HYBRID STATE ESTIMATION

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Introduction:

The modern power system is much complex and large due to ever growing demand of energy in present world. Thus the reliability of electric power supply becomes of utmost importance in today's time. We need information regarding control and protection of the power system to maintain higher level of reliability. This information(measurements) is provided to TSO (Transmission System Operator). These information (measurements) are obtained by system state estimation.

Traditionally the measurements include voltage magnitudes active and reactive power injections as well as active and reactive power-flows. These measurements are gathered by SCADA(Supervisory Control and Data Acquisition).The main entity of the SCADA system is RTU(Remote terminal Unit).RTU captures data through CT's and PT's which are installed through the transmission grid. Current and Potential Transformers capture voltage and current signals and give rms value of analog quantities. These quantities are given to the RTU and RTU further communicates it to the control centre where TSO is present. From the gathered data and its applications and its output Transmission System Operator takes corrective or preventive actions with the help of SCADA.

After the US blackout of 2003 where northeast part of us and central part of Canadian grid faced blackout. It almost took a week to fully restore. When assessment was done on this blackout it was documented that due to the lack of real time monitoring of the power system this took place.

Wide Area Monitoring System (WAMS) is now the most common measurement data source which is used in the modern power system. The main entity of the WAMS is PMU.PMU stands for Phasor Measurement Unit. PMU's captures analog data(through ct's and pt's) and gives phasor quantities. All these measurements are send to the control centre which in this case is PDC.PDC stands for phasor data concentrator. PMU'S are GPS-synchronized. All the measuring devices are at different geographical locations but when they send their measurements to control centre (PDC) then along with the phasor measurements they have time tag associated with them. Synchrophaser data is used in the modern state estimations which has enhanced its accuracy and performance. PMU measures voltage phasors of the buses at which it is installed and current phasors of all or some of the adjoining lines through that bus with very high accuracy. The placement of PMUs is done by using various algorithms and is called as optimal PMU placement. PMUs directly measures the system state which simplifies the mathematical formulation of state estimation in a linear form. The rate of reporting measurement data by the PMUs are almost 100 times faster than the RTUs which is very important for tracking system states in a highly dynamic power system network.

Due to the economic constraints PMU's are used in limited quantities. Thus only synchrophasor measurements are not enough for full state estimation execution. Thus in todays world both RTU measurements(SCADA) as well as PMU measurements(WAMS) are used for viable state estimation implementation. This integration of SCADA(RTU) and WAMS(PMU) measurements in state estimation is called as Hybrid State Estimation.

Algorithmic description of conventional state estimation:

1) **Measurements:** We have taken 59 measurements from IEEE-14 bus system using PSCAD.

$$Redundancy\ factor = \frac{Total\ no\ of\ measurement}{Number\ of\ state\ variable} = \frac{59}{27} = 2.185$$

$$Z = [V_{mag}, P_i, Q_i, P_{f,} Q_f]^T$$

2) **Weight matrix:** It is the inverse of covariance matrix

$$W = R^{-1}$$

$$R = \begin{bmatrix} \sigma_1^2 & 0 & 0 \\ 0 & . & 0 \\ . & . & \sigma_m^2 \end{bmatrix}$$

Where m, is number of measurements

3) **Initialization:**

Assume flat start,

$$V = [1,\ 1,\ 1,\ \dots\dots\dots..1]_{1\ X\ 14}^T$$

$$\mathrm{de}l = [0,\ 0,\ 0,\ \dots\dots\dots..1]_{1\ X\ 14}^T$$

State Vector,

$$X = [del(2{:}end); V]_{27\ X\ 1}$$

Conductance,

$$G = real(Ybus)$$

Susceptance,

$$B = imag(Ybus)$$

tolerance,

$$tol = 1e^{-4}$$

iteration,

$$iter = 1$$

4) **Loop starts for first iteration**

Residue, $$delta\_z = Z - f$$

Calculation of *f*

a) f_1 *calculated value of Voltage in terms of state variable,*

$$f_1 = \begin{bmatrix} V_1 & 0 & 0 \\ 0 & V_2 & 0 \\ 0 & 0 & V_{nvi} \end{bmatrix}_{nvi\ X\ nbus}$$

b) f_2, *calculated value of injected real power*

$$f_2 = [P_1;\ P_2;\ P_3;\ \dots\dots]_{(npi\ X\ 1)}$$

$$P_i = \sum_{k=1}^{nbus} V_i * V_k * [G_{ik} \cos(\delta_i - \delta_k) + B_{ik} \sin(\delta_i - \delta_k)]$$

