Unit 1: Linear Algebra

(**Book:** Advanced Engineering Mathematics by Jain and Iyengar,**Chapter-3**)

Topic:

Review of Matrices

Learning Outcomes:

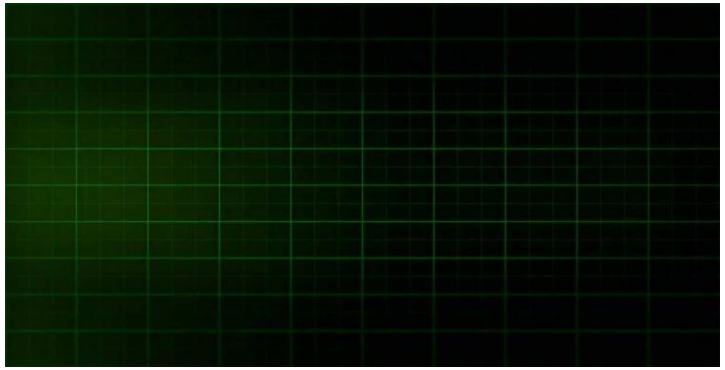
1. Uses of matrices in various fields and daily life.

2. Definition of matrix, Types of matrices .

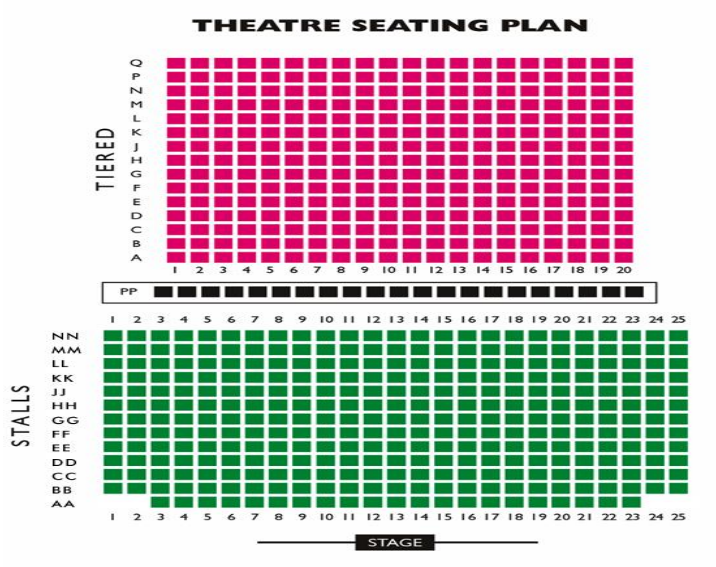
3. Matrix Operations (Addition, subtraction, multiplication, transpose, determinant)

You might have observed in routine:

Grid of Computer Screen



Online Booking of Cinema Hall



You might have observed in routine:

Republic Day Parade

Matrix Movie

Origin of the word “MATRIX”:

Matrices or matrix is commonly used its mathematics, but have you thought about how important it is or where you can use it?

Ever wondered where the word matrix came from?

Matrix is actually a Latin word used for the womb.

It is also used to express something that is formed or produced.

Uses of Matrices in Various Fields:

Encryption

In encryption, we use it to scramble data for security purpose to encode and to decode this data we need matrices. There is a key which helps encode and decode data which is generated by matrices.

Games especially 3D

They use it to alter the object, in 3d space. They use the 3d matrix to 2d matrix to convert it into the different objects as per requirement.

Economics and business

To study the trends of a business, shares and more. To create business models.

Uses of Matrices in Various Fields:

Construction

Have you seen some buildings are straight but sometimes architects try to change the outer structure of the building like the famous Burj Khalifa etc. This can be done with matrices.

A matrix is made of rows and columns you can change the number of rows and columns within a matrix.

Matrices can help support various historical structures

Dance – contra dance

It is used to organize complicated group dances.

Uses of Matrices in Various Fields:

Animation
It can help make animations in a more precise and perfect.

Physics

Matrices are applied in the study of electrical circuits, quantum mechanics and optics. It helps in the calculation of battery power outputs, resistor conversion of electrical energy into another useful energy. Therefore, matrices play a major role in calculations. Especially in solving the problems using Kirchhoff's laws of voltage and current. It helps in studying quantum physics and in using it.

