OBJECTIVE TYPE QUESTIONS

Choose the correct alternative:

1. The value of the integral $\iint xy\,(x+y)\,dx\,dy$ over the area between $y = x^2$ and $y = x$ is

   (*i*) $\frac{3}{56}$ (*ii*) $\frac{47}{56}$ (*iii*) $\frac{33}{56}$ (*iv*) $\frac{23}{56}$ **Ans.** (*i*)

2. The integral - $\iint\limits_{x^2+y^2\le 1} \frac{1}{\pi}(x^2+y^2)\,dx\,dy$ equals

   (*i*) 0 (*ii*) 1 (*iii*) 1/3 (*iv*) 1/2 **Ans.** (*iv*)

3. $\int \frac{\sqrt{\tan x}}{\sin x \cos x}\,dx$ is equal to

   (*i*) $2\sqrt{\tan x}+C$ (*ii*) $2\sqrt{\cot x}+C$ (*iii*) $\frac{\sqrt{\tan x}}{2}+C$ (*iv*) None of these

   Ans. (*i*)

4. Value of the integral $\int\limits_{-a}^{a}\int_0^{\sqrt{a^2-x^2}} dx\,dy$ is equal to

   (*i*) $4a$ (*ii*) $2a$ (*iii*) 0 (*iv*) None of these

   Ans. (*iv*)

5. The value of $\int_1^0\int_0^1 (x+y)\,dx\,dy$ is equal to

   (*i*) 1 (*ii*) −1 (*iii*) 2 (*iv*) 0 **Ans.** (*i*)

6. The value of $\int_0^1\int_0^x e^x\,dx\,dy$ is

   (*i*) −1 (*ii*) 0 (*iii*) 1 (*iv*) 2 **Ans.** (*iii*)

7. The value of $\int_{-a}^{a}\left[\int_0^x dy\right]dx$ is

   (*i*) 1 (*ii*) 2 (*iii*) 3 (*iv*) 0 **Ans.** (*iv*)

8. The value of $\int_0^1 dx\int_0^x e^{\frac{y}{x}}\,dy$ is

   (*i*) $\frac{1}{2}(e-1)$ (*ii*) $(e+1)$ (*iii*) $(e-1$ (*iv*) $\frac{1}{2}(e+1)$

   Ans. (*i*)

9. The value of $\int_0^{\pi}\int_0^{a(1-\cos\theta)} r^3\sin\theta\,dr\,d\theta$ is

   (*i*) $\frac{15}{16}a^4$ (*ii*) $\frac{8a^4}{5}$ (*iii*) a^4 (*iv*) $\frac{16}{15}$ **Ans.** (*ii*)

10. The value of $\int_0^{\pi}\left[\int_{2\sin\theta}^{4\sin\theta} r^3\,dr\right]d\theta$ is

    (*i*) 22.5 π (*ii*) 22 π (*iii*) 10.5 π (*iv*) π **Ans.** (*i*)

11. The value of integral $\int\limits_0^2\int\limits_0^x (x+y)\,dx\,dy$ is equal to

    (*i*) − 4 (*ii*) 3 (*iii*) 4 (*iv*) − 3 **Ans.** (*iii*)

12. $\int_3^{2a}\int_0^{\sqrt{2ax-x^2}} dx\,dy$ is equal to

(i) $\int_0^{\pi}\int_0^{2a\cos\theta} r\,dr\,d\theta$ (ii) $\int_0^{\frac{\pi}{2}}\int_0^{2a\cos\theta} r\,dr\,d\theta$

(iii) $\int_0^{\frac{\pi}{2}}\int_0^{2a\sin\theta} r\,dr\,d\theta$ (iv) None of these Ans. (iii)

13. The value of $\int_0^{\pi}\int_0^{a(1+\cos\theta)} r^2\sin\theta\,.\,d\theta\,dr$ is

(i) a^3 (ii) $\frac{4}{3}\pi^3$ (iii) $\frac{4}{3}a^3$ (iv) $\frac{1}{3}a^3$ Ans. (iii)

