ECE-249

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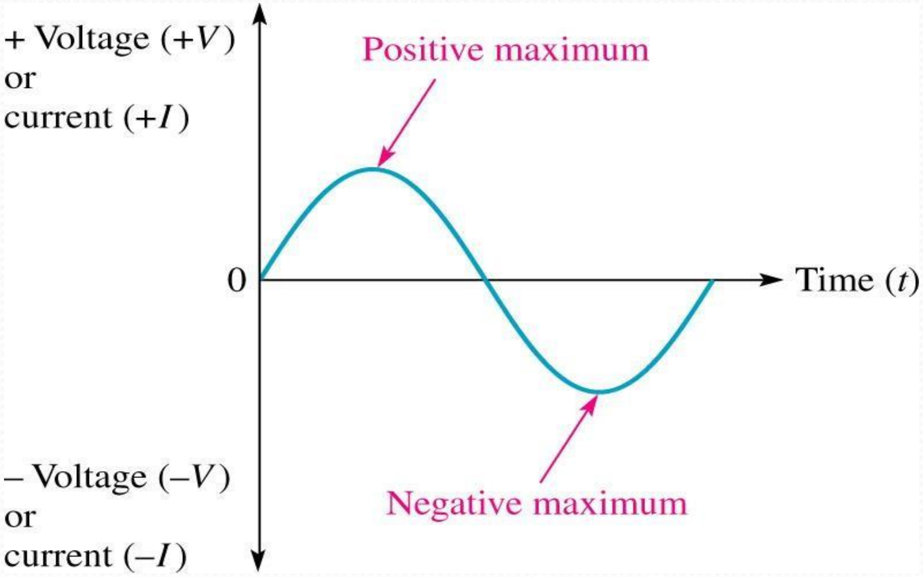
Unit-2

Fundamentals of A.C. circuits



Contents:

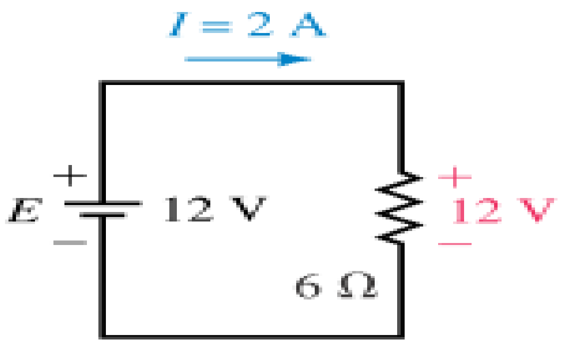
- Alternating Current
- Generating Ac Voltages
- Determination of Frequency (f) In The Ac Generator Fundamental
- Periodic Voltage Or Current Waveform
  - Average Value
  - Root Mean Square (RMS) Value
  - Average And RMS Values Of Sinusoidal Voltage Waveform
- Single-phase AC Supply
  - Purely Resistive Circuit (R Only)
  - Purely Inductive Circuit (L Only)
  - Purely Capacitive Circuit (C Only)

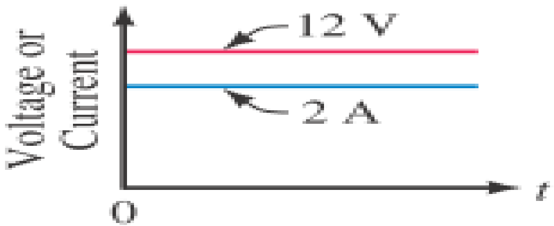
Cont:

- series A.C. circuit
  - R–L series A.C. circuit
  - R–C series A.C. circuit
  - R–L–C series A.C. circuit
- Three-phase AC Circuits
  - Generation of Three-phase Balanced Voltages
  - Delta(Δ)-Star(Y) conversion and Star-Delta conversion
  - Delta(Δ)-Star(Y) conversion and Star-Delta conversion impedance conduction

AC Fundamentals

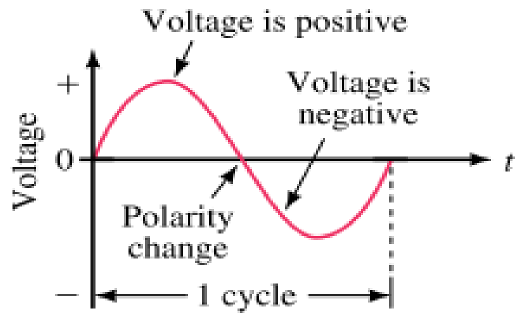
- Previously you learned that DC sources have fixed polarities and constant magnitudes and thus produce currents with constant value and unchanging direction





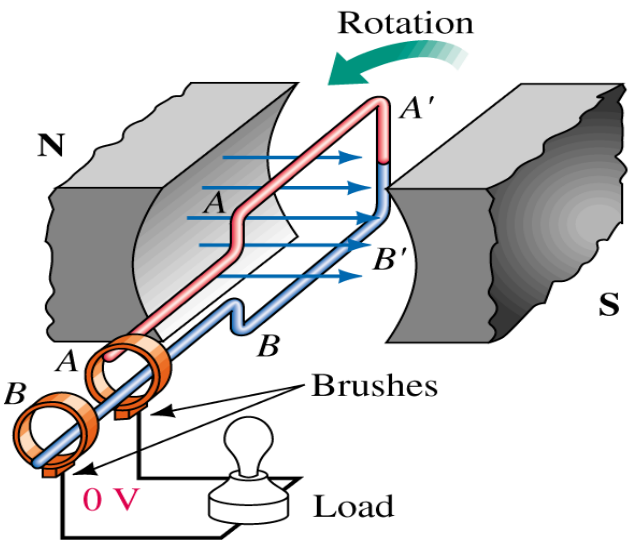
(b) Voltage and current versus time for dc

- In contrast, the voltages of ac sources alternate in polarity and vary in magnitude and thus produce currents that vary in magnitude and alternate in direction.

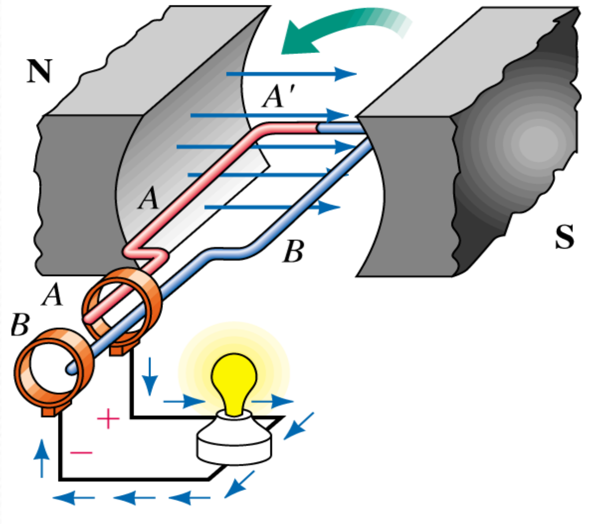


Generating AC Voltages

- Generators convert rotational energy to electrical energy.
- Generate alternative emf by rotating a coil within a stationary magnetic field.
- Another way to rotating magnetic field a within a stationary coil.



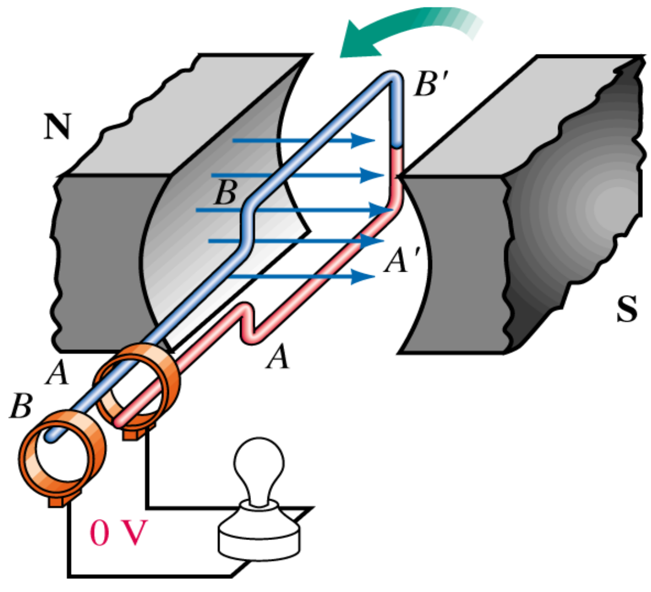
(a) 0° Position: Coil sides move parallel to flux lines. Since no flux is being cut, induced voltage is zero.



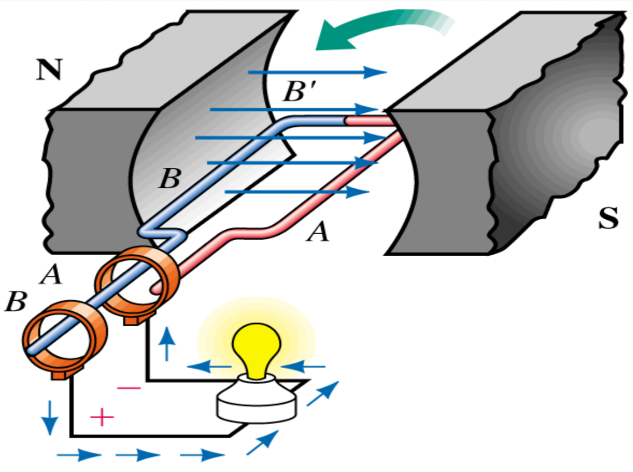
(b) 90° Position: Coil end *A* is positive with respect to *B*. Current direction is out of slip ring *A*.

Cont….

- The armature has an induced voltage, which is connected through slip rings and brushes to a load.
- The armature loops are wound on a magnetic core



(c) 180° Position: Coil again cutting no flux. Induced voltage is zero.

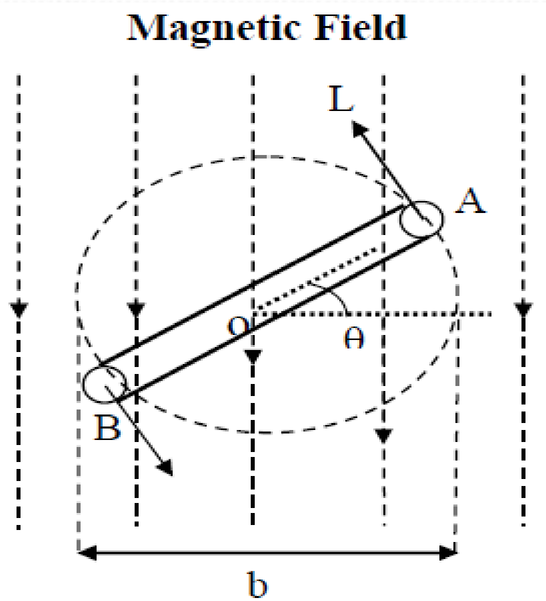


(d) 270° Position: Voltage polarity has reversed, therefore, current direction reverses.

- A multi-turn coil is placed inside a magnet with an air gap as shown in above Fig.
- The flux lines are from North Pole to South Pole. The coil is rotated at an angular speed,

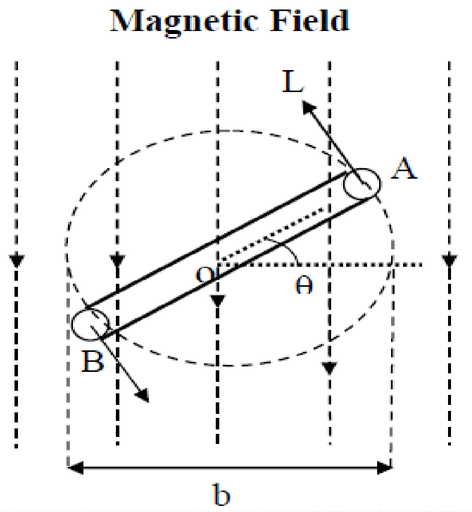
$$\omega = 2\,\pi\,n \text{ (rad/S)}$$

- N is rotating a rectangular coil in a counter clockwise direction with a angular velocity of ω radians per sec in a uniform magnetic field.
- The instant of coincidence of the plane of the coil with X-axis. At this instant max flux, Φ_{max} link with the coil
- The coil assume the position, as shown in fig , after moving the counter clockwise for t sec.
- The angle θ through which the coil has rotated in sec = ω t.



