1

Assignment-14

Satyam Singh EE20MTECH14015

 $\begin{subarray}{lll} Abstract — This assignment deals with linear transformation. \end{subarray}$

 $\epsilon = c \begin{pmatrix} -i \\ -1 \\ 1 \end{pmatrix} \tag{3.1.4}$

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https://github.com/satyam463/Assignment-14/blob/main/Assignment%2014.tex

Hence it holds the condition of singularity therefore T is not invertible.

1 Problem Statement

Let T be the unique linear operator on C^3 for which

$$T(\epsilon_1) = \begin{pmatrix} 1 & 0 & i \end{pmatrix}, T(\epsilon_2) = \begin{pmatrix} 0 & 1 & 1 \end{pmatrix},$$
$$T(\epsilon_3) = \begin{pmatrix} i & 1 & 0 \end{pmatrix}$$
(1.0.1)

Is T invertible?

2 INVERTIBLE LINEAR TRANSFORMATION

2.1 Properties

- 1. T must be non singular.
- 2. T must be linearly independent.
- 3. T must be 1:1 and onto.

3 Solution

Let ϵ_i is basis for C^3 such that $T(\epsilon_i)$ is basis for C^3

3.1 using property 1

T is said to be singular if

$$T(\epsilon) = 0 \implies \epsilon \neq 0$$
 (3.1.1)

now,

$$\begin{pmatrix} 1 & 0 & i \\ 0 & 1 & 1 \\ i & 1 & 0 \end{pmatrix} \epsilon = 0 \tag{3.1.2}$$

consider the row reduced matrix

$$\begin{pmatrix} 1 & 0 & i \\ 0 & 1 & 1 \\ i & 1 & 0 \end{pmatrix} \xrightarrow[R_3 \to R_3 - R_2]{R_3 \to R_3 - iR_1} \begin{pmatrix} 1 & 0 & i \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$
(3.1.3)