14. The value of integral $\int_0^1\int_{x^2}^{2-x} xy\,dx\,dy$ is equal to

(i) $\frac{3}{4}$ (ii) $\frac{3}{8}$ (iii) $\frac{3}{5}$ (iv) $\frac{3}{7}$ Ans. (ii)

15. The value of the integral $\int_0^{a/2}\int_0^{\sqrt{a^2-x^2}} dy\,dx$ is equal to

(i) πa^2 (ii) $\frac{\pi a^2}{8}$ (iii) $\frac{\pi a^2}{4}$ (iv) None of these

Ans. (iv)

Fill in the blanks:

16. $\int_1^{2}\int_0^{1}(x+y)\,dx\,dy$ = Ans.

17. $\int_0^{1}\int_0^{x} e^{x}\,dx\,dy$ = Ans. 1

18. $\int_{-a}^{a}\left[\int_0^{x} dy\right]dx$ = ... Ans. 0

19. $\int_0^{1}\int_{e^x}^{e}\frac{dy\,dx}{\log y}$ = ... Ans. $e-1$

20. $\int_0^{a}\int_y^{a}\frac{x\,dx\,dy}{x^2+y^2}$ = ... Ans. $\frac{\pi a}{4}$

21. $\int_0^{1}\int_{2y}^{2} e^{2x}\,dx\,dy$ = ... Ans. $\frac{e^4-1}{4}$

22. $\int_0^{1}\int_0^{\sqrt{1-x^2}} y^2\,dy\,dx$ = ... Ans. $\frac{\pi}{16}$

Match the following:

23. (a) $\int_0^{1}\int_0^{y} xye^{-x^2}\,dx\,dy$ (p) $\frac{1}{2}$

(b) $\int_0^{1}\int_0^{x^2} e^{\frac{y}{x}}\,dy\,dx$ (q) $\left[\frac{\pi a}{4}-a\tan^{-1}\frac{1}{a}\right]$

(c) $\int_1^{\log 8}\int_0^{\log y} e^{x+y}\,dx\,dy$ (r) $\frac{1}{4e}$

(d) $\int_0^{a}\int_{\frac{x}{a}}^{x}\frac{x\,dy\,dx}{x^2+y^2}$ (s) $8\log 8-16+e$

Ans. (a) → (r), (b) → (p), (c) → (s), (d) → (q)

24. (a) $\int_0^{\frac{\pi}{2}}\left[\int_0^{a\cos\theta} r\sqrt{a^2-r^2}\,dr\right]d\theta$ (p) $\frac{4}{3}a^3$

(b) $\int_0^{\pi}\int_0^{a(1-\cos\theta)} r^2\sin\theta\,dr\,d\theta$ (q) $\frac{21}{4}+e^4-e^3$

(c) $\int_1^{x}\int_3^{2}(xy+e^y)\,dy\,dx$ (r) $\frac{\pi a^3}{6}$

(d) $\int_0^{a}\int_0^{\sqrt{a^2-y^2}}\sqrt{a^2-x^2-y^2}\,dx\,dy$ (s) $\frac{a^3}{12}(3\pi-4)$

24. (a) $\int_0^{\frac{\pi}{2}} \left[\int_0^{a\cos\theta} r\sqrt{a^2 - r^2}\, dr \right] d\theta$ (p) $\frac{4}{3}a^3$

(b) $\int_0^{\pi} \int_0^{a(1-\cos\theta)} r^2 \sin\theta\, dr\, d\theta$ (q) $\frac{21}{4} + e^4 - e^3$

(c) $\int_1^x \int_3^2 (xy + e^y)\, dy\, dx$ (r) $\frac{\pi a^3}{6}$

(d) $\int_0^a \int_0^{\sqrt{a^2 - y^2}} \sqrt{a^2 - x^2 - y^2}\, dx\, dy$ (s) $\frac{a^3}{18}(3\pi - 4)$

Ans. (a) → (s), (b) → (p), (c) → (q), (d) → (r)

2. $\int_0^{\frac{\pi}{2}} \int_0^{\sqrt{\sin\theta}} r\, d\theta\, dr$ is equal to

(i) $\frac{1}{2}$ (ii) $-\frac{1}{2}$ (iii) 1 (iv) − 1 **Ans.** (i)

