Unit-1:
Network Theorem

SUPERPOSITION THEOREM

- If a circuit has two or more independent sources, one way to determine the value of a specific variable (voltage or current) is to use nodal or mesh analysis.
- Another way is to determine the contribution of each independent source to the variable and then add them up. The latter approach is known as the *superposition*.
- The idea of superposition rests on the linearity property.

STATEMENT

“The superposition principle states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone”.

- The principle of superposition helps us to analyze a linear circuit with more than one independent source by calculating the contribution of each independent source separately.
- However, to apply the superposition principle, we must keep two things in mind:

- 1. We consider one independent source at a time while all other independent sources are *turned off*. This implies that we replace every **voltage source by 0 V** (or a short circuit), and **every current source by 0 A** (or an open circuit). This way we obtain a simpler and more manageable circuit.
- 2. Dependent sources are left intact because they are controlled by circuit variables.

Procedure to Apply Superposition Principle/Theorem

- 1. Turn off all independent sources except one source.

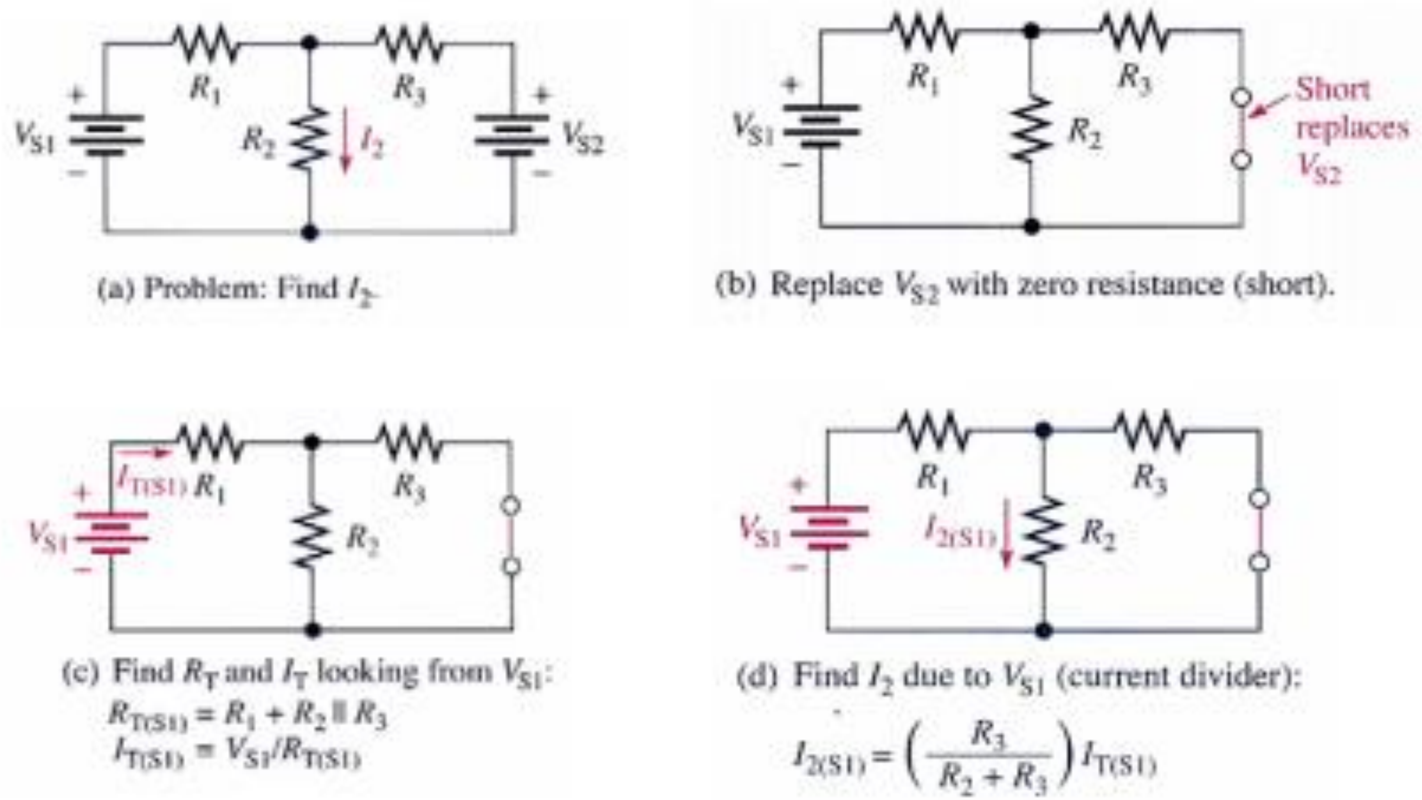
- 2. Find the output (voltage or current) due to that active source using any techniques such as Ohm's Law, KCL, KVL, Nodal/Mesh Analysis etc.

- 3. Repeat step 1 for each of the other independent sources.

- 4. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Superposition Theorem

The approach to superposition is demonstrated in the figure for a series-parallel circuit with two ideal voltage sources.



Superposition Theorem

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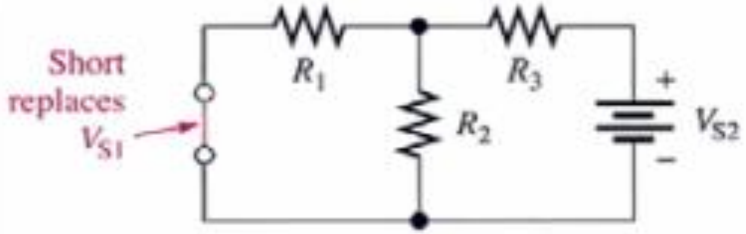
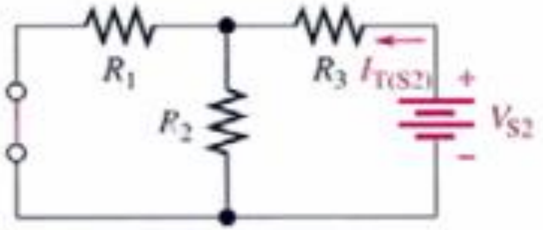
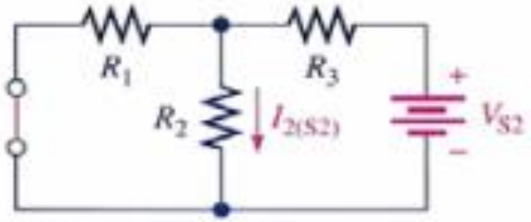


(e) Replace V_{S1} with zero resistance (short).

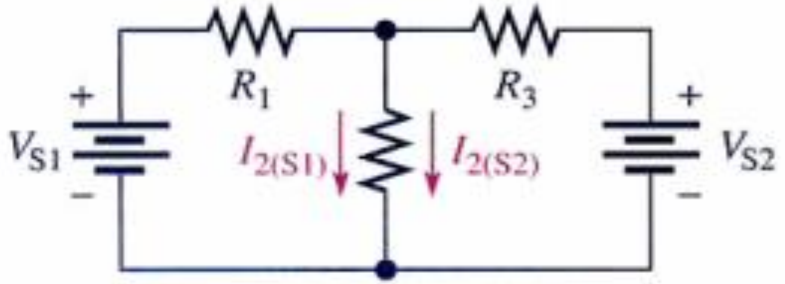


(f) Find R_T and I_T looking from V_{S2}:
$R_{T(S2)} = R_3 + R_1 \parallel R_2$
$I_{T(S2)} = V_{S2}/R_{T(S2)}$



(g) Find I_2 due to V_{S2}:

$$I_{2(S2)} = \left(\frac{R_1}{R_1 + R_2}\right) I_{T(S2)}$$

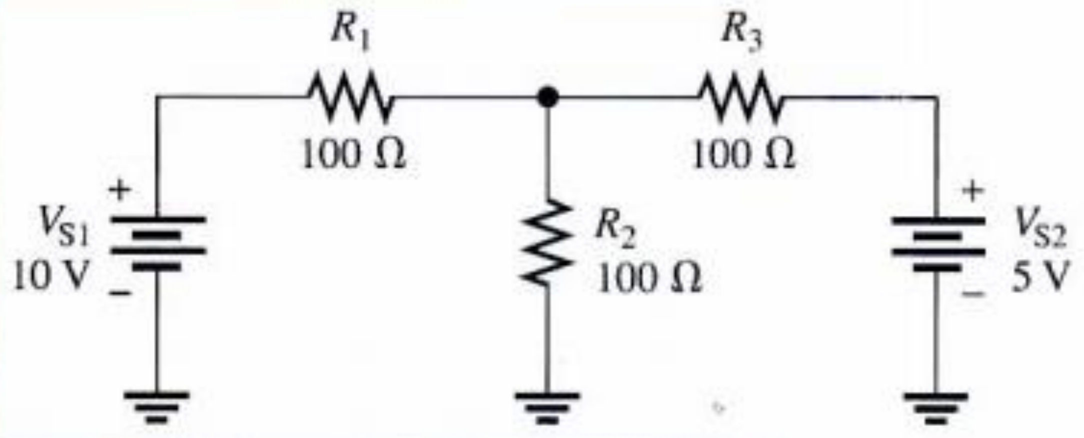


(h) Restore the original sources. Add $I_{2(S1)}$ and $I_{2(S2)}$ to get the actual I_2 (they are in same direction):

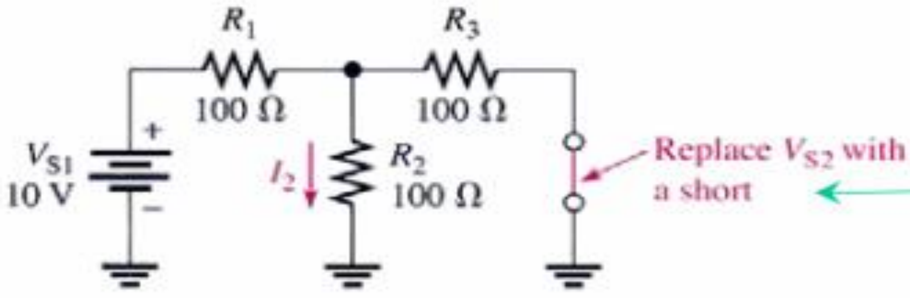
$$\boxed{I_2 = I_{2(S1)} + I_{2(S2)}}$$

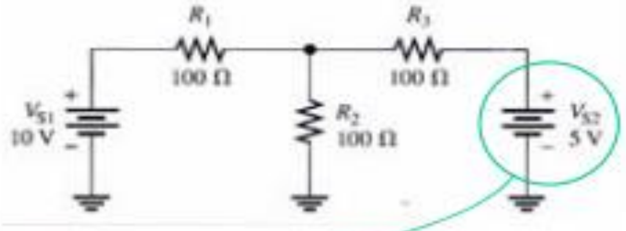
Superposition Theorem

EXAMPLE Use the superposition theorem to find the current through R_2.



