MCQs (Units -II, III)

1.The general solution of the differential equation $y'' - 4y = 0$ is

(a) $a\cos 2x + b\sin 2x$
(b) $ae^{-2x} + bxe^{2x}$
(c) $ae^{-2x} + bx^2e^{2x}$
(d) $ae^{-2x} + be^{2x}$

2. What is the Wronskian of x, x^2, x^3 ?

(a) $2x^3$
(b) x^3
(c) $3x^3$
(d) $4x^3$

3. If differential equation $y' = \frac{3y}{x}$ is normal over the interval I then I is

(a) $(-\infty, 2)$
(b) $(-\infty, \infty)$
(c) $(-2, \infty)$
(d) $(-\infty, 0) \cup (0, \infty)$

4. If $y'' + y = 32x^3$ then by the method of undetermined coefficient the assumed particular integral

$m^2+1=0$ $\quad$ $y_c = c_1\cos x + c_2\sin x$

[no modification] $\Rightarrow y_p = ax^3+bx^2+cx+d$

(a) $y_p(x) = c_1x^3 + c_2x^2 + c_3$
(b) $y_p(x) = c_1x^3 + c_2x^2 + c_3x + c_4$
(c) $y_p(x) = c_1x^3 + c_2$
(d) $y_p(x) = c_1x^2 + c_2x + c_3$

5. Particular integral for the differential equation $(D^2 + 9)y = \sin 3x$ is

(a) $\frac{x\cos 3x}{6}$
(b) $-\frac{x\cos 3x}{6}$
(c) $\frac{x\cos 3x}{3}$
(d) $\frac{x\cos 2x}{6}$

$y_p = \frac{1}{D^2+9}\sin 3x = x\frac{1}{2D}\sin 3x = \frac{x}{2}\int \sin 3x$

$-9+9=0$

6. The particular integral of the differential equation $y'' + y = \tan x$ if $y_1 = \cos x$, $y_2 = \sin x$ is $\quad W=1$

$y_p = -\cos x\int \frac{\sin x\tan x}{1}\,dx + \sin x\int \frac{\cos x\tan x}{1}\,dx$

$-\cos x\int\frac{\sin^2 x}{\cos x} + \sin x\int \sin x\,dx$

(a) $y_p(x) = \sin x - ln|\sec x + \tan x|$
(b) $y_p(x) = \cos x - ln|\sec x + \tan x|$
(c) $y_p(x) = -\cos x\, ln|\sec x + \tan x|$
(d) $y_p(x) = -ln|\sec x + \tan x|$

7. The general solution of the differential equation $x^2y'' + xy' - 4y = 0$ is

$\theta^2 - \cancel{\theta} + \cancel{\theta} - 4 = 0$ $\quad$ $\theta^2-4=0$ $\quad$ $\theta = 2, -2$ $\quad$ $y_c = c_1x^2 + c_2x^{-2}$

(a) $ax^2 + \frac{b}{x^2}$
(b) $ax + \frac{b}{x^2}$
(c) $ax^2 + \frac{b}{x}$
(d) $3x + 4/x^2$

8. For the system of the differential equations

$1\times(2D-4)y_1 + (3D+5)y_2 = 3t+2$ ①, $(D-2)y_1 + (D+1)y_2 = t$ ② $\times 2$, the value of y_2 is given by

(a) $ae^{3t} + \frac{1}{9}(3t+5)$
(b) $ae^{3t} - \frac{1}{9}(3t+5)$
(c) $ae^{-3t} - \frac{1}{9}(3t+5)$
(d) $ae^{-3t} + \frac{1}{9}(3t+5)$

$y_2 =$

① ⇒ $(2D-\cancel{4})y_1 + (3D+5)y_2 = 3t+2$

$(2D-4)y_1 + (2D+2)y_2 = 2t$

$(D+3)y_2 = t+2$

eliminating y_1 from eq ① & ②

$m+3=0$ $\quad$ $y_c = ae^{-3t}$

$y_p = \frac{1}{D+3}(t+2) = \frac{1}{3}\left(1+\frac{D}{3}\right)^{-1}(t+2)$

$y_p = \frac{1}{3}\left[1 - \frac{D}{3} + \frac{D^2}{9}\right)(t+2) = \frac{1}{3}\left[(t+2) - \frac{1}{3}(1)\right]$

