Assignment-8

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***Abstract*—This assignment deals with row equivalent matrices.**

Download tex file from

https://github.com/satyam463/Assignment−8/blob/main/Assignment%208%20.tex

1 Problem Statement

Prove that the following two matrices are not row equivalent

$$\begin{pmatrix} 2 & 0 & 0 \\ a & -1 & 0 \\ b & c & 3 \end{pmatrix}, \begin{pmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{pmatrix} \tag{1.0.1}$$

2 solution

Call the first matrix $\mathbf{A}$ and the second matrix $\mathbf{B}$. The matrix $\mathbf{A}'$ is row reduced echelon form of $\mathbf{A}$

$$\mathbf{A}' = \begin{pmatrix} 2 & 0 & 0 \\ a & -1 & 0 \\ b & c & 3 \end{pmatrix} \xleftrightarrow[R_2 \rightarrow R_2 - aR_1]{R_1 \rightarrow R_1/2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ b & c & 3 \end{pmatrix} \tag{2.0.1}$$

$$\xleftrightarrow[R_2 \leftarrow R_2 - cR_3]{R_3 \leftarrow R_3/3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & c & 3 \end{pmatrix} \tag{2.0.2}$$

$$\xleftrightarrow[R_3 \leftarrow R_3/3]{R_3 \leftarrow R_3 - cR_2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.0.3}$$

$$(\mathbf{A}')^T = \begin{pmatrix} 2 & a & b \\ 0 & -1 & c \\ 0 & 0 & 3 \end{pmatrix} \xleftrightarrow[R_2 \rightarrow -R_2]{R_1 \rightarrow R_1/2} \begin{pmatrix} 1 & \frac{a}{2} & \frac{b}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix} \tag{2.0.4}$$

$$\xleftrightarrow[R_2 \leftarrow -R_2]{R_3 \leftarrow R_3 - bR_1} \begin{pmatrix} 1 & \frac{a}{2} & \frac{b}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.0.5}$$

$$\xleftrightarrow[R_1 \leftarrow 2R_1 - bR2]{R_1 \leftarrow 2R_1 - aR_3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.0.6}$$

Now consider homogeneous equation

$$\mathbf{A}'\mathbf{X} = 0 \tag{2.0.7}$$

$$\implies rank(\mathbf{A}') + null(\mathbf{A}')^T = m \tag{2.0.8}$$

$$\implies rank(\mathbf{A}') = 3 - 0 = 3 \tag{2.0.9}$$

Therefore it has trivial solution i.e. (0,0,0)
The matrix $\mathbf{B}'$ is row reduced echelon form of $\mathbf{B}$

$$\mathbf{B}' = \begin{pmatrix} 1 & 1 & 2 \\ -2 & 0 & -1 \\ 1 & 3 & 5 \end{pmatrix} \xleftrightarrow[R_3 \rightarrow R_3 - R_1]{R_2 \rightarrow R_2 + 2R_1} \begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & 3 \\ 0 & 2 & 3 \end{pmatrix} \tag{2.0.10}$$

$$\xleftrightarrow[R_2 \leftarrow R_2/2]{R_3 \leftarrow R_3 - R_2} \begin{pmatrix} 1 & 1 & 2 \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{pmatrix} \tag{2.0.11}$$

$$\xleftrightarrow{R_1 \leftarrow R_1 - R_2} \begin{pmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{3}{2} \\ 0 & 0 & 0 \end{pmatrix} \tag{2.0.12}$$

$$(\mathbf{B}')^T = \begin{pmatrix} 1 & -2 & 1 \\ 1 & 0 & 3 \\ 2 & -1 & 5 \end{pmatrix} \xleftrightarrow[R_3 \rightarrow R_3 - 2R_1]{R_2 \rightarrow R_2 - R_1} \begin{pmatrix} 1 & -2 & 1 \\ 0 & 2 & 2 \\ 0 & 3 & 3 \end{pmatrix} \tag{2.0.13}$$

$$\xleftrightarrow[R_2 \leftarrow R_2/2]{R_3 \leftarrow R_3/3} \begin{pmatrix} 1 & -2 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \tag{2.0.14}$$

$$\xleftrightarrow[R_1 \leftarrow R_1 + 2R_2]{R_3 \leftarrow R_3 - R_2} \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \tag{2.0.15}$$

Now consider homogeneous equation

$$\mathbf{B}'\mathbf{X} = 0 \tag{2.0.16}$$

$$\implies rank(\mathbf{B}') + null(\mathbf{B}')^T = m \tag{2.0.17}$$

$$\implies rank(\mathbf{B}') = 3 - 1 = 2 \tag{2.0.18}$$

Therefore it has infinite many solution. Hence both $\mathbf{A}'$ and $\mathbf{B}'$ have different solutions , so it cannot be row equivalent.