from polar to rectangular:

- Polar form : $v_s = r \angle \phi$
- For x component x = r cos φ and y component y = r sin φ
- v(t) = x + jy
- V(t) = r cos φ + j r sin φ

Conversion from rectangular to polar :

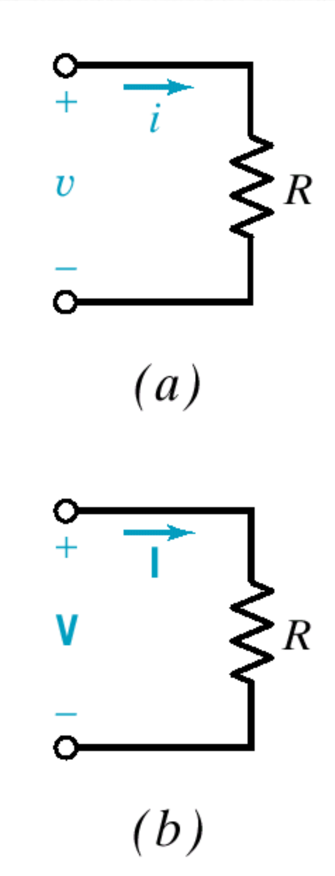
- For rectangular form v(t) = x + jy

$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2 + y^2} \qquad \phi = tan^{-1}\left(\frac{y}{x}\right)$$

- For polar form $v_s = r \angle \phi = \sqrt{x^2 + y^2} \angle tan^{-1}\left(\frac{y}{x}\right)$

Phasor Relationship for R, L, and C Elements



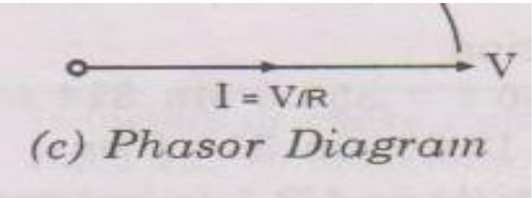
Time domain

$$v = Ri$$

Resistor

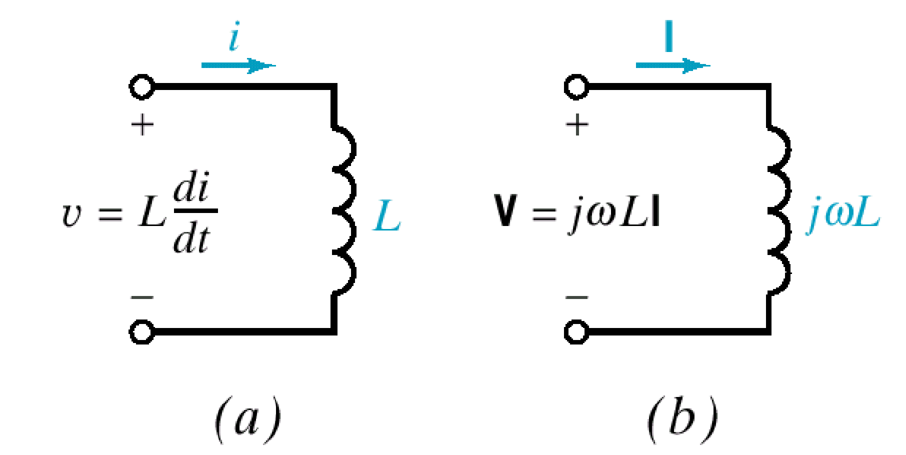
Frequency domain

$$\mathbf{V} = R\mathbf{I} \quad or \quad \mathbf{I} = \frac{\mathbf{V}}{R}$$



Voltage and current are *in phase.*

Inductor



$$\text{As } v = L\frac{di}{dt} = V_m \sin \omega t$$

$$di = \frac{V}{L}\sin(\omega t)\, dt$$

$$i = -\frac{V_m}{\omega L}\cos \omega t = \frac{V_m}{\omega L}\sin(\omega t - 90^\circ)$$

$$= I_m \sin(\omega t - 90^\circ)$$

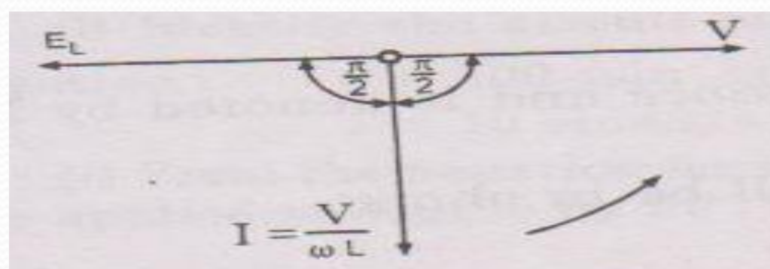
Time domain **Frequency domain**

$$v = L\frac{di}{dt}$$

$$\mathbf{V} = j\omega L\mathbf{I} \quad \text{or} \quad \mathbf{I} = \frac{\mathbf{V}}{j\omega L} = \frac{-j\mathbf{V}}{\omega L}$$

