

Unit-5: Combinational Circuit

Adders

Subtractor

Comparator

Parity Generator

Combinational Circuits

- output depends only on the present input
- The combinational circuit do not use any memory.
- The previous state of input does not have any effect on the present state of the circuit.



Half Adder

A combinational logic circuit with two inputs and two outputs.

The half adder circuit add two single bit Carry number

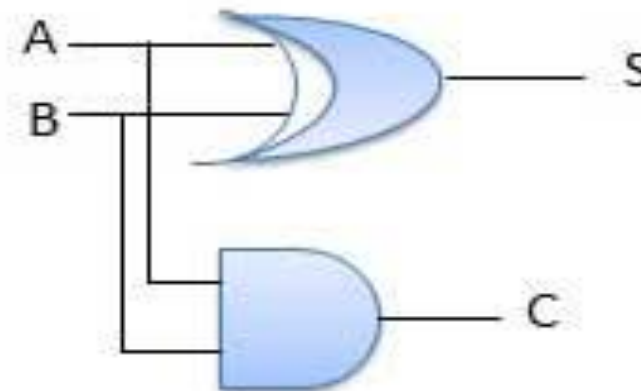
This circuit has two outputs **carry** and **sum**.



| Inputs | | Output | |
|--------|---|--------|---|
| A | B | S | C |
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 1 |

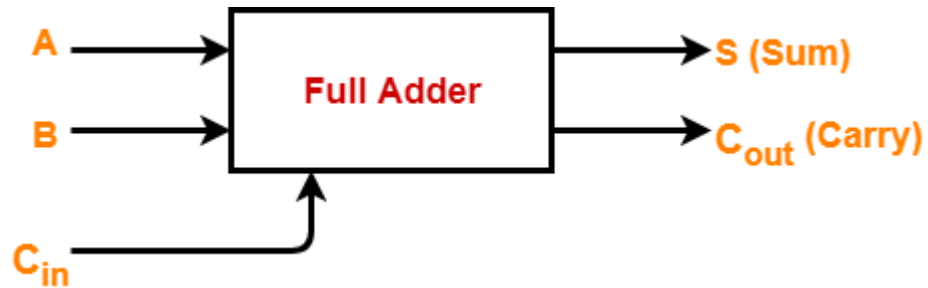
$$S(A, B) = \sum m(1, 2)$$

$$CY(A, B) = \sum m(3)$$



Full Adder (1-bit Adder)

A combinational logic circuit with 3 inputs and 2 outputs.
The Full adder circuit add 3 single bit Carry number
This circuit has two outputs **carry** and **sum**.



$$\text{Sum}(A, B, C) = \sum m(1, 2, 4, 7)$$

$$\text{Cout}(A, B, C) = \sum m(3, 5, 6, 7)$$

For S:

| | | BC_{in} | | | |
|-----|----------------|---------------------------------|----------------------|-----------|----------------------|
| | | $\overline{B}\overline{C}_{in}$ | $\overline{B}C_{in}$ | BC_{in} | $B\overline{C}_{in}$ |
| A | \overline{A} | | 1 | | 1 |
| | A | 1 | | 1 | |

| A | B | Cin | Sum | Cout |
|---|---|-----|-----|------|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 |

$$\overline{A}\overline{B}C + \overline{A}B\overline{C} + A\overline{B}\overline{C} + ABC$$

$$\overline{A}(\overline{B}C + B\overline{C}) + A(\overline{B}\overline{C} + BC)$$

