Assignment-14

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***Abstract*—This assignment deals with linear transformation.**

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https://github.com/satyam463/Assignment−14/blob/main/Assignment%2014.tex

1 Problem Statement

Let T be the unique linear operator on C^3 for which

$$T(\epsilon_1) = \begin{pmatrix}1 & 0 & i\end{pmatrix}, T(\epsilon_2) = \begin{pmatrix}0 & 1 & 1\end{pmatrix}, \quad T(\epsilon_3) = \begin{pmatrix}i & 1 & 0\end{pmatrix} \tag{1.0.1}$$

Is T invertible ?

2 invertible linear transformation

2.1 Properties

1. T must be non singular.
2. T must be linearly independent.
3. T must be 1:1 and onto.

3 Solution

Let ϵ_i is basis for C^3 such that $T(\epsilon_i)$ is basis for C^3

3.1 using property 1

T : $C^3 \to C^3$ is said to be singular if

$$T(\epsilon) = 0 \implies \epsilon \neq 0 \tag{3.1.1}$$

now,

$$\begin{pmatrix}1 & 0 & i\\ 0 & 1 & 1\\ i & 1 & 0\end{pmatrix}\epsilon = 0 \tag{3.1.2}$$

consider the row reduced matrix

$$\begin{pmatrix}1 & 0 & i\\ 0 & 1 & 1\\ i & 1 & 0\end{pmatrix} \xleftrightarrow[R_3\to R_3-R_2]{R_3\to R_3-iR_1} \begin{pmatrix}1 & 0 & i\\ 0 & 1 & 1\\ 0 & 0 & 0\end{pmatrix} \tag{3.1.3}$$

$$\epsilon = c\begin{pmatrix}-i\\ -1\\ 1\end{pmatrix} \tag{3.1.4}$$

Hence it holds the condition of singularity and T is not invertible .

$$T(\epsilon) = \mathbf{A}\epsilon \tag{3.1.5}$$

$$T(\epsilon) = \begin{pmatrix}1 & 0 & i\\ 0 & 1 & 1\\ i & 1 & 0\end{pmatrix}\epsilon \tag{3.1.6}$$

consider the row reduced matrix

$$\begin{pmatrix}1 & 0 & i\\ 0 & 1 & 1\\ i & 1 & 0\end{pmatrix} \xleftrightarrow[R_3\to R_3-R_2]{R_3\to R_3-iR_1} \begin{pmatrix}1 & 0 & i\\ 0 & 1 & 1\\ 0 & 0 & 0\end{pmatrix} \tag{3.1.7}$$

Therefore the rank = no. of pivot columns = 2 (less than no. of columns).Thus these are not linearly independent. Hence T is not invertible.