$y_p = \frac{1}{9}[3t+6-1] = \frac{1}{9}[3t+5]$

4+3
$4m^2+7m+3=0$
$4m^2+4m+3m+3=0$
$(4m+3)(m+1)=0$

9 If e^{kx} is solution of $4y'' + 7y' + 3y = 0$ then k is

a) 1 b) 2 c) ¾ d) −3/4

10. what is lowest order of differential equation whose particular integral is
$3\cos 2x + 5\sinh 3x$ $= 3\cos 2x + 0 \sin 2x + \frac{5}{2}e^{3x} - \frac{5}{2}e^{-3x}$

$\sinh 3x = \frac{1}{2}[e^{3x} - e^{-3x}]$

$m = \pm 2i, \pm 3$
$\cdot$ 4 roots

a) 2 b) 3 c) 4 d) 6

11. If $ay'' + by' + cy = 0$ be second order differential equation where a , b , c are functions of x and $a \neq 0$ on any interval then wronskian according to Abel's formula is

a) $w(x) = constant$ b) $w(x) = Ae^{\int \frac{-b}{a}dx}$ c) $w(x) = Ae^{\int \frac{b}{a}dx}$ d) None.

12. If $ay'' - by' + cy = 0$ be second order differential equation then its roots are real and distinct if

a) $b^2 - 4ac > 0$ b) $b^2 - 4ac = 0$ c) $b^2 - 4ac < 0$ d) both a and b

13. e^{-i3x}, e^{i3x} be solution of

a) $y'' + 6y' + 9y = 0$ b) $y'' - 6y' + y = 0$ c) $y'' + 9y = 0$ d) $y'' - 9y = 0$

If e^{-at}
$\frac{dy}{dt} - ay = f$
or
$ye^{-at} = \int f e^{-at}$

14. If $(D-a)y = X$ then $y_p = \frac{1}{D-a}X =$ $e^{ax}\int e^{-ax} X$ [If $\frac{1}{D+a}X = e^{-ax}\int e^{ax} X$]

a) $e^{ax}\int X e^{-ax}dx$ b) $e^{-ax}\int X e^{-ax}dx$ c) $e^{-ax}\int X e^{ax}dx$ d) $e^{ax}\int X e^{ax}dx$

15. Which of the following is true?

a) $\frac{1}{f(D)}\cos bx\, e^{ax} = e^{ax}\frac{1}{f(D+b)}\cos bx$ b) $\frac{1}{f(D^2)}\cos bx\, e^{ax} = \cos bx \frac{1}{f(-a^2)}e^{ax}$
c) $\frac{1}{f(D)}xe^{ax} = x\frac{1}{f(D+a)}e^{ax}$ d) $\frac{1}{f(D)}x^2e^{ax} = e^{ax}\frac{1}{f(D+a)}x^2$

Solve 1 + 2 to eliminate y_1

16. If $y_1(t), y_2(t)$ satisfy the equations $y_1' + 5y_2 = 0$ (1), $y_2' + y_1 = 0$ (2) and $y_2'' + by_2 = 0$ (3) is the second order differential equation satisfied by y_1 then what is the value of b

a) 5 b) −5 c) 3 d) −3

$b = -5$

eq (2) $\Rightarrow y_1 = -y_2'$
Put in (1) $(-y_2')' + 5y_2 = 0 \Rightarrow -y_2'' + 5y_2 = 0 \Rightarrow y_2'' - 5y_2 = 0$ compare it with (3) to find (b)

To find eq in y_1 eliminate y_2 from (1) + (2)

17. For a given system of linear differential equation $y_1' = 2y_1 + y_2$ (1), $y_2' = y_1 + 2y_2$ (2), the second order linear differential satisfied by the y_1 is

a) $y_1'' + 4y_1' + 3y_1 = 0$ b) $y_1'' - 4y_1' + 3y_1 = 0$ c) $y_1'' - 4y_1' - 3y_1 = 0$
(d) none

From (1) $y_2 = y_1' - 2y_1$ Put it in eq (2) $(y_1' - 2y_1)' = y_1 + 2(y_1' - 2y_1)$ and simplify

