

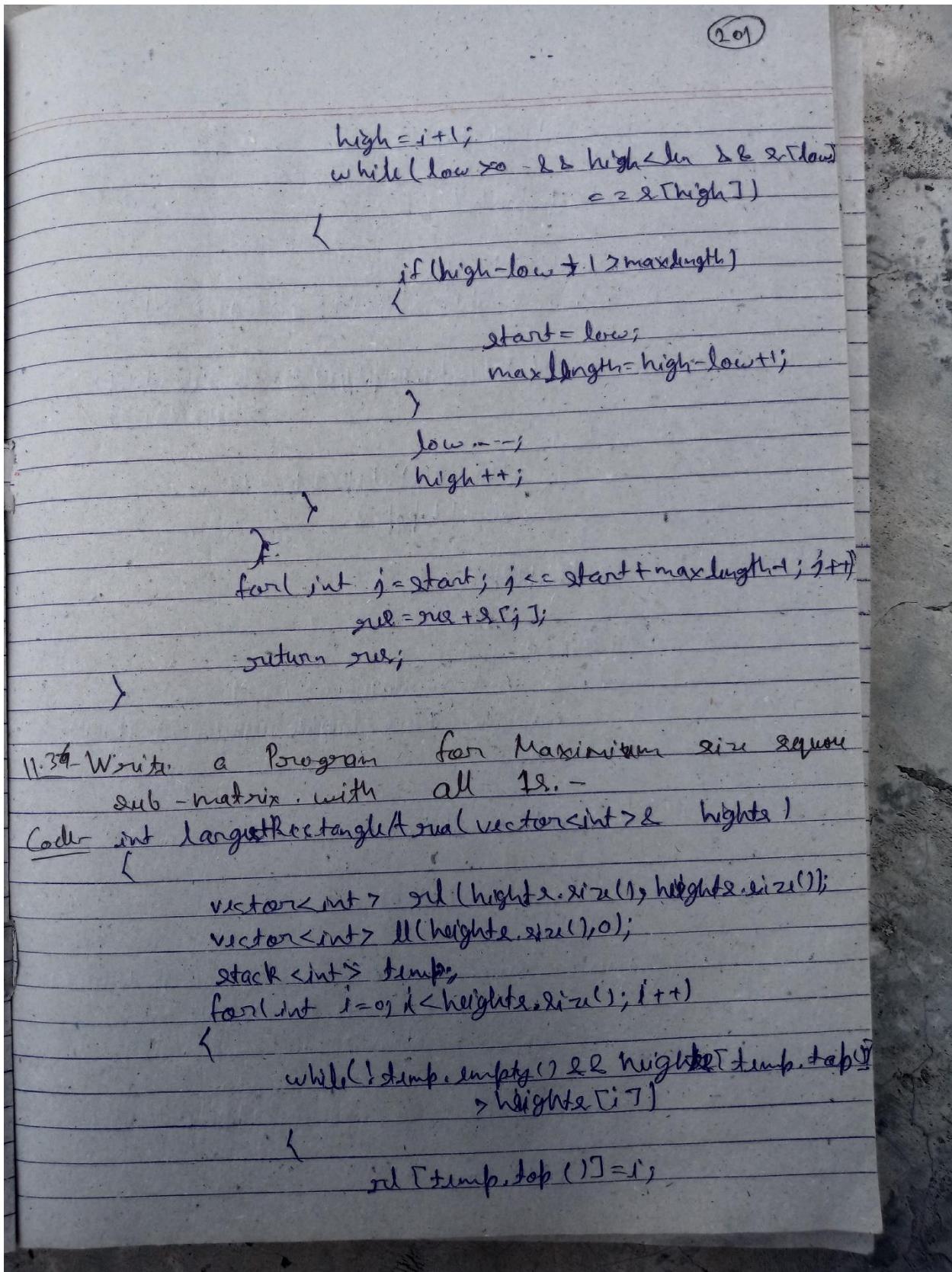
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Problem Solving

Applied Prep Course



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PART-3



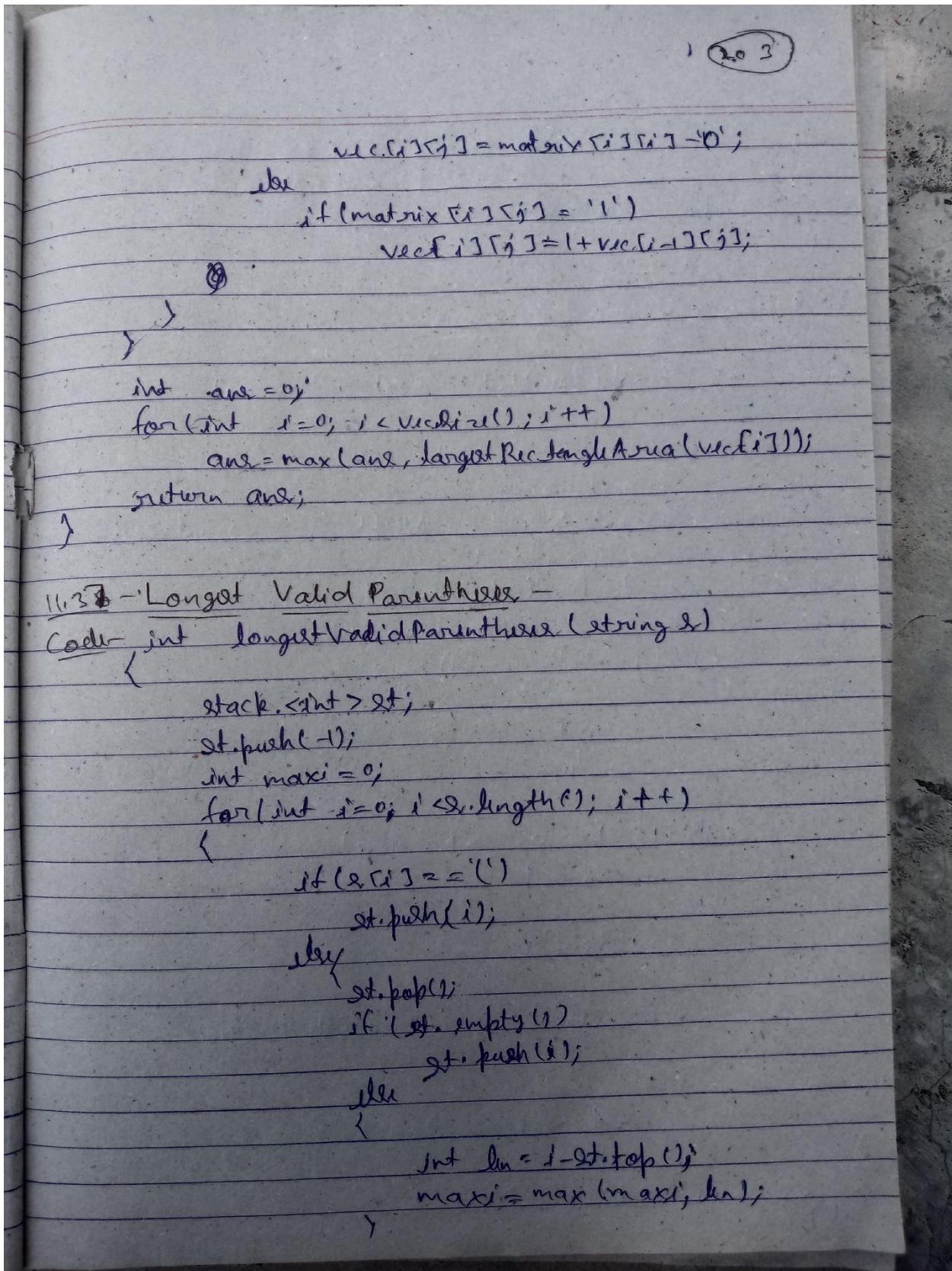
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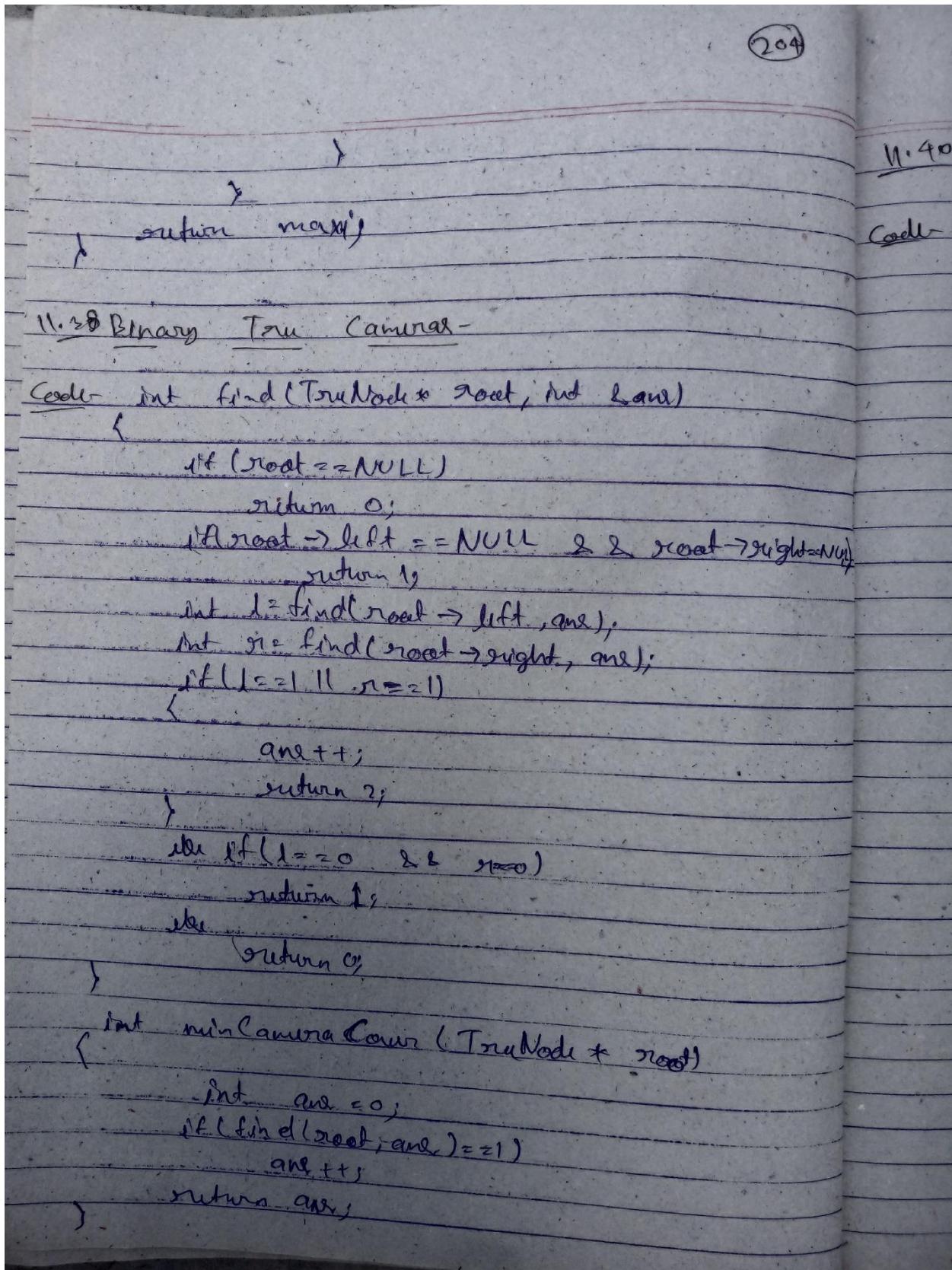
```

temp.pop();
}
temp.push(j);
}
stack<int> temp1;
for (int i = heights.size() - 1; i >= 0; i--) {
    while (!temp1.empty() && heights[temp1.top()] > heights[i])
        temp1.pop();
    if (temp1.top() == i + 1)
        temp1.pop();
    temp1.push(i);
}
int ans = 0;
for (int i = 0; i < n; i++) {
    ans = max(ans, heights[i] * (n - i - 1));
}
return ans;
}

int maximumRectangle(vector<vector<char>> &matrix) {
    if (matrix[0].size() == 0)
        return 0;
    vector<vector<int>> vec(matrix.size(), vector<int>(
        (matrix[0].size(), 0)));
    for (int i = 0; i < matrix.size(); i++)
        for (int j = 0; j < matrix[0].size(); j++)
            if (matrix[i][j] == 'A')
                vec[i][j] = 1;
    for (int j = 0; j < matrix[0].size(); j++)
        for (int i = 0; i < matrix.size(); i++) {
            if (vec[i][j] == 0)
                continue;
            if (i > 0 && vec[i - 1][j] == 1)
                vec[i][j] = vec[i - 1][j] + 1;
        }
    int ans = 0;
    for (int i = 0; i < matrix.size(); i++) {
        stack<int> temp;
        for (int j = 0; j < matrix[0].size(); j++) {
            while (!temp.empty() && vec[i][temp.top()] < vec[i][j])
                temp.pop();
            if (temp.empty())
                temp.push(j);
            else
                ans = max(ans, vec[i][temp.top()] * (j - temp.top()));
        }
    }
    return ans;
}

```





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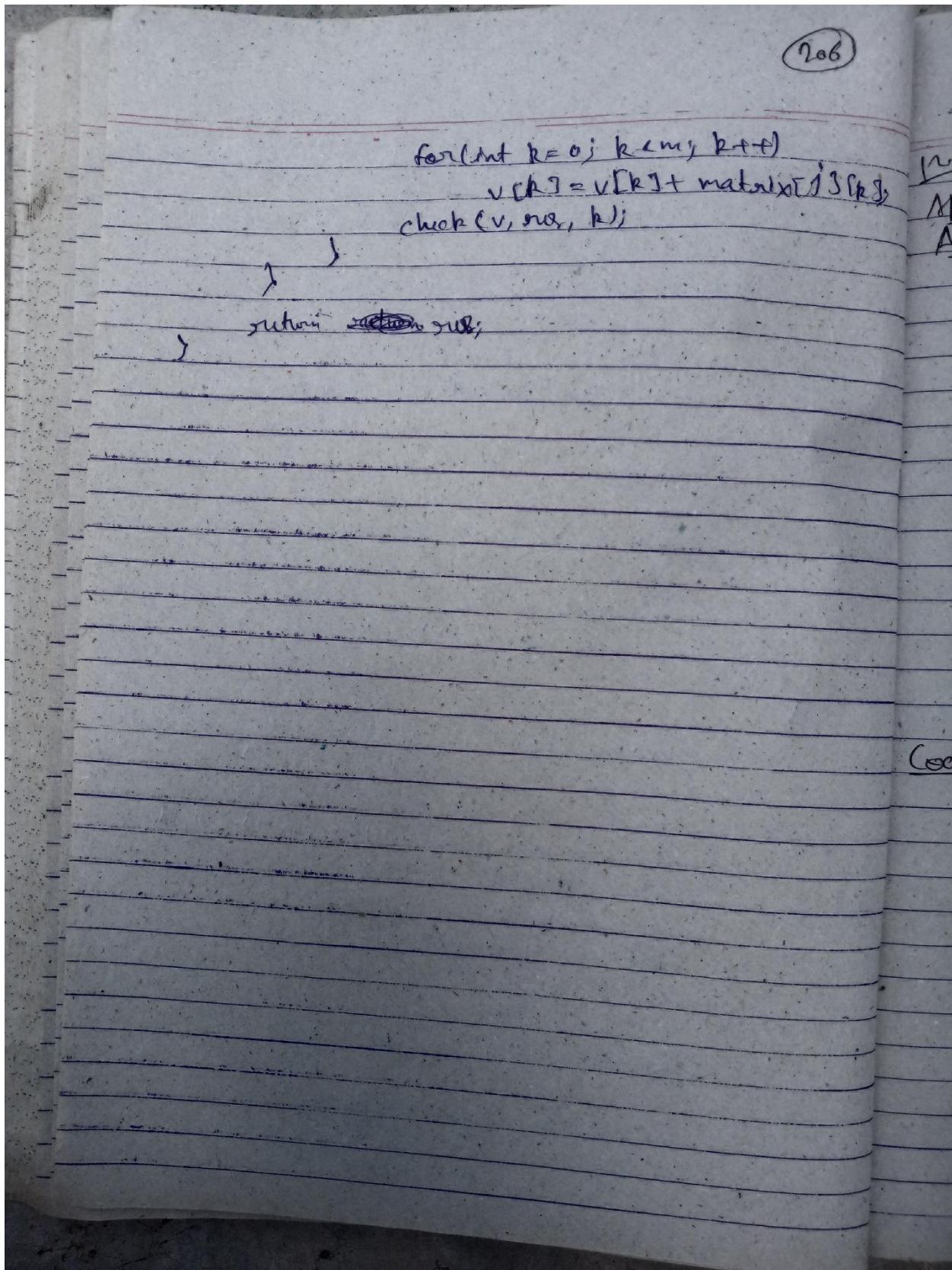
II.40. Max Sum Of Rectangle No Larger Than k -

```

    void check(vector<int> v, int &p, int k)
    {
        for(int i=0; i<v.size(); i++)
        {
            int s=v[i];
            for(int j=i+1; j<v.size(); j++)
            {
                if(s <= k)
                    p = max(p, s);
                s = s + v[j];
            }
            if(s <= k)
                p = max(p, s);
        }
        return p;
    }

    int maxSumSubmatrix(vector<vector<int>> matrix, int k)
    {
        int n=matrix[0].size(), m=INT_MIN;
        if(n==0)
            return 0;
        int m=matrix[0].size();
        if(m==0)
            return 0;
        for(int i=0; i<n; i++)
        {
            vector<int> v(m, 0);
            for(int j=i; j<n; j++)
            {

```



Problems on Backtracking

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1. Write a Program for N Queen Problem.

Approach Use Backtracking

Algorithm Go column by column from left to right.

② If n Queens are placed (or) all columns return.

③ Try placing a Queen in each row of the current column \rightarrow recursively

\hookrightarrow If you hit a deadend \rightarrow invalid node
 \rightarrow Parent node with all children invalid

\hookrightarrow Simple back track.

④ In case you are not able to place n queens in an $n \times n$ grid after trying out all possibilities return "Impossible".

Code vector<string> a;

vector<vector<string>> ans;

bool isvalid(int n, int row, int col)

{

 int total = 0;

 // row wise check

 for (int i = 0; i < n; i++)

 if (a[row][i] == 'Q')

 total++;

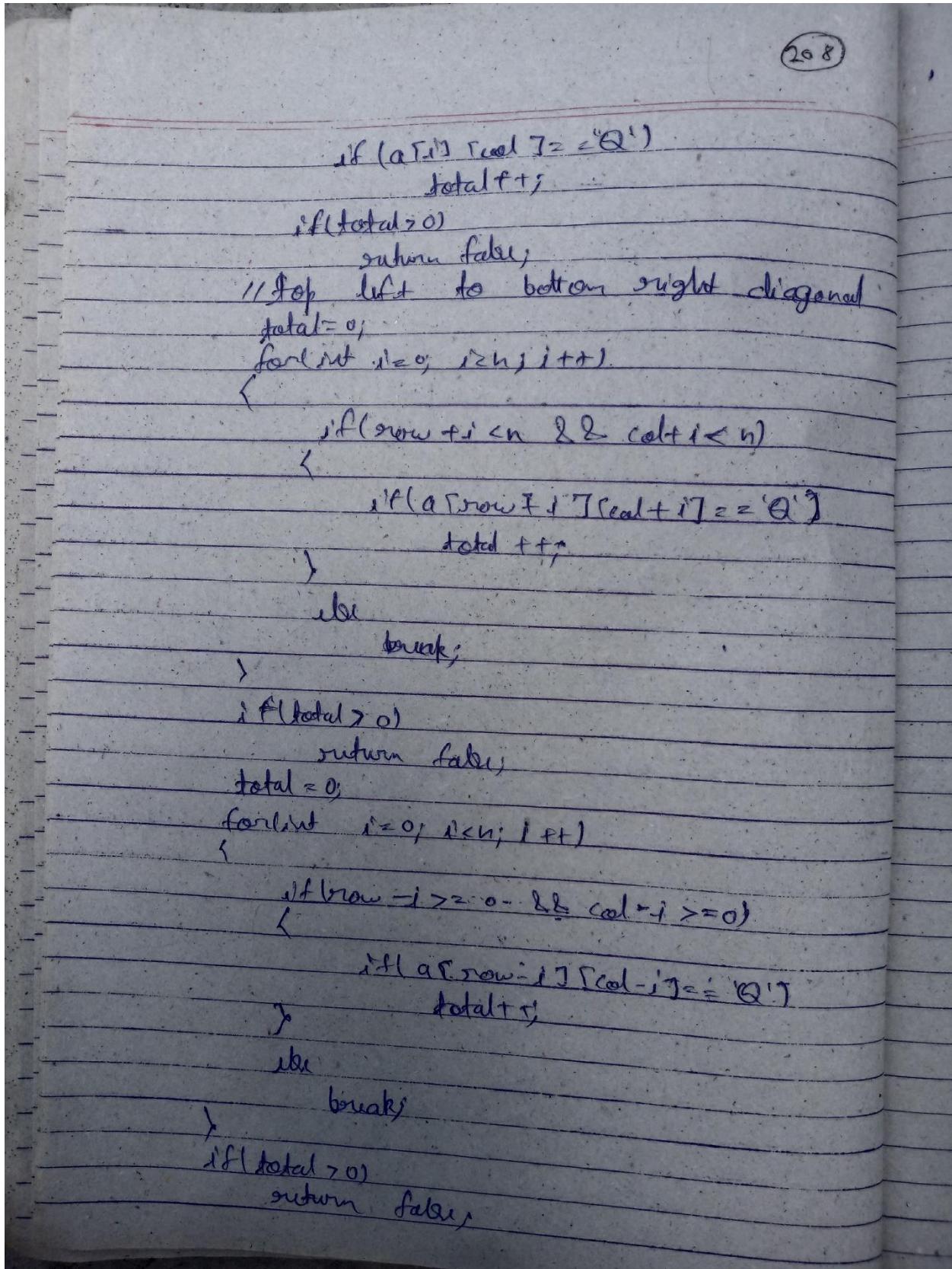
 if (total > 0)

 return false;

 // Column wise check

 total = 0;

 for (int i = 0; i < n; i++)

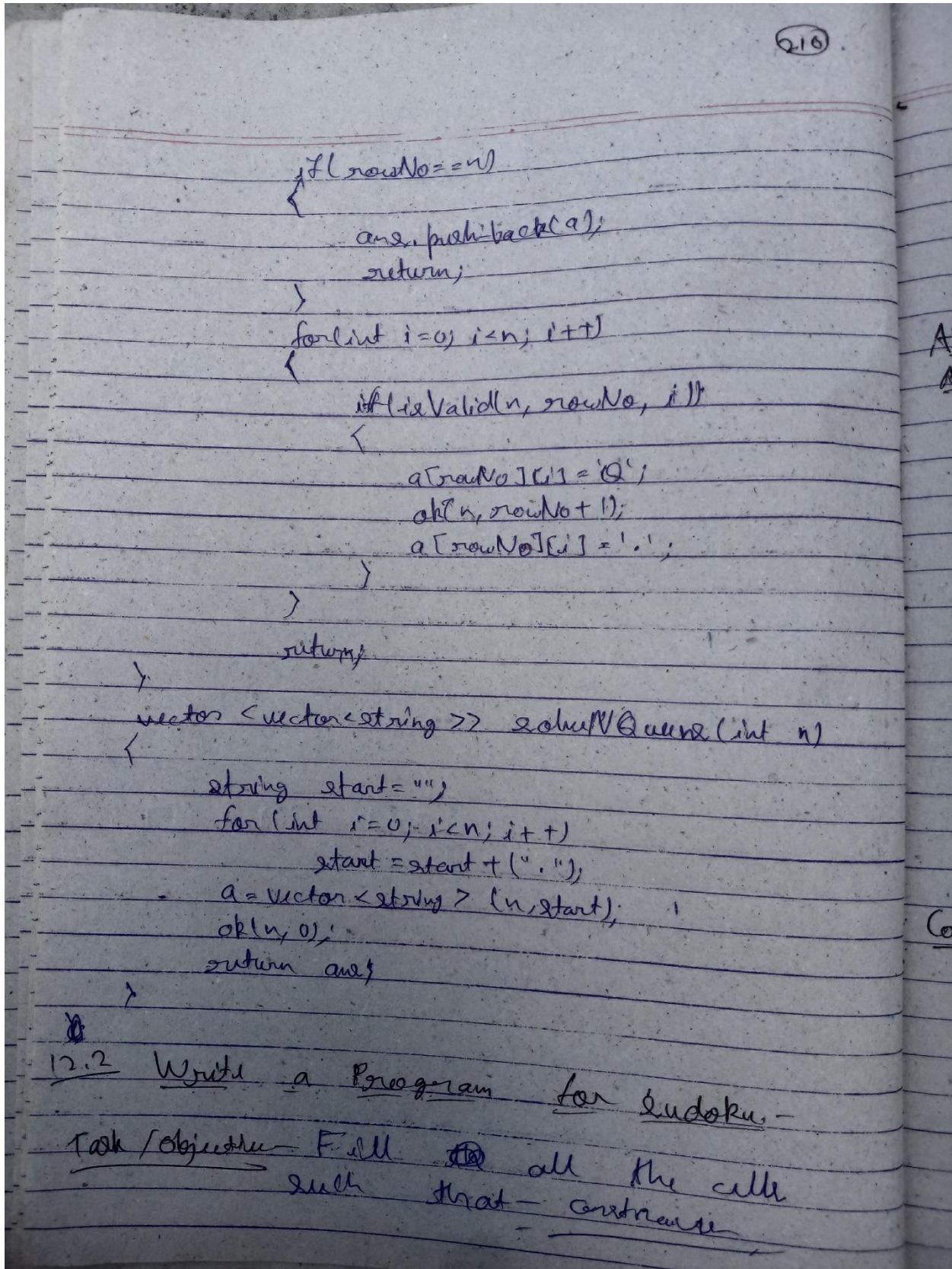


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```

// from top right to bottom left
total = 0;
for (int i=0; i<n; i++) {
    if (row+i < n && col-i >= 0) {
        if (a[row+i][col-i] == 'Q') {
            total++;
        } else {
            break;
        }
    }
    if (total > 0)
        return false;
    total = 0;
    for (int i=0; i<n; i++) {
        if (row-i >= 0 && col+i < n) {
            if (a[row-i][col+i] == 'Q') {
                total++;
            } else {
                break;
            }
        }
        if (total > 0)
            return false;
        return true;
    }
}

```



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- (a) Each row should contain 1 ton without repetition.
- (b) Each col should contain 1 ton without repetition.
- (c) Each ~~subgrid~~ should contain 1 ton without repetition.

Approach - Use Backtracking

- ① Make a list of all empty cells.
- ② Select an empty cell & place possible values from 1 ton one after another.
- ③ Recursively expand as long as you don't hit a dead-end.
 - ↳ Node is invalid.
 - ↳ Parent node where all children are invalid
- ④ Backtrack whenever you hit a dead-end & go back to the parent node & continue
- ⑤ Repeat 2-4 till all the cells are filled with valid values.

Code -

```
#include <iostream>
using namespace std;
#define UNASSIGNED 0
#define N 9
bool FindUnassignedLocation (int grid[N][N],
                             int& row, int& col);
bool IsSafe (int grid[N][N], int row, int col, int
             num);
```

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```

int row, col)
if (!FindUnassignedLocation(grid, row, col))
    return true;
for (int num = 1; num <= 9; num++)
{
    if (isValid(grid, row, col, num))
        grid[row][col] = num;
    if (SolveSudoku(grid))
        return true;
    grid[row][col] = UNASSIGNED;
}
return false;
}

bool FindUnassignedLocation(int grid[N][N], int
    row, int col)
{
    for (row = 0; row < N; row++)
        for (col = 0; col < N; col++)
            if (grid[row][col] == UNASSIGNED)
                return true;
    return false;
}

bool UsedInRow(int grid[N][N], int row, int num)
{
    for (int col = 0; col < N; col++)
        if (grid[row][col] == num)
            return true;
    return false;
}