Superposition Theorem





Solution **Step 1:** Replace V_{S2} with a short and find the current through R_2 due to voltage source V_{S1}, as shown in Figure 8–18. To find I_2, use the current-divider formula (Equation 6–6). Looking from V_{S1},

$$R_{T(S1)} = R_1 + \frac{R_3}{2} = 100\ \Omega + 50\ \Omega = 150\ \Omega$$

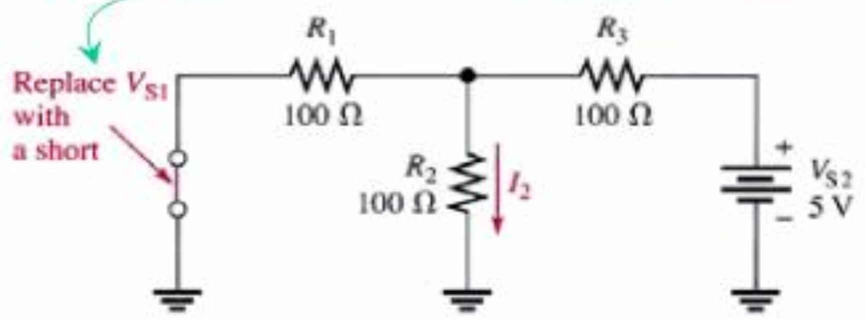
$$I_{T(S1)} = \frac{V_{S1}}{R_{T(S1)}} = \frac{10\text{ V}}{150\ \Omega} = 66.7\text{ mA}$$

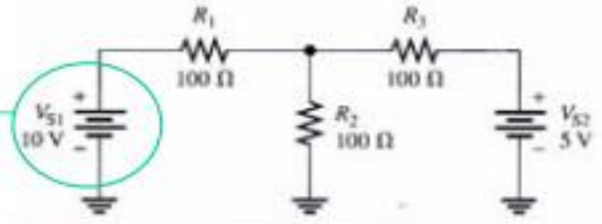
The current through R_2 due to V_{S1} is

$$I_{2(S1)} = \left(\frac{R_3}{R_2 + R_3}\right) I_{T(S1)} = \left(\frac{100\ \Omega}{200\ \Omega}\right) 66.7\text{ mA} = 33.3\text{ mA}$$

Note that this current is downward through R_2.

Superposition Theorem





Step 2: Find the current through R_2 due to voltage source V_{S2} by replacing V_{S1} with a short, as shown in Figure 8–19. Looking from V_{S2},

$$R_{T(S2)} = R_3 + \frac{R_1}{2} = 100\ \Omega + 50\ \Omega = 150\ \Omega$$

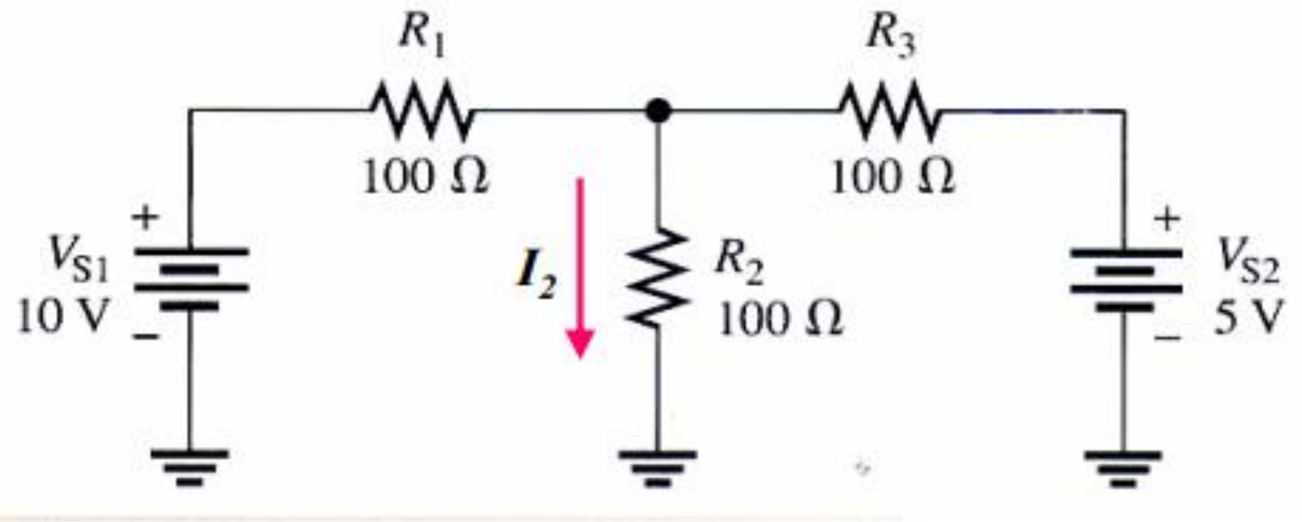
$$I_{T(S2)} = \frac{V_{S2}}{R_{T(S2)}} = \frac{5\ \text{V}}{150\ \Omega} = 33.3\ \text{mA}$$

The current through R_2 due to V_{S2} is

$$I_{2(S2)} = \left(\frac{R_1}{R_1 + R_2}\right) I_{T(S2)} = \left(\frac{100\ \Omega}{200\ \Omega}\right) 33.3\ \text{mA} = 16.7\ \text{mA}$$

Note that this current is downward through R_2.

Superposition Theorem

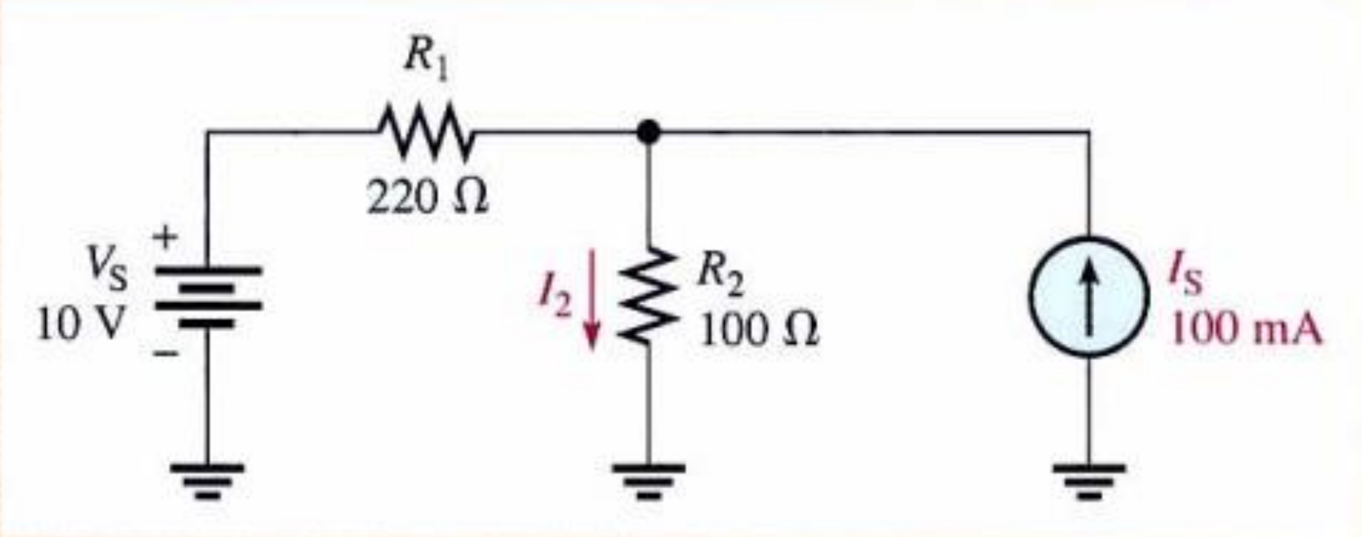


Step 3: Both component currents are downward through R_2, so they have the same algebraic sign. Therefore, add the values to get the total current through R_2.

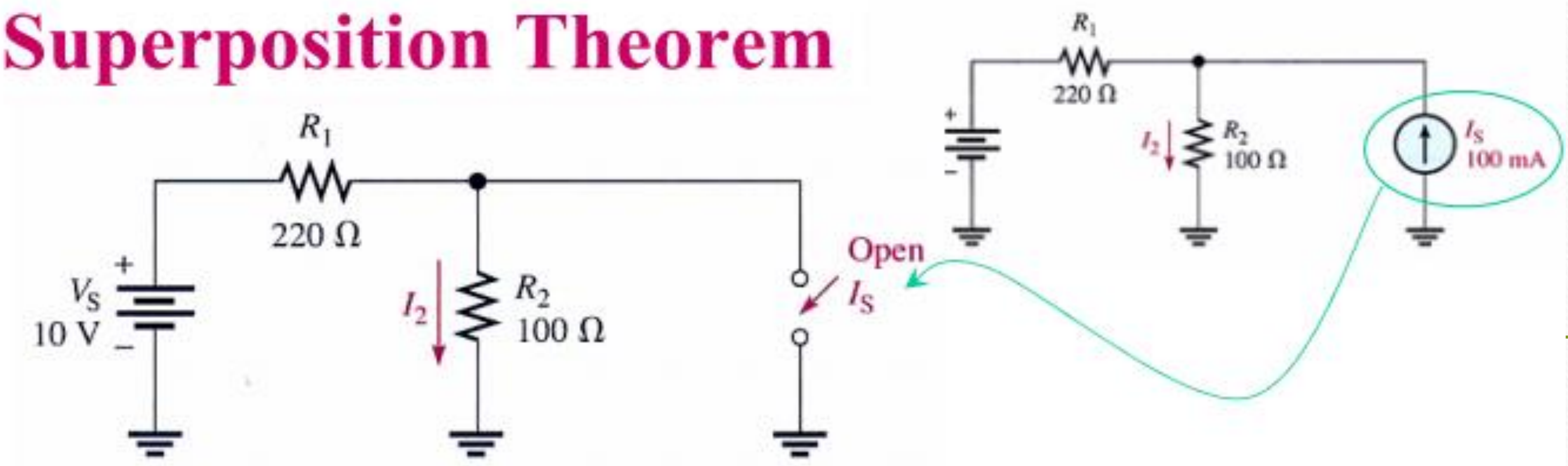
$$I_{2(\text{tot})} = I_{2(\text{S1})} + I_{2(\text{S2})} = 33.3\,\text{mA} + 16.7\text{mA} = \mathbf{50\,mA}$$

Superposition Theorem

EXAMPLE Find the current through R_2 in the circuit.



Superposition Theorem



Solution

Step 1: Find the current through R_2 due to V_S by replacing I_S with an open.