3. $\int_{-a}^{a} \left[\int_0^x dy \right] dx$ is equal to

(i) $-a$ (ii) a (iii) 0 (iv) $\frac{a}{2}$ **Ans.** (iii)

4. $\int_0^{2\pi} d\theta \int_0^1 e^{2r}\, dr$ is equal to

(i) $(e^2 - 1)$ (ii) $\frac{\pi}{2}(e^2 - 1)$ (iii) $\pi(e^2 - 1)$ (iv) $2\pi(e^2 - 1)$ **Ans.** (iii)

5. The transformations $x + y = u$, $y = uv$ transform the area element $dy\, dx$ into $|J|\, du\, dv$, where $|J|$ is equal to

(i) 1 (ii) u (iii) − 1 (iv) none of these **Ans.** (ii)

6. $\int_1^{\log 8} \int_0^{\log y} e^{x+y}\, dx\, dy =$

(i) 8 log 8 + 16 + e (ii) 8 log 8 − 16 − e

(iii) 8 log 8 − 16 + e (iv) log 8 − 16 + e **Ans.** (iii)

7. $\iint_D (x^2 + y^2)\, dx\, dy = ?$, where D is bounded by $y = x$ and $y^2 = 4x$.

(i) $\frac{768}{25}$ (ii) $\frac{768}{35}$ (iii) $\frac{708}{35}$ (iv) $\frac{68}{35}$ **Ans.** (ii)

8. $\iint_D x^3 y\, dxdy$, where D is the region enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in the first quadrant.

(i) $\frac{b^2a^4}{24}$ (ii) $\frac{b^3a^4}{24}$ (iii) $\frac{ba^4}{24}$ (iv) $\frac{b^2a^2}{24}$ **Ans.** (i)

9. $\int_0^3 \int_x^{4x - x^2} y\, dx\, dy =$

(i) $\frac{54}{7}$ (ii) $\frac{54}{17}$ (iii) $\frac{34}{5}$ (iv) 54 **Ans.** (iv)

10. $\int_0^1 \int_y^{10y} \sqrt{xy - y^2}\, dxdy =$ .....

(i) 6 (ii) 4 (iii) 5 (iv) 16 **Ans.** (i)

11. $\int_0^1 \int_{e^x}^{e} \frac{dxdy}{\log y} =$

(i) $e + 1$ (ii) $e - 1$ (iii) e (iv) e^{-1} **Ans.** (ii)

12. $\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y}\, dy\, dx$

(i) 1 (ii) 2 (iii) 3 (iv) 4 **Ans.** (i)

13. $\iint f(x, y)\,dx\,dy = J \iint f(r, \theta)\,dr\,d\theta$, where $J =$

(i) r^2 (ii) $\frac{\partial(x, y)}{\partial(r, \theta)}$ (iii) $\frac{\partial(r, \theta)}{\partial(x, y)}$ (iv) r, θ Ans. (ii)

On Changing the order of integration:

14. $\int_0^1 \int_0^x f(x, y)dy\,dx =$ ..... Ans. $\int_0^1 \int_y^1 f(x, y)\,dx\,dy$

15. $\int_0^a \int_0^{\sqrt{a^2 - x^2}} f(x, y)\,dx\,dy =$ ..... Ans. $\int_0^a \int_0^{\sqrt{a^2 - y^2}} f(x, y)\,dx\,dy$

16. $\int_0^\infty \int_0^x \frac{e^{-y}}{y}\,dx\,dy =$ ..... Ans. $\int_0^\infty \frac{e^{-y}}{y}\,dy \int_y^\infty dx$

17. $\int_0^2 \int_1^{e^x} dy\,dx =$ ..... Ans. $\int_1^{e^2} \int_{x = \log y}^2 dx\,dy$

18. The value of $\int_0^e d\theta \int_0^\theta e^{\frac{r}{\theta}}\,dr$ is ..... Ans. $\frac{e^2}{2}(e - 1)$

Indicate Ture/False for the following statements:

19. For $\int_0^\infty \int_x^\infty f(x\,y)\,dx\,dy$, the change of order of integration is

(i) $\int_0^\infty \int_0^\infty f(x\,y)\,dx\,dy$ True/False (ii) $\int_0^\infty \int_0^\infty f(x\,y)\,dx\,dy$ True/False

(iii) $\int_0^\infty \int_0^\infty f(x\,y)\,dx\,dy$, True/False (iv) $\int_0^\infty \int_0^x f(x\,y)\,dx\,dy$ True/False

(U.P.I. Sem., Dec. 2009) **Ans.** (i) False (ii) False (iii) True (iv) False

20. Match the following:

(i) $\iint dx\,dy$ (p) $3\left(\frac{\pi^2}{8} - 1\right)$

(ii) $\int_0^{\frac{\pi}{2}} \int_0^x yx \sin x\,dx\,dy$ (q) $\frac{\lfloor n}{2}$

(iii) $\int_0^{\frac{\pi}{4}} \int_0^1 r \cos^2\theta\,dr\,d\theta$ (r) $\iint r\,d\theta dr$

(iv) $\int_0^1 \int_0^\infty x^{n-1} e^{-x}\,y\,dx\,dy$ (s) $\frac{1}{16}[2 + \pi]$

Ans. (i) → (r), (ii) → (p), (iii) → (s), (iv) → (q)

3. If a circle $x^2 + y^2 = a^2$ is rotated about x-axis, the volume generated is

(i) πa^3 (ii) $2\pi a^2$ (iii) $\frac{4}{3}\pi a^3$ (iv) $\frac{2}{3}\pi a^3$ Ans. (iii)

4. If a circle in positive quadrant is rotated about y–axis is

(i) $\frac{4}{3}\pi a^3$ (ii) $\frac{2}{3}\pi a^3$ (iii) $4\pi a^3$ (iv) πa^3 Ans. (ii)

5. If the area enclosed by $y = x$, $y = 0$ and $x = a$ is revolved about x–axis, the volume generated is

(i) πa^3 (ii) $2\pi a^3$ (iii) $\frac{2\pi a^3}{3}$ (iv) $\frac{\pi}{3}a^3$ Ans. (iv)

6. The volume of the solid generated by revolving the segment of $x + y = 2a$ between the axes about x-axis is

(i) $\frac{8}{3}\pi a^3$ (ii) $8\pi a^3$ (iii) $\frac{1}{3}\pi a^3$ (vi) $\frac{4}{3}\pi a^3$ Ans. (i)

7. An area, being surrounded by a closed curve, revolves about a line. The volume of the solid of revolution shall depend on

(i) the length of the curve

(ii) path described by centre of gravity of the area

(i) $\frac{1}{3}\pi a^3$ (ii) $\frac{2}{3}\pi a^3$

7. An area, being surrounded by a closed curve, revolves about a line. The volume of the solid of revolution shall depend on
   (i) the length of the curve
   (ii) path described by centre of gravity of the area
   (iii) length of the line
   (iv) area enclosed
   Ans. (ii)

8. The area bounded by the circle $r = 4$ is
   (i) 16π (ii) 17π (iii) 18π (vi) 19π **Ans.** (i)

9. The area bounded by the cardioid $r = 2(1 + \cos\theta)$ is
   (i) 16π (ii) 6π (iii) 5π (vi) π **Ans.** (ii)

10. The formula of area in polar co-ordinates is
    (i) $\iint d\theta\, dr$ (ii) $\iint r^2 d\theta\, dr$ (iii) $\iint r d\theta\, dr$ (vi) $\iint \frac{1}{r} d\theta\, dr$
    Ans. (iii)

11. If A is the area under the curve $y = \sin x$ above x-axis in the interval $[0, \pi/4]$, then the area included between $y = \cos x$, and x-axis in the interval $[0, \pi/4]$ is given by
    (i) A (ii) $\pi/2 - A$ (iii) $1 - A$ (iv) None of these.
    Ans. (iii)

12. If A is the area under the curve $y = \sin x$, above x-axis st. $0 \le x \le \pi/2$, then the area under the curve $y = \sin 2x$, $0 \le x \le \pi/2$, is
    (i) A (ii) $2A$ (iii) $A/2$ (iv) $1 + A$ **Ans.** (i)

