Assignment-15

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Abstract—**This assignment deals with matrix linear transformation.**

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1 PROBLEM STATEMENT

Let T be the linear operator on C^2 defined by

$$T(x_1, x_2) = (x_1, 0) \tag{1.0.1}$$

Let β be the standard ordered basis for C^2 and

$$\beta' = \{\alpha_1, \alpha_2\} \tag{1.0.2}$$

be the ordered basis defined by

$$\alpha_1 = (1, i), \alpha_2 = (-i, 2) \tag{1.0.3}$$

What is the matrix of T relative to the pair β, β'?

2 SOLUTION

Transformation T from C^2 to C^2. Let

$$\beta = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{2.0.1}$$

$$\beta' = \begin{pmatrix} 1 & -i \\ i & 2 \end{pmatrix} \tag{2.0.2}$$

T is defined by

$$T(\mathbf{x}) = \mathbf{A}\mathbf{x} \tag{2.0.3}$$

$$T(\beta) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \beta \tag{2.0.4}$$

$$T(\beta) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{2.0.5}$$

To find relative matrix we will use row reduce augmented matrix.

$$\begin{pmatrix} 1 & -i & 1 & 0 \\ i & 2 & 0 & 0 \end{pmatrix} \tag{2.0.6}$$

$$\begin{pmatrix} 1 & -i & 1 & 0 \\ i & 2 & 0 & 0 \end{pmatrix} \xleftarrow[R_1 \to R_1 + iR_2]{R_2 \to R_2 - iR_1} \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & -i & 0 \end{pmatrix} \tag{2.0.7}$$

Hence the matrix of T in the order basis of β'

$$\mathbf{B} = \begin{pmatrix} 2 & 0 \\ -i & 0 \end{pmatrix} \tag{2.0.8}$$

Therefore matrix of relative to the pair β, β'

$$T(\beta) = \mathbf{A}\beta = \mathbf{B}\beta' = \begin{pmatrix} 2 & 0 \\ -i & 0 \end{pmatrix} \beta' \tag{2.0.9}$$