Notice that all of the current produced by V_S is through R_2. Looking from V_S,

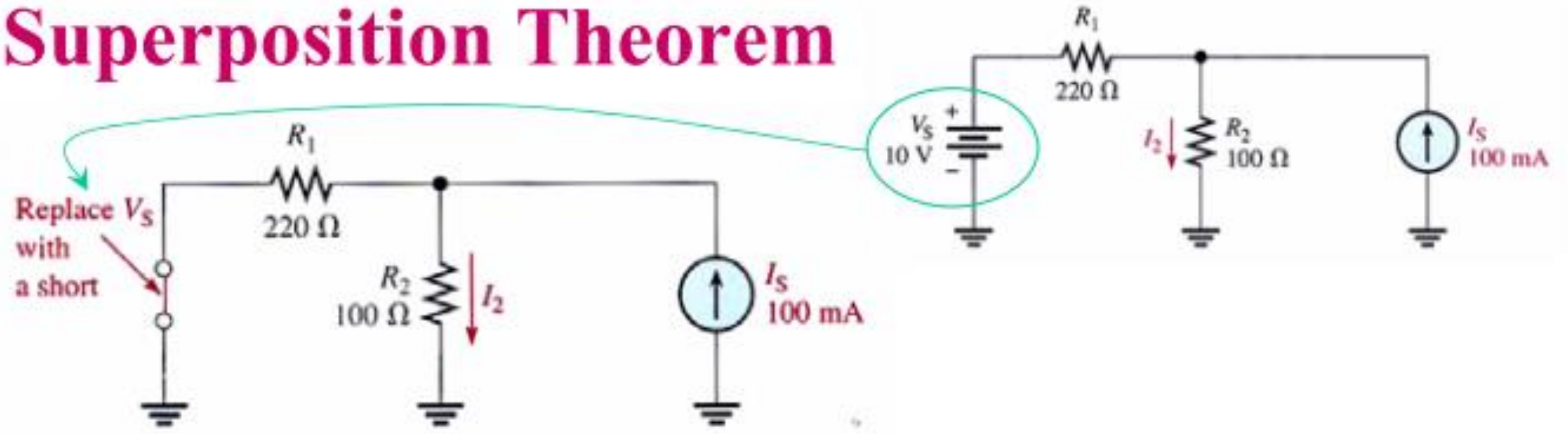
$$R_T = R_1 + R_2 = 320\ \Omega$$

The current through R_2 due to V_S is

$$I_{2(V_S)} = \frac{V_S}{R_T} = \frac{10\ \text{V}}{320\ \Omega} = 31.2\ \text{mA}$$

Note that this current is downward through R_2.

Superposition Theorem



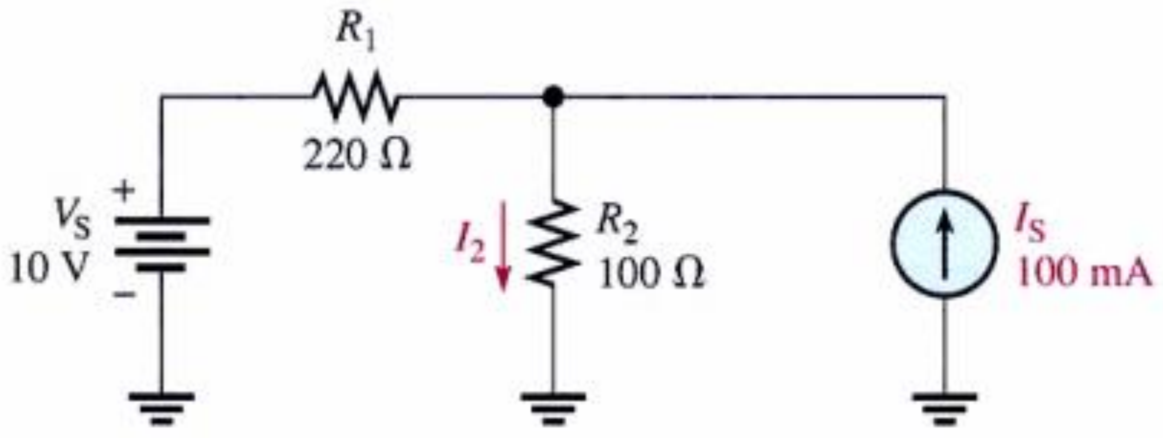
Step 2: Find the current through R_2 due to I_S by replacing V_S with a short.

Use the current-divider formula to determine the current through R_2 due to I_S.

$$I_{2(I_S)} = \left(\frac{R_1}{R_1 + R_2}\right)I_S = \left(\frac{220\ \Omega}{320\ \Omega}\right)100\ \text{mA} = 68.8\ \text{mA}$$

Note that this current also is downward through R_2.

Superposition Theorem

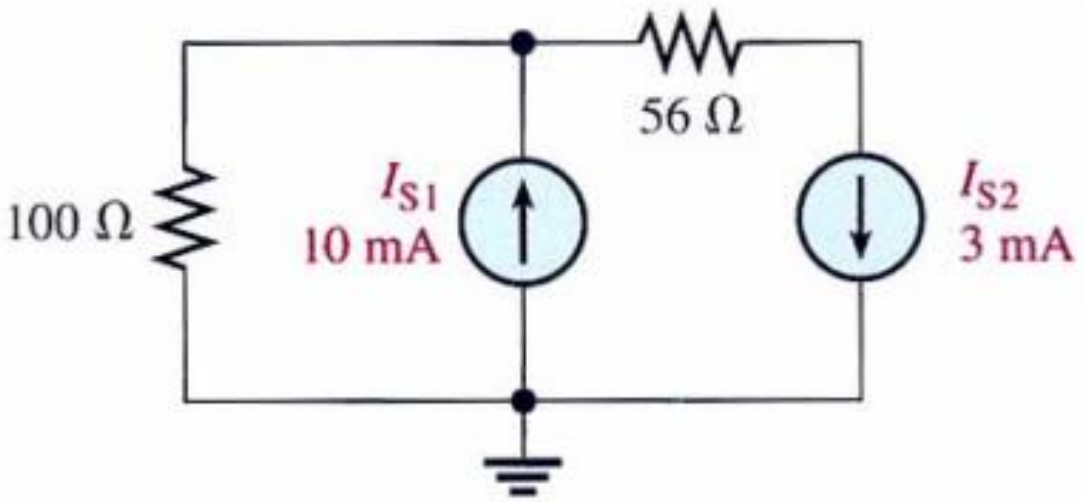


Step 3: Both currents are in the same direction through R_2, so add them to get the total.

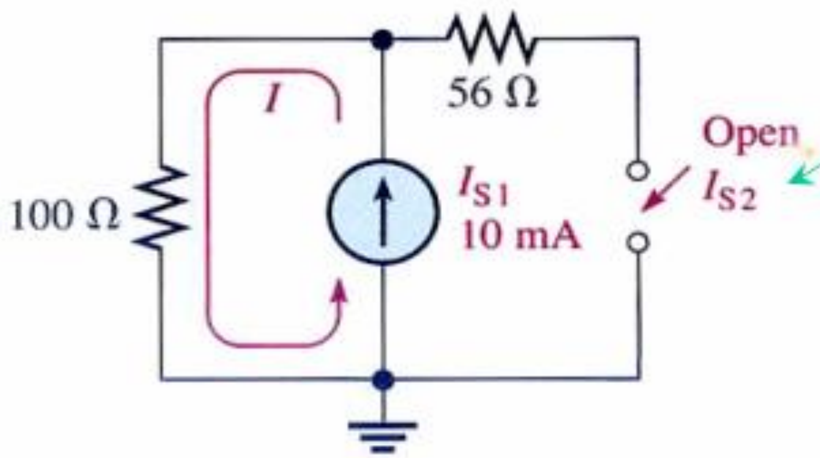
$$I_{2(\text{tot})} = I_{2(V_S)} + I_{2(I_S)} = 31.2\,\text{mA} + 68.8\,\text{mA} = \mathbf{100\,mA}$$

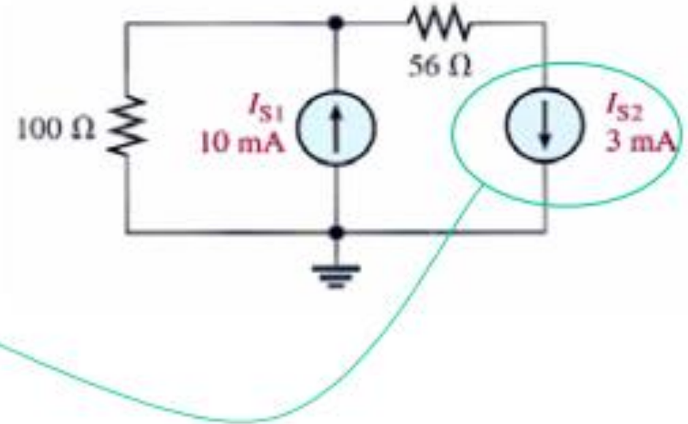
Superposition Theorem

EXAMPLE Find the current through the 100 Ω resistor.



Superposition Theorem



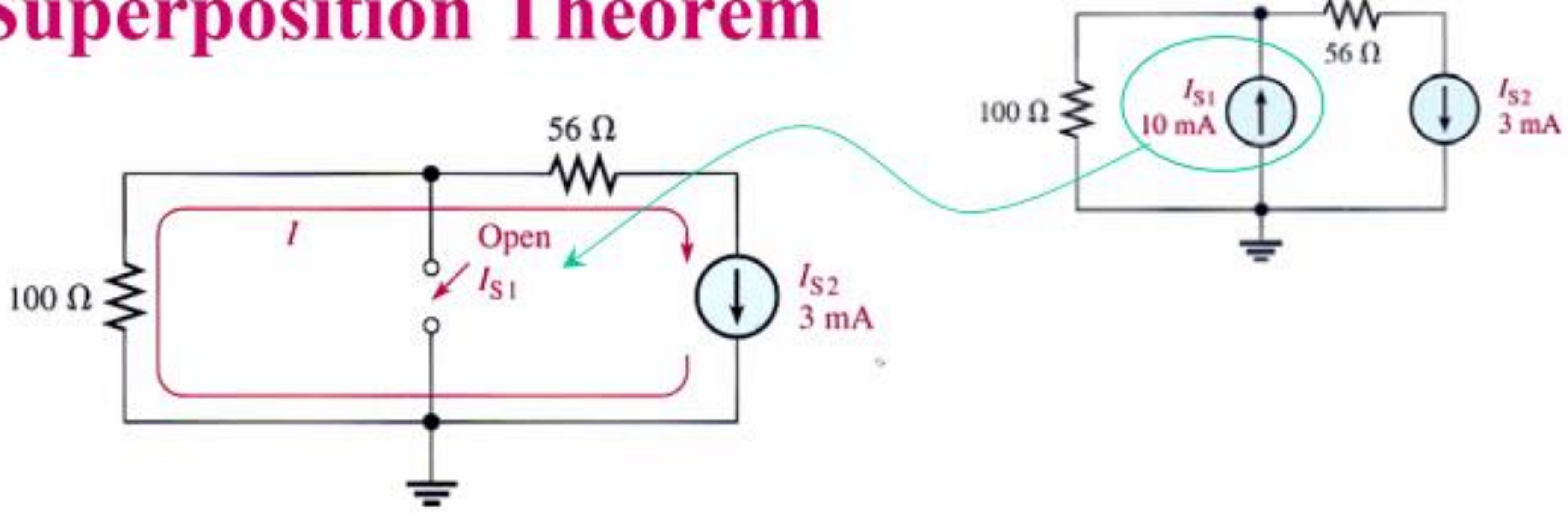


Solution

Step 1: Find the current through the 100 Ω resistor due to current source I_{S1} by replacing source I_{S2} with an open,

$$I_{S1} = 10\text{ mA}$$

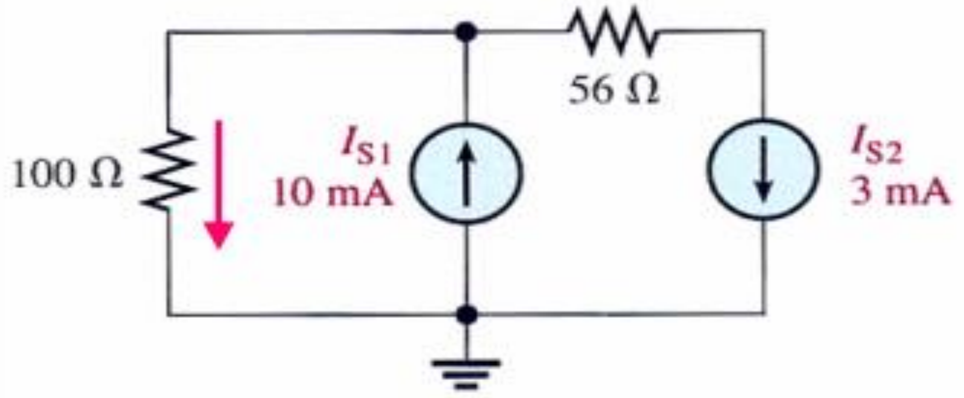
Superposition Theorem



Step 2: Find the current through the 100 Ω resistor due to source I_{S2} by replacing source I_{S1} with an open.

