# Assignment-15

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***Abstract*—This assignment deals with linear transformation.**

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https://github.com/satyam463/Assignment−15/blob/main/Assignment%2015.tex

## 1 Problem Statement

Let V be a vector space over the field of complex numbers , and suppose there is an isomorphism T of V onto $C^3$. Let $\alpha_1$ , $\alpha_2$ , $\alpha_3$ , $\alpha_4$ be vectors in V such that

$$T(\alpha_1) = \begin{pmatrix} 1 & 0 & i \end{pmatrix}, T(\alpha_2) = \begin{pmatrix} -2 & 1+i & 0 \end{pmatrix}, T(\alpha_3) = \begin{pmatrix} -1 & 1 & 1 \end{pmatrix}, T(\alpha_4) = \begin{pmatrix} \sqrt{2} & i & 3 \end{pmatrix} \tag{1.0.1}$$

Is $\alpha_1$ in the subspace spanned by $\alpha_2$ and $\alpha_3$.

## 2 Isomorphism

### 2.1 Properties

T : V → W is an isomorphism if
(1) T is one one.
(2) T is onto.

## 3 Solution

T is one one over $C^3$ if

$$T(\alpha) = 0 \implies \alpha = 0 \tag{3.0.1}$$

now,

$$\begin{pmatrix} 1 & -2 & \sqrt{2} \\ 0 & 1+i & i \\ i & 0 & 3 \end{pmatrix} \alpha = 0 \tag{3.0.2}$$

consider the row reduced matrix

$$\begin{pmatrix} 1 & -2 & \sqrt{2} \\ 0 & 1+i & i \\ i & 0 & 3 \end{pmatrix} \xleftrightarrow[R_3 \to iR_3]{R_3 \to R_3 - iR_1} \begin{pmatrix} 1 & -2 & \sqrt{2} \\ 0 & 1+i & i \\ 0 & -2 & \sqrt{2}+3i \end{pmatrix} \tag{3.0.3}$$

$$\xleftrightarrow[R_3 \leftarrow R_3 + R_2]{R_2 \leftarrow (1-i)R_2} \begin{pmatrix} 1 & -2 & \sqrt{2} \\ 0 & 2 & i+1 \\ 0 & 0 & \sqrt{2}+4i+1 \end{pmatrix} \tag{3.0.4}$$

$$\alpha = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \tag{3.0.5}$$

Therefore it holds the condition of one one and the rank = no. of pivot columns = 3 (equal to no. of columns).Thus the vectors are linearly independent hence it is onto . Since T is an isomorphoism ,

$$T(\alpha_1) = c_1 T(\alpha_2) + c_2 T(\alpha_3) \tag{3.0.6}$$

$c_1$ and $c_2$ are scalar .

$$\begin{pmatrix} 1 & 0 & i \end{pmatrix} = c_1 \begin{pmatrix} -2 & 1+i & 0 \end{pmatrix} + c_2 \begin{pmatrix} -1 & 1 & 1 \end{pmatrix} \tag{3.0.7}$$

$$\begin{pmatrix} 1 \\ 0 \\ i \end{pmatrix} = \begin{pmatrix} -2 & -1 \\ 1+i & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \tag{3.0.8}$$

Now we find $c_i$ by row reducing augmented matrix.

$$\begin{pmatrix} -2 & -1 & 1 \\ 1+i & 1 & 0 \\ 0 & 1 & i \end{pmatrix} \xleftrightarrow[R_2 \to R_3]{R_1 \to -R_1/2} \begin{pmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & i \\ 1+i & 1 & 0 \end{pmatrix} \tag{3.0.9}$$

$$\xleftrightarrow[R_3 \leftarrow R_3 - (1+i)R_1]{R_1 \leftarrow R_1 - R_2/2} \begin{pmatrix} 1 & 0 & \frac{-1-i}{2} \\ 0 & 1 & i \\ 0 & \frac{1-i}{2} & \frac{1+i}{2} \end{pmatrix} \tag{3.0.10}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 - (1-i)/2R_2} \begin{pmatrix} 1 & 0 & \frac{-1-i}{2} \\ 0 & 1 & i \\ 0 & 0 & 0 \end{pmatrix} \tag{3.0.11}$$

Therefore the coordinate matrix of the vector is

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} \frac{-1-i}{2} \\ i \end{pmatrix} \tag{3.0.12}$$

$$T(\alpha_1) = -\frac{1+i}{2} T(\alpha_2) + iT(\alpha_3) \tag{3.0.13}$$

Hence $\alpha_1$ belongs to the subspace spanned by $\alpha_2$ and $\alpha_3$.