$$\overline{A}(B \oplus C) + A(B \oplus C)$$

$$\text{Let } B \oplus C = D$$

$$\overline{A}D + A\overline{D}$$

$$A \oplus D$$

$$A \oplus B \oplus C$$

For Cout

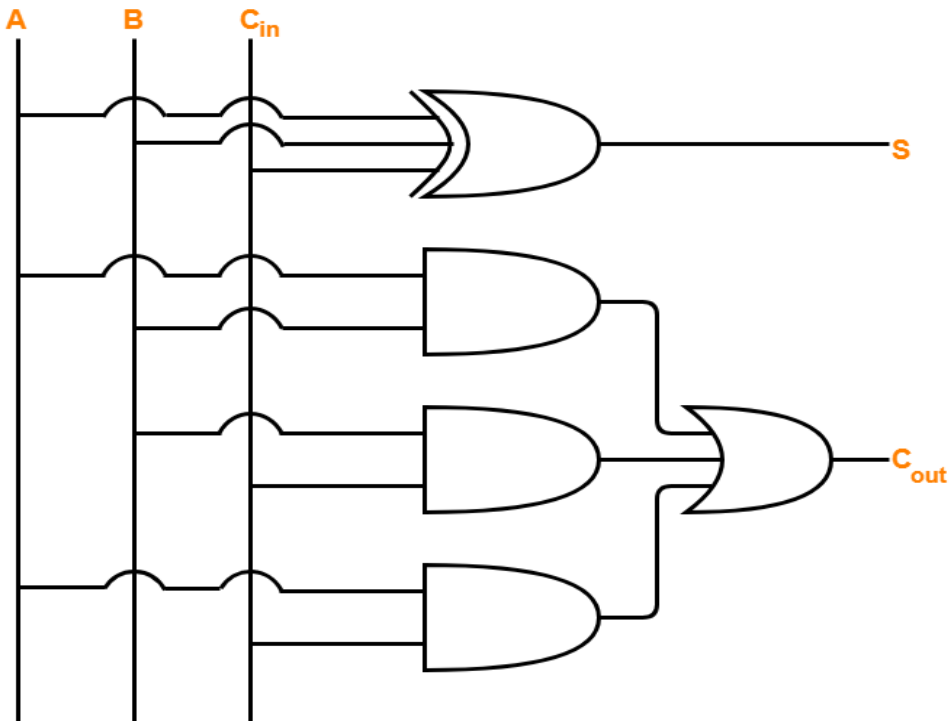
| | | | | | |
|----------------|-----------|---------------------------------|----------------------|-----------|----------------------|
| | BC_{in} | $\overline{B}\overline{C}_{in}$ | $\overline{B}C_{in}$ | BC_{in} | $B\overline{C}_{in}$ |
| \overline{A} | | | 1 | | |
| A | | 1 | 1 | 1 | |

$$\overline{A}BC + A\overline{B}C + ABC + AB\overline{C}$$

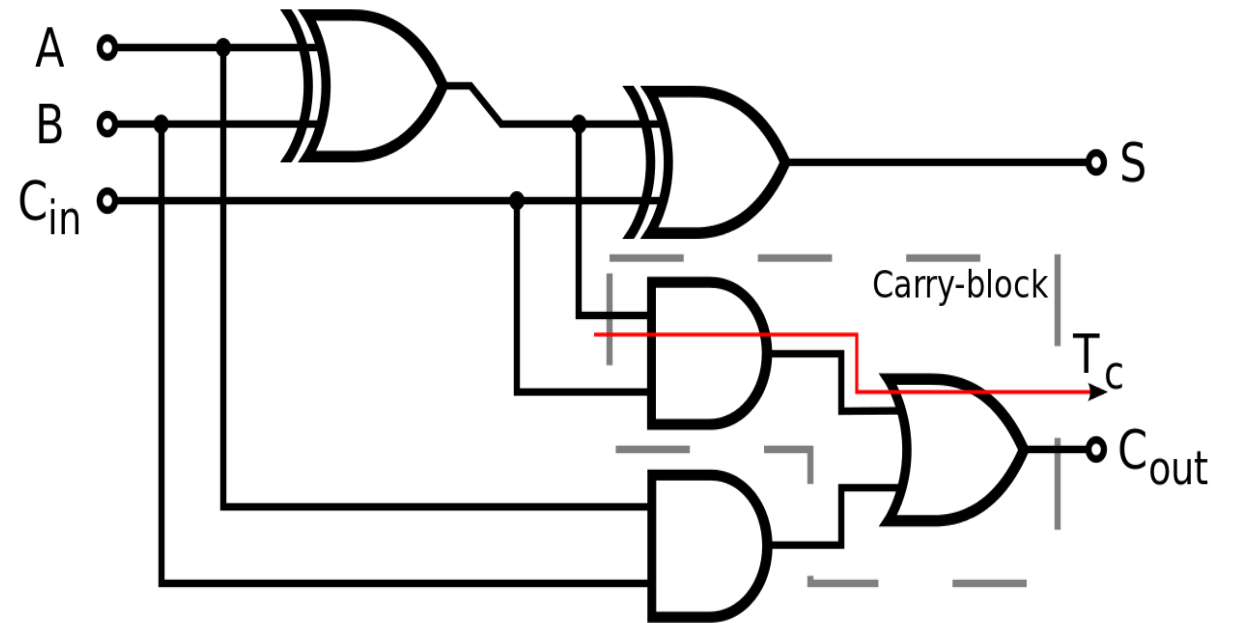
$$C(\overline{A}B + A\overline{B}) + AB(C + \overline{C})$$