$$I_{S2} = 3\text{ mA} \quad \text{(upward through the 100 Ω resistor.)}$$

Superposition Theorem



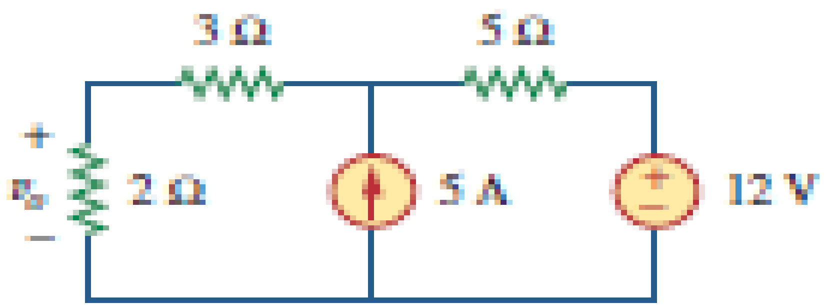
Step 3: To get the total current through the 100 Ω resistor, subtract the smaller current from the larger because they are in opposite directions. The resulting total current is in the direction of the larger current from source I_{S1}.

$$\begin{aligned} I_{100\Omega(\text{tot})} &= I_{100\Omega(I_{S1})} - I_{100\Omega(I_{S2})} \\ &= 10\,\text{mA} - 3\,\text{mA} = \mathbf{7\,mA} \end{aligned}$$

The resulting current is downward through the resistor.

PRACTICE PROBLEM

Using the superposition theorem, find v_o in the circuit of Fig. 4.8.



Answer: 7.4 V.

Thevenin’s Theorem

- Thevenin’s theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{Th} in series with a resistor R_{Th}, where V_{Th} is the open-circuit voltage at the terminals and R_{Th} is the input or equivalent resistance at the terminals when the independent sources are turned off.

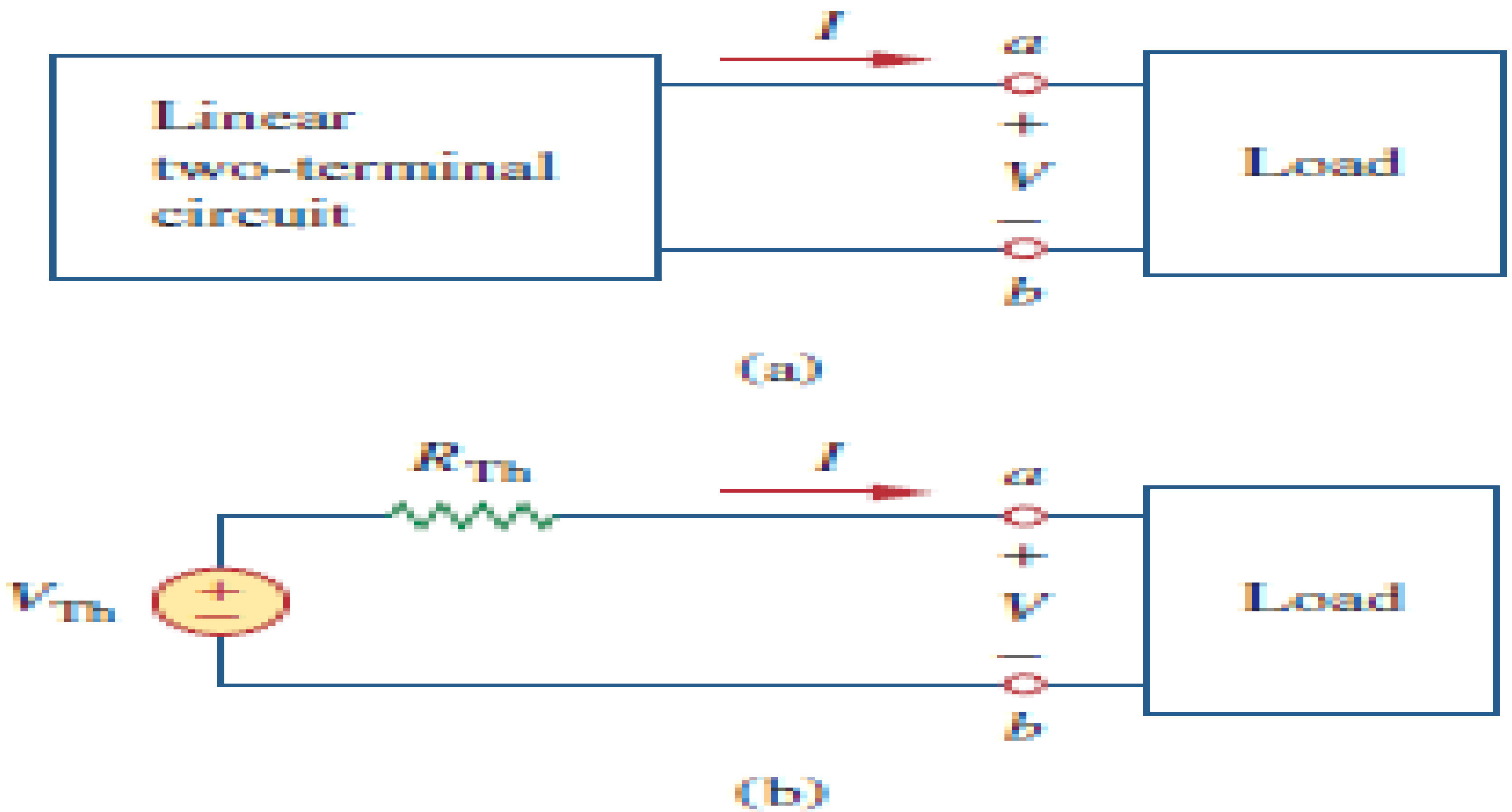
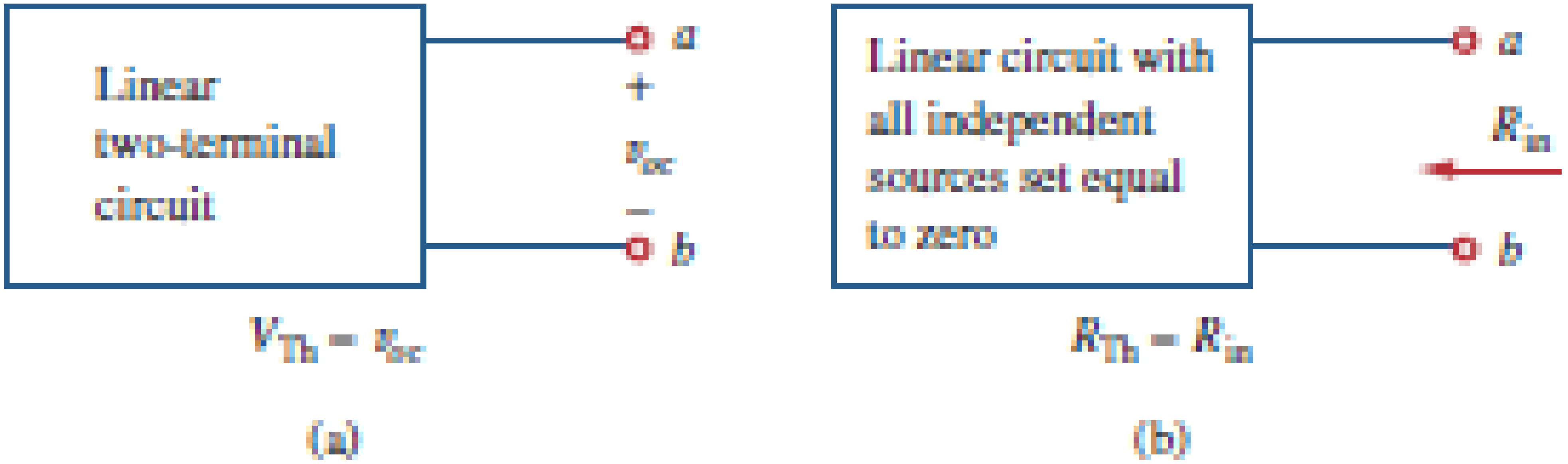


Figure 4.23
Replacing a linear two-terminal circuit by its Thevenin equivalent: (a) original circuit, (b) the Thevenin equivalent circuit.

$$V_{Th} = v_{oc} \tag{4.6}$$



Summary of Thevenin's Theorem

Step 1. Open the two terminals (remove any load) between which you want to find the Thevenin equivalent circuit.

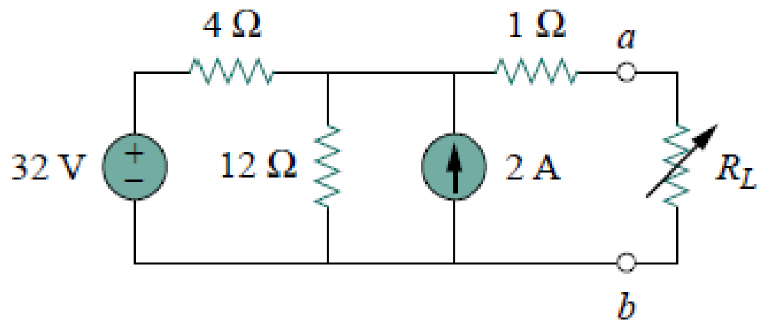
Step 2. Determine the voltage (V_{TH}) across the two open terminals.

Step 3. Determine the resistance (R_{TH}) between the two open terminals with all sources replaced with their internal resistances (ideal voltage sources shorted and ideal current sources opened).