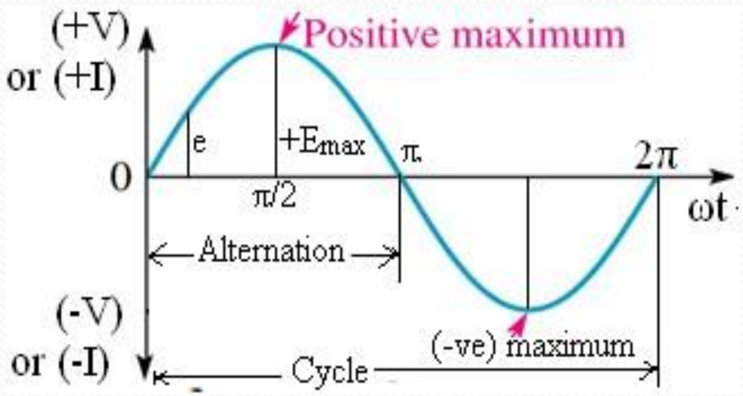
- The component of flux along perpendicular to the coil = Φ_{max} Cos ωt.
- Flux linkage of the coil at the instant = no. of turns on coil x linkage flux

  $N \; \Phi_{max} \cos \omega t$
- emf induced in a coil is equal to the rate of change of the flux linkage with minus sign .
- emf induced is max at any instant.



$$e = -\frac{d}{dt}[N\Phi_{max} cos\ \omega t] \quad = \quad N\Phi_{max}\frac{d}{dt}[-cos\ \omega t]$$

$$e = N\Phi_{max}\ \omega\ sin\ \omega t$$



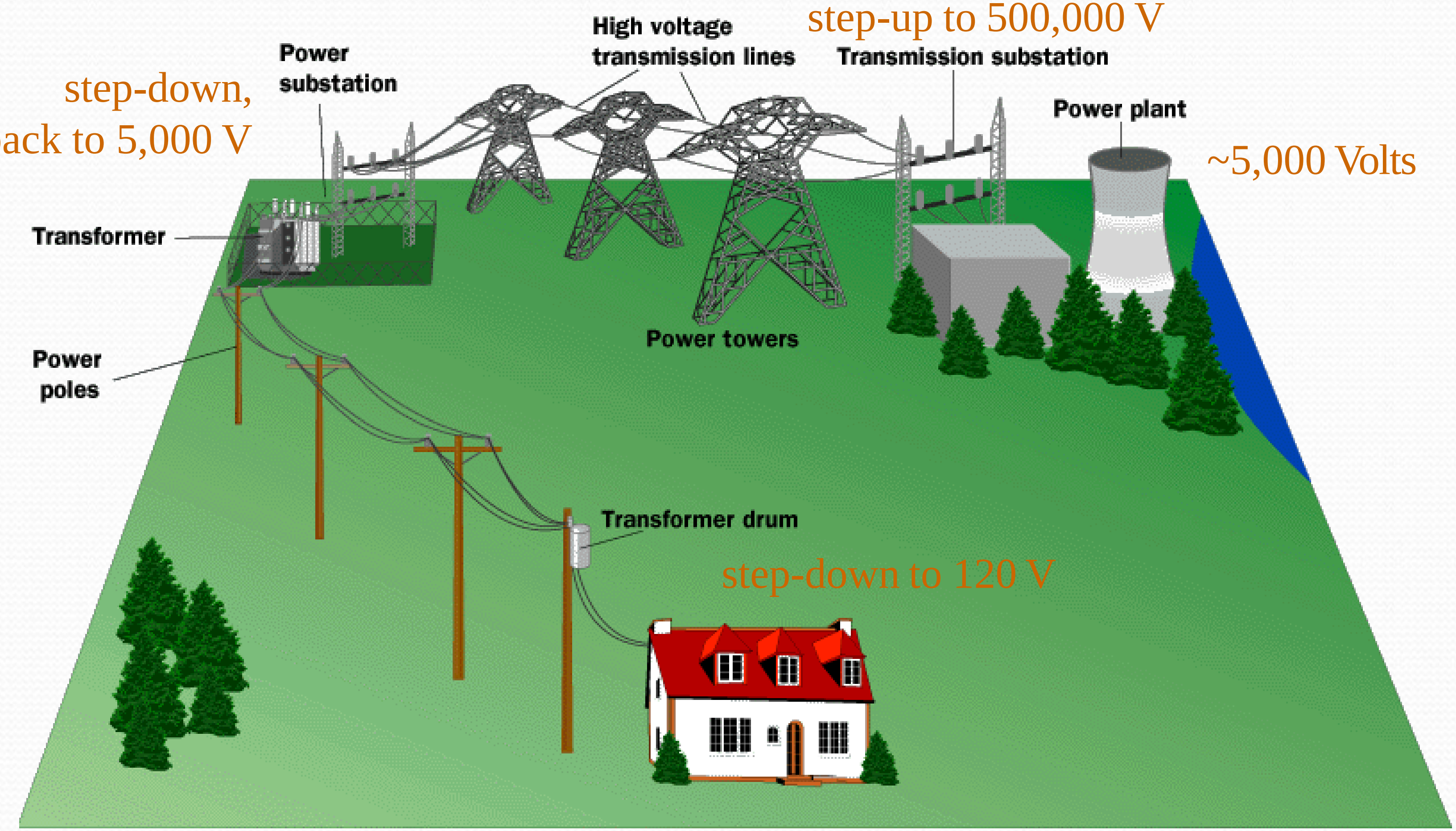
- Instantaneous emf , e = E_{max} sin ωt

A way to provide high efficiency, safe low voltage:

step-down, back to 5,000 V

step-up to 500,000 V

~5,000 Volts



step-down to 120 V

High Voltage Transmission Lines
Low Voltage to Consumers

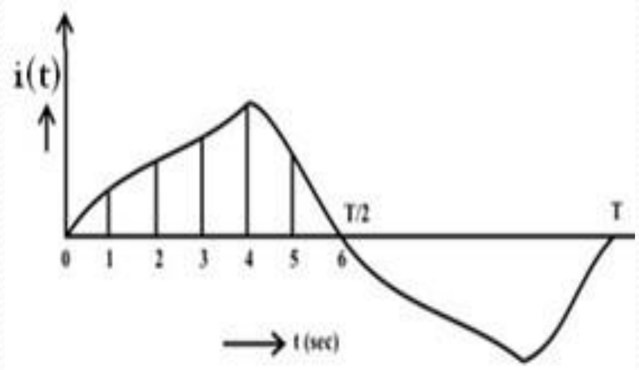
Periodic Voltage or Current Waveform

- Average value
  - The current waveform shown in Fig, is periodic in nature, with time period, T. It is positive for first half cycle, while it is negative for second half cycle.
  - The average value of the waveform, i(t) is defined as

$$I_{av} = \frac{Area\ over\ half\ cycle}{Time\ period\ of\ half\ cycle} = \frac{1}{T/2}\int_0^{T/2} i(t)\,dt = \frac{2}{T}\int_0^{T/2} i(t)\,dt$$

  - In this case, only half cycle, or half of the time period, is to be used for computing the average value, as the average value of the waveform over full cycle is zero (0).
  - If the half time period (T/2) is divided into 6 equal time intervals (ΔT)

$$I_{av} = \frac{(i_1 + i_2 + i_3 + \cdots i_6)\Delta T}{6 \cdot \Delta T} = \frac{(i_1 + i_2 + i_3 + \cdots i_6)}{6}$$



Root Mean Square (RMS) value

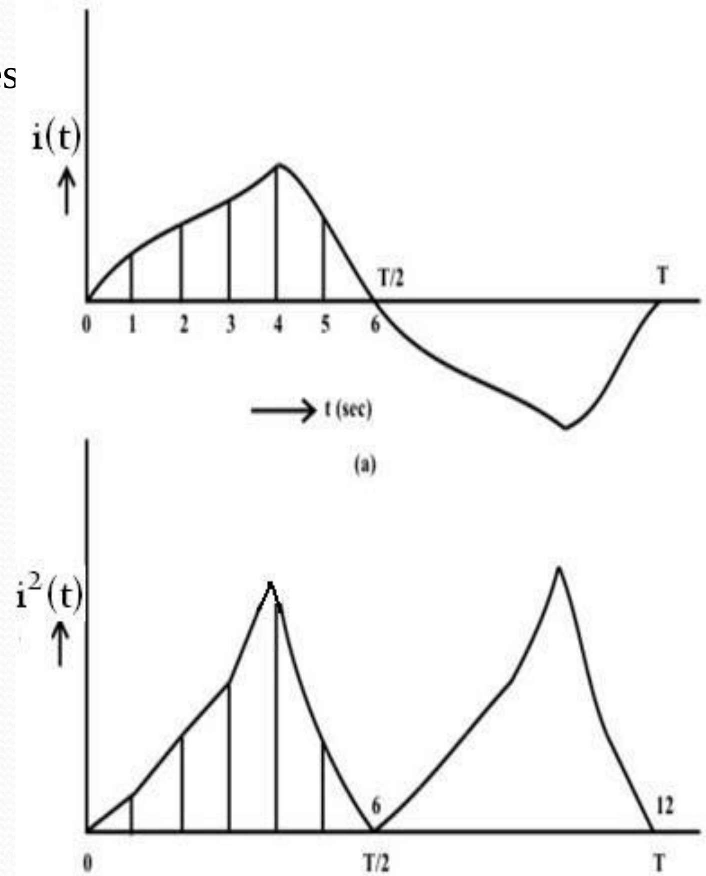
- For this current in half time period subdivided into 6 time intervals as given above, in the resistance R, the average value of energy dissipated is given by
  - Heat produce during interval = $I^2 R \frac{T}{n}$ joules

$$\propto \left[\frac{(i_1^2 + i_2^2 + i_3^2 + \cdots i_6^2)}{6} \right] R$$

$$I^2 R \,\Delta T = \left[\frac{(i_1^2 + i_2^2 + i_3^2 + \cdots i_n^2)\,\Delta T}{n \cdot} \right] R$$

$$I = \sqrt{\frac{(i_1^2 + i_2^2 + i_3^2 + \cdots i_n^2)\,\Delta T}{n \cdot \Delta T}} = \sqrt{\frac{\text{Area of } i^2 \text{ curve over half cycle}}{\text{Time period of half cycle}}}$$

$$= \sqrt{\frac{1}{T/2} \int_0^{T/2} i^2 \, dt} = \sqrt{\frac{2}{T} \int_0^{T/2} i^2 \, dt}$$



Average Values of Sinusoidal Voltage Waveform

- The average value of sine wave over the complete cycle is zero.

- The half wave value of sinusoidal current is

$$I = I_{max} \ \sin \omega t$$

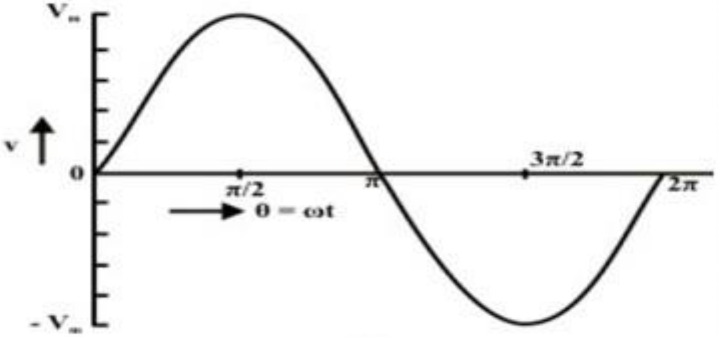
- For half cycle when ωt varies from 0 to π

$$I_{AV} = \frac{area\ of\ half\ cycle}{\pi}$$

$$I_{AV} = \frac{1}{\pi}\int_0^{\pi} i d(\omega t)$$

$$I_{AV} = \frac{1}{\pi}\int_0^{\pi} I_{max} \ \sin \omega t \ d(\omega t)$$

$$I_{AV} = \frac{I_{max}}{\pi}[-\cos \omega t] = \frac{2 \ I_{max}}{\pi} = 0.637 \ I_{max}$$



- Similarly

$$V_{AV} = \frac{2 \ V_{max}}{\pi} = 0.637 \ V_{max}$$

RMS Values of Sinusoidal Voltage Waveform

- The waveform of the voltage v(t), and the square of waveform, $v^2(t)$, are shown in figures 12.4a and 12.4b respectively.

Time period, $T = 1/f = (2\pi)/\omega$; in angle $(\omega T = 2\pi)$

Half time period, $T/2 = 1/(2f) = \pi/\omega$; in angle $(\omega T/2 = \pi)$

$v(\theta) = V_m \sin\theta \quad for \quad \pi \le \theta \le 0;$

$$V_{\text{rms}} = \left[\frac{1}{\pi}\int_0^{\pi} v^2\, d\theta\right]^{\frac{1}{2}} = \left[\frac{1}{\pi}\int_0^{\pi} V_m^2 \sin^2\theta\, d\theta\right]^{\frac{1}{2}} = \left[\frac{V_m^2}{\pi}\int_0^{\pi}\frac{1}{2}(1-\cos 2\theta)\, d\theta\right]^{\frac{1}{2}}$$

$$V_{\text{rms}} = \left[\frac{V_m^2}{2\pi}\left(\theta - \frac{1}{2}\sin 2\theta\right)\Big|_0^{\pi}\right]^{\frac{1}{2}} = \left[\frac{V_m^2}{2\pi}\pi\right]^{\frac{1}{2}} = \frac{V_m}{\sqrt{2}} = 0.707\, V_m$$

or, $V_m = \sqrt{2}\, V_{\text{rms}}$

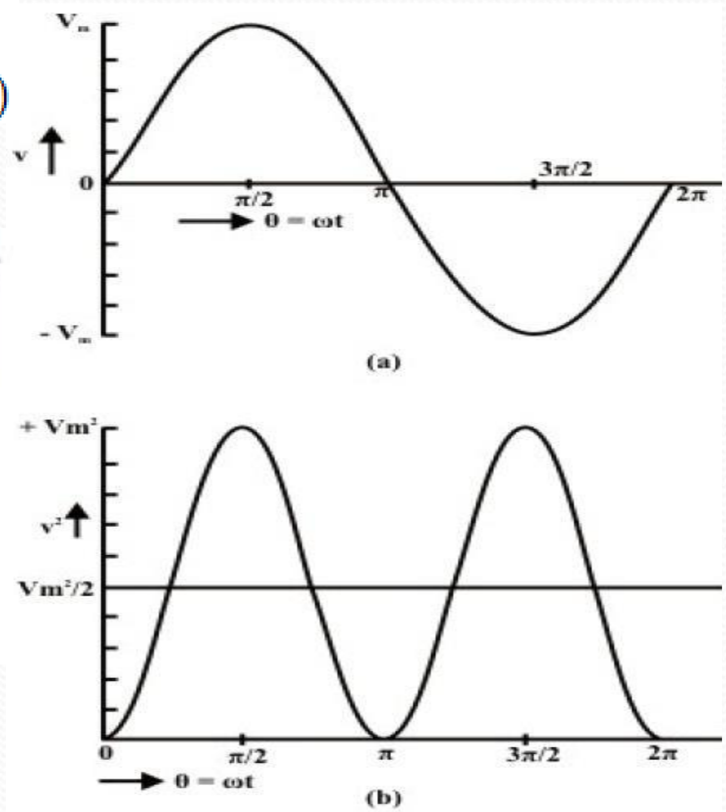


Fig. 12.4 Sinusoidal voltage waveform
(a) Voltage (v), (b) Square of voltage (v²)

- Similarly for current RMA value

$$I_{rms} = \frac{I_{max}}{\sqrt{2}} = 0.707\, I_{max}$$

Form factors

- The form factor of an alternating quantity is define as the ratio of RMS value to the average value.

$$\text{Form factor} = \frac{RMS\ value}{Average\ value} = \frac{0.707\ V_m}{0.637\ V_m} = 1.11$$

- **<u>Peak value:-</u>**
- The peck factor of an alternating quantity is define as the ratio of maximum value to the average value

$$\text{Peak factor (P.E.)} = \frac{Maximum\ value}{RMS\ value}$$

$$= \frac{Peak\ Value}{Peak\ Value/\sqrt{2}} = \sqrt{2}$$

$$\text{P.F.} = 1.414$$

- **<u>NOTE:-</u>**
  - The rms value is always greater than the average value.
  - Except for a rectangular waveform, in which case the heating effect remains constant, so that the average and the rms values are same

Question

Q1) determine the average value and RMS value of sinusoidal current of peak value 40A.