Geology
Matrices are used for taking seismic surveys.

Matrix - A rectangular array of variables or constants in horizontal rows and vertical columns enclosed in brackets.

Element - Each value in a matrix; either a number or a constant.

Dimension (Order) - Number of rows (m) by number of columns (n) of a matrix (m x n).

**A matrix is named by its dimensions (order).

A matrix is denoted by a bold capital letter and the elements within the matrix are denoted by lower case letters e.g. matrix [A] with elements a_{ij}

$$
\begin{array}{c}
n \text{ columns} \longrightarrow \\
\begin{array}{cccccc} \text{Column 1} & \text{Column 2} & & \text{Column } j & & \text{Column } n \\ \downarrow & \downarrow & & \downarrow & & \downarrow \end{array}
\end{array}
$$

$$
A=[a_{ij}]_{m\times n} = \begin{bmatrix}
a_{11} & a_{12} & \cdots & a_{1j} & \cdots & a_{1n} \\
a_{21} & a_{22} & \cdots & a_{2j} & \cdots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
a_{i1} & a_{i2} & \cdots & a_{ij} & \cdots & a_{in} \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
a_{m1} & a_{m2} & \cdots & a_{mj} & \cdots & a_{mn}
\end{bmatrix}
\begin{array}{l}
\leftarrow \text{Row 1} \\
\leftarrow \text{Row 2} \\
\\
\leftarrow \text{Row } i \\
\\
\leftarrow \text{Row } m
\end{array}
\Bigg\downarrow \; m \text{ rows}
$$

Types of Matrices:

Row Matrix

$$\begin{pmatrix} a & b & c \end{pmatrix}$$

Column Matrix
Vector Matrix

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

Zero Matrix
Null Matrix

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

Diagonal Matrix

$$\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$$

Scalar Matrix

$$\begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{pmatrix}$$

Unit Matrix

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Upper Triangular Matrix

$$\begin{pmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{pmatrix}$$

Lower Triangular Matrix

$$\begin{pmatrix} a & 0 & 0 \\ b & c & 0 \\ d & e & f \end{pmatrix}$$

Types of Matrices:

$$\begin{bmatrix} 1 & 3 \\ -4 & 7 \end{bmatrix}$$

Square matrix
2×2

$$\begin{bmatrix} a & b \\ c & d \\ e & f \end{bmatrix}$$

Rectangular matrix
3×2

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Zero matrix
3×5

$$\begin{bmatrix} 1 & 9 & -3 & 0 \end{bmatrix}$$

Row matrix
1×4

$$\begin{bmatrix} 1 \\ 2 \\ 6 \end{bmatrix}$$

Column matrix
3×1

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Identity matrix
3×3

Equal Matrices:

Two matrices are equal if:

They have the same dimensions

The corresponding entries are equal

$$\begin{bmatrix} 7 & 8 \\ \frac{1}{2} & \sqrt{16} \\ -7 & 4 \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 0.5 & 4 \\ -7 & 4 \end{bmatrix}$$

Each pair is equal:
7 = 7
8 = 8
½ = 0.5
Etc.

The definition of equal matrices can be used to the values when the elements of the matrices are the algebraic expressions.

Examples: Find the values for x and y

1. $\begin{bmatrix} 2x \\ 2x+3y \end{bmatrix} = \begin{bmatrix} y \\ 12 \end{bmatrix}$

* Since the matrices are equal, the corresponding elements are equal!

$2x = y$

$2x + 3y = 12$

* Form two linear equations.