Step 4. Connect V_{TH} and R_{TH} in series to produce the complete Thevenin equivalent for the original circuit.

Step 5. Replace the load removed in Step 1 across the terminals of the Thevenin equivalent circuit. You can now calculate the load current and load voltage using only Ohm's law. They have the same value as the load current and load voltage in the original circuit.

MCQ



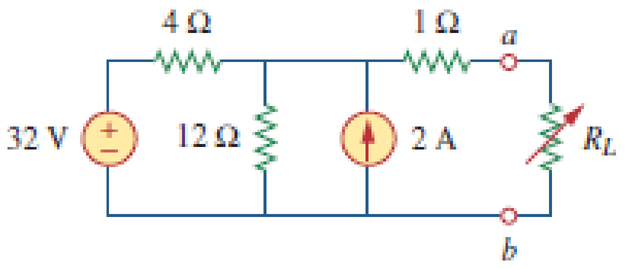
Find the Thevenin equivalent circuit, for the circuit shown in Fig. to the left of the terminals *a*-*b*, if RL=6 Ω.

(a) Rth= 4 ohm , Vth= 30 V

(b) Rth= 16 ohm , Vth= 30 V

(c) Rth= 17 ohm , Vth= 20 V

(d) Rth= 17 ohm , Vth= 30 V



Find the Thevenin equivalent circuit of the circuit shown in Fig. 4.27, to the left of the terminals a-b. Then find the current through $R_L = 6$, 16, and 36 Ω.

Solution:

We find R_{Th} by turning off the 32-V voltage source (replacing it with a short circuit) and the 2-A current source (replacing it with an open circuit). The circuit becomes what is shown in Fig. 4.28(a). Thus,

$$R_{Th} = 4 \parallel 12 + 1 = \frac{4 \times 12}{16} + 1 = 4\ \Omega$$

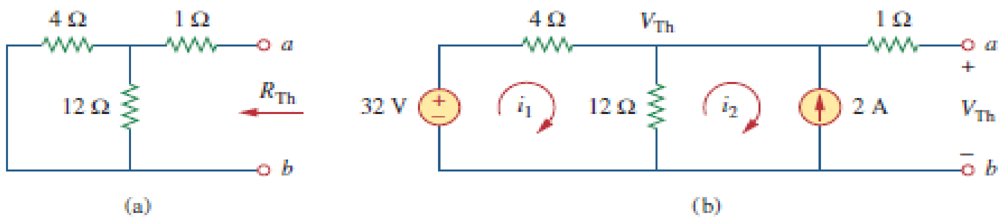


Figure 4.28
For Example 4.8: (a) finding R_{Th}, (b) finding V_{Th}.

To find V_{Th}, consider the circuit in Fig. 4.28(b). Applying mesh analysis to the two loops, we obtain

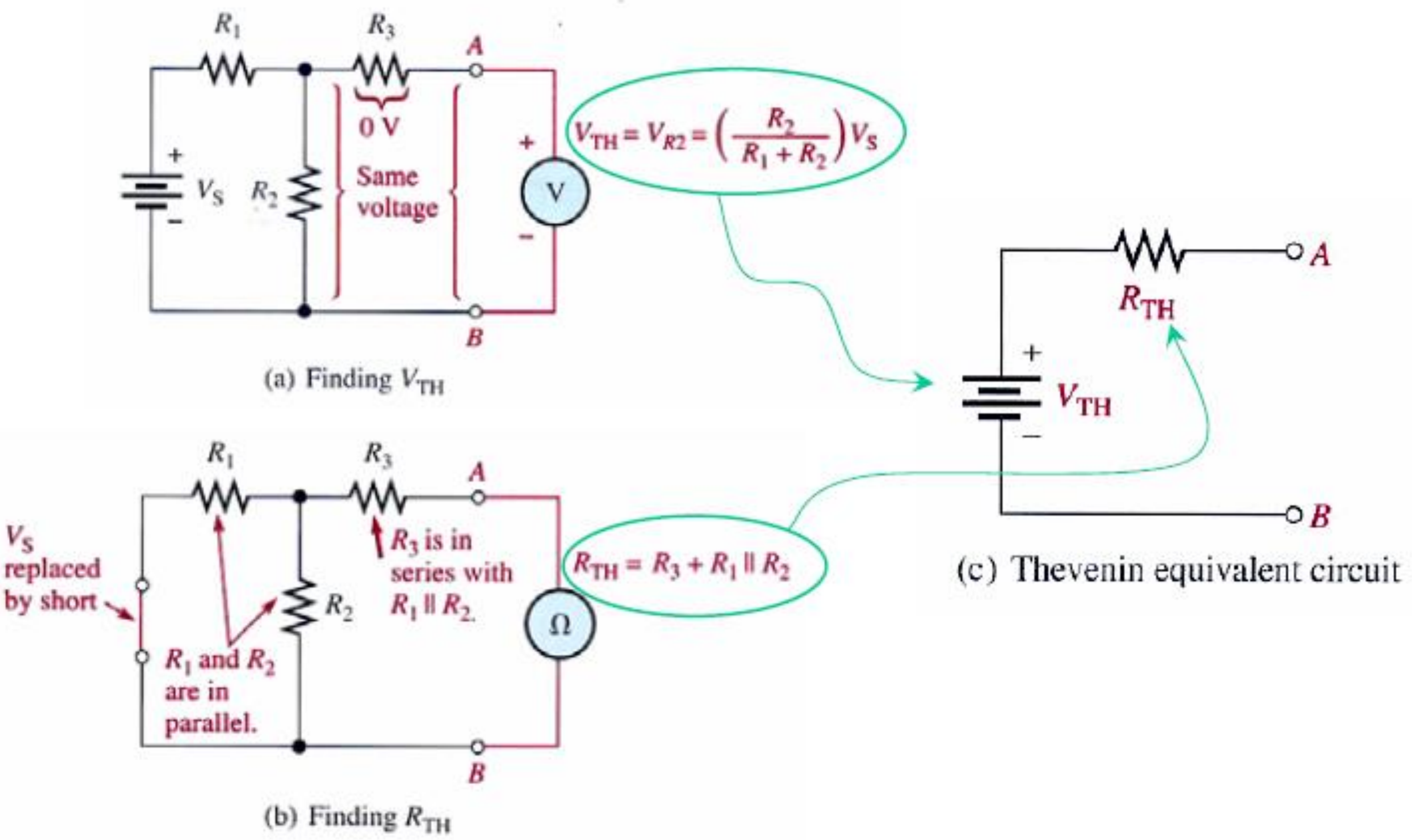
$$-32 + 4i_1 + 12(i_1 - i_2) = 0, \qquad i_2 = -2\text{ A}$$

Solving for i_1, we get $i_1 = 0.5$ A. Thus,

$$V_{Th} = 12(i_1 - i_2) = 12(0.5 + 2.0) = 30\text{ V}$$

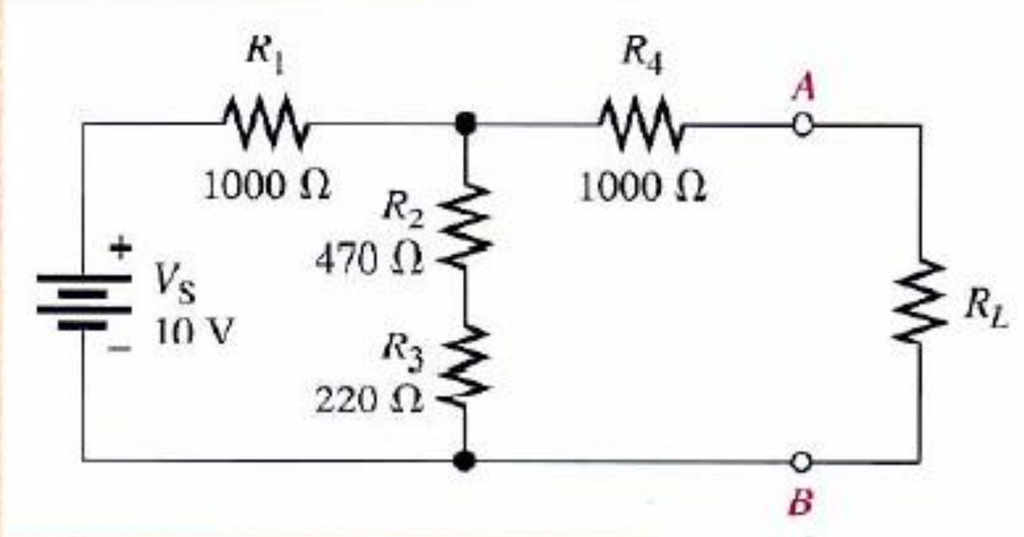
Thevenin's Theorem

Example of the simplification of a circuit by Thevenin's theorem.

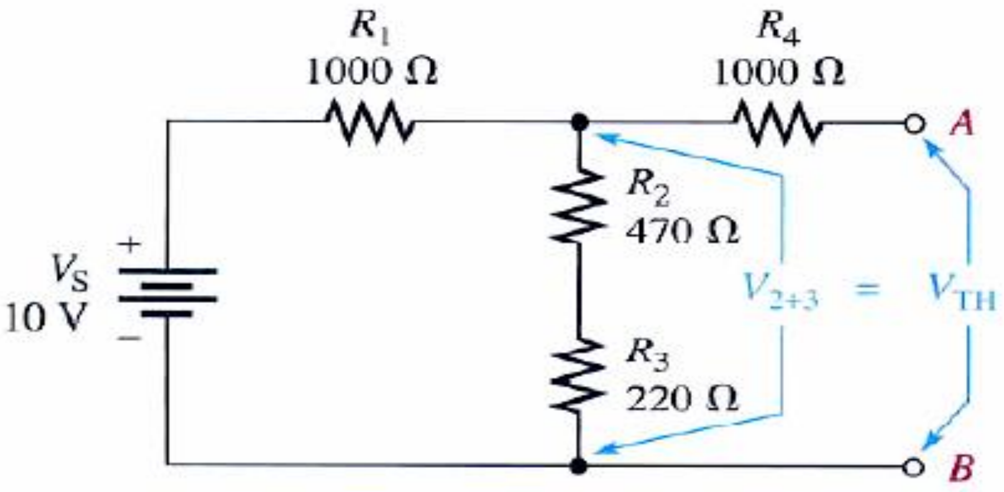


Thevenin's Theorem

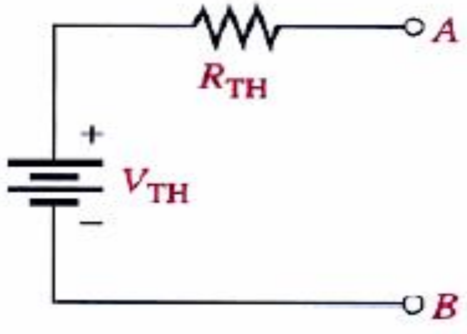
EXAMPLE Find the Thevenin equivalent circuit between A and B of the circuit.