$y_1'' - 2y_1' = y_1 + 2y_1' - 4y_1$
$y_1'' - 4y_1' + 3y_1 = 0$ option (b) ans

18. What are the characteristic roots of a homogeneous LDE having 4 + x e^{2x} as its particular solution?

a) 0, 2 b) 4, 2 c) 4, 2 ,2 d) 0, 2, 2

write missing terms Soln = $4 + xe^{2x} = 4e^{0x} + 0e^{2x} + 1.xe^{2x} \Rightarrow m = 0, 2, 2$

19. The general solution of y$^{(4)}$ - 2y''' = 0 is given by

[In ques 18 we need to write all missing terms to get idea of all roots]

Q19 $m^4 - 2m^3 = 0$ $m^3(m-2)=0$ $m = 0, 0, 0, 2$

$y = Ae^{0x} + Bxe^{0x} + Cx^2e^{0x} + De^{2x} = A + Bx + Cx^2 + De^{2x}$

a) A + B e^{2x} + Dxe^{2x} b) A + B + Ce^{2x} c) A + Bx + Cx^2 + De^{2x} d) A + Bx + Cx^2

where A, B, C and D are arbitrary constants.

20. The DE $x^2y'' - 4xy' + 6y = 0$ on $(0, \infty)$ has 2 linearly independent solutions.

2nd order has 2 LI soln

a) 2 b) 3 c) infinite d) Can't say

21. The general solution of the equation $4y'' - 4y' + y = 8e^{x/2}$ is given by

a) $Ae^{x/2} + Be^{x/2} + x^2e^{x/2}$
b) $Ae^{x/2} + Bxe^{x/2} + x^2e^{x/2}$
c) $Ae^{x/2} + Be^{x/2} + e^{x/2}$
d) $Ae^{x/2} + Bxe^{x/2} + xe^{x/2}$

$4m^2 - 4m + 1 = 0$ $(2m-1)^2 = 0$ $m = \frac{1}{2}, \frac{1}{2}$

$y_c = Ae^{\frac{1}{2}x} + Axe^{\frac{1}{2}x}$

$y_p = \frac{1}{4D^2 - 4D + 1} 8e^{x/2} = x^2 \frac{1}{8} 8e^{x/2} = x^2 e^{x/2}$

$f(D)$ $4(\frac{1}{2})^2 - 4(\frac{1}{2}) + 1 = 0$ $f'(D) = 8D - 4$ $f'(\frac{1}{2}) = 0$ $f''(D) = 8$ $f''(\frac{1}{2}) = 8$

22. If $(D^2 - 3D + 2)y = \cosh x$, then what will be the Complementary Factor?

$m^2 - 3m + 2 = 0$ $m = 1, 2$

a) $C_1e^{-x} + C_1e^{-2x}$ b) $C_1e^{x} + C_1e^{2x}$ c) $C_1e^{-\frac{x}{3}} + C_1e^{-\frac{2x}{3}}$ d) $\frac{e^{2x}}{5} + \frac{e^{-\frac{4x}{7}}}{7}$

23. If $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = e^{ax}V, V \text{ is a function of } x$, then Particular Integral will be written as $\frac{1}{f(0)} e^{ax} V =$

formula long

a) $\frac{1}{f(D+a)}V$ b) $e^{ax}\frac{1}{f(D+a)}V$ c) $e^{ax}\frac{1}{f(D-a)}V$ d) $e^{ax}\frac{1}{f(D^2+a^2)}V$

24. The auxiliary equation of $4x^2y'' + y = 0$ is

a) $4m^2 + 4m + 1 = 0$
b) $4m^2 + 1 = 0$
c) $4m^2 - 4m + 1 = 0$
d) $4m^2 - 4m - 1 = 0$

$4(\theta^2 - \theta) + 1 = 0$ $4\theta^2 - 4\theta + 1 = 0$ $4m^2 - 4m + 1 = 0$

25. The auxiliary equation of the second order Euler Cauchy differential equation $x^2y'' - 5xy' + 13y = 30x^2$ has roots

a) -3, 2
b) 3, 2
c) -3 ± 2i
d) 3 ± 2i

$\theta^2 - \theta - 5\theta + 13 = 0$ $\theta^2 - 6\theta + 13 = 0$ $\theta = \frac{+6 \pm \sqrt{-16}}{2} = \frac{+6 \pm 4i}{2}$

