# Assignment-17

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***Abstract*—This assignment deals with algebra of polynomials.**

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https://github.com/satyam463/Assignment−17/blob/main/Assignment%2017.tex

## 1 Problem Statement

Let F be a sub field of the complex numbers , and let A be the following $2 \times 2$ matrix over F.

$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix} \tag{1.0.1}$$

compute f(**A**) for the polynomial

$$f(x) = x^2 - x + 2 \tag{1.0.2}$$

## 2 Solution

consider

$$\left|\mathbf{A} - \lambda\mathbf{I}\right| = 0 \tag{2.0.1}$$

$$\begin{pmatrix} 2-\lambda & 1 \\ -1 & 3-\lambda \end{pmatrix} = 0 \tag{2.0.2}$$

The characteristics equation of the matrix

$$\lambda^2 - 5\lambda + 7 = 0 \tag{2.0.3}$$

The characteristics equation will satisfy its own matrix

$$\mathbf{A}^2 - 5\mathbf{A} + 7 = 0 \tag{2.0.4}$$

$$\mathbf{A}^2 = 5\mathbf{A} - 7\mathbf{I} \tag{2.0.5}$$

The given polynomial

$$f(x) = x^2 - x + 2 \tag{2.0.6}$$

$$f(\mathbf{A}) = \mathbf{A}^2 - \mathbf{A} + 2\mathbf{I} \tag{2.0.7}$$

substituting 2.0.5 in 2.0.7 we get

$$f(\mathbf{A}) = 5\mathbf{A} - 7\mathbf{I} - \mathbf{A} + 2\mathbf{I} \tag{2.0.8}$$

$$f(\mathbf{A}) = 4\mathbf{A} - 5\mathbf{I} \tag{2.0.9}$$

$$f(\mathbf{A}) = 4\begin{pmatrix} 2 & 1 \\ -1 & 3 \end{pmatrix} - 5\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{2.0.10}$$

$$f(\mathbf{A}) = \begin{pmatrix} 3 & 4 \\ -4 & 7 \end{pmatrix} \tag{2.0.11}$$