

## OBJECTIVE TYPE QUESTIONS

### A. Fill up the blanks

1. The formula for the Fourier coefficients  $a_n, b_n$  for  $f(x)$  in  $(-\pi, \pi)$  are \_\_\_\_\_.
2. If  $f(x)$  is an even function in  $(-\pi, \pi)$ , then the Fourier coefficients are  $a_n =$  \_\_\_\_\_,  $b_n =$  \_\_\_\_\_.
3. If  $f(x) = x^2 + x$  is expressed as a Fourier series in  $(-2, 2)$ , then  $f(2) =$  \_\_\_\_\_.
4. If the Fourier series for the function  $f(x) = \begin{cases} 0, & 0 < x < \pi \\ \sin x, & \pi < x < 2\pi \end{cases}$  is  $f(x) = -\frac{1}{\pi} + \frac{2}{\pi} \left[ \frac{\cos 2x}{1 \cdot 3} + \frac{\cos 4x}{3 \cdot 5} + \frac{\cos 6x}{5 \cdot 7} + \dots \right] + \frac{\sin x}{2}$ , then  $\frac{1}{1 \cdot 3} - \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} - \dots =$  \_\_\_\_\_.
5. The half-range sine series for  $f(x) = x$  in  $(0, \pi)$  is \_\_\_\_\_.
6. The Dirichlet's conditions for  $f(x)$  is  $c < x < c + 2\pi$  to have a Fourier series expansion are \_\_\_\_\_.
7. The value of  $f(2)$  in the half-range cosine series for  $f(x) = x^2$  in  $(0, 2)$  is \_\_\_\_\_.
8. The root mean square value of  $f(x) = x^2$  in  $(0, 6)$  is \_\_\_\_\_.
9. The half-range sine series for  $f(x) = x(\pi - x)$  in  $(0, \pi)$  is  $x(\pi - x) = \frac{8}{\pi} \left[ \frac{\sin x}{1^3} + \frac{\sin 3x}{3^3} + \frac{\sin 5x}{5^3} + \dots \right]$  then the value of  $\frac{1}{1^3} - \frac{1}{3^3} + \frac{1}{5^3} - \frac{1}{7^3} + \dots =$  \_\_\_\_\_.
10. The half-range cosine series for  $f(x) = (x - 1)^2$  in  $(0, 1)$  is  $f(x) = \frac{1}{3} + \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{n^2} \cos n\pi x$ , then the value of  $\sum_{n=1}^{\infty} \frac{1}{n^4}$  is \_\_\_\_\_.
11. The Fourier series for  $f(x) = x$  in  $(0, 2\pi)$  is  $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$ , then the value of  $\frac{a_0^2}{2} + \sum_{n=1}^{\infty} (a_n^2 + b_n^2)$  is \_\_\_\_\_.
12. If the half-range cosine series of  $f(x) = \begin{cases} \pi x, & 0 \leq x \leq 1 \\ \pi(2 - x), & 1 < x \leq 2 \end{cases}$  is  $f(x) = \frac{\pi}{2} - \frac{4}{\pi} \left[ \frac{\cos \pi x}{1^2} + \frac{\cos 3\pi x}{3^2} + \frac{\cos 5\pi x}{5^2} + \dots \right]$  then  $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots =$  \_\_\_\_\_.
13. If the Fourier series of  $f(x) = x(2\pi - x)$  in  $(0, 2\pi)$  is  $x(2\pi - x) = \frac{2\pi^2}{3} - 4 \sum_{n=1}^{\infty} \frac{\cos nx}{x^2}$ , then the sum of the series  $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots =$  \_\_\_\_\_.

14. If  $f(x)$  is discontinuous at  $x = a$ , then the sum of the Fourier series of  $f(x)$  when  $x = a$  is \_\_\_\_\_.
15. The Parseval's identity for the half-range cosine expansion of  $f(x)$  in  $(0, 1)$  is \_\_\_\_\_.