c) f_3, *calculated value of injected reactive power*

$$f_3 = [Q_1; Q_2; Q_3; \;\dots\dots]_{(nqi\,X\,1)}$$

$$Q_i = \sum_{k=1}^{nbus} V_i * V_k * [G_{ik} \sin(\delta_i - \delta_k) - B_{ik} \cos(\delta_i - \delta_k)]$$

d) f_4, *calculated value of real power flows*

$$f_4 = [P_{fl1}; P_{fl2}; P_{fl3}; \;\dots\dots]_{(npf\,X\,1)}$$

$$P_{fl} = -V_i^2 * G_{ik} - V_i * V_k * [-G_{ik} \cos(\delta_i - \delta_k) - B_{ik} \sin(\delta_i - \delta_k)]$$

e) f_5, *calculated value of reactive power flows*

$$f_5 = [Q_{fl1}; Q_{fl2}; Q_{fl3}; \;\dots\dots]_{(nqf\,X\,1)}$$

$$Q_{fl} = -V_i^2(-B_{ik} + b_{ik}) - V_i V_k[-G_{ik} \sin(\delta_i - \delta_k) + B_{ik} \cos(\delta_i - \delta_k)]$$

Now,

$$f = [f_1; f_2; f_3; f_4; f_5]$$

$$delta\_z = Z - f$$

Calculation of jacobian (H):

$$H = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \\ H_{31} & H_{32} \\ H_{41} & H_{42} \\ H_{51} & H_{52} \end{bmatrix}_{nmeas\,X\,nstate}$$

a) H_{11} is $\frac{dv}{d\delta}$ at X_0

$$H_{11} = \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{bmatrix}_{nvi\,X\,nbus-1}$$

H_{11} is a zero matrix

b) H_{12} is $\frac{dV_i}{dV_k}$

Order of H_{12} is $(nvi,\ nbus)$

Diagonal elements = 1

Off-diagonal elements = 0

c) H_{21} is $dP_i/d\delta$

Order of H_{21} is $(npi,\ nbus-1)$

Diagonal elements

$$\left.\frac{dP_i}{d\delta_k}\right|_{k=i} = \sum_{k=1}^{N} V_i V_k[-G_{ik}\sin(\delta_i - \delta_K) + B_{ik}\cos(\delta_i - \delta_k)] - V_i^2 B_{ii}$$

Off-Diagonal elements

$$\left.\frac{dP_i}{d\delta_k}\right|_{k\neq i} = V_i V_k[G_{ik}\sin(\delta_i - \delta_k) - B_{ik}\cos(\delta_i - \delta_k)$$

d) H_{22} is $\frac{dP_i}{dV_k}$

Order of H_{22} is $(npi,\ nbus)$

Diagonal Elements

$$\left.\frac{dP_i}{dV_k}\right|_{k=i} = \sum_{k=1}^{N} V_i\ [G_{ik}\cos(\delta_i - \delta_K) + B_{ik}\cos(\delta_i - \delta_k)] - V_i G_{ii}$$

Off-diagonal elements

$$\left.\frac{dP_i}{dV_k}\right|_{k\neq i} = V_i\ [G_{ik}\cos(\delta_i - \delta_k) + B_{ik}\cos(\delta_i - \delta_k)$$

e) H_{31} is $\frac{dQ}{d\delta}$

Order of H_{22} is $(nqi,\ nbus-1)$

Diagonal element,

$$\left.\frac{dQ_i}{d\delta_k}\right|_{k=i} = \sum_{k=1}^{N} V_i V_k\ [G_{ik}\cos(\delta_i - \delta_K) + B_{ik}\cos(\delta_i - \delta_k)] - V_i^2 G_{ii}$$

Off-diagonal elements,

$$\left.\frac{dP_i}{dV_k}\right|_{k\neq i} = V_i\ V_k[-G_{ik}\cos(\delta_i - \delta_k) - B_{ik}\cos(\delta_i - \delta_k)$$

f) H_{32} is $\frac{dQ}{dV}$

Order of H_{32} is $(nqi,\ nbus)$

Diagonal elements,

g) H_{41} is $\frac{dP_{fl}}{d\delta}$

Order = $(npf,\ nbus-1)$

$$\frac{dP_{ik}}{d\delta_i} = V_i V_k * [-G_{ik}\sin(\delta_i - \delta_k) + B_{ik}\cos(\delta_i - \delta_k)]$$

$$\frac{dP_{ik}}{d\delta_k} = -V_i V_k * [-G_{ik}\sin(\delta_i - \delta_k) + B_{ik}\cos(\delta_i - \delta_k)]$$