13. If A is the area under the curve $y = \cos x$, above x-axis, $0 \le x \le \pi/3$, then the area under the curve $y = \cos 2x$ in the same interval is
    (i) A (ii) $2A$ (iii) $A/2$ (iv) $\left(\frac{\sqrt{3}}{2}\right)A$
    Ans. (iii)

14. If A_1 and A_2 are the areas between the x-axis and the curves $y = \sin^n x$ and $y = \cos^n x$ in the interval $[0, \pi/2]$ respectively, then
    (i) $A_2 = 1 - A_1$ (ii) $A_2 = A_1$ (iii) $A_2 = 2A_1$ (iv) None of these.
    Ans. (ii)

15. The area bounded by the rectangular hyperbola $xy = c^2$, the axis of x, and the ordinates $x = c$ and $x = 2c$ is
    (i) $c^2 \log 2$ (ii) $c \log 2$ (iii) $2c \log 2$ (iv) None of these.
    Ans. (i)

16. The area bounded by the curve $x = 3 + \cos\theta$, $y = 4 \sin\theta$, is
    (i) 7π (ii) 2π (iii) 4π (iv) None of these.
    Ans. (ii)

17. The line which divides the area of curvilinear triangle bounded by $y = 2x - x^2$, $y = 0$, $x = 1$, into two equal areas, is
    (i) $y = x$ (ii) $y = x/3$ (iii) $y = 2x/3$ (iv) $y = 2x/5$
    Ans. (iii)

18. The area bounded by the two curves $y = x^2$, $y^2 = x$ is
    (i) $\frac{1}{3}$ (ii) 2/3 (iii) 4/3 (iv) None of these.
    Ans. (i)

19. The area common to the two ellipses $a^2x^2 + b^2y^2 = 1$, $b^2x^2 + a^2y^2 = 1$, where $0 < a < b$ is
    (i) $\frac{4}{ab}\tan^{-1}\frac{a}{b}$ (ii) $\frac{1}{ab}\tan^{-1}\frac{a}{b}$ (iii) $\frac{4}{ab}\tan^{-1}\frac{b}{a}$ (iv) None of these.
    Ans. (i)

20. The area enclosed by the curve $|x| + |y| = 2$ is
    (i) 2 (ii) 4 (iii) 8 (iv) None of these.
    Ans. (iii)

21. The area bounded by the line $y = x$, x-axis and the ordinates $x = -1$ and $x = 2$.
    (i) 2 (ii) 5 (iii) 5/2 (iv) None of these.
    Ans. (iii)

22. The area of a circle centred at (1, 2) and passing through (4, 6) is
    (i) 5π (ii) 10π (iii) 25π (iv) None of these.
    Ans. (iii)

Ans. (*iii*)

21. The area bounded by the line $y = x$, x-axis and the ordinates $x = -1$ and $x = 2$.

(*i*) 2 (*ii*) 5 (*iii*) 5/2 (*iv*) None of these.

Ans. (*iii*)

22. The area of a circle centred at (1, 2) and passing through (4, 6) is

(*i*) 5π (*ii*) 10π (*iii*) 25π (*iv*) None of these.

Ans. (*iii*)

23. The area between the parabola $y^2 = ax$ and its latus rectum is

(*i*) $\frac{a^2}{3}$ (*ii*) $\frac{a^2}{4}$ (*iii*) $\frac{4a^2}{3}$ (*iv*) $\frac{8a^2}{3}$ Ans. (*iii*)

24. The area bounded by the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ is

(*i*) 3π (*ii*) 4π (*iii*) 5π (*iv*) 6π Ans. (*iv*)

25. The area bounded by the circle $x^2 + y^2 = 16$ is

(*i*) 15π (*ii*) 16π (*iii*) 17π (*iv*) 18π Ans. (*ii*

EXERCISE 17.1

Evaluate the following :

1. $\int_1^1 \int_{-2}^{2} \int_{-3}^{3} dx\, dy\, dz$ (*M.U., II Semester 2002*) Ans. 48

2. $\int_0^4 \int_0^x \int_0^{x+y} z\, dz\, dy\, dx$ (*R.G.P.V. Bhopal I Sem. 2003*) Ans. 70

3. $\int_1^2 \int_0^1 \int_{-1}^{1} (x^2 + y^2 + z^2)\, dx\, dy\, dz$ Ans. 6

Ans. $\frac{\pi}{2}a^3$.