Solution:- $I_{max} = 40A$

$$I_{rms} = \frac{I_{max}}{\sqrt{2}}$$

$$I_{av} = 0.637\, I_{max}$$

Q2) write the instantaneous value for a 50Hz sinusoidal voltage supply for domestic purposes at 230V.

Solution:- given value $V_{rms} = 230$ V. , f = 50 Hz

$$V(t) = V_{max} \sin \omega t$$

$$V_{max} = \sqrt{2} \times V_{rms}$$

$$\omega = 2\pi f$$

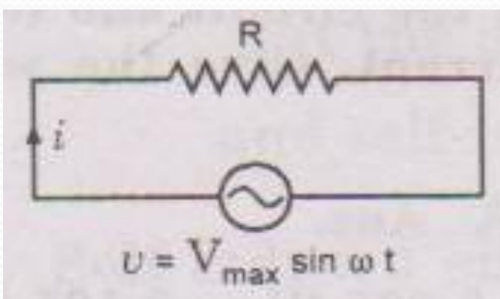
Single-phase AC Supply

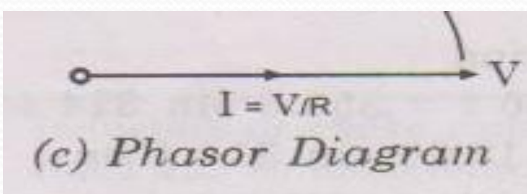
- Purely resistive circuit (R only)
  - The instantaneous value of the current though the circuit is given by

$$v = V_m \sin \omega t$$

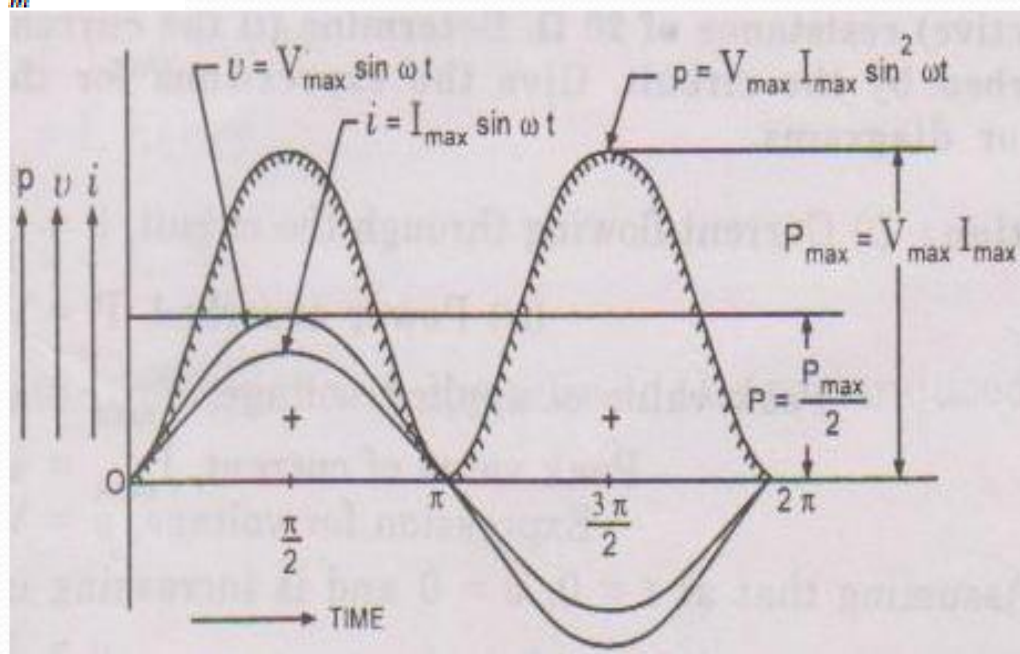
  - Im and Vm are the maximum values of current and voltage respectively

$$i = \frac{v}{R} = \frac{V_m}{R} \sin \omega t = I_m \sin \omega t$$





(c) Phasor Diagram



Purely inductive circuit (L only)

- For the circuit , the current i, is obtained by the procedure described here

$$\text{As } v = L\frac{di}{dt} = V_m \sin \omega t$$

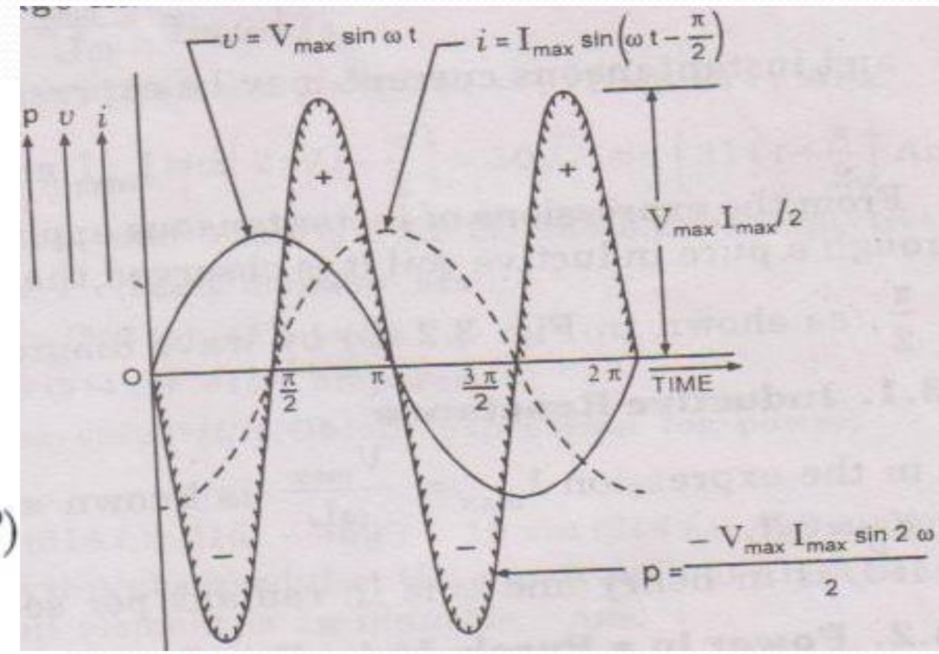
$$di = \frac{V}{L}\sin(\omega t)\, dt$$

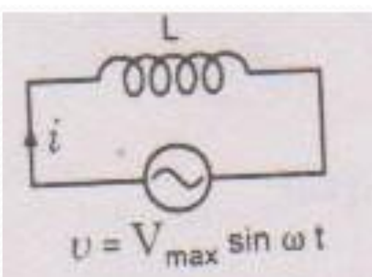
Integrating,

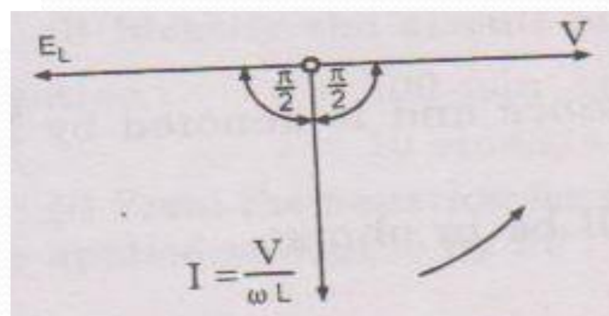
$$i = -\frac{V_m}{\omega L}\cos \omega t = \frac{V_m}{\omega L}\sin(\omega t - 90^\circ)$$

$$= I_m \sin(\omega t - 90^\circ)$$

-







(c) Phasor Diagram

Purely capacitive circuit (C only)

The current i, in the circuit (Fig. 14.3a), is,

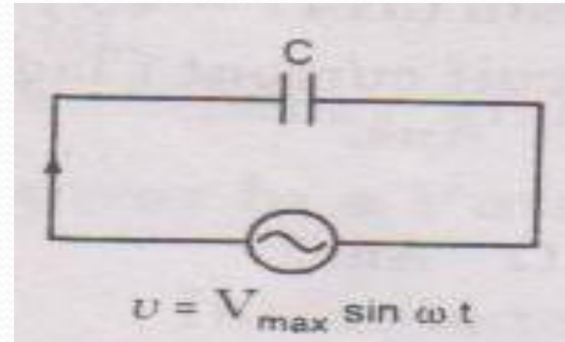
$$i = C\frac{dv}{dt}$$

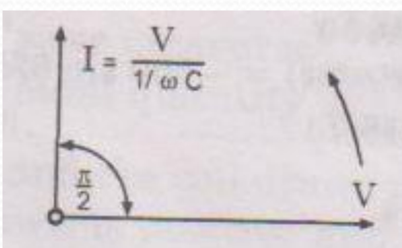
Substituting $v = V_m \sin \omega t$, i is