All other elements will be zero

h) H_{42} is $\frac{dP_{fl}}{dV}$

Order = $(npf,\ nbus)$

$\frac{dP_{ik}}{dV_i} = -V_k * [-G_{ik}\cos(\delta_i - \delta_k) - B_{ik}\sin(\delta_i - \delta_k)] - 2G_{ik}V_i$

$\frac{dP_{ik}}{dV_k} = -V_i * [-G_{ik}\cos(\delta_i - \delta_k) - B_{ik}\sin(\delta_i - \delta_k)]$

All other elements will be zero

i) H_{51} is $\frac{dQ_{fl}}{d\delta}$

Order = $(npf,\ nbus)$

$\frac{dQ_{ik}}{d\delta_i} = -V_iV_k * [-G_{ik}\cos(\delta_i - \delta_k) - B_{ik}\sin(\delta_i - \delta_k)]$

$\frac{dQ_{ik}}{d\delta_k} = V_iV_k * [-G_{ik}\cos(\delta_i - \delta_k) - B_{ik}\sin(\delta_i - \delta_k)]$

All other elements will be zero

$$\left.\frac{dQ_i}{dV_k}\right|_{k=i} = \sum_{k=1}^{N} V_k[G_{ik}\sin(\delta_i - \delta_K) - B_{ik}\cos(\delta_i - \delta_k)]$$

Off-diagonal elements,

$$\left.\frac{dQ_i}{dV_k}\right|_{k\neq i} = V_i\ [G_{ik}\sin(\delta_i - \delta_k) - B_{ik}\cos\ (\delta_i - \delta_k)$$

j) H_{52} is $\frac{dQ_{fl}}{dV}$

Order = $(npf,\ nbus - 1)$

$\frac{dQ_{ik}}{dV_i} = -V_k * [-G_{ik}\sin(\delta_i - \delta_k) + B_{ik}\cos(\delta_i - \delta_k)] - 2V_i(-B_{ik} + bpq_{ik})$

$\frac{dQ_{ik}}{dV_k} = -V_i * [-G_{ik}\sin(\delta_i - \delta_k) + B_{ik}\cos(\delta_i - \delta_k)]$