```

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```

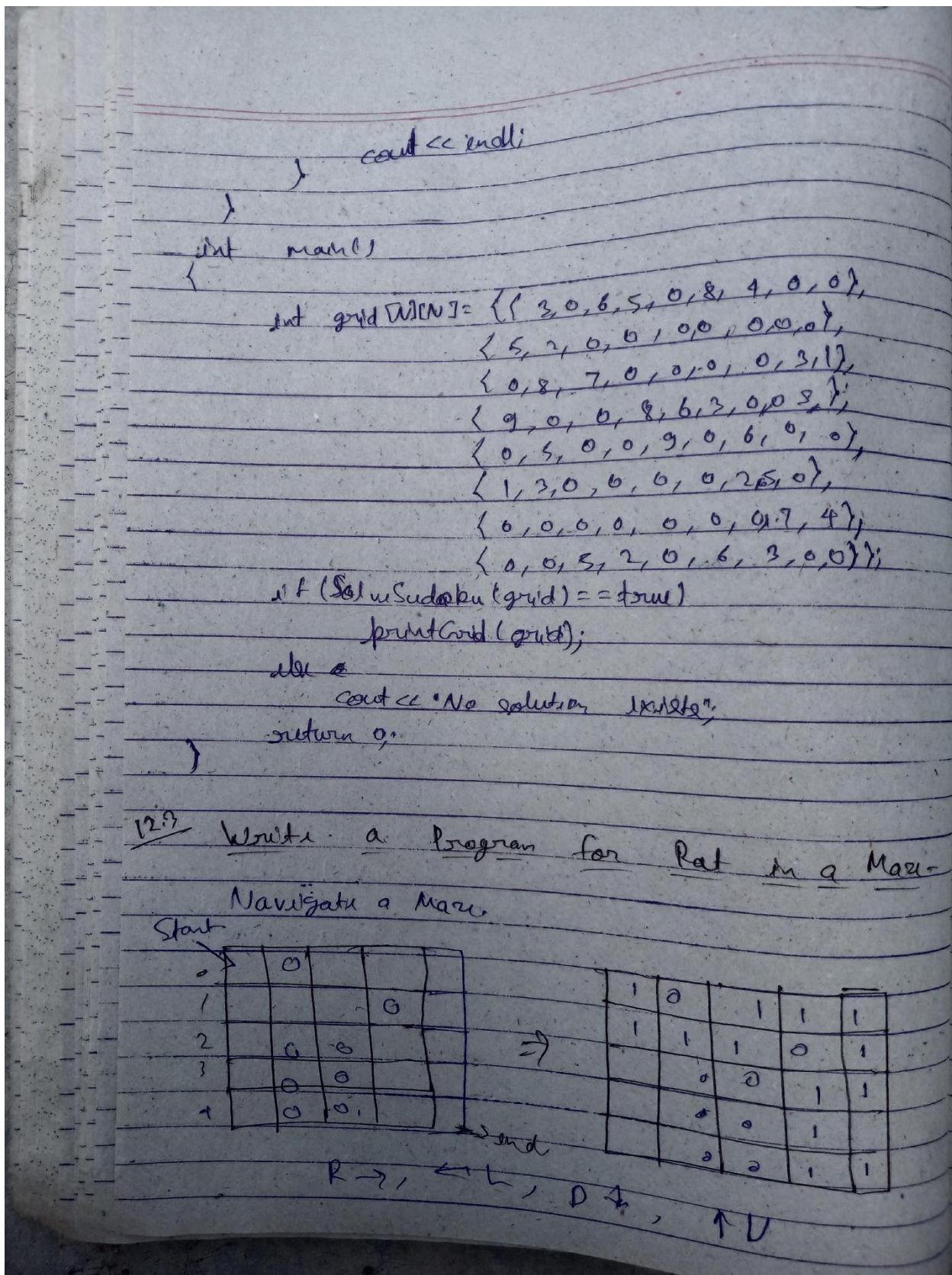
bool UsedInCol(int grid[N][N], int col, int num)
{
    for (int row = 0; row < N; row++)
        if (grid[row][col] == num)
            return true;
    return false;
}

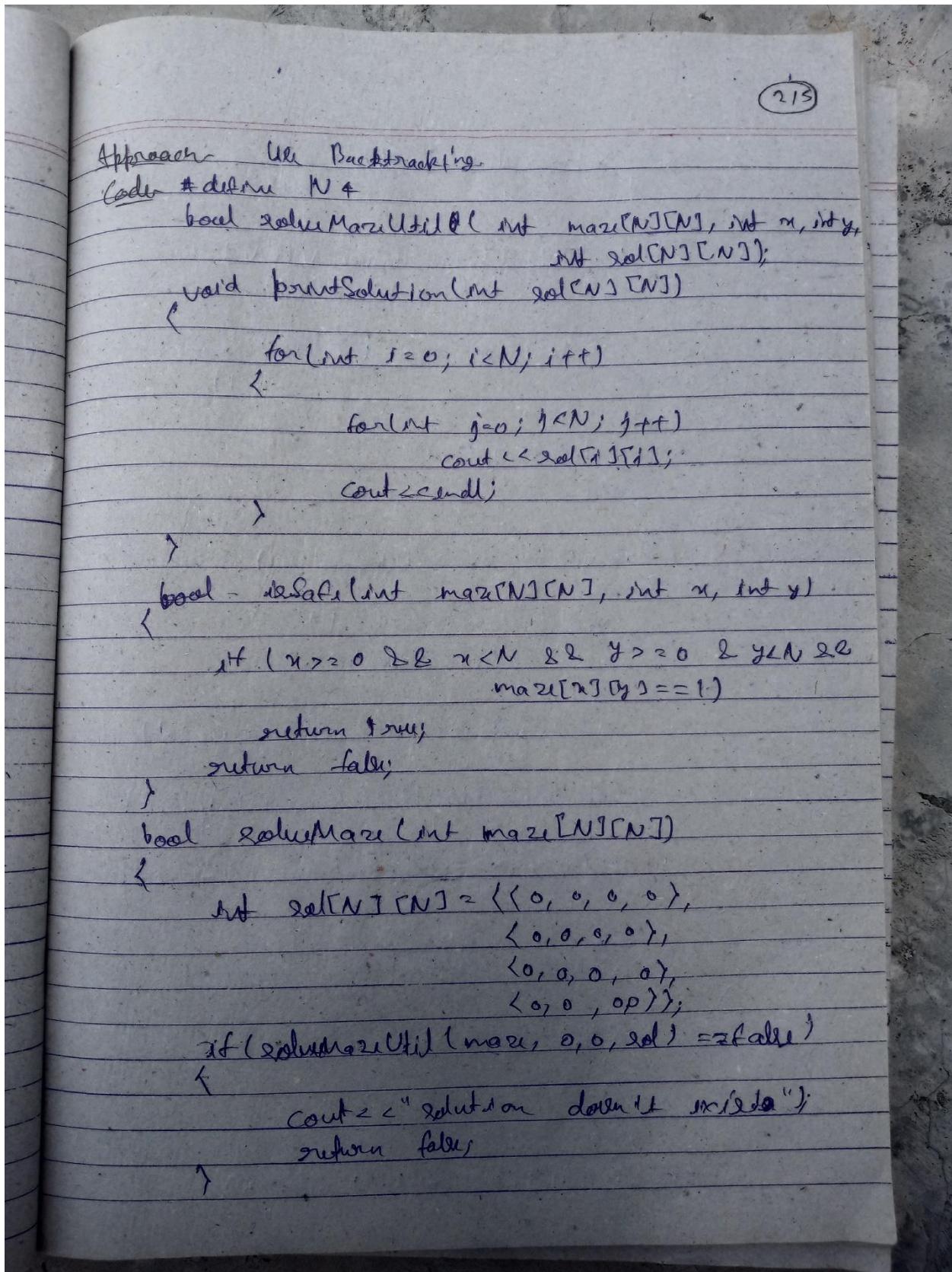
bool UsedInBox (int grid[N][N], int boxStartRow,
                int StartCol, int num)
{
    for (int row = 0; row < 3; row++)
        for (int col = 0; col < 3; col++)
            if (grid [row + boxStartRow][col + StartCol] == num)
                return true;
    return false;
}

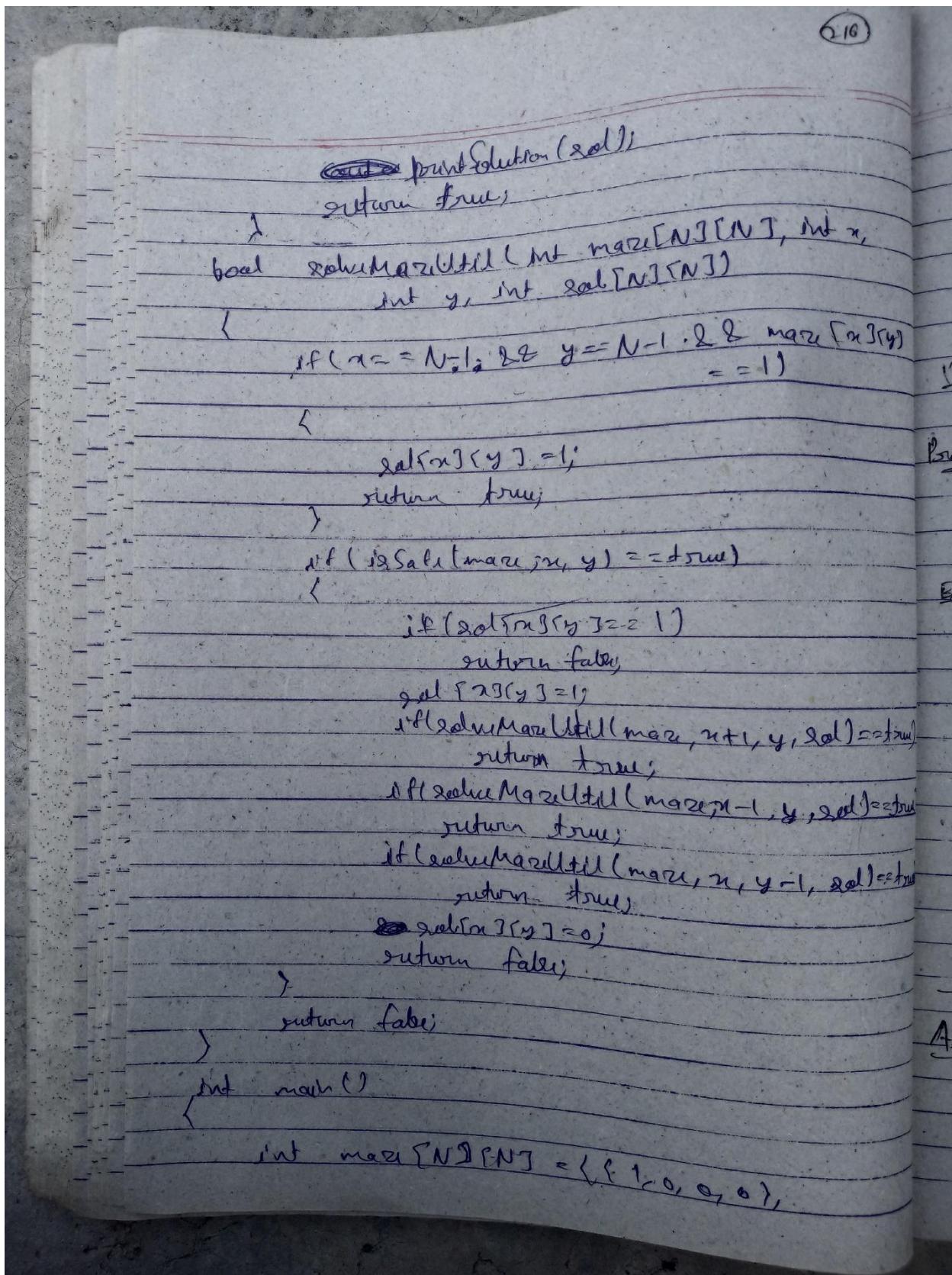
bool IsSafe (int grid[N][N], int row, int col,
             int num)
{
    return !UsedInRow (grid, row, num) & &
           !UsedInCol (grid, col, num) & &
           !UsedInBox (grid, row - row % 3, col - col % 3,
                       num) & & grid [row][col] != UNASSIGNED;
}

void printGrid (int grid[N][N])
{
    for (int row = 0; row < N; row++)
    {
        for (int col = 0; col < N; col++)
            cout << grid [row][col] << " ";
    }
}

```







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```

    {1, 1, 0, 1},
    {0, 1, 0, 0},
    {1, 1, 1, 1};
}

solveMaze(maze);
return 0;
}

```

Q2.4 - Letter Combinations of a phone number-

Problem Statement - Given a string containing digits from 2-9 inclusive, return all possible letter combinations that the number could represent.

Ex. I/P - "23" O/P - ["ad", "ae", "af", "bd", "be", "bf", "cd", "ce", "cf"].

Approach - Use Backtracking - $T(N) = \Theta(3^N 4^N) \approx \Theta(3^N \times 4^N)$

Algo - ① If there are no more digits to check, that means the current combination is completed.
 ② If there are still digits to check.

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- ~~(i) think~~
- (i) Iteration over the letter mapping.
 - (ii) the next available digit.
 - (iii) Append the current letter to current combination (combination + current letter).
 - (iv) Proceed to check next digit (combination, next digit & $i+1$).

Code - vector<string> letterCombinations(string digits)

vector<string> comb;

vector<string> char;

if (digit.size() == 0)

return comb;

char = getChar(digit[0]);

for (int i = 1; i < digit.size(); i++)

vector<string> temp = comb;

char = getChar(digit[i]);

comb.clear();

for (int j = 0; j < temp.size(); j++)

for (int k = 0; k < char.size(); k++)

comb.push_back(temp[j] + char[k]);

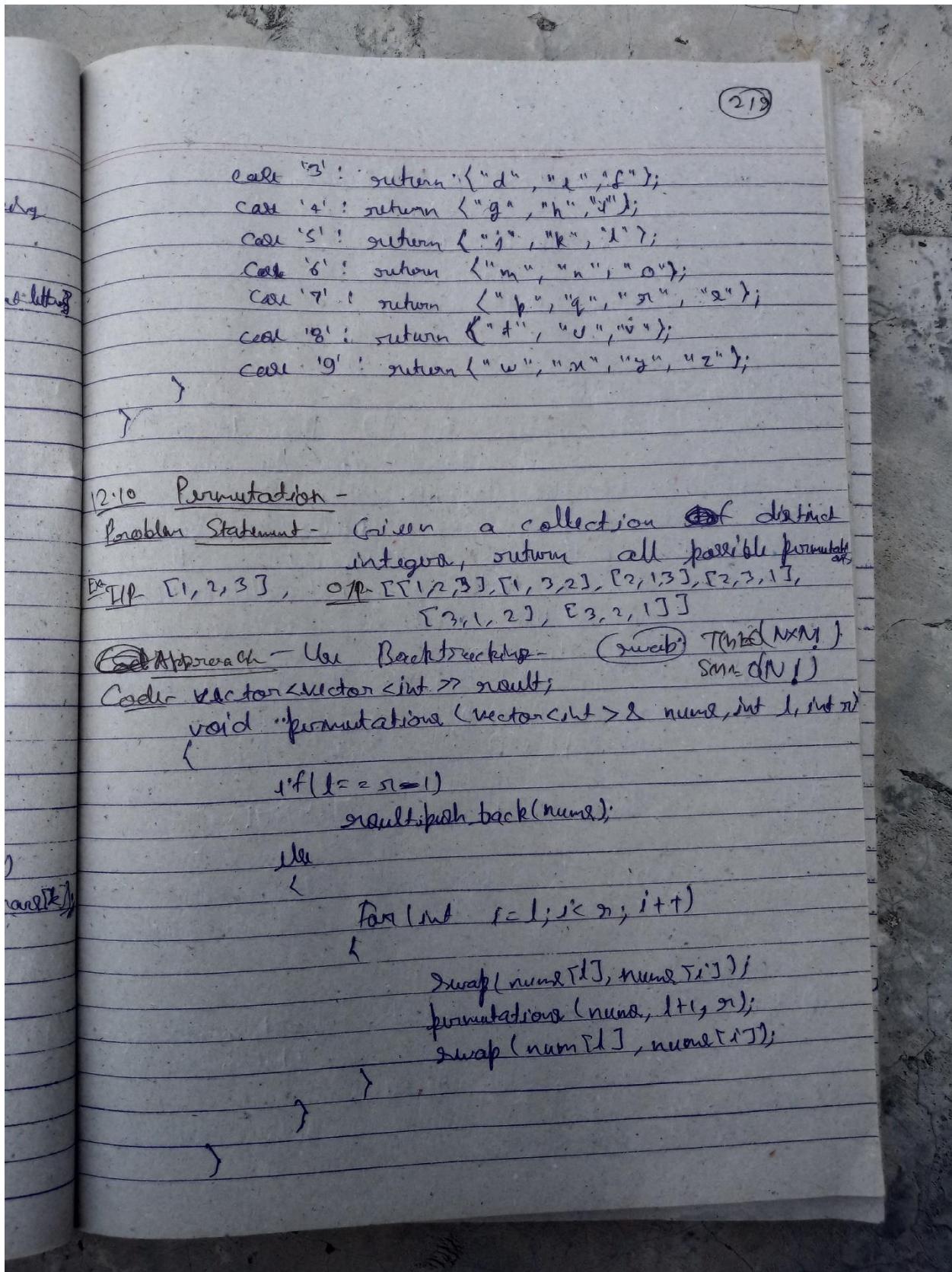
return comb;

vector<string> getChar(char num)

{

switch (num)

case '2': return {"a", "b", "c"};



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```

vector<vector<int>> permute(vector<int> &num)
{
    int n = num.size();
    permutations(&num, 0, n);
    return result;
}

```

12.11 Word Search -

Problem Statement - Given a 2D Board and a word, find if the word exists in the grid. The word can be constructed from letters of sequentially adjacent cell, where "adjacent" cells are those horizontally or vertically neighboring. The same letter cell may not be used more than once.

Ex board = [["A", "B", "C", "E"], ["S", "F", "C", "S"], ["A", "D", "E"]]
 Given word = "ABCCED"
 " " = "ABED" return true
 " " = "ABED" return false

Approach (or) Backtracking $T(n) = O(L \times 4^n), S(n) = O(n)$

Algorithm

Loop through each cell in the grid.
 For each cell we invoke backtracking function

Backtracking

- ① Base Condition - We found exact match of the word. i.e. $L(w) = L$
- ② Check the current status, before jumping into backtracking.

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- ④ If we found current char match then we will mark as visited.
 ⑤ Explore the 4 directional neighbors (horizontally or vertically).

Code book solve (vector<vector<char>>& board, int n, int c, int R, int C, start word, int idx)

```

if (word.size() == idx)
    return true;
if ((r < 0) || r >= R || c < 0 || c >= C || board[r][c] == '$' || board[r][c] != word[idx])
    return false;
char ch = board[r][c];
board[r][c] = '$';
int ans = false;
if (solve (board, n+1, c, R, C, word, idx+1))
    ans = true;
board[r][c] = ch;
return ans;
  
```

bool exist (vector<vector<char>>& board, string word)

```

int ROW = board.size(), COL = board[0].size();
if (ROW * COL < word.size())
    return 0;
for (int i = 0; i < ROW; i++)
    for (int j = 0; j < COL; j++)
        if (board[i][j] == word[0])
            if (solve (board, i, j, ROW, COL,
  
```

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return true;

return false;

12.15 Generate Parenthesis -

Problem Statement: Given n pairs of parentheses, write a function to generate all combinations of well-formed parentheses.

Ex- I/P - n = 3
O/P - $\{ "((()))", "((())", "(())()", "(()())", "()(())", "()((())" \}$

Approach: Use Backtracking -

Algo -

- ① If there is nothing on stack, the string is fully completed and now we can add into the resultant list.
- ② If there is open parenthesis available on stack try add it on.
 → Now you have one less open parenthesis but more close parenthesis to balance it out.
- ③ If there is a close parenthesis available on stack try and add it on.
 → Now you have one less close parenthesis.

Code:
~~vector<string> result;~~
 void printAllParenthesis(~~int~~ openStack, ~~int~~ closeStack, ~~string~~ str, ~~char~~ c);

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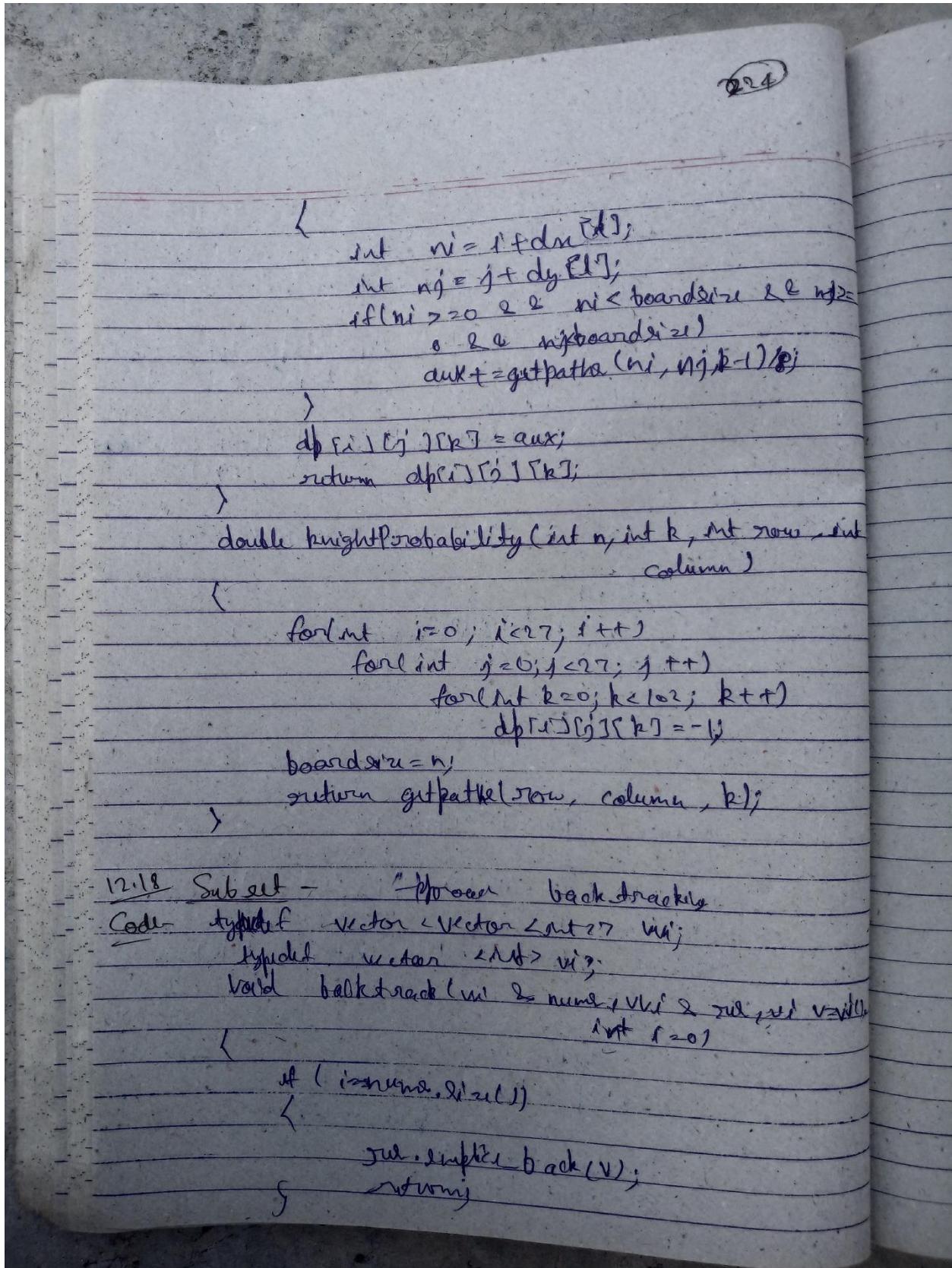
```

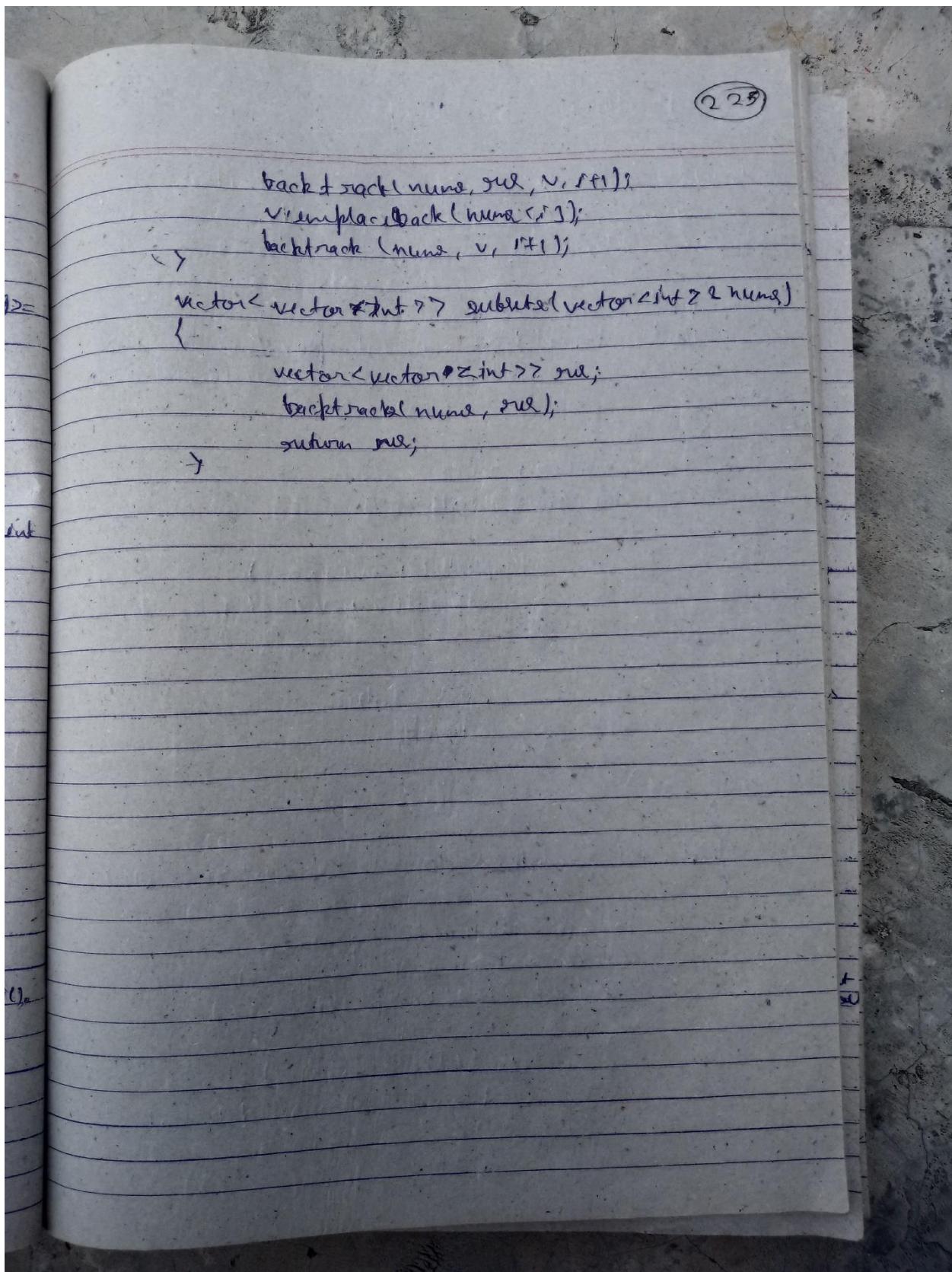
if (openStack == 0 & & closeStack == 0)
    result.push_back("");
if (openStack > 0)
    printAllParenthesis(openStack - 1, closeStack
                        , stack + 1, & = "(");
if (closeStack > 0)
    printAllParenthesis(openStack, closeStack - 1
                        , stack + 1, & = ")");
vector<string> generateParenthesis (int n)
{
    if (n)
        printAllParenthesis(n, 0, "");
    return result;
}

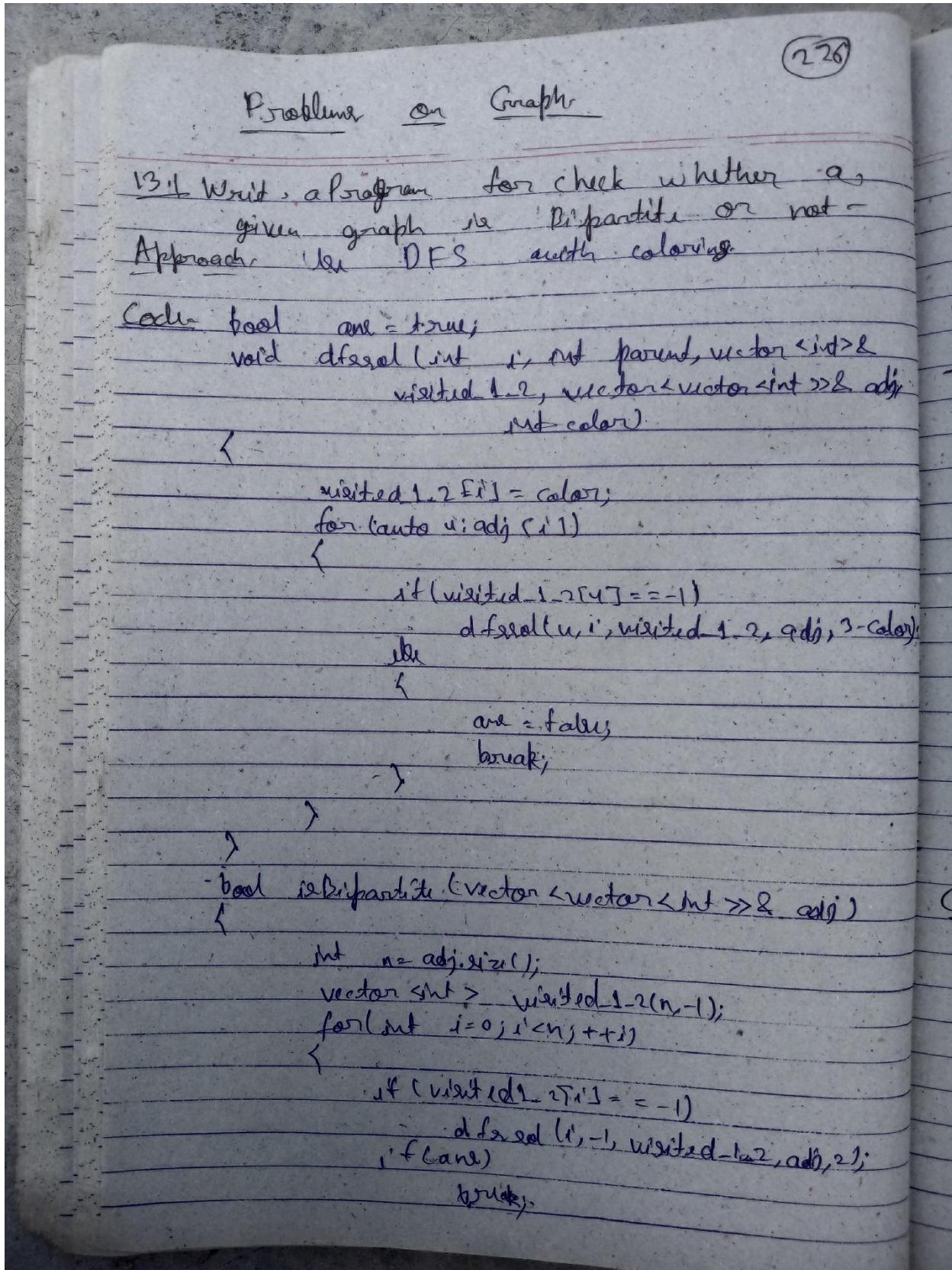
```

12.17 Knight Probability in Chessboard
Approach via DP.

Code = double dp[8][8][102];
int dx[8] = {-2, -2, -1, 1, 2, 2, 1, -1};
int dy[8] = {-1, 1, 2, 2, 1, -1, -2, -2};
int board[8];
double getpath(int i, int j, int k)
{
 if (k == 0)
 return 1;
 if (dp[i][j][k] != -1)
 return dp[i][j][k];
 double aux = 0.0;
 for (int l = 0; l < 8; l++)







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3.2 Clone Graph
 Approach: Use Hashmap, $T(n) = O(n)$, $S(n) = O(n)$
 Algo: ① Start traversing the graph from the given node.
 ② Maintain a hashtable to store the reference of the ~~copy~~ copy of all nodes that have already been visited and cloned.
 key: node of the original graph.
 value: Corresponding cloned node of the cloned graph.
 ③ If we don't find the node in hashtable, then create a copy of it and keep it in hashtable.
 ④ Now, make a recursive call for the neighbors of the node.

Code: unordered map<Node*, Node*> visited;
 Node* cloneGraph(Node* node)

```

if (!node)
  return node;
if (visited[node])
  return visited[node];
Node* new_node = new Node(*node->val);
visited[*node] = new_node;
if (node->neighbors != vector<Node*>())
  for (auto neighbor : node->neighbors)
    new_node->neighbors.push_back(cloneGraph(neighbor));
  }
  return new_node;
}
  
```

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```

vector<Node*> neighbors;
for (auto n : nodes) {
    neighbors.push_back(CloneGraph(n));
    new_node->neighbors = neighbors;
}
return new_node;

```

136 Rotting Oranges -

IP- [[2,1,1], [1,1,0], [0,1,1]]. OP- 4

x	1	1	x	x	1	x	x	x	x	x	x
1	1	0	x	1	0	x	x	0	x	x	0
0	1	1	0	1	1	1	1	1	0	x	1

Approach- Use BFS.