Thevenin's Theorem



The voltage from A to B is V_{TH} and equals V_{2+3}.

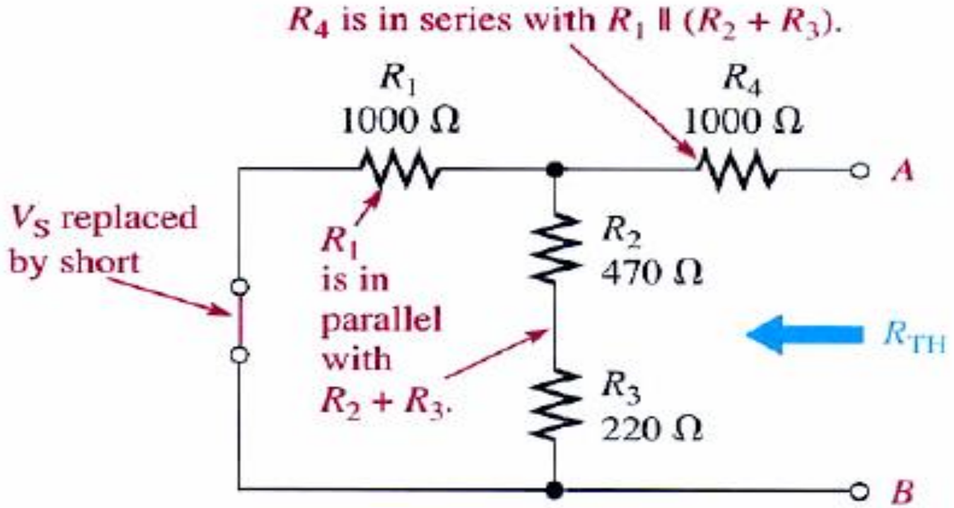


(c) Thevenin equivalent circuit

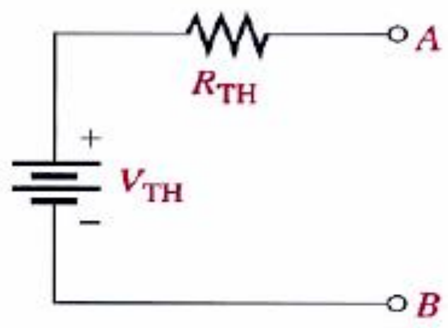
Solution First, remove R_L. Then V_{TH} equals the voltage across $R_2 + R_3$, because $V_4 = 0$ V since there is no current through it.

$$V_{TH} = \left(\frac{R_2 + R_3}{R_1 + R_2 + R_3}\right)V_S - \left(\frac{690\ \Omega}{1690\ \Omega}\right)10\text{ V} = \mathbf{4.08\ V}$$

Thevenin's Theorem



Looking from terminals A and B, R_4 appears in series with the combination of R_1 in parallel with $(R_2 + R_3)$.

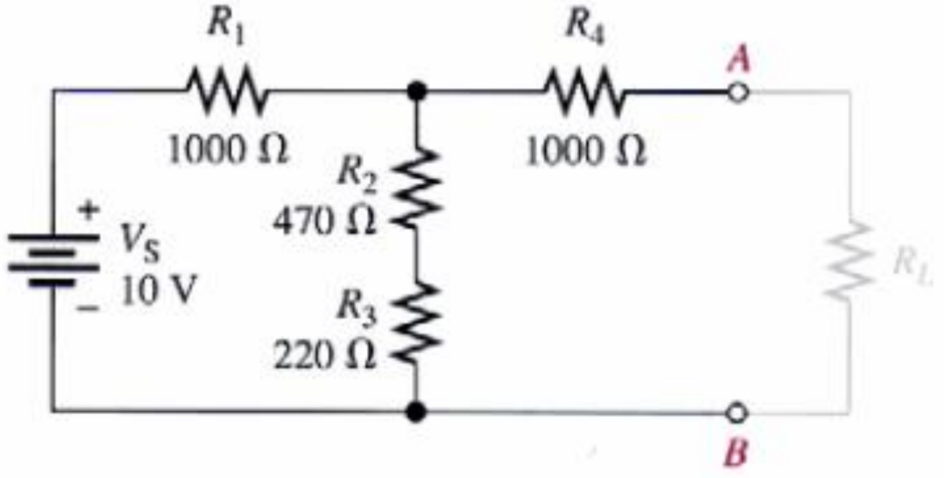


(c) Thevenin equivalent circuit

To find R_{TH}, first replace the source with a short to simulate a zero internal resistance. Then R_1 appears in parallel with $R_2 + R_3$, and R_4 is in series with the series-parallel combination of R_1, R_2, and R_3,

$$R_{TH} = R_4 + \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3} = 1000\ \Omega + \frac{(1000\ \Omega)(690\ \Omega)}{1690\ \Omega} = \mathbf{1410\ \Omega}$$

Thevenin's Theorem



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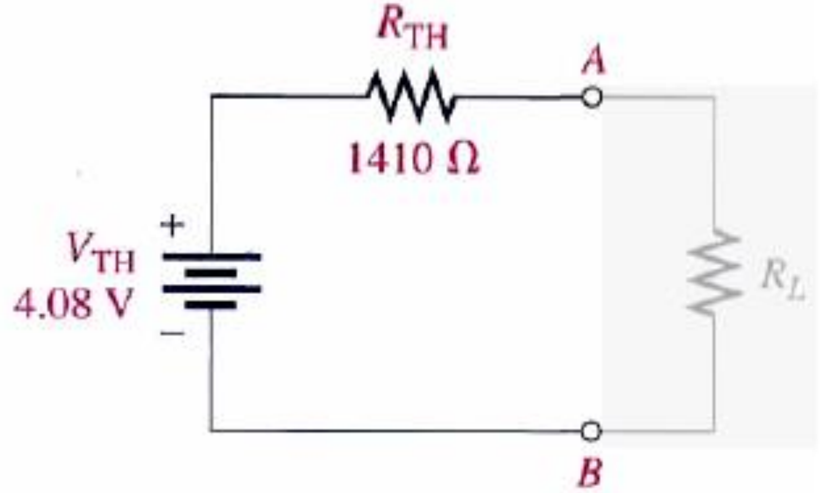
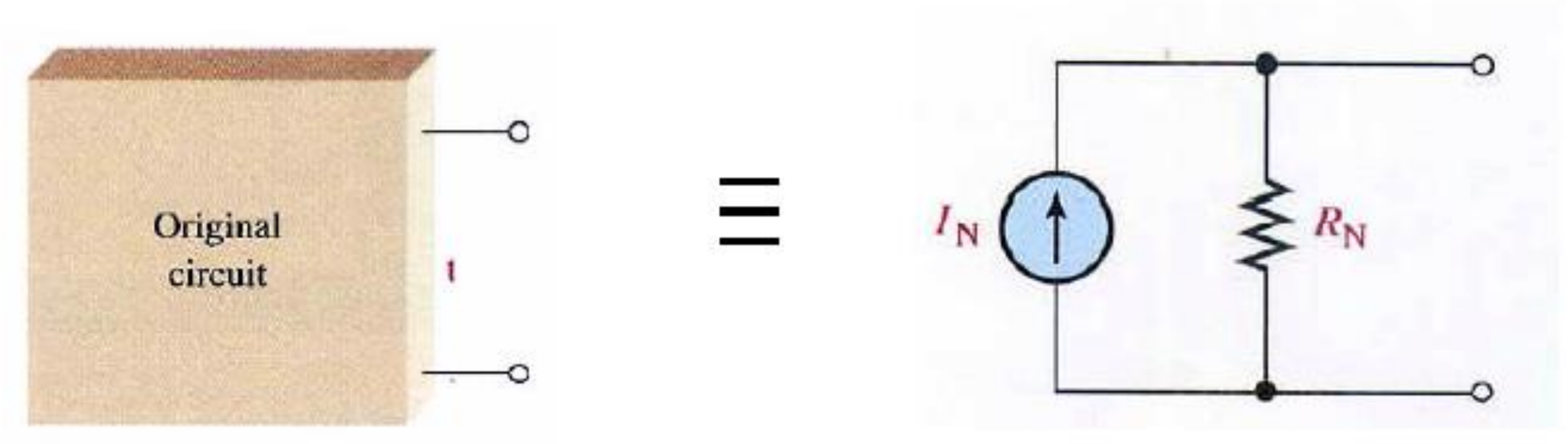


Thevenin equivalent circuit

Norton's Theorem

Like Thevenin's theorem, Norton's theorem provides a method of reducing a more complex circuit to a simpler equivalent form.



Norton's Theorem

Summary of Norton's Theorem

Step 1. Short the two terminals between which you want to find the Norton equivalent circuit.

Step 2. Determine the current (I_N) through the shorted terminals.

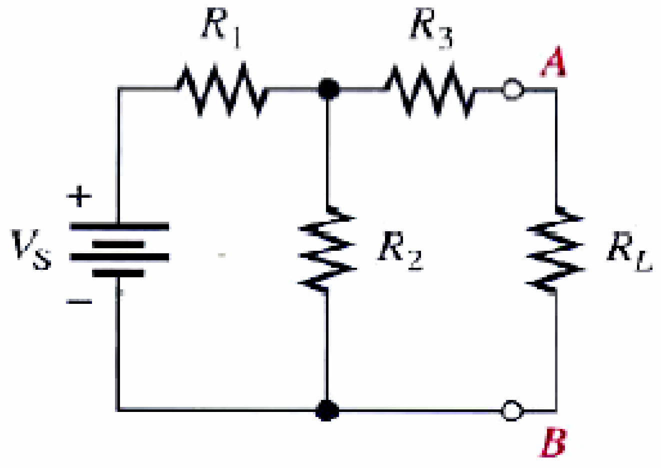
Step 3. Determine the resistance (R_N) between the two open terminals with all sources replaced with their internal resistances (ideal voltage sources shorted and ideal current sources opened). $R_N = R_{TH}$.

Step 4. Connect I_N and R_N in parallel to produce the complete Norton equivalent for the original circuit.

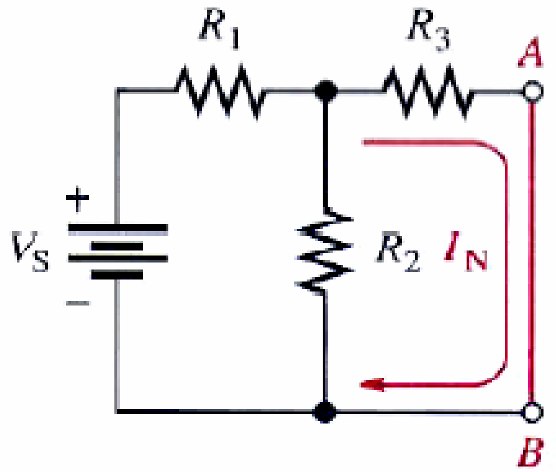
Norton's Theorem

Norton's Equivalent Current (I_N)

Norton's equivalent current (I_N) is the short-circuit current between two output terminals in a circuit.



