Assignment-2($EE5600$)

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***Abstract*—This assignment deals with basic linear form.**

Download tex file from

https://github.com/satyam463/EE5600Ass1/blob/main/Assignment2.tex

1 Problem Statement

1.1 Vector2, Example 4, Question No 6

Sketch the loci of the following equation

$$3x = y^2 - 9 \tag{1.1.1}$$

2 Solution

Consider given equation

$$y^2 - 3x - 9 = 0 \tag{2.0.1}$$

2.0.1 can be expressed as

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{2.0.2}$$

with parameters

$$\mathbf{V} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -\frac{3}{2} \\ 0 \end{pmatrix}, f = -9 \tag{2.0.3}$$

$$\left|V\right| = 0 \tag{2.0.4}$$

Thus, the given curve is a parabola. $\because$ $\mathbf{V}$ is diagonal and in standard form.Using

$$\mathbf{x} = \mathbf{P}\mathbf{y} + \mathbf{c}\,(\text{Affine Transformation}) \tag{2.0.5}$$

such that,

$$\mathbf{P}^T\mathbf{V}\mathbf{P} = \mathbf{D}\,(\text{EigenValue Decomposition}) \tag{2.0.6}$$

$$\mathbf{P} = \begin{pmatrix} \mathbf{p_1} & \mathbf{p_2} \end{pmatrix}, \mathbf{P}^T = \mathbf{P}^{-1} \tag{2.0.7}$$

The vertex of parabola can be given as $\mathbf{c}$

$$\begin{pmatrix} \mathbf{u}^T + \eta\mathbf{p_1}^T \\ V \end{pmatrix}\mathbf{c} = \begin{pmatrix} -f \\ \eta\mathbf{p_1} - \mathbf{u} \end{pmatrix} \tag{2.0.8}$$

where

$$\eta = \mathbf{p_1}^T\mathbf{u}, \mathbf{p_1} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2.0.9}$$

$$\begin{pmatrix} -3 & 0 \\ 0 & 0 \\ 0 & 1 \end{pmatrix}\mathbf{c} = \begin{pmatrix} 9 \\ 0 \\ 0 \end{pmatrix} \tag{2.0.10}$$

$$\begin{pmatrix} -3 & 0 \\ 0 & 1 \end{pmatrix}\mathbf{c} = \begin{pmatrix} 9 \\ 0 \end{pmatrix} \tag{2.0.11}$$

$$\mathbf{c} = \begin{pmatrix} -3 \\ 0 \end{pmatrix} \tag{2.0.12}$$

Using 2.0.5 and 2.0.7 we can the write

$$\mathbf{y} = \mathbf{P}^{-1}(\mathbf{x} - \mathbf{c}) \implies \mathbf{y} = \mathbf{P}^T(\mathbf{x} - \mathbf{c}) \tag{2.0.13}$$

$$\mathbf{x} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \implies \mathbf{y} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} \tag{2.0.14}$$

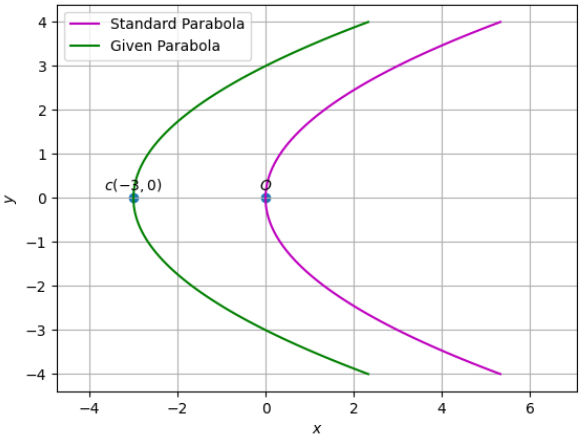

Fig. 0: Given parabola with vertex $\mathbf{c} = \begin{pmatrix} -3 \\ 0 \end{pmatrix}$ and Standard Parabola with vertex $\mathbf{O} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$