

Assignment-10

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Abstract—This assignment deals with vector spaces.

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<https://github.com/satyam463/Assignment-10/blob/main/Assignment%2010.tex>

1 PROBLEM STATEMENT

Let V be the set of all pairs (x, y) of real numbers and let F be the field of real numbers. Define

$$(x, y) + (x_1, y_1) = (x + x_1, y + y_1) \quad (1.0.1) \quad 5)$$

$$c(x, y) = (cx, y) \quad (1.0.2)$$

Is V with these operations, a vector space over the field of real numbers ?

2 SOLUTION

$V = \{(x, y) \mid x, y \in R\}$, consider

$$u = (x_1, y_1), v = (x_2, y_2), w = (x_3, y_3) \in V, a, b, c \in R$$

Axioms with respect to addition and scalar multiplication.

1)

$$u + v = (x_1, y_1) + (x_2, y_2) \quad (2.0.1)$$

$$= (x_1 + x_2, y_1 + y_2) \quad (2.0.2)$$

$$= (x_2 + x_1, y_2 + y_1) \quad (2.0.3)$$

$$= (x_2, y_2) + (x_1, y_1) = v + u \quad (2.0.4)$$

2)

$$u + (v + w) \quad (2.0.5)$$

$$= (x_1, y_1) + ((x_2, y_2) + (x_3, y_3)) \quad (2.0.6)$$

$$= (x_1, y_1) + ((x_2 + x_3), (y_2 + y_3)) \quad (2.0.7)$$

$$= (x_1 + (x_2 + x_3), y_1 + (y_2 + y_3)) \quad (2.0.8)$$

$$= ((x_1 + x_2) + x_3, (y_1 + y_2) + y_3) \quad (2.0.9)$$

$$= (x_1 + x_2, y_1 + y_2) + (x_3, y_3) \quad (2.0.10)$$

$$= (u + v) + w \quad (2.0.11)$$

$$u + \mathbf{0} = (x_1 + y_1) + (0, 0) \quad (2.0.12)$$

$$= (x_1 + 0, y_1 + 0) \quad (2.0.13)$$

$$= (x_1, y_1) = u \quad (2.0.14)$$

4)

$$u + (-u) = (x_1, y_1) + (-x_1, -y_1) \quad (2.0.15)$$

$$= (x_1 + (-x_1), y_1 + (-y_1)) \quad (2.0.16)$$

$$= (0, 0) = \mathbf{0} \quad (2.0.17)$$

$$1.u = 1.(x_1, y_1) = (1.x_1, 1.y_1) = u \quad (2.0.18)$$

6)

$$(ab).u = ab.(x_1, y_1) = ((ab)x_1, (ab)y_1) \quad (2.0.19)$$

$$= (a(bx_1), a(by_1)) = a(bx_1, by_1) \quad (2.0.20)$$

$$= ab(x_1, y_1) = a(b.u) \quad (2.0.21)$$

7)

$$c.(u + v) = c.((x_1, y_1) + (x_2, y_2)) \quad (2.0.22)$$

$$= c.((x_1 + x_2), (y_1 + y_2)) \quad (2.0.23)$$

$$= (c(x_1 + x_2), c(y_1 + y_2)) \quad (2.0.24)$$

$$= (cx_1 + cx_2, cy_1 + cy_2) \quad (2.0.25)$$

$$= (cx_1, cy_1) + (cx_2, cy_2) \quad (2.0.26)$$

$$= c.(x_1, y_1) + c.(x_2, y_2) \quad (2.0.27)$$

$$= c.u + c.v \quad (2.0.28)$$

8)

$$(a + b).u = (a + b).(x_1, y_1) \quad (2.0.29)$$

$$= ((a + b)x_1, (a + b)y_1) \neq a.u + b.u \quad (2.0.30)$$

Since V with the given operations the equation 2.0.30 contradicts the axioms of scalar multiplication. Hence it is not vector space over real number with these operations.