Assignment-11

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***Abstract*—This assignment deals with vector sub spaces.**

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https://github.com/satyam463/Assignment−11/blob/main/Assignment%2011.tex

1 Problem Statement

Let $\mathbf{W_1}$ and $\mathbf{W_2}$ be sub spaces of a vector space $\mathbf{V}$ such that the set-theoretic union of $\mathbf{W_1}$ and $\mathbf{W_2}$ is also a sub space.Prove that one of the spaces $\mathbf{W_i}$ is contained in the other.

2 Solution

Since $\mathbf{W_1}$ and $\mathbf{W_2}$ are sub spaces of a vector space $\mathbf{V}$ then we have $\mathbf{W_1} \cup \mathbf{W_2}$ is sub spaces of a vector space $\mathbf{V}$. Suppose

$$\mathbf{W_1} \not\subseteq \mathbf{W_2}, \mathbf{W_2} \not\subseteq \mathbf{W_1} \tag{2.0.1}$$

$$\exists u, v \in \mathbf{V}, u \in \mathbf{W_1} \setminus \mathbf{W_2}, v \in \mathbf{W_2} \setminus \mathbf{W_1} \tag{2.0.2}$$

1)

$$u, v \in \mathbf{W_1} \cup \mathbf{W_2} \implies u + v \in \mathbf{W_1} \cup \mathbf{W_2} \tag{2.0.3}$$

2)

$$u + v \in \mathbf{W_1}, (-u) + (u + v) \in \mathbf{W_1} \implies v \in \mathbf{W_1} \tag{2.0.4}$$

3)

$$u + v \in \mathbf{W_2}, (u + v) + (-v) \in \mathbf{W_2} \implies u \in \mathbf{W_2} \tag{2.0.5}$$

2.0.4 and 2.0.5 which contradicts the assumption.Hence $\mathbf{W_1} \subseteq \mathbf{W_2}$ or $\mathbf{W_2} \subseteq \mathbf{W_1}$