$m = 3 \pm 2i$

26. The characteristic equation of $4x^2y'' + y = 0$ has double root ½, what is its general solution?

a) $(A + B \ln x)x^{1/2}$
b) $(A + Bx)e^{x/2}$
c) $Ax^{1/2}$
d) $Ae^{x/2}$

$m = \frac{1}{2}, \frac{1}{2}$ if Cauchy Euler

$(A + Bt)e^{\frac{1}{2}t}$

$t = \log x$ $e^t = x$

m=1 m=-2

27. If m-1 and m+2 are factors of auxiliary equation of $y'' + y' - 2y = 0$ then general solution is

(a) $Ae^{-x} + Be^{2x}$ (b) $e^{x} + e^{-2x}$ (c) $Ae^{x} + Be^{-2x}$ (d) $e^{-x} + e^{2x}$

28. The Wronskian of the functions f(x)=secx and g(x)=tanx is

(a) 1 (b) -1 (c) secx (d) tanx

$\begin{vmatrix} \sec x & \tan x \\ \sec x \tan x & \sec^2 x \end{vmatrix} = \sec^3 x - \tan^2 x \sec x$

$\sec^3 x - (\sec^2 x - 1)\sec x$

$\cancel{\sec^3 x} - \cancel{\sec^3 x} + \sec x$

$m=-1 \quad m=2$

29. If e^{-x} and e^{2x} are solutions of $y''-y'-6y=0$ then roots of auxiliary equation are
(a) 1 and -2 (b) -1 and 2 (c) 1 and 2 (d) -1 and -2

you can check ans also

30. The general solution of $4y''-9y'+2y=0$ is

$y''-\frac{9}{4}y'+\frac{1}{2}y=0$ $\quad S=2+\frac{1}{4}=\frac{9}{4}$
$y''-Sy'+P=0$ $\quad P=2\cdot\frac{1}{4}=\frac{1}{2}$

(a) Ae^{2x} (b) $Ae^{2x}+Be^{\frac{x}{4}}$ (c) $(A+Bx)e^{2x}$ (d) $(A+B)e^{2x}$

31. The general solution of $4y''-4y'+17y=0$ is

$y''-y'+\frac{17}{4}y=0$ $\quad S=1$, $P=\frac{17}{4}$

Sometimes you can root check and ans quickly

$S=0$

(a) $Ae^{\frac{x}{2}}+Be^{\pm 2x}$ (b) $Ae^{2x}+Be^{-2x}$

(c) $e^{2x}\left(A\cos\frac{x}{2}+B\sin\frac{x}{2}\right)$ (d) $e^{\frac{x}{2}}(A\cos 2x+B\sin 2x)$ $\quad m=\frac{1}{2}\pm 2i \quad S=1$

$m=2\pm\frac{i}{2}$ $\quad S=4$

32. The general solution of $y'''+4y''+5y'+2y=0$ is

$S=-4$, $P=-2$ $\quad$ $-1,-2,-2$ $\quad S=-5$ $\quad S=-4$, $P=-2$

(a) $Ae^{-2x}+(B+Cx)e^{-x}$ $\quad -2,-1,-1$ (b) $Ae^{-x}+(B+Cx)e^{-2x}$

(c) $Ae^{-2x}+(B+C)xe^{-x}$ error (d) $(A+Bx)e^{-x}+(C+Dx)e^{2x}$

$-1\;-1\quad 2,2$ $\quad$ 4 roots eqn is 3 roots

33. If D is a differential operator then value of $\frac{1}{D}(e^{-2x}+Sm2x+4)$ $\quad$ Sin2x

$\int e^{-2x}+\sin 2x+4$

(a) $-\frac{e^{-2x}}{2}-\frac{Cos2x}{2}$ (b) $-\frac{e^{-2x}}{2}+\frac{Cos2x}{2}+4x$ (c) $-\frac{e^{-2x}}{2}-\frac{Cos2x}{2}+4x$

(d) $-\frac{e^{-2x}}{2}-\frac{Cos2x}{2}+4$

$\frac{1}{D^2+2D-3}e^{2x} \quad \frac{1}{4+4-3}e^{2x}$

34. Particular Integral of $y''+2y'-3y=e^{2x}$ is

(a) $-\frac{1}{5}e^{2x}$ (b) $\frac{1}{5}e^{2x}$ (c) $\frac{1}{5}$ (d) $-\frac{1}{5}$

$D^2+4=-4+4=0$

35. Particular Integral of $y''+4y=Sin2x$ is

discrepancy → error in options

(a) $-\frac{1}{2}\cos 2x$ (b) $\frac{x}{2}\cos 2x$ (c) $-\frac{x}{2}\cos 2x$ (d) $-x\cos 2x$

