DOUBLE INTEGRALS

1. $\int_1^2 \int_1^3 xy^2 \, dxdy$.

2. $\int_0^1 \int_x^{\sqrt{x}} (x^2 + y^2) \, dx \, dy$.

Example 7.1. *Evaluate* $\int_0^5 \int_0^{x^2} x(x^2 + y^2) \, dxdy$.

Ex. Find the area between $y = x^2$ and $y = x$

$\iint xy(x+y)\,dxdy$ over the area between $y = x^2$ and $y = x$.

1. The value of the integral $\iint xy\,(x+y)\,dx\,dy$ over the area between $y = x^2$ and $y = x$ is

(i) $\frac{3}{56}$ (ii) $\frac{47}{56}$ (iii) $\frac{33}{56}$ (iv) $\frac{23}{56}$

2. The integral $\iint\limits_{x^2+y^2 \le 1} \frac{1}{\pi}(x^2+y^2)\,dx\,dy$ equals

(i) 0 (ii) 1 (iii) [illegible] (iv) [illegible]

$x^2 + y^2 \le 1$

(i) 0 (ii) 1 (iii) 1/3 (iv) 1/2

4. Value of the integral $\int_{-a}^{a} \int_0^{\sqrt{a^2 - x^2}} dx\, dy$ is equal to

(i) $4a$ (ii) $2a$ (iii) 0 (iv) None of these

Ans.

5. The value of $\int_1^0 \int_0^1 (x+y)\, dx\, dy$ is equal to

(i) 1 (ii) −1 (iii) 2 (iv) 0 An

TRIPLE INTEGRALS

$$I = \boxed{\int_{x_1}^{x_2} \boxed{\int_{y_1(x)}^{y_2(x)} \boxed{\int_{z_1(x,y)}^{z_2(x,y)} f(x, y, z) dz} dy} dx}$$

1. $\displaystyle\int_0^a \int_0^b \int_0^c (x^2 + y^2 + z^2)\, dx\, dy\, dz.$ (Anna, 2009)

2. $\displaystyle\int_{-c}^{c} \int_{-b}^{b} \int_{-a}^{a} (x^2 + y^2 + z^2)\, dx\, dy\, dz$

(S.V.T.U., 2009 ; V.T.U.

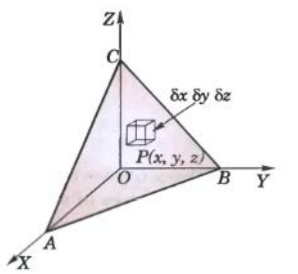
$\displaystyle\int_0^1 \int_{y^2}^1 \int_0^{1-x} x\, dz\, dx\, dy$

4. $\displaystyle\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\, dz\, dy\, dx.$

$\displaystyle\int_0^{\log 2} \int_0^x \int_0^{x+\log y} e^{x+y+z}\, dx\, dy\, dz.$

6. $\displaystyle\int_1^e \int_1^{\log y} \int_1^{e^x} \log z\, dz\, dx\, dy.$

Example 7.20. Calculate the volume of the solid bounded by the planes $x = 0, y = 0, x + y + z = a$ and $z = 0$.



Cylindrical Coordinates

The conversions for x and y are the same conversions that we used back when we were looking at polar coordinates. So, if we have a point in cylindrical coordinates the Cartesian coordinates can be found by using the following conversions.

$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$z = z$$

Evaluate $\iiint\limits_E z\,dV$ where E is the region between the two planes $x + y + z = 2$ and $x = 0$ and inside the cylinder $y^2 + z^2 = 1$.

Example 1 Evaluate $\iiint\limits_E y\,dV$ where E is the region that lies below the plane $z = x + 2$ above the xy-plane and between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

Example 2 Convert $\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}}\int_{x^2+y^2}^{\sqrt{x^2+y^2}} xyz\,dz\,dx\,dy$ into an integral in cylindrical coordinates.

4. Use a triple integral to determine the volume of the region below $z = 6 - x$, above $z = -\sqrt{4x^2 + 4y^2}$ inside the cylinder $x^2 + y^2 = 3$ with $x \le 0$.

Spherical Coordinates

$$x = \rho \sin\varphi \cos\theta$$
$$y = \rho \sin\varphi \sin\theta$$
$$z = \rho \cos\varphi$$

Evaluate $\iiint\limits_E 16z\,dV$ where E is the upper half of the sphere $x^2 + y^2 + z^2 = 1$.

12. Find the value of integral $\int_0^1 \int_{x^2}^{x} xy(x+y)dydx$.

a) $3/15$

b) $2/15$

c) $2/30$

d) $1/15$

$$\int_0^2 \int_0^1 4x^2y \, dy \, dx$$

a. 14/3

b. 15/3

c. 16/3

d. 4

$$\int_0^1 \int_{3x^4}^{5\sqrt{x}} dy \, dx$$

a. 32/15

b. 37/15

c. 39/15

d. 41/15

Change the order of integration $\int_0^{\infty}\int_x^{\infty} \frac{e^{-y}}{y} dxdy$

$\int\int_D 10xy\,dA$, where D is the portion between the circles of radius 4 and 10 and lies in first quadrant.