$$(A \oplus B)C + AB$$

$$C_{out} = AB + BC + AC$$



Full Adder Logic Diagram

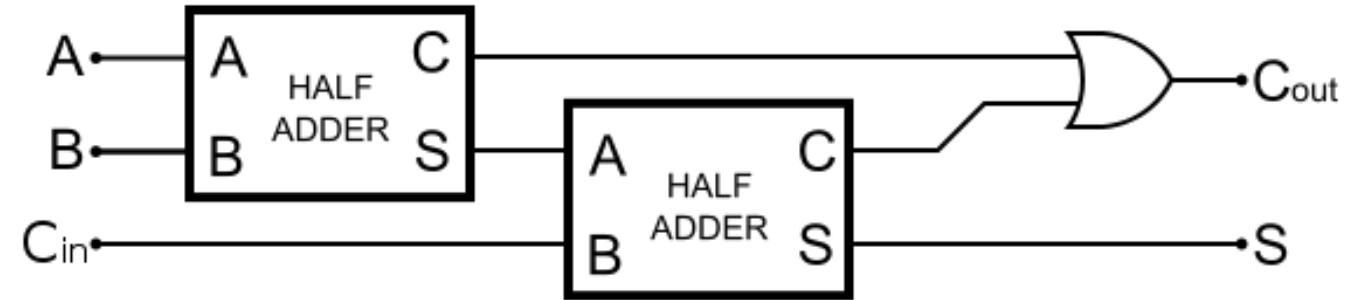


Full Adder using Half Adder

FA

$$\text{Sum} = A \oplus B \oplus C$$