+ (M+N) = O(N) SCN = O(N)

x	x	x
x	x	0
0	x	x

Code- int orangeRotting (vector<vector<int>>& grid)

```

int n = grid.size(), m = grid[0].size();
queue<pair<int, int>> q;
int fresh = 0;
for (int i = 0; i < n; i++)

```

```

    for (int j = 0; j < m; j++)
        if (grid[i][j] == 2)
            q.push({i, j});

```

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```

if (grid[0][0] == -1)
    fresh++;

int dx[4] = {0, 0, 1, -1};
int dy[4] = {1, -1, 0, 0};
int total_time = 0;
while (!q.empty())
{
    if (!fresh)
        break;
    int x = q.front();
    total_time++;
    while (x--)

        auto cur = q.front();
        q.pop();
        for (int dir = 0; dir < 4; dir++)
            int nx = cur.first + dx[dir], ny = cur.second + dy[dir];
            if (nx >= 0 && nx < n && ny >= 0 && ny < m && grid[nx][ny] == 1)
                fresh--;
                grid[nx][ny] = 2;
                q.push({nx, ny});
}
cout << (fresh ? -1 : total_time);
}

```

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13.7 - Number of Islands

Problem Statement - Given a 2d grid map of 1's (land) and 0's (water), count the number of islands. An island is surrounded by water and is formed by connecting adjacent lands horizontally or vertically. You may assume all four edges of the grid are all surrounded by water.

Ex IP

1	1	1	0
1	1	0	0
1	1	0	0
0	0	0	0

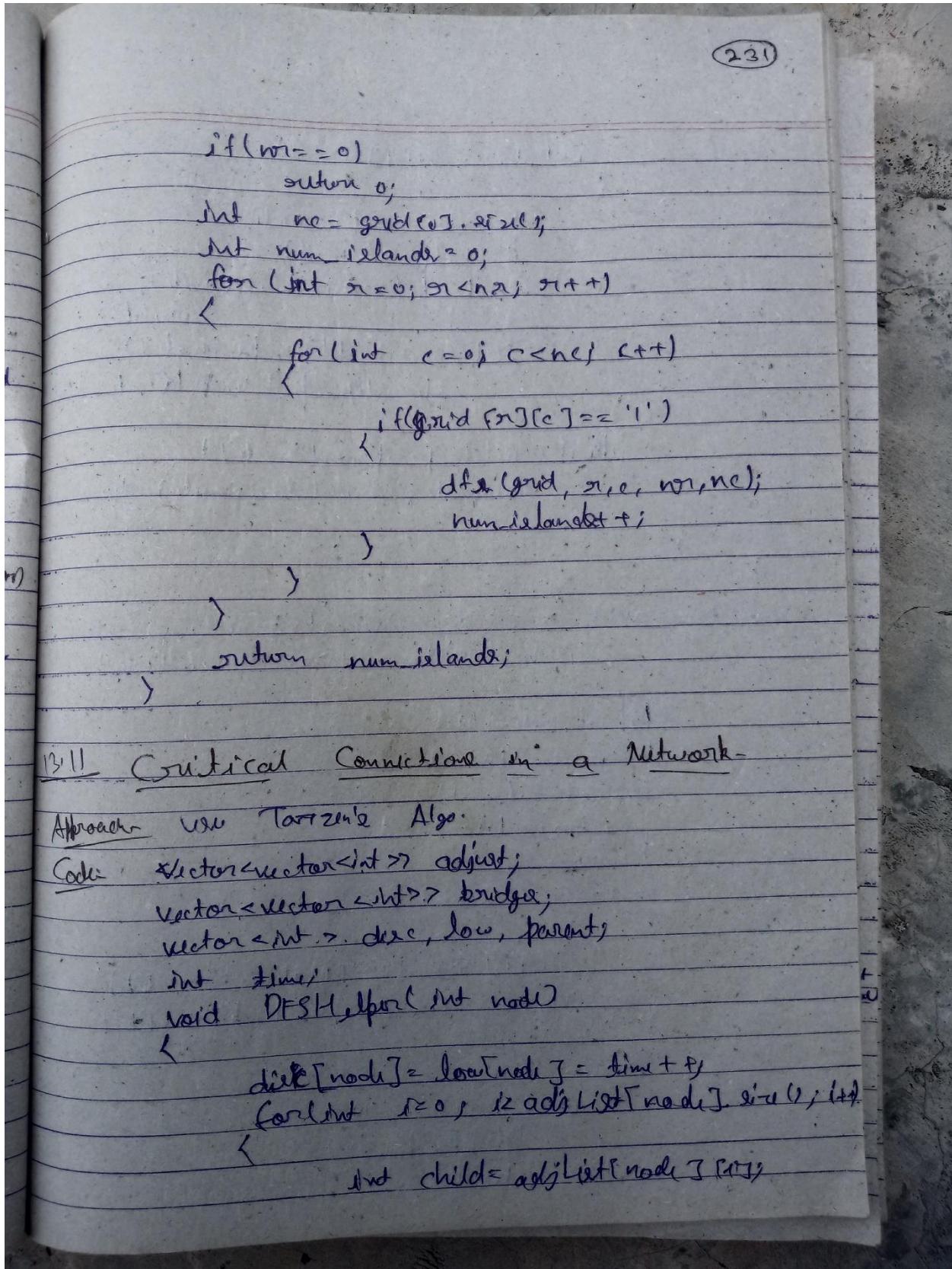
O/P = 1

Approach Use DFS. $T(n) = O(n \times m)$, $S(n) = O(n \times m)$

Code - void dfs(vector<vector<char>> &grid, int r, int c, int nr, int nc)

```

grid[r][c] = '0';
if (c-1 >= 0 and grid[r-1][c] == '1')
    dfelgrid, nr-1, c, nr, nc);
if (c+1 < nr and grid[r+1][c] == '1')
    dfs(grid, r+1, c, nr, nc);
if (c-1 >= 0 and grid[r][c-1] == '1')
    dfs(grid, r, nr, c-1, nr, nc);
if (c+1 < nc and grid[r][c+1] == '1')
    dfs(grid, r, nr, c+1, nr, nc);
}
int numIslands(vector<vector<char>> &grid)
{
    int nr = grid.size();
}
```



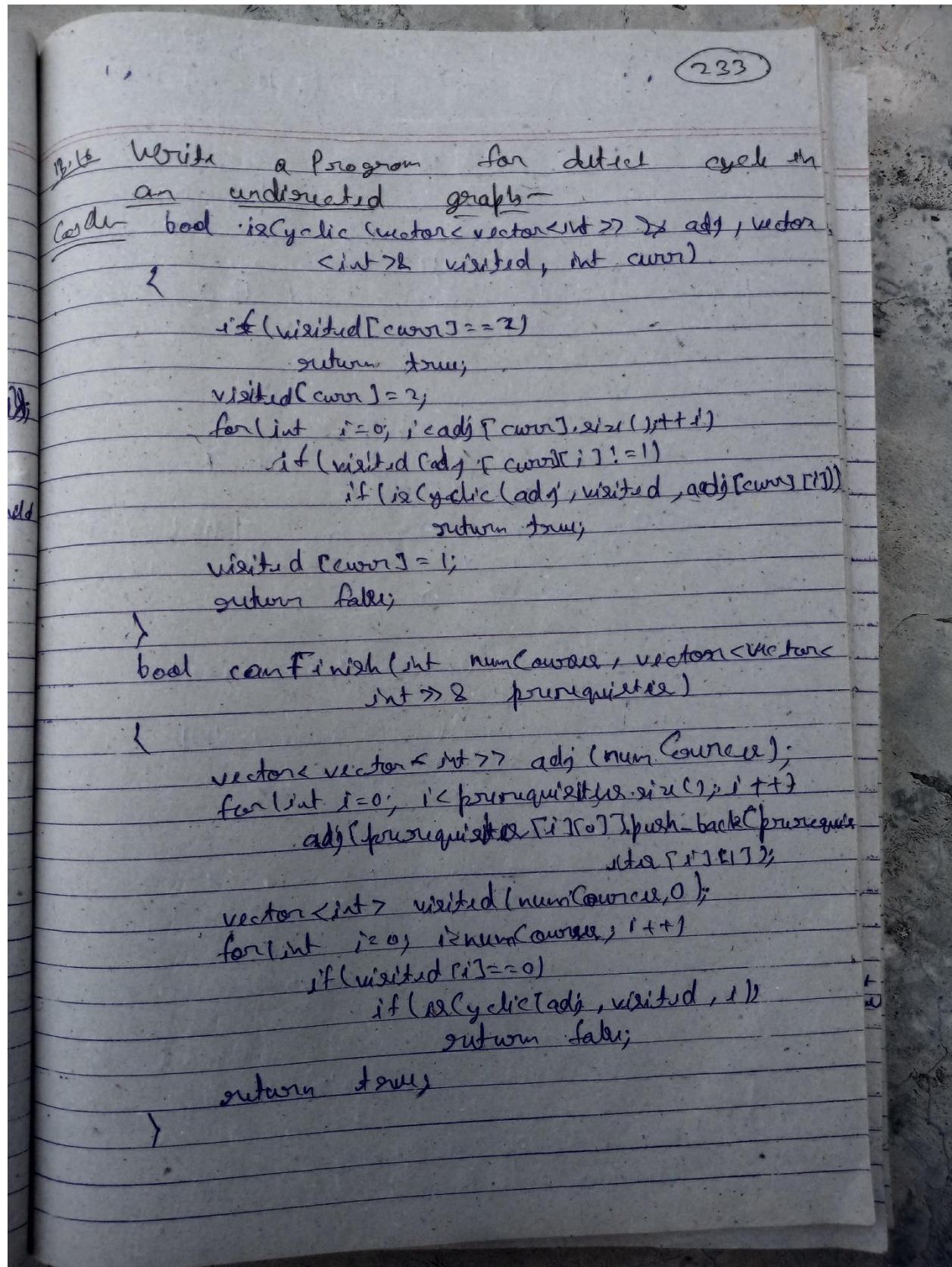
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```

if (discr[child] == -1)
{
    parent[child] = node;
    DFSHelper(child);
    low[node] = min(low[node], low[child]);
    if (low[child] > discr[node])
        bridge.push_back({node, child});
}
else if (child != parent[node])
    low[node] = min(low[node], discr[child]);
}

vector<vector<int>> criticalConnections(int n,
vector<vector<int>>& connections)
{
    adjList.resize(n);
    discr.resize(n, -1);
    low.resize(n, -1);
    parent.resize(n, -1);
    time = 0;
    for (auto & conn : connections)
    {
        adjList[conn[0]].push_back(conn[1]);
        adjList[conn[1]].push_back(conn[0]);
    }
    for (int i = 0; i < n; i++)
    {
        if (discr[i] == -1)
            DFSHelper(i);
    }
    return bridge;
}

```



Manipulation - 234

~~Maths and Bit Manipulation~~

14.1 Single Number - II

Problem Statement - Given a non-empty array of integers, every element appears three times except for one, which appears exactly once. Find that single one.

Ex- I/P: [2, 2, 3, 2] O/P: 3

Approach 1 Use mathematical formulae.

$$3 \times (\text{sum} + 99) - (102) = 2 \times \cancel{x}$$

$\cancel{\text{unique addition}}$

$$3 \times 100 - 102 = 2 \times n \quad T(n) = O(n)$$

$$298 = 2n \quad S(n) = O(n)$$

$$\cancel{298} \quad n = 99$$

Code - int singleNumber(vector<int>& nums)

```

    int sum = 0;
    for (auto num : nums)
        sum += num;
    return (3 * sum - numSum) / 2;
}

```

Approach 2 Use Hash Map $T(n) = O(n) \quad S(n) = O(n)$

out singleNumber (vector<int>& nums)

```

unordered_map<int, int> m;
for (auto num : nums)
    m[num]++;
    if (m[num] == 3)
        return num;
}

```

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```

m[num]++;
for (auto ele : m)
    if (ele.second == 1)
        return ele.first;
return 0;
}

Approach - 3 - Using Bitwise Operators  $T(n) = O(m), S(n) = O(1)$ 

Code for singleNumber(vector<int> &num)
{
    int one = 0, two = 0;
    for (auto num : num)
    {
        two = one & num;
        one ^= num;
        int non_three = ~ (one & two);
        one &= non_three;
        two &= non_three;
    }
    return one;
}

```

14.6 Number of 1 Bits -

Problem Statement - Write a function that takes an unsigned integer and returns the number of '1' bits it has (also known as the Hamming weight).

Approach - Bit Manipulation. $T(n) = O(32) = O(1), S(n) = O(1)$.

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Code int hammingWeight(uint32_t n)

```

    {
        int count = 0;
        while(n > 0)
        {
            count += n & 1;
            n >>= 1;
        }
        return count;
    }

```

Approach 2 Use Bit Manipulation $T(M) = O(1)$ since $M = 32$

Code int hammingWeight(uint32_t n)

```

    {
        int count = 0;
        while(n > 0)
        {
            n = n & (n - 1);
            count++;
        }
        return count;
    }

```

14.9 - Counting Bits

Problem Statement - Given a non negative integer number num. For every number i in the range $0 \leq i \leq num$, calculate the number of 1's in their binary representation and return them as an array.

Ex T[0-2] OR [0, 1, 1]

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Approach-1 $T(M) = O(n \cdot k)$. $S(M) = O(n)$

Code `int countOne(int n)`

```

int count = 0;
while (n > 0)
{
    n &= n - 1;
    count++;
}
return count;

```

`vector<int> countBit(int num);`

```

vector<int> result;
for (int i = 0; i < num; i++)
    result.push_back(countOne(i));
return result;

```

Approach-2 $T(M) = O(n)$. $S(M) = O(n)$

Code `vector<int> countBit(int num)`

```

vector<int> result(num + 1, 0);
for (int i = 1; i < num; i++)
    result[i] = result[i / 2] + (i % 2);
return result;

```

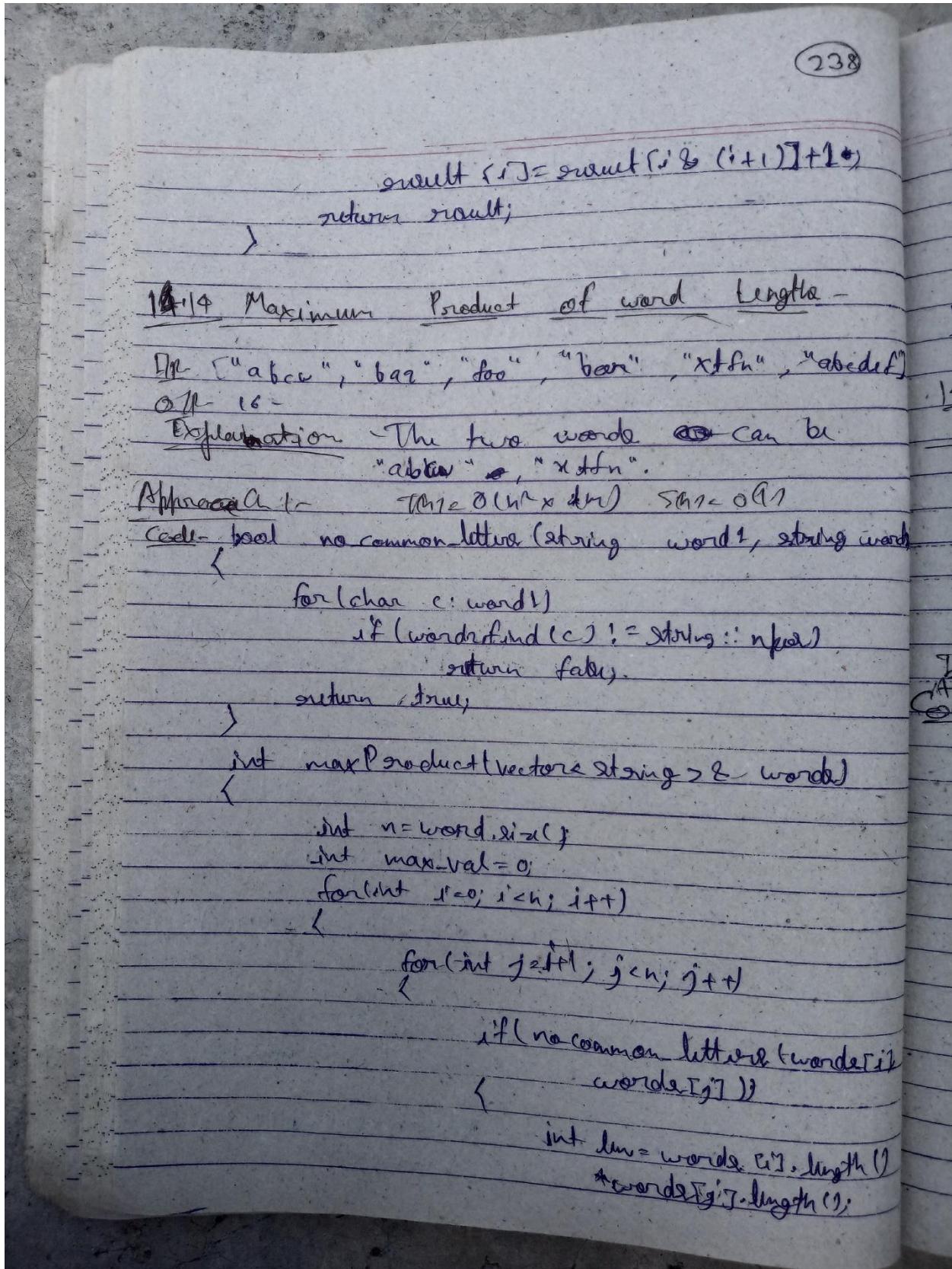
Approach-3 $T(M) = O(n)$. $S(M) = O(n)$

Code `vector<int> countBit(int num)`

```

vector<int> result(num + 1, 0);
for (int i = 1; i < num; i++)
    result[i] = result[i / 2] + (i % 2);

```



(229)

```

    max_val = max(max_val, m);
}

return max_val;
}

14.19 Total Hamming Distance -
Problem Statement - The Hamming distance between two integers is the number of positions at which the corresponding bits are different.

Given an integer array num, return the sum of Hamming distance between all the pairs of the integers integers in num.

The num = [4, 1, 2]  O(n^2)  T.M.C. O(n^2)  S.O.S.
Approach - int countbit(int n)
{
    int count = 0;
    while (n > 0)
    {
        n = n - 1;
        count += 1;
    }
    return count;
}

void totalHammingDistance (vector<int> &num)
{
    int result = 0;
    for (int i = 0; i < num.size() - 1; i++)
        for (int j = i + 1; j < num.size(); j++)
}

```

240

```

result = count & size (num[1] ^ num[0]);
return result;
}

Logic - 2
T(n) = O(3n) = O(n) S(n) = O(1)

Code - int totalHammingDistances (vector<int>& num)
{
    int n = num.size();
    int result = 0;
    for (int i = 0; i < 32; i++) {
        int count = 0;
        for (int j = 0; j < n; j++) {
            if (num[j] & (1 << i)) {
                count += 1;
            }
        }
        result += (count * (n - count));
    }
    return result;
}