All other elements will be zero

$$H = [H_{11}\ H_{12}; H_{21}\ H_{22}; H_{31}\ H_{32}; H_{41}\ H_{42}; H_{51}\ H_{52}]$$

Gain matrix,

$$G_m = H'R^{-1}H$$

Delta_x

$$dX = G_m^{-1}H'R^{-1}delta\_z$$

Tolerance,

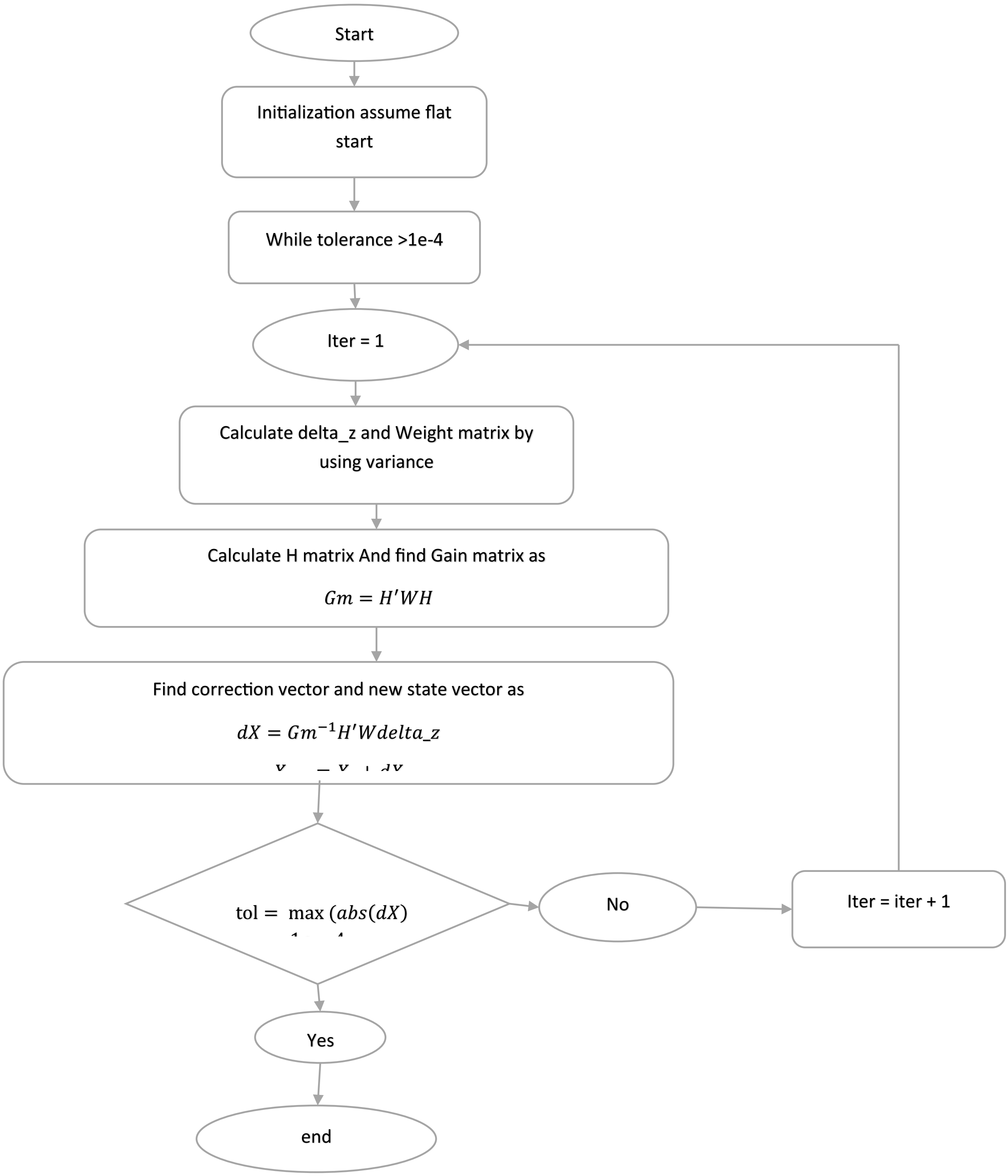
$$tol = \max\ (abs(dX))$$

When tolerance exceeds loop will end

New state vector,

$$X_{t+1} = X_t + dX$$

Flow chart of conventional state estimation:



Hybrid state estimation

Measurement: we use three type of measurements

a) We have used the direct state output from the conventional state estimation process discussed earlier as a measurement.
b) Voltage phasor measurement for different buses using PMU
c) Flow current measurement for different lines using PMU

For creating a measurement vector we break measurements into real part and imaginary part

$$Z = \left[V_{rse}\ \ V_{ise}\ \ V_{rpmu}\ \ V_{ipmu}\ I_{rpmu}\ I_{iPMU}\right]^T$$

Weight matrix: It is the inverse of covariance matrix

$$W = R^{-1}$$

$$R = \begin{bmatrix} \sigma_1^2 & 0 & 0 \\ 0 & . & 0 \\ . & . & \sigma_m^2 \end{bmatrix}$$

Where m, is number of measurements

State Vector:

$$X = [V_{real}\ V_{imag}]$$

The new measurement model is,

$$\begin{bmatrix} V_{rse} \\ V_{ise} \\ V_{rpmu} \\ V_{ipmu} \\ I_{rpmu} \\ I_{ipmu} \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \\ H_{31} & H_{32} \\ H_{41} & H_{42} \\ H_{51} & H_{52} \\ H_{61} & H_{62} \end{bmatrix} \begin{bmatrix} V_{real} \\ V_{imag} \end{bmatrix} + \begin{bmatrix} e_{rse}^V \\ e_{ise}^V \\ e_{rpmu}^V \\ e_{ipmu}^V \\ e_{rpmu}^I \\ e_{ipmu}^I \end{bmatrix}$$

$$Z = HX + error$$

The state estimate can be solved directly without iteration using Weighted least square solution as given below:

$$X = (H'WH)^{-1}(H'WZ) = G^{-1}H'WZ$$

Where G is the gain matrix

Calculation of element of H matrix

1) H_{11} it represents $\frac{dV_{rse}}{dV_{real}}$
Order of H_{11} is (nbus, nbus)

Diagonal elements of H_{11} = 1
All other elements = 0
H_{11}= Identity matrix of order (nbus, nbus)

2) H_{12} it represents $\frac{dV_{rse}}{dV_{imag}}$
Order of H_{12} is (nbus,nbus)

All elements =0

$$H_{12} = \text{null matrix of order (nbus,nbus)}$$

3) H_{21} it represents $\frac{dV_{ise}}{dV_{real}}$
   Order of H_{21} is (nbus,nbus)

