Assignment-15

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***Abstract*—This assignment deals with linear transformation.**

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https://github.com/satyam463/Assignment−15/blob/main/Assignment%2015.tex

1 Problem Statement

Let V be a vector space over the field of complex numbers , and suppose there is an isomorphism T of V onto C^3. Let α_1 , α_2 , α_3 , α_4 be vectors in V such that

$$T(\alpha_1) = \begin{pmatrix} 1 & 0 & i \end{pmatrix}, T(\alpha_2) = \begin{pmatrix} -2 & 1+i & 0 \end{pmatrix}, T(\alpha_3) = \begin{pmatrix} -1 & 1 & 1 \end{pmatrix}, T(\alpha_4) = \begin{pmatrix} \sqrt{2} & i & 3 \end{pmatrix} \tag{1.0.1}$$

Is α_1 in the subspace spanned by α_2 and α_3.

2 Isomorphism

2.1 Properties

T : V → W is an isomorphism if
(1) T is linear transformation.
(2) T is one one.
(3) T is onto.

3 Solution

Since T is an isomorphoism ,

$$T(\alpha_1) = c_1 T(\alpha_2) + c_2 T(\alpha_3) \tag{3.0.1}$$

c_1 and c_2 are scalar .

$$\begin{pmatrix} 1 & 0 & i \end{pmatrix} = c_1 \begin{pmatrix} -2 & 1+i & 0 \end{pmatrix} + c_2 \begin{pmatrix} -1 & 1 & 1 \end{pmatrix} \tag{3.0.2}$$

$$\begin{pmatrix} 1 \\ 0 \\ i \end{pmatrix} = \begin{pmatrix} -2 & -1 \\ 1+i & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} \tag{3.0.3}$$

Now we find c_i by row reducing augmented matrix.

$$\begin{pmatrix} -2 & -1 & 1 \\ 1+i & 1 & 0 \\ 0 & 1 & i \end{pmatrix} \xleftrightarrow[R_2 \to R_3]{R_1 \to -R_1/2} \begin{pmatrix} 1 & \frac{1}{2} & -\frac{1}{2} \\ 0 & 1 & i \\ 1+i & 1 & 0 \end{pmatrix} \tag{3.0.4}$$

$$\xleftrightarrow[R_3 \leftarrow R_3 - (1+i)R_1]{R_1 \leftarrow R_1 - R_2/2} \begin{pmatrix} 1 & 0 & \frac{-1-i}{2} \\ 0 & 1 & i \\ 0 & \frac{1-i}{2} & \frac{1+i}{2} \end{pmatrix} \tag{3.0.5}$$

$$\xleftrightarrow{R_3 \leftarrow R_3 - (1-i)/2R_2} \begin{pmatrix} 1 & 0 & \frac{-1-i}{2} \\ 0 & 1 & i \\ 0 & 0 & 0 \end{pmatrix} \tag{3.0.6}$$

Therefore the coordinate matrix of the vector is

$$\begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} \frac{-1-i}{2} \\ i \end{pmatrix} \tag{3.0.7}$$

$$T(\alpha_1) = -\frac{1+i}{2} T(\alpha_2) + iT(\alpha_3) \tag{3.0.8}$$

Hence α_1 belongs to the subspace spanned by α_2 and α_3.