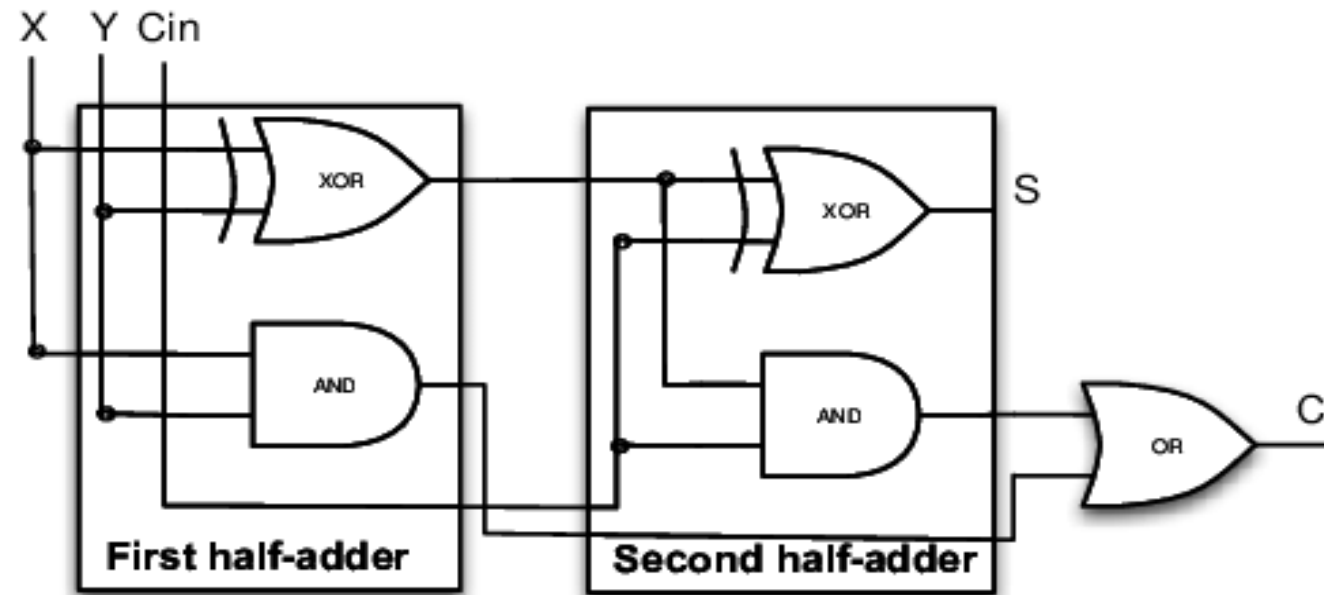
$$\text{Carry} = AB + (A \oplus B)C$$



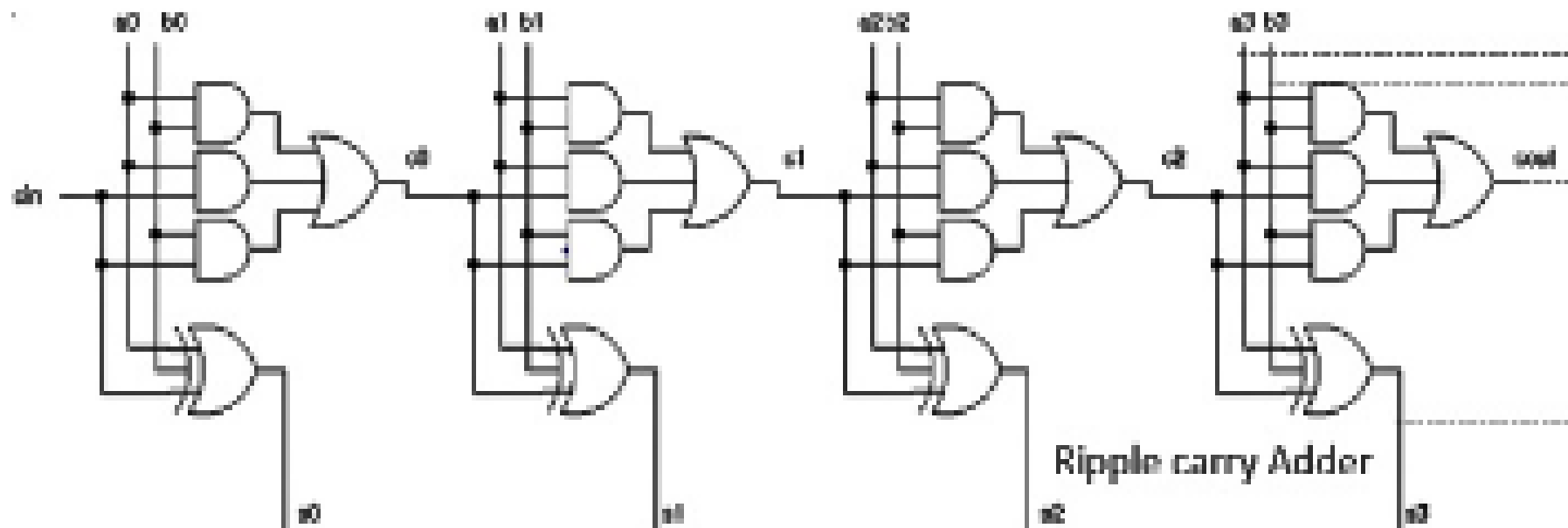
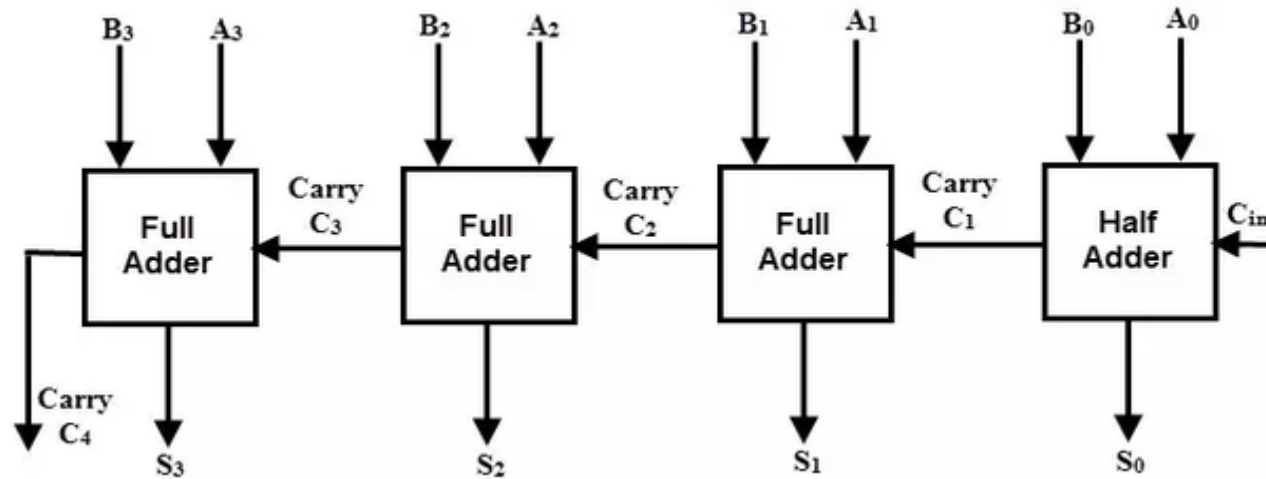
HA