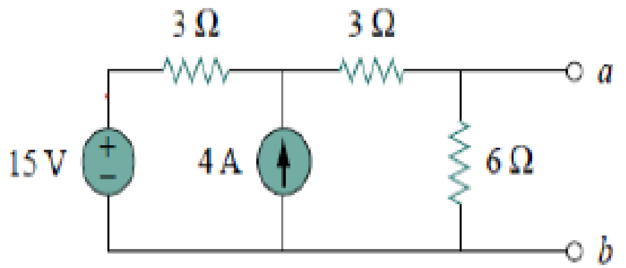
(a) Original circuit



(b) Short the terminals to get I_N.

MCQ

Find the Norton equivalent circuit for the circuit in Fig



(a) *RN* = 3 Ω, *IN* = 4.5 A

(b) *RN* = 6 Ω, *IN* = 4.5 A

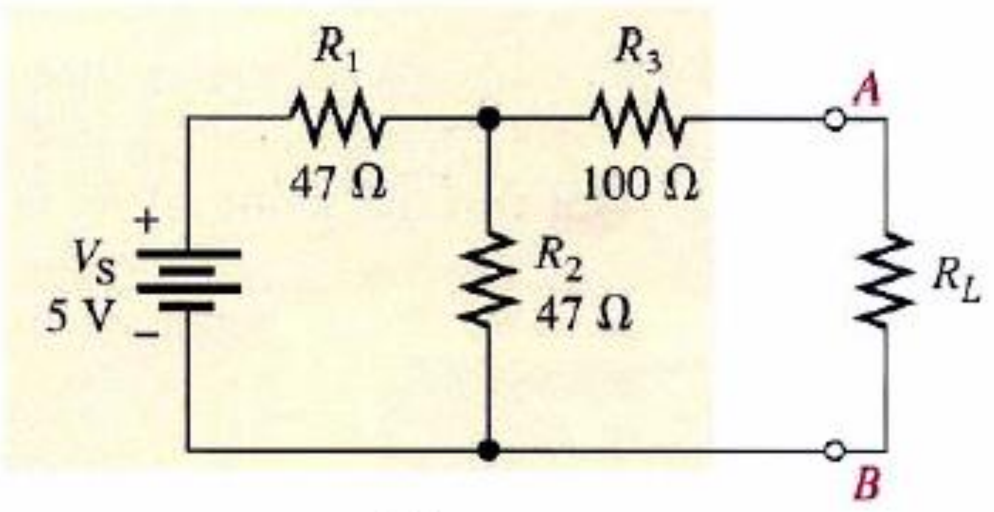
(c) *RN* = 3 Ω, *IN* = 1.5 A

(d) *RN* = 3 Ω, *IN* = 2.5 A

Norton's Theorem

Norton's Equivalent Current (I_N)

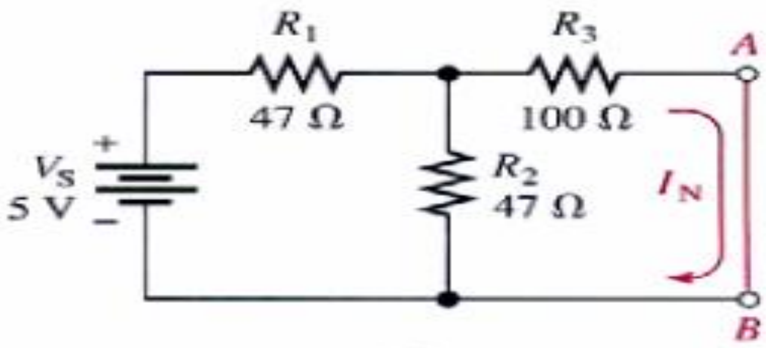
EXAMPLE Determine I_N for the circuit within the beige area.



(a)

Norton's Theorem

Norton's Equivalent Current (I_N)



(b)

Solution

Short terminals A and B. I_N is the current through the short. First, the total resistance seen by the voltage source is

$$R_T = R_1 + \frac{R_2 R_3}{R_2 + R_3} = 47\ \Omega + \frac{(47\ \Omega)(100\ \Omega)}{147\ \Omega} = 79\ \Omega$$

The total current from the source is

$$I_T = \frac{V_S}{R_T} = \frac{5\ \text{V}}{79\ \Omega} = 63.3\ \text{mA}$$

Now apply the current-divider formula to find I_N (the current through the short).

$$I_N = \left(\frac{R_2}{R_2 + R_3}\right) I_T = \left(\frac{47\ \Omega}{147\ \Omega}\right) 63.3\ \text{mA} = \mathbf{20.2\ mA}$$

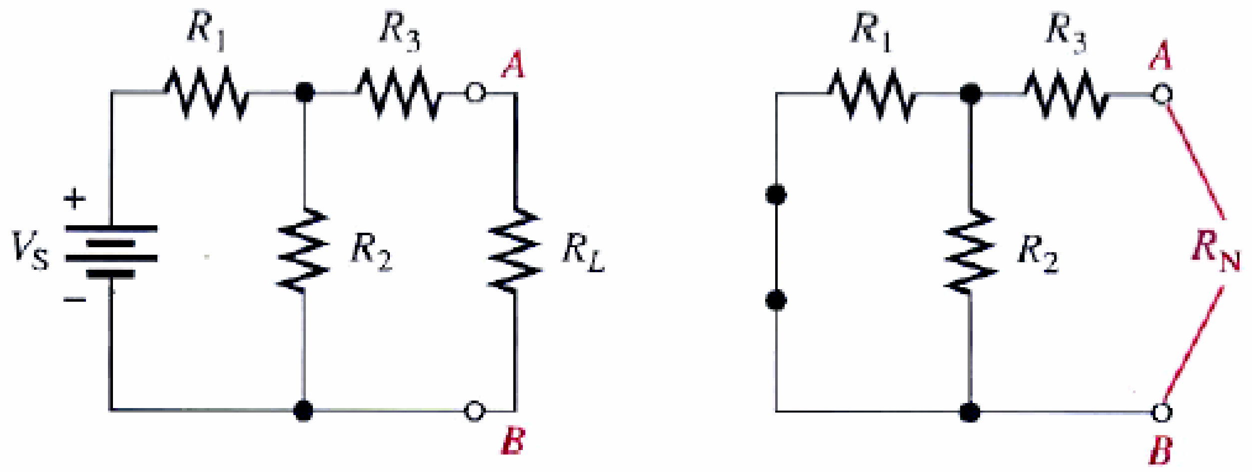
This is the value for the equivalent Norton current source.

Norton's Theorem

Norton's Equivalent Resistance (R_N)

Norton's equivalent resistance (R_N) is defined in the same way as R_{TH}.

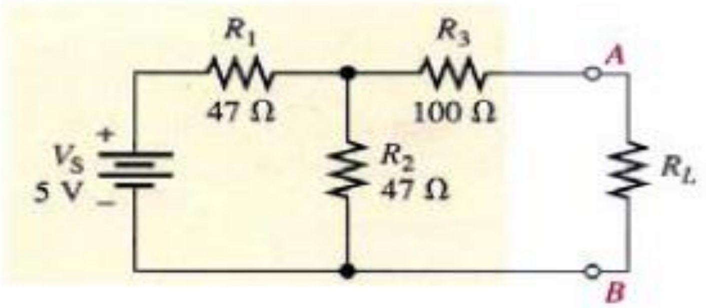
The Norton equivalent resistance, R_N, is the total resistance appearing between two output terminals in a given circuit with all sources replaced by their internal resistances.

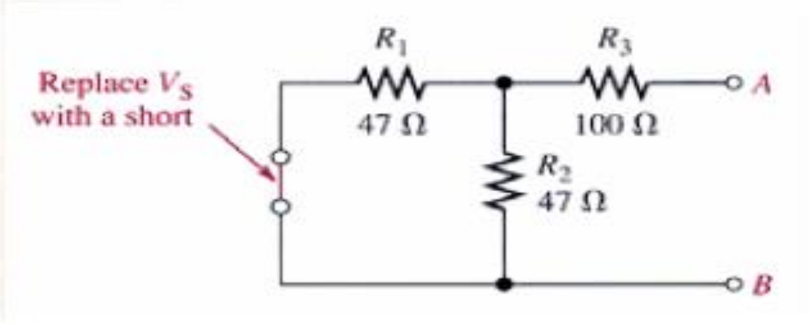


Norton's Theorem

Norton's Equivalent Resistance (R_N)

EXAMPLE Find R_N for the circuit within the beige area





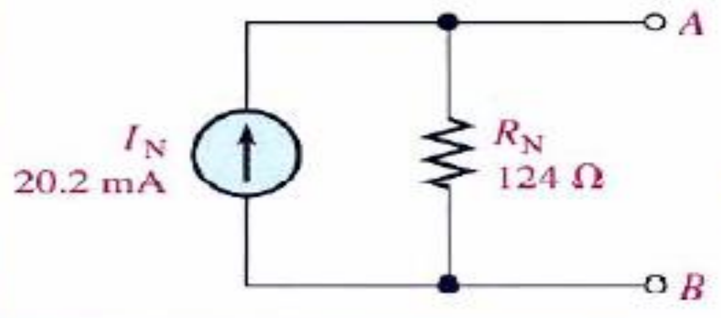
Solution

$$R_N = R_3 + \frac{R_1}{2} = 100\ \Omega + \frac{47\ \Omega}{2} = \mathbf{124\ \Omega}$$

Norton's Theorem

EXAMPLE Draw the complete Norton equivalent circuit for the original circuit that $I_N = 20.2\ \text{mA}$ and $R_N = 124\ \Omega$.

Solution

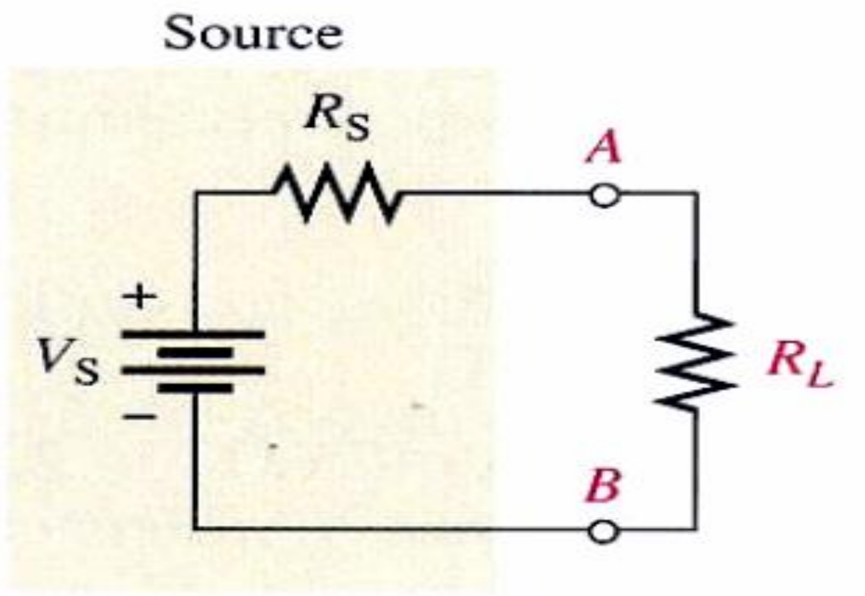


Maximum Power Transfer Theorem

The maximum power transfer theorem is important when you need to know the value of the load at which the most power is delivered from the source.