**B. Choose the correct answer**

- The value of the constant term in the Fourier series expansion of  $\cos^2 x$  in  $(-\pi, \pi)$  is  
 (a) 1 (b)  $\frac{1}{2}$  (c)  $\frac{\pi}{2}$  (d)  $\pi$
- The value of  $b_n$  in the Fourier series expansion of  $f(x) = x^2$  in  $(-\pi, \pi)$  is  
 (a) 0 (b)  $2\pi$  (c)  $\frac{\pi}{2}$  (d)  $\frac{\pi^2}{4}$
- The value of  $a_n$  in the Fourier series of  $f(x) = x - x^3$  in  $(-\pi, \pi)$  is  
 (a)  $\frac{\pi}{2}(2 - \pi^2)$  (b)  $\frac{\pi}{4}(2 - \pi^2)$  (c) 0 (d) None of these
- The Fourier of  $f(x) = \begin{cases} \sin x, & 0 \leq x \leq \pi \\ 0, & \pi < x \leq 2\pi \end{cases}$  of period  $2\pi$  is  

$$f(x) = \frac{1}{\pi} + \frac{1}{2}\sin x - \frac{2}{\pi} \left[ \frac{\cos 2x}{1 \cdot 3} + \frac{\cos 4x}{3 \cdot 5} + \frac{\cos 6x}{5 \cdot 7} + \dots \right]$$
, then the value of  $\frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \dots$  is  
 (a) 1 (b)  $\pi$  (c)  $\frac{1}{2}$  (d)  $\frac{\pi}{2}$
- The Fourier series of  $f(x) = x + x^2$  in  $(-\pi, \pi)$  is  $\frac{\pi^2}{3} + \sum_{n=1}^{\infty} (-1)^n \left[ \frac{4}{n^2} \cos nx - \frac{2}{n} \sin nx \right]$ , then the value of  $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$  is  
 (a)  $\frac{\pi - 2}{4}$  (b)  $\frac{\pi^2}{6}$  (c)  $\frac{\pi^2}{8}$  (d)  $\frac{\pi^2}{12}$
- If  $f(x) = 2x$  in  $(0, 4)$ , then the value of  $a_2$  in the Fourier series expansion of period 4 is  
 (a) 4 (b) 2 (c) 0 (d) 3
- The root mean square value of  $f(x) = 1 - x$  in  $0 < x < 1$  is  
 (a)  $\frac{1}{2}$  (b)  $\frac{1}{\sqrt{3}}$  (c)  $\frac{1}{\sqrt{2}}$  (d) 1
- If the Fourier series for  $f(x)$  in  $(0, 2\pi)$  is  $f(x) = \sum_{n=1}^{\infty} \frac{\sin nx}{n}$ , then the root mean value is  
 (a)  $\frac{\pi}{2\sqrt{3}}$  (b)  $\frac{\pi}{\sqrt{3}}$  (c)  $\frac{\pi}{3\sqrt{2}}$  (d)  $\frac{\pi^2}{\sqrt{3}}$
- The Fourier coefficient  $b_n$  for  $x \sin x$  in  $[-\pi, \pi]$  is  
 (a)  $\frac{1}{2}$  (b) 0 (c)  $\frac{\pi}{\sqrt{3}}$  (d)  $\frac{\pi}{3}$
- The Fourier series for  $f(x) = \begin{cases} -k, & -\pi < x < 0 \\ k, & 0 < x < \pi \end{cases}$  is  $f(x) = \frac{4k}{\pi} \left[ \sin x + \frac{1}{3} \sin 3x + \frac{1}{5} \sin 5x + \dots \right]$ , then the value of  $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$  is  
 (a)  $\frac{\pi}{6}$  (b)  $\frac{\pi^2}{6}$  (c)  $\frac{\pi}{4}$  (d)  $\frac{\pi^2}{4}$
- The half-range cosine series for  $f(x) = x$  in  $(0, \pi)$  is  $x = \frac{\pi}{2} - \frac{4}{\pi} \sum_{n \text{ odd}} \frac{\cos nx}{n^2}$ , then the value of  $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$  is  
 (a)  $\frac{\pi^2}{6}$  (b)  $\frac{\pi^2}{8}$  (c)  $\frac{\pi^2}{12}$  (d)  $\frac{\pi}{4}$