OBJECTIVE TYPE QUESTIONS

1. The volume of the integral $\iiint_E xyz\ dx\ dy\ dz$, over the domain E bounded by planes $x = 0$, $y = 0$, $x + y + z = 1$ is

(*i*) $\frac{1}{20}$ (*ii*) $\frac{1}{40}$ (*iii*) $\frac{1}{720}$ (*iv*) $\frac{1}{800}$ Ans. (*iii*)

2. The triple integral $\iiint_T dx\ dy\ dz$ gives

(*i*) Volume of region T (*ii*) Surface area of region T
(*ii*) Area of region T (*iv*) Density of region T (*A.M.I.E.T.E. 2002*)

Ans. (*i*)

3. The volume of the solid under the surface $az = x^2 + y^2$ and whose base R is the circle $x^2 + y^2 = a^2$ is given as

(*i*) $\frac{\pi}{2a}$ (*ii*) $\frac{\pi a^3}{2}$ Ans. (*ii*)

(*iii*) $\frac{4}{3}\pi a^3$ (*iv*) None of the above. [*U.P., I. Sem. Dec. 2008*]

4. The Value of integral $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x + y + z)\,dy\,dx\,dz$ is

(*i*) 2π (*ii*) 2 (*iii*) -2 (*iv*) 0 Ans. (*iv*)

5. The value of $\int_0^1\int_0^1\int_0^1 (x^2 + y^2 + z^2)\,dz\,dy\,dx$ is

(*i*) 1 (*ii*) 1/3 (*iii*) 2/3 (*iv*) 3 Ans. (*i*)

6. The volume of the sphere $r = 2$ is

(*i*) π (*ii*) 32π (*iii*) $\frac{\pi}{3}$ (*iv*) $\frac{32\pi}{3}$ Ans. (*iv*)

7. The volume of the cylinder $x^2 + y^2 = \frac{25}{4}$, $z = 4$ & $z = 0$ is

(*i*) 23 π (*ii*) 24 π (*iii*) 25 π (*iv*) 26 π Ans. (*iii*)

8. The volume of the cylinder $r = 16$, $z = 0$ and $z = 3$ is

(*i*) $\frac{768}{3}\pi$ (*ii*) 768 π (*iii*) 256 π (*iv*) 48 π Ans. (*ii*)

9. A triangle ABC is rotated about x-axis, where A (4, 3), B (0, 0) and C (8, 0). The volume of the solid generated is

(*i*) 6 π (*ii*) 12 π (*iii*) 24 π (*iv*) none of these Ans. (*iii*)

10. The volume of the solid generated by revolving the area bounded by $x = 1$, $x = 2$, $y = 1$, $y = 2$ about x-axis is

(*i*) 3 π (*ii*) 6 π (*iii*) π (*iv*) none of these Ans. (*i*)

11. In spherical coordinates, $dx\ dy\ dz$ is equal to

(*i*) $r\ d\theta\ d\phi\ dr$ (*ii*) $r \sin\theta\ d\theta\ d\phi\ dr$ (*iii*) $r^2 \sin\theta\ d\theta\ d\phi\ dr$ (*iv*) $r^2\ d\theta\ d\phi\ dr$ Ans. (*iii*)

12. The formula for calculating surface area is

(*i*) $S = \iint_A \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 \times \left(\frac{\partial z}{\partial y}\right)^2 + 1}\,dx\,dy$ (*ii*) $S = \iint_A \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 \times \left(\frac{\partial z}{\partial y}\right)^2}\,dx\,dy$

(*iii*) $S = \iint_A \left[\left(\frac{\partial z}{\partial x}\right)^2 \times \left(\frac{\partial z}{\partial y}\right)^2 + 1\right]dx\,dy$ (*iv*) $S = \iint_A \left[\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + \left(\frac{\partial z}{\partial z}\right)^2\right]dx\,dy$ Ans. (*i*)

