Assignment-11

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***Abstract*—This assignment deals with vector sub spaces.**

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https://github.com/satyam463/Assignment−11/blob/main/Assignment%2011.tex

1 Problem Statement

Let $\mathbf{W_1}$ and $\mathbf{W_2}$ be sub spaces of a vector space $\mathbf{V}$ such that the set-theoretic union of $\mathbf{W_1}$ and $\mathbf{W_2}$ is also a sub space.Prove that one of the spaces $\mathbf{W_i}$ is contained in the other.

2 Solution

Since $\mathbf{W_1}$ and $\mathbf{W_2}$ are sub spaces of a vector space $\mathbf{V}$ then we have $\mathbf{W_1} \cup \mathbf{W_2}$ is sub spaces of a vector space $\mathbf{V}$. Suppose

$$\mathbf{W_1} \subseteq \mathbf{W_2}, \mathbf{W_2} \subseteq \mathbf{W_1} \tag{2.0.1}$$

now,

$$\mathbf{W_1} \cup \mathbf{W_2} = \mathbf{W_2} \implies \mathbf{W_1} \subseteq \mathbf{W_2} \tag{2.0.2}$$

$$\mathbf{W_1} \cup \mathbf{W_2} = \mathbf{W_1} \implies \mathbf{W_2} \subseteq \mathbf{W_1} \tag{2.0.3}$$

2.0.2 and 2.0.3 which shows union of $\mathbf{W_1}$ and $\mathbf{W_2}$ are sub spaces if and only if one sub space is contained in other. Hence $\mathbf{W_1} \subseteq \mathbf{W_2}$ or $\mathbf{W_2} \subseteq \mathbf{W_1}$