All elements =0

$$H_{21} = \text{null matrix of order (nbus,nbus)}$$

4) H_{22} it represents $\frac{dV_{ise}}{dV_{imag}}$
   Order of H_{22} is (nbus, nbus)

Diagonal elements of H_{22} = 1
All other elements = 0
H_{22}= Identity matrix of order (nbus, nbus)

5) H_{31} it represents $\frac{dV_{rpmu}}{dV_{real}}$
   Order of H_{31} is $(nvi,\ nbus)$
   nvi = number of voltage measurement using pmu

   Each row i corresponds to pmu i and has all zeros except at the j th column corresponding to the index of pmu Ri V in the state vector.
   Eg : H_{31i} = [0 0 0 0 .... 1 .... 0]

6) H_{32} it represents $\frac{dV_{rpmu}}{dV_{imag}}$

   Order of H_{32} is $(nvi,\ nbus)$

   nvi = number of voltage measurement using pmu

All elements =0

$$H_{32} = \text{null matrix of order (nbus,nbus)}$$

7) H_{41} it represents $\frac{dV_{ipmu}}{dV_{real}}$

   Order of H_{41} is $(nvi,\ nbus)$

   nvi = number of voltage measurement using pmu

All elements =0

$$H_{41} = \text{null matrix of order (nbus,nbus)}$$

8) H_{42} it represents $\frac{dV_{ipmu}}{dV_{imag}}$
   Order of H_{31} is $(nvi,\ nbus)$
   nvi = number of voltage measurement using pmu

   Each row i corresponds to pmu i and has all zeros except at the j th column corresponding to the index of pmu Ri V in the state vector.
   Eg : H_{42i} = [0 0 0 0 .... 1 .... 0]

For calculating element of H matrix corresponding to flow current phasors first of all we have to derive formulae how the real and imaginary part of is depending on state variable for doing so we modelled the transmission line as a pie model

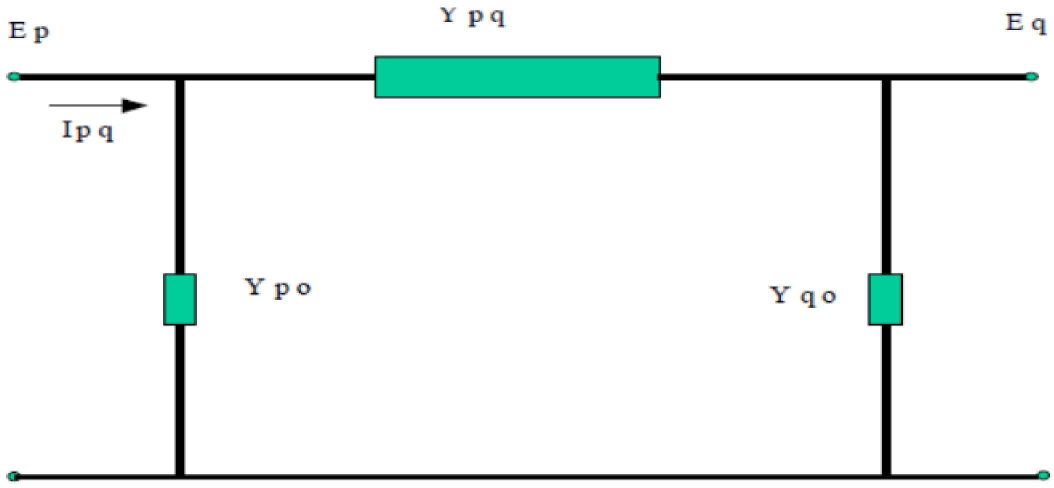


Figure 1 Pi model of transmission line

$$I_{PQ} = E_p * Y_{po} + (E_P - E_q) * Y_{PQ}$$

H_{51} is an is $(L * nbus)$matrix (L being the number of PMU current measurements) whose elements of will be purely conductances while H_{52} is an $(L * nbus)$matrix that has purely susceptance elements. Conversely, H_{61} is an $(L * nbus)$ matrix that has purely susceptance elements and those of the LxN matrix H_{62} will be purely conductance.

Steps for implementing hybrid state estimation for simulation

Step 1: Take measurements from PSCAD and we have assumed that pmu's are optimally placed at different location in our case of IEEE 14 bus system pmu's are placed at bus number 2,6 7 and 9. So the system is fully observable.

First we have taken the measurement from PSCAD and added the gaussian noise to them, for doing conventional state estimation we have taken the measurement similar to that RTU gives means only magnitude.