14.23 Pow(m, n) -

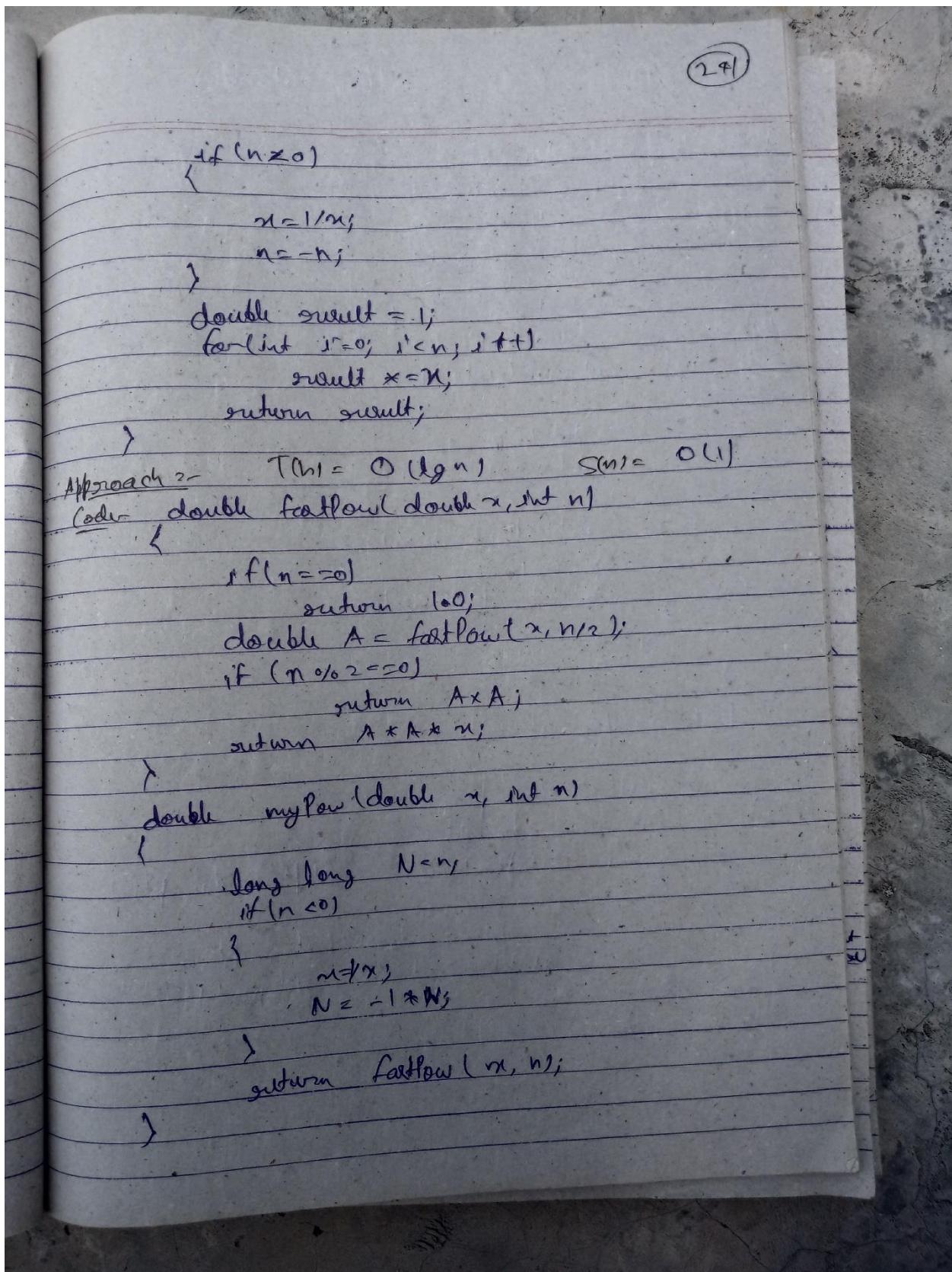
```

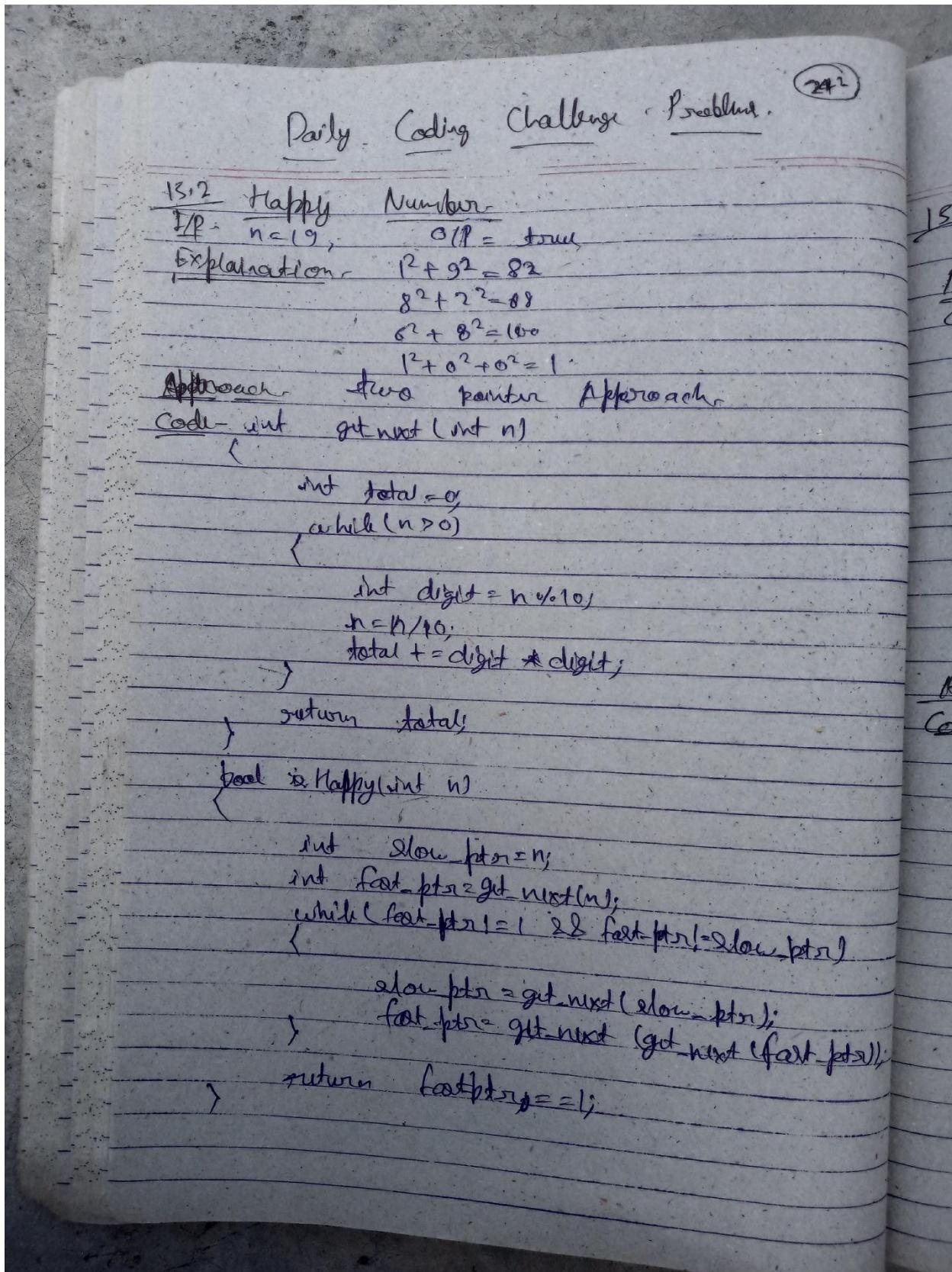
Problem Statement: Implement `pow(m, n)` which calculates m raised to the power n . ($\text{e.g. } x^n$).

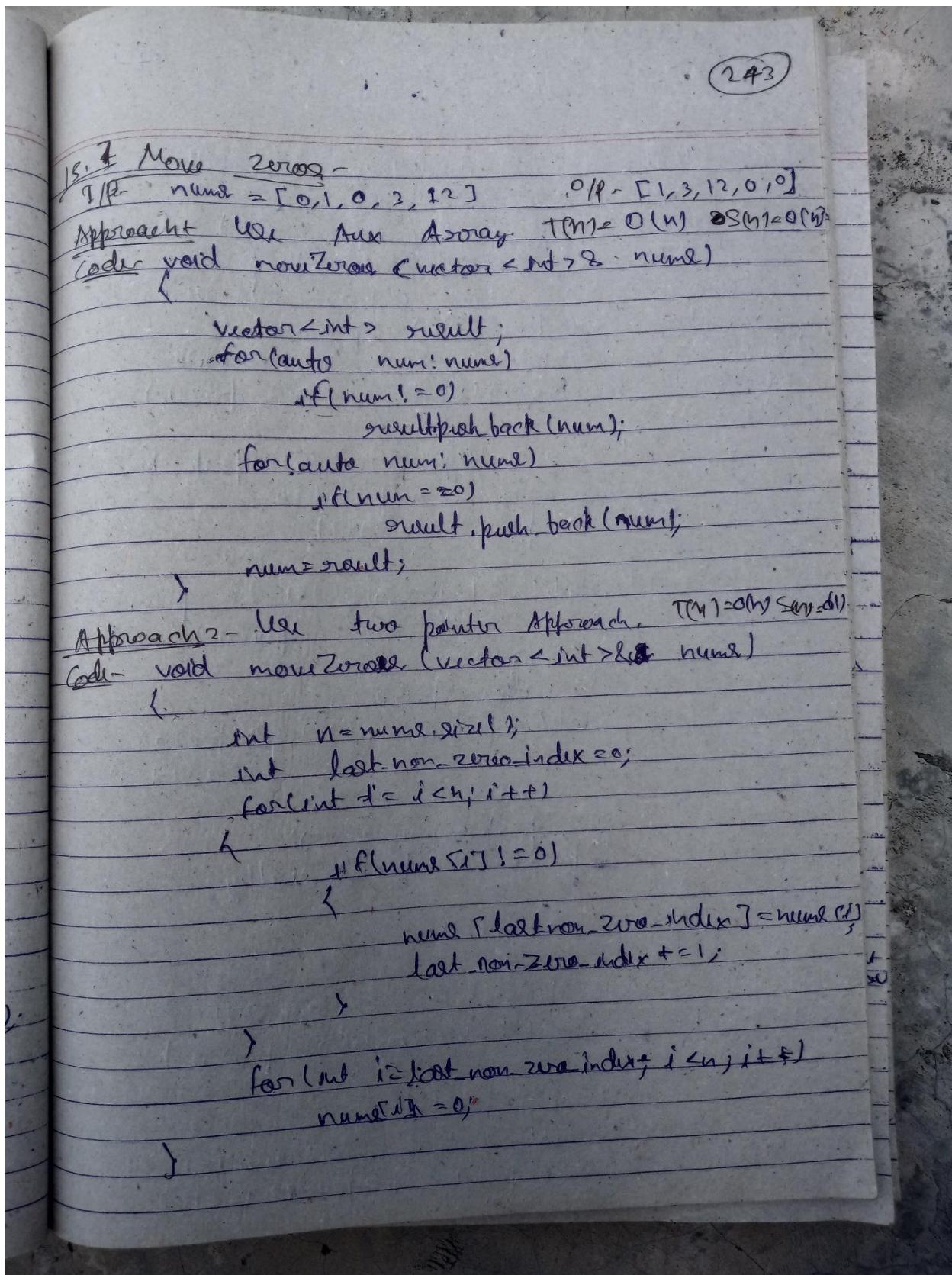
Exm I/P - $m = 2.00000$, $n = 10$
 O/P - 1024.00000

Approach - 1 - $T(n) = O(n)$, $S(n) = O(1)$

Code - `double myPow(double m, int n)`







Q44

15.11 Best Time to Buy and Sell Stock

Ex: SP = [7, 1, 5, 3, 6, 4], OP = ?

Explanation - Buy on day 1 (price = 1) and sell on day 3 (price = 3), profit = 3 - 1 = 2
 Then buy on day 4 (price = 3) and sell on day 5 (price = 6), profit = 6 - 3 = 3.

Approach - $T(n) = O(n)$ Since O(1)
Code: int maxProfit(vector<int>& price);

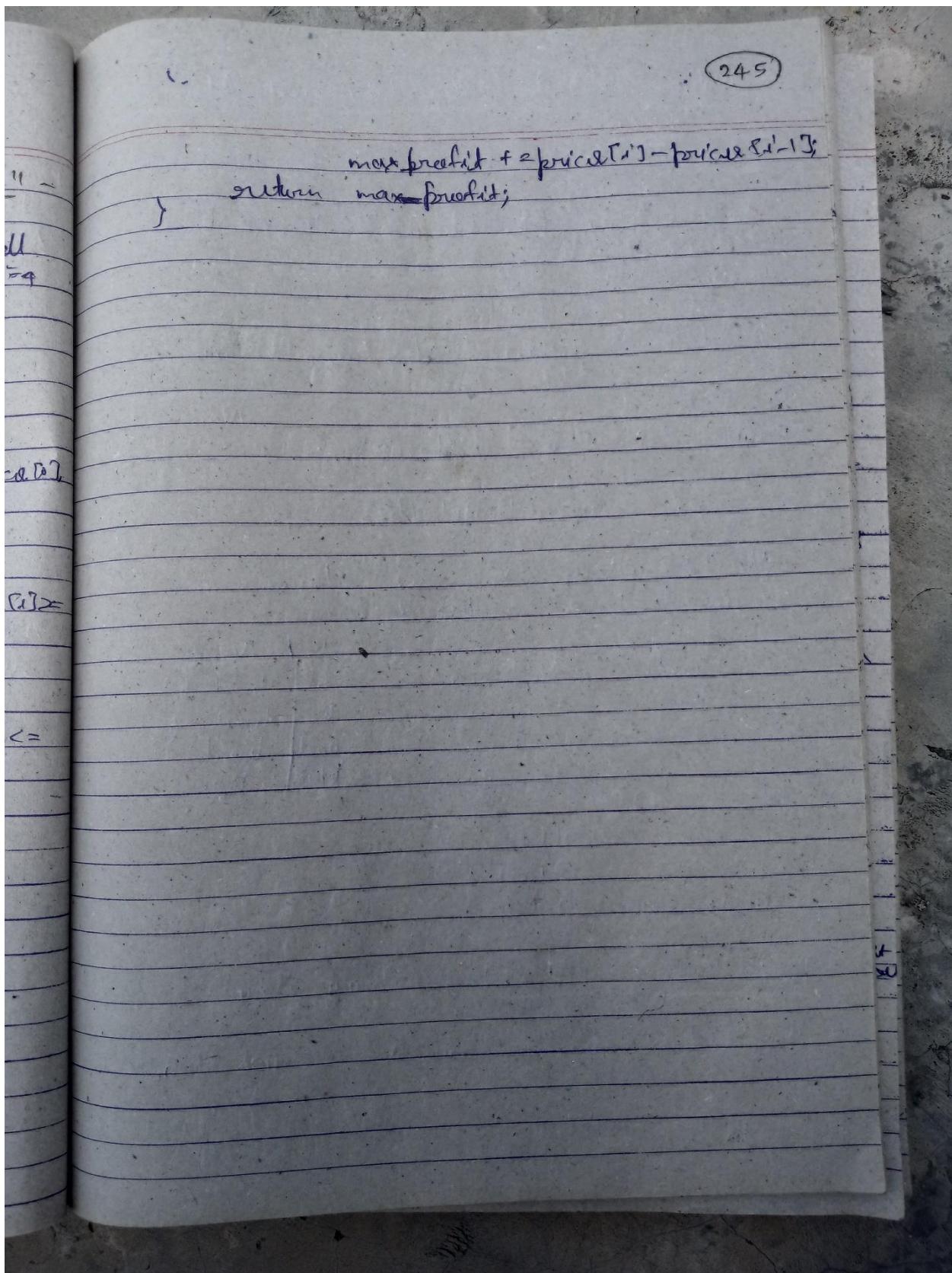
```

int i=0, valley = price[0], peak = price[0];
maxProfit = 0;
while(i < price.size() - 1)
{
    while(i < price.size() - 1 && price[i] >= price[i + 1])
        i++;
    int valley = price[i];
    while(i < price.size() - 1 && price[i] <= price[i + 1])
        i++;
    int peak = price[i];
    maxProfit += peak - valley;
}
return maxProfit;
  
```

Approach 2 - $T(n) = O(n)$ $S(n) = O(1)$
Code: int maxProfit(vector<int>& price);

```

int maxProfit = 0;
for(int i = 1; i < price.size(); i++)
{
    if(price[i] > price[i - 1])
        maxProfit += price[i] - price[i - 1];
}
  
```



(24)

Advanced Data Structures and Algorithms

Trie

II Trie -

Auto complete feature (of search bar)

Problem definition

$A = \{ \text{bad, abdomen, abacus, accuse, bared} \}$

$Q = "ab"$ Not sorted

Result = {abacus, abdomen}

Can you use HashTable -

No. of Words = n . is large.

Max. len of a word = m .

$w_1 = \text{bad} \rightarrow \text{prefix}$

$w_2 = \text{ban}$

$w_3 = \text{can}$

Space = $O(mn)$

Search Time = $O(1)$

Insert Time = $O(m)$

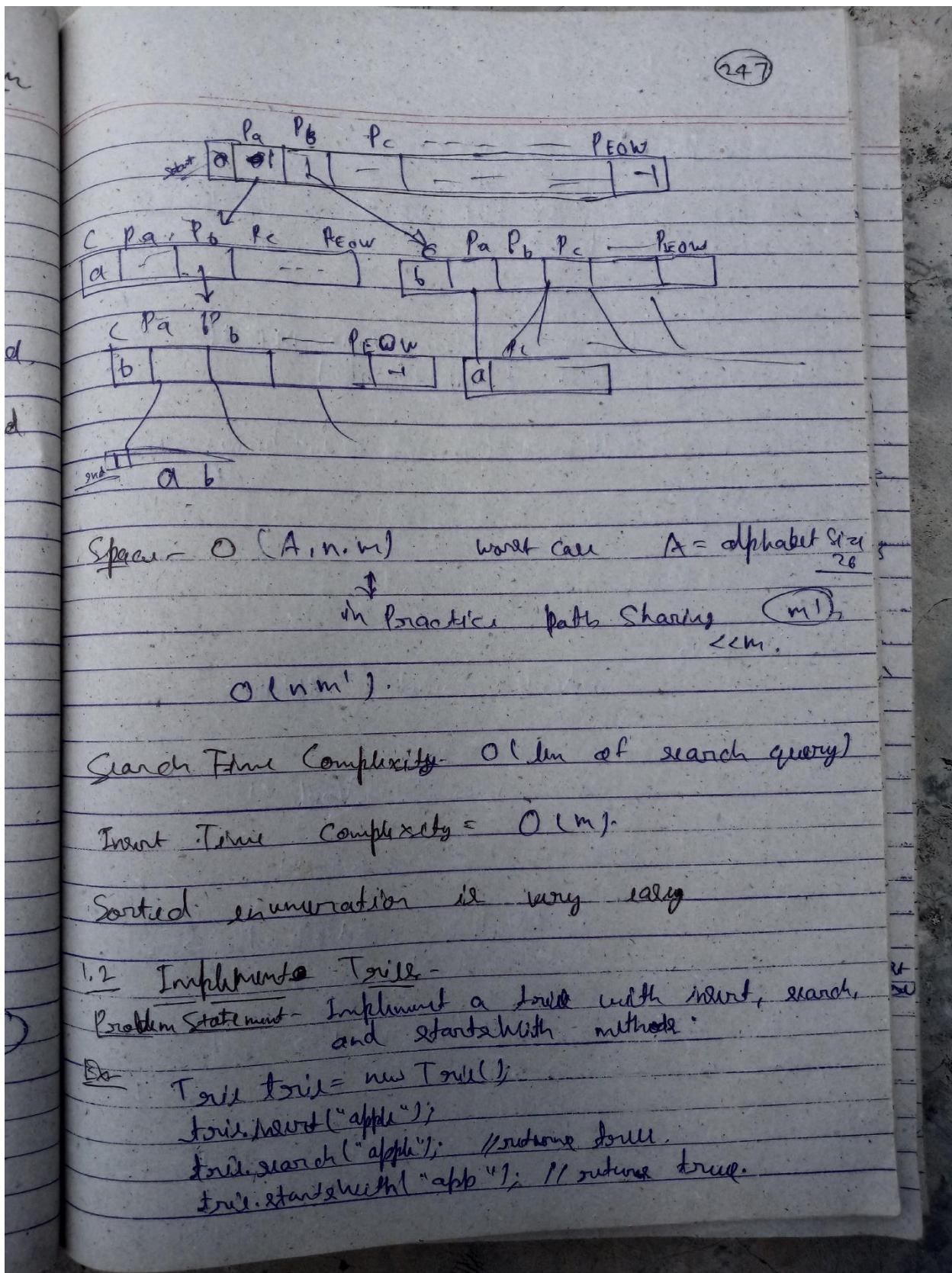
Deletion Time = $O(m)$

Prefix Tree - (done) - Key - Tree (x=26)

$A = \{ \text{bad, abdomen, abacus, accuse, bared, } \}$

B.

k	v
b	bad, ban
ba	bad, ban
bad	bad
ban	ban



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```

Code- #include <iostream>
using namespace std;
const int MAX_ALPHABET = 26 // only lowercase
struct TrieNode {

```

```

    bool finished;
    TrieNode* children[MAX_ALPHABET];
    TrieNode* finishedLabel;
}
```

```

    fill(&children[0], &children[MAX_ALPHABET - 1], NULL);
}
```

```

};
```

```

class Trie {

```

```

public:
```

```

    TrieNode* root = new TrieNode();
}
```

```

    Trie() {

```

```

        root = new TrieNode();
    }
}
```

```

    void insert(string word) {

```

```

        TrieNode* curNode = this->root;
        for (char ch : word) {

```

```

            if (!curNode->children[ch - 'a']) {

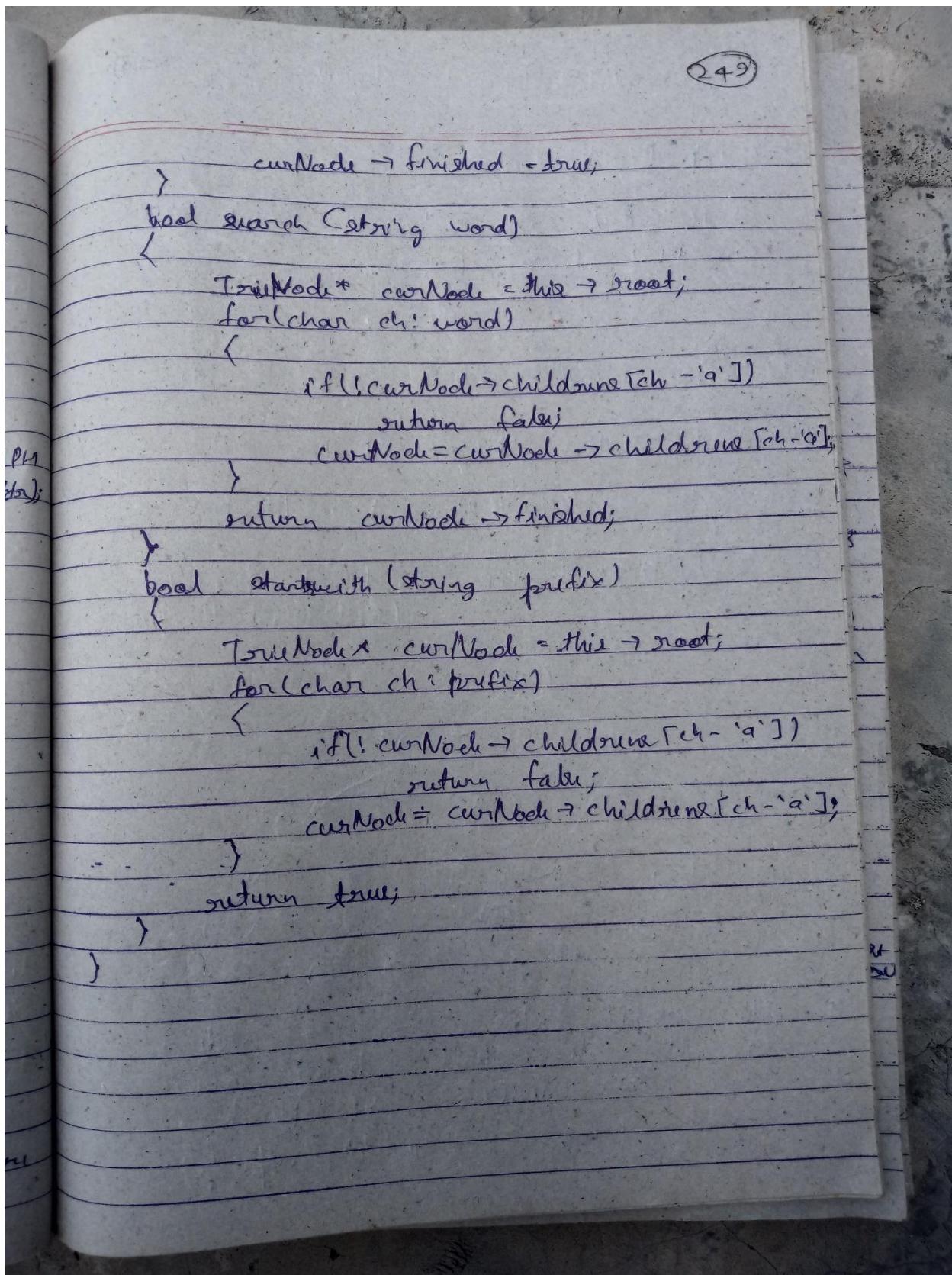
```

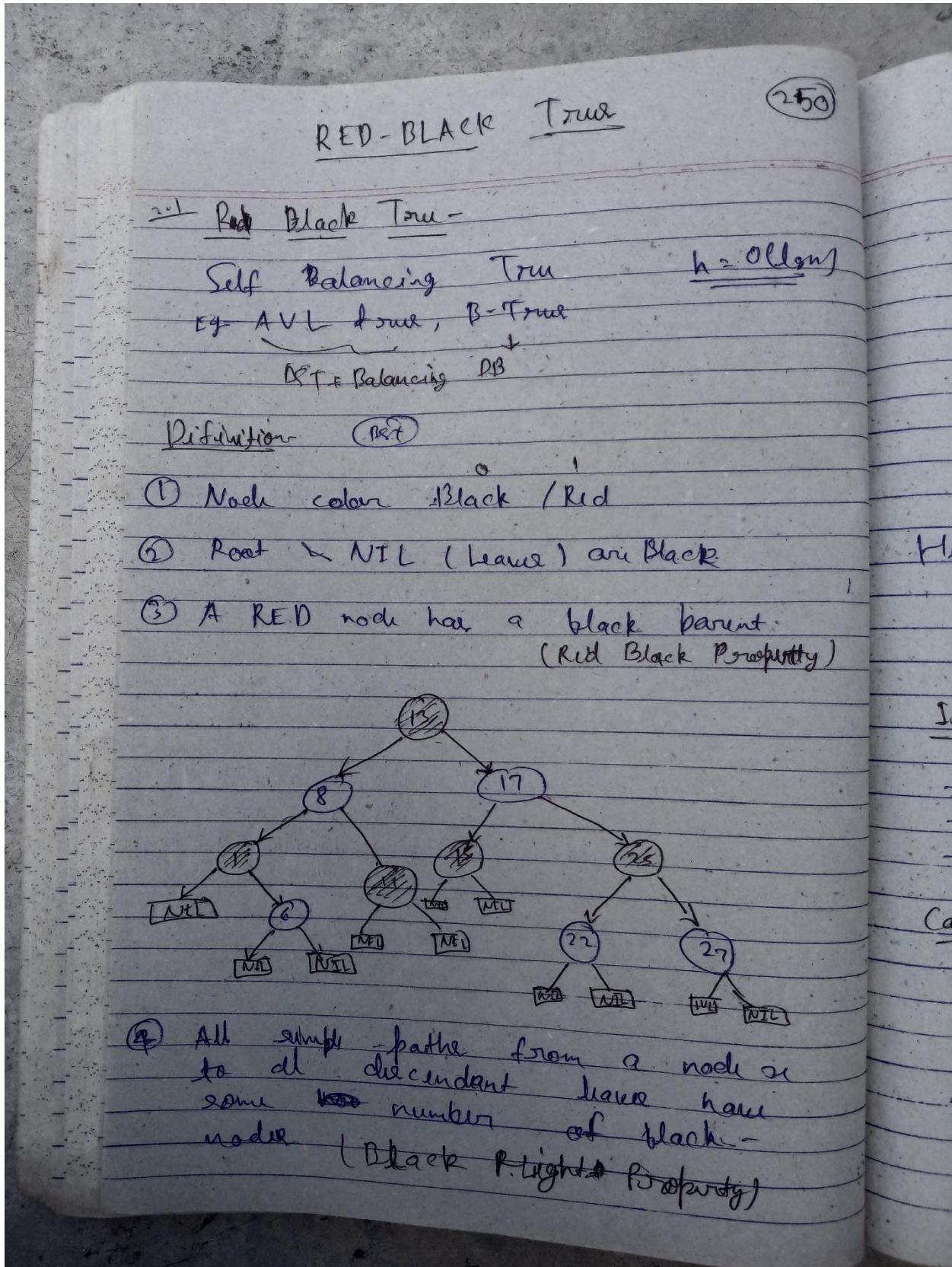
```

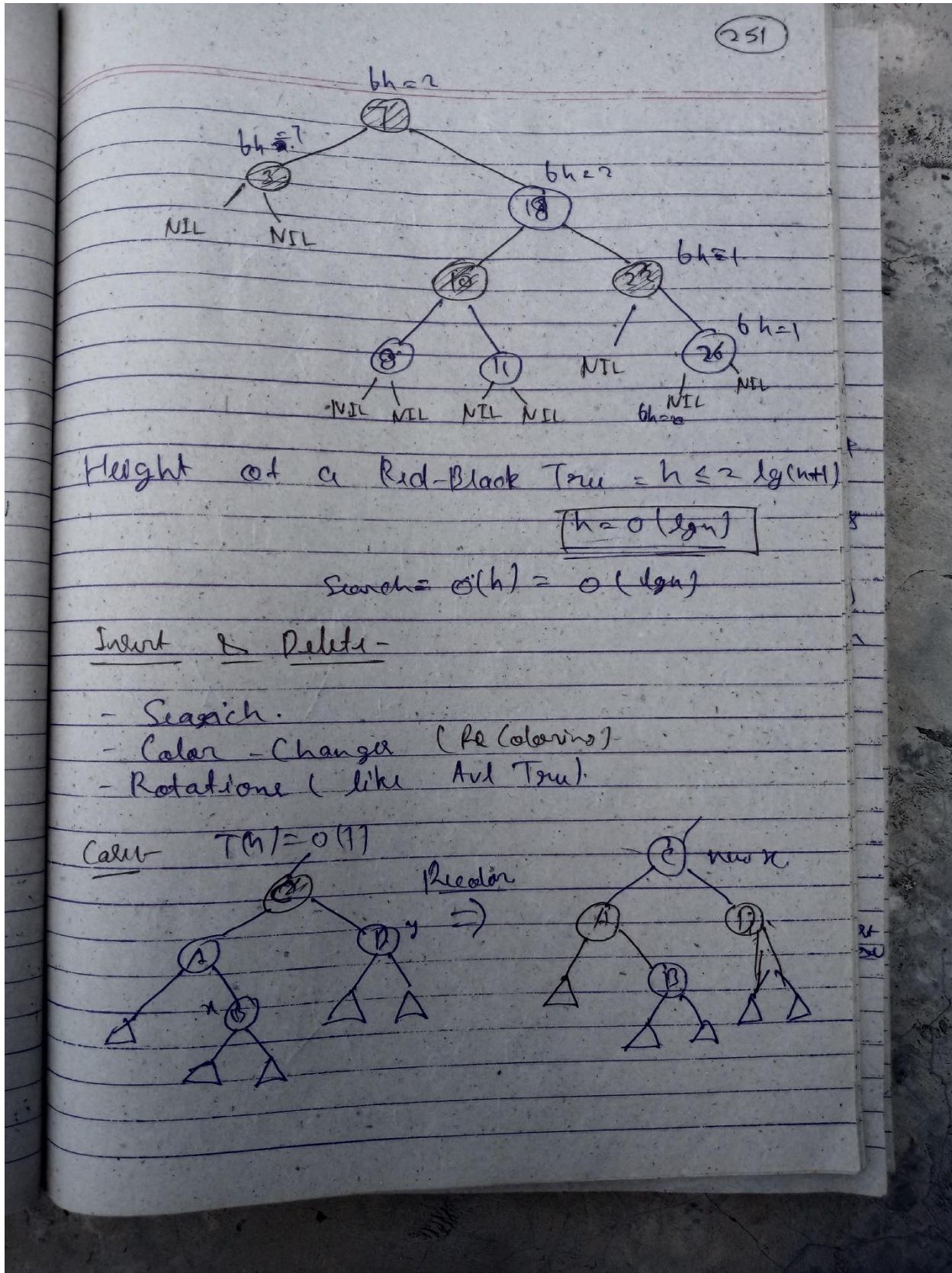
                curNode->children[ch - 'a'] = new
                TrieNode();
            }
        }
    }
}
```