$$\text{Sum} = A \oplus B$$

$$\text{Carry} = AB$$

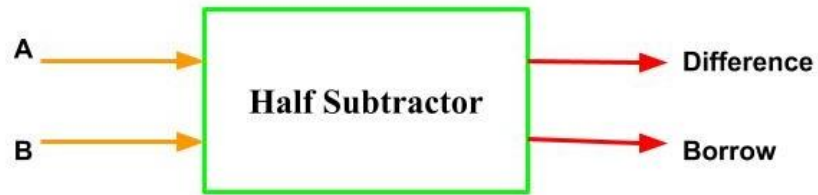


4-bit Ripple Carry Adder



Half Subtractor

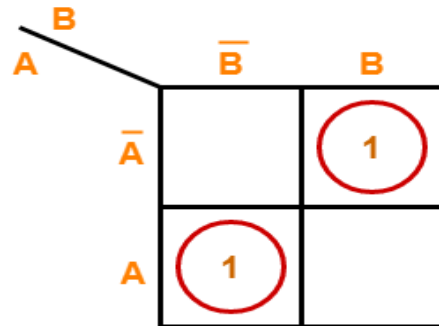
Combinational circuit perform binary Subtraction
Accepts 2 input and Two output **Difference** and **Borrow**



$$D(A, B) = \sum m(1, 2)$$

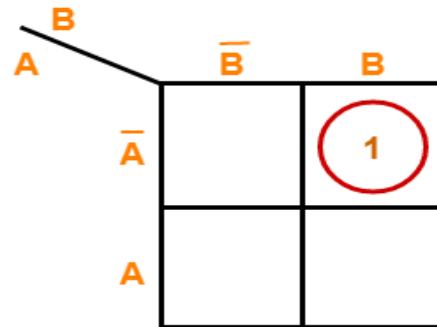
$$Br(A, B) = \sum m(1)$$

For D:



$$D = A \oplus B$$

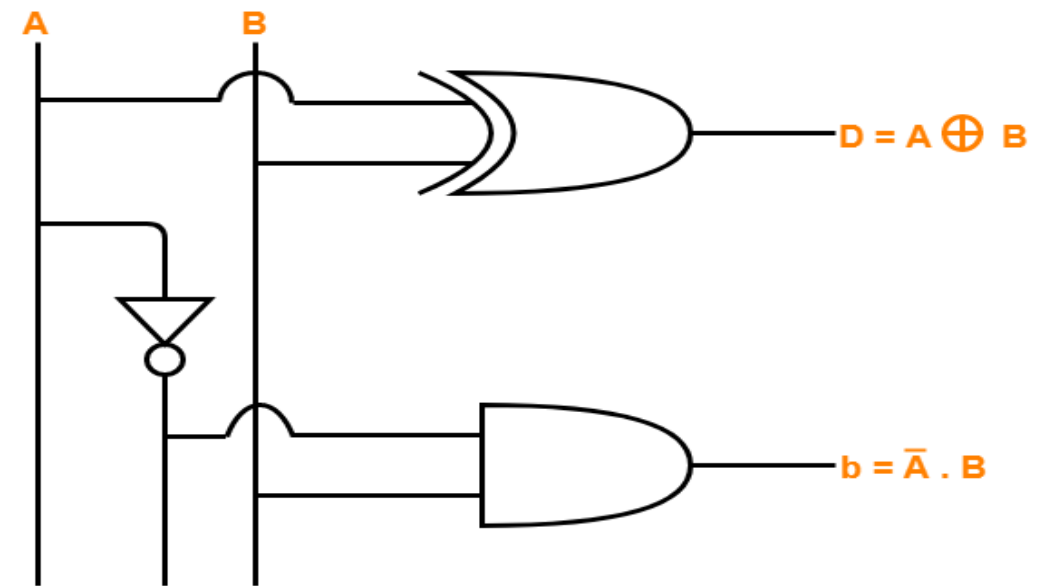
For b:



$$b = \bar{A} \cdot B$$

K Maps

| Inputs | | Outputs | |
|--------|---|----------------|------------|
| A | B | D (Difference) | b (Borrow) |
| 0 | 0 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |



Half Subtractor Logic Diagram

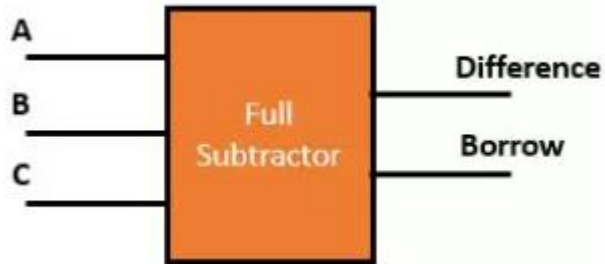
Full Subtractor

Performs subtraction of 3 bits

This circuit **has three inputs and two outputs**.