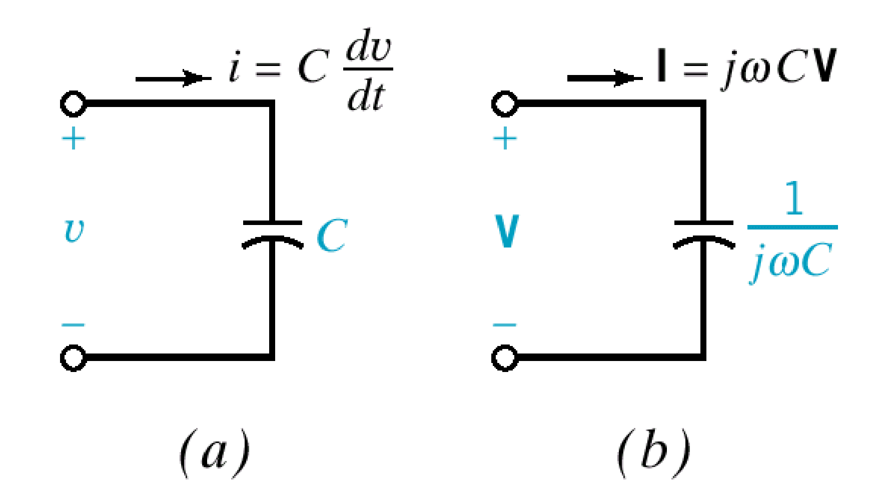
Voltage ***leads*** **current by** 90°

Current ***lags*** **voltage by** 90°



(c) Phasor Diagram

Capacitor



The current i, in the circuit

$$i = C\frac{dv}{dt}$$

$$i = C\frac{d}{dt}\left(V_m \sin\omega t\right) = \omega C V_m \cos\omega t$$

$$= \omega C V_m \sin(\omega t + 90^\circ)$$

$$= I_m \sin(\omega t + 90^\circ)$$

Time domain

$$V = \frac{1}{C}\int i(t)\,dt$$

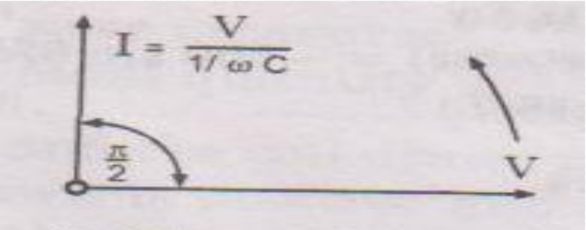
$$i = C\frac{dv}{dt}$$

Frequency domain

$$\mathbf{I} = j\omega C\mathbf{V} \quad \text{or} \quad \mathbf{V} = \frac{\mathbf{I}}{j\omega C} = \frac{-j\mathbf{I}}{\omega C}$$

Voltage *lags* current by 90°

Current *leads* voltage by 90°



(c) Phasor Diagram

Impedance and Admittance

Impedance is defined as the ratio of the **phasor voltage** to the **phasor current**.

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}}$$

Ohm's law in phasor notation

$$= \frac{V_m \angle \phi}{I_m \angle \beta} = \frac{V_m}{I_m} \angle \phi - \beta$$

phase $\theta = \phi - \beta$

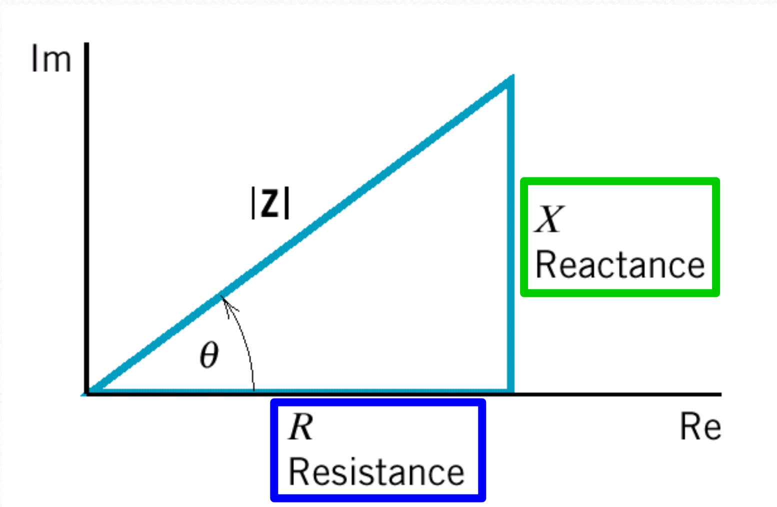
magnitude $|\mathbf{Z}|$

or

$$\mathbf{Z} = |\mathbf{Z}| \angle \theta \quad = Ze^{j\theta} \quad = R + jX$$

polar exponential rectangular

Graphical Representation of Impedance

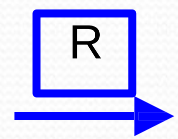


$$\mathbf{Z} = |\mathbf{Z}|\angle\theta$$

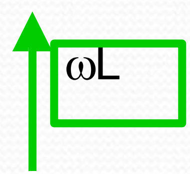
$$|\mathbf{Z}| = \sqrt{R^2 + X^2}$$

$$\theta = \tan^{-1}\frac{X}{R}$$

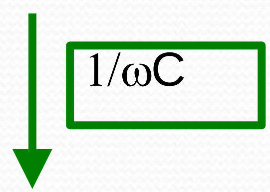
Resistor $\mathbf{Z} = R$



Inductor $\mathbf{Z} = j\omega L$



Capacitor $\mathbf{Z} = \dfrac{1}{j\omega C} = \dfrac{-j}{\omega C}$



Admittance is defined as the reciprocal of **impedance**.

In Polar form

$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = \frac{1}{|\mathbf{Z}|\angle\theta} = |\mathbf{Y}|\angle -\theta$$

In rectangular form

conductance

$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = \frac{1}{R + jX} = \frac{R - jX}{R^2 + X^2} = G + jB$$

susceptance

Resistor $\mathbf{Y} = G = \frac{1}{R}$ G

Inductor $\mathbf{Y} = \frac{1}{j\omega L}$ $1/\omega L$

Capacitor $\mathbf{Y} = j\omega C$ ωC