$$i = C\frac{d}{dt}\left(V_m \sin \omega t\right) = \omega C V_m \cos \omega t$$

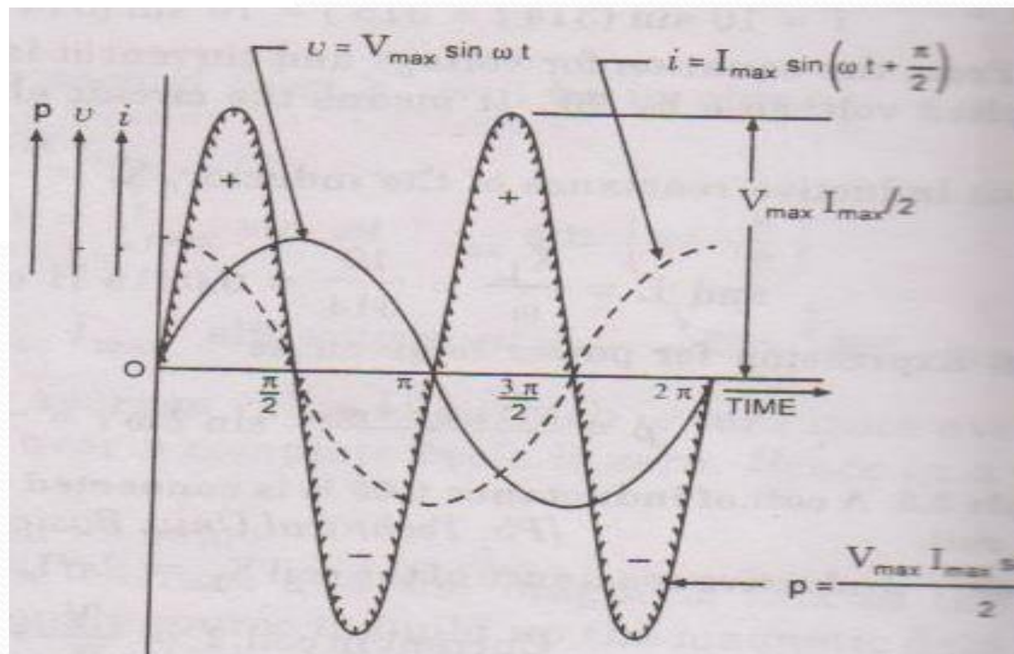
$$= \omega C V_m \sin(\omega t + 90°)$$

$$= I_m \sin(\omega t + 90°)$$





(c) Phasor Diagram



- From our earlier discussions we know that

$$v = V_m \sin(\omega t + \phi)$$

where V_m is the **peak voltage**

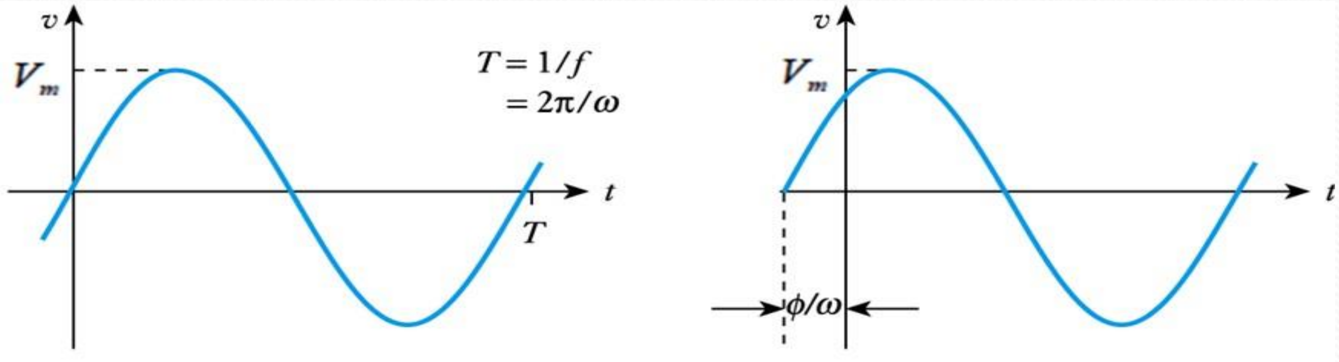
ω is the **angular frequency**

ϕ is the **phase angle**

- Since $\omega = 2\pi f$ it follows that the period T is given by

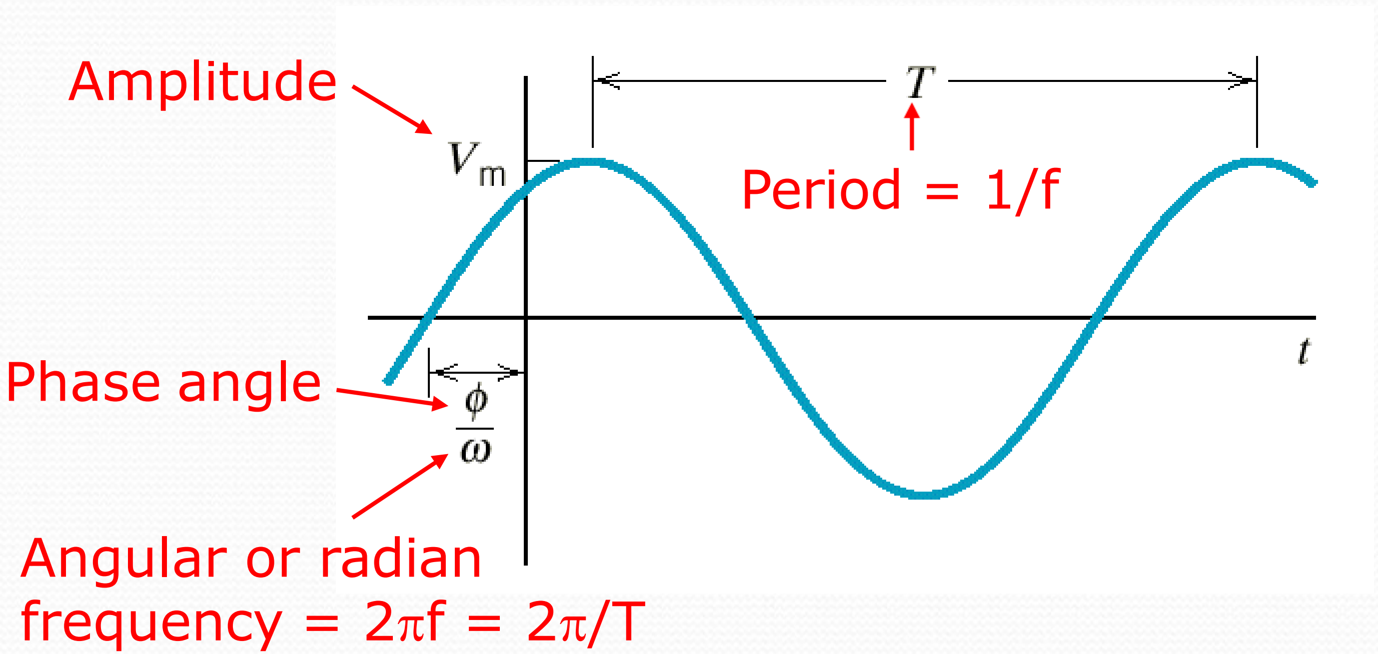
$$T = \frac{1}{f} = \frac{2\pi}{\omega}$$

- If ϕ is in radians, then a time delay t is given by ϕ/ω as shown below



(a) $v = V_m \sin(\omega t)$ (b) $v = V_m \sin(\omega t + \phi)$

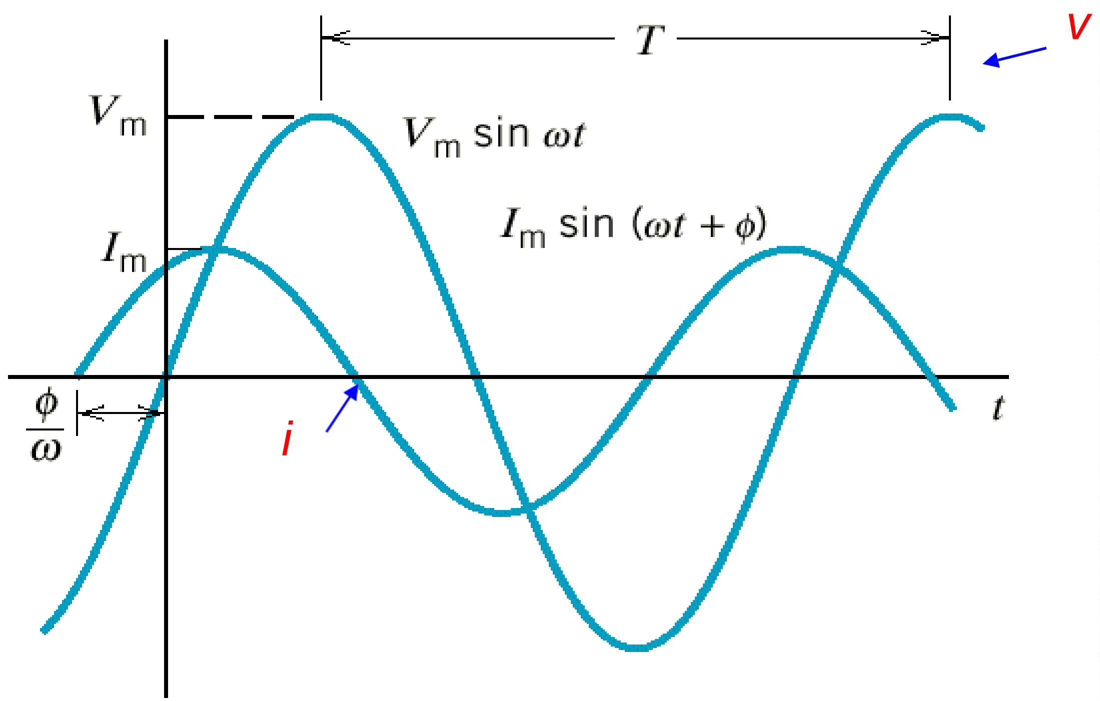
- Sinusoidal signals are characterised by their **magnitude**, their **frequency** and their **phase**

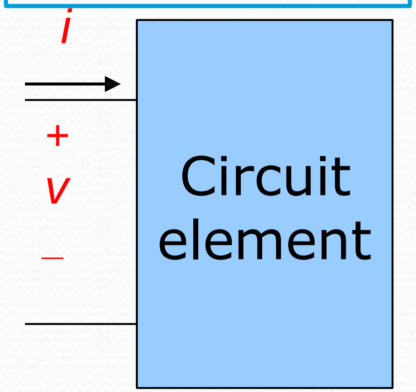


Example

$$\cos\theta = \sin\left(\theta + \frac{\pi}{2}\right)$$

$$\sin\theta = \cos\left(\theta - \frac{\pi}{2}\right)$$





Voltage and current of a circuit element.

The current *leads* the voltage by ϕ radians.

OR

The voltage *lags* the current by ϕ radians.

Mathematical representation of phasor

- Phasor can be representation in two way.
  - Rectangular form
  - Polar form

Polar form:-

- The instantaneous voltage $v_s = V_m \sin(\omega t + \phi)$ can be represented in polar form.

$$v_s = V_m \angle \phi$$

- For example v(t) = 20 sin(2πft + 60).
  - Then it represented in the polar form

$$v(t) = 20 \angle 60^\circ \text{ volt}$$

For Rectangular form:-

- The instantaneous voltage $v_s = V_m \sin(\omega t + \phi)$ can be represented in Rectangular form.

$$v(t) = x + jy$$

Where 'x' is x component of the phasor $= V_m \cos \varphi$

'y' is y component of the phasor $= V_m \sin \varphi$

- $V(t) = V_m \cos \varphi + j V_m \sin \varphi$

Example: $v(t) = 20 \sin(2\pi ft + 60)$.

$$v(t) = 20 \angle 60^\circ \text{ volt}$$

$$v(t) = 20 (\cos 60 + j \sin 60). = (10 + j17.32)$$

Conversion from polar to rectangular:

- Polar form : $v_s = r \angle \phi$
- For x component $x = r \cos \varphi$ and y component $y = r \sin \varphi$
- $v(t) = x + jy$
- $V(t) = r \cos \varphi + j r \sin \varphi$

Conversion from rectangular to polar :

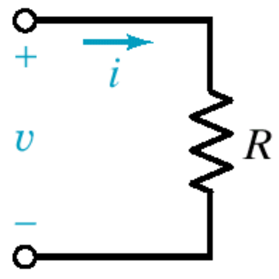
- For rectangular form $v(t) = x + jy$

$$r^2 = x^2 + y^2$$

$$r = \sqrt{x^2 + y^2} \qquad \phi = \tan^{-1}\left(\frac{y}{x}\right)$$

- For polar form $v_s = r \angle \phi = \sqrt{x^2 + y^2} \angle \tan^{-1}\left(\frac{y}{x}\right)$

Phasor Relationship for R, L, and C Elements



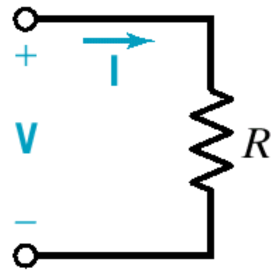
(a)

Time domain

$$v = Ri$$

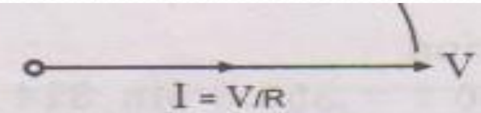
Resistor

Frequency domain



(b)

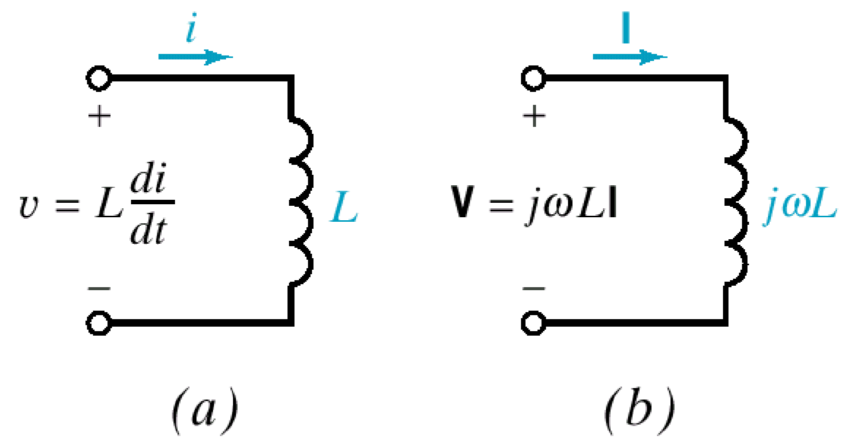
$$\mathbf{V} = R\mathbf{I} \quad or \quad \mathbf{I} = \frac{\mathbf{V}}{R}$$



(c) Phasor Diagram

Voltage and current are *in phase.*

Inductor



$$\text{As } v = L\frac{di}{dt} = V_m \sin \omega t$$

$$di = \frac{V}{L}\sin(\omega t)\, dt$$

$$i = -\frac{V_m}{\omega L}\cos \omega t = \frac{V_m}{\omega L}\sin(\omega t - 90°)$$

$$= I_m \sin(\omega t - 90°)$$

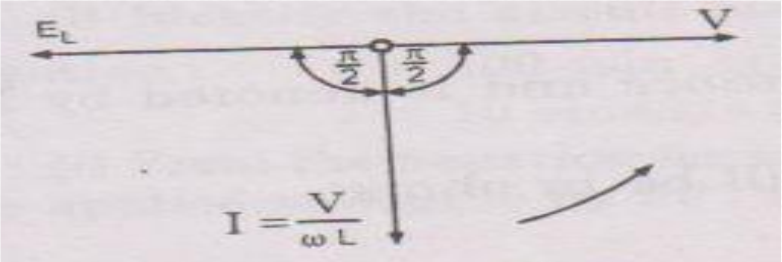
Time domain **Frequency domain**

$$v = L\frac{di}{dt}$$

$$\mathbf{V} = j\omega L\,\mathbf{I} \quad \text{or} \quad \mathbf{I} = \frac{\mathbf{V}}{j\omega L} = \frac{-j\mathbf{V}}{\omega L}$$

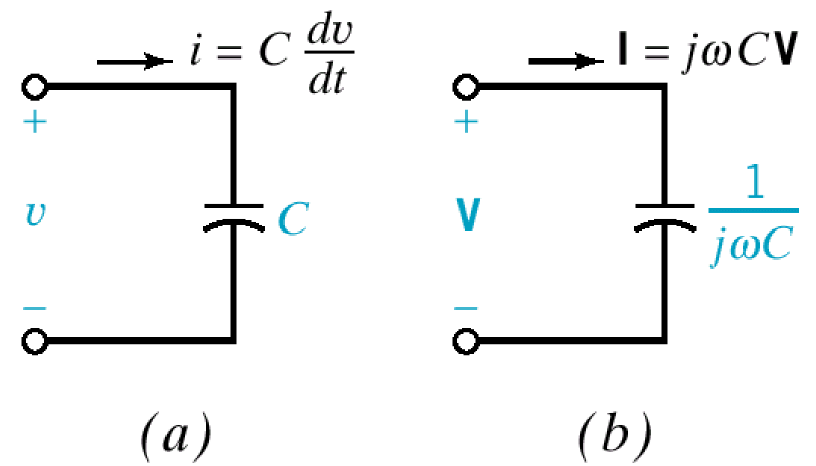
Voltage ***leads*** **current by** 90°

Current ***lags*** **voltage by** 90°



(c) Phasor Diagram

Capacitor



The current i, in the circuit

$$i = C\frac{dv}{dt}$$

$$i = C\frac{d}{dt}\left(V_m \sin\omega t\right) = \omega C V_m \cos\omega t$$

$$= \omega C V_m \sin(\omega t + 90°)$$

$$= I_m \sin(\omega t + 90°)$$

Time domain

$$V = \frac{1}{C}\int i(t)\,dt$$

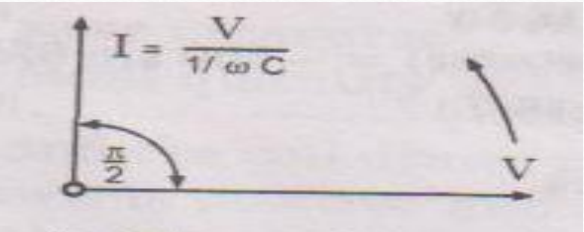
$$i = C\frac{dv}{dt}$$

Frequency domain

$$\mathbf{I} = j\omega C\mathbf{V} \quad \text{or} \quad \mathbf{V} = \frac{\mathbf{I}}{j\omega C} = \frac{-j\mathbf{I}}{\omega C}$$

Voltage *lags* current by $90°$

Current *leads* voltage by $90°$



(c) Phasor Diagram

Impedance and Admittance

Impedance is defined as the ratio of the **phasor voltage** to the **phasor current**.

