

Assignment-11

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Abstract—This assignment deals with linear dependence.

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<https://github.com/satyam463/Assignment-11/blob/main/Assignment%2011.tex>

1 PROBLEM STATEMENT

Are the vectors

$$\alpha_1 = (1, 1, 2, 4), \alpha_2 = (2, -1, -5, 2) \quad (1.0.1)$$

$$\alpha_3 = (1, -1, -4, 0), \alpha_4 = (2, 1, 1, 6) \quad (1.0.2)$$

linearly independent in \mathbb{R}^4

2 SOLUTION

consider the row reduced matrix

$$\begin{pmatrix} 1 & 1 & 2 & 4 \\ 2 & -1 & -5 & 2 \\ 1 & -1 & -4 & 0 \\ 2 & 1 & 1 & 6 \end{pmatrix} \xleftrightarrow[R_3 \rightarrow R_3 - R_1]{R_2 \rightarrow R_2 - 2R_1} \begin{pmatrix} 1 & 1 & 2 & 4 \\ 0 & -3 & -9 & -6 \\ 0 & -2 & -6 & -4 \\ 2 & 1 & 1 & 6 \end{pmatrix} \quad (2.0.1)$$

$$\xleftrightarrow[R_2 \leftarrow R_4]{R_4 \leftarrow R_4 - 2R_1} \begin{pmatrix} 1 & 1 & 2 & 4 \\ 0 & -1 & -3 & -2 \\ 0 & -2 & -6 & -4 \\ 0 & -3 & -9 & -6 \end{pmatrix} \quad (2.0.2)$$

$$\xleftrightarrow[R_2 \leftarrow -R_2]{R_4 \leftarrow R_2} \begin{pmatrix} 1 & 1 & 2 & 4 \\ 0 & 1 & 3 & 2 \\ 0 & -3 & -9 & -6 \\ 0 & -2 & -6 & -4 \end{pmatrix} \quad (2.0.3)$$

$$\xleftrightarrow[R_4 \leftarrow R_4 + 2R_2]{R_3 \leftarrow R_3 + 3R_2} \begin{pmatrix} 1 & 1 & 2 & 4 \\ 0 & 1 & 3 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (2.0.4)$$

Therefore the rank = no. of pivot columns = 2 (less than no. of columns). Thus the four vectors are not linearly independent.