The three inputs A, B and C, denote the minuend, subtrahend, and previous borrow, respectively.

The two outputs, D and Bout



| Input | | | Output | |
|-------|---|---|------------|--------|
| A | B | C | Difference | Borrow |
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |

$$\text{Sum}(A, B, C) = \sum m(1, 2, 4, 7)$$

$$\text{Bout}(A, B, C) = \sum m(1, 2, 3, 7)$$

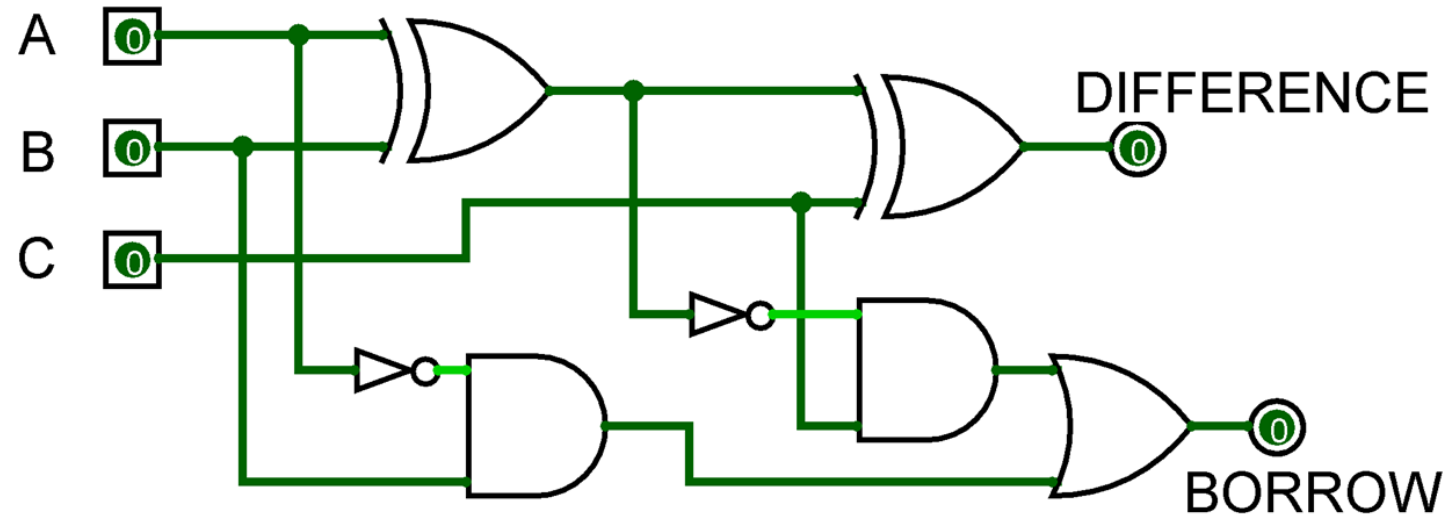
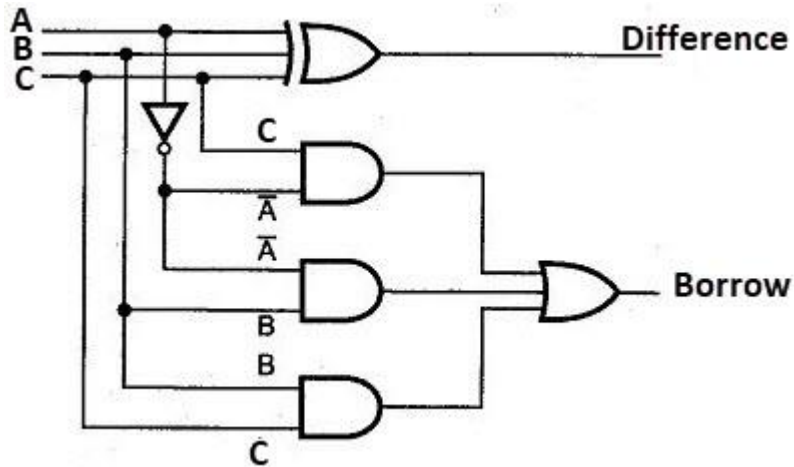
| A | BC | | | |
|---|----|----|----|----|
| | 00 | 01 | 11 | 10 |
| 0 | 0 | 1 | 0 | 1 |
| 1 | 1 | 0 | 1 | 0 |

