Assignment-10

Satyam Singh
EE20MTECH14015

***Abstract*—This assignment deals with vector spaces.**

Download tex file from

https://github.com/satyam463/Assignment−10/blob/main/Assignment%2010.tex

1 Problem Statement

Let V be the set of all pairs (x,y) of real numbers and let F be the field of real numbers. Define

$$(x,y)+(x_1,y_1)=(x+x_1,y+y_1) \tag{1.0.1}$$
$$c(x,y)=(cx,y) \tag{1.0.2}$$

Is V with these operations , a vector space over the field of real numbers ?

2 Solution

V={(x,y) $|x,y \in R$} , consider

$$u=(x_1,y_1), v=(x_2,y_2), w=(x_3,y_3) \in V, a,b,c \in R$$

Axioms with respect to addition and scalar multiplication.

1)

$$u+v=(x_1,y_1)+(x_2,y_2) \tag{2.0.1}$$
$$=(x_1+x_2,y_1+y_2) \tag{2.0.2}$$
$$=(x_2+x_1,y_2+y_1) \tag{2.0.3}$$
$$=(x_2,y_2)+(x_1,y_1)=v+u \tag{2.0.4}$$

2)

$$u+(v+w)=(x_1,y_1)+((x_2,y_2)+(x_3,y_3)) \tag{2.0.5}$$
$$=(x_1,y_1)+((x_2+x_3),(y_2+y_3)) \tag{2.0.6}$$
$$=(x_1+(x_2+x_3),y_1+(y_2+y_3)) \tag{2.0.7}$$
$$=((x_1+x_2)+x_3,(y_1+y_2)+y_3) \tag{2.0.8}$$
$$=(x_1+x_2,y_1+y_2)+(x_3,y_3) \tag{2.0.9}$$
$$=(u+v)+w \tag{2.0.10}$$

3)

$$u+\mathbf{0}=(x_1+y_1)+(0,0) \tag{2.0.11}$$
$$=(x_1+0,y_1+0) \tag{2.0.12}$$
$$=(x_1,y_1)=u \tag{2.0.13}$$

4)

$$u+(-u)=(x_1,y_1)+(-x_1,-y_1) \tag{2.0.14}$$
$$=(x_1+(-x_1),y_1+(-y_1)) \tag{2.0.15}$$
$$=(0,0)=\mathbf{0} \tag{2.0.16}$$

5)

$$1.u=1.(x_1,y_1)=(1.x_1,y_1)=u \tag{2.0.17}$$

6)

$$(ab).u=ab.(x_1,y_1)=((ab)x_1,y_1) \tag{2.0.18}$$
$$=(a(bx_1),y_1)=a(bx_1,y_1) \tag{2.0.19}$$
$$=ab(x_1,y_1)=a(b.u) \tag{2.0.20}$$

7)

$$c.(u+v)=c.((x_1,y_1)+(x_2,y_2)) \tag{2.0.21}$$
$$=c.((x_1+x_2),(y_1+y_2)) \tag{2.0.22}$$
$$=(c(x_1+x_2),(y_1+y_2)) \tag{2.0.23}$$
$$=(cx_1+cx_2,y_1+y_2) \tag{2.0.24}$$
$$=(cx_1,y_1)+(cx_2+y_2)) \tag{2.0.25}$$
$$=c.(x_1,y_1)+c.(x_2+y_2)) \tag{2.0.26}$$
$$=c.u+c.v \tag{2.0.27}$$

8)

$$(a+b).u=(a+b).(x_1,y_1) \tag{2.0.28}$$
$$=((a+b)x_1,y_1)\neq a.u+b.u \tag{2.0.29}$$

Since V with the given operations the equation 2.0.29 contradicts the axioms of scalar multiplication . Hence it is not vector space over real number with these operations.