Assignment-15

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***Abstract*—This assignment deals with matrix linear transformation.**

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https://github.com/satyam463/Assignment−13/blob/main/Assignment%2013.tex

1 Problem Statement

Let T be the linear operator on C^2 defined by

$$T\left(x_1, x_2\right) = \begin{pmatrix} x_1, 0 \end{pmatrix} \tag{1.0.1}$$

Let β be the standard ordered basis for C^2 and

$$\beta^{'} = \{\alpha_1, \alpha_2\} \tag{1.0.2}$$

be the ordered basis defined by

$$\alpha_1 = \begin{pmatrix} 1, i \end{pmatrix}, \alpha_2 = \begin{pmatrix} -i, 2 \end{pmatrix} \tag{1.0.3}$$

What is the matrix of T relative to the pair β , $\beta^{'}$?

2 Solution

Transformation T from C^2 to C^2. Let

$$\beta = \{e_1, e_2\} \implies e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{2.0.1}$$

$$\beta^{'} = \{\alpha_1, \alpha_2\} \implies \alpha_1 = \begin{pmatrix} 1 \\ i \end{pmatrix}, \alpha_2 = \begin{pmatrix} -i \\ 2 \end{pmatrix} \tag{2.0.2}$$

T is defined by

$$T\left(\mathbf{x}\right) = \mathbf{A}\mathbf{x} \tag{2.0.3}$$

$$T\left(\mathbf{x}\right) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\mathbf{x} \tag{2.0.4}$$

To find relative matrix we will use row reduce augmented matrix.

$$\begin{pmatrix} 1 & -i & 1 & 0 \\ i & 2 & 0 & 0 \end{pmatrix} \tag{2.0.5}$$

$$\begin{pmatrix} 1 & -i & 1 & 0 \\ i & 2 & 0 & 0 \end{pmatrix} \xleftrightarrow[R_1 \to R_1 + iR_2]{R_2 \to R_2 - iR_1} \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -i & 0 \end{pmatrix} \tag{2.0.6}$$

Therefore matrix of relative to the pair β , $\beta^{'}$

$$T\left(\beta\right) = \begin{pmatrix} 2 & 0 \\ -i & 0 \end{pmatrix}\beta^{'} \tag{2.0.7}$$