The **maximum power transfer** theorem is stated as follows:

For a given source voltage, maximum power is transferred from a source to a load when the load resistance is equal to the internal source resistance.



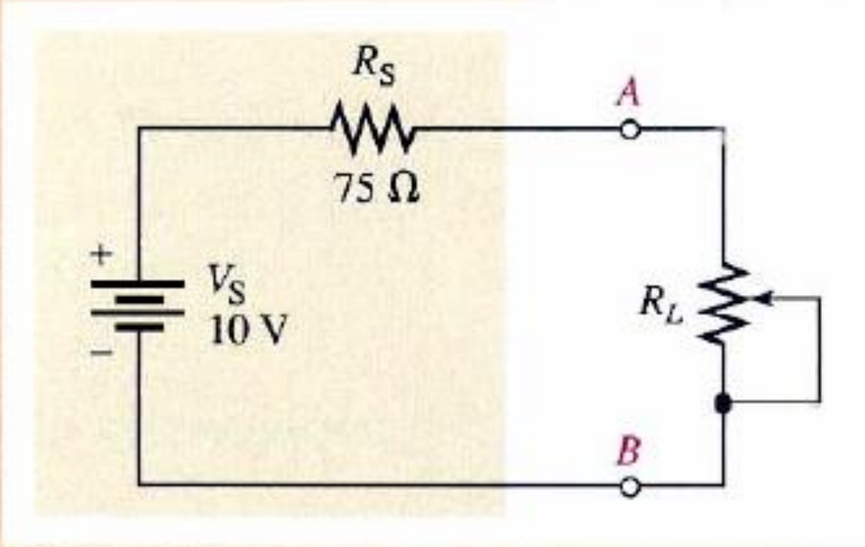
Maximum power is transferred to the load when $R_L = R_S$.

Maximum Power Transfer Theorem

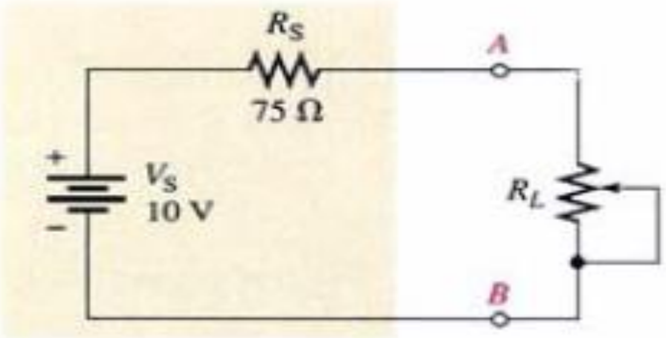
EXAMPLE The source has an internal source resistance of 75 Ω. Determine the load power for each of the following values of load resistance:

(a) 0 Ω (b) 25 Ω (c) 50 Ω (d) 75 Ω (e) 100 Ω (f) 125 Ω

Draw a graph showing the load power versus the load resistance.



Maximum Power Transfer Theorem



Solution

Use Ohm's law ($I = V/R$) and the power formula ($P = I^2R$) to find the load power, P_L, for each value of load resistance.

(a) For $R_L = 0\ \Omega$,

$$I = \frac{V_S}{R_S + R_L} = \frac{10\ \text{V}}{75\ \Omega + 0\ \Omega} = 133\ \text{mA}$$

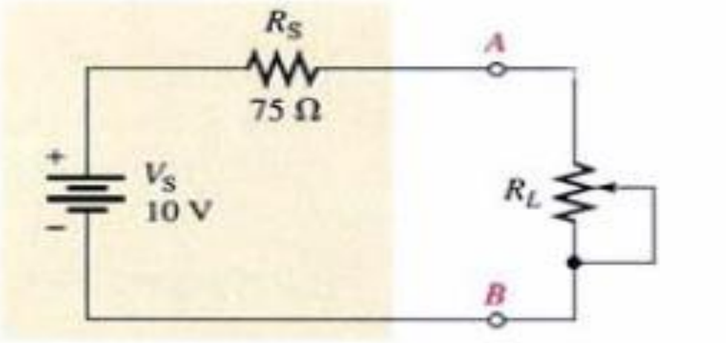
$$P_L = I^2R_L = (133\ \text{mA})^2(0\ \Omega) = \mathbf{0\ mW}$$

(b) For $R_L = 25\ \Omega$,

$$I = \frac{V_S}{R_S + R_L} = \frac{10\ \text{V}}{75\ \Omega + 25\ \Omega} = 100\ \text{mA}$$

$$P_L = I^2R_L = (100\ \text{mA})^2(25\ \Omega) = \mathbf{250\ mW}$$

Maximum Power Transfer Theorem



(c) For $R_L = 50\ \Omega$,

$$I = \frac{V_S}{R_S + R_L} = \frac{10\ \text{V}}{125\ \Omega} = 80\ \text{mA}$$

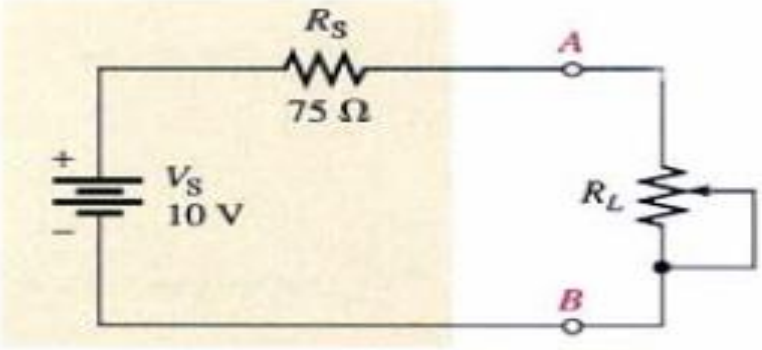
$$P_L = I^2R_L = (80\ \text{mA})^2(50\ \Omega) = \mathbf{320\ mW}$$

(d) For $R_L = 75\ \Omega$,

$$I = \frac{V_S}{R_S + R_L} = \frac{10\ \text{V}}{150\ \Omega} = 66.7\ \text{mA}$$

$$P_L = I^2R_L = (66.7\ \text{mA})^2(75\ \Omega) = \mathbf{334\ mW}$$

Maximum Power Transfer Theorem



(e) For $R_L = 100\ \Omega$,

$$I = \frac{V_S}{R_S + R_L} = \frac{10\ \text{V}}{175\ \Omega} = 57.1\ \text{mA}$$

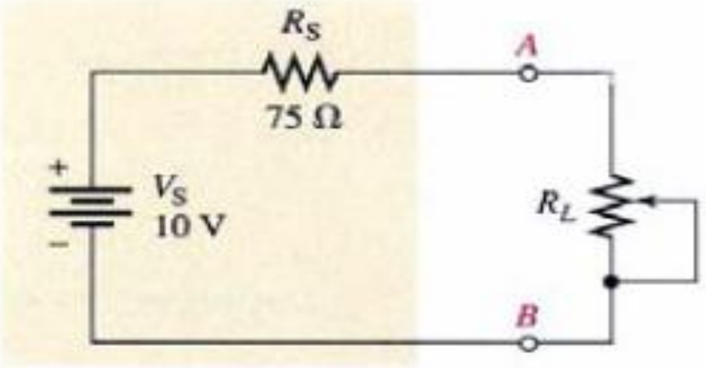
$$P_L = I^2R_L = (57.1\ \text{mA})^2(100\ \Omega) = \mathbf{326\ mW}$$

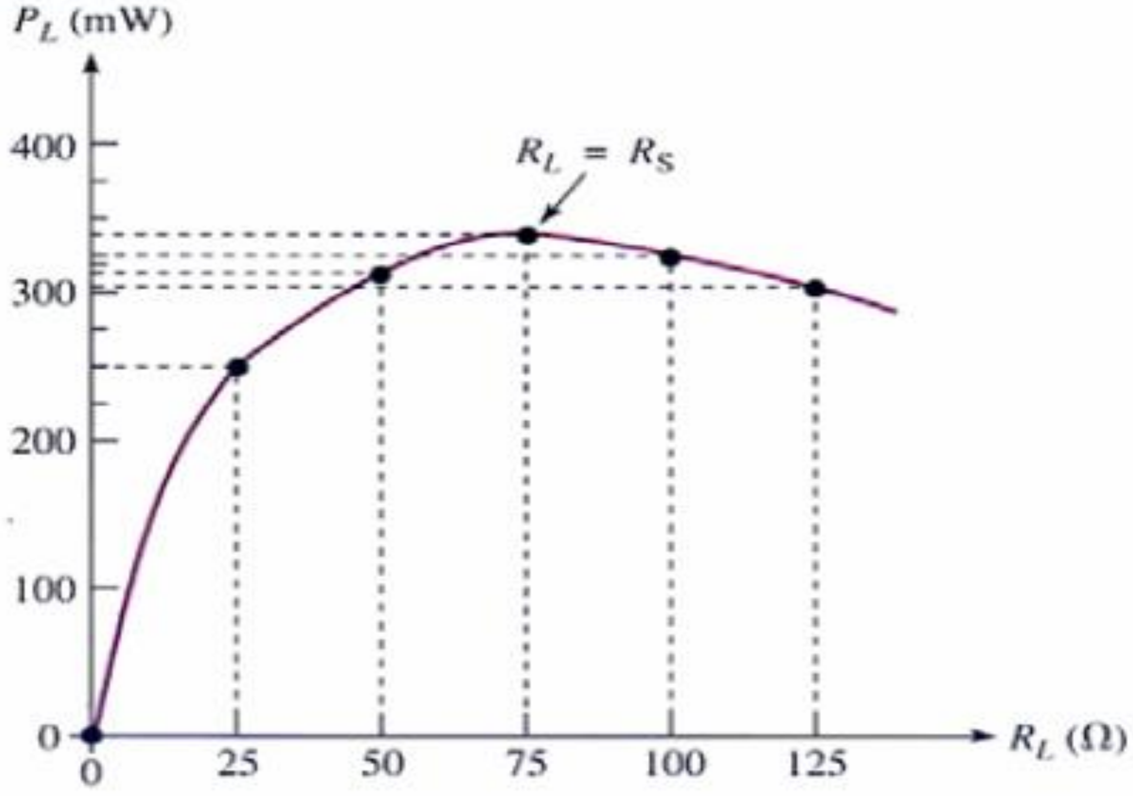
(f) For $R_L = 125\ \Omega$,

$$I = \frac{V_S}{R_S + R_L} = \frac{10\ \text{V}}{200\ \Omega} = 50\ \text{mA}$$

$$P_L = I^2R_L = (50\ \text{mA})^2(125\ \Omega) = \mathbf{313\ mW}$$

Maximum Power Transfer Theorem





The load power is greatest when $R_L = 75\ \Omega$, which is the same as the internal source resistance.