(i) π (ii) 32π

7. The volume of the cylinder $x^2 + y^2 = \frac{25}{4}$, $z = 4$ & $z = 0$ is
(i) 23 π (ii) 24 π (iii) 25 π (iv) 26 π Ans. (iii)

8. The volume of the cylinder $r = 16$, $z = 0$ and $z = 3$ is
(i) $\frac{768}{3}\pi$ (ii) 768 π (iii) 256 π (iv) 48 π Ans. (ii)

9. A triangle ABC is rotated about x-axis, where A (4, 3), B (0, 0) and C (8, 0). The volume of the solid generated is
(i) 6 π (ii) 12 π (iii) 24 π (iv) none of these Ans. (iii)

10. The volume of the solid generated by revolving the area bounded by
$x = 1$, $x = 2$, $y = 1$, $y = 2$ about x-axis is
(i) 3 π (ii) 6 π (iii) π (iv) none of these Ans. (i)

11. In spherical coordinates, $dx\,dy\,dz$ is equal to
(i) $r\,d\theta\,d\phi\,dr$ (ii) $r\sin\theta\,d\theta\,d\phi\,dr$ (iii) $r^2\sin\theta\,d\theta\,d\phi\,dr$ (iv) $r^2\,d\theta\,d\phi\,dr$ Ans. (iii)

12. The formula for calculating surface area is

(i) $S = \iint_A \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 \times \left(\frac{\partial z}{\partial y}\right)^2 + 1}\,dx\,dy$ (ii) $S = \iint_A \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 \times \left(\frac{\partial z}{\partial y}\right)^2}\,dx\,dy$

(iii) $S = \iint_A \left[\left(\frac{\partial z}{\partial x}\right)^2 \times \left(\frac{\partial z}{\partial y}\right)^2 + 1\right] dx\,dy$ (iv) $S = \iint_A \left[\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + \left(\frac{\partial z}{\partial z}\right)^2\right] dx\,dy$ Ans. (i)

13. The value of $\int_0^1 \int_0^x \int_0^{x+y} dx\,dy\,dz$ is
(i) 1 (ii) $\frac{1}{3}$ (iii) $\frac{1}{4}$ (iv) $\frac{1}{2}$ Ans. (iv)

14. The value of $\int_0^a dx \int_0^{\sqrt{a^2-x^2}} dy \int_0^{\sqrt{a^2-x^2-y^2}} dz$ is
(i) $4\pi a^2$ (ii) $\frac{\pi a^3}{6}$ (iii) $4\pi a^3$ (iv) $\frac{\pi}{3}a^2$ Ans. (ii)

15. The surface of the solid generated by revolving the area enclosed by curve $x^2 + y^2 = 16$ about $x = 4$ is
(i) 64 π (ii) 32 π^2 (iii) 32 π (iv) 64 π^2 Ans. (iv)

16. The value of intergral $\int_0^2 \int_1^3 \int_1^2 xy^2 z\,dx\,dy\,dz$ is equal to
(i) 22 (ii) 26 (iii) 5 (iv) 25 Ans. (ii)

Fill up the blanks:

17. $\int_0^{\pi} \int_0^{2\pi} \int_0^1 r^2 (r^2 \sin\theta\,d\theta\,d\phi\,dr) =$ ..... Ans. $\frac{4\pi}{5}$

18. In spherical coordinates $dx\,dy\,dz$ = ... Ans. $r^2\sin\theta\,dr\,d\theta\,d\phi$

19. $\int_{-1}^1 \int_{-2}^2 \int_{-3}^3 dx\,dy\,dz =$ .... Ans. 48

20. The formula for the volume in spherical coordinates is ..... Ans. $\iiint_V r^2 \sin\theta\,dr\,d\theta\,d\phi$

21. $\int_0^a \int_0^{\frac{bx}{a}} \int_0^{c+xy} dz\,dy\,dx =$ ..... Ans. $\frac{ab}{8}(4c + ab)$