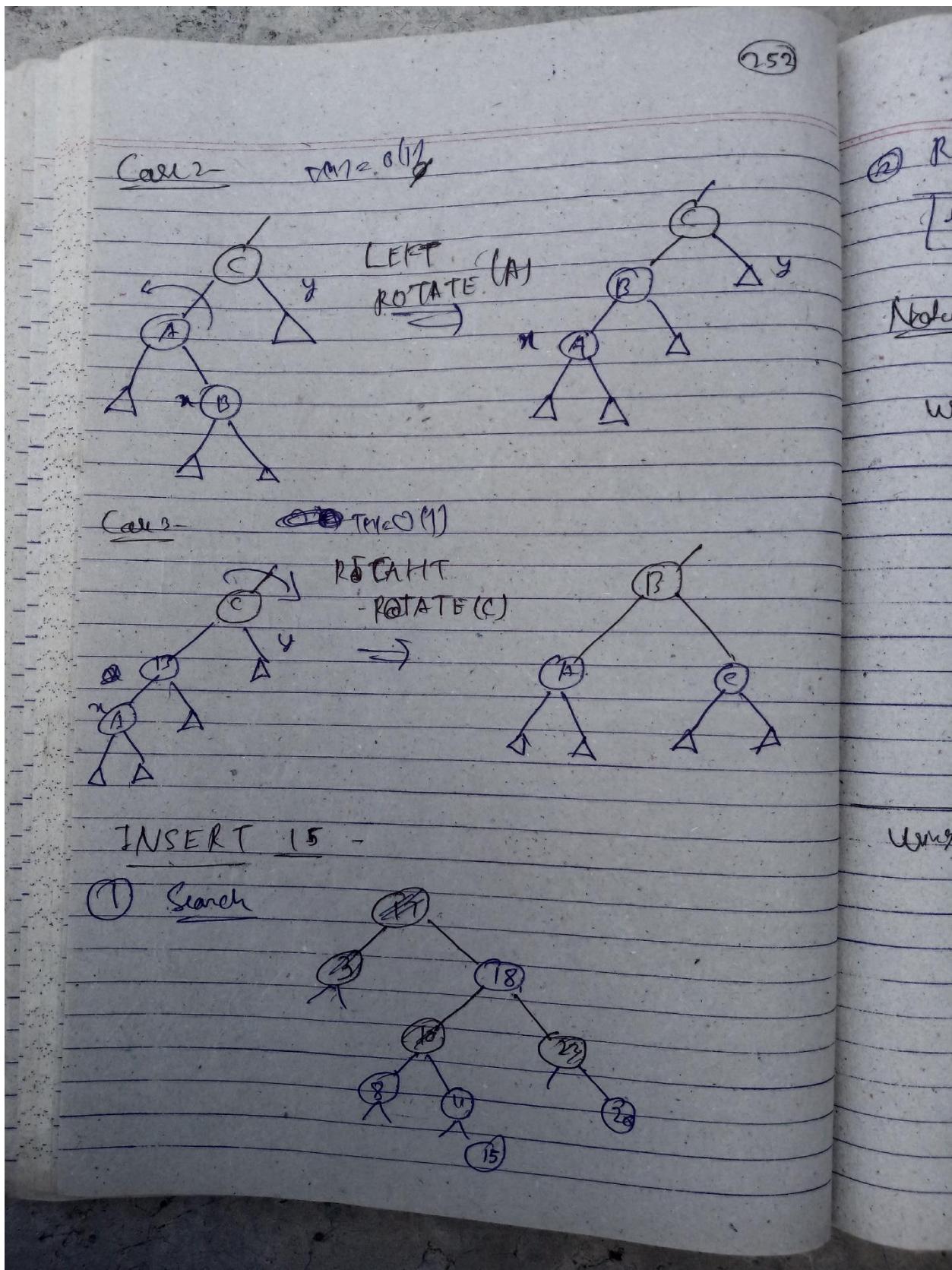
Applied Prep Course

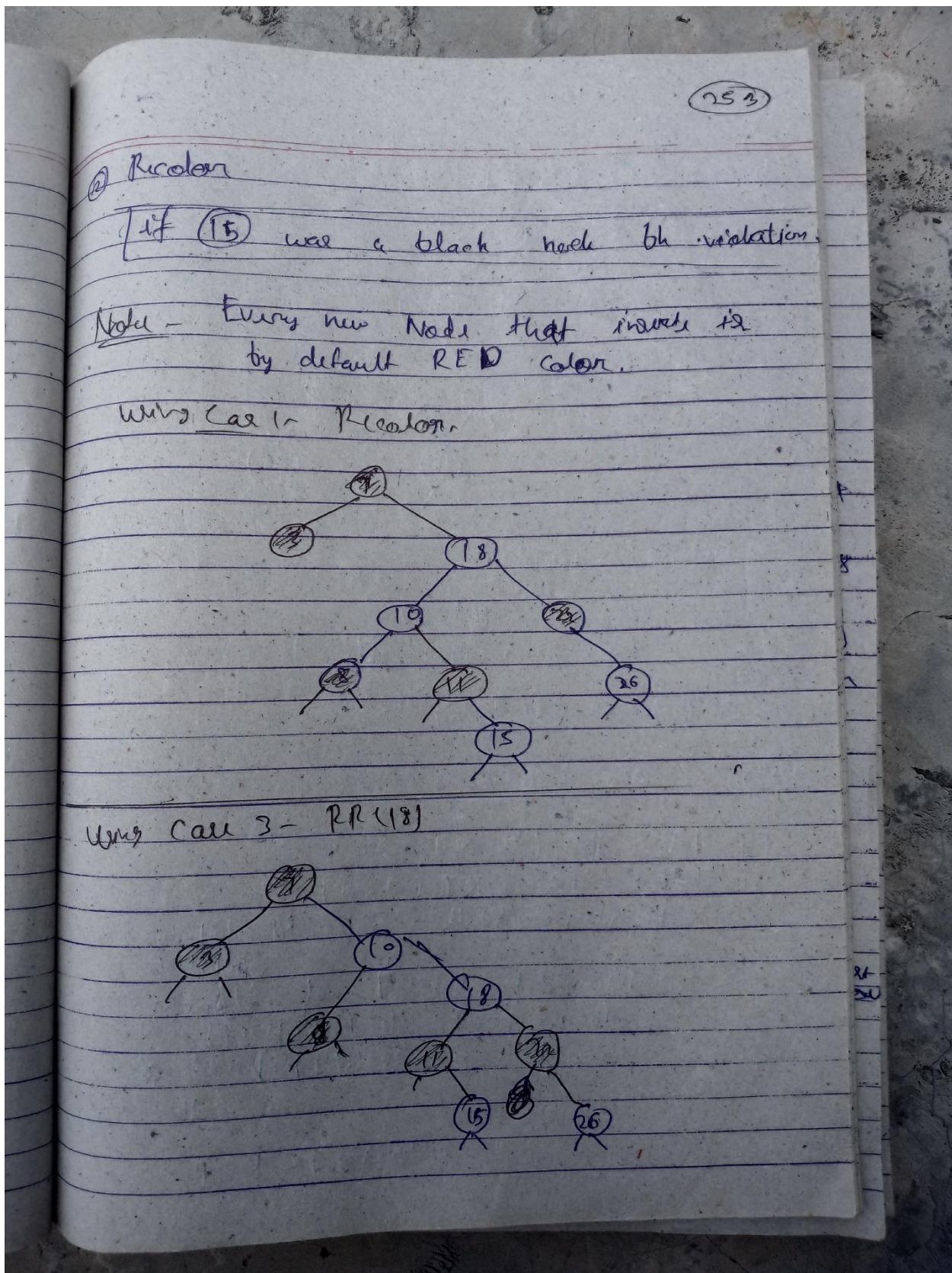
Page | 48

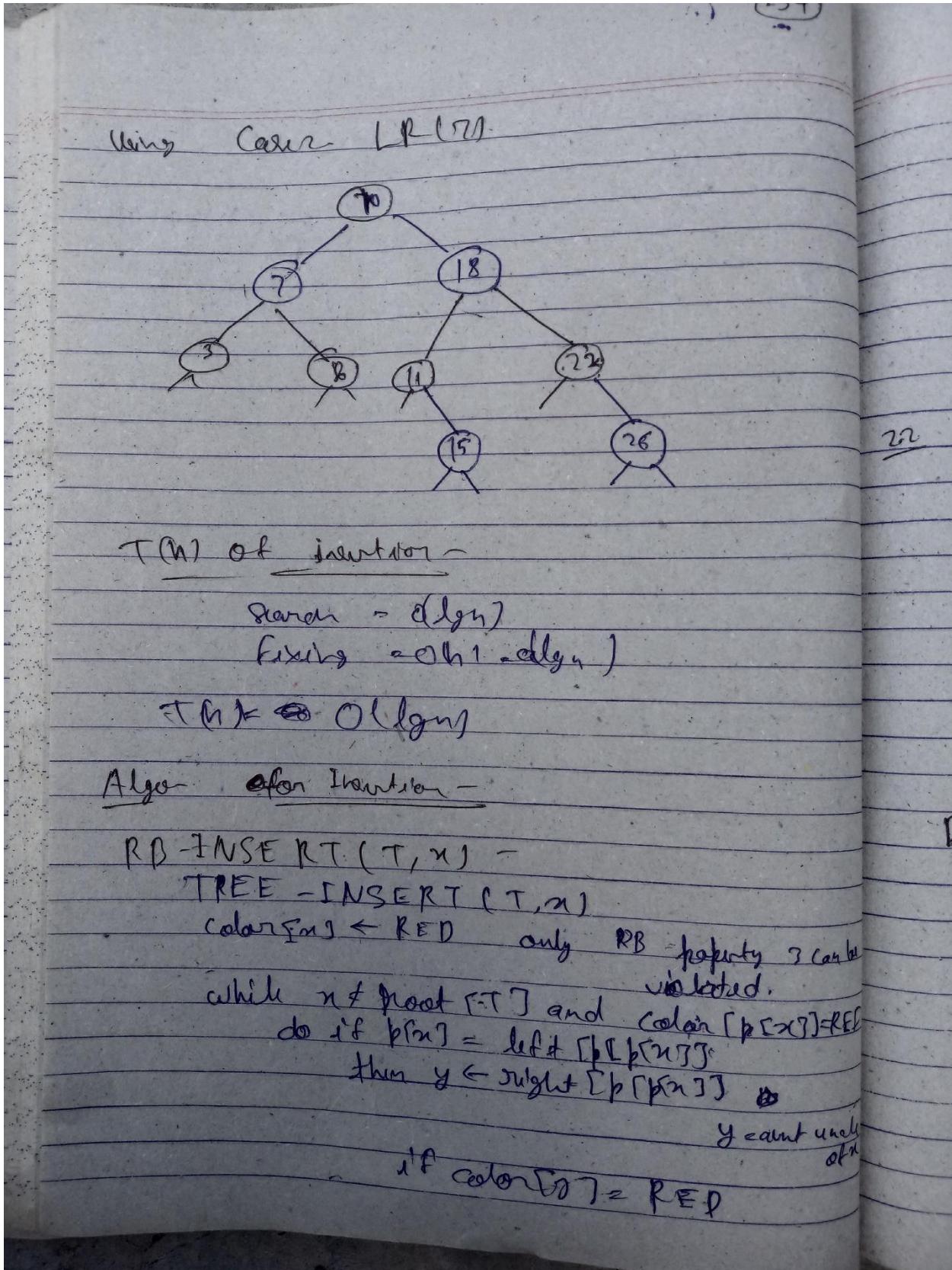












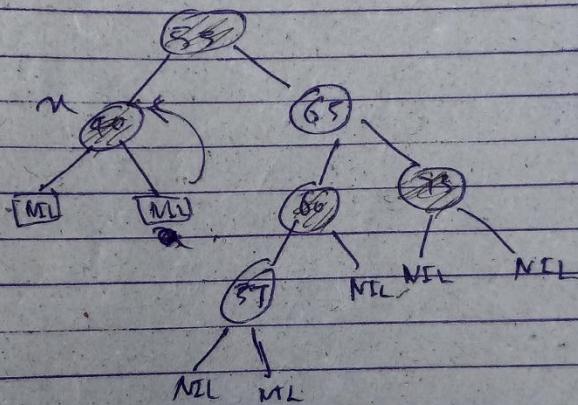
(253)

then (Case 1)

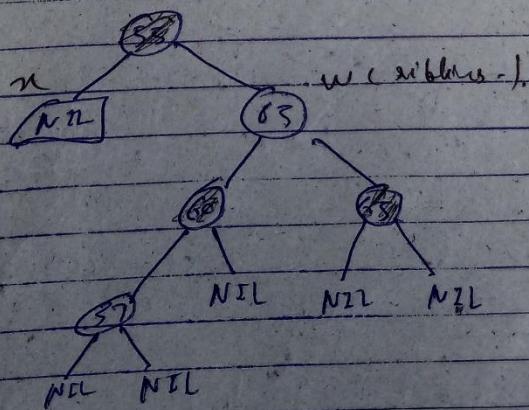
else if $n = \text{right}[p[n]]$

then (Case 2) - (Case 2 fails - Method Case 3)

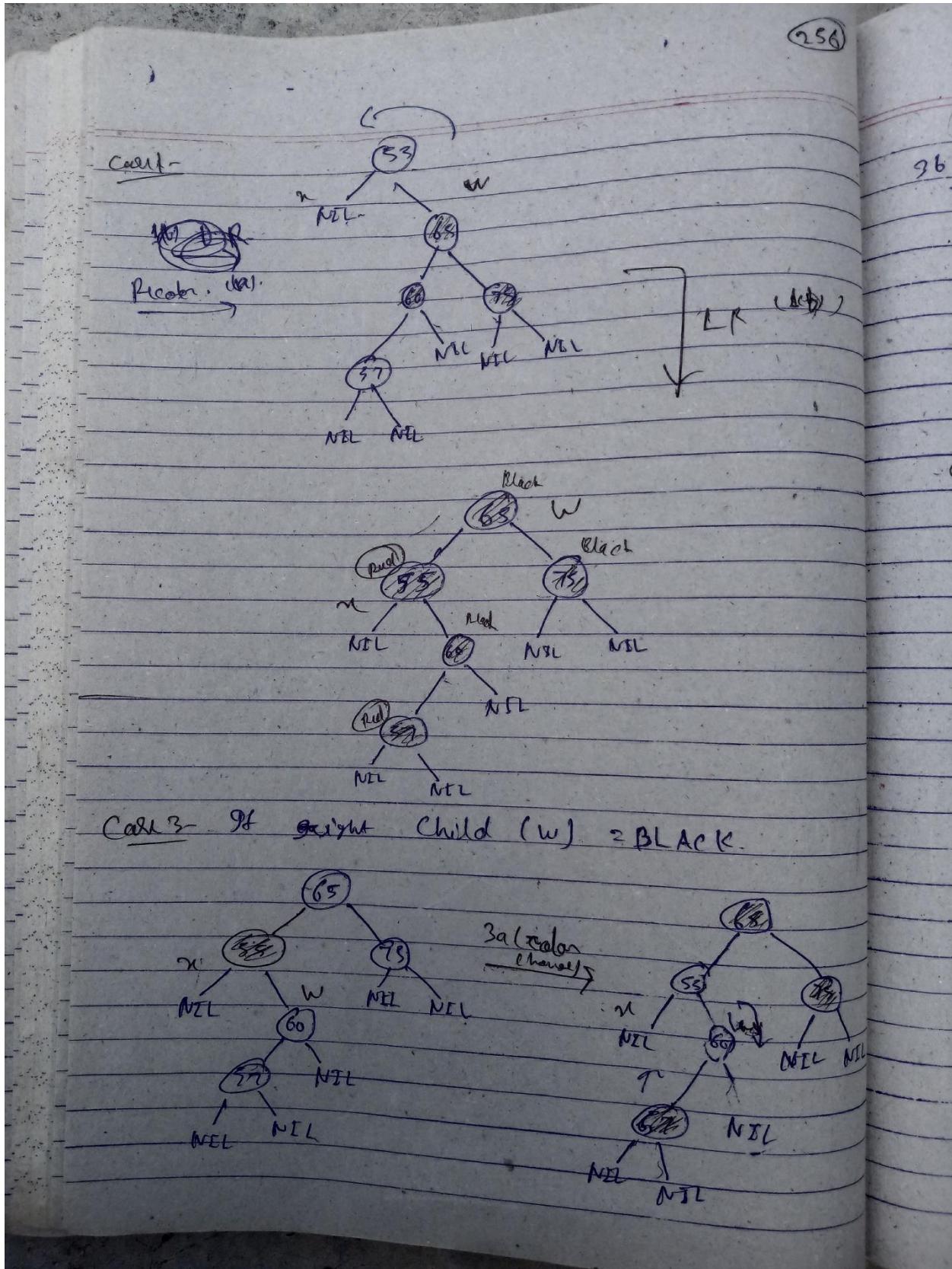
<Case 3>

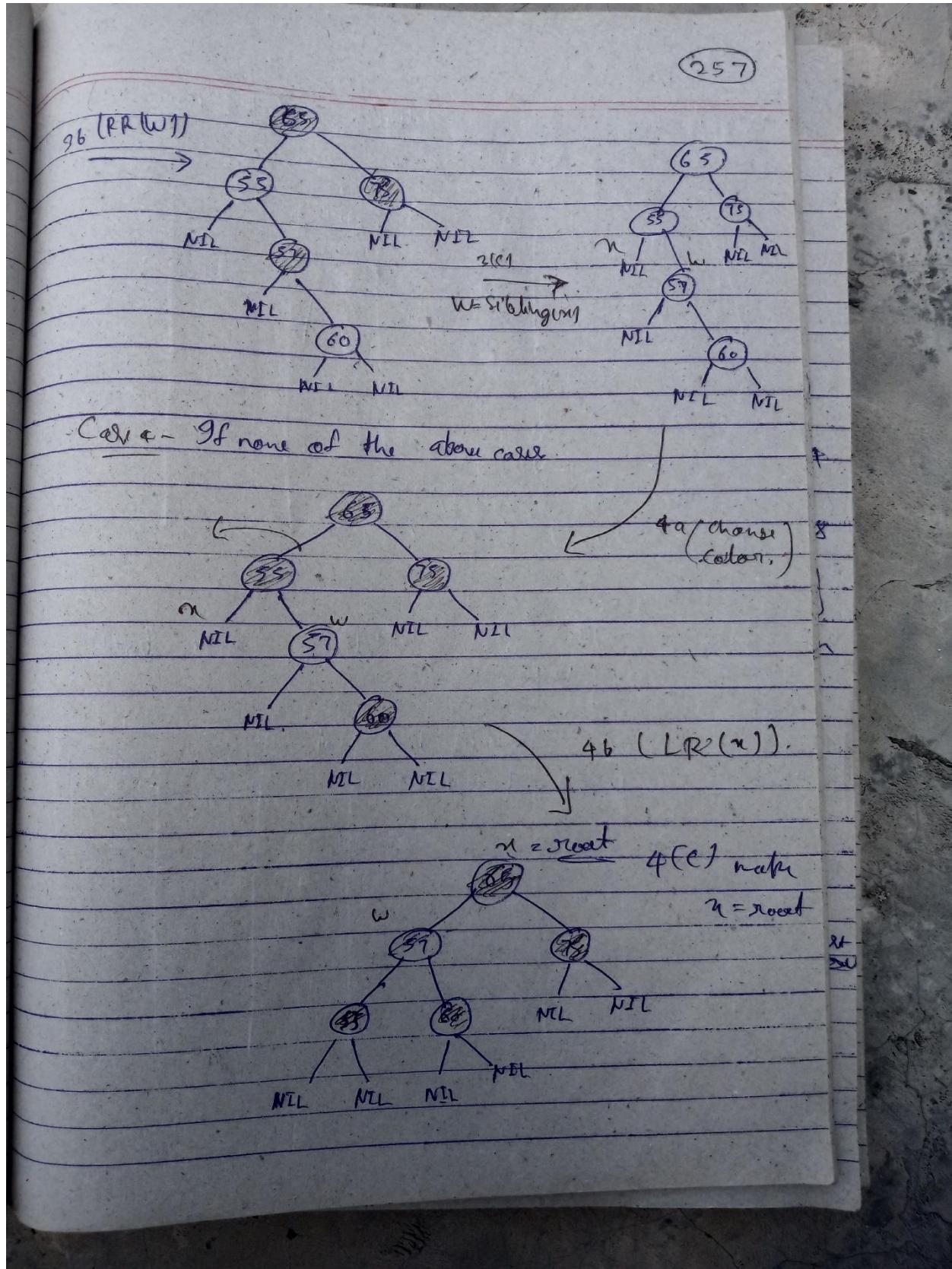
else <"then" clause with "left" and
"right" skip applied>
color [root[T]] ← BLACK.Red Black Tree - Deletion.

Delete - 40



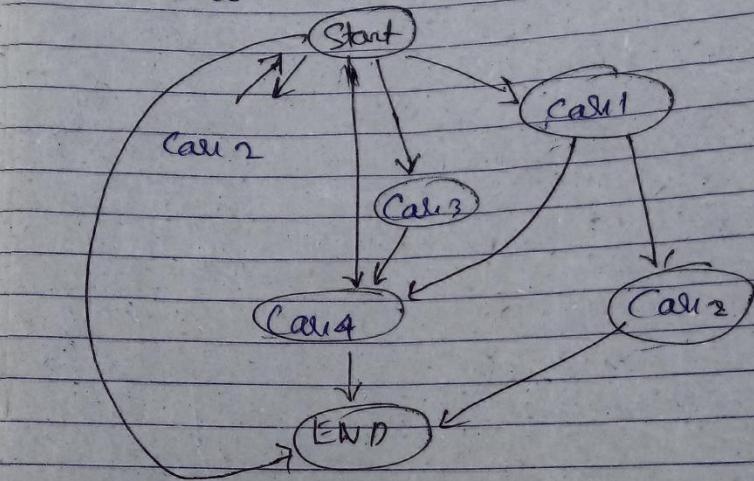
Warrant - Case 1 -





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Flow chart for deletion of RB Tree

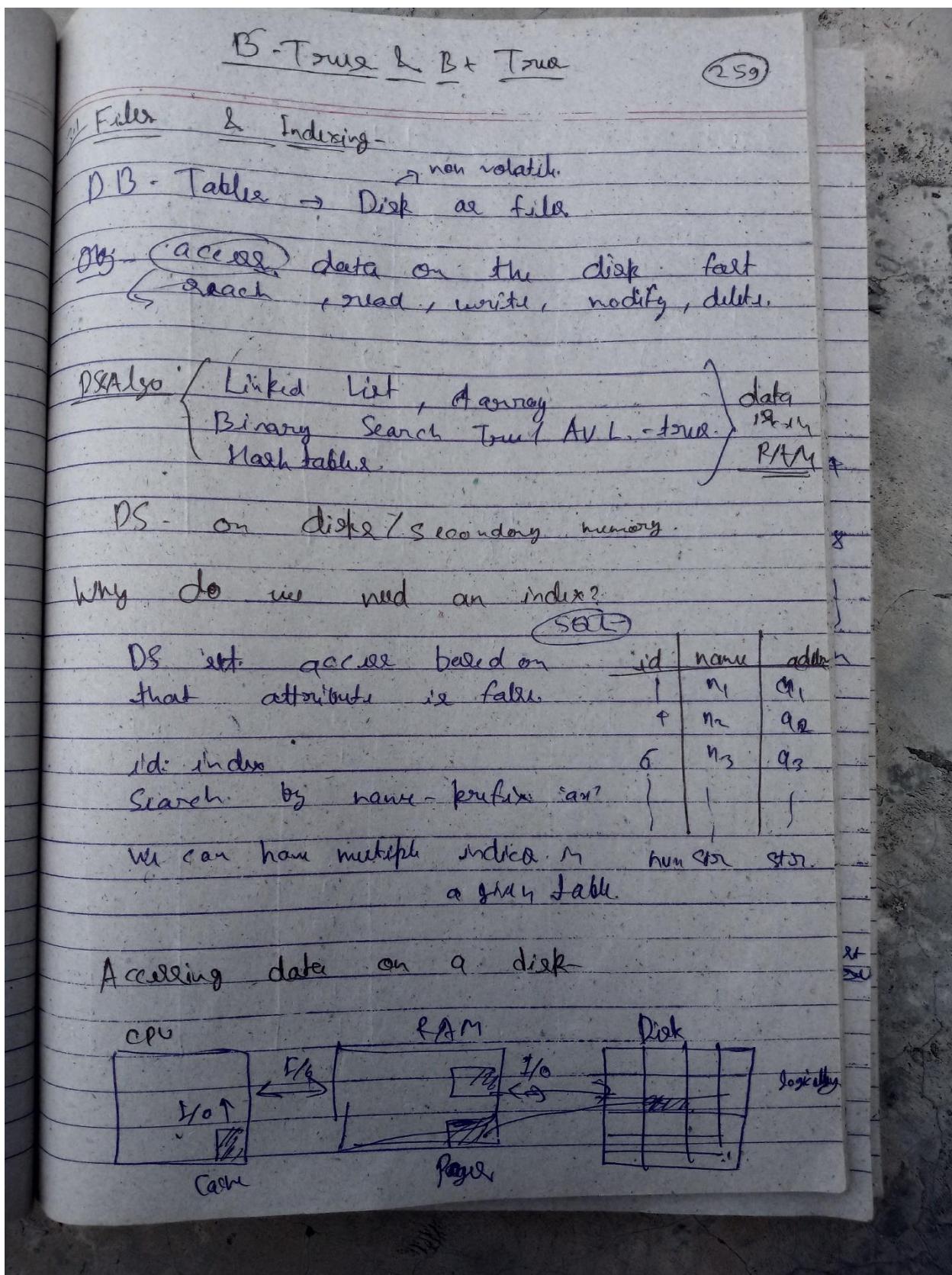


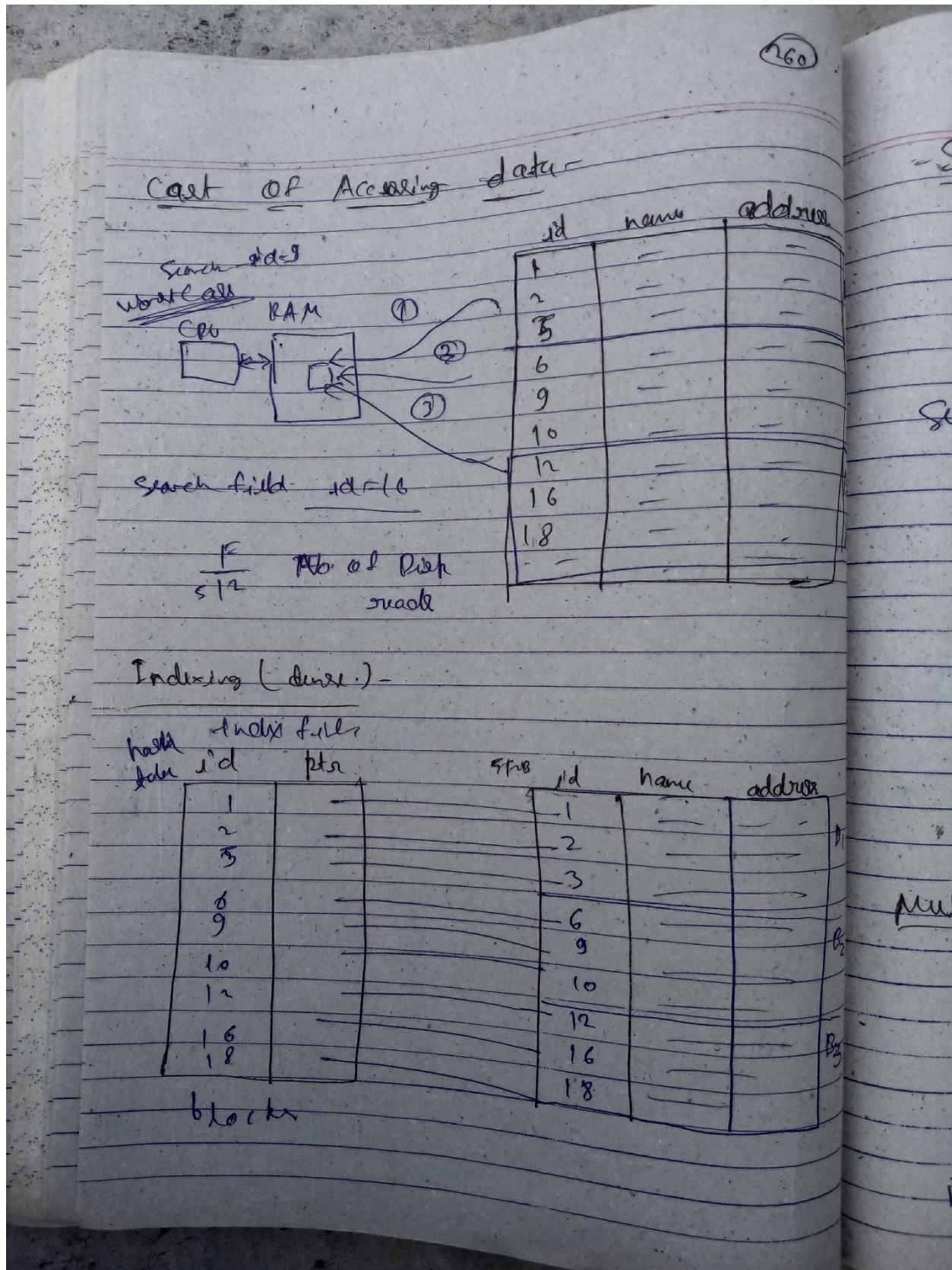
$$T(n) = O(\lg n)$$

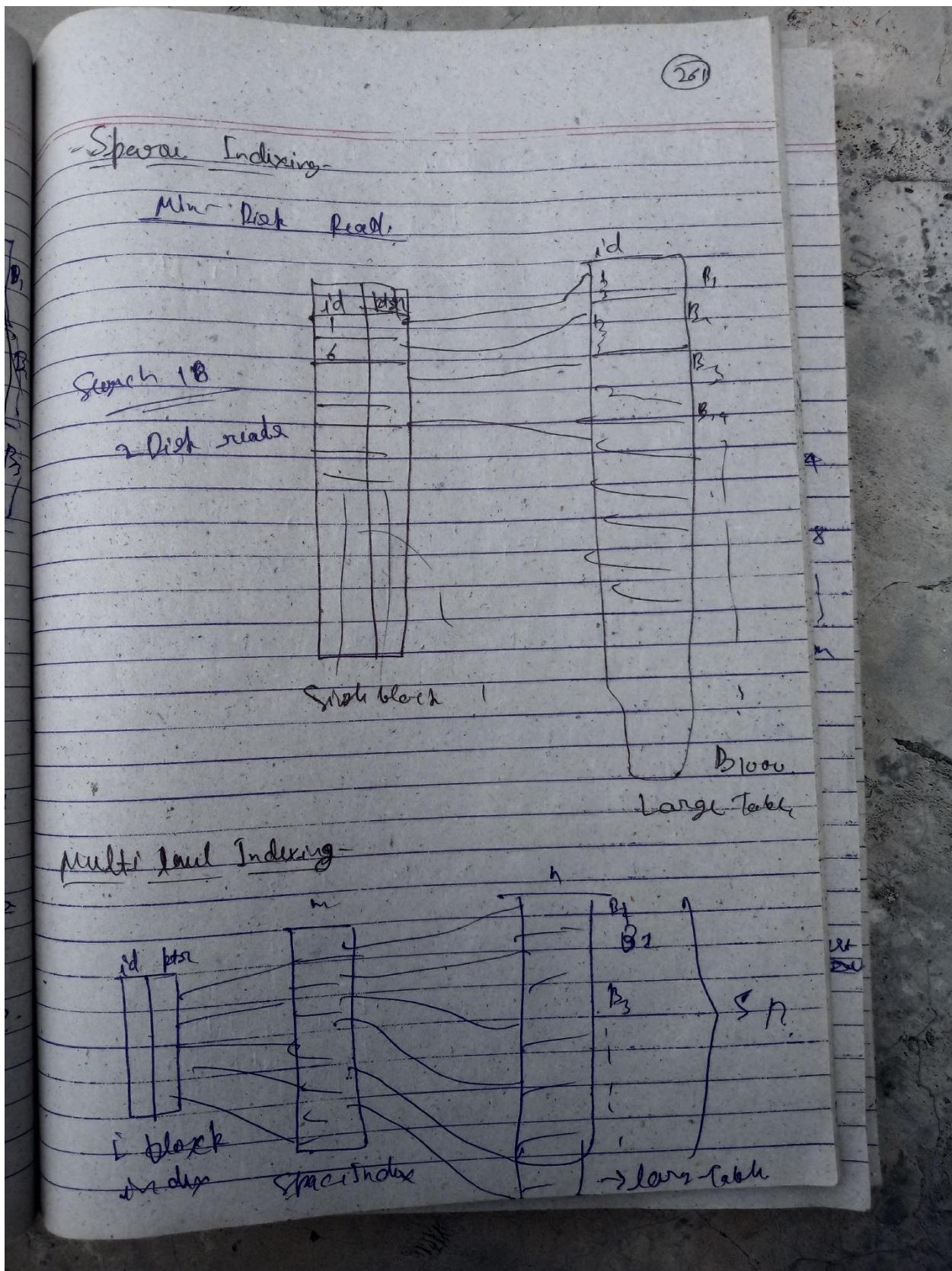
- Case 2 - if w's children are both BLACK:

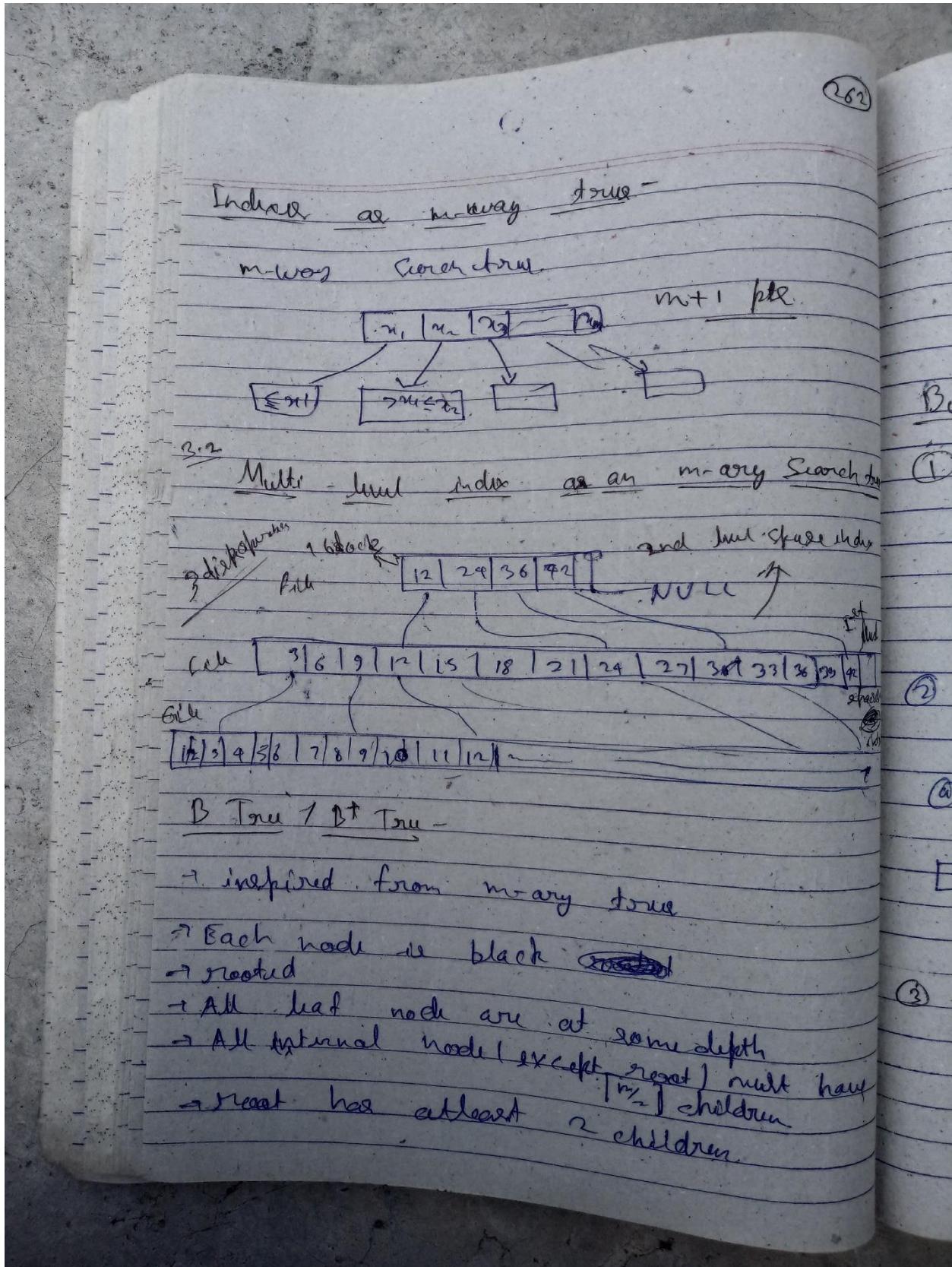
a. color(w) = red

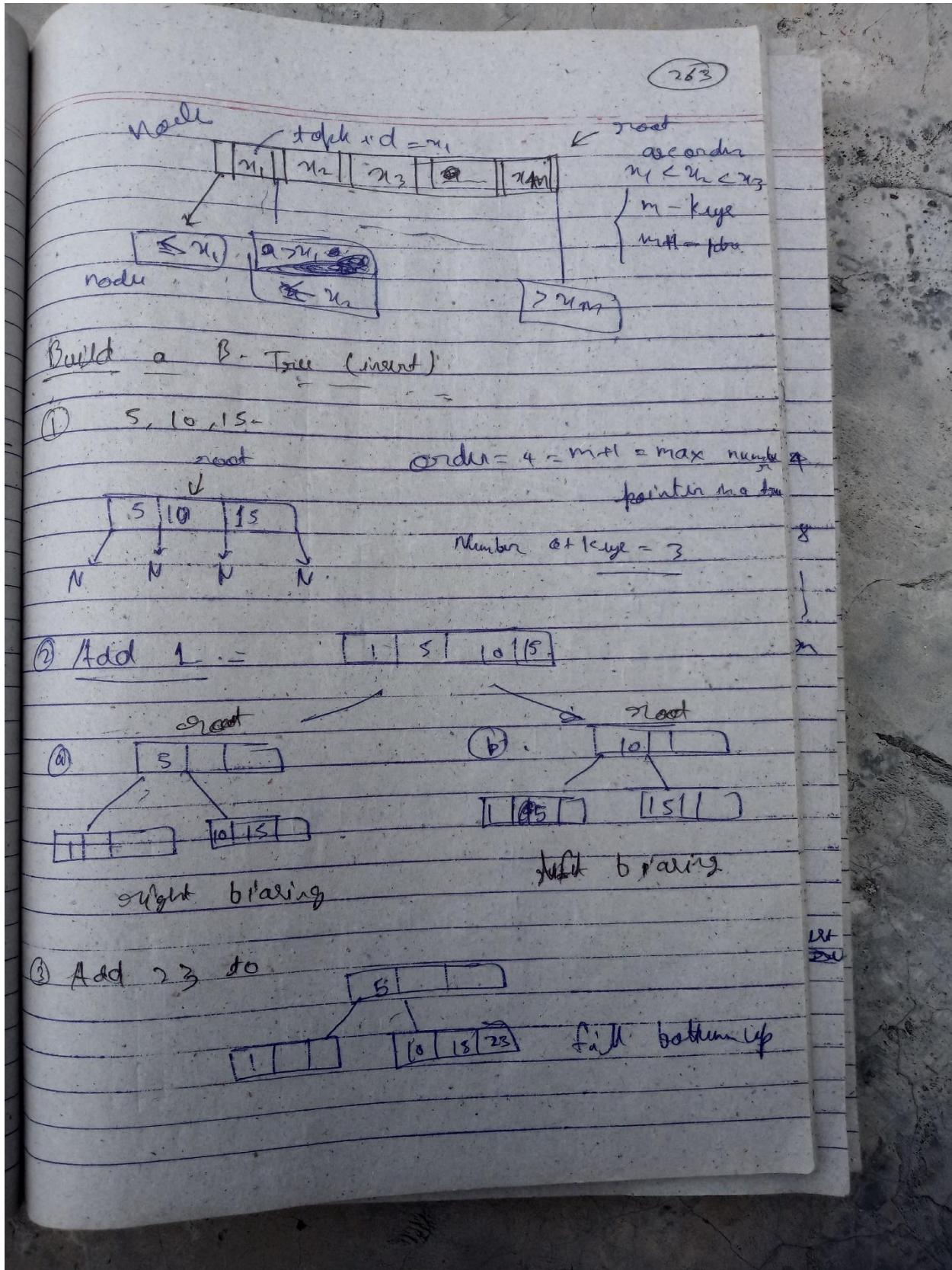
b. x = parent(w)





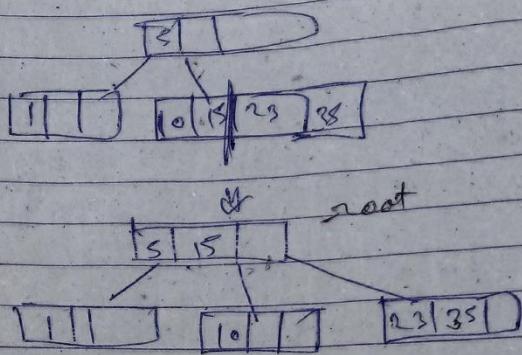




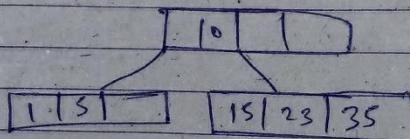


(264)

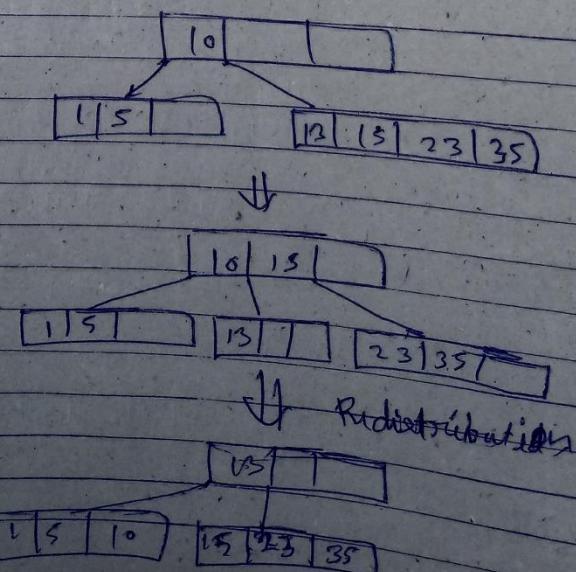
④ Add 35 to-



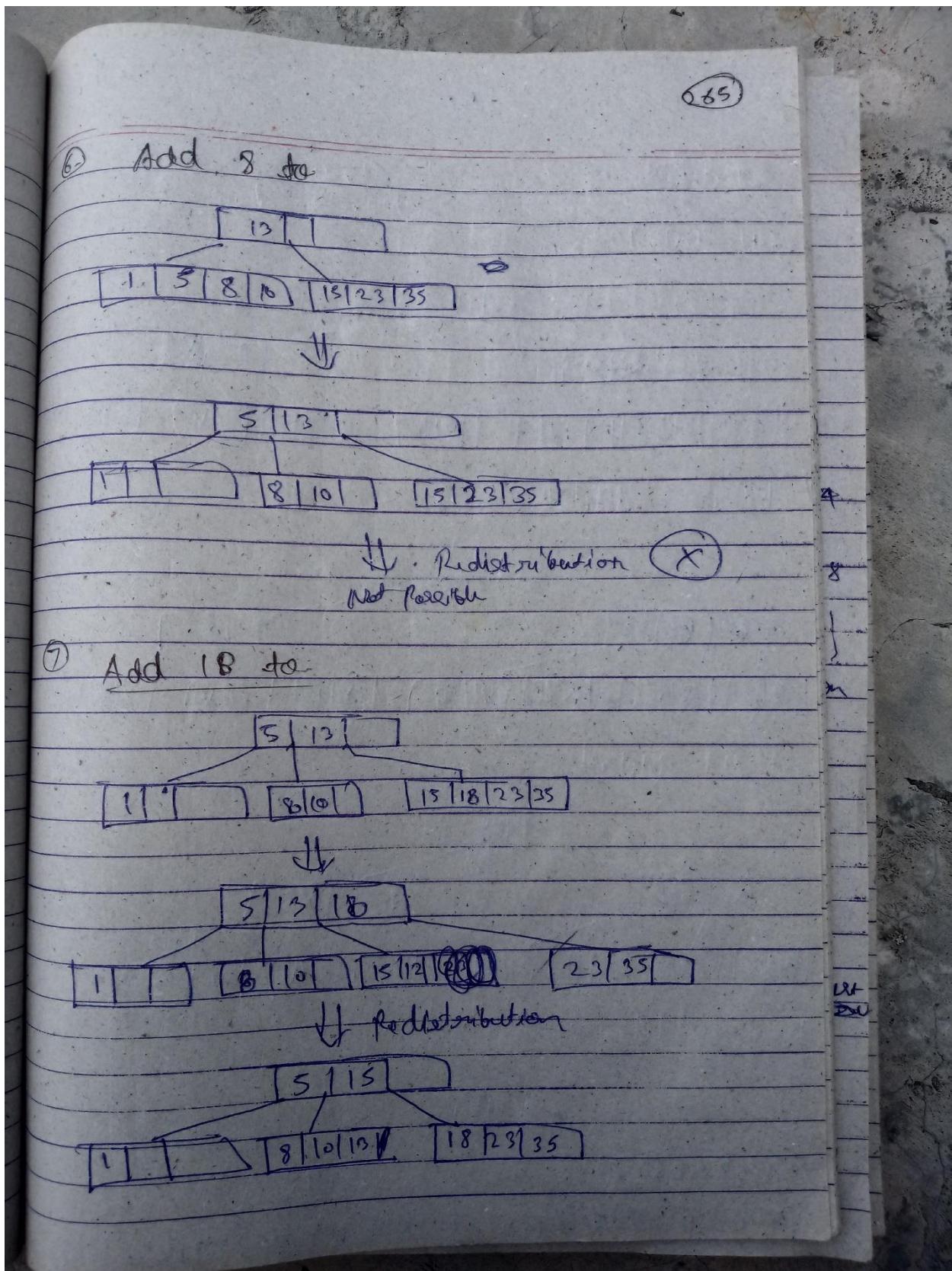
Redistributing keys ↓ to reduce no. of node.

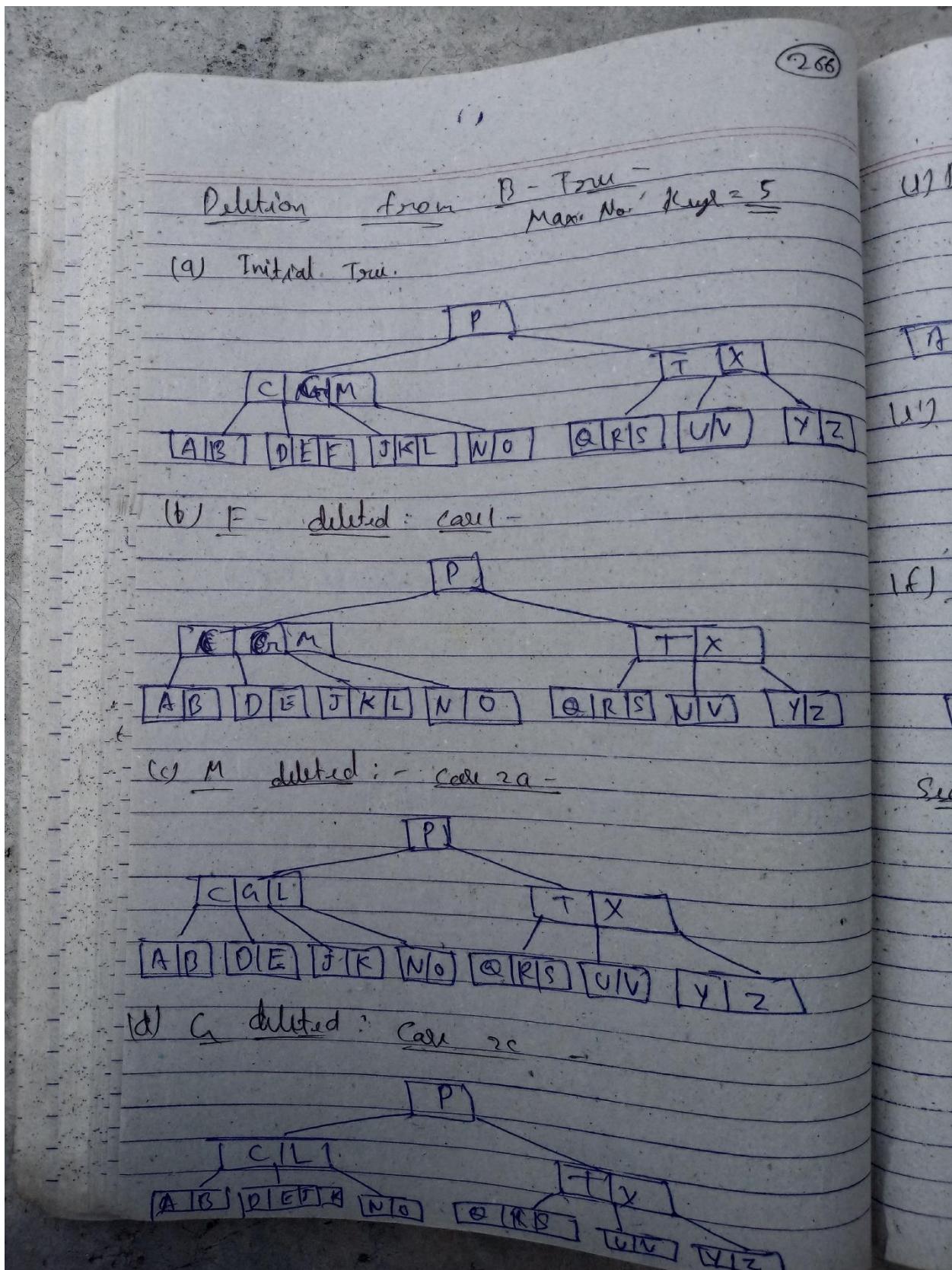


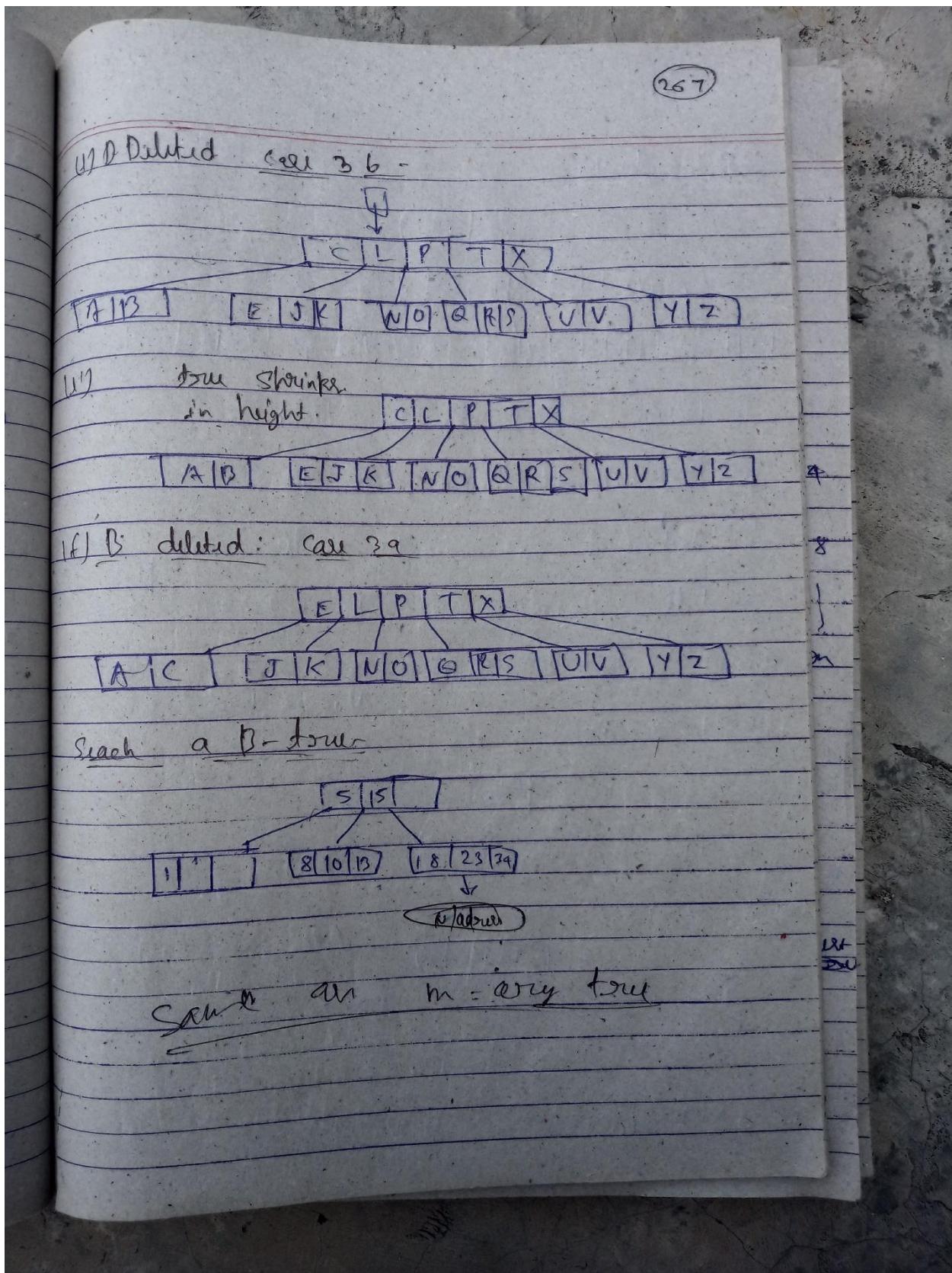
⑤ add 13 to-

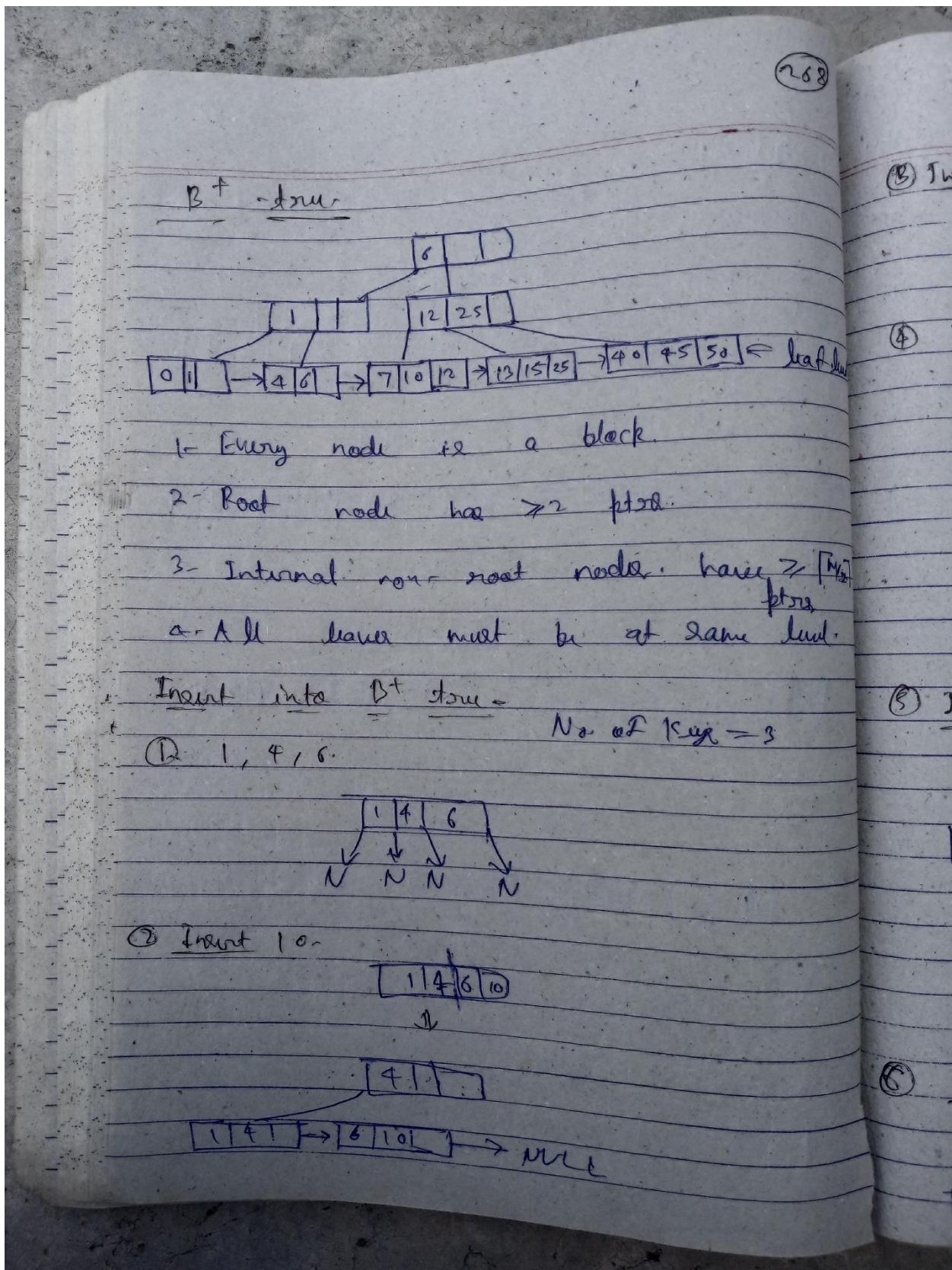


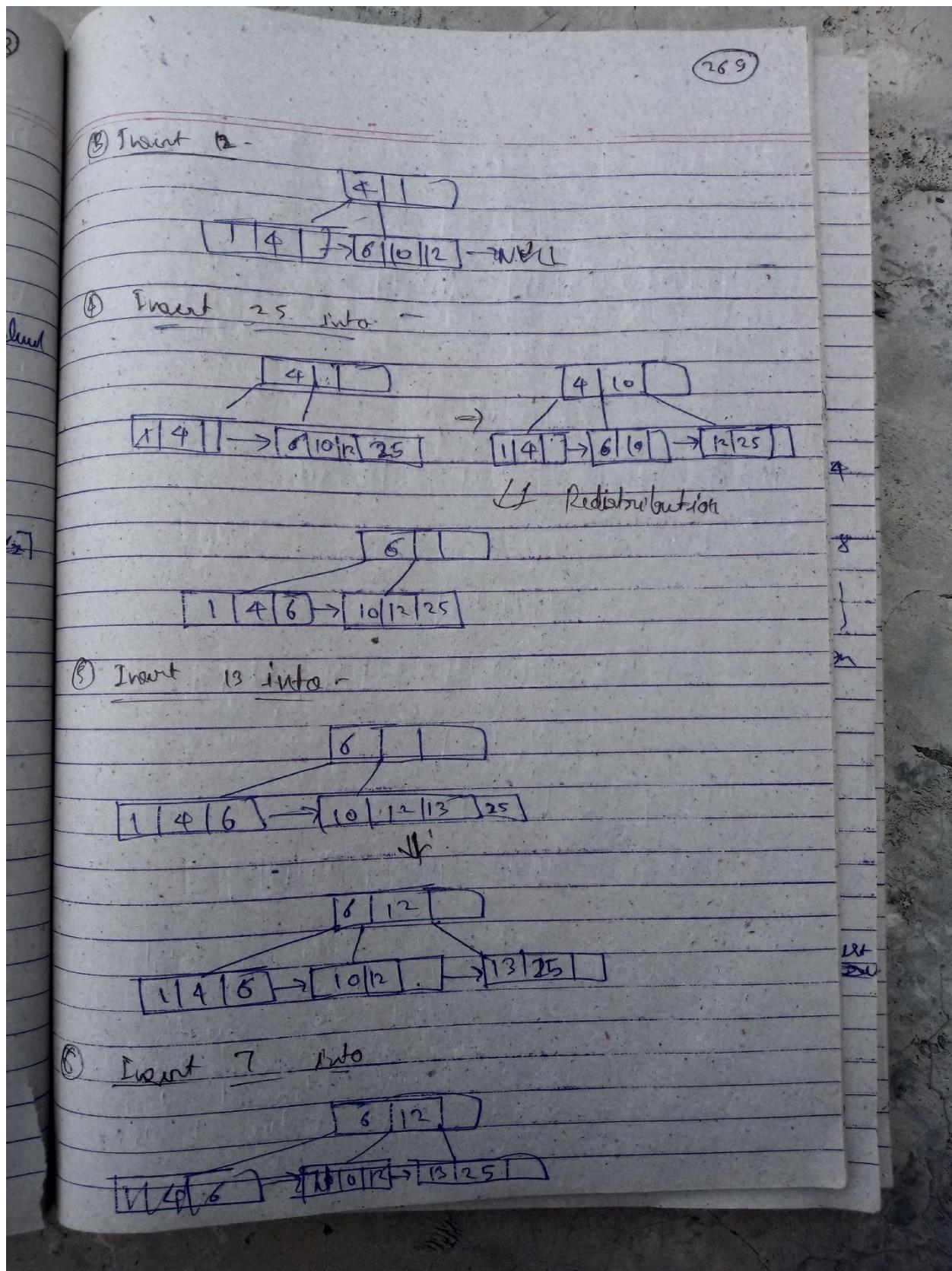
↓ Redistribution

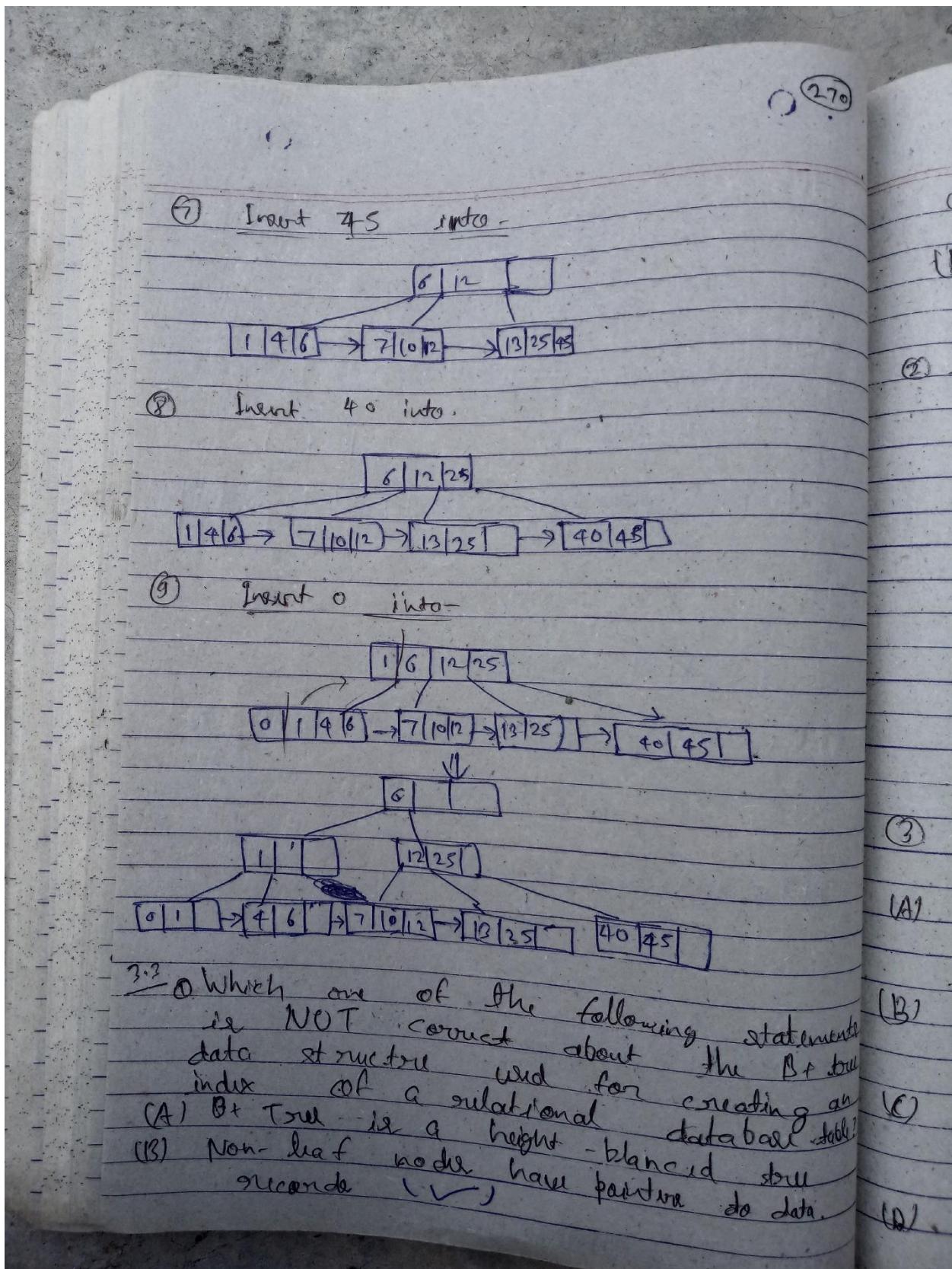












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- (C) Key values in each node are kept in sorted order ~~order~~
- (D) Each leaf node has a pointer to the next leaf node.