12. The half-range cosine series for  $f(x) = x(\pi - x)$  in  $0 < x < \pi$  is  $x(\pi - x) = \frac{\pi^2}{6} - \left[ \frac{\cos 2x}{1^2} + \frac{\cos 4x}{2^2} + \frac{\cos 6x}{3^2} + \dots \right]$ ,  
then the value of  $\sum_{n=1}^{\infty} \frac{1}{n^4} =$  (a)  $\frac{\pi^4}{8}$  (b)  $\frac{\pi^4}{96}$  (c)  $\frac{\pi^4}{90}$  (d)  $\frac{\pi^2}{90}$
13. If the Fourier series of  $f(x) = x(2l - x)$  is  $(0, 2l)$  of period  $2l$  is  $f(x) = \frac{2}{3}l^2 - \frac{4}{\pi^2}l^2 \sum_{n=1}^{\infty} \frac{1}{n^2} \cos\left(\frac{n\pi x}{l}\right)$ ,  
then the value of  $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$  is (a)  $\frac{\pi^2}{6}$  (b)  $\frac{\pi^2}{8}$  (c)  $\frac{\pi^2}{12}$  (d)  $\frac{\pi^2}{4}$
14. If  $x = \frac{l}{2} - \frac{4l}{\pi^2} \left( \cos \frac{\pi x}{l} + \frac{1}{3^2} \cos \frac{3\pi x}{l} + \frac{1}{5^2} \cos \frac{5\pi x}{l} + \dots \right)$  in  $0 < x < l$ ,  $f(x + 2l) = f(x)$ ,  
then the value of  $\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \dots$  is (a)  $\frac{\pi^2}{32}$  (b)  $\frac{\pi^4}{96}$  (c)  $\frac{\pi^4}{90}$  (d) None of these
15. If the half-range cosine series for  $f(x) = (x - 1)^2$ ,  $0 < x < 1$ , is  $f(x) = \frac{1}{3} + \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{1}{n^2} \cos n\pi x$ , then the value of  
 $\sum_{n=1}^{\infty} \frac{1}{n^4}$  is (a)  $\frac{\pi^4}{90}$  (b)  $\frac{\pi^4}{96}$  (c)  $\frac{\pi^2}{16}$  (d) None of these

### ANSWERS

#### A. Fill up the blanks

- $a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx \, dx$ ,  $n = 0, 1, 2, 3, \dots$ ,  $b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx$ ,  $n = 1, 2, 3, \dots$
- $a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx \, dx$ ,  $n = 0, 1, 2, \dots$  and  $b_n = 0$ ,  $3, 4$
- 4
- $\frac{\pi - 2}{4}$
- $2 \left[ \sin x - \frac{1}{2} \sin 2x + \frac{1}{3} \sin 3x - \dots \right]$
- Refer definition 17.3, page 17.2.
- 4
- $\frac{l^2}{\sqrt{5}}$
- $\frac{\pi^3}{32}$
- $\frac{\pi^4}{90}$
- $\frac{8\pi^2}{3}$
- $\frac{\pi^2}{8}$
- $\frac{\pi^2}{6}$
- $\frac{1}{2} [f(a-) + f(a+)]$
- $\int_0^1 [f(x)^2] \, dx = \frac{a_0^2}{4} + \frac{1}{2} \sum_{n=1}^{\infty} a_n^2$

#### B. Choose the correct answer

- (b)
- (a)
- (c)
- (c)
- (b)
- (c)
- (b)
- (a)
- (b)
- (c)
- (c)
- (b)
- (a)