Name of measurement	Bus Numbers	Number of measurement
Voltage	2,6,7,9,10,11,12,13,14	9
Real power injection	10,11,12,13,14	5
Reactive Power injection	10,11,12,13,14	5
Real power flow	In all lines	20
Reactive Power flow	In all lines	20

Second, we take measurement from PSCAD similar to that PMU gives means phasor quantities.

Name	Standard deviation
Voltages	0.03
Injected power	0.01
Power flow	0.008

Step 2: Apply conventional state estimation and find buses voltage magnitude and angle at each bus

Step 3: Use these states and pmu measurement in measurement vector for applying hybrid state estimator

Step 4: Calculate the different elements of H matrix

Step 5: Using this equation find the state vector.

$$X = (H'WH)^{-1}(H'WZ) = G^{-1}H'WZ$$

Simulation Result:

Conventional state estimation

Bus Number	Voltage magnitude (pu)		Magnitude error
	True Value	Estimated Value	
1	1.06	1.0858	0.0258
2	1.045	1.0712	0.0262
3	1.01	1.0373	0.0273
4	1.01319	1.0449	0.031706
5	1.01657	1.0433	0.026725
6	1.07	1.0377	0.0323
7	1.0457	1.0516	0.005901
8	1.08	1.0814	0.0014
9	1.03052	1.0424	0.011875
10	1.02992	1.0354	0.005482
11	1.04613	1.0343	0.011831
12	1.05326	1.0234	0.02986
13	1.04664	1.0204	0.026236
14	1.01927	1.0169	0.002372

Figure 2 Absolute error of volatge magnitude

Bus Number	Angle magnitude (pu)		Angle error
	True Value	Estimated Value	
1	0	0	0
2	-4.98912	-4.6078	0.38132
3	-12.749208	-11.1002	1.64901
4	-10.241967	-8.1078	2.13417
5	-8.760083	-9.3917	0.631617
6	-14.446916	-13.9325	0.51442
7	-13.236818	-11.6286	1.60822
8	-13.236818	-11.6204	1.61642
9	-14.820122	-13.3537	1.46642
10	-15.036036	-13.7277	1.30834

11	-14.858072	-13.8995	0.95857
12	-15.297285	-14.7081	0.58918
13	-15.331292	-14.7555	0.57579
14	-16.071737	-14.9477	1.12404

