

Assignment-12

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Abstract—This assignment deals with basis of a vector.

Download tex file from

<https://github.com/satyam463/Assignment-12/blob/main/Assignment%2012.tex>

1 PROBLEM STATEMENT

Find the coordinate matrix of the vector $\begin{pmatrix} 1 & 0 & 1 \end{pmatrix}$ in the basis of C^3 consisting of the vectors $\begin{pmatrix} 2i & 1 & 0 \end{pmatrix}$, $\begin{pmatrix} 2 & -1 & 1 \end{pmatrix}$, $\begin{pmatrix} 0 & 1+i & 1-i \end{pmatrix}$ in that order.

2 SOLUTION

$$\begin{pmatrix} 1 & 0 & 1 \end{pmatrix} = \alpha_1 \begin{pmatrix} 2i & 1 & 0 \end{pmatrix} + \alpha_2 \begin{pmatrix} 2 & -1 & 1 \end{pmatrix} + \alpha_3 \begin{pmatrix} 0 & 1+i & 1-i \end{pmatrix} \quad (2.0.1)$$

$$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2i & 2 & 0 \\ 1 & -1 & 1+i \\ 0 & 1 & 1-i \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} \quad (2.0.2)$$

Now we find α_i by row reducing augmented matrix.

$$\begin{pmatrix} 2i & 2 & 0 & 1 \\ 1 & -1 & 1+i & 0 \\ 0 & 1 & 1-i & 1 \end{pmatrix} \xrightarrow[R_2 \rightarrow R_2 - 2iR_3]{R_1 \rightarrow R_2} \begin{pmatrix} 1 & -1 & 1+i & 0 \\ 0 & 2+2i & 2-2i & 1 \\ 0 & 1 & 1-i & 1 \end{pmatrix} \quad (2.0.3)$$

$$\xrightarrow[R_3 \leftarrow R_3 - R_2]{R_2 \leftarrow R_2/2} \begin{pmatrix} 1 & -1 & 1+i & 0 \\ 0 & 1+i & 1-i & \frac{1}{2} \\ 0 & -i & 0 & \frac{1}{2} \end{pmatrix} \quad (2.0.4)$$

Therefore the coordinate matrix of the vector is

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \begin{pmatrix} \frac{-1-i}{2} \\ \frac{i}{2} \\ \frac{3+i}{4} \end{pmatrix} \quad (2.0.5)$$