

Assignment-7

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Abstract—This assignment deals with subspace of vector field.

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<https://github.com/satyam463/Assignment-7/blob/master/assignment%207.tex>

1 PROBLEM STATEMENT

Verify that the set of complex numbers numbers described in the form of $x + y\sqrt{2}$ where x and y are rational is a sub-field of \mathbb{C} .

2 SOLUTION

Lets consider the subfield $S = x + y\sqrt{2}$ We must verify that S meets the following two conditions:

$$0, 1 \in S \quad (2.0.1)$$

$$\text{If } a, b \in S, a + b, -a, ab, a^{-1} \in S \quad (2.0.2)$$

Throughout let

$$a = x + y\sqrt{2}, b = w + z\sqrt{2} \quad (2.0.3)$$

$$\text{if } x = 0 = y = 0 \in \mathbb{Q}, a = 0 + \sqrt{2}.0 = 0, 0 \in S \quad (2.0.4)$$

$$\text{if } x = 1, y = 0, a = 1 + \sqrt{2}.0 = 1, 1 \in S \quad (2.0.5)$$

Hence 2.0.1 is verified .

$$a + b = x + y\sqrt{2} + w + z\sqrt{2} = b + a \quad (2.0.6)$$

$$-a = -x - y\sqrt{2}, x, y \in \mathbb{Q} \text{ so } -x, -y \in \mathbb{Q}, a \in S \quad (2.0.7)$$

$$ab = (x + y\sqrt{2})(w + z\sqrt{2}) = ba, ab \in S \quad (2.0.8)$$

$$a^{-1}a = (x + y\sqrt{2})^{-1}(x + y\sqrt{2}) = 1, a^{-1} \in S \quad (2.0.9)$$

Hence 2.0.2 is verified .