Figure 3 Absolute error in Angle magnitude

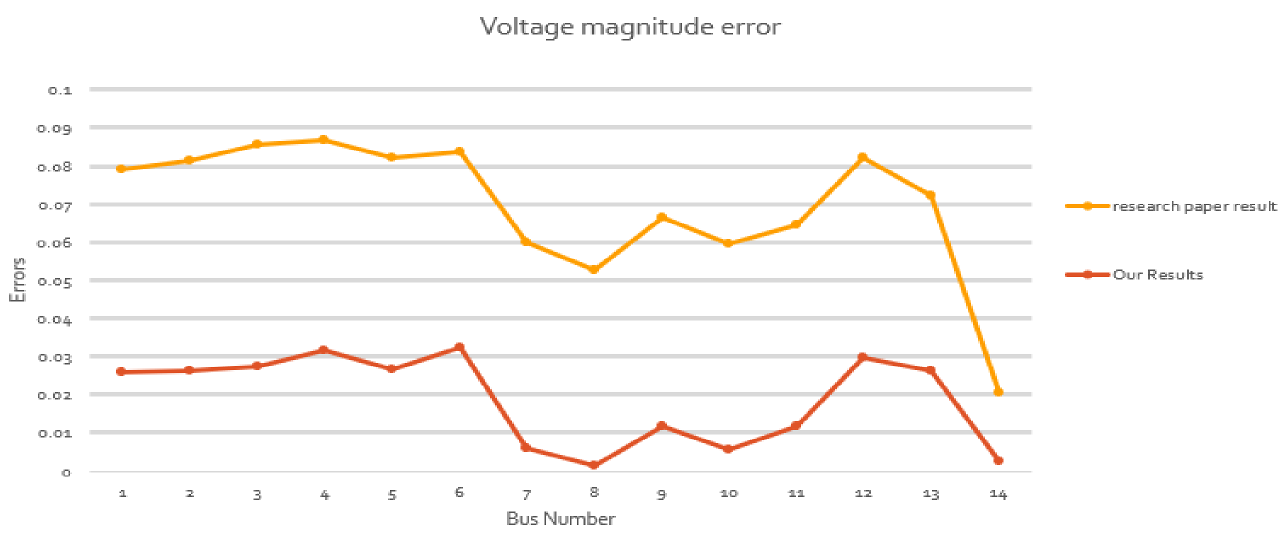


Figure 4 Error in voltage magnitude

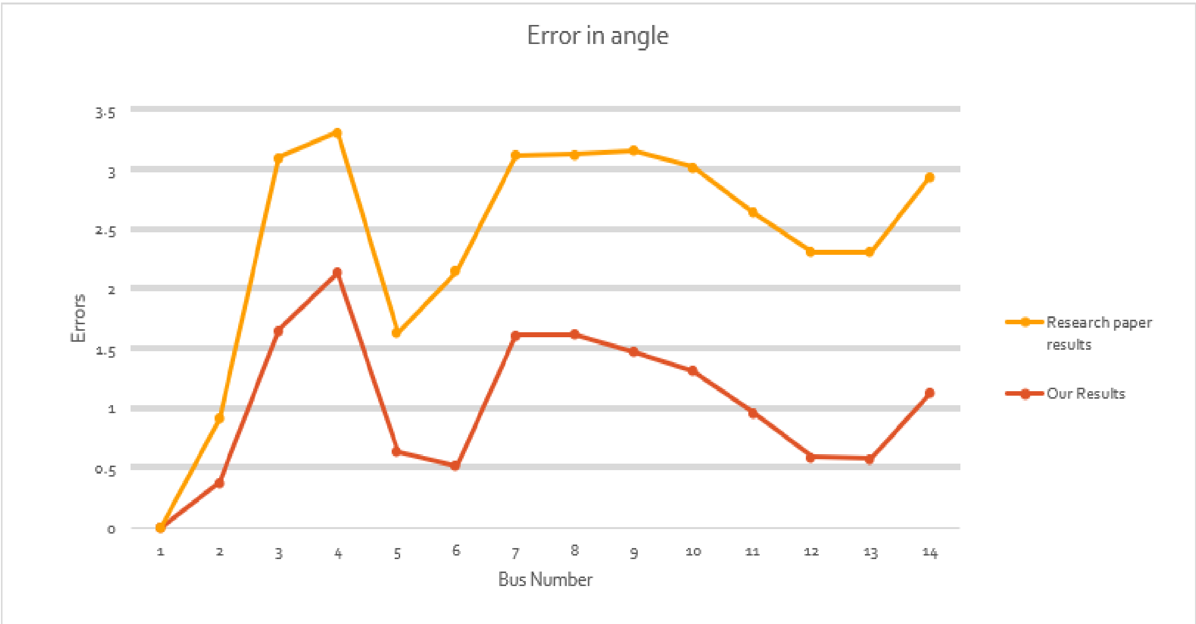


Figure 5 Error in angle magnitude

Hybrid state estimation

Bus Number	Voltage magnitude (pu)		Magnitude error
	True Value	Estimated Value	
1	1.06	1.05841406	0.001585943
2	1.045	1.04500002	2.08781E-08
3	1.01	1.00441574	0.00558426
4	1.01319	1.00820461	0.004985392
5	1.01657	1.01168949	0.00488051
6	1.07	1.06999998	2.3272E-08
7	1.0457	1.04568749	1.25067E-05
8	1.08	1.0799839	1.61018E-05
9	1.03052	1.03051157	8.43385E-06
10	1.02992	1.02990487	1.51261E-05
11	1.04613	1.04613888	8.87832E-06
12	1.05326	1.05325958	4.16051E-07
13	1.04664	1.04663793	2.06849E-06
14	1.01927	1.01925337	1.66306E-05

Figure 6 Absolute error in voltage magnitude

Bus Number	Angle magnitude (pu)		Angle error
	True Value	Estimated Value	
1	0	0	0
2	-4.98912	-4.99045845	0.001338
3	-12.749208	-12.7188085	0.030399
4	-10.241967	-10.1781569	0.06381
5	-8.760083	-8.69034601	0.069737
6	-14.446916	-14.4442598	0.002656
7	-13.236818	-13.2371533	0.000336

8	-13.236818	-13.2370885	0.000271
9	-14.820122	-14.8205364	0.000414
10	-15.036036	-15.0363498	0.000314
11	-14.858072	-14.8552731	0.002799
12	-15.297285	-15.294584	0.002701
13	-15.331292	-15.3285097	0.002782
14	-16.071737	-16.0726328	0.000896