$x\,\frac{1}{2D}\sin 2x$

$-\frac{x}{4}\cos 2x = \frac{x}{2}\int \sin 2x$

36. If D is a linear differential operator then $\frac{1}{f(D)}.e^{-\alpha x}=$

$y=e^{-\alpha x}$

Put $D=-\alpha$

(a) $\frac{1}{f(-\alpha^2)}.e^{-\alpha x}$, $f(-\alpha^2)\neq 0$ (b) $\frac{1}{f(D-\alpha)}.e^{-\alpha x}$

(c) $\frac{1}{f(\alpha^2)}.e^{-\alpha x}$, $f(\alpha^2)\neq 0$ (d) $\frac{1}{f(-\alpha)}.e^{-\alpha x}$, $f(-\alpha)\neq 0$

37. In method of undetermined coefficients if complimentary function $y_c=Ae^{-x}+(B+Cx)e^{2x}$ of equation $y'''-3y''+4y=e^{2x}$ then choice of particular integral will be

e^{2x} repeats twice

(a) cxe^{2x} (b) cx^2e^{2x} (c) ce^{2x} (d) $c_1e^{-x}+C_2e^{2x}$

38. In method of undetermined coefficients. If complementary function $y_c=ACos2x+BSin2x$ of equation $y''+4y=Sin2x$ then choice of particular integral
(a) c_1 xCos2x+ c_2Sin2x (b) c_1Cos2x+ c_2xSin2x

$y_p = x\,(c_1\cos 2x + c_2\sin 2x)$

x is to multiply on its parts of C.F.

(c) x[c_1Cos2x+ c_2Sin2x] (d) c_1Cos2x+ c_2Sin2x

39. General solution of $x^2y''+xy'-4y=0$ $\theta^2-\theta+\theta-4=0$ $\theta^2-4=0$ $\theta=2,-2$

(a) $y=Ax+Bx^{-2}$ (b) $y=Ax^2+Bx^{-2}$ (c) $y=Ax^2+Bx$ (d) $y=Ax^{-2}+Bx^{-2}$

40. General solution of $x^2y''+3xy'+10y=0$ $\theta^2-\theta+3\theta+10=0$ $\theta^2+2\theta+10=0$ $\theta=-1\pm 3i$

$t=\log x$

$e^t=x$

(a) $y=x[A\cos(3\log x)+B\sin(3\log x)]$ (b) $y=x[A\cos(\log x)+B\sin(\log x)]$

(c) $y=x^{-1}[A\cos(\log x)+B\sin(\log x)]$ (d) $y=e^{-t}[A\cos 3t+B\sin 3t]$

$y=x^{-1}[A\cos(3\log x)+B\sin(3\log x)]$

41. General solution of system of simultaneous equations $y_1'=-2y_1+y_2$ (1) $\Rightarrow y_2=y_1'+2y_1$ (3)

Same type discussion in Q 17

after elimination we get

$y_2'=y_1-2y_2$ (2) $y_1''+4y_1'+3y_1=0$ $y_1=Ae^{-t}+Be^{-3t}$ Put in (1) Put in (3) $\rightarrow$ to get y_2 soln

(a) $y_1=Ae^{-t}+Be^{3t},\ y_2=Ae^{t}-Be^{3t}$ (b) $y_1=Ae^{-t}+Be^{3t},\ y_2=Ae^{t}-Be^{3t}$

(c) $y_1=Ae^{-t}+Be^{-3t},\ y_2=Ae^{t}+Be^{-3t}$ (d) $y_1=Ae^{-t}+Be^{-3t},\ y_2=Ae^{t}-Be^{-3t}$