Possible Answers:

1000

2000

6

1218

2) Evaluate : $\int_1^2 \int_0^x (x + 2y)\,dy\,dx$

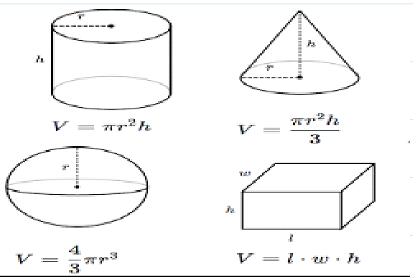
3) Evaluate : $\int_1^2 \int_0^1 (x^2 + y^2)\,dx\,dy$

4) Evaluate : $\int_0^1 \int_0^{x^2} e^{\frac{y}{x}}\,dx\,dy$

5) Evaluate : $\int_1^2 \int_0^x \frac{dx\,dy}{x^2 + y^2}$

6) Evaluate : : $\int_0^a \int_0^b (x^2 + y^2)\,dx\,dy$

7)Evaluate : $\int_0^4 \int_0^{\sqrt{y}} xy\,dx\,dy$

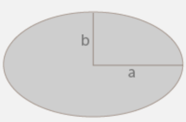


Area A $= \pi$ ab

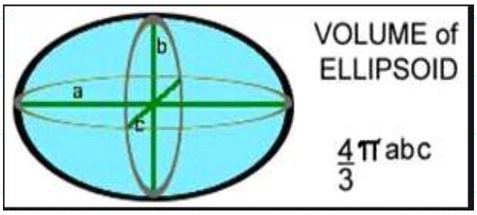
Perimeter P $= 2\pi\sqrt{\frac{a^2 + b^2}{2}}$

a ⟶ Major axis length

b ⟶ Minor axis length



Ellipse



1) The area of region bounded by the circle $x^2+y^2=a^2$ is

a) $\Pi^2 a$ unit   b) Πa^2 unit^2

c) Πa^3 unit^3 d) Πa unit

2) The area of the ellipse $\frac{x^2}{16}+\frac{x^2}{9}=1$ is

a) 12Π unit^2 b) 12 unit

c) 12Π unit^3 d) 12Π

3) The volume of the sphere $x^2+y^2+z^2=a^2$ is

a) Πa^2 unit^2 b) Πa^3 unit

c) $\frac{4}{3}\Pi a^3$ unit^3 d) Πa unit

4) The Volume of ellipsoid $\frac{x^2}{1}+\frac{y^2}{4}+\frac{z^2}{9}=1$ is

a) $\frac{4}{3}\Pi$ unit   b) $\frac{4}{3}\Pi^2$ unit^2

c) $\frac{24}{3}\Pi$ unit^3 d) $\frac{12}{3}\Pi$ unit^3

1. The value of $\int_0^1 \int_0^x \int_0^{x+y} xyz\, dz\, dy\, dx$ is given by ____

a) 17/144
b) 16/72
c) 17/72
d) 15/144

2. The integral value of $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\, dz\, dy\, dx$ is given by ____

a) $= \frac{1}{3}(e^{4a}+6e^{2a}+8e^{a}+3)$
b) $= \frac{1}{3}(e^{4a}-6e^{2a}+4e^{a}+3)$
c) $= \frac{1}{8}(e^{4a}-6e^{2a}+8e^{a}-3)$
d) 0

3. The integral value of $\int_0^{\frac{\pi}{2}} \int_0^{a\sin\theta} \int_0^{r} r\, dr\, d\theta\, dz$ is ____

a) 0.5
b) 0.25
c) 1
d) 0

5. The integral of $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x+y+z)\, dy\, dx\, dz$ is given by ______

a) 0
b) 1
c) 0.25
d) 4

4. The integral value of $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{dzdydx}{(1+x+y+z)^3}$ is given by____

a) $log\sqrt{2}-\frac{7}{16}$
b) $log\sqrt{4}+\frac{5}{32}$
c) $log\sqrt{2}-\frac{5}{16}$
d) $log\sqrt{4}-\frac{6}{32}$

1. Find the value of $\int_0^1 \int_0^2 \int_1^2 xy^2z^3 dxdydz$..

a) 2
b) 3
c) 4
d) 5

EXERCISE 2.5

Use double integration in the following questions:

1. Find the area bounded by $y = x - 2$ and $y^2 = 2x + 4$. **Ans.** 18.

2. Find the area between the circle $x^2 + y^2 = a^2$ and the line $x + y = a$ in the first quadrant. **Ans.** $(\pi - 2)a^2/4$

3. Find the area of a plate in the form of quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. **Ans.** $\frac{\pi ab}{4}$

4. Find the area included between the curves $y^2 = 4a(x + a)$ and $y^2 = 4b(b - x)$. **Ans.** $\frac{8\sqrt{ab}}{3}$

(A.M.I.E.T.E., Summer 2001)

5. Find the area bounded by (*a*) $y^2 = 4 - x$ and $y^2 = x$. **Ans.** $\frac{16\sqrt{2}}{3}$

(*b*) $x - 2y + 4 = 0$, $x + y - 5 = 0$, $y = 0$ *(A.M.I.E., Winter 2001)* **Ans.** $\frac{27}{2}$

6. Find the area enclosed by the leminscate $r^2 = a^2 \cos 2\theta$. **Ans.** a^2

7. Find the area common to the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = 2ax$. **Ans.** $\left[\frac{\pi}{3} - \frac{\sqrt{3}}{4}\right]a^2$

8. Find the area included between the curves $y = x^2 - 6x + 3$ and $y = 2x + 9$.

(A.M.I.E., Summer 2001) **Ans.** $\frac{88\sqrt{22}}{3}$

9. Determine the area of region bounded by the curves $xy = 2$, $4y = x^2$, $y = 4$. **Ans.** $\frac{28}{3} - 4\log 2$

(U.P. I Semester 2003)