

# Assignment-11

Satyam Singh  
EE20MTECH14015

**Abstract—This assignment deals with vector sub spaces.**

Download tex file from

<https://github.com/satyam463/Assignment-11/blob/main/Assignment%2011.tex>

## 1 PROBLEM STATEMENT

Let  $\mathbf{W}_1$  and  $\mathbf{W}_2$  be sub spaces of a vector space  $\mathbf{V}$  such that the set-theoretic union of  $\mathbf{W}_1$  and  $\mathbf{W}_2$  is also a sub space. Prove that one of the spaces  $\mathbf{W}_i$  is contained in the other.

## 2 SOLUTION

Since  $\mathbf{W}_1$  and  $\mathbf{W}_2$  are sub spaces of a vector space  $\mathbf{V}$  then we have  $\mathbf{W}_1 \cup \mathbf{W}_2$  is sub spaces of a vector space  $\mathbf{V}$

Let  $a \in \mathbf{W}_1$

$$\mathbf{W}_2 \subseteq \mathbf{W}_1 \implies \mathbf{W}_1 \cup \mathbf{W}_2 = \mathbf{W}_1 \quad (2.0.1)$$

$$\mathbf{W}_1 \subseteq \mathbf{W}_2 \implies \mathbf{W}_1 \cup \mathbf{W}_2 = \mathbf{W}_2 \quad (2.0.2)$$

Therefore  $a \in \mathbf{W}_2$ , 2.0.1 2.0.2 which shows union of  $\mathbf{W}_1$  and  $\mathbf{W}_2$  are sub spaces if one sub space is contained in other. Hence  $\mathbf{W}_2 \subseteq \mathbf{W}_1$  or  $\mathbf{W}_1 \subseteq \mathbf{W}_2$