42. The roots of auxiliary equation of $x^2y''+xy'-9y=0$ are $\theta^2-\theta+\theta-9=0$ $\theta^2-9=0$ $\theta=3,-3$

a)3,3 b) −3,−3 c) −3, 3 d) None of these

$a_0=1\neq 0$ $x^2-9>0$ $(-\infty,-3)\cup(3,\infty)$

43.The equation $y''+xy'+6y=\ln(x^2-9)$ is normal in any of the subintervals of

a) $(3,\infty)$ b) $(0,\infty)$ c) $(0,4)$ d) $(-\infty,-3)\cup(3,\infty)$

44. A2x+B(6x+3)+C(3x+2)=0 will be linearly independent when

a) A=B=C b) A=B=C=0 c) Wronskian(A,B,C)=0 d) A=B=C=1

$-2+1$

45. General solution of $y''-y'-2y=0$ is $m^2-m-2=0$ $m=2,-1$

a)$Ae^{2x}+Be^{x}$ b)$Ae^{2x}+Be^{-x}$ c) $Ae^{-2x}+Be^{x}$ d)$Ae^{-2x}+Be^{x}$

check ans

46. General solution of $4y''+8y'-5y=0$ is given by $y''+\frac{8}{4}y'-\frac{5}{4}y=0$

a)$Ae^{x/2}+Be^{-5x/2}$ b)$Ae^{-x/2}+Be^{-5x/2}$ c)$Ae^{x/2}+Be^{5x/2}$ d)$Ae^{-x/2}+Be^{5x/2}$

$S=-2$ × $S=-3$ × $S=3$ × $S=2$ $S=-\frac{8}{4}$ $P=-\frac{5}{4}$ $S=-2$

47. Particular integral for $y''+16y=\cos 4x$ is

$\frac{1}{D^2+16}\cos 4x$ $-16+16=0$

a) $-2x\sin 4x$ b) $-\frac{x\sin 4x}{8}$ c) $\frac{x\sin 4x}{8}$ d) None of these $x\frac{1}{2D}\cos 4x$ $\frac{x}{2}\frac{\sin 4x}{4}$

48. The diff. equation of $(D^2+6D+9)y=50e^{2x}$ has P.I. $\frac{1}{25}50e^{2x}$

a) $\frac{2}{3}e^{2x}$ b) $2e^{2x}$ c) e^{2x} d) None of these

$y_c=c_1e^{2x}+c_2e^{-2x}$

49.By method of undetermined coefficient , the choice of particular integral for $y''-4y=5e^{-2x}$ is

a) Ce^{-2x} b) Cxe^{-2x} c) Cx^2e^{-2x} d) Cx^3e^{-2x} $y_p=x(Ce^{-2x})$

50. By variation of parameters $y''+4y=\cos 2x$,the value of wronskian is

a)1 b) 2 c) 4 d) none of these

$m^2+16=0 \quad m=\pm 4i$

51. General solution of $y''+16y=12e^{-2x}$ is

a) $Ae^{-4x}+Be^{4x}+\frac{3}{5}e^{-2x}$ b) $A\cos 4x+B\sin 4x+\frac{3}{5}e^{-2x}$ c) $Ae^{4x}+Bxe^{4x}+e^{-2x}$ d) none of these

52. The solution of diff. equation $x^2y''+xy'+y=0$ is

$\theta^2-\cancel{\theta}+\cancel{\theta}+1=0 \Rightarrow \theta^2+1=0$

$\theta=\pm i$

$t=\ln x$

a) $c_1\cos(\ln x)+c_2\sin(\ln x)$ b) $c_1x+c_2x^2$ c) $c_1\cos x+c_2\sin x$ d) None of these

$c_1\cos t+c_2\sin t$

53.By elimination, solution of y_2 in the simultaneous system $(D+3)y_1+(3D+23)y_2=e^{-2t}$ *and* $(D+2)y_1+(4D+14)y_2=e^{2t}$ will be

a)$Be^{-t}+Ae^{4t}-e^{2t}$ b) $Be^{-t}+Ae^{4t}-\frac{5}{6}e^{2t}$ c) 0 d) Does not exist

54. $x^2y''+xy'-4y=0$ has auxiliary equation

$\theta^2-\cancel{\theta}+\cancel{\theta}-4=0 \qquad \theta^2-4=0$

a) m²+2m-4=0 b) m²-4=0 c) m²-2m-4=0 d) m²+4=0

$(D+3)y_1+(3D+23)y_2=e^{-2t} \rightarrow \times(D+2)$

eliminate y_1

$(D+2)y_1+(4D+14)y_2=e^{2t} \rightarrow \times(D+3)$

and Substitute

$(D^2-3D-4)y_2=+5e^{2t} \qquad y_p=\frac{1}{4-6-4}\,5e^{2t}$

$y_2=c_1e^{-t}+c_2e^{4t}-\frac{5}{6}e^{2t}$