

Assignment-12

Satyam Singh
EE20MTECH14015

Abstract—This assignment deals with basis of a vector.

Download tex file from

<https://github.com/satyam463/Assignment-12/blob/main/Assignment%2012.tex>

1 PROBLEM STATEMENT

Find the coordinate matrix of the vector $\begin{pmatrix} 1 & 0 & 1 \end{pmatrix}$ in the basis of C^3 consisting of the vectors $\begin{pmatrix} 2i & 1 & 0 \end{pmatrix}$, $\begin{pmatrix} 2 & -1 & 1 \end{pmatrix}$, $\begin{pmatrix} 0 & 1+i & 1-i \end{pmatrix}$ in that order.

2 SOLUTION

consider

$$P = \begin{pmatrix} 2i & 2 & 0 \\ 1 & -1 & 1+i \\ 0 & 1 & 1-i \end{pmatrix} \quad (2.0.1)$$

Now we find P^{-1} by Gauss Jordan row reduction.

$$\begin{pmatrix} 2i & 2 & 0 & 1 & 0 & 0 \\ 1 & -1 & 1+i & 0 & 1 & 0 \\ 0 & 1 & 1-i & 0 & 0 & 1 \end{pmatrix} \quad (2.0.2)$$

$$\xleftrightarrow[R_2 \leftarrow R_2 - 2iR_3]{R_1 \leftarrow R_2} \begin{pmatrix} 1 & -1 & 1+i & 0 & 1 & 0 \\ 0 & 2+2i & -2-2i & 1 & -2 & 0 \\ 0 & 1 & 1-i & 0 & 0 & 1 \end{pmatrix} \quad (2.0.3)$$

$$\xleftrightarrow[R_2 \leftarrow R_2 - iR_3]{R_2 \leftarrow R_2/2} \begin{pmatrix} 1 & -1 & 1+i & 0 & 1 & 0 \\ 0 & 1+i & -1-i & \frac{1}{2} & -1 & 0 \\ 0 & 1 & 1-i & 0 & 0 & 1 \end{pmatrix} \quad (2.0.4)$$

$$\xleftrightarrow[R_3 \leftarrow R_3 - R_2]{R_1 \leftarrow R_1 + R_3} \begin{pmatrix} 1 & 0 & 2 & 0 & 1 & 1 \\ 0 & 1 & -2-2i & \frac{1}{2} & -1 & -i \\ 0 & 0 & 3+i & \frac{-1}{2} & 1 & 1+i \end{pmatrix} \quad (2.0.5)$$

$$\xleftrightarrow[R_2 \leftarrow R_2 + R_3]{R_1 \leftarrow R_1 - R_3} \begin{pmatrix} 1 & 0 & -1-i & \frac{1}{2} & 0 & -i \\ 0 & 1 & 1-i & 0 & 0 & 1 \\ 0 & 0 & 3+i & \frac{-1}{2} & 1 & 1+i \end{pmatrix} \quad (2.0.6)$$

$$\xleftrightarrow[R_2 \leftarrow R_2 + R_3]{R_1 \leftarrow R_1 + R_3} \begin{pmatrix} 1 & 0 & 2 & 0 & 1 & 1 \\ 0 & 1 & 4 & \frac{-1}{2} & 1 & 2+i \\ 0 & 0 & 3+i & \frac{-1}{2} & 1 & 1+i \end{pmatrix} \quad (2.0.7)$$

$$\xleftrightarrow[R_1 \leftarrow R_1 - 2R_3]{R_3 \leftarrow R_3/3+i} \begin{pmatrix} 1 & 0 & 0 & \frac{1}{3+i} & \frac{1+i}{3+i} & \frac{1-i}{3+i} \\ 0 & 1 & 4 & \frac{-1}{2} & 1 & 2+i \\ 0 & 0 & 1 & \frac{1}{6+2i} & \frac{1}{3+i} & \frac{1+i}{3+i} \end{pmatrix} \quad (2.0.8)$$

$$\xleftrightarrow{R_2 \leftarrow R_2 - 4R_3} \begin{pmatrix} 1 & 0 & 0 & \frac{1}{3+i} & \frac{1+i}{3+i} & \frac{1-i}{3+i} \\ 0 & 1 & 0 & \frac{1-i}{3+i} & \frac{-1+i}{3+i} & \frac{1+i}{3+i} \\ 0 & 0 & 1 & \frac{1}{6+2i} & \frac{1}{3+i} & \frac{1+i}{3+i} \end{pmatrix} \quad (2.0.9)$$

Therefore the coordinate matrix of the vector is

$$P^{-1} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{3+i} & \frac{1+i}{3+i} & \frac{1-i}{3+i} \\ \frac{1-i}{3+i} & \frac{-1+i}{3+i} & \frac{1+i}{3+i} \\ \frac{1}{6+2i} & \frac{1}{3+i} & \frac{1+i}{3+i} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1-i}{2} \\ \frac{3+i}{5} \\ \frac{5}{2+i} \end{pmatrix} \quad (2.0.10)$$