$$\begin{aligned}
 \text{Difference} &= \overline{A} \overline{B} C + \overline{A} B \overline{C} + A \overline{B} \overline{C} + ABC \\
 &= C (\overline{A} \overline{B} + AB) + \overline{C} (\overline{A} B + A \overline{B}) \\
 &= C (A \odot B) + \overline{C} (A \oplus B) \\
 &= C (A \oplus B) + \overline{C} (A \oplus B) \\
 &= C \oplus (A \oplus B)
 \end{aligned}$$

| | | BC | | | |
|---|---|----|----|----|----|
| A | | 00 | 01 | 11 | 10 |
| | 0 | 0 | 1 | 1 | 1 |
| | 1 | 0 | 0 | 1 | 0 |

$$\begin{aligned}
 \text{Bout} &= A'B'C + A'BC' + A'BC + ABC \\
 &= C(AB + A'B') + A'B(C + C') \\
 &= C(A \text{ XNOR } B) + A'B \\
 &= C(A \text{ XOR } B)' + A'B
 \end{aligned}$$

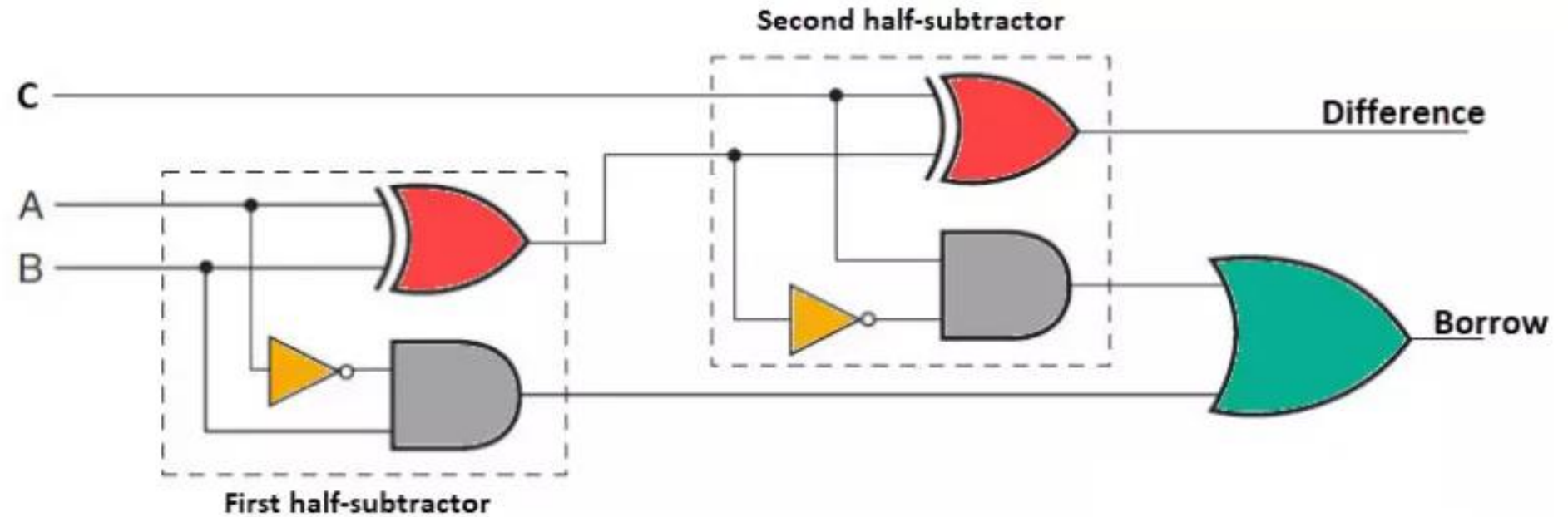
$$\begin{aligned}
 \text{Borrow} &= \overline{A} \overline{B} C + \overline{A} B \overline{C} + \overline{A} BC + ABC \\
 &= \overline{A} B + \overline{A} C + BC
 \end{aligned}$$