- (2) In a B+ tree, if the search-key value is 8 byte long, the block size is 512 byte and the block pointer is 2 bytes, then the maximum order of the B+ tree is $m+1 = 52$

$$\begin{array}{c} \boxed{m+1} \\ \downarrow \quad \downarrow \\ m \text{ keys} \\ \text{order: } m+1 \text{ pointers} \\ 8 \\ 8 \\ 8 \\ 8 \\ 8 \\ 8 \end{array}$$

$$\begin{aligned} 8m + 2(m+1) &\leq 512 \\ 10m + 2 &\leq 512 \\ 10m &\leq 510 \\ m &\leq 51 \end{aligned}$$

- (3) B+ Trees are considered BALANCED.

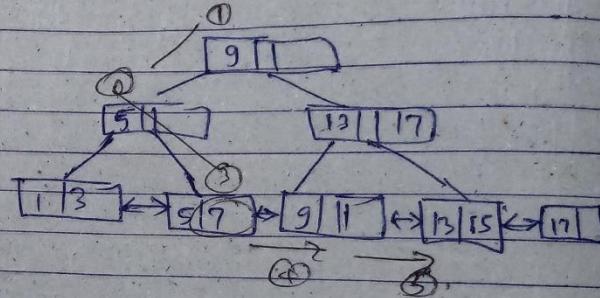
because -

- (A) the length of the paths from the root to all leaf nodes are all equal. (✓)
- (B) the lengths of the paths from the root to all leaf nodes differ from each other by at most 1.
- (C) the number of children of any two non-leaf sibling nodes differ by at most 1.
- (D) the number of records in any two leaf nodes differ by at most 1.

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④ With reference to the B+ tree index of order 3 shown below, the minimum number of nodes (including the root node) that must be fetched in order to satisfy the following query - "Get all records with a search key greater than or equal to 7 and less than 15" is -

- (A) 4
- (B) 5 (✓)
- (C) 6
- (D) 7



- (A)
- (B)
- (C)
- (D)

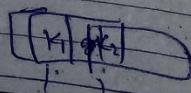
⑤ Consider a B+ tree in which the maximum number of keys in a node is 5. What is the minimum number of keys in any non-root node?

- 1- 1
- 2- 2 (✓)
- 3- 3
- 4- 4

$$m = 5$$

$$\text{No. of keys} = m + 1 = \text{order}$$

$\left(\frac{5}{2}\right)$ 5 per



Span, Range Multi index

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The order of a leaf node in a B+ tree is the maximum number of key-value, data record pointer pairs it can hold. Given that the block size is 1K byte, data records pointer is 7 bytes long, the value field is 9 bytes long and a block pointer is 6 bytes long, what is the order of the leaf node?

(A) 63 (B) 64 (C) 67 (D) 68

~~to 243 - 7 >~~

$1917 \#m + 6 \leq 1024$

$16m \leq 1018$

$m \leq 63$

Terminology related to index -

Primary & Secondary Index

key, non-key.

Clustered vs Unclustered Index -

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Ternary Search Tree

Code Class Node

```

class Node {
    char data;
    boolean isEnd;
    Node left, center, right;
}

public Node (char data,
              boolean isEnd)

```

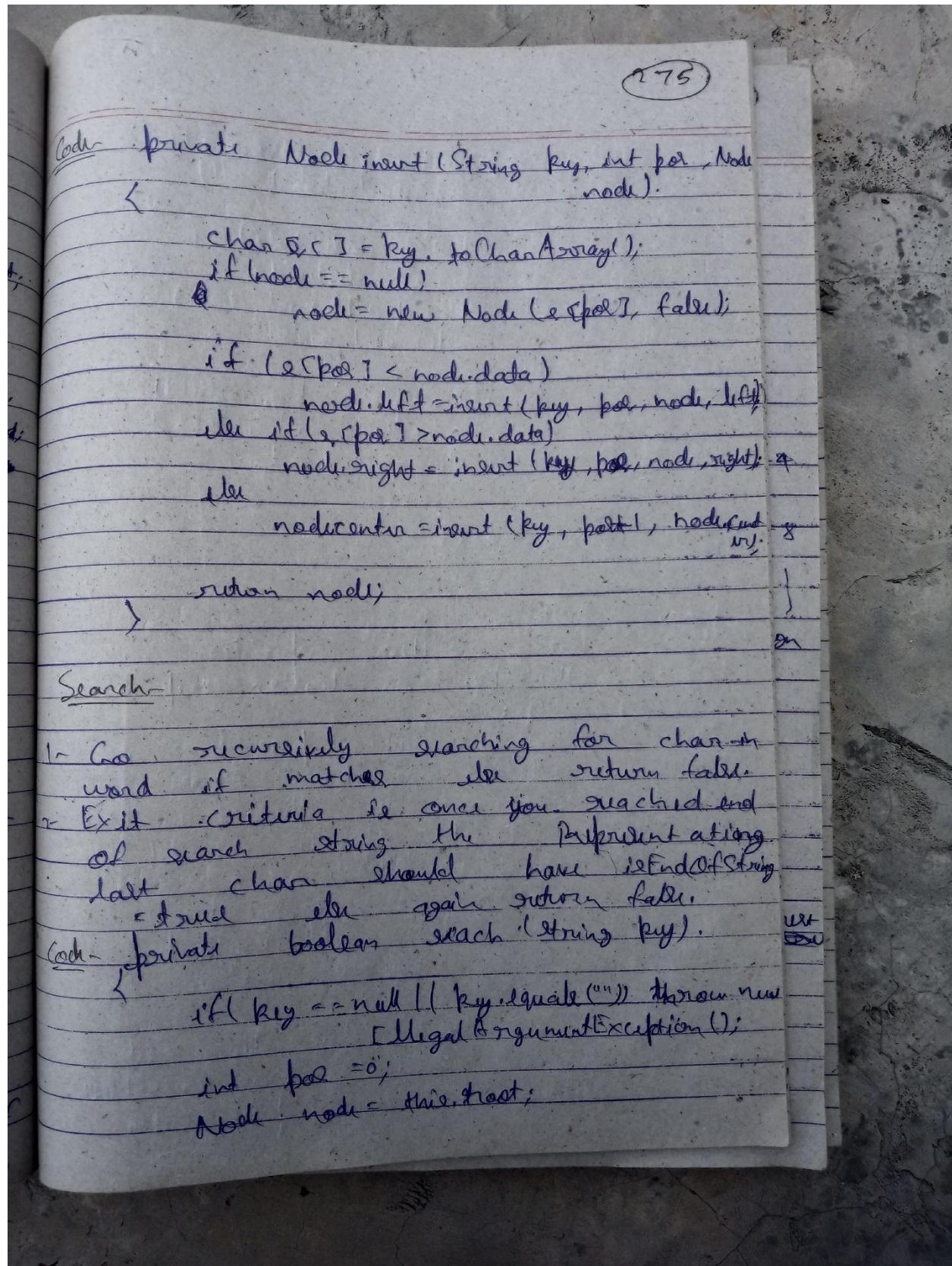
```

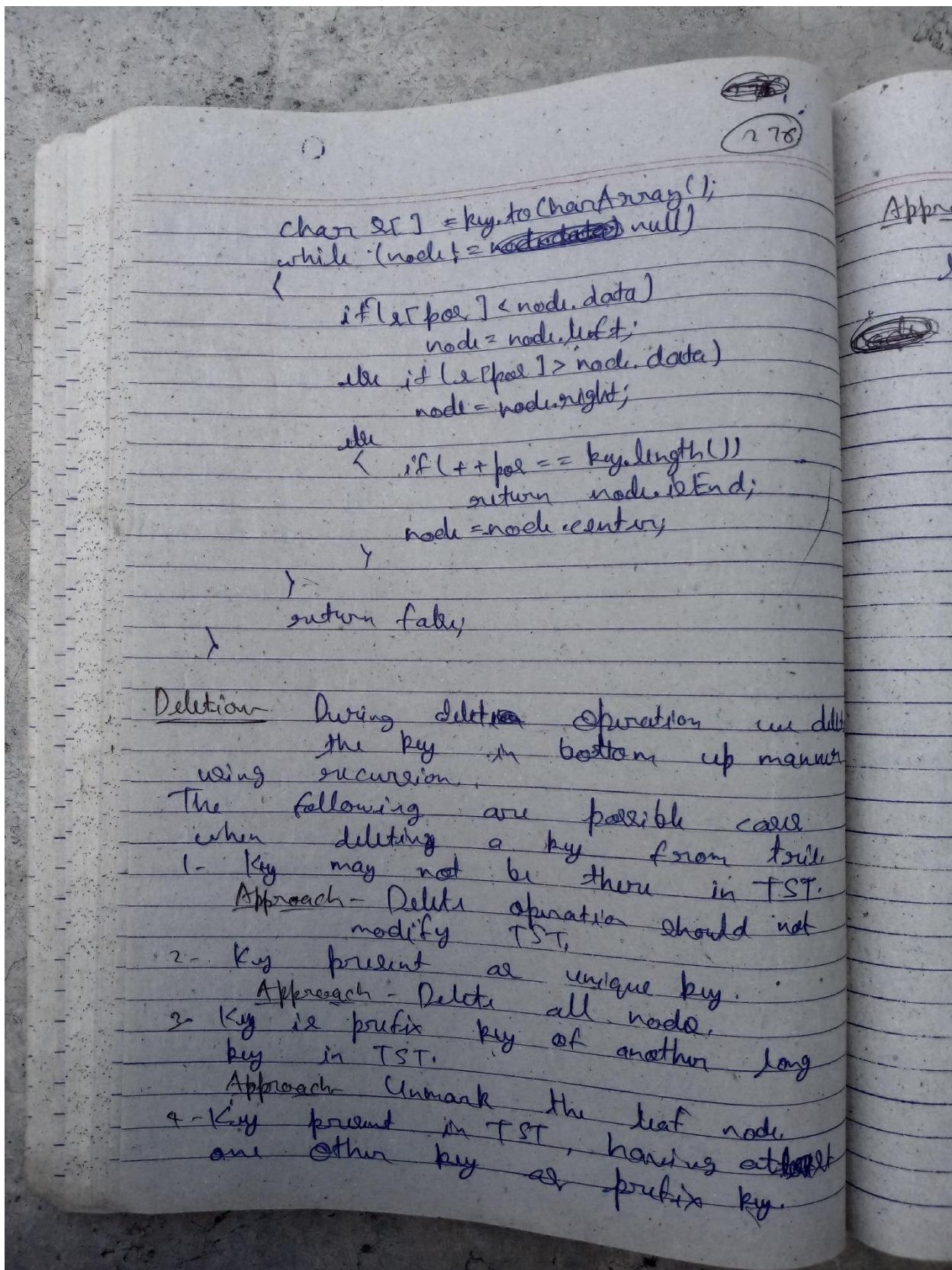
this.data = data;
this.isEnd = isEnd;
left = right = center
= null;

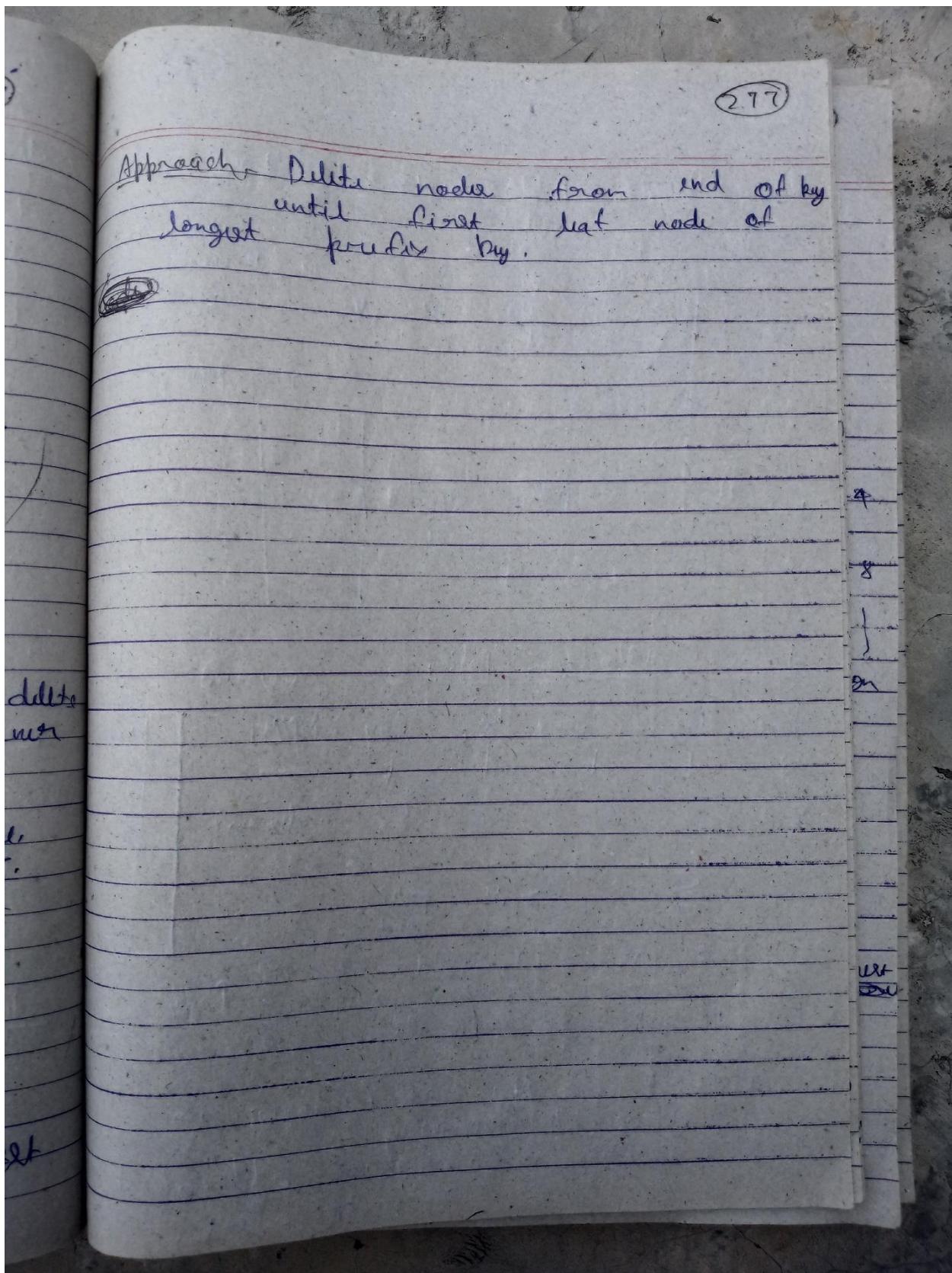
```

Invention

- 1- If Tree is empty, then insert the element.
- 2- If current character of word is smaller than root's character, then insert this word in left subtree of root.
- 3- If current character of word is greater than root's character, then insert this word in right subtree of root.
- 4- If current character of word is same as root's character.
 - (a) Insert character as root, equal.
 - (b) Check if the last character of the word, then set isEndofString = true.







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51 Page Replacement Strategies -

Page reference strings { L: length
n: # unique pages
B_i: VA_i, VA_{i+1}, VA_{i+2}...VA_{i+n} }

Page faults → Pre-demand page.

	Sequence page numbers
P ₁	f ₀ f ₁ f ₂ f ₃ f ₄

main-memory

Frame Allocation Policy

P₁, P₂, P₃, P₄, P₅ : MRU/MFU

S₁ S₂ S₃ S₄ S₅ demand

$\sum S_i = S > M$ FAP used

Let P₁, P₂, P₃, P₄, P₅

M = 4 = No. of frames in Physical Memory

a_i = No. of frames allocated to P_i

	Physical Mem
1	1
2	1
3	1
4	1
5	1

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	S_i	a_i^*	a_i^* = $\frac{S_i}{s} \times M$	
Page 1	11	8	equal alloc.	
Page 2	4	8		Prob alloc.
Page 3	20	8		some
Page 4	13	8		
Page 5	30	8		

$S = 80$ M May Not be equal to M .

$S = \sum S_i$

Prob. : $a_i^* = \frac{S_i}{s} \times M$

Eg - Reference string = 7, 0, 1, 2, 6, 3, 0, 4, 2, 3, 6, 3, 2, 1, 2, 0, 1, 7, 0, 1
 n=6 L=20. Set of unique page = {7, 0, 1, 2, 3, 4}

Page - demand - Pages. Snap.

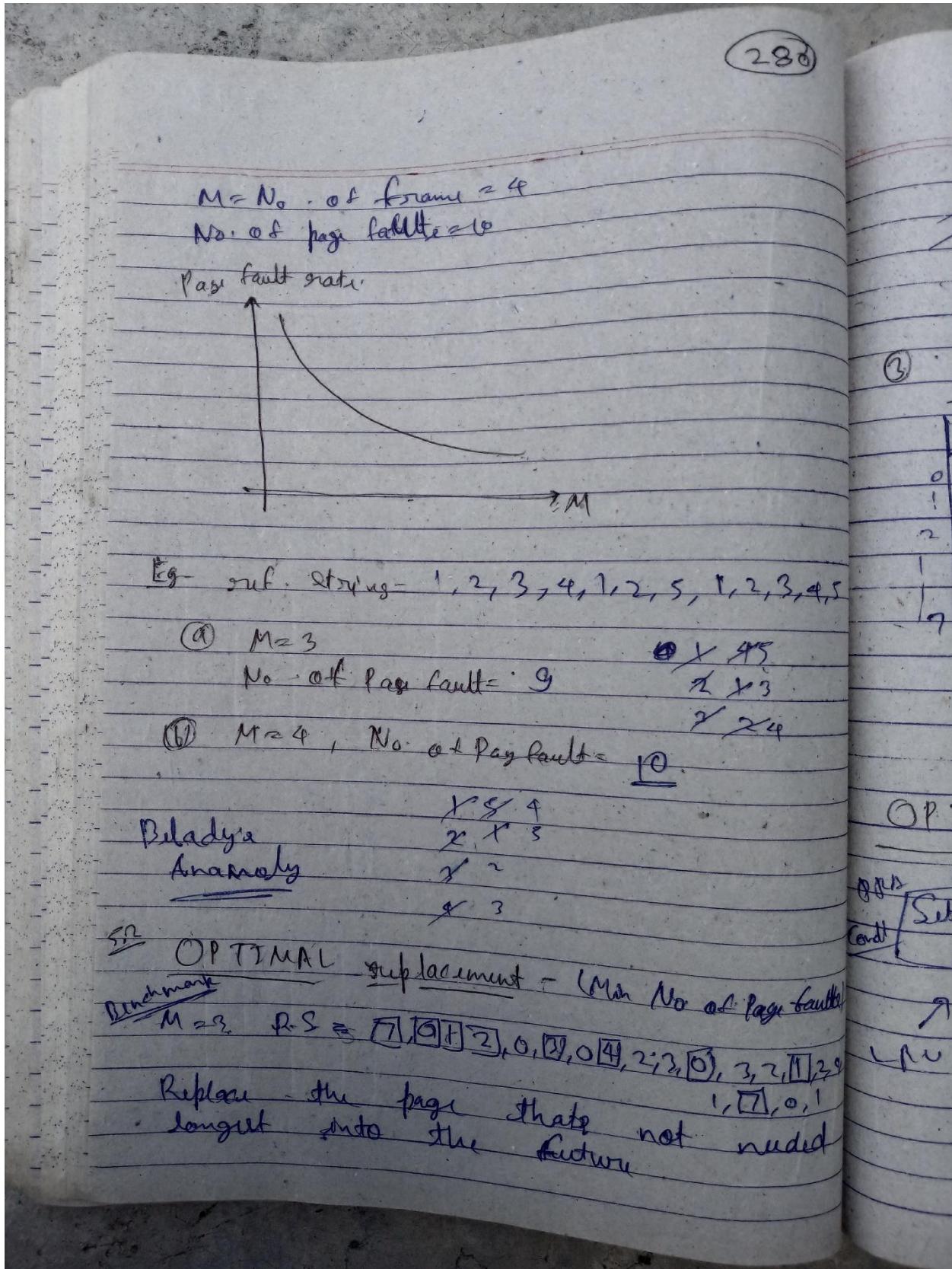
Page - Replacement Algorithms

+ First-in-First-out (FIFO) -

String - 7, 0, 1, 2, 6, 3, 0, 4, 2, 3, 6, 3, 2, 0, 1, 7, 0, 1
 SW/I TOL → Time of Loading/At

7	2	2	4	4	4	0	0	0	7	9	7	
0	0	3	3	3	2	2	2	1	1	1	0	0
1	1	1	1	1	0	0	3	3	3	2	2	1

PT of buf No. of faults = 15



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$\begin{matrix} x & x & ? \\ 0 & \rightarrow & 0 \\ & 3 & 1 \end{matrix}$ { No. Beladje's Anomaly }
 No. of page Fault = 9

Drawback - Not implementable.

③ Least Recently Used (LRU) -

	F	VI	AT	TOP	Time of Reference
0	f ₁			~	
1	f ₂			~	M = 3
2					
3					
4					
5	f ₀			~	
6					
7					

1 2 3 4 5 6 7 8
 1 2 3 4 5 6 7 8
 1 2 3 4 5 6 7 8
 1 2 3 4 5 6 7 8

No. of page Faults = 12

f_0 f_1 f_2	 X 4 0 1 3 0 2 7	Implementation - Stack + linked-list
-------------------------	---------------------------	---

OPTIMAL & LRU :- Stack algorithms [No. Beladje's anomaly]

AND
 AND
 Set of Page in memory with n-frames ⊂ Set of Page in memory with m-frames

OUT
 OUT

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(4) Most Recently Used (MRU) - Not widely used

M = 2,
 $R = [101112], 0, 1, 0, 4, 2, 3, 6, 3, 2, 1, 3, 0, 1, 2, 0$

Time ↑

2	0
0	2, 0, 4
X	1, 0, 3, 2, 1, 2, 0, 1

No. of Page Faults = 16

(5) Counting / Frequency algo → Least Frequently Used
 → Most frequently used

f VI AT TOP FC → Freq Count.

0						
1						
2						
3						
4						
5						
6						
7						

Updating the FC.
 for every memory reference.

PT. at. P_i

(6) LRU - Approximation algo (Very widely used)

(7) f VI AT R R = m pages

0	1	3	1
1	0	-	-
2	1	2	0
3	2	1	0
4	1	0	1

epoch 1 2 3 4 5 6 7 8

Calculation of PT of last page

At start of new epoch make R bits + pages

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P_0				
P_1				
P_2				
P_3				

$R + P_0 + P_1 + P_2 + P_3$

Problem :- All out-bit = 1.

(A) Second Chance / Clock - algorithm

- Just like the inferior-bit method.
- If $R=1$ & Page then FIFO on Arrival-time into main memory.

(B) Enhanced Second Chance - [Modified Second Chance]

PT	L	VTF	R	M
P_1	1	0	0	0
P_2	1	0	1	0
P_3	1	1	0	0
P_4	1	1	1	1

→ 1st [Chance - Page : disk - n]
 [Dirty Page : disk - w + disk - n].
 → 2nd
 → 3rd
 → 4th

ref string length = L 0, 1, 2, 3, ... L
 No. of unique - page = n. 0, 1, 0, 1, ... n-1
 No. of allocated to $P_i = m$. n-2, ...
 Calculate the lower & upper-bound on the no. of page faults using any page-replacement algo

Max. No. of Page faults = L

Min. No. of Page faults = n. M, N

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Q- Which page is victimized when page fault occurs?

