Assignment-15

Satyam Singh
EE20MTECH14015

***Abstract*—This assignment deals with matrix linear transformation.**

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https://github.com/satyam463/Assignment−13/blob/main/Assignment%2013.tex

1 Problem Statement

Let T be the linear operator on C^2 defined by

$$T(x_1, x_2) = \left(x_1, 0\right) \tag{1.0.1}$$

Let β be the standard ordered basis for C^2 and

$$\beta^{'} = \{\alpha_1, \alpha_2\} \tag{1.0.2}$$

be the ordered basis defined by

$$\alpha_1 = \left(1, i\right), \alpha_2 = \left(-i, 2\right) \tag{1.0.3}$$

What is the matrix of T relative to the pair $\beta^{'}$, β ?

2 Solution

Transformation T from C^2 to C^2. Let

$$\beta = \{e_1, e_2\} \implies e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.1}$$

$$\beta^{'} = \{\alpha_1, \alpha_2\} \implies \alpha_1 = \begin{pmatrix} 1 \\ i \end{pmatrix}, \alpha_2 = \begin{pmatrix} -i \\ 2 \end{pmatrix} \tag{2.0.2}$$

T is defined by

$$T(\mathbf{x}) = \mathbf{A}\mathbf{x} \tag{2.0.3}$$

$$T(\mathbf{x}) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\mathbf{x} \tag{2.0.4}$$

consider $\beta^{'}$ under T

$$T(\alpha_{\mathbf{1}}) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ i \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.5}$$

$$T(\alpha_{\mathbf{2}}) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} -i \\ 2 \end{pmatrix} = \begin{pmatrix} -i \\ 0 \end{pmatrix} \tag{2.0.6}$$

we have to write $T(\alpha_{\mathbf{1}})$ and $T(\alpha_{\mathbf{2}})$ as linear combinations of e_1 , e_2

$$T(\alpha_{\mathbf{1}}) = 1e_1 + 0e_2 \tag{2.0.7}$$

$$T(\alpha_{\mathbf{2}}) = -ie_1 + 0e_2 \tag{2.0.8}$$

Therefore matrix of relative to the pair $\beta^{'}$, β

$$T\left(\beta^{'}\right) = \begin{pmatrix} 1 & -i \\ 0 & 0 \end{pmatrix}\beta \tag{2.0.9}$$