* Solve the system using substitution.

$$\begin{aligned} 2x &= y \\ y + 3y &= 12 \end{aligned} \longrightarrow \begin{aligned} 4y &= 12 \\ y &= 3 \end{aligned} \longrightarrow \begin{aligned} 2x &= 3 \\ x &= \frac{3}{2} \end{aligned}$$

Matrix Addition and Subtraction:

- Only possible for matrices of same dimension
- Add/subtract matrices element-by-element
- Addition example: $\mathbf{C} = \mathbf{A}+\mathbf{B}$

$$\begin{bmatrix} 1 & 3 & -2 \\ 4 & 2 & 3 \\ -1 & 5 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 2 & -1 \\ 3 & 1 & 2 \\ 4 & 3 & -2 \end{bmatrix} = \begin{bmatrix} 6 & 5 & -3 \\ 7 & 3 & 5 \\ 3 & 8 & 2 \end{bmatrix}$$

- Subtraction example: $\mathbf{C} = \mathbf{A}-\mathbf{B}$

$$\begin{bmatrix} 4 & 2 & -1 \\ 5 & 3 & -3 \end{bmatrix} - \begin{bmatrix} 1 & 4 & 2 \\ -4 & 3 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -2 & -3 \\ 9 & 0 & -6 \end{bmatrix}$$

Matrix Multiplication:

Multiplication Conformability

- Regular Multiplication
- To multiply two matrices A and B:
- # of columns in A = # of rows in B
- Multiply: A (m x n) by B (n by p)

Multiplication General Formula

$$C_{ij} = \sum_{k=1}^{n} A_{ik} \times B_{kj}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & \\ & \end{bmatrix} \qquad 1\times7 + 2\times9 + 3\times11 = 58$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & 64 \\ & \end{bmatrix} \qquad 1\times8 + 2\times10 + 3\times12 = 64$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & 64 \\ 139 & \end{bmatrix} \qquad 4\times7 + 5\times9 + 6\times11 = 139$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \times \begin{bmatrix} 7 & 8 \\ 9 & 10 \\ 11 & 12 \end{bmatrix} = \begin{bmatrix} 58 & 64 \\ 139 & 154 \end{bmatrix} \qquad 4\times8 + 5\times10 + 6\times12 = 154$$

Matrix multiplication is not commutative: $AB \neq BA$

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 4 & 4 \\ 10 & 8 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 7 & 10 \end{bmatrix}$$

Transpose of a Matrix:

The transpose of a matrix can be defined as an operator which can switch the rows and column indices of a matrix i.e. it flips a matrix over its diagonal. If A is a given matrix, then its transpose is dented as A^T

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}^T = \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

Transpose of a matrix

Geeky Circle

$$\underset{A_{(3x2)}}{\begin{bmatrix} 3 & 2 \\ 5 & 4 \\ 7 & 10 \end{bmatrix}} \qquad \underset{A(\text{Transpose})_{(2x3)}}{\begin{bmatrix} 3 & 5 & 7 \\ 2 & 4 & 10 \end{bmatrix}}$$

Symmetric and Skew- Symmetric matrices:

- Symmetric: $\mathbf{A}^T = \mathbf{A}$.
- Skew-symmetric: $\mathbf{A}^T = -\mathbf{A}$.
- Examples:

$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 0 \\ -1 & 0 & 5 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$$

symmetric　　　　skew-symmetric

*Diagonal entries of a skew-symmetric matrix are all zeros.

Hermitian and Skew- Hermitian matrices (For complex elements):

A complex matrix A is said to be:

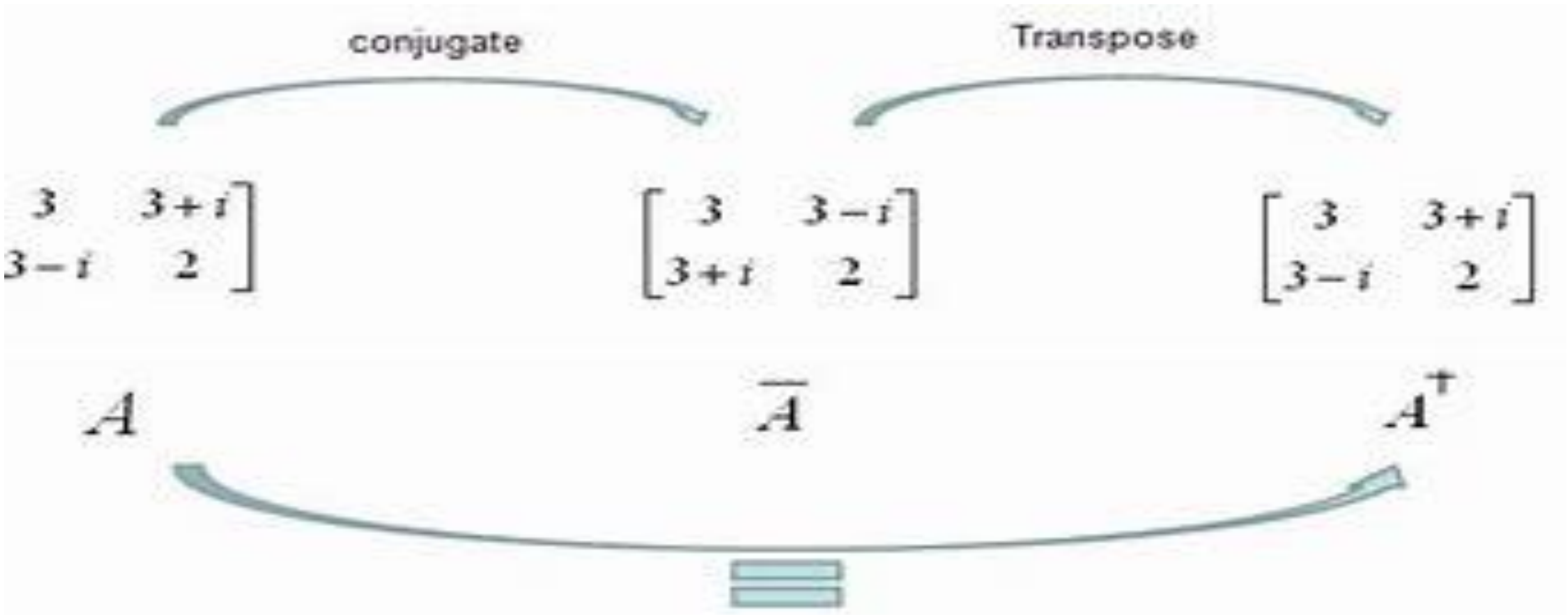
Hermitian if $\bar{A}^T = A$

Skew-Hermitian if $\bar{A}^T = -A$

*Diagonal entries of a Hermitian matrix are all reals.

*Diagonal entries of a skew-Hermitian matrix are purely imaginary or all zeros.

Example: Hermitian Matrix



Determinant of a Matrix:

In linear algebra, the determinant is a **scalar value** that can be computed from the elements of a square matrix and reveals certain properties of described by the matrix.

The determinant of a matrix A is denoted det (A), det A, or |A|.

Geometrically, it can be viewed as the volume of the *n*-dimensional parallelepiped spanned by the column or row vectors of the matrix.

How to calculate Determinant of a Square Matrix:

Determinant of a Matrix

The determinant of a 2 x 2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ is $ad - bc = 0$

The determinant of a 3 x 3 matrix $\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$ is

$$a_1 \bullet \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2 \bullet \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3 \bullet \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} = 0$$

$$a_1 (b_2 c_3 - b_3 c_2) - a_2 (b_1 c_3 - b_3 c_1) + a_3 (b_1 c_2 - b_2 c_1) = 0$$

How to calculate Determinant of a Square Matrix:

For a 2 × 2 matrix:

$$\det\begin{bmatrix} -5 & -4 \\ -2 & -3 \end{bmatrix} = (-5)(-3) - (-4)(-2)$$
$$= 15 - 8$$
$$= 7 \checkmark$$

For a 3 × 3 matrix:

$$\det\begin{bmatrix} 2 & -3 & 1 \\ 2 & 0 & -1 \\ 1 & 4 & 5 \end{bmatrix} = 2 \cdot \det\begin{bmatrix} 0 & -1 \\ 4 & 5 \end{bmatrix} - (-3) \cdot \det\begin{bmatrix} 2 & -1 \\ 1 & 5 \end{bmatrix} + 1 \cdot \det\begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix}$$
$$= 2[0 - (-4)] + 3[10 - (-1)] + 1[8 - 0]$$
$$= 2(0 + 4) + 3(10 + 1) + 1(8)$$
$$= 2(4) + 3(11) + 8$$
$$= 8 + 33 + 8$$
$$= 49 \checkmark$$