

Assignment-10

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Abstract—This assignment deals with vector spaces.

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<https://github.com/satyam463/Assignment-10/blob/main/Assignment%2010.tex>

1 PROBLEM STATEMENT

Let V be the set of all pairs (x, y) of real numbers and let F be the field of real numbers. Define

$$(x, y) + (x_1, y_1) = (x + x_1, y + y_1) \quad (1.0.1)$$

$$c(x, y) = (cx, y) \quad (1.0.2)$$

Is V with these operations, a vector space over the field of real numbers ?

2 SOLUTION

$V = \{(x, y) \mid x, y \in R\}$, consider $u = (x_1, y_1) \in V, a, b, c \in R$. Axioms with respect to addition and scalar multiplication.

1)

$$(a + b).u = (a + b).(x_1, y_1) \quad (2.0.1)$$

$$= ((a + b)x_1, y_1) \neq a.u + b.u \quad (2.0.2)$$

Since V with the given operations the equation 2.0.2 contradicts the axioms of scalar multiplication. Hence it is not vector space over real number with these operations.