

Assignment-14

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Abstract—This assignment deals with linear transformation.

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<https://github.com/satyam463/Assignment-14/blob/main/Assignment%2014.tex>

$$\epsilon = c \begin{pmatrix} -i \\ -1 \\ 1 \end{pmatrix} \quad (3.1.4)$$

Hence it holds the condition of singularity and T is not invertible .

1 PROBLEM STATEMENT

Let T be the unique linear operator on C^3 for which

$$\begin{aligned} T(\epsilon_1) &= \begin{pmatrix} 1 & 0 & i \end{pmatrix}, T(\epsilon_2) = \begin{pmatrix} 0 & 1 & 1 \end{pmatrix}, \\ T(\epsilon_3) &= \begin{pmatrix} i & 1 & 0 \end{pmatrix} \end{aligned} \quad (1.0.1)$$

Is T invertible ?

2 INVERTIBLE LINEAR TRANSFORMATION

2.1 Properties

1. T must be non singular.
2. T must be linearly independent.
3. T must be 1:1 and onto.

3 SOLUTION

Let ϵ_i is basis for C^3 such that $T(\epsilon_i)$ is basis for C^3

3.1 using property 1

T is said to be singular if

$$T(\epsilon) = 0 \implies \epsilon \neq 0 \quad (3.1.1)$$

now,

$$\begin{pmatrix} 1 & 0 & i \\ 0 & 1 & 1 \\ i & 1 & 0 \end{pmatrix} \epsilon = 0 \quad (3.1.2)$$

consider the row reduced matrix

$$\begin{pmatrix} 1 & 0 & i \\ 0 & 1 & 1 \\ i & 1 & 0 \end{pmatrix} \xleftrightarrow[R_3 \rightarrow R_3 - R_2]{R_3 \rightarrow R_3 - iR_1} \begin{pmatrix} 1 & 0 & i \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad (3.1.3)$$

$$T(\epsilon) = A\epsilon \quad (3.1.5)$$

$$T(\epsilon) = \begin{pmatrix} 1 & 0 & i \\ 0 & 1 & 1 \\ i & 1 & 0 \end{pmatrix} \epsilon \quad (3.1.6)$$

consider the row reduced matrix

$$\begin{pmatrix} 1 & 0 & i \\ 0 & 1 & 1 \\ i & 1 & 0 \end{pmatrix} \xleftrightarrow[R_3 \rightarrow R_3 - R_2]{R_3 \rightarrow R_3 - iR_1} \begin{pmatrix} 1 & 0 & i \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad (3.1.7)$$

Therefore the rank = no. of pivot columns = 2 (less than no. of columns). Thus these are not linearly independent. Hence T is not invertible.