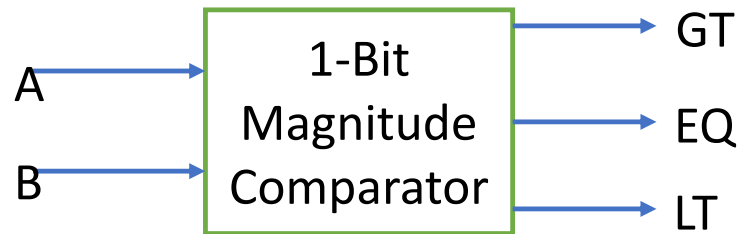
Full Subtractor using Half Subtractor

FS
 $\text{Sum} = A \oplus B \oplus C$
 $\text{Carry} = A'B + (A \oplus B)'C$

HS
 $\text{Sum} = A \oplus B$
 $\text{Carry} = A'B$



1-BIT Magnitude Comparator

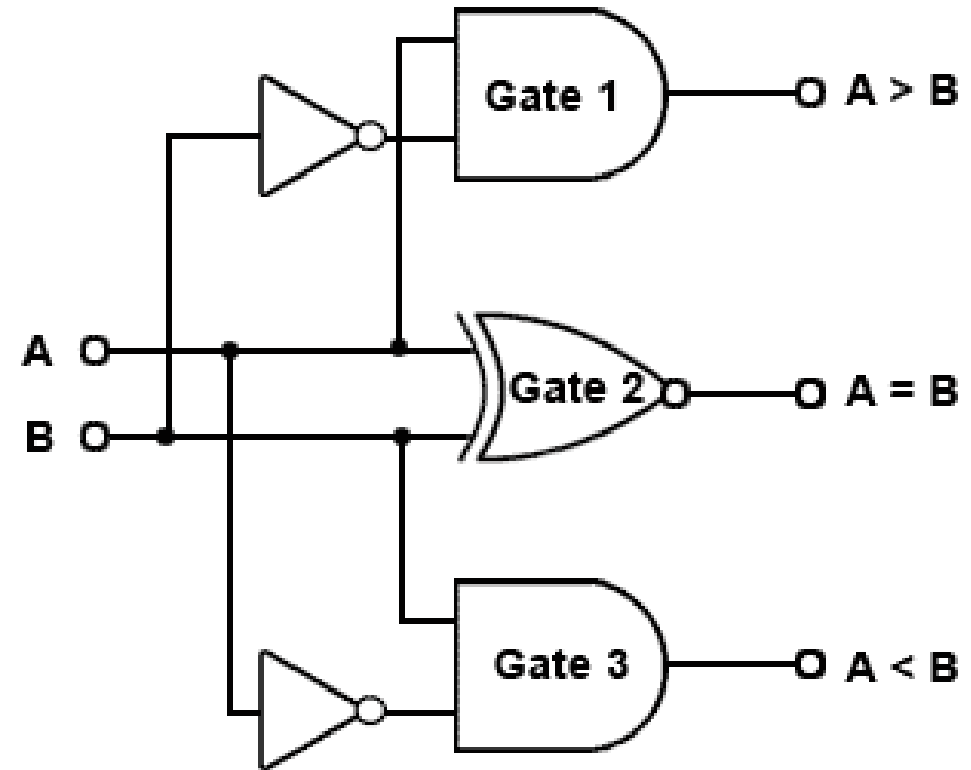


| | | EQ | | |
|--------|---|---------|---------|---------|
| Inputs | | GT | Outputs | LT |
| A | B | $A > B$ | $A = B$ | $A < B$ |
| 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 |

$$GT = AB'$$

$$EQ = A'B' + AB$$

$$LT = A'B$$



Parity Generator Circuit

Even Parity Generator

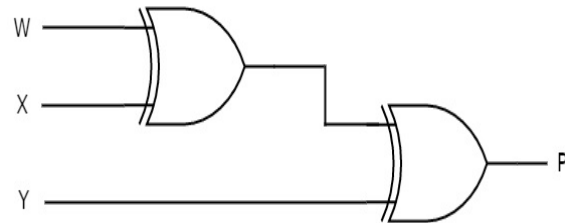
If odd number of ones present in the input, then even parity bit, P should be '1' so that the resultant word contains even number of ones.

| Binary Input WXY | Even Parity bit P |
|------------------|-------------------|
| 000 | 0 |
| 001 | 1 |
| 010 | 1 |
| 011 | 0 |
| 100 | 1 |
| 101 | 0 |
| 110 | 0 |
| 111 | 1 |

$$P = W'X'Y + W'XY' + WX'Y' + WXY$$

$$\Rightarrow P = W'(X'Y + XY') + W(X'Y' + XY)$$

$$\Rightarrow P = W'(X \oplus Y) + W(X \oplus Y)' = W \oplus X \oplus Y$$



Odd Parity Generator

If even number of ones present in the input, then odd parity bit, P should be '1' so that the resultant word contains odd number of ones

| Binary Input WXY | Odd Parity bit P |
|------------------|------------------|
| 000 | 1 |
| 001 | 0 |
| 010 | 0 |
| 011 | 1 |
| 100 | 0 |
| 101 | 1 |
| 110 | 1 |
| 111 | 0 |

