Assignment-12

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***Abstract*—This assignment deals with basis of a vector.**

Download tex file from

https://github.com/satyam463/Assignment−12/blob/main/Assignment%2012.tex

1 Problem Statement

Find the coordinate matrix of the vector $\begin{pmatrix}1 & 0 & 1\end{pmatrix}$ in the basis of C^3 consisting of the vectors $\begin{pmatrix}2i & 1 & 0\end{pmatrix}$, $\begin{pmatrix}2 & -1 & 1\end{pmatrix}$, $\begin{pmatrix}0 & 1+i & 1-i\end{pmatrix}$ in that order.

2 Solution

$$\begin{pmatrix}1 & 0 & 1\end{pmatrix} = \alpha_1\begin{pmatrix}2i & 1 & 0\end{pmatrix} + \alpha_2\begin{pmatrix}2 & -1 & 1\end{pmatrix} + \alpha_3\begin{pmatrix}0 & 1+i & 1-i\end{pmatrix} \tag{2.0.1}$$

$$\begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}2i & 2 & 0\\1 & -1 & 1+i\\0 & 1 & 1-i\end{pmatrix}\begin{pmatrix}\alpha_1\\\alpha_2\\\alpha_3\end{pmatrix} \tag{2.0.2}$$

consider

$$\mathbf{P} = \begin{pmatrix}2i & 2 & 0\\1 & -1 & 1+i\\0 & 1 & 1-i\end{pmatrix} \tag{2.0.3}$$

$$\mathbf{P^{-1}}\begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}\alpha_1\\\alpha_2\\\alpha_3\end{pmatrix} \tag{2.0.4}$$

Now we find $\mathbf{P^{-1}}$ by Gauss Jordan row reduction.

$$\begin{pmatrix}2i & 2 & 0 & 1 & 0 & 0\\1 & -1 & 1+i & 0 & 1 & 0\\0 & 1 & 1-i & 0 & 0 & 1\end{pmatrix} \tag{2.0.5}$$

$$\xleftrightarrow[R_2\leftarrow R_2-2iR_3]{R_1\leftarrow R_2}\begin{pmatrix}1 & -1 & 1+i & 0 & 1 & 0\\0 & 2+2i & -2-2i & 1 & -2 & 0\\0 & 1 & 1-i & 0 & 0 & 1\end{pmatrix} \tag{2.0.6}$$

$$\xleftrightarrow[R_2\leftarrow R_2-iR_3]{R_2\leftarrow R_2/2}\begin{pmatrix}1 & -1 & 1+i & 0 & 1 & 0\\0 & 1+i & -1-i & \frac{1}{2} & -1 & 0\\0 & 1 & 1-i & 0 & 0 & 1\end{pmatrix} \tag{2.0.7}$$

$$\xleftrightarrow[R_3\leftarrow R_3-R_2]{R_1\leftarrow R_1+R_3}\begin{pmatrix}1 & 0 & 2 & 0 & 1 & 1\\0 & 1 & -2-2i & \frac{1}{2} & -1 & -i\\0 & 0 & 3+i & \frac{-1}{2} & 1 & 1+i\end{pmatrix} \tag{2.0.8}$$

$$\xleftrightarrow[R_2\leftarrow R_2+R_3]{R_1\leftarrow R_1-R_3}\begin{pmatrix}1 & 0 & -1-i & \frac{1}{2} & 0 & -i\\0 & 1 & 1-i & 0 & 0 & 1\\0 & 0 & 3+i & \frac{-1}{2} & 1 & 1+i\end{pmatrix} \tag{2.0.9}$$

$$\xleftrightarrow[R_2\leftarrow R_2+R_3]{R_1\leftarrow R_1+R_3}\begin{pmatrix}1 & 0 & 2 & 0 & 1 & 1\\0 & 1 & 4 & \frac{-1}{2} & 1 & 2+i\\0 & 0 & 3+i & \frac{-1}{2} & 1 & 1+i\end{pmatrix} \tag{2.0.10}$$

$$\xleftrightarrow[R_1\leftarrow R_1-2R_3]{R_3\leftarrow R_3/3+i}\begin{pmatrix}1 & 0 & 0 & \frac{1}{3+i} & \frac{1+i}{3+i} & \frac{1-i}{3+i}\\0 & 1 & 4 & \frac{-1}{2} & 1 & 2+i\\0 & 0 & 1 & \frac{-1}{6+2i} & \frac{1}{3+i} & \frac{1+i}{3+i}\end{pmatrix} \tag{2.0.11}$$

$$\xleftrightarrow{R_2\leftarrow R_2-4R_3}\begin{pmatrix}1 & 0 & 0 & \frac{1}{3+i} & \frac{1+i}{3+i} & \frac{1-i}{3+i}\\0 & 1 & 0 & \frac{1-i}{3+i} & \frac{-1+i}{3+i} & \frac{1+i}{3+i}\\0 & 0 & 1 & \frac{-1}{6+2i} & \frac{1}{3+i} & \frac{1+i}{3+i}\end{pmatrix} \tag{2.0.12}$$

Therefore the coordinate matrix of the vector is

$$\begin{pmatrix}\alpha_1\\\alpha_2\\\alpha_3\end{pmatrix} = \mathbf{P^{-1}}\begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}\frac{1}{3+i} & \frac{1+i}{3+i} & \frac{1-i}{3+i}\\\frac{1-i}{3+i} & \frac{-1+i}{3+i} & \frac{1+i}{3+i}\\\frac{-1}{6+2i} & \frac{1}{3+i} & \frac{1+i}{3+i}\end{pmatrix}\begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}\frac{1-i}{2}\\\frac{3+i}{5}\\\frac{2+i}{5}\end{pmatrix} \tag{2.0.13}$$