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}}$$

Ohm's law in phasor notation

phase $\theta = \phi - \beta$

$$= \frac{V_m \angle \phi}{I_m \angle \beta} = \frac{V_m}{I_m} \angle \phi - \beta$$

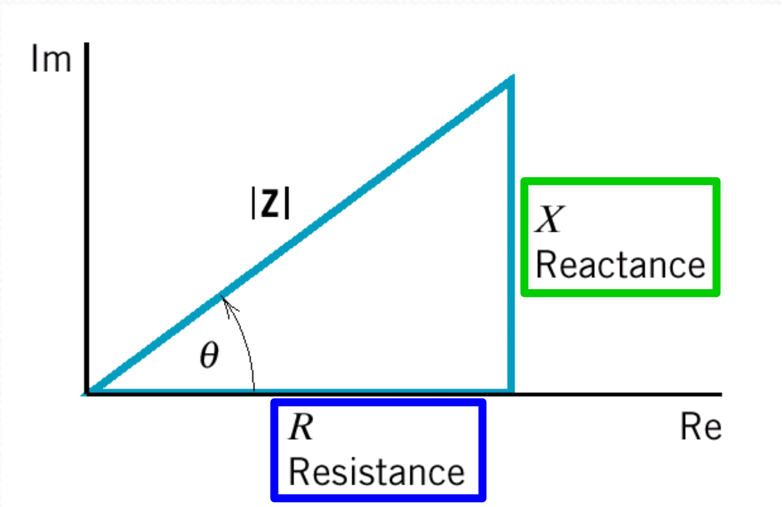
magnitude $|\mathbf{Z}|$

or

$$\mathbf{Z} = |\mathbf{Z}| \angle \theta \qquad = Ze^{j\theta} \qquad = R + jX$$

polar exponential rectangular

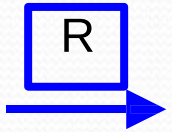
Graphical Representation of Impedance



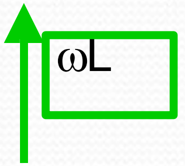
$$\mathbf{Z} = |\mathbf{Z}| \angle \theta$$

$$|\mathbf{Z}| = \sqrt{R^2 + X^2}$$

$$\theta = \tan^{-1} \frac{X}{R}$$

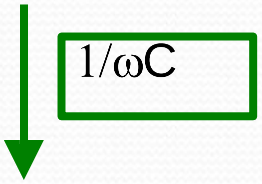


Resistor $\mathbf{Z} = R$



Inductor $\mathbf{Z} = j\omega L$

Capacitor $\mathbf{Z} = \dfrac{1}{j\omega C} = \dfrac{-j}{\omega C}$



Admittance is defined as the reciprocal of **impedance**.

In Polar form

$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = \frac{1}{|\mathbf{Z}|\angle\theta} = |\mathbf{Y}|\angle -\theta$$

In rectangular form

conductance

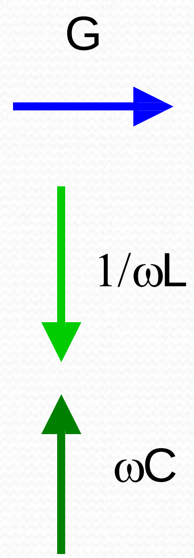
$$\mathbf{Y} = \frac{1}{\mathbf{Z}} = \frac{1}{R + jX} = \frac{R - jX}{R^2 + X^2} = G + jB$$

susceptance

Resistor $\quad \mathbf{Y} = G = \dfrac{1}{R}$

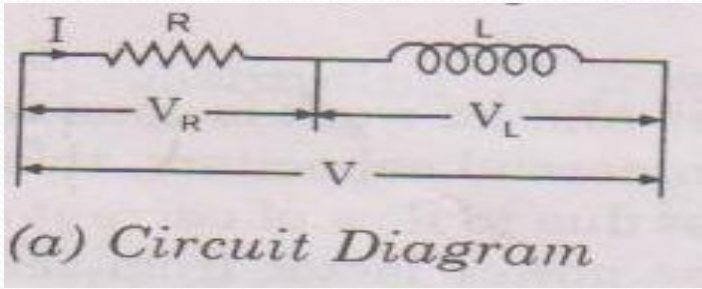
Inductor $\quad \mathbf{Y} = \dfrac{1}{j\omega L}$

Capacitor $\quad \mathbf{Y} = j\omega C$



AC circuit with series element

- **The series R-L Circuit**
  - We consider the ac circuit consisting of resistance of R and inductance of L connecting in series.



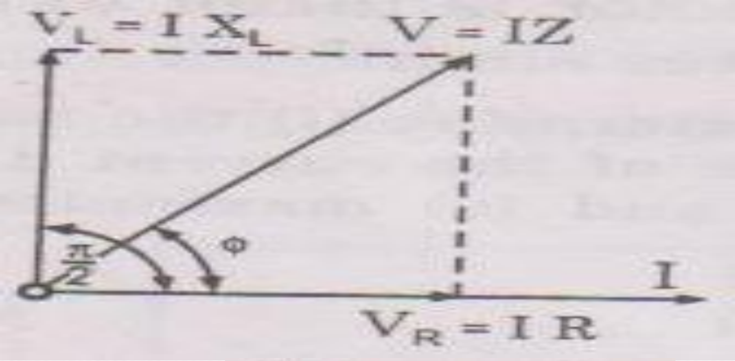
(a) Circuit Diagram

$V = V_R + V_L$

$V_R = I\ R$

$V_L = I\ X_L$

$V = I\ R + I\ X_L$ where $X_L = \omega L = 2\pi f L$



(c) Phasor Diagram

- Vector sum of the V_R and V_L

$$V = \sqrt{V_R^2 + V_L^2} \qquad V = \sqrt{(IR)^2 + (IX_L)^2}$$

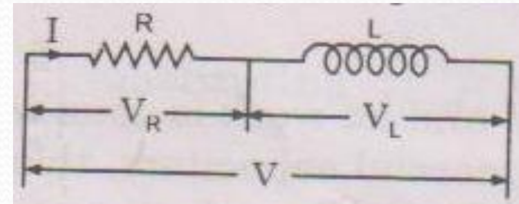
$$V = \sqrt{I^2(R^2 + X_L{}^2)} = I\sqrt{(R^2 + X_L{}^2)}$$

$$|Z| = \sqrt{(R^2 + X_L{}^2)} = Impedence\ of\ the\ circuit$$

$$V = I\,|Z| \qquad \phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

Impedance of R-L series circuit

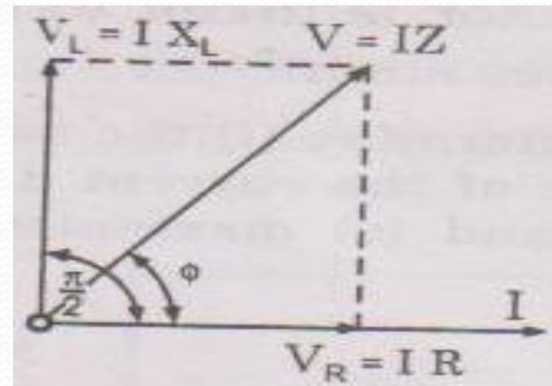
- Impedance of R-L series circuit is expressed in the rectangular form as
  - $Z = R + j X_L$
- Expressed in the polar form as



(a) Circuit Diagram

$$\mathbf{Z} = |\mathbf{Z}| \angle \phi \qquad \phi = \tan^{-1}\left(\frac{X_L}{R}\right)$$

$$|Z| = \sqrt{(R^2 + X_L{}^2)} = \textit{Impedence of the circuit}$$



- Expression for current:
  - The current through the R-L circuit is

$$i(t) = \frac{V(t)}{Z} = \frac{V\angle\phi}{|Z|\angle\phi} = \frac{V\angle{-\phi}}{|Z|}$$

Let $\frac{V}{|Z|} = I_m$

- $i(t) = I_m \angle{-\phi}$ Amp.
- instantaneous current $i(t) = I_m \sin(\omega t - \phi)$

Power in Resistance - inductance circuit

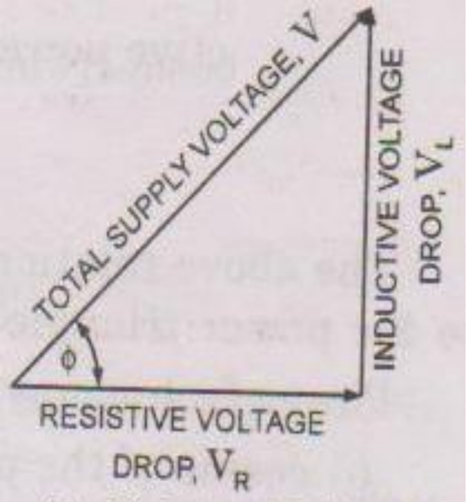
- Instantaneous power,

$$P = v\,i \; = V_{max}\sin\omega t \;X\; I_{max}\sin(\omega t - \varphi)$$

$$= \frac{1}{2}V_{max}X\;I_{max}[2\sin\omega t\;.\sin(\omega t - \varphi)]$$

$$= \frac{1}{2}V_{max}X\;I_{max}[\cos\varphi - \cos\,(2\omega t - \varphi)\,]$$

$$= \frac{1}{2}V_{max}\;I_{max}\cos\varphi - \frac{1}{2}V_{max}\;I_{max}\cos\,(2\omega t - \varphi)$$



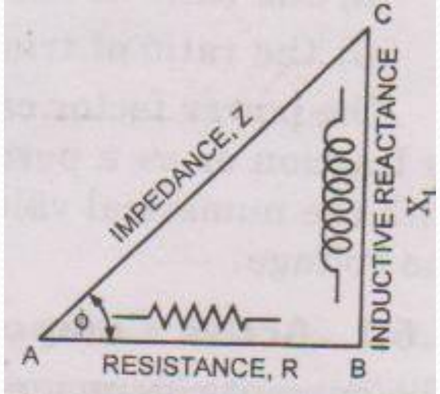
(a) Voltage Triangle

- Since average value of pulsating component $\frac{1}{2}V_{max}\;I_{max}\cos(2\omega t - \varphi)$ over the complete cycle is zero.

- The average power of the circuit

$$P = constnt\;component\; = \frac{1}{2}V_{max}\;I_{max}\cos\varphi$$

$$= \frac{V_{max}}{\sqrt{2}}\frac{I_{max}}{\sqrt{2}}\;\cos\varphi = VI\cos\varphi$$

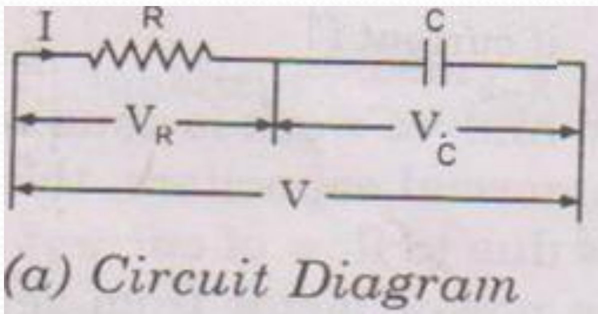


(b) Impedance Triangle

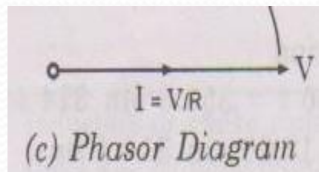
- V & I are the RMS value of voltage & current and φ is the phase angle between applied voltage & current.

