Assignment-8

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***Abstract*—This assignment deals with row equivalent matrices.**

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https://github.com/satyam463/Assignment−8/blob/main/Assignment%208%20.tex

1 Problem Statement

Prove that the following two matrices are not row equivalent

$$\begin{pmatrix}2 & 0 & 0\\ a & -1 & 0\\ b & c & 3\end{pmatrix},\begin{pmatrix}1 & 1 & 2\\ -2 & 0 & -1\\ 1 & 3 & 5\end{pmatrix} \tag{1.0.1}$$

2 solution

Call the first matrix **A** and the second matrix **B**. The matrix $\mathbf{A}'$ is row reduced echelon form of **A**

$$\mathbf{A}' = \begin{pmatrix}2 & 0 & 0\\ a & -1 & 0\\ b & c & 3\end{pmatrix} \xleftrightarrow[R_2\to R_2-aR_1]{R_1\to R_1/2} \begin{pmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ b & c & 3\end{pmatrix} \tag{2.0.1}$$

$$\xleftrightarrow[R_2\leftarrow -R_2]{R_3\leftarrow R_3-bR_1} \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & c & 3\end{pmatrix} \tag{2.0.2}$$

$$\xleftrightarrow[R_3\leftarrow R_3/3]{R_3\leftarrow R_3-cR_2} \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix} \tag{2.0.3}$$

Now consider homogeneous equation

$$\mathbf{A}'\mathbf{X} = 0 \tag{2.0.4}$$

$$\implies rank(\mathbf{A}') = 3 = n(order of matrix) \tag{2.0.5}$$

Therefore it has trivial solution i.e. (0,0,0)
Now the matrix $\mathbf{B}'$ is row reduced echelon form of **B**

$$\mathbf{B}' = \begin{pmatrix}1 & 1 & 2\\ -2 & 0 & -1\\ 1 & 3 & 5\end{pmatrix} \xleftrightarrow[R_3\to R_3-R_1]{R_2\to R_2+2R_1} \begin{pmatrix}1 & 1 & 2\\ 0 & 2 & 3\\ 0 & 2 & 3\end{pmatrix} \tag{2.0.6}$$

$$\xleftrightarrow[R_2\leftarrow R_2/2]{R_3\leftarrow R_3-R_2} \begin{pmatrix}1 & 1 & 2\\ 0 & 1 & \frac{3}{2}\\ 0 & 0 & 0\end{pmatrix} \tag{2.0.7}$$

$$\xleftrightarrow{R_1\leftarrow R_1-R_2} \begin{pmatrix}1 & 0 & \frac{1}{2}\\ 0 & 1 & \frac{3}{2}\\ 0 & 0 & 0\end{pmatrix} \tag{2.0.8}$$

Now consider homogeneous equation

$$\mathbf{B}'\mathbf{X} = 0 \tag{2.0.9}$$

$$\implies rank(\mathbf{B}') = 2 < n(order of matrix) \tag{2.0.10}$$

Therefore it has infinite many solution. Hence both $\mathbf{A}'$ and $\mathbf{B}'$ have different solutions , so it cannot be row equivalent.