Assignment-18

Satyam Singh
EE20MTECH14015

***Abstract*—This assignment deals with Lagrange interpolation.**

Download tex file from

https://github.com/satyam463/Assignment−18/blob/main/Assignment%2018.tex

1 Problem Statement

Use the Lagrange interpolation formula to find a polynomial f with real coefficients such that f has degree ≤ 3 and f(−1)=-6 , f(0)=2 , f(1)=-2 , f(2)=6.

2 Solution

Let the required polynomial be

$$f(x) = a + bx + cx^2 + dx^3 \tag{2.0.1}$$

Testing points are

$$t_0 = -1, t_1 = 0, t_2 = 1, t_3 = 2 \tag{2.0.2}$$

Substituting the values of testing points in 2.0.1 we form matrix equation

$$\begin{pmatrix} 1 & -1 & 1 & -1 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} -6 \\ 2 \\ -2 \\ 6 \end{pmatrix} \tag{2.0.3}$$

By row reducing augmented matrix we get

$$\begin{pmatrix} 1 & -1 & 1 & -1 & -6 \\ 1 & 0 & 0 & 0 & 2 \\ 1 & 1 & 1 & 1 & -2 \\ 1 & 2 & 4 & 8 & 6 \end{pmatrix} \tag{2.0.4}$$

$$\xleftrightarrow[R_3 \leftarrow R_3 - R_1]{R_2 \leftarrow R_2 - R_1} \begin{pmatrix} 1 & -1 & 1 & -1 & -6 \\ 0 & 1 & -1 & 1 & 8 \\ 0 & 2 & 0 & 2 & 4 \\ 1 & 2 & 4 & 8 & 6 \end{pmatrix} \tag{2.0.5}$$

$$\xleftrightarrow[R_3 \leftarrow R_3/2]{R_4 \leftarrow R_4 - R_1} \begin{pmatrix} 1 & -1 & 1 & -1 & -6 \\ 0 & 1 & -1 & 1 & 8 \\ 0 & 1 & 0 & 1 & 2 \\ 0 & 3 & 3 & 9 & 12 \end{pmatrix} \tag{2.0.6}$$

$$\xleftrightarrow[R_3 \leftarrow R_3 - R_2]{R_4 \leftarrow R_4/3} \begin{pmatrix} 1 & -1 & 1 & -1 & -6 \\ 0 & 1 & -1 & 1 & 8 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 1 & 1 & 3 & 4 \end{pmatrix} \tag{2.0.7}$$

$$\xleftrightarrow[R_4 \leftarrow R_4 - R_2]{R_1 \leftarrow R_1 + R_2} \begin{pmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & -1 & 1 & 8 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 2 & 2 & -4 \end{pmatrix} \tag{2.0.8}$$

$$\xleftrightarrow[R_4 \leftarrow R_4 - R_2]{R_1 \leftarrow R_1 + R_2} \begin{pmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & -1 & 1 & 8 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 2 & 2 & -4 \end{pmatrix} \tag{2.0.9}$$

$$\xleftrightarrow[R_2 \leftarrow R_2 + R_3]{R_4 \leftarrow R_4/2} \begin{pmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 1 & 1 & -2 \end{pmatrix} \tag{2.0.10}$$

$$\xleftrightarrow[R_2 \leftarrow R_2 - R_4]{R_4 \leftarrow R_4 - R_3} \begin{pmatrix} 1 & 0 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & -6 \\ 0 & 0 & 0 & 1 & 4 \end{pmatrix} \tag{2.0.11}$$

Therefore ,

$$\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} 2 \\ -2 \\ -6 \\ 4 \end{pmatrix} \tag{2.0.12}$$

Hence required polynomial is

$$f(x) = 2 - 2x - 6x^2 + 4x^3 \tag{2.0.13}$$