The series R-C Circuit

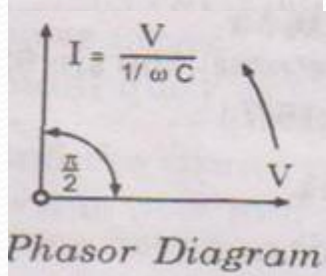
- We consider the ac circuit consisting of resistance of R and inductance of L connecting in series



(a) Circuit Diagram



(c) Phasor Diagram



Phasor Diagram

- Voltage drop across R , $V_R = I\,R$
  - (V_R is in phase with I)
- Voltage drop across C , $V_c = I\,X_c$
  - (V_C lags I by 90)

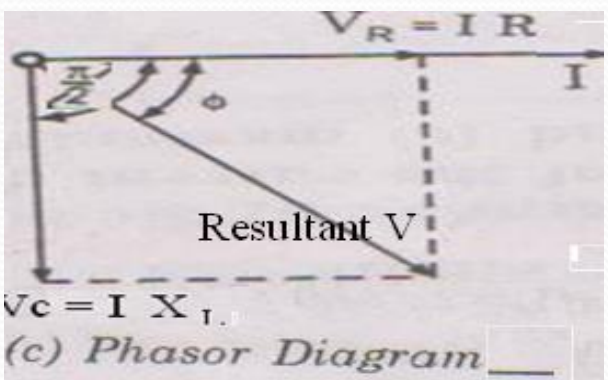
Applying KVL for RC series circuit

$V = V_R + V_c$

$V_R = I\ R$ and $V_C = I\ X_C$

$V = I\ R + I\ X_C$ where $X_C = 1/\omega L = 1/2\pi f\,C$

Vector sum of V_R and V_L



(c) Phasor Diagram

$$V = \sqrt{V_R^2 + V_C^2} \qquad V = \sqrt{(IR)^2 + (IX_c)^2}$$

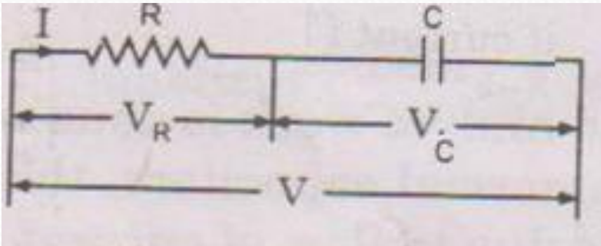
$$V = \sqrt{I^2(R^2 + X_c^2)} = I\sqrt{(R^2 + X_c^2)}$$

$$|Z| = \sqrt{(R^2 + X_c^2)} = Impedence\ of\ the\ circuit$$

$$V = I\,|Z|$$

Impedance of R-C series circuit

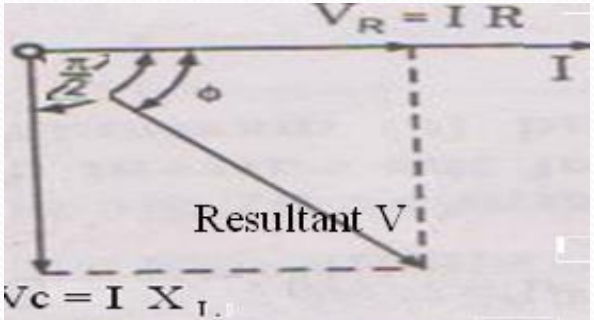
- Impedance of R-C series circuit is expressed in the rectangular form as
  - Z = R - j XC
- Expressed in the polar form as



(a) Circuit Diagram

$$\mathbf{Z} = |\mathbf{Z}|\angle\phi \qquad \phi = \tan^{-1}\left(\frac{-X_c}{R}\right)$$

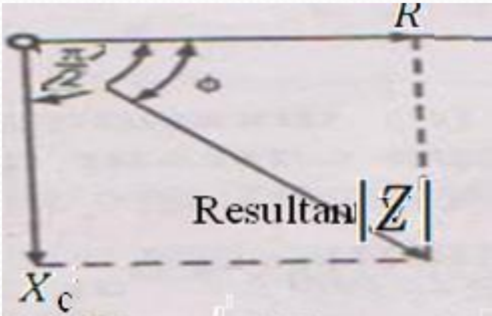
$$|Z| = \sqrt{(R^2 + X_c{}^2)} = Impedence\ of\ the\ circuit$$



(c) Phasor Diagram

- Expression for current:
  - The current through the R-L circuit is

$$i(t) = \frac{V(t)}{Z} = \frac{V\angle\phi}{|Z|\angle-\phi} = \frac{V\angle\phi}{|Z|} \qquad \text{Let } \frac{V}{|Z|} = I_m$$

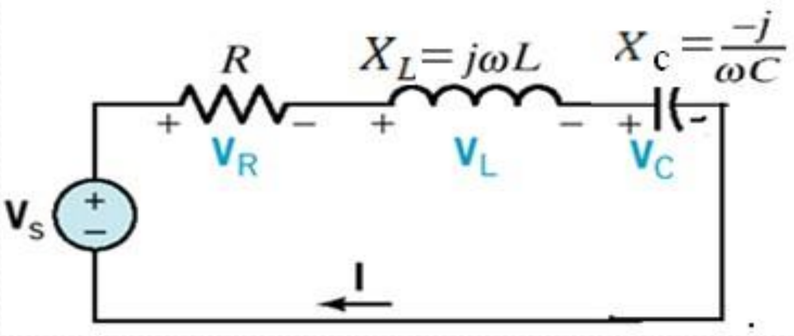


(c) Phasor Diagram

- i(t) = Im $\angle\phi$ Amp.
- instantaneous current i(t) = Im sin (ωt + ϕ)

Series R-L-C Circuit

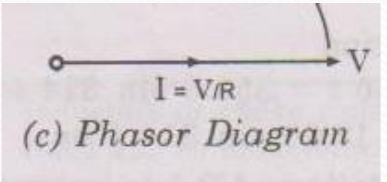
A **Phasor Diagram** is a graphical representation of phasors and their relationship on the *complex plane*.

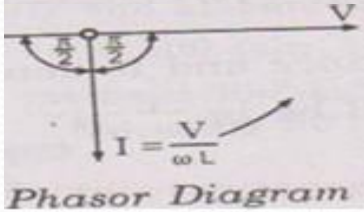


$$\mathbf{V}_s = \mathbf{V}_R + \mathbf{V}_L + \mathbf{V}_C$$

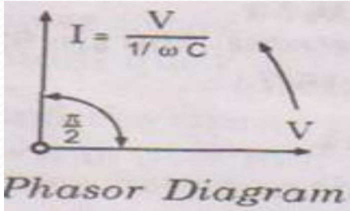
The voltage phasors are

$$\begin{cases} \mathbf{V}_R = R\mathbf{I} = RI\angle 0^\circ \\ \mathbf{V}_L = j\omega L\mathbf{I} = \omega LI\angle 90^\circ \\ \mathbf{V}_C = \dfrac{-j}{\omega C}\mathbf{I} = \dfrac{I}{\omega C}\angle -90^\circ \end{cases}$$





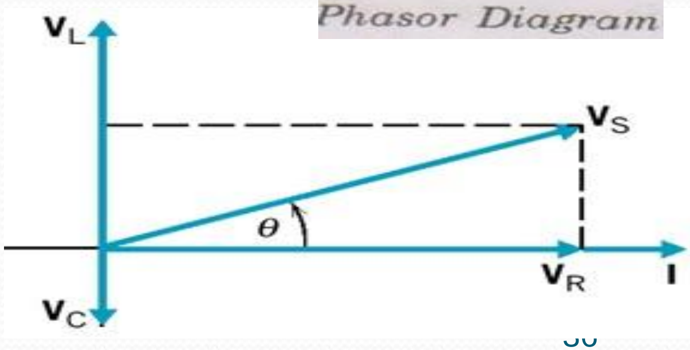
- Applied voltage V, being equal to the phasor vector sum of V_R, V_L & V_C is given in magnitude



$$V = \sqrt{V_R^2 + (V_L - V_c)^2} = \sqrt{(IR)^2 + (IX_L - IX_c)^2}$$

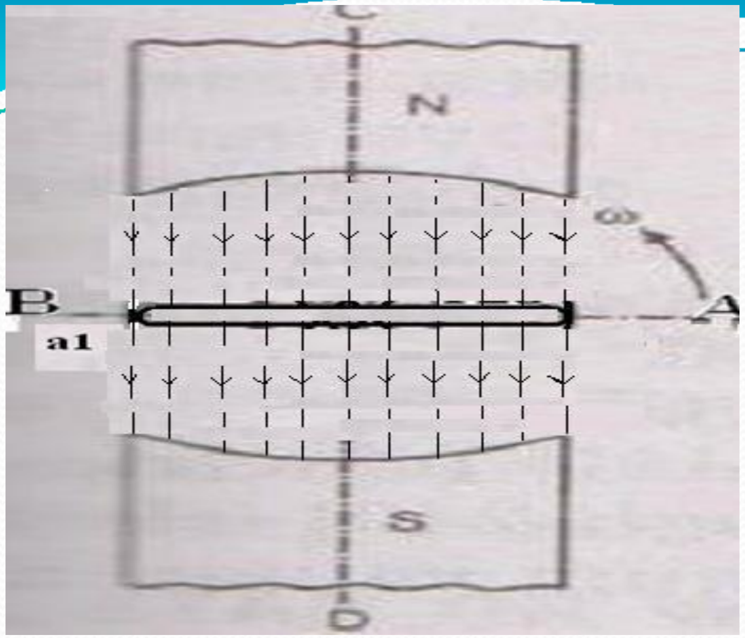
$$= I\sqrt{(R)^2 + (X_L - X_c)^2}$$

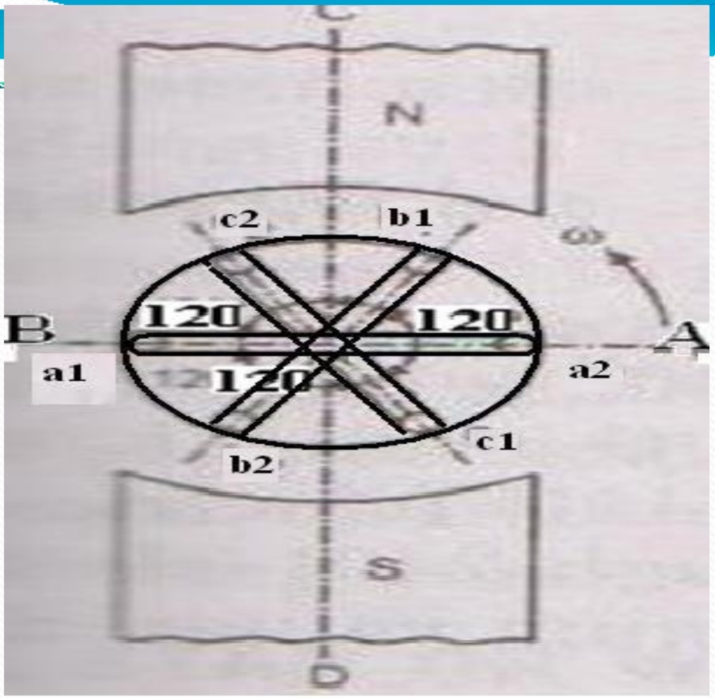
$$|Z| = \sqrt{(R)^2 + (X_L - X_c)^2} = \textit{Impedence of the circuit}$$

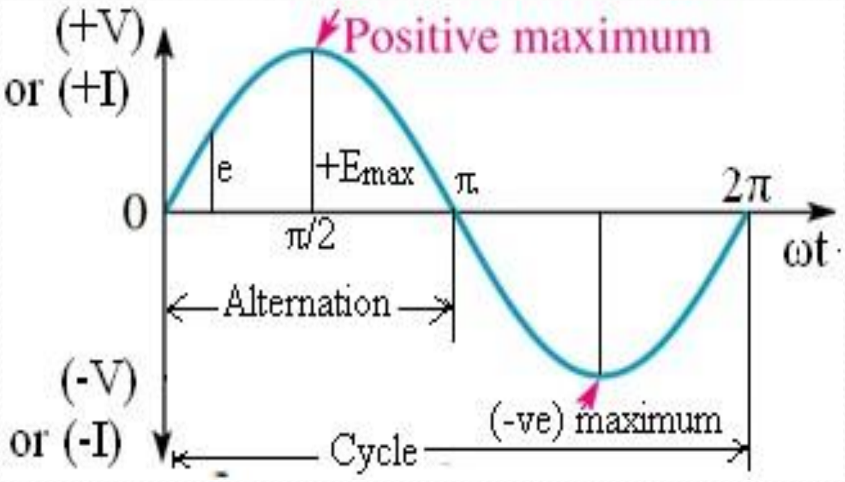


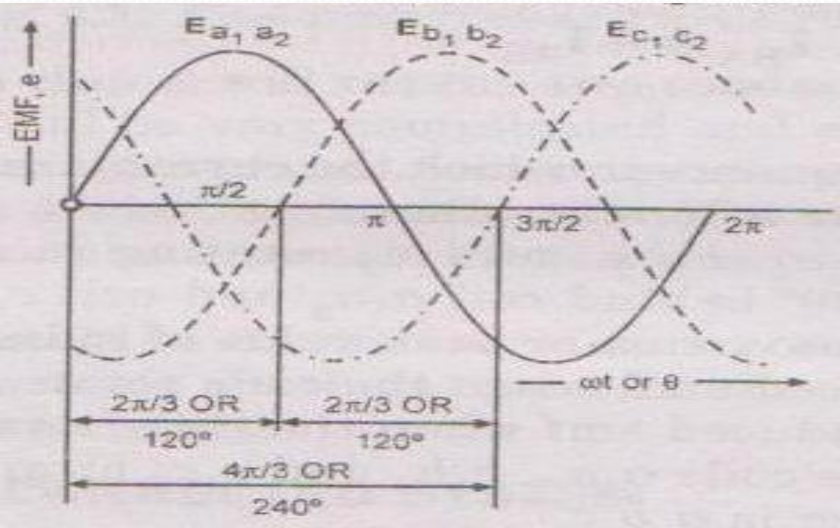
$$V = I\,|Z| \qquad \phi = \tan^{-1}\left(\frac{X_L - X_c}{R}\right)$$