	TOL	TLR	R	M
P ₀	126	279	0	0
P ₁	230	260	1	0
P ₂	120	272	1	1
P ₃	180	280	1	1

A-T

(A) FIFO: P₂ is replaced
 (B) LRU: P₁ is replaced
 (C) Second chance: P₀ is replaced
 (D) Enhanced Second Chance: P₀ is replaced

53 LRU Cache

Time = O(1) Space = O(capacity)

Approach via Doubly Linked List with Hash Map

Cache Struct Node

```

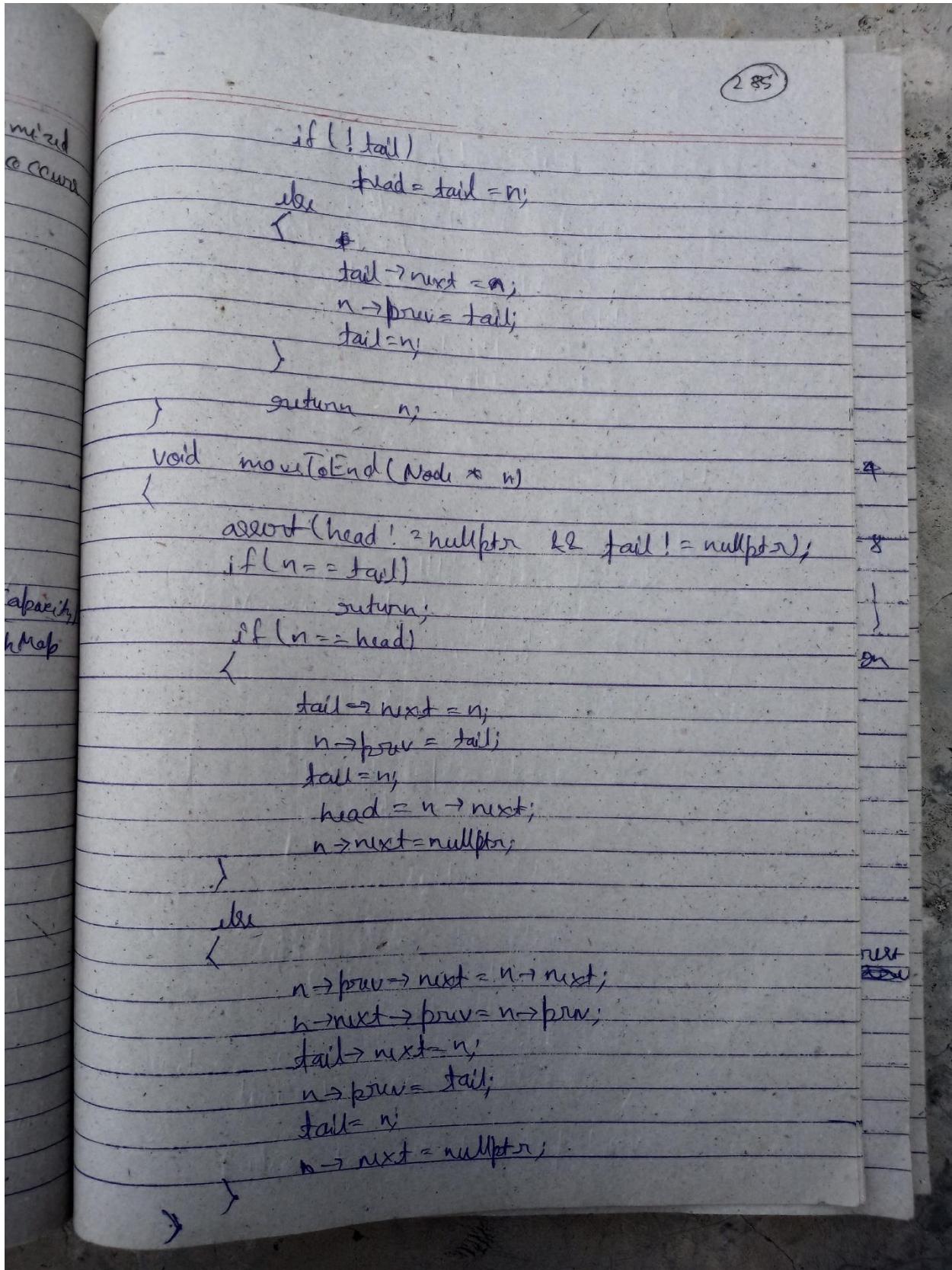
int key;
Node* prev = nullptr;
Node* next = nullptr;
Node (int key) : key(key) {}
  
```

class DoublyLinkedList

```

public:
  Node* head = nullptr;
  Node* tail = nullptr;
  void append(int key)
  {
    Node* new_node (key);
  }
  
```

Node* no new Node (key);



(286)

```

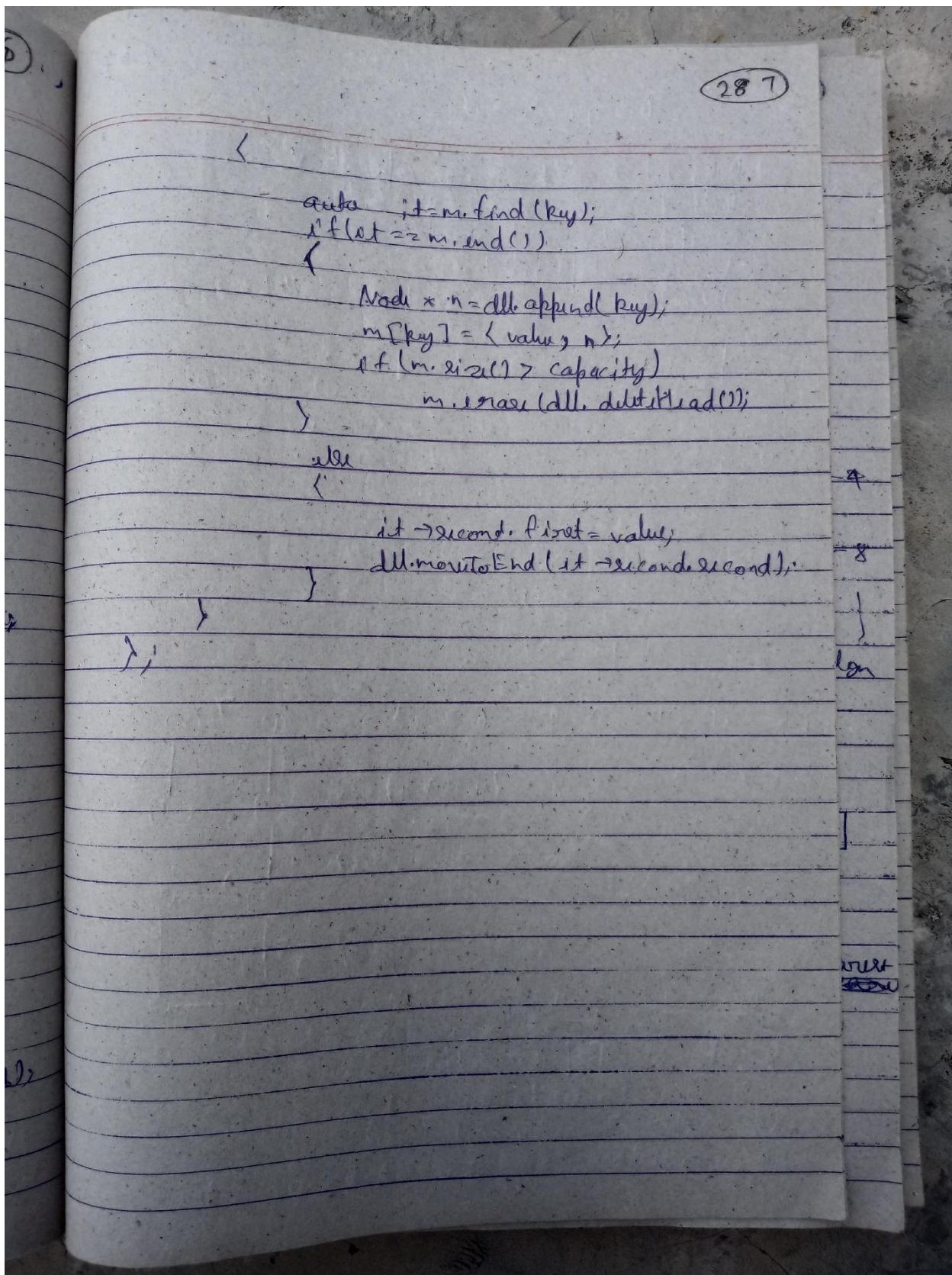
int deleteHead()
{
    assert (head != nullptr);
    Node* newHead = head->next;
    int delkey = head->key;
    delete head;
    head = newHead;
    return delkey;
}

class LRUCache
{
private:
    DoubleLinkedList *dll;
    unordered_map<int, pair<int, Node*>> m;
    int capacity;
public:
    LRUCache (int capacity)
    {
        assert (capacity > 0);
        capacity = -capacity;
    }

    int get (int key)
    {
        auto st = m.find (key);
        if (st == m.end ())
            return -1;
        dll.moveToEnd (st->second);
        return st->second.value;
    }

    void put (int key, int value)
    {
        if (m.find (key) != m.end ())
            m[key].value = value;
        else
            m[key] = {value, dll.insert (key)};
    }
}

```



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Disjoint Sets

Q1. Disjoint Sets & operations

- Detecting a cycle
- Graphical Representation
- Array Representation
- Weighted union and collapsing Find.

Operations

- Find
- Union

$S_1 \cap S_2 = \emptyset$
 $S_1 \cup S_2 = \{10, 20, 30, 40, 50, 60, 70, 80\}$

Cycle $(10, 30)$

Ex $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$

$S_1 = \{1, 2\}$
 $S_2 = \{3, 4\}$
 $S_3 = \{5, 6\}$
 $S_4 = \{7, 8\}$

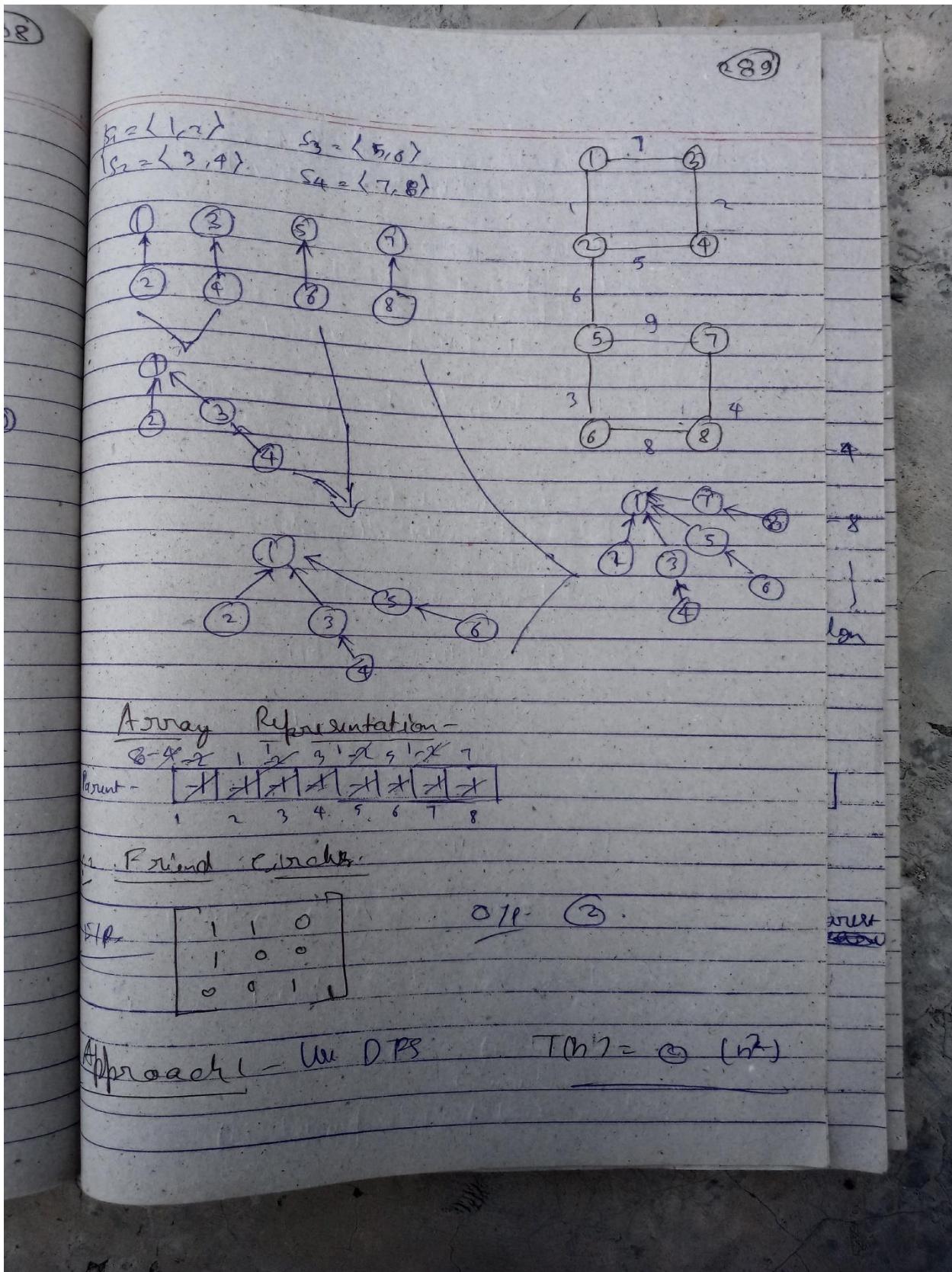
$S_{12} = \{1, 2, 3, 4, 5, 6, 7, 8\}$

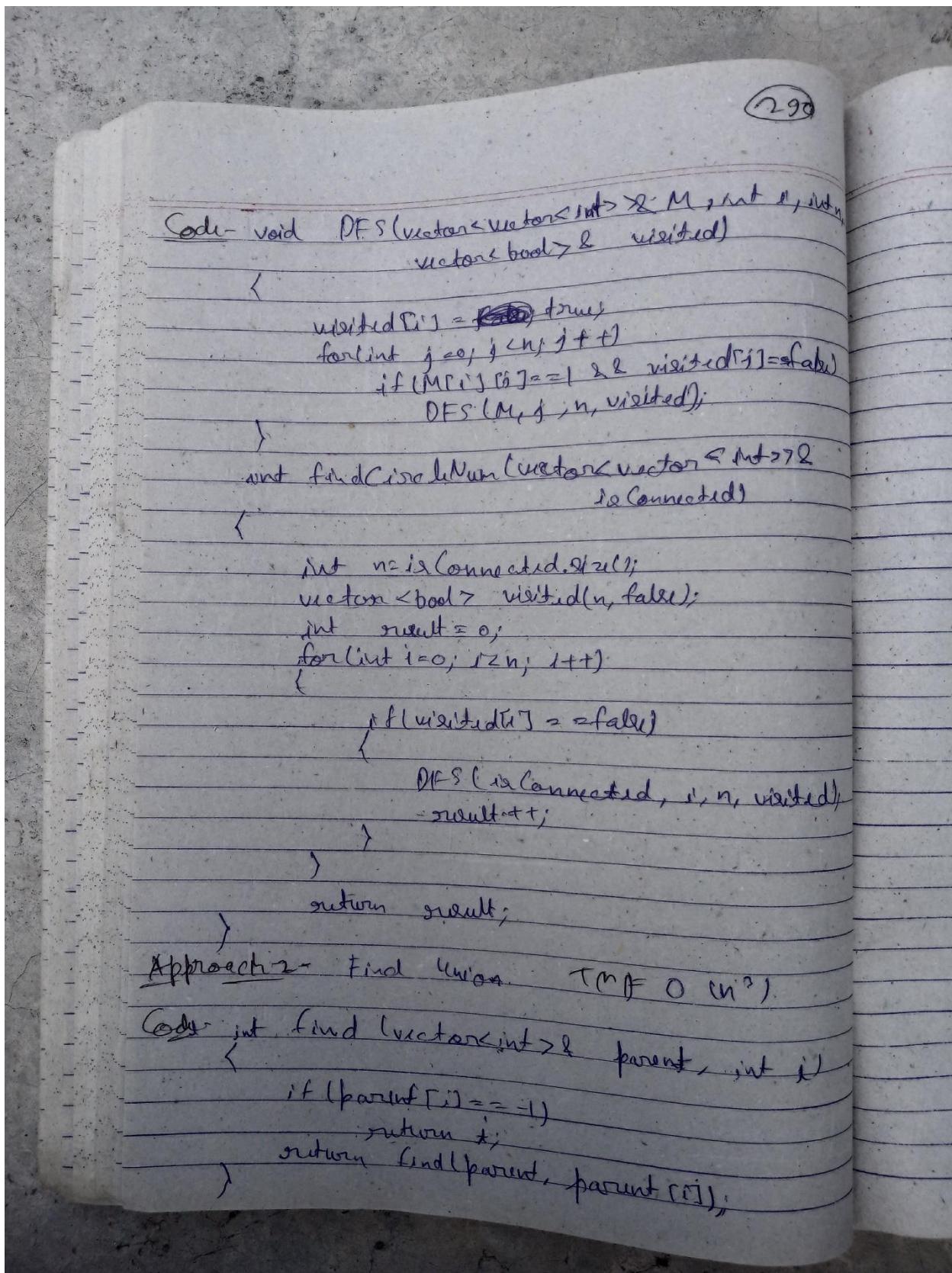
Union Find Algo

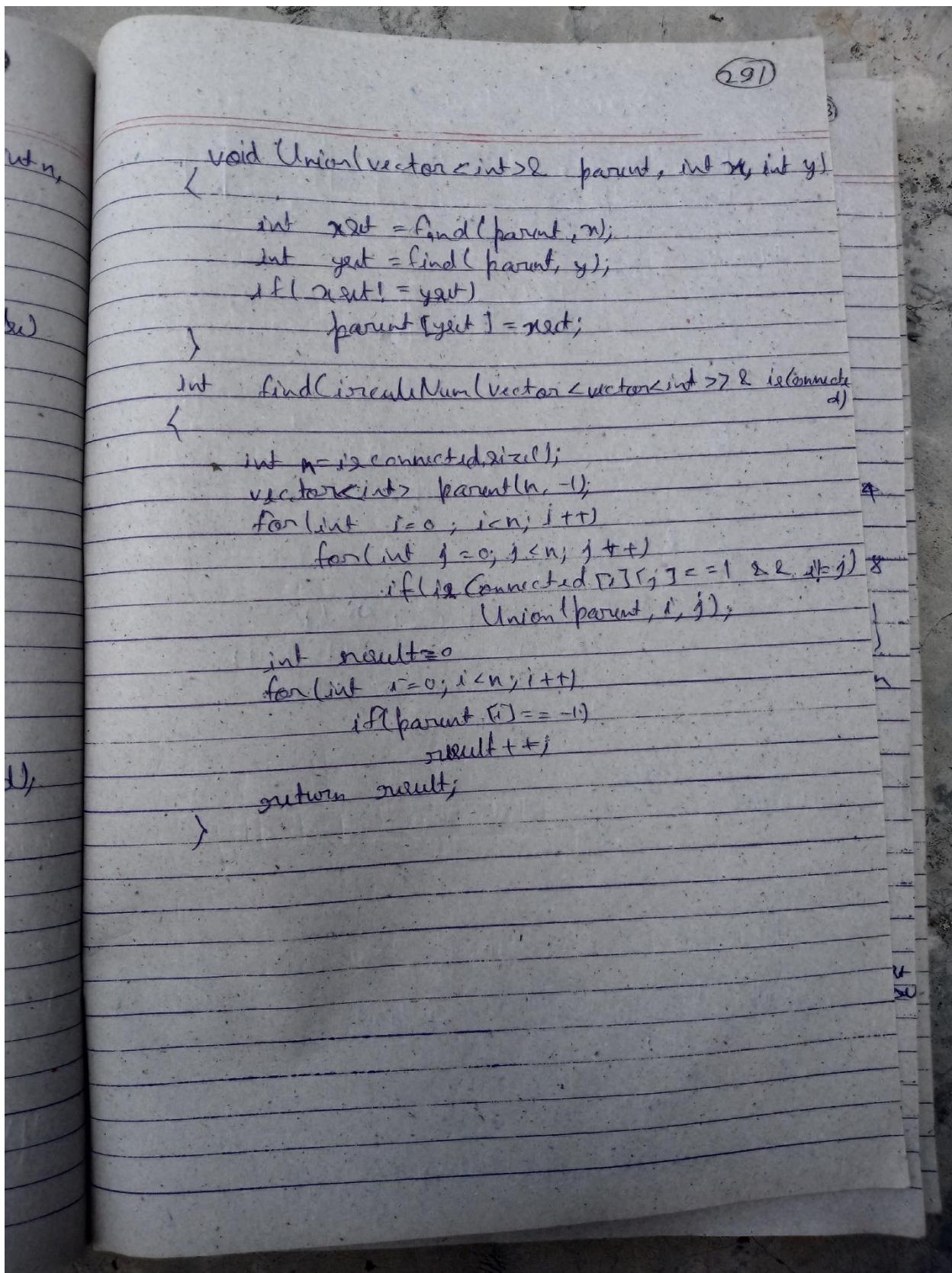
Graphical Representation

$U = \{1, 2, 3, 4, 5, 6, 7, 8\}$

① ② ③ ④ ⑤ ⑥ ⑦ ⑧







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Segment Tree

7.1 Range Queries
Range Update.

Diagram illustrating a Segment Tree node. The node is represented by a rectangle divided into four quadrants. The top-left quadrant contains a question mark, representing a query. The bottom-right quadrant contains a plus sign, representing an update operation. The top-right and bottom-left quadrants are empty. Labels include 'Q' above the top-left quadrant, 'L' and 'R' below the top-right quadrant, and 'update' next to the plus sign.

Naive Approach

for, in range (l, r):
style // takes input

if type = 1:
sum =
// take input L and R
for j in range ($L, R+1$):
 $sum += A[j]$
print sum

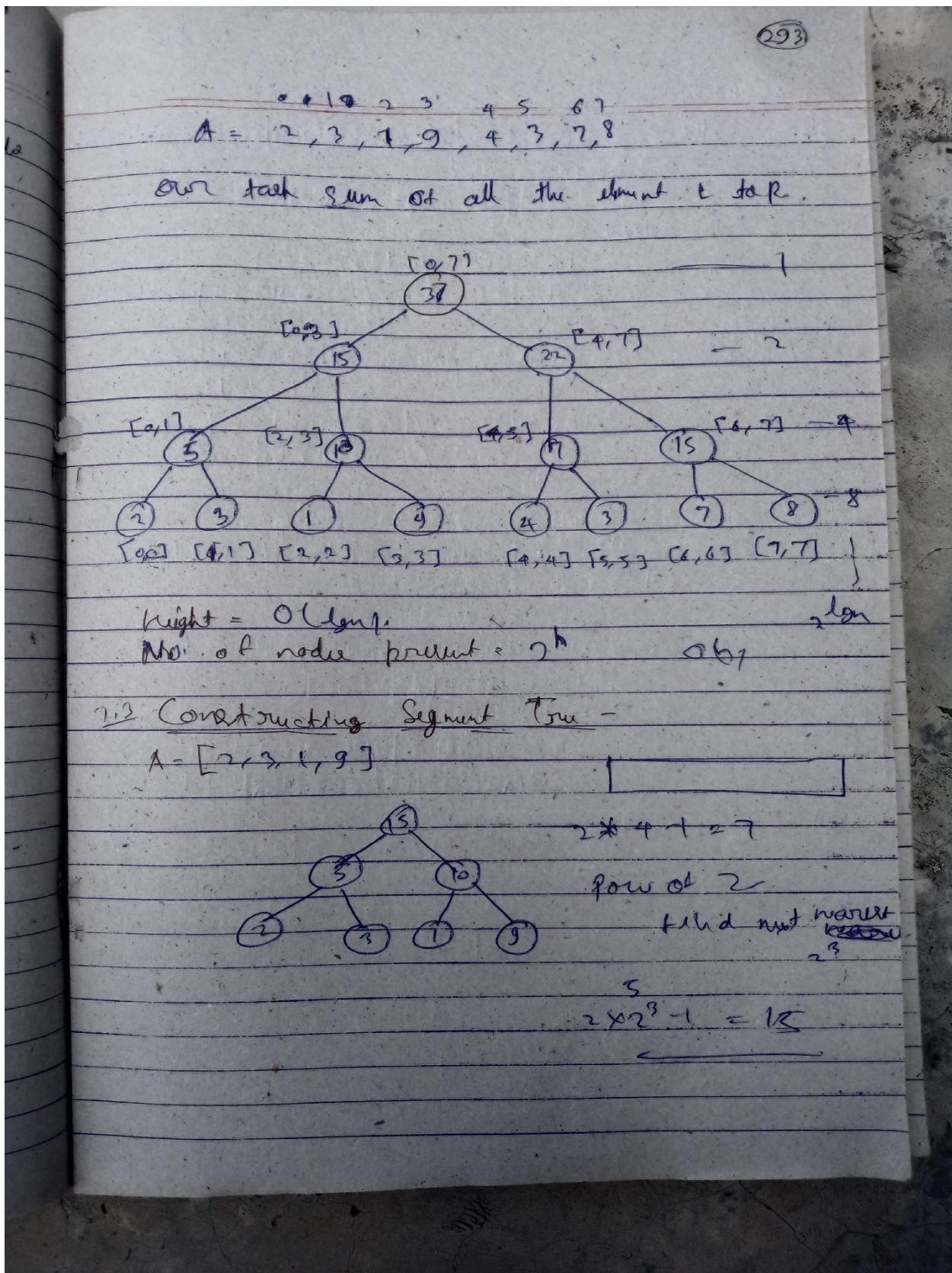
if type = 2:
// take input x.
 $A[i] = x$
 $A[3] = 6$

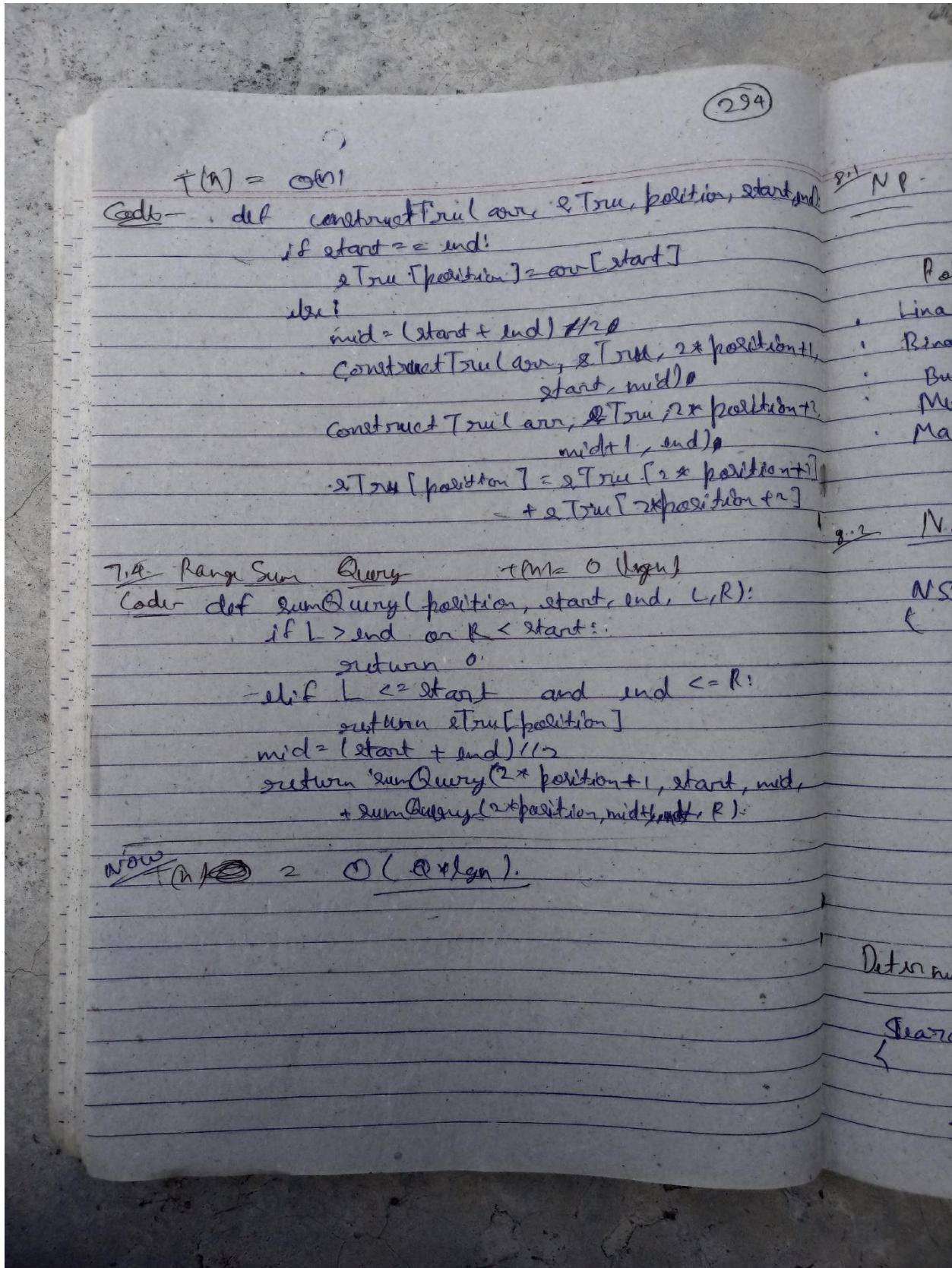
7.2 Structure of segment tree-

Fully Binary Tree. $O(n)$

Segment
↳
Binary tree

Store the interval for segment 1.





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Complexity Class

NP-Hard and NP-Complete

Polynomial Time <ul style="list-style-type: none"> Linear Search = n Binary Search = $\log n$ Bubble Sort = n^2 Merge Sort = $n \log n$ Matrix chain multiplication = n^3 	Exponential Time <ul style="list-style-type: none"> 0/1 knapsack = 2^n Satellite Problem = 2^n Travelling Salesman = 2^n Graph coloring = 2^n
--	--

Non Deterministic Algorithm

NSearch(A, n, key)

```

i = choose(1)           = O(n)
if (key == A[i])
{
    write(i)             = O(1)
    success()             = O(1)
}
write(0)                 = O(1)
failure()                = O(1)
}
= O(n)
  
```

Deterministic Polynomial algorithm

Search(A, n, key)

```

for i = 0 to n
{
    if (key == A[i])
        return i
}
return -1
  
```

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```

if (key == A[w])
    front(D)
} print(element not found)
  
```

P	NP
\downarrow	\downarrow
Linear BFS Merge	DFS Sub

$P \subseteq NP$.

8.3 Satisfiability - CNF Boolean

$$\pi_1 = \langle x_1, x_2, x_3 \rangle$$

$$CNF = (x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee x_2 \vee x_3)$$

If we can't solve and exponential time problem in polynomial time then one find the relationship between two exponential time problems.

$\overset{P}{\circ}$ $\overset{NP}{\circ}$ $\overset{NP\text{-Hard}}{\circ}$

\cap

$NP\text{-Complete}$