Figure7 Absolute error in angle magnitude

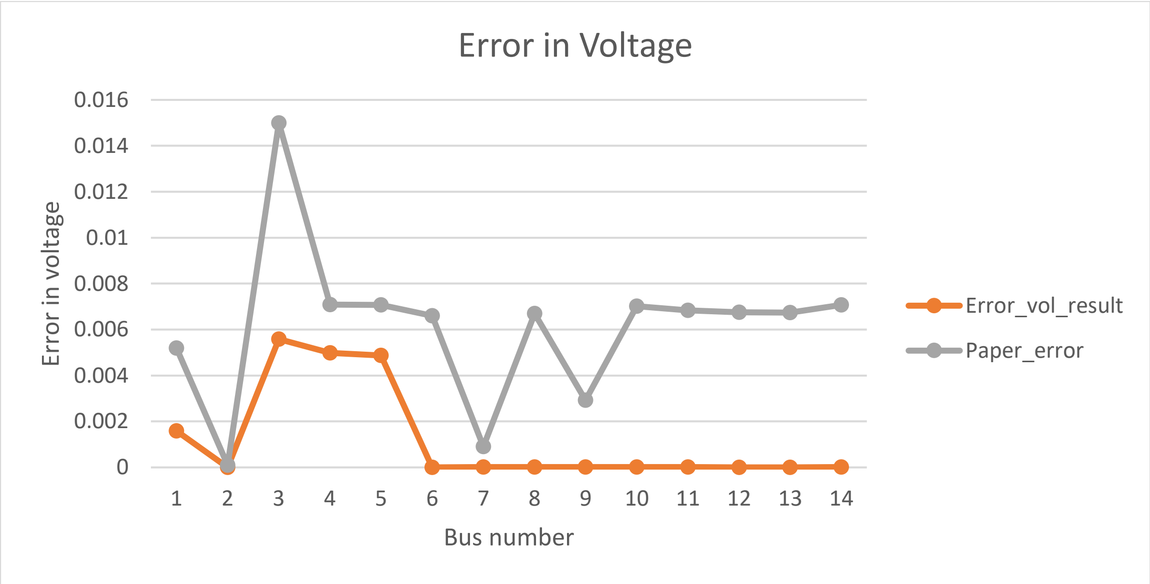


Figure 8 Error in voltage magnitude

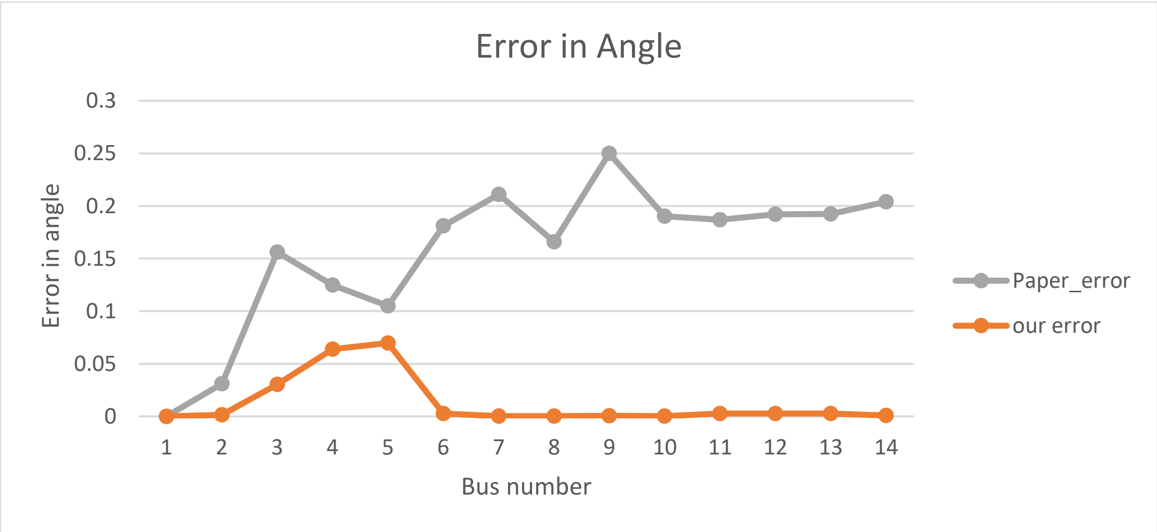


Figure 9 Error in angle magnitude

Conclusion

Conventional as well as synchronised phasor measurements are used for modified state estimation algorithm. By comparing the result of conventional state estimation and hybrid state estimation we observe that the results of hybrid state estimation are more accurate and better than the conventional state estimation. For validating these we have used IEEE14 Bus system and simulated this on PSCAD software and took the measurement from there in order to mimic the measurement of RTU and PMU we introduce gaussian error in the measurements and used that measurements in MATLAB where we have developed the code for Conventional state estimation and hybrid state estimation.