Three-phase System





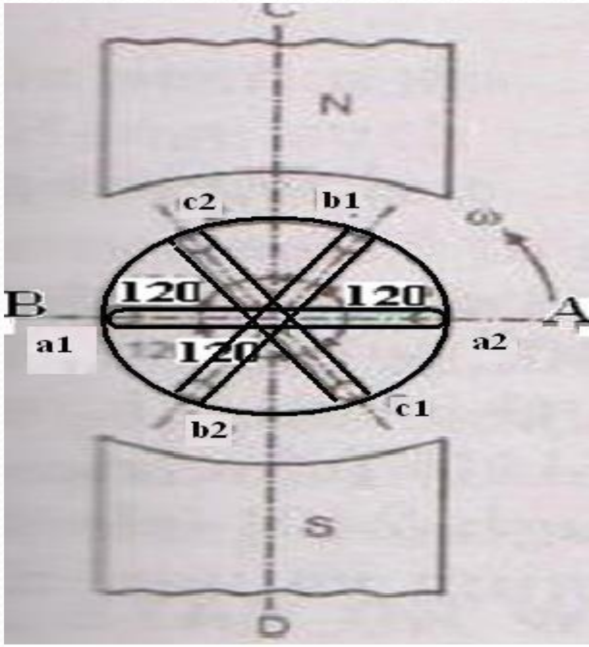




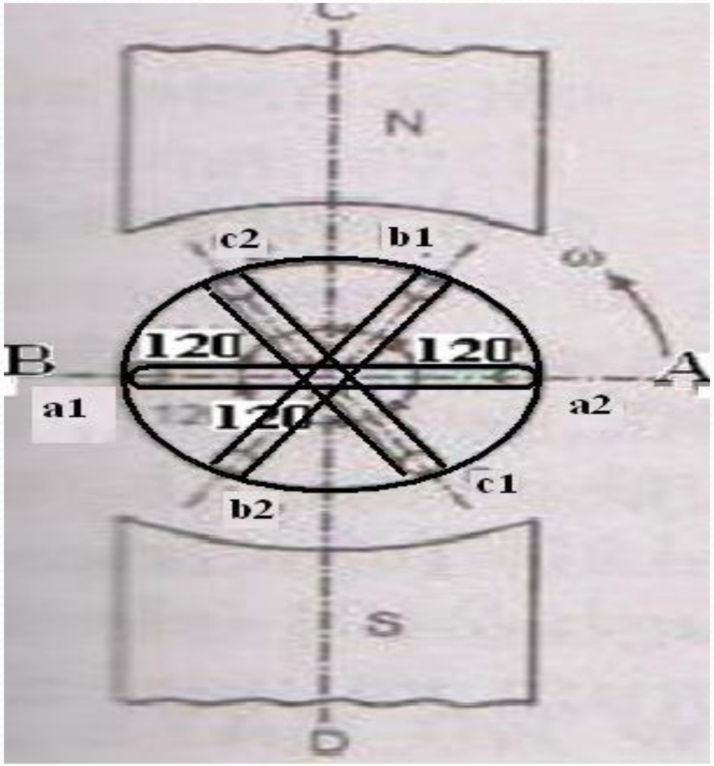
- Single phase generator
- Three phase generator

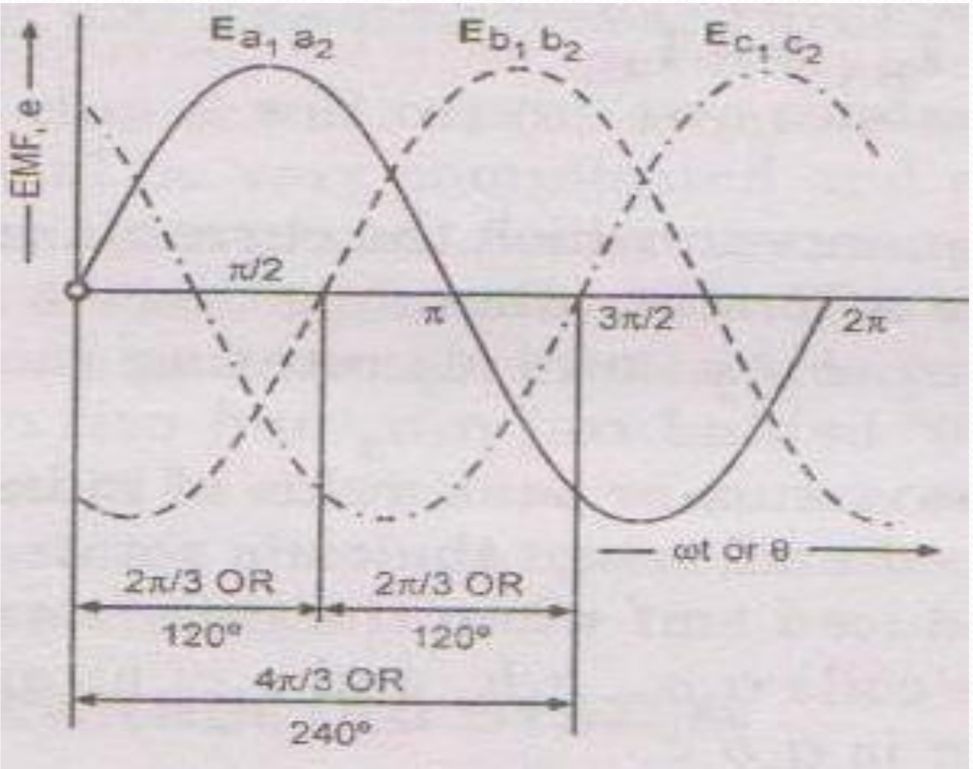
Three-phase AC Circuits

- Generation of Three-phase Balanced Voltages
  - Three windings, with equal no. of turns in each one, are used, so as to obtain **equal voltage in magnitude** in all three phases.
  - Also to obtain a balanced three-phase voltage, the windings are to be placed at an electrical angle of 120 with each other.
- Contraction:-
  - consider three identical coil "a1a2", "b1 b2", and "c1c2" are mounted in same axis but displaced from each other by 120 rotating in counter clockwise direction in bipolar magnetic field as shown in fig.



- When a coil "a1 a2" is the position AB the induced emf in this coil is zero.
- At that time 'b1 b2' is 120 behind the coil 'a1 a2' so the emf is induced in this coil as approaching its maximum negative (-ve) value .
- At the same time 'c1 c2' is 240 behind the coil 'a1 a2' so the emf is induced in this coil as approaching its maximum positive (+ve) value and is decreasing





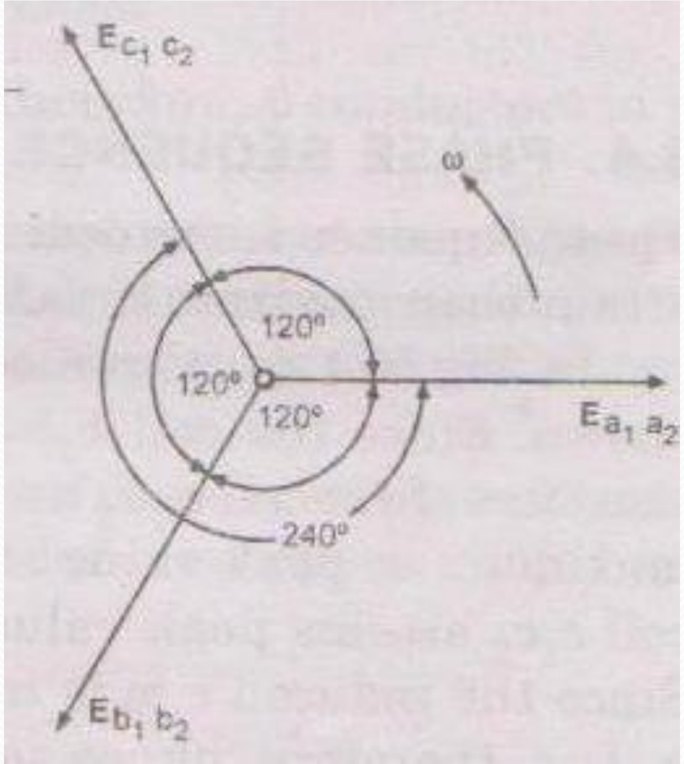
Some important point

- The induced emf in each of the coil are of same magnitude.
- The induced emf in each of the coil are of same frequency and Waveform (sinusoidal wave).
- The instantaneous value of the emf induced in the coil "a1 a2" , "b1 b2" and " c1 c2"is given as below.

$$e_{a_1a_2} = E_{max}\sin\omega t$$

$$e_{b_1b_2} = E_{max}\sin(\omega t - 120^\circ)$$

$$e_{c_1c_2} = E_{max}\sin(\omega t - 240^\circ)$$
$$= E_{max}\sin(\omega t + 120^\circ)$$



- Note:- if t = 0 corresponding to the instant then voltage & emf of the coil "a1 a2" pass through zero & increasing in positive direction.

Balance Three Phase Star Connection

- The connection diagram of a star (Y)-connected three-phase system is shown in Fig.

- Three phase voltages (E_P) are:

$$e_{RN} = E_m \sin\theta\,; \qquad e_{YN} = E_m \sin(\theta - 120°)\,;$$

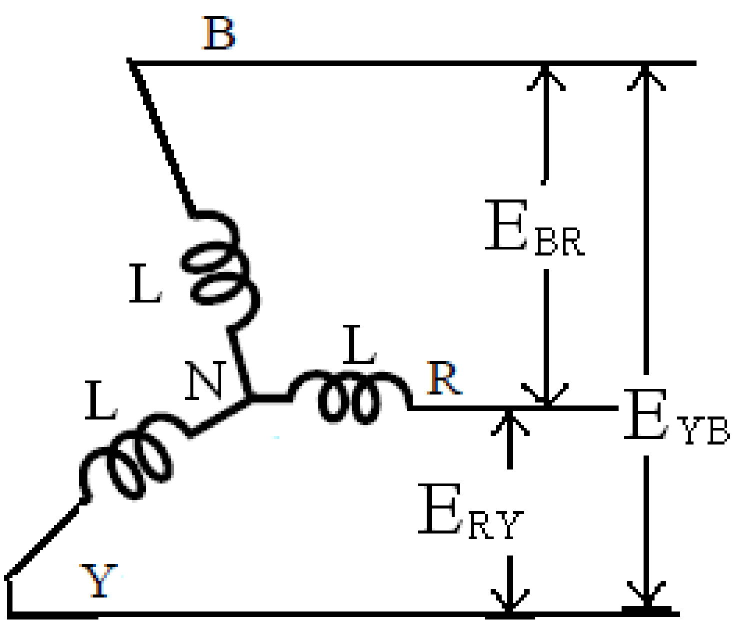
$$e_{BN} = E_m \sin(\theta - 240°) = E_m \sin(\theta + 120°)$$

$$E_{RN} \angle 0° = E\,(1.0 + j0.0)\,;$$

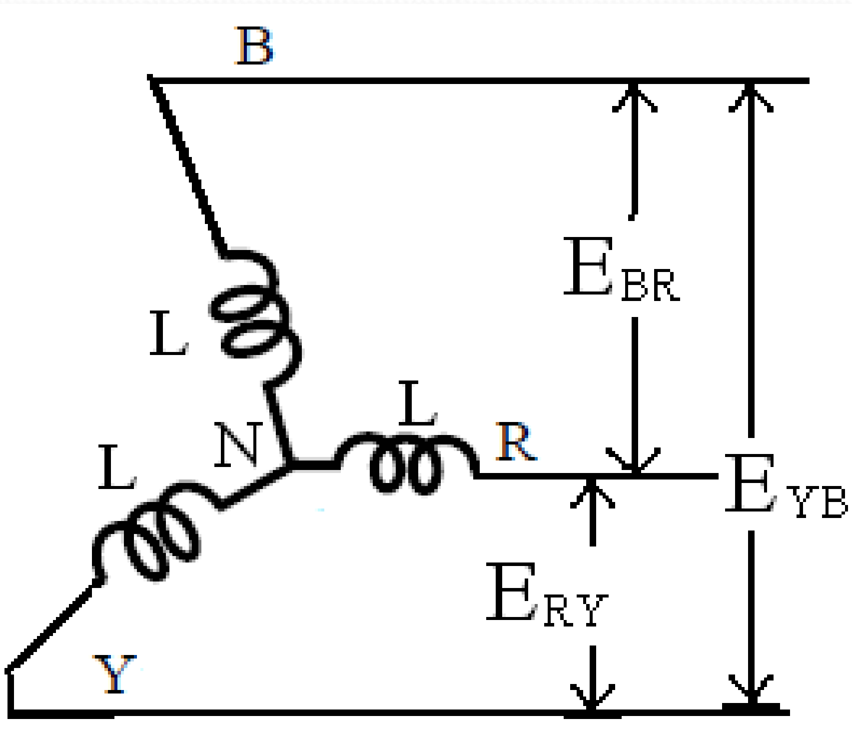
$$E_{YN} \angle -120° = E\,(-0.5 - j0.866)\,;$$

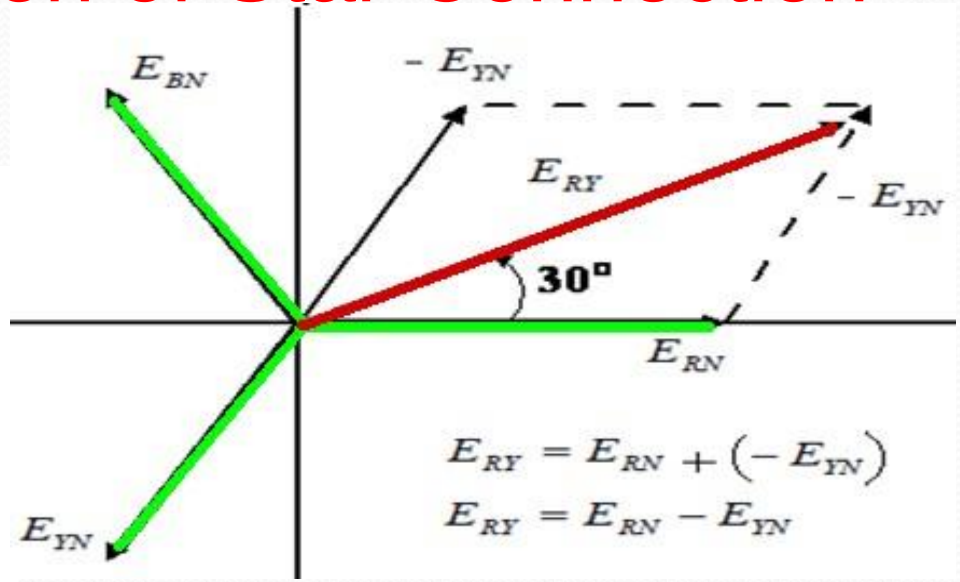
$$E_{BN} \angle +120° = E\,(-0.5 + j0.866)\,.$$

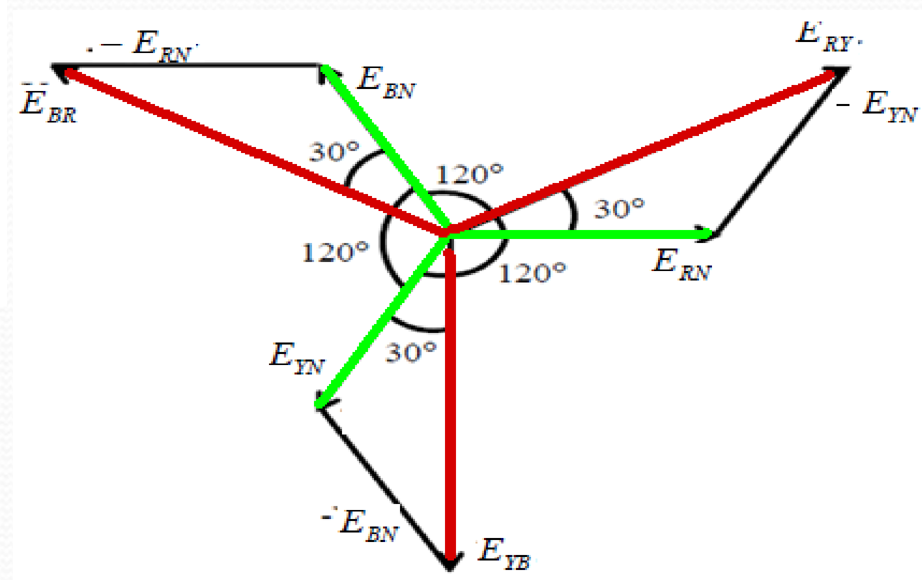
- Three line voltages (E_L) are:
  - E_{RY}
  - E_{RB}
  - E_{YB}



Phase Representation of Star Connection







Relation between Phase and Line Voltages for Star Connection

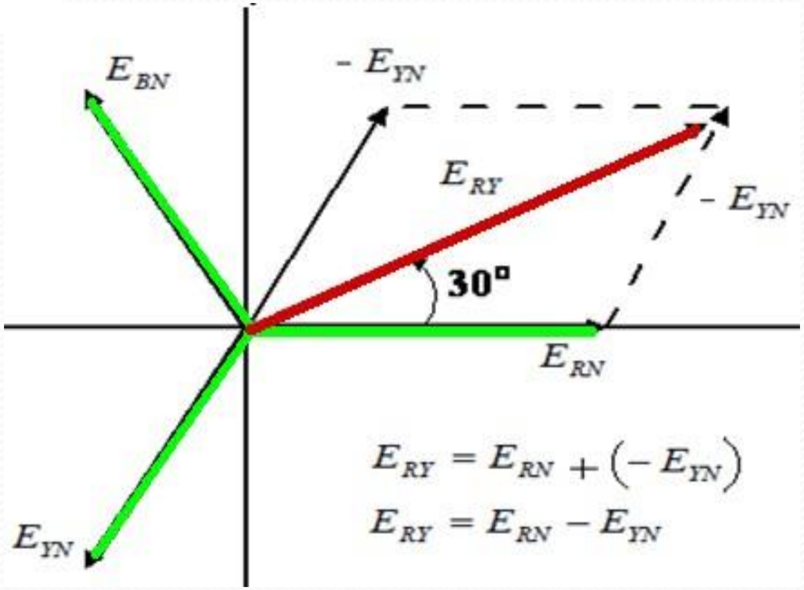
- Three line voltages are obtained by the following procedure. The line voltage, E_{RY} is

$$E_{RY} = E_{RN} - E_{YN} \qquad = E\angle 0^\circ - E\angle -120^\circ$$

$$= E\,[(1 + j0) - (-0.5 - j0.866)]$$

$$= E\,(1.5 + j0.866) \;=\; \sqrt{3}\,E\angle 30^\circ$$

$$E_L = E_{RY} = \sqrt{3}\,E\angle 30^\circ = \sqrt{3}\,E_P\angle 30^\circ$$



- The magnitude of the line voltage, is E_{RY} *is* $\sqrt{3}$ *times the magnitude of the phase voltage* E_{RN}.
- E_{RY} lead the E_{RN} by 30 degree

Cont…

- Similarly other line voltages **E_{YB}** as shown in brief

$$E_{YB} = E_{YN} - E_{BN} \qquad = E\angle -120^\circ - E\angle +120^\circ$$

$$E_{YB} = \sqrt{3}E\angle -90^\circ$$

  - The magnitude of the line voltage, is E_{YB} *is* $\sqrt{3}$ *times the magnitude of the phase voltage* E_{YN}.
  - E_{RY} lead the E_{RN} by 90 degree

- Similarly other line voltages **E_{YB}** as shown in brief

$$E_{BR} = E_{BN} - E_{RN} \qquad = E\angle +120^\circ - E\angle 0^\circ$$

$$E_{BR} = \sqrt{3}E\angle +150^\circ$$

- So, the three line voltages are balanced, with their magnitudes being equal, and the phase angle being displaced from each other in sequence by 30.

Relation between the Phase and Line Current for Star Connection

- In the star connected system each line connector is connected to separate phase, so current flowing through the lines and phase

  - Lines Current I_L (I_{RY}, I_{RB}, I_{YB}) = Phase Current I_P (I_{BN}, I_{YN}, I_{RN})

- If the phase current has a phase difference of φ with the phase voltage, then

  - Power output per phase = $E_P \; I_P \cos\varphi$
  - Total power output phase = $3 \; E_P \; I_P \cos\varphi$

$$\therefore \; E_P = \frac{E_L}{\sqrt{3}}.$$

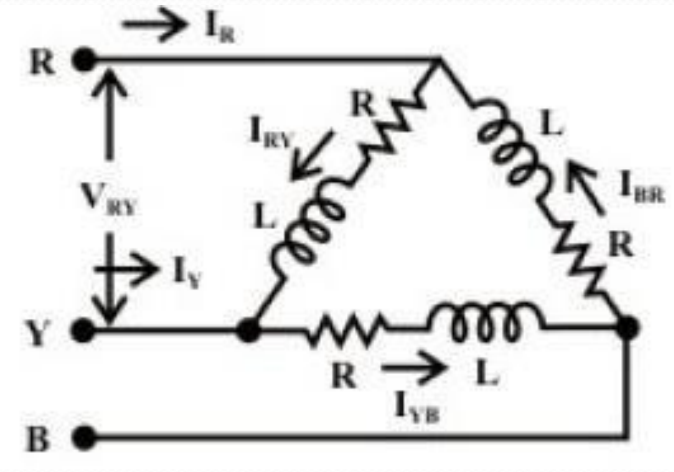
$$= \frac{3 \, E_L}{\sqrt{3}} \, I_L \cos\varphi$$

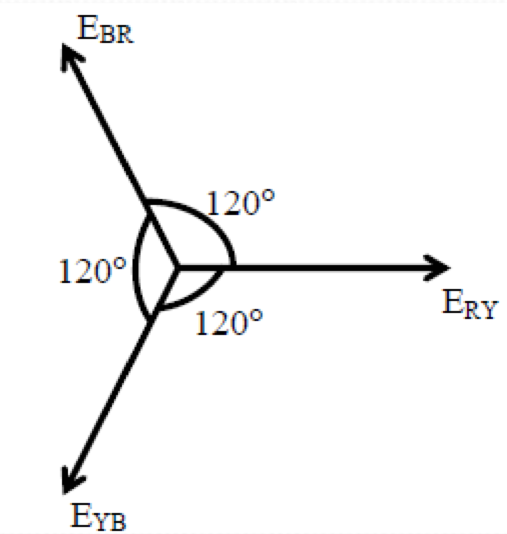
$$= \sqrt{3} \, E_L \, I_L \cos\varphi$$

Currents for Circuits with Balanced Load (Delta-connected)

-

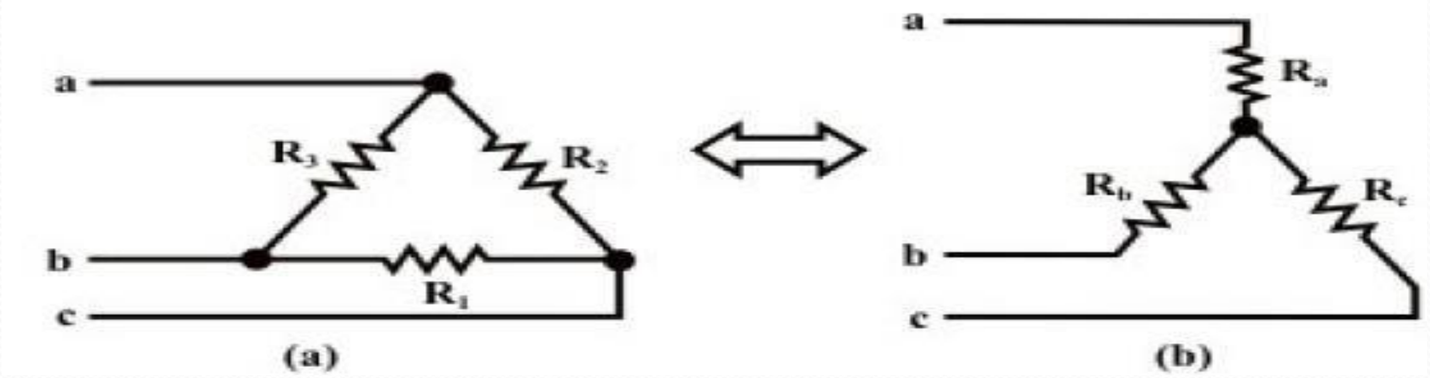
$$E_{RY} = E\angle 0° \; ; \qquad E_{YB} = E\angle -120° \; ; \qquad E_{BR} = E\angle +120°$$





Delta(Δ)-Star(*Y*) *conversion and Star-Delta conversion*

- The formulas for delta-star conversion, using resistance



$$R_a = \frac{R_2 R_3}{R_1 + R_2 + R_3} \qquad R_b = \frac{R_3 R_1}{R_1 + R_2 + R_3} \qquad R_c = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

- The formulas for star-delta conversion, using resistance, are

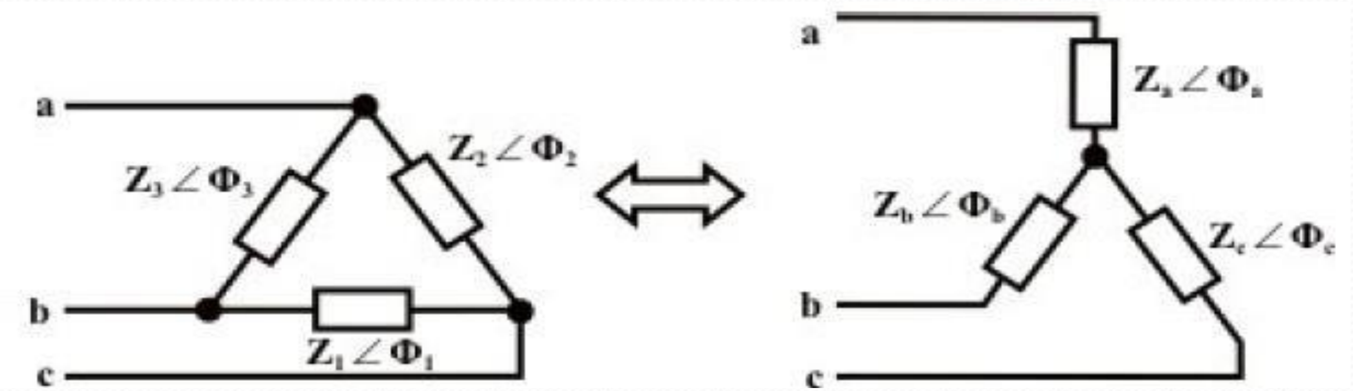
$$R_1 = R_b + R_c + \frac{R_b R_c}{R_a} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_a}$$

$$R_2 = R_c + R_a + \frac{R_c R_a}{R_b} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_b}$$

$$R_3 = R_a + R_b + \frac{R_a R_b}{R_c} = \frac{R_a R_b + R_b R_c + R_c R_a}{R_c}$$

For impedance conduction

- The formulas for delta-star conversion, using impedances.



$$Z_a = \frac{Z_2 Z_3}{Z_1 + Z_2 + Z_3} \qquad Z_b = \frac{Z_3 Z_1}{Z_1 + Z_2 + Z_3} \qquad Z_c = \frac{Z_1 Z_2}{Z_1 + Z_2 + Z_3}$$

- The formulas for delta-star conversion, using impedance, are

$$Z_1 = Z_b + Z_c + \frac{Z_b Z_c}{Z_a} = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_a} \qquad Z_2 = Z_c + Z_a + \frac{Z_c Z_a}{Z_b} = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_b}$$

$$Z_3 = Z_a + Z_b + \frac{Z_a Z_b}{Z_c} = \frac{Z_a Z_b + Z_b Z_c + Z_c Z_a}{Z_c}$$

Thank You